Exercise 1

November 26, 2021

Machine Learning Lab

Lab 03

Exercise 1

Importing Packages

[1]:
```
import matplotlib.pyplot as plt     #Importing Matplotlib
import numpy as np                  #Importing Numpy
```

Defining Rosenbrock Function

[2]:
```
#Defining Constant values of Rosenbrock Function
a = 1
b = 100
```

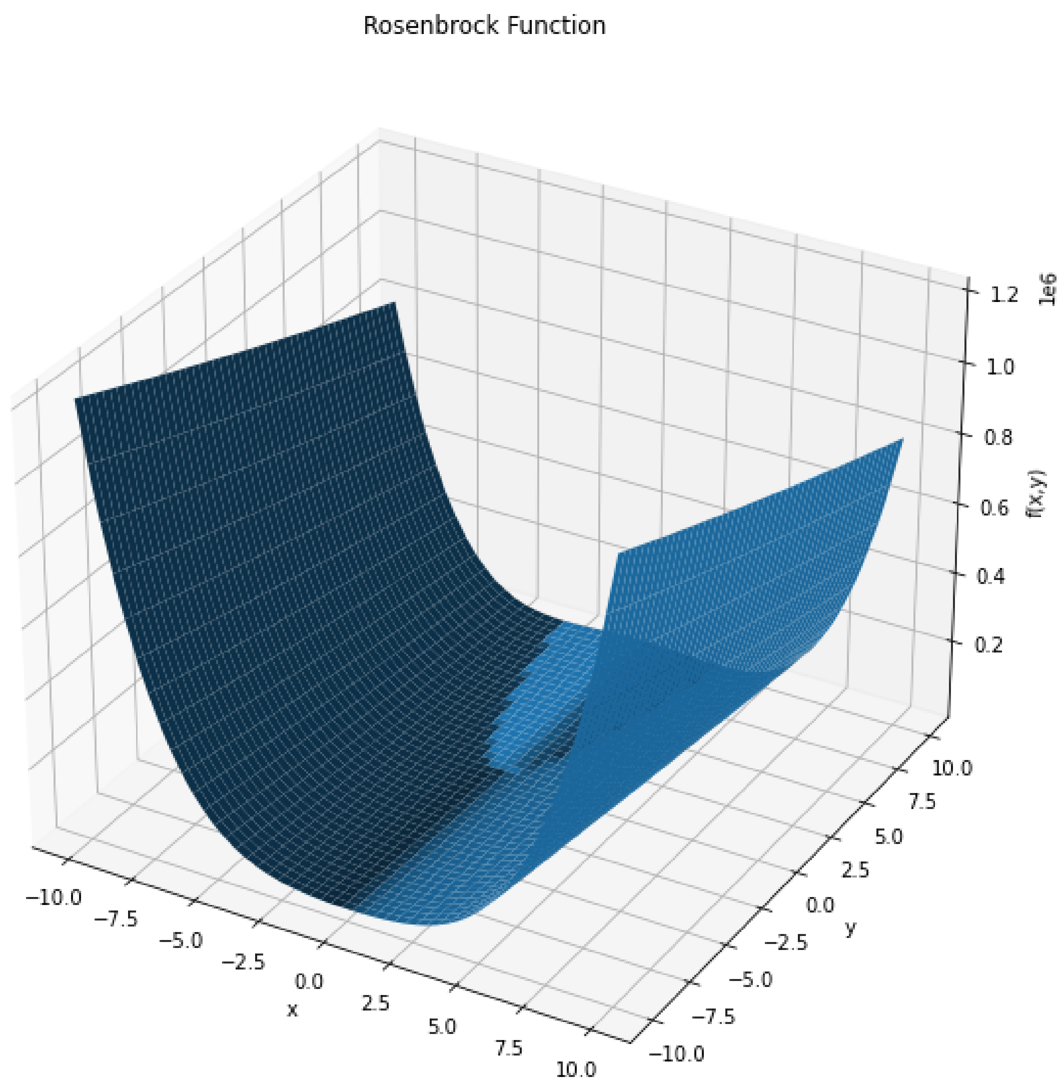
[3]:
```
rosenbrock_func = lambda x , y : (a - x)**2 + b * (y - x**2)**2
```

