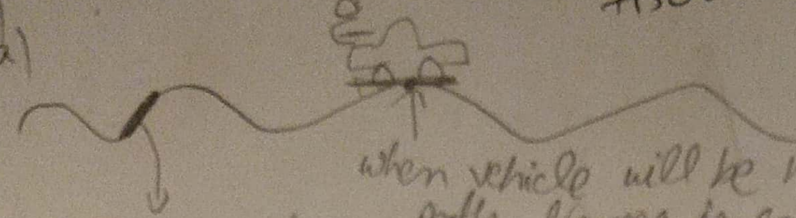
Ex. Sheet 1 :— SAMEED QUAIS 715861 23-04-2021

Q1)
a)

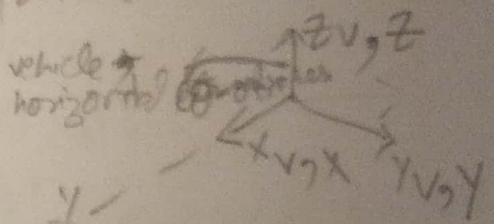


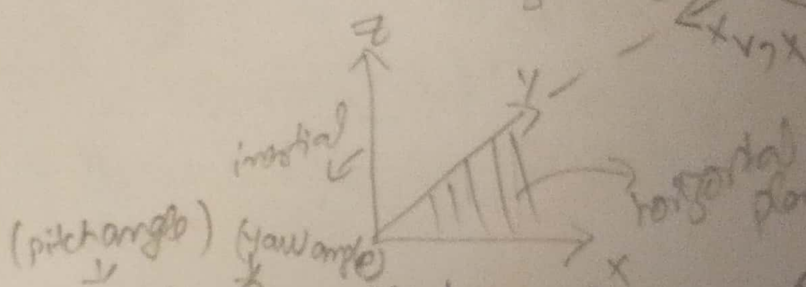
Here on slope vehicle frame & path frame will be diff.

when vehicle will be here. the vehicle frame & path frame is same. ~~bed~~

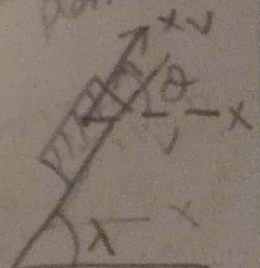
But if vehicle is tracking path perfectly than even in the case of non-straight path both frame will be same & if tracking imperfectly it will not be same then.

b) when inertial frame coincides with the vehicle reference frame. In such case horizontal frame will become same to vehicle frame so x y z & x_v & y_v & z_v coincides.

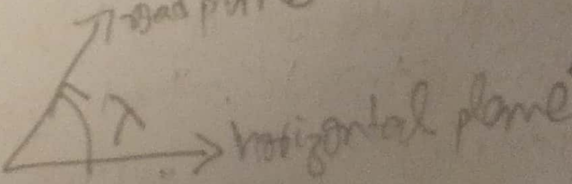




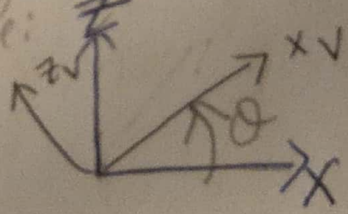
So In such a case θ & ϕ will be zero.



c) Road slope angle:



vehicle pitch angle:



- horizontal frame
- vehicle frame

i.e $\theta = 0, \phi = 0$

If vehicle's suspension is not deformed then x_v is parallel to road plane (x) then road slope angle & vehicle pitch angle will be same i.e θ.

If vehicle's suspension is deformed then x_v will not be parallel to x then slope angle & vehicle pitch angle will not be equal anymore.

