

ETF Casestudy

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0. Load Libraries

```

library("quantmod")

## Loading required package: xts
## Loading required package: zoo
##
## Attaching package: 'zoo'
## The following objects are masked from 'package:base':
##   as.Date, as.Date.numeric
## Loading required package: TTR
## Registered S3 method overwritten by 'quantmod':
##   method      from
##   as.zoo.data.frame zoo

library("tseries")
library("ggplot2")
library("reshape2")
library("tidyverse")

## -- Attaching core tidyverse packages ----- tidyverse 2.0.0 --
## v dplyr     1.1.4    v readr     2.1.5
## vforcats   1.0.0    v stringr   1.5.1
## v lubridate 1.9.3    v tibble    3.2.1
## v purrr    1.0.2    v tidyrr    1.3.1

## -- Conflicts ----- tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::first()  masks xts::first()
## x dplyr::lag()    masks stats::lag()
## x dplyr::last()   masks xts::last()
## i Use the conflicted package (<http://conflicted.r-lib.org/>) to force all conflicts to become errors

library("tidyquant")

## -- Attaching core tidyquant packages ----- tidyquant 1.0.9 --
## v PerformanceAnalytics 2.0.4
## -- Conflicts ----- tidyquant_conflicts() --
## x zoo::as.Date()           masks base::as.Date()
## x zoo::as.Date.numeric()   masks base::as.Date.numeric()
## x dplyr::filter()          masks stats::filter()
## x dplyr::first()           masks xts::first()

```

```
## x dplyr::lag()                         masks stats::lag()
## x dplyr::last()                          masks xts::last()
## x PerformanceAnalytics::legend()        masks graphics::legend()
## x quantmod::summary()                   masks base::summary()
## i Use the conflicted package (<http://conflicted.r-lib.org/>) to force all conflicts to become errors
library("car")

## Loading required package: carData
##
## Attaching package: 'car'
##
## The following object is masked from 'package:dplyr':
##   recode
##
## The following object is masked from 'package:purrr':
##   some
library("ggfortify")
library("glmnet")

## Loading required package: Matrix
##
## Attaching package: 'Matrix'
##
## The following objects are masked from 'package:tidyverse':
##   expand, pack, unpack
##
## Loaded glmnet 4.1-8
```

1. Load data into R session

1.1 Daily Stock Price Data from Yahoo

Apply quantmod(sub-package TTR) function: getYahooData

Returns historical data for any symbol at the website

<http://finance.yahoo.com>

1.1.1 Set start and end date for collection in YYYYMMDD (numeric) format

```
date.start<-"2000-01-01"  
date.end<-"2024-09-20"
```

1.1.2 Input list of ETF symbols

```
options(stringsAsFactors=FALSE)  
list.etf0.labels0<-read.table(file="spyders_symbols_labels.csv",sep=",", header=FALSE)  
list.etf0<-as.character(list.etf0.labels0[,1])  
  
for (etf0 in list.etf0){  
  #etf0<-"XLB"  
  getSymbols(etf0, from=date.start, to=date.end)  
  print(etf0)  
}  
  
## [1] "SPY"  
## [1] "MDY"  
## [1] "QQQ"  
## [1] "XLB"  
## [1] "XLV"  
## [1] "XLP"  
## [1] "XLY"  
## [1] "XLE"  
## [1] "XLF"  
## [1] "XLI"  
## [1] "XLK"  
## [1] "XLU"  
## [1] "DIA"
```

1.1.3 Construct matrix of closing prices

```
etf0.pmat0<-matrix(NA, nrow=NROW(SPY),ncol=length(list.etf0))  
dimnames(etf0.pmat0)<-list(dimnames(SPY)[[1]], list.etf0)  
# Create zoo objects of just Adjusted prices  
for (j.etf0 in c(1:length(list.etf0))){  
  #j.etf0<-3  
  j.etf0.name0<-list.etf0[j.etf0]  
  obj0.0<-get(list.etf0[j.etf0])[,6]  
  dimnames(obj0.0)[[2]]<-j.etf0.name0  
  obj0.0.name0<-paste(j.etf0.name0,".0",sep="")  
  print(obj0.0.name0)
```

```

    assign(obj0.0$name0, obj0.0)
}

## [1] "SPY.0"
## [1] "MDY.0"
## [1] "QQQ.0"
## [1] "XLB.0"
## [1] "XLV.0"
## [1] "XLP.0"
## [1] "XLY.0"
## [1] "XLE.0"
## [1] "XLF.0"
## [1] "XLI.0"
## [1] "XLK.0"
## [1] "XLU.0"
## [1] "DIA.0"

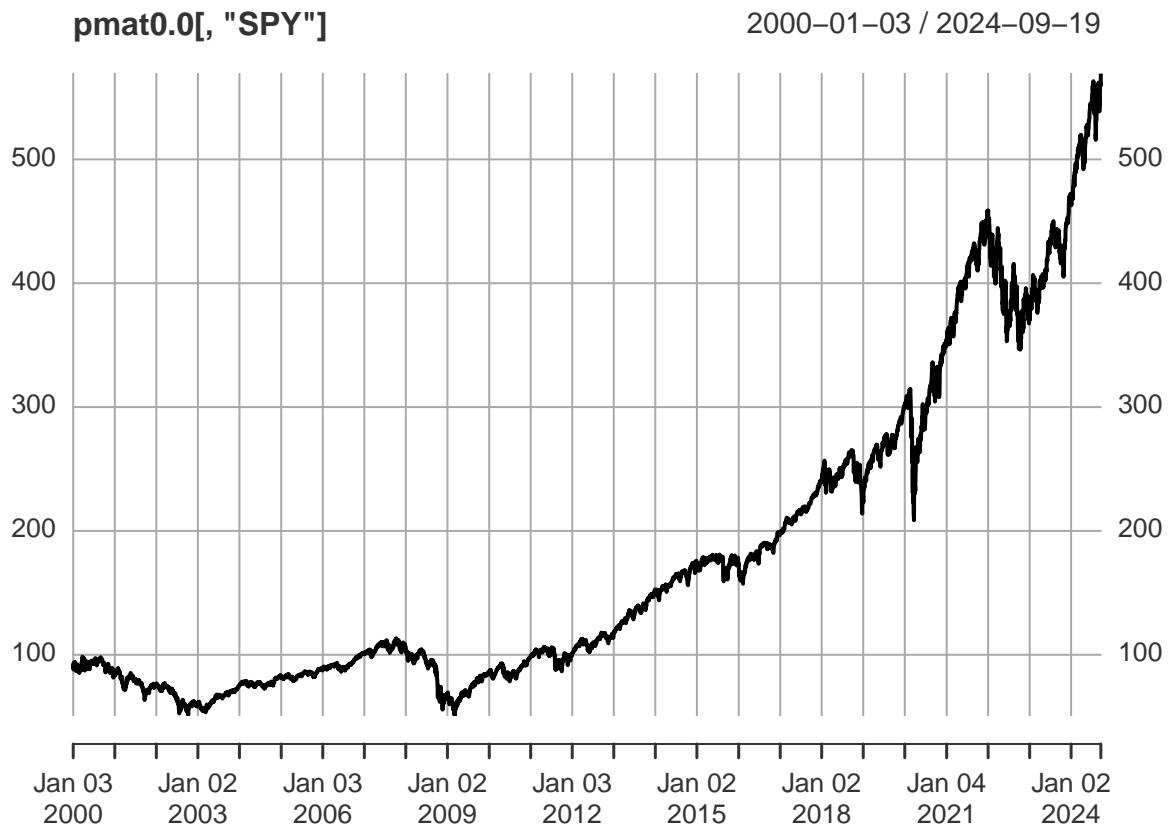
list.etf0.0<-paste(list.etf0,".0",sep="")

pmat0.0<-eval(parse(text=paste("merge(",
                                paste(list.etf0.0,collapse=","),
                                ")", collapse="")))

```

1.1.4 Plot time series of daily SPY prices

```
plot(pmat0.0[, "SPY"])
```



```
chartSeries(pmat0.0[, "XLF"])
```



1.2 Weekly Stock Price Data

1.2.1 Create matrix of weekly closing prices

```
pmat0.0.1.weekly<-to.weekly(pmat0.0[,1])
pmat0.0.weekly.coredata<-matrix(NA, nrow=nrow(pmat0.0.1.weekly), ncol=ncol(pmat0.0))

for (jcol0 in c(1:ncol(pmat0.0))){
  pmat0.0.weekly.coredata[,jcol0]<- to.weekly(pmat0.0[,jcol0])[,4]
}

dim(pmat0.0.weekly.coredata)

## [1] 1290    13
length(time(pmat0.0.1.weekly))

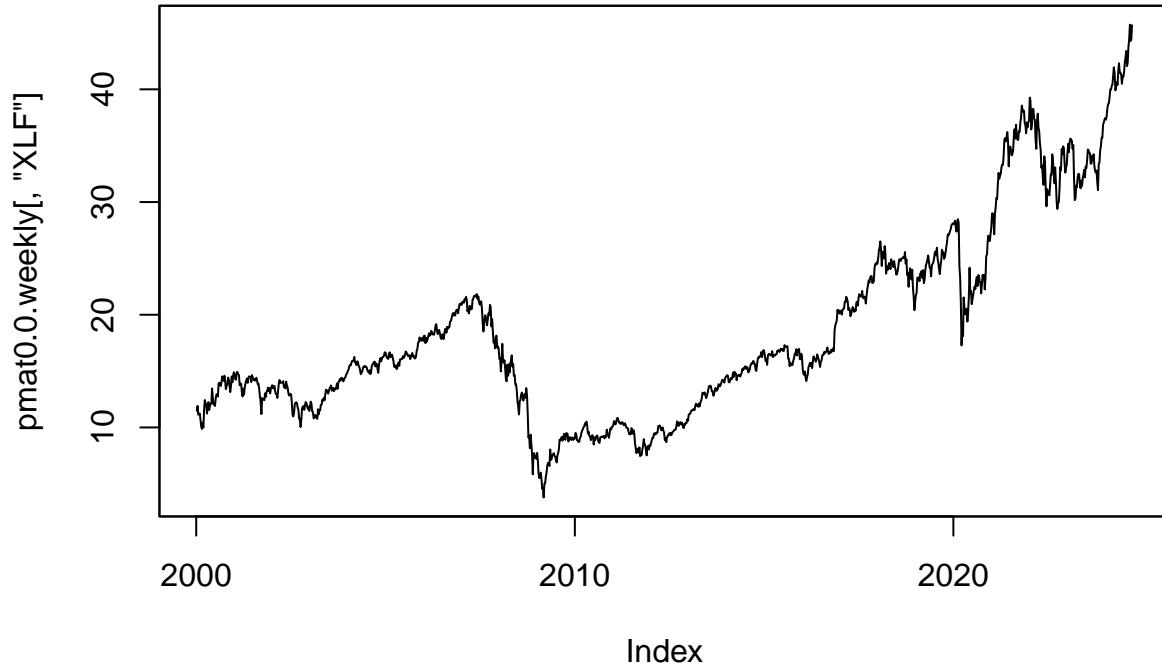
## [1] 1290
as.numeric(time(pmat0.0.1.weekly)[1:5])

## [1] 10963 10970 10977 10984 10991
pmat0.0.weekly<-zoo(pmat0.0.weekly.coredata, order.by=time(pmat0.0.1.weekly))
```

```
dimnames(pmat0.0.weekly)[[2]] <- dimnames(pmat0.0)[[2]]
```

1.2.2 Create matrix of weekly log returns

```
plot(pmat0.0.weekly[, "XLF"])
```



```
head(pmat0.0.weekly)
```

```
##          SPY      MDY      QQQ      XLB      XLV      XLP      XLY
## 2000-01-07 93.20474 60.77375 76.63248 15.80843 20.71107 13.61024 22.95816
## 2000-01-14 93.98414 61.94609 79.50619 15.29558 22.30757 13.48810 22.64126
## 2000-01-21 92.36544 62.27434 81.95415 14.31486 22.19897 13.02570 21.70226
## 2000-01-28 86.88984 59.03870 73.22663 13.51409 21.02603 12.73780 20.28206
## 2000-02-04 91.18630 61.12546 82.72588 13.48710 21.99261 12.68545 21.08020
## 2000-02-11 88.68842 61.19578 85.04080 12.50639 21.30840 12.04856 20.27033
##          XLE      XLF      XLI      XLK      XLU      DIA
## 2000-01-07 14.88110 11.47026 18.86840 38.36602 11.60620 67.18431
## 2000-01-14 14.87281 11.91383 18.62753 39.77654 11.38846 68.34761
## 2000-01-21 15.41936 11.12786 18.19596 40.34076 11.77115 65.66428
## 2000-01-28 14.14408 11.17456 17.34287 36.81446 11.15752 62.51753
## 2000-02-04 13.94533 11.24459 17.41313 40.71688 11.44784 63.88174
## 2000-02-11 13.63065 10.70765 16.35930 40.90496 11.07835 60.82587
```

```
rmat0.0.weekly <- na.omit(diff(log(pmat0.0.weekly)))
```

Table frequency of return horizon in days. Note that September 11, 2001 corresponds to 11.

```
table(diff(time(rmat0.0.weekly)))
```

```
##
##      3      6      7      8     11
##      1    39 1209    38      1
```

```

cor(rmat0.0.weekly)

##          SPY        MDY        QQQ        XLB        XLV        XLP        XLY
## SPY 1.0000000 0.9263244 0.8406530 0.8169412 0.8048856 0.7133526 0.8851434
## MDY 0.9263244 1.0000000 0.7857591 0.8350609 0.7038070 0.6111273 0.8456266
## QQQ 0.8406530 0.7857591 1.0000000 0.5875579 0.6467435 0.4414167 0.7333358
## XLB 0.8169412 0.8350609 0.5875579 1.0000000 0.6174494 0.5939057 0.7566491
## XLV 0.8048856 0.7038070 0.6467435 0.6174494 1.0000000 0.6477534 0.6718096
## XLP 0.7133526 0.6111273 0.4414167 0.5939057 0.6477534 1.0000000 0.6295337
## XLY 0.8851434 0.8456266 0.7333358 0.7566491 0.6718096 0.6295337 1.0000000
## XLE 0.6713572 0.7040841 0.4252876 0.7034908 0.4825468 0.4657262 0.5259733
## XLF 0.8404444 0.8232555 0.5853957 0.7296194 0.6391777 0.5685931 0.7733551
## XLI 0.9091362 0.9002693 0.7081750 0.8512075 0.7054340 0.6662999 0.8287766
## XLK 0.8542522 0.7790903 0.9640273 0.6072689 0.6348147 0.4588322 0.7419041
## XLU 0.6166184 0.5736058 0.3873537 0.5073337 0.5512258 0.6180840 0.4970907
## DIA 0.9556731 0.8819201 0.7305154 0.8294260 0.7928100 0.7558884 0.8572597
##          XLE        XLF        XLI        XLK        XLU        DIA
## SPY 0.6713572 0.8404444 0.9091362 0.8542522 0.6166184 0.9556731
## MDY 0.7040841 0.8232555 0.9002693 0.7790903 0.5736058 0.8819201
## QQQ 0.4252876 0.5853957 0.7081750 0.9640273 0.3873537 0.7305154
## XLB 0.7034908 0.7296194 0.8512075 0.6072689 0.5073337 0.8294260
## XLV 0.4825468 0.6391777 0.7054340 0.6348147 0.5512258 0.7928100
## XLP 0.4657262 0.5685931 0.6662999 0.4588322 0.6180840 0.7558884
## XLY 0.5259733 0.7733551 0.8287766 0.7419041 0.4970907 0.8572597
## XLE 1.0000000 0.5858364 0.6812261 0.4344610 0.4907898 0.6729492
## XLF 0.5858364 1.0000000 0.8241749 0.5965313 0.4832339 0.8299180
## XLI 0.6812261 0.8241749 1.0000000 0.7224326 0.5653708 0.9232702
## XLK 0.4344610 0.5965313 0.7224326 1.0000000 0.4106982 0.7606385
## XLU 0.4907898 0.4832339 0.5653708 0.4106982 1.0000000 0.6176370
## DIA 0.6729492 0.8299180 0.9232702 0.7606385 0.6176370 1.0000000

dim(rmat0.0.weekly)

## [1] 1289   13

