Problem 1

Restricted Boltzmann Machine:

We denote the visible features as **v** and the hidden features as **h** .The network of weights between them is **W** with the biases **c** and **b**.

Defining the energy over v and h as:

$$E(\mathbf{v}, \mathbf{h}) = -\mathbf{h}^\top \mathbf{W} \mathbf{v} - \mathbf{c}^\top \mathbf{v} - \mathbf{b}^\top \mathbf{h}$$
$$= -\sum_j \sum_k W_{j,k} h_j v_k - \sum_k c_k v_k - \sum_j b_j h_j$$

This translates directly to the following free energy formula: $\mathcal{F}(v) = -c' v - \sum_i \log \sum_{h_i} e^{h_i (b_i + W_i v)}$

Joint probability over v and h is:

$$p(\mathbf{v}, \mathbf{h}) = \exp(-E(\mathbf{v}, \mathbf{h}))/Z$$
$$= \exp(\mathbf{h}^T \mathbf{W} \mathbf{v} + \mathbf{c}^T \mathbf{v} + \mathbf{b}^T \mathbf{h})/Z$$
$$= \exp\left(\mathbf{h}^\top \mathbf{W} \mathbf{v}\right) \exp(\mathbf{c}^T \mathbf{v}) \exp(\mathbf{b}^T \mathbf{h})/Z$$
$$= \prod_j \prod_k \exp(W_{j,k} h_j v_k\big) \prod_k \exp(c_k v_k) \prod_j \exp(b_j h_j)$$

Because of the specific structure of RBMs, visible and hidden units are conditionally independent. Thus we can say:

$$p(h|v) = \prod_i p(h_i|v)$$
$$p(v|h) = \prod_j p(v_j|h)$$

RBMs with binary units: $P(h_i = 1|v) = \sigma(b_i + W_i v)$

$$P(v_j = 1|h) = \sigma(c_j + W_j' h)$$

Thus the Free energy term reduces to:

$$\mathcal{F}(v) = -c' v - \sum_i \log(1 + e^{(b_i + W_i v)})$$

We aim to maximise the log likelihood:

$$\frac{1}{N} \sum_n \log p(\mathbf{v}^{(n)})$$

N represents no. of training examples.

$$p(\mathbf{v}) = \sum_{\mathbf{h} \in \{0,1\}^H} p(\mathbf{v}, \mathbf{h}) = \sum_{\mathbf{h} \in \{0,1\}^H} \exp(-E(\mathbf{v}, \mathbf{h}))/Z$$

Assuming θ be the trainable weights and biases(W,c and b) for stochastic gradient descent we will require:

$$\frac{\partial \mathbf{-log\ p(v;\theta)}}{\partial\theta} = -\frac{\partial \mathbf{log} \sum_{\mathbf{h}} \mathbf{e^{-E(v,h)}}}{\partial\theta} + \frac{\partial \mathbf{log\ Z}}{\partial\theta}$$

For the postive phase:

$$-\frac{\partial log \sum_h e^{-E(v,h)}}{\partial\theta} = -\frac{1}{\sum_h e^{-E(v,h)}} \sum_h \frac{\partial e^{-E(v,h)}}{\partial\theta} = -\frac{1}{\sum_h e^{-E(v,h)}} \sum_h e^{-E(v,h)} \frac{\partial -E(v,h)}{\partial\theta}$$

After moving the first fraction into the summation, we keep applying the chain rule so to cancel out the negative sign.

$$\sum_h \frac{e^{-E(v,h)}}{\sum_h e^{-E(v,h)}} \frac{\partial E(v,h)}{\partial\theta}$$

Dividing Z from numerator and denominator of the first fraction simutaneouly :

$$\sum_h \frac{\frac{e^{-E(v,h)}}{Z}}{\frac{\sum_h e^{-E(v,h)}}{Z}} \frac{\partial E(v,h)}{\partial\theta} = \sum_h \frac{P(v,h)}{P(v)} \frac{\partial E(v,h)}{\partial\theta} = \sum_h P(h|v) \frac{\partial E(v,h)}{\partial\theta}$$

Thus the final form of the positive phase is:

$$E_{h\sim P(h|v)} \frac{\partial E(v,h)}{\partial\theta}$$

For the negative phase :

$$\frac{\partial log\ Z}{\partial\theta} = \frac{1}{Z}\frac{\partial Z}{\partial\theta}$$

After substituting $Z = \sum_{x,h} e^{-E(x,h)}$ and applying chain rule:

$$= \frac{1}{Z} \sum_{h',v'} \frac{\partial e^{-E(h',v')}}{\partial\theta} = \frac{1}{Z} \sum_{h',v'} e^{-E(h',v')} \frac{\partial -E(h',v')}{\partial\theta} = -\frac{1}{Z} \sum_{h',v'} e^{-E(h',v')} \frac{\partial E(h',v')}{\partial\theta}$$

Moving the first fraction into summation so to use joint distribution:

$$= -\sum_{h',v'} \frac{e^{-E(h',v')}}{Z} \frac{\partial E(h',v')}{\partial\theta} = -\sum_{h',v'} P(h',v') \frac{\partial E(h',v')}{\partial\theta}$$

We get the final form of negative phase to be:

$$-E_{h',v'\sim P(h,v)} \frac{\partial E(h',v')}{\partial\theta}$$

We get the following learning rule:

$$\frac{\partial \mathbf{-log\ P(v;\theta)}}{\partial\theta} = \mathbf{E_{h\sim P(h|v)}} \frac{\partial \mathbf{E(v,h)}}{\partial\theta} - \mathbf{E_{h',v'\sim P(h,v)}} \frac{\partial \mathbf{E(h',v')}}{\partial\theta}$$

The parameter updation rule thus becomes:

$$\theta(t) = \theta(t-1) + \eta \frac{\partial log\ P(v)}{\partial\theta(t-1)}$$

where η is the learning rate.

Even though the positive phase is tractible, the negative phase is intractible as there are infinite free choices over v and h. Thus we use**Contrastive Divergence Algorithm** to solve this problem.

Replace estimation over $(\mathbf{v})$ by a point estimation at $\tilde{\mathbf{v}} \sim p(\mathbf{v})$ obtained from Gibbs sampling starting at $\mathbf{v}^{(t)}$

sample $(\mathbf{h}^1 \ \sim p(\mathbf{h}|\mathbf{v}^{(t)})$

sample $(\mathbf{v}^1 \ \sim p(\mathbf{v}|\mathbf{h}^1)$

This goes on till..

sample $(\mathbf{h}^k \ \sim p(\mathbf{h}|\mathbf{v}^{(k-1)})$

sample $(\mathbf{v}\tilde{\sim} p(\mathbf{v}|\mathbf{h}^{k-1})$

Thus we can write:

$$\mathbf{\frac{\partial - log\ P(v;\theta)}{\partial\theta} = E_h[\frac{\partial E(v^{(t)},h)}{\partial\theta}|v^{(t)}] - E_{v,h}[\frac{\partial E(v,h)}{\partial\theta}]}$$
$$\mathbf{= E_h[\frac{\partial E(v^{(t)},h)}{\partial\theta}|v^{(t)}] - E_h[\frac{\partial E(\tilde{v},h)}{\partial\theta}]}$$

This solves the intractible problem and we can go ahead with parameter updation.

The final update rules are:

$$\mathbf{W} = \mathbf{W} + \alpha(\mathbf{h}(\mathbf{v}^{(t)})\mathbf{v}^{(t)^T} \ - \mathbf{h}(\tilde{\mathbf{v}})\tilde{\mathbf{v}}^T)$$

$$\mathbf{b} = \mathbf{b} + \alpha(\mathbf{h}(\mathbf{v}^{(t)}) - \mathbf{h}(\tilde{\mathbf{v}}))$$

$$\mathbf{c} = \mathbf{c} + \alpha(\mathbf{v}^{(t)} - \tilde{\mathbf{v}})$$

where $\mathbf{h}(\mathbf{v}) = p(\mathbf{h}|\mathbf{v})$.

In [0]:
```
import matplotlib.pyplot as plt
import numpy as np
import tensorflow as tf
import os
import keras.datasets
```

In [4]:
```
print(tf.__version__)
```

```
2.2.0-rc3
```

In [0]:
```
import random
import gzip, struct

class DataSet:
    batch_index = 0

    def __init__(self, flag, batch_size = None, one_hot = False, se
```

```
ed = 0):

        if flag=='train':
            (X,Y),(_,_)= keras.datasets.mnist.load_data()
        if flag=='test':
            (_,_),(X,Y)= keras.datasets.mnist.load_data()

        shape = X.shape
        X = X.reshape([shape[0], shape[1] * shape[2]])
        self.X = X.astype(np.float)/255
        self.size = self.X.shape[0]
        if batch_size == None:
            self.batch_size = self.size
        else:
            self.batch_size = batch_size
        # abandom last few samples
        self.batch_num = int(self.size / self.batch_size)
        # shuffle samples
        np.random.seed(seed)
        np.random.shuffle(self.X)
        np.random.seed(seed)
        np.random.shuffle(Y)
        self.one_hot = one_hot
        if one_hot:
            y_vec = np.zeros((len(Y), 10), dtype=np.float)
            for i, label in enumerate(Y):
                y_vec[i, Y[i]] = 1.0
            self.Y = y_vec
        else:
            self.Y = Y


    def next_batch(self):
        start = self.batch_index * self.batch_size
        end = (self.batch_index + 1) * self.batch_size
        self.batch_index = (self.batch_index + 1) % self.batch_num
        if self.one_hot:
            return self.X[start:end, :], self.Y[start:end, :]
        else:
            return self.X[start:end, :], self.Y[start:end]

    def sample_batch(self):
        index = random.randrange(self.batch_num)
        start = index * self.batch_size
        end = (index + 1) * self.batch_size
        if self.one_hot:
            return self.X[start:end, :], self.Y[start:end, :]
        else:
            return self.X[start:end, :], self.Y[start:end]
```

In [6]:
```
mnist = DataSet('test', 2)
print('batch index: %d' % mnist.batch_index)
X, Y = mnist.next_batch()
print('batch index: %d' % mnist.batch_index)
print('X:')
print(X)
print('Y:')
print(Y)
```

```
Downloading data from https://s3.amazonaws.com/img-datasets/mnist.
npz
11493376/11490434 [==============================] - 1s 0us/step
batch index: 0
batch index: 1
X:
[[0. 0. 0. ... 0. 0. 0.]
 [0. 0. 0. ... 0. 0. 0.]]
Y:
[8 7]
```

In [0]:
```
def weight(shape, name='weights'):
    return tf.Variable(tf.random.truncated_normal(shape, stddev=0.1
), name=name)

def bias(shape, name='biases'):
    return tf.Variable(tf.constant(0.1, shape=shape), name=name)
```

In [0]:
```
class RBM:
    i = 0 # fliping index for computing pseudo likelihood

    def __init__(self, n_hidden, n_visible=784, k=30, momentum=Fals
e):
        self.n_visible = n_visible
        self.n_hidden = n_hidden
        self.k = k

        self.lr = tf.compat.v1.placeholder(tf.float32)
        if momentum:
            self.momentum = tf.compat.v1.placeholder(tf.float32)
        else:
            self.momentum = 0.0
        self.w = weight([n_visible, n_hidden], 'w')
        self.hb = bias([n_hidden], 'hb')
        self.vb = bias([n_visible], 'vb')

        self.w_v = tf.Variable(tf.zeros([n_visible, n_hidden]), dty
pe=tf.float32)
        self.hb_v = tf.Variable(tf.zeros([n_hidden]), dtype=tf.floa
t32)
        self.vb_v = tf.Variable(tf.zeros([n_visible]), dtype=tf.flo
at32)
```

```
    def propup(self, visible):
        pre_sigmoid_activation = tf.matmul(visible, self.w) + self.
hb
        return tf.nn.sigmoid(pre_sigmoid_activation)

    def propdown(self, hidden):
        pre_sigmoid_activation = tf.matmul(hidden, tf.transpose(sel
f.w)) + self.vb
        return tf.nn.sigmoid(pre_sigmoid_activation)

    def sample_h_given_v(self, v_sample):
        h_props = self.propup(v_sample)
        h_sample = tf.nn.relu(tf.sign(h_props - tf.random.uniform(t
f.shape(h_props))))
        return h_sample

    def sample_v_given_h(self, h_sample):
        v_props = self.propdown(h_sample)
        v_sample = tf.nn.relu(tf.sign(v_props - tf.random.uniform(t
f.shape(v_props))))
        return v_sample

    def CD_k(self, visibles):
        # k steps gibbs sampling
        v_samples = visibles
        h_samples = self.sample_h_given_v(v_samples)
        for i in range(self.k):
            v_samples = self.sample_v_given_h(h_samples)
            h_samples = self.sample_h_given_v(v_samples)

        h0_props = self.propup(visibles)
        w_positive_grad = tf.matmul(tf.transpose(visibles), h0_prop
s)
        w_negative_grad = tf.matmul(tf.transpose(v_samples), h_samp
les)
        w_grad = (w_positive_grad - w_negative_grad) / tf.compat.v1
.to_float(tf.shape(visibles)[0])
        hb_grad = tf.reduce_mean(h0_props - h_samples, 0)
        vb_grad = tf.reduce_mean(visibles - v_samples, 0)
        return w_grad, hb_grad, vb_grad

    def learn(self, visibles):
        w_grad, hb_grad, vb_grad = self.CD_k(visibles)
        # compute new velocities
        new_w_v = self.momentum * self.w_v + self.lr * w_grad
        new_hb_v = self.momentum * self.hb_v + self.lr * hb_grad
        new_vb_v = self.momentum * self.vb_v + self.lr * vb_grad
        # update parameters
        update_w = tf.compat.v1.assign(self.w, self.w + new_w_v)
        update_hb = tf.compat.v1.assign(self.hb, self.hb + new_hb_v
)
        update_vb = tf.compat.v1.assign(self.vb, self.vb + new_vb_v
)
```

```
        # update velocities
        update_w_v = tf.compat.v1.assign(self.w_v, new_w_v)
        update_hb_v = tf.compat.v1.assign(self.hb_v, new_hb_v)
        update_vb_v = tf.compat.v1.assign(self.vb_v, new_vb_v)

        return [update_w, update_hb, update_vb, update_w_v, update_
hb_v, update_vb_v]

    def sampler(self, visibles, steps=200):
        v_samples = visibles
        for step in range(steps):
            v_samples = self.sample_v_given_h(self.sample_h_given_v
(v_samples))
        return v_samples

    def free_energy(self, visibles):
        first_term = tf.matmul(visibles, tf.reshape(self.vb, [tf.sh
ape(self.vb)[0], 1]))
        second_term = tf.reduce_sum(tf.math.log(1 + tf.exp(self.hb
+ tf.matmul(visibles, self.w))), axis=1)
        return - first_term - second_term

    def pseudo_likelihood(self, visibles):
        x = tf.round(visibles)
        x_fe = self.free_energy(x)
        split0, split1, split2 = tf.split(x, [self.i, 1, tf.shape(x
)[1] - self.i - 1], 1)
        xi = tf.concat([split0, 1 - split1, split2], 1)
        self.i = (self.i + 1) % self.n_visible
        xi_fe = self.free_energy(xi)
        return tf.reduce_mean(self.n_visible * tf.math.log(tf.nn.si
gmoid(xi_fe - x_fe)), axis=0)
```

In [0]:
```
def show_images(images, size):


    img = (images + 1.0) / 2.0
    h, w = img.shape[1], img.shape[2]


    merge_img = np.zeros((h * size[0], w * size[1]))


    for idx, image in enumerate(images):
        i = idx % size[1]
        j = idx // size[1]
        merge_img[j*h:j*h+h, i*w:i*w+w] = image


    merge_img = (merge_img * 255 / np.max(merge_img)).astype('uint8
')
    plt.imshow(merge_img,cmap='gray')
    plt.show()

    return
```

In [0]:
```
def train(train_data,hidden_nodes,epoches):

    x = tf.compat.v1.placeholder(tf.float32, shape=[None, 784])
    noise_x, _ = train_data.sample_batch()
    # noise_x = tf.random_normal([train_data.batch_size, 784])
    rbm = RBM(hidden_nodes)
    step = rbm.learn(x)
    sampler = rbm.sampler(x)

    saver = tf.compat.v1.train.Saver()

    with tf.compat.v1.Session() as sess:
        init = tf.compat.v1.global_variables_initializer()
        sess.run(init)
        #mean_cost = []
        epoch = 1
        for i in range(epoches * train_data.batch_num):
            # draw samples
            if i % train_data.batch_num == 0:
                samples = sess.run(sampler, feed_dict = {x: noise_x
})
                samples = samples.reshape([train_data.batch_size, 2
8, 28])
                show_images(samples, [8, 8])

            batch_x, _ = train_data.next_batch()
            sess.run(step, feed_dict = {x: batch_x, rbm.lr: 0.1})

            if i is not 0 and train_data.batch_index is 0:
```

```
                print('Epoch ', epoch)
                epoch += 1


        print('Testing !')



        x_test,_=test_data.sample_batch()

        x_test1=np.copy(x_test)
        x_test2=np.copy(x_test)
        x_test3=np.copy(x_test)


        l1,l2,l3=[],[],[]
        for j in range(len(x_test)):
            i,= np.nonzero(x_test[j])
            ix1 = np.random.choice(i, int(np.floor(0.2 * len(i))),r
eplace=False)
            ix2 = np.random.choice(i, int(np.floor(0.5 * len(i))),r
eplace=False)
            ix3 = np.random.choice(i, int(np.floor(0.8 * len(i))),r
eplace=False)

            l1.append(ix1)
            l2.append(ix2)
            l3.append(ix3)

        print('After removing 20% pixels')

        j=0
        for ix in l1:
            for loc in ix:
              x_test1[j][loc]=0.0
            j+=1

        samples = sess.run(sampler, feed_dict = {x: x_test1})
        samples = samples.reshape([train_data.batch_size, 28, 28])
        show_images(samples, [8, 8])


        print('After removing 50% pixels')

        j=0
        for ix in l2:
            for loc in ix:
              x_test2[j][loc]=0.0
            j+=1

        samples = sess.run(sampler, feed_dict = {x: x_test2})
        samples = samples.reshape([train_data.batch_size, 28, 28])
        show_images(samples, [8, 8])
```

```
        print('After removing 80% pixels')

        j=0
        for ix in l3:
            for loc in ix:
              x_test3[j][loc]=0.0
            j+=1

        samples = sess.run(sampler, feed_dict = {x: x_test3})
        samples = samples.reshape([train_data.batch_size, 28, 28])
        show_images(samples, [8, 8])
```

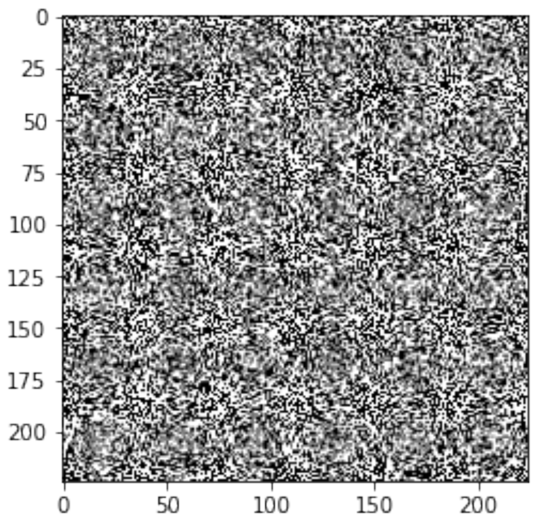
In [0]:
```
train_data = DataSet('train', batch_size=64, one_hot=True)
test_data = DataSet('test', batch_size=64, one_hot=True)
```

20 hidden nodes training.

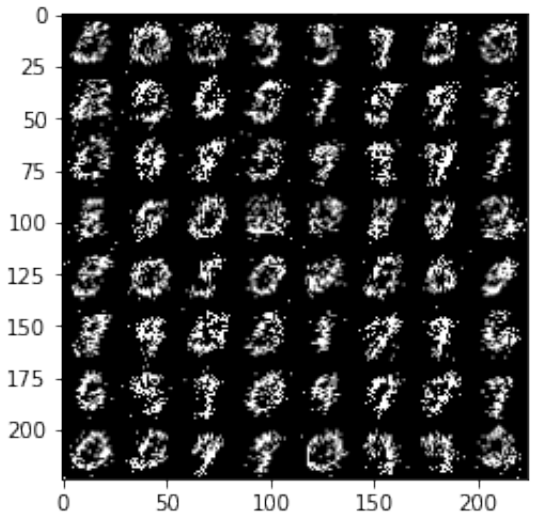
Followed by testing: removing 20%, 50% and 80% pixels.

In [20]:
```
tf.compat.v1.disable_eager_execution()
train(train_data, 20,30)
```

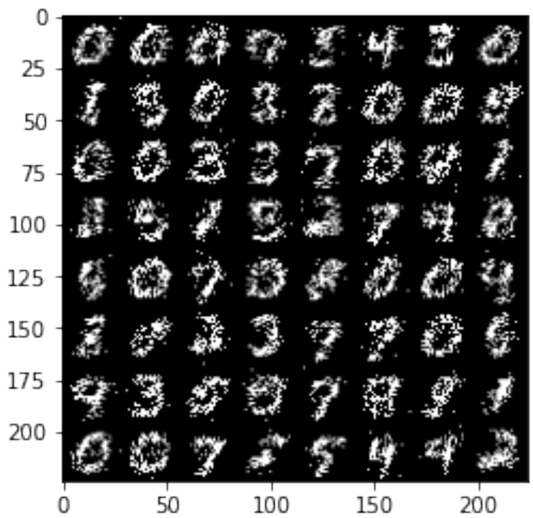
```
WARNING:tensorflow:From /usr/local/lib/python3.6/dist-packages/ten
sorflow/python/ops/resource_variable_ops.py:1666: calling BaseReso
urceVariable.__init__ (from tensorflow.python.ops.resource_variabl
e_ops) with constraint is deprecated and will be removed in a futu
re version.
Instructions for updating:
If using Keras pass *_constraint arguments to layers.
WARNING:tensorflow:From <ipython-input-8-9bfcc859e862>:51: to_floa
t (from tensorflow.python.ops.math_ops) is deprecated and will be
removed in a future version.
Instructions for updating:
Use `tf.cast` instead.
```



