

$$\bar{n}_1' = (2x_1+1)(2u_1+1) - 2\Delta_1 \Delta x_{12} + \alpha_1$$

$$\bar{n}_2' = \underbrace{(2x_1'+1)(2x_1'+1)}_{=:(2x_2+1)}(2u_2+1) - 2\Delta_2 \Delta x_{12} + \alpha_2$$

$$\Rightarrow \bar{n}_1' = \bar{n}_2'$$

$$(2x_1+1)(2u_1+1) - 2\Delta_1 \Delta x_{12} + \alpha_1 = (2x_1+1)(2x_1'+1)(2u_2+1) - 2\Delta_2 \Delta x_{12} + \alpha_2$$

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$$\Leftrightarrow (2x_1+1)(2u_1+1) = (2x_1+1)(2x_1'+1)(2u_2+1) - 2\Delta_2 \Delta x_{12} + 2\Delta_1 \Delta x_{12} + \alpha_2 - \alpha_1$$

$$\Leftrightarrow (2u_1+1) = (2x_1+1)(2u_2+1) + 2 \frac{(-\Delta_2 + \Delta_1) \Delta x_{12}}{2x_1+1} + \frac{\alpha_2 - \alpha_1}{2x_1+1}$$

$$\alpha_2 = \alpha_2' \Delta x_{12}$$

$$\alpha_1 = \alpha_1' \Delta x_{12}$$

$$\Rightarrow + \frac{2(-\Delta_2 + \Delta_1) \Delta x_{12} + (\alpha_2' - \alpha_1') \Delta x_{12}}{2x_1+1}$$

$$= \Delta x_{12} \cdot \frac{2(-\Delta_2 + \Delta_1) + (\alpha_2' - \alpha_1')}{2x_1+1}$$

$$+ \frac{2(-\Delta_2 + \Delta_1) \Delta x_{12} + \alpha_2 - \alpha_1}{2x_1+1}$$

$$\text{I. } (2x_1+1)$$

$$\text{II. } (2x_2+1) = (2x_1+1)(2x_1'+1) = 2 \cdot (2x_1x_1' + x_1 + x_1') + 1 = x_2$$

$$\begin{aligned} \frac{x_2 > x_1}{\Rightarrow \Delta x_{12} = x_2 - x_1} &= (2x_1x_1' + x_1 + x_1') - x_1 \\ &= 2x_1x_1' + \cancel{x_1} + x_1' - \cancel{x_1} \\ &= 2x_1x_1' + x_1' \end{aligned}$$

$$\boxed{\Delta x_{12} = (2x_1+1)x_1'}$$

$\Rightarrow$  This is integer divisible by  $(2x_1+1)$ !

$$\Rightarrow (*) = 2 \frac{(-\Delta_2 + \Delta_1) \Delta x_{12}}{2x_1+1} = 2 \frac{(-\Delta_2 + \Delta_1) (2x_1+1)x_1'}{2x_1+1}$$

$$= 2(-\Delta_2 + \Delta_1)x_1 \Rightarrow \boxed{2u_1+1 = (2x_1+1)(2u_2+1) + 2(-\Delta_2 + \Delta_1)x_1}$$

$$2u_1 + 1 = (2x_1 + 1)(2u_2 + 1) + 2(-\Delta_2 + \Delta_1)x_1$$

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Be  $u_2 = (-\Delta_2 + \Delta_1 + 1)x_1$

