15 - rozkład wykładniczy

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X ma rozkład wykładniczy z parametrem θ

Dystrybuanta

$$F(x) = \begin{cases} 1 - e^{-\theta x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

Gęstość

$$f(x) = \begin{cases} \theta e^{-\theta x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

Momenty

$$\int x e^{cx} dx = \frac{1}{c} x e^{cx} - \frac{1}{c^2} e^{cx} + C \qquad \int u\, dv = uv - \int v\, du$$

$$\mathbb{E}[X] = \int_0^\infty x \theta e^{-\theta x} dx = \theta \cdot \frac{1}{\theta^2}(-\theta x - 1) e^{-\theta x} \Big|_0^\infty = \frac{1}{\theta}$$

$$\mathbb{E}[X^k] = \frac{k!}{\theta^k}$$

$$Var(X) = \mathbb{E}[X^2] - \mathbb{E}[X] = \frac{1}{\theta^2}$$

Funkcja tworząca momenty

$$M_X(t) = \mathbb{E}[e^{tX}] = \int_0^\infty e^{tx} \cdot \theta e^{-\theta x} dx = \frac{\theta}{t-\theta} e^{(t-\theta)x} \Big|_0^\infty = \frac{\theta}{\theta - t} = \frac{1}{1 - \frac{t}{\theta}}$$

Bez pamięci

$$\mathbb{P}[X > s+t \mid X > t] = \mathbb{P}[X > s]$$

$$\mathbb{P}[X > s+t \mid X > t] = \frac{\mathbb{P}[X > s+t \cap X > s]}{\mathbb{P}[X > s]} = \frac{e^{-\theta(s+t)}}{e^{-\theta s}} = e^{-\theta t}$$

Jedyny taki rozkład ciągły. Zmienne można resetować.

Minimum n niezależnych prób

$X_1, X_2, \dots, X_n \quad X_i \sim Exp[\theta_i]$

$$\min(X_1, X_2, \dots, X_n) \sim Exp\left[\sum_{i=1}^n \theta_i\right]$$

$$\mathbb{P}[\min(X_1, X_2, \dots, X_n) = X_i] = \frac{\theta_i}{\sum_{j=1}^n \theta_j}$$

$X_1 \sim Exp[\theta_1]$, $X_2 \sim Exp[\theta_2]$

$$\mathbb{P}[\min(X_1, X_2) > x] = e^{-\theta_1 x} \cdot e^{-\theta_2 x} = e^{-(\theta_1 + \theta_2)x}$$

$$\mathbb{P}[X_1 < X_2] = \int_0^\infty \theta_2 e^{-\theta_2 x} \mathbb{P}[X_1 \leq x] dx = \int_0^\infty \theta_2 e^{-\theta_2 x}(1 - e^{-\theta_1 x}) dx$$

$$= 1 - e^{\theta_2 x} \Big|_0^\infty - \theta_2 \left(1 - e^{-(\theta_1+\theta_2)x}\right) \Big|_0^\infty = 1 - \frac{\theta_2}{\theta_1 + \theta_2} = \frac{\theta_1}{\theta_1 + \theta_2}$$