```

1.2.3 Create separate objects for sector etfs and index etfs

```

rmat0.0.weekly.sectoretfs<-rmat0.0.weekly[,c(4:12)]
list.sectoretfs<-dimnames(rmat0.0.weekly.sectoretfs)[[2]]
list.sectoretfs.labels<-list.ETF0.labels0[4:12,2]
list.sectoretfs.labels.0<-paste(list.sectoretfs.labels,"(",list.sectoretfs,")",sep="")

rmat0.0.weekly.indexetfs<-rmat0.0.weekly[,c(1:3, 13)]

list.indexetfs<-dimnames(rmat0.0.weekly.indexetfs)[[2]]

head(rmat0.0.weekly.indexetfs)

##          SPY        MDY        QQQ        DIA
## 2000-01-14  0.008327424  0.019106545  0.03681378  0.01716685
## 2000-01-21 -0.017373142  0.005284925  0.03032506 -0.04005145
## 2000-01-28 -0.061111787 -0.053356329 -0.11260074 -0.04910812
## 2000-02-04  0.048263591  0.034735339  0.12197327  0.02158654
## 2000-02-11 -0.027775413  0.001149762  0.02759872 -0.04901837

```

```

## 2000-02-18 -0.024636530 -0.013889528 -0.01449766 -0.02053414
list.sectoretfs<-dimnames(rmat0.0.weekly.sectoretfs)[[2]]
list.sectoretfs.labels<-list.etf0.labels0[4:12,2]
list.sectoretfs.labels.0<-paste(list.sectoretfs.labels,"(",list.sectoretfs,")",sep="")
rmat0.0.weekly.indexetfs<-rmat0.0.weekly[,c(1:3, 13)]
list.indexetfs<-dimnames(rmat0.0.weekly.indexetfs)[[2]]

```

1.3 Save data objects for casestudy in R data file

```

list.obj.tosave0<-c(
  "pmat0.0.weekly",
  "rmat0.0.weekly.sectoretfs",
  "rmat0.0.weekly.indexetfs",
  "list.sectoretfs",
  "list.sectoretfs.labels",
  "list.sectoretfs.labels.0",
  "list.indexetfs")

save(file="casestudy_1_0_etfs.RData", list=list.obj.tosave0)

```

2. Analysis of returns for specific period (2013-2024)

2.1 Subset data objects for specific period

```
period0.startyear0<-"2013"
period0.endyear0<-"2024"
period0.startdate0<-as.Date(paste(period0.startyear0,"-01-01",sep=""))
period0.enddate0<-as.Date(paste(period0.endyear0,"-12-31",sep=""))
period0.label0<-paste("Period: ", period0.startyear0, " - ", period0.endyear0, sep="")

rmat0.0.weekly.sectoretfs.period0<-window(rmat0.0.weekly.sectoretfs,
                                             start = period0.startdate0,
                                             end=period0.enddate0)

rmat0.0.weekly.indexetfs.period0<-window(rmat0.0.weekly.indexetfs,
                                             start = period0.startdate0,
                                             end=period0.enddate0)

tail(rmat0.0.weekly.sectoretfs.period0)

##          XLB         XLV         XLP         XLY         XLE
## 2024-08-16  0.02244277  0.019085932  0.017036621  0.049188915  0.0117857438
## 2024-08-23  0.02356905  0.016736262  0.016384650  0.025553898 -0.0008846825
## 2024-08-30  0.01657698  0.011194744  0.008153447 -0.004000112  0.0097980571
## 2024-09-06 -0.04768237 -0.020891006  0.005800569 -0.025548052 -0.0594680988
## 2024-09-13  0.03067392  0.014253292  0.011263029  0.053790186 -0.0048951823
## 2024-09-19  0.02187566 -0.001538103 -0.015368279  0.023921903  0.0386184091
##          XLF         XLI         XLK         XLU
## 2024-08-16  0.031800263  0.02141377  0.07389253  0.011212570
## 2024-08-23  0.014966262  0.01802974  0.01149650  0.013079003
## 2024-08-30  0.029058301  0.01672209 -0.01634106  0.011469413
## 2024-09-06 -0.032214287 -0.04329417 -0.07739868 -0.004993404
## 2024-09-13  0.004954907  0.03650103  0.07789781  0.034059585
## 2024-09-19  0.026165580  0.02682535  0.01432260 -0.006642900

tail(rmat0.0.weekly.indexetfs.period0)

##          SPY         MDY         QQQ         DIA
## 2024-08-16  0.039221464  0.026451262  0.053219670  0.02971464
## 2024-08-23  0.014009032  0.028103817  0.010408147  0.01243647
## 2024-08-30  0.002753516 -0.001537826 -0.007801207  0.01057932
## 2024-09-06 -0.042251072 -0.049590953 -0.059652666 -0.02861231
## 2024-09-13  0.039284067  0.031976940  0.057698098  0.02555637
## 2024-09-19  0.015834531  0.029501289  0.016731358  0.01547228
```

2.2 Define y0 equal to weekly returns of specific sector etf (XLP)

```
y0.name="XLP (Consumer Staples ETF)"
y0.symbol="XLP"
y0<-rmat0.0.weekly.sectoretfs.period0$XLP
```

The following R code can be copied above to change the choice of sector etf

```
if (FALSE) {
# 1.1 Change to XLK Technology:
```

```

# Define y0 equal to weekly returns of sector etf XLK
y0.name="XLK (Technology ETF)"
y0.symbol="XLK"
y0<-rmat0.O.weekly.sectoretfs.period0$XLK

# 1.2 Change to XLI Industrials:
# Define y0 equal to weekly returns of sector etf XLI
y0.name="XLI (Industrials ETF)"
y0.symbol="XLI"
y0<-rmat0.O.weekly.sectoretfs.period0$XLI

}

```

2.3 Define df0 equal to data frame with y0 and index etfs

```
df0<-data.frame(cbind(y0,rmat0.O.weekly.indexetfs.period0))
```

2.4 Print out summary statistics for weekly returns

2.4.1 Annual means, volatilities

```

apply(df0,2,summary)

##           y0          SPY          MDY          QQQ          DIA
## Min.   -0.116194529 -0.157189170 -0.202701342 -0.119356994 -0.187241821
## 1st Qu. -0.007267913 -0.007898118 -0.010483530 -0.010700447 -0.007719788
## Median   0.003452617  0.004176716  0.003299196  0.004865223  0.003425232
## Mean     0.001926583  0.002634949  0.002100222  0.003474893  0.002332767
## 3rd Qu.  0.012286899  0.014691387  0.015611677  0.019312326  0.013328769
## Max.    0.065588245  0.114145893  0.172891253  0.091102251  0.119922619

mean.annual<-apply(df0,2,mean)*52
vol.annual<-sqrt(52*apply(df0,2,var))
data.frame(mean=mean.annual, vol=vol.annual)

##           mean         vol
## y0  0.1001823 0.1370785
## SPY 0.1370173 0.1607707
## MDY 0.1092116 0.1957695
## QQQ 0.1806944 0.1893826
## DIA 0.1213039 0.1628826

```

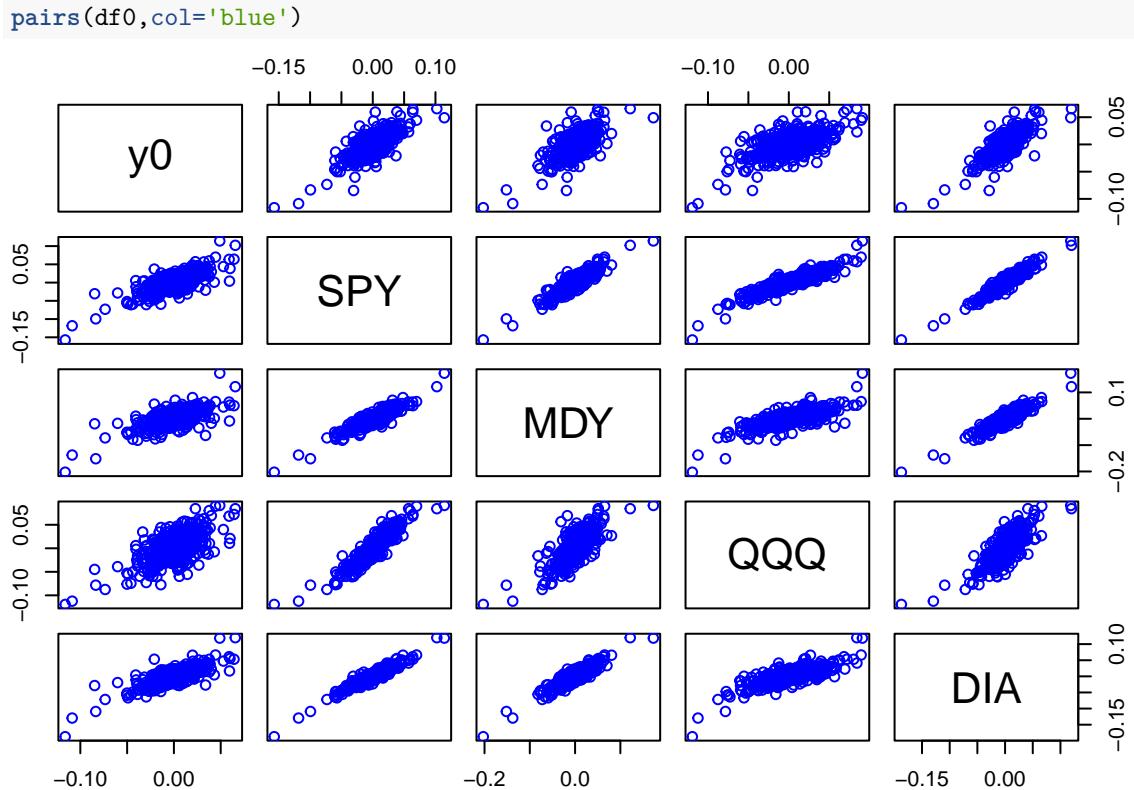
2.4.2 Pairwise correlations and pairs plots

```

cor(df0)

##           y0          SPY          MDY          QQQ          DIA
## y0  1.0000000 0.7526003 0.6552977 0.6243527 0.7768283
## SPY 0.7526003 1.0000000 0.9175267 0.9181598 0.9540900
## MDY 0.6552977 0.9175267 1.0000000 0.7689490 0.9092160
## QQQ 0.6243527 0.9181598 0.7689490 1.0000000 0.7977508
## DIA 0.7768283 0.9540900 0.9092160 0.7977508 1.0000000

```



2.5 Compute/plot cumulative returns

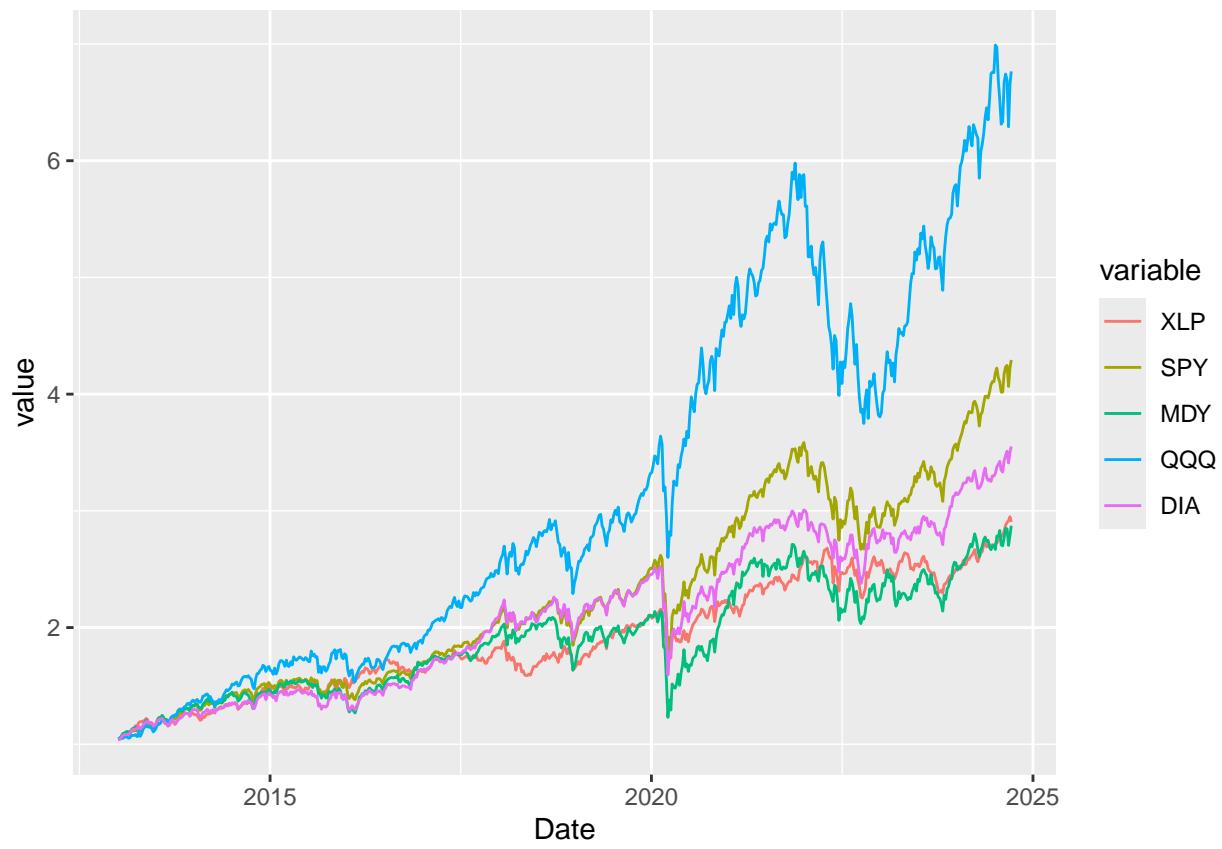
```
# Convert df0 to cumulative return series
# All starting at 1.

df0cumret<-data.frame(exp(apply(log(1+df0),2,cumsum)))
names(df0cumret)

## [1] "y0"   "SPY"  "MDY"  "QQQ"  "DIA"
names(df0cumret)[1]<-y0.symbol
# Rename y0 to actual symbol for plotting
# Add Date variable (after computing cumulative returns)
df0cumret$Date<-time(rmat0.0.weekly.indexetfs.period0)

# Apply melt function from reshape2 package for using ggplot
meltdf0cumret <- melt(df0cumret,id="Date")
#names(meltdf)
#meltdf$variable

gg0<-ggplot(meltdf0cumret,aes(x=Date,y=value,colour=variable,group=variable)) +
  geom_line()
print(gg0)
```