$$\begin{aligned} \Rightarrow 2u_1 + 1 &= (2x_1 + 1) [2(-\Delta_2 + \Delta_1 + 1)x_1 + 1] + 2(-\Delta_2 + \Delta_1)x_1 \\ &= (2x_1 + 1) 2(-\Delta_2 + \Delta_1 + 1)x_1 + (2x_1 + 1) \\ &\quad + 2(-\Delta_2 + \Delta_1)x_1 \\ &= (2x_1 + 1) \cdot 2(-\Delta_2 + \Delta_1 + 1)x_1 \\ &\quad + 2(-\Delta_2 + \Delta_1 + 1)x_1 + 1 \\ &= 2[(2x_1 + 1)(-\Delta_2 + \Delta_1 + 1)x_1 + (-\Delta_2 + \Delta_1 + 1)x_1] + 1 \\ &= 2[(-\Delta_2 + \Delta_1 + 1) \underbrace{x_1(2x_1 + 1 + 1)}_{= x_1(2x_1 + 2)}] + 1 \\ &= x_1(2x_1 + 2) \\ &= 2x_1x_1 + 2x_1 \\ &= 2x_1x_1 + x_1 + x_1 = x_2 \end{aligned}$$

$\Rightarrow$  For this case follows:  $x_1 = x_1'$

General case

$$2u_1 + 1 = (2x_1 + 1)(2u_2 + 1) + 2(-\Delta_2 + \Delta_1)x_1$$

$$\Leftrightarrow 2u_1 = (2x_1 + 1)(2u_2 + 1) + 2(-\Delta_2 + \Delta_1)x_1 - 1$$

$$u_1 = \frac{1}{2} [(2x_1 + 1)(2u_2 + 1) + 2(-\Delta_2 + \Delta_1)x_1 - 1]$$

$$= \frac{1}{2} [ \underbrace{(2x_1 + 1)(2u_2 + 1) - 1}_{\text{odd number}} + (-\Delta_2 + \Delta_1)x_1 ]$$

$$\Rightarrow \frac{1}{2} ((2x_1 + 1)(2u_2 + 1) - 1)$$

is an integer number, too.

$\Rightarrow k_2$  is an arbitrary number  $\in \mathbb{N}$



$$\bar{u}_1' = (2x_1+1)(2u_1+1) - 2\Delta_1 \Delta x_{12}$$

$$\bar{u}_2' = (2x_2+1)(2u_2+1) - 2\Delta_2 \Delta x_{12}$$

$$\Rightarrow u_1 = u_2 + \frac{\Delta x_{12}(2u_2+1) + \Delta x_{12}(-\Delta_2 + \Delta_1)}{2x_1+1}$$

$$= u_2 + \Delta x_{12} \frac{(2u_2+1) + (-\Delta_2 + \Delta_1)}{2x_1+1}$$

$$\text{Be } u_2 \text{ mit } (-\Delta_2 + \Delta_1)$$

$$\left[ \begin{array}{l} \text{NR:} \\ u_1 = u_2 + \Delta x_{12} \frac{(-\Delta_2 + \Delta_1) \cdot (2x_1+1)(2x_1'+1) + (2x_1+1)}{2x_1+1} \end{array} \right.$$

$$u_1 = u_2 + \Delta x_{12} \cdot [(-\Delta_2 + \Delta_1) \cdot (2x_1'+1) + 1]$$

$$\frac{(-\Delta_2 + \Delta_1)(2x_1+1)(2x_1'+1) + (2x_1+1)}{2x_1+1} \stackrel{!}{=} \frac{(2u_2+1) + (-\Delta_2 + \Delta_1)}{2x_1+1}$$

$$\Leftrightarrow (-\Delta_2 + \Delta_1)(2x_1+1)(2x_1'+1) + (2x_1+1) = (2u_2+1) + (-\Delta_2 + \Delta_1)$$

$$\Leftrightarrow (-\Delta_2 + \Delta_1)(2x_1+1)(2x_1'+1) + (2x_1+1) - (-\Delta_2 + \Delta_1) = 2u_2+1$$

$$\Leftrightarrow (-\Delta_2 + \Delta_1) [(2x_1+1)(2x_1'+1) - 1] + (2x_1+1) = 2u_2+1$$

