Exercise 2 and 3

November 26, 2021

Machine Learning Lab

Lab 03

0.1 Exercise 2: Linear Regression with Gradient Descent

0.1.1 Part A: (Datasets)

Importing Packages

[1]:
```
import pandas as pd                     #Importing Pandas
import numpy as np                      #Importing Numpy
import matplotlib.pyplot as plt         #Importing Matplotlib
```

Reading Wine Quality Dataset

[2]:
```
wine_quality = pd.read_csv('winequality-red.csv',delimiter=';')
wine_quality.head()
```

[2]:
```
   fixed acidity  volatile acidity  citric acid  residual sugar  chlorides  \
0            7.4              0.70         0.00             1.9      0.076
1            7.8              0.88         0.00             2.6      0.098
2            7.8              0.76         0.04             2.3      0.092
3           11.2              0.28         0.56             1.9      0.075
4            7.4              0.70         0.00             1.9      0.076

   free sulfur dioxide  total sulfur dioxide  density    pH  sulphates  \
0                 11.0                  34.0   0.9978  3.51       0.56
1                 25.0                  67.0   0.9968  3.20       0.68
2                 15.0                  54.0   0.9970  3.26       0.65
3                 17.0                  60.0   0.9980  3.16       0.58
4                 11.0                  34.0   0.9978  3.51       0.56

   alcohol  quality
0      9.4        5
1      9.8        5
2      9.8        5
3      9.8        6
4      9.4        5
```

Reading Airfare and Demand Dataset

[3]:
```
airfare_demand = pd.read_fwf('airq402.data',header=None)
airfare_demand.columns = ['City1','City2','Average Fare1','Distance','Average␣
↪weekly passengers',
                         'market leading airline','market share','Average␣
↪fare2','Low price airline',
                         'market share','price']
airfare_demand.head()
```

[3]:
```
  City1 City2  Average Fare1  Distance  Average weekly passengers  \
0   CAK   ATL         114.47       528                     424.56
1   CAK   MCO         122.47       860                     276.84
2   ALB   ATL         214.42       852                     215.76
3   ALB   BWI          69.40       288                     606.84
4   ALB   ORD         158.13       723                     313.04

  market leading airline  market share  Average fare2 Low price airline  \
0                     FL         70.19         111.03                FL
1                     FL         75.10         123.09                DL
2                     DL         78.89         223.98                CO
3                     WN         96.97          68.86                WN
4                     UA         39.79         161.36                WN

   market share   price
0         70.19  111.03
1         17.23  118.94
2          2.77  167.12
3         96.97   68.86
4         15.34  145.42
```

Reading Parkison Dataset

[4]:
```
parkison = pd.read_csv('parkinsons_updrs.data')
parkison.head()
```

[4]:
```
   subject#  age  sex  test_time  motor_UPDRS  total_UPDRS  Jitter(%)  \
0         1   72    0     5.6431       28.199       34.398    0.00662
1         1   72    0    12.6660       28.447       34.894    0.00300
2         1   72    0    19.6810       28.695       35.389    0.00481
3         1   72    0    25.6470       28.905       35.810    0.00528
4         1   72    0    33.6420       29.187       36.375    0.00335

   Jitter(Abs)  Jitter:RAP  Jitter:PPQ5  …  Shimmer(dB)  Shimmer:APQ3  \
0     0.000034     0.00401      0.00317  …        0.230       0.01438
1     0.000017     0.00132      0.00150  …        0.179       0.00994
2     0.000025     0.00205      0.00208  …        0.181       0.00734
3     0.000027     0.00191      0.00264  …        0.327       0.01106
4     0.000020     0.00093      0.00130  …        0.176       0.00679
```

```
   Shimmer:APQ5  Shimmer:APQ11  Shimmer:DDA       NHR     HNR     RPDE  \
0       0.01309        0.01662      0.04314  0.014290  21.640  0.41888
1       0.01072        0.01689      0.02982  0.011112  27.183  0.43493
2       0.00844        0.01458      0.02202  0.020220  23.047  0.46222
3       0.01265        0.01963      0.03317  0.027837  24.445  0.48730
4       0.00929        0.01819      0.02036  0.011625  26.126  0.47188

       DFA      PPE
0  0.54842  0.16006
1  0.56477  0.10810
2  0.54405  0.21014
3  0.57794  0.33277
4  0.56122  0.19361

[5 rows x 22 columns]
```

Convert any non-numeric values to numeric values. For example you can replace a country name with an integer value or more appropriately use hot-one encoding. [Hint: use pandas.get_dummies].Please explain your solution.

[5]:
```python
wine_quality.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 1599 entries, 0 to 1598
Data columns (total 12 columns):
 #   Column                Non-Null Count  Dtype  
---  ------                --------------  -----  
 0   fixed acidity         1599 non-null   float64
 1   volatile acidity      1599 non-null   float64
 2   citric acid           1599 non-null   float64
 3   residual sugar        1599 non-null   float64
 4   chlorides             1599 non-null   float64
 5   free sulfur dioxide   1599 non-null   float64
 6   total sulfur dioxide  1599 non-null   float64
 7   density               1599 non-null   float64
 8   pH                    1599 non-null   float64
 9   sulphates             1599 non-null   float64
 10  alcohol               1599 non-null   float64
 11  quality               1599 non-null   int64  
dtypes: float64(11), int64(1)
memory usage: 150.0 KB
```

We can see from the Wine Quality Dataset info that No Column is Non-Numeric, so we skip Hot One Encoding here

[6]:
```python
airfare_demand.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 1000 entries, 0 to 999
```

```
Data columns (total 11 columns):
 #   Column                     Non-Null Count  Dtype  
---  ------                     --------------  -----  
 0   City1                      1000 non-null   object 
 1   City2                      1000 non-null   object 
 2   Average Fare1              1000 non-null   float64
 3   Distance                   1000 non-null   int64  
 4   Average weekly passengers  1000 non-null   float64
 5   market leading airline     1000 non-null   object 
 6   market share               1000 non-null   float64
 7   Average fare2              1000 non-null   float64
 8   Low price airline          1000 non-null   object 
 9   market share               1000 non-null   float64
 10  price                      1000 non-null   float64
dtypes: float64(6), int64(1), object(4)
memory usage: 86.1+ KB
```

Here we see that the Columns : 0,1,5,8 are Non-Numeric Columns, so converting these columns into suitable Hot one Encoding

[7]:
```
#Converting Non-Numeric Columns to to numeric Columns using Hot one encoding␣
↪scheme
airfare_demand_encoded = pd.
↪get_dummies(airfare_demand,columns=['City1','City2','market leading␣
↪airline','Low price airline'])
airfare_demand_encoded.head()
```

[7]:
```
   Average Fare1  Distance  Average weekly passengers  market share  \
0         114.47       528                     424.56         70.19   
1         122.47       860                     276.84         75.10   
2         214.42       852                     215.76         78.89   
3          69.40       288                     606.84         96.97   
4         158.13       723                     313.04         39.79   

   Average fare2  market share   price  City1_ABQ  City1_ACY  City1_ALB  …  \
0         111.03         70.19  111.03          0          0          0  …   
1         123.09         17.23  118.94          0          0          0  …   
2         223.98          2.77  167.12          0          0          1  …   
3          68.86         96.97   68.86          0          0          1  …   
4         161.36         15.34  145.42          0          0          1  …   

   Low price airline_G4  Low price airline_HP  Low price airline_NK  \
0                     0                     0                     0   
1                     0                     0                     0   
2                     0                     0                     0   
3                     0                     0                     0   
4                     0                     0                     0   
```

```
   Low price airline_NW  Low price airline_SY  Low price airline_TZ  \
0                     0                     0                     0
1                     0                     0                     0
2                     0                     0                     0
3                     0                     0                     0
4                     0                     0                     0

   Low price airline_UA  Low price airline_US  Low price airline_WN  \
0                     0                     0                     0
1                     0                     0                     0
2                     0                     0                     0
3                     0                     0                     1
4                     0                     0                     1

   Low price airline_YX
0                     0
1                     0
2                     0
3                     0
4                     0

[5 rows x 217 columns]
```

[8]:
```
parkison.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 5875 entries, 0 to 5874
Data columns (total 22 columns):
 #   Column          Non-Null Count  Dtype
---  ------          --------------  -----
 0   subject#        5875 non-null   int64
 1   age             5875 non-null   int64
 2   sex             5875 non-null   int64
 3   test_time       5875 non-null   float64
 4   motor_UPDRS     5875 non-null   float64
 5   total_UPDRS     5875 non-null   float64
 6   Jitter(%)       5875 non-null   float64
 7   Jitter(Abs)     5875 non-null   float64
 8   Jitter:RAP      5875 non-null   float64
 9   Jitter:PPQ5     5875 non-null   float64
 10  Jitter:DDP      5875 non-null   float64
 11  Shimmer         5875 non-null   float64
 12  Shimmer(dB)     5875 non-null   float64
 13  Shimmer:APQ3    5875 non-null   float64
 14  Shimmer:APQ5    5875 non-null   float64
 15  Shimmer:APQ11   5875 non-null   float64
 16  Shimmer:DDA     5875 non-null   float64
 17  NHR             5875 non-null   float64
```

```
 18  HNR             5875 non-null   float64
 19  RPDE            5875 non-null   float64
 20  DFA             5875 non-null   float64
 21  PPE             5875 non-null   float64
dtypes: float64(19), int64(3)
memory usage: 1009.9 KB
```

We can see from the Parkisons Dataset info that No Column is Non-Numeric, so we skip Hot One Encoding here

If required drop out the rows with missing values or NA. In next lectures we will handle sparse data, which will allow us to use records with missing values.

