Lab 9

January 21, 2022

Machine Learning Lab

Lab 09

0.0.1 Importing Packages

[295]:
```
import pandas as pd                             #Importing Pandas
import numpy as np                              #Importing Numpy
import matplotlib.pyplot as plt                 #Importing Matplotlib
from sklearn.datasets import make_moons         #Importing Moon Dataset
```

0.0.2 Exercise 1: Implement Decision Tree

Read and split data into three parts train,validation and test (70%, 15% and 15% respectively).

Reading the Dataset using Pandas

[296]:
```
#Dataset Path
iris_dataset_path = 'Datasets/Iris/iris.data'
```

[297]:
```
#Reading the dataset using pandas CSV function
iris = pd.read_csv(iris_dataset_path, header=None,␣
↪names=['sepal_length','sepal_width','petal_length','petal_width','class'])

#Storing the header values of the dataset for further use
iris_headers =␣
↪['sepal_length','sepal_width','petal_length','petal_width','class']

#Displaying initial Dataset values
iris.head()
```

[297]:

	sepal_length	sepal_width	petal_length	petal_width	class
0	5.1	3.5	1.4	0.2	Iris-setosa
1	4.9	3.0	1.4	0.2	Iris-setosa
2	4.7	3.2	1.3	0.2	Iris-setosa
3	4.6	3.1	1.5	0.2	Iris-setosa
4	5.0	3.6	1.4	0.2	Iris-setosa

Function to split the Dataset into Train, Validation and Test based on Respective Sizes

[298]:
```python
def split_dataset(dataset, train_size = 0.7, validation_size = 0.15, test_size␣
↪= 0.15):
    total_rows = len(dataset)
    dataset = dataset.sample(frac = 1).reset_index(drop=True)
    train_set = dataset.sample(frac = train_size)
    dataset = dataset.drop(train_set.index)

    validation_set = dataset[:int(validation_size * total_rows)]
    dataset = dataset.drop(validation_set.index)
    test_set = dataset.copy()

    train_set = train_set.to_numpy()
    validation_set = validation_set.to_numpy()
    test_set = test_set.to_numpy()

    return train_set, validation_set, test_set
```

Splitting the Iris Dataset using given Set Sizes

[299]:
```python
iris_train, iris_validation, iris_test = split_dataset(iris)
```

Desigining different parts of Decision Tree

Class to represent the condition inside each node in a decision tree

[300]:
```python
class Question:
    #Initializer/Constructor Function
    def __init__(self, column_name, column, value):
        #Feature Name
        self.column_name = column_name

        #Feature Column Index
        self.column = column

        #Feature Value
        self.feature_value = value

        #Is the feature value numeric or Categorical
        self.feature_numeric = isinstance(self.feature_value, int) or␣
↪isinstance(self.feature_value, float)

    #Function to evaluate an instance on the current condition
    def evaluate(self, row):
        #Extracting the value at column of given instance
        value = row[self.column]

        #If the value is numerical than check for less than equal to condition
        if isinstance(value, int) or isinstance(value, float):
```

```
            return value <= self.feature_value

        #If the value is categorical than check for equals condition
        return value == self.feature_value

    #Function to split the dataset into left dataset and right dataset based on␣
↪the current condition
    def apply_split(self, dataset):
        #List to contain the left and right dataset
        left_partition, right_partition = [], []

        #Iterating through all instances and evaluating the condition
        for row in dataset:
            #If true, then appending that instance into left dataset
            if self.evaluate(row):
                left_partition.append(row)
            #If False, then appending that instance into right dataset
            else:
                right_partition.append(row)

        #Returning the Left and Right Datasets
        return np.array(left_partition), np.array(right_partition)

    #Function to print the condition in a formatted way
    def to_string(self):
        #If feature is numeric than print <= else ==
        check_condition = '<=' if self.feature_numeric else '=='

        #Returning the formatted condition String
        return 'check for {} {} {}'.format(self.column_name, check_condition,␣
↪self.feature_value)
```

Function to Return the Dominant/Most occuring Class in the Dataset

[301]:
```
def dominant_class(dataset, target_column):
    #Extracting only the class column from the dataset
    target_values = dataset[:,target_column]

    #Extracting the unique classes and their counts
    unique_values, counts = np.unique(target_values, return_counts=True)
    dominant_class = unique_values[np.argmax(counts)]

    #Returning the unique classes and their counts
    return (target_values == dominant_class).sum(), dominant_class
```

Function to calculate the Misclassification Rate on a Node/Condition

```
[302]: def MCR(dataset, target_column, question):
    #Splitting the dataset into left and right dataset based on the condition␣
 ↪of the node
    left_data, right_data = question.apply_split(dataset)

    #If any of the left/right dataset is empty then return the highest MCR␣
 ↪which is 1
    if len(left_data) == 0 or len(right_data) == 0:
        return 1

    #Extracting the dominant class and its count from the left dataset
    left_dominant_count, left_dominant_class = dominant_class(left_data,␣
 ↪target_column)

    #Extracting the dominant class and its count from the Right dataset
    right_dominant_count, right_dominant_class = dominant_class(right_data,␣
 ↪target_column)

    #Returning the MCR = ((|LD| - |LD_dominant_class_count|) + (|RD| -␣
 ↪|RD_dominant_class_count|)) / |D|
    return ((len(left_data) - left_dominant_count) + (len(right_data) -␣
 ↪right_dominant_count)) / len(dataset)
```

Function to calculate the Cross entropy of the Dataset

```
[303]: def cross_entropy(dataset, target_column):
    #Initializing the entropy to 0
    entropy = 0

    #Extracting the unique classes and its count from the class column
    unique_values, count= np.unique(dataset[:,target_column],␣
 ↪return_counts=True)

    #Iterating over all unique classes
    for index, unique_val in enumerate(unique_values):
        #Calculating Pt = |unique_class| / |D|
        pt = count[index] / len(dataset)

        #Shanon Entropy = Summation(Pt * log(Pt))
        entropy += (pt * np.log2(pt))

    #Returning the negative of entropy
    return -1 * entropy
```

Function to calculate the Information Gain

```
[304]: def information_gain(dataset, left_dataset, right_dataset, target_column):
    #Calculating the Cross Entropy of the whole Dataset
```

```
    h_d = cross_entropy(dataset, target_column)

    #Calculating the Weighted entropy of the Left Dataset
    left_hd = (len(left_dataset)/len(dataset)) * cross_entropy(left_dataset,␣
↪target_column)

    #Calculating the Weighted entropy of the Right Dataset
    right_hd = (len(right_dataset)/len(dataset)) * cross_entropy(right_dataset,␣
↪target_column)

    #Returning the Information Gain = H(D) - (W_H(LD) + W_H(RD))
    return h_d - (left_hd + right_hd)
```

Function to calculate the average values between two consecutive numbers in the column

[305]:
```
def numerical_splitting_average(values):
    #Initializing the list
    V = []

    #Iterating through each values in pair
    for i in range(len(values) - 1):
        #Calculating the average between two consecutive numbers
        V.append((values[i] + values[i+1])/2)

    #Returning the calculated List V
    return V
```

Function to find the Best split based on MCR value

[306]:
```
def best_fit_mcr(dataset, headers, target_column):
    #Initializing the best MCR to 1 and best Condition to None
    best_mcr = 1
    best_question = None

    #Iterating through all feature columns
    for i in range(len(dataset[0])):

        #We have to skip the class column
        if i == target_column:
            continue

        #Extracting the unique values from the selected column
        unique_values = np.unique(dataset[:,i])

        #If the column values are numeric than calculating the list containing␣
↪the average values between two consecutive numbers
```

```
        if isinstance(unique_values[0], int) or isinstance(unique_values[0],␣
↪float):
            unique_values = numerical_splitting_average(unique_values)

        #Iterating through all unique values
        for unique_val in unique_values:

            #Creating a condition with selected feature and unique value
            question = Question(headers[i], i, unique_val)

            #Calculating the MCR value for that condition
            mcr = MCR(dataset, target_column, question)

            #If we find a better MCR then swapping it with the previous best
            if mcr <= best_mcr:
                best_mcr = mcr
                best_question = question

    #Returning the Best MCR and Best Condition
    return best_mcr, best_question
```

Function to find the Best split based on Information Gain value

[307]:
```
def best_fit_gain(dataset, headers, target_column):
    #Initializing the best Information Gain to 0 and best Condition to None
    best_gain = 0
    best_question = None

    #Iterating through all feature columns
    for i in range(len(dataset[0])):
        #We have to skip the class column
        if i == target_column:
            continue

        #Extracting the unique values from the selected column
        unique_values = np.unique(dataset[:,i])

        #If the column values are numeric than calculating the list containing␣
↪the average values between two consecutive numbers
        if isinstance(unique_values[0], int) or isinstance(unique_values[0],␣
↪float):
            unique_values = numerical_splitting_average(unique_values)

        #Iterating through all unique values
        for unique_val in unique_values:

            #Creating a condition with selected feature and unique value
```

```
            question = Question(headers[i], i, unique_val)

            #Calculating the Information Gain value for that condition
            left_data , right_data = question.apply_split(dataset)
            gain = information_gain(dataset, left_data, right_data,␣
↪target_column)

            #If we find a better Information Gain then swapping it with the␣
↪previous best
            if gain >= best_gain:
                best_gain = gain
                best_question = question

    #Returning the Best Information Gain and Best Condition
    return best_gain, best_question
```

Class to Represent a Node in the Tree

[308]:
```
class Node:
    #Initializer/Constructor Function
    def __init__(self, question, left_node, right_node, is_leaf, leaf_class):
        #Condition of the Node
        self.question = question

        #Left Child Node
        self.left_node = left_node

        #Right Child Node
        self.right_node = right_node

        #If the Node is a Leaf Node
        self.is_leaf = is_leaf

        #If it is Leaf Node then its output value
        self.leaf_class = leaf_class
```

Function to Learn a Decision Tree from the Dataset

[361]:
```
def learn_decision_tree(dataset, headers, target_column, evaluation_metric =␣
↪'mcr', max_depth = 2):

    #Calculating the Best Condition based on Either MCR or Information Gain
    eval_val , question = best_fit_mcr(dataset, headers, target_column) if␣
↪evaluation_metric == 'mcr' else best_fit_gain(dataset, headers,␣
↪target_column)

    #At each decision step (or split) presenting the probability of each class␣
↪using Bar Plot
```

```
    unique_values, counts = np.unique(dataset[:,target_column],␣
↪return_counts=True)
    plt.bar(unique_values, counts)
    plt.title('Probabilities of each class at Level: {}'.format(max_depth))
    plt.xlabel('Classes')
    plt.ylabel('Count')
    plt.show()

    if max_depth == 0:
        #Extracting the Dominant Class
        unique_values, counts = np.unique(dataset[:,target_column],␣
↪return_counts=True)
        dominant_class = unique_values[np.argmax(counts)]

        #Returning the Node
        return Node(question, None, None, True, dominant_class)

    #If MCR is 1 or Information Gain is 0 or the Question is Null, then it is␣
↪Leaf node
    if (evaluation_metric == 'mcr' and eval_val == 1) or (evaluation_metric ==␣
↪'gain' and eval_val == 0) or question == None:
        #Extracting the Dominant Class
        unique_values, counts = np.unique(dataset[:,target_column],␣
↪return_counts=True)
        dominant_class = unique_values[np.argmax(counts)]

        #Returning the Leaf Node
        return Node(question, None, None, True, dominant_class)

    #Split the Data into Left and Right Dataset based on Best Condition
    left_data, right_data = question.apply_split(dataset)

    #Creating a Left Child Node using Recursive call
    left_node = learn_decision_tree(left_data, headers, target_column,␣
↪evaluation_metric, max_depth - 1)

    #Creating a Right Child Node using Recursive call
    right_node = learn_decision_tree(right_data, headers, target_column,␣
↪evaluation_metric, max_depth - 1)

    #Returning the Node which is not Leaf and contains Left and Right Nodes
    return Node(question, left_node, right_node, False, '')
```

Function to Print the Tree

[428]:
```
def print_tree(root, spaces = ''):
```

```
    #The the node is Leaf node, than printing its predicted class with spaces␣
↪indicating depth
    if root.is_leaf:
        print(spaces + 'Leaf Node: ' + str(root.leaf_class))
        return

    #Printing the Node Condition
    print(spaces + root.question.to_string())

    #Printing the Left Node if the Condition is True
    print(spaces + 'True: ')
    print_tree(root.left_node, spaces + '  ')

    #Printing the Right Node if the Condition is False
    print(spaces + 'False: ')
    print_tree(root.right_node, spaces + '  ')
```

Function to Predict the Class of the Instance Row using Decision Tree

[363]:
```
def predict_class(tree , row):
    #If the Node is leaf, then returning the leaf class
    if tree.is_leaf:
        return tree.leaf_class

    #If the Node condition is True, then predicting the class with the left Node
    if tree.question.evaluate(row):
        return predict_class(tree.left_node, row)
    #If the Node condition is False, then predicting the class with the Right␣
↪Node
    else:
        return predict_class(tree.right_node, row)
```

Function to Predict the Classes of the entire Dataset using Decision Tree

[364]:
```
def predict_classes(tree, dataset):
    #Initializing the empty list for classes
    predicted_classes = []

    #Iterating through all the rows in the dataset
    for row in dataset:
        #Appending the predicted class to the list
        predict_classes.append(predict_class(tree, row))

    #Returning the Predicted Class list
    return predicted_classes
```

Function the Calculate the Accuracy of the Decision Tree

[365]:
```python
def accuracy(tree, dataset, target_column):
    #Initializing the Correct prediction to 0
    correct = 0

    #Iterating through all instances
    for row in dataset:
        #Checking if the actual class is equal to the predicted class
        if row[target_column] == predict_class(tree, row):
            correct += 1

    #Returning the Accuracy = correct Result / Total Rows
    return correct/len(dataset)
```

Part A: Basic working with MCR

Defining an appropriate stopping criteria i.e. max depth

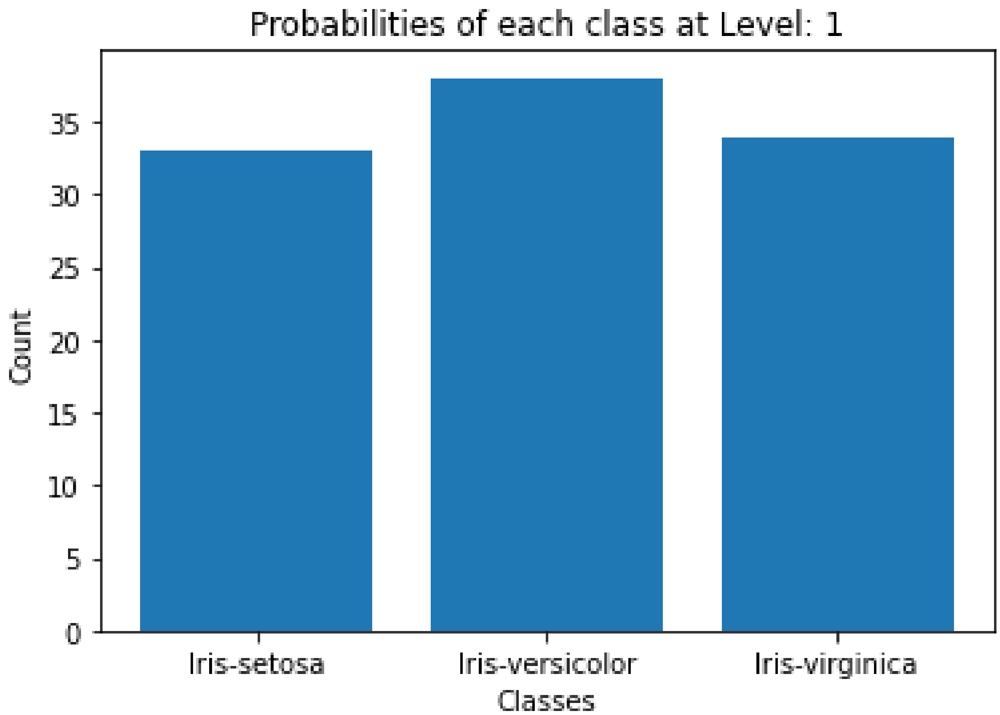
[366]:
```python
max_depth_list = [1,2,3,4,5]
```

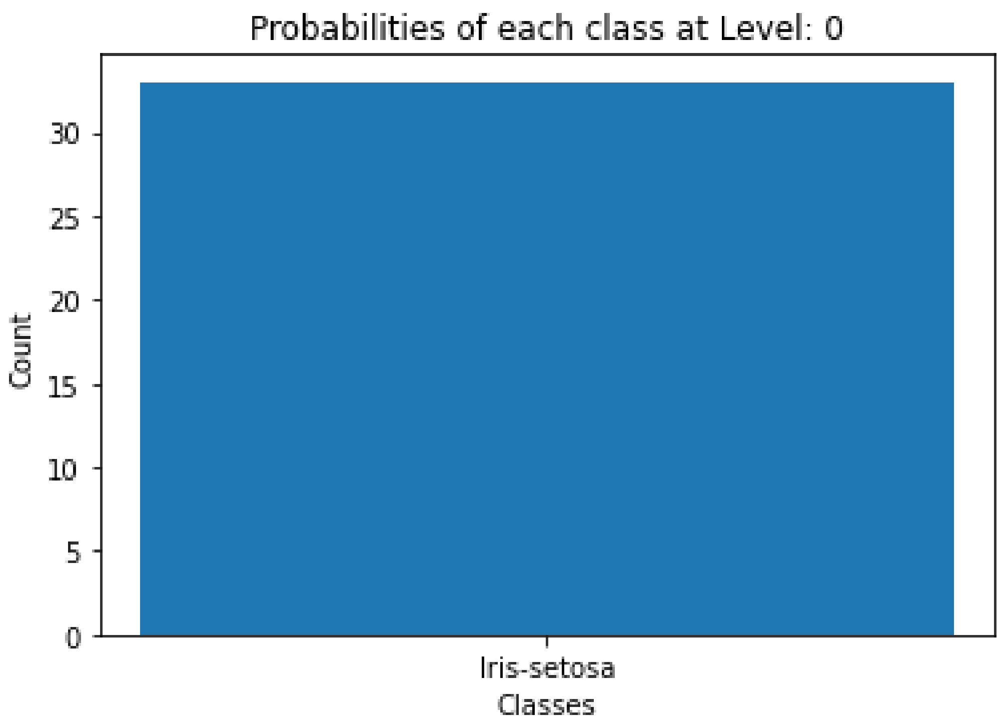
Applying Grid Search on Max-Depth, Printing the Resultant Tree and Displaying the Accuarcy and Cross Entropy Loss

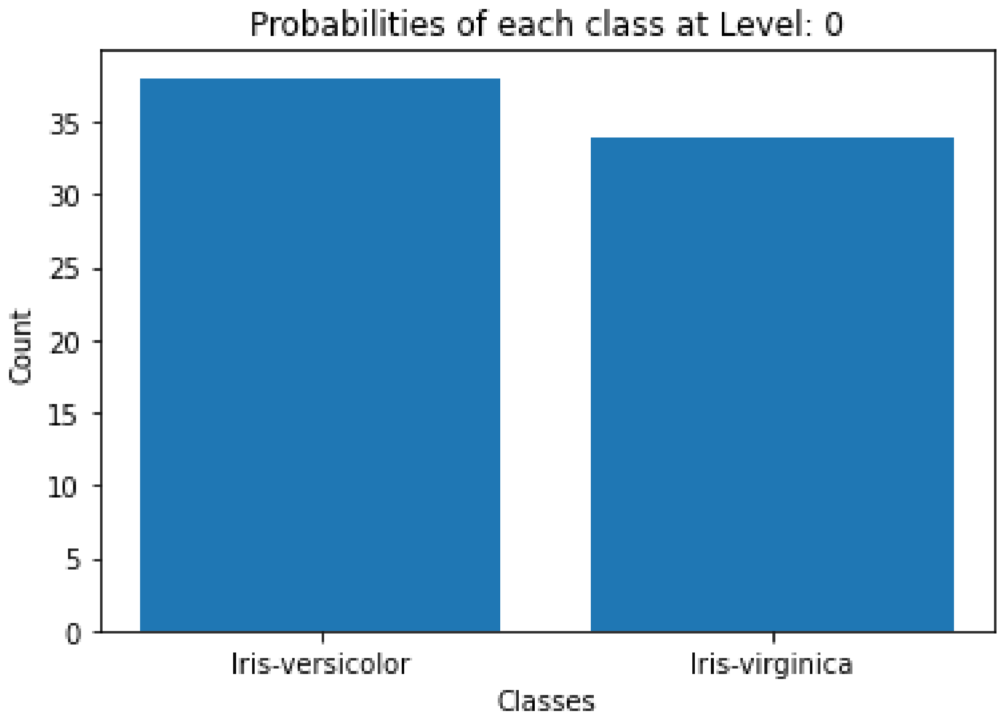
[ ]:
```python
for max_depth in max_depth_list:
    print('--------------------------------------------------------------\n')
    print('Learning Decision Tree with Misclassification Rate and Max Depth:␣
↪{}\n'.format(max_depth))
    root = learn_decision_tree(iris_train, iris_headers, 4, 'mcr' , max_depth)
    print('Printing the Tree using a breath first tree traversal:\n')
    print_tree(root)
    print('\nAccuracy: {}\n'.format(accuracy(root, iris_validation, 4)))
    print('Cross Entropy Loss: {}\n'.format(-1 * np.log(accuracy(root,␣
↪iris_validation, 4))))
```

```
--------------------------------------------------------------

Learning Decision Tree with Misclassification Rate and Max Depth: 1
```


Probabilities of each class at Level: 1
Count
0
5
10
15
20
25
30
35
Iris-setosa
Iris-versicolor
Iris-virginica
Classes


