Tutorial-2

1.
```
void fun (int n){
    int j=1, i=0;
    while (i<n){
        i = i+j;
        j++;
    }
}
```

→ Values after execution

1st time → $i = 1$

2nd time → $i = 1+2$

3rd time → $i = 1+2+3$

4th time → $i = 1+2+3+4$

for ith time → $i = (1+2+3+ \ldots i) < n$

$$\Rightarrow \frac{i(i+1)}{2} < n$$

$$\Rightarrow i^2 < n$$

$$\Rightarrow i = \sqrt{n}$$

Time complexity = $O(\sqrt{n})$

2.
```
int fib (int n){
    if (n <= 1)
        return n;
    return fib(n-1) + fib(n-2);
}
```

* Recurrance Relation

$$F(n) = F(n-1) + F(n-2)$$

Let $T(n)$ denote the time complexity of $F(n)$.

In $F(n-1)$ and $F(n-2)$ time will be $T(n-1)$ and $T(n-2)$. We have one more addition to sum our results

For $n > 1$

$$T(n) = T(n-1) + T(n-2) + 1 \quad \text{—} ①$$

For $n = 0$ & $n = 1$, no addition occurs

$$\therefore T(0) = T(1) = 0$$

Let $T(n-1) \approx T(n-2)$ — ②

Adding ② in ①

$$T(n) = T(n-1) + T(n-1) + 1$$
$$= 2 \times T(n-1) + 1$$

Using backward substitution

$$\therefore T(n-1) = 2 \times T(n-2) + 1$$
$$T(n) = 2 \times [2 \times T(n-2) + 1] + 1$$
$$= 4T(n-2) + 3$$

We can substitute

$$T(n-2) = 2 \times T(n-3) + 1$$
$$T(n) = 8 \times T(n-3) + 7$$

General equation -

$$T(n) = 2^k \times T(n-k) + (2^k - 1) \quad \text{—} ③$$

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for $T(0)$

$n-k=0 \Rightarrow k=n$

Substituting values in ③

$$T(n) = 2^n \times T(0) + 2^n - 1$$
$$= 2^n + 2^n - 1$$

$$\boxed{T(n) = O(2^n)}$$

Space complexity → $O(N)$

Reason - The function calls are executed sequentially. Sequential execution guarantess that the stack size will never exceed the depth of calls for first $F(n-1)$ it will create N stack.

3.(i) $O(n \log n)$ -

```
# include <iostream>
using namespace std;
int partition (int arr[], int s, int e){
    int pivot = arr[s];
    int count =0;
    for (int i= s; i<= e ; i++){
        if (arr [i] <= pivot)
            count ++;
    }
```

```
int pivot_ind = s+ count;
swap (arr [pivot_ind], arr [s]);
int i = s, j = e;
while (i< pivot_ind && j > pivot_ind){
    while (arr [i] <= pivot)
        i++;
    while (arr [j] > pivot)
        j--;
    if (i< pivot_ind && j> pivot_ind){
        swap(arr[i++], arr[j--]);
}
return pivot_ind;
}
void quick (int arr [], int s, int e){
    if (s == e)
        return;
    int p = partition (arr, s, e);
    quicksort (arr, s, p-1);
    quicksort (arr, p+1, e);
}
int main(){
    int arr[] = {6, 8, 5, 2, 1}
    int n = 5;
    quicksort (arr, 0, n-1);
    return 0;
}
```

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(ii) $O(N^3)$

```
int main(){
    int n=10;
    for(int i=0; i<n; i++){
        for (int j=0; j<n; j++){
            for(int k=0; k<n; k++){
                printf("*");
            }
        }
    }
    return 0;
}
```

(iii) $O(\log \log n)$ -

```
int countPrimes(int n){
    if (n<2)
        return 0;
    bool * non-prime = new bool [n];
    non-prime[1]= true;
    int numnonPrime =1;
    for (int i=2; i<n; i++){
        if (nonPrime[i])
            continue;
        int j= i*2;
        while (j<n) {
            if (!nonprime [j]) {
                nonprime[j] = true;
                numnonprime++;
            }
```

