Ex Sheet 3

Ex 1

a) The understeer gradient is the difference b/w the ratios of the load fraction & cornering stiffness b/w the front & rear wheels.

$$K = \frac{W_f}{C_{\alpha,f}} - \frac{W_r}{C_{\alpha,r}}$$

b) For understeering vehicle the point where the steering angle passes two times the kinematic steering angle ($2\delta_{kin}$) for the first time is defined as a special point, the velocity corresponding to the lateral acc in the special point is called characteristic velocity.

For the oversteering vehicle, the point where the steering angle passes the zero mark is defined as a particular lateral acceleration and the speed that corresponds to that lateral acceleration is defined as the critical velocity. So, characteristic speed is for the under-steering vehicle, critical speed is for the over-steering vehicle.

c) $\delta_{dyn} = \delta_{kin} + \alpha_f - \alpha_r$

~~for understeering if $\alpha_f > 0$~~

For understeering $\Rightarrow$ The vehicle is still stable

For oversteering $\Rightarrow$ The driver feels that vehicle seems to want to slow around & ultimately to travel in reverse.

d) Yes.

e) β

LBM → non zero bank angle

$$\frac{d}{dt}\begin{bmatrix} v_{lat} \\ \omega \end{bmatrix} = A\begin{bmatrix} v_{lat} \\ \omega \end{bmatrix} + B\delta + B_{bank}\beta$$

$A \in \mathbb{R}^{2\times2}$

$B, B_{bank} \in \mathbb{R}^{2\times1}$

e) Considering the bank angle lateral dynamics will be:

$$m(\dot{v}_{lat} + \omega v_{lon}) = F_{r} + F_{f} + \underbrace{F_{bank}}_{= mg\sin(\beta)}$$

Yaw dynamics is not effected by the bank angle.

$$\frac{d}{dt}\begin{bmatrix} v_{lat} \\ \omega \end{bmatrix} = \underbrace{\begin{bmatrix} & \\ & \end{bmatrix}}_{\hat{=} A}\begin{bmatrix} v_{lat} \\ \omega \end{bmatrix} + \underbrace{\begin{bmatrix} \\ \end{bmatrix}}_{= B}\delta + \underbrace{\begin{bmatrix} g \\ 0 \end{bmatrix}}_{= B_{Bank}}\beta$$

Ex 2:

a)

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Ex 2:- $m = 1900\,kg$ $l = 2.40\,m$ $I_z = 2900\,kg\,m^2$

a) $K = \frac{W_f}{C_{\alpha,f}} - \frac{W_r}{C_{\alpha,r}}$ where $W_f = \frac{l_r}{l}$, $W_r = \frac{l_f}{l}$

Car A) $K = \frac{0.60}{100{,}000} - \frac{0.4}{100{,}000} = 2 \times 10^{-6}$

Car B) $K = \frac{0.65}{150{,}000} - \frac{0.35}{150{,}000} = 2 \times 10^{-6}$

Car C) $K = \frac{0.6}{100{,}000} - \frac{0.4}{150{,}000} = 3.33 \times 10^{-6}$

Car D) $K = \frac{0.45}{150{,}000} - \frac{0.55}{180{,}000} = -6.66 \times 10^{-7}$