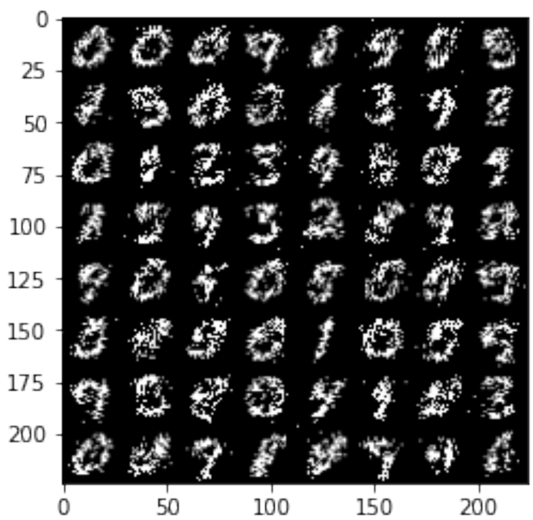
Epoch 1



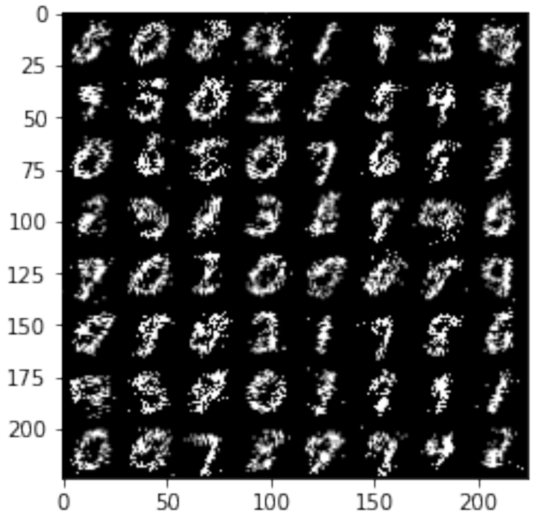
Epoch 2



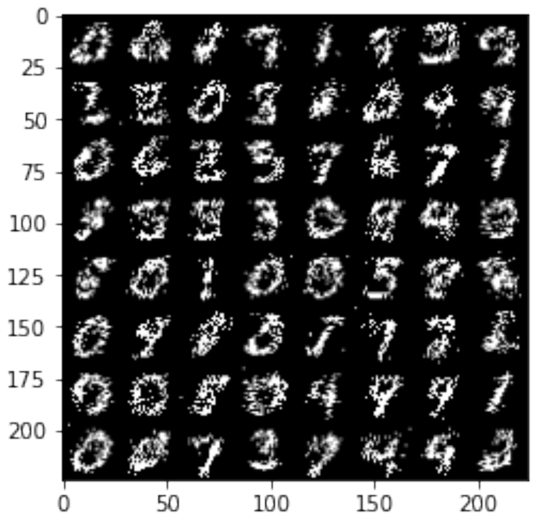
Epoch 3



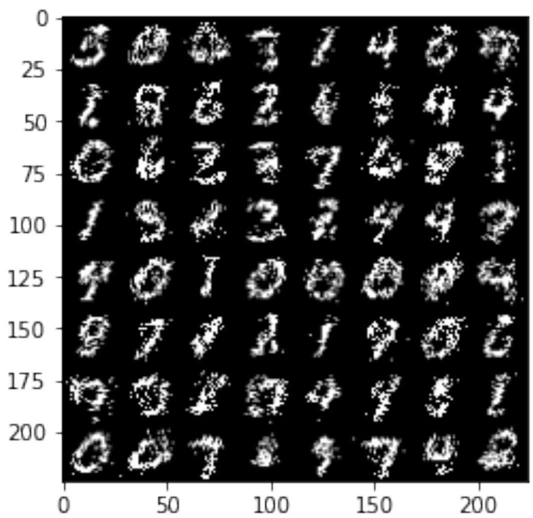
Epoch 4



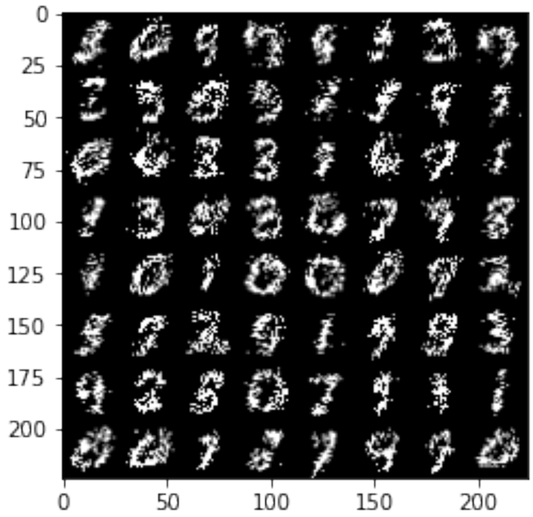
Epoch 5



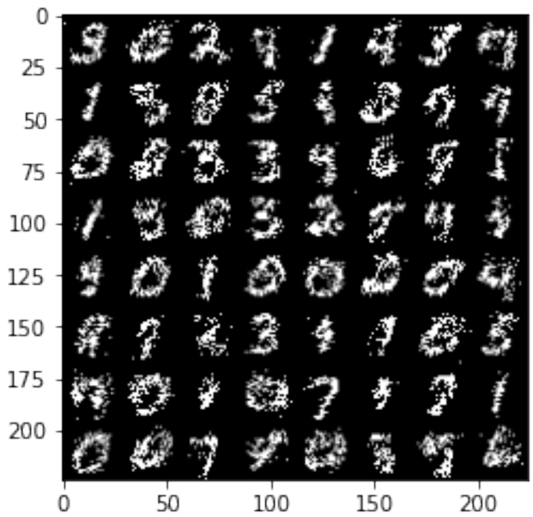
Epoch 6



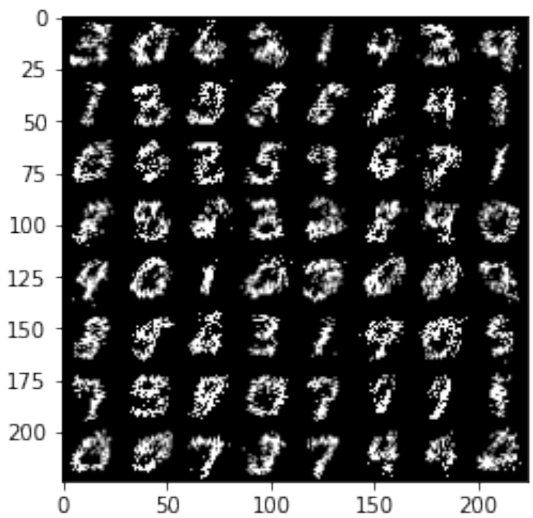
Epoch 7



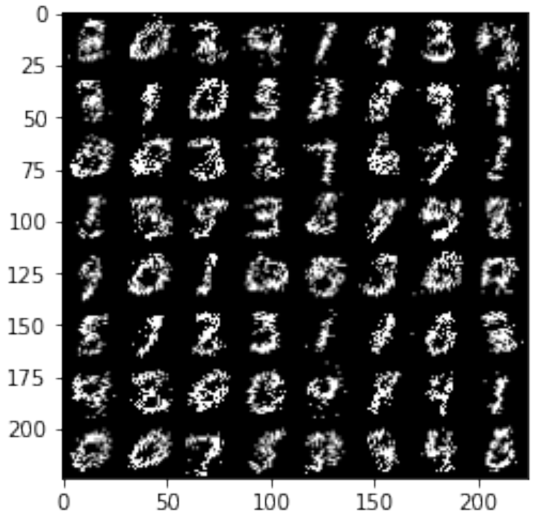
Epoch 8



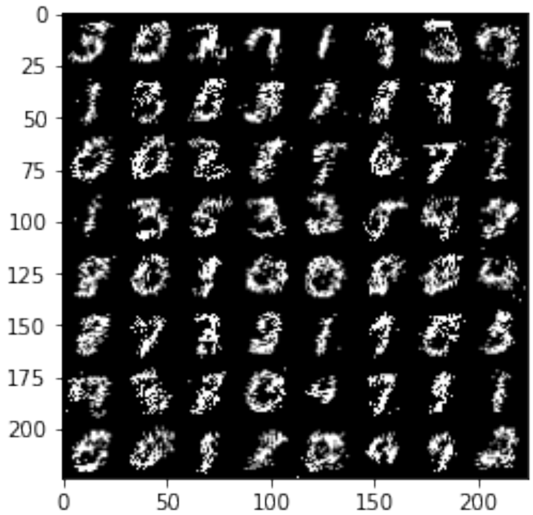
Epoch 9



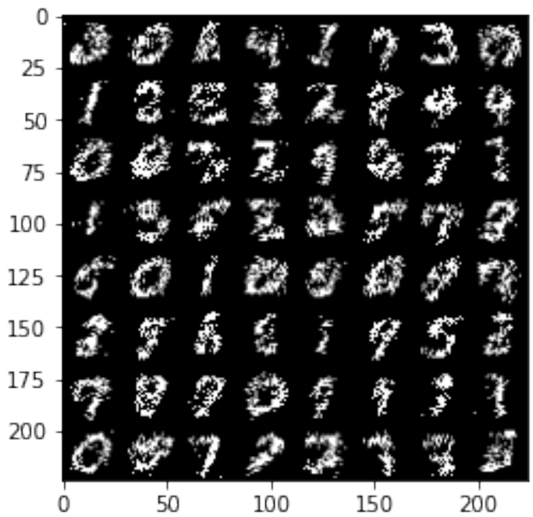
Epoch 10



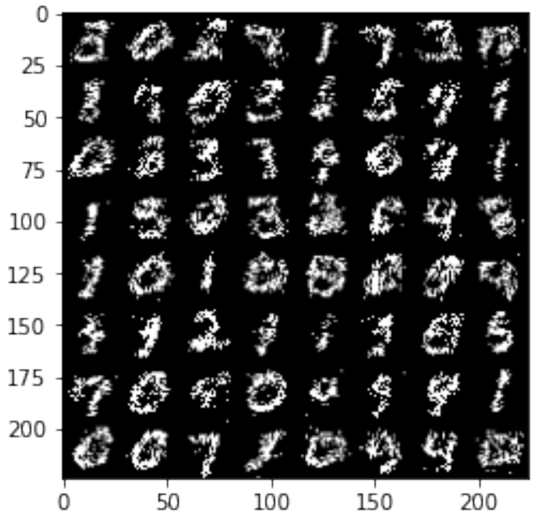
```
Epoch  11
```



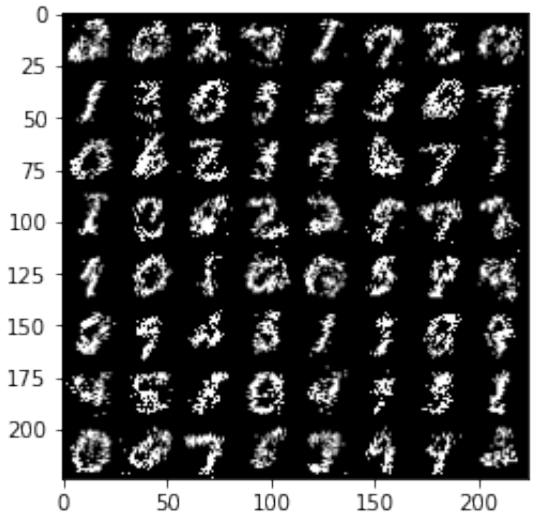
```
Epoch  12
```



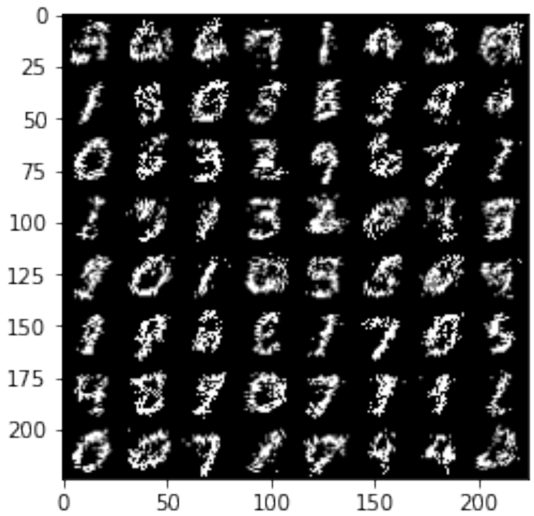
```
Epoch  13
```



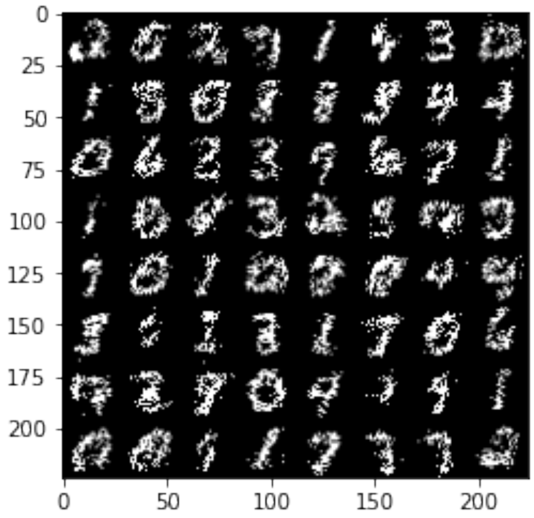
Epoch 14



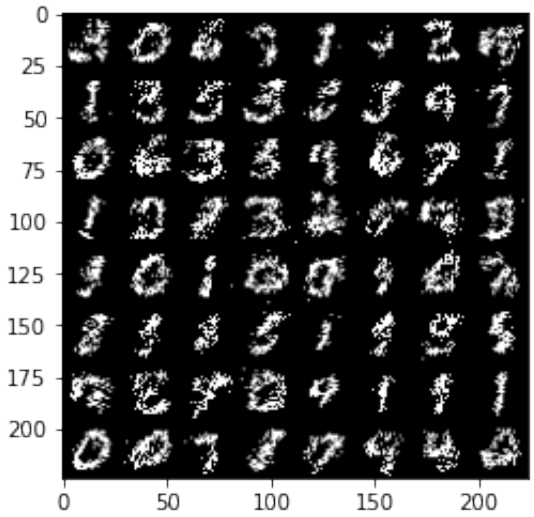
Epoch 15



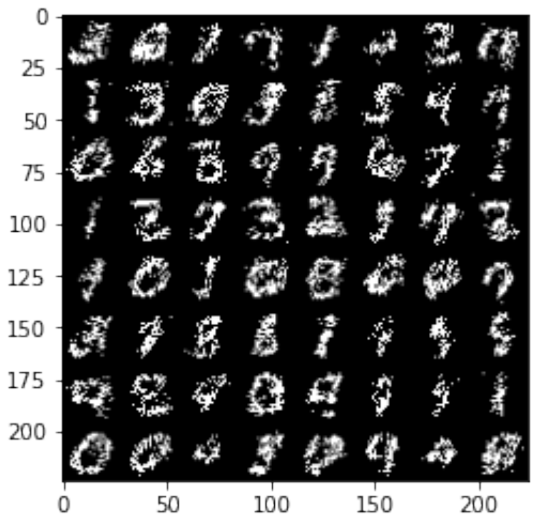
Epoch 16



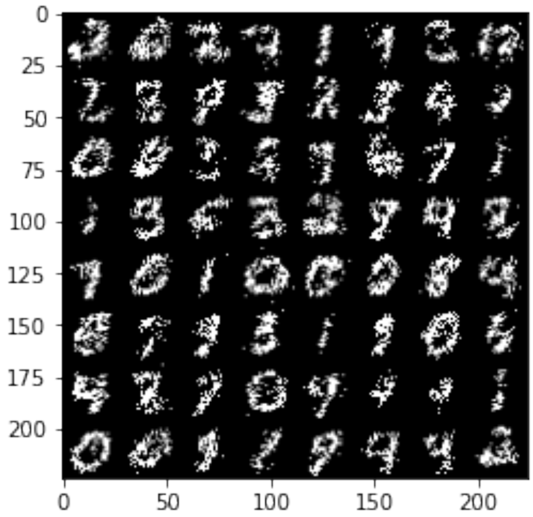
Epoch 17



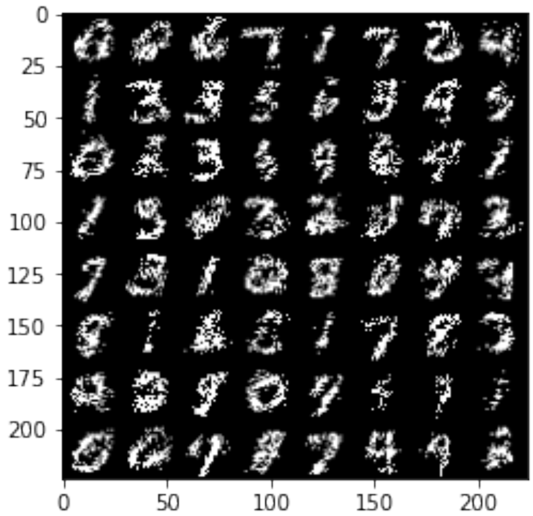
Epoch 18



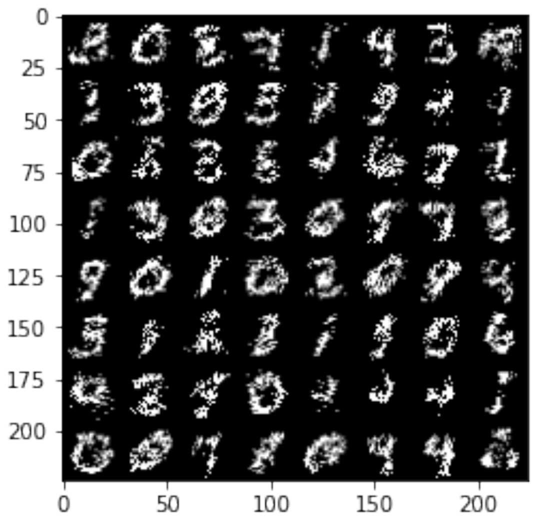
Epoch 19



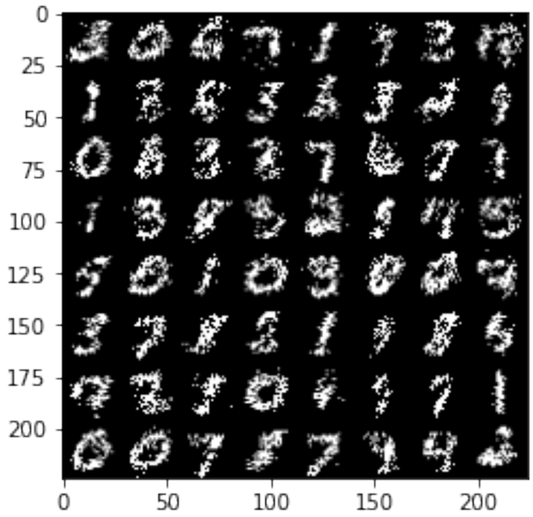
Epoch 20



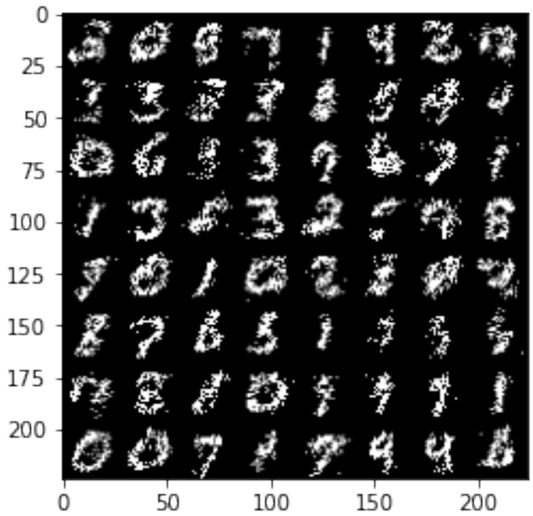
Epoch 21



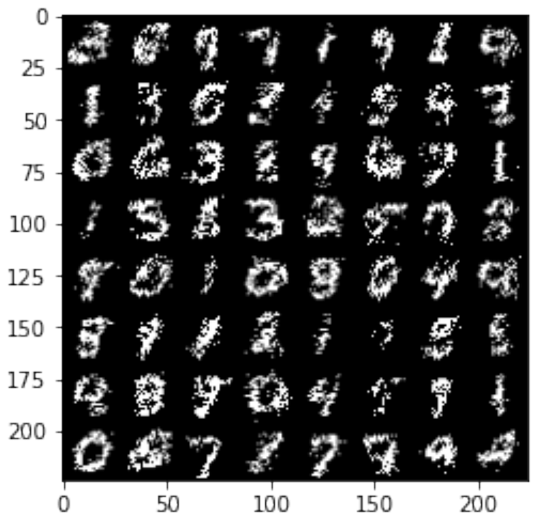
Epoch 22



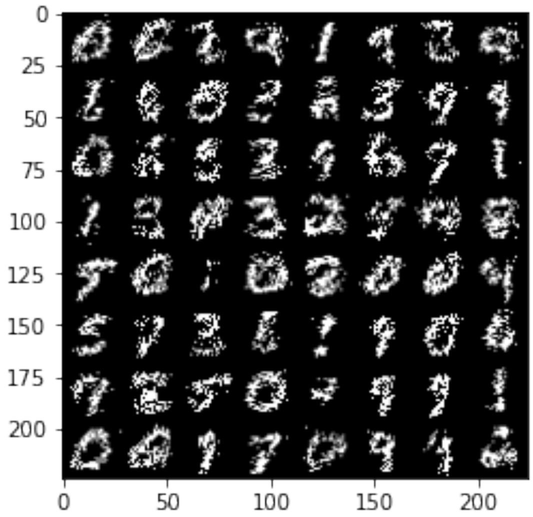
Epoch 23



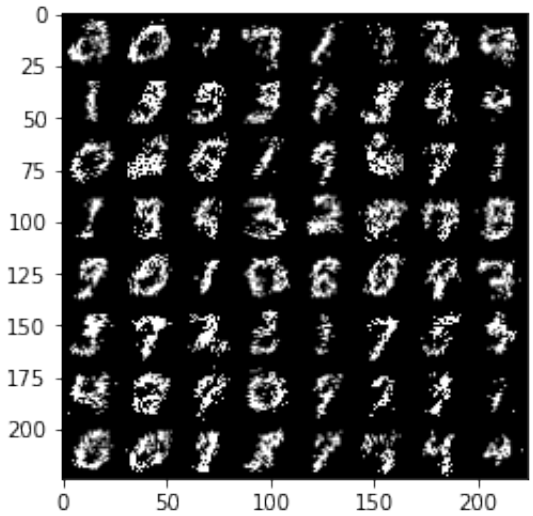
Epoch 24



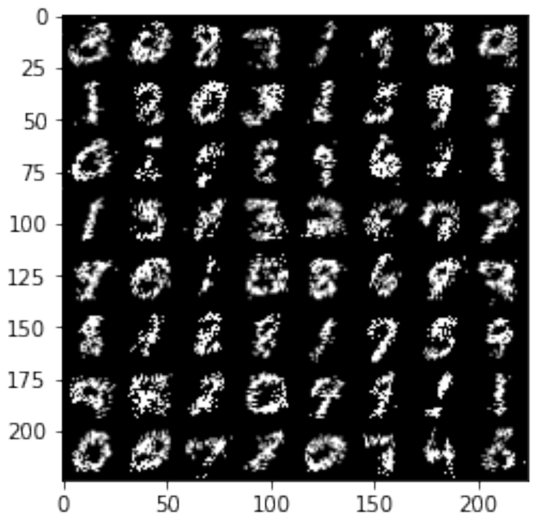
Epoch 25



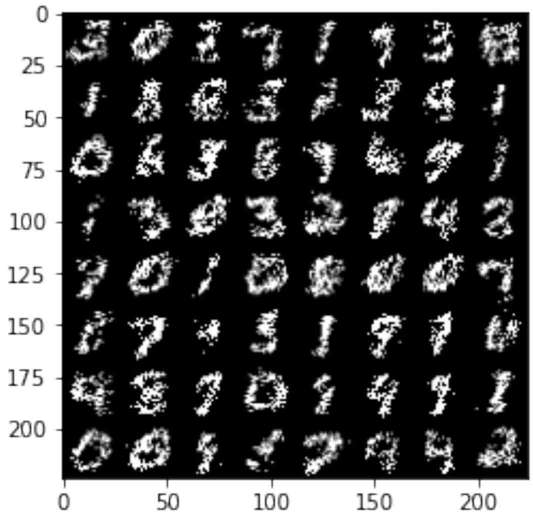
Epoch 26



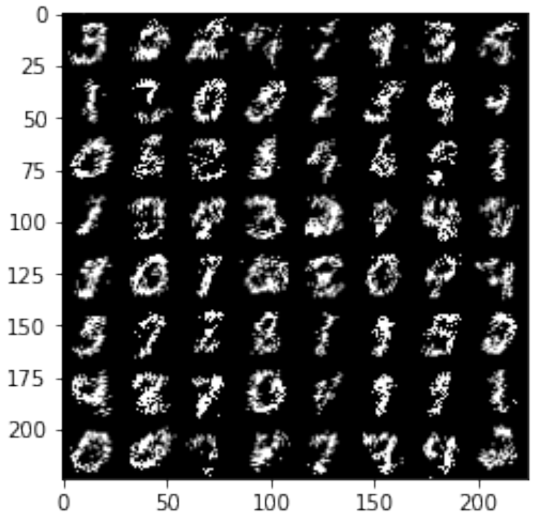
Epoch 27



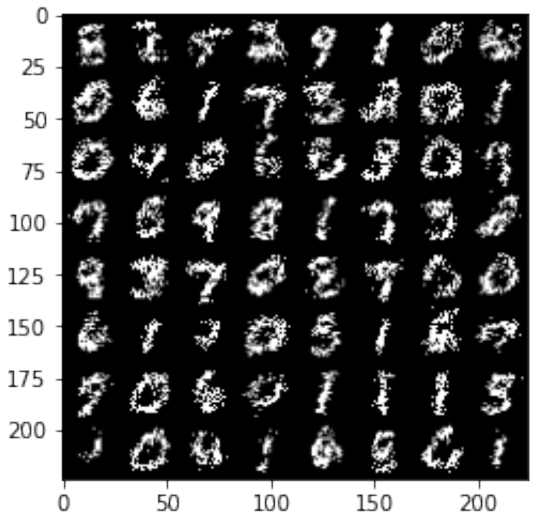
Epoch 28



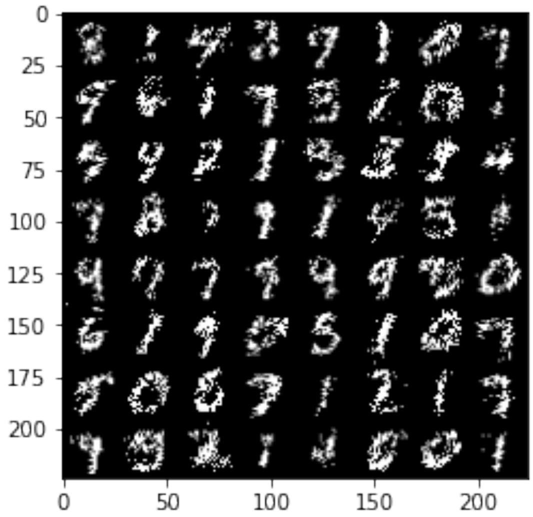

```
Epoch  29
```



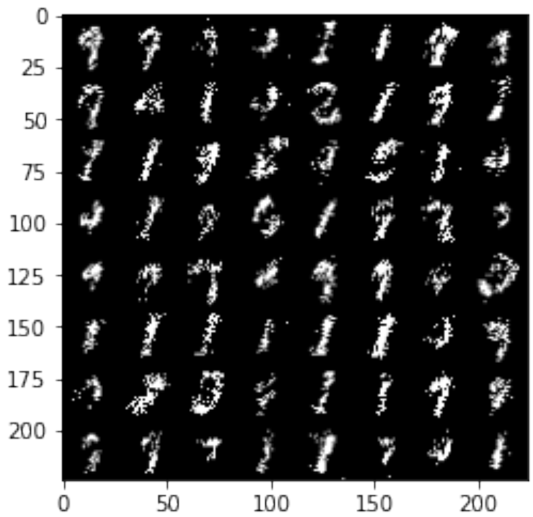

```
Epoch  30
Testing !
After removing 20% pixels
```




```
After removing 50% pixels
```



