

Be $x_1 = x_2 + \Delta x_{12}$, $\Delta x_{12} \in \mathbb{Z}^-$, $x_2 > x_1$

$$0 = (2x_2 + 1)(2u_2 + 1) + 2\Delta x_{12}\Delta_2$$

$$= (2x_2 + 1)(2u_2 + 1)$$

Be $x_1 = x_2 - \Delta x_{12}$

$$0 = (2x_2 + 1)(2u_2 + 1) - 2\Delta x_{12}\Delta_2$$

$$= (2x_2 + 1)(2u_2 + 1) + \cancel{2\Delta x_{12}\Delta_2} + 2\Delta x_{12}(2u_2 + 1) + 2\Delta x_{12}\Delta_1$$

Be $(2x_2 + 1) = (2x_1 + 1)(2x_1' + 1)$

$$(2x_1 + 1)(2x_1' + 1)$$

$$0 = \cancel{(2x_2 + 1)(2u_2 + 1)} - 2\Delta x_{12}\Delta_2$$

$$= (2x_1 + 1)(2x_1' + 1)(2u_1 + 1) + 2\Delta x_{12}(2u_1 + 1) + 2\Delta x_{12}\Delta_1$$

$$(2x_1 + 1)(2x_1' + 1)(2u_1 + 1)$$

$$= \cancel{(2x_2 + 1)(2u_2 + 1)} - 2\Delta x_{12}\Delta_2$$

$$= (2x_1 + 1)(2x_1' + 1)(2u_2 + 1) - 2\Delta x_{12}\Delta_2$$

$$+ 2\Delta x_{12}(2u_1 + 1) + 2\Delta x_{12}\Delta_1$$

$$2u_1 + 1 = (2u_2 + 1) + 2\Delta x_{12} \frac{(-\Delta_2 + \Delta_1) + (2u_2 + 1)}{(2x_1 + 1)(2x_1' + 1)}$$

Be $u_1 = (-\Delta_2 + \Delta_1 + \alpha)(2x_1x_1' + x_1 + x_1') + \gamma$

$$\Rightarrow 2u_1 + 1 = (2u_2 + 1) + 2\Delta x_{12} \frac{(-\Delta_2 + \Delta_1) + 2 \cdot (-\Delta_2 + \Delta_1 + \alpha)(2x_1x_1' + x_1 + x_1') + 2\gamma + 1}{(2x_1 + 1)(2x_1' + 1)}$$

$$= (2u_2 + 1) + 2\Delta x_{12} \frac{(-\Delta_2 + \Delta_1) + 2 \cdot (-\Delta_2 + \Delta_1)(2x_1x_1' + x_1 + x_1') + 2\alpha(2x_1x_1' + x_1 + x_1') + 2\gamma + 1}{(2x_1 + 1)(2x_1' + 1)}$$

$$+ \frac{2\gamma + 1}{(2x_1 + 1)(2x_1' + 1)}$$

$$= (2u_2 + 1) + 2\Delta x_{12} (-\Delta_2 + \Delta_1) \frac{1 + 2(2x_1x_1' + x_1 + x_1')}{(2x_1 + 1)(2x_1' + 1)}$$

$$+ 2\Delta x_{12} \frac{2\alpha(2x_1x_1' + x_1 + x_1') + 2\gamma + 1}{(2x_1 + 1)(2x_1' + 1)}$$



$$= (2u_2 + 1) + 2\Delta x_{112} (-\Delta_2 + \Delta_1) \cdot 1$$

$$+ 2\Delta x_{112} \frac{2 \cdot \alpha (2x_1 x_1' + x_1 + x_1') + 2\gamma + 1}{(2x_1 + 1)(2x_1' + 1)}$$

