Rozdział 3

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Füredi Theorem (2015)

Any n-vertex K_{k+1}-free graph G with $e(T_k(n))-t$ edges contains a k-partite subgraph G' with $e(G') \geq e(G)-t$.

Proof: We form partition by taking vertex with highest degree, putting its non-neighbours to one component and iterating on its neighbours. Then by double counting edges we obtain desired bound.

Erdős - Simonovits Stability Theorem (1966)

For any $\varepsilon > 0$ and graph F s.t. $\chi(F) = k+1$ there exists $\delta > 0$ s.t. any large enough F-free graph with at least $\frac{n^2}{2}\left(1-\frac{1}{k}\right) - \delta n^2$ edges can be transformed into $T_k(n)$ by adding or deleting at most εn^2 edges.

Proof: Szemerédi.

Asymptotic structure theorem

If G is extremal n-vertex graph for F with $\chi(F) = k+1$, then $\delta(G) = \left(1-\frac{1}{k}\right)n + o(n)$

Proof: From Erdős-Simonovits stability theorem we can transform $T_k(n)$ to G by flipping $o(n^2)$ edges.
We count how many neighbours of on $T_k(n)$ component were removed and obtain desired bound.

Andrásfai-Erdős-Sós (1974)

Every n-vertex K_{k+1}-free graph with $\delta(G) \geq \left(1-\frac{3}{3k-1}\right)n$ is k-partite.

Proof: discharging + „symetrization".