Title: Primes matrix Date: 2018-03-23 01:30 Category: Math Tags: primes, number theory Author: Samdney

Some time ago, I already wrote about representation ideas of primes and we saw that we run in troubles with this. Today, I want to present you a similar approach.

Let's start again with our equation

$$x_{ij} = (2x_i + 1)\, x_j + x_i y_{ij} = \left(2x_{ij} + 1\right)$$

and the following represenation

x_{1j} |00010010010010010010 ... x_{2j} |00000010000100001000 ... $x_{(1,2),j}$|11101101101001100101 ...

The first line is given by $x_{1j} = 3x_j + 1 = 4, 7, 10, 13, 16, 19$. If we look a the numbers from 1 to 20 (from left to right), we represent all numbers which are generated by x_{1j}, by '1' and the other numbers by '0'. In the second line, we do the same for $x_{2j} = 5x_j + 2 = 7, 12, 17$.

In the third line we see $x_{(1,2),j}$, which represents all numbers which are not element of x_{1j} and not element of x_{2j} by '1' and the others by '0'. So we can write

$$x_{(1,2),j} = \overline{x_{1j}} \cdot \overline{x_{2j}}$$

. Ok. What can we do with this, now?

At first, we look at x_{1j} and x_{2j}. We will rewrite them to matrices $X^{n\times n}_{(1)}$ and $X^{n\times n}_{(2)}$. This matrices, all of the same size $n \times n$, have the numbers from the representation above as diagonal entries. All other entries are '0'.

$$X^{n\times n}_{(1)} := \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} = \left(x_{(1),kj}\right)_{k=1,\ldots,n\ \ ,j=1,\ldots,n} \quad \delta_{kj}$$

$$X_{(2)}^{n\times n} := \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} = \left(x_{(2),kj}\right)_{k=1,\ldots,n\ \ ,j=1,\ldots,n}\ \ \delta_{kj}$$

Here are

$$x_{(i),kj} := \begin{cases} 1 & \text{if } k = (2x_i + 1)\, x_l + x_i \\ 0 & \text{else} \end{cases}$$

and

$$\delta_{kj} := \begin{cases} 1 & \text{if } k = j \\ 0 & \text{else} \end{cases}$$

With this, we get $\overline{X_{(i)}^{n\times n}}$ by

$$\overline{X_{(i)}^{n\times n}} = \mathbb{1}_n - X_{(i)}^{n\times n}$$

For an arbitrary number $i = a, \ldots, b$, $a, b \in \mathbb{N}$, $a \leq b$, of equations x_{ij} we receive

$$X_{(a,\ldots,b)}^{n\times n} = \prod_{i=a}^{b} \left(\mathbb{1}_n - X_{(i)}^{n\times n}\right)$$

and so

$$x_{(a,\ldots,b),kj} = \prod_{i=a}^{b} \left(1 - x_{(i),kj}\right) \delta_{kj}$$

We received a matrix with ‘1’ entries at the places $j = k$ which represent primes and else ‘0’.