Probabilities of each class at Level: 0
Count
0
5
10
15
20
25
30
Iris-setosa
Classes




```
Printing the Tree using a breath first tree traversal:

check for petal_width <= 0.75
True:
  Leaf Node: Iris-setosa
False:
  Leaf Node: Iris-versicolor

Accuracy: 0.6363636363636364

Cross Entropy Loss: 0.45198512374305727

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Learning Decision Tree with Misclassification Rate and Max Depth: 2
```
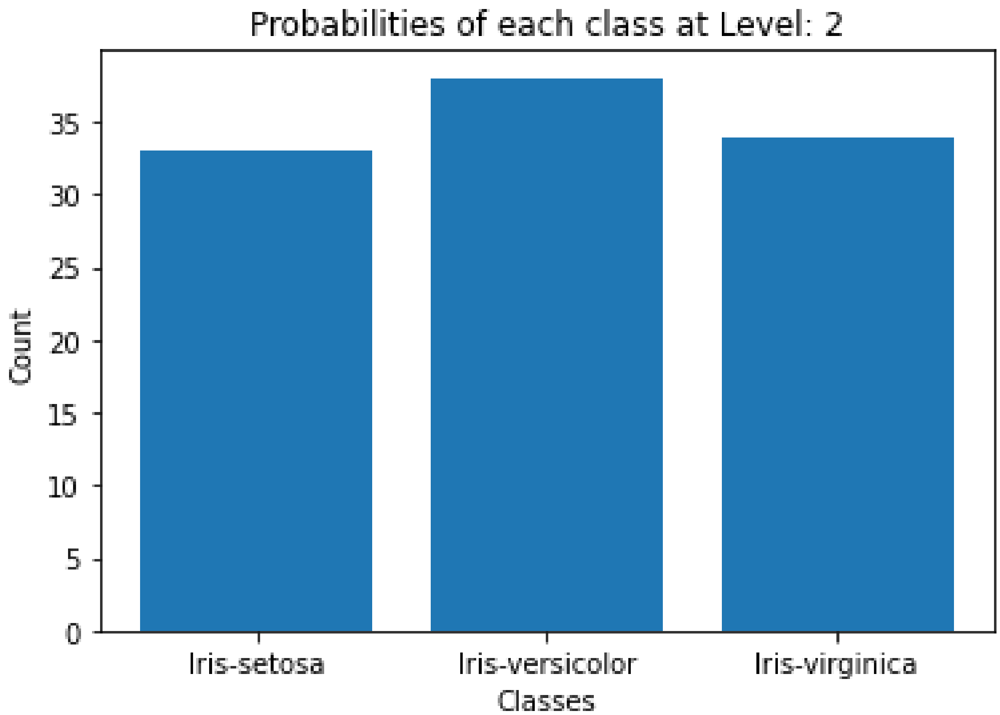
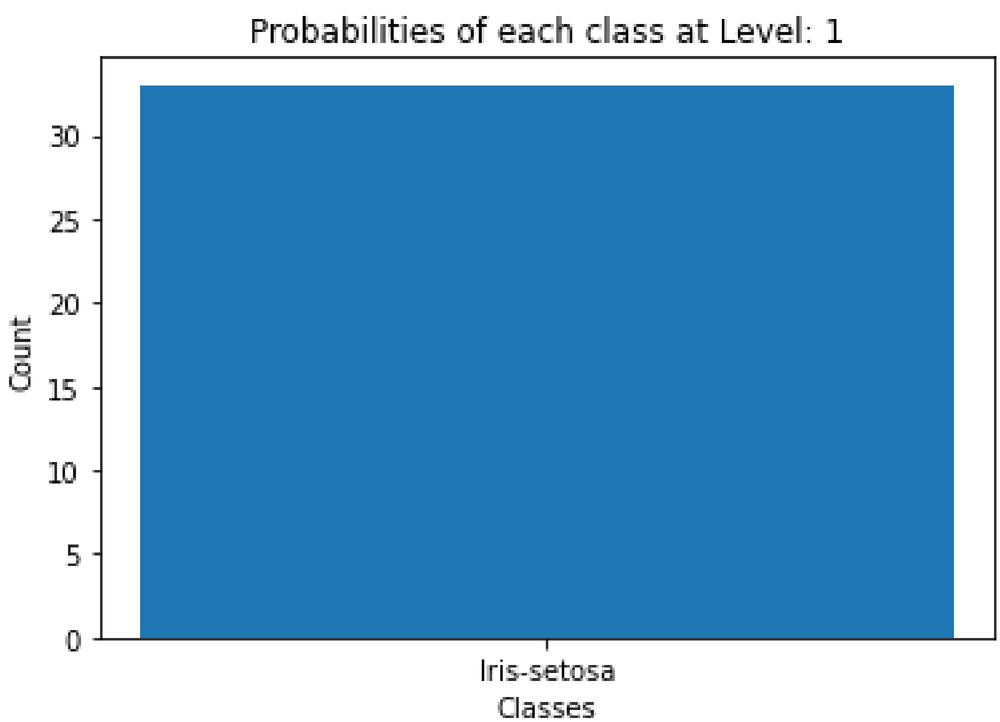
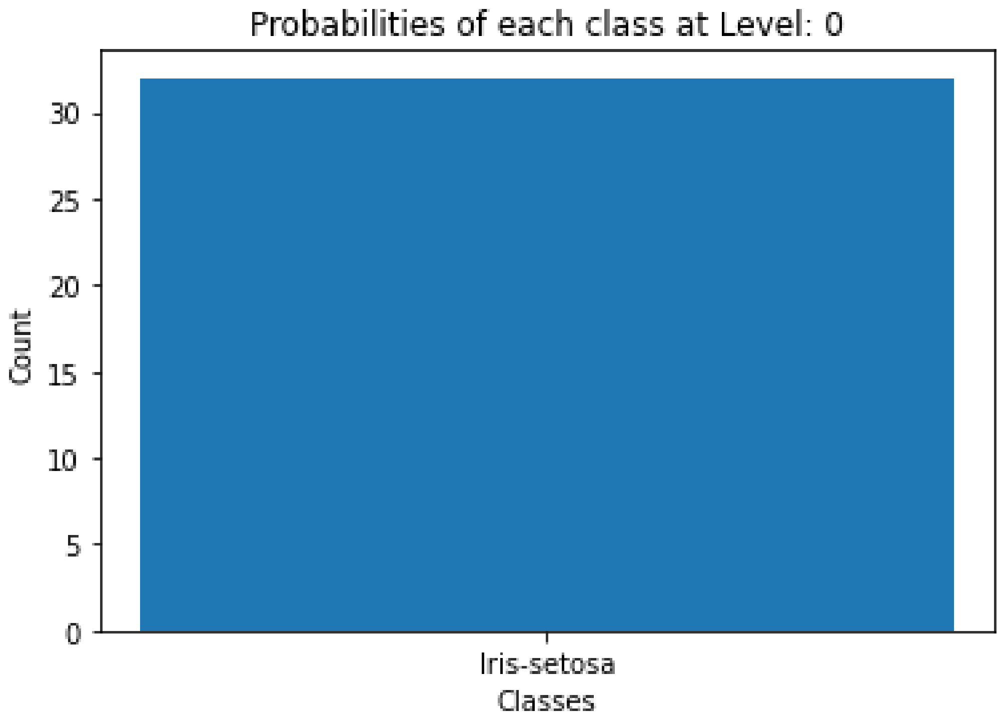
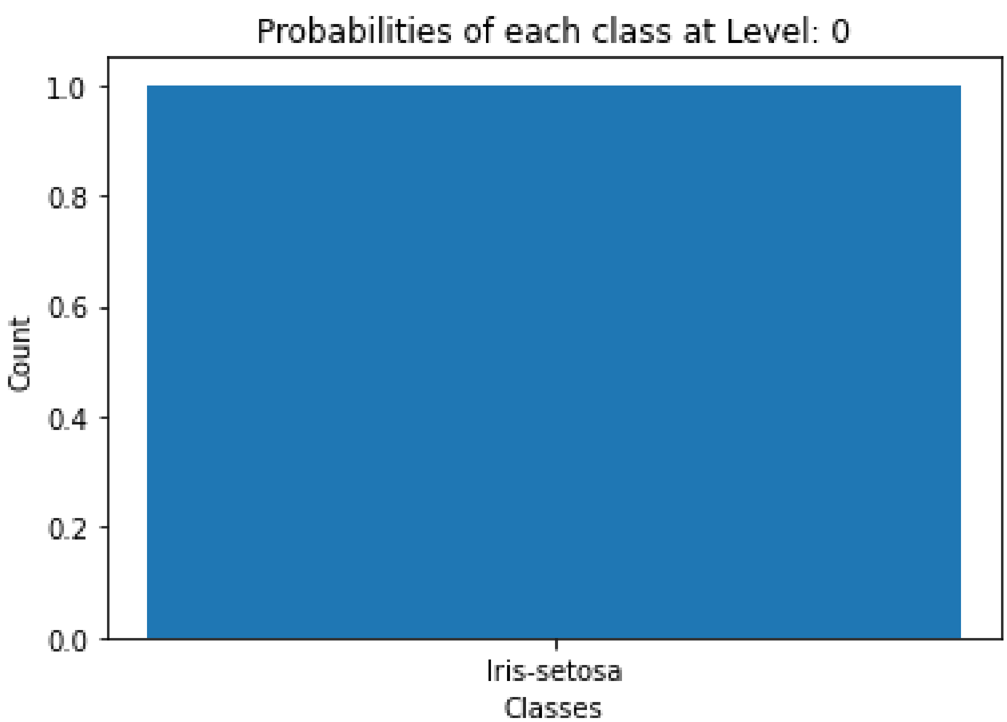



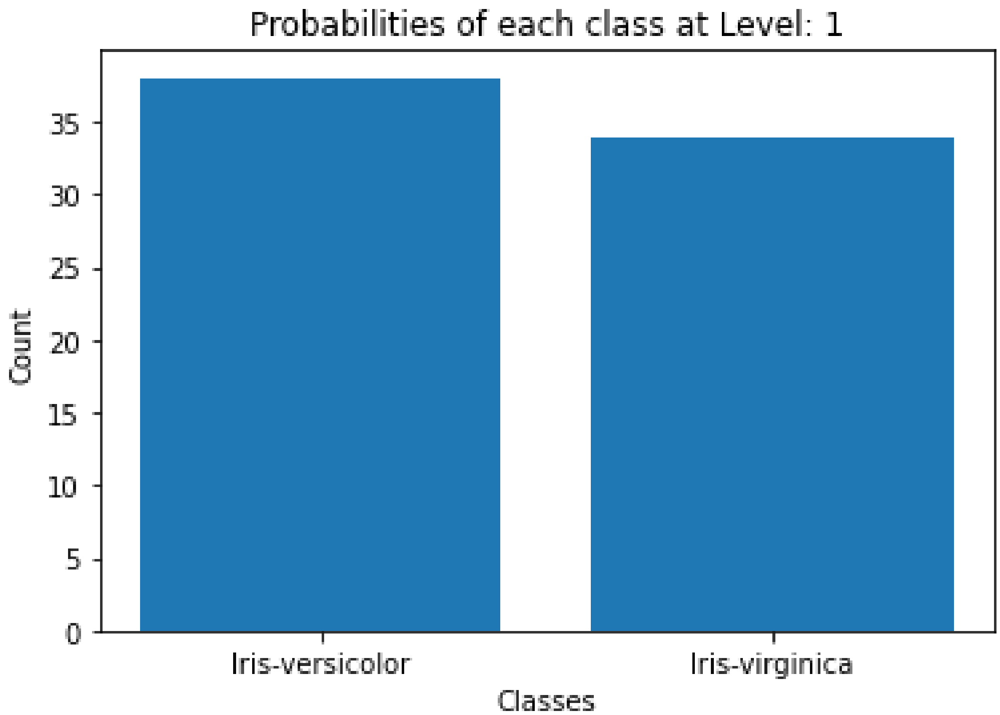
Probabilities of each class at Level: 2
Count
Classes
Iris-setosa
Iris-versicolor
Iris-virginica
0
5
10
15
20
25
30
35

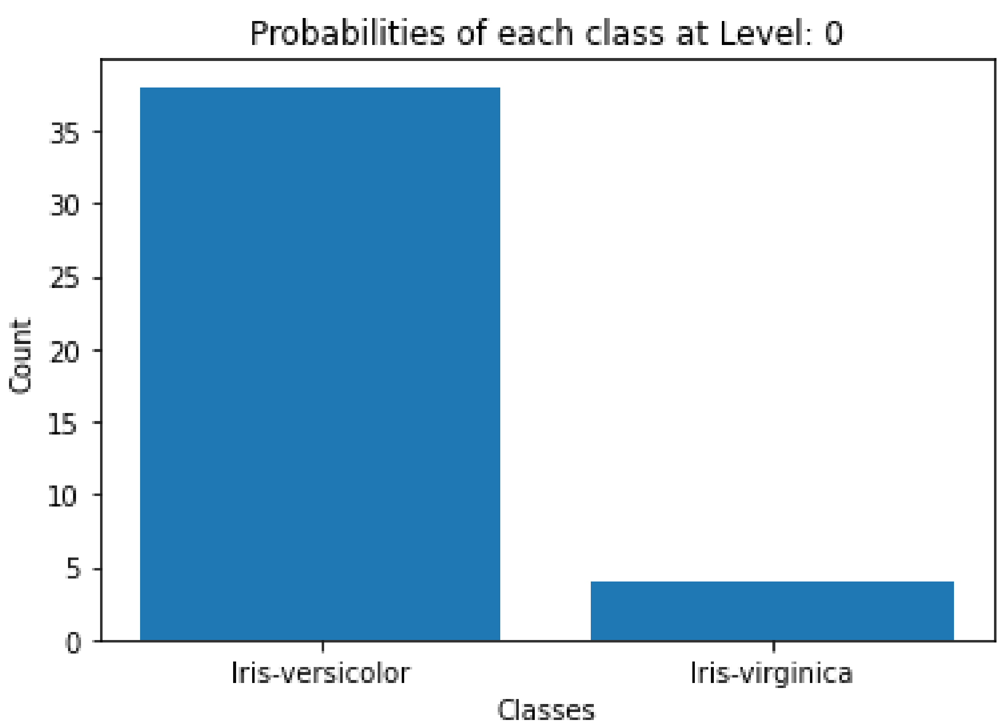


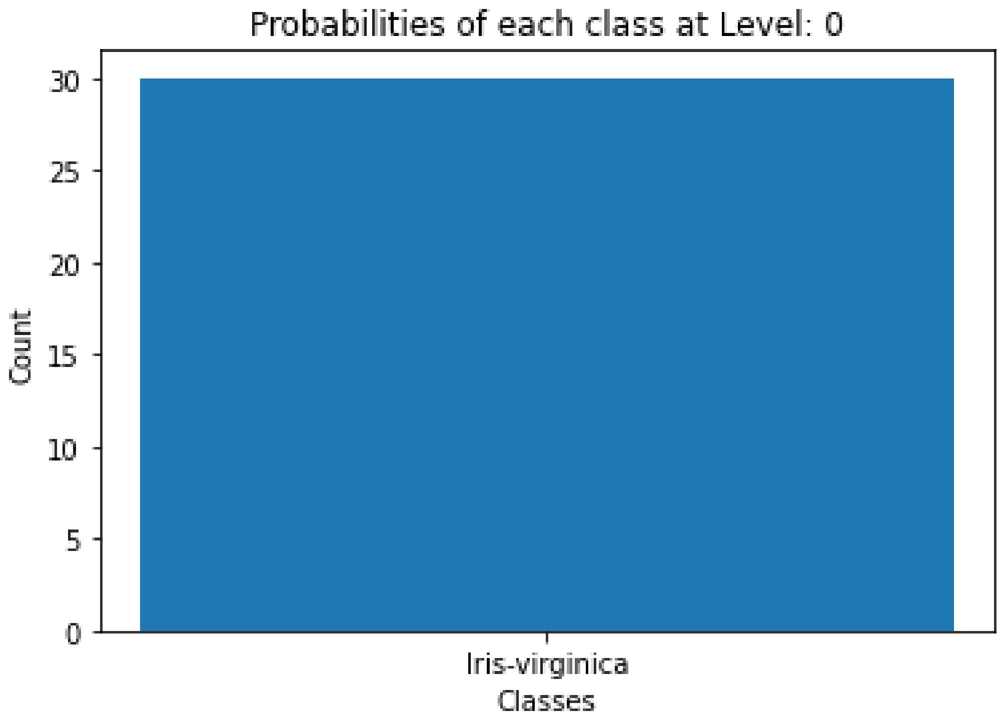
Probabilities of each class at Level: 1
Count
Classes
Iris-setosa
0
5
10
15
20
25
30


Probabilities of each class at Level: 0
Count
0
5
10
15
20
25
30
Iris-setosa
Classes


Probabilities of each class at Level: 0
Count
0.0
0.2
0.4
0.6
0.8
1.0
Iris-setosa
Classes


Probabilities of each class at Level: 1
Count
Iris-versicolor
Iris-virginica
Classes


Probabilities of each class at Level: 0
Count
Iris-versicolor
Iris-virginica
Classes




```
Printing the Tree using a breath first tree traversal:

check for petal_width <= 0.75
True:
  check for petal_width <= 0.45
  True:
    Leaf Node: Iris-setosa
  False:
    Leaf Node: Iris-setosa
False:
  check for petal_width <= 1.75
  True:
    Leaf Node: Iris-versicolor
  False:
    Leaf Node: Iris-virginica

Accuracy: 0.9545454545454546

Cross Entropy Loss: 0.04652001563489282

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Learning Decision Tree with Misclassification Rate and Max Depth: 3
```
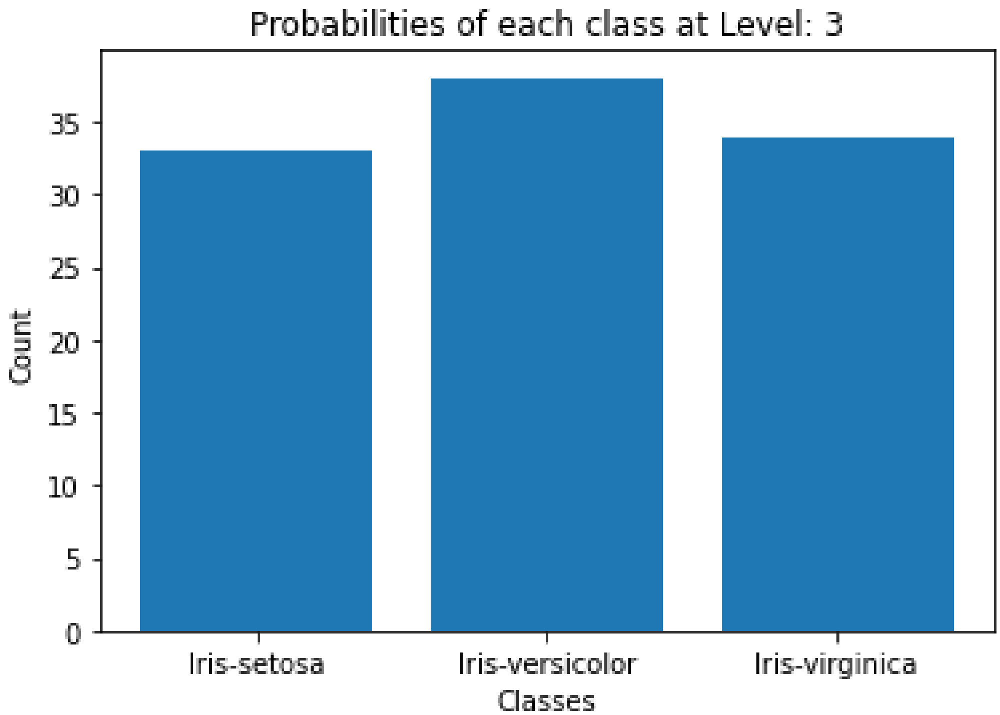
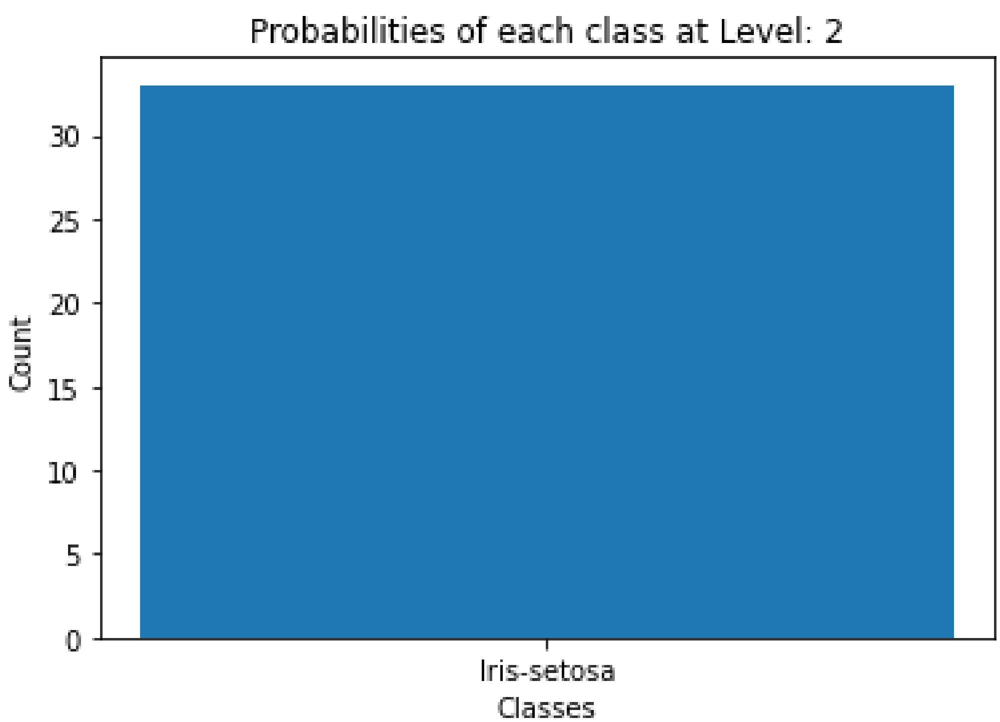
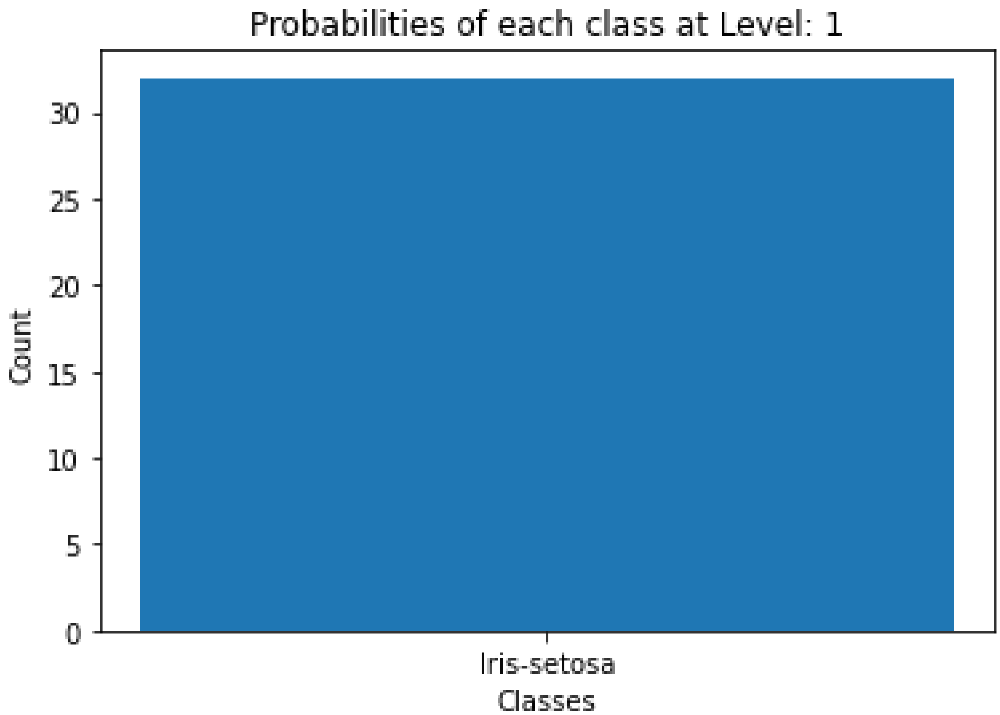
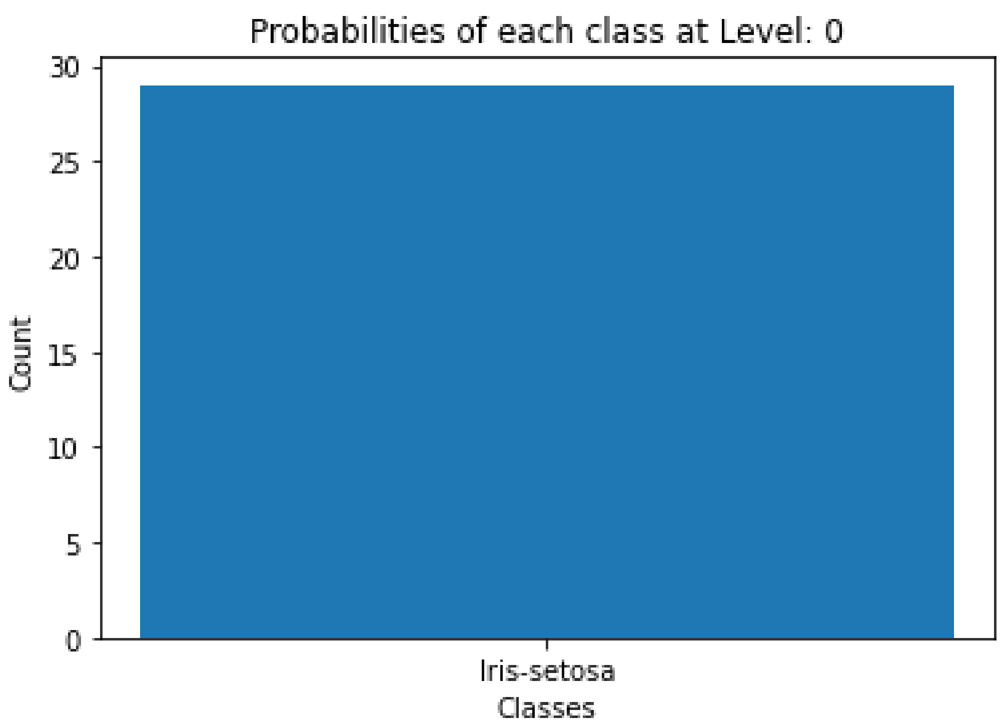
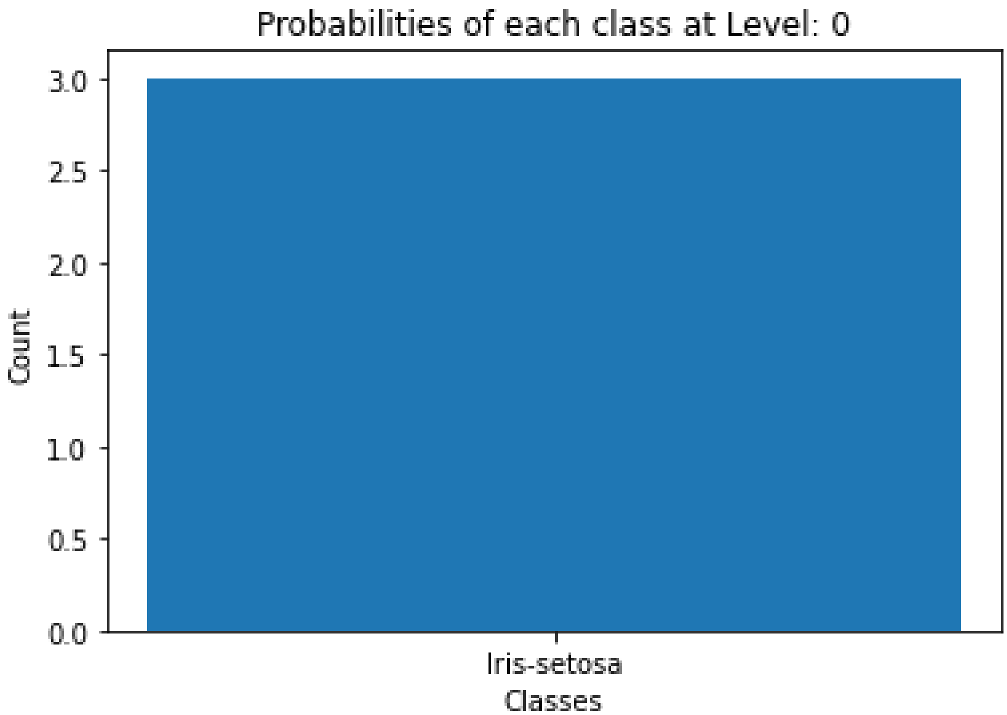
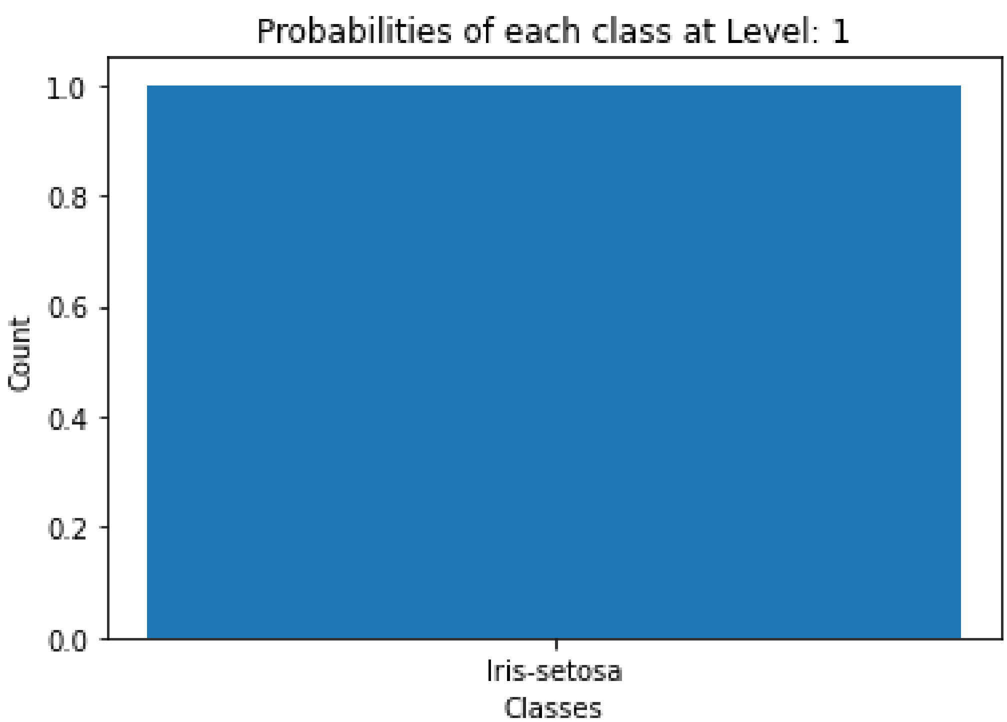


