

# First recursion steps

Assume, we know:  $p_1 = 3, p_2 = 5$

$$\Rightarrow \text{I. } \bar{n}_1 = 3(2k_1 + 1) - 2\Delta_1, \Delta_1 \in \{1, 2\}, k_1, k_2 \in \mathbb{N}$$

$$\text{II. } \bar{n}_2 = 5(2k_2 + 1) - 2\Delta_2, \Delta_2 \in \{1, 2, 3, 4\}$$

$$n_1: 9, 15, 21, 27, 33, 39, \dots$$

$$n_2: 15, 25, 35, 45, 55, 65, \dots$$

$$n_3 = (2(x_2 + 1) + 1) = 2 \cdot 3 + 1 = 7: 21, 35, 49, 63, 77, 91, \dots$$

\*  $\Rightarrow$  From the knowledge of  $p_1 = 3$  and  $p_2 = 5 \in \mathbb{P}$ , ( $\mathbb{P}$  set of Primes)

$$1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, \dots$$

$$\Rightarrow \text{our allowed values: } k_1 = 1, 2, \Delta_1 \in \{1, 2\}$$

$$k_2 = 1, \Delta_2 \in \{1, 2, 3, 4\}$$

$$\Rightarrow \begin{cases} k_1 = 1, \Delta_1 = \{1\} \\ k_1 = 2, \Delta_1 = \{1, 2\} \\ k_2 = 1, 2, \Delta_2 = \{1, 2, 3, 4\} \end{cases}$$

$$\rightarrow \text{Since } \Delta x_{1,2} = 1 \Rightarrow \bar{n}_1' = \bar{n}_1 \text{ and } \bar{n}_2' = \bar{n}_2$$

$$\Rightarrow \bar{n}_1' = \bar{n}_2'$$

$$\bar{n}_1 = \bar{n}_2$$

$$3(2k_1 + 1) - 2\Delta_1 = 5(2k_2 + 1) - 2\Delta_2$$

$$\Leftrightarrow 0 = 5(2k_2 + 1) - 3(2k_1 + 1) - 2\Delta_2 + 2\Delta_1$$

$$\dots$$

$$\Rightarrow \begin{cases} k_1 = (2x_2 + 1)z_{1,2} + (-\Delta_2 + \Delta_1 + 1)x_2 \\ k_2 = (2x_1 + 1)z_{1,2} + (-\Delta_2 + \Delta_1 + 1)x_1 \end{cases}$$

$$\Rightarrow \begin{cases} k_1 = 5z_{1,2} + (-\Delta_2 + \Delta_1 + 1) \cdot 2 \\ k_2 = 3z_{1,2} + (-\Delta_2 + \Delta_1 + 1) \cdot 1 \end{cases}$$

$$\Rightarrow \bar{n}_1 = 3(2k_1 + 1) - 2\Delta_1$$

$$= 3[2 \cdot (5z_{1,2} + (-\Delta_2 + \Delta_1 + 1) \cdot 2) + 1] - 2\Delta_1$$

$$\bar{n}_1' = \bar{n}_1 = 3 \cdot 2 \cdot 5 \cdot z_{1,2} + 3 \cdot 2 \cdot 2(-\Delta_2 + \Delta_1 + 1) + 3 - 2\Delta_1$$