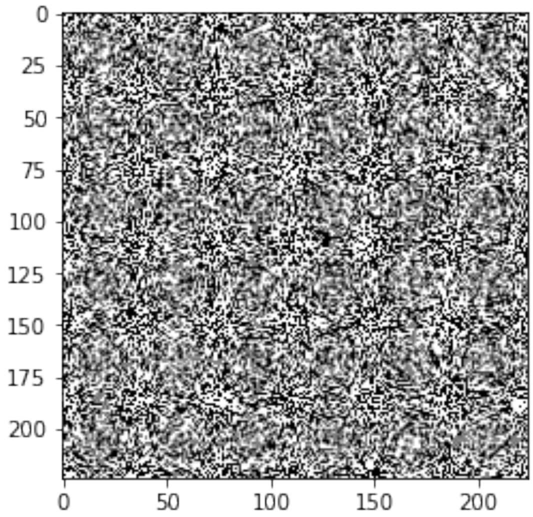
After removing 80% pixels



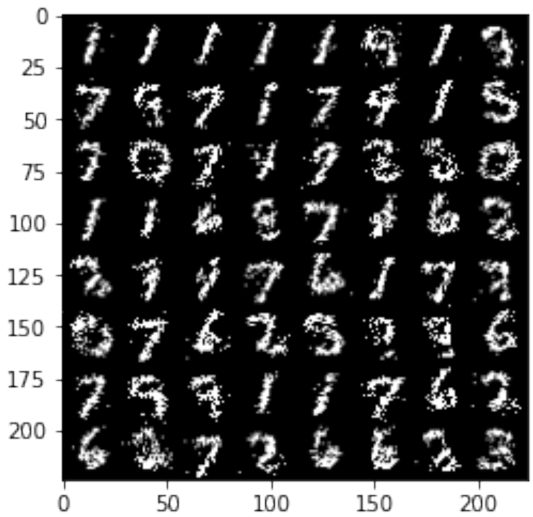
100 hidden nodes training.

Followed by testing: removing 20%, 50% and 80% pixels.

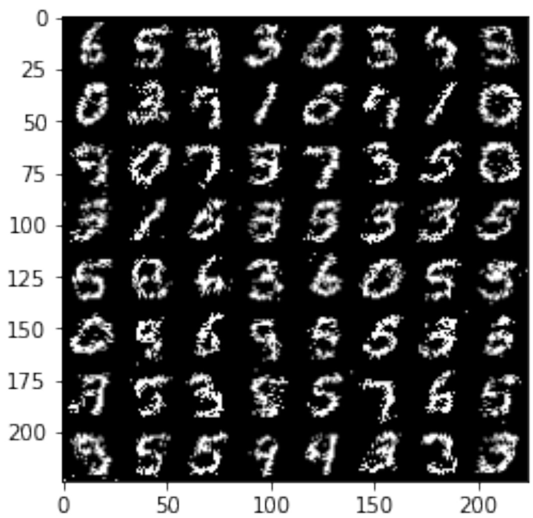
In [21]:
```
tf.compat.v1.disable_eager_execution()
train(train_data, 100,30)
```