Plotting a 3D Plot of Rosenbrock Function

[4]:
```
#Creating x and y vectors for plotting Rosenbrock function within a specific␣
 ↪range
x = np.linspace(-10,10)
y = np.linspace(-10,10)
X, Y = np.meshgrid(x, y)

#Applying Rosenbrock function to all values of vector x and y to get values for␣
 ↪f(x,y)
fxy = np.array(rosenbrock_func(X,Y))
```

[5]:
```
#Plotting Rosenbrock Function as a 3D Surface
fig = plt.figure(figsize=(10,10))
ax = fig.add_subplot(111, projection='3d')
ax.plot_surface(X, Y, fxy)
ax.set_xlabel('x')
ax.set_ylabel('y')
ax.set_zlabel('f(x,y)')
ax.set_title('Rosenbrock Function')
plt.show()
```



Derive the partial gradients. The Partial Derivative of Rosenbrock Function with Respect to X:

$\frac{\partial f}{\partial x} = 2(a-x)(-1) + 2b(y-x^2)(-2x)$
$\frac{\partial f}{\partial x} = -2(a-x) - 4bx(y-x^2)$

The Partial Derivative of Rosenbrock Function with Respect to Y:

$\frac{\partial f}{\partial y} = 2b(y-x^2)(1)$
$\frac{\partial f}{\partial y} = 2b(y-x^2)$

Convert the function and gradient of this function into equivalent code representation.

Partial Derivative of Rosenbrock Function with respect to X

[6]:
```
dx = lambda x , y : -2 * (a - x) -4*x*b * (y - x**2)
```

Partial Derivative of Rosenbrock Function with respect to Y

[7]:
```
dy = lambda x , y : 2*b * (y - x**2)
```

Optimize the function with Gradient Descent. Set the appropriate hyperparameters like initial valus of (x,y) and the steplength through trial and error.

[8]:
```
#Defining Hyperparameters for Gradient Descent
alpha = 0.00001
epsilon = 10**-4
total_iteration = 3000
```

[9]:
```
#Initializing x and y vector for storing all intermediate values of x and y so␣
↪that to use for plotting
x_arr = []
y_arr = []
```

Function to perform Gradient Descent on X and Y

[10]:
```
def gradient_descent(x,y,total_iter):
    global x_arr, y_arr
    for i in range(total_iter):
        #Updating the value of x and y based on gradient direction
        x_ = x - alpha * dx(x,y)
        y_ = y - alpha * dy(x,y)

        #Checking the stopping Condition
        if rosenbrock_func(x,y) - rosenbrock_func(x_,y_) < epsilon:
            x_arr.append(x_)
            y_arr.append(y_)
            return (x_,y_)

        #Swapping the new values of x and y with previous values
        x , y = x_ , y_

        #Appending the new x and y values in x and y arrays for plotting
        x_arr.append(x_)
        y_arr.append(y_)

    #Raising Exception if after total Iterations, our Gradient Descent didn't␣
↪Converge
    raise Exception('Not Converged in {} many Iterations'.format(total_iter))
```

[11]:
```
x_ , y_ = gradient_descent(-7,-5,total_iteration)
print('The Minima Point after {} many Iteration is:\n({},{})'.
↪format(total_iteration,x_,y_))
```

```
The Minima Point after 3000 many Iteration is:
(0.031151356204668527,-0.012073605080919696)
```