3. Regression of Sector ETF on Index ETFs

3.1 Fit regression of y0 on index etf returns

```
fit<-lm(y0 ~ ., data=df0); fit.summary<-summary(fit)

fit.summary

## 
## Call:
## lm(formula = y0 ~ ., data = df0)
## 
## Residuals:
##       Min     1Q Median     3Q    Max 
## -0.063859 -0.006274  0.000033  0.007041  0.048673 
## 
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept) 0.0002070  0.0004662   0.444   0.657    
## SPY         1.0069903  0.1632673   6.168 1.27e-09 *** 
## MDY        -0.3683498  0.0483853  -7.613 1.03e-13 *** 
## QQQ        -0.3142920  0.0640313  -4.908 1.18e-06 *** 
## DIA         0.3995081  0.0908524   4.397 1.29e-05 *** 
## ---        
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 
## 
## Residual standard error: 0.01141 on 607 degrees of freedom
## Multiple R-squared:  0.6418, Adjusted R-squared:  0.6394 
## F-statistic: 271.9 on 4 and 607 DF,  p-value: < 2.2e-16

# Note list components of output from summary()
names(fit.summary)

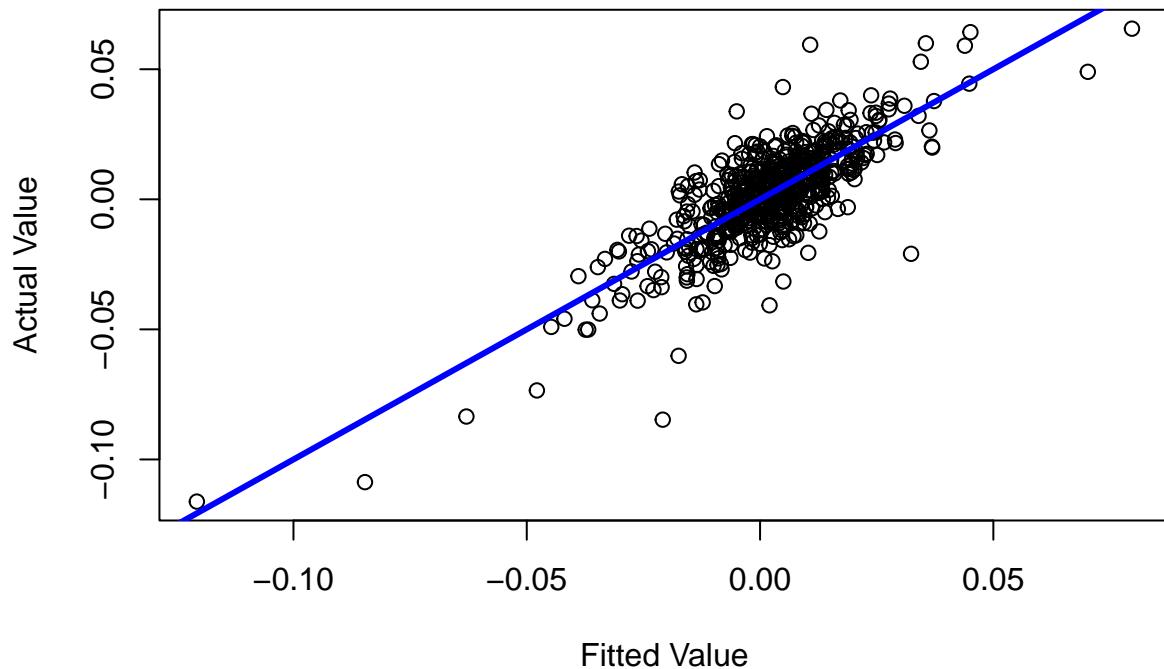
## [1] "call"          "terms"        "residuals"      "coefficients" 
## [5] "aliased"       "sigma"        "df"            "r.squared"      
## [9] "adj.r.squared" "fstatistic"    "cov.unscaled" 

# print method for fit.summary is condensed
```

3.2 Plot Actual versus Fitted Values

```
plot(fit$fitted.values, df0$y0,
      xlab="Fitted Value",
      ylab = "Actual Value",
      main=paste(c("Regression Model","\n", y0.name,
      "\nR-Squared =", as.character(round(fit.summary$r.squared,digits=2))),,
      collapse="\n"))
abline(a=0,b=1,col="blue",lwd=3)
```

Regression Model
XLP (Consumer Staples ETF)
R-Squared =0.64

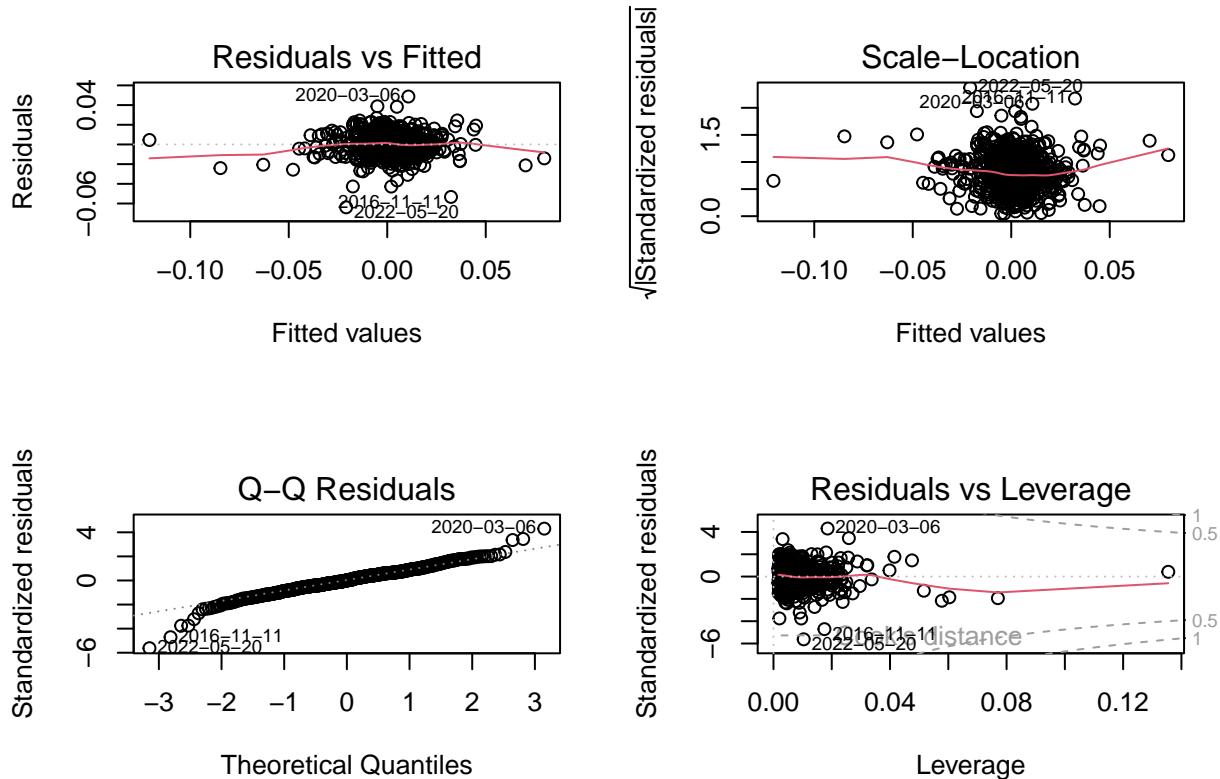


4. Use R package car to analyse residuals/influence

```
library(car)
```

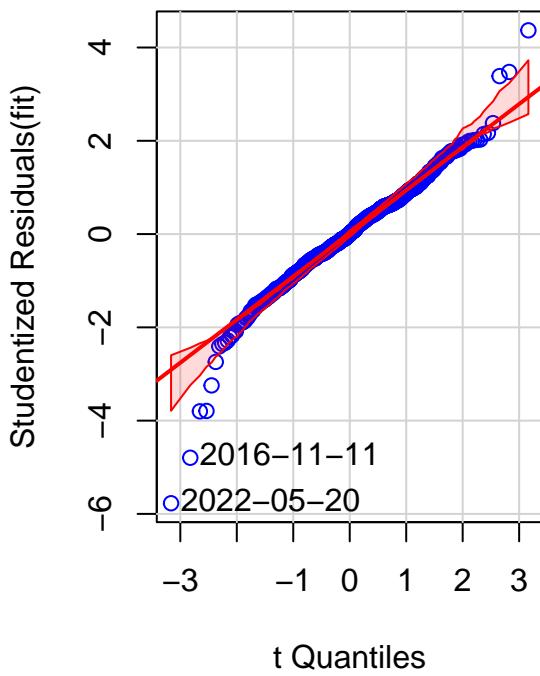
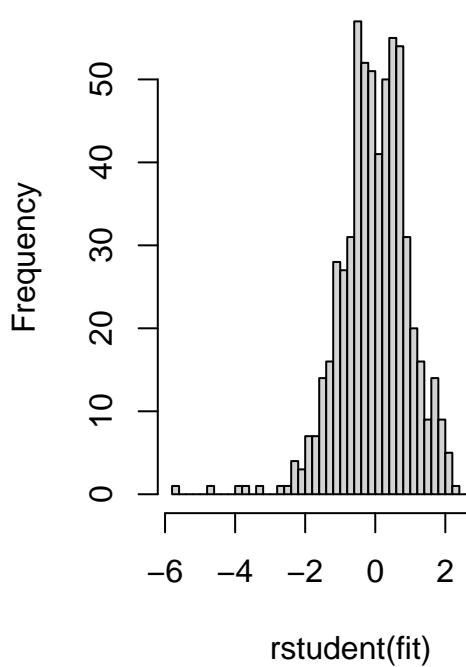
4.1 plot() method gives 4-panel plots of residuals and Leverage

```
oldpar=par(no.readonly=TRUE)
layout(matrix(c(1,2,3,4),2,2)) # optional 4 graphs/page
plot(fit) ; par(oldpar,no.readonly=TRUE)
```



```
## 4.2 studres() Studentized residuals and qqPlot() -
#      plot histogram and t-dist QQ plot
par(mfcol=c(1,2));
hist(rstudent(fit),breaks=50);
qqPlot(fit,col="blue",col.lines="red")
```

Histogram of rstudent(fit)

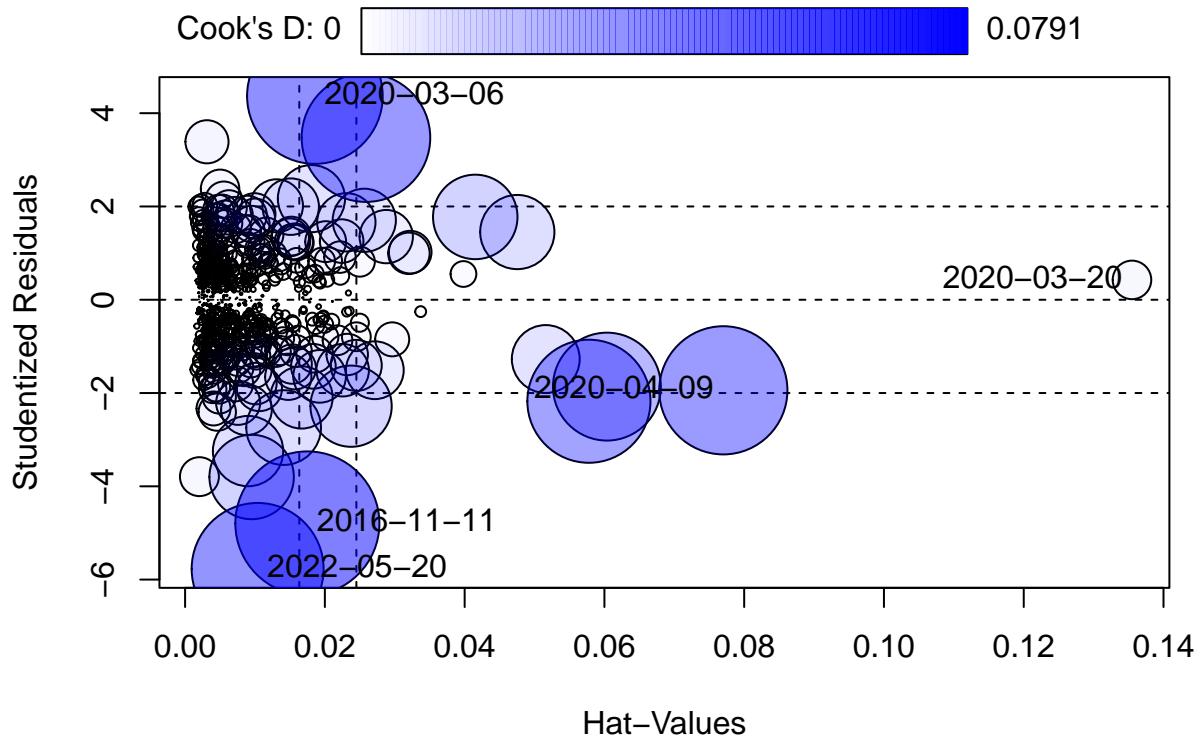


```
## 2016-11-11 2022-05-20  
##          202          490
```

```
#
```

4.3 influencePlot() output

```
par(mfcol=c(1,1))  
  
influencePlot(lm(y0~., data=df0))
```

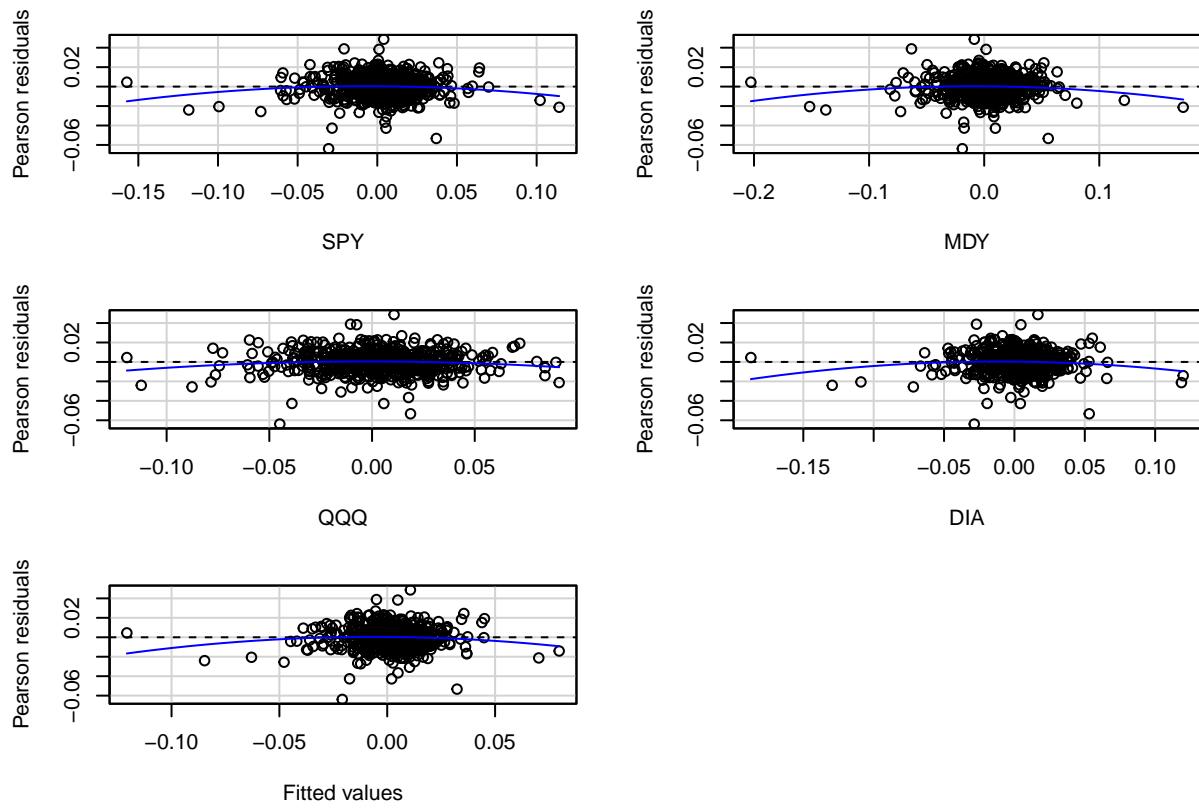


```
##           StudRes      Hat     CookD
## 2016-11-11 -4.7964026 0.01749576 0.079066753
## 2020-03-06  4.3678362 0.01857384 0.070123119
## 2020-03-20  0.4249459 0.13549138 0.005667955
## 2020-04-09 -1.9418203 0.07701466 0.062639579
## 2022-05-20 -5.7714057 0.01037971 0.066341866
```

