

P_k - path of length k , S_k - star on k vertices

Theorem

$$ex(n, S_k) \leq \frac{1}{2}(k-2)n$$

Proof: trivial.

Theorem (Erdős-Kollai, 1959)

$$ex(n, P_k) \leq \frac{k-2}{2} \cdot n \quad (\text{sharp when } k-1 \mid n, \text{ disjoint union of } K_{k-1})$$

Proof: induction on n - there exists a path on $\min(n, 2\delta+1)$ vertices.

T_k - tree on k vertices

Conjecture (Erdős-Sós)

$$ex(n, T_k) \leq \frac{k-2}{2} \cdot n$$

Works for some particular classes of trees + large enough k .

Hall's Theorem (1935)

Bipartite graph (A, B) contains matching incident to every vertex in A

iff $\forall S \subseteq A: |N(S)| \geq |S|$

Avoiding matchings (Erdős-Kollai 1959)

$$ex(n, k \cdot K_2) \leq \max\left(\binom{2k-1}{2}, \binom{k-1}{2} + (k-1)(n-k+1)\right)$$

Proof: Induction on n .