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```
            j+ = i;
        }
    }
    return (n-1) - numnonPrime;
}
```

4. $T(n) = T(n/4) + T(n/2) + cn^2$

Using master's theorem -

we can assume $T(n/2) >= T(n/4)$

Equation can be rewritten as

$T(n) <= 2T(n/2) + Cn^2$

$\Rightarrow T(n) <= O(n^2)$

$\Rightarrow T(n) = O(n^2)$

Also $T(n) >= cn^2 \Rightarrow T(n) >= O(n^2)$

$\Rightarrow T(n) = \Omega(n^2)$

$\because T(n) = O(n^2)$ and $T(n) = \Omega(n^2)$

$T(n) = O(n^2)$

5.
```
int fun (int n) {
    for (int i=1; i<=n; i++) {
        for (int j=1; j<n; j+= i) {
            // some O(1) task
        }
    }
}
```

for $i=1$, inner loop is executed n times
for $i=2$, inner loop is executed $n/2$ times.
for $i=3$, inner loop is executed $n/3$ times.

It is forming a series -

$$\Rightarrow n + \frac{n}{2} + \frac{n}{3} + \cdots + \frac{n}{n}$$

$$\Rightarrow n\left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}\right)$$

$$\Rightarrow n \times \sum_{k=1}^{n} \frac{1}{k}$$

$$\Rightarrow n \times \log n$$

Time Complexity $= O(n \log n)$

6. for (int i=2; i<=n; i= pow(i,k)) {
   // some O(1) expressions
}

with iterations -
i take values
for 1 st iteration $\rightarrow 2$
for 2nd iteration $\rightarrow 2^k$
for 3rd iteration $\rightarrow (2^k)^k$
⋮

for n iteration $\rightarrow 2^{k \log k(\log(n))}$

∵ last term must be less than or equal to n.

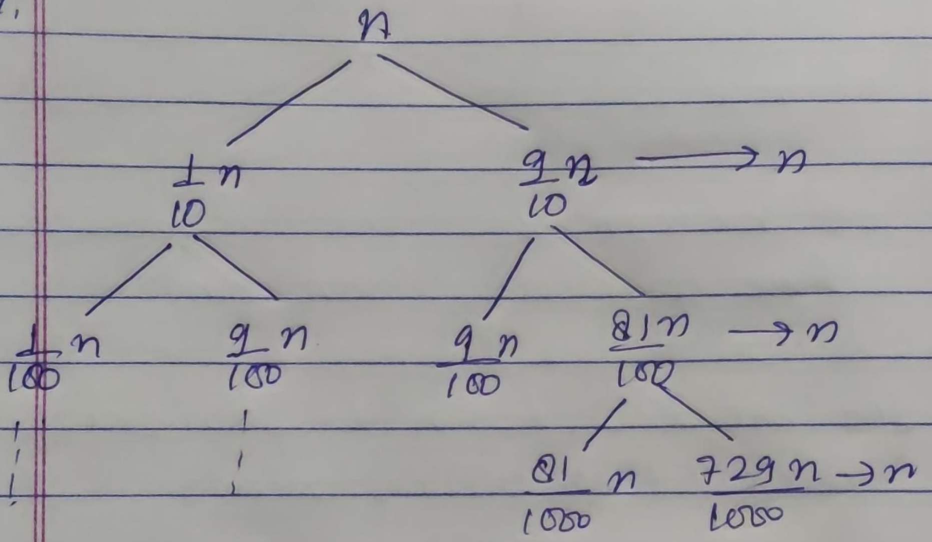
$$2^{k^{\log_k(\log(n))}} = 2^{\log n} = n$$

Each iteration takes constant times

∴ Total iteration = $\log_k(\log(n))$

Time complexity = $O(\log(\log(n)))$

7.



If we split in this manner

Recurrance Relation

$$T(n) = T\left(\frac{9n}{10}\right) + T\left(\frac{n}{10}\right) + O(n)$$

When first branch is of size $9n/10$ & second one is $n/10$.

Showing the above using recursion tree approach calculating values.

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At 1st level, value $= n$

At 2nd level, value $= \frac{9n}{10} + \frac{n}{10} = n$

Value remains same at all levels i.e. n

Time complexity = summation of value

$= O(n \times \log \log n)$ (upper bound)

$= \Omega(n \log_{10} n)$ (lower bound)

$\Rightarrow \boxed{O(n \log n)}$

8. (a) $100 < \log(\log n) < \log(n) < \sqrt{n} < n\log(n) <$
$\log^{\wedge}2(n) < \log(n!) < n^2 < 2^n < n! < 4^n < 2^{2^n}$

(b) $1 < \log(\log(n)) < \sqrt{\log(n)} < \log(n) < 2\log(n) <$
$\log(2n) < n < n\log(n) < \log(\sqrt{n}) < 2n < 4n < n^2 <$
$n! < 2(2^n)$

(c) $96 < \log_8(n) < n\log_6(n) < \log_2(n) < n\log_2(n)$
$< \log(n!) < 5n < 8n^2 < 7n^3 < n! < (8)^{2n}$