from talk w/ Daji.

→ 2 thms.

① when all vertices are 3 deg. then worst case is 3/5.

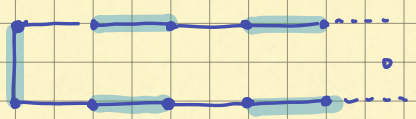
② If you have 2 & 3-deg vertices (and max 2 1-deg. endpoint vertices) then you get

$\lceil (\#\text{I's} \cdot 1/3) + (\#\text{paths} \cdot \text{path prob}) \rceil$ OPT.

not sure exactly true but intuition.

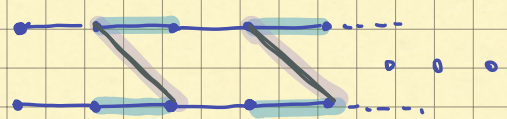
"I-lemma": Given a $x \cdot OPT$ matching for max 2-deg graph, we get a worst case $1/3 \cdot x \cdot OPT$ matching for 3-deg.

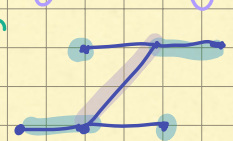
- 2 deg. matching.



Worst case, adversary gives this graph w/ middle edges first in the permutation:

- 2 deg. matching. – 10 v's matched
- 3 deg matching – 6 v's matched.





→ $1/3 \cdot x \cdot OPT$.

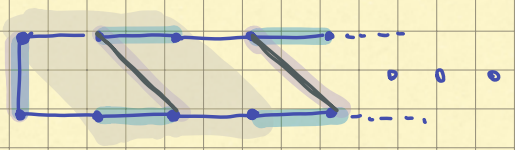
(because worst case, 4/6 vertices that were in matching can no longer be in the matching).

↳ Let this be refered to as the "I-structure".

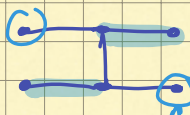
Thrm. 2: If you have 2 & 3-deg vertices then you get $\left[(\#\,I\text{'s} \cdot 1/3) + (\#\,\text{paths} \cdot \text{path prob})\right]$ OPT.

Note: we are not accounting for 1-deg endpoints.

Now, take the "I-structure":



→ OPT:

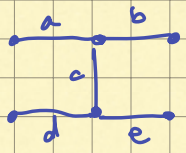


→ worst:

maybe these are covered in other I-structures.

↪ each I gives a 1/3 × OPT worse solution (I-lemma).

What is the chance of getting worst case ordering in an I-structure?

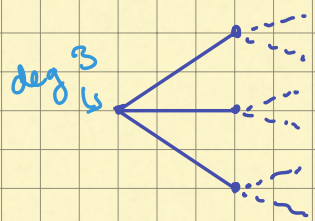


↳ $\Pr[c \text{ comes first in ordering}] = 1/5$.

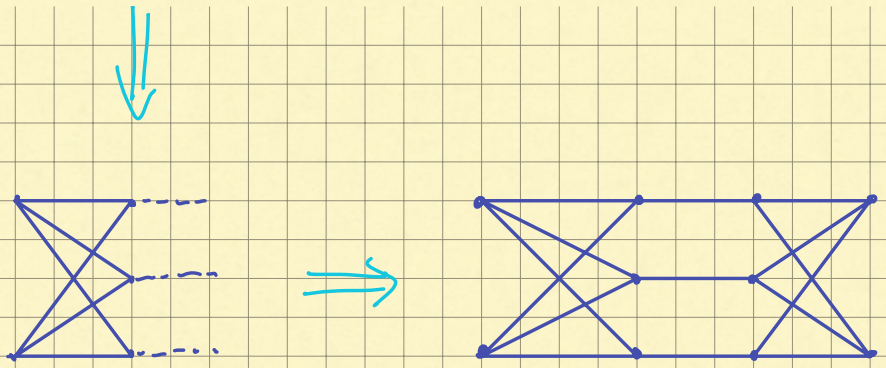
∴ we get 1/3 w/ prob 1/5. for I-structures, and the rest are paths, so same analysis there.

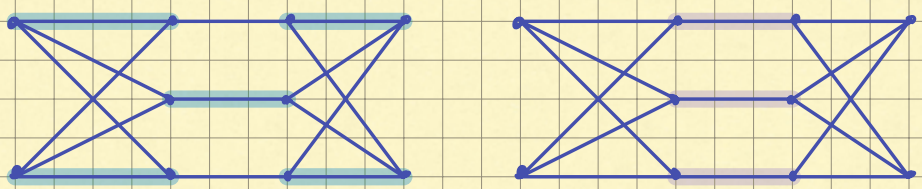
Thrm 1: if all vertices have degree 3, then the worst case is 3/5 · OPT.

for the construction, consider:



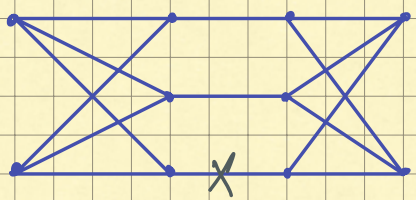
Ok, now let's try to make it as bad as possible...





OPT. → 16 Worst → 6

⚠ We need some sort of counting arg. to say that this is the worst.



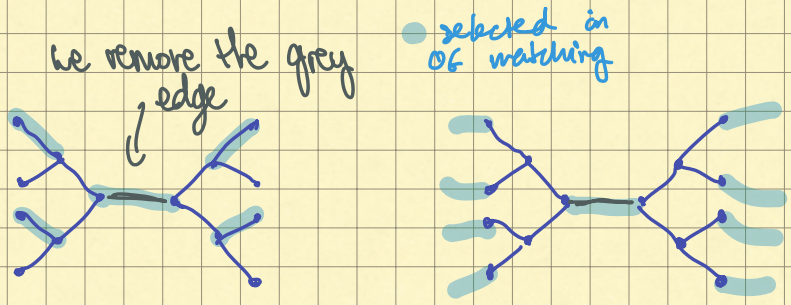
then, removing 1 edge

either the edge is not in our worst-case selection
↓
matching doesn't change.

OR

the edge IS in the worst case matching.
↓
Now we have 2 deg. 2 vertices in our graph.

2 bounding cases:



Worst case:
⇒ we lose 2 vertices from the matching

Best case:
⇒ we add 4 vertices to the matching

→ Can we argue on probability of each case?