Recursive structure of primes reloaded. Now with mod 2, too.[*]

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Abstract.

Contents

1 Introduction

In this working document I will make a revision of my work [1]. Now for mod 2 and not mod 6 which makes all so easy. Sometimes I'm so stupid. wtf!

2 Road map

Since the main steps will be the same like in [1] I will only make a few comments. If you need more information please read the arXiv paper at first.

To future comments: Yes, I know it's no 'professional' math, but why I should use abstract math if it's also possible to explain issues with easy tools. This here is to be only a first impression. Not the solution of all! After this you can construct your buildings, too!

Personal road map for this document:

1. Formulation of the important points for mod 2.

2. Consequences from this.

3. ...

3 What we already know ...

Short repetition of the properties of primes which we already know and we will use for our work.

1. Q: What are prime numbers $\mathbb{P}$?
   A: A prime number $p \in \mathbb{P}$ is a integer number lager than one which has no positive integer divisors apart from 1 and itself.
   $\Rightarrow$ More mathematical: $p \in \mathbb{Z} : \gcd(n,p) = 1, \forall n \neq p \in \mathbb{N}$

2. The set of primes: $\mathbb{P} := \{2, 3, 5, 7, 11, 13, \dots\}$

3. The number 2 is the only even prime.

In the following work we will always ignore the prime 2!

$\Rightarrow$ Hence, for all primes $p \in \mathbb{P} \setminus \{2\}$ we know that we are able to write them as $p_n := 2n + 1$, $n \in \mathbb{N}_0$. Q: For which n we receive primes?

Now if we look at the set of all odd integer divisible numbers $p_{nn\prime} \in \mathbb{N}$. We recognise

$$
\begin{aligned}
p_{nn\prime} &:= (2n + 1)(2n\prime + 1) \\
&= 4nn\prime + 2n + 2n\prime + 1 \\
&= 2 \left(\underbrace{2nn\prime + n + n\prime}_{=:N_{n,n\prime}} \right) + 1,
\end{aligned}
\tag{1}
$$

$\forall n, n\prime \in \mathbb{N}_0$. Since the case $a \cdot 1 = 1 \cdot a = a$ isn't interesting for use we will change our domain of definition from $n, n\prime \in \mathbb{N}_0$ to $n, n\prime \in \mathbb{N}$.
Additionally we will see that it make sense to expand p_n to p_z with the set $\mathbb{Z}$. In this case we will write

$$
\begin{aligned}
p_{z,z\prime} &:= (2z + 1)(2z\prime + 1) \\
&= 4zz\prime + 2z + 2z\prime + 1 \\
&= 2 \left(\underbrace{2zz\prime + z + z\prime}_{=:Z_{z,z\prime}} \right) + 1,
\end{aligned}
\tag{2}
$$

$\forall z, z\prime \in \mathbb{Z}$. Here we are also not interested in the cases $a \cdot 1 = 1 \cdot a = a$ and $a \cdot (-1) = (-1) \cdot a = -a$. So we use $z, z\prime \in \mathbb{Z} \setminus \{-1, 0\}$.

4 Intersection

Ok. Now we have an equation to describe all integer divisible numbers of $\mathbb{Z}$. So we also write

$$
\begin{aligned}
Z_{z,z\prime} &= 2zz\prime + z + z\prime \\
&= (2z + 1) z\prime + z \\
&= (2z\prime + 1) z + z\prime.
\end{aligned}
\tag{3}
$$

For example, the second line can be interpreted as the equation which gives us all $Z_{z,z\prime}$ which belongs to numbers which are the product of $(2z + 1) V$, $V \in \mathbb{Z}$.

We use this to write the following for all numbers which are not integer divisible by $(2z + 1)$:

$$
Z_{z,z\prime} = (2z + 1) z\prime + z + \chi_z,
\tag{4}
$$

$\chi_z \in [1, (2z + 1) - 1]$. For the intersection of two of this equations follows

$$
\begin{aligned}
0 &= Z_{z_i,z_{i\prime}} - Z_{z_j,z_{j\prime}} \\
\\
&= (2z_i + 1) z_i\prime - (2z_j + 1) z_j\prime + \underbrace{z_i + \chi_{z_i}}_{=:\kappa_{z_i}} - \left(\underbrace{z_j + \chi_{z_j}}_{=:\kappa_{z_j}} \right) \\
\\
&= (2z_i + 1) z_i\prime - (2z_j + 1) z_j\prime + \kappa_{z_i} - \kappa_{z_j},
\end{aligned}
\tag{5}
$$

$\forall i, j \in \mathbb{N} : i \neq j$.

