Recursive structure of primes reloaded. Now with mod 2, too.*

Carolin Zöbelein†

April 8, 2016

Abstract.

Contents

1 Introduction

In this working document I will make a revision of my work [1]. Now for mod 2 and not mod 6 which makes all so easy. Sometimes I'm so stupid. wtf!

*Keywords: integer divisible numbers, prime numbers, recursive structure, mod 2;

†*E-mail address:* contact@carolin-zoebelein.de, *PGP Fingerprint*: D4A7 35E8 D47F 801F 2CF6 2BA7 927A FD3C DE47 E13B; *URL:* http://www.carolin-zoebelein.de

2 Road map

Since the main steps will be the same like in [1] I will only make a few comments. If you need more information please read the arXiv paper at first.

To future comments: Yes, I know it's no 'professional' math, but why I should use abstract math if it's also possible to explain issues with easy tools. This here is to be only a first impression. Not the solution of all! After this you can construct your buildings, too!

Personal road map for this document:

1. Formulation of the important points for mod 2.
2. Consequences from this.
3. ...

3 What we already know ...

Short repetition of the properties of primes which we already know and we will use for our work.

1. Q: What are prime numbers $\mathbb{P}$?
   A: A prime number $p \in \mathbb{P}$ is a integer number lager than one which has no positive integer divisors apart from 1 and itself.
   $\Rightarrow$ More mathematical: $p \in \mathbb{Z} : \gcd(n, p) = 1, \forall n \neq p \in \mathbb{N}$
2. The set of primes: $\mathbb{P} := \{2, 3, 5, 7, 11, 13, \dots\}$
3. The number 2 is the only even prime.

In the following work we will always ignore the prime 2!

$\Rightarrow$ Hence, for all primes $p \in \mathbb{P} \setminus \{2\}$ we know that we are able to write them as $p_n := 2n + 1$, $n \in \mathbb{N}_0$. Q: For which n we receive primes?

Now if we look at the set of all odd integer divisible numbers $p_{nn\prime} \in \mathbb{N}$. We recognise

$$\begin{aligned} p_{nn\prime} &:= (2n+1)(2n\prime+1) \\ &= 4nn\prime + 2n + 2n\prime + 1 \\ &= 2\left(\underbrace{2nn\prime + n + n\prime}_{=:N_{n,n\prime}}\right) + 1, \end{aligned} \tag{1}$$

$\forall n, n\prime \in \mathbb{N}_0$. Since the case $a \cdot 1 = 1 \cdot a = a$ isn't interesting for use we will change our domain of definition from $n, n\prime \in \mathbb{N}_0$ to $n, n\prime \in \mathbb{N}$.

Additionally we will see that it make sense to expand p_n to p_z with the set $\mathbb{Z}$. In this case we will write

$$\begin{aligned} p_{z,z\prime} &:= (2z+1)\,(2z\prime+1) \\ &= 4zz\prime + 2z + 2z\prime + 1 \\ &= 2\left(\underbrace{2zz\prime + z + z\prime}_{=:Z_{z,z\prime}}\right) + 1, \end{aligned} \tag{2}$$

$\forall z, z\prime \in \mathbb{Z}$. Here we are also not interested in the cases $a \cdot 1 = 1 \cdot a = a$ and $a \cdot (-1) = (-1) \cdot a = -a$. So we use $z, z\prime \in \mathbb{Z} \setminus \{-1, 0\}$.

4 Intersection

Ok. Now we have an equation to describe all integer divisible numbers of $\mathbb{Z}$. So we also write

$$\begin{aligned} Z_{z,z\prime} &= 2zz\prime + z + z\prime \\ &= (2z+1)\,z\prime + z \\ &= (2z\prime+1)\,z + z\prime. \end{aligned} \tag{3}$$

For example, the second line can be interpreted as the equation which gives us all $Z_{z,z\prime}$ which belongs to numbers which are the product of $(2z+1)\,V$, $V \in \mathbb{Z}$.
We use this to write the following for all numbers which are not integer divisible by $(2z+1)$:

$$Z_{z,z\prime} = (2z+1)\,z\prime + z + \chi_z, \tag{4}$$

$\chi_z \in [1, (2z+1) - 1]$. For the intersection of two of this equations follows

$$\begin{aligned} 0 &= Z_{z_i,z_i\prime} - Z_{z_j,z_j\prime} \\ &= (2z_i+1)\,z_i\prime - (2z_j+1)\,z_j\prime + \underbrace{z_i + \chi_{z_i}}_{=:\kappa_{z_i}} - \left(\underbrace{z_j + \chi_{z_j}}_{=:\kappa_{z_j}}\right) \\ &= (2z_i+1)\,z_i\prime - (2z_j+1)\,z_j\prime + \kappa_{z_i} - \kappa_{z_j}, \end{aligned} \tag{5}$$

$\forall i, j \in \mathbb{N} : i \neq j$.

