Rozdział 5

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Erdős - Rademacher problem

Minimum number of copies of K_s among n-vertex graphs with e edges:

$$g_s(n,e) = \min\{N_s(G) : v(G)=n,\ e(G)=e\}$$

Asymptotic version: $\frac{e(G_n)}{\binom{n}{2}} \to \lambda$

$$g_3(\lambda) = \lim_{n\to\infty} \frac{g_3\left(n, \lambda\binom{n}{2}\right)}{\binom{n}{3}}$$

This limit exists and is independent from sequence $e(G_n)$.

Due to Moon-Moser $g_3(n,e) \geq \frac{e}{3}\left(\frac{4e}{n} - n\right) = \frac{e}{3}\left(\frac{4e-n^2}{n}\right)$

$$N_3 \geq \frac{N_2}{2^2-1}\left(\frac{2^2 N_2}{N_1} - n\right)$$

and asymptotically: $g_3(\lambda) \geq \frac{\lambda\binom{n}{2}}{3}\left(\frac{4\lambda\binom{n}{2}}{n} - n\right) \cdot \frac{1}{\binom{n}{3}}$

$$\sim \frac{\lambda n^2}{3\cdot 2}\left(2\lambda n - n\right)\cdot\frac{6}{n^3} = \lambda(2\lambda - 1)$$