Research
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RESEARCH NOTES

Primes (part 02): Recursion - First-step valid solutions

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Abstract

This notes gives a short overview over the first-recursion-step valid solutions.

Content

First recursion steps

Assume, we know: $p_1 = 3 \,,\, p_2 = 5$

$$\Rightarrow \text{I. } \bar{n}_1 = 3(2k_1+1) - 2\Delta_1 \quad , \Delta_1 \in \{1,2\} \qquad , k_1, k_2 \in \mathbb{N}$$
$$\text{II. } \bar{n}_2 = 5(2k_2+1) - 2\Delta_2 \quad , \Delta_2 \in \{1,2,3,4\}$$

n_1: 9, 15, 21, 27, 33, 39, ...

n_2: 15, 25, 35, 45, 55, 65, ...

$n_3 = (2(x_2+1)+1) = 2\cdot 3+1 = 7$: 21, 35, 49, 63, 77, 91, ...

$\Rightarrow$ From the knowledge of $p_1=3$ and $p_2=5 \in \mathbb{P}$, ($\mathbb{P}$ set of Primes)

1, 2, 3 | 4, 5 | 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, ...

$\Rightarrow$ our allowed values: $k_1 = 1,2 \;,\; \Delta_1 \in \{1, \overset{k_1=1}{2}\}$

$k_2 = 1 \;,\; \Delta_1 \in \{1,2,3,4\}$

$$\Rightarrow \left\{ \begin{array}{l} k_1 = 1 \;,\; \Delta_1 = \{1\} \\ k_1 = 2 \;,\; \Delta_1 = \{1,2\} \\ k_2 = 1,2 \;,\; \Delta_2 = \{1,2,3,4\} \end{array} \right.$$

$\rightarrow$ Since $\Delta x_{1,2} = 1 \Rightarrow \bar{n}_1' = \bar{n}_1$ and $\bar{n}_2' = \bar{n}_2$

$$\Rightarrow \bar{n}_1' = \bar{n}_2'$$
$$\bar{n}_1 = \bar{n}_2$$
$$3(2k_1+1) - 2\Delta_1 = 5(2k_2+1) - 2\Delta_2$$
$$\Leftrightarrow \qquad 0 = 5(2k_2+1) - 3(2k_1+1) - 2\Delta_2 + 2\Delta_1$$

$$\Rightarrow \left\| \begin{array}{l} k_1 = (2x_2+1)z_{1,2} + (-\Delta_2+\Delta_1+1)x_2 \\ k_2 = (2x_1+1)z_{1,2} + (-\Delta_2+\Delta_1+1)x_1 \end{array} \right.$$

$$\Rightarrow \left\| \begin{array}{l} k_1 = 5z_{1,2} + (-\Delta_2+\Delta_1+1)\cdot 2 \\ k_2 = 3z_{1,2} + (-\Delta_2+\Delta_1+1)\cdot 1 \end{array} \right.$$

$$\Rightarrow \quad \bar{n}_1 = 3(2k_1+1) - 2\Delta_1$$
$$= 3\left[2\cdot(5z_{1,2} + (-\Delta_2+\Delta_1+1)\cdot 2) + 1\right] - 2\Delta_1$$

$$\bar{n}_1' = \bar{n}_1 = 3\cdot 2\cdot 5\cdot z_{1,2} + 3\cdot 2\cdot 2(-\Delta_2+\Delta_1+1) + 3 - 2\Delta_1$$

$$k_1 = (2x_2+1)\, z_{1,2} + (-\Delta_2 + \Delta_1 + 1)\, x_2$$
$$k_2 = (2x_1+1)\, z_{1,2} + (-\Delta_2 + \Delta_1 + 1)\, x_1$$

$$\Rightarrow \quad z_{1,2} = \frac{1}{(2x_2+1)} \left[k_1 - (-\Delta_2 + \Delta_1 + 1)\, x_2 \right] \quad (*1)$$
$$\Leftrightarrow \quad z_{1,2} = \frac{1}{(2x_1+1)} \left[k_2 - (-\Delta_2 + \Delta_1 + 1)\, x_1 \right] \quad (*2)$$