5 Solution of intersection equation

Assume $| (2z_i + 1) | < | (2z_j + 1) |$ and $(2z_i + 1) \perp (2z_j + 1)$. Be $z_j = z_i + \Delta z_{i,j} : \Delta z_{i,j} \in \mathbb{N}$ we can write

$$
\begin{aligned}
0 &= (2z_i + 1) z_i\prime - (2z_j + 1) z_j\prime + \kappa_{\left(z_i, \chi_{z_i}\right)} - \kappa_{\left(z_j, \chi_{z_j}\right)} \\
&= (2z_i + 1) (z_i\prime - z_j\prime) - 2\Delta z_j + \chi_{z_i} - \chi_{z_j},
\end{aligned}
\tag{6}
$$

with $\chi_{z_i} \in [1, 2z_i]$ and $\chi_{z_j} \in [1, 2(z_i + \Delta z_{i,j})]$. Like in [1] we receive for $z_j\prime$

$$
z_j\prime^{\Delta z_{i,j}} = (2z_i + 1) Y_{j \circ i} - \left(\chi_{z_i} - \chi_{z_j} \right) z_i \prod_{k=2}^{\Delta z_{i,j}} \left(1 + \frac{2}{k} z_i \right)
\tag{7}
$$

and in the same way for $z_i = z_j - \Delta z_{i,j}$ we receive for $z_i\prime$

$$
z_i\prime^{\Delta z_{i,j}} = (2z_j + 1) Y_{j \circ i} - \left(\chi_{z_i} - \chi_{z_j} \right) z_j \prod_{k=2}^{\Delta z_{i,j}} \left(1 + \frac{2}{k} z_j \right).
\tag{8}
$$

3

Finally we can write

$$f\left(z_{i,j}, \Delta z_{i,j}\right) := \prod_{k=2}^{\Delta z_{i,j}} \left(1 + \frac{2}{k} z_{i,j}\right) = \frac{\Gamma\left(2z_{i,j} + \Delta z_{i,j} + 1\right)}{\Gamma\left(\Delta z_{i,j} + 1\right)\Gamma\left(2z_{i,j} + 2\right)} \tag{9}$$

Possible ways to rewrite:

$$\frac{1}{B\left(\Delta z_{i,j} + 1, 2z_{i,j} + 2\right)} = \frac{\Gamma\left(\Delta z_{i,j} + 2z_{i,j} + 3\right)}{\Gamma\left(\Delta z_{i,j} + 1\right)\Gamma\left(2z_{i,j} + 2\right)}$$
$$= \frac{\left(\Delta z_{i,j} + 2z_{i,j} + 2\right)\left(\Delta z_{i,j} + 2z_{i,j} + 1\right)\Gamma\left(\Delta z_{i,j} + 2z_{i,j} + 1\right)}{\Gamma\left(\Delta z_{i,j} + 1\right)\Gamma\left(2z_{i,j} + 2\right)}$$
$$= \left(\Delta z_{i,j} + 2z_{i,j} + 2\right)\left(\Delta z_{i,j} + 2z_{i,j} + 1\right) f_{\left(z_{i,j}, \Delta z_{i,j}\right)}$$
$$f\left(z_{i,j}, \Delta z_{i,j}\right) = \frac{1}{B\left(\Delta z_{i,j} + 1, 2z_{i,j} + 2\right)\left(\Delta z_{i,j} + 2z_{i,j} + 2\right)\left(\Delta z_{i,j} + 2z_{i,j} + 1\right)} \tag{10}$$

with the Beta function $B\left(x, y\right)$ and $\Gamma\left(x + 1\right) = x\Gamma\left(x\right)$. Properties and approximations for $B\left(x, y\right)$:

1. $B\left(x, y\right) = B\left(y, x\right)$

2. If x and y are large:
$$B(x, y) \approx \sqrt{2\pi} \frac{x^{x - \frac{1}{2}} y^{y - \frac{1}{2}}}{(x + y)^{x + y - \frac{1}{2}}}$$

3. If x large and y fixed:
$$B(x, y) \approx \Gamma\left(y\right) x^{-y}$$

We will see if this approximations are useful.

6 ...

Until now all was a repetition of [1]. Question: How does the way with 2 make all easier now?

References

[1] ZÖBELEIN, C.: The recursive structure of the distribution of primes. In: *ArXiv e-prints, 1411.2824 [math.GM]* (2014), November