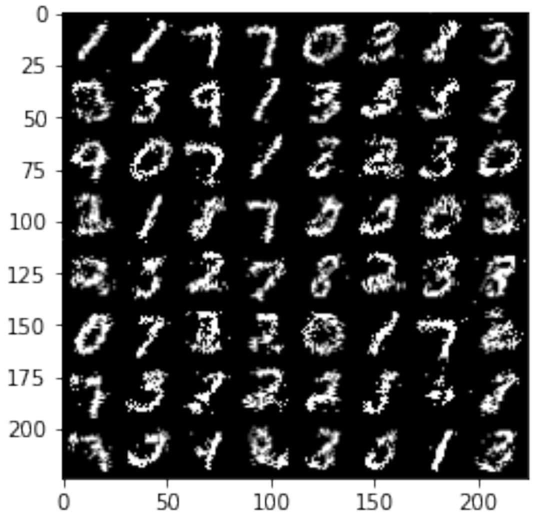

```
Epoch  1
```



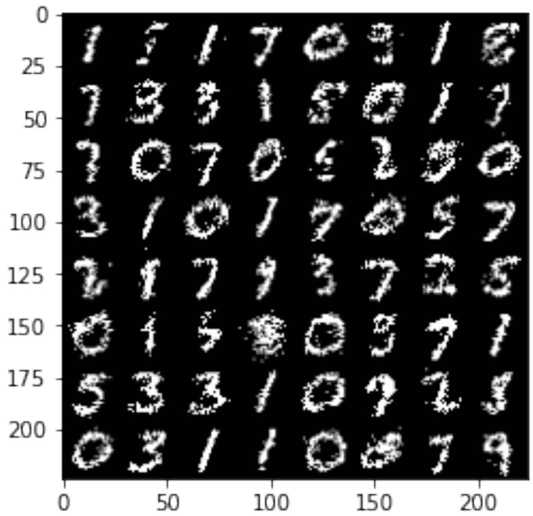

```
Epoch  2
```



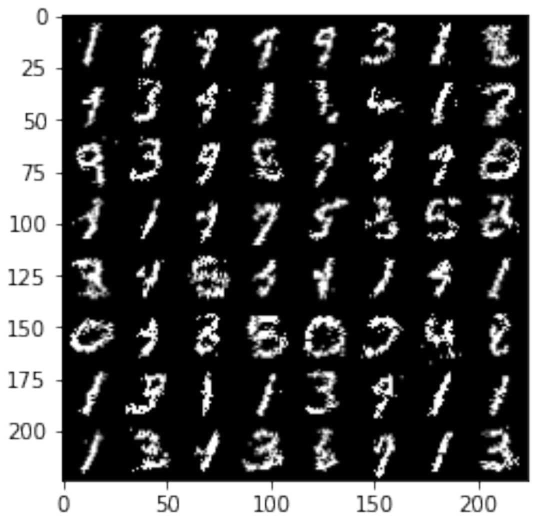

```
Epoch  3
```



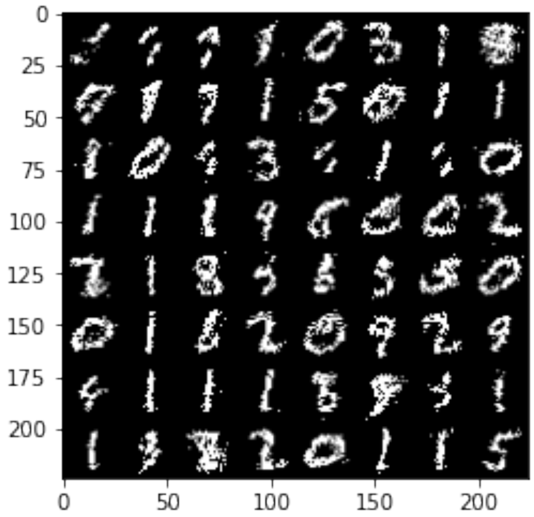
Epoch 4



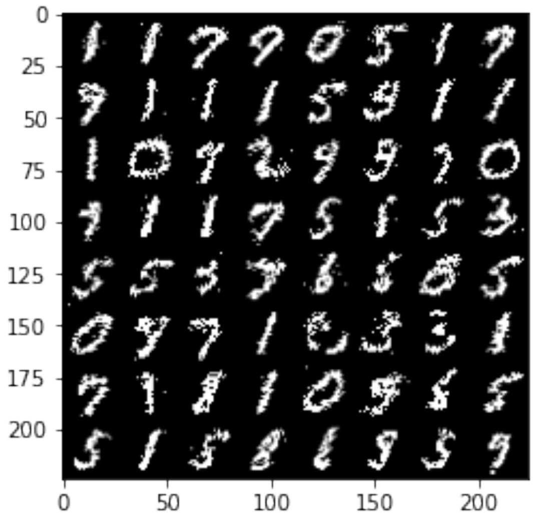
Epoch 5



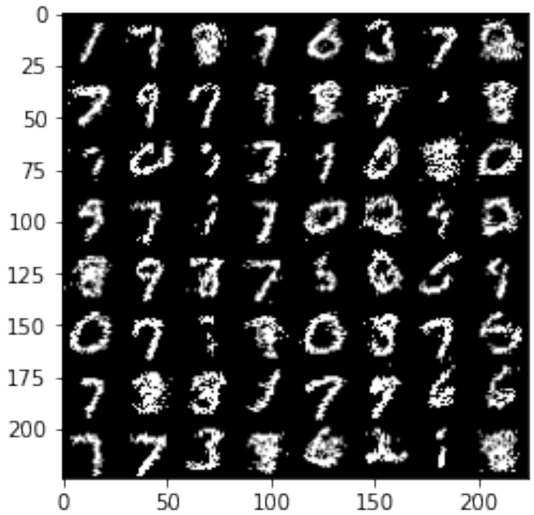
Epoch 6



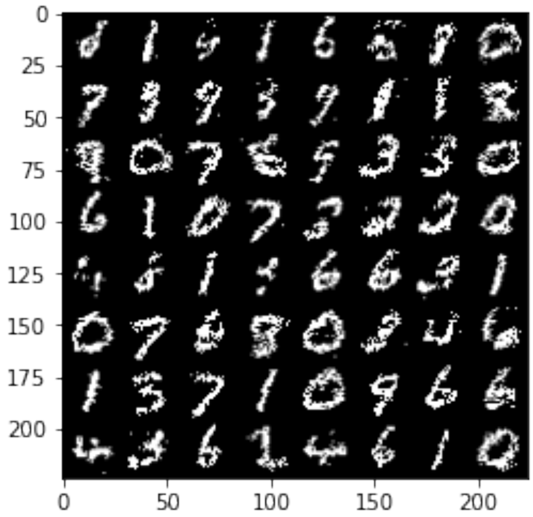
Epoch 7



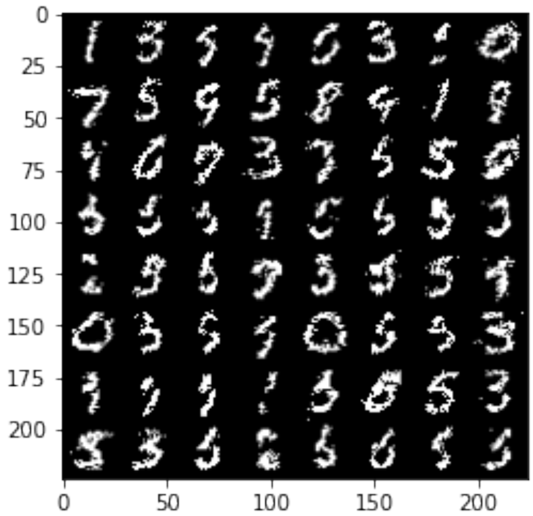
Epoch 8



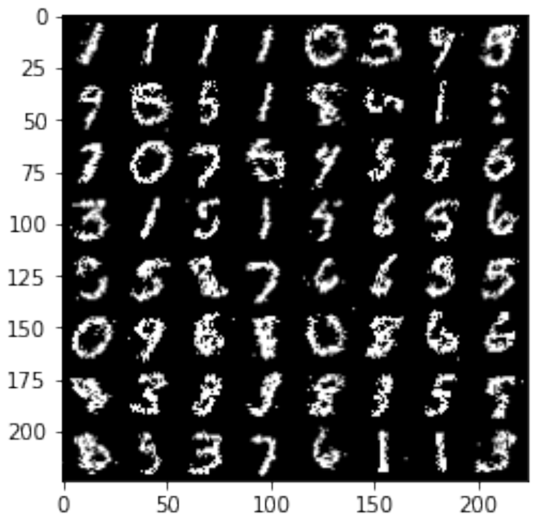
Epoch 9



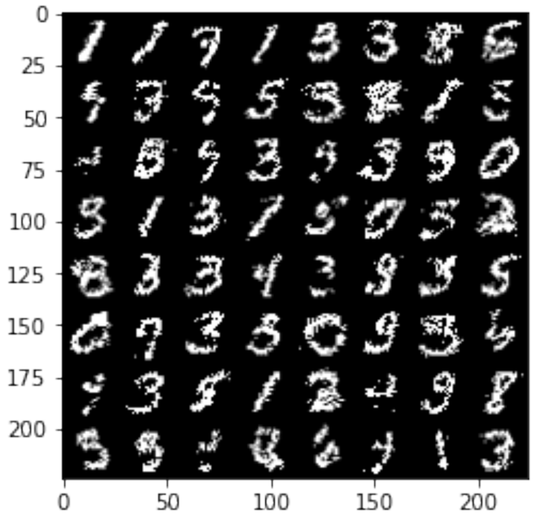
Epoch 10



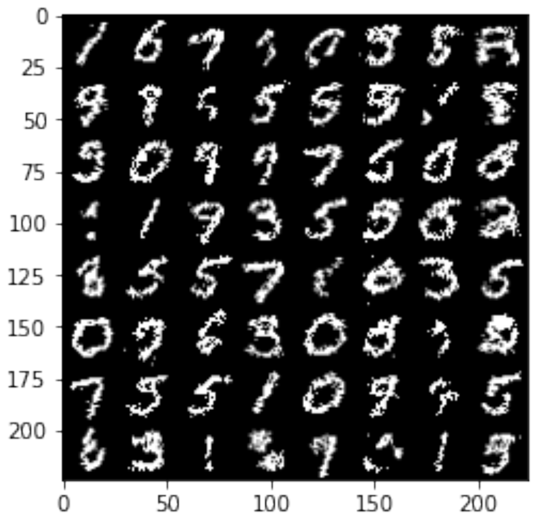
Epoch 11



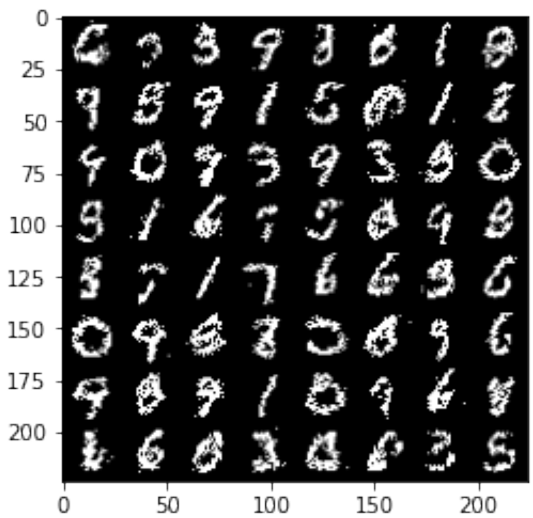
Epoch 12



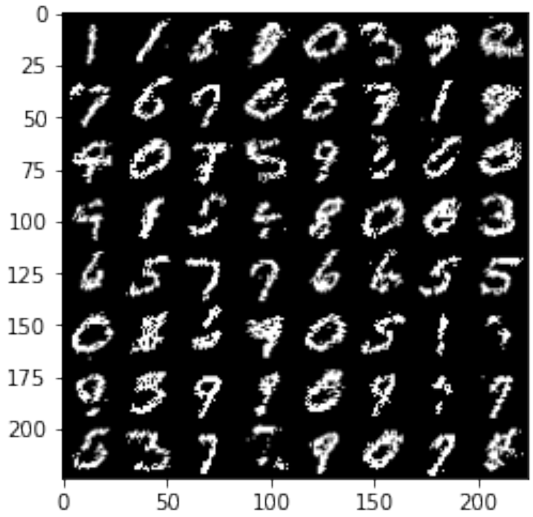
Epoch 13



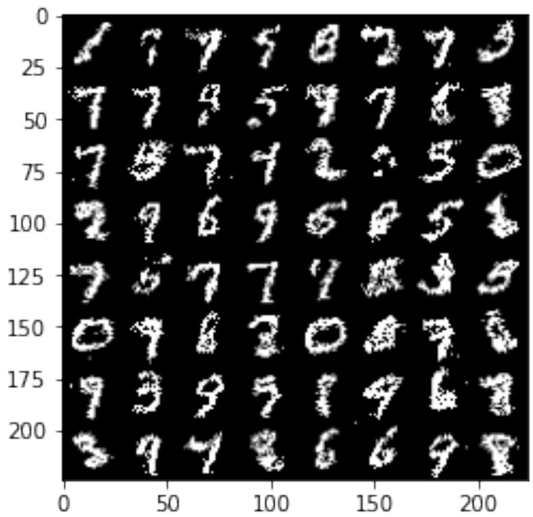
Epoch 14



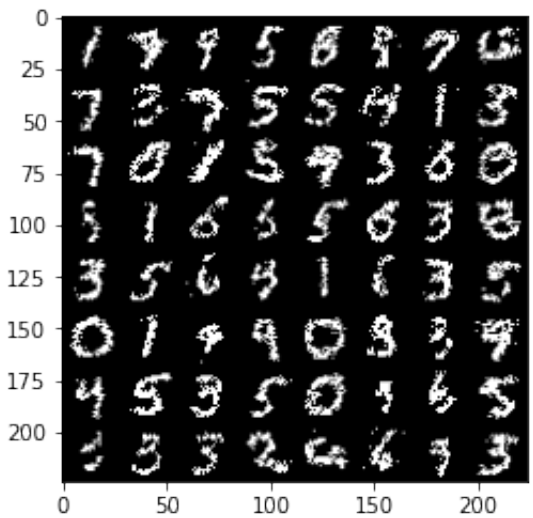
Epoch 15



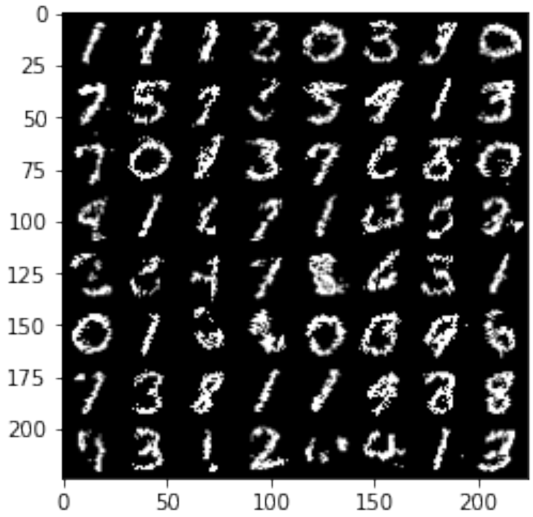
Epoch 16



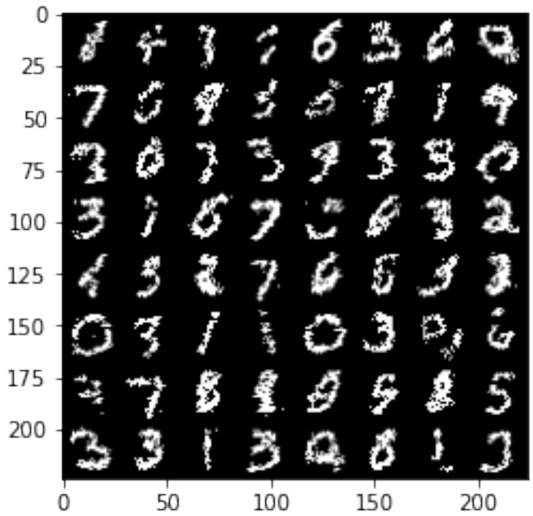
Epoch 17



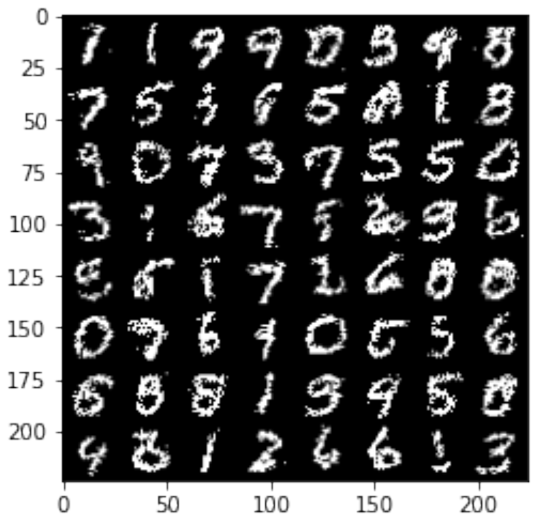
Epoch 18



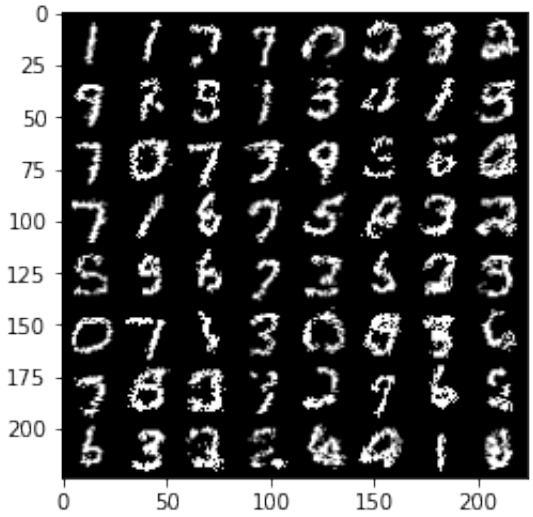
Epoch 19



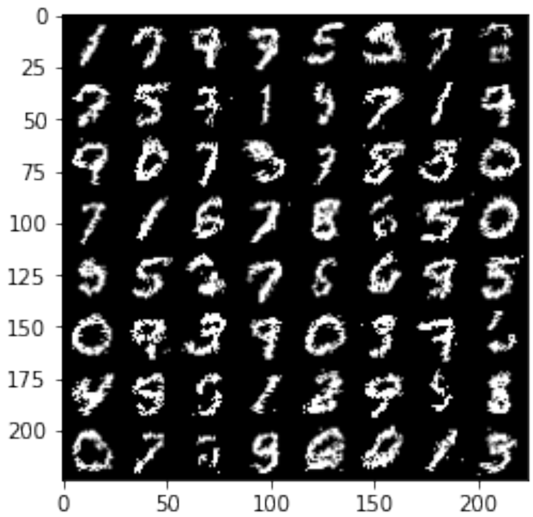
Epoch 20



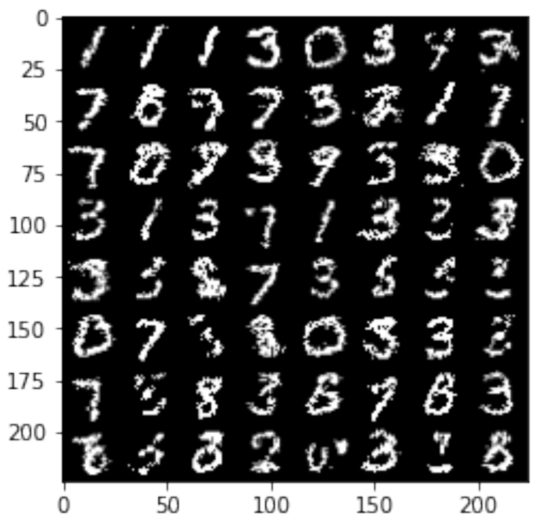
Epoch 21



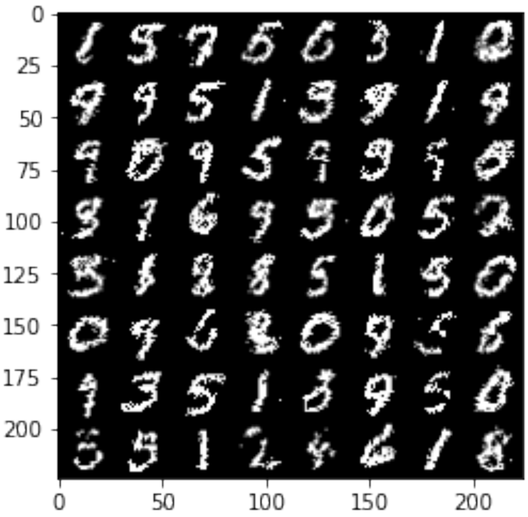
Epoch 22



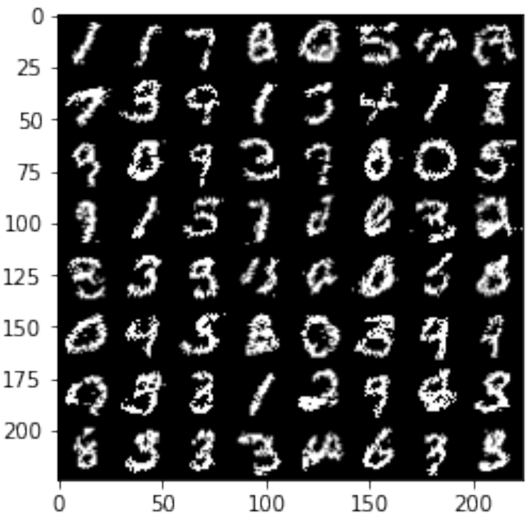
Epoch 23



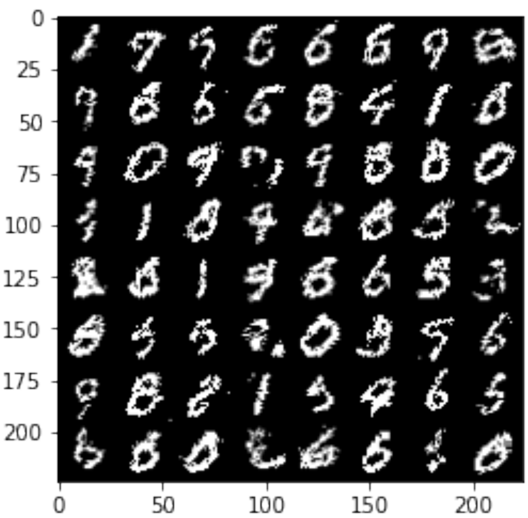
Epoch 24



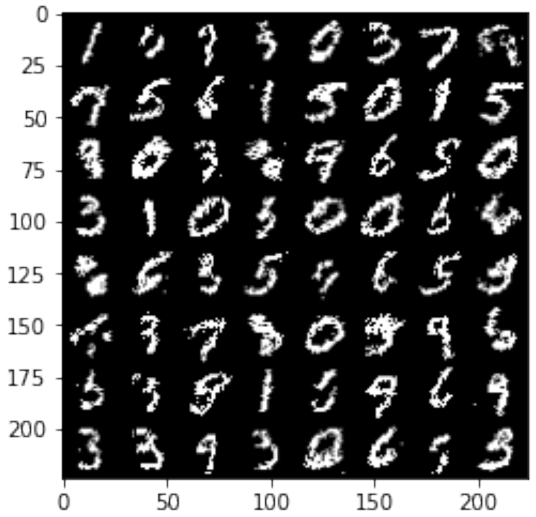
Epoch 25



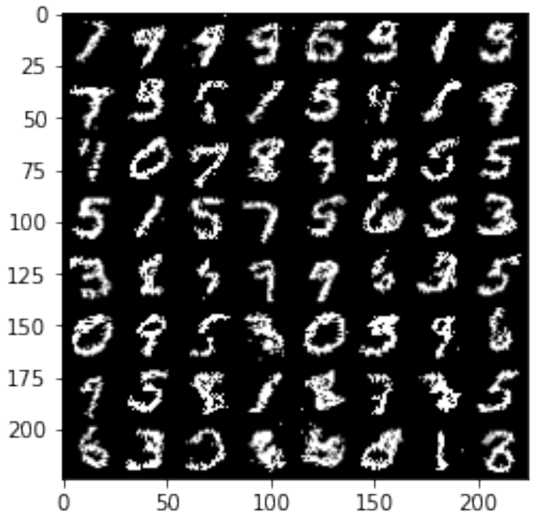
Epoch 26



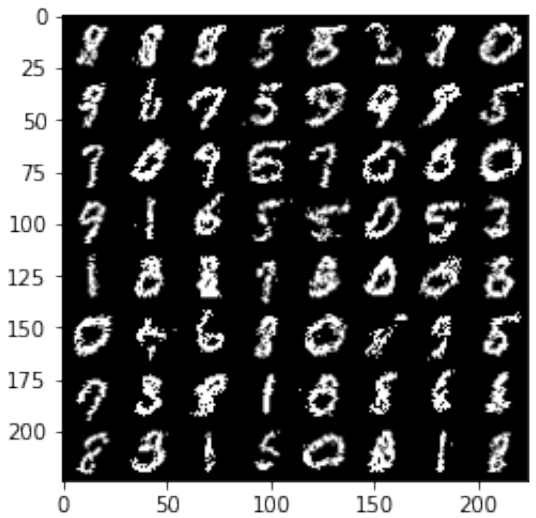
Epoch 27



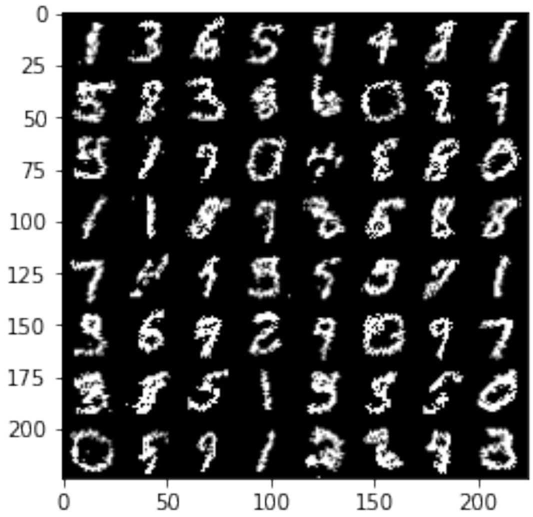
Epoch 28



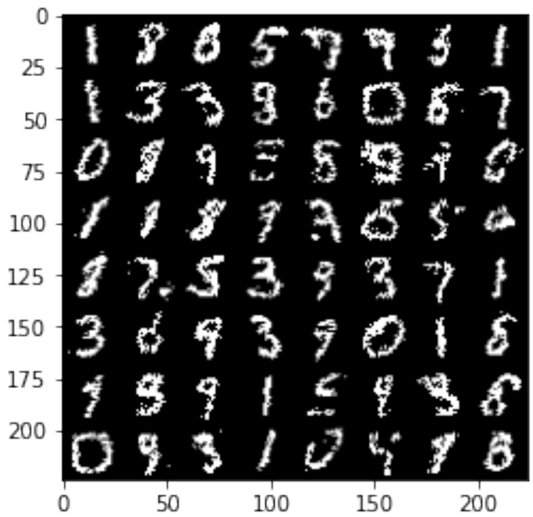
Epoch 29



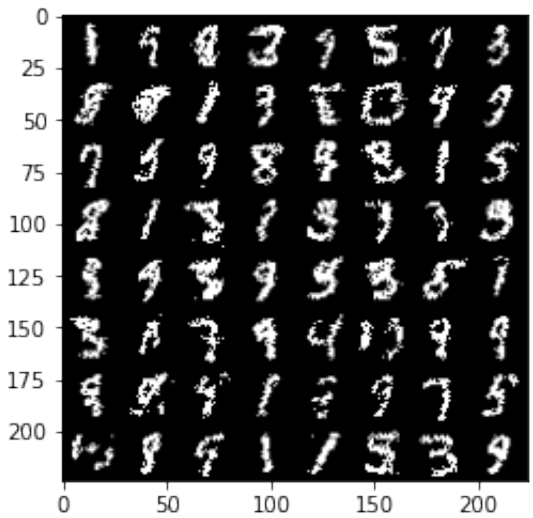

```
Epoch  30
Testing !
After removing 20% pixels
```



After removing 50% pixels



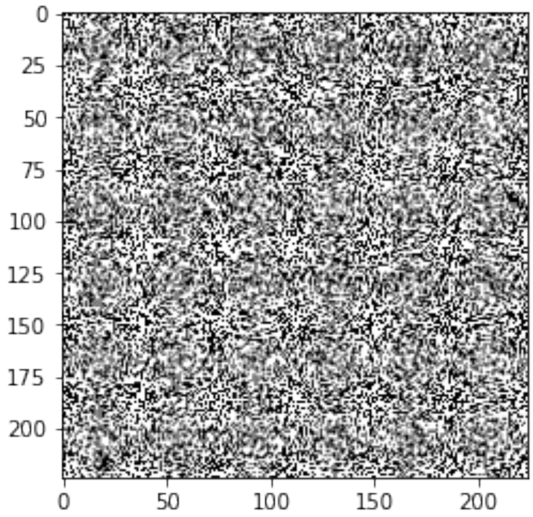
After removing 80% pixels



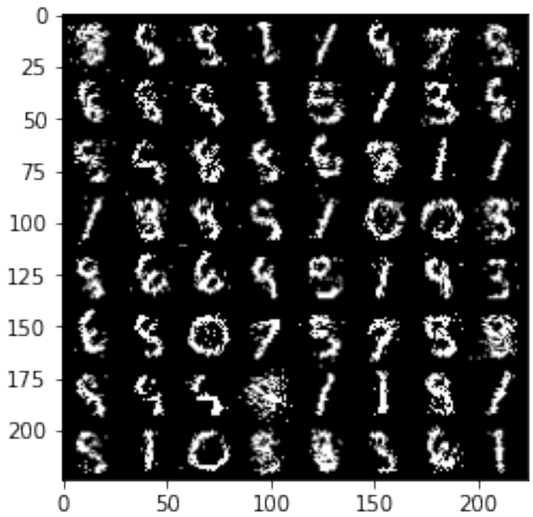
500 hidden nodes training.

Followed by testing: removing 20%, 50% and 80% pixels.

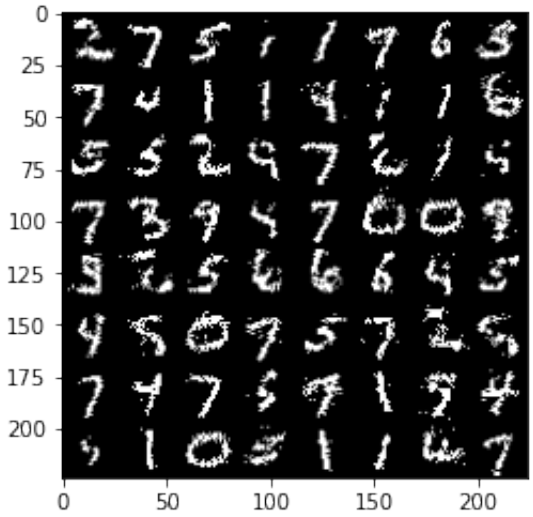
In [22]:
```
tf.compat.v1.disable_eager_execution()
train(train_data, 500,30)
```