4.4 residualPlots()

Visually test for curvature in linear regression terms.

```
residualPlots(lm(y0~., data=df0), tests=FALSE)
```



Print out test results

```
# Test results (for curvature in plots)
residualPlots(lm(y0~., data=df0), tests=TRUE, plot=FALSE)
```

```
##           Test stat Pr(>|Test stat|) 
## SPY        -2.2498  0.02482 *  
## MDY        -2.2815  0.02287 *  
## QQQ        -2.0270  0.04310 *  
## DIA        -2.3455  0.01932 *  
## Tukey test -2.2004  0.02778 *  
## ---                                 
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

5. Compare Regression Using Z-scores of Predictors

5.1 Reproduce Linear Regression on Predictors

```
# df0: data frame with security and indexes
#   y0
#   Regress y0 on indexes

# Add arguments to get x matrix and y
fit<-lm(y0 ~., data=df0,x=TRUE,y=TRUE);
fit.summary<-summary(fit)
fit.summary

##
## Call:
## lm(formula = y0 ~ ., data = df0, x = TRUE, y = TRUE)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.063859 -0.006274  0.000033  0.007041  0.048673
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 0.0002070  0.0004662   0.444   0.657
## SPY         1.0069903  0.1632673   6.168 1.27e-09 ***
## MDY        -0.3683498  0.0483853  -7.613 1.03e-13 ***
## QQQ        -0.3142920  0.0640313  -4.908 1.18e-06 ***
## DIA         0.3995081  0.0908524   4.397 1.29e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.01141 on 607 degrees of freedom
## Multiple R-squared:  0.6418, Adjusted R-squared:  0.6394
## F-statistic: 271.9 on 4 and 607 DF,  p-value: < 2.2e-16
fit$coefficients

## (Intercept)      SPY      MDY      QQQ      DIA
## 0.0002070038 1.0069902798 -0.3683498406 -0.3142920210  0.3995081430

# Output argument x has matrix of predictor variables
head(fit$x)

##
## (Intercept)      SPY      MDY      QQQ      DIA
## 2013-01-04      1 0.044280965 0.049746267 0.043715341 3.793252e-02
## 2013-01-11      1 0.004771257 0.001196815 0.009410744 4.910796e-03
## 2013-01-18      1 0.008530791 0.015020782 -0.002828828 1.118619e-02
## 2013-01-25      1 0.012861058 0.022542269 -0.001044416 1.841951e-02
## 2013-02-01      1 0.006567324 0.004198834 0.009802910 8.047782e-03
## 2013-02-08      1 0.003695823 0.007057868 0.004865621 -7.152686e-05
#
```

5.2 Repeat regression with Z-scores of index etfs

```
# The R function scale outputs z scores of data vectors
df0.zscore<-data.frame(cbind(y0,apply(rmat0.0.weekly.indexetfs.period0,2,scale)))
fit<-lm(y0 ~ ., data=df0.zscore, x=TRUE,y=TRUE); fit.summary<-summary(fit)
fit.summary

##
## Call:
## lm(formula = y0 ~ ., data = df0.zscore, x = TRUE, y = TRUE)
##
## Residuals:
##       Min     1Q Median     3Q    Max 
## -0.063859 -0.006274  0.000033  0.007041  0.048673 
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept)  0.0019266  0.0004614   4.175 3.41e-05 ***
## SPY          0.0224507  0.0036400   6.168 1.27e-09 ***
## MDY         -0.0100001  0.0013136  -7.613 1.03e-13 ***
## QQQ         -0.0082541  0.0016816  -4.908 1.18e-06 ***
## DIA          0.0090240  0.0020522   4.397 1.29e-05 ***
## ---        
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.01141 on 607 degrees of freedom
## Multiple R-squared:  0.6418, Adjusted R-squared:  0.6394 
## F-statistic: 271.9 on 4 and 607 DF,  p-value: < 2.2e-16
```

Note the two regressions have identical results for t-stats, R-Square.

6. PCA of Explanatory Variables

6.1 Extract scaled x matrix (except intercept)

```
x=fit$x[,-1]
cor(x)

##          SPY        MDY        QQQ        DIA
## SPY 1.0000000 0.9175267 0.9181598 0.9540900
## MDY 0.9175267 1.0000000 0.7689490 0.9092160
## QQQ 0.9181598 0.7689490 1.0000000 0.7977508
## DIA 0.9540900 0.9092160 0.7977508 1.0000000

var(x)

##          SPY        MDY        QQQ        DIA
## SPY 1.0000000 0.9175267 0.9181598 0.9540900
## MDY 0.9175267 1.0000000 0.7689490 0.9092160
## QQQ 0.9181598 0.7689490 1.0000000 0.7977508
## DIA 0.9540900 0.9092160 0.7977508 1.0000000
```

6.2 Compute PCA of x

The R function princomp() is used below. Note that other functions performing PCA are available, e.g. prcomp().

```
#library(ggplot2)
#library(ggfortify)
x.princomp<-princomp(x)
summary(x.princomp)

## Importance of components:
##                 Comp.1    Comp.2    Comp.3    Comp.4
## Standard deviation   1.9053417 0.51045028 0.30208710 0.106400323
## Proportion of Variance 0.9090671 0.06524648 0.02285149 0.002834889
## Cumulative Proportion 0.9090671 0.97431362 0.99716511 1.000000000
```

Note available output from princomp() is a named list:

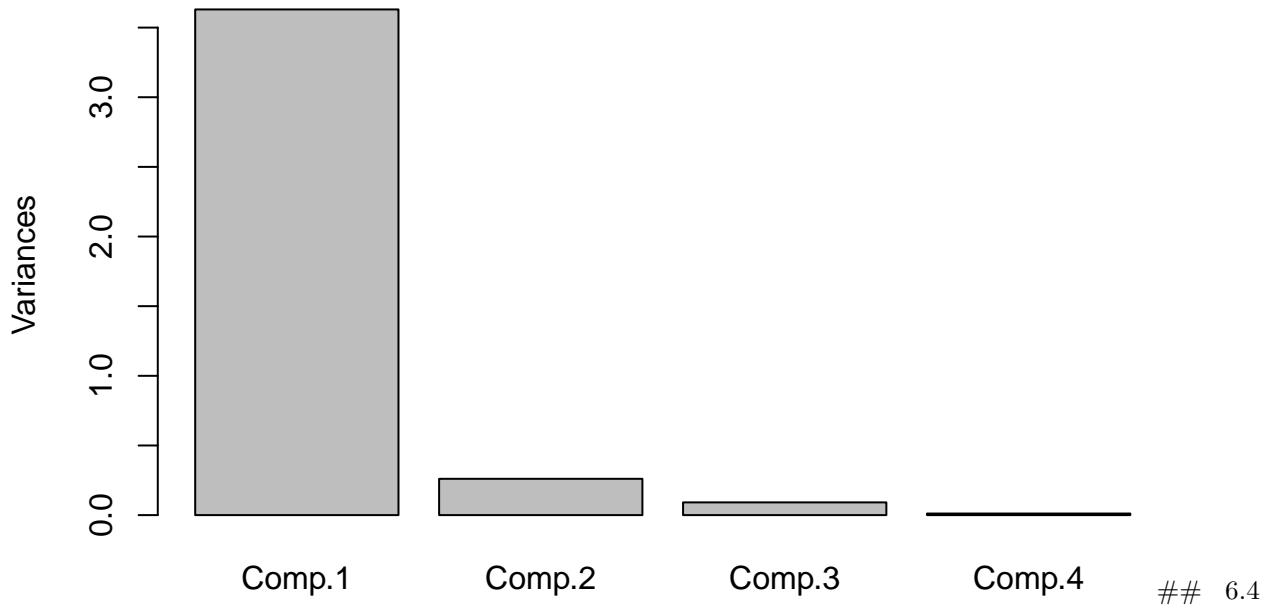
```
names(x.princomp)

## [1] "sdev"      "loadings"   "center"     "scale"      "n.obs"      "scores"     "call"
```

6.3 The Screeplot: Barplot of Principal Component Variances

```
screeplot(x.princomp)
```

x.princomp



Loadings of Principal Components

Printing out the loadings matrix (all values and with cutoff):

```
# Print out loadings
x.princomp$loadings
```



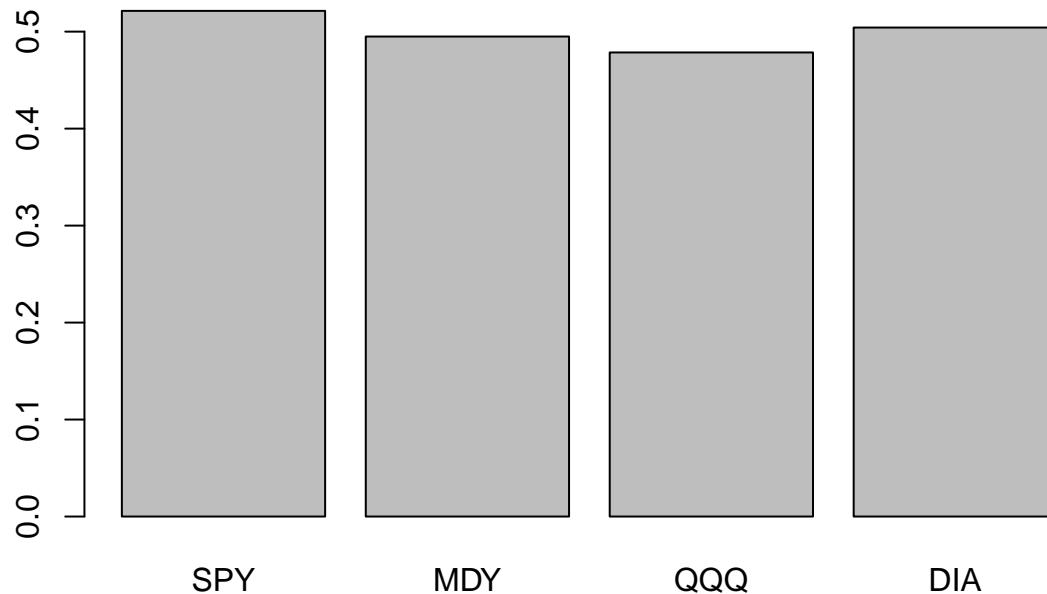
```
##  
## Loadings:  
##      Comp.1 Comp.2 Comp.3 Comp.4  
## SPY    0.521     0.145   0.837  
## MDY    0.495 -0.497  -0.699 -0.140  
## QQQ    0.479    0.792  -0.155 -0.346  
## DIA    0.504 -0.346   0.683 -0.400  
##  
##           Comp.1 Comp.2 Comp.3 Comp.4  
## SS loadings     1.00    1.00    1.00    1.00  
## Proportion Var  0.25    0.25    0.25    0.25  
## Cumulative Var 0.25    0.50    0.75    1.00  
# Get all values with no cutoffs
print(x.princomp$loadings,cutoff=0.)
```



```
##  
## Loadings:  
##      Comp.1 Comp.2 Comp.3 Comp.4  
## SPY    0.521   0.079   0.145   0.837  
## MDY    0.495 -0.497  -0.699 -0.140  
## QQQ    0.479   0.792  -0.155 -0.346  
## DIA    0.504 -0.346   0.683 -0.400  
##  
##           Comp.1 Comp.2 Comp.3 Comp.4  
## SS loadings     1.00    1.00    1.00    1.00  
## Proportion Var  0.25    0.25    0.25    0.25
```

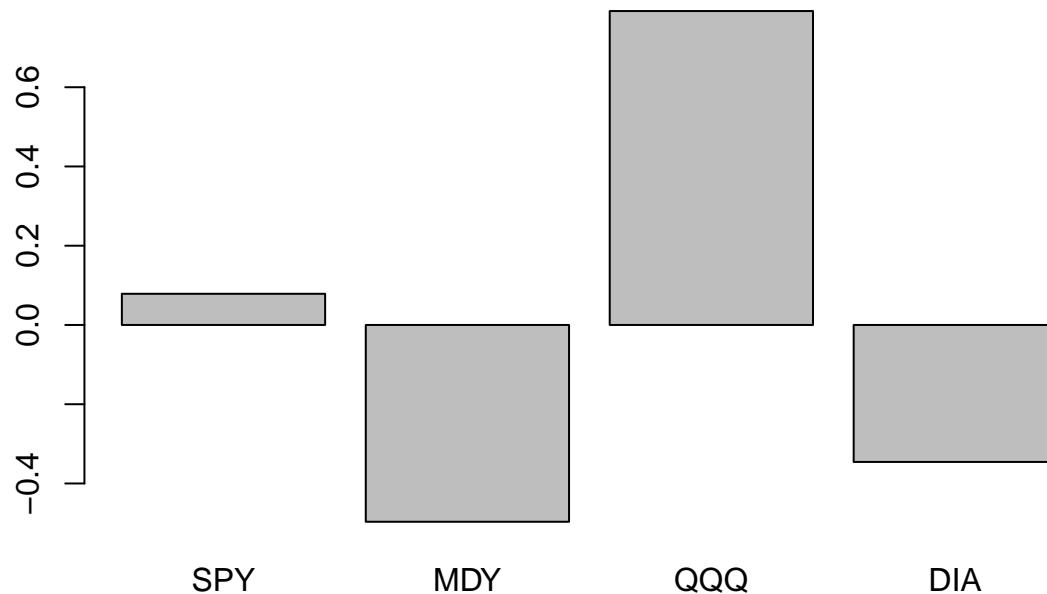
```
## Cumulative Var 0.25 0.50 0.75 1.00  
Plotting the loadings of each Principal Component variable  
# Plot loadings of PC1-PC4  
  
