$$\cancel{\Leftrightarrow 2(2x_1x_2+x_1+x_2)(1-z_{1,2}) = -2(1-z_{1,2})+1+2\hat{\Delta}_{1,2}+\gamma}$$

$$\cancel{\Leftrightarrow 2(2x_1x_2+x_1+x_2)(1-z_{1,2})+(1-z_{1,2}) = -(1-z_{1,2})+1+2\hat{\Delta}_{1,2}+\gamma}$$

$$\cancel{\Leftrightarrow \left(2(2x_1x_2+x_1+x_2)+1\right)(1-z_{1,2}) = -1+z_{1,2}+1+2\hat{\Delta}x_{1,2}+\gamma}$$

$$\cancel{\Leftrightarrow (2x_1+1)(2x_2+1)(1-z_{1,2}) = z_{1,2}+2\tilde{\Delta}x_{1,2}+\gamma}$$

$$\Leftrightarrow 2(2x_1x_2+x_1+x_2)-2\hat{\Delta}x_{1,2}+1 = \left(2(2x_1x_2+x_1+x_2)+1\right)z_{1,2}+\gamma$$

$$\Leftrightarrow (2x_1+1)(2x_2+1)-2\hat{\Delta}x_{1,2} = (2x_1+1)(2x_2+1)z_{1,2}+\gamma$$

$$\Leftrightarrow (2x_1+1)(2x_2+1)-(2x_1+1)(2x_2+1)z_{1,2} = 2\hat{\Delta}x_{1,2}+\gamma$$

$$\Leftrightarrow (2x_1+1)(2x_2+1)(1-z_{1,2}) = 2\tilde{\Delta}x_{1,2}+\gamma$$

$$\Leftrightarrow (2x_1+1)(2x_2+1)(1-z_{1,2}) - \gamma = 2\tilde{\Delta}x_{1,2}$$

→ We know that we have for odd solutions $\tilde{n} = 2\tilde{x}+1$,
always $2\Delta_i^\circ$. $\Delta_i^\circ \in \frac{1}{2}\{1,..,2x_i\}$

$\Delta_i \in$

$$\Leftrightarrow \tilde{\Delta}x_{1,2} = \underbrace{\underbrace{(2x_1+1)(2x_2+1)(1-z_{1,2})\cdot\tfrac{1}{2}}_{①}}_{\text{this part is always: } =(2x_1+1)\cdot u \text{ respectively } =(2x_2+1)u'} - \underbrace{\tfrac{1}{2}\gamma}_{②} \qquad ✱$$

(eq. 20)

we can resubstitute this, with

$$\frac{(1-z_{1,2})}{2} = z^{new}_{1,2}$$

for all $z_{1,2}$ odd

this part is always:

respectively $= (2x_1+1)\cdot u$

$= (2x_2+1)u'$

✱✱ $\forall u_1,u_2 \in \mathbb{N}$

$\tilde{\Delta}x_{1,2} \neq (2x_1+1)u_1$

and $\tilde{\Delta}x_{1,2} \neq (2x_2+1)u_2$

⇒ so we can always use our $\Delta\tilde{x}_{1,2}$ from recursion for the next step, in the valid range

<u>To ②:</u> → From the deffinition of our original equation $\bar{n}_i$,
we know that all results are between
$(2x_i+1)u$ and $(2x_i+1)(u+1)$, $\forall u \in \mathbb{N}$.

excluded $(2x_i+1)u$ $\forall u \in \mathbb{N}$

<u>and</u> → From this follows, that also all values
from $\bar{n}_{ij}$, are between $(2x_1+1)(2x_2+1)u$
and $(2x_1+1)(2x_2+1)(u+1)$
$\forall u \in \mathbb{N}$,

$(2x_i+1)(2x_2+1)u$ excluded $\forall u \in \mathbb{N}$

⟹ all values of ✱ are not
element of $(2x_1+1)$ times table <u>and</u> not
element of $(2x_2+1)$ times table <u>and</u> not element of
$(2x_1+1)(2x_2+1)$ times table ⟹ ✱✱