Recursive structure of primes reloaded. Now with mod 2, too.*

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Abstract.

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1 Introduction

In this working document I will make a revision of my work [1]. Now for mod 2 and not mod 6 which makes all so easy. Sometimes I'm so stupid. wtf!

2 Road map

Since the main steps will be the same like in [1] I will only make a few comments. If you need more information please read the arXiv paper at first.

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To future comments: Yes, I know it's no 'professional' math, but why I should use abstract math if it's also possible to explain issues with easy tools. This here is to be only a first impression. Not the solution of all! After this you can construct your buildings, too!

Personal road map for this document:

1. Formulation of the important points for mod 2.
2. Consequences from this.
3. ...

3 What we already know ...

Short repetition of the properties of primes which we already know and we will use for our work.

1. Q: What are prime numbers $\mathbb{P}$?
   A: A prime number $p \in \mathbb{P}$ is a integer number lager than one which has no positive integer divisors apart from 1 and itself.
   $\Rightarrow$ More mathematical: $p \in \mathbb{Z} : \gcd(n, p) = 1, \forall n \neq p \in \mathbb{N}$
2. The set of primes: $\mathbb{P} := \{2, 3, 5, 7, 11, 13, \dots\}$
3. The number 2 is the only even prime.

In the following work we will always ignore the prime 2!

$\Rightarrow$ Hence, for all primes $p \in \mathbb{P} \setminus \{2\}$ we know that we are able to write them as $p_n := 2n + 1$, $n \in \mathbb{N}_0$. Q: For which n we receive primes?

Now if we look at the set of all odd integer divisible numbers $p_{nn\prime} \in \mathbb{N}$. We recognise

$$\begin{aligned} p_{nn\prime} &:= (2n+1)(2n\prime+1) \\ &= 4nn\prime + 2n + 2n\prime + 1 \\ &= 2\left(\underbrace{2nn\prime + n + n\prime}_{=:N_{n,n\prime}}\right) + 1, \end{aligned} \tag{1}$$

$\forall n, n\prime \in \mathbb{N}_0$. Since the case $a \cdot 1 = 1 \cdot a = a$ isn't interesting for use we will change our domain of definition from $n, n\prime \in \mathbb{N}_0$ to $n, n\prime \in \mathbb{N}$.
Additionally we will see that it make sense to expand p_n to p_z with the set $\mathbb{Z}$. In this case we will write

$$\begin{aligned} p_{z,z\prime} &:= (2z+1)(2z\prime+1) \\ &= 4zz\prime + 2z + 2z\prime + 1 \\ &= 2\left(\underbrace{2zz\prime + z + z\prime}_{=:Z_{z,z\prime}}\right) + 1, \end{aligned} \tag{2}$$

$\forall z, z\prime \in \mathbb{Z}$. Here we are also not interested in the cases $a \cdot 1 = 1 \cdot a = a$ and $a \cdot (-1) = (-1) \cdot a = -a$. So we use $z, z\prime \in \mathbb{Z} \setminus \{-1, 0\}$.

4 Intersection

Ok. Now we have an equation to describe all integer divisible numbers of $\mathbb{Z}$. So we also write

$$\begin{aligned} Z_{z,z\prime} &= 2zz\prime + z + z\prime \\ &= (2z+1)\, z\prime + z \\ &= (2z\prime + 1)\, z + z\prime . \end{aligned} \tag{3}$$

For example, the second line can be interpreted as the equation which gives us all $Z_{z,z\prime}$ which belongs to numbers which are the product of $(2z+1)\, V$, $V \in \mathbb{Z}$.
We use this to write the following for all numbers which are not integer divisible by $(2z+1)$:

$$Z_{z,z\prime} = (2z+1)\, z\prime + z + \chi_z , \tag{4}$$

$\chi_z \in [1, (2z+1) - 1]$. For the intersection of two of this equations follows

$$\begin{aligned} 0 &= Z_{z_i, z_i\prime} - Z_{z_j, z_j\prime} \\ &= (2z_i+1)\, z_i\prime - (2z_j+1)\, z_j\prime + \underbrace{z_i + \chi_{z_i}}_{=:\kappa_{z_i}} - \left(\underbrace{z_j + \chi_{z_j}}_{=:\kappa_{z_j}} \right) \\ &= (2z_i+1)\, z_i\prime - (2z_j+1)\, z_j\prime + \kappa_{z_i} - \kappa_{z_j} , \end{aligned} \tag{5}$$

$\forall i, j \in \mathbb{N} : i \neq j$.

5 Solution of intersection equation

Assume $|\,(2z_i+1)\,| < |\,(2z_j+1)\,|$ and $(2z_i+1) \perp (2z_j+1)$. Be $z_j = z_i + \Delta z_{i,j} : \Delta z_{i,j} \in \mathbb{N}$ we can write

$$\begin{aligned} 0 &= (2z_i+1)\, z_i\prime - (2z_j+1)\, z_j\prime + \kappa_{\left(z_i, \chi_{z_i}\right)} - \kappa_{\left(z_j, \chi_{z_j}\right)} \\ &= (2z_i+1)\, (z_i\prime - z_j\prime) - 2\Delta z_j + \chi_{z_i} - \chi_{z_j} , \end{aligned} \tag{6}$$

with $\chi_{z_i} \in [1, 2z_i]$ and $\chi_{z_j} \in [1, 2\,(z_i + \Delta z_{i,j})]$. Like in [1] we receive for $z_j\prime$

$$z_j\prime^{\Delta z_{i,j}} = (2z_i+1)\, Y_{j \circ i} - \left(\chi_{z_i} - \chi_{z_j}\right) z_i \prod_{k=2}^{\Delta z_{i,j}} \left(1 + \frac{2}{k} z_i\right) \tag{7}$$

and in the same way for $z_i = z_j - \Delta z_{i,j}$ we receive for z_i'

$$z_i\prime^{\Delta z_{i,j}} = (2z_j+1)\, Y_{j \circ i} - \left(\chi_{z_i} - \chi_{z_j}\right) z_j \prod_{k=2}^{\Delta z_{i,j}} \left(1 + \frac{2}{k} z_j\right) . \tag{8}$$

References

[1] Zöbelein, C.: The recursive structure of the distribution of primes. In: *ArXiv e-prints, 1411.2824 [math.GM]* (2014), November