barplot(x.princomp$loadings[, 1], main="PC1 Loadings")
```

PC1 Loadings



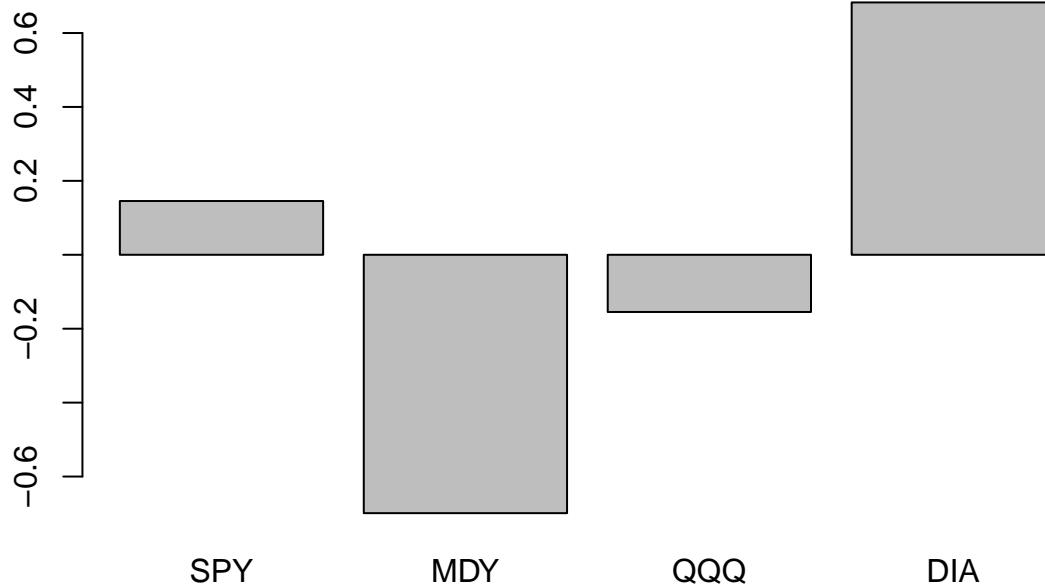
```
barplot(x.princomp$loadings[, 2], main="PC2 Loadings")
```

PC2 Loadings



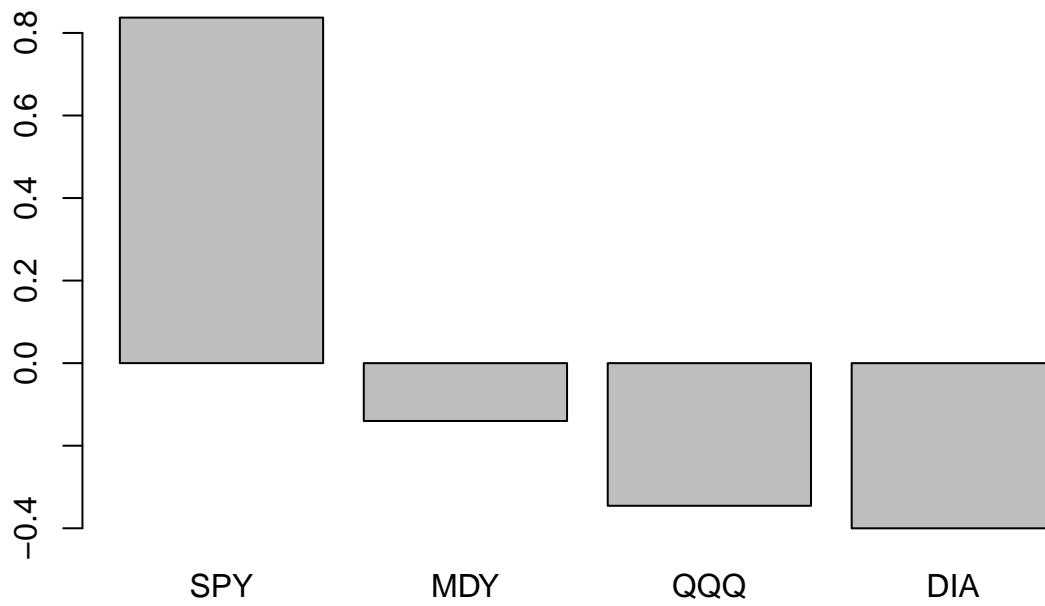
```
barplot(x.princomp$loadings[, 3], main="PC3 Loadings")
```

PC3 Loadings



```
barplot(x.princomp$loadings[,4], main="PC4 Loadings")
```

PC4 Loadings



7. Regressions on PC variables

7.1 Univariate regressions

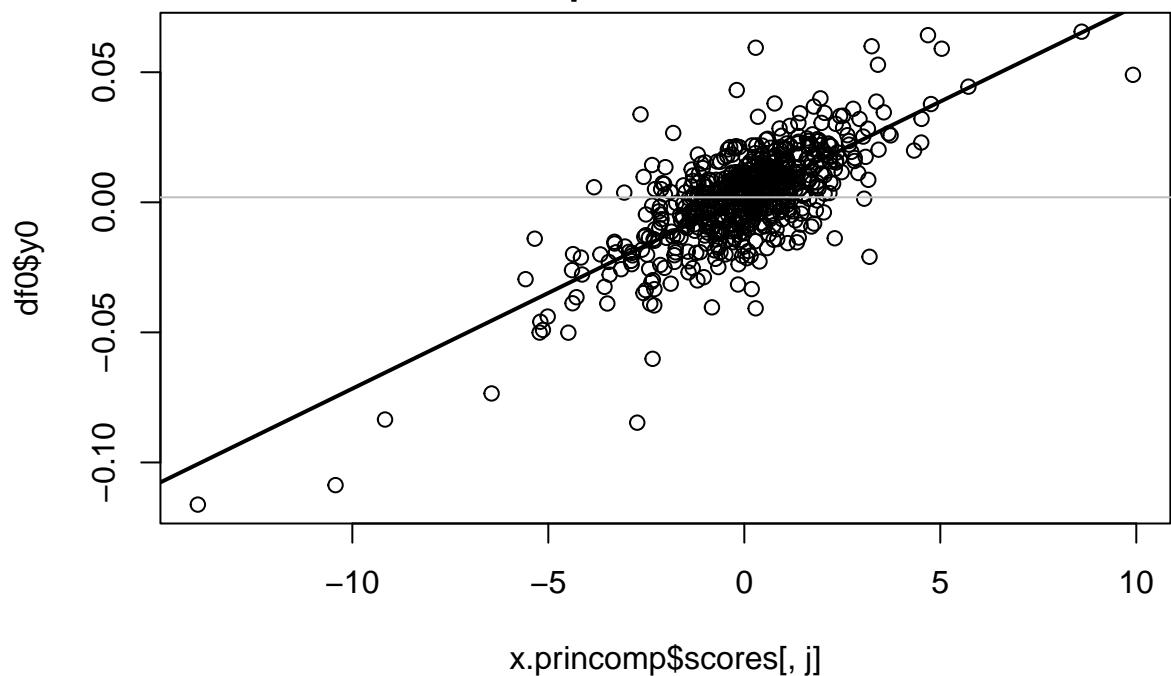
```
# Loop over all PC Variables
for (j in 1:NCOL(x.princomp$scores)){
  lm.j<-lm(df0$y0 ~ x.princomp$scores[,j])
  lm.j.summary<-summary(lm.j)

  # Use lsfit() to compute p-value s
  lsfit.j<-lsfit(x=x.princomp$scores[,j], y=y0)
  lsfit.j.print<-ls.print(lsfit.j)

  plot(x.princomp$scores[,j], df0$y0,main=c(" \n \n "))
  abline(lm.j, col=j,lwd=2)
  abline(h=mean(df0$y0),col='gray')
  title(main=paste(c("y0 ~ pc",as.character(j),
    "\n R-Squared = ", as.character(round(lm.j.summary$r.squared,digits=3)),
    "\n p-value = ", as.character(lsfit.j.print$summary[1,6])),
    collapse=""))
}

## Residual Standard Error=0.0128
## R-Square=0.5445
## F-statistic (df=1, 610)=729.3205
## p-value=0
##
##           Estimate Std. Err t-value Pr(>|t|)
## Intercept   0.0019   5e-04  3.7121   2e-04
## X          0.0074   3e-04 27.0059   0e+00
```

$y0 \sim pc1$
R-Squared = 0.545
p-value = 0

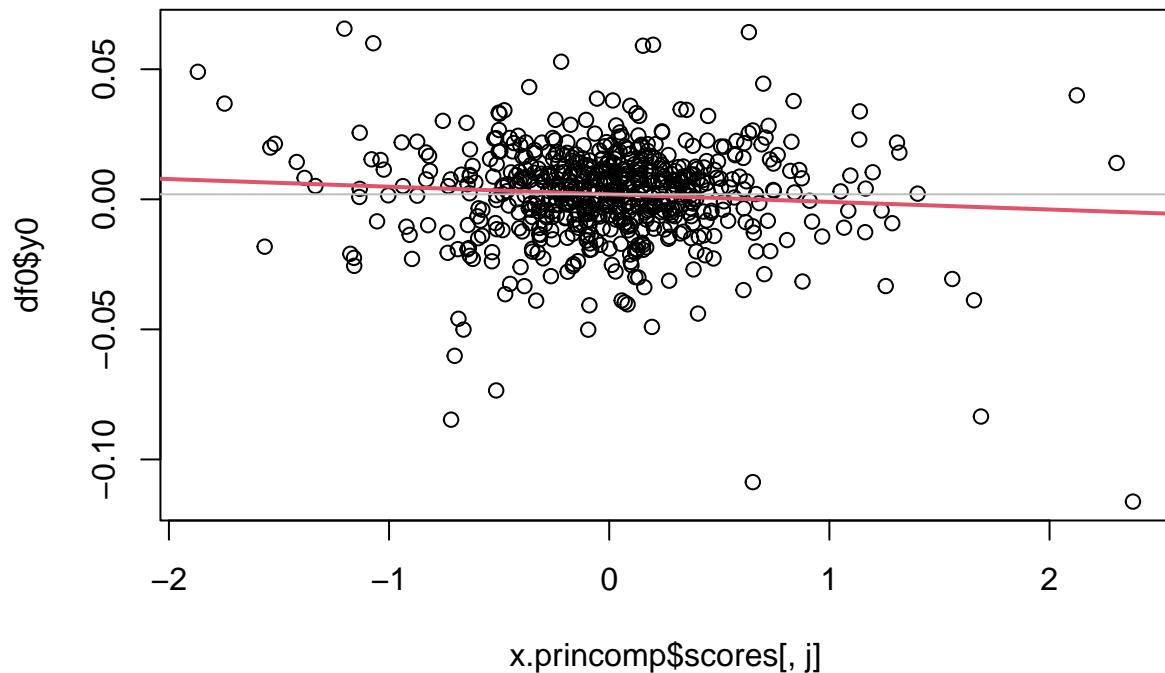


```

## Residual Standard Error=0.019
## R-Square=0.0062
## F-statistic (df=1, 610)=3.797
## p-value=0.0518
##
##             Estimate Std. Err t-value Pr(>|t|)
## Intercept    0.0019   0.0008  2.5130   0.0122
## X          -0.0029   0.0015 -1.9486   0.0518

```

$y0 \sim pc2$
R-Squared = 0.006
p-value = 0.0518

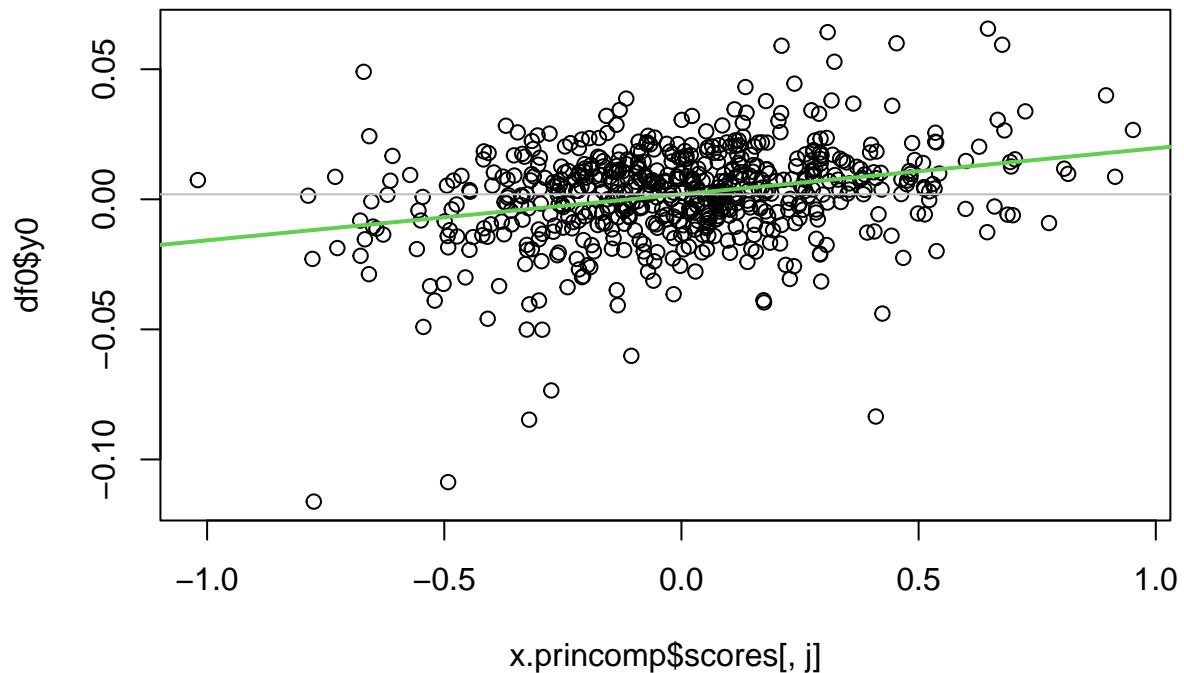


```

## Residual Standard Error=0.0183
## R-Square=0.0792
## F-statistic (df=1, 610)=52.4679
## p-value=0
##
##             Estimate Std. Err t-value Pr(>|t|)
## Intercept    0.0019   0.0007  2.6107  0.0093
## X            0.0177   0.0024  7.2435  0.0000

```

$y0 \sim pc3$
R-Squared = 0.079
p-value = 0

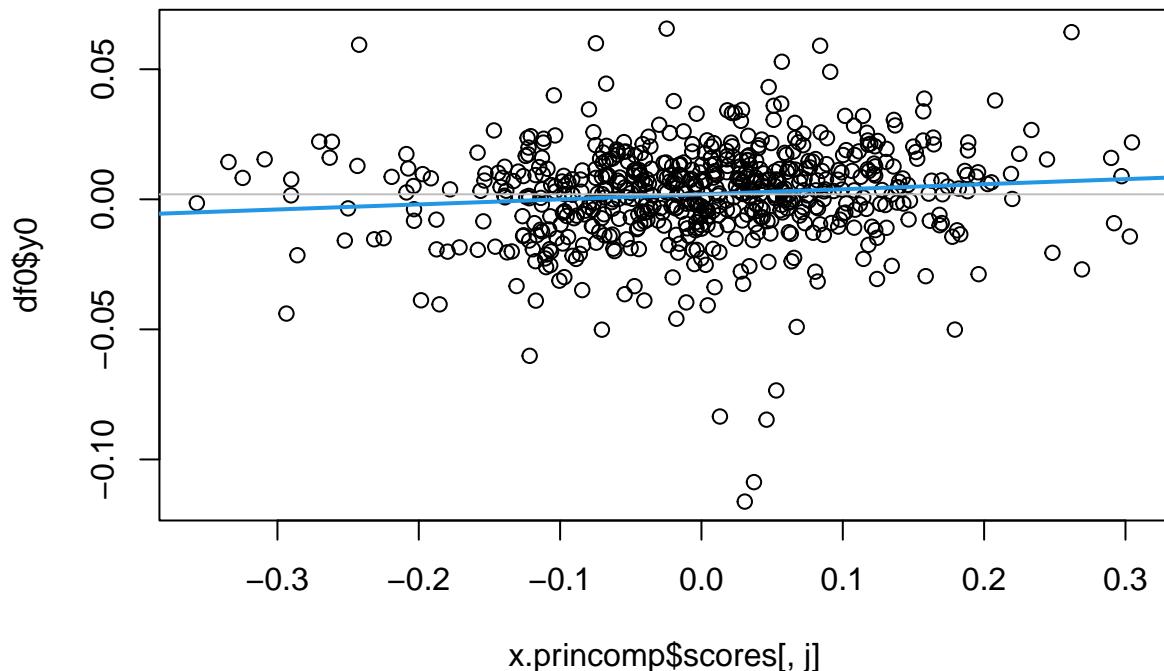


```

## Residual Standard Error=0.0189
## R-Square=0.0119
## F-statistic (df=1, 610)=7.319
## p-value=0.007
##
##             Estimate Std. Err t-value Pr(>|t|)
## Intercept    0.0019   0.0008  2.5202   0.012
## X            0.0194   0.0072  2.7054   0.007

```

y0 ~ pc4
R-Squared = 0.012
p-value = 0.007



7.2 Regression model on all PC variables

```

pcregfit<-lm(df0$y0 ~ x.princomp$scores)
summary(pcregfit)

