Case: $\Delta x_{1,2} = 1$:

$$\Rightarrow \frac{2 \cdot 1 \cdot x_1(-\Delta_2+\Delta_1) + (-\Delta_2+\Delta_1)}{2x_1+1}$$

$$= \frac{2x_1(-\Delta_2+\Delta_1) + (-\Delta_2+\Delta_1)}{2x_1+1}$$

$$= \frac{(-\Delta_2+\Delta_1)\cdot(2x_1+1)}{2x_1+1}$$

$$= (-\Delta_2+\Delta_1) \quad \checkmark$$

Case: $\Delta x_{1,2} > 1$:

$$\Rightarrow \frac{2 \cdot \Delta x_{1,2} \cdot x_1(-\Delta_2+\Delta_1) + (-\Delta_2+\Delta_1)}{2x_1+1}$$

$$= ?$$

, we see, in this case, we have a problem to find integer solutions.

Let's Assume instead of (eq.4), we have

New!

$$\frac{2\Delta x_{1,2}\, x_1(-\Delta_2+\Delta_1) + \Delta x_{1,2}(-\Delta_2+\Delta_1)}{2x_1+1} \qquad \text{(eq.5)}$$

Now, we can solve our intersection:

$$y_1 = y_2 + \frac{2\Delta x_{1,2} y_2 + (-\Delta_2+\Delta_1)\cdot \Delta x_{1,2}}{2x_1+1}$$

with $y_2 := x_1(-\Delta_2+\Delta_1) + (2x_1+1)z_{1,2}$, $z_{1,2} \in \mathbb{Z}$

$y_1 = x_2\cdot(-\Delta_2+\Delta_1) + (2x_2+1)z_{1,2}$

Test: $y_1 = x_1(-\Delta_2+\Delta_1) + (2x_1+1)z_{1,2} + \frac{2\Delta x_{1,2} x_1(-\Delta_2+\Delta_1)}{2x_1+1}$

$$+ \frac{2\Delta x_{1,2}(2x_1+1)z_{1,2}}{(2x_1+1)} + \frac{(-\Delta_2+\Delta_1)\Delta x_{1,2}}{2x_1+1}$$

$$= x_1(-\Delta_2+\Delta_1) + (2x_1+1)z_{1,2} + 2\Delta x_{1,2} z_{1,2} + \frac{(-\Delta_2+\Delta_1)\Delta x_{1,2}(2x_1+1)}{2x_1+1}$$

because of $x_2 = x_1 + \Delta x_{1,2}$

$$= x_2(-\Delta_2+\Delta_1) + (2x_2+1)z_{1,2} \quad \checkmark$$