Probabilities of each class at Level: 3
Count
0
5
10
15
20
25
30
35
Iris-setosa
Iris-versicolor
Iris-virginica
Classes

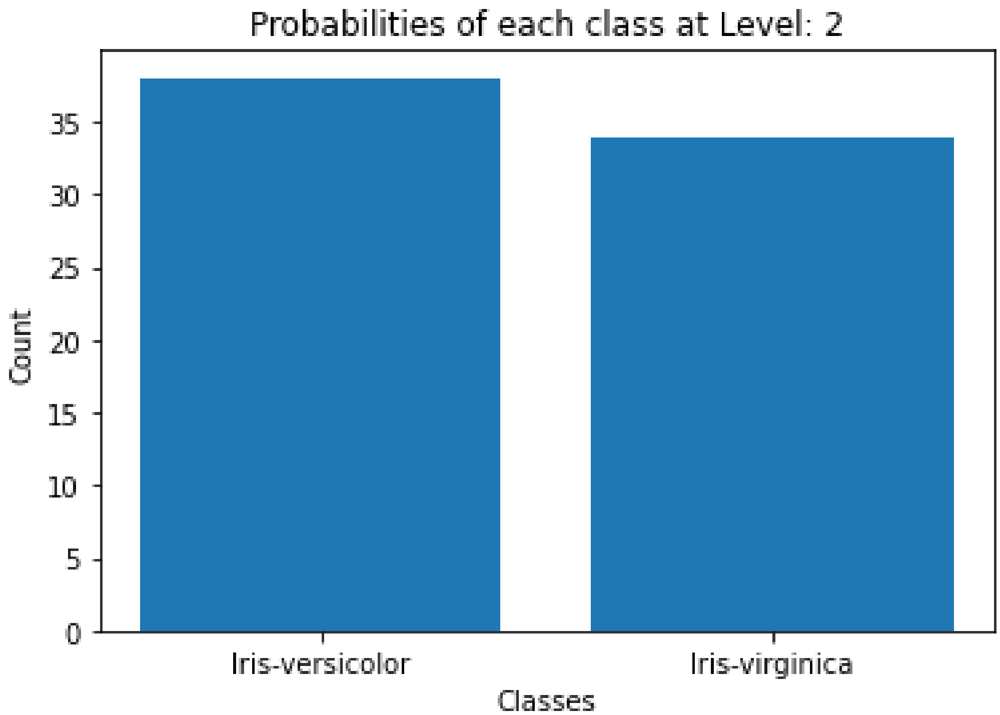

Probabilities of each class at Level: 2
Count
0
5
10
15
20
25
30
Iris-setosa
Classes

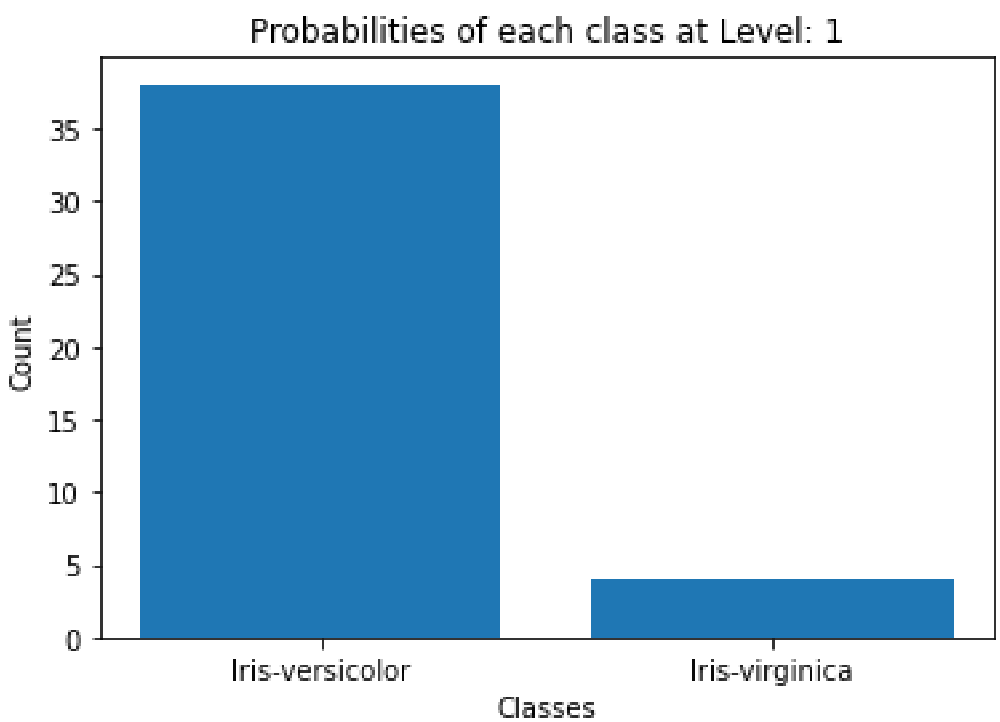

Probabilities of each class at Level: 1
Count
0
5
10
15
20
25
30
Iris-setosa
Classes

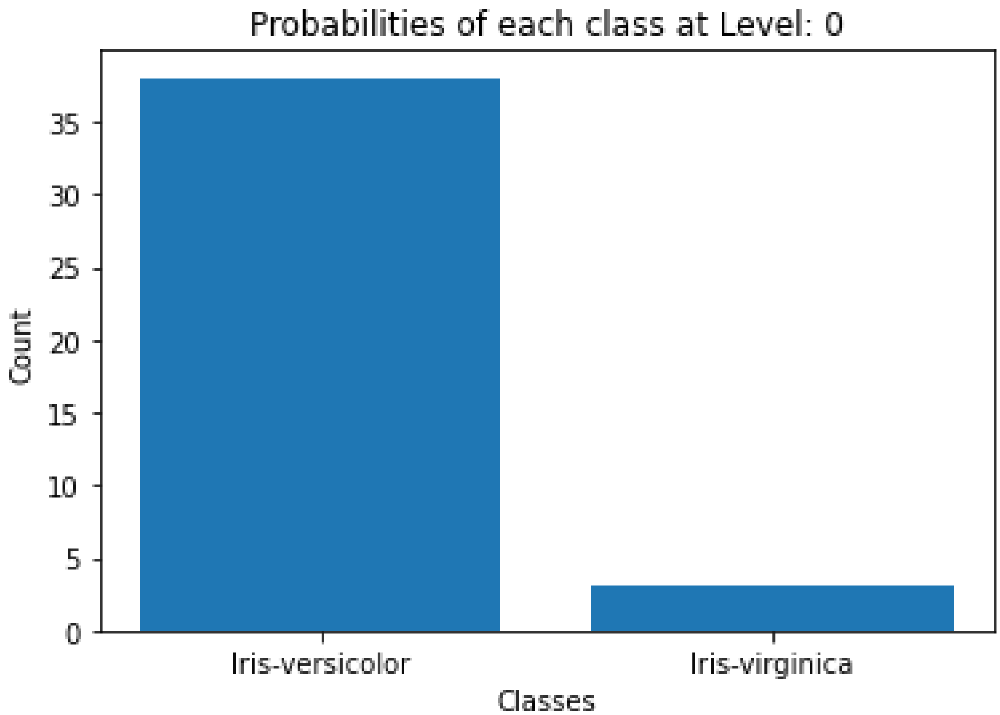

Probabilities of each class at Level: 0
Count
0
5
10
15
20
25
30
Iris-setosa
Classes

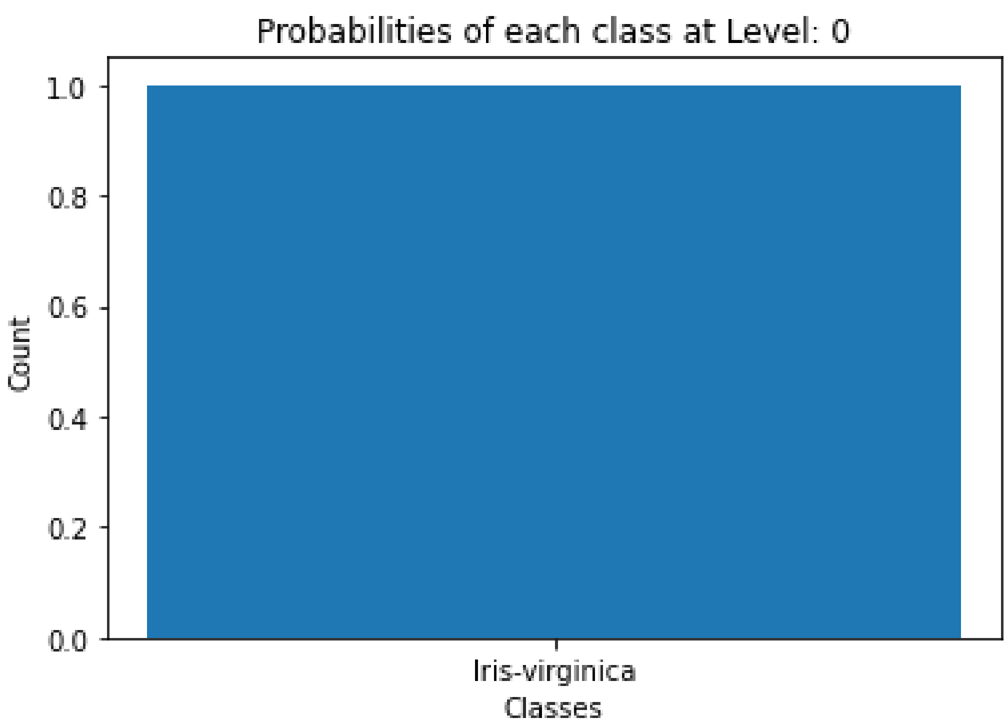

Probabilities of each class at Level: 0
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Count
Iris-setosa
Classes

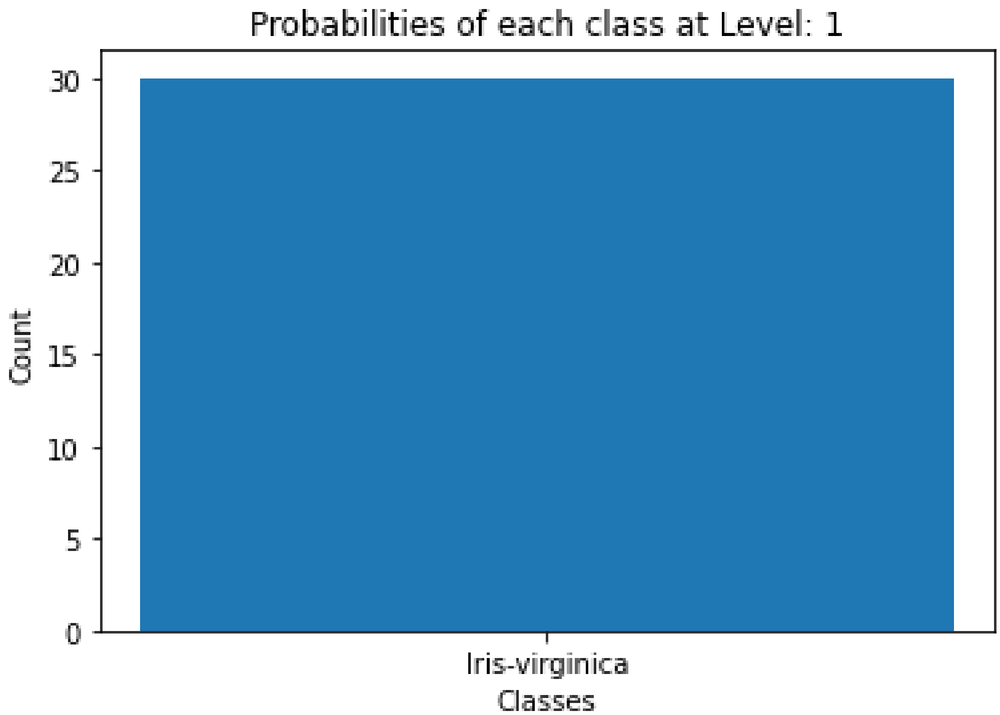

Probabilities of each class at Level: 1
1.0
0.8
0.6
0.4
0.2
0.0
Count
Iris-setosa
Classes

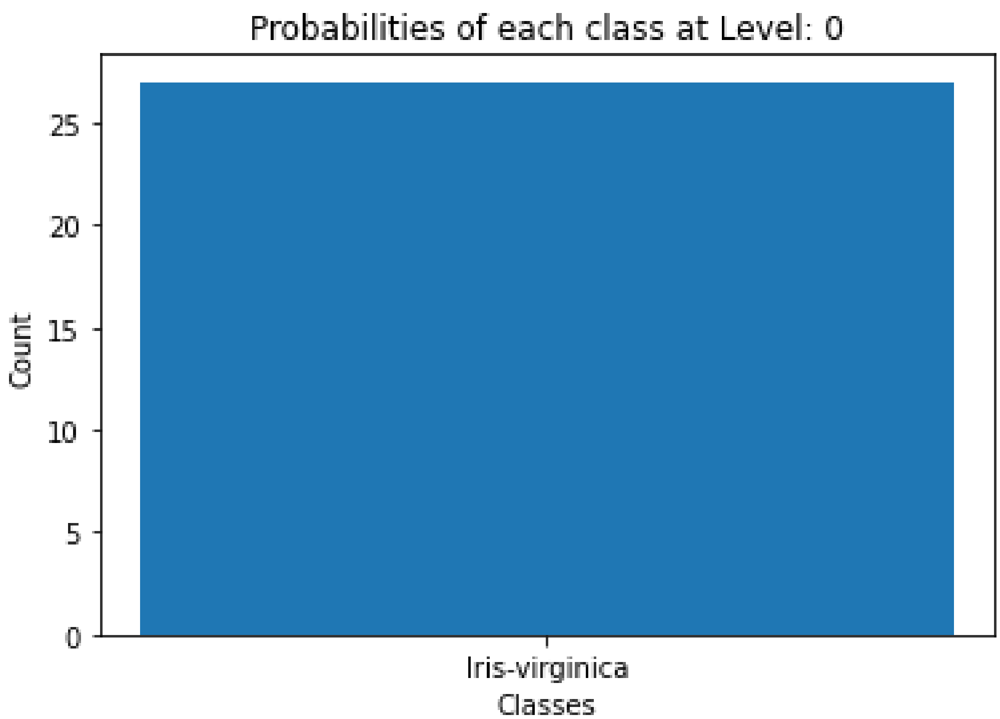

Probabilities of each class at Level: 2
Count
35
30
25
20
15
10
5
0
Iris-versicolor
Iris-virginica
Classes

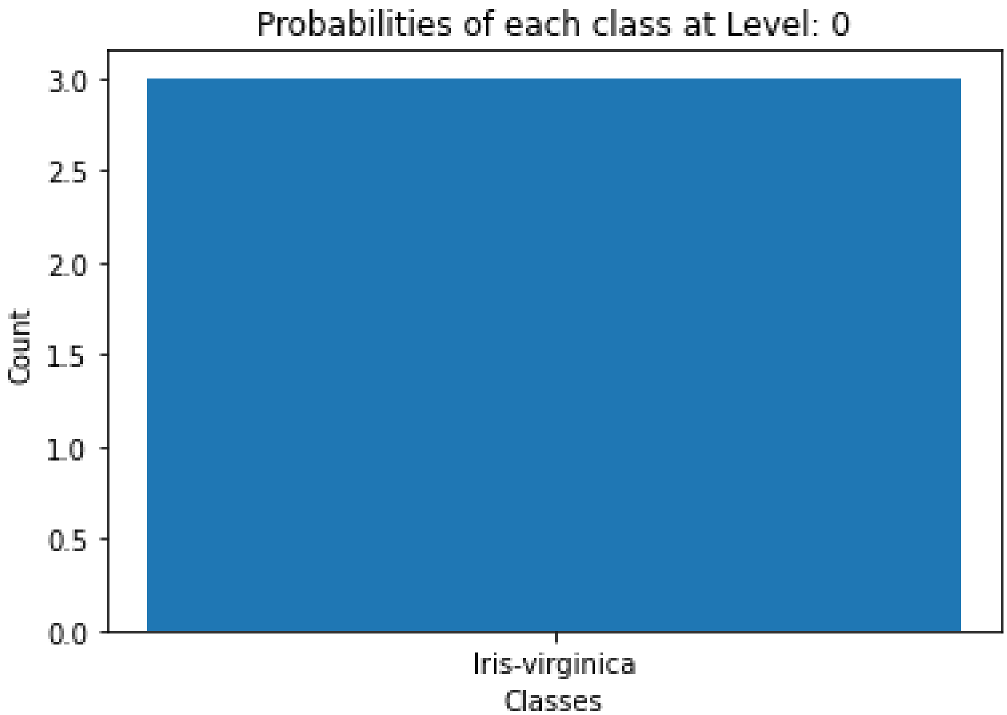

Probabilities of each class at Level: 1
Count
35
30
25
20
15
10
5
0
Iris-versicolor
Iris-virginica
Classes


Probabilities of each class at Level: 0
Count
0
5
10
15
20
25
30
35
Iris-versicolor
Iris-virginica
Classes


Probabilities of each class at Level: 0
Count
0.0
0.2
0.4
0.6
0.8
1.0
Iris-virginica
Classes

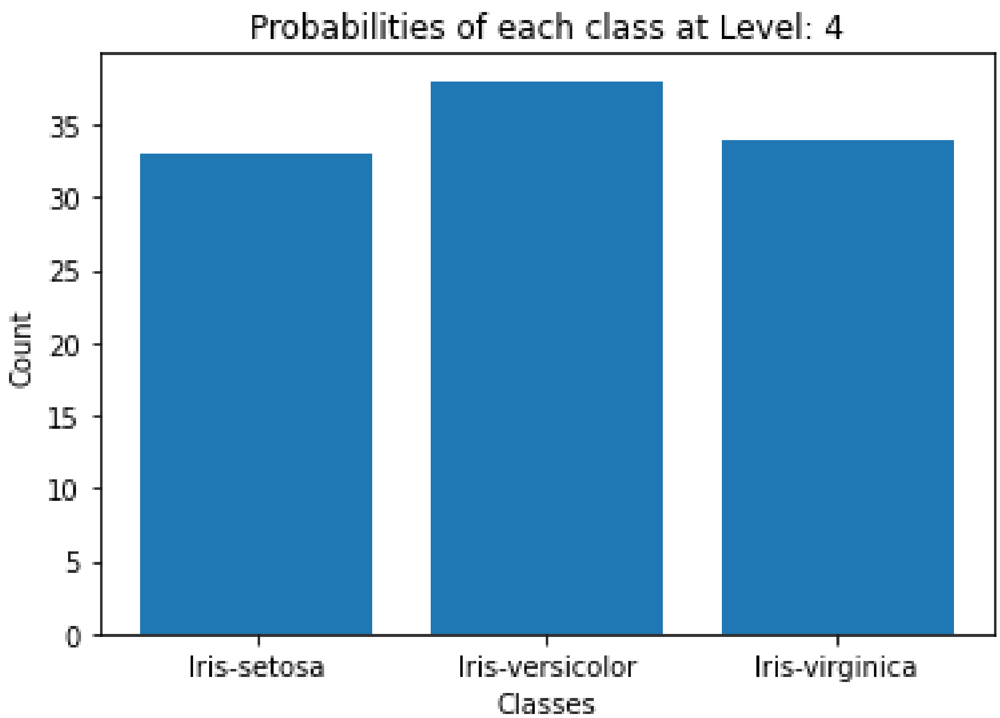

Probabilities of each class at Level: 1
Count
30
25
20
15
10
5
0
Iris-virginica
Classes

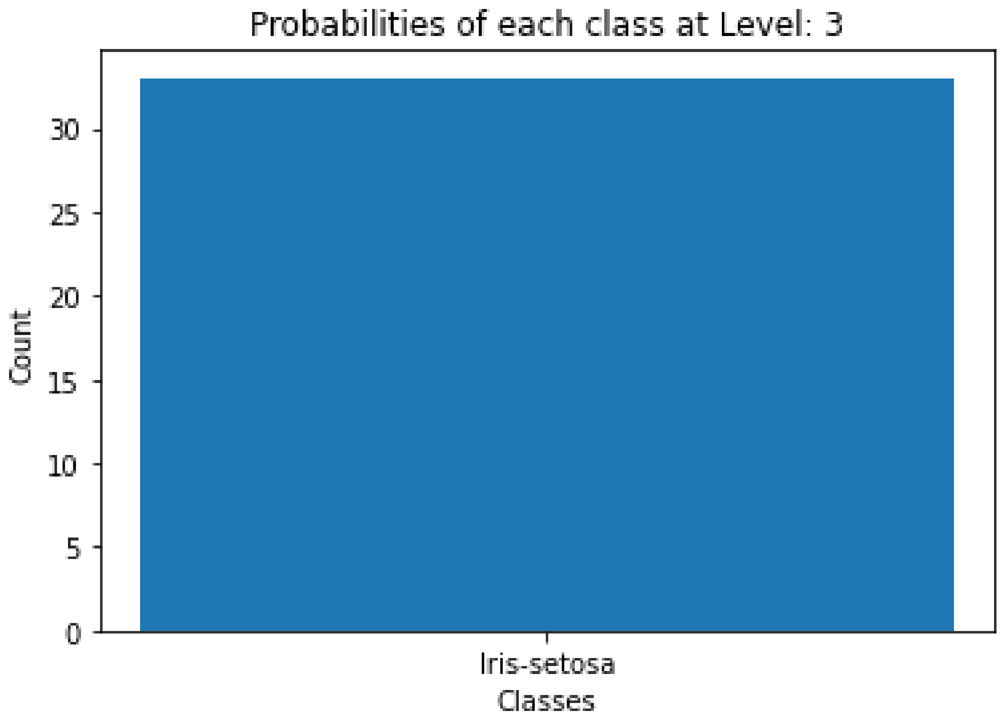

Probabilities of each class at Level: 0
Count
25
20
15
10
5
0
Iris-virginica
Classes

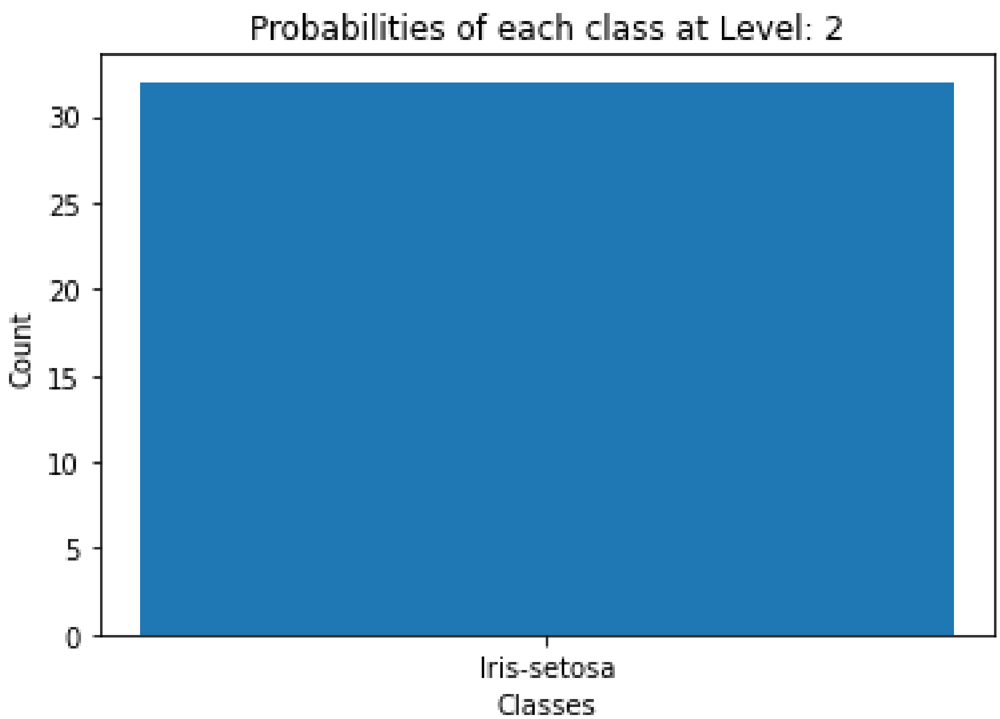


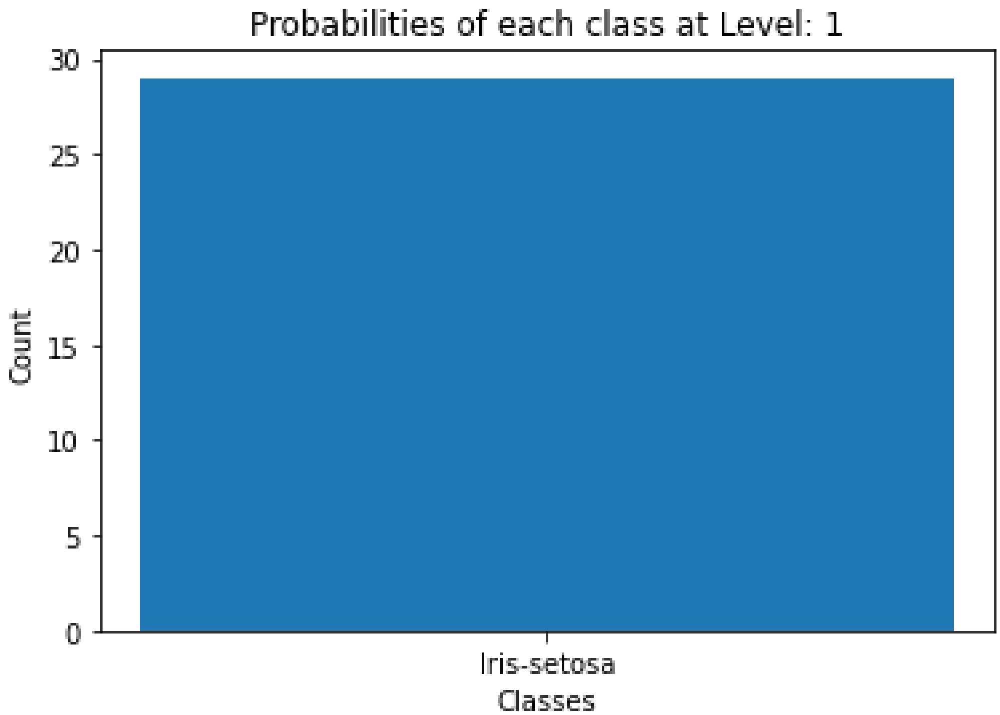

```
Printing the Tree using a breath first tree traversal:

check for petal_width <= 0.75
True:
  check for petal_width <= 0.45
  True:
    check for petal_width <= 0.35
    True:
      Leaf Node: Iris-setosa
    False:
      Leaf Node: Iris-setosa
  False:
    Leaf Node: Iris-setosa
False:
  check for petal_width <= 1.75
  True:
    check for petal_length <= 5.35
    True:
      Leaf Node: Iris-versicolor
    False:
      Leaf Node: Iris-virginica
  False:
    check for petal_width <= 2.45
    True:
```

