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Operations Research

Minimizing Cost for Transportation

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Abstract

Transportation is arguably one of the most crucial pillars of our modern-day world, whether it's commuting to work or having goods delivered to one's nearby store. However, unoptimized routes can lead to time waste and thus lead to an exponential rise in costs. As such, minimizing transportation costs is the main area of research. Linear programming models offer powerful tools to optimize transportation systems, addressing problems like delivery truck scheduling, airline routing, and public transportation optimization. This report will examine three research papers that utilize linear programming models in different transportation problems around the world. We will compare their approaches and how each one deals with their specific scenario and lastly conclude on solving one of these models using simplex method and apply sensitivity analysis accordingly.

Literature Review

The first paper revolves around J & J Essential Products (Pvt.) Ltd. in Bangladesh, a company that delivers mosquito coils from its three warehouses in Dhaka, Chittagong and Bogra to seven distributors' warehouses in Barisal, Chittagong, Dhaka, Rajshahi, Rangpur, Sylhet and Khulna without considering the optimal quantity. The company aims to minimize transportation costs for mosquito coil shipments between their warehouses and distributor warehouses. As such, this paper explores how linear programming can be used to find optimal quantities to ship from each company warehouse to each distributor with relevant costs, warehouse supply, and distributor demand put into consideration. The model also assumes a consistent supply and a fixed transportation cost per unit; It also puts forth constraints that address each company warehouse capacity limitations to each of the seven distributors so that the total quantity shipped from a warehouse cannot exceed its storage capacity, as well as the demand requirements of each of the distributors' location so that the total quantity shipped to a distributor meets or exceeds their demand accordingly [1].

The second paper follows a similar pattern to the first where the paper revolves around Albert David Company (ADC), a company that delivers products from its three factories in Madideep, Gajiabad and Calcutta to three depots in Barisal, Raipur, and Mumbai. The company also aims to minimize transportation costs for its shipments between their warehouses and their depots. As such, this paper applies linear programming methods such as dual simplex, two phase and big M methods to find optimal quantities to ship from each of the company's plants to each depot with relevant costs, warehouse supply, and distributor demand and compare the results. The model considers relevant factors such as transportation costs, supply levels at each plant, and demand requirements at each depot. The model incorporates constraints that ensure that the total quantity shipped does not exceed the available supply for each plant as well as the total quantity received by each depot meets its demand at the bare minimum [2].

The third paper follows a similar pattern to the first two, but it follows a more abstract methodology where it uses an example from Albania where a company needs to distribute products from two factories to nine distributors. The company aims to minimize transportation costs for its shipments between their warehouses and their depots. as such, this paper outlines how to solve this problem using the north-west corner rule and mentions least cost method and Vogel's approximation method to find optimal quantities to ship from each of the company's factories to each distributor with relevant costs, warehouse supply, and distributor demand and compare the results. The model considers the number of units shipped from each origin to each destination as the main decision variables of the model. The model incorporates constraints that ensure that the total number of units shipped from each factory cannot exceed its capacity as well as the total number of units received by each distributor must meet its demand [3].

Comparison

All three papers address minimizing transportation costs for a product being shipped from various origins to multiple destinations. They all have identical generic minimization models and attempt to solve them using linear programming methods with all models being subject to ensuring supply capacity at origins, warehouses or factories, is not exceeded and demand at destinations, distributors or depots, is met. The decision variable in all cases is the quantity shipped from each origin to each destination only differing in their numbers since the first paper has 21 decision variables, the second paper has 9 while the third paper has 18 decision variables. Additionally, the results differ for each one of the papers as they tackle the issue in their respective circumstances. Paper 1 concluded that Vogel's method was the most optimal. Paper 2 is written about theory, which proves that the optimal solution exists, and it makes the comparison of some methods find a most cost-efficient dual simplex for the company Albert David. Lastly, paper 3 does not give details of the findings.

Research Paper	Decision Variables	Constraints	Solving Method
Paper 1	21	11	Excel Solver
Paper 2	9	7	Not mentioned
Paper 3	18	12	North-West Least-Cost Vogel's Approximation

Paper 1: Transportation Cost Optimization Using Linear Programming

Source: https://www2.kuet.ac.bd/icmiee2014/wp-content/uploads/2015/02/ICMIEE-PI-140224.pdf

The first paper's linear programming formulation consists of one multi-indexed parameter representing the cost per unit and one multi-indexed decision variable both consisting of two subscripts, i (warehouse) and j (destination), representing the amount of units shipped from origin (i) to destination (j) for all i=1…3 and j=1…7.

