23SCIB04I

Operations Research

Minimizing Cost for Plant-Based Meal for Brain Function

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Abstract:

This report attempts to find a solution to the challenge of meeting the daily nutritional for brain function by crafting a dietary plan based on proper scientific research. Inspired by the Harvard University research highlighting the link between specific nutrients and cognitive health, we constructed a model focusing on brain-supporting nutrients, such PUFAs and vitamin Bs, and chose food items that fit the criteria accordingly. Adhering to budgetary constraints of the local Egyptian economy and to avoid the high costs associated with meat-based diets, the food items selected were specifically plant-based foods, such as spinach and broccoli, and to create an accurate estimate we calculated their corresponding costs in the market in 100-gram serving. Furthermore, the constraints were formed based on the scientific data of the daily nutritional requirements of an adult male. Thus, the goal of this model is to determine the optimal number of servings for each plant-based food item that minimizes the total cost, while satisfying minimum daily requirements for essential brain-supporting nutrients [1].

Nutrients	Spinach	Broccoli	Kale	Strawberry	Kiwi	Avocado	Walnut (Abu Auf)	Dark Chocolate (Corona)
PUFA (mg)	165	112	673	155	287	1820	47200	1260
Vitamin B6 (mg)	0.195	0.2	0.147	0.047	0.063	0.257	0.537	0.038
Vitamin B1 (mg)	0.078	0.063	0.113	0.024	0.027	0.067	0.341	0.034
Vitamin C (mg)	28	64.9	93.4	58.8	92.7	10	1.3	0
Vitamin E (mg)	2	1.45	0.66	0.29	1.46	2.07	0.7	0.59
Magnesium (mg)	79	21	33	13	17	29	272	228
Zinc (mg)	0.53	0.45	0.39	0.14	0.14	0.64	1.69	3.31
Potassium (mg)	558	293	348	153	312	485	441	715
Sodium (mg)	79	41	53	1	3	7	2	20
Calcium (mg)	99	40	254	16	34	12	98	73
Choline (mg)	19.3	18.7	0.5	5.7	7.8	14.2	39.2	0

Table 1: Food Items and Their Nutritional Value in 100-gram servings

Formulation:

The linear programming formulation can be written in the following general form where it consists of one parameter representing the cost (Egyptian pounds per 100 grams serving) and a decision variable both representing the number of 100 grams servings of food (i) for all i=1…8.

$$Minmize\ f = \sum_{i=1}^{8} C(i)X(i)$$

Upon dissolving the sigma notation of the decision variables, we get 8 unique decision variables that, along with the cost of each food item per 100-gram servings, result in the following explicit objective function:

$Z_{(Min)} = 0.8 * X_{(1)} + 3.3 * X_{(2)} + 0.55 * X_{(3)} + 3 * X_{(4)} + 20 * X_{(5)} + 28.8 * X_{(6)} + 130 * X_{(7)} + 51 * X_{(8)}$

Then there are the constraints which are divided into two categories: ones that represent the minimum required intake per day for adult male for each nutrient found in the food items, and ones that denotes a maximum number of serving for each food item as to avoid ridiculous amounts of one type of food per day. Thus, the objective function is subject to the following:

Minimum Required Intake Constraints (in mg):

- $165*X(1) + 112*X(2) + 673*X(3) + 155*X(4) + 287*X(5) + 1820*X(6) + 47200*X(7) + 1260*X(8) >= 500,000$ (mg)
- $0.195*X(1) + 0.2*X(2) + 0.147*X(3) + 0.047*X(4) + 0.063*X(5) + 0.257*X(6) + 0.537*X(7) + 0.038*X(8) >= 1.7$ (mg)
- $0.78 * X(1) + 0.63 * X(2) + 0.113 * X(3) + 0.024 * X(4) + 0.027 * X(5) + 0.067 * X(6) + 0.341 * X(7) + 0.034 * X(8) >= 1.2$ (mg)

