Group No : *SM11*

Project Report

School of Engineering and Applied Science (SEAS), Ahmedabad University
B.Tech(CSE) Semester IV: Probability and Stochastic Processes (MAT 277)

Project Title: *Probability of failure model in mechanical component because of fatigue*

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Abstract:
The objective of this project is to calculate the probability of fatigue failure of different components/materials. All structures and objects have a finite limit of durability after which they are bound to fail. Fatigue is one of the most common forms of failure in mechanical components. This report includes the basic idea of the probabilistic approach, the reason for using such an approach and its coding in Python so that it is easy for any newbie to understand the topic. Along with this, the report also includes application based ideas of how this model could be used in the real world and how it would help others. The probabilistic concepts used in this report are Gaussian Random Variable, Probability Distribution function and Cumulative Distribution function of a Normal Random Variable. Thus a brief knowledge of each of these concepts is essential. The final results include a graphical representation of probability of fatigue failure in different metals based on their surrounding temperature and other atmospheric conditions.

In order to achieve the goal, we have used the following concepts of probability and stochastic processes:

- Gaussian random variable
- Joint probability distribution function
- Joint cumulative distribution function
- Error function to find the probability of gaussian rv

Probabilistic Model/PSP concept:
Justification of these psp concepts(how did we use it) + explanation of base article + how we used our base article to do our project's modelling

Coding excluding shared code:
Explaining code + errors + results of diff materials

Innovation:
Effect of external factors on different materials + user centric innovation

Inferences
Conclusion + what did we achieve + how this will affect the end user(making it user centric)

Contribution of Team members(Technical/Non-technical):

Technical:

Tasks	Sameep Vani	Malav Doshi	Khushi Shah	Kairavi Shah	Kashvi Gandhi
Simplification of the base article	1	1	1	1	1
Mathematical understanding and explanation	1	1	0	1	0
Coding of the probabilistic part	1	0	1	1	1
Frontend Coding and other detailing in the code	1	1	1	0	1

Non-technical:

Tasks	Sameep Vani	Malav Doshi	Khushi Shah	Kairavi Shah	Kashvi Gandhi
Concept Map 1	0	1	0	1	1
Existing work compilation x	1	1	1	1	0
Citation	0	0	1	1	1
Compilation of the Report	1	1	1	1	1
Final creation in Latex and Submission	1	1	1	1	1

References:

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[3]. Ph.D., M. A. S. H., & Aliofkhazraei, M. (2016). Chapter 13 - Fatigue failure in aircraft structural components. In *Handbook of Materials Failure Analysis with Case Studies from the Aerospace and Automotive Industries* (1st ed., pp. 261–277). Butterworth-Heinemann.

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[10]. *Normal Distribution Function -- from Wolfram MathWorld*. (2018, March 7). Normal Distribution Function. https://mathworld.wolfram.com/NormalDistributionFunction.html

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Probabilistic Concepts Used and Their Justification:

In order to achieve our objective, we have used the following concepts of probability and stochastic processes:

- Gaussian random variable
- Probability distribution function
- Cumulative distribution function
- Q function(Error function to find the probability of gaussian rv)

The base article used for this project report, “Probability of failure model in mechanical component because of fatigue” gives us an overview of the concept of fatigue failure in mechanical components [2]. The article answers questions like how the failure conditions occur due to damage accumulation and mathematically proves that when damage accumulation gets more than critical damage, failure is bound to occur.

The linear model of damage accumulation, also known as the Palmgren-Miner’s rule, defines damage as the ratio of number of cycles of operation to number of cycles to failure at any given stress level [9]. This formula is used assuming that there is no initial damage. It is also assumed that failure occurs when total damage accumulation reaches unity. The number of cycles to failure at any given stress level can be obtained from the S-N curve model. However, in order to come up with a realistic estimate of reliability of any given product, it is necessary to adopt a probabilistic approach. It is, therefore, important to treat the damage accumulation measure under consideration.

To advance our understanding about fatigue failure we have assumed and used the properties of a Gaussian Random Variable. According to the research works done by Wang and Coit, at any specific time, there exists a distribution of degradation measures considering the population of similar components. Since damage accumulation is also a measure of degradation, Wang and Coit’s [9] reasoning can be applied here as well. Hence, damage accumulation follows a certain probability distribution and the expected value and variability of any damage accumulation measure will increase with usage time. Furthermore, it has also been shown that under constant random amplitude loading conditions normal distribution provides a good fit to fatigue failure data. Hence, here the damage accumulation is modeled as a nonstationary probabilistic process assuming that fatigue life follows normal distribution.