<u>with:</u> $k_1 = 1 \quad \Delta_1 = \{1\}$ and $k_1 = 2 \quad \Delta_1 = \{1, 2\}$

$$(*1): \quad z_{1,2}(k_1 = 1, \Delta_1 = 1) = \frac{1}{5} \left[1 - (-\Delta_2 + 1 + 1) \cdot 2 \right]$$

$\Delta_2 = \{1, 2, 3, 4\}$

$\lceil\rceil$, $\lfloor\rfloor$ solutions

$$= \frac{1}{5} \left[1 - (-\Delta_2 + 2) \cdot 2 \right]$$
$$= \frac{1}{5} \left[1 + 2\Delta_2 - 4 \right]$$

$$\boxed{z_{1,2}(k_1 = 1, \Delta_1 = 1) = \frac{1}{5} \left[2\Delta_2 - 3 \right]}$$

$\underline{z_{1,2} = 0} : \Delta_2 = 1$
$z_{1,2} = 0 : \Delta_2 = 2$
$z_{1,2} = 0 : \Delta_2 = 3$
$z_{1,2} = 1 : \Delta_2 = 4$

$\Delta_2 = 1: \quad z_{1,2} = -\frac{1}{5}$
$\Delta_2 = 2: \quad z_{1,2} = \frac{1}{5}$
$\Delta_2 = 3: \quad z_{1,2} = \frac{3}{5}$
$\Delta_2 = 4: \quad z_{1,2} = \frac{5}{5} = 1$

$$z_{1,2}(k_1 = 2, \Delta_1 = 1) = \frac{1}{5} \left[2 - (-\Delta_2 + 1 + 1) \cdot 2 \right]$$
$$= \frac{1}{5} \left[2 + 2\Delta_2 - 4 \right]$$

$$\boxed{z_{1,2}(k_1 = 2, \Delta_1 = 1) = \frac{1}{5} \left[2\Delta_2 - 2 \right]}$$

$z_{1,2} = 0 : \Delta_2 = 1$
$z_{1,2} = 0 : \Delta_2 = 2$
$z_{1,2} = 0 : \Delta_2 = 3$
$z_{1,2} = 1 : \Delta_2 = 4$

$\Delta_2 = 1: \quad z_{1,2} = \frac{0}{5} = 0$
$\Delta_2 = 2: \quad z_{1,2} = \frac{2}{5}$
$\Delta_2 = 3: \quad z_{1,2} = \frac{4}{5}$
$\Delta_2 = 4: \quad z_{1,2} = \frac{6}{5}$

$$z_{1,2}(k_1 = 2, \Delta_1 = 2) = \frac{1}{5} \left[2 - (-\Delta_2 + 2 + 1) \cdot 2 \right]$$
$$= \frac{1}{5} \left[2 + 2\Delta_2 - 6 \right]$$

$$\boxed{z_{1,2}(k_1 = 2, \Delta_1 = 2) = \frac{1}{5} \left[2\Delta_2 - 4 \right]}$$

$\underline{z_{1,2} = 0} : \Delta_2 = 1$
$z_{1,2} = 0 : \Delta_2 = 2$
$z_{1,2} = 0 : \Delta_2 = 3$
$z_{1,} \quad : \Delta_2 = 4$

$\Delta_2 = 1: \quad z_{1,2} = -\frac{2}{5}$
$\Delta_2 = 2: \quad z_{1,2} = \frac{0}{5} = 0$
$\Delta_2 = 3: \quad z_{1,2} = \frac{2}{5}$
$\Delta_2 = 4: \quad z_{1,2} = \frac{4}{5}$

with $u_2=1$ $\Delta_2=\{1,2,3,4\}$ and $u_2=2$ $\Delta_2=\{1,2,3,4\}$

$(*^2)$: $z_{1,2}(u_2=1,\Delta_2=1) = \frac{1}{3}[1-(-1+\Delta_1+1)\cdot 1]$

$\left.\begin{array}{l} z_{1,2}=0 : \Delta_1=1 \\ z_{1,2}=0 : \Delta_1=2 \end{array}\right\}$ $\boxed{z_{1,2}(u_2=1,\Delta_2=1) = \frac{1}{3}[1-\Delta_1]}$ $\left\{\begin{array}{l} \Delta_1=1: z_{1,2}=\frac{0}{3}=0 \\ \Delta_1=2: z_{1,2}=-\frac{1}{3} \end{array}\right.$

