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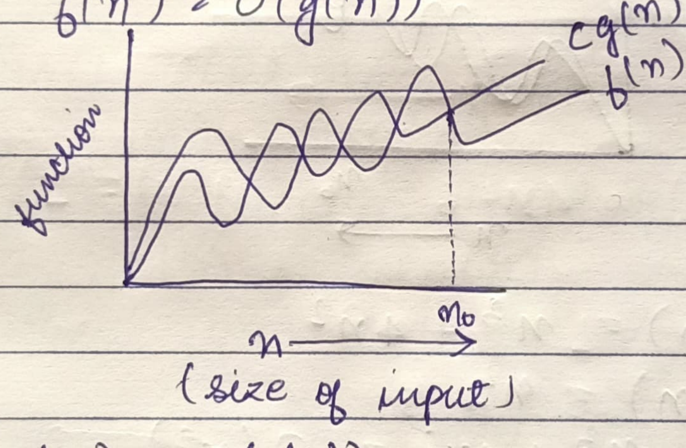
Tutorial - 1

1. Asymptotic Notation - They are the mathematical notations used to describe the running time of an algorithm when the input tends towards a particular value or a limiting value.

Different asymptotic notations -

i Big $O(n)$

$f(n) = O(g(n))$



$f(n) = O(g(n))$

iff

$f(n) \leq cg(n)$

$\forall\ n \geq n_0$

for some constant, $c > 0$

$g(n)$ is "tight" upper bound of $f(n)$.

ex → $f(n) = n^2 + n$

$g(n) = n^3$

$n^2 + n \leq c.n^3$

$n^2 + n = O(n^3)$

ii Big Omega (Ω)

$$f(n) = \Omega(g(n))$$

g(n) is "tight" ~~upper and~~ lower bound of function f(n)

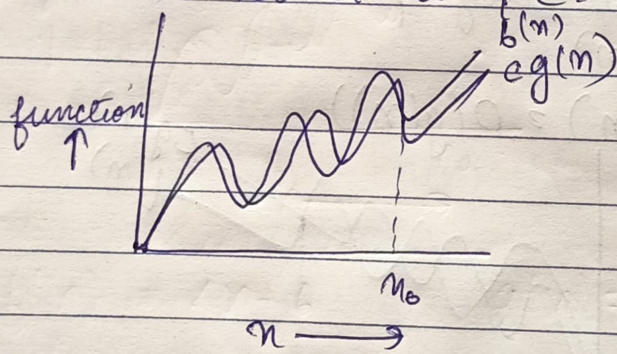
$$f(n) = \Omega(g(n))$$

iff

$$f(n) \geq c\,g(n)$$

$$\forall\, n \geq n_0$$

for some constant $c > 0$



Ex -

$$f(n) = n^3 + 4n^2$$

$$g(n) = n^2$$

$$n^3 + 4n^2 = \Omega(n^2)$$

iii Big Theta (Θ)

$$f(n) = \Theta(g(n))$$

g(n) is both "tight" upper and lower bound of function f(n)

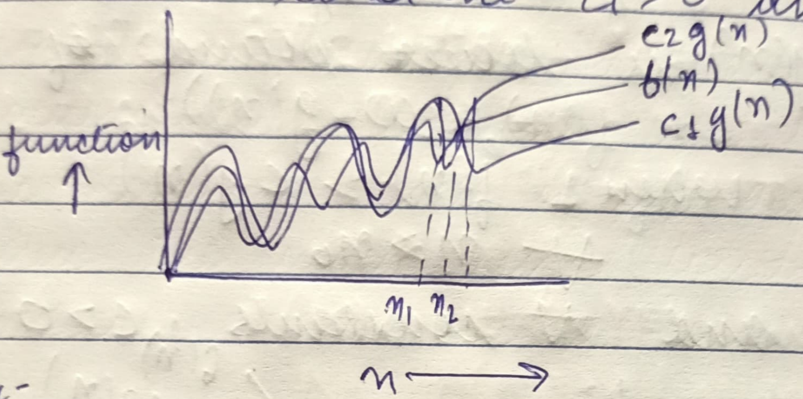
$$f(n) = \Theta(g(n))$$

iff

$$c_1 g(n) \leq f(n) \leq c_2 g(n)$$

$\forall\ n \geq \max(n_1, n_2)$

for some constant $c_1 > 0$ and $c_2 > 0$



ex-

$3n+2 = \Theta(n)$ as $3n+2 \geq 3n$ and

$3n+2 \leq 4n$ for n, $k_1 = 3$, $k_2 = 4$ & $n_0 = 2$

iv) Small $o(\theta)$ -

$$f(n) = o(g(n))$$

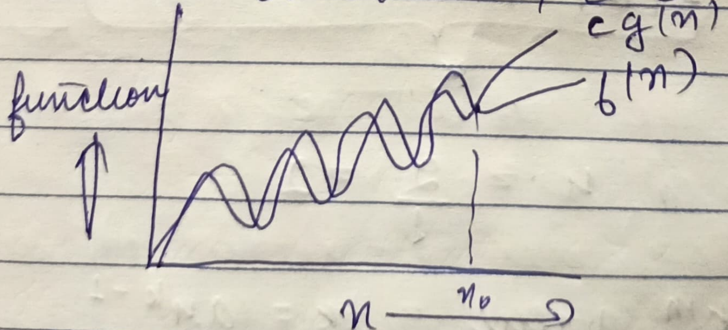
$g(n)$ is upper bound of function $f(n)$