$\Rightarrow \alpha = 1$ $\Rightarrow \gamma = 0$

$$= (2u_2 + 1) + 2\Delta x_{112} (-\Delta_2 + \Delta_1) + 2\Delta x_{112} \cdot 1$$

$$= (2u_2 + 1) + 2\Delta x_{112} (-\Delta_2 + \Delta_1 + 1)$$

~~$$= (2u_2 + 1) + 2\Delta x_{112} (-\Delta_2 + \Delta_1 + 1)$$~~

$$\Rightarrow u_2 = (-\Delta_2 + \Delta_1 + 1)(2x_1 x_1' + x_1 + x_1')$$

$$= [(2x_1 + 1)(2x_1' + 1) \frac{1}{2} - 1]$$

$$\text{Be } u_2 = (-\Delta_2 + \Delta_1 + \alpha)(2x_1 y_1 + x_1 + y_1) + \gamma$$

$$\Rightarrow 2u_1 + 1 = (2u_2 + 1) + 2\Delta x_{112} \frac{(-\Delta_2 + \Delta_1) + 2 \cdot (-\Delta_2 + \Delta_1 + \alpha)(2x_1 y_1 + x_1 + y_1) + 2\gamma + 1}{(2x_1 + 1)(2x_1' + 1)}$$

$$= (2u_2 + 1) + 2\Delta x_{112} \frac{(-\Delta_2 + \Delta_1) + 2 \cdot (-\Delta_2 + \Delta_1 + \alpha)(2x_1 y_1 + x_1 + y_1) + 2\gamma + 1}{(2x_1 + 1)(2x_1' + 1)}$$

$$= (2u_2 + 1) + 2\Delta x_{112} (-\Delta_2 + \Delta_1) \frac{1 + 2 \cdot (2x_1 y_1 + x_1 + y_1)}{(2x_1 + 1)(2x_1' + 1)} + 2\Delta x_{112} \frac{2\alpha(2x_1 y_1 + x_1 + y_1) + 2\gamma + 1}{(2x_1 + 1)(2x_1' + 1)}$$

$$+ 2\Delta x_{112} \frac{2\alpha(2x_1 y_1 + x_1 + y_1) + 2\gamma + 1}{(2x_1 + 1)(2x_1' + 1)}$$

$$= (2u_2 + 1) + 2\Delta x_{112} (-\Delta_2 + \Delta_1) \frac{2 \cdot ((2x_1 + 1)(2y_1 + 1) - 1) \frac{1}{2} + 1}{(2x_1 + 1)(2x_1' + 1)}$$

$$+ 2\Delta x_{112} \frac{2\alpha((2x_1 + 1)(2y_1 + 1) - 1) \frac{1}{2} + 2\gamma + 1}{(2x_1 + 1)(2x_1' + 1)}$$

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$$(2x_1 + 1)(2x_1' + 1)$$

~~$$= (2x_1 + 1)(2x_1' + 1)$$~~

$$= 2 \cdot (2x_1 x_1' + x_1 + x_1') + 1$$



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$$= (2u_2 + 1) + 2\Delta x_{1,2} (-\Delta_2 + \Delta_1) \frac{(2x_1 + 1)(2y_1 + 1) - 1 + 1}{(2x_1 + 1)(2x_1^* + 1)}$$

$$+ 2\Delta x_{1,2} \frac{\alpha (2x_1 + 1)(2y_1 + 1) + 2x_1 + 1 - \alpha}{(2x_1 + 1)(2x_1^* + 1)}$$

Be $\gamma = 0$ and $\alpha = 1$

$$= (2u_2 + 1) + 2\Delta x_{1,2} (-\Delta_2 + \Delta_1) \frac{(2x_1 + 1)(2y_1 + 1)}{(2x_1 + 1)(2x_1^* + 1)}$$

$$+ 2\Delta x_{1,2} \frac{(2x_1 + 1)(2y_1 + 1)}{(2x_1 + 1)(2x_1^* + 1)}$$

$$= (2u_2 + 1) + 2\Delta x_{1,2} (-\Delta_2 + \Delta_1) \frac{2y_1 + 1}{2x_1^* + 1} + 2\Delta x_{1,2} \frac{2y_1 + 1}{2x_1^* + 1}$$

$$= (2u_2 + 1) + 2\Delta x_{1,2} \frac{2y_1 + 1}{2x_1^* + 1} (-\Delta_2 + \Delta_1 + 1)$$

$$\uparrow$$

$$x_1 = x_2 - \Delta x_{1,2}$$

$$\Delta x_{1,2} = x_2 - x_1$$

$$x_1 + \Delta x_{1,2} = x_2 = ((2x_1 + 1)(2x_1^* + 1) - 1)^{\frac{1}{2}}$$

$$= (2u_2 + 1) + 2 \cdot (x_2 - x_1) \frac{2y_1 + 1}{2x_1^* + 1} (-\Delta_2 + \Delta_1 + 1)$$

