The Effects of Mass and Impact Parameter on Asteroid Capture

R. Herd and S. Craig

INTRODUCTION

Typically, when an asteroid approaches a planet it either enters the planets atmosphere and burns up or it's momentum carries it past the planet. However, under certain conditions the asteroid could be captured by the planets gravity and maintain an orbit. In this experiment we tested which combinations of mass and impact parameter would lead to an asteroid being captured by Earth. **Given these conditions tested what would be the extrema of the impact parameter and what would be the largest possible mass in this particular range of impact parameters?**

METHODS

The following table explains the ranges used for both variables in our simulation with their associated step size.

	Lower Bound	Upper Bound	Step Size
Impact Parameter	6.4e7 (m)	3.0e7 (m)	1000 (m)
Mass	10 (kg)	200 (kg)	10 (kg)

Table 1: Iteration bounds.

The upper bound of the impact parameter was determined once no successful orbits were achieved for all masses. An orbit was deemed successful if the orbital time was less than 36 weeks and made one full rotation around the Earth.

The net force on the asteroid is a combination of the gravitational force and the drag force opposing the direction of the asteroids velocity when it is within the Earth's atmosphere.

$$\vec{F}^{net} = -G\frac{Mm}{|\,\vec{r}\,|^2}\hat{r} - \frac{1}{2}c_d\rho A\,|\,\vec{v}\,|^2\,\Theta(|\,\vec{r}\,| - a)\hat{v}$$

Where Θ is a step function and a = 1e8 m. [1], A= cross sectional area of the asteroid, m = mass of the asteroid, M=mass of Earth, and cd= drag coefficient with value of 0.7 [2]. The vectors are represented on Figure 1.

To accurately calculate air density (ρ) we found ρ as a function of altitude and temperature [3], while using an approximation that the atmosphere is an ideal gas, thus we extracted ρ from the ideal gas law.

In order to ensure our simulation remained as general as possible, we calculated the asteroids initial velocity at its starting position based on the gravitational potential energy. Thus it could be approximated that the asteroid was coming from infinity. With this generalization and the spherical symmetry of the Earth, the direction the asteroid approaches the Earth and the Earth's orientation does not influence the results.

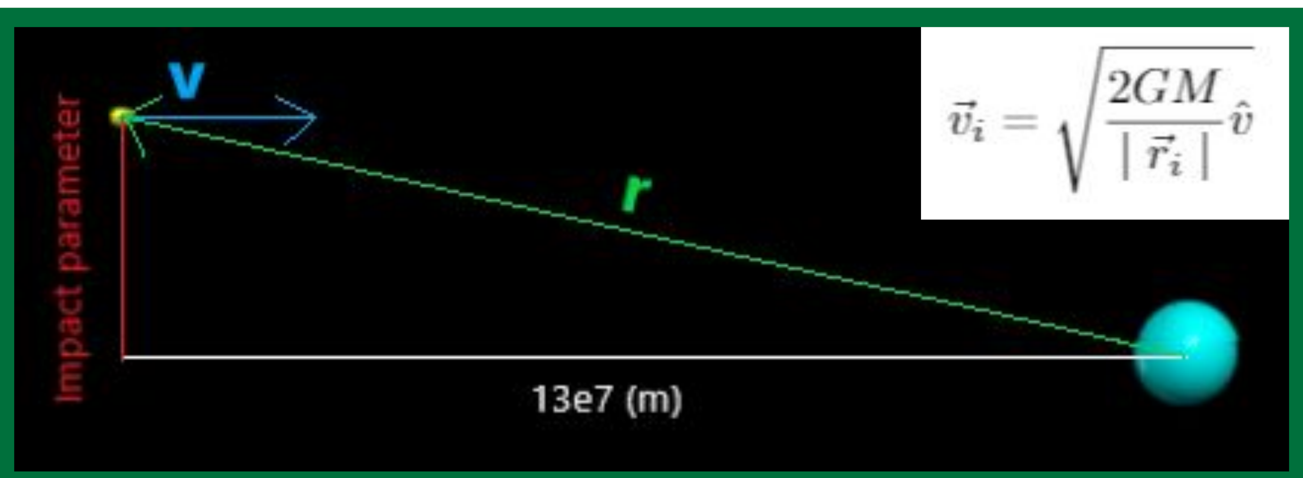

Figure 1: A visual representation of the asteroid in relation to Earth as well as the initial velocity equation. Note the bolded v and r are vectors.