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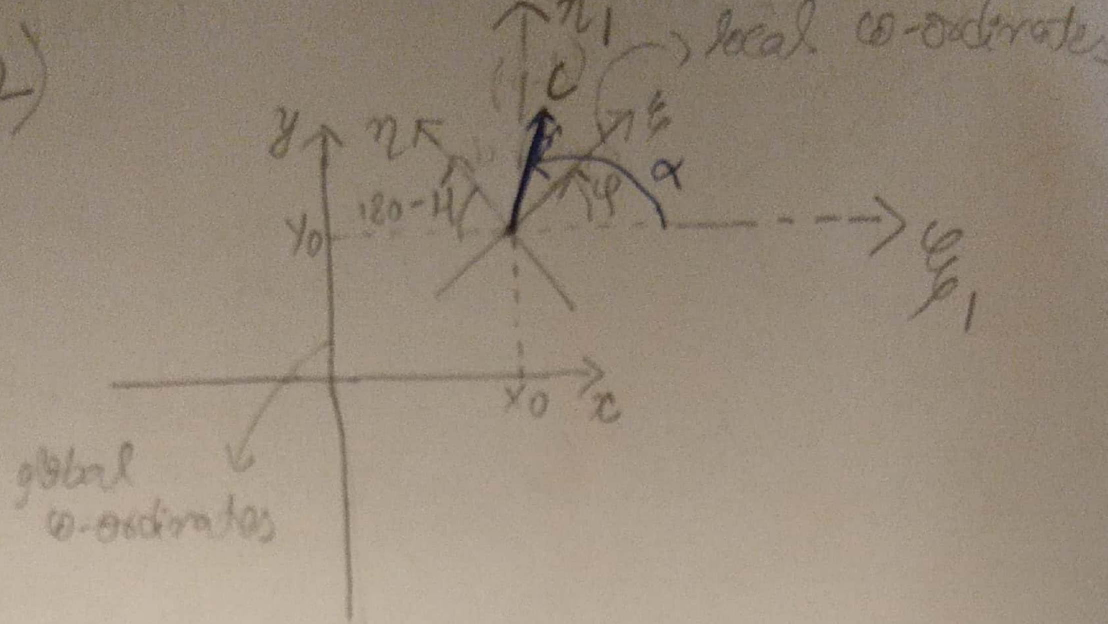
Q2)

a)



⇒ Given: Given the co-ordinates of c in (ξ_1, η_1)

⇒ To find: c w.r.t x & y co-ordinates.

$\xi_1 = r \sin\alpha$

$\eta_1 = r\cos\alpha$

First transform in $\xi\eta$-coordinate.

$$\eta = r\cos(\alpha-\varphi) = r(\cos\alpha\cos\varphi + \sin\alpha\sin\varphi)$$
$$= \underbrace{r\cos\alpha}_{\eta_1}\cos\varphi + \underbrace{r\sin\alpha}_{\xi_1}\sin\varphi$$

$$\xi = r\sin(\alpha-\varphi) = \underbrace{r\sin(\alpha)}_{\xi_1}\cos\varphi - \underbrace{r\cos\alpha}_{\eta_1}\sin\varphi$$

Then:
$$\begin{bmatrix}\eta\\ \xi\end{bmatrix} = \begin{bmatrix}\cos\varphi & \sin\varphi\\ -\sin\varphi & \cos\varphi\end{bmatrix}\begin{bmatrix}\eta_1\\ \xi_1\end{bmatrix}$$

So in xy co-ordinate it will be -

$$x = x_0 + \xi\eta$$

$$x = x_0 + \begin{bmatrix}\cos\varphi & \sin\varphi\\ -\sin\varphi & \cos\varphi\end{bmatrix}^{-1}\begin{bmatrix}\eta\\ \xi\end{bmatrix}$$

$$x = x_0 + \begin{bmatrix}\cos\varphi & -\sin\varphi\\ \sin\varphi & \cos\varphi\end{bmatrix}\begin{bmatrix}\eta\\ \xi\end{bmatrix}$$

$$x = x_0 - \sin\varphi\, \ell + \cos\varphi\, \eta$$
$$y = y_0 + \cos\varphi\, \ell + \sin\varphi\, \eta$$

b) From above

$$\begin{bmatrix} x - x_0 \\ y - y_0 \end{bmatrix} = \begin{bmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{bmatrix} \begin{bmatrix} \eta \\ \ell \end{bmatrix}$$

$$\begin{bmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{bmatrix}^{-1} \begin{bmatrix} x - x_0 \\ y - y_0 \end{bmatrix} = \begin{bmatrix} \eta \\ \ell \end{bmatrix}$$

$$\begin{bmatrix} \cos\varphi & \sin\varphi \\ \sin\varphi & \cos\varphi \end{bmatrix} \begin{bmatrix} x - x_0 \\ y - y_0 \end{bmatrix} = \begin{bmatrix} \eta \\ \ell \end{bmatrix}$$

$$\eta = (x - x_0) \cdot \cos\varphi + (y - y_0) \cdot \sin\varphi$$
$$\ell = (x - x_0) \cdot \sin\varphi + (y - y_0) \cdot \cos\varphi$$

