VII → Be given

$$n_1 = (2x_1 + 1)y_1 + \alpha \qquad x_i, y_i \in \mathbb{N}$$

and $$n_2 = (2x_2 + 1)y_2 + \beta$$

with $x_2 > x_1$.

→ So, we we can write $x_2 = x_1 + \Delta x_{1,2}$, $\Delta x_{1,2} \in \mathbb{N}$

$$\Rightarrow \quad n_1 = (2x_1 + 1)y_1 + \alpha$$

$$n_2 = (2 \cdot (x_1 + \Delta x_{1,2}) + 1) y_2 + \beta$$

→ Now, let's assume:

$$\boxed{\Delta x_{1,2} = (2x_1 + 1)}$$

$$\begin{aligned}
\Rightarrow n_2 &= (2 \cdot (x_1 + (2x_1 + 1)) + 1) y_2 + \beta \\
&= (2 \cdot (x_1 + 2x_1 + 1) + 1) y_2 + \beta \\
&= (2 \cdot (3x_1 + 1) + 1) y_2 + \beta \\
&= (6x_1 + 2 + 1) y_2 + \beta \\
&= (6x_1 + 3) y_2 + \beta \\
&= 3 \cdot (2x_1 + 1) y_2 + \beta \\
n_2 &= (2x_1 + 1)(\underbrace{3y_2}_{=:\tilde{y}_2}) + \beta \\
&= (2x_1 + 1)\tilde{y}_2 + \beta
\end{aligned}$$

→ Now, look at

$$(2\tilde{x} + 1) = \tilde{n}_{1,2} = \underbrace{(2x_1 + 1)(2x_2 + 1)}_{\substack{= 2 \cdot 2x_1x_2 + 2x_1 + 2x_2 + 1 \\ = 2 \cdot (2x_1x_2 + x_1 + x_2) + 1}} z_{1,2} + \underbrace{(2x_1 + 1)(-\Delta_2 + \Delta_1)x_2 - \Delta x_{1,2}\Delta_1}_{=:\gamma}$$

Be $\tilde{x} = 2x_1x_2 + x_1 + x_2 - \tilde{\Delta}x_{12}$

$$\Rightarrow 2 \cdot (2x_1x_2 + x_1 + x_2 - \tilde{\Delta}x_{12}) + 1 = (2(2x_1x_2 + x_1 + x_2) + 1) z_{1,2} + \gamma$$

$$\Leftrightarrow 2(2x_1x_2 + x_1 + x_2) - 2\tilde{\Delta}x_{12} + 1 = (2(2x_1x_2 + x_1 + x_2) + 1) z_{1,2} + \gamma$$

$$\Leftrightarrow 2(2x_1x_2 + x_1 + x_2) - 2(2x_1x_2 + x_1 + x_2) z_{1,2} = 2z_{1,2} + 2\tilde{\Delta}x_{12} - 1 + \gamma$$

$$\Leftrightarrow \cancel{2(2x_1x_2 + x_1 + x_2) \cdot (1 - z_{1,2}) = 2(z_{1,2} + \tilde{\Delta}_{1,2}) - 1 + \gamma}$$

$$\Leftrightarrow \cancel{2(2x_1x_2 + x_1 + x_2)(1 - z_{1,2}) = 2 \cdot (z_{1,2} - 1) + 1 + 2\tilde{\Delta}_{1,2} + \gamma}$$