[9]:
```
wine_quality.isnull().sum()
```

[9]:
```
fixed acidity           0
volatile acidity        0
citric acid             0
residual sugar          0
chlorides               0
free sulfur dioxide     0
total sulfur dioxide    0
density                 0
pH                      0
sulphates               0
alcohol                 0
quality                 0
dtype: int64
```

There are No Empty and NA values in Wine Quality Dataset

[10]:
```
airfare_demand_encoded.isnull().sum()
```

[10]:
```
Average Fare1               0
Distance                    0
Average weekly passengers   0
market share                0
Average fare2               0
                           ..
Low price airline_TZ        0
Low price airline_UA        0
Low price airline_US        0
Low price airline_WN        0
Low price airline_YX        0
Length: 217, dtype: int64
```

There are also No Empty and NA values in Airfare Demand Dataset

[11]:
```
parkison.isnull().sum()
```

[11]:
```
subject#         0
age              0
sex              0
test_time        0
motor_UPDRS      0
total_UPDRS      0
Jitter(%)        0
Jitter(Abs)      0
Jitter:RAP       0
Jitter:PPQ5      0
Jitter:DDP       0
Shimmer          0
Shimmer(dB)      0
Shimmer:APQ3     0
Shimmer:APQ5     0
Shimmer:APQ11    0
Shimmer:DDA      0
NHR              0
HNR              0
RPDE             0
DFA              0
PPE              0
dtype: int64
```

There are also No Empty and NA values in Parkisons Dataset

Split the dataset into 80% Train set and 20% Test set.

Function to Split any Dataframe into Training Set and Test Set

[12]:
```
def split_dataset(dataset,label):
    #Creating X matrix by removing the label/Target column
    X = dataset.drop(label,axis=1)
    #Creating Y vector which include only the Label/Target Column
    Y = dataset[label].to_numpy()

    #Adding a Bias Column in the X Matrix
    X = np.append(np.ones(shape=(len(X),1)),X,axis=1)

    #Calculating number of rows to be copied in the Training Set according to␣
 ↪80:20 ratio
    total_training_rows = int(len(X)*0.8)

    #Splitting the Dataset into Training set and Test Set based on calculated␣
 ↪rows
    X_train , Y_train = X[:total_training_rows,:] , Y[:total_training_rows].
 ↪reshape(-1,1)
```

```
    X_test , Y_test = X[total_training_rows:,:] , Y[total_training_rows:].
↪reshape(-1,1)

    return X_train,X_test,Y_train,Y_test
```

Splitting Wine Quality Dataset

[13]:
```
wine_Xtrain , wine_Xtest , wine_Ytrain , wine_Ytest =␣
↪split_dataset(wine_quality,'quality')
```

Splitting Airfare Demand Dataset

[14]:
```
airfare_Xtrain , airfare_Xtest , airfare_Ytrain , airfare_Ytest =␣
↪split_dataset(airfare_demand_encoded,'price')
```

Splitting Parkison Dataset

[15]:
```
parkison_Xtrain , parkison_Xtest , parkison_Ytrain , parkison_Ytest =␣
↪split_dataset(parkison,'total_UPDRS')
```

0.1.2 Part B: Linear Regression with Real-World Data

[16]:
```
#Initializing arrays to store loss difference and RMSE values in different␣
↪number of Iterations
loss_difference_values = np.array([])
rmse_values = np.array([])
```

Function to Calculate Loss Between Actual Y and Predicted Y

[17]:
```
def loss_function(X,Y,B):
    # L(B) = summation((y-ypred)^2)
    return np.sum(np.square(np.subtract(Y,X @ B)))
```

Function to Calculate the Loss Difference based on Old Beta Values and New Beta Values

[18]:
```
def loss_difference(X,Y,B_old,B_new):
    # |L(B_old) - L(B_new)|
    return np.abs(loss_function(X,Y,B_old) - loss_function(X,Y,B_new))
```

Function to Calculate the RMSE Loss between Actual Y and Predicted Y

[19]:
```
def rmse(X,Y,B):
    # RMSE = square_root(summation((y-ypred)^2)/N)
    return np.sqrt(np.sum(np.square(np.subtract(Y , X @ B)))/len(X))
```

Function which returns the gradient of Loss function

[20]:
```
def dL(X,Y,B):
    # Derivation of Loss = -2 * X^t * (y - ypred)
    return -2 * (X.T @ (Y - X @ B))
```

Function to Minimize Gradient Descent based on Total Iterations and Learning Rate

[21]:
```
def minimize_GD(X,Y,X_test,Y_test,imax,mu):
    #Using global arrays to store loss difference and rmse values for different␣
 ↪Iterations
    global loss_difference_values , rmse_values

    #Emptying both Loss difference and RMSE arrays
    loss_difference_values , rmse_values = np.array([]) , np.array([])

    #Initializing beta with Zeros
    beta = np.zeros(shape=(len(X[0]),1))
    for i in range(imax):
        #Calculating new Beta values from previous beta values and gradient␣
 ↪descent direction
        #Beta = Beta - learning_rate * gradient Descent based on Beta
        beta_ = beta - mu * dL(X,Y,beta)

        #Appending Loss difference between between previous and new Beta
        loss_difference_values = np.
 ↪append(loss_difference_values,loss_difference(X,Y,beta,beta_))

        #Appending RMSE loss between actual Y and Predicted Y
        rmse_values = np.append(rmse_values,rmse(X_test,Y_test,beta_))

        #Copying new Beta value to old Beta value for Further Calculation
        beta = np.copy(beta_)
    return beta_
```

Function to Plot Loss Difference and RMSE Loss

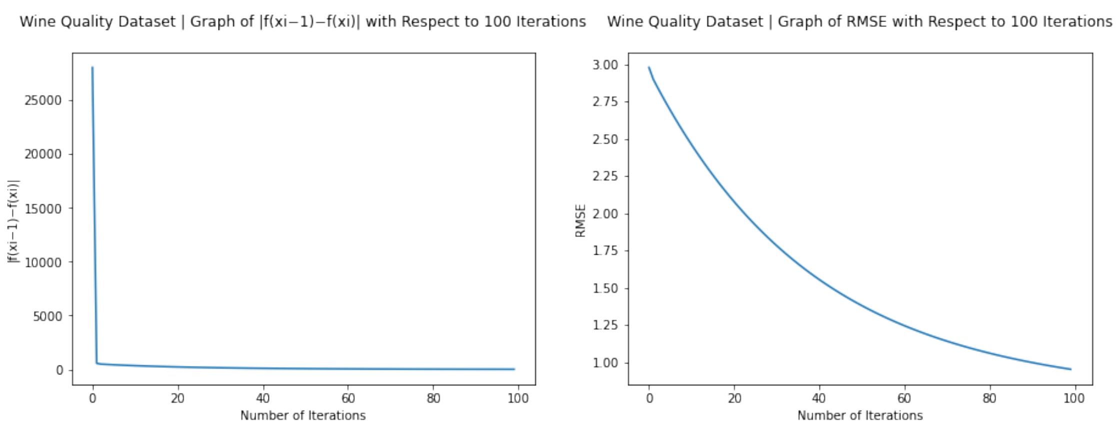
[22]:
```
def plot_loss(xvalues , loss_difference , rmse ,title_graph1 ,title_graph2␣
 ↪,xlabel_graph1 ,xlabel_graph2 ,ylabel_graph1,ylabel_graph2):
    #Plotting Loss Difference Graph
    fig = plt.figure(figsize=(15,5))
    ax = fig.add_subplot(121)
    ax.plot(xvalues,loss_difference)
    ax.set_title(title_graph1)
    ax.set_xlabel(xlabel_graph1)
    ax.set_ylabel(ylabel_graph1)

    #Plotting RMSE Loss
    ax1 = fig.add_subplot(122)
    ax1.plot(xvalues,rmse)
    ax1.set_title(title_graph2)
    ax1.set_xlabel(xlabel_graph2)
    ax1.set_ylabel(ylabel_graph2)
```