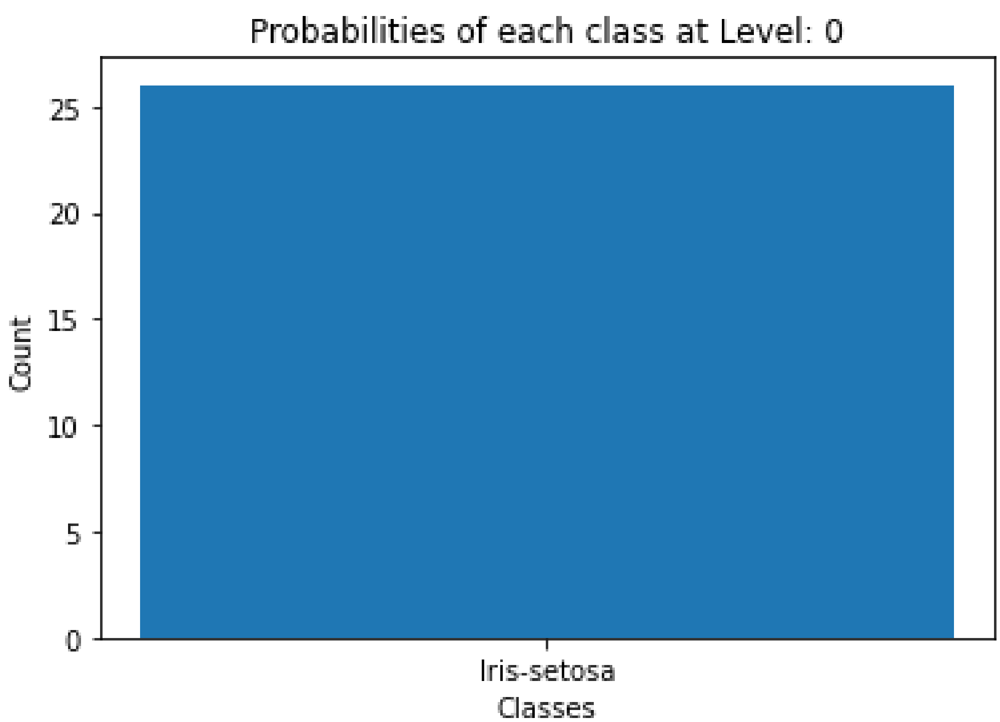
```
      Leaf Node: Iris-virginica
    False:
      Leaf Node: Iris-virginica

Accuracy: 1.0

Cross Entropy Loss: -0.0

--------------------------------------------------------------

Learning Decision Tree with Misclassification Rate and Max Depth: 4
```

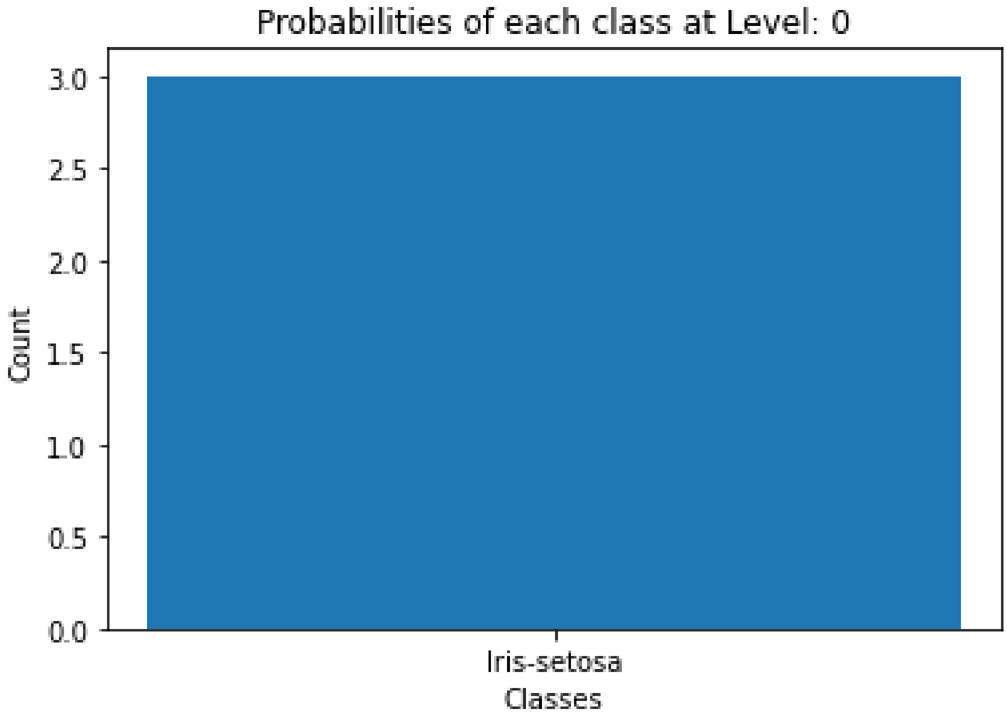


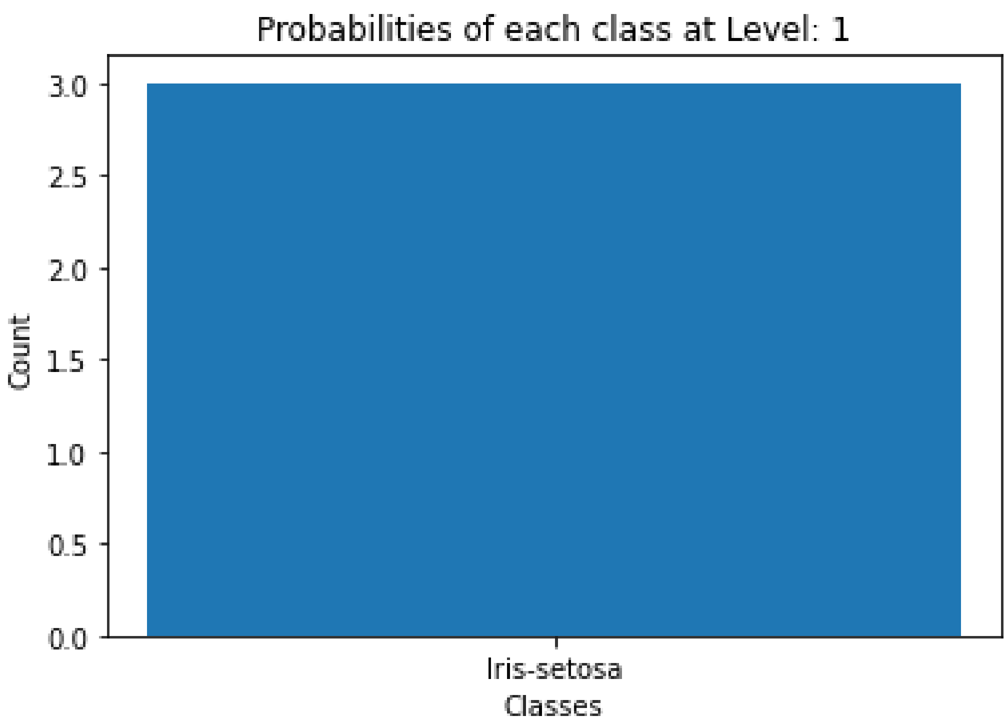

Probabilities of each class at Level: 3
Count
0
5
10
15
20
25
30
Iris-setosa
Classes

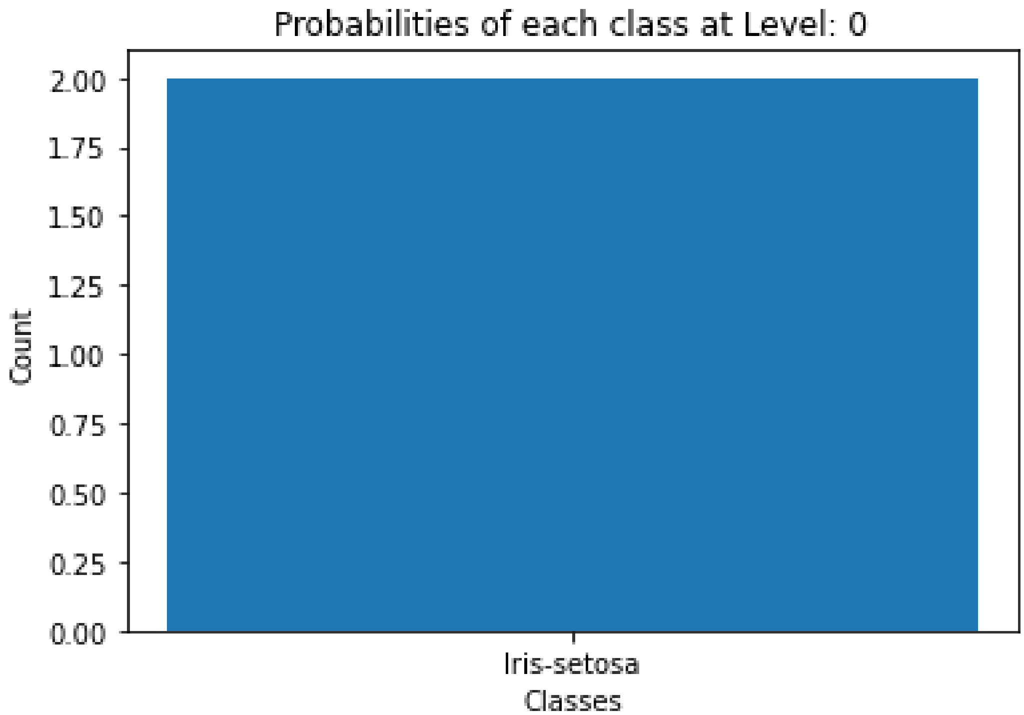

Probabilities of each class at Level: 2
Count
0
5
10
15
20
25
30
Iris-setosa
Classes

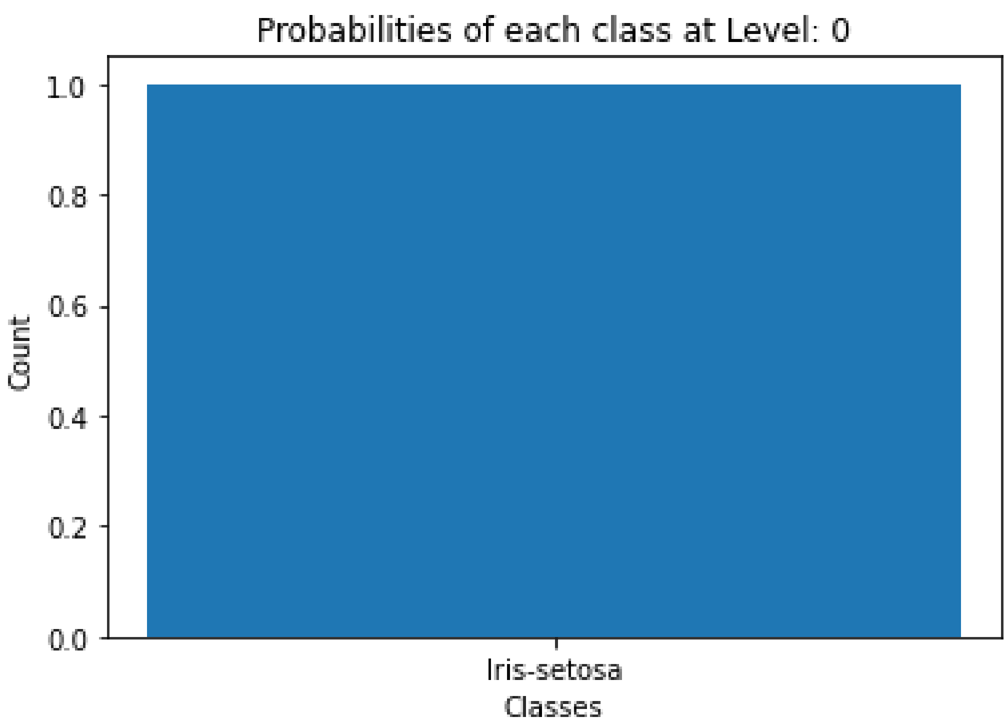

Probabilities of each class at Level: 1
Count
Iris-setosa
Classes
0
5
10
15
20
25
30

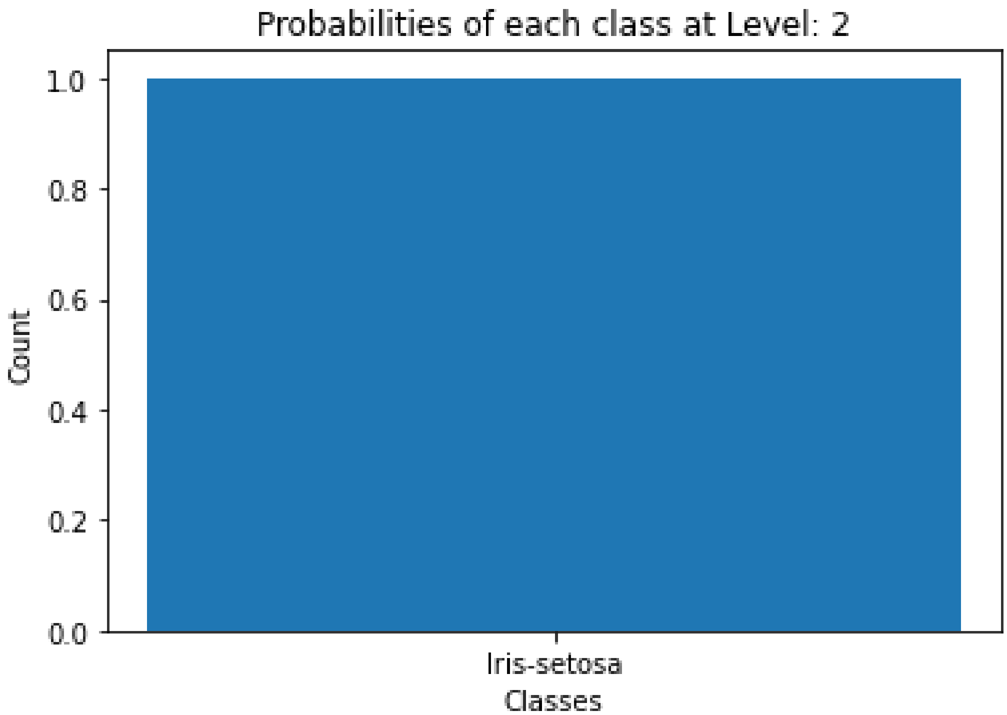

Probabilities of each class at Level: 0
Count
Iris-setosa
Classes
0
5
10
15
20
25

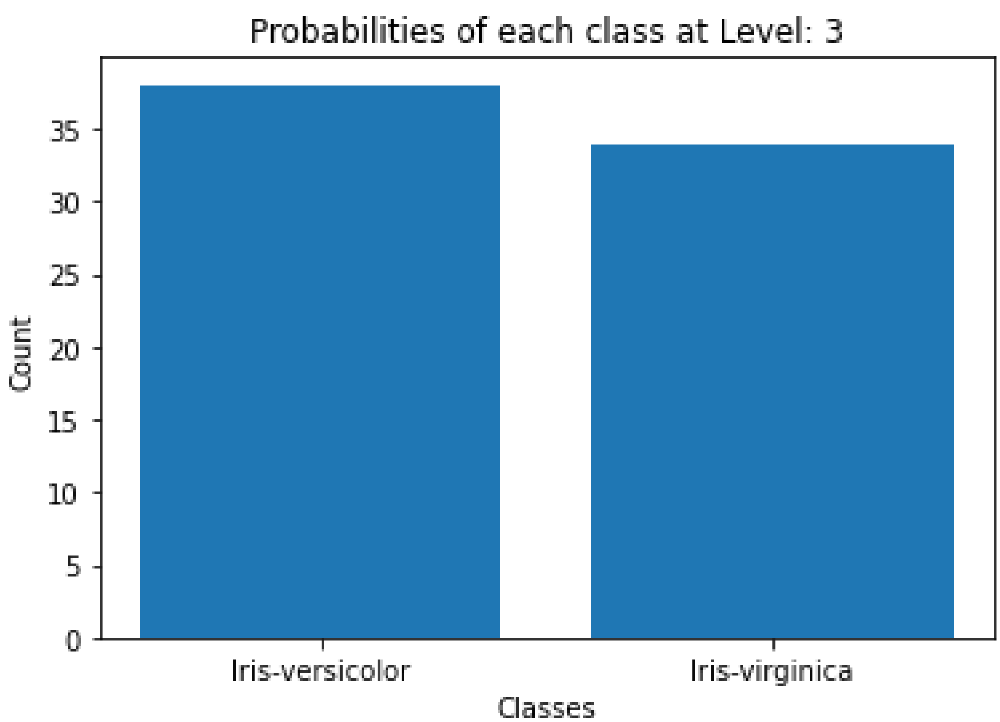

Probabilities of each class at Level: 0
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Count
Iris-setosa
Classes

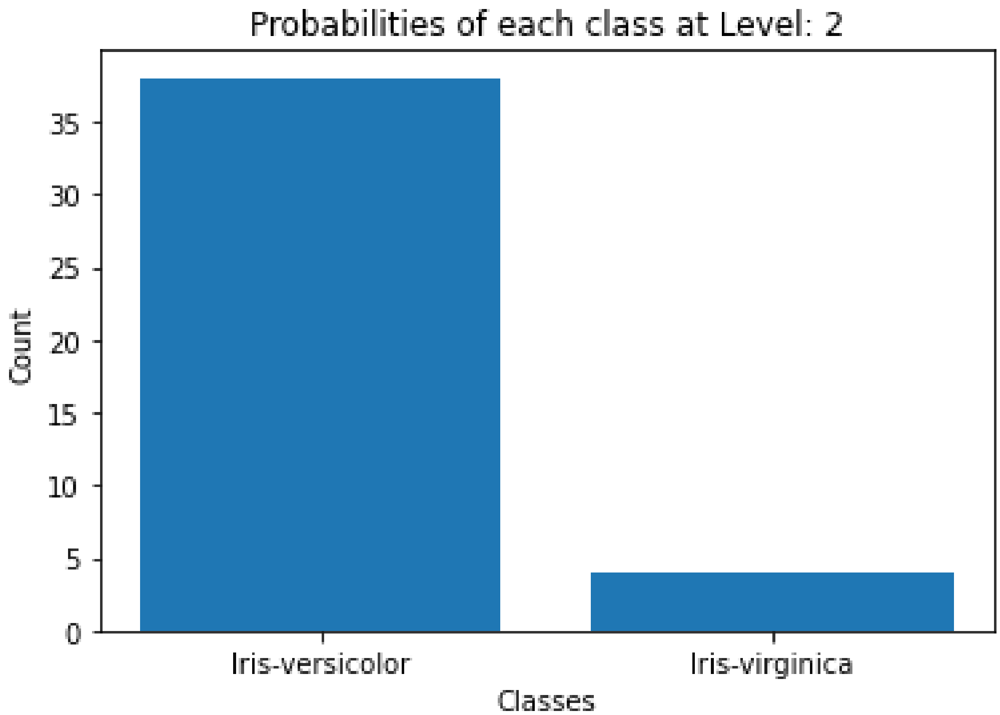

Probabilities of each class at Level: 1
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Count
Iris-setosa
Classes

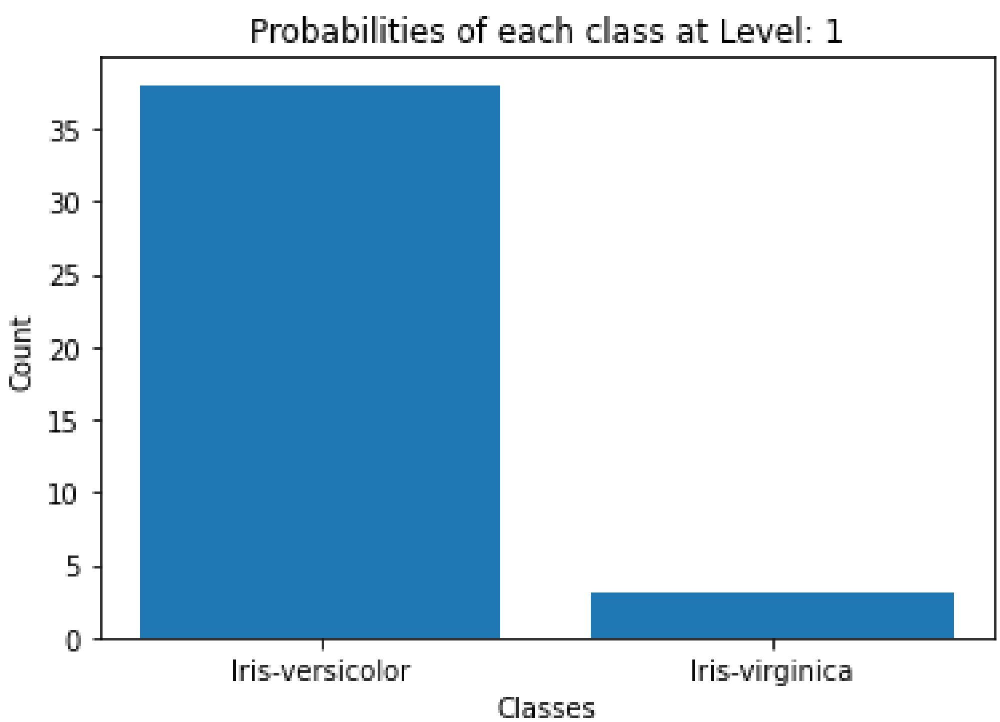

Probabilities of each class at Level: 0
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
Count
Iris-setosa
Classes

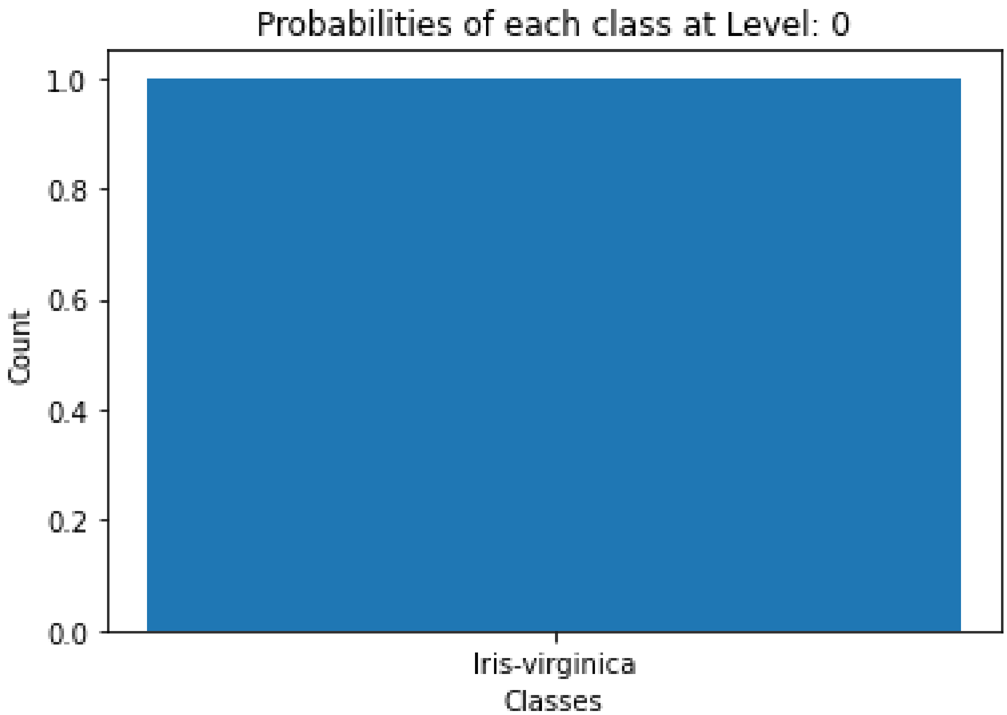

Probabilities of each class at Level: 0
1.0
0.8
0.6
0.4
0.2
0.0
Count
Iris-setosa
Classes

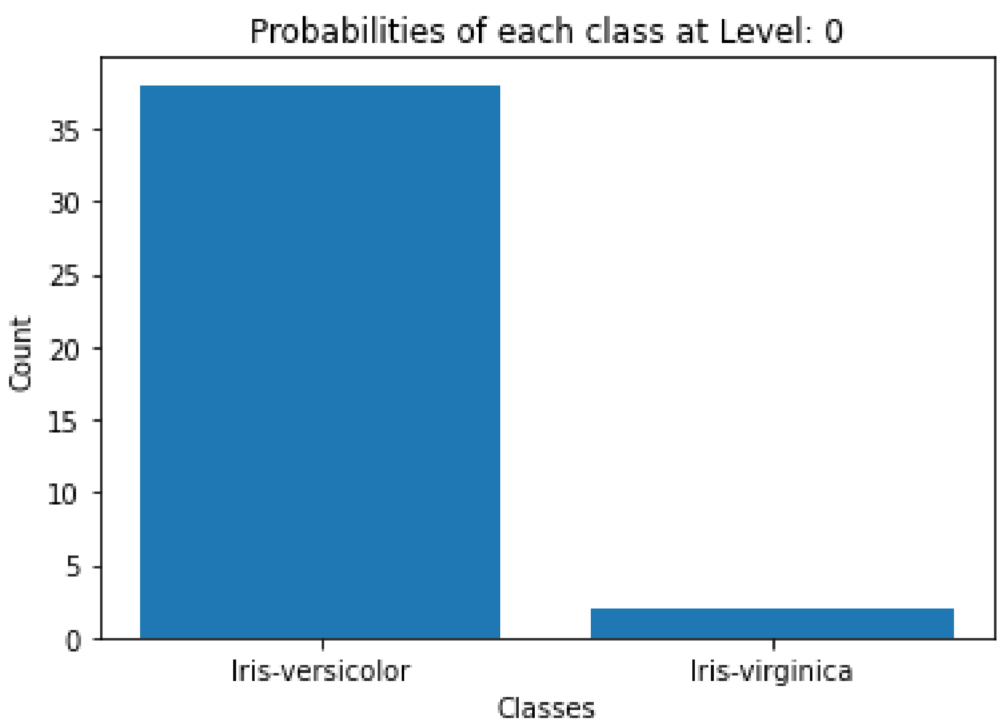

Probabilities of each class at Level: 2
1.0
0.8
0.6
0.4
0.2
0.0
Count
Iris-setosa
Classes