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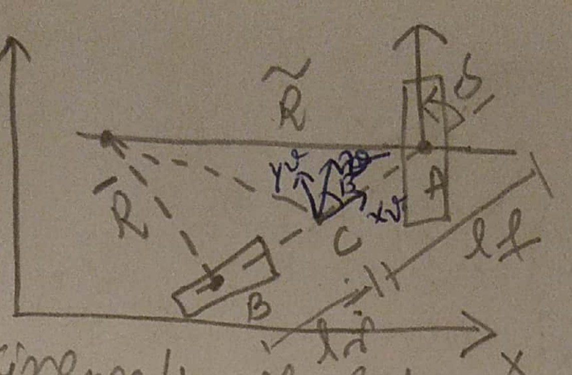


As in front wheel drive $\delta_r = 0$

Kinematics of lateral vehicle motions

i) Tire slip angles of front & rear wheel is zero b/c actual velocity of wheel's centre point A & B in above diag are in the same direction as the wheel's orientation.
This is a good assumption for low speed motion (less than $5\frac{m}{s}$) so in that case the lateral force generated by the tires is small. To derive on any circular road of radius R, the total lateral forces (centripetal forces) from both tires is

$$\frac{mV^2}{R}$$ which varies quadratically with the speed V and is small at low speeds.

so when V is small, it can be assumed that the velocity vector at each wheel is in the direction of wheel.

ii) The two front & the two rare wheels are lumped into 1 front & 1 rear wheel for bicycle kinematic model.

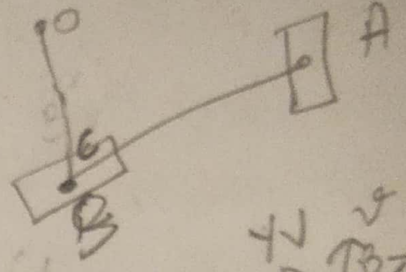
iii) Assuming front wheel drive so no rear steering ($\delta_r = 0$).

iv) vehicle velocity 'ϑ' is perpendicular to the connection b/w the point & the instantaneous centre of rotation 'O'.

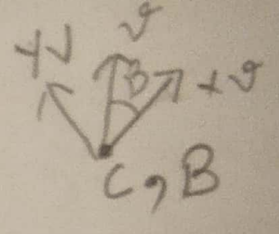
v) vehicle ref point 'C' is at the centre of vehicle (equidistant from front & rear axle)

vi) 'v' is assumed to be constant

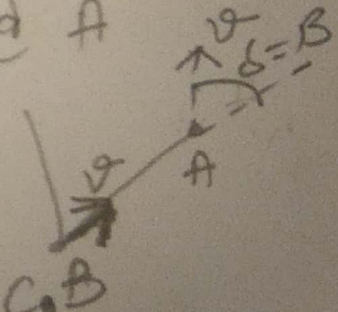
b) $l_r \in [0, l]$



As $\delta_r = 0$ so if vehicle ref point 'C' lies on the centre of rear axle, so $l_r = l$ and $l_f = 0$ then at this point A: $\delta = \beta$ and @ B $\delta = 0$ so lateral velocity will be 'O' for this case when B & C co-incides.

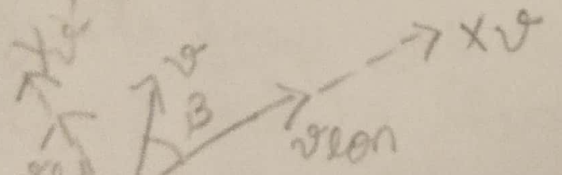


If C & B coincides then @ A



@ C, B
$\delta = 0$

c)



$\vartheta_{lon} = V\cos\beta$

$\vartheta_{lat} = V\sin\beta$

$\beta = a\tan\left(\frac{l_r \tan\delta}{l_r + l_f}\right)$

d) ϑ is constant & $\delta \neq 0$

as $v = r \times w = rw\sin\theta$ ∵ As ϑ is always perp to $\bar{R}$ & $\tilde{R}$

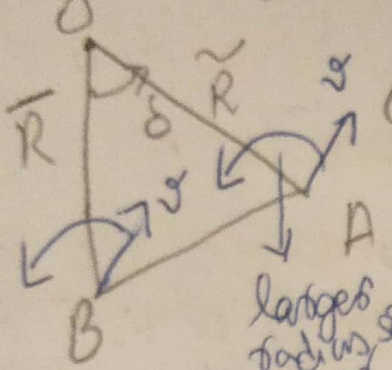
$\vartheta = rw\sin 90^\circ = rw$ ∴ So it means