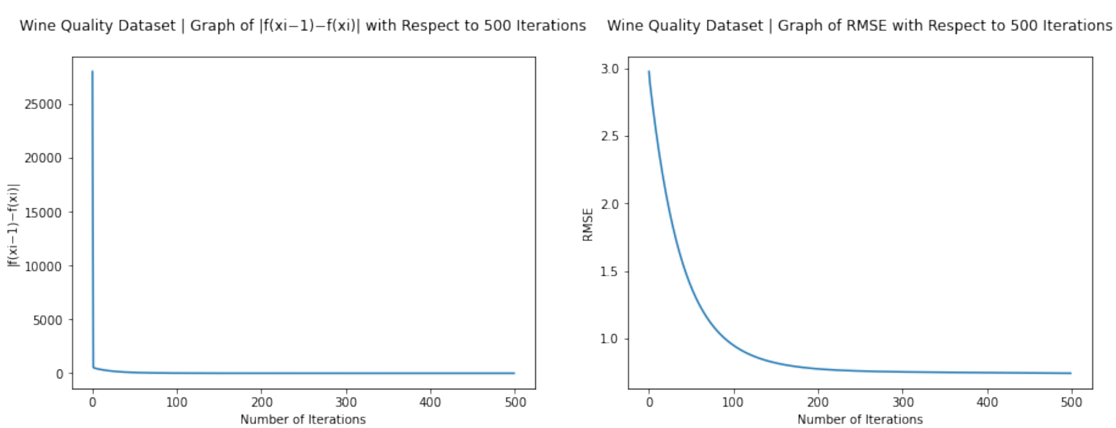
Applying Different Iteration and Different mu values for the Optimum values

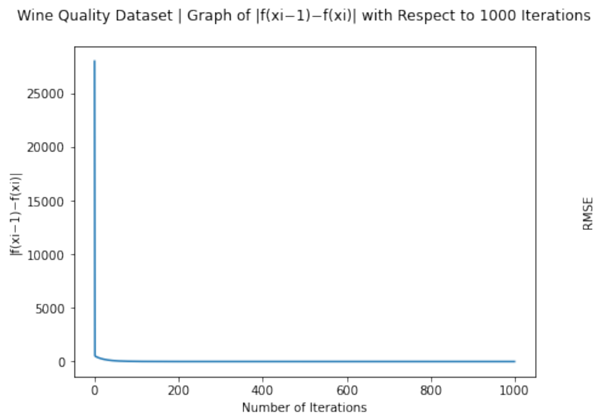
```
[23]: imax_d = [100,500,1000]
mu_wine = [0.0000001,0.00000001,0.000000001]
mu_parkison = [0.00000001,0.000000001,0.0000000001]
mu_airfare = [0.000000000001,0.0000000000001,0.00000000000001]
```

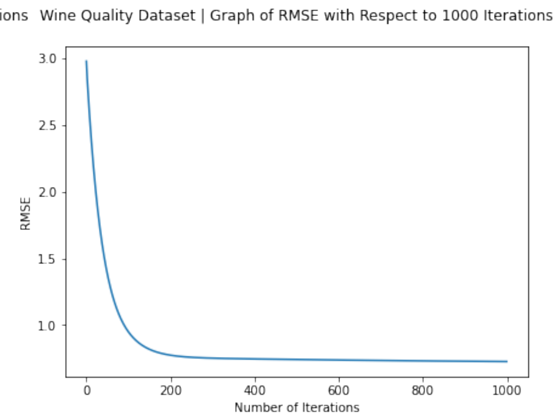
Applying Different Iterations on Wine Quality Dataset with mu=0.0000001

```
[24]: for i in imax_d:
    #Calculating new beta using Gradient Descent using mu = 0.0000001
    beta = minimize_GD(wine_Xtrain,wine_Ytrain,wine_Xtest,wine_Ytest,i,0.
 ↪0000001)

    #Plotting the Loss and RMSE with different Iterations
    plot_loss([j for j in range(i)],loss_difference_values,rmse_values,
              'Wine Quality Dataset | Graph of |f(xi-1)-f(xi)| with Respect to␣
 ↪{} Iterations\n'.format(i),
              'Wine Quality Dataset | Graph of RMSE with Respect to {}␣
 ↪Iterations\n'.format(i),
              'Number of Iterations','Number of␣
 ↪Iterations','|f(xi-1)-f(xi)|','RMSE')
```



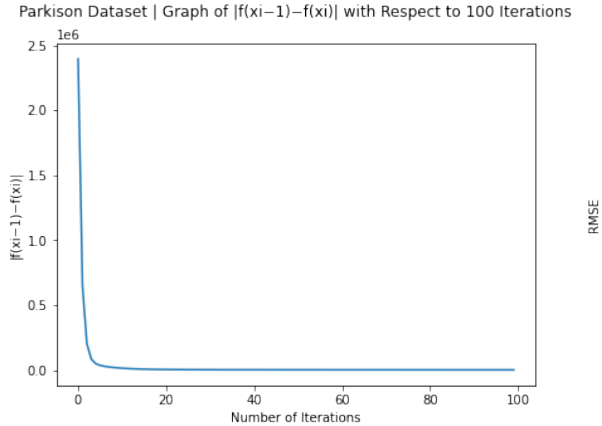


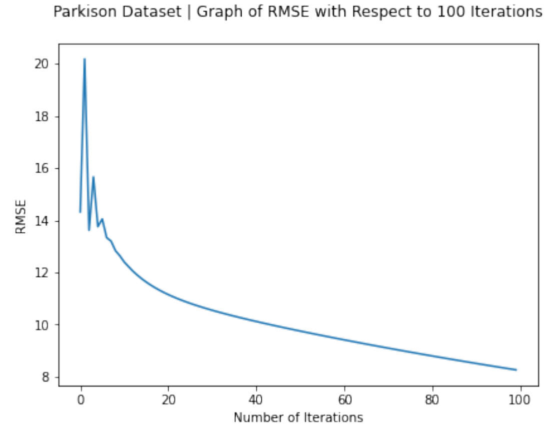


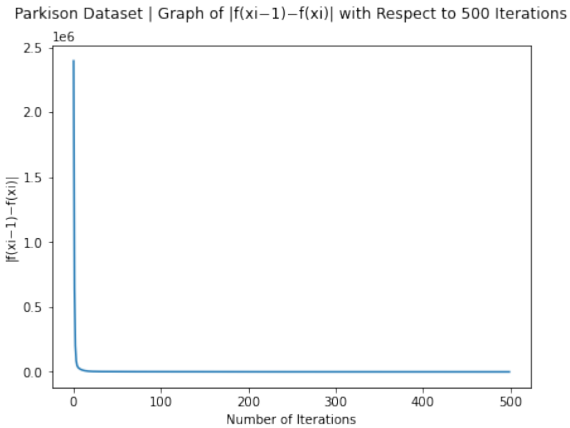


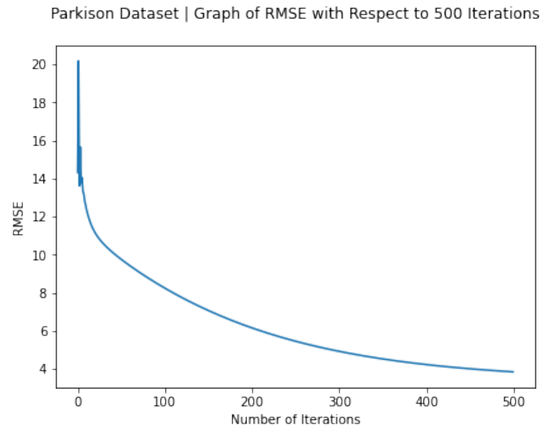
Applying Different Iterations on Parkison Dataset with mu=0.00000001

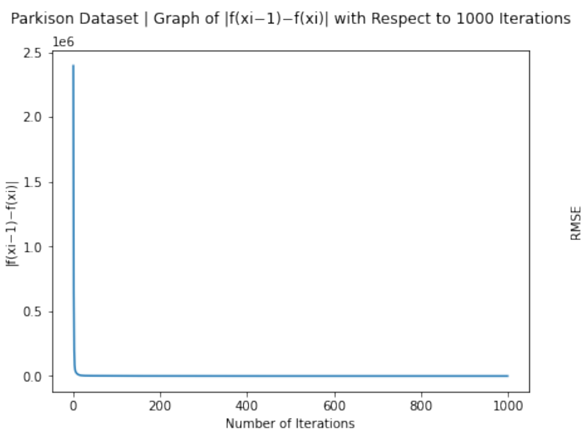
[25]:
```
for i in imax_d:
    #Calculating new beta using Gradient Descent using mu = 0.00000001
    beta =␣
 ↪minimize_GD(parkison_Xtrain,parkison_Ytrain,parkison_Xtest,parkison_Ytest,i,0.
 ↪00000001)

    #Plotting the Loss and RMSE with different Iterations
    plot_loss([j for j in range(i)],loss_difference_values,rmse_values,
              'Parkison Dataset | Graph of |f(xi-1)-f(xi)| with Respect to {}␣
 ↪Iterations\n'.format(i),
              'Parkison Dataset | Graph of RMSE with Respect to {}␣
 ↪Iterations\n'.format(i),
              'Number of Iterations','Number of␣
 ↪Iterations','|f(xi-1)-f(xi)|','RMSE')
```