$z_{1,2}(u_2=1,\Delta_2=2) = \frac{1}{3}[1-(-2+\Delta_1+1)\cdot 1]$

$= \frac{1}{3}[1+1-\Delta_1]$

$\left.\begin{array}{l} z_{1,2}=0 : \Delta_1=1 \\ z_{1,2}=0 : \Delta_1=2 \end{array}\right\}$ $\boxed{z_{1,2}(u_2=1,\Delta_2=2) = \frac{1}{3}[2-\Delta_1]}$ $\left\{\begin{array}{l} \Delta_1=1: z_{1,2}=\frac{1}{3} \\ \Delta_1=2: z_{1,2}=\frac{0}{3}=0 \end{array}\right.$

$z_{1,2}(u_2=1,\Delta_2=3) = \frac{1}{3}[1-(-3+\Delta_1+1)\cdot 1]$

$= \frac{1}{3}[1+2-\Delta_1]$

$\left.\begin{array}{l} z_{1,2}=0 : \Delta_1=1 \\ z_{1,2}=0 : \Delta_1=2 \end{array}\right\}$ $\boxed{z_{1,2}(u_2=1,\Delta_2=3) = \frac{1}{3}[3-\Delta_1]}$ $\left\{\begin{array}{l} \Delta_1=1: z_{1,2}=\frac{2}{3} \\ \Delta_1=2: z_{1,2}=\frac{1}{3} \end{array}\right.$

$z_{1,2}(u_2=1,\Delta_2=4) = \frac{1}{3}[1-(-4+\Delta_1+1)\cdot 1]$

$= \frac{1}{3}[1+3-\Delta_1]$

$\left.\begin{array}{l} z_{1,2}=1 : \Delta_1=1 \\ z_{1,2}=0 : \Delta_1=2 \end{array}\right\}$ $\boxed{z_{1,2}(u_2=1,\Delta_2=4) = \frac{1}{3}[4-\Delta_1]}$ $\left\{\begin{array}{l} \Delta_1=1: z_{1,2}=\frac{3}{3}=1 \\ \Delta_1=2: z_{1,2}=\frac{2}{3} \end{array}\right.$

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$z_{1,2}(u_2=2,\Delta_2=1) = \frac{1}{3}[2-(-1+\Delta_1+1)\cdot 1]$

$\left.\begin{array}{l} z_{1,2}=0 : \Delta_1=1 \\ z_{1,2}=0 : \Delta_1=2 \end{array}\right\}$ $\boxed{z_{1,2}(u_2=2,\Delta_2=1) = \frac{1}{3}[2-\Delta_1]}$ $\left\{\begin{array}{l} \Delta_1=1: z_{1,2}=\frac{1}{3} \\ \Delta_1=2: z_{1,2}=\frac{0}{3}=0 \end{array}\right.$

$z_{1,2}(u_2=2,\Delta_2=2) = \frac{1}{3}[2-(-2+\Delta_1+1)\cdot 1]$

$= \frac{1}{3}[2+1-\Delta_1]$

$\left.\begin{array}{l} z_{1,2}=0 : \Delta_1=1 \\ z_{1,2}=0 : \Delta_1=2 \end{array}\right\}$ $\boxed{z_{1,2}(u_2=2,\Delta_2=2) = \frac{1}{3}[3-\Delta_1]}$ $\left\{\begin{array}{l} \Delta_1=1: z_{1,2}=\frac{2}{3} \\ \Delta_1=2: z_{1,2}=\frac{1}{3} \end{array}\right.$

