UNIVERSITÄT ZU LÜBECK
INSTITUTE FOR ELECTRICAL
ENGINEERING IN MEDICINE

RO5500 – Vehicle Dynamics & Control

Exercise Sheet 1

Summer semester 2021 April 16, 2021

The exercises may be solved individually or in small groups. All calculations and simulations should be performed in Matlab / Simulink.
Hand-in of solutions: Friday, April 30, via Moodle

On this exercise sheet, we will start to look at some basics of vehicle models. These models are important for control design, and they also help with developing a fundamental understanding of vehicle dynamics. After the first exercise sheet, you should be familiar with the most important coordinate systems and the kinematic bicycle model. You will also start to build your first vehicle simulation.

Exercise 1 (Coordinate systems)

When working with vehicle dynamics, it is important to have a solid understanding of the basic terminology. The coordinate systems are the basis for the definition of fundamental notions such as the vehicle's velocity, acceleration and the yaw, pitch, roll angles.

a) Suppose a vehicle is tracking a given reference path. Describe a scenario where the vehicle frame is the same as the path frame, and one where these two coordinate frames are different.

b) Under which assumption(s) are the vehicle coordinates the same as the horizontal coordinates?

c) What is the difference between the road slope angle and the vehicle pitch angle, and where does it come from?

Exercise 2 (Coordinate transformations)

In vehicle dynamics, we use several different reference frames. Clearly, it is important to be able to transform the coordinates from one reference frame into another. Particularly often one has to transform the coordinates between some *global reference frame* (e.g., inertial frame) and some *local reference frame* (e.g., vehicle frame or path frame).

Let's look at the planar case, where (x, y) denote the coordinates of a global reference frame and (ξ, η) those of the local reference frame. Suppose the origin of a local reference frame has the coordinates (x_0, y_0) in terms of the global coordinates and its axes are rotated by an angle $\varphi \in [0, 2\pi)$, as shown in Figure 1. The goal is to derive the transformation for a reference point C between the two coordinate systems.

a) Suppose the coordinates (ξ, η) of the reference point C are given. Find an expression for its coordinates (x, y) in terms of ξ, η, x_0, y_0, φ.

b) Suppose the coordinates (x, y) of the reference point C are given. Find an expression for its coordinates (ξ, η) in terms of x, y, x_0, y_0, φ.

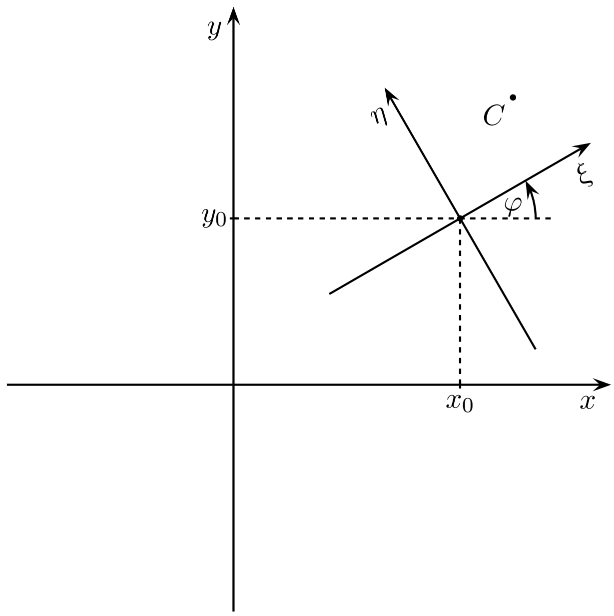

Figure 1: Illustration of global and local coordinates.