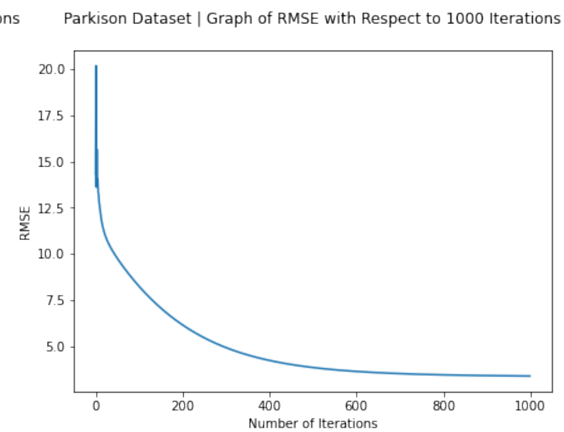






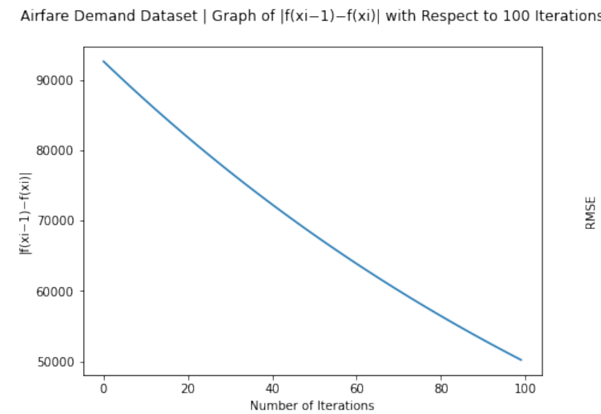


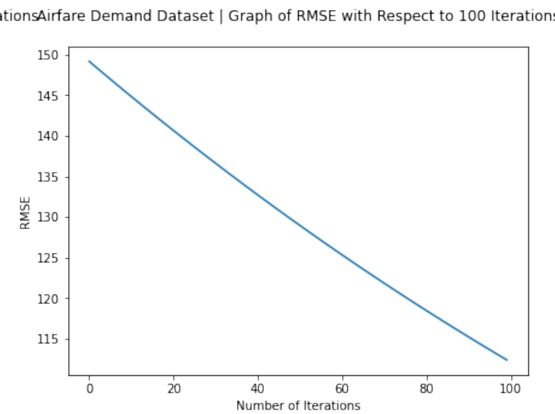


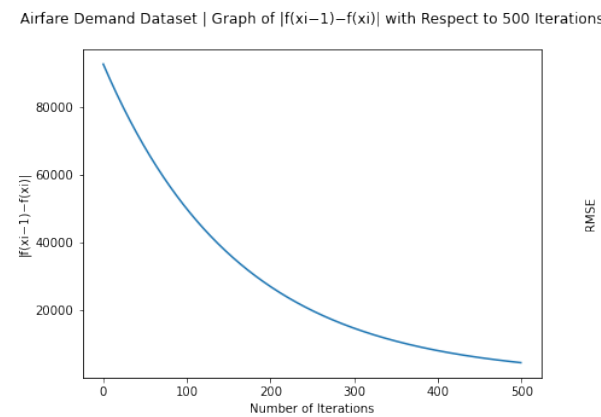


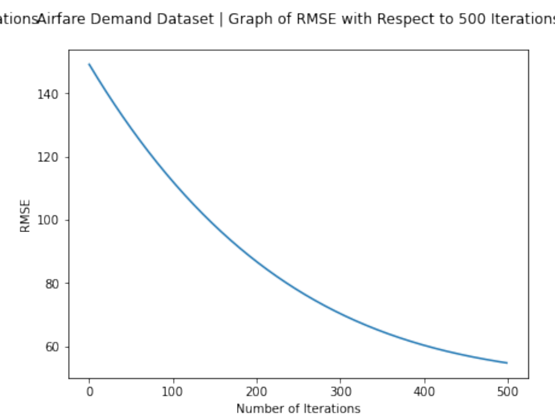
Applying Different Iterations on Airfare Demand Dataset with mu=0.000000000001

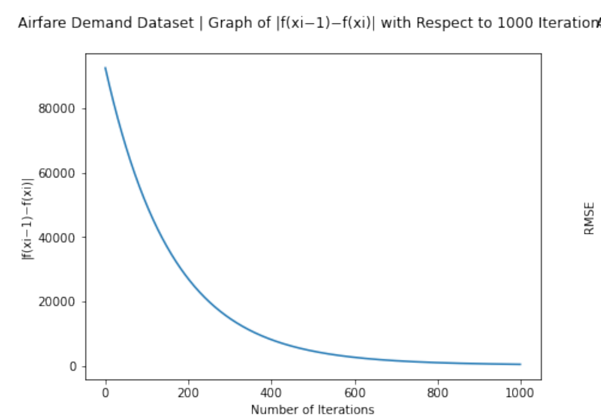
[26]:
```
for i in imax_d:
    #Calculating new beta using Gradient Descent using mu = 0.000000000001
    beta = minimize_GD(airfare_Xtrain,airfare_Ytrain,airfare_Xtest,airfare_Ytest,i,0.000000000001)

    #Plotting the Loss and RMSE with different Iterations
    plot_loss([j for j in range(i)],loss_difference_values,rmse_values,
              'Airfare Demand Dataset | Graph of |f(xi-1)-f(xi)| with Respect to {} Iterations\n'.format(i),
              'Airfare Demand Dataset | Graph of RMSE with Respect to {} Iterations\n'.format(i),
              'Number of Iterations','Number of Iterations','|f(xi-1)-f(xi)|','RMSE')
```

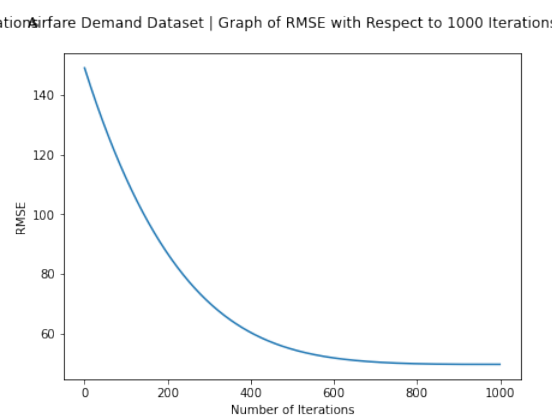








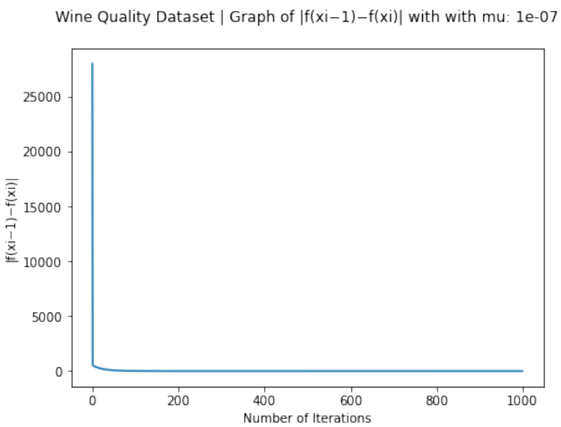


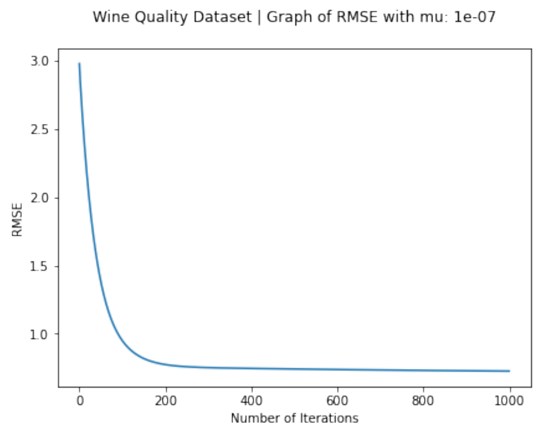


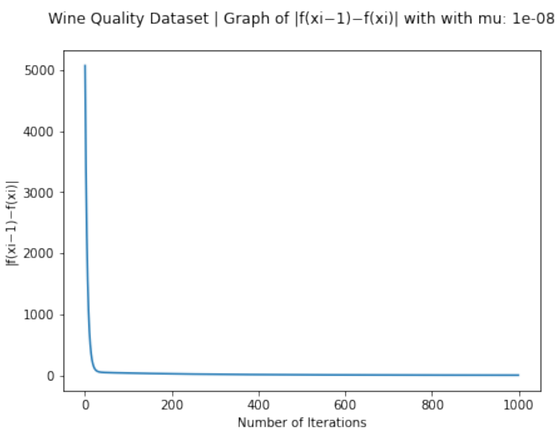
From the above plots and visualization we can see that the minimum loss occurs after 1000 iterations.So selecting imax = 1000 for further experimentation

