Tutorial - 4

1. $T(n) = 3T(n/2) + n^2$

$a = 3$, $b = 2$, $f(n) = n^2$

$\because$ a & b are ~~co~~ constant and $f(n)$ is a true function.

$\therefore$ Master's theorem is applicable.

$c = \log_b a$

$= \log_2 3 = 1.58$

$\Rightarrow n^c = n^{1.58}$

which is $n^2 > n^{1.58}$

$\therefore$ Case 3 is applied here

$\boxed{T(n) = 8(n^2)}$

2. $T(n) = 4T(n/2) + n^2$

$a = 4$ $b = 2$ $f(n) = n^2$

$\therefore$ a & b are const. and $f(n)$ is a positive function.

$\therefore$ Master's theorem is applicable

$c = \log_b a$

$= \log_2 4 = \log_2 2^2 = 2\log_2 2 = 2$

$\therefore n^c = n^2$

$\because n^c = f(n)$

$\therefore$ Case 2 is applied here

$\boxed{T(n) = 8(n^2 \log n)}$

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3. $T(n) = T(n/2) + 2^n$

$a = 1 \quad b = 2 \quad f(n) = 2^n$

$\because$ a & b are constant and f(n) is a +ve func^n $\therefore$ Masters theorem is applicable.

$c = \log_b a = \log_2 1$

$= 0$

$\Rightarrow n^c = n^0 = 1$

$\because f(n) > n^c$

$\therefore$ Case 3 is applied here

$\boxed{T(n) = \Theta(2^n)}$

4. $T(n) = 2^n T(n/2) + n^n$

$a = 2^n, \; b = 2, \; f(n) = n^n$

$\therefore$ a is not constant, its value depends on n.

$\therefore$ Master's theorem is not applicable here.

5. $T(n) = 16T(n/4) + n$

$a = 16, \; b = 4, \; f(n) = n$

$\therefore$ a & b are constant and f(n) is a +ve func^n

$c = \log_b a$

$= \log_4 16 = \log_4 4^2 = 2\log_4 4 = 2$

$\Rightarrow n^c = n^2$

$\because f(n) < n^c$

$\therefore$ Case 1 is applied here

$\boxed{T(n) = \Theta(n^2)}$

6. $T(n) = 2T(n/2) + n\log n$

$a = 2,\ b = 2,\ f(n) = n\log n$

$\therefore$ a & b are constant and $f(n)$ is a +ve funcⁿ.

$\therefore c = \log_b a = \log_2 2 = 1$

$n^c = n^1 = n$

$\because f(n) > n^c$

$\therefore$ Case 3 is applied

$\boxed{T(n) = \Theta(n\log n)}$

7. $T(n) = 2T(n/2) + n/\log n$

$a = 2,\ b = 2,\ f(n) = n/\log n$

$\because$ a and b are constant & $f(n)$ is a +ve function

$c = \log_b a$

$= \log_2 2 = 1$

$n^c = n = n$

$\because$ non-polynomial difference b/w $f(n)$ & n^c

$\therefore$ Master's theorem is not applicable

~~$c = \log_b a = \log_4 2 = 0.50$~~

~~$n^c = n^{0.50}$~~

~~$\therefore f(n) > n^c$~~

~~$\therefore$ Case 3 is applicable~~

~~$\boxed{T(n) = \Theta(n^{0.50})}$~~

8. $T(n) = 2T(n/4) + n^{0.51}$

$a = 2, \quad b = 4, \quad f(n) = n^{0.51}$

$\because$ a & b are constant & f(n) is a +ve function

$\therefore$ Master's theorem is applicable

$c = \log_b a = \log_4 2 = 0.50$

$n^c = n^{0.50}$

$\because f(n) > n^c$

$\therefore$ Case 3 is applicable

$\boxed{T(n) = \Theta(n^{0.50})}$

9. $T(n) = 0.5T(n/2) + 1/n$

$a = 0.5, \quad b = 2, \quad f(n) = 1/n$

$\because a < 1$

Master's theorem is not applicable

10. $T(n) = 16T(n/4) + n!$

$a = 16, \quad b = 4, \quad f(n) = n!$

$\because$ a & b are constant & f(n) is a +ve function.

Master's theorem is applicable.

