

Gradient Descent

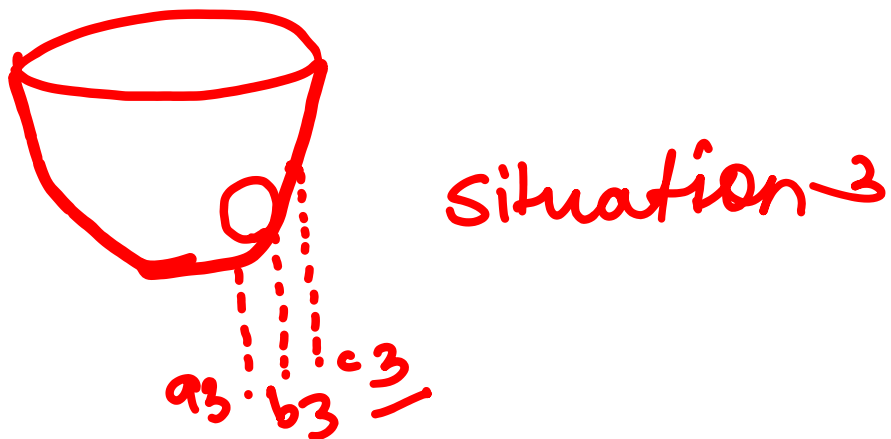
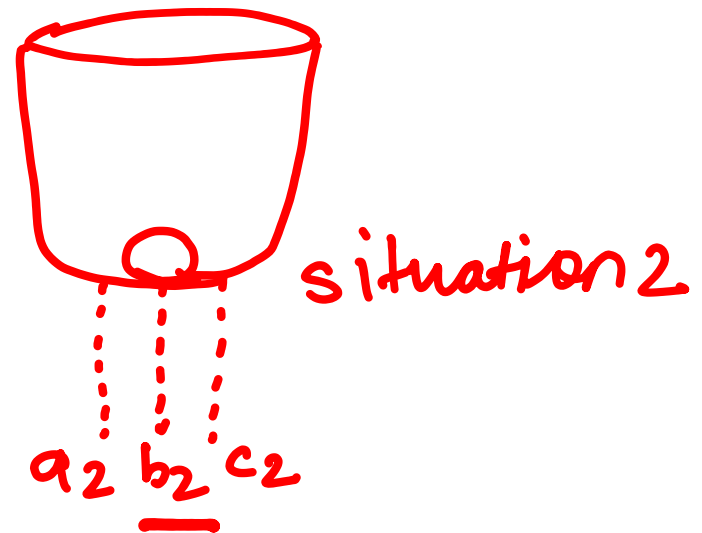
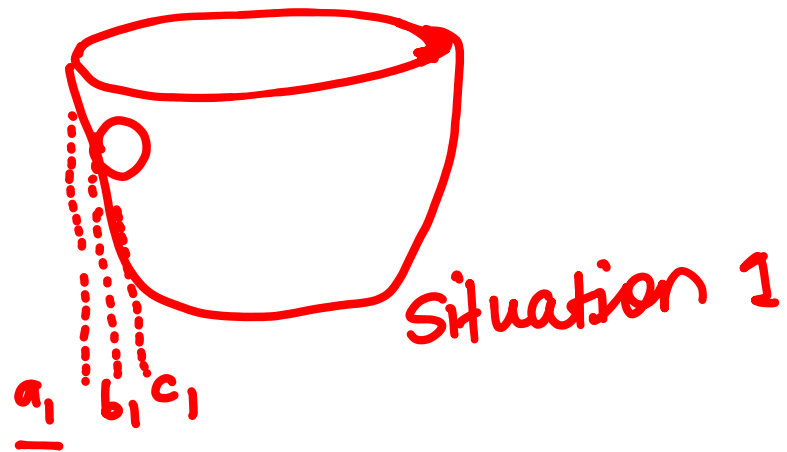
$$y = \theta_0 + \theta_1 x \quad \dots \quad (\theta_0, \theta_1 = \text{Regr}^n \text{ param...})$$

