Private research project

The recursively calculation of prime numbers.

Draft/(Working) paper

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Abstract.

Roadmap

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1 Introduction

In the following paper, I will show that prime numbers can be calculated recursively. I will start with the suggestion of descriptions itself, over different perspectives on this problem, until the final explanation of caculating prime numbers in the most efficient way, as a result from this considerations.

Let's start with the definition of prime numbers itself.

Definition 1.0.1 (Prime numbers) *Every natural number greater than one which has no positive integer divisors apart from one and itself is called Prime Number or just only Prime.*

Be $\mathcal{P}$ the set of all prime numbers p. So we can write

$$\mathcal{P} := \{p \in \mathbb{N}_{>1} \mid \forall n \in \mathbb{N}_{>1} \setminus \{p\} : \; n \nmid p\}.$$

Hence, the first prime numbers are $\mathcal{P} := \{2, 3, 5, 7, 11, 13, 17, 19, 23, \dots\}$.

2 Odd-Divisible Numbers

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At first, for the description of prime numbers, we have to look at the set of divisible numbers. Since, apart from 2, all prime numbers are odd, we will only analyze this numbers. In the whole paper, we will ignore the prime number 2, because we will see, this makes a lot easier.

2.1 Basic description: Odd-Numbers

Be given the set of all odd natural numbers $y \in \mathbb{N}_{>1}$ through

$$y_i\left(x_i\right) := 2x_i + 1, \tag{2.1}$$

with $x, i \in \mathbb{N}$. If we expand the definition set of x to $\mathbb{Z}$, we also know

$$\begin{aligned} y\left(0\right) &= 1 && (2.2) \\ \text{and} \quad y(-x) &= 2\left(-x\right) + 1 \\ &= -\left(2x - 1\right) \\ &= -\left(2\left(x-1\right) + 1\right) \\ &= -y\left(x-1\right). && (2.3) \end{aligned}$$

Later, we will see that this properties can be very useful.

2.2 Basic description: Odd-Divisible Numbers

Next, we look at all odd-divisible numbers. We know, they can't have a factor which is a multiple of 2. Hence, we get an equation which describes all odd-divisible numbers by

$$\begin{aligned} y_{i,j}(x_i, x_j) &= y_i(x_i) \cdot y_j(x_j) \\ &= (2x_i + 1)(2x_j + 1) \\ &= 2^2 x_i x_j + 2x_i + 2x_j + 1 \\ &= 2\left(\underbrace{2x_i x_j + x_i + x_j}_{=:x_{i,j}}\right) + 1 \\ &= y_{i,j}(x_{i,j}). \end{aligned} \tag{2.4}$$

If we expand again our sets to $\mathbb{Z}$, we receive additional cases. At first, assume at one factor is $y(0) = 1$. We see directly

$$\begin{aligned} y_{0,j}(0, x_j) &= y_0(0) \cdot y_j(x_j) \\ &= 1 \cdot (2x_j + 1) \\ &= 2x_j + 1 \\ &= y_j(x_j) \end{aligned} \tag{2.5}$$

$$\text{respectively} \quad y_{i,0}(x_i, 0) = y_i(x_i). \tag{2.6}$$

Next, assume we have one factor with $y(-x)$.

$$\begin{aligned} y_{i,j}(-x_i, x_j) &= y_i(-x_i) \cdot y_j(x_j) \\ &= (2(-x_i) + 1)(2x_j + 1) \\ &= -2^2 x_i x_j - 2x_i + 2x_j + 1 \\ &= -(2(2x_i x_j + x_i - x_j - 1) + 1) \\ &= -(2(2x_i x_j + x_i - 2x_j + x_j - 1) + 1) \\ &= -(2(2(x_i - 1)x_j + (x_i - 1) + x_j) + 1) \\ &= -y_i(x_i - 1) \cdot y_j(x_j) \end{aligned} \tag{2.7}$$

$$\text{respectively} \quad y_{i,j}(x_i, -x_j) = -y_i(x_i) \cdot y_j(x_j - 1) \tag{2.8}$$

In the case of both factors, we have

$$\begin{aligned}
y_{i,j}\left(-x_i, -x_j\right) &= y_i\left(-x_i\right) \cdot y_j\left(-x_j\right) \\
&= \left(2\left(-x_i\right)+1\right)\left(2\left(-x_j\right)+1\right) \\
&= \left(2x_i - 1\right)\left(2x_j - 1\right) \\
&= 2^2 x_i x_j - 2x_i - 2x_j + 1 \\
&= 2\left(2x_i x_j - x_i - x_j\right) + 1 \\
&= \left(2x_i - 2 + 1\right)\left(2x_j - 2 + 1\right) \\
&= \left(2\left(x_i - 1\right)+1\right)\left(2\left(x_j - 1\right)+1\right) \\
&= \left(-1\right) y_i\left(x_i - 1\right)\left(-1\right) y_j\left(x_j - 1\right) \\
&= y_{i,j}\left(x_i - 1, x_j - 1\right). \qquad (2.9)
\end{aligned}$$

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