$$Minimize\ f = \sum_{i=1}^{3}\sum_{j=1}^{7} C(i,j)X(i,j)$$

Upon dissolving the sigma notation of the decision variable into the combination of the 3 warehouses and 7 distributors, we get 21 unique decision variables of the problem that, along with the given cost of each warehouse-destination groupings, result in the following explicit objective function:

$$Z_{(Min)} = 15 * X_{(1,1)} + 160 * X_{(2,1)} + 100 * X_{(3,1)} + 160 * X_{(1,2)} + 12 * X_{(2,2)} + 260 * X_{(3,2)} + 154 * X_{(1,3)} + 315 * X_{(2,3)} + 56 * X_{(3,3)} + 245 * X_{(1,4)} + 410 * X_{(2,4)} + 190 * X_{(3,4)} + 130 * X_{(1,5)} + 290 * X_{(2,5)} + 58 * X_{(3,5)} + 125 * X_{(1,6)} + 427 * X_{(2,6)} + 204 * X_{(3,6)} + 215 * X_{(1,7)} + 375 * X_{(2,7)} + 160 * X_{(3,7)}$$

The other component discussed in the paper is the constraints set on the data where A_i, representing the constraint on the storage capacity of mosquito coils for all i=1…3, and B_j, representing the demand for mosquito coils for all j=1…7, and finally the nonnegativity constraint characterizing the overall constraints of the paper in the following forms:

$$\sum_{i=1}^{3} X(i,j) \leq a(i)$$

$$\sum_{j=1}^{7} X(i,j) \geq b(j)$$

$$X(i,j) \geq 0$$

Comparable to the objective function, these constraints can be further dissolved into more explicit and specific variables, ones that represent the available quantities of mosquito coils at each origin to all destination warehouses, and other constraints that represent the total demand at each destination from all origin warehouses. As such, they are denoted as follows:

- *X(1, 1) + X(1, 2) + X(1, 3) + X(1, 4) + X(1, 5) + X(1, 6) + X(1, 7) <= 3980 (Dhaka Destination)*
- *X(2, 1) + X(2, 2) + X(2, 3) + X(2, 4) + X(2, 5) + X(2, 6) + X(2, 7) <= 1785 (Chittagong Destination)*
- *X(3, 1) + X(3, 2) + X(3, 3) + X(3, 4) + X(3, 5) + X(3, 6) + X(3, 7) <= 4856 (Bogra Destination)*
- *X(1, 1) + X(2, 1) + X(3, 1) >= 1168 (Dhaka Distributor)*
- *X(1, 2) + X(2, 2) + X(3, 2) >= 1560 (Chittagong Distributor)*
- *X(1, 3) + X(2, 3) + X(3, 3) >= 1439 (Rangpur Distributor)*
- *X(1, 4) + X(2, 4) + X(3, 4) >= 986 (Barisal Distributor)*
- *X(1, 5) + X(2, 5) + X(3, 5) >= 1658 (Rajshahi Distributor)*
- *X(1, 6) + X(2, 6) + X(3, 6) >= 2035 (Sylhet Distributor)*
- *X(1, 7) + X(2, 7) + X(3, 7) >= 1159 (Khulna Distributor)*
- *X(1, 1), X(2, 1) , X(3, 1) , X(1, 2) , X(2, 2) , X(3, 2) , X(1, 3) , X(2, 3) , X(3, 3) , X(1, 4) , X(2, 4) , X(3, 4) , X(1, 5) , X(2, 5) X(3, 5) , X(1, 6) , X(2, 6) , X(3, 6) , X(1, 7) , X(2, 7) , X(3, 7) >= 0 (Nonnegativity)*

i. Solving Paper 1:

Optimized Decision Variables		
X(1, 1): 1168	*X(2, 1): 0*	*X(3, 1): 0*
X(1, 2): 0	*X(2, 2): 1560*	*X(3, 2): 0*
X(1, 3): 0	*X(2, 3): 0*	*X(3, 3): 1439*
X(1, 4): 0	*X(2, 4): 0*	*X(3, 4): 986*
X(1, 5): 0	*X(2, 5): 0*	*X(3, 5): 1658*
X(1, 6): 2035	*X(2, 6): 0*	*X(3, 6): 0*
X(1, 7): 386	*X(2, 7): 0*	*X(3, 7): 773*

Optimal Minimum Transportation Cost: $861373

ii. Dual Form:

The dual form is an alternative way to represent and solve the same problem from a different perspective. If the primal problem aims to either maximize or minimize an objective function, the dual problem aims for the opposite to find the optimal values of where both problems are equal. As such, to prepare for the dual formulation, we need to standardize the constraints in the appropriate direction according to the objective function, which in that case is a minimization problem. This means ensuring all constraints are expressed as "greater than or equal to" inequalities with exception for the nonnegativity constraint. Any constraints with "less than or equal to" can be converted by multiplying both sides by negative one as the following:

