

Recursive structure of primes reloaded. Now with mod 2, too.*

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Abstract.

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1 Introduction

In this working document I will make a revision of my work [1]. Now for mod 2 and not mod 6 which makes all so easy. Sometimes I'm so stupid. wtf!

2 Road map

Since the main steps will be the same like in [1] I will only make a few comments. If you need more information please read the arXiv paper at first.

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To future comments: Yes, I know it's no 'professional' math, but why I should use abstract math if it's also possible to explain issues with easy tools. This here is to be only a first impression. Not the solution of all! After this you can construct your buildings, too!

Personal road map for this document:

1. Formulation of the important points for mod 2.
2. Consequences from this.
3. ...

3 What we already know ...

Short repetition of the properties of primes which we already know and we will use for our work.

1. Q: What are prime numbers \mathbb{P} ?

A: A prime number $p \in \mathbb{P}$ is a integer number larger than one which has no positive integer divisors apart from 1 and itself.

\Rightarrow More mathematical: $p \in \mathbb{Z} : \gcd(n, p) = 1, \forall n \neq p \in \mathbb{N}$

2. The set of primes: $\mathbb{P} := \{2, 3, 5, 7, 11, 13, \dots\}$
3. The number 2 is the only even prime.

In the following work we will always ignore the prime 2!

\Rightarrow Hence, for all primes $p \in \mathbb{P} \setminus \{2\}$ we know that we are able to write them as $p_n := 2n + 1, n \in \mathbb{N}_0$. Q: For which n we receive primes?

Now if we look at the set of all odd integer divisible numbers $p_{nn'}$ $\in \mathbb{N}$. We recognise

$$\begin{aligned} p_{nn'} &:= (2n + 1)(2n' + 1) \\ &= 4nn' + 2n + 2n' + 1 \\ &= 2 \left(\underbrace{2nn' + n + n'}_{=: N_{n,n'}} \right) + 1, \end{aligned} \tag{1}$$

$\forall n, n' \in \mathbb{N}_0$. Since the case $a \cdot 1 = 1 \cdot a = a$ isn't interesting for use we will change our domain of definition from $n, n' \in \mathbb{N}_0$ to $n, n' \in \mathbb{N}$.

Additionally we will see that it make sense to expand p_n to p_z with the set \mathbb{Z} . In this case we will write

$$\begin{aligned} p_{z,z'} &:= (2z + 1)(2z' + 1) \\ &= 4zz' + 2z + 2z' + 1 \\ &= 2 \left(\underbrace{2zz' + z + z'}_{=: Z_{z,z'}} \right) + 1, \end{aligned} \tag{2}$$

$\forall z, z' \in \mathbb{Z}$. Here we are also not interested in the cases $a \cdot 1 = 1 \cdot a = a$ and $a \cdot (-1) = (-1) \cdot a = -a$. So we use $z, z' \in \mathbb{Z} \setminus \{-1, 0\}$.

4 Intersection

Ok. Now we have an equation to describe all integer divisible numbers of \mathbb{Z} . So we also write

$$\begin{aligned} Z_{z,z'} &= 2zz' + z + z' \\ &= (2z + 1)z' + z \\ &= (2z' + 1)z + z'. \end{aligned} \quad (3)$$

For example, the second line can be interpreted as the equation which gives us all $Z_{z,z'}$ which belongs to numbers which are the product of $(2z + 1)V$, $V \in \mathbb{Z}$.

We use this to write the following for all numbers which are not integer divisible by $(2z + 1)$:

$$Z_{z,z'} = (2z + 1)z' + z + \chi_z, \quad (4)$$

$\chi_z \in [1, (2z + 1) - 1]$. For the intersection of two of this equations follows

$$\begin{aligned} 0 &= Z_{z_i,z_i'} - Z_{z_j,z_j'} \\ &= (2z_i + 1)z_i' - (2z_j + 1)z_j' + \underbrace{z_i + \chi_{z_i}}_{=: \kappa_{z_i}} - \left(\underbrace{z_j + \chi_{z_j}}_{=: \kappa_{z_j}} \right) \\ &= (2z_i + 1)z_i' - (2z_j + 1)z_j' + \kappa_{z_i} - \kappa_{z_j}, \end{aligned} \quad (5)$$

$\forall i, j \in \mathbb{N} : i \neq j$.