5 Solution of intersection equation

Assume $|\,(2z_i+1)\,| < |\,(2z_j+1)\,|$ and $(2z_i+1) \perp (2z_j+1)$. Be $z_j = z_i + \Delta z_{i,j} : \Delta z_{i,j} \in \mathbb{N}$ we can write

$$\begin{aligned} 0 &= (2z_i+1)\,z_i\prime - (2z_j+1)\,z_j\prime + \kappa_{(z_i,\chi_{z_i})} - \kappa_{(z_j,\chi_{z_j})} \\ &= (2z_i+1)\,(z_i\prime - z_j\prime) - 2\Delta z_j + \chi_{z_i} - \chi_{z_j}, \end{aligned} \tag{6}$$

with $\chi_{z_i} \in [1, 2z_i]$ and $\chi_{z_j} \in [1, 2(z_i + \Delta z_{i,j})]$. Like in [1] we receive for $z_j\prime$

$$z_j\prime^{\Delta z_{i,j}} = (2z_i + 1) Y_{j\circ i} - \left(\chi_{z_i} - \chi_{z_j}\right) z_i \prod_{k=2}^{\Delta z_{i,j}} \left(1 + \frac{2}{k} z_i\right) \tag{7}$$

and in the same way for $z_i = z_j - \Delta z_{i,j}$ we receive for z_i'

$$z_i\prime^{\Delta z_{i,j}} = (2z_j + 1) Y_{j\circ i} - \left(\chi_{z_i} - \chi_{z_j}\right) z_j \prod_{k=2}^{\Delta z_{i,j}} \left(1 + \frac{2}{k} z_j\right). \tag{8}$$

Finally we can write

$$f(z_{i,j}, \Delta z_{i,j}) := \prod_{k=2}^{\Delta z_{i,j}} \left(1 + \frac{2}{k} z_{i,j}\right) = \frac{\Gamma(2z_{i,j} + \Delta z_{i,j} + 1)}{\Gamma(\Delta z_{i,j} + 1)\Gamma(2z_{i,j} + 2)} \tag{9}$$

Possible ways to rewrite:

$$\begin{aligned}
\frac{1}{B(\Delta z_{i,j} + 1, 2z_{i,j} + 2)} &= \frac{\Gamma(\Delta z_{i,j} + 2z_{i,j} + 3)}{\Gamma(\Delta z_{i,j} + 1)\Gamma(2z_{i,j} + 2)} \\
&= \frac{(\Delta z_{i,j} + 2z_{i,j} + 2)(\Delta z_{i,j} + 2z_{i,j} + 1)\Gamma(\Delta z_{i,j} + 2z_{i,j} + 1)}{\Gamma(\Delta z_{i,j} + 1)\Gamma(2z_{i,j} + 2)} \\
&= (\Delta z_{i,j} + 2z_{i,j} + 2)(\Delta z_{i,j} + 2z_{i,j} + 1) f_{(z_{i,j}, \Delta z_{i,j})} \\
f(z_{i,j}, \Delta z_{i,j}) &= \frac{1}{B(\Delta z_{i,j} + 1, 2z_{i,j} + 2)(\Delta z_{i,j} + 2z_{i,j} + 2)(\Delta z_{i,j} + 2z_{i,j} + 1)}
\end{aligned} \tag{10}$$

with the Beta function $B(x, y)$ and $\Gamma(x + 1) = x\Gamma(x)$. Properties and approximations for $B(x, y)$:

1. $B(x, y) = B(y, x)$
2. If x and y are large:
$$B(x,y) \approx \sqrt{2\pi} \frac{x^{x-\frac{1}{2}} y^{y-\frac{1}{2}}}{(x+y)^{x+y-\frac{1}{2}}}$$
3. If x large and y fixed:
$$B(x,y) \approx \Gamma(y)\, x^{-y}$$

We will see if this approximations are useful.

6 Solution step by step

Until now all was a repetition of [1]. Question: How does the way with 2 will make all easier now? For this we will make the examination step by step and discuss all advantages or problems, now.

6.1 The Beginning

Let's start! Be $z_1 = 1$

$$\begin{aligned} Z_{1,z_1\prime} &= (2 \cdot 1 + 1)\, z_1\prime + 1 + \chi_1 \\ &= 3 z_1\prime + 1 + \chi_1, \end{aligned} \tag{11}$$

with $\chi_1 \in [1, 2 \cdot 1] = [1, 2]$.

References

[1] Zöbelein, C.: The recursive structure of the distribution of primes. In: *ArXiv e-prints, 1411.2824 [math.GM]* (2014), November