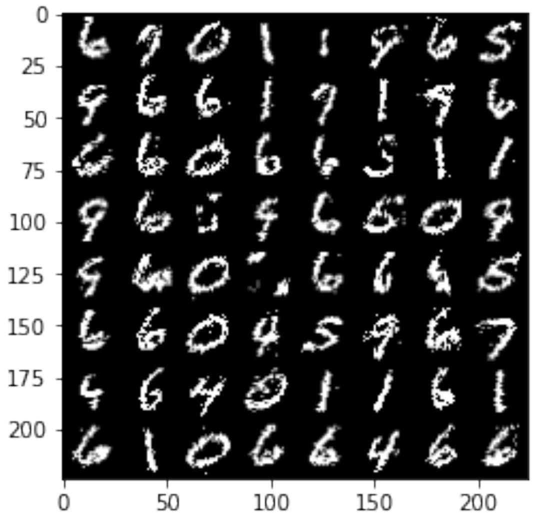
```
Epoch  1
```



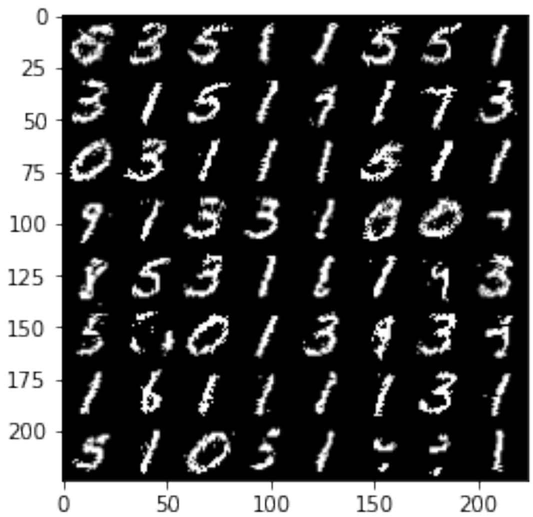
```
Epoch  2
```



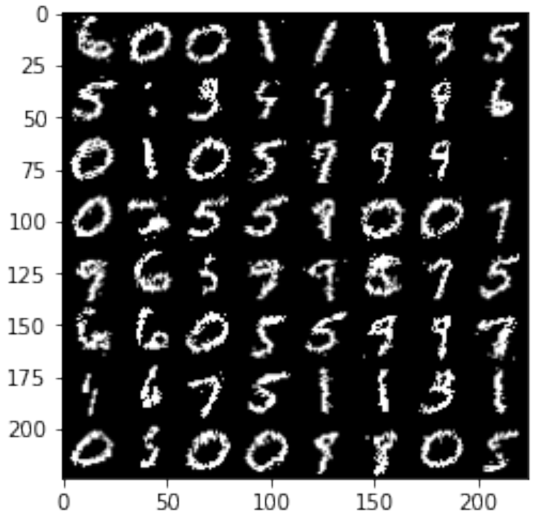
Epoch 3



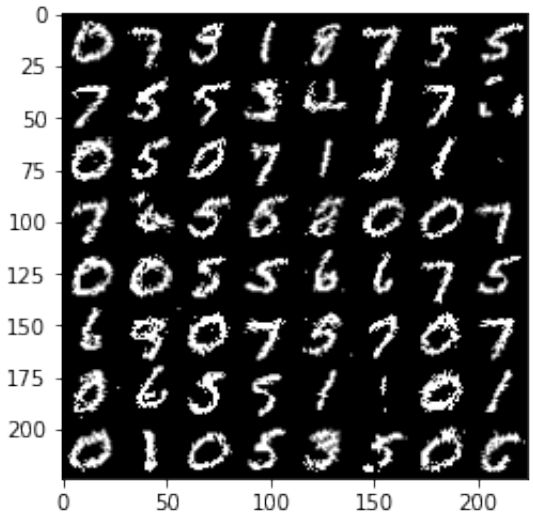
Epoch 4



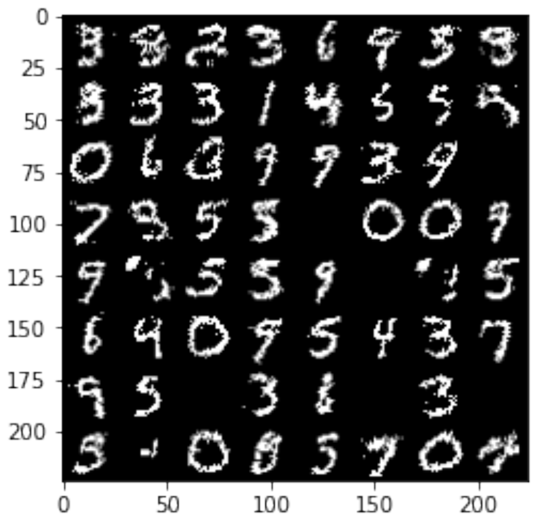
Epoch 5



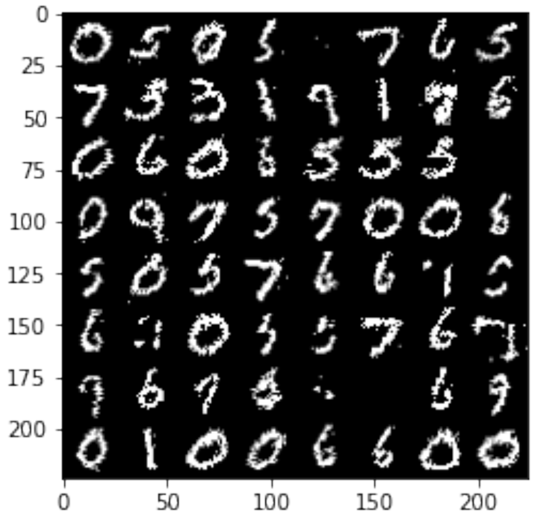
Epoch 6



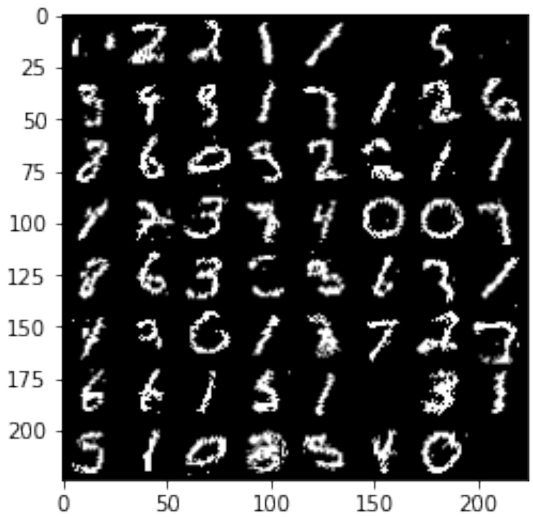
Epoch 7



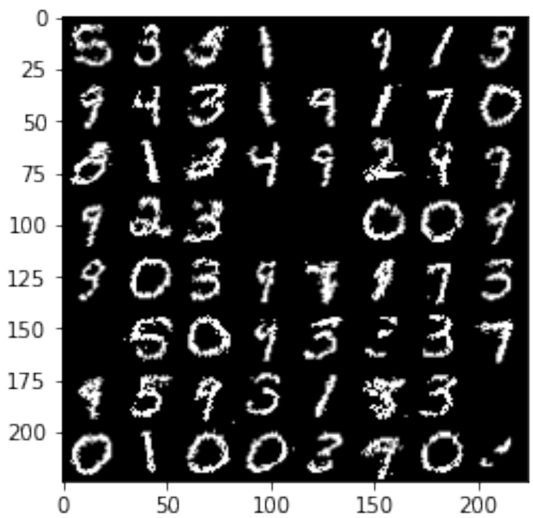
Epoch 8



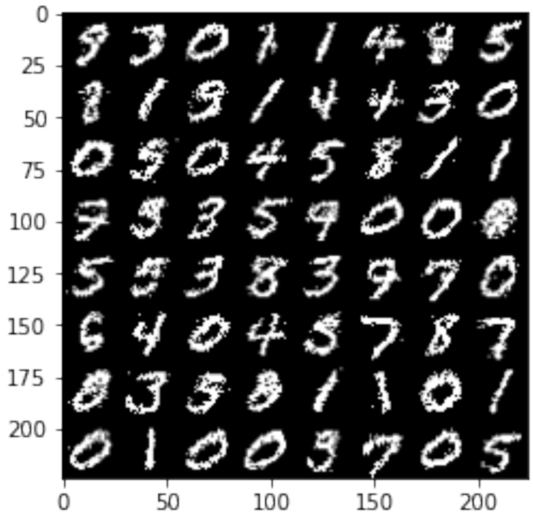
Epoch 9



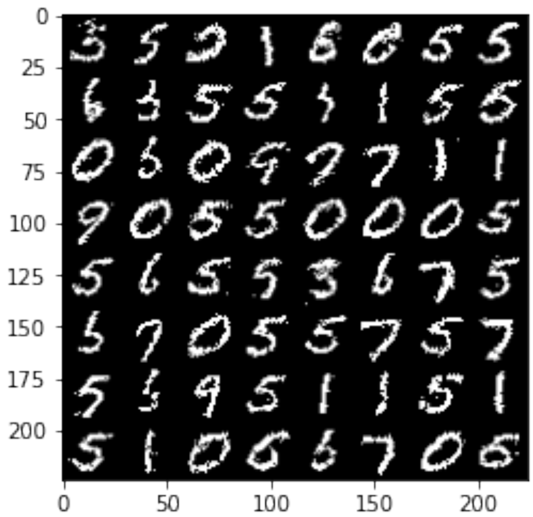
Epoch 10



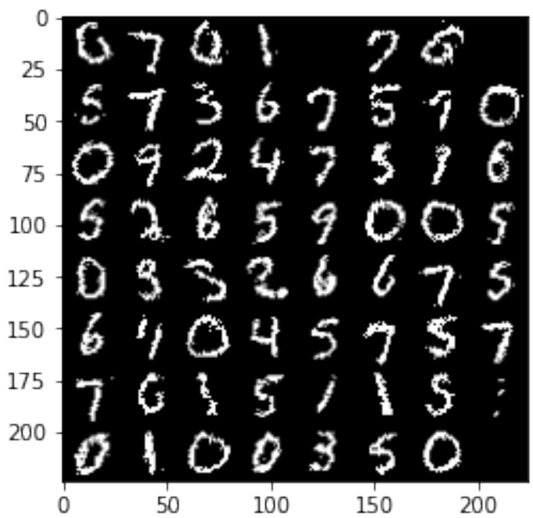
Epoch 11



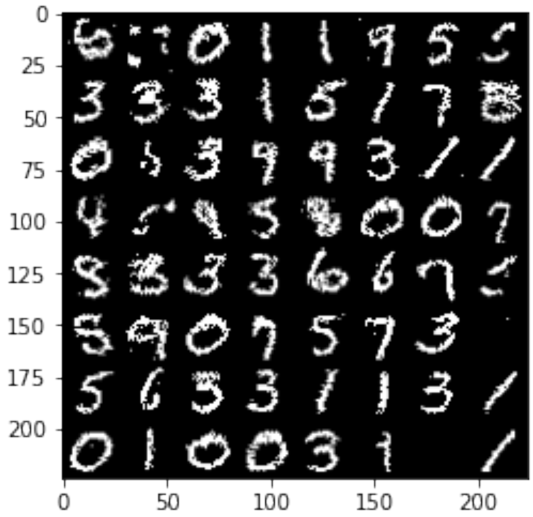
Epoch 12



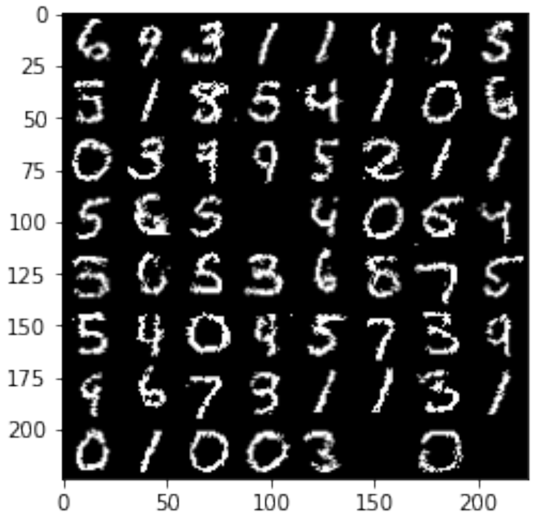
Epoch 13



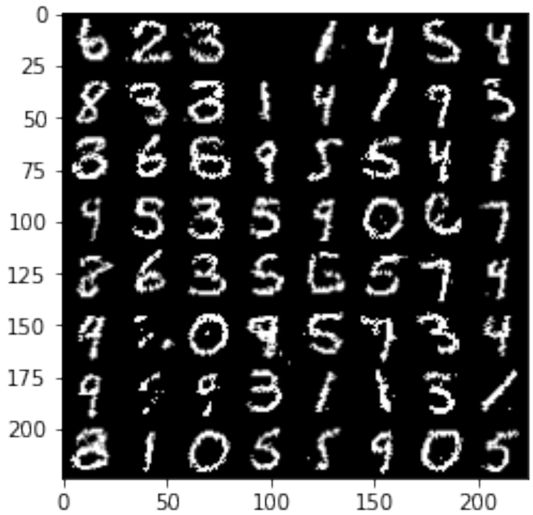
Epoch 14



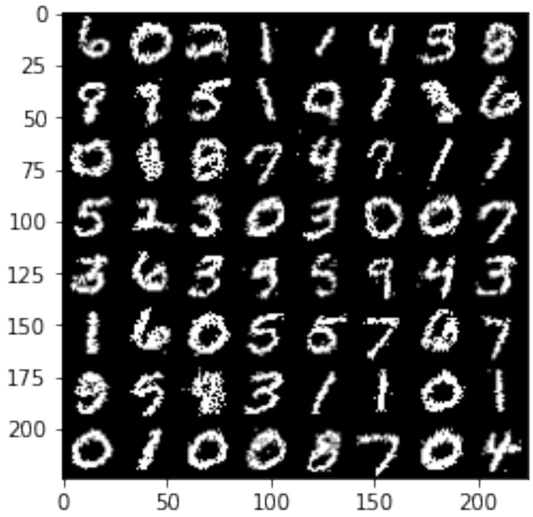
Epoch 15



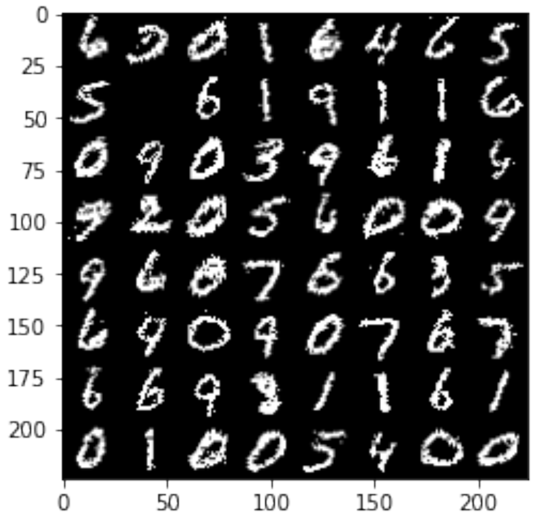
Epoch 16



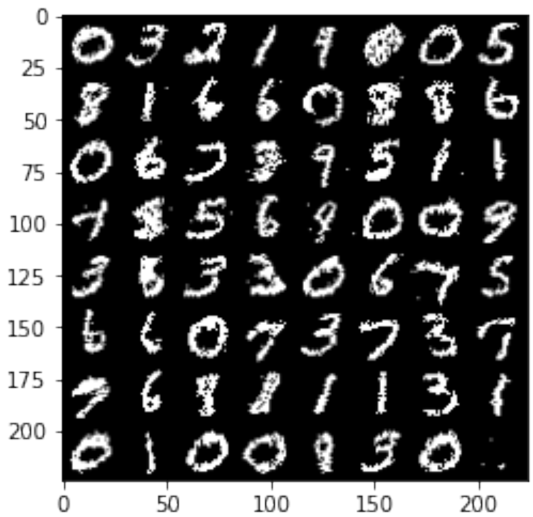
Epoch 17



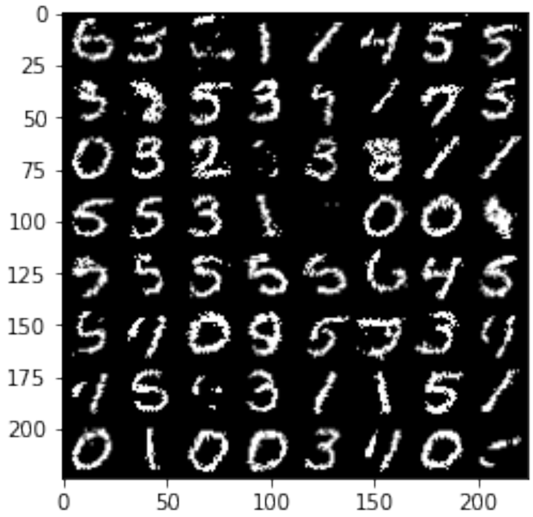
Epoch 18



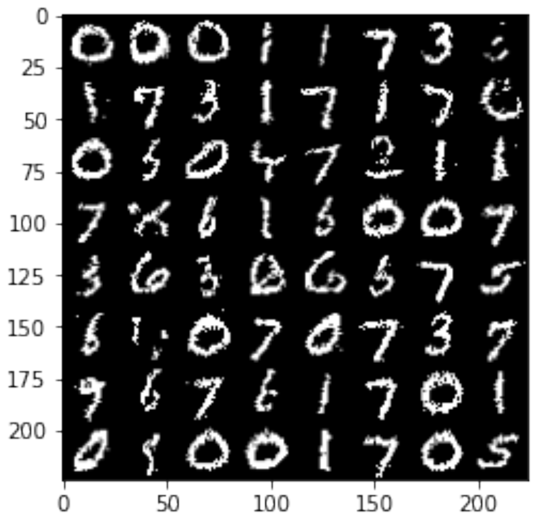
Epoch 19



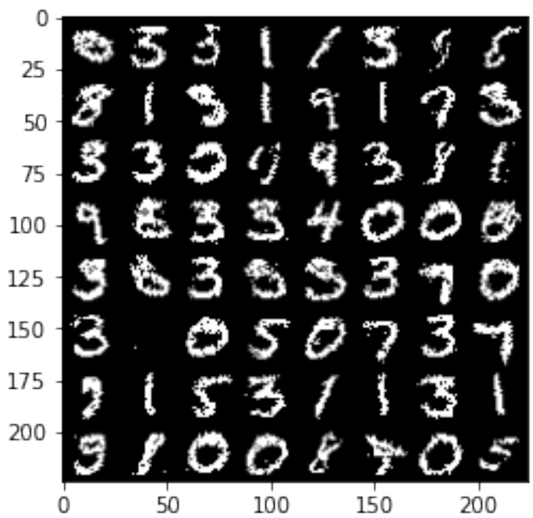
Epoch 20



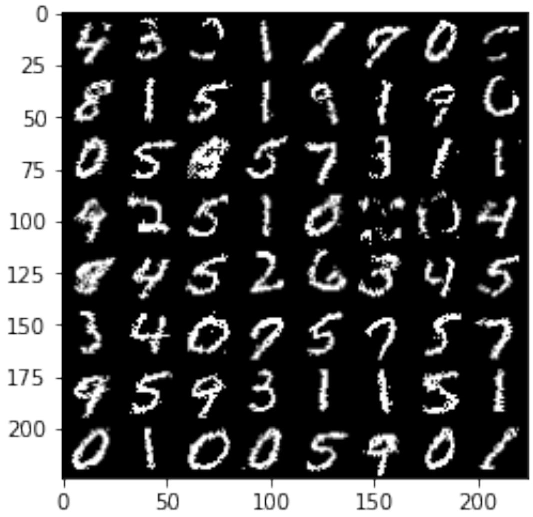
Epoch 21



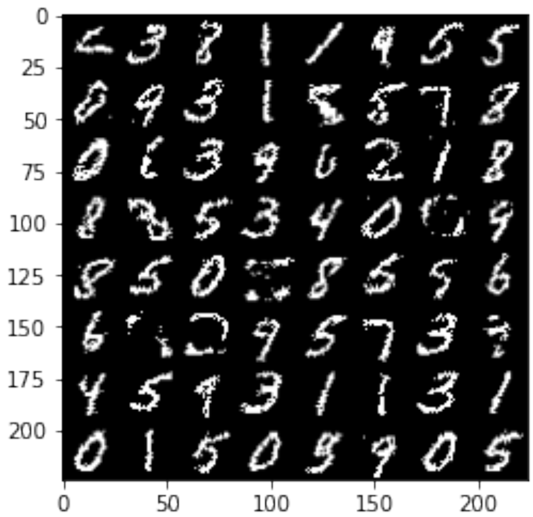
Epoch 22



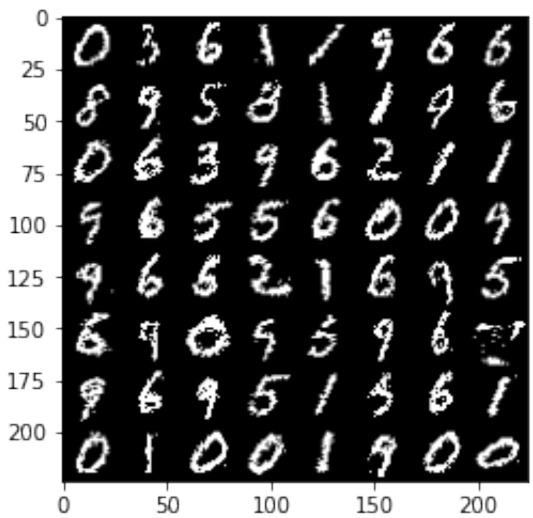
Epoch 23



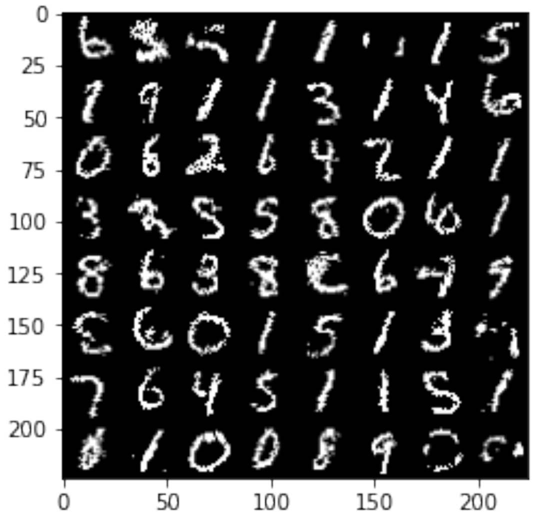
Epoch 24



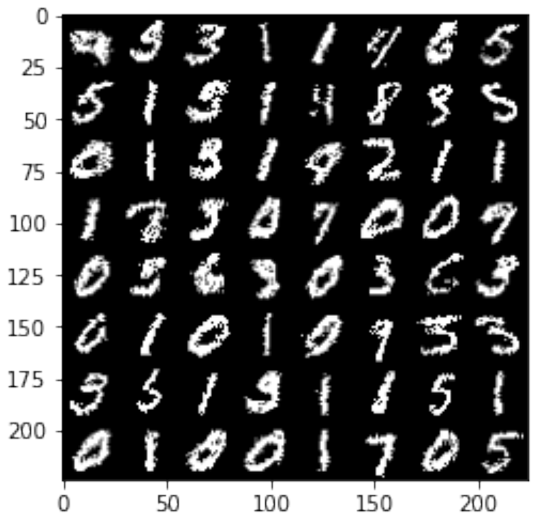
Epoch 25



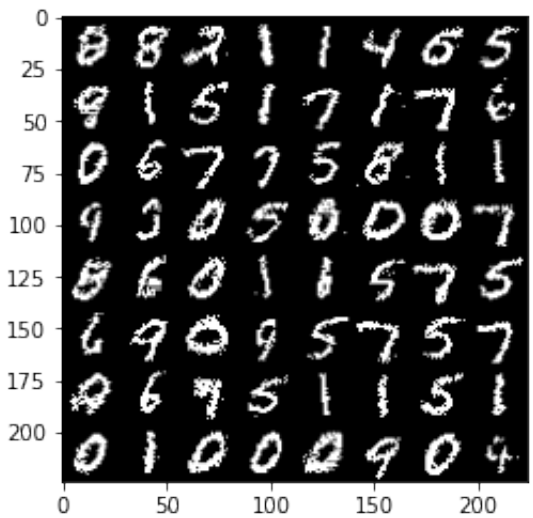
Epoch 26



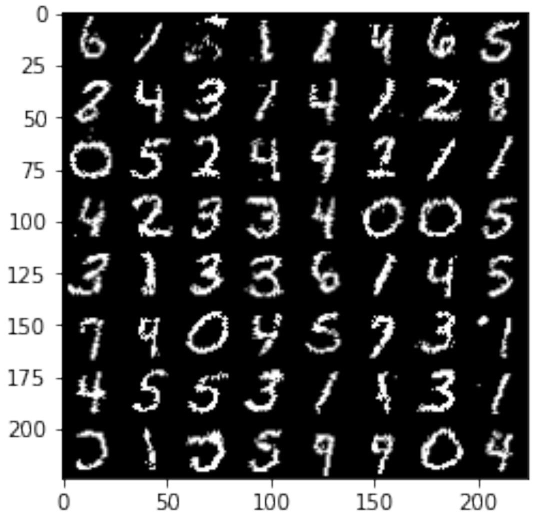
Epoch 27



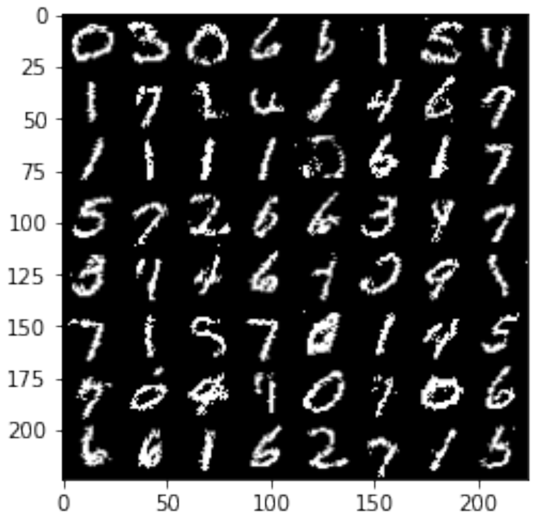
Epoch 28



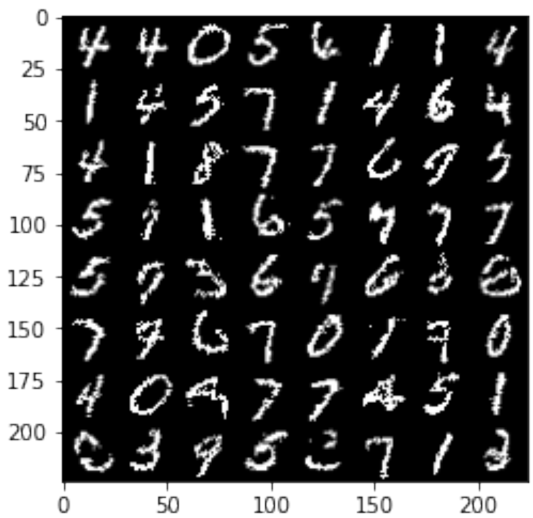
Epoch 29



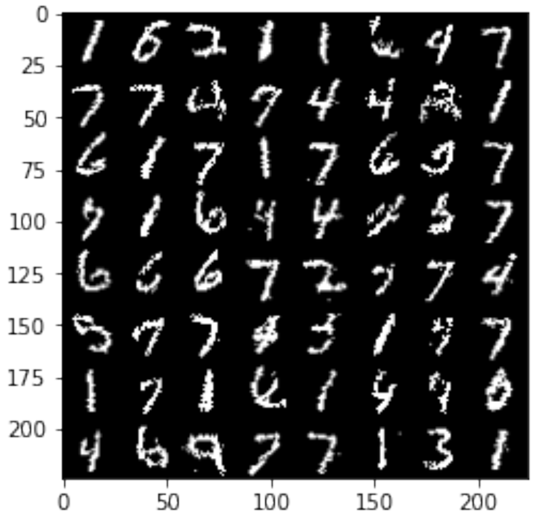
```
Epoch  30
Testing !
After removing 20% pixels
```



```
After removing 50% pixels
```



```
After removing 80% pixels
```



Problem 2

VAE derivation:

Let,

X: data that we want to model
z : latent variable
$P(X)$: probability distribution of the data
$P(z)$: probability distribution of latent variable
$P(X|z)$: distribution of generating data given latent variable

Our objective here is to model the data, hence we want to find P(X).Using the law of probability, we could find it in relation with z as follows:

$P(X) = \int P(X|z)P(z)dz$

But the problem arises since the integral is intractible. So our aim is to approximate $P(z|X)$ by $Q(z|X)$ using variational inference.

let's say we want to infer P(z|X) using Q(z|X).The KL divergence then formulated as follows:

$D_{KL}Q(z|X)\|P(z|X) = \sum_z Q(z|X)\ \log \frac{Q(z|X)}{P(z|X)}$ Using Bayes Rule:
$= E[\log \frac{Q(z|X)}{P(z|X)}]$
$= E[\log Q(z|X) - \log P(z|X)]$

$D_{KL}Q(z|X)\|P(z|X) = E[\log Q(z|X) - \log \frac{P(X|z)P(z)}{P(X)})]$
$= E[\log Q(z|X) - \log P(X|z) - \log P(z) + \log P(X)]$

As P(X) is independent of z we move it outside the expectation term, this gives us the following:

$$D_{KL}[Q(z|X)\|P(z|X)] = E[\log Q(z|X) - \log P(X|z) - \log P(z)] + \log P(X)$$

$$D_{KL}[Q(z|X)\|P(z|X)] - \log P(X) = E[\log Q(z|X) - \log P(X|z) - \log P(z)]$$

On looking at the right hand side of the equation, we would notice that it could be rewritten as another KL divergence:

$$\log P(X) - D_{KL}[Q(z|X)\|P(z|X)] = E[\log P(X|z) - (\log Q(z|X) - \log P(z))]$$
$$= E[\log P(X|z)] - E[\log Q(z|X) - \log P(z)]$$
$$= E[\log P(X|z)] - D_{KL}[Q(z|X)\|P(z)]$$

Thus we get the **VAE objective function** as:

$$\log P(X) - D_{KL}[Q(z|X)\|P(z|X)] = E[\log P(X|z)] - D_{KL}[Q(z|X)\|P(z)]$$

We have the following 3 steps:

1. Learn $Q(z|X)$ by transforming data X into z i.e. the Encoder module.
2. Learn $P(z)$ i.e. distribution of latent space.
3. Generate new data from z $P(X|z)$ i.e. the Decoder module.

Maximizing $E[\log P(X|z)]$ is a maximum likelihood estimation. However for the other term $D_{KL}[Q(z|X)\|P(z)]$, $P(z)$ is the latent variable distribution. When sampling $P(z)$ the easiest choice is N(0,1). Hence, $Q(z|X)$ must be close to N(0,1). Also then, the KL divergence between those two distribution could be computed in closed form.

$$D_{KL}[N(\mu(X), \Sigma(X))\|N(0, 1)] = \frac{1}{2}(\sum_k \Sigma(X) + \sum_k \mu^2(X) - \sum_k 1 - \log \prod_k \Sigma(X))$$
$$= \frac{1}{2}\sum_k(\Sigma(X) + \mu^2(X) - 1 - \log \Sigma(X))$$

In practice, however, it's better to model $\sigma(X)$ as $\log \sigma(X)$, as it is more numerically stable to take exponent compared to computing log. Hence, our final KL divergence term is:

$$D_{KL}[N(\mu(X), \Sigma(X))\|N(0, 1)] = \frac{1}{2}\sum_k(\exp(\Sigma(X)) + \mu^2(X) - 1 - \Sigma(X))$$

Reparameterization trick is used to divert the non-differentiable operation out of the network, so that, even though we still involve a thing that is non-differentiable, at least it is out of the network, hence the network could still be trained. The reparameterization trick is as follows. If we sample from a standard normal distribution, we could convert it to any Gaussian we want if we know the mean and the variance. Hence we could implement our sampling operation of z by:

$$z = \mu(X) + \Sigma^{\frac{1}{2}}(X)\epsilon$$

where $\epsilon \sim N(0, 1)$

During backpropagation, we don't care anymore with the sampling process, as it is now outside of the network, i.e. doesn't depend on anything in the net, hence the gradient won't flow through it.

In [0]:
```python
from keras.layers import Lambda, Input, Dense
from keras.models import Model
from keras.datasets import mnist
from keras.losses import mse, binary_crossentropy
from keras.utils import plot_model
from keras import backend as K

import numpy as np
import matplotlib.pyplot as plt
import argparse
import os
```

In [0]:
```python
def sampling(args):

    z_mean, z_log_var = args
    batch = K.shape(z_mean)[0]
    dim = K.int_shape(z_mean)[1]
    # by default, random_normal has mean = 0 and std = 1.0
    epsilon = K.random_normal(shape=(batch, dim))
    return z_mean + K.exp(0.5 * z_log_var) * epsilon
```

In [0]:
```
def plot_results(models,
                 data,
                 batch_size=128,
                 model_name="vae_mnist"):

    encoder, decoder = models
    x_test, y_test = data

    # display a 30x30 2D manifold of digits
    n = 30
    digit_size = 28
    figure = np.zeros((digit_size * n, digit_size * n))
    # linearly spaced coordinates corresponding to the 2D plot
    # of digit classes in the latent space
    grid_x = np.linspace(-3, 3, n)
    grid_y = np.linspace(-3, 3, n)[::-1]

    for i, yi in enumerate(grid_y):
        for j, xi in enumerate(grid_x):
            z_sample = np.array([[xi, yi]])
            x_decoded = decoder.predict(z_sample)
            digit = x_decoded[0].reshape(digit_size, digit_size)
            figure[i * digit_size: (i + 1) * digit_size,
                   j * digit_size: (j + 1) * digit_size] = digit

    plt.figure(figsize=(10, 10))
    start_range = digit_size // 2
    end_range = (n - 1) * digit_size + start_range + 1
    pixel_range = np.arange(start_range, end_range, digit_size)
    sample_range_x = np.round(grid_x, 1)
    sample_range_y = np.round(grid_y, 1)
    plt.xticks(pixel_range, sample_range_x)
    plt.yticks(pixel_range, sample_range_y)
    plt.imshow(figure, cmap='Greys_r')
    plt.show()
```

In [0]:
```
def vae(latent_dim):

  inputs = Input(shape=input_shape, name='encoder_input')
  x = Dense(intermediate_dim, activation='relu')(inputs)
  z_mean = Dense(latent_dim, name='z_mean')(x)
  z_log_var = Dense(latent_dim, name='z_log_var')(x)


  z = Lambda(sampling, output_shape=(latent_dim,), name='z')([z_mea
n, z_log_var])

  # instantiate encoder model
  encoder = Model(inputs, [z_mean, z_log_var, z], name='encoder')
  encoder.summary()
```

```
  # build decoder model
  latent_inputs = Input(shape=(latent_dim,), name='z_sampling')
  x = Dense(intermediate_dim, activation='relu')(latent_inputs)
  outputs = Dense(original_dim, activation='sigmoid')(x)

  # instantiate decoder model
  decoder = Model(latent_inputs, outputs, name='decoder')
  decoder.summary()


  # instantiate VAE model
  outputs = decoder(encoder(inputs)[2])
  vae = Model(inputs, outputs, name='vae_mlp')
  vae.summary()


  models = (encoder, decoder)
  data = (x_test, y_test)

  reconstruction_loss = mse(inputs, outputs)
  reconstruction_loss *= original_dim
  kl_loss = 1 + z_log_var - K.square(z_mean) - K.exp(z_log_var)
  kl_loss = K.sum(kl_loss, axis=-1)
  kl_loss *= -0.5
  vae_loss = K.mean(reconstruction_loss + kl_loss)
  vae.add_loss(vae_loss)
  vae.compile(optimizer='adam')
  vae.summary()

  vae.fit(x_train,epochs=epochs,batch_size=batch_size,validation_da
ta=(x_test, None))

  if latent_dim==2:
    plot_results(models,data,batch_size=batch_size,model_name="vae_
mlp")

  return vae
```