- 28 * X(1) + 64.9 * X(2) + 93.4 * X(3) + 58.8 * X(4) + 92.7 * X(5) + 10 * X(6) + 1.3 * X(7) + 0 * X(8) >= 90 (mg)
- 2 * X(1) + 1.45 * X(2) + 0.66 * X(3) + 0.29 * X(4) + 1.46 * X(5) + 2.07 * X(6) + 0.7 * X(7) + 0.59 * X(8) >= 15 (mg)
- 79 * X(1) + 21 * X(2) + 33 * X(3) + 13 * X(4) + 17 * X(5) + 29 * X(6) + 272 * X(7) + 228 * X(8) >= 420 (mg)
- 0.53 * X(1) + 0.45 * X(2) + 0.39 * X(3) + 0.14 * X(4) + 0.14 * X(5) + 0.64 * X(6) + 1.69 * X(7) + 3.31 * X(8) >= 11 (mg)
- 558 * X(1) + 293 * X(2) + 348 * X(3) + 153 * X(4) + 312 * X(5) + 485 * X(6) + 441 * X(7) + 715 * X(8) >= 4700 (mg)
- 79 * X(1) + 41 * X(2) + 53 * X(3) + 1 * X(4) + 3 * X(5) + 7 * X(6) + 2 * X(7) + 20 * X(8) >= 2300 (mg)
- 99 * X(1) + 40 * X(2) + 254 * X(3) + 16 * X(4) + 34 * X(5) + 12 * X(6) + 98 * X(7) + 73 * X(8) >= 1300 (mg)
- 19.3 * X(1) + 18.7 * X(2) + 0.5 * X(3) + 5.7 * X(4) + 7.8 * X(5) + 14.2 * X(6) + 39.2 * X(7) + 0 * X(8) >= 550 (mg)

Maximum Number of Serving Constraints (in 100-gram servings):

- X(1) <= 25 (100 gram servings)
- X(2) <= 20 (100 gram servings)
- X(3) <= 30 (100 gram servings)
- X(4) , X(5) , X(6) , X(7) , X(8) <= 10 (100 gram servings)
- X(1), X(2) , X(3) , X(4) , X(5) , X(6) , X(7) , X(8) >= 0 (Nonnegativity)

	Decision Variables	Constraints
Formulation	8	19

Solving:

Optimized Decision Variables							
X1	X2	X3	X4	X5	X6	X7	X8
25	0	30	0	0	2.024725	10	0

Optimal Minimum Meal Cost: £ 1394.81

Dual Form:

Much like was done in part 1, to prepare for the dual formulation, we must standardize the constraints in the appropriate direction according to the minimization objective function. This means ensuring all constraints are expressed as "greater than or equal to" inequalities apart from the nonnegativity constraint. Any constraints with "less than or equal to" can be converted by multiplying both sides by negative one as the following:

- *(X(1) <= 25) * -1*
- *(X(2) <= 20) * -1*
- *(X(3) <= 30) * -1*
- *(X(4) , X(5) , X(6) , X(7) , X(8) <= 10) * -1*

So, the primal form we are working is as follows:

Objective Function (Minimize):

$Z_{(Min)} = 0.8 * X_{(1)} + 3.3 * X_{(2)} + 0.55 * X_{(3)} + 3 * X_{(4)} + 20 * X_{(5)} + 28.8 * X_{(6)} + 130 * X_{(7)} + 51 * X_{(8)}$

s.t.

$165*X(1) + 112*X(2) + 673*X(3) + 155*X(4) + 287*X(5) + 1820*X(6) + 47200*X(7) + 1260*X(8) >= 500,000$ (mg)

$0.195*X(1) + 0.2*X(2) + 0.147*X(3) + 0.047*X(4) + 0.063*X(5) + 0.257*X(6) + 0.537*X(7) + 0.038*X(8) >= 1.7$ (mg)

$0.78 * X(1) + 0.63 * X(2) + 0.113 * X(3) + 0.024 * X(4) + 0.027 * X(5) + 0.067 * X(6) + 0.341 * X(7) + 0.034 * X(8) >= 1.2$ (mg)

$28 * X(1) + 64.9 * X(2) + 93.4 * X(3) + 58.8 * X(4) + 92.7 * X(5) + 10 * X(6) + 1.3 * X(7) + 0 * X(8) >= 90$ (mg)

$2 * X(1) + 1.45 * X(2) + 0.66 * X(3) + 0.29 * X(4) + 1.46 * X(5) + 2.07 * X(6) + 0.7 * X(7) + 0.59 * X(8) >= 15$ (mg)

$79 * X(1) + 21 * X(2) + 33 * X(3) + 13 * X(4) + 17 * X(5) + 29 * X(6) + 272 * X(7) + 228 * X(8) >= 420$ (mg)

$0.53 * X(1) + 0.45 * X(2) + 0.39 * X(3) + 0.14 * X(4) + 0.14 * X(5) + 0.64 * X(6) + 1.69 * X(7) + 3.31 * X(8) >= 11$ (mg)

$558 * X(1) + 293 * X(2) + 348 * X(3) + 153 * X(4) + 312 * X(5) + 485 * X(6) + 441 * X(7) + 715 * X(8) >= 4700$ (mg)

*79 * X(1) + 41 * X(2) + 53 * X(3) + 1 * X(4) + 3 * X(5) + 7 * X(6) + 2 * X(7) + 20 * X(8) >= 2300 (mg)*