## 
## Call:
## lm(formula = df0$y0 ~ x.princomp$scores)
## 
## Residuals:
##      Min       1Q   Median       3Q      Max 
## -0.063859 -0.006274  0.000033  0.007041  0.048673 
## 
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept) 0.0019266  0.0004614   4.175 3.41e-05 ***
## x.princomp$scoresComp.1 0.0073563  0.0002422  30.377 < 2e-16 ***
## x.princomp$scoresComp.2 -0.0029266  0.0009039  -3.238 0.00127 ** 
## x.princomp$scoresComp.3  0.0176948  0.0015274  11.585 < 2e-16 ***
## x.princomp$scoresComp.4  0.0194375  0.0043366   4.482 8.83e-06 ***
## ---      
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## 
## Residual standard error: 0.01141 on 607 degrees of freedom
## Multiple R-squared:  0.6418, Adjusted R-squared:  0.6394 
## F-statistic: 271.9 on 4 and 607 DF,  p-value: < 2.2e-16

```

```
pcregfit.summary<-summary(pcregfit)
```

7.3 Use PCA regression to recompute regression parameters

```
#      on original variables
pcbetavec<-as.numeric(pcregfit$coefficients[-1])
x.princomp$loadings

##
## Loadings:
##      Comp.1 Comp.2 Comp.3 Comp.4
## SPY    0.521     0.145   0.837
## MDY    0.495   -0.497   -0.699  -0.140
## QQQ    0.479     0.792   -0.155  -0.346
## DIA    0.504   -0.346    0.683  -0.400
##
##      Comp.1 Comp.2 Comp.3 Comp.4
## SS loadings     1.00    1.00    1.00    1.00
## Proportion Var  0.25    0.25    0.25    0.25
## Cumulative Var 0.25    0.50    0.75    1.00
betaFromPCA<-x.princomp$loadings %*% as.matrix(pcbetavec)
fit$coefficients

##  (Intercept)          SPY          MDY          QQQ          DIA
##  0.001926583  0.022450726 -0.010000091 -0.008254136  0.009023990
```

7.4 Compute regression parameter based on only first 3 pc vars

```
pcbetavec

## [1]  0.007356254 -0.002926641  0.017694763  0.019437483
pcbetavec123<-0*pcbetavec
pcbetavec123[1:3]<-pcbetavec[1:3]

betaFromPC123<-x.princomp$loadings %*% as.matrix(pcbetavec123)
```

7.5 Compute regression parameter based on significant pc vars

The code below computes the regression parameter (original scale) using only the statistically significant pc variables, as determined by those that have abs t stat > sqrt(2).

```
ind.tokeep<-as.numeric(
  abs(pcregfit.summary$coefficients[-1,3])>sqrt(2))

pcbetavectokeep<-pcbetavec*ind.tokeep

betaFromPCTokeep<-x.princomp$loadings %*% as.matrix(pcbetavectokeep)
```

7.6 Create table of betas from these three fits

```
tab.betas<-cbind(betaFromPCA, betaFromPC123, betaFromPCTokeep)
dimnames(tab.betas)[[2]]<-c("LS", "PC123", "PCsignif")
```

```

print(tab.betas)

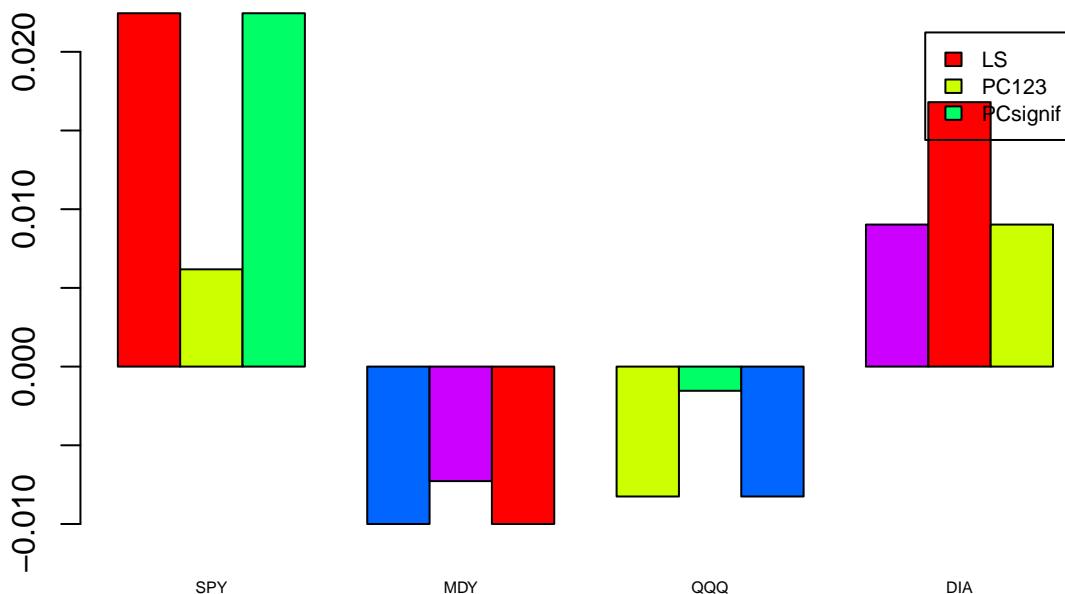
##          LS      PC123     PCs signif
## SPY  0.022450726  0.006178644  0.022450726
## MDY -0.010000091 -0.007275983 -0.010000091
## QQQ -0.008254136 -0.001537180 -0.008254136
## DIA  0.009023990  0.016801960  0.009023990

# 5. Compare coefficients from fits including LASSO and Ridge

barplot(t(as.matrix(tab.betas)), beside=TRUE,
        col=rainbow(5), cex.names=.5,
        legend=TRUE, args.legend=list(cex=.7))
title(main="Regression Coefficients By Method")

```

Regression Coefficients By Method



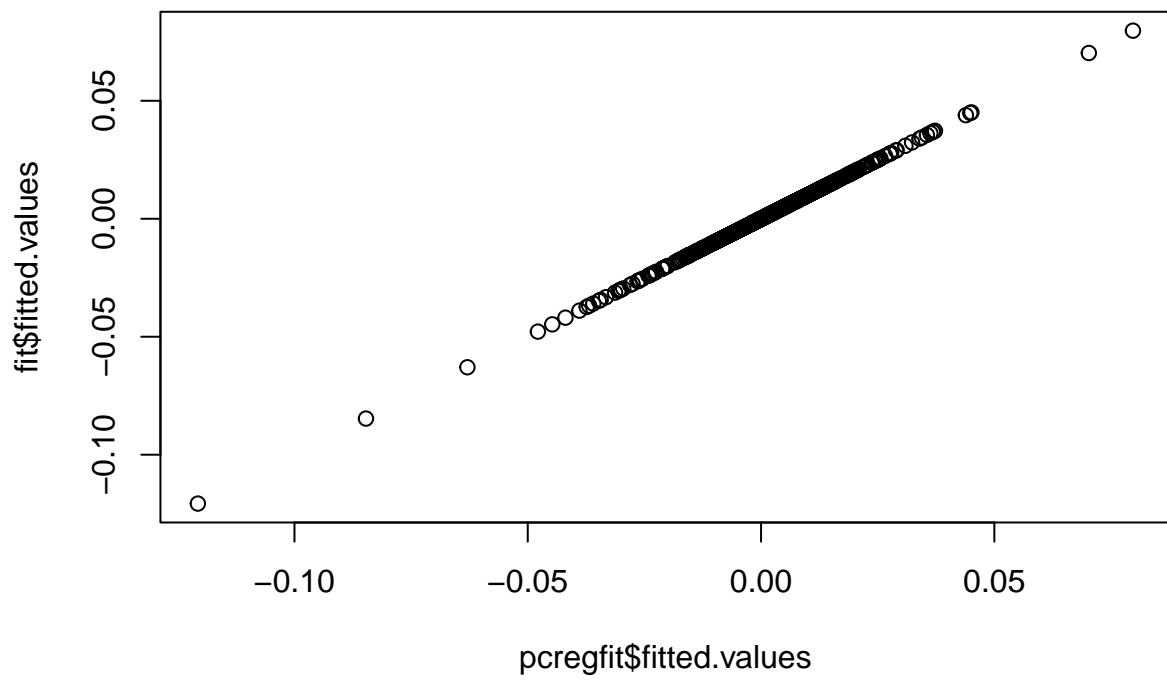
7.7 Demonstrate equality of LS and PCA Regression Fitted Values

```

# pcregfit equals LS fit

par(mfcol=c(1,1))
plot(pcregfit$fitted.values, fit$fitted.values)

```



8. Ridge and Lasso Regression Fits

8.1 Load glmnet package for ridge/lasso regressions

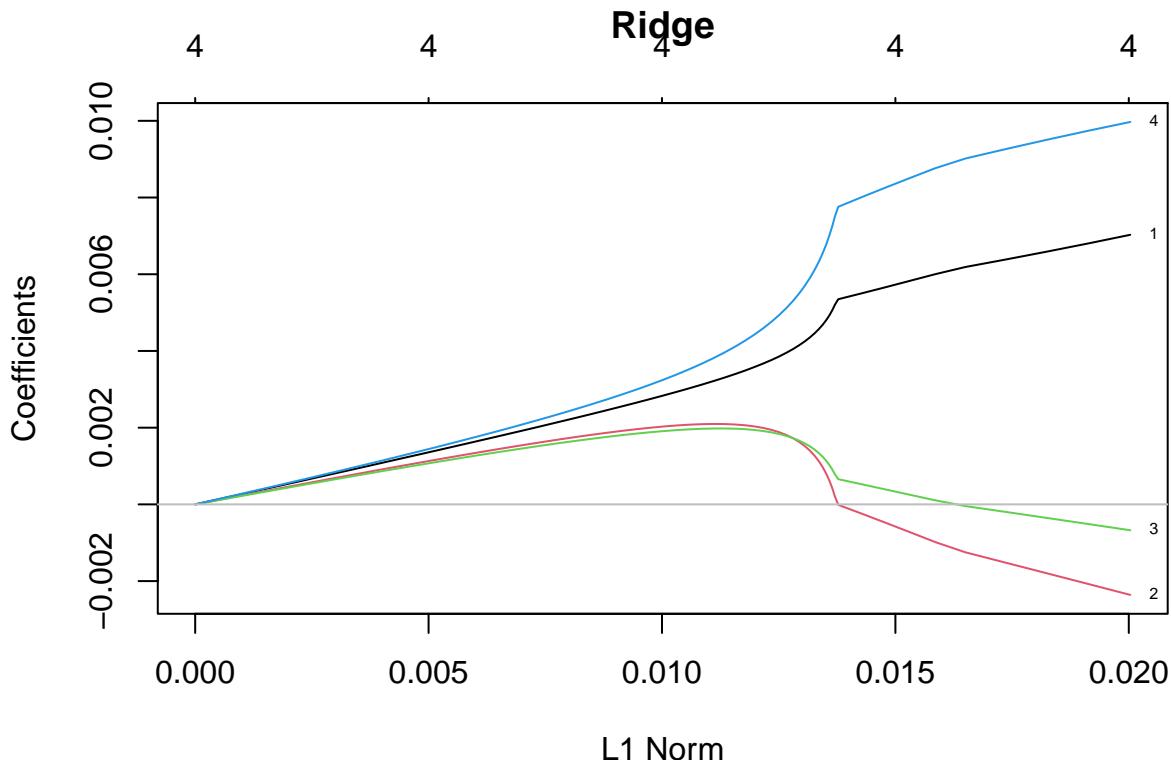
```
library(glmnet)
```

8.2 Define y vector and x matrix for ridge/lasso fits

```
y=fit$y  
x=fit$x[, -1]
```

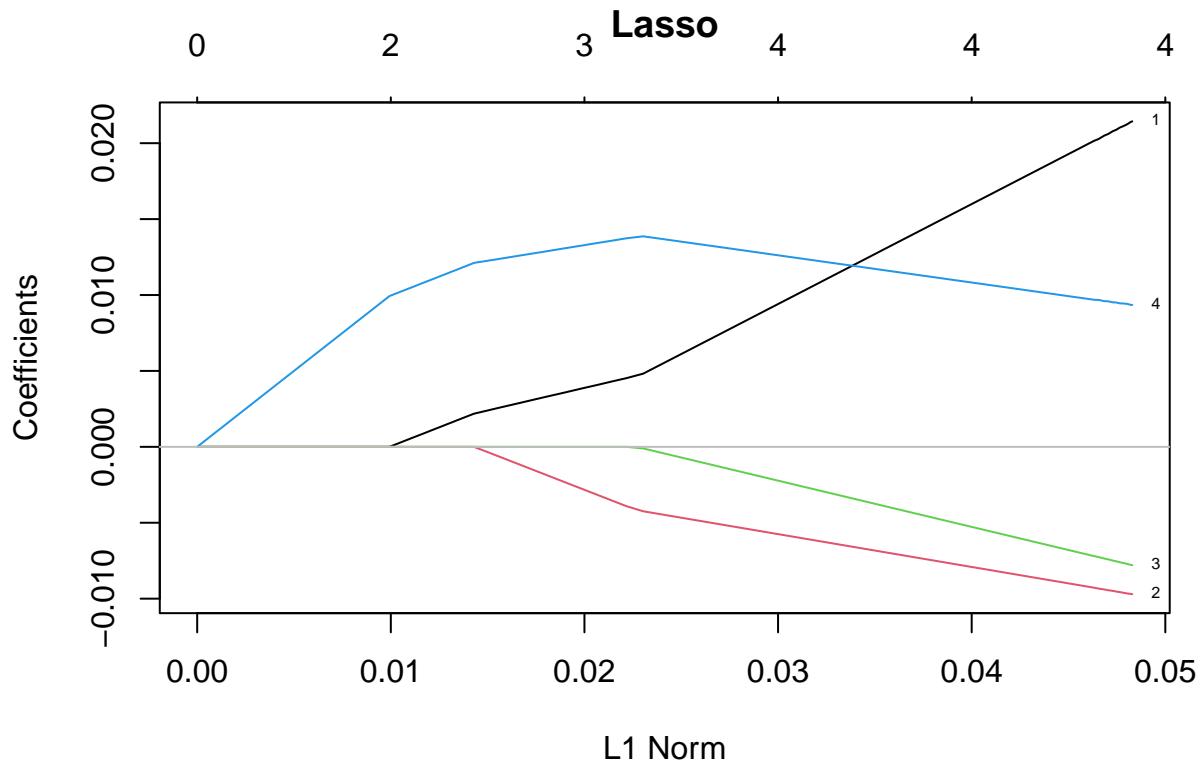
8.3 Plot coefficient trace of ridge regression model

```
y.glmnet.ridge<-glmnet(x,y, alpha=0)  
# alpha=0 for Ridge  
plot(y.glmnet.ridge, label=TRUE, main="Ridge")  
abline(h=0,col='gray')
```



8.4 Plot coefficient trace of lasso regression model —

```
y.glmnet.lasso<-glmnet(x,y, alpha=1)  
# alpha=1 for LASSO  
plot(y.glmnet.lasso, label=TRUE,main="Lasso")  
abline(h=0,col='gray')
```



8.5 Apply Cross Validation to choose ridge parameters

```

lambdas=10^seq(3.,-5, by=-.1)
summary(y.glmnet.ridge)

##          Length Class      Mode
## a0         100  -none-   numeric
## beta       400   dgCMatrix S4
## df         100  -none-   numeric
## dim         2   -none-   numeric
## lambda     100  -none-   numeric
## dev.ratio  100  -none-   numeric
## nulldev     1   -none-   numeric
## npasses     1   -none-   numeric
## jerr        1   -none-   numeric
## offset      1   -none-   logical
## call         4   -none-   call
## nobs        1   -none-   numeric
#
# Cross-validation estimates of prediction error
# ridge case (alpha=0)
y.glmnet.ridge<-glmnet(x,y,alpha=0, lambda=lambdas)
summary(y.glmnet.ridge)

```