$z_{1,2}(u_2=2,\Delta_2=3) = \frac{1}{3}[2-(-3+\Delta_1+1)\cdot 1]$

$= \frac{1}{3}[2+2-\Delta_1]$

$\left.\begin{array}{l} z_{1,2}=1 : \Delta_1=1 \\ z_{1,2}=0 : \Delta_1=2 \end{array}\right\}$ $\boxed{z_{1,2}(u_2=2,\Delta_2=3) = \frac{1}{3}[4-\Delta_1]}$ $\left\{\begin{array}{l} \Delta_1=1: z_{1,2}=\frac{3}{3}=1 \\ \Delta_1=2: z_{1,2}=\frac{2}{3} \end{array}\right.$

$$z_{1,2}(u_2=2, \Delta_2=4) = \frac{1}{3}\left[2 - (-4+\Delta_1+1)\cdot 1\right]$$

$$= \frac{1}{3}\left[2+3-\Delta_1\right]$$

$: \Delta_1 = 1$

$z_{1,2} = 1$

$: \Delta_1 = 2$

$z_{1,2} = 1$

$$\boxed{z_{1,2}(u_2=2, \Delta_2=4) = \frac{1}{3}\left[5-\Delta_1\right]}$$

$\Delta_1 = 1 : z_{1,2} = 4/3$

$\Delta_1 = 2 : z_{1,2} = 3/3$

$= 1$

$$3\cdot 2\cdot 2\,(-\Delta_2+\Delta_1+1)+3-2\Delta_1 \quad , \Delta_1=\{1,2\}$$

$$=\underbrace{12\,(-\Delta_2+\Delta_1+1)+3-2\Delta_1}_{(*)} \quad \Delta_2=\{1,2,3,4\}$$

Δ_1	1	1	1	1	2	2	2	2
Δ_2	1	2	3	4	1	2	3	4
(*)	$\boxed{13}$	1	−11	$\boxed{-23}$	23	$\boxed{11}$	−1	−13
	(3)			(1)		(2)		

→ $12\cdot(-1+1+1)+3-2\cdot 1 = 12\cdot 1+3-2 = 12+1 = 13$

→ $12\cdot(-2+1+1)+3-2\cdot 1 = 12\cdot 0+3-2 = 3-2 = 1$

→ $12\cdot(-3+1+1)+3-2\cdot 1 = 12\cdot(-1)+3-2 = -12+1 = -11$

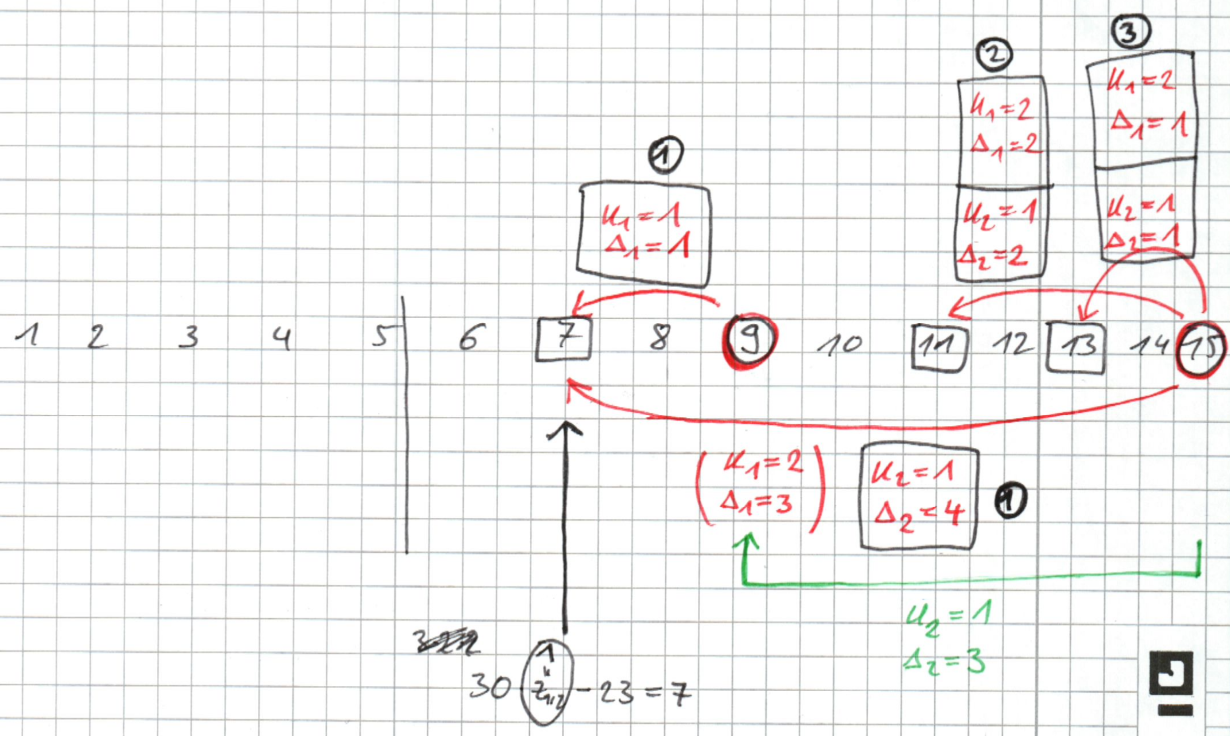
→ $12\cdot(-4+1+1)+3-2\cdot 1 = 12\cdot(-2)+3-2 = -24+1 = -23$