$$= (2u_2 + 1) + 2 \cdot \left[((2x_1 + 1)(2x_1^* + 1) - 1)^{\frac{1}{2}} - x_1 \right] \frac{2y_1 + 1}{2x_1^* + 1} (-\Delta_2 + \Delta_1 + 1)$$

$$= (2u_2 + 1) + \left[(2x_1 + 1)(2x_1^* + 1) - 1 - 2x_1 \right] \frac{2y_1 + 1}{2x_1^* + 1} (-\Delta_2 + \Delta_1 + 1)$$

$$= (2u_2 + 1) + \frac{(2x_1 + 1)(2x_1^* + 1)(2y_1 + 1) - (2x_1 + 1)(2y_1 + 1)}{2x_1^* + 1} (-\Delta_2 + \Delta_1 + 1)$$

$$= (2u_2 + 1) + \left[(2x_1 + 1)(2y_1 + 1) - \frac{(2x_1 + 1)(2y_1 + 1)}{2x_1^* + 1} \right] \cdot (-\Delta_2 + \Delta_1 + 1)$$

$$= (2u_2 + 1) + (2x_1 + 1)(2y_1 + 1) \cdot (-\Delta_2 + \Delta_1 + 1)$$

$$- \frac{(2x_1 + 1)(2y_1 + 1)}{2x_1^* + 1}$$

$$- \frac{(2x_1 + 1)(2x_1^* + 1)(2\hat{y}_1 + 1)}{2x_1^* + 1}$$

$$- (2x_1 + 1)(2\hat{y}_1 + 1)$$

$$\text{Be } y_1 = 2x_1^* \hat{y}_1 + x_1^* + \hat{y}_1$$

$$= ((2x_1^* + 1)(2\hat{y}_1 + 1) - 1)^{\frac{1}{2}}$$

$$2y_1 + 1 = (2x_1^* + 1)(2\hat{y}_1 + 1)$$

$$= 2(2x_1^* \hat{y}_1 + x_1^* + \hat{y}_1) + 1$$

$$\Rightarrow K_2 = (-\Delta_2 + \Delta_1 + 1) (2x_1 (2x_1' \tilde{y}_1 + x_1' + \tilde{y}_1) + x_1 + (2x_1' \tilde{y}_1 + x_1' + \tilde{y}_1))$$

$$\tilde{y}_1 \in \mathbb{N}$$

$$= (-\Delta_2 + \Delta_1 + 1) \cancel{(2^2 x_1 x_1' \tilde{y}_1 + 2 x_1 x_1' + 2 \tilde{y}_1 + x_1 + 2 x_1' \tilde{y}_1 + x_1' + \tilde{y}_1)}$$

$$K_2 = (-\Delta_2 + \Delta_1 + 1) (2^2 x_1 x_1' \tilde{y}_1 + 2 x_1 x_1' + 3 \tilde{y}_1 + 2 x_1' \tilde{y}_1 + x_1 + x_1')$$

$$= (-\Delta_2 + \Delta_1 + 1) (2x_1 \tilde{y}_1 + x_1 + \tilde{y}_1)$$

$$= (-\Delta_2 + \Delta_1 + 1) ((2x_1 + 1)(2\tilde{y}_1 + 1) - 1)^{\frac{1}{2}}$$

$$= (-\Delta_2 + \Delta_1 + 1) \left[(2x_1 + 1) \left(2((2x_1' + 1)(2\tilde{y}_1' + 1) - 1)^{\frac{1}{2}} + 1 \right) - 1 \right]^{\frac{1}{2}}$$

$$= (-\Delta_2 + \Delta_1 + 1) \left[(2x_1 + 1) \left((2x_1' + 1)(2\tilde{y}_1' + 1) - 1 + 1 \right) - 1 \right]^{\frac{1}{2}}$$

$$K_2 = (-\Delta_2 + \Delta_1 + 1) \left[(2x_1 + 1)(2x_1' + 1)(2\tilde{y}_1 + 1) - 1 \right]^{\frac{1}{2}}$$

$$\cancel{(-\Delta_2 + \Delta_1 + 1)}$$