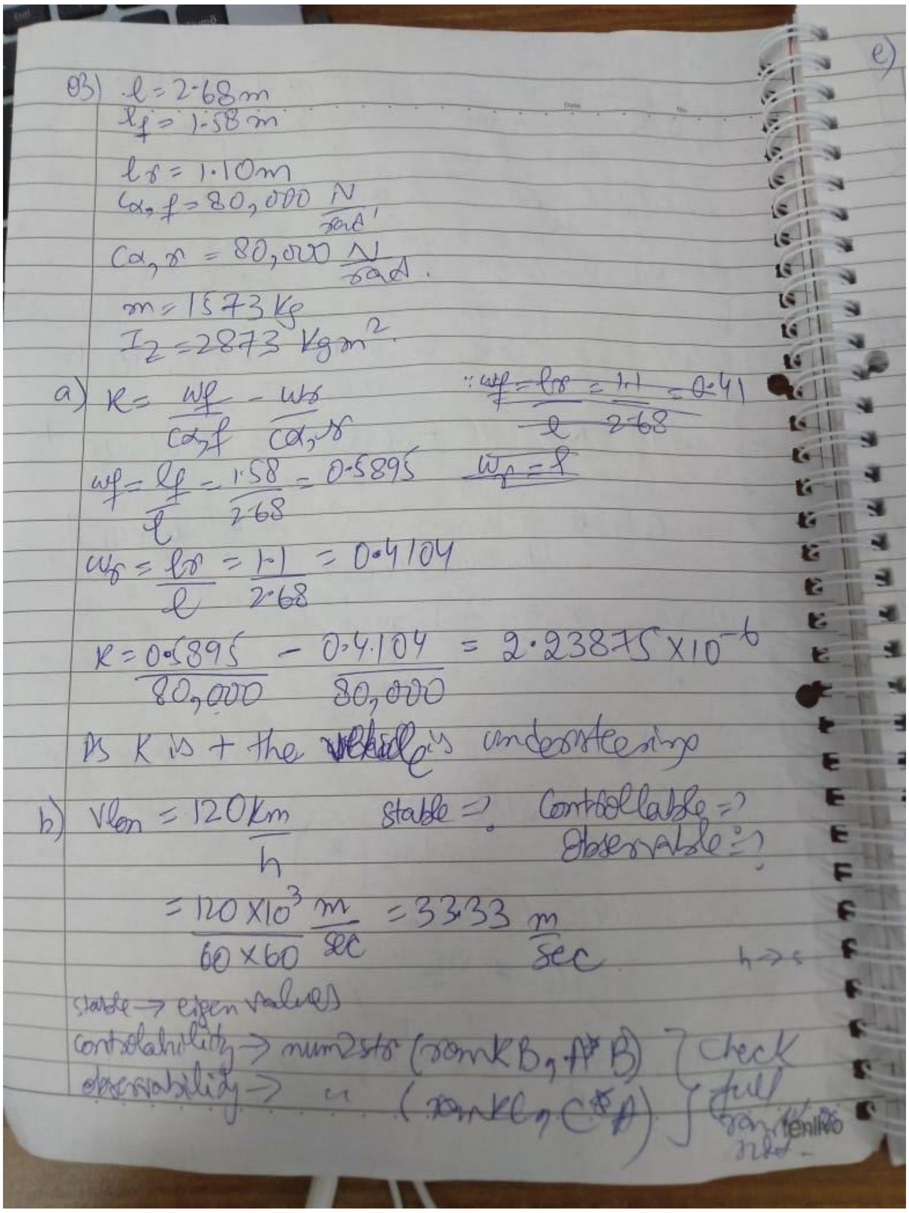
Car e) $K = \frac{0.5}{140{,}000} - \frac{0.5}{160{,}000} = 4.464 \times 10^{-7}$

$\rightarrow$ 5m & 60m

b-e) In matlab

Ex3

a-b)



Q3) $l = 2.68\,m$

$l_f = 1.58\,m$

$l_r = 1.10\,m$

$C_{\alpha,f} = 80{,}000\ \frac{N}{rad}$

$C_{\alpha,r} = 80{,}000\ \frac{N}{rad}$

$m = 1573\,kg$

$I_z = 2873\,kg\,m^2$

a) $K = \frac{W_f}{C_{\alpha,f}} - \frac{W_r}{C_{\alpha,r}}$ $\quad$ $\cancel{\because W_f = \frac{l_r}{l} = \frac{1.1}{2.68} = 0.41}$