$$\Leftrightarrow (-\Delta_2 + \Delta_1) \left[ \underbrace{(2x_1+1)(2x_1'+1) - 1}_{2(2x_1x_1' + x_1 + x_1') + 1 - 1} \right] \cdot \frac{1}{2} + x_1 = u_2$$

$$\Leftrightarrow (-\Delta_2 + \Delta_1) [2(2x_1x_1' + x_1 + x_1')] \cdot \frac{1}{2} + x_1 = u_2$$

$$\Leftrightarrow (-\Delta_2 + \Delta_1) (2x_1x_1' + x_1 + x_1') + x_1 = u_2$$

$$\Leftrightarrow (-\Delta_2 + \Delta_1) [(2x_1+1)x_1' + x_1] + x_1 = u_2$$

$$\begin{aligned} \Leftrightarrow & \underbrace{(-\Delta_2 + \Delta_1)(2x_1+1)x_1'}_{= (-\Delta_2 + \Delta_1)2x_1x_1'} + x_1(-\Delta_2 + \Delta_1 + 1) = u_2 \\ & = (-\Delta_2 + \Delta_1)2x_1x_1' + (-\Delta_2 + \Delta_1)x_1 \\ & = x_1' [(-\Delta_2 + \Delta_1)2x_1 + (-\Delta_2 + \Delta_1)] \\ & = x_1' [(-\Delta_2 + \Delta_1)(2x_1 + 1)] \end{aligned}$$

$$\Leftrightarrow (-\Delta_2 + \Delta_1)(2x_1x_1' + (-\Delta_2 + \Delta_1)x_1' + x_1(-\Delta_2 + \Delta_1) + x_1) = u_2$$

$$\Leftrightarrow (-\Delta_2 + \Delta_1) \left( \frac{2x_1x_1' + x_1' + x_1}{x_2} \right) + x_1 = u_2$$

$$K_2 = (-\Delta_2 + \Delta_1)x_2 + x_1$$

$$2x_1x_1' + x_1 + x_1' = (2x_1 + 1)x_1' + x_1$$

Test

$$= \Delta x_{12} \frac{2(-\Delta_2 + \Delta_1)x_2 + 2x_1 + 1 + (-\Delta_2 + \Delta_1)}{2x_1 + 1}$$

$$= \Delta x_{12} \frac{2(-\Delta_2 + \Delta_1)(2x_1 + 1)x_1' + 2(-\Delta_2 + \Delta_1)x_1 + 2x_1 + 1 + (-\Delta_2 + \Delta_1)}{2x_1 + 1}$$

$$= \Delta x_{12} \left( 2(-\Delta_2 + \Delta_1)x_1' + (-\Delta_2 + \Delta_1) \frac{2x_1 + 1 + 2x_1 + 1}{2x_1 + 1} \right)$$

$$= 2$$

$$\Rightarrow K_1 = \Delta x_{12} \left( 2(-\Delta_2 + \Delta_1)x_1' + (-\Delta_2 + \Delta_1) \cdot 2 \right)$$

$$+ (-\Delta_2 + \Delta_1)x_2 + x_1$$

$$= (x_2 - x_1) \left( (-\Delta_2 + \Delta_1)(2x_1' + 2) \right) + (-\Delta_2 + \Delta_1)x_2 + x_1$$

$$K_1 = x_2(-\Delta_2 + \Delta_1)(2x_1' + 2) - x_1(-\Delta_2 + \Delta_1)(2x_1' + 2) + (-\Delta_2 + \Delta_1)x_2 + x_1$$

$$K_1 = (-\Delta_2 + \Delta_1) \left[ \underbrace{x_2(2x_1' + 2) - x_1(2x_1' + 2)}_{\text{part 22) d'après 22)}} + x_2 \right] + x_1$$

~~part 22) d'après 22)~~

$$= 2x_1'x_2 + 2x_2 - 2x_1'x_1 - 2x_1 + x_2$$

$$= 2x_1'x_2 + 3x_2 - 2x_1'x_1 - 2x_1$$