

$$\bar{n}_1' = (2x_1+1)(2u_1+1) - 2\Delta_1 \Delta x_{12} + \alpha_1$$

$$\bar{n}_2' = \underbrace{(2x_1'+1)(2x_1'+1)}_{=: (2x_2+1)} (2u_2+1) - 2\Delta_2 \Delta x_{12} + \alpha_2$$

$$\Rightarrow \bar{n}_1' = \bar{n}_2'$$

$$(2x_1+1)(2u_1+1) - 2\Delta_1 \Delta x_{12} + \alpha_1 = (2x_1+1)(2x_1'+1)(2u_2+1) - 2\Delta_2 \Delta x_{12} + \alpha_2$$

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$$\Leftrightarrow (2x_1+1)(2u_1+1) = (2x_1+1)(2x_1'+1)(2u_2+1) - 2\Delta_2 \Delta x_{12} + 2\Delta_1 \Delta x_{12} + \alpha_2 - \alpha_1$$

$$\Leftrightarrow (2u_1+1) = (2x_1+1)(2u_2+1) + 2 \frac{(-\Delta_2 + \Delta_1) \Delta x_{12}}{2x_1+1} + \frac{\alpha_2 - \alpha_1}{2x_1+1}$$

$$\alpha_2 = \alpha_2' \Delta x_{12}$$

$$\alpha_1 = \alpha_1' \Delta x_{12}$$

$$\Rightarrow + \frac{2(-\Delta_2 + \Delta_1) \Delta x_{12} + (\alpha_2' - \alpha_1') \Delta x_{12}}{2x_1+1}$$

$$= \Delta x_{12} \cdot \frac{2(-\Delta_2 + \Delta_1) + (\alpha_2' - \alpha_1')}{2x_1+1}$$

$$+ \frac{2(-\Delta_2 + \Delta_1) \Delta x_{12} + \alpha_2 - \alpha_1}{2x_1+1}$$

$$\text{I. } (2x_1+1)$$

$$\text{II. } (2x_2+1) = (2x_1+1)(2x_1'+1) = 2 \cdot (2x_1x_1' + x_1 + x_1') + 1 = x_2$$

$$\begin{aligned} \Rightarrow \Delta x_{12} &= x_2 - x_1 = (2x_1x_1' + x_1 + x_1') - x_1 \\ &= 2x_1x_1' + \cancel{x_1} + x_1' - \cancel{x_1} \\ &= 2x_1x_1' + x_1' \end{aligned}$$

$$\boxed{\Delta x_{12} = (2x_1+1)x_1'}$$

\Rightarrow This is integer divisible by $(2x_1+1)$!

$$\Rightarrow (*) = 2 \frac{(-\Delta_2 + \Delta_1) \Delta x_{12}}{2x_1+1} = 2 \frac{(-\Delta_2 + \Delta_1) (2x_1+1)x_1'}{2x_1+1}$$

$$= 2(-\Delta_2 + \Delta_1)x_1 \Rightarrow \boxed{2u_1+1 = (2x_1+1)(2u_2+1) + 2(-\Delta_2 + \Delta_1)x_1}$$