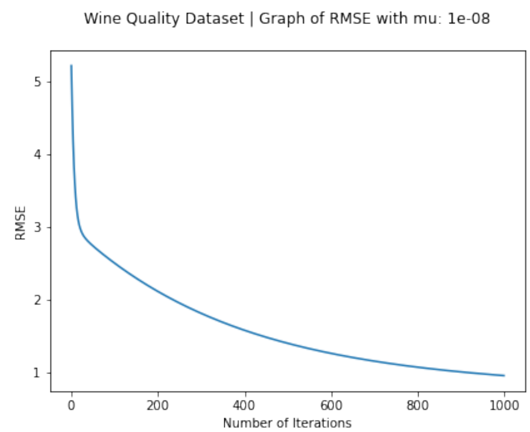
Applying Different mu values on Wine Quality Dataset with imax = 1000

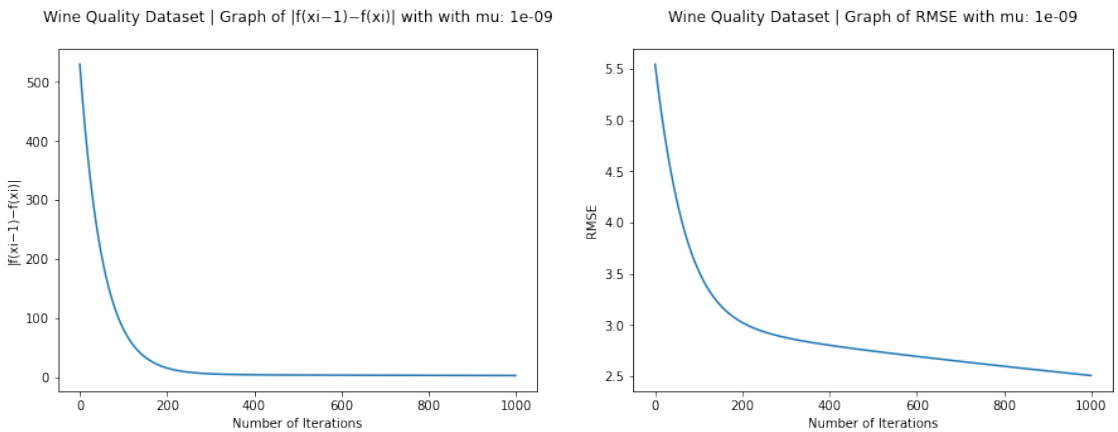
[27]:
```
for mu in mu_wine:
    #Calculating new beta using Gradient Descent using imax = 1000
    beta = minimize_GD(wine_Xtrain,wine_Ytrain,wine_Xtest,wine_Ytest,1000,mu)

    #Plotting the Loss and RMSE with different mu values
    plot_loss([i for i in range(1000)],loss_difference_values,rmse_values,
              'Wine Quality Dataset | Graph of |f(xi-1)-f(xi)| with with mu:␣
↪{}\n'.format(mu),
              'Wine Quality Dataset | Graph of RMSE with mu: {}\n'.format(mu),
              'Number of Iterations','Number of␣
↪Iterations','|f(xi-1)-f(xi)|','RMSE')
```





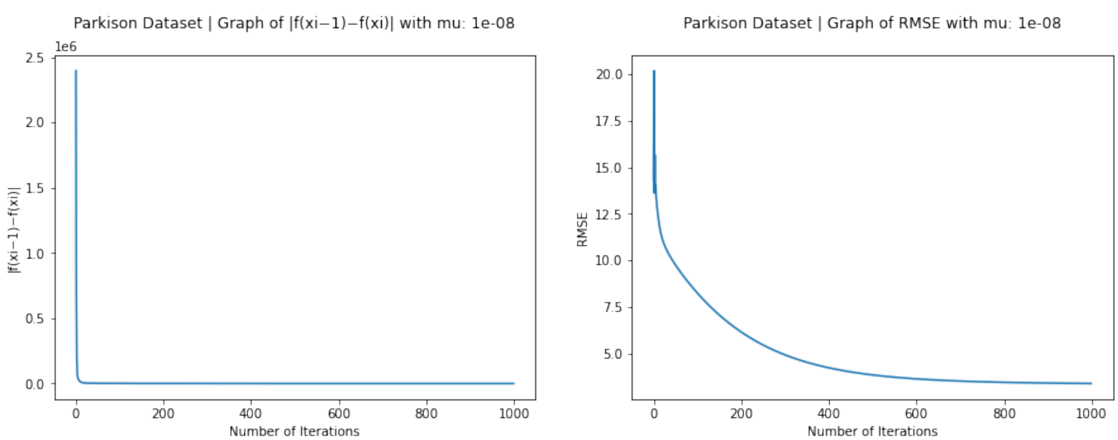


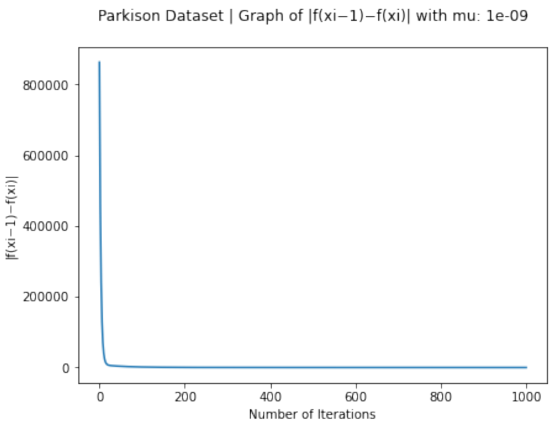


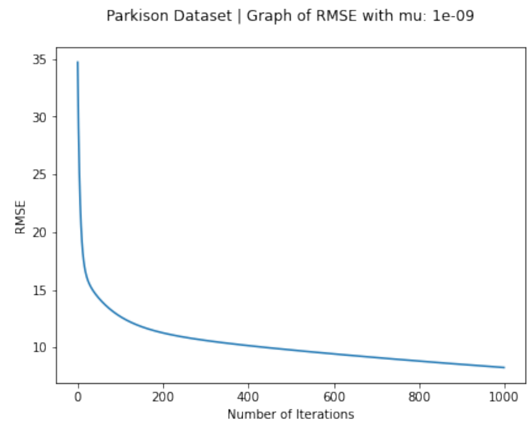


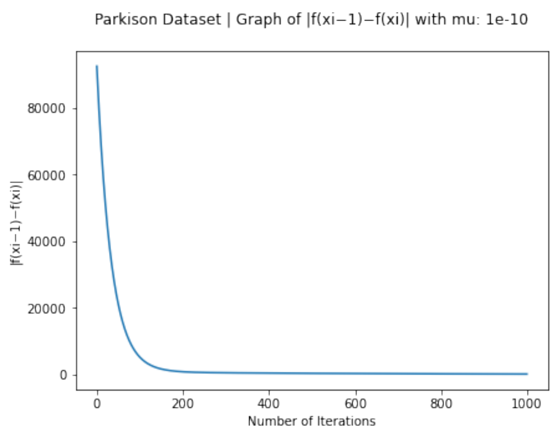
Applying Different mu values on Parkison Dataset with imax = 1000

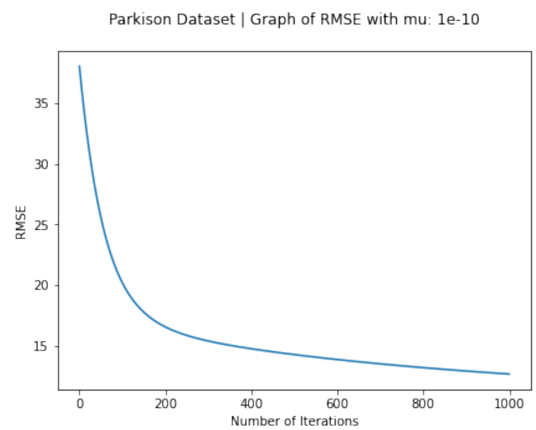
[28]:
```python
for mu in mu_parkison:
    #Calculating new beta using Gradient Descent using imax = 1000
    beta = minimize_GD(parkison_Xtrain,parkison_Ytrain,parkison_Xtest,parkison_Ytest,1000,mu)

    #Plotting the Loss and RMSE with different mu values
    plot_loss([i for i in range(1000)],loss_difference_values,rmse_values,
              'Parkison Dataset | Graph of |f(xi-1)-f(xi)| with mu: {}\n'.format(mu),
              'Parkison Dataset | Graph of RMSE with mu: {}\n'.format(mu),
              'Number of Iterations','Number of Iterations','|f(xi-1)-f(xi)|','RMSE')
```





