$$\begin{aligned} &\mathrm{I}\quad \overline{n}_1 = (2x_1+1)(2u_1+1) - 2\Delta x_{1/2}\Delta_1 \\ &\mathrm{II}\quad \overline{n}_2 = (2x_2+1)(2u_2+1) - 2\Delta x_{1/2}\Delta_2 \end{aligned} \qquad \text{(eq. 1)}$$

Be $x_2 > x_1$ $\quad (x_2 = x_1 + \Delta x_{1/2})$

$$x_1 = x_2 - \Delta x_{1/2}$$
$$\Delta x_{1/2} = x_2 - x_1$$

1. Substitute: $x_2 = x_1 + \Delta x_{1/2}$

(eq2) $$0 = (2x_2+1)(2u_2+1) - (2x_1+1)(2u_1+1) + 2\Delta x_{1/2}(-\Delta_2+\Delta_1)$$

$$0 = (2(x_1+\Delta x_{1/2})+1)(2u_2+1) - (2x_1+1)(2u_1+1) + 2\Delta x_{1/2}(-\Delta_2+\Delta_1)$$
$$= (2x_1+1)(2u_2+1) + 2\Delta x_{1/2}(2u_2+1) - (2x_1+1)(2u_1+1) + 2\Delta x_{1/2}(-\Delta_2+\Delta_1)$$

$$(2x_1+1)(2u_1+1) = (2x_1+1)(2u_2+1) + 2\Delta x_{1/2}(2u_2+1) + 2\Delta x_{1/2}(-\Delta_2+\Delta_1)$$

(eq3.) $$(2u_1+1) = (2u_2+1) + 2\Delta x_{1/2}\,\frac{\cancel{(2u_2+1)+2\Delta x_{1/2}}\ (2u_2+1)+(-\Delta_2+\Delta_1)}{(2x_1+1)}$$

Case: $\Delta x_{1/2} = n\cdot(2x_1+1)$ $\quad (n\in\mathbb{N})$ $\quad$ (That is equal to $(2x_2+1) = (2x_1+1)(2n+1)$)

$$(2u_1+1) = (2u_2+1) + 2\cdot n\cdot\cancel{(2x_1+1)}\,\frac{(2u_2+1)+(-\Delta_2+\Delta_1)}{\cancel{2x_1+1}}$$

$$(2u_1+1) = (2u_2+1) + 2\cdot n\cdot(2u_2+1) + (-\Delta_2+\Delta_1)$$

Solve this for u_2: $(2u_2+1)$

$$(2u_1+1) = (2u_2+1)(2n+1) + (-\Delta_2+\Delta_1)$$

$$(2u_2+1) = \frac{(2u_1+1)-(-\Delta_2+\Delta_1)}{\cancel{2u_1+\Delta_1}\ \ 2n+1}$$

We need integer solutions for this!

$\Rightarrow$ Be $u_1 = (-\Delta_2+\Delta_1-1)(2ny_1+n+y_1)$

$$(2u_2+1) = \frac{2\cdot(-\Delta_2+\Delta_1-1)(2ny_1+n+y_1)+1-(-\Delta_2+\Delta_1)}{2n+1}$$

$$= \frac{2\cdot(-\Delta_2+\Delta_1-1)((2n+1)(2y_1+1)-1)\frac{1}{2}+1-(-\Delta_2+\Delta_1)}{2n+1}$$