Given be $\bar{n}_i' = (2x_i+1)y_i' - \Delta x_{i,j}\Delta_i$ with $\Delta_i \in \{1,\dots,2x_i\}$

from $\bar{n}_i = (2x_i+1)y_i - \Delta_i$, $x_i, y_i \in \mathbb{N}$

Now since this function shall start with $y_i \geq 2$ –

But, we would still like to have $\mathbb{N}$ as definition set for x_i and y_i, we do a substitution.

$$\boxed{y_i = \tilde{y}_i + 1 \quad , \quad \tilde{y}_i \in \mathbb{N}} \Leftrightarrow \boxed{\tilde{y}_i = y_i - 1}$$

$$\Rightarrow \bar{n}_i = (2x_i+1)(\tilde{y}_i+1) - \Delta_i$$

$$\Rightarrow \boxed{\bar{n}_i = (2x_i+1)\tilde{y}_i + (2x_i+1) - \Delta_i} \quad \text{(eq.11)}$$

and for

$\underline{\bar{n}_i'}$ $\cancel{\tilde{y}_i' = \tilde{y}_i + \Delta}$

$$y_i' = y_i + \frac{\Delta_i(-1+\Delta x_{i,j})}{2x_i+1}$$

$$\tilde{y}_i' + 1 = \tilde{y}_i + 1 + \text{〃 〃 〃}$$

$$\Leftrightarrow \tilde{y}_i' = \tilde{y}_i + \frac{\Delta_i(-1+\Delta x_{i,j})}{2x_i+1}$$

$\Rightarrow$ The same relationship like before

$$\Rightarrow \bar{n}_i' = (2x_i+1)(\tilde{y}_i'+1) - \Delta x_{i,j}\Delta_i$$

$$\Rightarrow \boxed{\bar{n}_i' = (2x_i+1)\tilde{y}_i' + (2x_i+1) - \Delta x_{i,j}\Delta_i} \quad \text{(eq.12)}$$

<u>Intersection</u>:

$$\bar{n}_1' = (2x_1+1)\tilde{y}_1' + (2x_1+1) - \Delta x_{1,2}\Delta_1$$

$$\bar{n}_2' = (2x_2+1)\tilde{y}_2' + (2x_2+1) - \Delta x_{1,2}\Delta_2$$

$$(2x_1+1)\tilde{y}_1' + (2x_1+1) - \Delta x_{1,2}\Delta_1 = (2x_2+1)\tilde{y}_2' + (2x_2+1) - \Delta x_{1,2}\Delta_2$$

$$\cancel{\tilde{y}_1' = \frac{2}{2x_1+1}[(2x_2+1)\tilde{y}_2']} \qquad = 2x_1 + 1 + 2\Delta x_{1,2} - 2x_1 - 1$$

$$0 = (2x_2+1)\tilde{y}_2' - (2x_1+1)\tilde{y}_1' + \overbrace{(2x_2+1) - (2x_1+1)} - \Delta x_{1,2}\Delta_2 + \Delta x_{1,2}\Delta_1$$

Be $x_2 = x_1 + \Delta x_{1,2}$

$$0 = (2x_1+1)\tilde{y}_2' + 2\Delta x_{1,2}\tilde{y}_2' - (2x_1+1)\tilde{y}_1' + 2\Delta x_{1,2} - \Delta x_{1,2}\Delta_2 + \Delta x_{1,2}\Delta_1$$