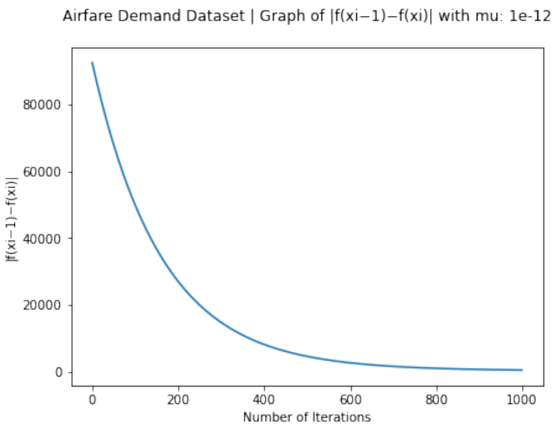


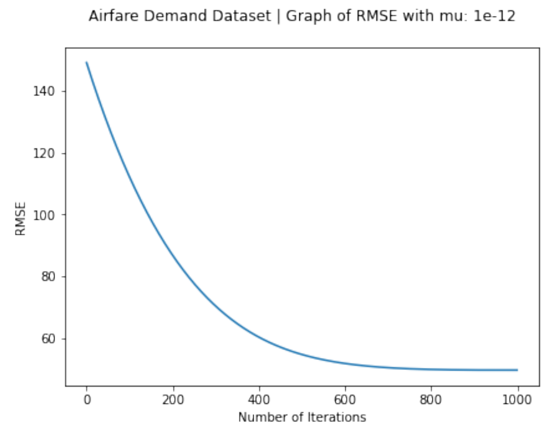


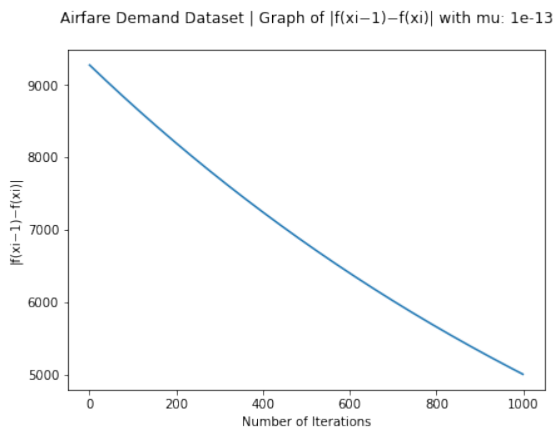


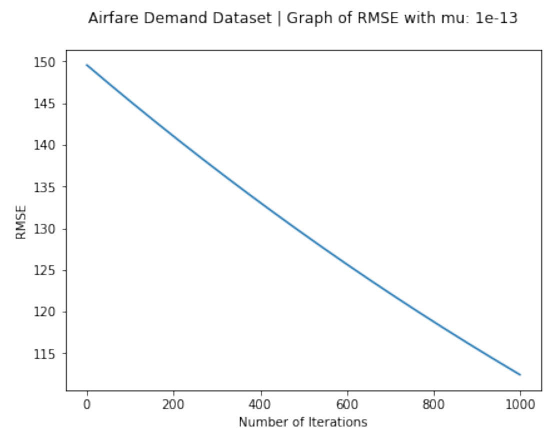
Applying Different mu values on Airfare Demand Dataset with imax = 1000

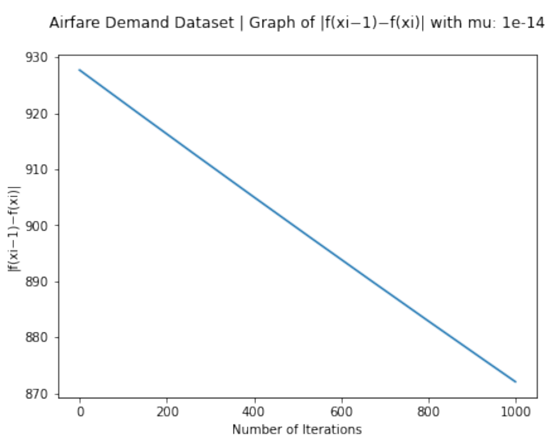
[29]:
```
for mu in mu_airfare:
    #Calculating new beta using Gradient Descent using imax = 1000
    beta = minimize_GD(airfare_Xtrain,airfare_Ytrain,airfare_Xtest,airfare_Ytest,1000,mu)

    #Plotting the Loss and RMSE with different mu values
    plot_loss([i for i in range(1000)],loss_difference_values,rmse_values,
              'Airfare Demand Dataset | Graph of |f(xi-1)-f(xi)| with mu: {}\n'.format(mu),
              'Airfare Demand Dataset | Graph of RMSE with mu: {}\n'.format(mu),
              'Number of Iterations','Number of Iterations','|f(xi-1)-f(xi)|','RMSE')
```

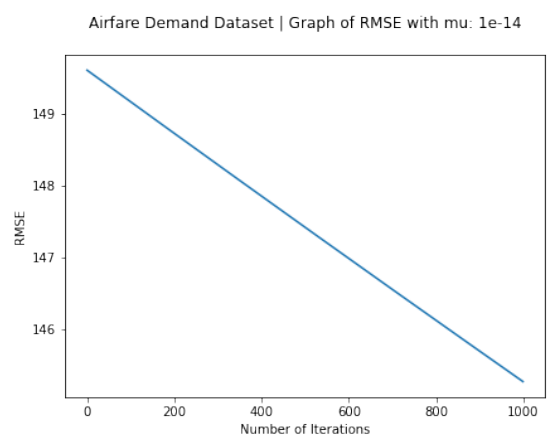

Airfare Demand Dataset | Graph of |f(xi−1)−f(xi)| with mu: 1e-12
|f(xi−1)−f(xi)|
0
20000
40000
60000
80000
0
200
400
600
800
1000
Number of Iterations


Airfare Demand Dataset | Graph of RMSE with mu: 1e-12
RMSE
60
80
100
120
140
0
200
400
600
800
1000
Number of Iterations


Airfare Demand Dataset | Graph of |f(xi−1)−f(xi)| with mu: 1e-13
|f(xi−1)−f(xi)|
5000
6000
7000
8000
9000
0
200
400
600
800
1000
Number of Iterations


Airfare Demand Dataset | Graph of RMSE with mu: 1e-13
RMSE
115
120
125
130
135
140
145
150
0
200
400
600
800
1000
Number of Iterations


Airfare Demand Dataset | Graph of |f(xi−1)−f(xi)| with mu: 1e-14
|f(xi−1)−f(xi)|
870
880
890
900
910
920
930
0
200
400
600
800
1000
Number of Iterations


Airfare Demand Dataset | Graph of RMSE with mu: 1e-14
RMSE
146
147
148
149
0
200
400
600
800
1000
Number of Iterations

0.2 Exercise 3: Steplength Control for Gradient Descent

Function to Plot Loss Difference and RMSE

[30]:
```
def plot_stepsize_iteration(xvalues1, xvalues2 , loss_difference , rmse␣
↪,title_graph1 ,title_graph2 ,
                            xlabel_graph1 ,xlabel_graph2␣
↪,ylabel_graph1,ylabel_graph2):
    #Plotting Total Iteration vs Loss Difference Graph
    fig = plt.figure(figsize=(15,5))
    ax = fig.add_subplot(121)
    ax.plot(xvalues1,loss_difference)
    ax.set_title(title_graph1)
    ax.set_xlabel(xlabel_graph1)
    ax.set_ylabel(ylabel_graph1)

    #Plotting different MU values vs RMSE Loss
    ax1 = fig.add_subplot(122)
    ax1.plot(xvalues2,rmse)
    ax1.set_title(title_graph2)
    ax1.set_xlabel(xlabel_graph2)
    ax1.set_ylabel(ylabel_graph2)
```