- *(X(1, 1) + X(1, 2) + X(1, 3) + X(1, 4) + X(1, 5) + X(1, 6) + X(1, 7) <= 3980) * -1*
- *(X(2, 1) + X(2, 2) + X(2, 3) + X(2, 4) + X(2, 5) + X(2, 6) + X(2, 7) <= 1785) * -1*
- *(X(3, 1) + X(3, 2) + X(3, 3) + X(3, 4) + X(3, 5) + X(3, 6) + X(3, 7) <= 4856) * -1*

So, the primal form we are working is as follows:

Objective Function (Minimize):

$Z_{(Min)} = 15 * X_{(1,1)} + 160 * X_{(2,1)} + 100 * X_{(3,1)} + 160 * X_{(1,2)} + 12 * X_{(2,2)} + 260 * X_{(3,2)} + 154 * X_{(1,3)} + 315 * X_{(2,3)} + 56 * X_{(3,3)} + 245 * X_{(1,4)} + 410 * X_{(2,4)} + 190 * X_{(3,4)} + 130 * X_{(1,5)} + 290 * X_{(2,5)} + 58 * X_{(3,5)} + 125 * X_{(1,6)} + 427 * X_{(2,6)} + 204 * X_{(3,6)} + 215 * X_{(1,7)} + 375 * X_{(2,7)} + 160 * X_{(3,7)}$

s.t.

-X(1, 1) - X(1, 2) - X(1, 3) - X(1, 4) - X(1, 5) - X(1, 6) - X(1, 7) >= -3980 (Dhaka Destination)

-X(2, 1) - X(2, 2) - X(2, 3) - X(2, 4) - X(2, 5) - X(2, 6) - X(2, 7) >=- 1785 (Chittagong Destination)

-X(3, 1) - X(3, 2) - X(3, 3) - X(3, 4) - X(3, 5) - X(3, 6) - X(3, 7) >= -4856 (Bogra Destination)

X(1, 1) + X(2, 1) + X(3, 1) >= 1168 (Dhaka Distributor)

X(1, 2) + X(2, 2) + X(3, 2) >= 1560 (Chittagong Distributor)

X(1, 3) + X(2, 3) + X(3, 3) >= 1439 (Rangpur Distributor)

X(1, 4) + X(2, 4) + X(3, 4) >= 986 (Barisal Distributor)

X(1, 5) + X(2, 5) + X(3, 5) >= 1658 (Rajshahi Distributor)

X(1, 6) + X(2, 6) + X(3, 6) >= 2035 (Sylhet Distributor)

X(1, 7) + X(2, 7) + X(3, 7) >= 1159 (Khulna Distributor)

X(1, 1), X(2, 1) , X(3, 1) , X(1, 2) , X(2, 2) , X(3, 2) , X(1, 3) , X(2, 3) , X(3, 3) , X(1, 4) , X(2, 4) , X(3, 4) , X(1, 5) , X(2, 5) X(3, 5) , X(1, 6) , X(2, 6) , X(3, 6) , X(1, 7) , X(2, 7) , X(3, 7) >= 0 (Nonnegativity)

We begin by constructing the dual form of the objective function where we multiply the right-hand side coefficients of the primal constraints by the new dual variables and add them accordingly:

$$Z_{(Max)} = -3980 * u1 - 1785 * u2 - 4856 * u3 + 1168 * v1 + 1560 * v2 + 1439 * v3 + 986 * v4 + 1658 * v5 + 2035 * v6 + 1159 * v7$$

We then construct a matrix in the same order as the primal objective function by taking left-hand side coefficients of each of the decision variables as they appear in each primal constraint which results in the following:

$X_{(1,1)}$	$X_{(2,1)}$	$X_{(3,1)}$	$X_{(1,2)}$	$X_{(2,2)}$	$X_{(3,2)}$	$X_{(1,3)}$	$X_{(2,3)}$	$X_{(3,3)}$	$X_{(1,4)}$	$X_{(2,4)}$	$X_{(3,4)}$	$X_{(1,5)}$	$X_{(2,5)}$	$X_{(3,5)}$	$X_{(1,6)}$	$X_{(2,6)}$	$X_{(3,6)}$	$X_{(1,7)}$	$X_{(2,7)}$	$X_{(3,7)}$
-1	0	0	-1	0	0	-1	0	0	-1	0	0	-1	0	0	-1	0	0	-1	0	0
0	-1	0	0	-1	0	0	-1	0	0	-1	0	0	-1	0	0	-1	0	0	-1	0
0	0	-1	0	0	-1	0	0	-1	0	0	-1	0	0	-1	0	0	-1	0	0	-1
1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1