*99 * X(1) + 40 * X(2) + 254 * X(3) + 16 * X(4) + 34 * X(5) + 12 * X(6) + 98 * X(7) + 73 * X(8) >= 1300 (mg)*

*19.3 * X(1) + 18.7 * X(2) + 0.5 * X(3) + 5.7 * X(4) + 7.8 * X(5) + 14.2 * X(6) + 39.2 * X(7) + 0 * X(8) >= 550 (mg)*

-X(1) >= -25

-X(2) >= -20

-X(3) >= -30

-X(4) , -X(5) , -X(6) , -X(7) , -X(8) >= -10

X(1), X(2) , X(3) , X(4) , X(5) , X(6) , X(7) , X(8) >= 0 (Nonnegativity)

We begin by constructing the dual form of the objective function where we multiply the right-hand side coefficients of the primal constraints by the new dual variables and add them accordingly:

$Z_{(Max)}$ *= 500000* u1 + 1.7*u2 + 1.2*u3 + 90* u4 + 15* u5 + 420*u6 + 11* u7 + 4700*u8 + 2300* u9 + 1300*u10 + 550* u11 - 25*v1 - 20*v2 - 30*v3 - 10*v4 - 10*v5 - 10*v6 - 10*v7 - 10*v8*

We then construct a matrix in the same order as the primal objective function by taking left-hand side coefficients of each of the decision variables as they appear in each primal constraint. This results in the following coefficient matrix:

$$\begin{bmatrix}
165 & 112 & 673 & 155 & 287 & 1820 & 47200 & 1260 \\
0.195 & 0.2 & 0.147 & 0.047 & 0.063 & 0.257 & 0.537 & 0.038 \\
0.078 & 0.063 & 0.113 & 0.024 & 0.027 & 0.067 & 0.341 & 0.34 \\
28 & 64.9 & 93.4 & 58.8 & 92.7 & 10 & 13 & 0 \\
2 & 1.45 & 0.66 & 0.29 & 1.46 & 2.07 & 0.7 & 0.59 \\
79 & 21 & 33 & 13 & 17 & 29 & 272 & 228 \\
0.53 & 0.45 & 0.39 & 0.14 & 0.14 & 0.64 & 1.69 & 3.31 \\
558 & 293 & 348 & 153 & 312 & 485 & 441 & 715 \\
79 & 41 & 53 & 1 & 3 & 7 & 2 & 20 \\
99 & 40 & 254 & 16 & 34 & 12 & 98 & 73 \\
19.3 & 18.7 & 0.5 & 5.7 & 7.8 & 14.2 & 39.2 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1
\end{bmatrix}$$

We then get the transpose of the matrix from earlier and multiply it by the new dual variables and equate it to the cost matrix from primal objective function:

$$\begin{bmatrix}
165 & 0.195 & 0.078 & 28 & 2 & 79 & 0.53 & 558 & 79 & 99 & 19.3 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
112 & 0.2 & 0.063 & 64.9 & 1.45 & 21 & 0.45 & 293 & 41 & 40 & 18.7 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
673 & 0.147 & 0.113 & 93.4 & 0.66 & 33 & 0.39 & 348 & 53 & 254 & 0.5 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
155 & 0.047 & 0.024 & 58.8 & 0.29 & 13 & 0.14 & 153 & 1 & 16 & 5.7 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
287 & 0.063 & 0.027 & 92.7 & 1.46 & 17 & 0.14 & 312 & 3 & 34 & 7.8 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
1820 & 0.257 & 0.067 & 10 & 2.07 & 29 & 0.64 & 485 & 7 & 12 & 14.2 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
47200 & 0.537 & 0.341 & 13 & 0.7 & 272 & 1.69 & 441 & 2 & 98 & 39.2 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
1260 & 0.038 & 0.34 & 0 & 0.59 & 228 & 3.31 & 715 & 20 & 73 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1
\end{bmatrix} * \begin{bmatrix} u1 \\ u2 \\ u3 \\ u4 \\ u5 \\ u6 \\ u7 \\ u8 \\ u9 \\ u10 \\ u11 \\ v1 \\ v2 \\ v3 \\ v4 \\ v5 \\ v6 \\ v7 \\ v8 \end{bmatrix} = \begin{bmatrix} 0.8 \\ 3.3 \\ 0.55 \\ 3 \\ 20 \\ 28.8 \\ 130 \\ 51 \end{bmatrix}$$

Lastly, we construct the new constraints from the multiplication to get the following dual constraints:

- *165*u1 + 0.195*u2 + 0.078*u3+ 28*u4 + 2*u5+ 79*u6 + 0.53*u7 + 558*u8 + 79*u9 + 99*u10 + 19.3*u11 -v1 <= 0.8*

- $112*u1 + 0.2 *u2 + 0.063*u3+ 64.9*u4 + 1.45*u5+ 21*u6 + 0.45*u7 + 293*u8 + 41*u9 + 40*u10 + 18.7*u11 -v2 <= 3.3$
- $673*u1 + 0.147u2 + 0.113*u3 + 93.4*u4 + 0.66*u5 + 33*u6 + 0.39*u7 + 348*u8 + 53*u9 + 254*u10 + 0.5*u11 -v3 <= 0.55$
- $155*u1 + 0.046*u2 + 0.024*u3+ 58.8*u4 + 0.29*u5+ 13*u6 + 0.14*u7 + 153*u8 + u9 + 16*u10 + 5.7*u11 -v4 <= 3$
- $287*u1 + 0.063*u2 + 0.027*u3+ 92.7*u4 + 1.46*u5+ 17*u6 + 0.14*u7 + 312*u8 + 3*u9 + 34*u10 + 7.8*u11 -v5 <= 20$
- $1820*u1 + 0.257*u2 + 0.067*u3+ 10*u4 + 2.07*u5+ 29*u6 + 0.64*u7 + 485*u8 + 7*u9 + 12*u10 + 14.2*u11 -v6 <= 28.8$
- $47200*u1 + 0.537*u2 + 0.341*u3+ 1.3*u4 + 0.7*u5+ 272*u6 + 1.69*u7 +441*u8 + 2*u9 + 98*u10 + 39.2*u11 -v7 <= 130$
- $1260*u1 + 0.038*u2 + 0.034*u3+ 0.59*u5+ 228*u6 + 3.31*u7 + 715*u8 + 20*u9 + 73*u10 -v8 <= 51$

In conclusion, the final dual form is as follows:

Objective Function (Maximize):

$Z_{(Max)} = 500000* u1 + 1.7*u2 + 1.2*u3 + 90* u4 + 15* u5 + 420*u6 + 11* u7 + 4700*u8 + 2300* u9 + 1300*u10 + 550* u11 - 25*v1 - 20*v2 - 30*v3 - 10*v4 - 10*v5 - 10*v6 - 10*v7 - 10*v8$

s.t.

- *165*u1 + 0.195*u2 + 0.078*u3+ 28*u4 + 2*u5+ 79*u6 + 0.53*u7 + 558*u8 + 79*u9 + 99*u10 + 19.3*u11 -v1 <= 0.8*
- *112*u1 + 0.2 *u2 + 0.063*u3+ 64.9*u4 + 1.45*u5+ 21*u6 + 0.45*u7 + 293*u8 + 41*u9 + 40*u10 + 18.7*u11 -v2 <= 3.3*
- *673*u1 + 0.147u2 + 0.113*u3 + 93.4*u4 + 0.66*u5 + 33*u6 + 0.39*u7 + 348*u8 + 53*u9 + 254*u10 + 0.5*u11 -v3 <= 0.55*
- *155*u1 + 0.046*u2 + 0.024*u3+ 58.8*u4 + 0.29*u5+ 13*u6 + 0.14*u7 + 153*u8 + u9 + 16*u10 + 5.7*u11 -v4 <= 3*
- *287*u1 + 0.063*u2 + 0.027*u3+ 92.7*u4 + 1.46*u5+ 17*u6 + 0.14*u7 + 312*u8 + 3*u9 + 34*u10 + 7.8*u11 -v5 <= 20*
- *1820*u1 + 0.257*u2 + 0.067*u3+ 10*u4 + 2.07*u5+ 29*u6 + 0.64*u7 + 485*u8 + 7*u9 + 12*u10 + 14.2*u11 -v6 <= 28.8*
- *47200*u1 + 0.537*u2 + 0.341*u3+ 1.3*u4 + 0.7*u5+ 272*u6 + 1.69*u7 +441*u8 + 2*u9 + 98*u10 + 39.2*u11 -v7 <= 130*
- *1260*u1 + 0.038*u2 + 0.034*u3+ 0.59*u5+ 228*u6 + 3.31*u7 + 715*u8 + 20*u9 + 73*u10 -v8 <= 51*

- *u1, u2, u3, u4, u5, u6, u7, u8, u9, u10, u11, v1, v2, v3, v4, v5, v6, v7, v8 >= 0*

References

[1] Harvard Medical School, "Foods linked to better brainpower," *Harvard Health*, Mar. 06, 2021. https://www.health.harvard.edu/healthbeat/foods-linked-to-better-brainpower