RESULTS

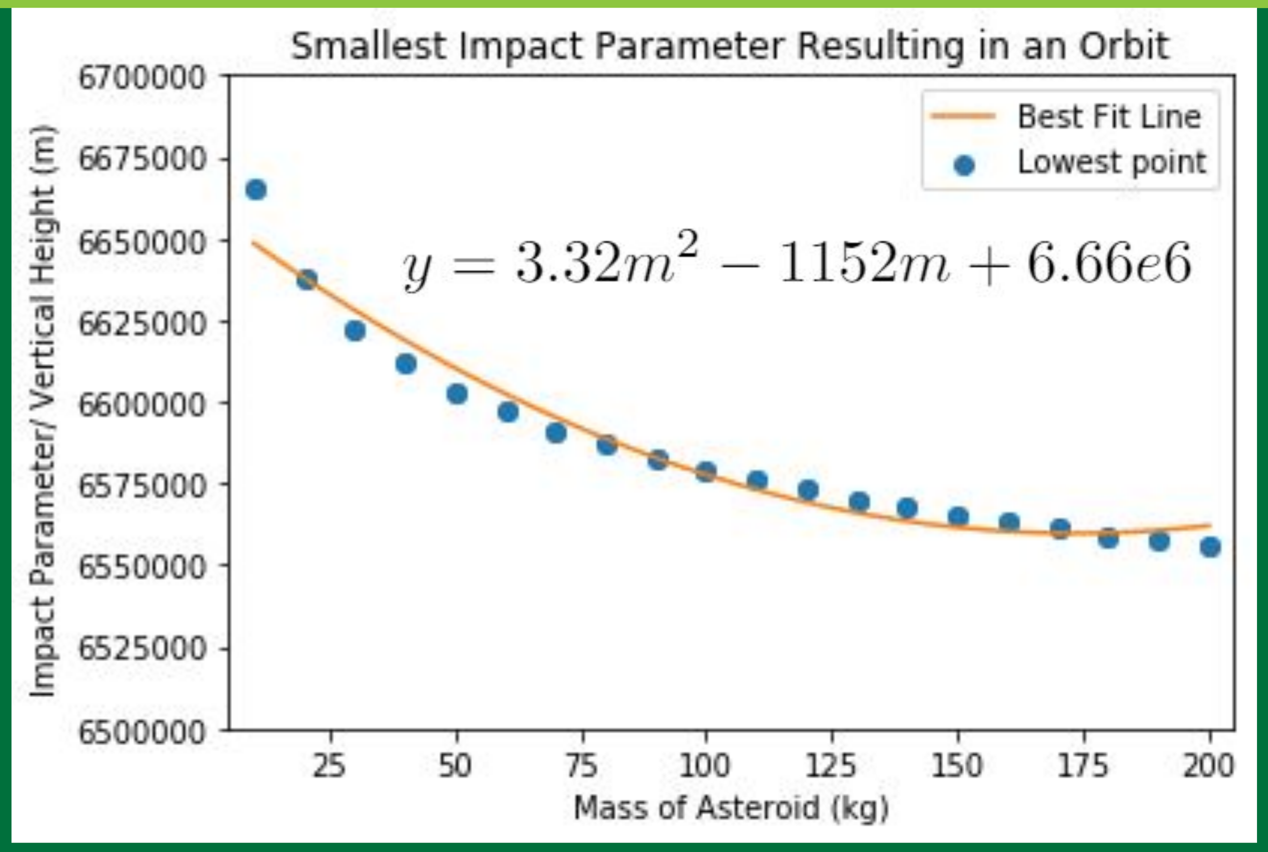

Figure 2: Plotting the smallest impact parameter for each mass that will result in an orbit.