Function to calculate optimum learning rate using Backtracking Algorithm

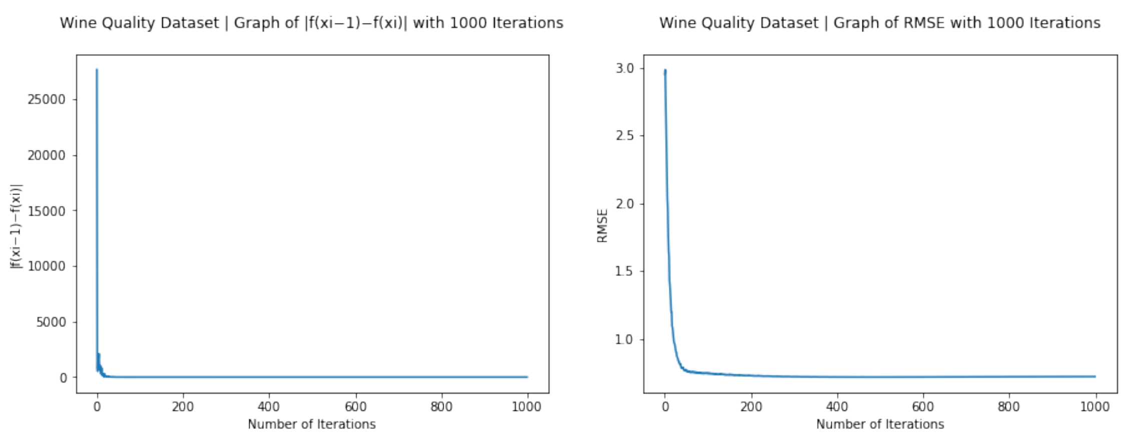
[31]:
```
def stepsize_backtracking(X,Y,B):
    mu = 1          #Starting Learning rate value
    alpha = 0.1     #Value for Alpha
    beta = 0.5      #Value for Beta

    #Iterate until the following condition is meet:
    #f (x) − f(x − µ f (x)) < µ f (x)T  f (x)
    while((loss_function(X,Y,B) - loss_function(X,Y,B+(mu*-1*dL(X,Y,B))) <␣
↪(alpha * mu * (dL(X,Y,B).T @ dL(X,Y,B))))):
        mu = mu * beta
    return mu
```

Function to Minimize Gradient Descent using Backtracking algorithm

[32]:
```
def minimize_GD_backtracking(X,Y,X_test,Y_test,imax):
    #Creating and Initializing arrays to store loss difference and RMSE
    global loss_difference_values , rmse_values
    loss_difference_values , rmse_values = np.array([]) , np.array([])

    #Initializing beta with 0s
    beta = np.zeros(shape=(len(X[0]),1))
    for i in range(imax):
        #Calculating new Beta using Gradient Descent and backtracking algorithm
        beta_ = beta - stepsize_backtracking(X,Y,beta) * dL(X,Y,beta)
```

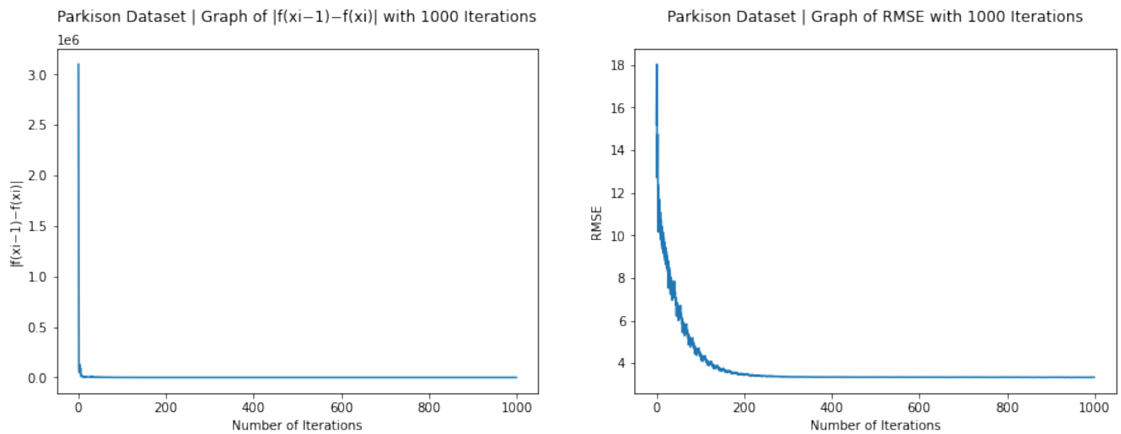
```
        #Calculating Loss difference between old Beta and New Beta
        loss_difference_values = np.
 ↪append(loss_difference_values,loss_difference(X,Y,beta,beta_))

        #Calculating RMSE loss based on new Beta
        rmse_values = np.append(rmse_values,rmse(X_test,Y_test,beta_))

        #Copying New beta to Old Beta for Next Iteration
        beta = np.copy(beta_)
    return beta_
```

[33]:
```
#Calculating new beta using Gradient Descent using imax = 1000
beta =␣
 ↪minimize_GD_backtracking(wine_Xtrain,wine_Ytrain,wine_Xtest,wine_Ytest,1000)

#Plotting the Loss and RMSE with different mu values
plot_stepsize_iteration([i for i in range(1000)],[j for j in␣
 ↪range(1000)],loss_difference_values,rmse_values,
          'Wine Quality Dataset | Graph of |f(xi-1)-f(xi)| with 1000␣
 ↪Iterations\n'.format(i),
          'Wine Quality Dataset | Graph of RMSE with 1000 Iterations\n'.
 ↪format(i),
          'Number of Iterations','Number of␣
 ↪Iterations','|f(xi-1)-f(xi)|','RMSE')
```



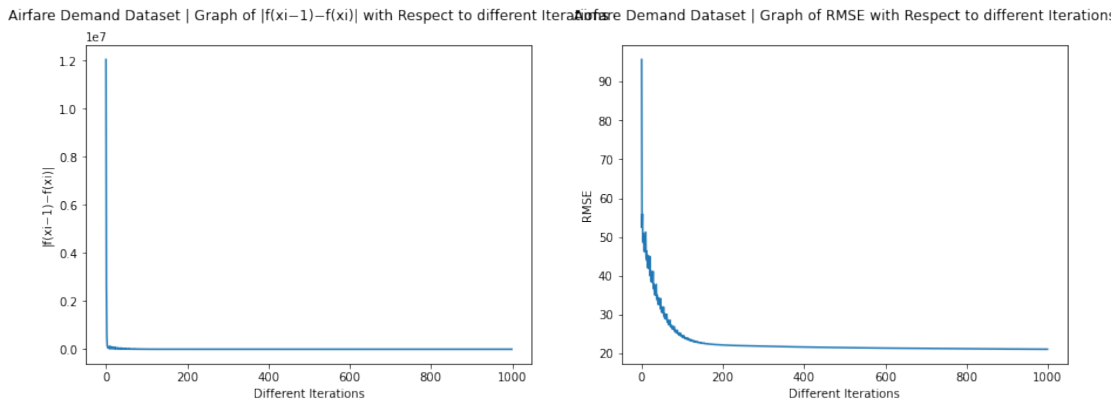
[34]:
```
#Calculating new beta using Gradient Descent using imax = 1000
beta =␣
 ↪minimize_GD_backtracking(parkison_Xtrain,parkison_Ytrain,parkison_Xtest,parkison_Ytest,1000

#Plotting the Loss and RMSE with different mu values
```

```
plot_stepsize_iteration([i for i in range(1000)],[j for j in␣
↪range(1000)],loss_difference_values,rmse_values,
          'Parkison Dataset | Graph of |f(xi-1)-f(xi)| with 1000 Iterations\n'.
↪format(i),
          'Parkison Dataset | Graph of RMSE with 1000 Iterations\n'.format(i),
          'Number of Iterations','Number of␣
↪Iterations','|f(xi-1)-f(xi)|','RMSE')
```



[35]:

```
#Calculating new beta using Gradient Descent using imax = 1000
beta =␣
↪minimize_GD_backtracking(airfare_Xtrain,airfare_Ytrain,airfare_Xtest,airfare_Ytest,1000)

#Plotting the Loss and RMSE with different mu values
plot_stepsize_iteration([i for i in range(1000)],[j for j in␣
↪range(1000)],loss_difference_values,rmse_values,
          'Airfare Demand Dataset | Graph of |f(xi-1)-f(xi)| with Respect to␣
↪different Iterations\n'.format(i),
          'Airfare Demand Dataset | Graph of RMSE with Respect to different␣
↪Iterations\n'.format(i),
          'Different Iterations','Different␣
↪Iterations','|f(xi-1)-f(xi)|','RMSE')
```



