

# Recursive structure of primes reloaded. Now with mod 2, too.\*

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Abstract.

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## 1 Introduction

In this working document I will make a revision of my work [1]. Now for mod 2 and not mod 6 which makes all so easy. Sometimes I'm so stupid. wtf!

## 2 Road map

Since the main steps will be the same like in [1] I will only make a few comments. If you need more information please read the arXiv paper at first.

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To future comments: Yes, I know it's no 'professional' math, but why I should use abstract math if it's also possible to explain issues with easy tools. This here is to be only a first impression. Not the solution of all! After this you can construct your buildings, too!

Personal road map for this document:

1. Formulation of the important points for mod 2.
2. Consequences from this.
3. ...

### 3 What we already know ...

Short repetition of the properties of primes which we already know and we will use for our work.

1. Q: What are prime numbers  $\mathbb{P}$ ?

A: A prime number  $p \in \mathbb{P}$  is a integer number larger than one which has no positive integer divisors apart from 1 and itself.

$\Rightarrow$  More mathematical:  $p \in \mathbb{Z} : \gcd(n, p) = 1, \forall n \neq p \in \mathbb{N}$

2. The set of primes:  $\mathbb{P} := \{2, 3, 5, 7, 11, 13, \dots\}$
3. The number 2 is the only even prime.

**In the following work we will always ignore the prime 2!**

$\Rightarrow$  Hence, for all primes  $p \in \mathbb{P} \setminus \{2\}$  we know that we are able to write them as  $p_n := 2n + 1, n \in \mathbb{N}_0$ . Q: For which  $n$  we receive primes?

Now if we look at the set of all odd integer divisible numbers  $p_{nn'}$   $\in \mathbb{N}$ . We recognise

$$\begin{aligned} p_{nn'} &:= (2n + 1)(2n' + 1) \\ &= 4nn' + 2n + 2n' + 1 \\ &= 2 \left( \underbrace{2nn' + n + n'}_{=: N_{n,n'}} \right) + 1, \end{aligned} \tag{1}$$

$\forall n, n' \in \mathbb{N}_0$ . Since the case  $a \cdot 1 = 1 \cdot a = a$  isn't interesting for use we will change our domain of definition from  $n, n' \in \mathbb{N}_0$  to  $n, n' \in \mathbb{N}$ .

Additionally we will see that it make sense to expand  $p_n$  to  $p_z$  with the set  $\mathbb{Z}$ . In this case we will write

$$\begin{aligned} p_{z,z'} &:= (2z + 1)(2z' + 1) \\ &= 4zz' + 2z + 2z' + 1 \\ &= 2 \left( \underbrace{2zz' + z + z'}_{=: Z_{z,z'}} \right) + 1, \end{aligned} \tag{2}$$

$\forall z, z' \in \mathbb{Z}$ . Here we are also not interested in the cases  $a \cdot 1 = 1 \cdot a = a$  and  $a \cdot (-1) = (-1) \cdot a = -a$ . So we use  $z, z' \in \mathbb{Z} \setminus \{-1, 0\}$ .

## 4 Intersection

Ok. Now we have an equation to describe all integer divisible numbers of  $\mathbb{Z}$ . So we also write

$$\begin{aligned} Z_{z,z'} &= 2zz' + z + z' \\ &= (2z + 1)z' + z \\ &= (2z' + 1)z + z'. \end{aligned} \quad (3)$$

For example, the second line can be interpreted as the equation which gives us all  $Z_{z,z'}$  which belongs to numbers which are the product of  $(2z + 1)V$ ,  $V \in \mathbb{Z}$ .

We use this to write the following for all numbers which are not integer divisible by  $(2z + 1)$ :

$$Z_{z,z'} = (2z + 1)z' + z + \chi_z, \quad (4)$$

$\chi_z \in [1, (2z + 1) - 1]$ . For the intersection of two of this equations follows

$$\begin{aligned} 0 &= Z_{z_i,z_i'} - Z_{z_j,z_j'} \\ &= (2z_i + 1)z_i' - (2z_j + 1)z_j' + \underbrace{z_i + \chi_{z_i}}_{=: \kappa_{z_i}} - \left( \underbrace{z_j + \chi_{z_j}}_{=: \kappa_{z_j}} \right) \\ &= (2z_i + 1)z_i' - (2z_j + 1)z_j' + \kappa_{z_i} - \kappa_{z_j}, \end{aligned} \quad (5)$$

$\forall i, j \in \mathbb{N} : i \neq j$ .

## 5 Solution of intersection equation

Assume  $|2z_i + 1| < |2z_j + 1|$  and  $(2z_i + 1) \perp (2z_j + 1)$ . Be  $z_j = z_i + \Delta z_{i,j} : \Delta z_{i,j} \in \mathbb{N}$  we can write

$$\begin{aligned} 0 &= (2z_i + 1)z_i' - (2z_j + 1)z_j' + \kappa_{(z_i, \chi_{z_i})} - \kappa_{(z_j, \chi_{z_j})} \\ &= (2z_i + 1)(z_i' - z_j') - 2\Delta z_j + \chi_{z_i} - \chi_{z_j}, \end{aligned} \quad (6)$$

with  $\chi_{z_i} \in [1, 2z_i]$  and  $\chi_{z_j} \in [1, 2(z_i + \Delta z_{i,j})]$ . Like in [1] we receive for  $z_j'$

$$z_j'^{\Delta z_{i,j}} = (2z_i + 1)Y_{j \circ i} - (\chi_{z_i} - \chi_{z_j})z_i \prod_{k=2}^{\Delta z_{i,j}} \left( 1 + \frac{2}{k} z_i \right) \quad (7)$$

and in the same way for  $z_i = z_j - \Delta z_{i,j}$  we receive for  $z_i'$

$$z_i'^{\Delta z_{i,j}} = (2z_j + 1)Y_{j \circ i} - (\chi_{z_i} - \chi_{z_j})z_j \prod_{k=2}^{\Delta z_{i,j}} \left( 1 + \frac{2}{k} z_j \right). \quad (8)$$

Finally we can write

$$\prod_{k=2}^{\Delta z_{i,j}} \left(1 + \frac{2}{k} z_{i,j}\right) = \frac{\Gamma(2z_{i,j} + \Delta z_{i,j} + 1)}{\Gamma(\Delta z_{i,j} + 1) \Gamma(2z_{i,j} + 2)} \quad (9)$$

## References

- [1] ZÖBELEIN, C.: The recursive structure of the distribution of primes. In: *ArXiv e-prints*, 1411.2824 [*math.GM*] (2014), November