Exercise 3 (Kinematic bicycle model)

In fact, the kinematic bicycle model is one of the most frequently used vehicle models, e.g., for path planning or path tracking control. This is mainly due to its simplicity, despite providing a fairly good representation of the nonlinear dynamics of a vehicle. Hence it is very important to have a thorough understanding of this fundamental model.

a) State the main assumptions that are made in the derivation of the kinematic bicycle model.

b) Despite the assumption of no slip at the front and rear tires, the lateral velocity of the kinematic bicycle model is not zero, in general. Suppose the vehicle reference point is selected on the bicycle model, and let $\xi \in [0, l]$ denote the location of the reference point between the rear axle ($\xi = 0$) and the front axle ($\xi = l$). What is the single location for the vehicle's reference point such that the lateral velocity of the vehicle is always zero?

c) Derive an expression for the vehicle's lateral velocity v_{lat} and longitudinal velocity v_{lon} in terms of the vehicle velocity v and the steering angle δ.

d) Suppose the kinematic bicycle model runs with a constant velocity v and steering angle $\delta \neq 0$. What is the shape of the path traced by the midpoint of the front bicycle wheel and the midpoint of the rear bicycle wheel?

e) Which one the two traced paths described in d) has the smaller radius?

Exercise 4 (Path tracking with kinematic bicycle model)

Let's start with building our own vehicle simulation based on a kinematic bicycle model, and use it to design of a path tracking controller. Before you start, please make sure a recent version of Matlab is installed on your computer, and that you also have an installation of Simulink. If you are not familiar with Matlab / Simulink, let us know (or try to find some tutorials online).

a) Implement the differential equations of motion of the kinematic bicycle model by using elementary blocks in Simulink. Assume a wheelbase of $l = 2\,\mathrm{m}$ and a constant speed of $10\,\frac{\mathrm{m}}{\mathrm{s}}$. The vehicle reference point should be chosen as the center of the rear axle.

For vehicle path tracking, a common approach is to use a so-called **lookahead controller**. The feedback is based on *lookahead lateral deviation* $e_{\mathrm{la}}(t)$ of the vehicle, which is the coordinate $e(t)$ in the path frame at some *lookahead distance* Δs ahead on the path. Let's assume that lookahead controller is a simple P-controller on the lookahead lateral deviation, i.e., it assigns the steering angle as

$$\delta(t) = K_{\mathrm{la}} e_{\mathrm{la}}(t) \;, \tag{1}$$

where $K_{\mathrm{la}} \in \mathbb{R}$ is a constant feedback gain. By a first-order approximation, the lookahead lateral deviation can be expressed as

$$e_{\mathrm{la}}(t) \approx e(t) + \Delta s\, \tan\bigl(\Delta\psi(t)\bigr) \approx e(t) + \Delta s\, \Delta\psi(t) \;. \tag{2}$$

Here $e(t)$ is the current lateral deviation of the vehicle and $\Delta\psi(t)$ is the current difference between the vehicle's yaw angle $\psi(t)$ and the current yaw angle of the path.

b) Assume the reference path is a straight line, given by the inertial X-axis. Draw a small diagram that illustrates the approximation formula for the lateral deviation in (2). Describe in which types of situations this approximation will become poor.

c) Given a positive lookahead distance $\Delta s > 0$, what is a necessary condition on the parameter K_{la} in order for the closed-loop system to be stable?

d) Linearize the kinematic bicycle model around the X-axis, assuming small angle approximations and a constant velocity v.

e) Show that the approximative lookahead controller (1),(2) is equivalent to a PD controller for the linearized system, i.e.,

$$\delta(t) = K_{\mathrm{p}} e(t) + K_{\mathrm{d}} \dot{e}(t) \tag{3}$$

where $e(t)$ is the vehicle's lateral deviation from the path. Determine the feedback gains K_{p} and K_{d} in terms of K_{la} and Δs.

f) Apply the Laplace transform to the closed-loop system, i.e., the linearized kinematic bicycle model together with the approximative lookahead controller (1),(2). Calculate values for K_{la} and Δs that give a rise time of $2\,\mathrm{s}$ and a peak overshoot of $10\,\%$.

g) Now implement this approximative lookahead controller (1),(2) on the nonlinear kinematic bicycle model designed in part a). Make sure the initial condition of the model is selected close to the reference path. How do you judge the performance of the controller?