ω is always constant which means R & $\tilde{R}$ are constant too

$\omega = \dfrac{v \leftarrow \text{const}}{r \leftarrow \text{const}}$

then the path traced by the midpoint of front bicycle & rear bicycle wheel will be both circle. (As r is constant only in case of circular path).

e) From large Δ_{OBA} :-



largest radius so largest path

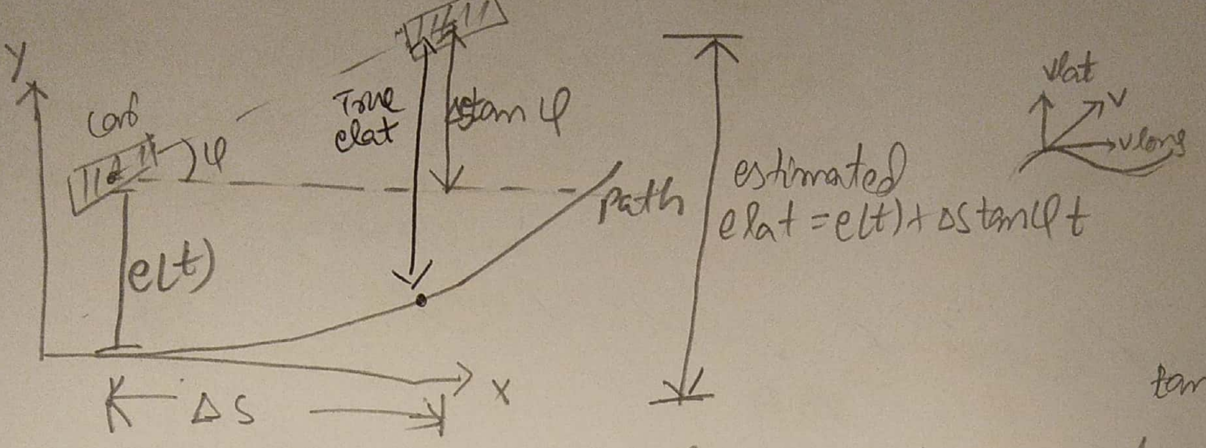
$\cos\delta = \dfrac{\bar{R}}{\tilde{R}}$ As max value of $\cos\delta$ can be 1 so

$\cos\delta \leq 1$

$\dfrac{|\bar{R}|}{\tilde{R}} \leq 1 \Rightarrow |\bar{R}| \leq |\tilde{R}|$

Therefore magnitude of $\tilde{R}$ will be greater than $\bar{R}$ so the path by rear wheel will be smaller than front wheel, Q4

b) $e_{lat} \approx e(t) + \Delta s \tan(\Delta\varphi(t)) \approx e(t) + \Delta s\, \Delta\varphi(t)$



tan = P/B

If path is not a straight line as shown above then True elat will be different than estimated.

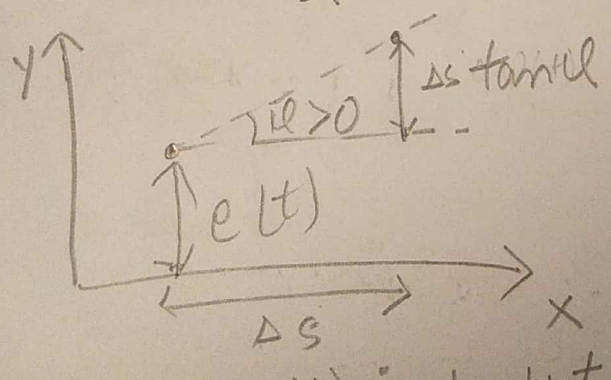
c) In our case the close loop stable means the system should follow desired path. we need to check sign of $e_{la}(t)$ in the formula as Δs is given positive

$$\delta(t) = K_{la}\, e_{la}(t)$$

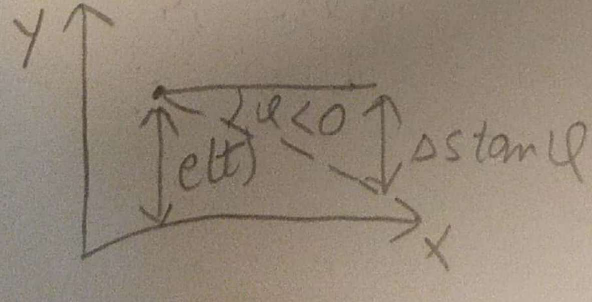
$$\downarrow$$

$$e_{la}(t) = e(t) + \Delta s\, \Delta\varphi(t) \quad \text{— A)}$$

i) so there can be a case 1:- $e(t)$ is + & $\varphi(t)$ is +, the car is moving away from X.