To find the probability of fatigue failure we need to find [2],

$$P(Z(n) \leq 0)$$

Where Z(n) is a function of ultimate strength, fatigue limit, fatigue strength fraction, alternating stress, and mean stress. The function Z(n) is written as:

$$Z(n) = 1 - \frac{n}{\left(\frac{S_{nf}}{a}\right)^{\frac{1}{b}}} \qquad (1)$$

Here, the corrected alternating stress Snf is calculated using the Goodman equation.

$$\frac{S_a}{S_{nf}} + \frac{S_m}{S_u} = 1 \tag{2}$$

Sa is the alternating stress, Su is ultimate strength and Sm is the mean stress. The variables 'a' and 'b' in equation (1) are calculated using the mathematical equation proposed by Basquin.

$$N_f = \left(\frac{S_{nf}}{a}\right)^{\frac{1}{b}}$$

$$a = \frac{(f \times S_u)^2}{S_e}$$

$$b = -\frac{1}{3}\log_{10}\left(\frac{f \times S_u}{S_e}\right)$$

where Snf is corrected alternating stress, Su is ultimate strength, Se is fatigue limit and f is fatigue strength fraction [2].

The error function erf of x is

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt.$$

The error function is essentially identical to the standard normal cumulative distribution function, denoted Φ, also named norm(x), as they differ only by scaling and translation. Consequently, the error function is also closely related to the Q-function, which is the tail probability of the standard normal distribution [10].

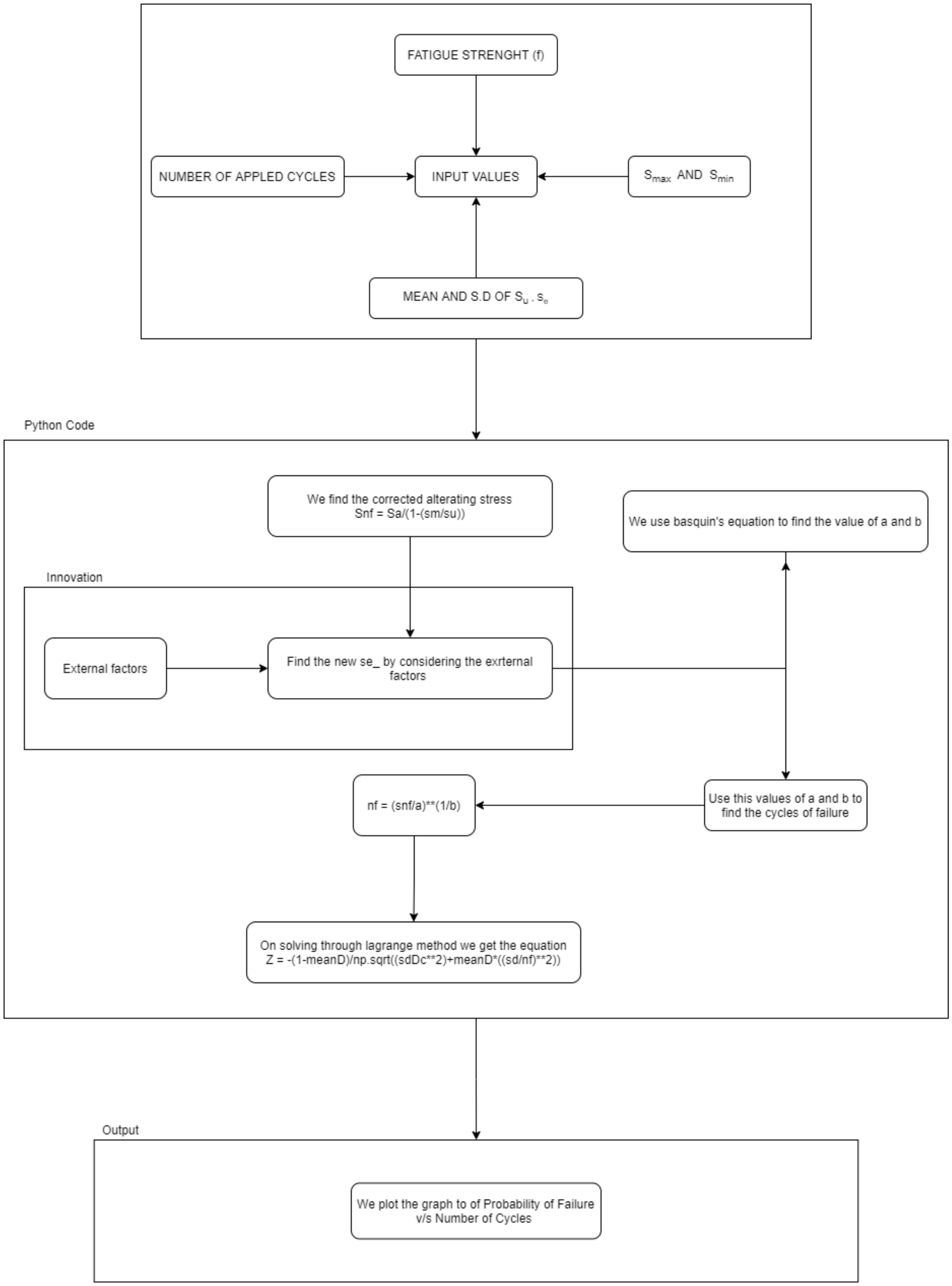
In order to find the probability of this Normal distribution, we compute the Cumulative Distribution function. For the same we have used the formula