$W_f = \frac{l_f}{l} = \frac{1.58}{2.68} = 0.5895$ $\quad$ $W_f = ?$

$W_r = \frac{l_r}{l} = \frac{1.1}{2.68} = 0.4104$

$K = \frac{0.5895}{80{,}000} - \frac{0.4104}{80{,}000} = 2.23875 \times 10^{-6}$

As K is + the vehicle is understeering

b) $V_{lon} = 120\,\frac{km}{h}$ $\quad$ stable => Controllable => Observable =>

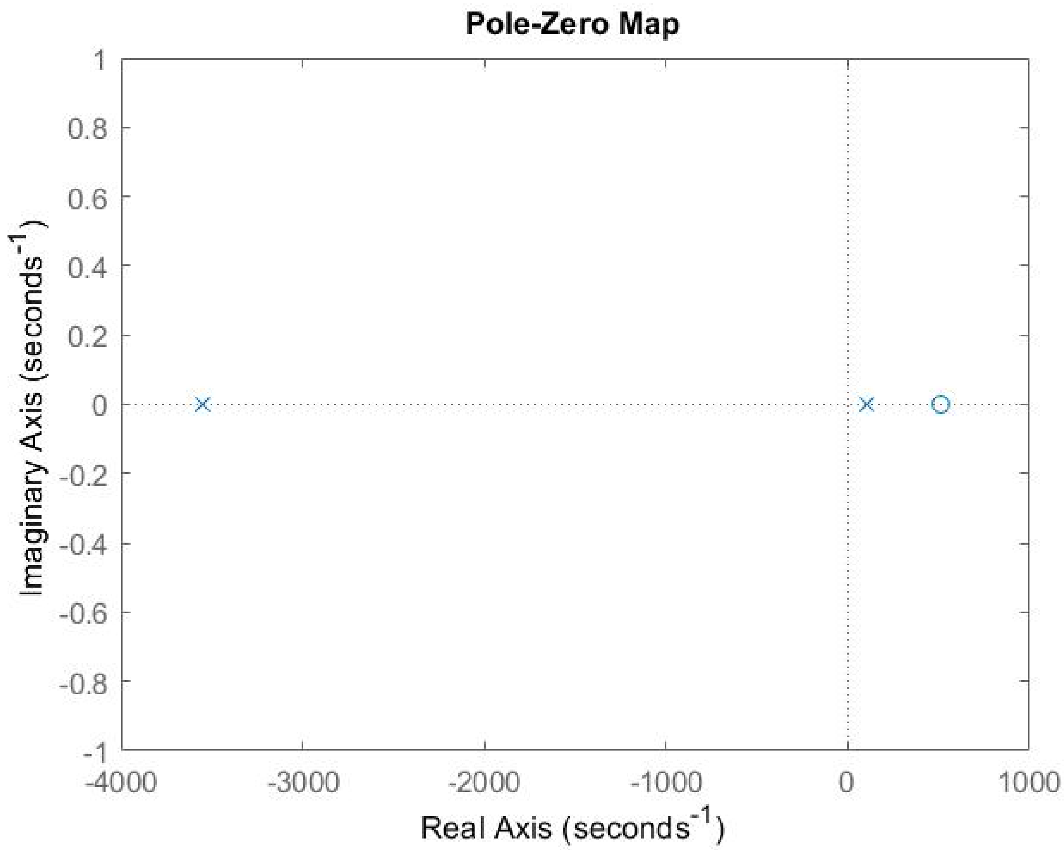
$= \frac{120 \times 10^3}{60 \times 60}\,\frac{m}{sec} = 33.33\,\frac{m}{sec}$ $\quad$ h→s

stable → eigen values

controlability → num2str (rank B, A*B) } check

observability → " (rank C, C*A) } full rank

From pole zero map of open system it can be seen that pole-zero lies in right half plane so it the system is open loop unstable. On the other hand system is controllable and observable (from matlab).



c) In matlab

d) As the system is controllable(proved in part b) so it can be stabilized using feedback.

e)

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e) $\delta(t) = K\omega(t) \quad \forall K \in \mathbb{R}$

$$u(t) = Ky(t) = KCx(t) = \underbrace{\begin{bmatrix} 0 & K \end{bmatrix}}_{\hat{=}K} \underbrace{\begin{bmatrix} v_{lat} \\ \omega \end{bmatrix}}_{= x(t)}$$

$$\dot{x} = Ax + Bu = (A + B \cdot K)x$$

Examine eigenvalues of $A + B \cdot K \Rightarrow \det(s - A - BK) = 0$

$$BK = \begin{bmatrix} \frac{C_{\alpha,f}}{m} \\ \frac{C_{\alpha,f}\, l_f}{I_z} \end{bmatrix} \begin{bmatrix} 0 & K \end{bmatrix} = \begin{bmatrix} 0 & K\frac{C_{\alpha,f}}{m} \\ 0 & K\frac{C_{\alpha,f}\, l_f}{I_z} \end{bmatrix}$$