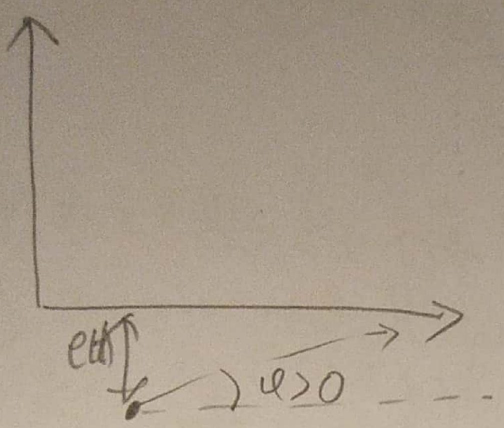


ii) case 2:- $e(t)$ is + but φ is −, the car is moving towards X



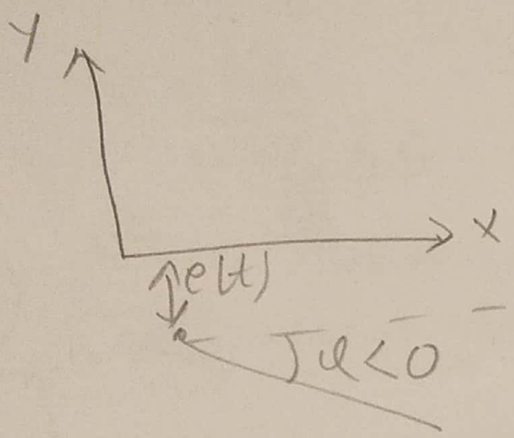
iii) Case 3:-

e(t) is $-$, θ is +, the car is moving towards X.



iv) Case 4:-

e(t) is $-$, θ is also $-$ the car is moving away from X.



so in order to want car to move towards x to acheive stability

- For Case 1:-
  elat $\rightarrow$ + so $\boxed{K_{la} < 0}$ to reach towards x
- For Case 2:-
  elat $\rightarrow$ + If $e(t) > \Delta s$ $\boxed{\text{so } K_{la} < 0}$
  $-$ If $e(t) < \Delta s$ so $K_{la} > 0$
- For Case 3:- elat $\rightarrow$ + If $e(t) > \Delta s$ so $K_{la} > 0$
  $-$ If $e(t) < \Delta s$ $\boxed{\text{so } K_{la} < 0}$
- For Cas 4:- elat $\rightarrow$ $-$ $\boxed{\text{so } K_{la} < 0}$

d) linearize the kinematic bicycle model around the x-axis, assuming small angle approximations & constant velocity v

: $\Delta\psi$ is small

v = constant

If $\psi \approx 0$ so $e_{lat} = e(t) + 0 \Rightarrow e_{lat} \approx e(t)$

so around x-axis the model will linearize &

$e(t) = y(t) - \underbrace{y_{ref}(t)}_{0} \Rightarrow e(t) = y(t)$

From kinematic bycycle equations:-

$\dot{y}(t) = v \sin(\psi + B)$

$\dot{\psi}(t) = \frac{V}{L} \sin(\delta(t))$

Replace $y(t)$ by $e(t)$ and using small angle approximation $\sin\theta \approx \theta$

$\dot{e}(t) = v(\psi(t) + \delta(t))$

$\dot{\psi}(t) = \frac{V}{L}\delta(t)$

e) $e_{lat} = e(t) + \Delta s \, \Delta\psi(t)$

As the path is x-axis, the yaw angle of the path is zero so $\Delta\psi(t) = \psi(t) - \underbrace{\psi_{ref}(t)}_{0} \Rightarrow \Delta\psi(t) = \psi(t)$

$e_{lat} = e(t) + \Delta s \, \psi(t)$

From the linearized model we know:-

$\dot{e}(t) = v(\psi(t) + \delta(t)) \Rightarrow \psi(t) = \frac{\dot{e}(t)}{v} - \delta(t)$

so: $e_{lat}(t) = e(t) + \Delta s \, \psi(t) = e(t) + \Delta s\left(\frac{\dot{e}(t)}{v} - \delta(t)\right)$

so from eq 1):- $\delta_t = K_{la}\left[e(t) + \frac{\Delta s\, \dot{e}(t)}{V} - \Delta_s\, \delta(t)\right]$

$$\delta_t = K_{la}\, e(t) + \frac{K_{la}\, \Delta s\, \dot{e}(t)}{V} - K_{la}\, \Delta_s\, \delta(t)$$