Function to calculate optimum learning rate using Bold Driver Algorithm

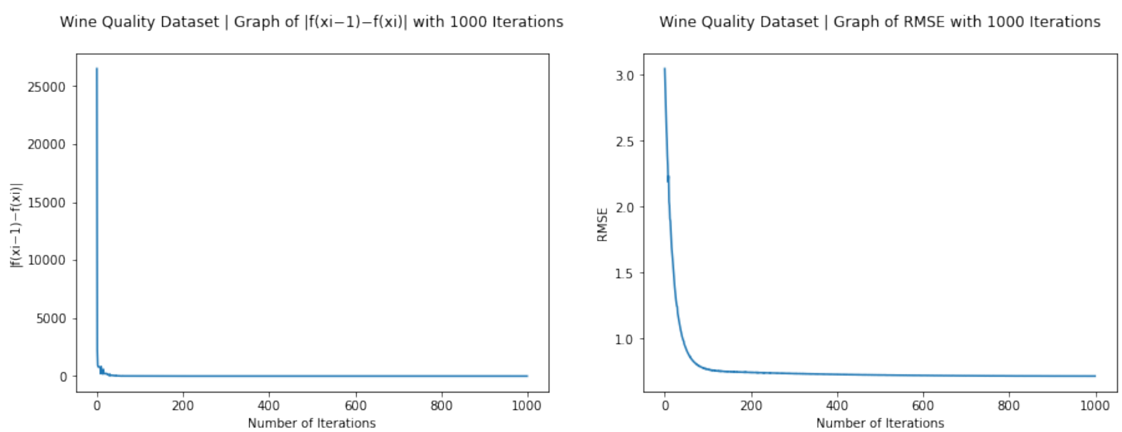
```
[36]: def steplength_bolddriver(X,Y,B,alpha_old,alpha_plus,alpha_minus):
          #Increasing alpha value using alpha+
          alpha = alpha_old*alpha_plus

          #Iterating until following condition is meet:
          #f(x) − f(x + µd)   0
          while(loss_function(X,Y,B) - loss_function(X,Y,B - (alpha * dL(X,Y,B))) <=␣
      ↪0):
              #Slowly Increasing alpha using alpha-
              alpha = alpha * alpha_minus
          return alpha
```

Function to Minimize Gradient Descent using Bold Driver algorithm

```
[37]: def minimize_GD_bolddriver(X,Y,X_test,Y_test,imax):
          #Creating and Initializing arrays to store loss difference and RMSE
          global loss_difference_values , rmse_values
          loss_difference_values , rmse_values = np.array([]) , np.array([])

          #Initializing beta with 0s
          theta = np.zeros(shape=(len(X[0]),1))
          alpha = 1
          for i in range(imax):
              #Calculating alpha based bold driver algorithm
              alpha = steplength_bolddriver(X,Y,theta,alpha,1.1,0.5)

              #Calculating new Beta using Gradient Descent and backtracking algorithm
              theta_ = theta - alpha * dL(X,Y,theta)

              #Calculating Loss difference between old Beta and New Beta
              loss_difference_values = np.
      ↪append(loss_difference_values,loss_difference(X,Y,theta,theta_))
```

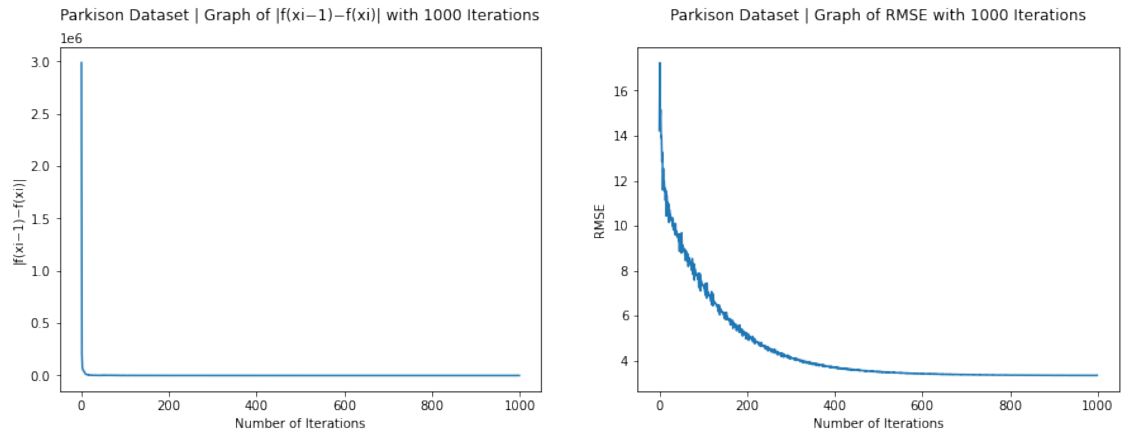
```
        #Calculating RMSE loss based on new Beta
        rmse_values = np.append(rmse_values,rmse(X_test,Y_test,theta_))

        #Copying New beta to Old Beta for Next Iteration
        theta = np.copy(theta_)
    return theta_
```

[38]:
```
#Calculating new beta using Gradient Descent using imax = 1000
beta =␣
↪minimize_GD_bolddriver(wine_Xtrain,wine_Ytrain,wine_Xtest,wine_Ytest,1000)

#Plotting the Loss and RMSE with different mu values
plot_stepsize_iteration([i for i in range(1000)],[j for j in␣
↪range(1000)],loss_difference_values,rmse_values,
          'Wine Quality Dataset | Graph of |f(xi-1)-f(xi)| with 1000␣
↪Iterations\n'.format(i),
          'Wine Quality Dataset | Graph of RMSE with 1000 Iterations\n'.
↪format(i),
          'Number of Iterations','Number of␣
↪Iterations','|f(xi-1)-f(xi)|','RMSE')
```

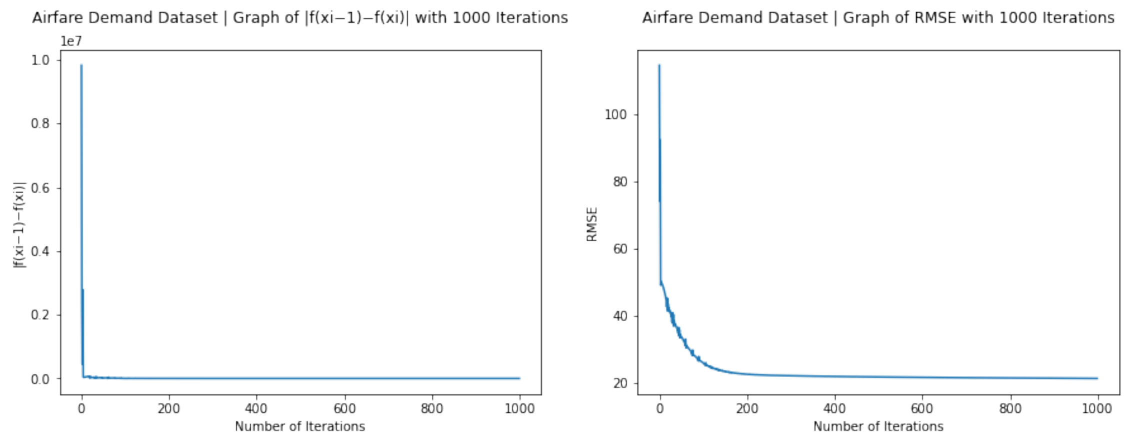


[39]:
```
#Calculating new beta using Gradient Descent using imax = 1000
beta =␣
↪minimize_GD_bolddriver(parkison_Xtrain,parkison_Ytrain,parkison_Xtest,parkison_Ytest,1000)

#Plotting the Loss and RMSE with different mu values
plot_stepsize_iteration([i for i in range(1000)],[j for j in␣
↪range(1000)],loss_difference_values,rmse_values,
          'Parkison Dataset | Graph of |f(xi-1)-f(xi)| with 1000 Iterations\n'.
↪format(i),
```

```
            'Parkison Dataset | Graph of RMSE with 1000 Iterations\n'.format(i),
            'Number of Iterations','Number of␣
↪Iterations','|f(xi-1)-f(xi)|','RMSE')
```



[40]:
```
#Calculating new beta using Gradient Descent using imax = 1000
beta =␣
↪minimize_GD_bolddriver(airfare_Xtrain,airfare_Ytrain,airfare_Xtest,airfare_Ytest,1000)

#Plotting the Loss and RMSE with different mu values
plot_stepsize_iteration([i for i in range(1000)],[j for j in␣
↪range(1000)],loss_difference_values,rmse_values,
            'Airfare Demand Dataset | Graph of |f(xi-1)-f(xi)| with 1000␣
↪Iterations\n'.format(i),
            'Airfare Demand Dataset | Graph of RMSE with 1000 Iterations\n'.
↪format(i),
            'Number of Iterations','Number of␣
↪Iterations','|f(xi-1)-f(xi)|','RMSE')
```



Based on Backtracking and Bold Driver Algorithm performance, I think Bold driver did a bit better job as compared to Backtracting as it converges faster than the later But with a very smaller margin.The margin is so small that we can use any of these algorithm without any significant performance drop.