WIKIPEDIA

Character (mathematics)

In mathematics, a **character** is (most commonly) a special kind of function from a group to a field (such as the complex numbers). There are at least two distinct, but overlapping meanings.[1] Other uses of the word "character" are almost always qualified.

Contents

Multiplicative character

A **multiplicative character** (or **linear character**, or simply **character**) on a group G is a group homomorphism from G to the multiplicative group of a field (Artin 1966), usually the field of complex numbers. If G is any group, then the set Ch(G) of these morphisms forms an abelian group under pointwise multiplication.

This group is referred to as the character group of G. Sometimes only *unitary* characters are considered (thus the image is in the unit circle); other such homomorphisms are then called *quasi-characters*. Dirichlet characters can be seen as a special case of this definition.

Multiplicative characters are linearly independent, i.e. if $\chi_1, \chi_2, \ldots, \chi_n$ are different characters on a group G then from $a_1\chi_1 + a_2\chi_2 + \ldots + a_n\chi_n = 0$ it follows that $a_1 = a_2 = \cdots = a_n = 0$.

Character of a representation

The **character of a representation** φ of a group G on a finite-dimensional vector space V over a field F is the trace of the representation φ (Serre 1977). In general, the trace is not a group homomorphism, nor does the set of traces form a group. The characters of one-dimensional representations are identical to one-dimensional representations, so the above notion of multiplicative character can be seen as a special case of higher-dimensional characters. The study of representations using characters is called "character theory" and one dimensional characters are also called "linear characters" within this context.

See also

- Dirichlet character
- Harish-Chandra character
- Hecke character
- Infinitesimal character
- Alternating character

References

1. "character in nLab" (https://ncatlab.org/nlab/show/character). *ncatlab.org*. Retrieved 2017-10-31.

- Artin, Emil (1966), *Galois Theory*, Notre Dame Mathematical Lectures, number 2, Arthur Norton Milgram (Reprinted Dover Publications, 1997), ISBN 978-0-486-62342-9 Lectures Delivered at the University of Notre Dame
- Serre, Jean-Pierre (1977), *Linear Representations of Finite Groups*, Graduate Texts in Mathematics, **42**, Translated from the second French edition by Leonard L. Scott, New York-Heidelberg: Springer-Verlag, doi:10.1007/978-1-4684-9458-7, ISBN 0-387-90190-6, MR 0450380

External links

- Hazewinkel, Michiel, ed. (2001) [1994], "Character of a group", *Encyclopedia of Mathematics*, Springer Science+Business Media B.V. / Kluwer Academic Publishers, ISBN 978-1-55608-010-4
- "Character of a group representation". *PlanetMath*.