So, we have our new equations:

New!

$$\left.\begin{array}{l} \text{I}': \quad \bar{n}_1' = (2x_1+1)\,y_1' - \Delta x_{1,2}\,\Delta_1 \quad , \Delta_1 \in \{1,\dots,2x_1\} \\ \text{II}': \quad \bar{n}_2' = (2x_2+1)\,y_2' - \Delta x_{1,2}\,\Delta_2 \quad , \Delta_2 \in \{1,\dots,2x_2\} \end{array}\right\} \quad \text{(eq. 6)}$$

For the intersection $\bar{n}_1' = \bar{n}_2$,

we have the solutions:

$$\left.\begin{array}{l} \text{III}'. \quad y_1' = x_2\cdot(-\Delta_2+\Delta_1) + (2x_2+1)\,z_{1,2} \quad , z_{1,2} \in \mathbb{Z} \\ \text{IV}'. \quad y_2' = x_1\cdot(-\Delta_2+\Delta_1) + (2x_1+1)\,z_{1,2} \end{array}\right\} \quad \text{(eq. 7)}$$

and hence finally:

$$\bar{n}_{1,2}' = (2x_1+1)\,y_1' - \Delta x_{1,2}\Delta_1$$

$$= (2x_1+1)\left[x_2(-\Delta_2+\Delta_1) + (2x_2+1)\,z_{1,2}\right] - \Delta x_{1,2}\Delta_1$$

$$\boxed{\bar{n}_{1,2}' = (2x_1+1)(2x_2+1)\,z_{1,2} + \underbrace{(2x_1+1)(-\Delta_2+\Delta_1)x_2 - \Delta x_{1,2}\Delta_1}_{(*)}} \quad \text{(eq. 81)}$$

$$\bar{n}_{1,2}' = (2x_2+1)\,y_2' - \Delta x_{1,2}\Delta_2$$

$$= (2x_2+1)\left[x_1\cdot(-\Delta_2+\Delta_1) + (2x_1+1)\,z_{1,2}\right] - \Delta x_{1,2}\Delta_2$$

$$\boxed{\bar{n}_{1,2}' = (2x_1+1)(2x_2+1)\,z_{1,2} + \underbrace{(2x_2+1)(-\Delta_2+\Delta_1)x_1 - \Delta x_{1,2}\Delta_2}_{(**)}} \quad \text{(eq. 82)}$$

~~$(*) = (2x_1x_2 + x_2)(-\Delta_2+\Delta_1) - \Delta x_{1,2}\Delta_1$~~

$*\,(x_1 = x_2 - \Delta x_{1,2})$

With $x_2 = x_1 + \Delta x_{1,2}$, it follows:

$$(**) = (2(x_1+\Delta x_{1,2})+1)(-\Delta_2+\Delta_1)\overbrace{(x_2-\Delta x_{1,2})} - \Delta x_{1,2}\Delta_2$$

$$\left(= (2x_1+1)(-\Delta_2+\Delta_1)x_1 + 2\Delta x_{1,2}(-\Delta_2+\Delta_1)(x_2-\Delta x_{1,2}) - \Delta x_{1,2}\Delta_2\right),$$

$$= (2x_1+1)(-\Delta_2+\Delta_1)(x_2-\Delta x_{1,2}) + 2\Delta x_{1,2}(-\Delta_2+\Delta_1)(x_2-\Delta x_{1,2}) - \Delta x_{1,2}\Delta_2$$

$$= (2x_1+1)(-\Delta_2+\Delta_1)x_2 - (2x_1+1)(-\Delta_2+\Delta_1)\Delta x_{1,2} + 2\Delta x_{1,2}(-\Delta_2+\Delta_1)(\overbrace{x_2-\Delta x_{1,2}}^{x_1}) - \Delta x_{1,2}\Delta_2$$