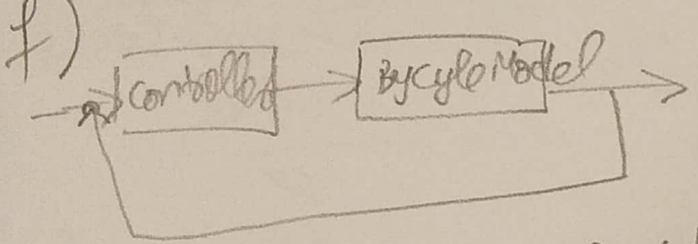
$$\delta_t (1 + K_{la} \Delta_s) = K_{la}\, e(t) + \frac{K_{la}\, \Delta s\, \dot{e}(t)}{V}$$

$$\delta_t = \frac{K_{la}\, e(t)}{1 + K_{la} \Delta s} + \frac{K_{la}\, \Delta s\, \dot{e}(t)}{(1 + K_{la} \Delta s) V}$$

compairing with equation of PD in eq 3

$$K_p = \frac{K_{la}}{1 + K_{la} \Delta S} \qquad K_d = \frac{K_{la}\, \Delta S}{V(1 + K_{la} \Delta S}$$

f)



$v = 10\ m/s$

$l = 2\ m$

$$\dot{e}(t) = 10\, \psi(t) + 10 \cdot \delta(t)$$

$$\dot{\psi}(t) = 5 \cdot \delta(t)$$

In S.S:- $\begin{bmatrix} \dot{e}(t) \\ \dot{\psi}(t) \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 10 \\ 0 & 0 \end{bmatrix}}_{A} \begin{bmatrix} e(t) \\ \psi(t) \end{bmatrix} + \underbrace{\begin{bmatrix} 10 \\ 5 \end{bmatrix}}_{B} \delta(t)$

$$y(t) = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C} \begin{bmatrix} e(t) \\ \psi(t) \end{bmatrix}$$

$$G(s) = C(sI - A)^{-1} B = \begin{bmatrix} 1 & 0 \end{bmatrix} \left(\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 10 \\ 0 & 0 \end{bmatrix} \right)^{-1} \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$

$$= \underset{1\times 2}{\begin{bmatrix} 1 & 0 \end{bmatrix}} \underset{2\times 2}{\begin{bmatrix} s & +10 \\ 0 & s \end{bmatrix}} \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 10s & 50 \\ 0 & 5s \end{bmatrix}$$

$$G(s) = \begin{bmatrix} 10 & 50 \end{bmatrix}$$

Laplace transform of the control input:-

$$C(s) = K_p + K_d s$$

$$H(s) = \frac{C(s)\,G(s)}{1 + C(s)\,G(s)} = \frac{\left(\frac{50}{s^2} + \frac{10}{s}\right) \times \left(K_p + K_d s\right)}{1 + \left(\frac{50}{s^2} + \frac{10}{s}\right)\left(K_p + K_d s\right)}$$

$$= \frac{\frac{50K_p}{s^2} + \frac{10K_p}{s} + \frac{50}{s}K_d + \frac{10}{1}K_d}{\frac{\cancel{s^2}\, 50K_p + 10K_p s + 50 s K_d + 10 s^2 K_d}{s^2}} = \frac{\frac{50K_p + 10K_p s + 50 K_d s + 10 s^2 K_d}{\cancel{s^2}}}{\frac{s^2 + 50K_p + 10K_p s + 50 s K_d + 10 s^2 K_d}{\cancel{s}}}$$

$$= \frac{s^2 K_d 10 + s(50K_d + 10K_p) + 50K_p}{s^2(1 + 10K_d) + s(10K_p + 50K_d) + 50K_p}$$

Comparing with standard 2nd order equation

$$s^2 + 2\zeta\omega_n s + \omega_n^2$$

$K_p = ?$ $K_d = ?$ $t_r = 2\,\text{sec}$ $O\% = 10\%$

$$t_r = \frac{\pi}{2\omega_n\sqrt{1-\zeta^2}} \qquad O\% = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100$$

from calculator

$\zeta = 0.5912$ $\omega_n = 0.9738$

$$\frac{50K_p}{1+10K_d} = \omega_n^2 \quad \ldots$$

$K_d = 0.0238$

$K_p = 0.0235$

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