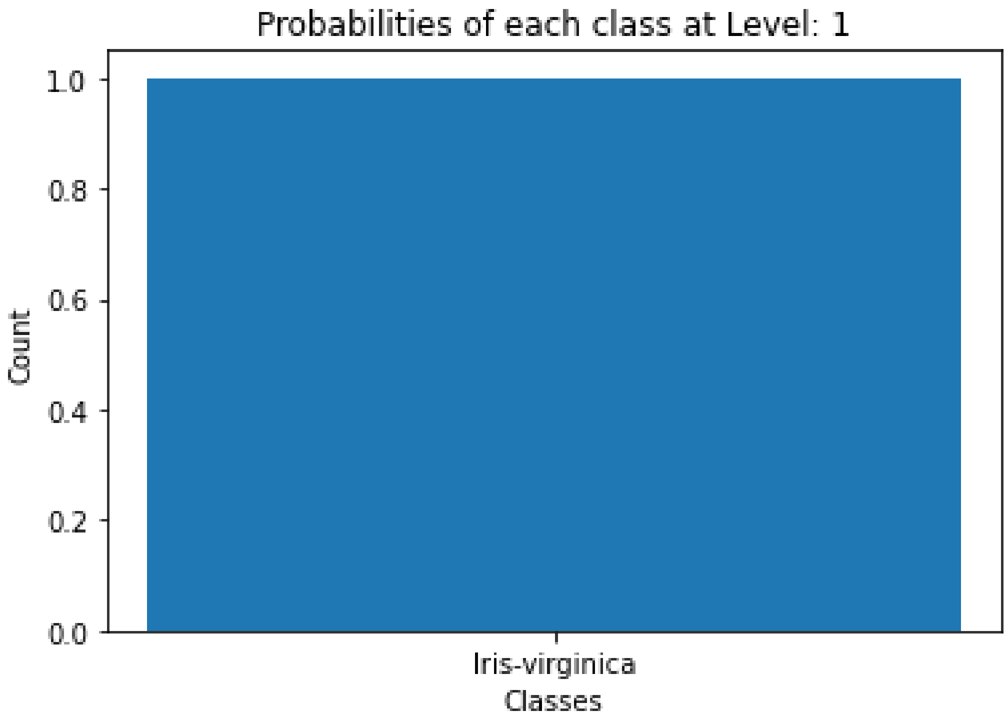

Probabilities of each class at Level: 3
35
30
25
20
15
10
5
0
Count
Iris-versicolor
Iris-virginica
Classes

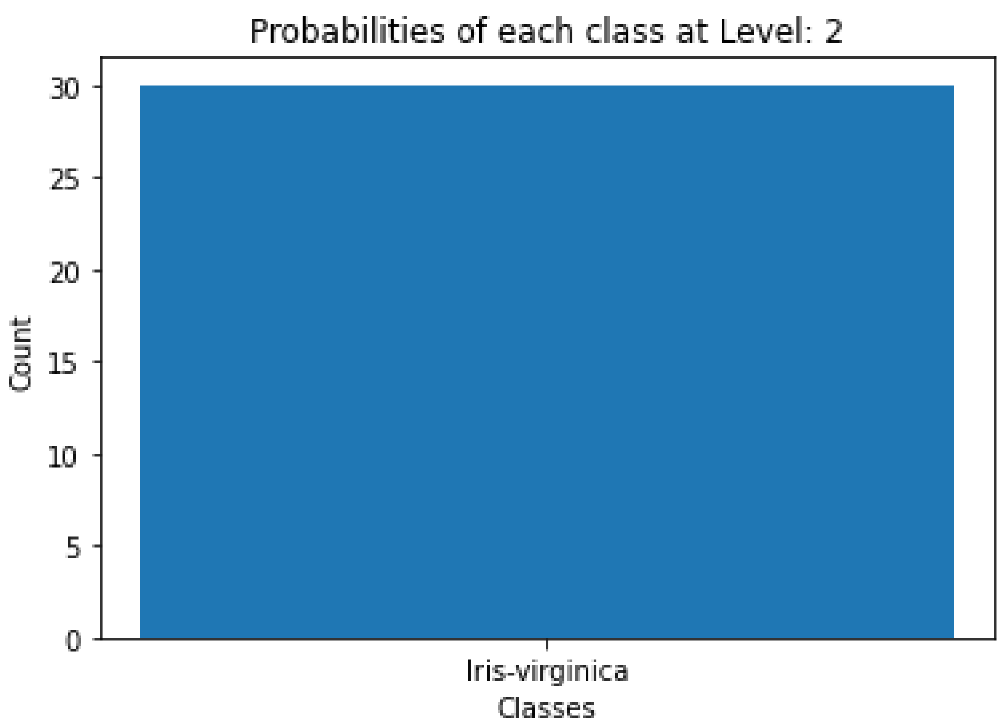

Probabilities of each class at Level: 2
35
30
25
20
15
10
5
0
Count
Iris-versicolor
Iris-virginica
Classes

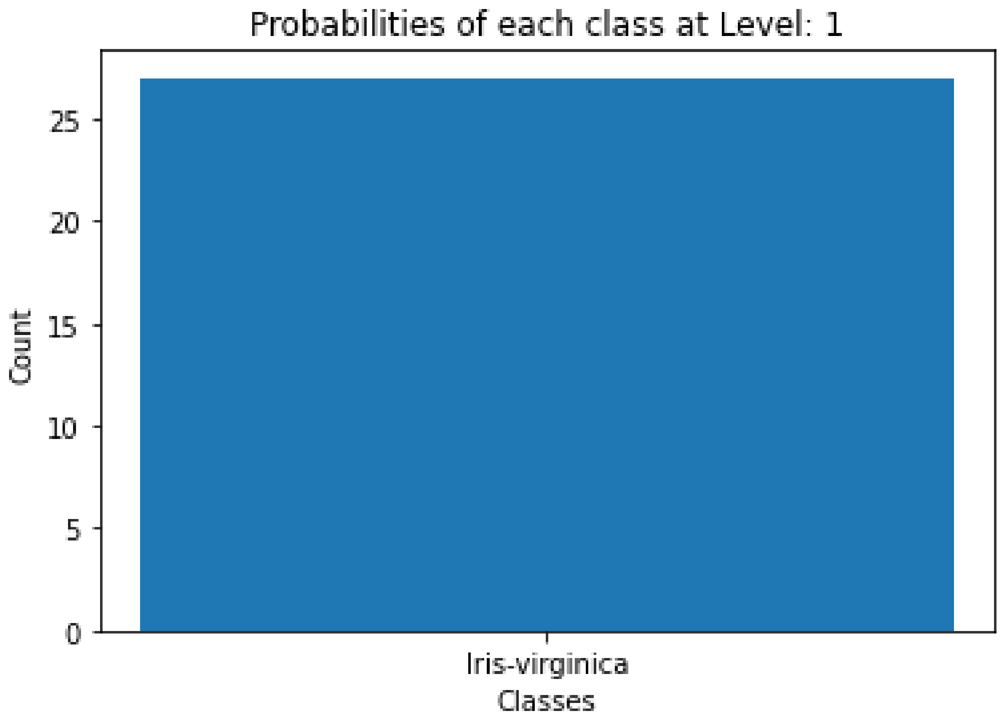

Probabilities of each class at Level: 1
35
30
25
20
15
10
5
0
Count
Iris-versicolor
Iris-virginica
Classes

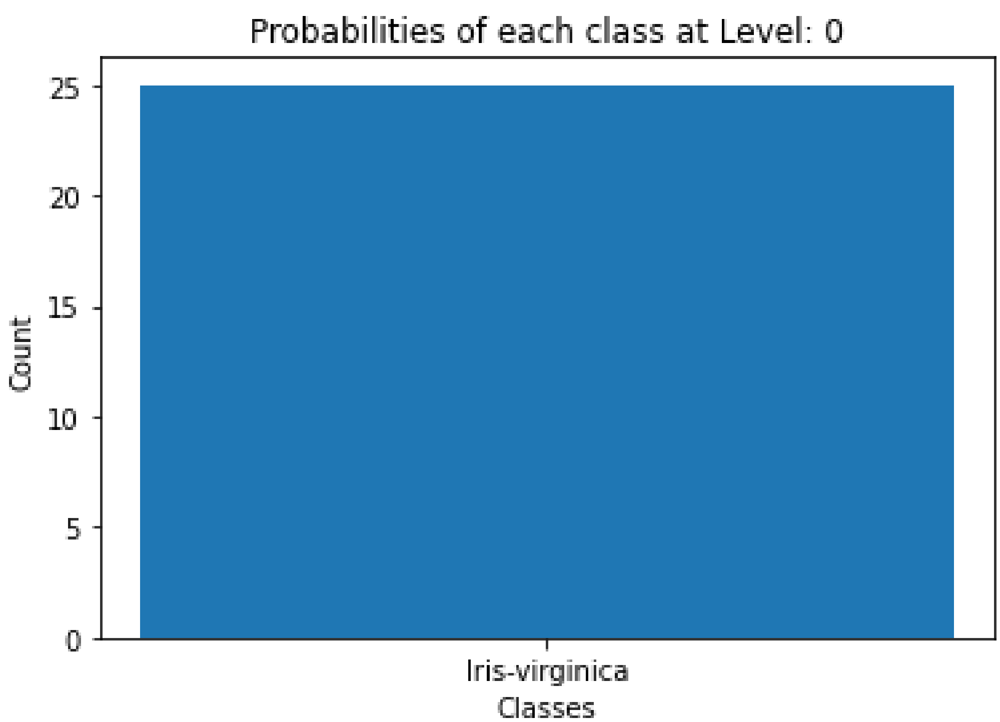

Probabilities of each class at Level: 0
1.0
0.8
0.6
0.4
0.2
0.0
Count
Iris-virginica
Classes

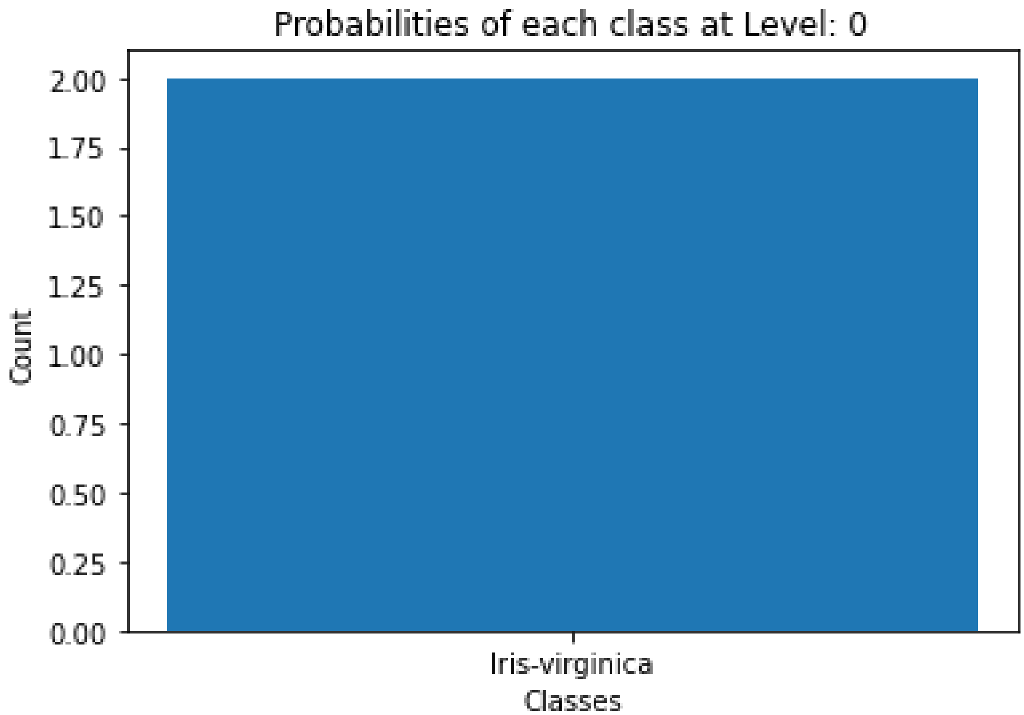

Probabilities of each class at Level: 0
35
30
25
20
15
10
5
0
Count
Iris-versicolor
Iris-virginica
Classes


Probabilities of each class at Level: 1
1.0
0.8
0.6
0.4
0.2
0.0
Count
Iris-virginica
Classes

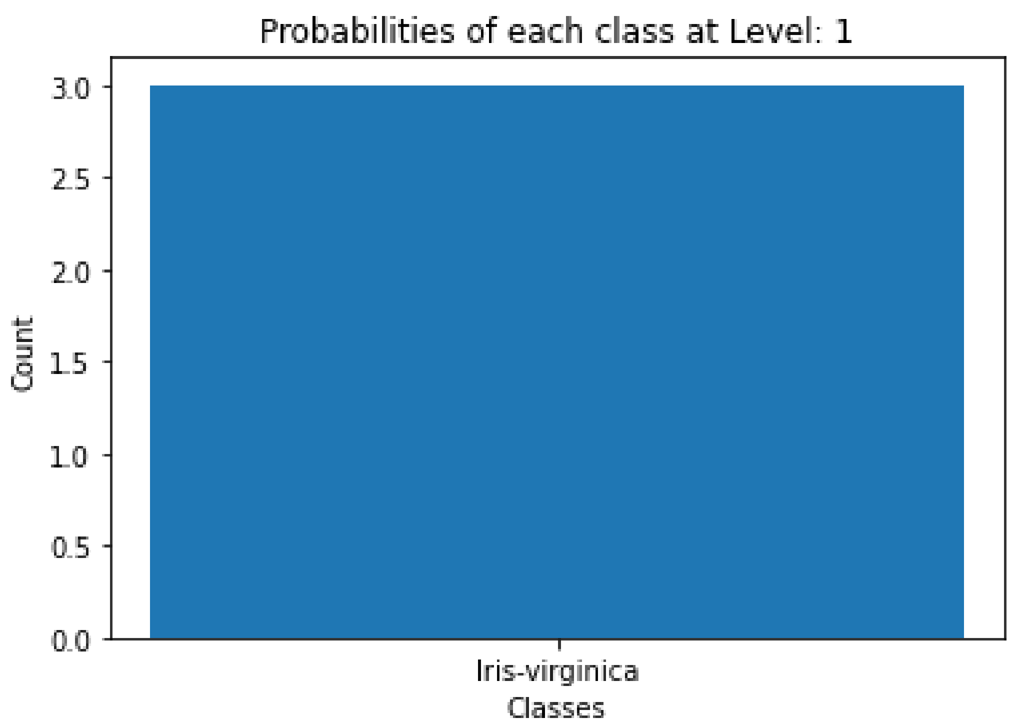

Probabilities of each class at Level: 2
30
25
20
15
10
5
0
Count
Iris-virginica
Classes


Probabilities of each class at Level: 1
Count
25
20
15
10
5
0
Iris-virginica
Classes

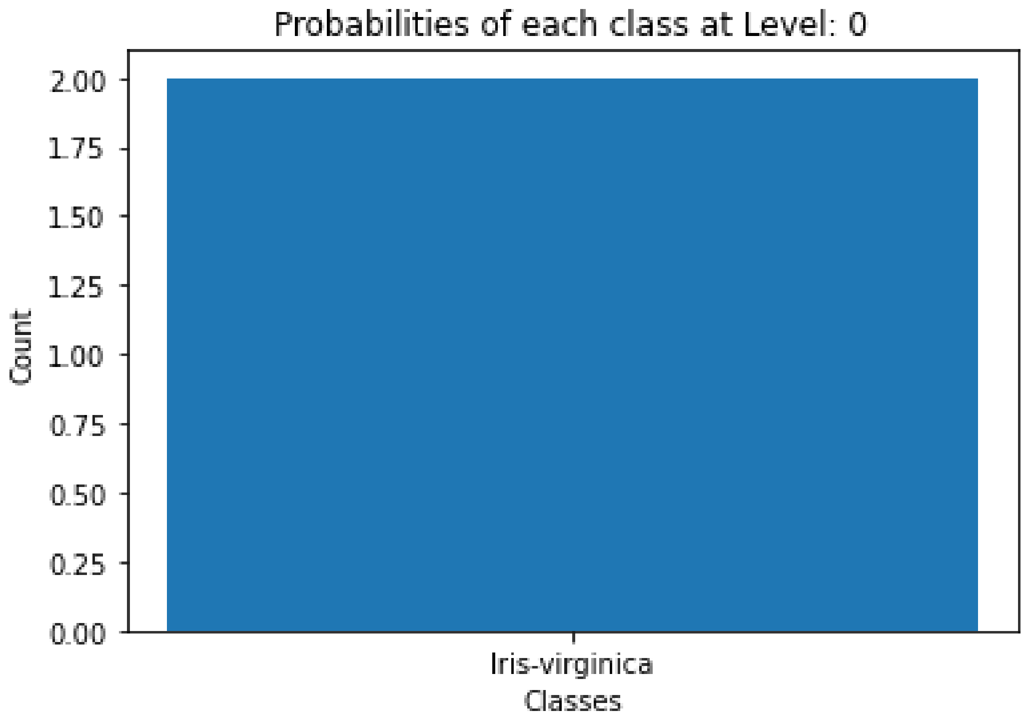

Probabilities of each class at Level: 0
Count
25
20
15
10
5
0
Iris-virginica
Classes



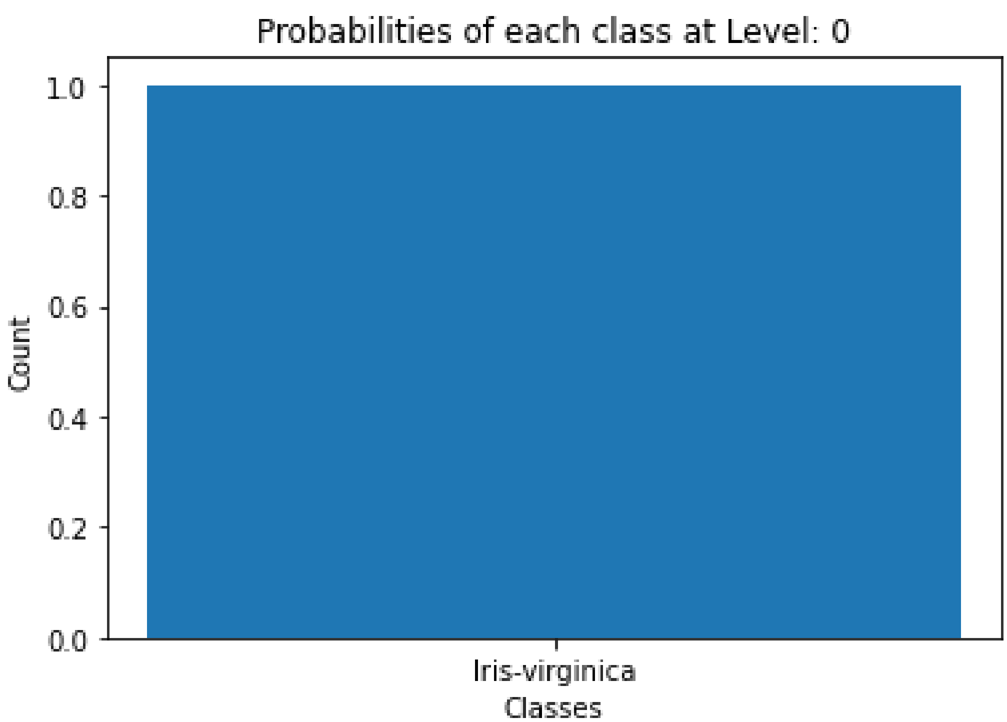
Probabilities of each class at Level: 0
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
Count
Iris-virginica
Classes



Probabilities of each class at Level: 1
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Count
Iris-virginica
Classes



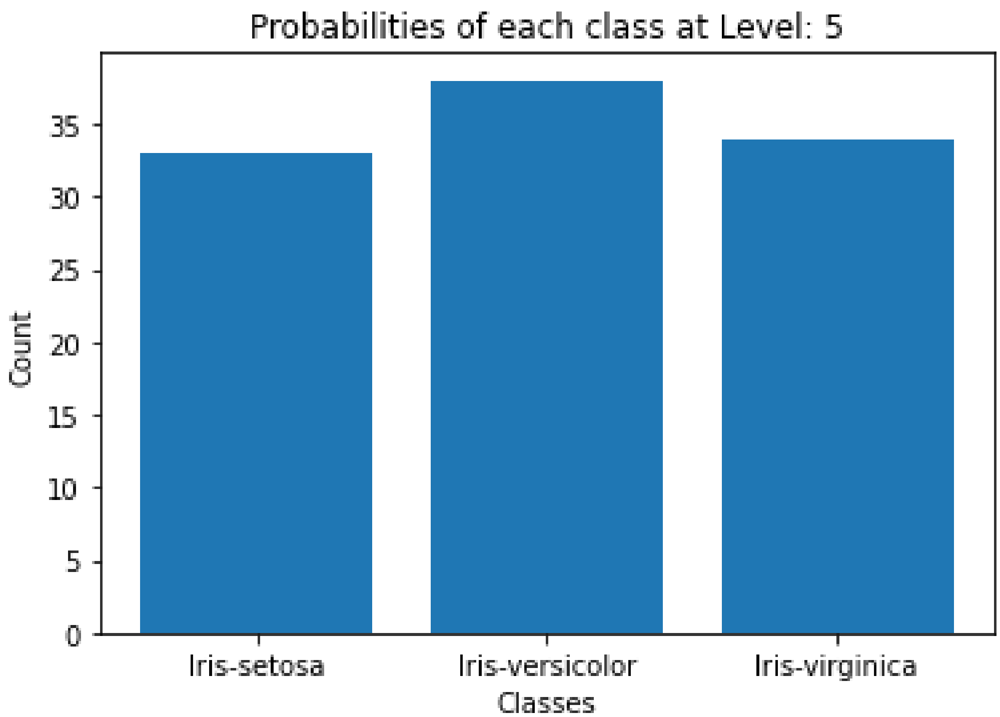
Probabilities of each class at Level: 0
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
Count
Iris-virginica
Classes

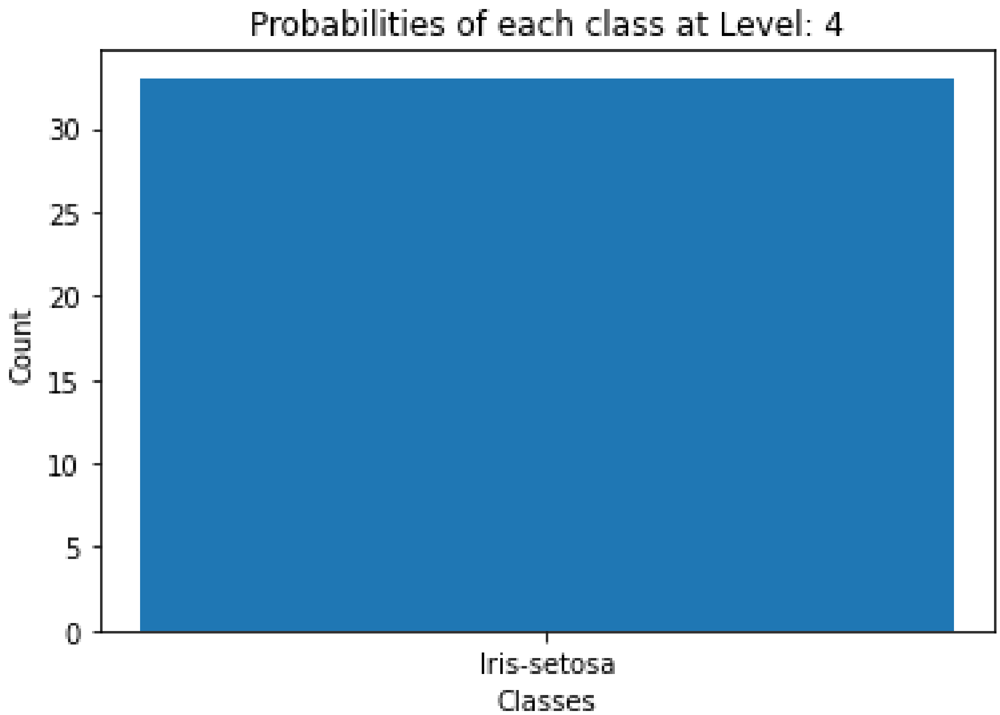


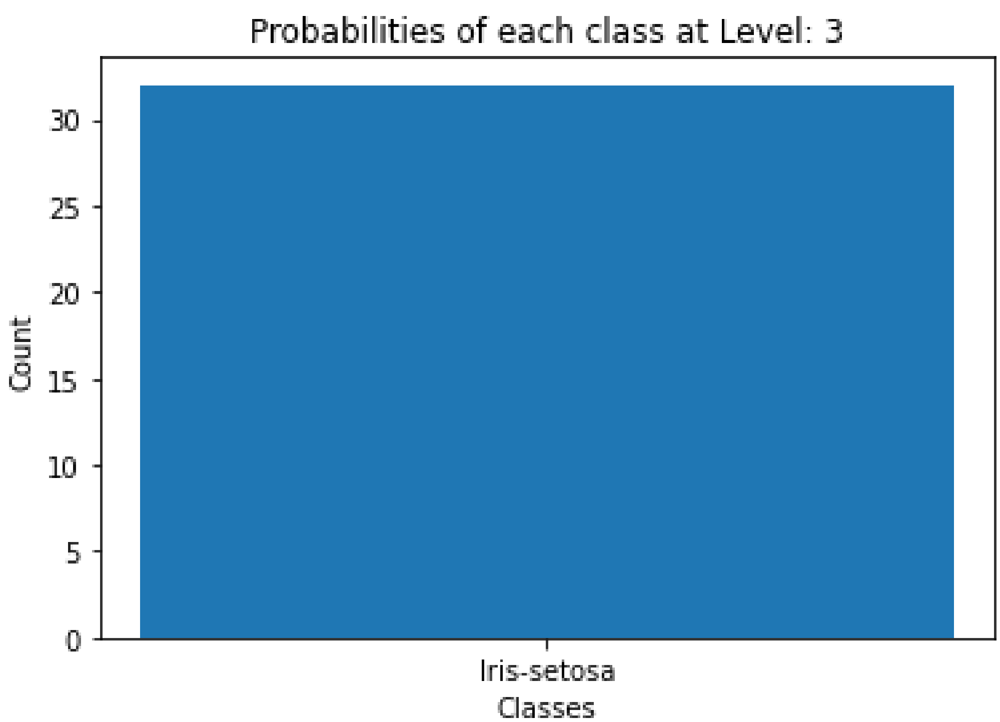
Probabilities of each class at Level: 0
1.0
0.8
0.6
0.4
0.2
0.0
Count
Iris-virginica
Classes