In [0]:
```
# MNIST dataset
(x_train, y_train), (x_test, y_test) = mnist.load_data()

image_size = x_train.shape[1]
original_dim = image_size * image_size
x_train = np.reshape(x_train, [-1, original_dim])
x_test = np.reshape(x_test, [-1, original_dim])
x_train = x_train.astype('float32') / 255
x_test = x_test.astype('float32') / 255

# network parameters
input_shape = (original_dim, )
intermediate_dim = 256
batch_size = 128
latent_dim = 2
epochs = 30
```

In [0]:
```
original_dim,intermediate_dim
```

Out[0]:
```
(784, 256)
```

Code units=2 and generated images by varying latent variables from -3 to 3

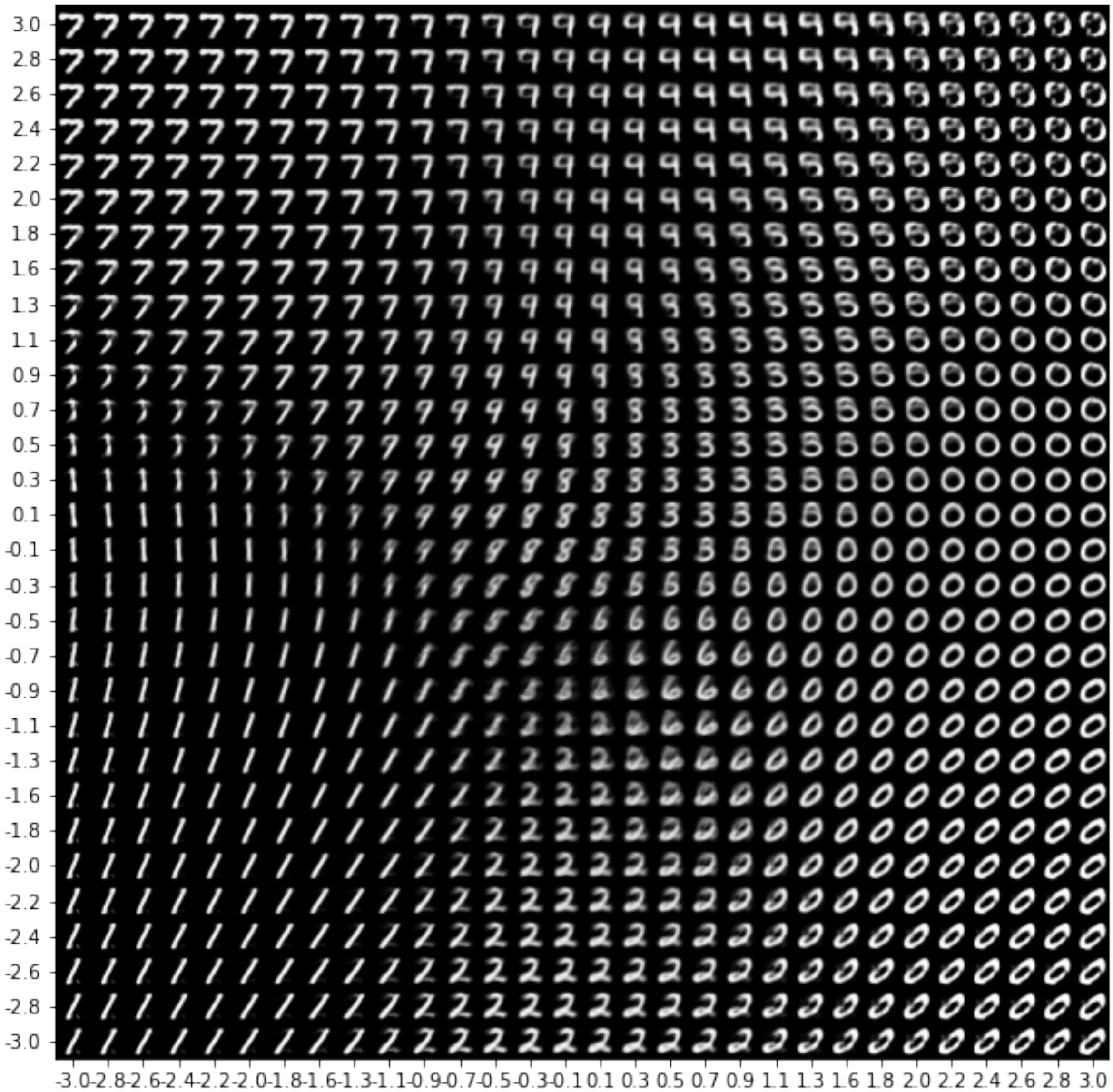
In [0]:
```
model=vae(2)
```

```
Model: "encoder"
__________________________________________________________________
________________________________
Layer (type)                    Output Shape         Param #     C
onnected to
==================================================================
================================
encoder_input (InputLayer)      (None, 784)          0
__________________________________________________________________
________________________________
dense_1 (Dense)                 (None, 256)          200960      e
ncoder_input[0][0]
__________________________________________________________________
________________________________
z_mean (Dense)                  (None, 2)            514         d
ense_1[0][0]
__________________________________________________________________
________________________________
z_log_var (Dense)               (None, 2)            514         d
ense_1[0][0]
__________________________________________________________________
________________________________
z (Lambda)                      (None, 2)            0           z
_mean[0][0]
                                                                 z
_log_var[0][0]
```

```
================================================================================================
Total params: 201,988
Trainable params: 201,988
Non-trainable params: 0
________________________________________________________________________________________________
Model: "decoder"
_________________________________________________________________
Layer (type)                 Output Shape              Param #   
=================================================================
z_sampling (InputLayer)      (None, 2)                 0         
_________________________________________________________________
dense_2 (Dense)              (None, 256)               768       
_________________________________________________________________
dense_3 (Dense)              (None, 784)               201488    
=================================================================
Total params: 202,256
Trainable params: 202,256
Non-trainable params: 0
_________________________________________________________________
Model: "vae_mlp"
_________________________________________________________________
Layer (type)                 Output Shape              Param #   
=================================================================
encoder_input (InputLayer)   (None, 784)               0         
_________________________________________________________________
encoder (Model)              [(None, 2), (None, 2), (N 201988    
_________________________________________________________________
decoder (Model)              (None, 784)               202256    
=================================================================
Total params: 404,244
Trainable params: 404,244
Non-trainable params: 0
_________________________________________________________________
Model: "vae_mlp"
_________________________________________________________________
Layer (type)                 Output Shape              Param #   
=================================================================
encoder_input (InputLayer)   (None, 784)               0         
_________________________________________________________________
encoder (Model)              [(None, 2), (None, 2), (N 201988    
_________________________________________________________________
decoder (Model)              (None, 784)               202256    
=================================================================
Total params: 404,244
Trainable params: 404,244
Non-trainable params: 0
_________________________________________________________________
```

```
/usr/local/lib/python3.6/dist-packages/keras/engine/training_utils
.py:819: UserWarning: Output decoder missing from loss dictionary.
We assume this was done on purpose. The fit and evaluate APIs will
not be expecting any data to be passed to decoder.
  'be expecting any data to be passed to {0}.'.format(name))

Train on 60000 samples, validate on 10000 samples
Epoch 1/30
60000/60000 [==============================] - 3s 58us/step - loss
: 55.0477 - val_loss: 45.6592
Epoch 2/30
60000/60000 [==============================] - 2s 33us/step - loss
: 44.0658 - val_loss: 43.2927
Epoch 3/30
60000/60000 [==============================] - 2s 33us/step - loss
: 42.7544 - val_loss: 42.5227
Epoch 4/30
60000/60000 [==============================] - 2s 33us/step - loss
: 42.1759 - val_loss: 41.9824
Epoch 5/30
60000/60000 [==============================] - 2s 33us/step - loss
: 41.7146 - val_loss: 41.5494
Epoch 6/30
60000/60000 [==============================] - 2s 33us/step - loss
: 41.3244 - val_loss: 41.1091
Epoch 7/30
60000/60000 [==============================] - 2s 33us/step - loss
: 40.9802 - val_loss: 40.8690
Epoch 8/30
60000/60000 [==============================] - 2s 34us/step - loss
: 40.6434 - val_loss: 40.5432
Epoch 9/30
60000/60000 [==============================] - 2s 33us/step - loss
: 40.3415 - val_loss: 40.3838
Epoch 10/30
60000/60000 [==============================] - 2s 33us/step - loss
: 40.0907 - val_loss: 40.0736
Epoch 11/30
60000/60000 [==============================] - 2s 33us/step - loss
: 39.8978 - val_loss: 39.9238
Epoch 12/30
60000/60000 [==============================] - 2s 33us/step - loss
: 39.6876 - val_loss: 39.7079
Epoch 13/30
60000/60000 [==============================] - 2s 33us/step - loss
: 39.5086 - val_loss: 39.6411
Epoch 14/30
60000/60000 [==============================] - 2s 33us/step - loss
: 39.3637 - val_loss: 39.4656
Epoch 15/30
60000/60000 [==============================] - 2s 33us/step - loss
: 39.2222 - val_loss: 39.2878
Epoch 16/30
```

```
60000/60000 [==============================] - 2s 33us/step - loss
: 39.0595 - val_loss: 39.2510
Epoch 17/30
60000/60000 [==============================] - 2s 33us/step - loss
: 38.9537 - val_loss: 39.1338
Epoch 18/30
60000/60000 [==============================] - 2s 32us/step - loss
: 38.8503 - val_loss: 39.0146
Epoch 19/30
60000/60000 [==============================] - 2s 33us/step - loss
: 38.7450 - val_loss: 38.9454
Epoch 20/30
60000/60000 [==============================] - 2s 33us/step - loss
: 38.6451 - val_loss: 38.8135
Epoch 21/30
60000/60000 [==============================] - 2s 32us/step - loss
: 38.5462 - val_loss: 38.7438
Epoch 22/30
60000/60000 [==============================] - 2s 33us/step - loss
: 38.4631 - val_loss: 38.7279
Epoch 23/30
60000/60000 [==============================] - 2s 33us/step - loss
: 38.3945 - val_loss: 38.5867
Epoch 24/30
60000/60000 [==============================] - 2s 32us/step - loss
: 38.3223 - val_loss: 38.6438
Epoch 25/30
60000/60000 [==============================] - 2s 33us/step - loss
: 38.2469 - val_loss: 38.4917
Epoch 26/30
60000/60000 [==============================] - 2s 33us/step - loss
: 38.1658 - val_loss: 38.5023
Epoch 27/30
60000/60000 [==============================] - 2s 33us/step - loss
: 38.1239 - val_loss: 38.4634
Epoch 28/30
60000/60000 [==============================] - 2s 32us/step - loss
: 38.0551 - val_loss: 38.4107
Epoch 29/30
60000/60000 [==============================] - 2s 33us/step - loss
: 38.0106 - val_loss: 38.5096
Epoch 30/30
60000/60000 [==============================] - 2s 33us/step - loss
: 37.9651 - val_loss: 38.3257
```

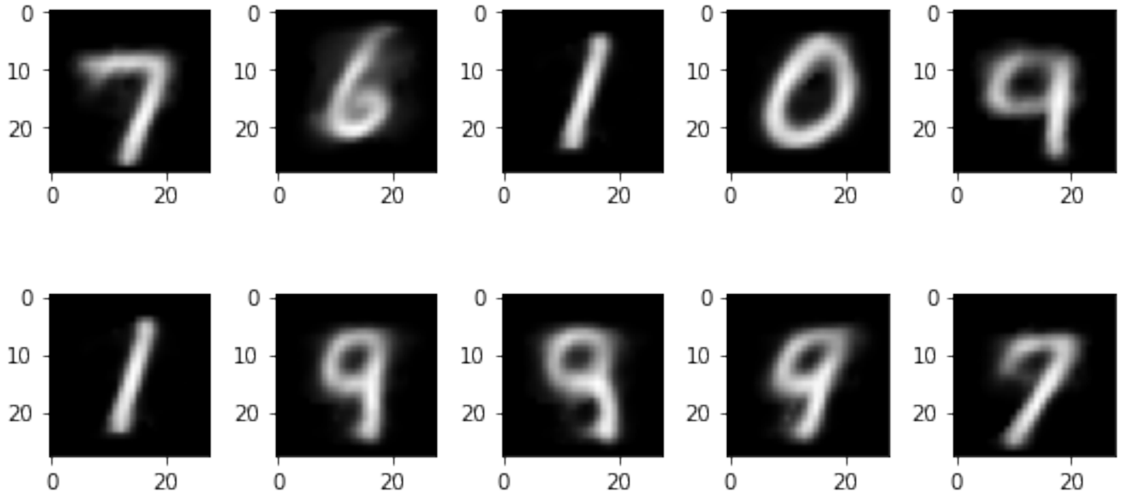


In [0]:
```
inference=model.predict(x_test)
```

In [0]:
```
inference.shape
```

Out[0]: (10000, 784)

In [0]:
```
num_row = 2
num_col = 5
# plot images
fig, axes = plt.subplots(num_row, num_col, figsize=(1.5*num_col,2*n
um_row))
for i in range(10):
    ax = axes[i//num_col, i%num_col]
    ax.imshow(inference[i].reshape(28,28), cmap='gray')
plt.tight_layout()
plt.show()
```



Code units=8

In [0]:
```
model=vae(8)
```

```
Model: "encoder"
______________________________________________________________________
____________________________________
Layer (type)                    Output Shape         Param #     C
onnected to
======================================================================
====================================
encoder_input (InputLayer)      (None, 784)          0
______________________________________________________________________
____________________________________
dense_4 (Dense)                 (None, 256)          200960      e
ncoder_input[0][0]
______________________________________________________________________
____________________________________
z_mean (Dense)                  (None, 8)            2056        d
ense_4[0][0]
______________________________________________________________________
____________________________________
z_log_var (Dense)               (None, 8)            2056        d
ense_4[0][0]
______________________________________________________________________
```

```
________________________________________________________________
z (Lambda)                      (None, 8)            0           z
_mean[0][0]
                                                                 z
_log_var[0][0]
==================================================================
================================
Total params: 205,072
Trainable params: 205,072
Non-trainable params: 0
__________________________________________________________________
________________________________
Model: "decoder"
_________________________________________________________________
Layer (type)                 Output Shape              Param #   
=================================================================
z_sampling (InputLayer)      (None, 8)                 0         
_________________________________________________________________
dense_5 (Dense)              (None, 256)               2304      
_________________________________________________________________
dense_6 (Dense)              (None, 784)               201488    
=================================================================
Total params: 203,792
Trainable params: 203,792
Non-trainable params: 0
_________________________________________________________________
Model: "vae_mlp"
_________________________________________________________________
Layer (type)                 Output Shape              Param #   
=================================================================
encoder_input (InputLayer)   (None, 784)               0         
_________________________________________________________________
encoder (Model)              [(None, 8), (None, 8), (N 205072    
_________________________________________________________________
decoder (Model)              (None, 784)               203792    
=================================================================
Total params: 408,864
Trainable params: 408,864
Non-trainable params: 0
_________________________________________________________________
Model: "vae_mlp"
_________________________________________________________________
Layer (type)                 Output Shape              Param #   
=================================================================
encoder_input (InputLayer)   (None, 784)               0         
_________________________________________________________________
encoder (Model)              [(None, 8), (None, 8), (N 205072    
_________________________________________________________________
decoder (Model)              (None, 784)               203792    
=================================================================
Total params: 408,864
Trainable params: 408,864
Non-trainable params: 0
```

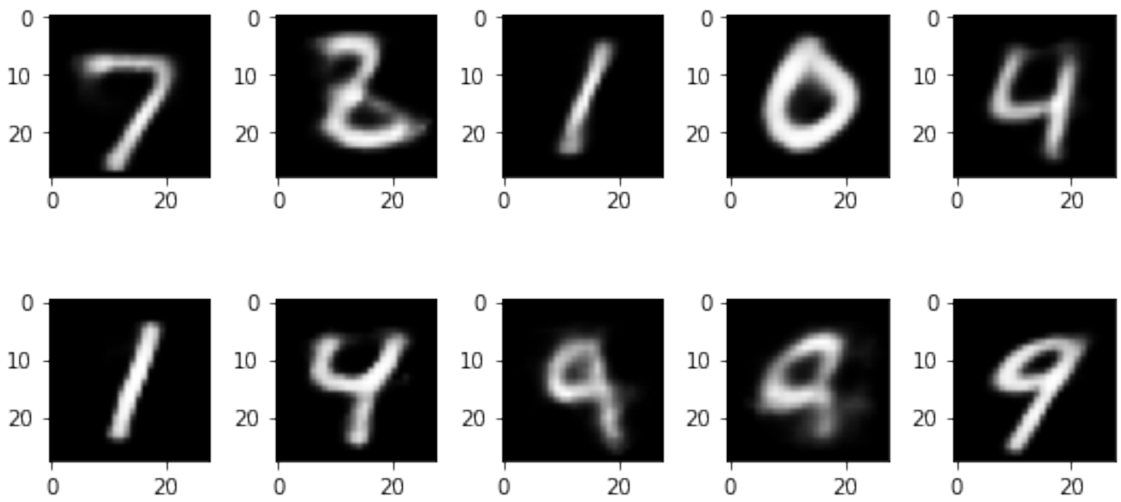
```
/usr/local/lib/python3.6/dist-packages/keras/engine/training_utils
.py:819: UserWarning: Output decoder missing from loss dictionary.
We assume this was done on purpose. The fit and evaluate APIs will
not be expecting any data to be passed to decoder.
  'be expecting any data to be passed to {0}.'.format(name))

Train on 60000 samples, validate on 10000 samples
Epoch 1/30
60000/60000 [==============================] - 4s 61us/step - loss
: 48.4368 - val_loss: 37.5004
Epoch 2/30
60000/60000 [==============================] - 2s 33us/step - loss
: 36.1297 - val_loss: 34.7652
Epoch 3/30
60000/60000 [==============================] - 2s 32us/step - loss
: 34.4447 - val_loss: 33.6796
Epoch 4/30
60000/60000 [==============================] - 2s 33us/step - loss
: 33.5777 - val_loss: 33.0398
Epoch 5/30
60000/60000 [==============================] - 2s 33us/step - loss
: 32.9973 - val_loss: 32.4594
Epoch 6/30
60000/60000 [==============================] - 2s 33us/step - loss
: 32.5824 - val_loss: 32.2101
Epoch 7/30
60000/60000 [==============================] - 2s 33us/step - loss
: 32.2896 - val_loss: 32.0000
Epoch 8/30
60000/60000 [==============================] - 2s 33us/step - loss
: 31.9871 - val_loss: 31.7186
Epoch 9/30
60000/60000 [==============================] - 2s 33us/step - loss
: 31.7781 - val_loss: 31.4970
Epoch 10/30
60000/60000 [==============================] - 2s 32us/step - loss
: 31.5954 - val_loss: 31.2821
Epoch 11/30
60000/60000 [==============================] - 2s 32us/step - loss
: 31.4011 - val_loss: 31.1281
Epoch 12/30
60000/60000 [==============================] - 2s 32us/step - loss
: 31.2556 - val_loss: 31.1085
Epoch 13/30
60000/60000 [==============================] - 2s 32us/step - loss
: 31.1052 - val_loss: 30.8670
Epoch 14/30
60000/60000 [==============================] - 2s 32us/step - loss
: 30.9381 - val_loss: 30.8271
Epoch 15/30
60000/60000 [==============================] - 2s 32us/step - loss
: 30.8391 - val_loss: 30.5673
```

```
Epoch 16/30
60000/60000 [==============================] - 2s 32us/step - loss
: 30.7304 - val_loss: 30.5606
Epoch 17/30
60000/60000 [==============================] - 2s 32us/step - loss
: 30.6023 - val_loss: 30.4539
Epoch 18/30
60000/60000 [==============================] - 2s 32us/step - loss
: 30.5075 - val_loss: 30.2782
Epoch 19/30
60000/60000 [==============================] - 2s 32us/step - loss
: 30.4349 - val_loss: 30.2428
Epoch 20/30
60000/60000 [==============================] - 2s 32us/step - loss
: 30.3516 - val_loss: 30.1365
Epoch 21/30
60000/60000 [==============================] - 2s 33us/step - loss
: 30.2616 - val_loss: 30.1102
Epoch 22/30
60000/60000 [==============================] - 2s 32us/step - loss
: 30.1911 - val_loss: 30.0771
Epoch 23/30
60000/60000 [==============================] - 2s 32us/step - loss
: 30.1357 - val_loss: 29.9404
Epoch 24/30
60000/60000 [==============================] - 2s 32us/step - loss
: 30.0499 - val_loss: 29.9513
Epoch 25/30
60000/60000 [==============================] - 2s 32us/step - loss
: 29.9813 - val_loss: 29.8946
Epoch 26/30
60000/60000 [==============================] - 2s 32us/step - loss
: 29.9199 - val_loss: 29.8648
Epoch 27/30
60000/60000 [==============================] - 2s 32us/step - loss
: 29.8733 - val_loss: 29.6888
Epoch 28/30
60000/60000 [==============================] - 2s 32us/step - loss
: 29.8457 - val_loss: 29.7381
Epoch 29/30
60000/60000 [==============================] - 2s 32us/step - loss
: 29.7829 - val_loss: 29.6970
Epoch 30/30
60000/60000 [==============================] - 2s 32us/step - loss
: 29.7382 - val_loss: 29.6226
```

In [0]:

```
inference=model.predict(x_test)
```

In [0]:
```python
num_row = 2
num_col = 5
# plot images
fig, axes = plt.subplots(num_row, num_col, figsize=(1.5*num_col,2*n
um_row))
for i in range(10):
    ax = axes[i//num_col, i%num_col]
    ax.imshow(inference[i].reshape(28,28), cmap='gray')
plt.tight_layout()
plt.show()
```



Code units=16

In [0]:
```python
model=vae(16)
```

```
Model: "encoder"
__________________________________________________________________
__________________________________
Layer (type)                    Output Shape         Param #     C
onnected to
==================================================================
==================================
encoder_input (InputLayer)      (None, 784)          0
__________________________________________________________________
__________________________________
dense_7 (Dense)                 (None, 256)          200960      e
ncoder_input[0][0]
__________________________________________________________________
__________________________________
z_mean (Dense)                  (None, 16)           4112        d
ense_7[0][0]
__________________________________________________________________
__________________________________
z_log_var (Dense)               (None, 16)           4112        d
ense_7[0][0]
__________________________________________________________________
```

