

Notes: Complex boolean functions

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DRAFT

Abstract

We examine the boolean functions **NEG**, **AND** and **OR** on the field of complex numbers \mathbb{C} with boolean values $\{i, 1\}$ and the map between them and the classical definition with $\{0, 1\}$.

Keywords: XXX, YYY, ZZZ

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Preamble

The following content is a sketch for discussion purposes only, without warranty for mathematical completeness.

1 Introduction

In this short notes, we will do an analysis of boolean functions on the field of complex numbers \mathbb{C} for a specific defined case regarding the values *True* and *False*, based on the work of Ryan ODonnell [1], Zilong Wang and Guang Gong [2] and Zhiwei Zhang [3] [4].

2 Preview

Now, a short preview of the final results. Given are integer boolean values $x \in \{0, 1\}$ and complex values $z \in \mathbb{C}$. For the boolean cases **NEG**, **AND** and **OR** we will use the two complex boolean values $\{i, 1\}$.

We will denote the inverse boolean value of x by \bar{x} respectively for z by \bar{z} . For the complex conjugate of z , we will write z^c .

In table 1, you can see the values for **NEG** of x and z , which gives us several statements.

Theorem 2.1 (**NEG** of x). *The inverse boolean value of x , generated by $\text{NEG}(x)$, is given by $\bar{x} = 1 - x$.*

Theorem 2.2 (**NEG** of z). *The inverse boolean value of z , generated by $\text{NEG}(z)$, is given by $\bar{z} = (1 + i) - z$ and by $\bar{z} = z^3 i$.*

For **AND** and **OR** we need an helper boolean function, called *generator*.

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Table 1: NEG

x	$-x$	\bar{x}	z	$-z$	\bar{z}
0	0	1	i	-i	1
1	-1	0	1	-1	i

Theorem 2.3 (Generator g of (x_i, x_j)). *The generator g of (x_i, x_j) , which maps $(0, 0) \mapsto 1$, $(0, 1) \mapsto 0$, $(1, 0) \mapsto 0$ and $(1, 1) \mapsto 0$, is given by*

$$g(x_i, x_j) = \left((x_i x_j + (x_i + x_j) - 1) \frac{1}{2} - x_i x_j \right) (-2). \quad (1)$$

Theorem 2.4 (Generator g of (z_i, z_j)). *The generator g of (z_i, z_j) , which maps $(i, i) \mapsto i$, $(i, 1) \mapsto 0$, $(1, i) \mapsto 0$ and $(1, 1) \mapsto 0$, is given by*

$$g(z_i, z_j) = (z_i z_j + (z_i + z_j) - 1) \frac{1}{2} - z_i z_j. \quad (2)$$

In table 2, you can see the values for **AND** of (x_i, x_j) and (z_i, z_j) .

Table 2: AND

x_i	x_j	$x_i x_j$	$x_i \wedge x_j$	z_i	z_j	$z_i z_j$	$z_i \wedge z_j$
0	0	0	0	i	i	-1	i
0	1	0	0	i	1	i	i
1	0	0	0	1	i	i	i
1	1	1	1	1	1	1	1

Theorem 2.5 (AND of (x_i, x_j)). *The boolean value of $x_i \wedge x_j$, generated by $AND(x_i, x_j)$, is given by*

$$x_i \wedge x_j = x_i x_j. \quad (3)$$

Theorem 2.6 (AND of (z_i, z_j)). *The boolean value of $z_i \wedge z_j$, generated by $AND(z_i, z_j)$, is given by*

$$z_i \wedge z_j = z_i z_j + (1 - i) g(z_i, z_j). \quad (4)$$

Finally, in table 3, you can see the values for **OR** of (x_i, x_j) and (z_i, z_j) .

Table 3: OR

x_i	x_j	$x_i + x_j$	$x_i \vee x_j$	z_i	z_j	$z_i + z_j$	$z_i \vee z_j$
0	0	0	0	i	i	2i	i
0	1	1	1	i	1	1 + i	1
1	0	1	1	1	i	1 + i	1
1	1	2	1	1	1	2	1

Theorem 2.7 (OR of (x_i, x_j)). *The boolean value of $x_i \vee x_j$, generated by $OR(x_i, x_j)$, is given by*

$$x_i \vee x_j = (x_i + x_j) - g(\bar{x}_i, \bar{x}_j). \quad (5)$$

Theorem 2.8 (OR of (z_i, z_j)). *The boolean value of $z_i \vee z_j$, generated by $OR(z_i, z_j)$, is given by*

$$z_i \vee z_j = (z_i + z_j) - i + (1 + i)g(\bar{z}_i, \bar{z}_j). \quad (6)$$

3 Conclusion

We will see, that our results will be helpful in future works.

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