$$\frac{1}{2}\left[1 + \mathrm{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right]$$

where x is the random variable, μ is the mean of the distribution and σ is the standard deviation.

Coding excluding the shared code

1. Modelled the probability distribution taking into consideration some of the extra external factors such as temperature and surface finishing.
2. We displayed the CDF of the random variable using error function instead of Q function. We did it using inbuilt python libraries.


FATIGUE STRENGHT (f)
NUMBER OF APPLED CYCLES
INPUT VALUES
S_{max} AND S_{min}
MEAN AND S.D OF S_u . s_e
Python Code
We find the corrected alterating stress
Snf = Sa/(1-(sm/su))
We use basquin's equation to find the value of a and b
Innovation
External factors
Find the new se_ by considering the exrternal factors
Use this values of a and b to find the cycles of failure
nf = (snf/a)**(1/b)
On solving through lagrange method we get the equation
Z = -(1-meanD)/np.sqrt((sdDc**2)+meanD*((sd/nf)**2))
Output
We plot the graph to of Probability of Failure v/s Number of Cycles

In our efforts of innovating something new in the existing model, we have tried taking the example of aircrafts. We know that a failure in the component of the same can be fatal So we cannot afford a failure in that. Hence our model produces an output which is a graph which shows the comparison between the flight models in terms of probability of failure and number of cycles. Also it considers some external factors such as temperature.

Innovation:

We also innovated our equation in a way

1. **Surface factor(k_a)** : It depends on a) quality of the finish of the actual part, b) tensile strength of the material of that part. k_a is low for rough surfaces and close to 1 for polished/smooth surfaces [8].

2. **Size factor(k_b)** :
   a) For bending and torsion
   It is directly proportional to d^x where x= -0.107 for d in range (0.11,2) and x= -0.157 for d in range (2,10). (Note: d is calculated in inches) [8].
   b) For axial loading
   k_b=1.
   c) For shapes with height and breadth
   An effective dimension d_e is obtained where d_e is proportional to square root of (hb), i.e. square root of area [8].

3. **Loading factor(k_c)** : It differs with tensile strength.
   Values for k_c are considered to be a) 1 for bending, b) 0.85 for axial and c) 0.59 for torsion [8].

4. **Temperature factor(k_d)** : The ultimate strength of the material drops with increase in temperature. The fatigue limit reduces with reduction in ultimate strength and thus at elevated temperatures [8].

5. **Reliability factor(k_e)** : Reliability of 50% is considered. Hence, $\mathbf{k_e}$ = 1.

Inference:

Our suggested model of probabilistic computing is salient and can be used in our society in order to find out the endurance limit of a metal before installation. It aims at helping the users get an idea of the time period for which the machine would work perfectly before further servicing or replacement of one or more components of the machinery. Other uses of this model include the analysis of different material for other research work such as done by our group considering two different propellers used in the day. The scope of such analytical work is inbound and thus this model could be used for that.

Using our program code, we generated the graphs of probability of fatigue failure to compare two aircraft propellers and their fatigue limits.

We have found three graphs where we have considered different materials which are used by different aircrafts and we can infer from them which aircraft shows a lower probability of failure of the propeller at a higher number of cycles making it easier to compare.