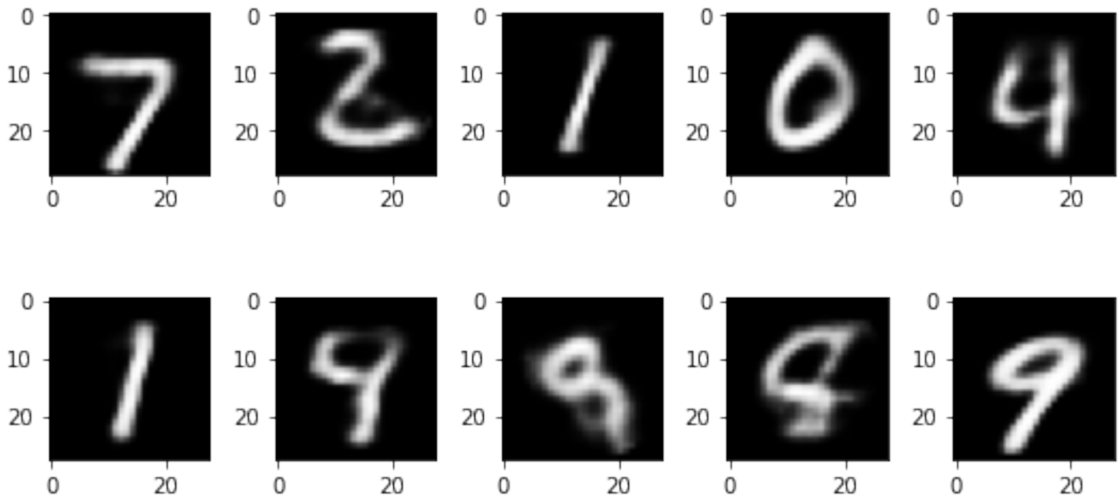
```
________________________________
z (Lambda)                      (None, 16)           0           z
_mean[0][0]
                                                                 z
_log_var[0][0]
==================================================================
================================
Total params: 209,184
Trainable params: 209,184
Non-trainable params: 0
__________________________________________________________________
________________________________
Model: "decoder"
_________________________________________________________________
Layer (type)                 Output Shape              Param #
=================================================================
z_sampling (InputLayer)      (None, 16)                0
_________________________________________________________________
dense_8 (Dense)              (None, 256)               4352
_________________________________________________________________
dense_9 (Dense)              (None, 784)               201488
=================================================================
Total params: 205,840
Trainable params: 205,840
Non-trainable params: 0
_________________________________________________________________
Model: "vae_mlp"
_________________________________________________________________
Layer (type)                 Output Shape              Param #
=================================================================
encoder_input (InputLayer)   (None, 784)               0
_________________________________________________________________
encoder (Model)              [(None, 16), (None, 16),  209184
_________________________________________________________________
decoder (Model)              (None, 784)               205840
=================================================================
Total params: 415,024
Trainable params: 415,024
Non-trainable params: 0
_________________________________________________________________
Model: "vae_mlp"
_________________________________________________________________
Layer (type)                 Output Shape              Param #
=================================================================
encoder_input (InputLayer)   (None, 784)               0
_________________________________________________________________
encoder (Model)              [(None, 16), (None, 16),  209184
_________________________________________________________________
decoder (Model)              (None, 784)               205840
=================================================================
Total params: 415,024
Trainable params: 415,024
Non-trainable params: 0
```

```
/usr/local/lib/python3.6/dist-packages/keras/engine/training_utils
.py:819: UserWarning: Output decoder missing from loss dictionary.
We assume this was done on purpose. The fit and evaluate APIs will
not be expecting any data to be passed to decoder.
  'be expecting any data to be passed to {0}.'.format(name))

Train on 60000 samples, validate on 10000 samples
Epoch 1/30
60000/60000 [==============================] - 4s 62us/step - loss
: 49.0297 - val_loss: 37.7708
Epoch 2/30
60000/60000 [==============================] - 2s 32us/step - loss
: 36.0786 - val_loss: 34.4709
Epoch 3/30
60000/60000 [==============================] - 2s 33us/step - loss
: 33.9707 - val_loss: 33.1215
Epoch 4/30
60000/60000 [==============================] - 2s 32us/step - loss
: 32.9443 - val_loss: 32.2671
Epoch 5/30
60000/60000 [==============================] - 2s 32us/step - loss
: 32.3295 - val_loss: 31.7344
Epoch 6/30
60000/60000 [==============================] - 2s 33us/step - loss
: 31.9197 - val_loss: 31.4109
Epoch 7/30
60000/60000 [==============================] - 2s 33us/step - loss
: 31.6239 - val_loss: 31.1337
Epoch 8/30
60000/60000 [==============================] - 2s 32us/step - loss
: 31.3720 - val_loss: 31.0695
Epoch 9/30
60000/60000 [==============================] - 2s 31us/step - loss
: 31.1935 - val_loss: 30.9267
Epoch 10/30
60000/60000 [==============================] - 2s 32us/step - loss
: 31.0544 - val_loss: 30.7454
Epoch 11/30
60000/60000 [==============================] - 2s 32us/step - loss
: 30.9226 - val_loss: 30.6007
Epoch 12/30
60000/60000 [==============================] - 2s 31us/step - loss
: 30.7965 - val_loss: 30.4270
Epoch 13/30
60000/60000 [==============================] - 2s 32us/step - loss
: 30.6800 - val_loss: 30.3863
Epoch 14/30
60000/60000 [==============================] - 2s 31us/step - loss
: 30.5851 - val_loss: 30.3613
Epoch 15/30
60000/60000 [==============================] - 2s 31us/step - loss
: 30.5006 - val_loss: 30.2732
```

```
Epoch 16/30
60000/60000 [==============================] - 2s 31us/step - loss
: 30.4565 - val_loss: 30.1469
Epoch 17/30
60000/60000 [==============================] - 2s 31us/step - loss
: 30.3848 - val_loss: 30.0676
Epoch 18/30
60000/60000 [==============================] - 2s 32us/step - loss
: 30.2877 - val_loss: 30.1163
Epoch 19/30
60000/60000 [==============================] - 2s 31us/step - loss
: 30.2353 - val_loss: 30.0230
Epoch 20/30
60000/60000 [==============================] - 2s 31us/step - loss
: 30.1928 - val_loss: 29.9477
Epoch 21/30
60000/60000 [==============================] - 2s 32us/step - loss
: 30.1131 - val_loss: 30.0213
Epoch 22/30
60000/60000 [==============================] - 2s 32us/step - loss
: 30.1035 - val_loss: 29.8594
Epoch 23/30
60000/60000 [==============================] - 2s 32us/step - loss
: 30.0254 - val_loss: 29.8712
Epoch 24/30
60000/60000 [==============================] - 2s 32us/step - loss
: 29.9937 - val_loss: 29.8370
Epoch 25/30
60000/60000 [==============================] - 2s 32us/step - loss
: 29.9424 - val_loss: 29.7841
Epoch 26/30
60000/60000 [==============================] - 2s 31us/step - loss
: 29.9131 - val_loss: 29.7074
Epoch 27/30
60000/60000 [==============================] - 2s 32us/step - loss
: 29.8772 - val_loss: 29.6718
Epoch 28/30
60000/60000 [==============================] - 2s 32us/step - loss
: 29.8316 - val_loss: 29.6844
Epoch 29/30
60000/60000 [==============================] - 2s 33us/step - loss
: 29.7857 - val_loss: 29.6533
Epoch 30/30
60000/60000 [==============================] - 2s 32us/step - loss
: 29.7461 - val_loss: 29.5284
```

In [0]:
```
inference=model.predict(x_test)
```

In [0]:
```
num_row = 2
num_col = 5
# plot images
fig, axes = plt.subplots(num_row, num_col, figsize=(1.5*num_col,2*n
um_row))
for i in range(10):
    ax = axes[i//num_col, i%num_col]
    ax.imshow(inference[i].reshape(28,28), cmap='gray')
plt.tight_layout()
plt.show()
```



Problem 3

In [0]:
```
from keras.layers import Dense, Input
from keras.layers import Conv2D, Flatten, Lambda
from keras.layers import Reshape, Conv2DTranspose
from keras.models import Model
from keras.datasets import mnist
from keras.losses import mse, binary_crossentropy
from keras import backend as K

import numpy as np
import matplotlib.pyplot as plt
import argparse
import os
```

```
Using TensorFlow backend.
```

Load MNIST data

In [0]:
```
(x_train, y_train), (x_test, y_test) = mnist.load_data()

image_size = x_train.shape[1]

train_len = 10000
test_len = 1000
x_train = np.reshape(x_train[:train_len], [-1, image_size, image_si
ze, 1])
x_test = np.reshape(x_test[:test_len], [-1, image_size, image_size,
1])
x_train = x_train.astype('float32') / 255
x_test = x_test.astype('float32') / 255
y_test = y_test[:test_len]
```

```
Downloading data from https://s3.amazonaws.com/img-datasets/mnist.
npz
11493376/11490434 [==============================] - 1s 0us/step
```

In [0]:
```
def sampling(args):
    z_mean, z_log_var = args
    batch = K.shape(z_mean)[0]
    dim = K.int_shape(z_mean)[1]
    # by default, random_normal has mean=0 and std=1.0
    epsilon = K.random_normal(shape=(batch, dim))
    return z_mean + K.exp(0.5 * z_log_var) * epsilon
```

In [0]:
```
def plot_results(encoder, decoder, x_test, y_test, batch_size=128):

    # display a 30x30 2D manifold of digits
    n = 30
    digit_size = 28
    figure = np.zeros((digit_size * n, digit_size * n))
    # linearly spaced coordinates corresponding to the 2D plot
    # of digit classes in the latent space
    grid_x = np.linspace(-4, 4, n)
    grid_y = np.linspace(-4, 4, n)[::-1]

    for i, yi in enumerate(grid_y):
        for j, xi in enumerate(grid_x):
            z_sample = np.array([[xi, yi]])
            x_decoded = decoder.predict(z_sample)
            digit = x_decoded[0].reshape(digit_size, digit_size)
            figure[i * digit_size: (i + 1) * digit_size,
                   j * digit_size: (j + 1) * digit_size] = digit

    plt.figure(figsize=(10, 10))
    start_range = digit_size // 2
    end_range = n * digit_size + start_range + 1
    pixel_range = np.arange(start_range, end_range, digit_size)
    sample_range_x = np.round(grid_x, 1)
    sample_range_y = np.round(grid_y, 1)
    plt.xticks(pixel_range, sample_range_x)
    plt.yticks(pixel_range, sample_range_y)
    plt.xlabel("z[0]")
    plt.ylabel("z[1]")
    plt.imshow(figure, cmap='Greys_r')
    plt.show()
```

Hyperparameters

In [0]:
```
input_shape = (image_size, image_size, 1)
batch_size = 32
kernel_size = 3
filters = 16
latent_dim = 2
epochs = 30
use_mse = True
```

Encoder with CNN

In [0]:
```
inputs = Input(shape=input_shape, name='encoder_input')
x = inputs
for i in range(2):
    filters *= 2
    x = Conv2D(filters=filters,
               kernel_size=kernel_size,
               activation='relu',
               strides=2,
               padding='same')(x)

# shape info needed to build decoder model
shape = K.int_shape(x)

# generate latent vector Q(z|X)
x = Flatten()(x)
x = Dense(16, activation='relu')(x)
z_mean = Dense(latent_dim, name='z_mean')(x)
z_log_var = Dense(latent_dim, name='z_log_var')(x)

# use reparameterization trick to push the sampling out as input
# note that "output_shape" isn't necessary with the TensorFlow back
end
z = Lambda(sampling, output_shape=(latent_dim,), name='z')([z_mean,
z_log_var])

# instantiate encoder model
encoder = Model(inputs, [z_mean, z_log_var, z], name='encoder')
encoder.summary()
```

```
Model: "encoder"
__________________________________________________________________________________________________
Layer (type)                    Output Shape         Param #     C
onnected to
==================================================================================================
encoder_input (InputLayer)      (None, 28, 28, 1)    0
__________________________________________________________________________________________________
conv2d_1 (Conv2D)               (None, 14, 14, 32)   320         e
ncoder_input[0][0]
__________________________________________________________________________________________________
conv2d_2 (Conv2D)               (None, 7, 7, 64)     18496       c
onv2d_1[0][0]
__________________________________________________________________________________________________
flatten_1 (Flatten)             (None, 3136)         0           c
onv2d_2[0][0]
__________________________________________________________________________________________________
dense_1 (Dense)                 (None, 16)           50192       f
latten_1[0][0]
__________________________________________________________________________________________________
z_mean (Dense)                  (None, 2)            34          d
ense_1[0][0]
__________________________________________________________________________________________________
z_log_var (Dense)               (None, 2)            34          d
ense_1[0][0]
__________________________________________________________________________________________________
z (Lambda)                      (None, 2)            0           z
_mean[0][0]
                                                                 z
_log_var[0][0]
==================================================================================================
Total params: 69,076
Trainable params: 69,076
Non-trainable params: 0
__________________________________________________________________________________________________
```

Decoder with CNN

In [0]:
```
latent_inputs = Input(shape=(latent_dim,), name='z_sampling')
x = Dense(shape[1] * shape[2] * shape[3], activation='relu')(latent
_inputs)
x = Reshape((shape[1], shape[2], shape[3]))(x)

# use Conv2DTranspose to reverse the conv layers from the encoder
for i in range(2):
    x = Conv2DTranspose(filters=filters,
                        kernel_size=kernel_size,
                        activation='relu',
                        strides=2,
                        padding='same')(x)
    filters //= 2

outputs = Conv2DTranspose(filters=1,
                          kernel_size=kernel_size,
                          activation='sigmoid',
                          padding='same',
                          name='decoder_output')(x)

# instantiate decoder model
decoder = Model(latent_inputs, outputs, name='decoder')
decoder.summary()
```

```
Model: "decoder"
_________________________________________________________________
Layer (type)                 Output Shape              Param #
=================================================================
z_sampling (InputLayer)      (None, 2)                 0
_________________________________________________________________
dense_2 (Dense)              (None, 3136)              9408
_________________________________________________________________
reshape_1 (Reshape)          (None, 7, 7, 64)          0
_________________________________________________________________
conv2d_transpose_1 (Conv2DTr (None, 14, 14, 64)        36928
_________________________________________________________________
conv2d_transpose_2 (Conv2DTr (None, 28, 28, 32)        18464
_________________________________________________________________
decoder_output (Conv2DTransp (None, 28, 28, 1)         289
=================================================================
Total params: 65,089
Trainable params: 65,089
Non-trainable params: 0
_________________________________________________________________
```

VAE

In [0]:
```
outputs = decoder(encoder(inputs)[2])
vae = Model(inputs, outputs, name='vae')
```

In [0]:
```
if use_mse:
    reconstruction_loss = mse(K.flatten(inputs), K.flatten(outputs)
)
else:
    reconstruction_loss = binary_crossentropy(K.flatten(inputs),
                                              K.flatten(outputs))

reconstruction_loss *= image_size * image_size
kl_loss = 1 + z_log_var - K.square(z_mean) - K.exp(z_log_var)
kl_loss = K.sum(kl_loss, axis=-1)
kl_loss *= -0.5
vae_loss = K.mean(reconstruction_loss + kl_loss)
vae.add_loss(vae_loss)
```

In [0]:
```
vae.compile(optimizer='rmsprop')
vae.summary()
```

```
Model: "vae"
_________________________________________________________________
Layer (type)                 Output Shape              Param #
=================================================================
encoder_input (InputLayer)   (None, 28, 28, 1)         0
_________________________________________________________________
encoder (Model)              [(None, 2), (None, 2), (N 69076
_________________________________________________________________
decoder (Model)              (None, 28, 28, 1)         65089
=================================================================
Total params: 134,165
Trainable params: 134,165
Non-trainable params: 0
_________________________________________________________________

/usr/local/lib/python3.6/dist-packages/keras/engine/training_utils
.py:819: UserWarning: Output decoder missing from loss dictionary.
We assume this was done on purpose. The fit and evaluate APIs will
not be expecting any data to be passed to decoder.
  'be expecting any data to be passed to {0}.'.format(name))
```

In [0]:
```
vae.fit(x_train,epochs=epochs,batch_size=batch_size,validation_data
=(x_test, None))
```

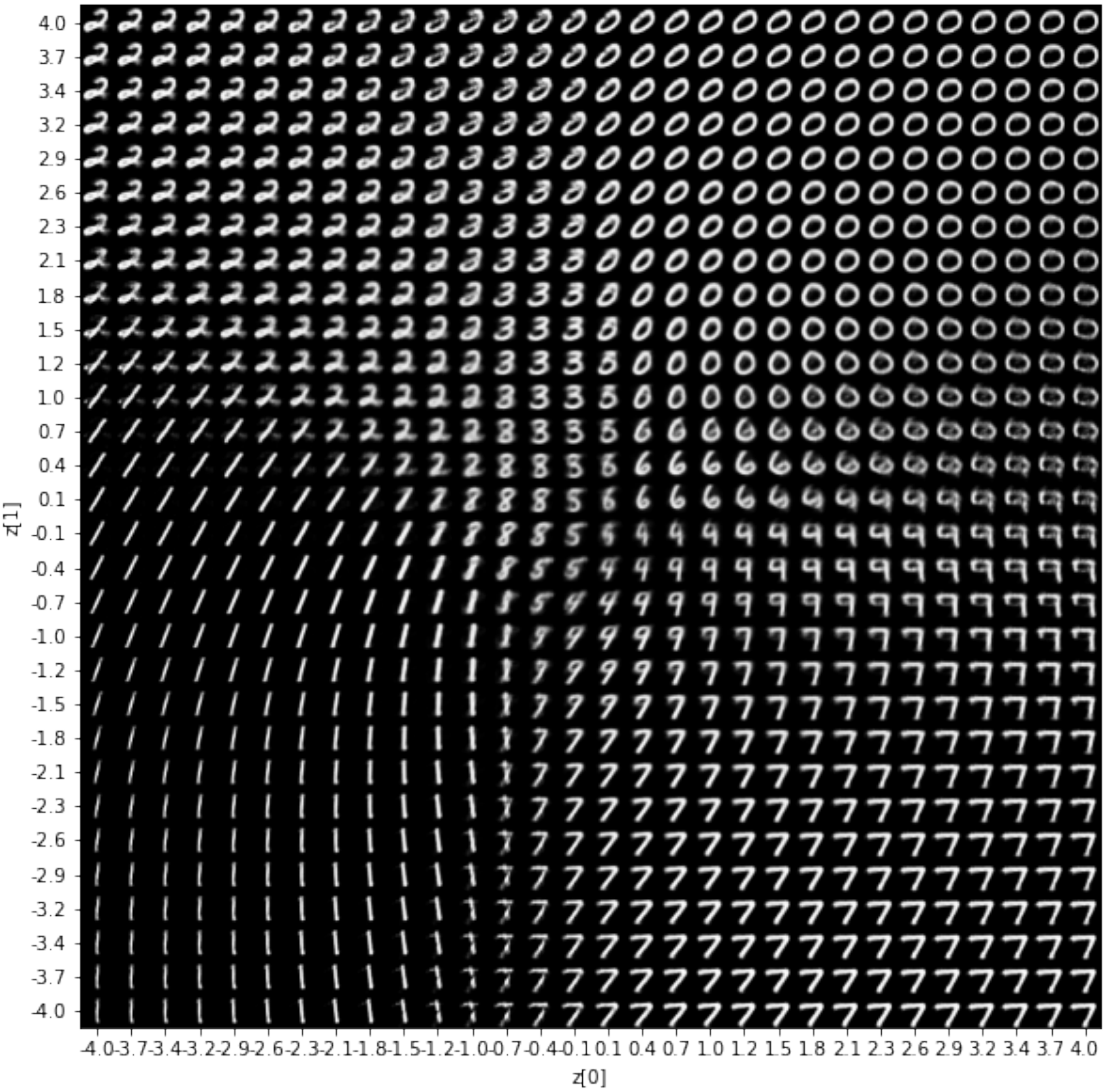
```
Train on 10000 samples, validate on 1000 samples
Epoch 1/30
10000/10000 [==============================] - 8s 827us/step - los
s: 53.3987 - val_loss: 44.6226
Epoch 2/30
10000/10000 [==============================] - 2s 191us/step - los
s: 44.6895 - val_loss: 42.9474
Epoch 3/30
10000/10000 [==============================] - 2s 189us/step - los
s: 43.3248 - val_loss: 43.7992
Epoch 4/30
```

```
10000/10000 [==============================] - 2s 192us/step - los
s: 42.6410 - val_loss: 42.1521
Epoch 5/30
10000/10000 [==============================] - 2s 191us/step - los
s: 42.1135 - val_loss: 42.8118
Epoch 6/30
10000/10000 [==============================] - 2s 193us/step - los
s: 41.5962 - val_loss: 41.5368
Epoch 7/30
10000/10000 [==============================] - 2s 193us/step - los
s: 41.1892 - val_loss: 40.6660
Epoch 8/30
10000/10000 [==============================] - 2s 190us/step - los
s: 40.7682 - val_loss: 40.8434
Epoch 9/30
10000/10000 [==============================] - 2s 189us/step - los
s: 40.4491 - val_loss: 41.0781
Epoch 10/30
10000/10000 [==============================] - 2s 189us/step - los
s: 40.1137 - val_loss: 40.3688
Epoch 11/30
10000/10000 [==============================] - 2s 188us/step - los
s: 39.8048 - val_loss: 39.9240
Epoch 12/30
10000/10000 [==============================] - 2s 190us/step - los
s: 39.5393 - val_loss: 39.9892
Epoch 13/30
10000/10000 [==============================] - 2s 189us/step - los
s: 39.3133 - val_loss: 39.5158
Epoch 14/30
10000/10000 [==============================] - 2s 188us/step - los
s: 39.1134 - val_loss: 40.5779
Epoch 15/30
10000/10000 [==============================] - 2s 190us/step - los
s: 38.8457 - val_loss: 39.6660
Epoch 16/30
10000/10000 [==============================] - 2s 187us/step - los
s: 38.6564 - val_loss: 39.4518
Epoch 17/30
10000/10000 [==============================] - 2s 191us/step - los
s: 38.5165 - val_loss: 40.3426
Epoch 18/30
10000/10000 [==============================] - 2s 188us/step - los
s: 38.3603 - val_loss: 39.3279
Epoch 19/30
10000/10000 [==============================] - 2s 188us/step - los
s: 38.2308 - val_loss: 39.6475
Epoch 20/30
10000/10000 [==============================] - 2s 190us/step - los
s: 38.0641 - val_loss: 40.2713
Epoch 21/30
10000/10000 [==============================] - 2s 190us/step - los
s: 37.9180 - val_loss: 39.0292
```

```
Epoch 22/30
10000/10000 [==============================] - 2s 189us/step - los
s: 37.7748 - val_loss: 38.8527
Epoch 23/30
10000/10000 [==============================] - 2s 190us/step - los
s: 37.7265 - val_loss: 39.0614
Epoch 24/30
10000/10000 [==============================] - 2s 193us/step - los
s: 37.6059 - val_loss: 39.2262
Epoch 25/30
10000/10000 [==============================] - 2s 189us/step - los
s: 37.5234 - val_loss: 39.3531
Epoch 26/30
10000/10000 [==============================] - 2s 189us/step - los
s: 37.4753 - val_loss: 39.8027
Epoch 27/30
10000/10000 [==============================] - 2s 189us/step - los
s: 37.2946 - val_loss: 39.0632
Epoch 28/30
10000/10000 [==============================] - 2s 196us/step - los
s: 37.2517 - val_loss: 39.3951
Epoch 29/30
10000/10000 [==============================] - 2s 196us/step - los
s: 37.1672 - val_loss: 39.3845
Epoch 30/30
10000/10000 [==============================] - 2s 198us/step - los
s: 37.0517 - val_loss: 40.3112
```

Out[0]:
```
<keras.callbacks.callbacks.History at 0x7f12102de438>
```

In [0]:
```
plot_results(encoder, decoder, x_test, y_test, batch_size=batch_siz
e)
```


z[1]
z[0]
4.0
3.7
3.4
3.2
2.9
2.6
2.3
2.1
1.8
1.5
1.2
1.0
0.7
0.4
0.1
-0.1
-0.4
-0.7
-1.0
-1.2
-1.5
-1.8
-2.1
-2.3
-2.6
-2.9
-3.2
-3.4
-3.7
-4.0
-4.0 -3.7 -3.4 -3.2 -2.9 -2.6 -2.3 -2.1 -1.8 -1.5 -1.2 -1.0 -0.7 -0.4 -0.1 0.1 0.4 0.7 1.0 1.2 1.5 1.8 2.1 2.3 2.6 2.9 3.2 3.4 3.7 4.0

Problem 4

Generative Adversarial Network consists of two models: a **generative model** G and a **discriminative model** D.The discriminator is responsible for distinguishing between theactual and generated data while the generator tries createing data in a way to fool the discriminator.

On using cross entropy loss function:

$$L(y, \hat{y}) = (y \log \hat{y}) + ((1 - y) \log(1 - \hat{y}))$$

While training the discriminator label of data coming from $P_{data}(x)$ is y=1 and $\hat{y} = D(x)$ thus the loss we get is:

$$L(D(x), 1) = \log(D(x))$$

Since the data for the generator is fake, therefore y=0 and $\hat{y} = D(G(z))$

So $L(D(G(z)), 0) = \log(1 - D(G(z)))$

The Objective of discriminator is to classify real and fake data.

The loss function of the generator model is :-

$$L^{(G)} = min[\log(D(x)) + \log(1 - D(G(z)))]$$

The loss function of the Discriminator model is :-

$$L^{(D)} = max[\log(D(x)) + \log(1 - D(G(z)))]$$

Therefore the **Combined loss function** is:

$$L = min_G max_D [E_{x \sim P_{data}(x)}[\log(D(x))] + E_{z \sim P(z)}[\log(1 - D(G(z)))]]$$

Thus, we can update the discriminator model based on ascending the stochastic gradient descent while update the generator model by descending the stochastic gradient descent.

