Rozdział 13

Saturday, 1 February 2025 18:59

Graph limits

Density

density of graph H in G $d(H,G)$ is probability that induced graph on random $v(H)$ vertices in G is isomorphic to H.

Convergence

A sequence G_n of graphs is convergent iff for any graph H $\lim d(H,G_n)$ is convergent.

Remark

All of the above can be equivalently defined via homomorphisms.

Kernel

Bounded, symmetric function $W: [0,1]^2 \to \mathbb{R}$ $W(x,y) = W(y,x)$

Graphon

Kernel with values in $[0,1]$.

Density in Graphon

$$d(H,W) = \frac{v(H)!}{|Aut(H)|} \int_{[0,1]^{|H|}} \prod_{i,j \in E(H)} W(x_i,x_j) \prod_{i,j \notin E(H)} 1 - W(x_i,x_j) \, dx_1 \cdots dx_{|H|}$$

Graph Limit

Graphon is a limit of convergent series G_n iff for any H: $\lim d(H,G_n) = d(H,W)$.

Weak isomorphism

Two graphons are weakly isomorphic iff all densities are the same.

Cut norm

$$\|W\|_\square = \sup_{S,T \subset [0,1]} \left| \int_{S \times T} W(x,y) \, dx \, dy \right|$$

$$d_\square(U,W) = \|U - W\|_\square$$

$$\delta_\square(U,W) = \inf_{\varphi \in S_{[0,1]}} d_\square(U, W^\varphi)$$

invertible maps

Theorem Lovász

Two graphons are weakly isomorphic iff $\delta_\square(U,W) = 0$

$\widetilde{W}_0$ space

Space of graphons identified by weak isomorphism.

$\widetilde{W}_0$ is compact.

Theorem

For any convergent sequence of graphs there exists a corresponding graphon.