$$f(n) = o(g(n))$$

when $f(n) < cg(n)$

$\forall\ n > n_0$

and $\forall$ constants, $c > 0$



ex→ $f(n) = n^2$

$g(n) = n^3$

$n^2 = o(n^3)$

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v Small omega (ω)

$$f(n) = \omega(g(n))$$

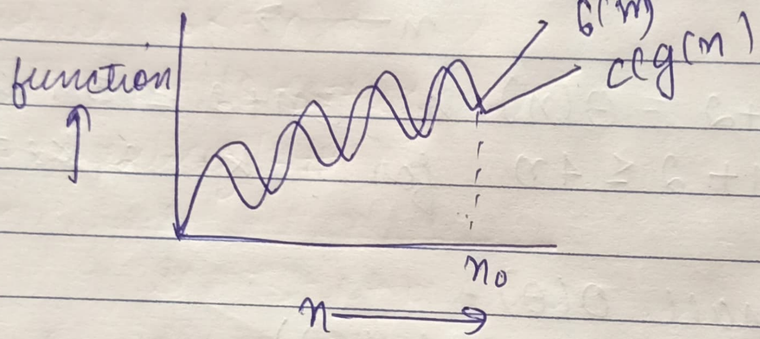
$g(n)$ is lower bound of $f(n)$

$$f(n) = \omega(g(n))$$

when $f(n) > c\,g(n)$

$\forall\ n > n_0$

and $\forall$ constants $c > 0$



$f(n) = 4n+6$ $\qquad$ $g(n) = (1)$

2. for (i = 1 to n)
{ i = i * 2 ; }

→ $i = \underbrace{1, 2, 4, 8, 16 \cdots\cdots n}_{k}$ (G.P) — $O(k)$

$a = 1$, $r = \frac{2}{1} = 2$

GP k^{th} value $= t_k = ar^{k-1}$

$$n = 1 \times 2^{k-1}.$$

$$n = 2^{k-1}$$

$$n = \frac{2^k}{2}$$

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$$2n = 2^k$$
$$\log(2n) = k\log 2$$
$$k = \log_2 2n$$
$$k = \log_2 2 + \log_2 n$$
$$k = 1 + \log n$$
Time comp $= O(1 + \log_2 n)$
$= O(\log_2 n)$

3. $T(n) = 3T(n-1)$ — ①

Let $n = n-1$

$T(n-1) = 3T(n-2)$ — ②

Put ② in ①

$T(n) = 3 \times 3T(n-2)$ — ③

Put $n = n-2$

$T(n-2) = 3T(n-3)$ — ④

Put ④ in ③

$T(n) = 3 \times 3 \times 3T(n-3)$ — ⑤

$T(n) = 3^n T(n-n) = 3^n T(0)$

$= 3^n$

$= O(3^n)$

4. $T(n) = 2T(n-1) - 1$

$= 2(2T(n-2) - 1) - 1$

$= 2^2(T(n-2)) - 2^1 - 1$

$= 2^3 T(n-3) - 2^2 - 2^1 - 2^0$

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$= 2^n T(n-n) - 2^{n-1} - 2^{n-2} - 2^{n-3}$
$\quad \cdots - 2^2 - 2^1 - 2^0$
$= 2^n - 2^{n-1} - 2^{n-2} - 2^{n-3} \cdots 2^2 - 2^1 - 2^0$
$= 2^n - (2^n - 1)$
$T(n) = 1$

5.
```
int i=1, s=1;
while (s<=n) {
    i++; s=s+i;
    printf("#");
}
```

$s_i = s_{i-1} + i$

i is incrementing by one step
s in incrementing by value of i
Following will be values after few iterations -
$\Rightarrow i = 2, \; s = 3$ 1st iteration
$\Rightarrow i = 3, \; s = 6$ 2nd iteration
$\Rightarrow i = 4, \; s = 10$ 3rd iteration
Let the value of n be k.
Values of $s \Rightarrow 1, 3, 6, 10 \cdots$
s represents a series of sum of first ~~n~~ n natural numbers
for $i = k, \; s = \frac{k(k+1)}{2}$

for stopping loop.