```

##          Length Class      Mode
## a0         81   -none-   numeric
## beta       324   dgCMatrix S4
## df         81  -none-   numeric
## dim         2   -none-   numeric

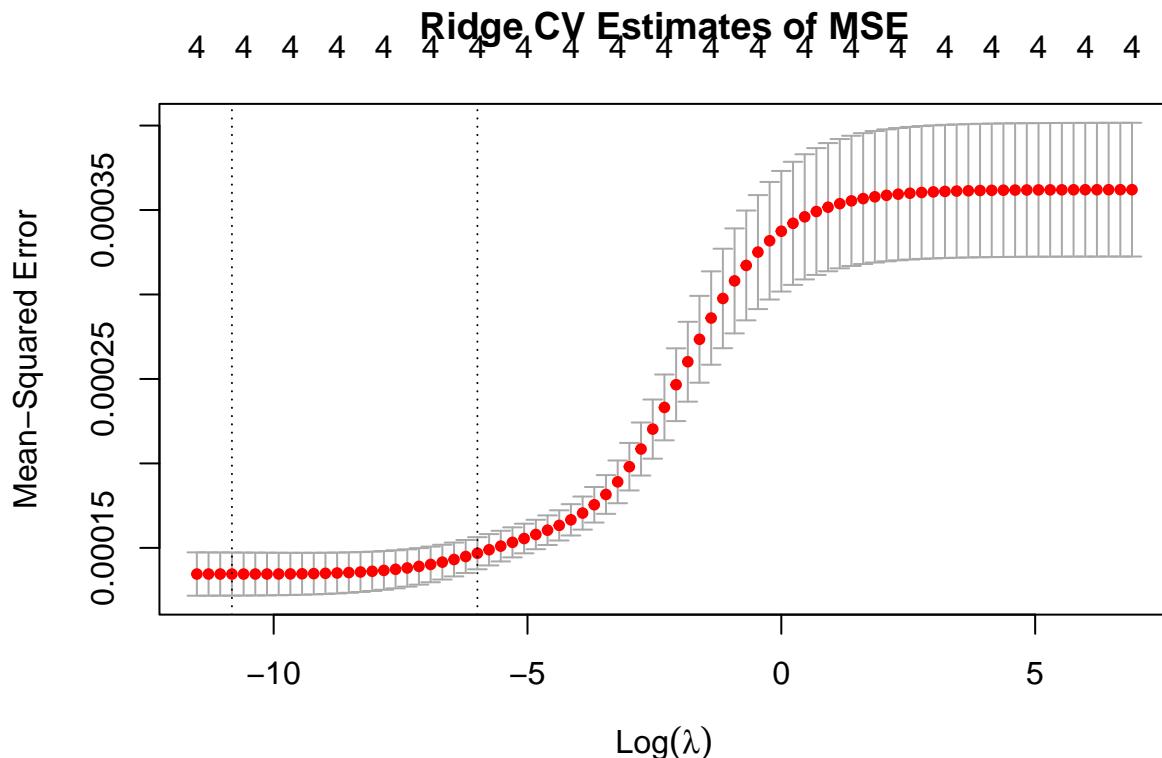
```

```

## lambda      81    -none-    numeric
## dev.ratio   81    -none-    numeric
## nulldev     1    -none-    numeric
## npasses     1    -none-    numeric
## jerr        1    -none-    numeric
## offset       1    -none-    logical
## call         5    -none-    call
## nobs        1    -none-    numeric

y.cv.glmnet.ridge<-cv.glmnet(x,y,alpha=0, lambda=lambdas)
plot(y.cv.glmnet.ridge, main="Ridge CV Estimates of MSE")

```



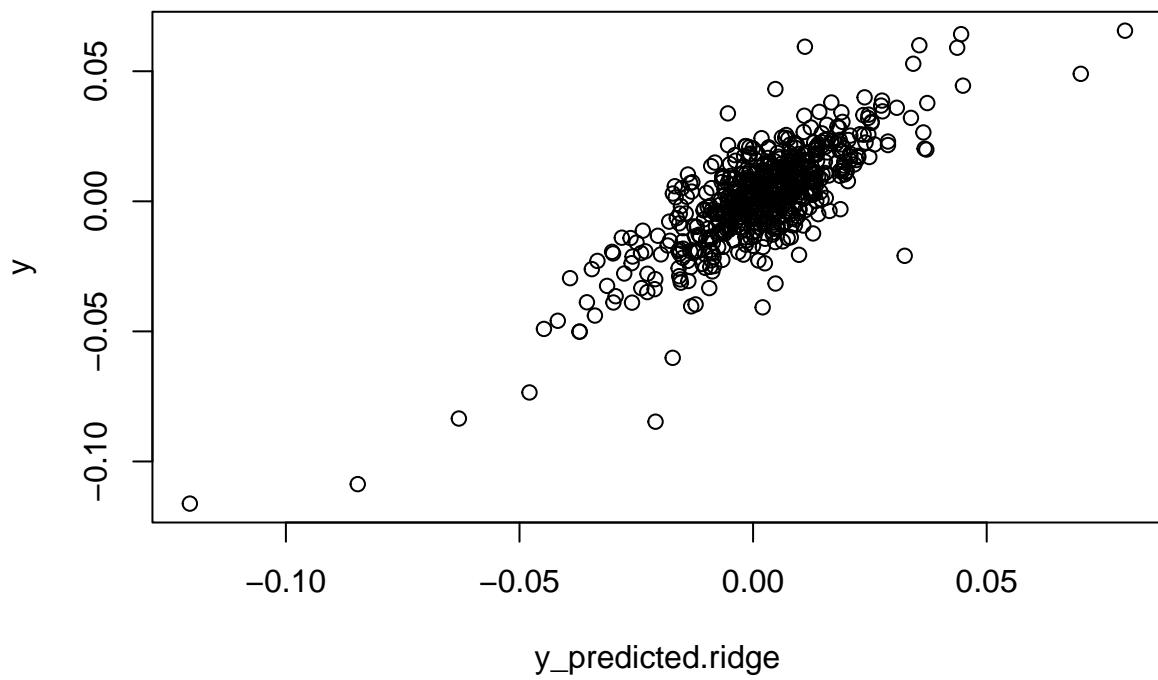
```

optlambda.ridge<-y.cv.glmnet.ridge$lambda.min
glmnet.ridgefit<-y.cv.glmnet.ridge$glmnet.fit
y_predicted.ridge<- predict(glmnet.ridgefit,s=optlambda.ridge,newx=x)

```

Plot Observed vs Fitted for Ridge Regression

```
plot(y_predicted.ridge, y)
```



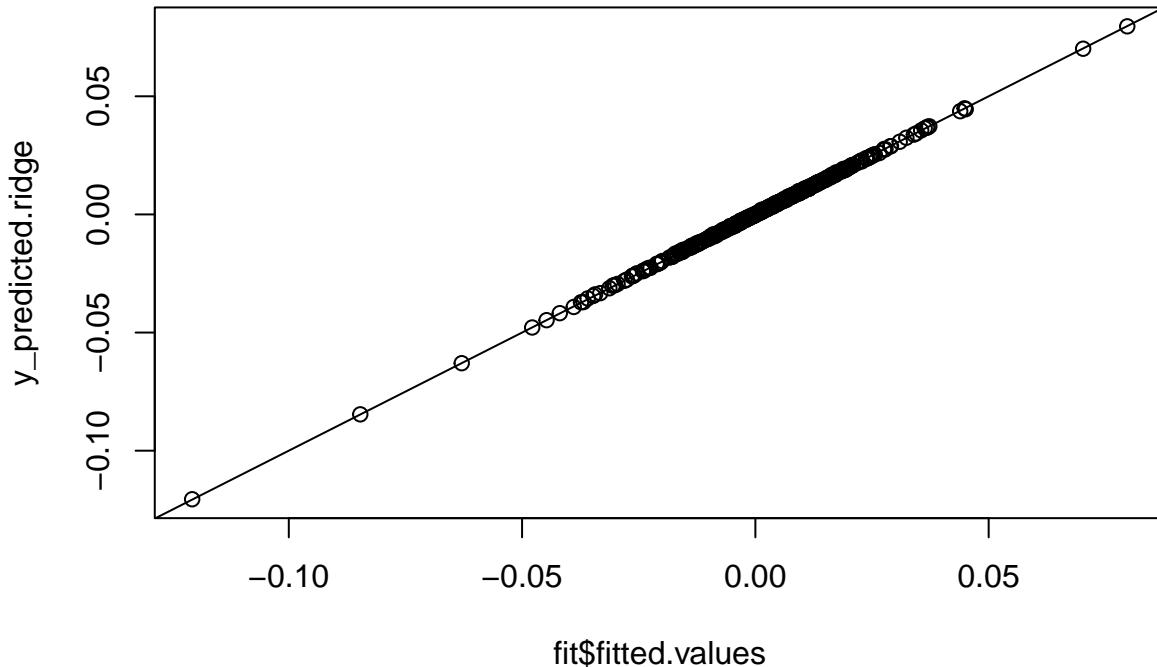
```

cor(y_predicted.ridge,y)^2

##          [,1]
## s1 0.6416741
# Compare ridge fitted to ls fitted
plot(fit$fitted.values, y_predicted.ridge,
      main="Ridge Regression\n(Shrinks to Mean)")
abline(a=0,b=1)

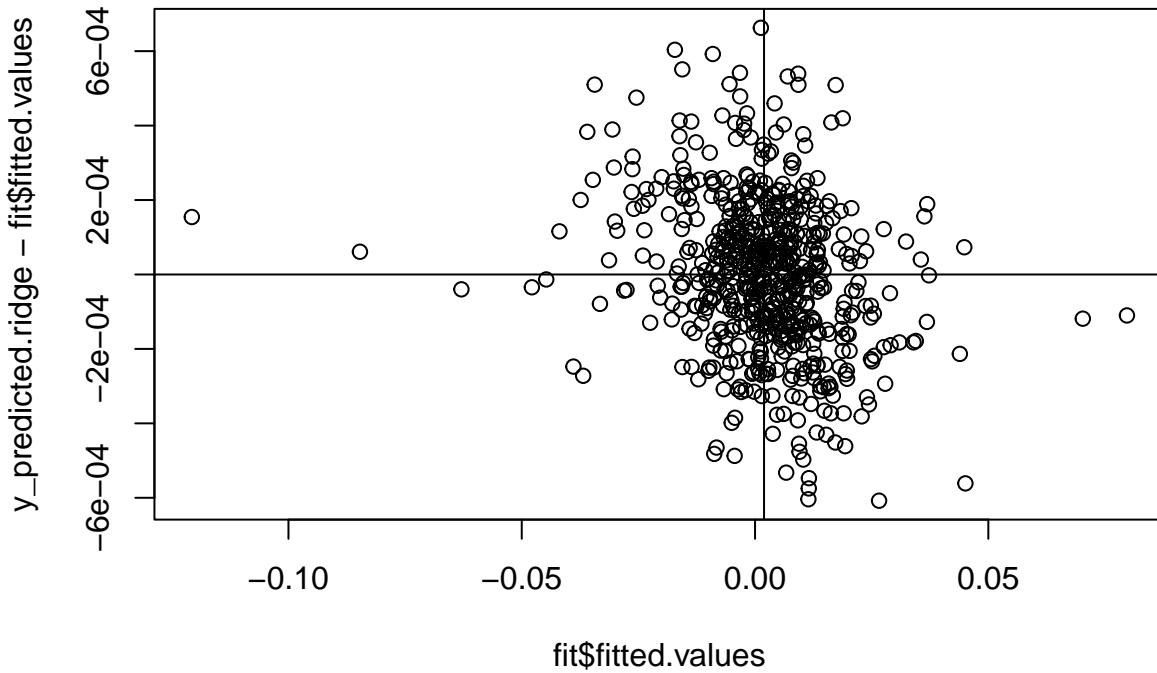
```

Ridge Regression (Shrinks to Mean)



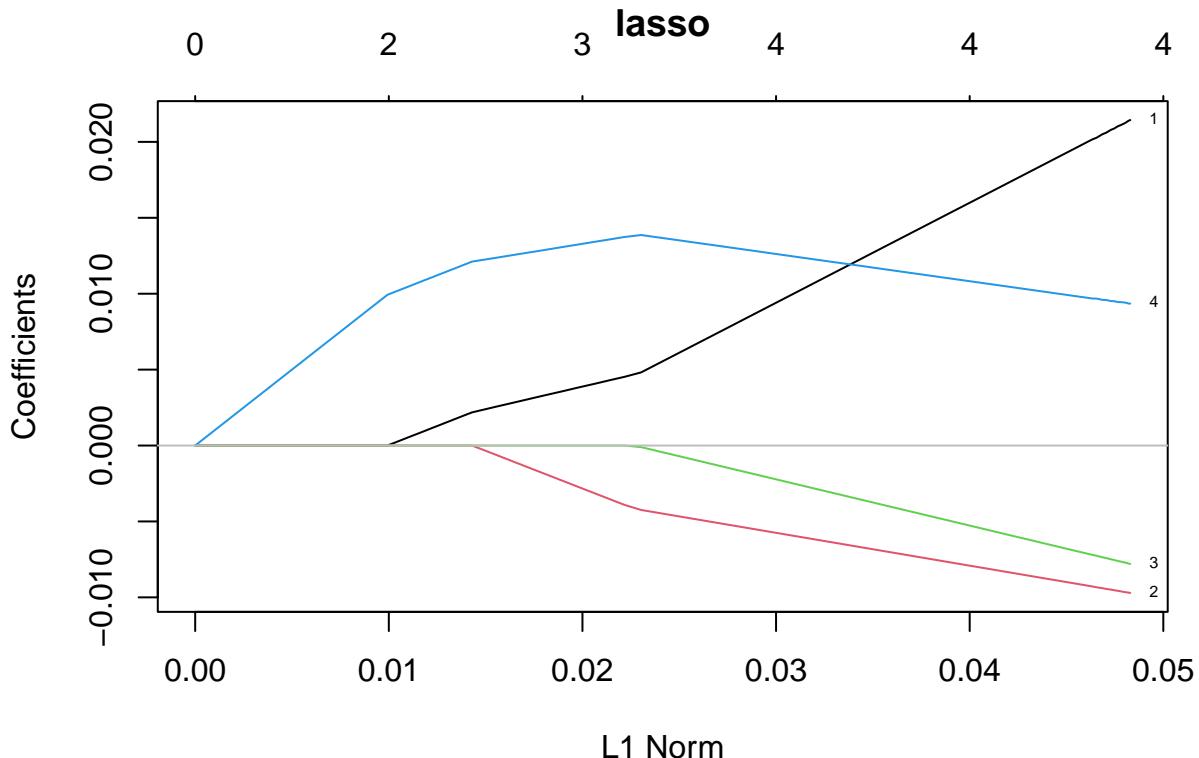
```
plot(fit$fitted.values, y_predicted.ridge - fit$fitted.values,
      main="Ridge Regression \n(Shrinks to Mean)")
abline(h=0)
abline(v=mean(fit$fitted.values))
```

Ridge Regression (Shrinks to Mean)



8.6 Apply Cross Validation to choose lasso parameters

```
y.glmnet.lasso<-glmnet(x,y, alpha=1)
# alpha=1 for lasso
plot(y.glmnet.lasso, label=TRUE, main="lasso")
abline(h=0,col='gray')
```