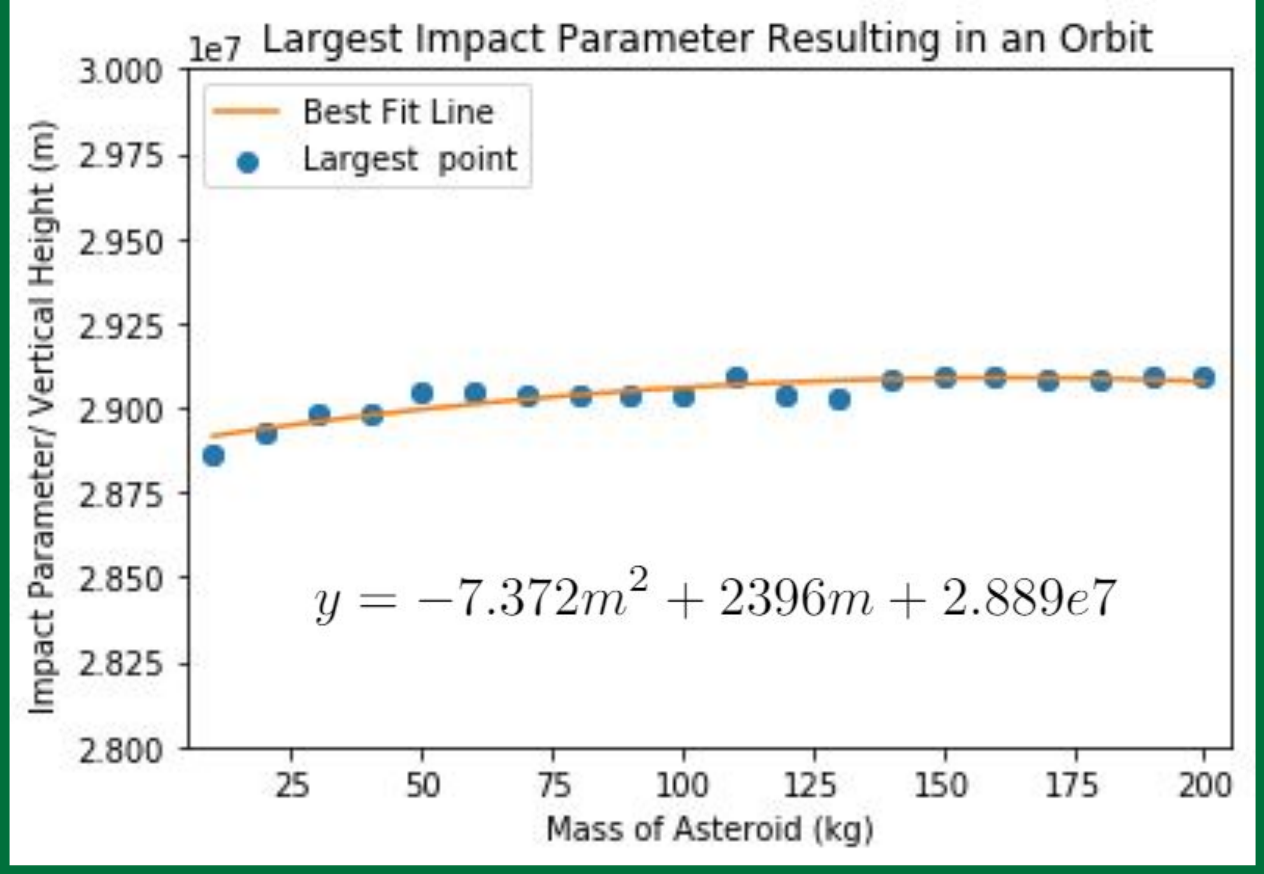

Figure 3: Plotting the largest impact parameter for each mass that will result in an orbit.

