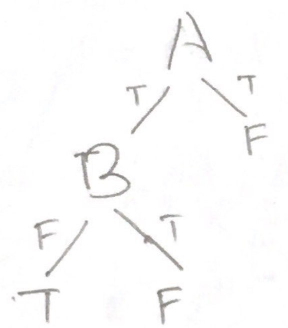
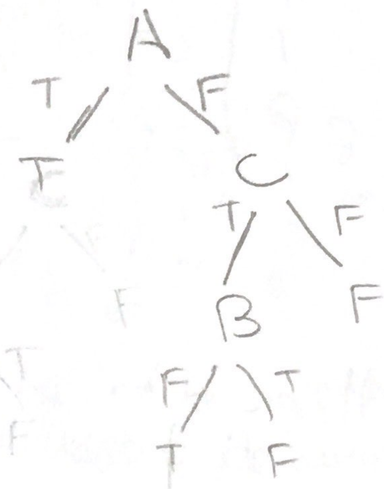
PS 1

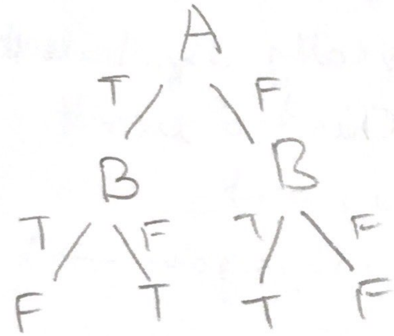
a) $A \wedge \neg B$



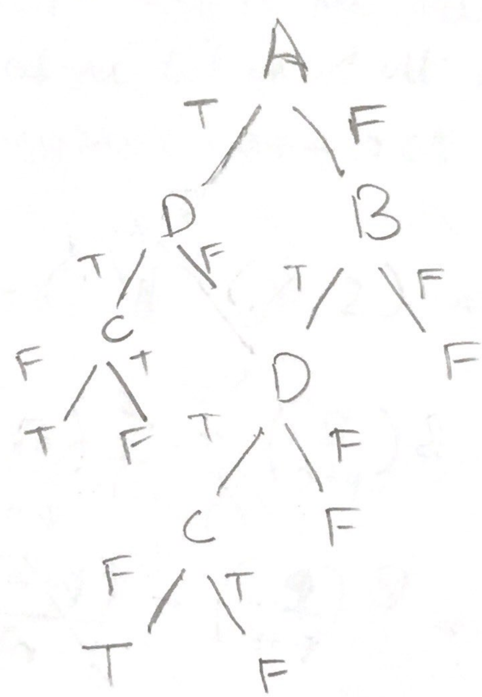
b) $A \vee (\neg B \wedge C)$



c) $A \oplus B$



d) $(A \vee B) \wedge (\neg C \wedge D)$



Ans 2 a)

$$E(S) = -\frac{3}{6}\log\left(\frac{3}{6}\right) - \frac{3}{6}\log\left(\frac{3}{6}\right)$$

$$= \log\left(\frac{3}{6}\right)(-1)$$

$$= \left(\log \frac{1}{2}\right)(-1)$$

$$= (\log 1 - \log 2)(-1)$$

$$= \log 2$$

$$= 1$$

Ans 3 d)

$$E(a_2=T) = -\frac{2}{4}\log\left(\frac{2}{4}\right) - \frac{2}{4}\log\left(\frac{2}{4}\right)$$

$$= \log\left(\frac{2}{4}\right)(-1) = \log 2 = 1$$

$$E(a_2=F) = -\frac{1}{2}\log\left(\frac{1}{2}\right) - \frac{1}{2}\left(\log\frac{1}{2}\right) = 1$$

Information

$$\text{Gain} = 1 - \frac{4}{6}(1) - \frac{2}{6}(1) = 1 - \frac{2}{3} - \frac{1}{3} = 0$$

Ans 3 If the data set is increasing in an accurate way and covers all possible output then it is return a different tree which will be logically equivalent to the set logical representation. This tree won't be the same but is logically equivalent.

As $n \to \infty$, decision tree information gain $\to 1$

Ans 4

$$\text{Gain}(S, X_j) = H(S) - \sum_k \frac{|S_k|}{|S|} H(S_k)$$

$$\text{Gain}: B\left(\frac{p}{p+n}\right) - \sum_k \frac{(p_k+n_k)}{p+n} B\left(\frac{p_k}{p_k+n_k}\right)$$

$$= B\left(\frac{p}{p+n}\right) - \frac{\cancel{(p+n)}}{\cancel{(p+n)}} B\left(\frac{p_k}{p_k+n_k}\right)$$

But we know

$$\sum_k p_k = p$$

$$\sum n_k = n$$

$$= 0$$

A5

a)

X_1	X_2	X_3	Y
0	0	0	0
0	1	0	1
1	0	0	1
1	1	0	0
0	0	1	1
0	1	1	0
1	0	1	0
1	1	1	1

b)