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$$\frac{k(k+1)}{2} > n \Rightarrow \frac{k^2+k}{2} > n$$

$$T(n) = O(\sqrt{n})$$

6.
```
void function(int n){
    int i, count = 0;
    for(i = 1; i*i <= n; i++)
        count++;
}
```

$i = 1, 2, 3 \ldots n$

$i^2 = 1, 4, 9 \ldots n$

So $i^2 <= n$ or $i <= \sqrt{n}$

$a_k = a + (k-1)d$

$a = 1 \quad d = 1$

$a_k <= \sqrt{n}$

$\sqrt{n} = 1 + (k-1) \cdot 1$

$\sqrt{n} = k$

$T(n) = O(\sqrt{n})$

7.
```
void function(int n){
    int i, j, k, count = 0;
    for(i = n/2; i <= n; i++)
        for(j = 1; j <= n; j = j*2)
            for(k = 1; k <= n; k = k*2)
                count++;
            }
        }
    }
}
```

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$i = n/2$ $\quad j = \log_2 n$ $\quad k = \log_2 n$

$\left(\frac{n}{2}+1\right)$ times $\quad \log_2 n \quad \log_2 n$

$O(i * j * k) = O\left(\left(\frac{n}{2}+1\right) * \log_2 n * \log_2 n\right)$

$= O\left(\left(\frac{n}{2}+1\right) \times (\log n)^2\right)$

$T(n) = O(n (\log n)^2)$

8. function (int n) {
    if (n==1) return;
    for (i=1 to n){
        for (j=1 to n) {
            print ("*");
        }
        function (n-3);
    }

$T(n) = T(n-3) + n^2$ —— ①

$T(1) = 1$ —— ②

put $n = n-3$ in ①

$T(n-3) = T(n-6) + (n-3)^2$ —— ③

Put ③ in ①

$T(n) = T(n-6) + (n-3)^2 + n^2$ —— ④

put $n = n-6$ in ①

$T(n-6) = T(n-9) + (n-6)^2$ —— ⑤

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Put (5) in (4)

$$T(n) = T(n-9) + (n-6)^2 + (n-3)^2 + n$$

Generalising

$$T(n) = T(n-3k) + (n-3(k-1))^2 + (n-3(k-2))^2 + \cdots + n^2$$

Let $n-3k = 1$

$$\frac{n-1}{3} = k$$

$$T(n) = T(1) + \left(n-3\left(\frac{n-1}{3} - 1\right)\right)^2 + \left(n-3\left(\frac{n-1}{3}\right)\right)^2 + \cdots n^2$$

$$T(n) = T(1) + (n-((n-1)-3])^2 + [n-(n-1-6)) + (n-(n-1-9))^2 + \cdots n^2$$

$$T(n) = 1 + (3+1)^2 + (6+1)^2 + \cdots n^2$$

$$T(n) = 1^2 + 4^2 + 7^2 \cdots n^2$$

$$T(n) = n^2 + \cdots 1$$

$$\boxed{Tn = O(n^2)}$$

9. void function (int n) {
```
        for (i=1 to n) {
            for (j=1; j<=n; j=j+i) {
                printf ("*");
            }
        }
}
```

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for $i = 1$, $j \to n$ times
for $i = 2$, $j = 1 + 3 + 5 - - - + n$

$a_n = a + (k-1)d$

$a = 1$ $\quad d = 2$

$n = 1 + (k-1) \times 2$

$\frac{n-1}{2} = k - 1$

$k = \frac{n-1+1}{2}$

$\boxed{k = \frac{n+1}{2}}$ No. of terms

for $i = 2$, $j \to \frac{n+1}{2}$ times

for $i = 3$, $j = 1 + 4 + 7 + - - - n$

$n = 1 + (k-1) \times 3$

$\boxed{\frac{n-1}{3} + 1 = k}$

for $i = 3$, $j = \frac{n+2}{3}$ times

Generalising
for $i = n$, $j = \frac{n+k-1}{k}$ times

Time complexity is

$$\underbrace{n + \frac{n+1}{2} + \frac{n+2}{3} + - - - + \frac{n+k-1}{k}}_{n \text{ terms}}$$

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General term $= \frac{n+k-1}{k}$

$$\sum_{k=1}^{n} \frac{n+k-1}{k} = \frac{\sum_{1}^{n} n + \sum_{1}^{n} k - \sum 1}{k}$$

$$\Rightarrow \frac{\frac{n(n+1)}{2} + nk - n}{k}$$

$$\Rightarrow \frac{\frac{n^2+n}{2} + nk - n}{k}$$

$$T(n) = \frac{\frac{n^2+n}{2} + nk - n}{k}$$

Negtecting constant terms

$$\boxed{T(n) = O(n^2)}$$

10. as given n^k d c^n
relation b/w n^k d c^n is

$n^k = O(c^n)$

as $n^k \leq ac^n$

$\forall\ n \geq n_0$ d some constant $a > 0$

for $n_0 = 1$

$c = 2$

$\Rightarrow 1^k \leq a2$

$n_0 = 1$ d $c = 2$