Rozdział 1

Sunday, 26 January 2025 22:01

Cauchy - Schwarz

Dowolne ciągi a_i, b_i:

$$\left(\sum_i a_i b_i\right)^2 \leq \left(\sum_i a_i^2\right) \cdot \left(\sum_i b_i^2\right)$$

Mantel's Theorem (1907)

Maximum number of edges in triangle-free n-vertex graph is $\left\lfloor \frac{n^2}{4} \right\rfloor$.

Proof:

Remove an edge + induction / double counting edges via degrees + CS.

Turán's Theorem (1941)

Maximum number of edges in K_{k+1}-free n-vertex graph is $\left(1 - \frac{1}{k}\right)\frac{n^2}{2}$

Proof:

Remove K_k + induction / show that it's complete multipatite - induction on k.

Turán Graph

Maximal K_{k+1}-free n-vertex graph is $T_k(n)$

This graph is unique.