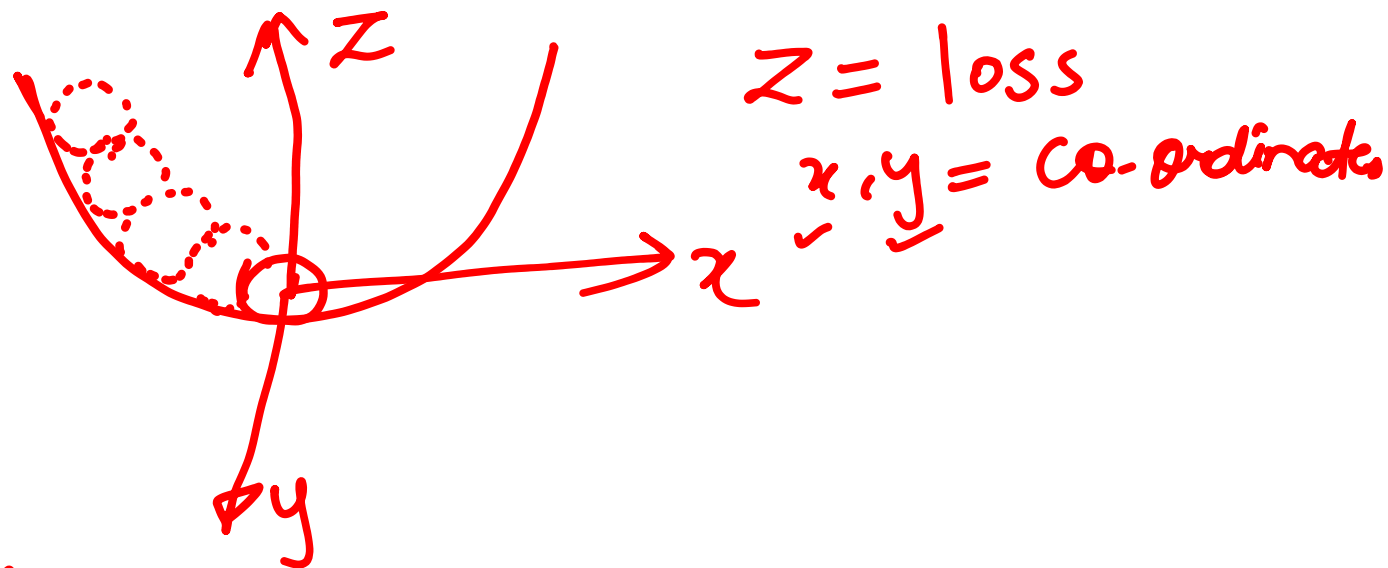
→ It is used to estimate Regr^n parameters.

Why GD over OLS

→ OLS is expensive in terms of complexity (it involved matrix inversion)

→ OLS fails on non linear trends.





Minimum of funⁿ

$$f(x) = x^4 - 10x^2 + 9 \quad \text{--- ①}$$

Step 1 find derivative of given funⁿ

$$\frac{d(f(x))}{dx} = \frac{d}{dx}(x^4 - 10x^2 + 9)$$

$$\frac{d}{dx}(f(x)) = 4x^3 - 20x$$

to find minima we need to equate it to 0

$$4x^3 - 20x = 0$$

$$4x^3 - 20x = 0$$

$$4x^2 = 20$$

1

$$x^2 = 5$$

$$x = \sqrt{5}$$

$$x = \pm 2.25$$



Iterative method \rightarrow G.D

$$f(x) = x^4 - 10x^2 + 9$$

consider initial value of $x = 5$

Learning rate $\alpha = 0.001$

> gradient of any funⁿ is given by

$$\begin{aligned}\frac{d}{dx}(f(x)) &= \frac{d}{dx}(x^4 - 10x^2 + 9) \\ &= \underline{4x^3 - 20x}\end{aligned}$$

$$\begin{array}{lcl}
 \text{1st iteration} & & \\
 \hline
 x=5 & \text{gradient} & = 4(5)^3 - 20(5) \\
 & & = 4(125) - 100 \\
 & & = \underline{\underline{50 \cdot 400}}
 \end{array}$$

We need to decrease x by d times gradient

$$\begin{aligned}
 \text{New } x &= \text{prev}(x) - d(4x^3 - 20x) \\
 &= 5 - 0.001(400) \\
 &= 5 - 0.4
 \end{aligned}$$

$$\boxed{\text{New } x = 4.6}$$

2nd iteration

$$\begin{aligned}\text{New value of } x &= \text{prev}(x) - \alpha(4x^3 - 20x) \\ &= 4.6 - 0.001(4(4.6)^3 - 20(4.6)) \\ &= 4.6 - 0.001(389.34 - 92) \\ &= 4.6 - 0.001(297.34) \\ &= 4.6 - 0.297\end{aligned}$$

$$\begin{aligned}\text{New value of } x &= 4.30\end{aligned}$$

$$\sqrt{5} = \underline{\underline{2.24}}$$