In [0]:
```
import numpy as np
import torch
import matplotlib.pyplot as plt
from torchvision import datasets
import torchvision.transforms as transforms

# number of subprocesses to use for data loading
num_workers = 0
# how many samples per batch to load
batch_size = 64

# convert data to torch.FloatTensor
transform = transforms.ToTensor()

# get the training datasets
train_data = datasets.MNIST(root='data', train=True,
                                   download=True, transform=transfo
rm)

# prepare data loader
train_loader = torch.utils.data.DataLoader(train_data, batch_size=b
atch_size,
                                           num_workers=num_workers)
```

```
Downloading http://yann.lecun.com/exdb/mnist/train-images-idx3-uby
te.gz to data/MNIST/raw/train-images-idx3-ubyte.gz


Extracting data/MNIST/raw/train-images-idx3-ubyte.gz to data/MNIST
/raw
Downloading http://yann.lecun.com/exdb/mnist/train-labels-idx1-uby
te.gz to data/MNIST/raw/train-labels-idx1-ubyte.gz


Extracting data/MNIST/raw/train-labels-idx1-ubyte.gz to data/MNIST
/raw
Downloading http://yann.lecun.com/exdb/mnist/t10k-images-idx3-ubyt
e.gz to data/MNIST/raw/t10k-images-idx3-ubyte.gz


Extracting data/MNIST/raw/t10k-images-idx3-ubyte.gz to data/MNIST/
raw
Downloading http://yann.lecun.com/exdb/mnist/t10k-labels-idx1-ubyt
e.gz to data/MNIST/raw/t10k-labels-idx1-ubyte.gz


Extracting data/MNIST/raw/t10k-labels-idx1-ubyte.gz to data/MNIST/
raw
Processing...
Done!
```

Discriminator

In [0]:

```
import torch.nn as nn
import torch.nn.functional as F

class Discriminator(nn.Module):

    def __init__(self, input_size, hidden_dim, output_size):
        super(Discriminator, self).__init__()

        # define hidden linear layers
        self.fc1 = nn.Linear(input_size, hidden_dim*4)
        self.fc2 = nn.Linear(hidden_dim*4, hidden_dim*2)
        self.fc3 = nn.Linear(hidden_dim*2, hidden_dim)

        # final fully-connected layer
        self.fc4 = nn.Linear(hidden_dim, output_size)

        # dropout layer
        self.dropout = nn.Dropout(0.3)


    def forward(self, x):
        # flatten image
        x = x.view(-1, 28*28)
        # all hidden layers
        x = F.leaky_relu(self.fc1(x), 0.2) # (input, negative_slope
=0.2)
        x = self.dropout(x)
        x = F.leaky_relu(self.fc2(x), 0.2)
        x = self.dropout(x)
        x = F.leaky_relu(self.fc3(x), 0.2)
        x = self.dropout(x)
        # final layer
        out = self.fc4(x)

        return out
```

Generator

In [0]:
```
class Generator(nn.Module):

    def __init__(self, input_size, hidden_dim, output_size):
        super(Generator, self).__init__()

        # define hidden linear layers
        self.fc1 = nn.Linear(input_size, hidden_dim)
        self.fc2 = nn.Linear(hidden_dim, hidden_dim*2)
        self.fc3 = nn.Linear(hidden_dim*2, hidden_dim*4)

        # final fully-connected layer
        self.fc4 = nn.Linear(hidden_dim*4, output_size)

        # dropout layer
        self.dropout = nn.Dropout(0.3)

    def forward(self, x):
        # all hidden layers
        x = F.leaky_relu(self.fc1(x), 0.2) # (input, negative_slope
=0.2)
        x = self.dropout(x)
        x = F.leaky_relu(self.fc2(x), 0.2)
        x = self.dropout(x)
        x = F.leaky_relu(self.fc3(x), 0.2)
        x = self.dropout(x)
        # final layer with tanh applied
        out = torch.tanh(self.fc4(x))

        return out
```

Hyperparameters

In [0]:
```
# Discriminator hyperparams

# Size of input image to discriminator (28*28)
input_size = 784
# Size of discriminator output (real or fake)
d_output_size = 1
# Size of last hidden layer in the discriminator
d_hidden_size = 32

# Generator hyperparams

# Size of latent vector to give to generator
z_size = 100
# Size of discriminator output (generated image)
g_output_size = 784
# Size of first hidden layer in the generator
g_hidden_size = 32
```

GAN model

In [0]:
```
D = Discriminator(input_size, d_hidden_size, d_output_size)
G = Generator(z_size, g_hidden_size, g_output_size)


print(D)
print()
print(G)
```

```
Discriminator(
  (fc1): Linear(in_features=784, out_features=128, bias=True)
  (fc2): Linear(in_features=128, out_features=64, bias=True)
  (fc3): Linear(in_features=64, out_features=32, bias=True)
  (fc4): Linear(in_features=32, out_features=1, bias=True)
  (dropout): Dropout(p=0.3, inplace=False)
)

Generator(
  (fc1): Linear(in_features=100, out_features=32, bias=True)
  (fc2): Linear(in_features=32, out_features=64, bias=True)
  (fc3): Linear(in_features=64, out_features=128, bias=True)
  (fc4): Linear(in_features=128, out_features=784, bias=True)
  (dropout): Dropout(p=0.3, inplace=False)
)
```

In [0]:
```
# Calculate losses
def real_loss(D_out, smooth=False):
    batch_size = D_out.size(0)
    # label smoothing
    if smooth:
        # smooth, real labels = 0.9
        labels = torch.ones(batch_size)*0.9
    else:
        labels = torch.ones(batch_size) # real labels = 1

    # numerically stable loss
    criterion = nn.BCEWithLogitsLoss()
    # calculate loss
    loss = criterion(D_out.squeeze(), labels)
    return loss

def fake_loss(D_out):
    batch_size = D_out.size(0)
    labels = torch.zeros(batch_size) # fake labels = 0
    criterion = nn.BCEWithLogitsLoss()
    # calculate loss
    loss = criterion(D_out.squeeze(), labels)
    return loss
```

In [0]:
```
import torch.optim as optim

# Optimizers
lr = 0.002

# Create optimizers for the discriminator and generator
d_optimizer = optim.Adam(D.parameters(), lr)
g_optimizer = optim.Adam(G.parameters(), lr)
```

Training

In [0]:
```
# training hyperparams
num_epochs = 50

# keep track of loss and generated, "fake" samples
samples = []
losses = []

print_every = 400

# Get some fixed data for sampling. These are images that are held
# constant throughout training, and allow us to inspect the model's
performance
sample_size=16
fixed_z = np.random.uniform(-1, 1, size=(sample_size, z_size))
fixed_z = torch.from_numpy(fixed_z).float()

# train the network
D.train()
G.train()
for epoch in range(num_epochs):

    for batch_i, (real_images, _) in enumerate(train_loader):

        batch_size = real_images.size(0)

        ## Important rescaling step ##
        real_images = real_images*2 - 1  # rescale input images fro
m [0,1) to [-1, 1)

        d_optimizer.zero_grad()

        # 1. Train with real images

        # Compute the discriminator losses on real images
        # smooth the real labels
        D_real = D(real_images)
        d_real_loss = real_loss(D_real, smooth=True)

        # 2. Train with fake images
```

```
        # Generate fake images
        z = np.random.uniform(-1, 1, size=(batch_size, z_size))
        z = torch.from_numpy(z).float()
        fake_images = G(z)

        # Compute the discriminator losses on fake images
        D_fake = D(fake_images)
        d_fake_loss = fake_loss(D_fake)

        # add up loss and perform backprop
        d_loss = d_real_loss + d_fake_loss
        d_loss.backward()
        d_optimizer.step()


        g_optimizer.zero_grad()

        # 1. Train with fake images and flipped labels

        # Generate fake images
        z = np.random.uniform(-1, 1, size=(batch_size, z_size))
        z = torch.from_numpy(z).float()
        fake_images = G(z)

        # Compute the discriminator losses on fake images
        # using flipped labels!
        D_fake = D(fake_images)
        g_loss = real_loss(D_fake) # use real loss to flip labels

        # perform backprop
        g_loss.backward()
        g_optimizer.step()

        # Print some loss stats
        if batch_i % print_every == 0:
            # print discriminator and generator loss
            print('Epoch [{:5d}/{:5d}] | d_loss: {:6.4f} | g_loss: 
{:6.4f}'.format(
                    epoch+1, num_epochs, d_loss.item(), g_loss.item
()))


    ## AFTER EACH EPOCH##
    # append discriminator loss and generator loss
    losses.append((d_loss.item(), g_loss.item()))

    # generate and save sample, fake images
    G.eval() # eval mode for generating samples
    samples_z = G(fixed_z)
    samples.append(samples_z)
    G.train() # back to train mode
```

```
Epoch [    1/   50] | d_loss: 1.4199 | g_loss: 0.6122
Epoch [    1/   50] | d_loss: 1.6515 | g_loss: 1.4794
```

```
Epoch [    1/   50] | d_loss: 1.0764 | g_loss: 2.1332
Epoch [    2/   50] | d_loss: 1.1537 | g_loss: 1.1083
Epoch [    2/   50] | d_loss: 1.2909 | g_loss: 0.6969
Epoch [    2/   50] | d_loss: 1.6127 | g_loss: 0.5472
Epoch [    3/   50] | d_loss: 1.2701 | g_loss: 0.8140
Epoch [    3/   50] | d_loss: 1.1618 | g_loss: 0.9189
Epoch [    3/   50] | d_loss: 1.2258 | g_loss: 2.5967
Epoch [    4/   50] | d_loss: 1.2586 | g_loss: 1.2860
Epoch [    4/   50] | d_loss: 1.0651 | g_loss: 1.5339
Epoch [    4/   50] | d_loss: 1.0524 | g_loss: 1.2302
Epoch [    5/   50] | d_loss: 1.1280 | g_loss: 1.0989
Epoch [    5/   50] | d_loss: 1.2278 | g_loss: 1.2629
Epoch [    5/   50] | d_loss: 1.4973 | g_loss: 0.7280
Epoch [    6/   50] | d_loss: 1.2898 | g_loss: 1.1424
Epoch [    6/   50] | d_loss: 1.3041 | g_loss: 0.9609
Epoch [    6/   50] | d_loss: 1.4286 | g_loss: 0.8839
Epoch [    7/   50] | d_loss: 1.1804 | g_loss: 1.1715
Epoch [    7/   50] | d_loss: 1.1365 | g_loss: 1.2973
Epoch [    7/   50] | d_loss: 1.0273 | g_loss: 2.4373
Epoch [    8/   50] | d_loss: 1.2099 | g_loss: 1.6809
Epoch [    8/   50] | d_loss: 1.3877 | g_loss: 1.2304
Epoch [    8/   50] | d_loss: 1.2133 | g_loss: 2.4702
Epoch [    9/   50] | d_loss: 1.1091 | g_loss: 1.0650
Epoch [    9/   50] | d_loss: 1.2882 | g_loss: 1.0777
Epoch [    9/   50] | d_loss: 1.2394 | g_loss: 1.6353
Epoch [   10/   50] | d_loss: 1.3832 | g_loss: 1.0669
Epoch [   10/   50] | d_loss: 1.2868 | g_loss: 1.0736
Epoch [   10/   50] | d_loss: 1.3039 | g_loss: 0.7582
Epoch [   11/   50] | d_loss: 1.2426 | g_loss: 1.6015
Epoch [   11/   50] | d_loss: 1.0400 | g_loss: 1.0918
Epoch [   11/   50] | d_loss: 1.2344 | g_loss: 1.1228
Epoch [   12/   50] | d_loss: 1.1862 | g_loss: 1.5529
Epoch [   12/   50] | d_loss: 1.0059 | g_loss: 1.6859
Epoch [   12/   50] | d_loss: 1.1994 | g_loss: 1.0489
Epoch [   13/   50] | d_loss: 1.2884 | g_loss: 0.8636
Epoch [   13/   50] | d_loss: 1.2170 | g_loss: 0.8706
Epoch [   13/   50] | d_loss: 1.3117 | g_loss: 1.1369
Epoch [   14/   50] | d_loss: 1.2062 | g_loss: 1.4767
Epoch [   14/   50] | d_loss: 1.3353 | g_loss: 1.1355
Epoch [   14/   50] | d_loss: 1.3380 | g_loss: 0.8093
Epoch [   15/   50] | d_loss: 1.3188 | g_loss: 1.0626
Epoch [   15/   50] | d_loss: 1.2285 | g_loss: 1.0403
Epoch [   15/   50] | d_loss: 1.2061 | g_loss: 1.3532
Epoch [   16/   50] | d_loss: 1.1674 | g_loss: 1.1758
Epoch [   16/   50] | d_loss: 1.2690 | g_loss: 1.5118
Epoch [   16/   50] | d_loss: 1.1016 | g_loss: 1.4641
Epoch [   17/   50] | d_loss: 1.1594 | g_loss: 1.3556
Epoch [   17/   50] | d_loss: 1.3024 | g_loss: 1.1883
Epoch [   17/   50] | d_loss: 1.3620 | g_loss: 0.9748
Epoch [   18/   50] | d_loss: 1.2076 | g_loss: 1.1343
Epoch [   18/   50] | d_loss: 1.3130 | g_loss: 0.9361
Epoch [   18/   50] | d_loss: 1.2873 | g_loss: 1.1706
Epoch [   19/   50] | d_loss: 1.5852 | g_loss: 1.2933
```

```
Epoch [   19/   50] | d_loss: 1.1843 | g_loss: 1.0082
Epoch [   19/   50] | d_loss: 1.3097 | g_loss: 1.1157
Epoch [   20/   50] | d_loss: 1.2634 | g_loss: 0.9743
Epoch [   20/   50] | d_loss: 1.1606 | g_loss: 1.1301
Epoch [   20/   50] | d_loss: 1.3761 | g_loss: 1.1775
Epoch [   21/   50] | d_loss: 1.2424 | g_loss: 0.9947
Epoch [   21/   50] | d_loss: 1.1746 | g_loss: 1.0627
Epoch [   21/   50] | d_loss: 1.3331 | g_loss: 0.9228
Epoch [   22/   50] | d_loss: 1.2118 | g_loss: 1.1222
Epoch [   22/   50] | d_loss: 1.1306 | g_loss: 1.6337
Epoch [   22/   50] | d_loss: 1.2765 | g_loss: 1.1879
Epoch [   23/   50] | d_loss: 1.2515 | g_loss: 0.9422
Epoch [   23/   50] | d_loss: 1.1771 | g_loss: 1.3238
Epoch [   23/   50] | d_loss: 1.4102 | g_loss: 1.1210
Epoch [   24/   50] | d_loss: 1.2850 | g_loss: 1.0056
Epoch [   24/   50] | d_loss: 1.2490 | g_loss: 1.0430
Epoch [   24/   50] | d_loss: 1.3396 | g_loss: 0.8730
Epoch [   25/   50] | d_loss: 1.2385 | g_loss: 1.2466
Epoch [   25/   50] | d_loss: 1.2371 | g_loss: 1.5061
Epoch [   25/   50] | d_loss: 1.3642 | g_loss: 0.8888
Epoch [   26/   50] | d_loss: 1.3380 | g_loss: 1.1839
Epoch [   26/   50] | d_loss: 1.1741 | g_loss: 1.0695
Epoch [   26/   50] | d_loss: 1.3976 | g_loss: 1.0299
Epoch [   27/   50] | d_loss: 1.3432 | g_loss: 0.8521
Epoch [   27/   50] | d_loss: 1.1745 | g_loss: 1.0786
Epoch [   27/   50] | d_loss: 1.3620 | g_loss: 1.0966
Epoch [   28/   50] | d_loss: 1.2902 | g_loss: 1.0683
Epoch [   28/   50] | d_loss: 1.2502 | g_loss: 1.3421
Epoch [   28/   50] | d_loss: 1.2578 | g_loss: 1.2000
Epoch [   29/   50] | d_loss: 1.3181 | g_loss: 1.1824
Epoch [   29/   50] | d_loss: 1.2371 | g_loss: 0.8801
Epoch [   29/   50] | d_loss: 1.4353 | g_loss: 0.9504
Epoch [   30/   50] | d_loss: 1.3651 | g_loss: 1.2938
Epoch [   30/   50] | d_loss: 1.1633 | g_loss: 1.0334
Epoch [   30/   50] | d_loss: 1.3691 | g_loss: 1.0443
Epoch [   31/   50] | d_loss: 1.2872 | g_loss: 1.0678
Epoch [   31/   50] | d_loss: 1.3934 | g_loss: 1.0067
Epoch [   31/   50] | d_loss: 1.3867 | g_loss: 1.0341
Epoch [   32/   50] | d_loss: 1.2284 | g_loss: 1.1083
Epoch [   32/   50] | d_loss: 1.1970 | g_loss: 0.9522
Epoch [   32/   50] | d_loss: 1.3643 | g_loss: 0.9842
Epoch [   33/   50] | d_loss: 1.3168 | g_loss: 0.9101
Epoch [   33/   50] | d_loss: 1.2395 | g_loss: 0.9652
Epoch [   33/   50] | d_loss: 1.3533 | g_loss: 1.0110
Epoch [   34/   50] | d_loss: 1.2560 | g_loss: 0.9041
Epoch [   34/   50] | d_loss: 1.2547 | g_loss: 1.1997
Epoch [   34/   50] | d_loss: 1.3714 | g_loss: 1.0845
Epoch [   35/   50] | d_loss: 1.2783 | g_loss: 1.1826
Epoch [   35/   50] | d_loss: 1.2055 | g_loss: 1.1693
Epoch [   35/   50] | d_loss: 1.4162 | g_loss: 0.9825
Epoch [   36/   50] | d_loss: 1.2335 | g_loss: 1.0916
Epoch [   36/   50] | d_loss: 1.3161 | g_loss: 1.1513
Epoch [   36/   50] | d_loss: 1.3667 | g_loss: 0.9204
```

```
Epoch [   37/   50] | d_loss: 1.3909 | g_loss: 0.9699
Epoch [   37/   50] | d_loss: 1.1925 | g_loss: 0.8976
Epoch [   37/   50] | d_loss: 1.2214 | g_loss: 1.1316
Epoch [   38/   50] | d_loss: 1.2759 | g_loss: 1.4615
Epoch [   38/   50] | d_loss: 1.2687 | g_loss: 1.1092
Epoch [   38/   50] | d_loss: 1.3101 | g_loss: 1.0101
Epoch [   39/   50] | d_loss: 1.2209 | g_loss: 1.0353
Epoch [   39/   50] | d_loss: 1.3207 | g_loss: 0.8350
Epoch [   39/   50] | d_loss: 1.3460 | g_loss: 1.0167
Epoch [   40/   50] | d_loss: 1.2809 | g_loss: 1.1309
Epoch [   40/   50] | d_loss: 1.1904 | g_loss: 1.3867
Epoch [   40/   50] | d_loss: 1.2163 | g_loss: 0.9536
Epoch [   41/   50] | d_loss: 1.2880 | g_loss: 1.2844
Epoch [   41/   50] | d_loss: 1.1887 | g_loss: 1.1490
Epoch [   41/   50] | d_loss: 1.3812 | g_loss: 0.9678
Epoch [   42/   50] | d_loss: 1.2835 | g_loss: 1.0684
Epoch [   42/   50] | d_loss: 1.3359 | g_loss: 0.9492
Epoch [   42/   50] | d_loss: 1.2633 | g_loss: 0.8281
Epoch [   43/   50] | d_loss: 1.2822 | g_loss: 0.9608
Epoch [   43/   50] | d_loss: 1.2924 | g_loss: 1.2194
Epoch [   43/   50] | d_loss: 1.3848 | g_loss: 1.0217
Epoch [   44/   50] | d_loss: 1.1986 | g_loss: 1.0440
Epoch [   44/   50] | d_loss: 1.2622 | g_loss: 1.1229
Epoch [   44/   50] | d_loss: 1.3113 | g_loss: 0.8368
Epoch [   45/   50] | d_loss: 1.2967 | g_loss: 0.9703
Epoch [   45/   50] | d_loss: 1.2194 | g_loss: 1.1371
Epoch [   45/   50] | d_loss: 1.3074 | g_loss: 1.0420
Epoch [   46/   50] | d_loss: 1.2673 | g_loss: 1.1737
Epoch [   46/   50] | d_loss: 1.1960 | g_loss: 1.2485
Epoch [   46/   50] | d_loss: 1.3350 | g_loss: 1.2967
Epoch [   47/   50] | d_loss: 1.3494 | g_loss: 1.0837
Epoch [   47/   50] | d_loss: 1.2017 | g_loss: 1.0485
Epoch [   47/   50] | d_loss: 1.3004 | g_loss: 0.9711
Epoch [   48/   50] | d_loss: 1.2758 | g_loss: 0.9524
Epoch [   48/   50] | d_loss: 1.2735 | g_loss: 1.1616
Epoch [   48/   50] | d_loss: 1.3285 | g_loss: 1.1261
Epoch [   49/   50] | d_loss: 1.1585 | g_loss: 1.2917
Epoch [   49/   50] | d_loss: 1.2306 | g_loss: 1.2467
Epoch [   49/   50] | d_loss: 1.2748 | g_loss: 1.0468
Epoch [   50/   50] | d_loss: 1.2563 | g_loss: 0.9532
Epoch [   50/   50] | d_loss: 1.2174 | g_loss: 0.9255
Epoch [   50/   50] | d_loss: 1.2864 | g_loss: 0.8795
```

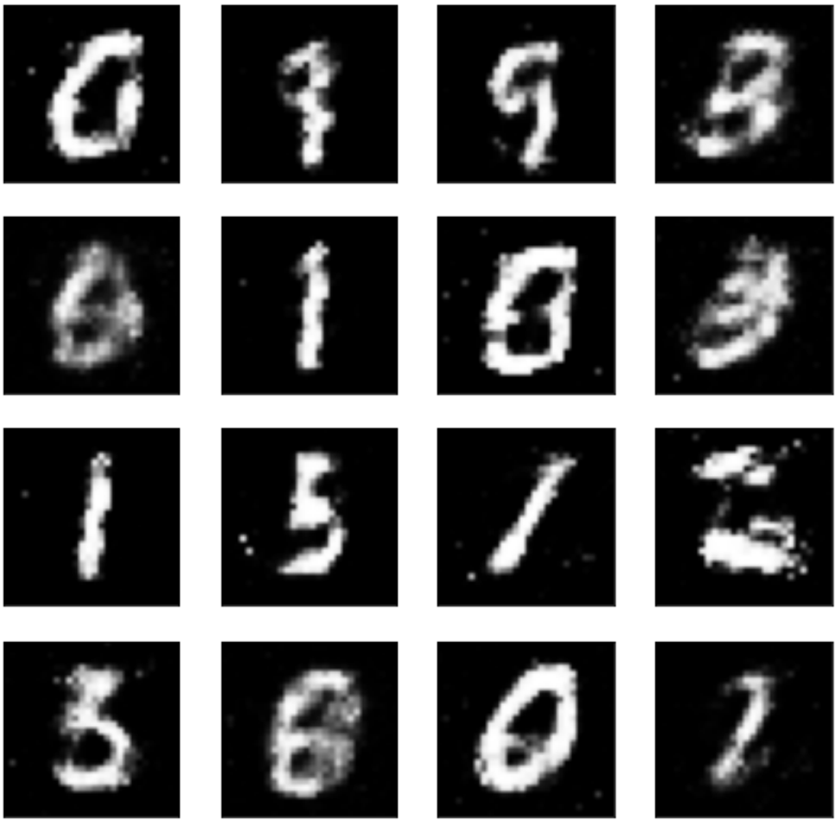
In [0]:
```
losses = np.array(losses)
plt.plot(losses.T[0], label='Discriminator')
plt.plot(losses.T[1], label='Generator')
plt.title("Training Losses")
plt.legend()
```

Out[0]: <matplotlib.legend.Legend at 0x7f224e8afbe0>

In [0]:
```
def view_samples(epoch, samples):
    fig, axes = plt.subplots(figsize=(7,7), nrows=4, ncols=4, share
y=True, sharex=True)
    for ax, img in zip(axes.flatten(), samples[epoch]):
        img = img.detach()
        ax.xaxis.set_visible(False)
        ax.yaxis.set_visible(False)
        im = ax.imshow(img.reshape((28,28)), cmap='Greys_r')
```

In [0]:
```
# -1 indicates final epoch's samples (the last in the list)
view_samples(-1, samples)
```

In [0]:
```
# randomly generated, new latent vectors
sample_size=16
rand_z = np.random.uniform(-1, 1, size=(sample_size, z_size))
rand_z = torch.from_numpy(rand_z).float()

G.eval() # eval mode
# generated samples
rand_images = G(rand_z)

# 0 indicates the first set of samples in the passed in list
# and we only have one batch of samples, here
view_samples(0, [rand_images])
```



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