We then get the transpose of the matrix from earlier and multiply it by the new dual variables and equate it to the cost matrix from primal objective function:

$$\begin{bmatrix} -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} u1 \\ u2 \\ u3 \\ v1 \\ v2 \\ v3 \\ v4 \\ v5 \\ v6 \\ v7 \end{bmatrix} = \begin{bmatrix} 15 \\ 160 \\ 100 \\ 160 \\ 12 \\ 260 \\ 154 \\ 315 \\ 56 \\ 245 \\ 410 \\ 190 \\ 130 \\ 290 \\ 58 \\ 125 \\ 427 \\ 204 \\ 215 \\ 375 \\ 160 \end{bmatrix}$$

Lastly, we construct the new constraints from the multiplication to get the following dual constraints:

- u1 + v1 <= 15	*- u2 + v3 <= 315*	*- u3 + v5 <= 58*
- u2 + v1 <= 160	*- u3 + v3 <= 56*	*- u1 + v6 <= 125*
- u3 + v1 <= 100	*- u1 + v4 <= 245*	*- u2 + v6 <= 427*
- u1+v2 <= 160	*- u2 + v4 <= 410*	*- u3 + v6 <= 204*
- u2 + v2 <= 12	*- u3 + v4 <= 190*	*- u1 + v7 <= 215*
- u3 + v2 <= 260	*- u1 + v5 <= 130*	*- u2 + v7 <= 375*
- u1 + v3 <= 154	*- u2 + v5 <= 290*	*- u3 + v7 <= 160*

In conclusion, the final dual form is as follows:

Objective Function (Maximize):

$$Z_{(Max)} = -3980 * u1 - 1785 * u2 - 4856 * u3 + 1168 * v1 + 1560 * v2 + 1439 * v3 + 986 * v4 + 1658 * v5 + 2035 * v6 + 1159 * v7$$

s.t.

$- u1 + v1 <= 15$	$- u2 + v3 <= 315$	$- u3 + v5 <= 58$
$- u2 + v1 <= 160$	$- u3 + v3 <= 56$	$- u1 + v6 <= 125$
$- u3 + v1 <= 100$	$- u1 + v4 <= 245$	$- u2 + v6 <= 427$
$- u1+v2 <= 160$	$- u2 + v4 <= 410$	$- u3 + v6 <= 204$
$- u2 + v2 <= 12$	$- u3 + v4 <= 190$	$- u1 + v7 <= 215$
$- u3 + v2 <= 260$	$- u1 + v5 <= 130$	$- u2 + v7 <= 375$
$- u1 + v3 <= 154$	$- u2 + v5 <= 290$	$- u3 + v7 <= 160$

$u1, u2, u3, v1, v2, v3, v4, v5, v6, v7 >= 0$

iii. Sensitivity Analysis:

Objective Coefficients Analysis:

Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$X_{(1, 1)}$	1168	0	15	140	15
$X_{(2, 1)}$	0	145	160	1E+30	145
$X_{(3, 1)}$	0	140	100	1E+30	140
$X_{(1, 2)}$	0	148	160	1E+30	148
$X_{(2, 2)}$	1560	0	12	148	12
$X_{(3, 2)}$	0	303	260	1E+30	303
$X_{(1, 3)}$	0	43	154	1E+30	43
$X_{(2, 3)}$	0	204	315	1E+30	204
$X_{(3, 3)}$	1439	0	56	43	111
$X_{(1, 4)}$	0	0	245	1E+30	0
$X_{(2, 4)}$	0	165	410	1E+30	165
$X_{(3, 4)}$	986	0	190	0	245
$X_{(1, 5)}$	0	17	130	1E+30	17
$X_{(2, 5)}$	0	177	290	1E+30	177
$X_{(3, 5)}$	1658	0	58	17	113
$X_{(1, 6)}$	2035	0	125	134	125
$X_{(2, 6)}$	0	302	427	1E+30	302
$X_{(3, 6)}$	0	134	204	1E+30	134
$X_{(1, 7)}$	386	0	215	0	55
$X_{(2, 7)}$	0	160	375	1E+30	160
$X_{(3, 7)}$	773	0	160	55	0

Constraint RHS Analysis:

Name	Final Value	Shadow Price	Constraint RHS	Allowable Increase	Allowable Decrease
Constraint 1 LHS	3589	0	3980	1E+30	391
Constraint 2 LHS	1560	0	1785	1E+30	225
Constraint 3 LHS	4856	-55	4856	386	391
Constraint 4 LHS	1168	15	1168	391	1168
Constraint 5 LHS	1560	12	1560	225	1560
Constraint 6 LHS	1439	111	1439	391	386
Constraint 7 LHS	986	245	986	391	386
Constraint 8 LHS	1658	113	1658	391	386
Constraint 9 LHS	2035	125	2035	391	2035
Constraint 10 LHS	1159	215	1159	391	386

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