Title: Let's calculate primes! Part III - Intersection of times tables: sum formula
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In the post before, we looked at the intersection ot times tables and its problems.
Now we will look at the belonging sum formula, which is given by

$$\sum_{x_i=l'}^{u'} \sum_{x_j=l}^{u} 10^{(2x_i+1)x_j+x_i}$$

and with our results before as

$$\sum_{x_i=l'}^{u'} \left( -\frac{10^{(2x_i+1)l+x_i} - 10^{(2x_i+1)(u+1)+x_i}}{10^{2x_i+1}-1} \right)$$

.

We are not able to solve this second sum analytical, but it can be interesting
to look a bit around this sum. I had a look what wolframalpha.com is able
to tell me about our first sum result and I found one thing which catched my
attention.

$$\int -\frac{10^x \left( 10^{l(2x+1)} - 10^{(u+1)(2x+1)} \right)}{10^{2x+1}-1} \mathrm{d}x$$

$$= \frac{10^x \left( \frac{10^{2lx+l} {}_2F_1\left(1,l+\frac{1}{2},l+\frac{3}{2},10^{2x+1}\right)}{2l+1} - \frac{10^{(u+1)(2x+1)} {}_2F_1\left(1,u+\frac{3}{2},u+\frac{5}{2},10^{2x+1}\right)}{2u+3} \right)}{\log(10)} + constant,$$

with

$$_2F_1\left(a,b;c;x\right)$$

is the hypergeometric function.

It's deeply interesting that we are able to find an integral which is connected
to the Gamma-Function over the hypergeometric function and hence to primes.
Althougth we are not able to do both sums exactly, this gives us an hint, that we
are maybe able to do the first calculation with a sum and the second calculation
with the help of an integration.

At the moment, we can't use this information, but we should keep this in mind.