→ $12\cdot(-1+2+1)+3-2\cdot 2 = 12\cdot 2+3-4 = 24-1 = 23$

→ $12\cdot(-2+2+1)+3-2\cdot 2 = 12\cdot 1+3-4 = 12-1 = 11$

→ $12\cdot(-3+2+1)+3-2\cdot 2 = 12\cdot 0+3-4 = 0-1 = -1$

→ $12\cdot(-4+2+1)+3-2\cdot 2 = 12\cdot(-1)+3-4 = -12-1 = -13$



$\Rightarrow$ For which cases, do we have:

$$\boxed{u_1 - (-\Delta_2 + \Delta_1 + 1) \cdot x_2 = (2x_2 + 1) \cdot m \,, \quad m \in \mathbb{Z}}$$

?

$$k_1 - (-\Delta_2 + \Delta_1 + 1) \cdot 2 = 5 \cdot m$$

$$\frac{u_1 - (-\Delta_2 + \Delta_1 + 1) \cdot 2}{5} = m \quad ?$$

$$m = \frac{u_1 - (-\Delta_2 + \Delta_1 + 1) x_2}{2x_2 + 1} = \frac{u_1 + (-\Delta_1 + \Delta_2 - 1) x_2}{2x_2 + 1}$$

$$= \frac{(-\Delta_1 + \Delta_2 - 1) x_2 + u_1}{2x_2 + 1}$$

fulfield, if:

$$(-\Delta_1 + \Delta_2 - 1) x_2 + u_1 = (2x_2 + 1)(2\gamma + 1) \,, \quad \gamma \in \mathbb{N}$$

$$= 2 \cdot (2x_2\gamma + x_2 + \gamma) + 1$$

$$(-\Delta_1 + \Delta_2 - 1) x_2 + u_1 = 2^2 x_2 \gamma + 2x_2 + 2\gamma + 1$$

$$(-\Delta_1 + \Delta_2 - 1) x_2 - 2^2 x_2 \gamma - 2x_2 = 2\gamma + 1 - u_1$$

$$x_2 (-\Delta_1 + \Delta_2 - 1) - 2^2\gamma - 2) = 2\gamma + 1 - u_1$$

$$(-\Delta_1 + \Delta_2 - 1) x_2 = 2\gamma + 1 - u_1 + 2^2 x_2 \gamma - 2x_2$$

$$\boxed{(-\Delta_1 + \Delta_2 - 1) = \frac{(2\gamma + 1 - u_1)}{x_2} + 2^2\gamma - 2}$$