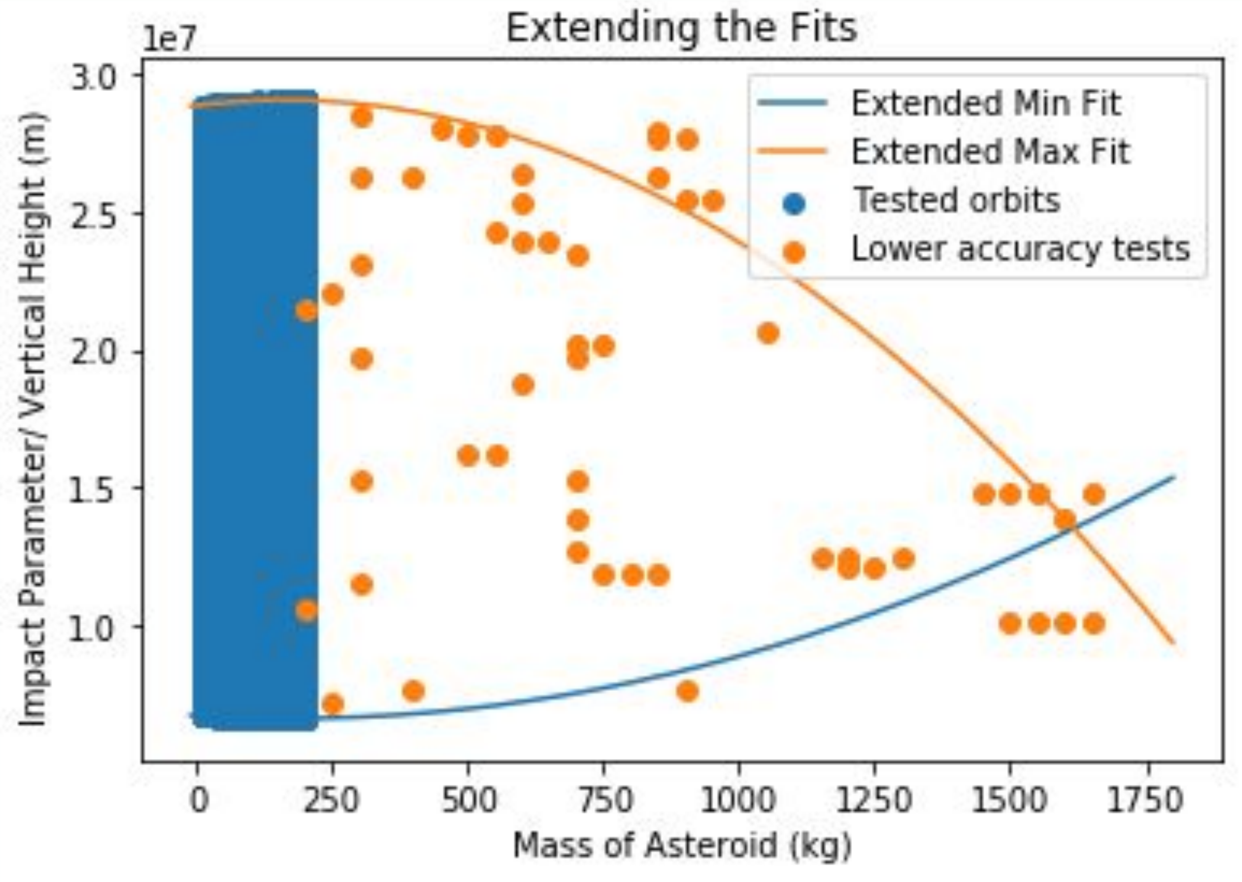

Figure 4: Plotting the lower accuracy data with our extended best fit lines to observe if the trend holds for larger masses.

DISCUSSION

With around 24,000 points tested for each mass we established just under 6,000 combinations that result in a successful orbit.

From Figure 2 we can see that our minimum graph appears to follow an exponential decay trend, however this cannot be the case. If it did exponentially decay then as the mass approaches infinity, the impact parameter would approach zero. This violates our conditions as the asteroid would have collided with the Earth, therefore a parabolic fit would be more appropriate.

We made a similar discovery for fitting the maximum data. As the impact parameter increases so does the initial velocity, which would be the same for every mass, however, once we increase the mass the momentum of the asteroid would carry it past the planet and thus not orbit.

To further test these fit lines we drastically reduced the step sizes of our simulation due to the high run-times and collected data. The data from the lower accuracy plots appear to follow the trend that the best fit lines predict. Extending these fits we extracted the following data by taking the second derivative with respect to mass and equating to zero.

	Mass(kg)	Impact Parameter (m)
Largest Possible Impact Parameter	163	2.91e7
Smallest Possible Impact Parameter	174	6.56e6
Largest Possible Mass	1617	1.35e7

Table 2: Extrema of impact parameter resulting in an orbit, with associated mass, and largest mass that will orbit in the impact parameter range investigated.

CONCLUSIONS

Through the use of a large quantity of simulations investigated the relationship between the mass and impact parameter of an asteroid approaching Earth and how these conditions affect the possibility of asteroid capture. After collecting the combinations of mass and impact parameter that resulted in one full revolution around Earth within 36 weeks, we extracted an equation for the minimum and maximum impact parameter in terms of mass. We were able to further explore and extract the extrema of the impact parameter and maximum mass that result in an orbit (Table 2). The best fit lines were extended to confirm our theory more simulations with increased step sizes over a larger range of values produced a majority of successful orbits that were in agreement with our equations (Figure 4).

REFERENCES

1. https://web.archive.org/web/20170517071138/https://scied.ucar.edu/shortcontent/exosphere-overview
2. Z. Ceplecka and D.O. ReVelle, M&PS 40, 35- 54 (2005)
3. https://www.grc.nasa.gov/WWW/K-12/airplane/atmosmet.html

UNIVERSITY OF THE FRASER VALLEY