$c = \log_b a$

$= \log_4 16 = \log_4 4^2 = 2\log_4 4 = 2$

$n^c = n^2$

$\because f(n) > n^c$

$\therefore$ Case 3 is applied here

$\boxed{T(n) = \Theta(n!)}$

11. $T(n) = 4T(n/2) + \log n$

$a = 4$, $b = 2$, $f(n) = \log n$

$\therefore$ a & b are constant & f(n) is a +ve function

$\therefore$ Master's theorem is applicable

$c = \log_b a = \log_2 4 = \log_2 2^2$
$= 2\log_2 2 = 2$

$n^c = n^2$

$\because f(n) < n^c$

$\therefore$ case 1 is applied

~~$c = \log_b a = \log_2 4 =$~~

$\boxed{T(n) = \Theta(n^2)}$

12. $T(n) = \sqrt{n}\,T(n/2) + \log n$

$a = \sqrt{n}$, $b = 2$, $f(n) = \log n$

$\therefore$ a is not constant

$\therefore$ Master's theorem is not applicable.

13. $T(n) = 3T(n/2) + n$

$a = 3$, $b = 2$, $f(n) = n$

$\therefore$ a & b are constant & f(n) is a +ve function

$\therefore$ Master's theorem is applicable

$c = \log_b a = \log_2 3 = 01.58$

$\therefore n^c = n^{1.58}$

$f(n) < n^c$.

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$\therefore$ case 1 is applied here
$$\boxed{T(n) = \Theta(n^{1.58})}$$

14. $T(n) = 3T(n/3) + \sqrt{n}$

$a = 3$, $b = 3$, $f(n) = \sqrt{n}$

$\therefore$ a & b are constant & f(n) is a +ve function

$\therefore$ Master's theorem is applicable

$c = \log_b a = \log_3 3 = 1$

$n^c = n^1 = n$

$\because f(n) < n^c$

$\therefore$ Case 1 is applicable
$$\boxed{T(n) = \Theta(n)}$$

15. $T(n) = 4T(n/2) + n$

$a = 4$, $b = 2$, $f(n) = n$

$\therefore$ a & b are constant and f(n) is a +ve function

$\therefore$ Master's theorem is applicable here.

$c = \log_b a = \log_2 4 = \log_2 2^2 = 2\log_2 2 = 2$

$n^c = n^2$

$\because f(n) < n^c$

$\therefore$ Case 1 is applicable here
$$\boxed{T(n) = \Theta(n^2)}$$

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16. $T(n) = 3T(n/4) + n \log n$

$a = 3,\ b = 4,\ f(n) = n \log n$

$\therefore$ a & b are constant & f(n) is a +ve function

$\therefore$ Master's theorem is applicable here.

$c = \log_b a = \log_4 3 = 0.79$

$n^c = n^{0.79}$

$\therefore f(n) > n^c$

$\therefore$ Case 3 is applicable here

$\Rightarrow \boxed{T(n) = \Theta(n \log n)}$

17. $T(n) = 3T(n/3) + n/2$

$a = 3,\ b = 3,\ f(n) = n/2$

$\therefore$ a, b are constant & f(n) is a +ve function

$\therefore$ Master's theorem is applicable here.

$c = \log_b a = \log_3 3 = 1$

$n^c = n^1 = n$

$\therefore f(n) = n^c$

$\therefore$ Case 2 is applied here

$\boxed{T(n) = n \log n}$

18. $T(n) = 6T(n/3) + n^2 \log n$

$a = 6,\ b = 3,\ f(n) = n^2 \log n$

$\therefore$ a & b are constant & f(n) is a +ve function.

$\therefore$ Master's theorem is applicable here

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$c = \log_b a = \log_3 6 = 1.63$ ∴

$n^c = n^{1.63}$

$\therefore f(n) > n^c$

→ case 3 is applied here

$\boxed{T(n) = \Theta(n^2 \log n)}$

19. $T(n) = 4T(n/2) + n/\log n$

$a = 4,\ b = 2,\ f(n) = n/\log n$

∴ a & b are constant and f(n) is a +ve function

∴ Master's theorem is application here

$c = \log_b a = \log_2 4 = \log_2 2^2 = 2\log_2 2 = 2$

$n^c = n^2$

$\therefore f(n) < n^c$

∴ case 1 is applied here

$\boxed{T(n) = \Theta(n^2)}$

20. $T(n) = 64T(n/8) - n^2 \log n$

∴ a & b are constant but f(n) is a -ve function.

∴ Master's theorem is not applicable here.

21. $T(n) = 7T(n/3) + n^2$

$a = 7,\ b = 3,\ f(n) = n^2$

∴ a & b are constant & f(n) is +ve function.

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∴ Master's theorem is applied here.

$c = \log_b a = \log_3 7 = 1.77$

$n^c = n^{1.77}$

∴ $f(n) > n^c$

∴ Case 3 is applied here

$$\boxed{T(n) = O(n^2)}$$

22: $T(n) = T(n/2) + n(2 - \cos n)$

∴ $f(n)$ is not regular function

∴ Master's theorem does not applied here.