

Problem Set 2

Simulation Study

Step 1: Generate x_1, x_2, \dots, x_n from Uniform(0,1). Compute $\mu_i = \alpha + \beta x_i$, $i = 1, 2, \dots, n$. Take $\alpha = -2$, $\beta = 1.5$.

Step 2: Generate Y_i from $N(\mu_i, \sigma^2)$, $i = 1, 2, \dots, n$. Take $\sigma^2 = 4$.

Step 3: Based on the data (Y_i, x_i) , $i = 1, 2, \dots, n$, compute the maximum likelihood estimate of α and β . Call them $\hat{\alpha}$, $\hat{\beta}$. Assume σ^2 is known.

Step 4: Compute Fisher (Expected) Information matrix and hence compute the standard error of $\hat{\beta}$.

Step 5: Compute Wald's interval for β using the analytical expression of standard error from Step 4.

Step 6: Compute simulated standard error, empirical bias, empirical mean square error, empirical coverage of β , empirical level and power for the test $H_0 : \beta = 0$ versus $H_1 : \beta = 1$.

Repeat steps 1-6, $R = 1000$ times. Prepare a table to report the average values of the following:

estimate of β , standard error of the estimator (both analytical and simulated), empirical bias and mean square error of the estimator, confidence interval and empirical coverage, level and power of the test.

Report the values for various choices of $n = 20, 50, 100, 500, 1000$.