5 Solution of intersection equation

Assume $|(2z_i + 1)| < |(2z_j + 1)|$ and $(2z_i + 1) \perp (2z_j + 1)$. Be $z_j = z_i + \Delta z_{i,j} : \Delta z_{i,j} \in \mathbb{N}$ we can write

$$\begin{aligned} 0 &= (2z_i + 1)z_i' - (2z_j + 1)z_j' + \kappa_{(z_i, \chi_{z_i})} - \kappa_{(z_j, \chi_{z_j})} \\ &= (2z_i + 1)(z_i' - z_j') - 2\Delta z_j + \chi_{z_i} - \chi_{z_j}, \end{aligned} \quad (6)$$

with $\chi_{z_i} \in [1, 2z_i]$ and $\chi_{z_j} \in [1, 2(z_i + \Delta z_{i,j})]$. Like in [1] we receive for z_j'

$$z_j'^{\Delta z_{i,j}} = (2z_i + 1)Y_{j \circ i} - (\chi_{z_i} - \chi_{z_j})z_i \prod_{k=2}^{\Delta z_{i,j}} \left(1 + \frac{2}{k}z_i \right) \quad (7)$$

and in the same way for $z_i = z_j - \Delta z_{i,j}$ we receive for z_i'

$$z_i'^{\Delta z_{i,j}} = (2z_j + 1)Y_{j \circ i} - (\chi_{z_i} - \chi_{z_j})z_j \prod_{k=2}^{\Delta z_{i,j}} \left(1 + \frac{2}{k}z_j \right). \quad (8)$$

Finally we can write

$$f(z_{i,j}, \Delta z_{i,j}) := \prod_{k=2}^{\Delta z_{i,j}} \left(1 + \frac{2}{k} z_{i,j}\right) = \frac{\Gamma(2z_{i,j} + \Delta z_{i,j} + 1)}{\Gamma(\Delta z_{i,j} + 1) \Gamma(2z_{i,j} + 2)} \quad (9)$$

Possible ways to rewrite:

$$\begin{aligned} \frac{1}{B(\Delta z_{i,j} + 1, 2z_{i,j} + 2)} &= \frac{\Gamma(\Delta z_{i,j} + 2z_{i,j} + 3)}{\Gamma(\Delta z_{i,j} + 1) \Gamma(2z_{i,j} + 2)} \\ &= \frac{(\Delta z_{i,j} + 2z_{i,j} + 2)(\Delta z_{i,j} + 2z_{i,j} + 1) \Gamma(\Delta z_{i,j} + 2z_{i,j} + 1)}{\Gamma(\Delta z_{i,j} + 1) \Gamma(2z_{i,j} + 2)} \\ &= (\Delta z_{i,j} + 2z_{i,j} + 2)(\Delta z_{i,j} + 2z_{i,j} + 1) f_{(z_{i,j}, \Delta z_{i,j})} \\ f(z_{i,j}, \Delta z_{i,j}) &= \frac{1}{B(\Delta z_{i,j} + 1, 2z_{i,j} + 2)(\Delta z_{i,j} + 2z_{i,j} + 2)(\Delta z_{i,j} + 2z_{i,j} + 1)} \end{aligned} \quad (10)$$

with the Beta function $B(x, y)$ and $\Gamma(x+1) = x\Gamma(x)$. Properties and approximations for $B(x, y)$:

1. $B(x, y) = B(y, x)$
2. If x and y are large:

$$B(x, y) \approx \sqrt{2\pi} \frac{x^{x-\frac{1}{2}} y^{y-\frac{1}{2}}}{(x+y)^{x+y-\frac{1}{2}}}$$

3. If x large and y fixed:

$$B(x, y) \approx \Gamma(y) x^{-y}$$

6 ...

Until now all was a repetition of [1]. Question: How does the way with 2 make all easier now?

References

- [1] ZÖBELEIN, C.: The recursive structure of the distribution of primes. In: *ArXiv e-prints*, 1411.2824 [math.GM] (2014), November