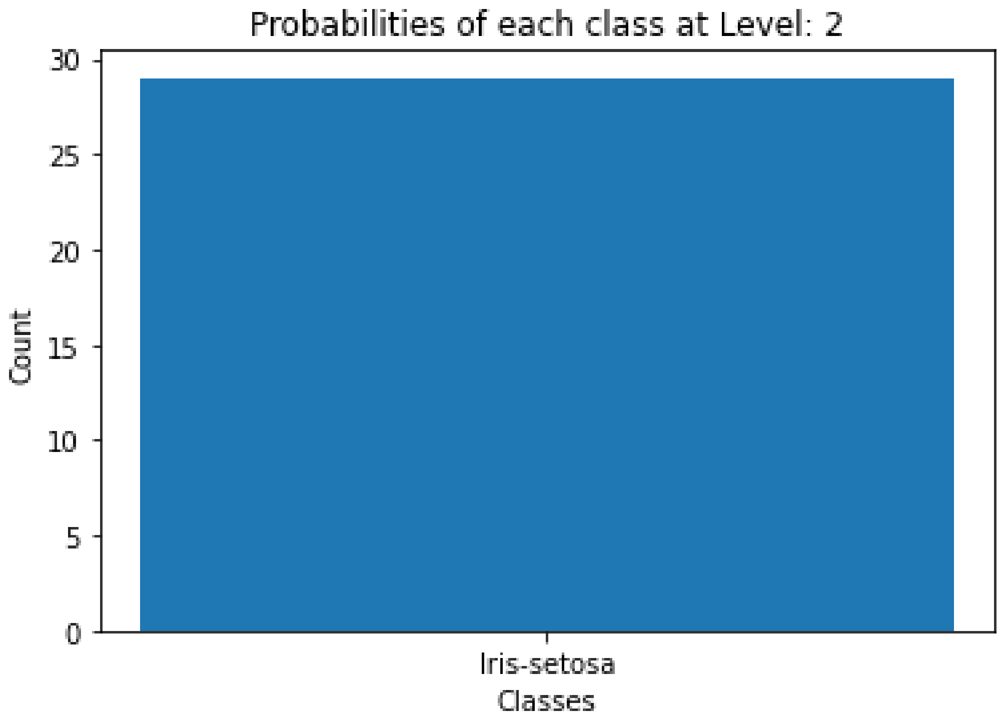
```
Printing the Tree using a breath first tree traversal:

check for petal_width <= 0.75
True:
  check for petal_width <= 0.45
  True:
    check for petal_width <= 0.35
    True:
      check for petal_width <= 0.25
      True:
        Leaf Node: Iris-setosa
      False:
        Leaf Node: Iris-setosa
    False:
      check for petal_length <= 1.65
      True:
        Leaf Node: Iris-setosa
      False:
        Leaf Node: Iris-setosa
  False:
    Leaf Node: Iris-setosa
False:
  check for petal_width <= 1.75
  True:
    check for petal_length <= 5.35
    True:
      check for sepal_length <= 4.95
      True:
        Leaf Node: Iris-virginica
      False:
        Leaf Node: Iris-versicolor
    False:
      Leaf Node: Iris-virginica
  False:
    check for petal_width <= 2.45
    True:
      check for petal_width <= 2.3499999999999996
      True:
        Leaf Node: Iris-virginica
      False:
        Leaf Node: Iris-virginica
    False:
      check for petal_length <= 6.05
      True:
        Leaf Node: Iris-virginica
      False:
        Leaf Node: Iris-virginica
```

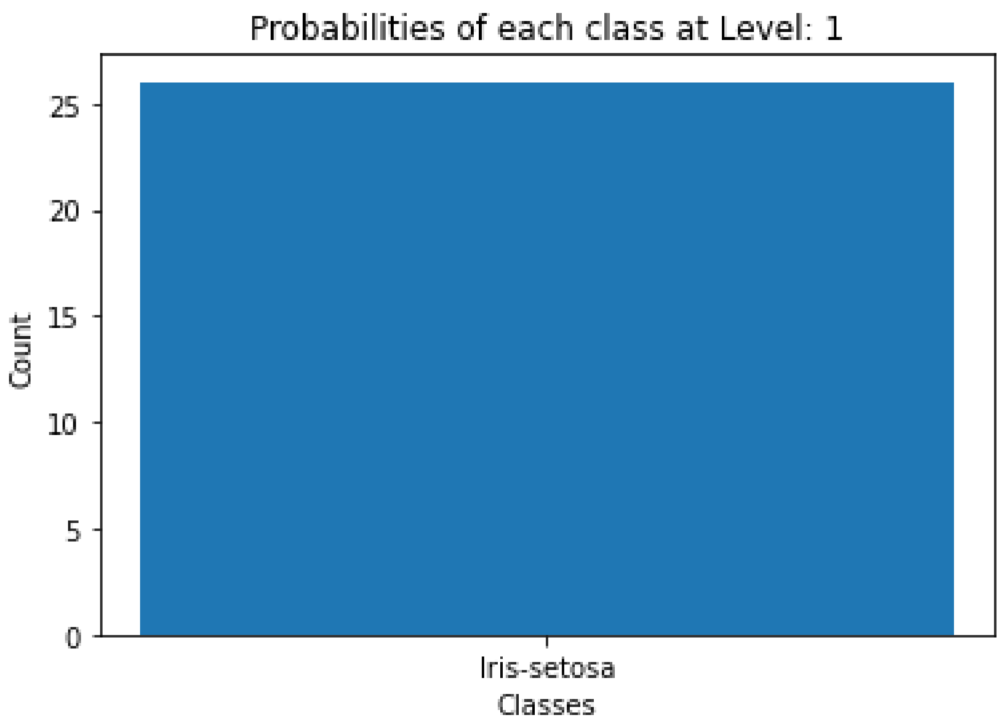
```
Accuracy: 0.9545454545454546

Cross Entropy Loss: 0.04652001563489282

--------------------------------------------------------------

Learning Decision Tree with Misclassification Rate and Max Depth: 5
```

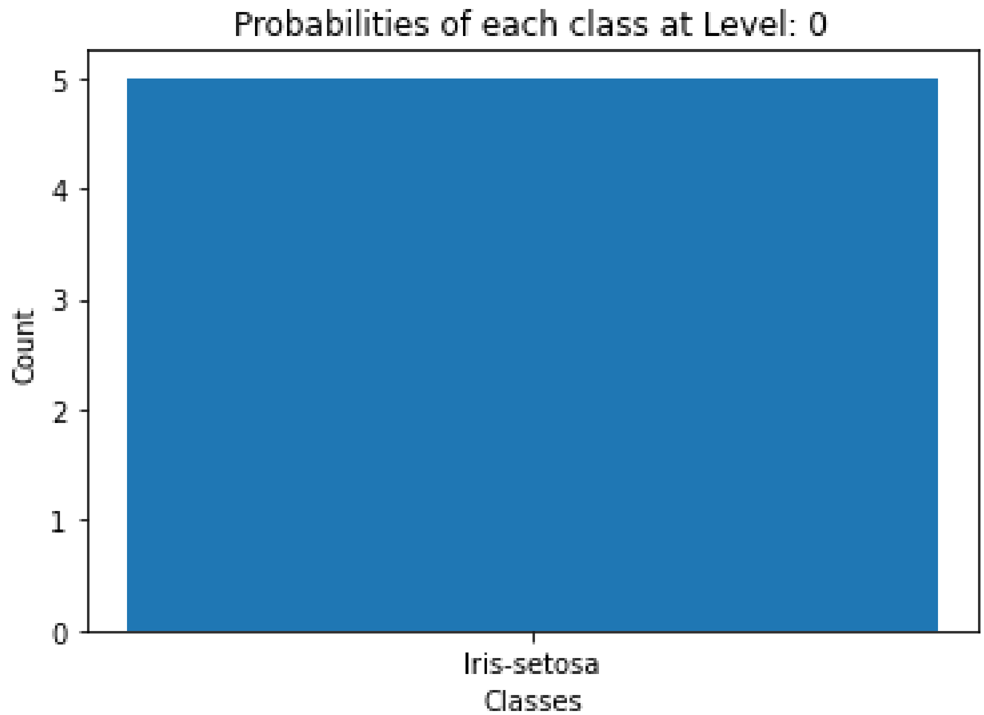


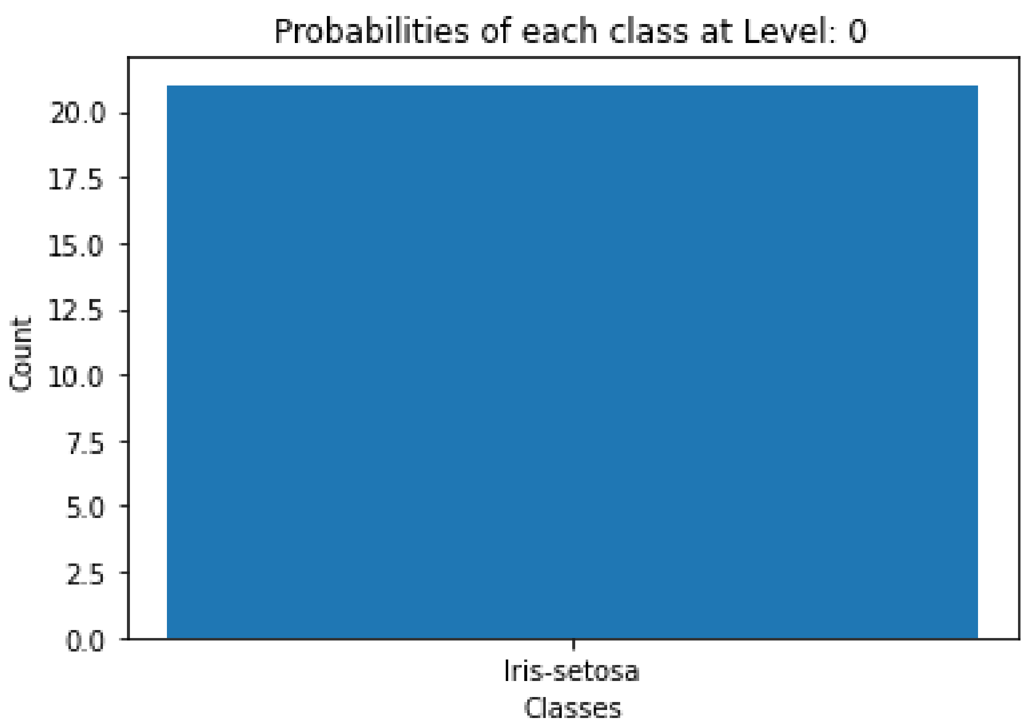

Probabilities of each class at Level: 4
30
25
20
15
10
5
0
Count
Iris-setosa
Classes

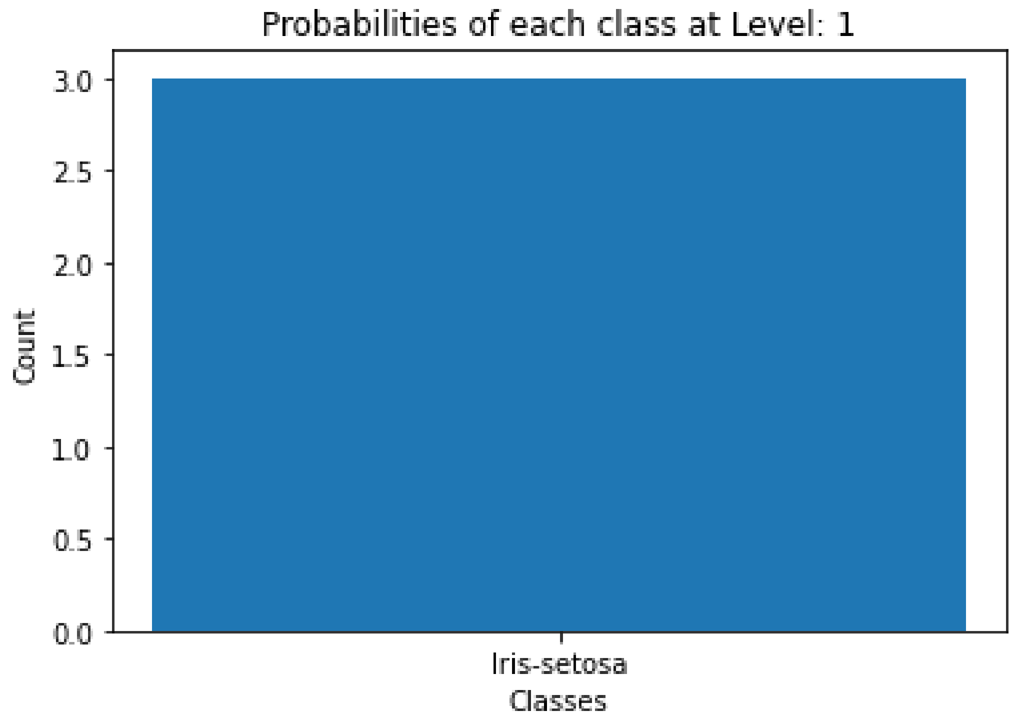

Probabilities of each class at Level: 3
30
25
20
15
10
5
0
Count
Iris-setosa
Classes


Probabilities of each class at Level: 2
Count
Iris-setosa
Classes

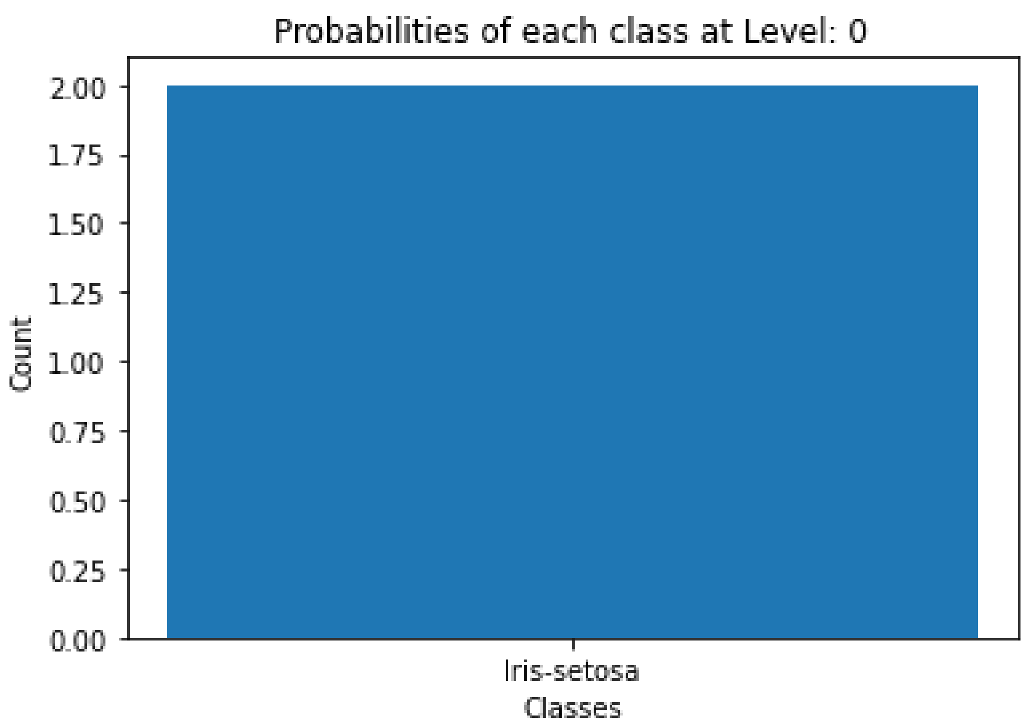

Probabilities of each class at Level: 1
Count
Iris-setosa
Classes


Probabilities of each class at Level: 0
5
4
3
2
1
0
Count
Iris-setosa
Classes

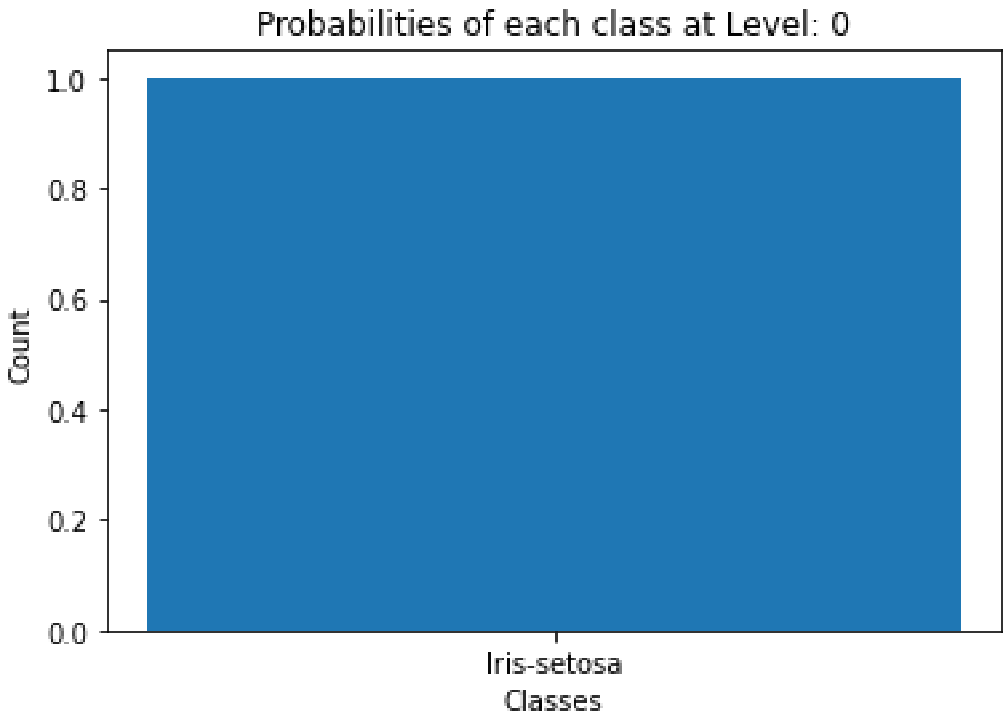

Probabilities of each class at Level: 0
20.0
17.5
15.0
12.5
10.0
7.5
5.0
2.5
0.0
Count
Iris-setosa
Classes

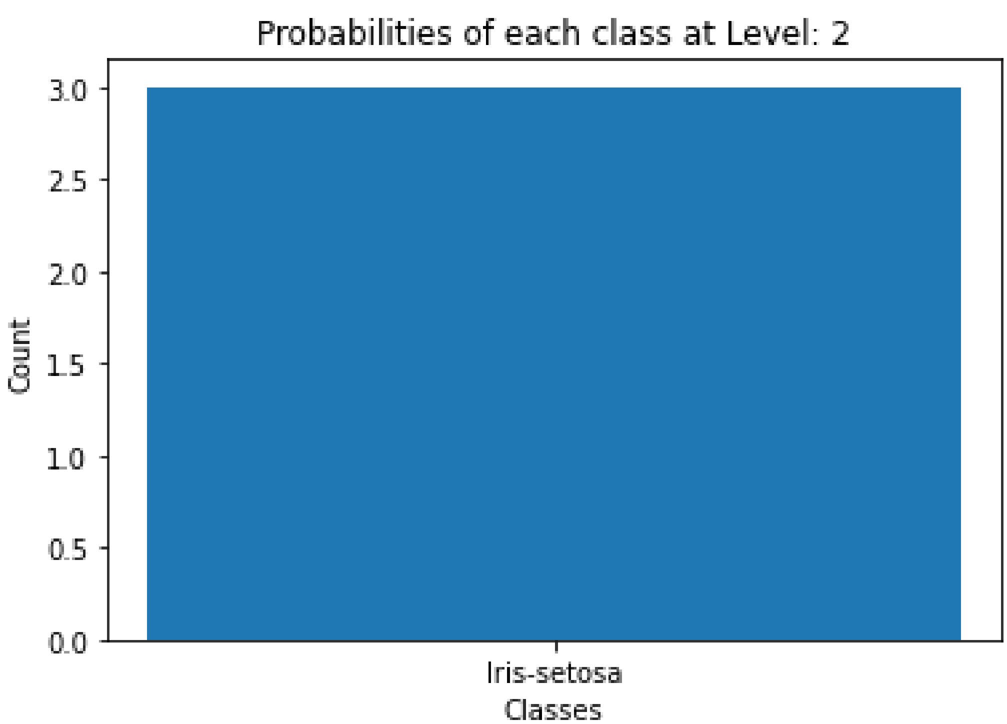


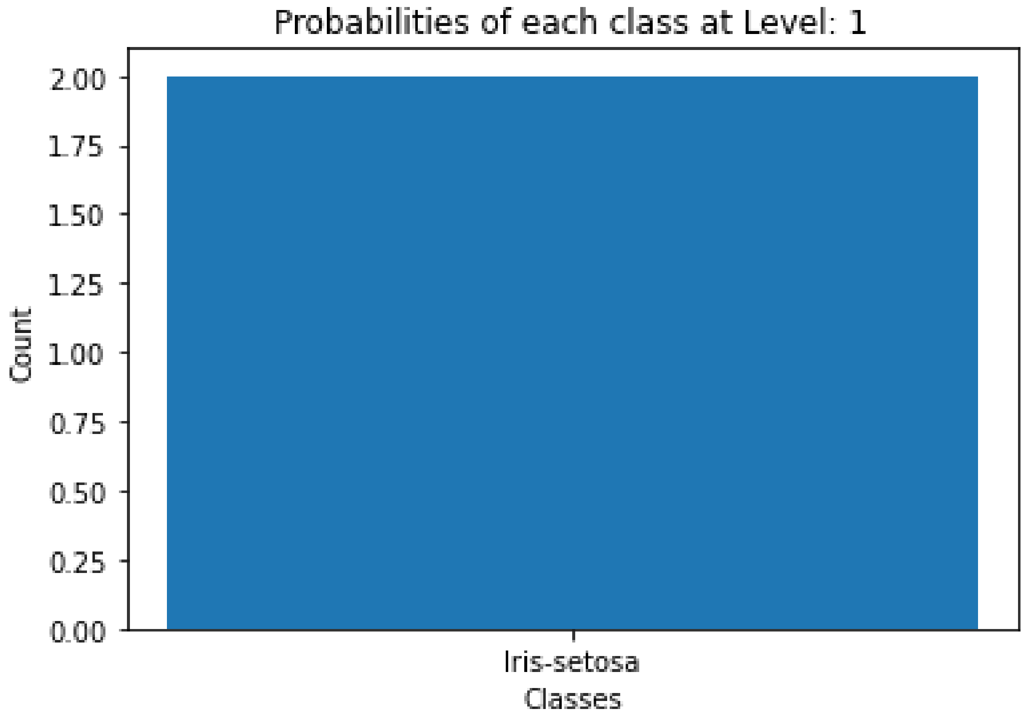
Probabilities of each class at Level: 1
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Count
Iris-setosa
Classes

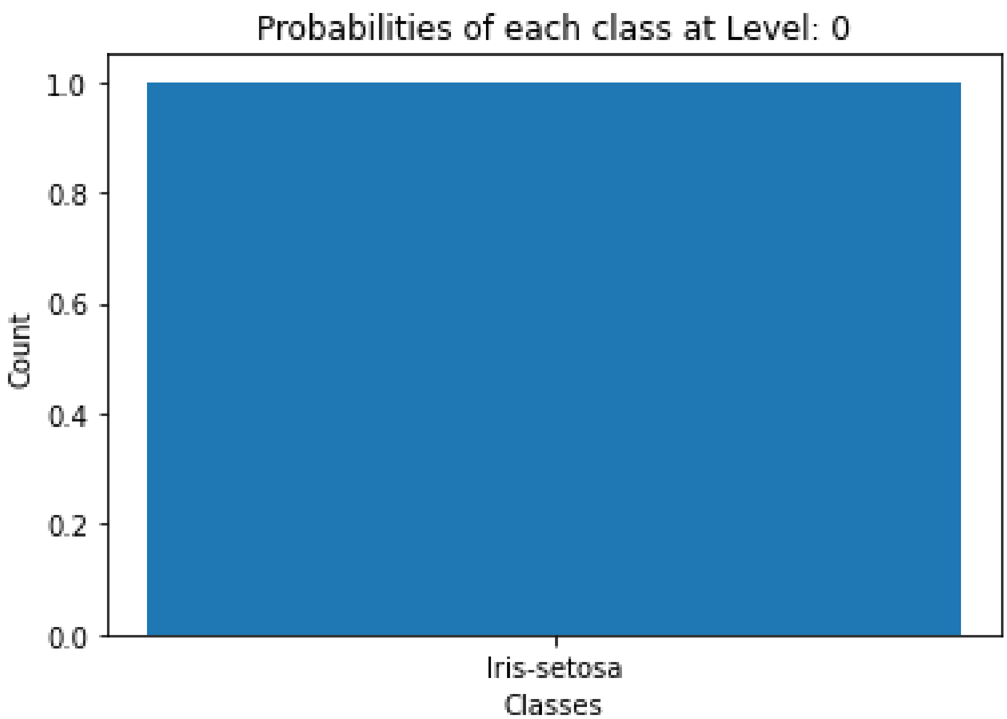


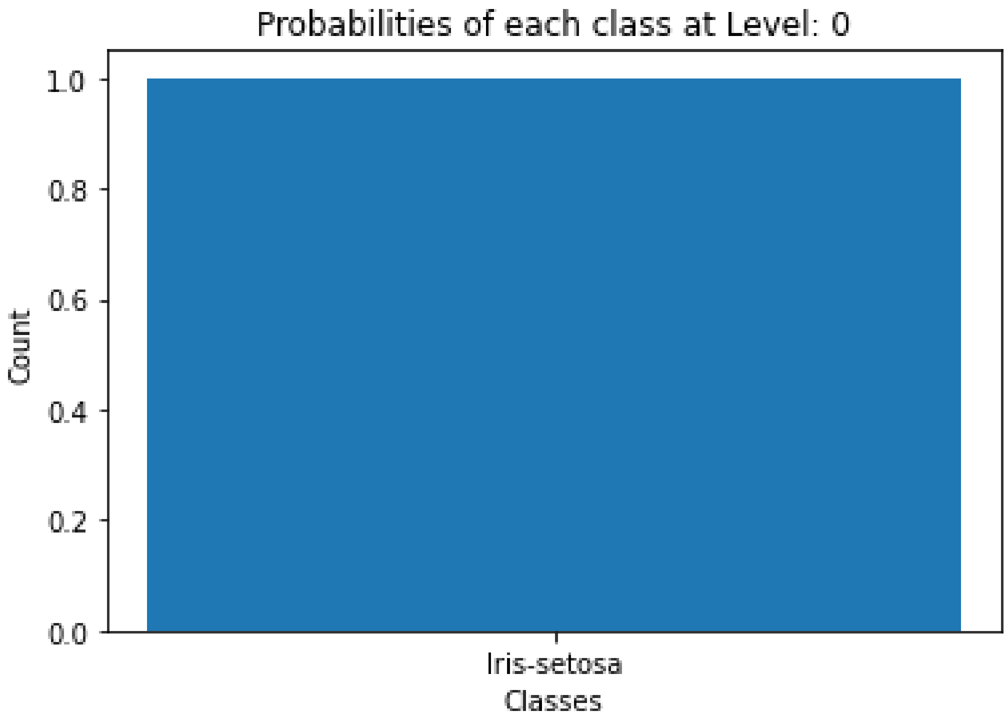
Probabilities of each class at Level: 0
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
Count
Iris-setosa
Classes

