WELCOME

Travelling Salesman Problem (TSP)

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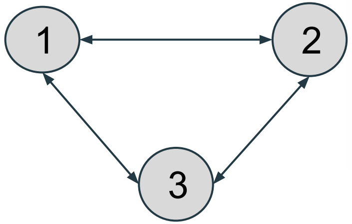
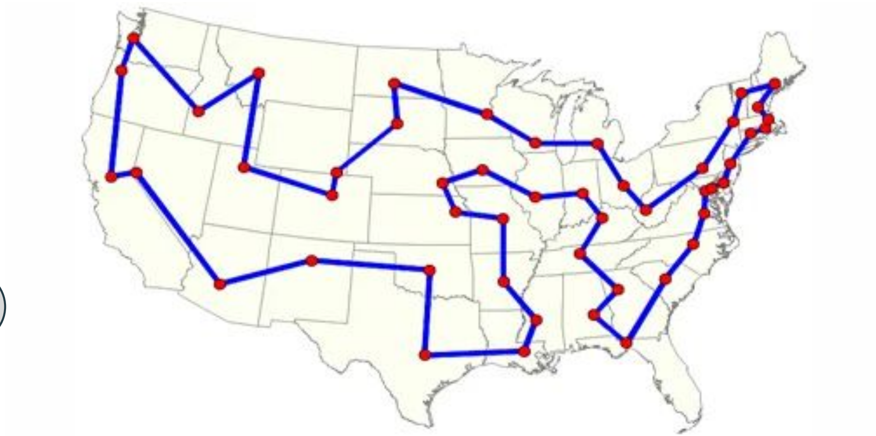
TSP Introduction

- ➔ Shortest possible route that visits every city exactly once and returns to the starting point.
- ➔ Cost of tour = Sum of the cost of edges on the tour.
- ➔ Find a tour of minimum cost.
- ➔ Minimum weight Hamiltonian Cycle.
- ➔ Possible no. of graph:

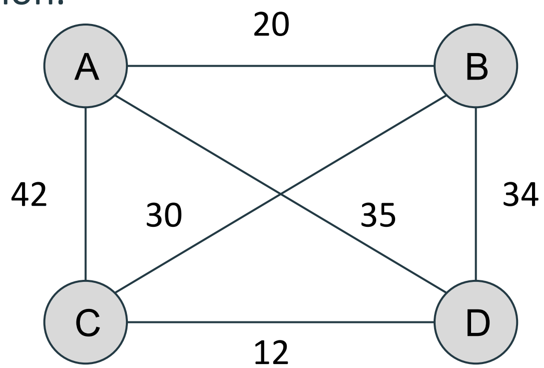
$(n-1)!/2$





Symmetric TSP

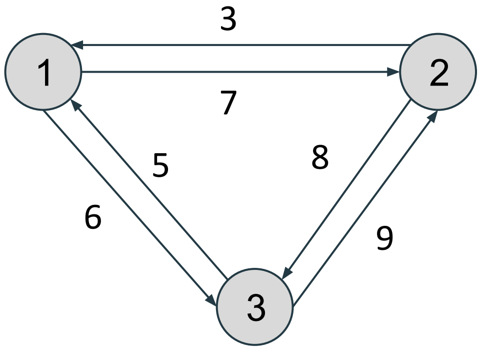
- Distance between two cities is same from both direction.
- Forms an undirected graph.
- Halves the number of possible solutions.



Symmetric TSP with four cities

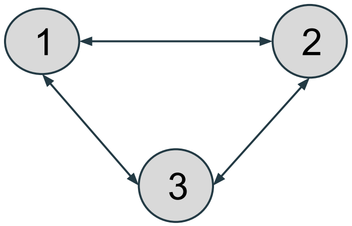
Asymmetric TSP

- Paths may not exist in both directions or the distances might be different.
- Forms an directed graph.
- Examples:
  - One-way streets
  - Airfares



Traveling Salesman Problem

Minimum cost tour for the below traveling salesperson problem:

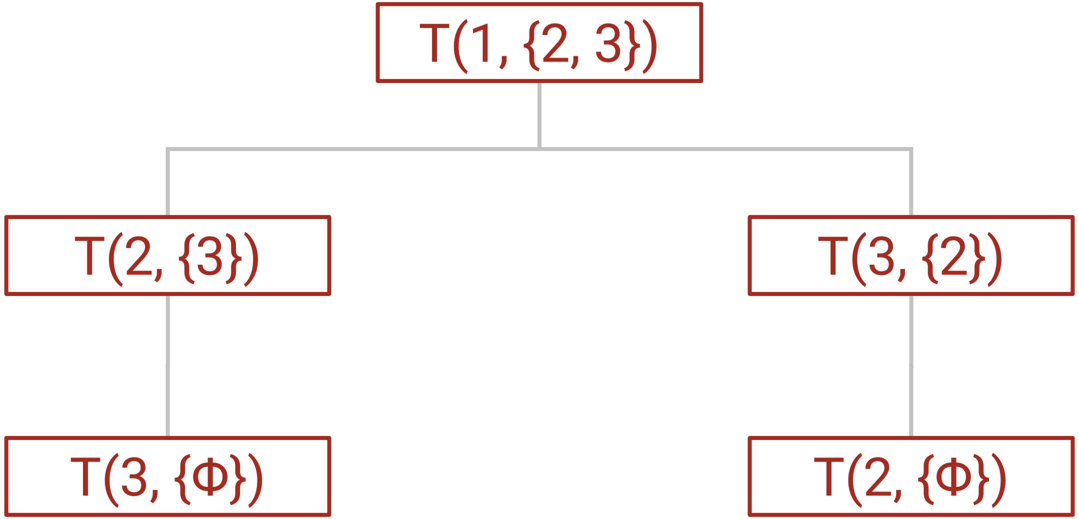


The cost adjacency matrix : $\begin{bmatrix} 0 & 5 & 7 \\ 4 & 0 & 6 \\ 8 & 3 & 0 \end{bmatrix}$

Formula:

$$g(1, V-\{1\}) = \min_{2 \le k \le n}\{ (C_{1K} + g(k, V - \{1,k\}) \}$$

Hierarchical Representation of the given problem:



Solution of the TSP using Dynamic programming [Part I]

$g(1, \{2,3\}) = \min \{ C_{12} + g(2,\{3\}), C_{13} + g(3,\{2\}) \}$ ------------------------- (1)

Where, C = Cost

i.e C_{12} = Cost of distance from vertex 1 to 2

$$\begin{bmatrix} 0 & 5 & 7 \\ 4 & 0 & 6 \\ 8 & 3 & 0 \end{bmatrix}$$

$g(2, \{3\}) = \min \{ C_{23} + g(3,\{\emptyset\}) \}$

$= \min \{6 + 8\} = 14$

$g(3, \{2\}) = \min \{ C_{32} + g(2, \{\emptyset\}) \}$

$= \min \{3 + 4\} =7$

Solution of the TSP using Dynamic programming [Part II]

Substituting values in (1);

g(1, {2,3}) = min {5 + 14, 7 + 7}

= min {19, 14}

g(1, {2,3}) = 14

So, the optimum tour for the given graph has length = 14

Therefore, the optimum tour is = 1, 3, 2, 1.

$$\begin{bmatrix} 0 & 5 & 7 \\ 4 & 0 & 6 \\ 8 & 3 & 0 \end{bmatrix}$$

Time and Space Complexity

Space Complexity

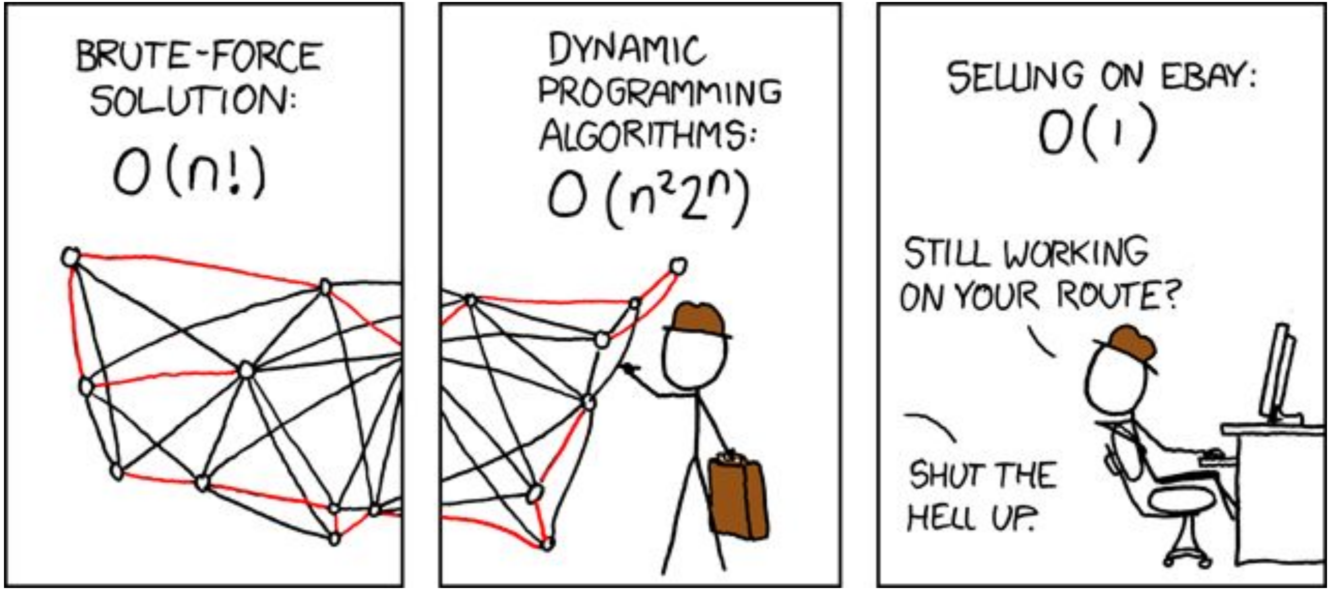
➔ $O(n2^n)$

Time Complexity

T.C. = no. of unique sub-position * time taken by each sub-problem

➔ $O(n2^n) * O(n)$
➔ $O(n^2 2^n)$

Time Complexity In Short



THANK YOU