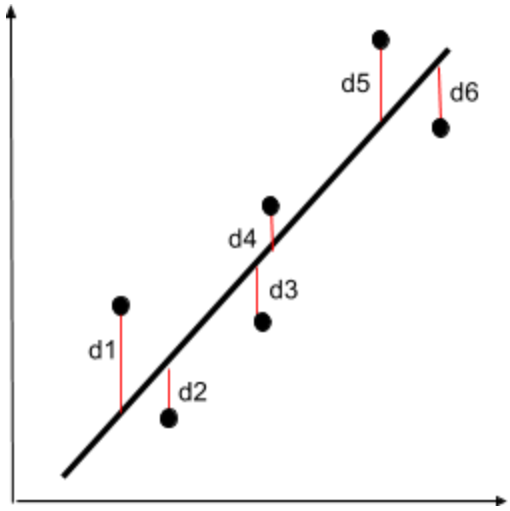
Linear Regression and OLS

Points about linear regression: a line equation has 1 variable and 2 parameters

y = ax + b

Thus the number of degrees of freedom of the model is 1.

Ordinary Least Square Method:
calculates the best-fit line for the observed data by minimizing the sum of the squares of the vertical deviations from each data point to the line.



So the idea is to chose a and b such that the
sum of distances between y_predicted - y_observed is minimum.

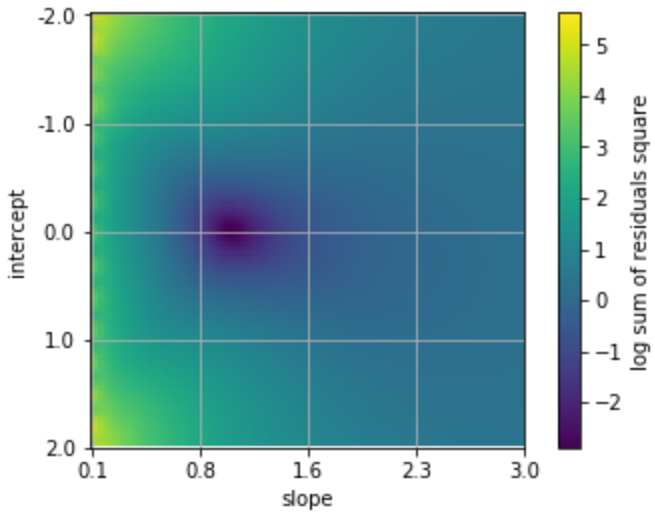
a and b $\Rightarrow$ min $\sum_{i=1}^{n} \left| a\,x_i + b - y_i \right|$

This can be done
Numerically:

1. the simplest minimization algorithm is brute force: choose a reasonable range for each of the N variable in the minimization (here 2: the parameters of the line equation) and explore the Ndimensional (here 2dimensiona, i.e. a surface) parameter space. Very expensive computationally, and will only get you as close to the minimum as the step size you chose to explore the space

   (This is the first code I ever wrote by the way, years ago in C)

```
a = -99e9 #some obviously irreasonable value
B = -99e9
residualsSum = np.zeros((100,100))
for i,aa in enumerate(np.linspace(-2, 2, 100)):
    for j,bb in enumerate(np.linspace(0.1, 3, 100)):
        residualsSum[i,j] = $\sum_{i=1}^{n} \left| aa\, x_i + bb - y_i \right|$
        if residualsSum[i,j] < minVal:
            minVal = residualsSum[i,j]
            a = aa
            b = bb
```



2. Use a wiser minimization algorithm which explores the space looking for a minimum by following a strategy, for example only moving downhill.

```
def residuals(pars, data, model, error=None):
    '''Calculates the residuals between model and data
    Arguments:
        data: series - 1D array of datapoints
        model: series - 1D array of model predictions
        error (optional): series - 1D array of errors on data
    Returns:
        sum of residuals square
    '''
    residuals = (line(data, pars) - model)**2
    if error:
        residuals = residuals / error**2
```

```
    return residuals.sum()
p0 = (0.5,1000)
#here the model is the female income given the male income: a regression line
##important!! never work with very large numbers! always try to reduce your numbers to
~unity when solving numerically
result = minimize(residuals, p0, args=(all_m/1e4, all_f/1e4))
a,b = result.x
```

Analytically: for this simple problem the linear algebra is solvable. The sum of the residual square is written in metrix form as

$S = \sum_{i=1}^{n} (y_i - \sum_{j=1}^{2} x_{ij}\ \theta_j)^2$ Which is what we want to minimize and where y are the observations, the dependent variables, x the location on independent variables (which may be more than one for an multiolinear regression, leading to a double index for x, but is 1 in our current case), and θ = (a,b) the parameters. Note that even with 1 independent variable to solve the problem analytically the n values of the independent variable need to be encoded in a 2D vecrot, which it what statsmodels.api.add_constant() does! The vector of x's (x = (0,0.5,1,1.5,2,2.5) for example) needs to look like:

```
x = [[ 1. ,  0. ],
     [ 1. ,  0.5],
     [ 1. ,  1. ],
     [ 1. ,  1.5],
     [ 1. ,  2. ],
     [ 1. ,  2.5]]
```

Full derivation is beyond the scope here, bu to minimize we take the derivative and set it to 0 $\delta r_i / \delta \theta_j = -X_{ij}$ are the partial derivatives we want to set to 0, with this **X** matrix we can write the problem as $(X^T X)\widehat{\theta} = X^T\ \widehat{y}$ or: $\widehat{\theta} - (X^T X)^{-1}\ X^T\ \widehat{y}$ = 0 (which we can solve as long as we can invert $(X^T X)$.