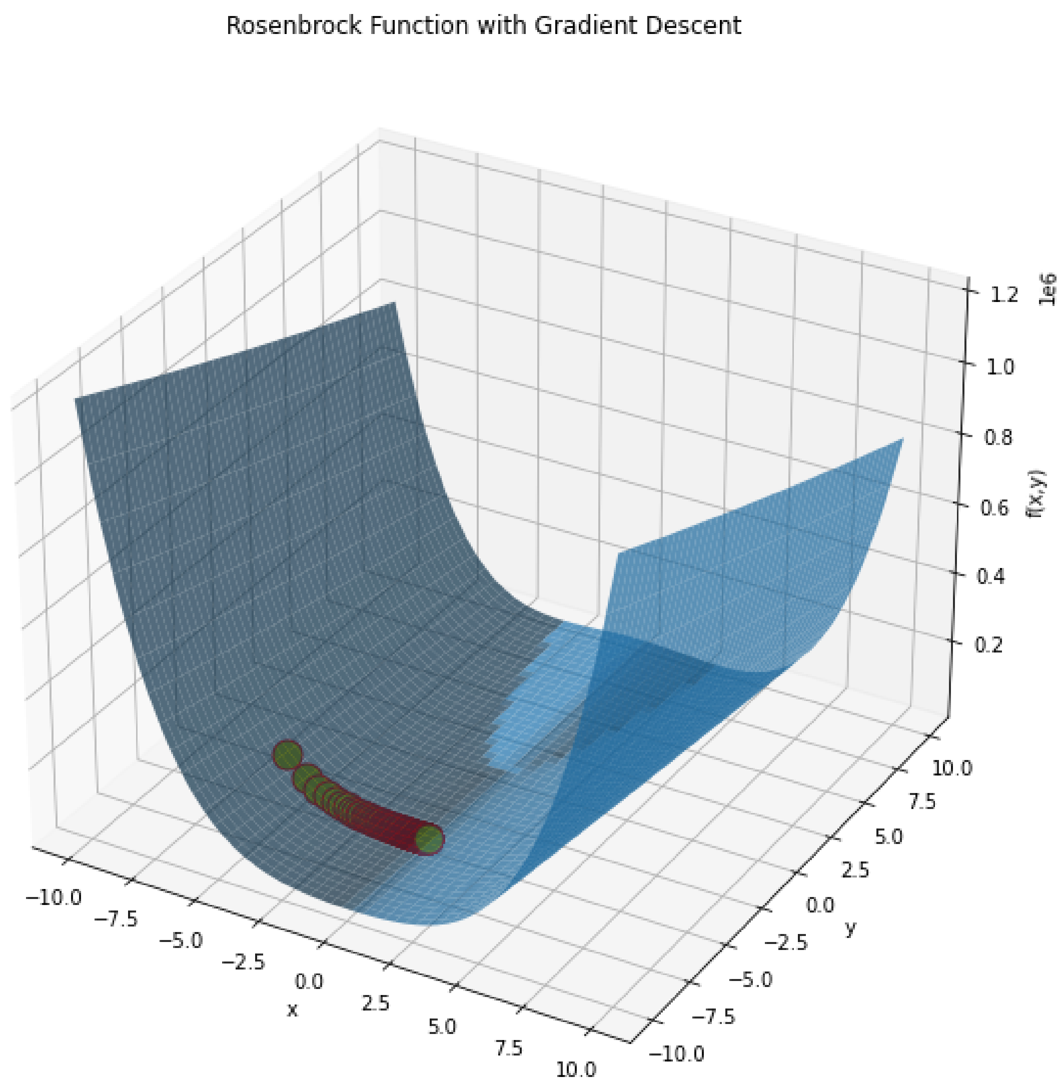
Visualize the trajectory on the same 3D plot. This trajectory should ideally lead to the function minimum, starting off with (x = 10, y = 10) for example.

[12]:
```python
#Preparing x and y array for plotting
X_arr, Y_arr = np.meshgrid(np.array(x_arr), np.array(y_arr))
fxy_ = np.array(rosenbrock_func(X_arr,Y_arr))
```

[13]:
```python
fig = plt.figure(figsize=(10,10))
ax = fig.add_subplot(111, projection='3d')
#Plotting the Original Rosenbrock Function
ax.plot_surface(X, Y, fxy,alpha=0.7)

#Plotting each intermediate x and y value to show Gradient Descent Convergence␣
↪to Minima
for i in range(len(X_arr[0])):
    ax.plot(X_arr[0][i], Y_arr[0][i], fxy_[0][i] , markerfacecolor='y',␣
↪markeredgecolor='r', marker='o', markersize=15)

ax.set_xlabel('x')
ax.set_ylabel('y')
ax.set_zlabel('f(x,y)')
ax.set_title('Rosenbrock Function with Gradient Descent')
plt.show()
```



Rosenbrock Function with Gradient Descent
1e6
1.2
1.0
0.8
0.6
0.4
0.2
f(x,y)
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
x
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
y