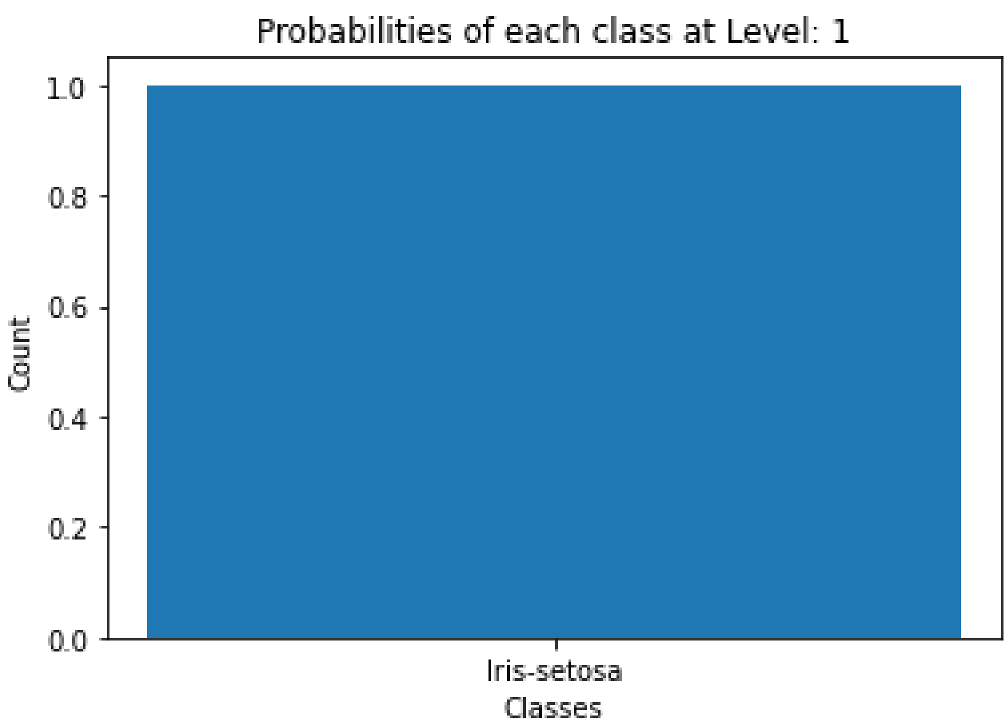

Probabilities of each class at Level: 0
1.0
0.8
0.6
0.4
0.2
0.0
Count
Iris-setosa
Classes

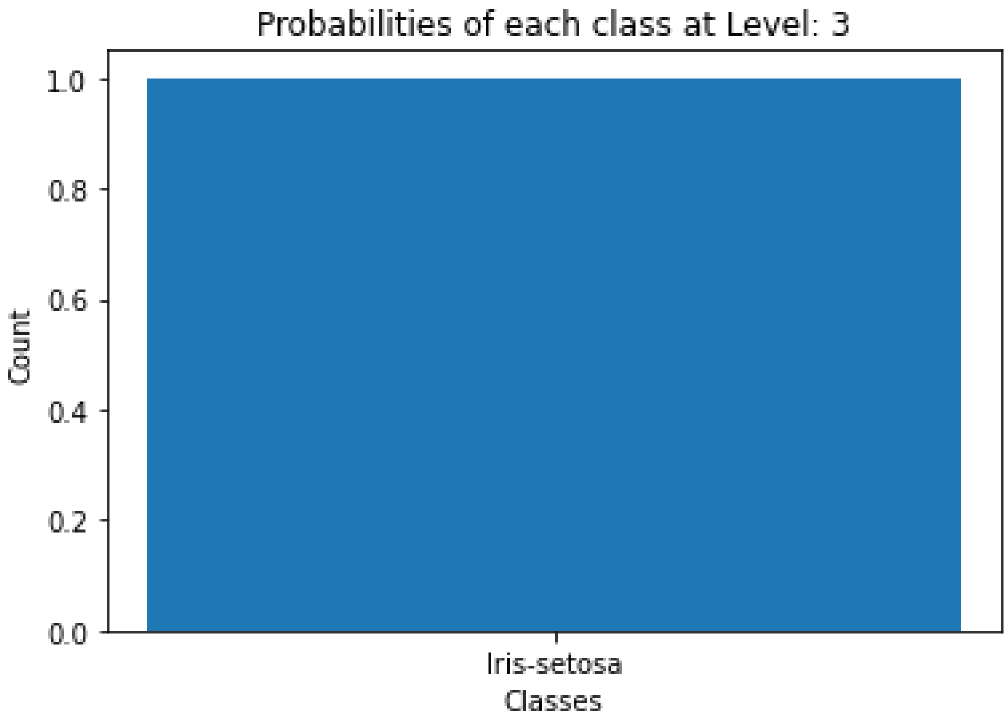

Probabilities of each class at Level: 2
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Count
Iris-setosa
Classes

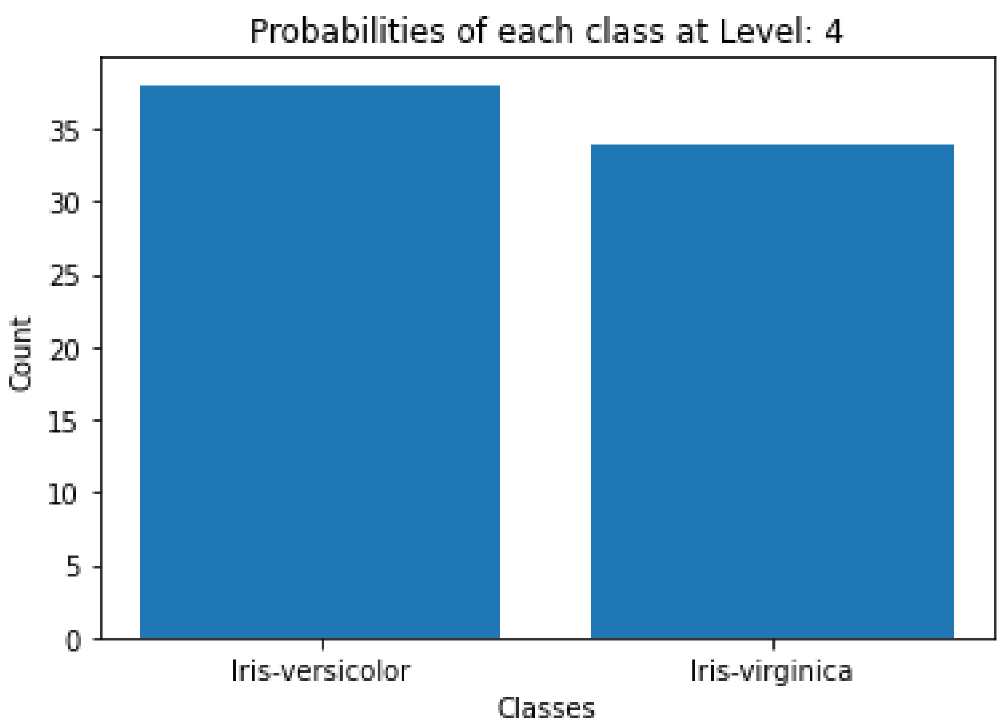

Probabilities of each class at Level: 1
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
Count
Iris-setosa
Classes

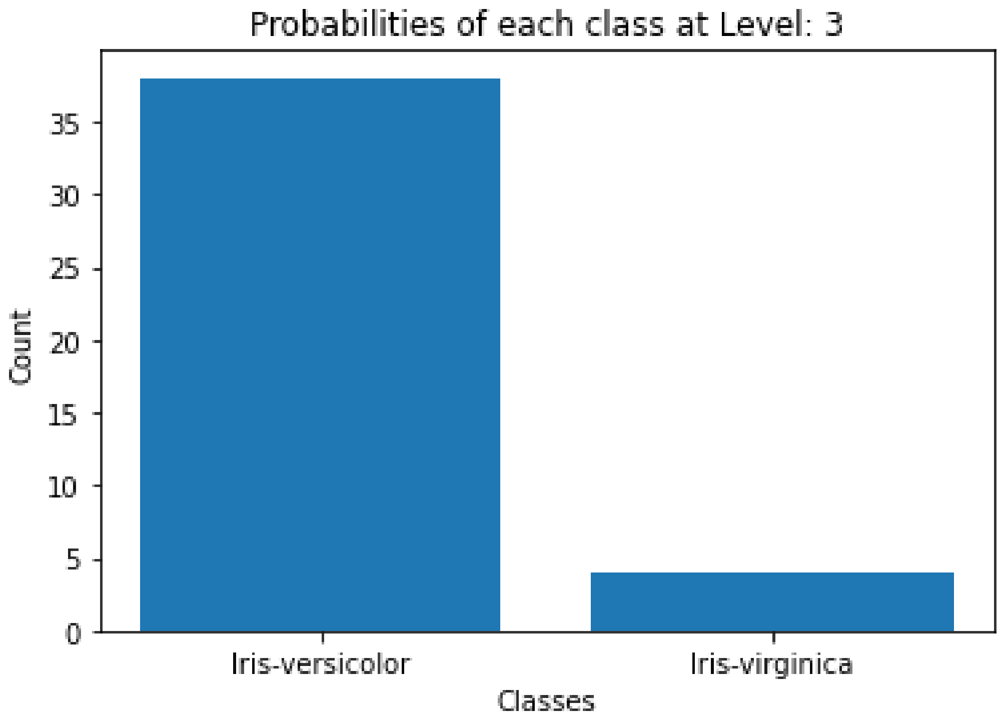

Probabilities of each class at Level: 0
1.0
0.8
0.6
0.4
0.2
0.0
Count
Iris-setosa
Classes

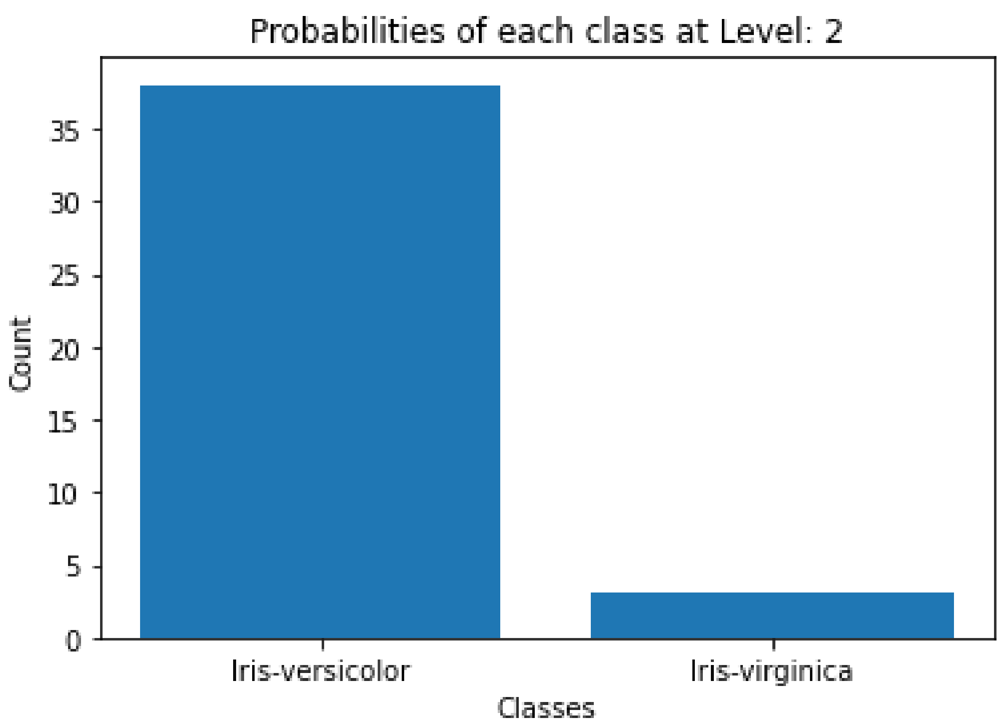

Probabilities of each class at Level: 0
1.0
0.8
0.6
0.4
0.2
0.0
Count
Iris-setosa
Classes

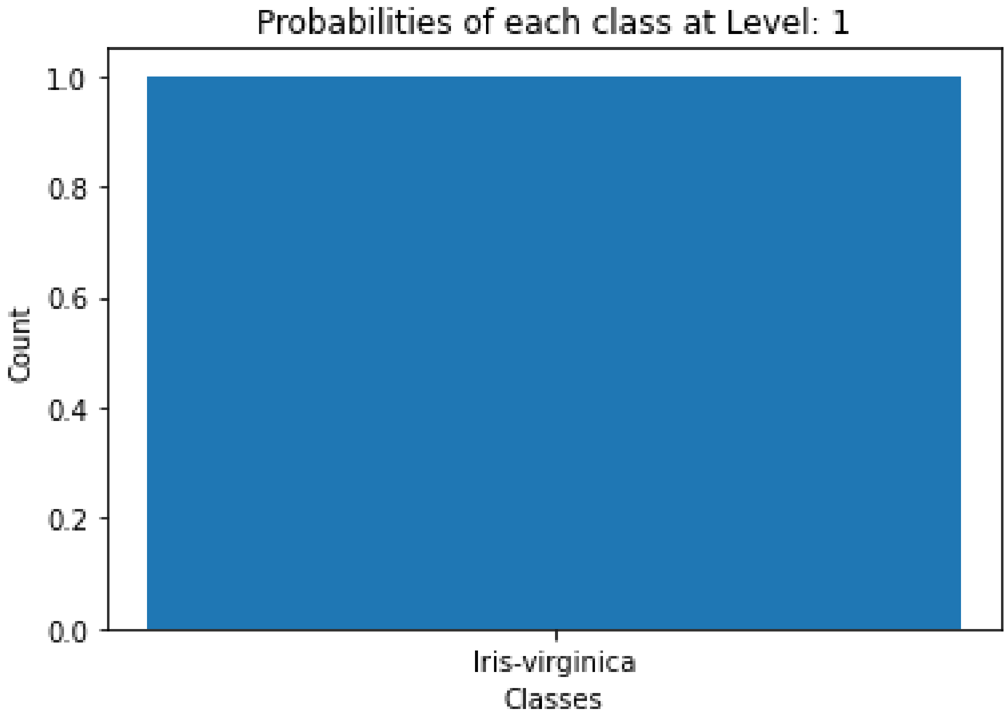

Probabilities of each class at Level: 1
1.0
0.8
0.6
0.4
0.2
0.0
Count
Iris-setosa
Classes

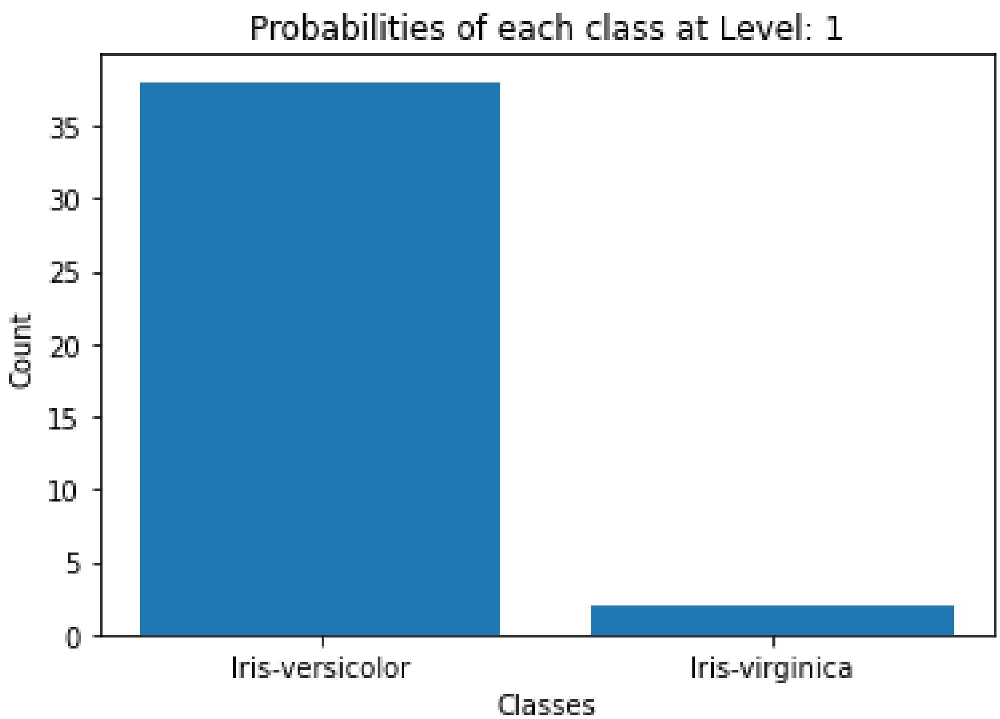

Probabilities of each class at Level: 3
1.0
0.8
0.6
0.4
0.2
0.0
Count
Iris-setosa
Classes

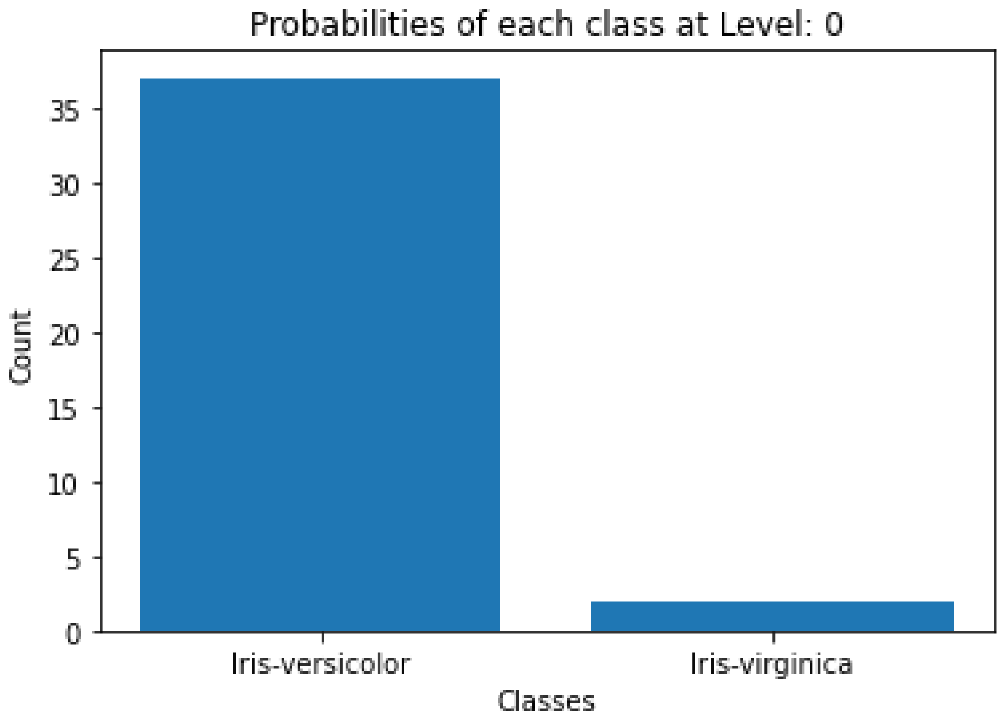

Probabilities of each class at Level: 4
35
30
25
20
15
10
5
0
Count
Iris-versicolor
Iris-virginica
Classes

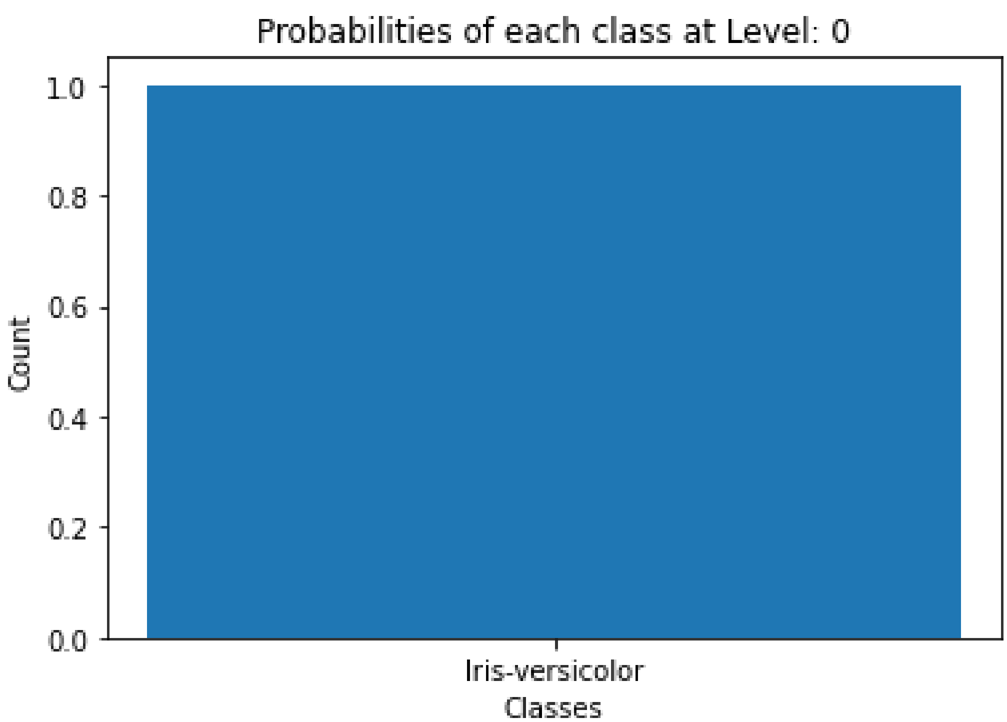

Probabilities of each class at Level: 3
Count
0
5
10
15
20
25
30
35
Iris-versicolor
Iris-virginica
Classes

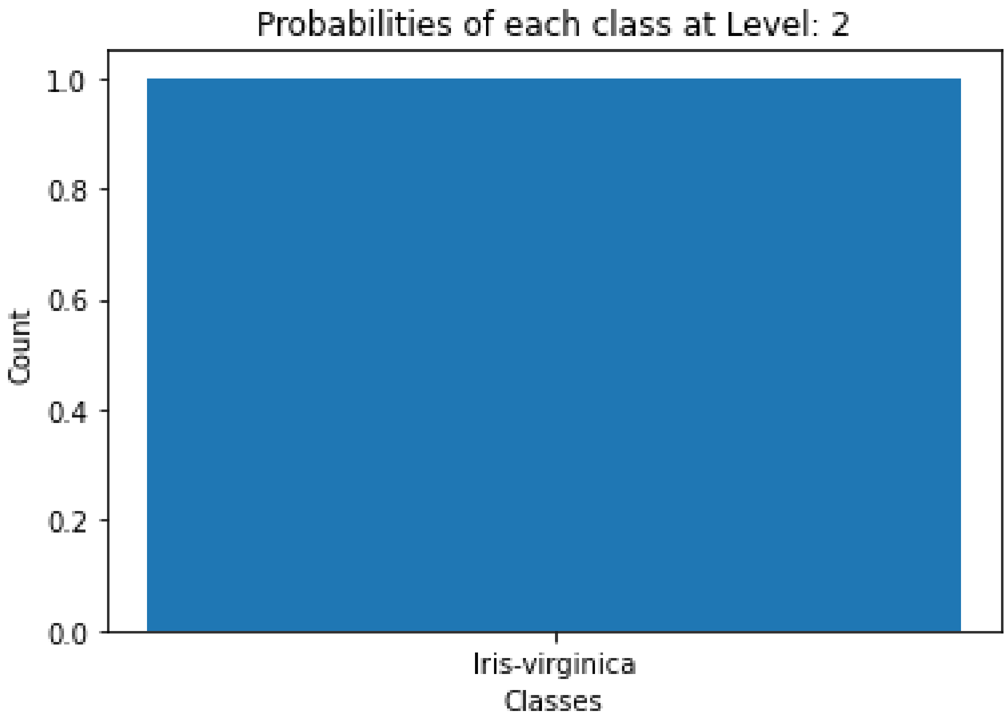

Probabilities of each class at Level: 2
Count
0
5
10
15
20
25
30
35
Iris-versicolor
Iris-virginica
Classes