$$\det\left(\begin{bmatrix} s + \frac{C_{\alpha,r} + C_{\alpha,f}}{m v_{lon}} - 0 & \frac{-C_{\alpha,r} l_r + C_{\alpha,f} l_f + v_{lon}}{m v_{lon}} - K\frac{C_{\alpha,f}}{m} \\ -\frac{C_{\alpha,r} l_r + C_{\alpha,f} l_f}{I_z v_{lon}} - 0 & s + \frac{C_{\alpha,r} l_r^2 + C_{\alpha,f} l_f^2}{I_z v_{lon}} - K\frac{C_{\alpha,f} l_f}{I_z} \end{bmatrix}\right) = 0$$

$$\left(s + \frac{C_{\alpha,r} + C_{\alpha,f}}{m v_{lon}}\right)\left(s + \frac{C_{\alpha,r} l_r^2 + C_{\alpha,f} l_f^2}{I_z v_{lon}} - K\frac{C_{\alpha,f} l_f}{I_z}\right) -$$

$$\left[\frac{C_{\alpha,r} l_r + C_{\alpha,f} l_f}{m v_{lon}} - v_{lon} + K\frac{C_{\alpha,f}}{m}\right]\left[\frac{C_{\alpha,r} l_r - C_{\alpha,f} l_f}{I_z v_{lon}}\right] = 0$$

$$s^2 + \left(\frac{C_{\alpha,r} + C_{\alpha,f}}{m v_{lon}} + \frac{C_{\alpha,r} l_r^2 + C_{\alpha,f} l_f^2}{I_z v_{lon}} - K\frac{C_{\alpha,f} l_f}{I_z}\right)s -$$

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$$\left[\left(\frac{C_{\alpha,r} l_r + C_{\alpha,f} l_f}{m v_{lon}} - v_{lon}\right)\frac{C_{\alpha,r} l_r + C_{\alpha,f} l_f}{I_z v_{lon}}\right]$$

$$+\frac{(C_{\alpha,r}+C_{\alpha,f})(C_{\alpha,r} l_r^2 + C_{\alpha,f} l_f^2)}{m I_z v_{lon}^2} - \frac{C_{\alpha,r}+C_{\alpha,f}}{m v_{lon}} K$$

$$\frac{C_{\alpha,f} l_f}{I_z} - K\frac{C_{\alpha,f}}{m} = 0$$

$$b = \frac{I_z(C_{\alpha,r}+C_{\alpha,f}) + m(C_{\alpha,r} l_r^2 + C_{\alpha,f} l_f^2)}{m I_z v_{lon}} - K\frac{C_{\alpha,f} l_f}{I_z}$$

$$c = \frac{m v_{lon}^2 (l_r C_{\alpha,r} - l_f C_{\alpha,f}) + C_{\alpha,f} C_{\alpha,r} l^2}{m I_z v_{lon}^2}$$

$$-K\left[\frac{(C_{\alpha,f}+C_{\alpha,r}) l_f C_{\alpha,f} + C_{\alpha,f}(l_r C_{\alpha,r} - l_f C_{\alpha,f})}{m I_z v_{lon}}\right]$$

$$= \frac{m v_{lon}^2 (l_r C_{\alpha,r} - l_f C_{\alpha,f}) + C_{\alpha,f} C_{\alpha,r} l^2}{m I_z v_{lon}^2}$$

$$-K\frac{C_{\alpha,f} C_{\alpha,r} l}{m I_z v_{lon}}$$

proved.

f)

f) for condition 1:-

$$K < \frac{I_z(C_{\alpha,r} + C_{\alpha,f}) + m(C_{\alpha,r} l_r^2 + C_{\alpha,f} l_f^2)}{m I_z v_{lon}}$$

for condition 2:-

$$K < \frac{m v_{lon}^2 (l_r C_{\alpha,r} - l_f C_{\alpha,f}) + C_{\alpha,f} C_{\alpha,r} l^2}{m I_z v_{lon}^2}$$

$$K < \frac{m v_{lon}^2 (l_r C_{\alpha,r} - C_{\alpha,f} l_f) + C_{\alpha,f} C_{\alpha,r} l^2}{m I_z v_{lon}^2}$$

Inputting values in matlab:-

$K < 0.1397$

$K < -6.7115 \times 10^{10}$

g)