```
lambdas=10^seq(3.,-5, by=-.1)
summary(y.glmnet.lasso)
```

```
##          Length Class      Mode
## a0            79  -none-   numeric
## beta         316 dgCMatrix S4
## df            79  -none-   numeric
## dim           2   -none-   numeric
## lambda        79  -none-   numeric
## dev.ratio    79  -none-   numeric
## nulldev       1   -none-   numeric
## npasses       1   -none-   numeric
## jerr           1   -none-   numeric
## offset         1   -none-   logical
## call           4   -none-   call
## nobs           1   -none-   numeric

#
# Cross-validation estimates of prediction error
# lasso case (alpha=1)
y.glmnet.lasso<-glmnet(x,y,alpha=1, lambda=lambdas)
```

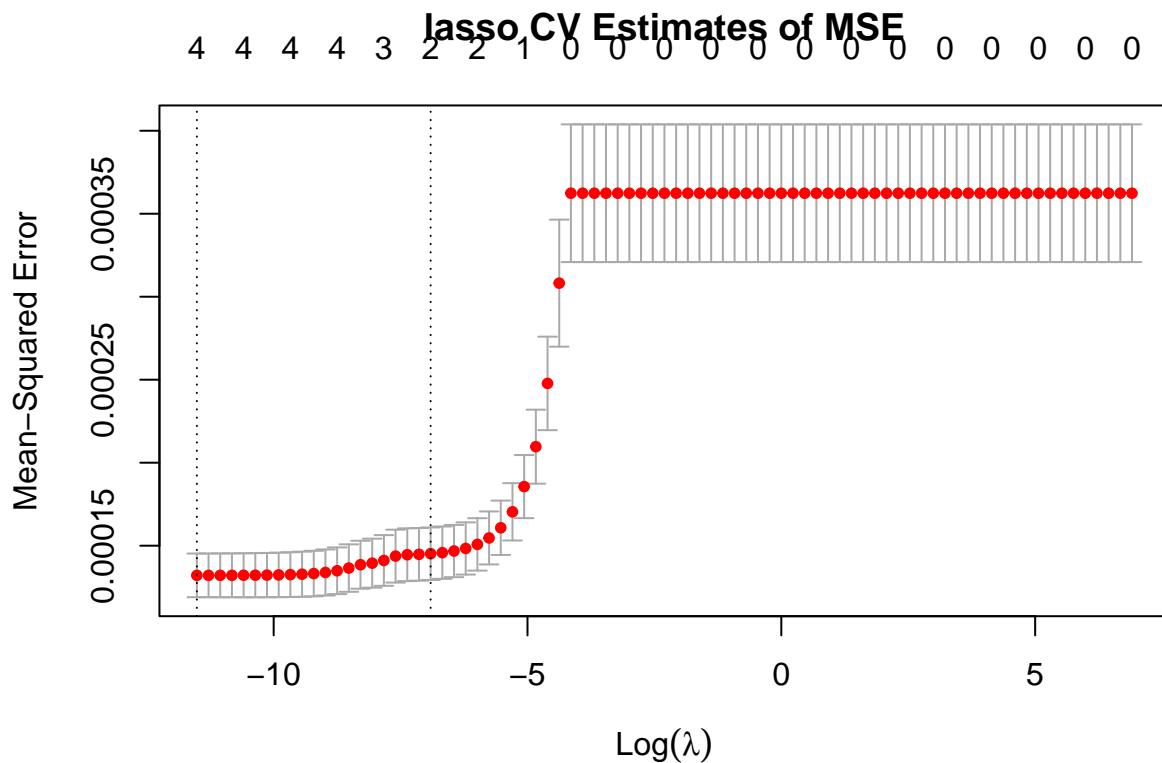
```

summary(y.glmnet.lasso)

##          Length Class      Mode
## a0           81   -none-   numeric
## beta        324    dgCMatrix S4
## df            81   -none-   numeric
## dim            2   -none-   numeric
## lambda        81   -none-   numeric
## dev.ratio     81   -none-   numeric
## nulldev         1   -none-   numeric
## npasses         1   -none-   numeric
## jerr            1   -none-   numeric
## offset           1   -none-   logical
## call             5   -none-   call
## nobs            1   -none-   numeric

y.cv.glmnet.lasso<-cv.glmnet(x,y,alpha=1, lambda=lambdas)
plot(y.cv.glmnet.lasso, main="lasso CV Estimates of MSE")

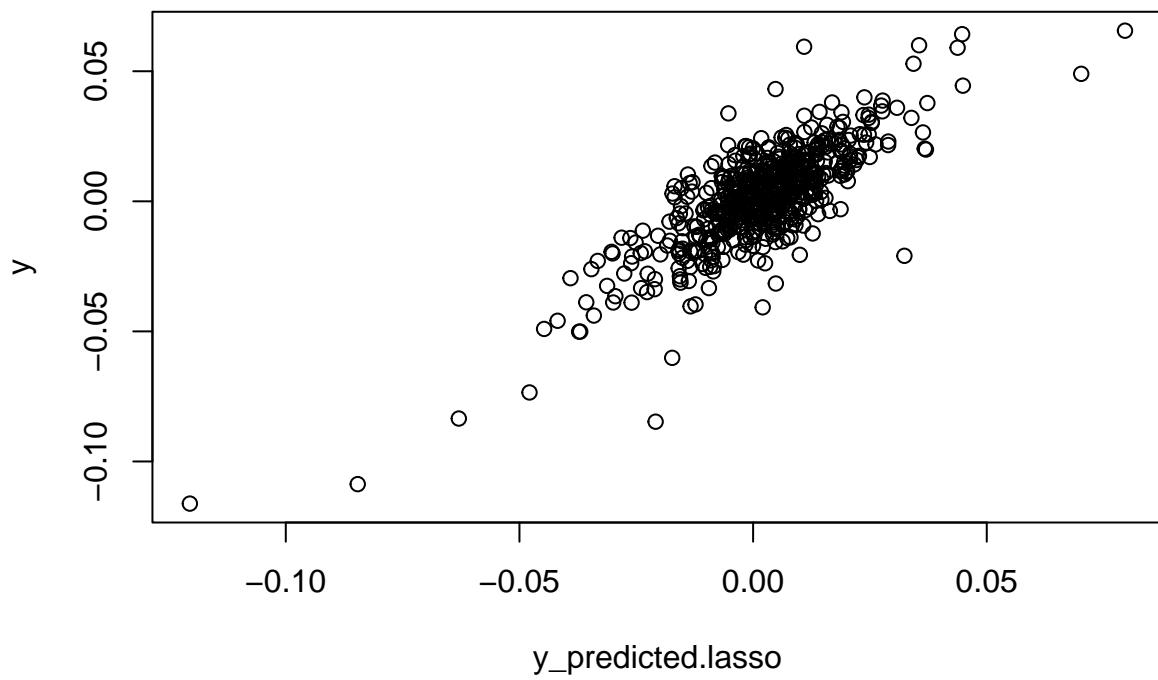
```



```

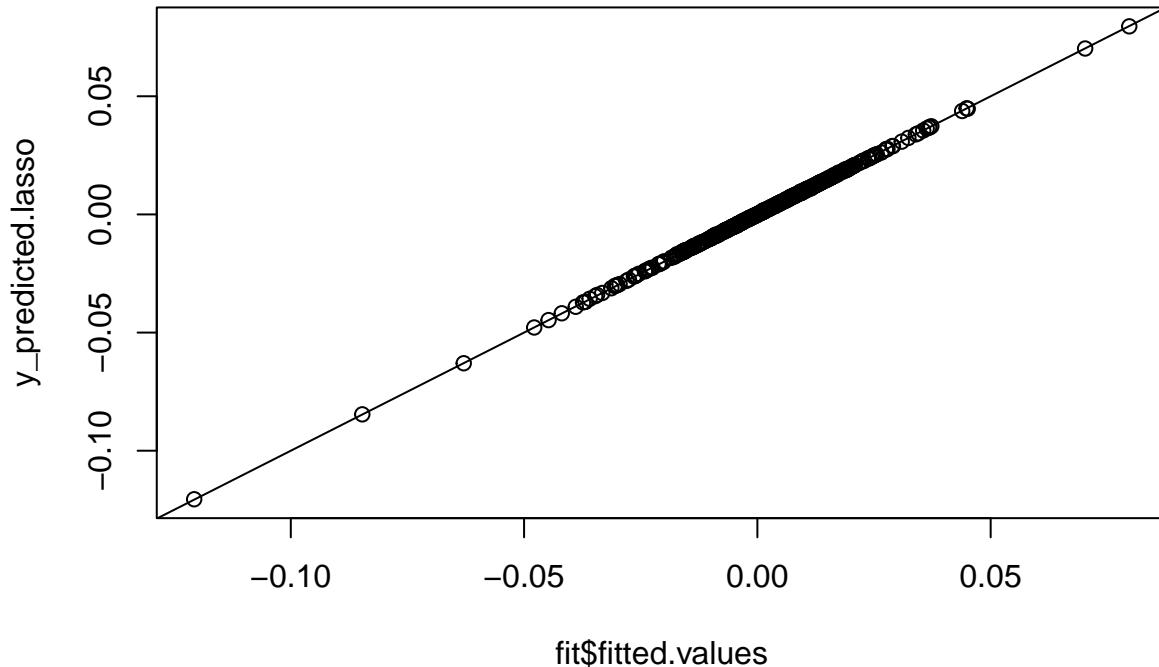
optlambda.lasso<-y.cv.glmnet.lasso$lambda.min
glmnet.lassofit<-y.cv.glmnet.lasso$glmnet.fit
y_predicted.lasso<- predict(glmnet.lassofit,s=optlambda.lasso,newx=x)
#
plot(y_predicted.lasso, y)

```



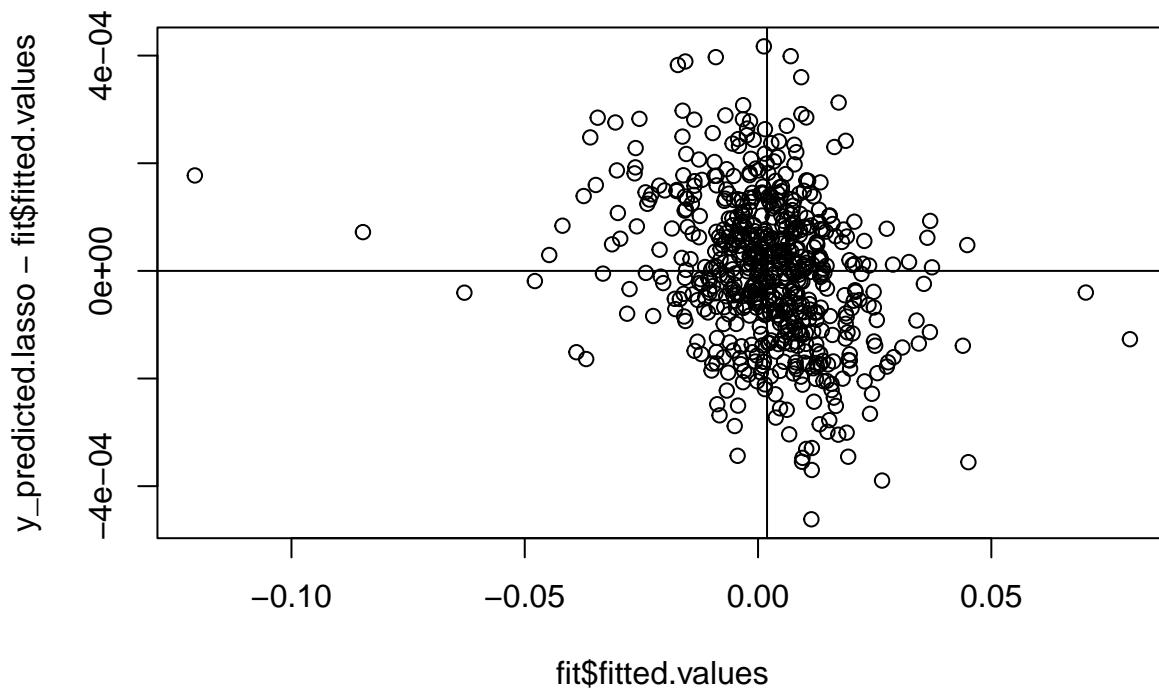
```
cor(y_predicted.lasso,y)^2  
## [1] 0.6417391  
# Compare lasso fitted to ls fitted  
plot(fit$fitted.values, y_predicted.lasso,  
      main="Lasso Regression \n(Shrinks to Mean)")  
abline(a=0,b=1)
```

Lasso Regression (Shrinks to Mean)



```
plot(fit\$fitted.values, y_predicted.lasso - fit\$fitted.values,
      main="Lasso Regression \n(Shrinks to Mean)")
abline(h=0,v=mean(fit\$fitted.values))
```

Lasso Regression (Shrinks to Mean)



```

## 8.7 Compare coefficients from fits including LASSO and Ridge ----

tab.betas<-cbind(betaFromPCA, betaFromPC123, betaFromPCtokeep)
dimnames(tab.betas)[[2]]<-c("LS", "PC123", "PCsignif")
print(tab.betas)

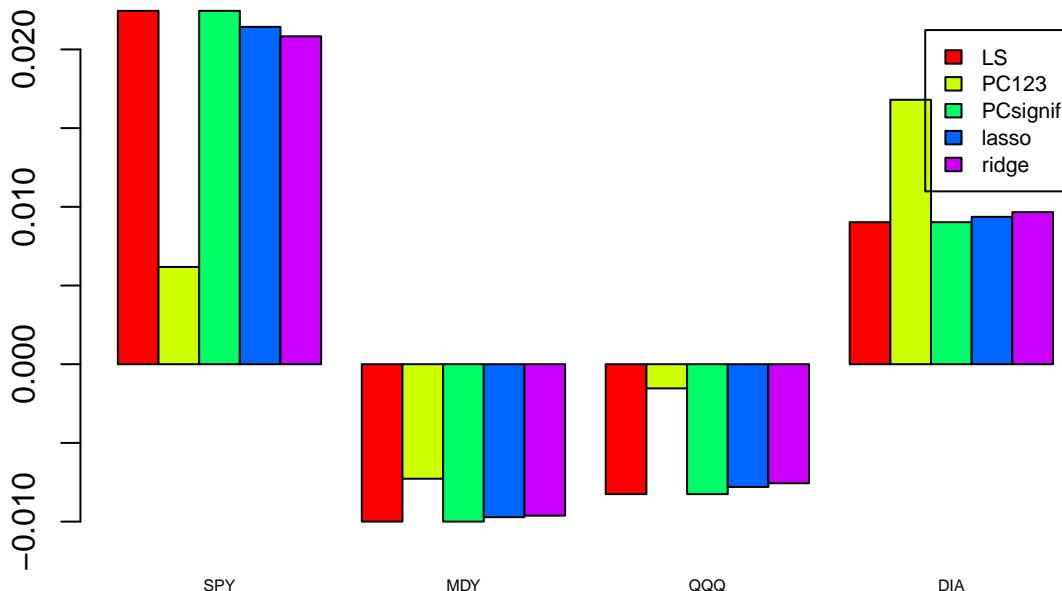
##          LS      PC123     PCsignif
## SPY  0.022450726  0.006178644  0.022450726
## MDY -0.010000091 -0.007275983 -0.010000091
## QQQ -0.008254136 -0.001537180 -0.008254136
## DIA  0.009023990  0.016801960  0.009023990

tab.betas2<-data.frame(cbind(tab.betas,
                                lasso=coef(y.cv.glmnet.lasso,s="lambda.min")[-1],
                                ridge=coef(y.cv.glmnet.ridge,s="lambda.min")[-1]))

barplot(t(as.matrix(tab.betas2)), beside=TRUE,
        col=rainbow(5), cex.names=.5,
        legend=TRUE, args.legend=list(cex=.7))
title(main="Regression Coefficients By Method")

```

Regression Coefficients By Method



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