

$$2u_1 + 1 = (2x_1 + 1)(2u_2 + 1) + 2(-\Delta_2 + \Delta_1)x_1$$

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Be $u_2 = (-\Delta_2 + \Delta_1 + 1)x_1$

$$\begin{aligned} \Rightarrow 2u_1 + 1 &= (2x_1 + 1) [2(-\Delta_2 + \Delta_1 + 1)x_1 + 1] + 2(-\Delta_2 + \Delta_1)x_1 \\ &= (2x_1 + 1) 2(-\Delta_2 + \Delta_1 + 1)x_1 + (2x_1 + 1) \\ &\quad + 2(-\Delta_2 + \Delta_1)x_1 \\ &= (2x_1 + 1) \cdot 2(-\Delta_2 + \Delta_1 + 1)x_1 \\ &\quad + 2(-\Delta_2 + \Delta_1 + 1)x_1 + 1 \\ &= 2[(2x_1 + 1)(-\Delta_2 + \Delta_1 + 1)x_1 + (-\Delta_2 + \Delta_1 + 1)x_1] + 1 \\ &= 2[(-\Delta_2 + \Delta_1 + 1) \underbrace{x_1(2x_1 + 1 + 1)}_{= x_1(2x_1 + 2)}] + 1 \\ &= 2x_1(2x_1 + 2) \\ &= 2x_1x_1 + 2x_1 \\ &= 2x_1x_1 + x_1 + x_1 = x_2 \end{aligned}$$

\Rightarrow For this case follows: $x_1 = x_1'$

General case

$$2u_1 + 1 = (2x_1 + 1)(2u_2 + 1) + 2(-\Delta_2 + \Delta_1)x_1$$

$$\Leftrightarrow 2u_1 = (2x_1 + 1)(2u_2 + 1) + 2(-\Delta_2 + \Delta_1)x_1 - 1$$

$$u_1 = \frac{1}{2} [(2x_1 + 1)(2u_2 + 1) + 2(-\Delta_2 + \Delta_1)x_1 - 1]$$

$$= \frac{1}{2} [\underbrace{(2x_1 + 1)(2u_2 + 1) - 1}_{\text{odd number}} + (-\Delta_2 + \Delta_1)x_1]$$

$$\Rightarrow \frac{1}{2} ((2x_1 + 1)(2u_2 + 1) - 1)$$

is an integer number, too.

$\Rightarrow k_2$ is an arbitrary number $\in \mathbb{N}$