$$NDCG@K = \frac{DCG@K}{IDCG@K} = \frac{\sum\limits_{i=1}^{k \ (actual \ order)} \frac{Gains}{log_2(i+1)}}{\sum\limits_{i=1}^{k \ (ideal \ order)} \frac{Gains}{log_2(i+1)}}$$

Example of nDCG@10 Calculation

Let's consider a query for which the relevance of the top 10 documents is as follows:

Rank (i)	Relevance Score (rel$_i$)
1	3
2	2
3	3
4	0
5	1
6	2
7	0
8	1
9	0
10	2

Step 1: Calculate DCG@10

DCG@10 is calculated by summing the relevance scores of the documents at each rank, discounted by the rank's logarithm. The formula is:

$$DCG@10 = \frac{2^{rel_1} - 1}{\log_2(1+1)} + \frac{2^{rel_2} - 1}{\log_2(2+1)} + \cdots + \frac{2^{rel_{10}} - 1}{\log_2(10+1)}$$

For this example, let's calculate the DCG for each position:

- DCG at rank 1: $\frac{2^3 - 1}{\log_2(1+1)} = \frac{7}{1} = 7$
- DCG at rank 2: $\frac{2^2 - 1}{\log_2(2+1)} = \frac{3}{1.585} \approx 1.89$
- DCG at rank 3: $\frac{2^3 - 1}{\log_2(3+1)} = \frac{7}{2} = 3.5$

Step 2: Calculate IDCG@10

IDCG is the DCG calculated with the ideal ranking. To get the ideal ranking, sort the relevance scores in descending order:

Rank (i)	Relevance Score (rel_i)
1	3
2	3
3	3
4	2
5	2
6	2
7	1
8	1
9	0
10	0

Now calculate **IDCG@10** in the same way as DCG, using the ideal ranking:

- IDCG at rank 1: $\frac{2^3-1}{\log_2(1+1)} = 7$
- IDCG at rank 2: $\frac{2^3-1}{\log_2(2+1)} = 3.5$
- IDCG at rank 3: $\frac{2^3-1}{\log_2(3+1)} = 3.5$
- IDCG at rank 4: $\frac{2^2-1}{\log_2(4+1)} = 1.89$
- IDCG at rank 5: $\frac{2^2-1}{\log_2(5+1)} = 1.07$
- IDCG at rank 6: $\frac{2^2-1}{\log_2(6+1)} = 1.07$
- IDCG at rank 7: $\frac{2^1-1}{\log_2(7+1)} = 0.387$
- IDCG at rank 8: $\frac{2^1-1}{\log_2(8+1)} = 0.316$

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