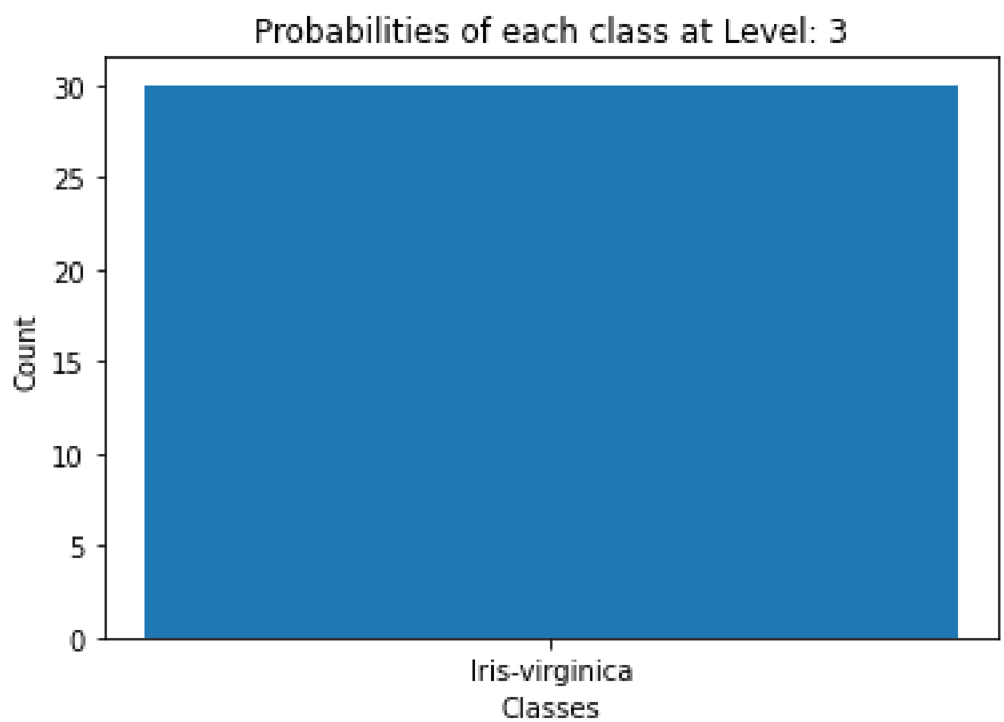

Probabilities of each class at Level: 1
1.0
0.8
0.6
0.4
0.2
0.0
Count
Iris-virginica
Classes

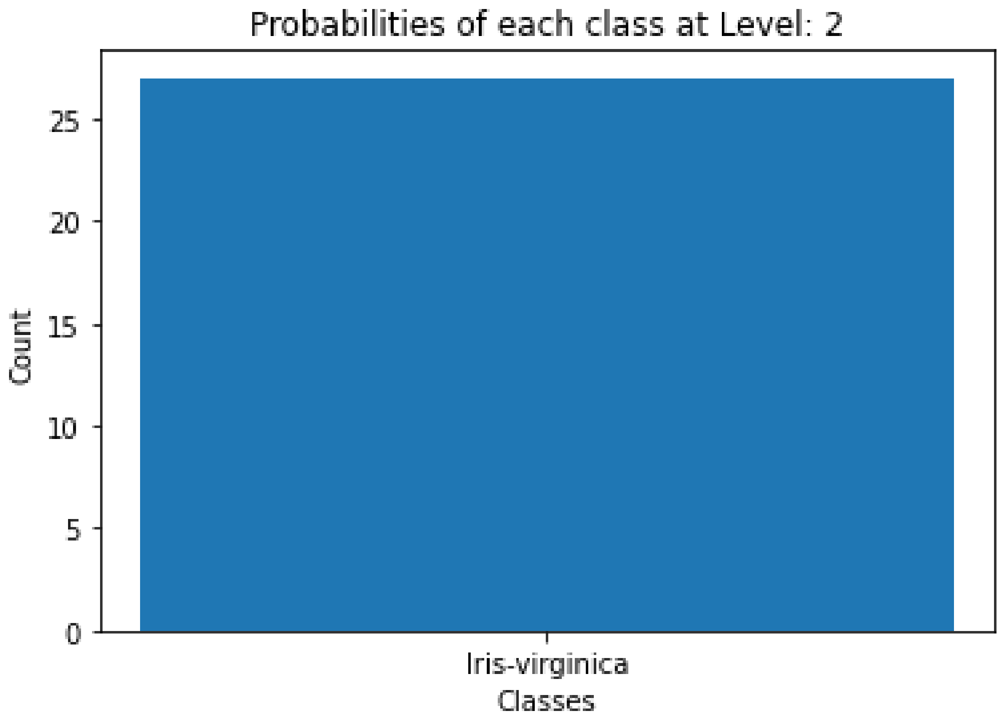

Probabilities of each class at Level: 1
35
30
25
20
15
10
5
0
Count
Iris-versicolor
Iris-virginica
Classes

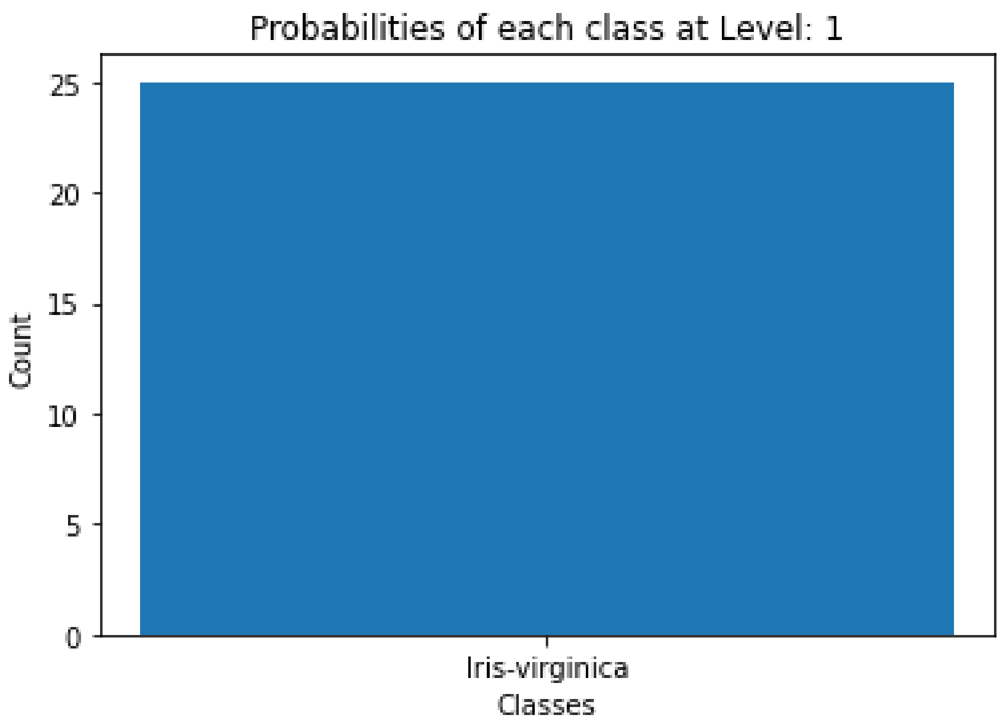

Probabilities of each class at Level: 0
35
30
25
20
15
10
5
0
Count
Iris-versicolor
Iris-virginica
Classes

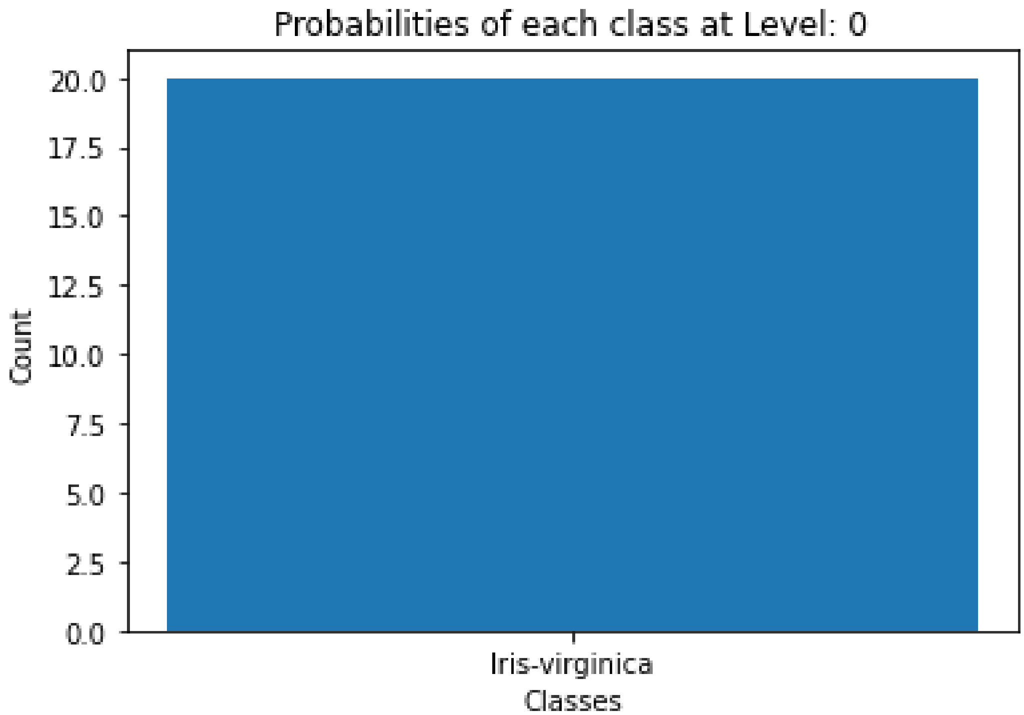

Probabilities of each class at Level: 0
1.0
0.8
0.6
0.4
0.2
0.0
Count
Iris-versicolor
Classes

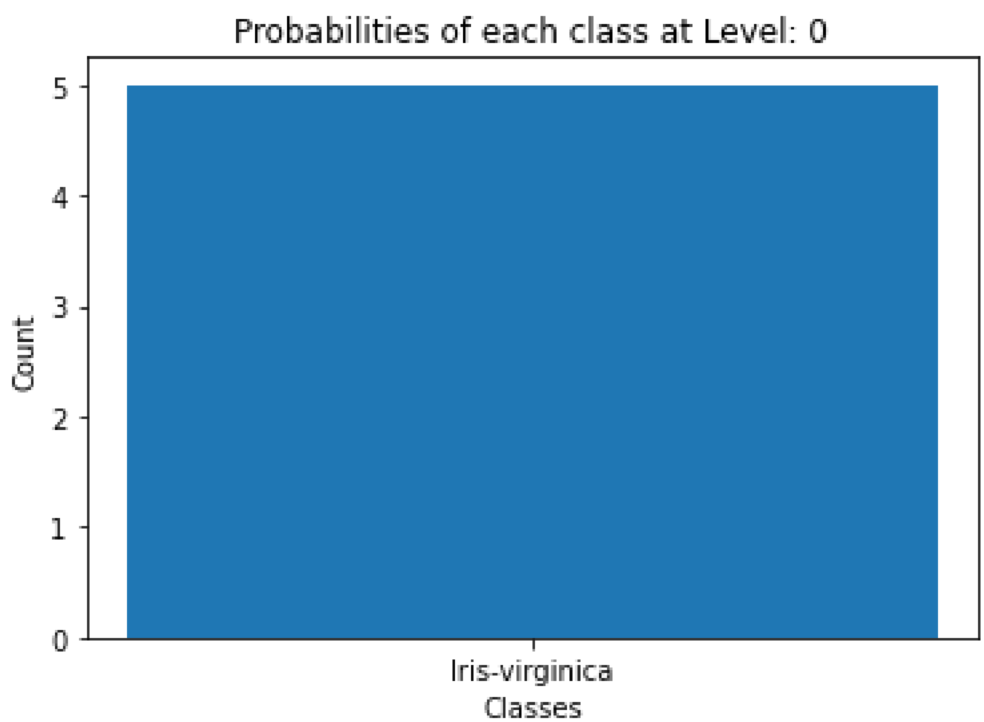

Probabilities of each class at Level: 2
1.0
0.8
0.6
0.4
0.2
0.0
Count
Iris-virginica
Classes

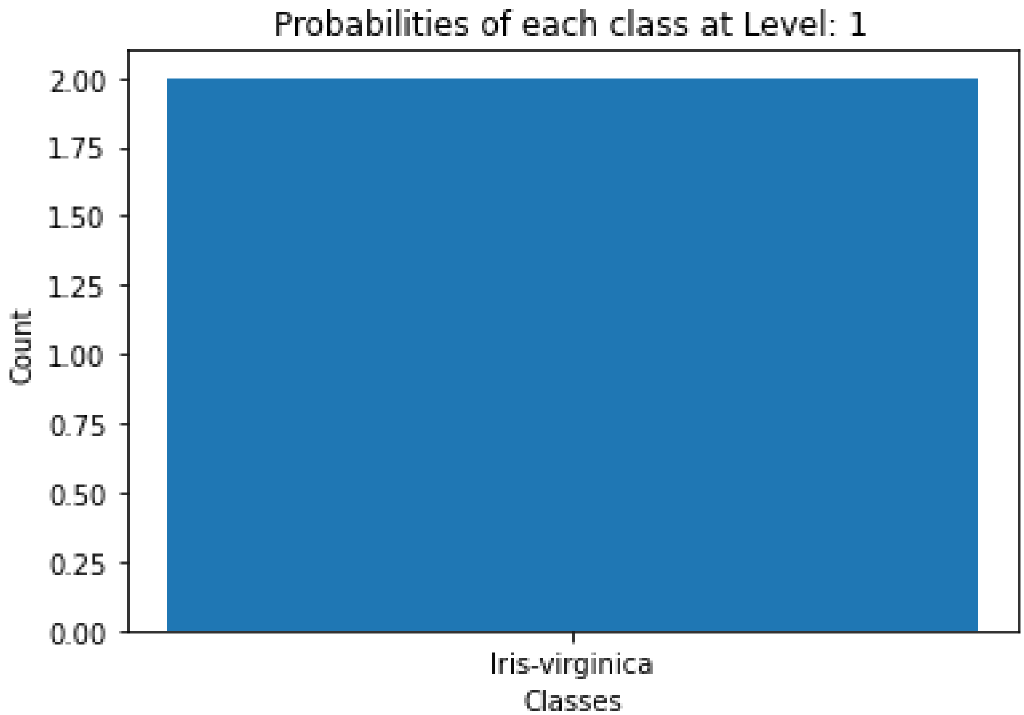

Probabilities of each class at Level: 3
30
25
20
15
10
5
0
Count
Iris-virginica
Classes

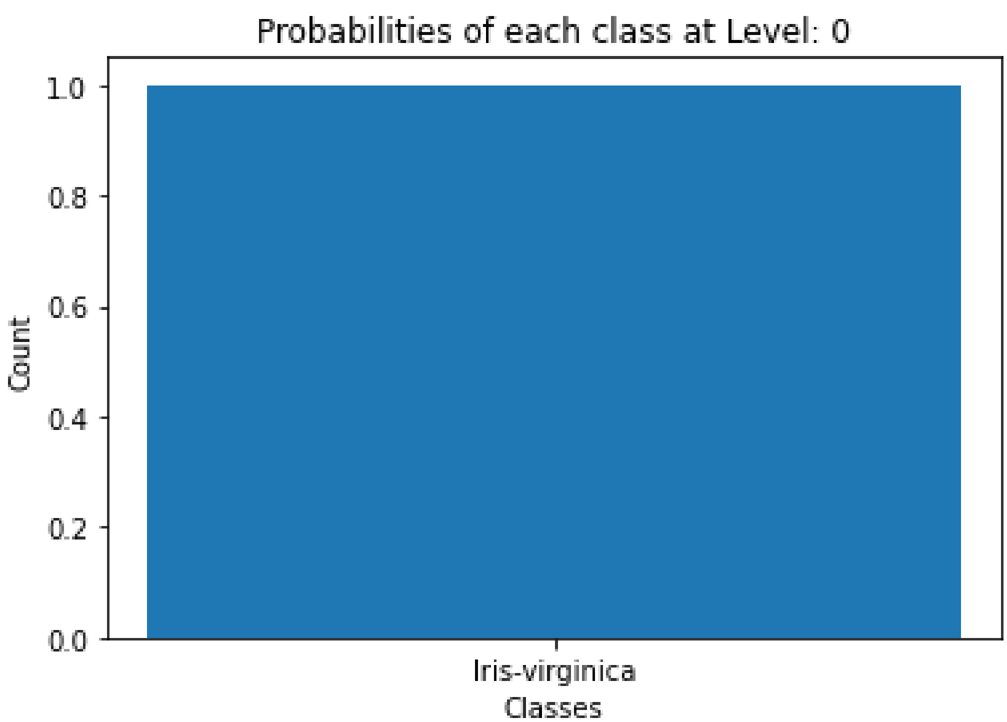

Probabilities of each class at Level: 2
Count
0
5
10
15
20
25
Iris-virginica
Classes

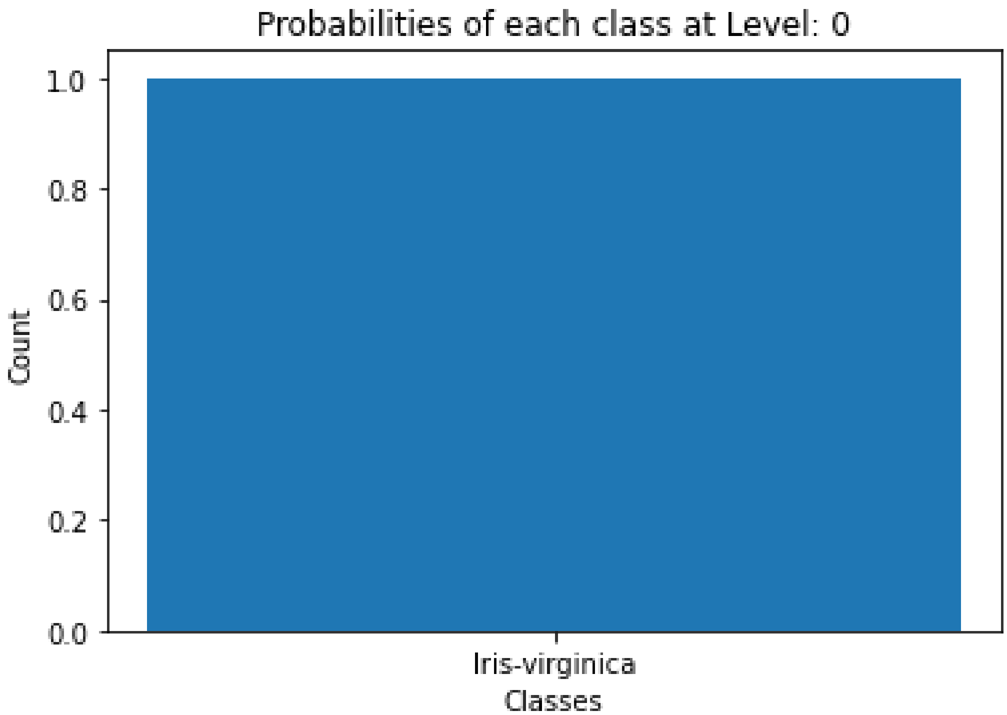

Probabilities of each class at Level: 1
Count
0
5
10
15
20
25
Iris-virginica
Classes

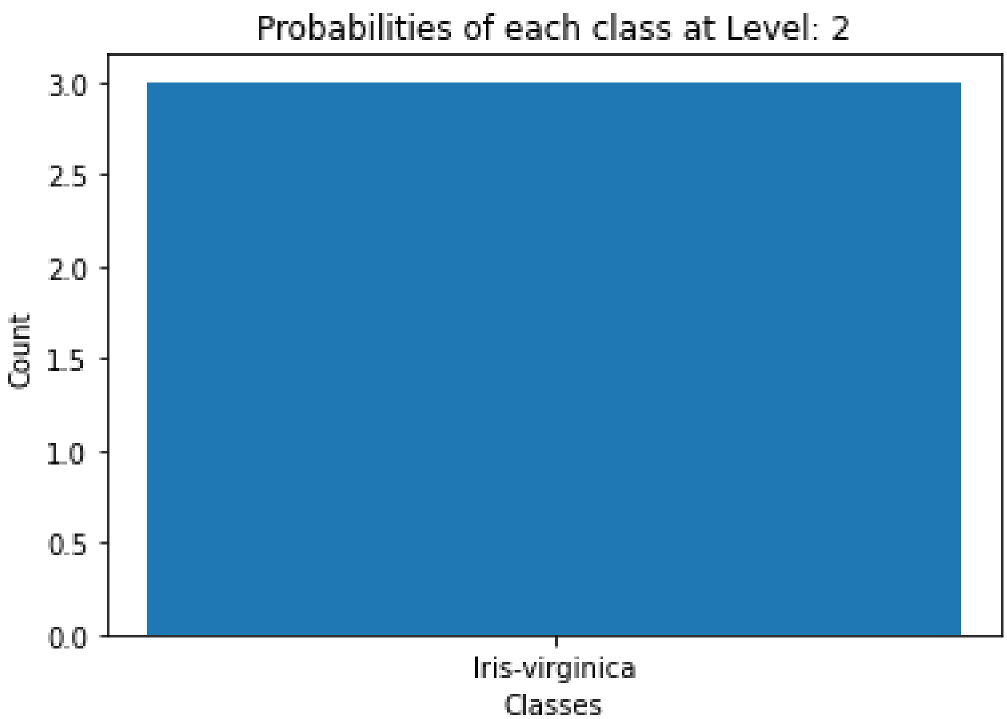

Probabilities of each class at Level: 0
20.0
17.5
15.0
12.5
10.0
7.5
5.0
2.5
0.0
Count
Iris-virginica
Classes

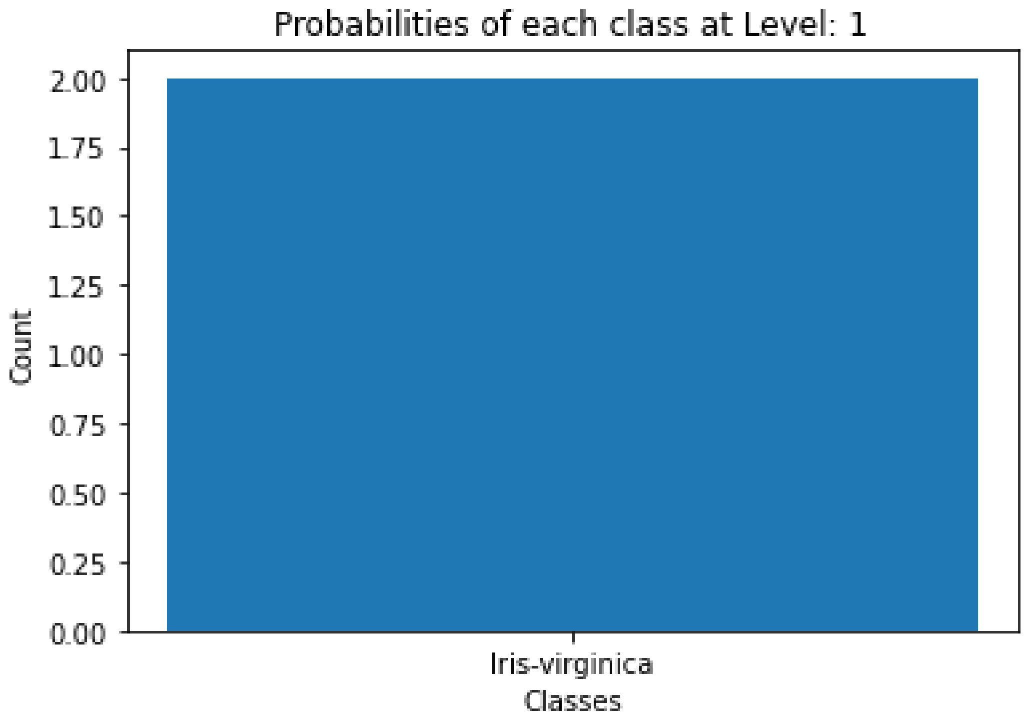

Probabilities of each class at Level: 0
5
4
3
2
1
0
Count
Iris-virginica
Classes

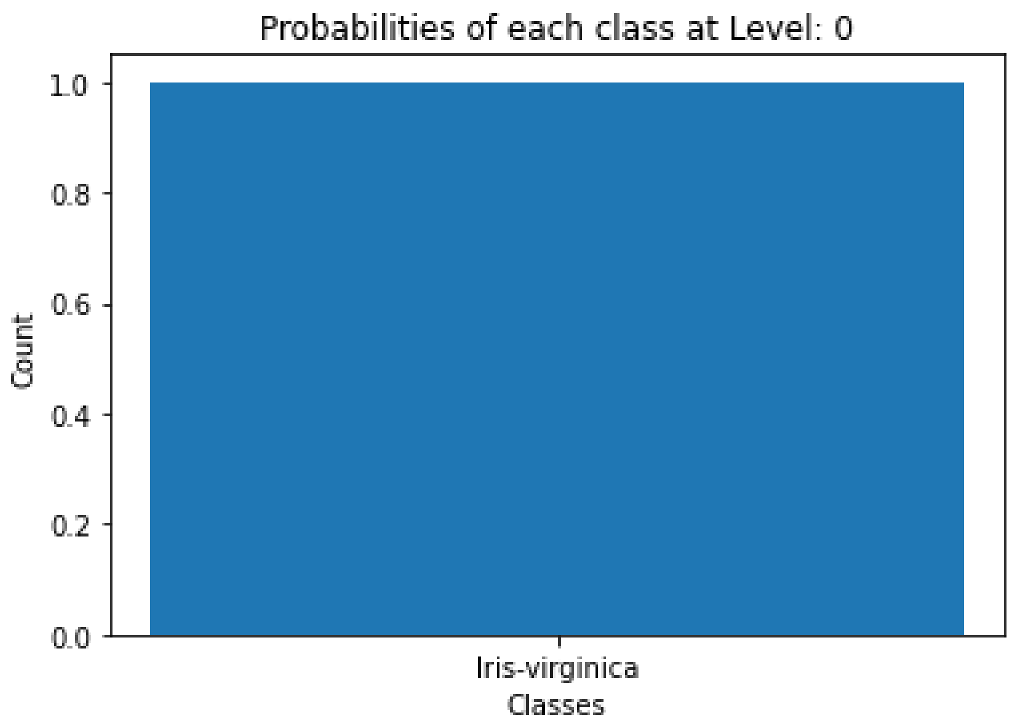

Probabilities of each class at Level: 1
Count
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
Iris-virginica
Classes

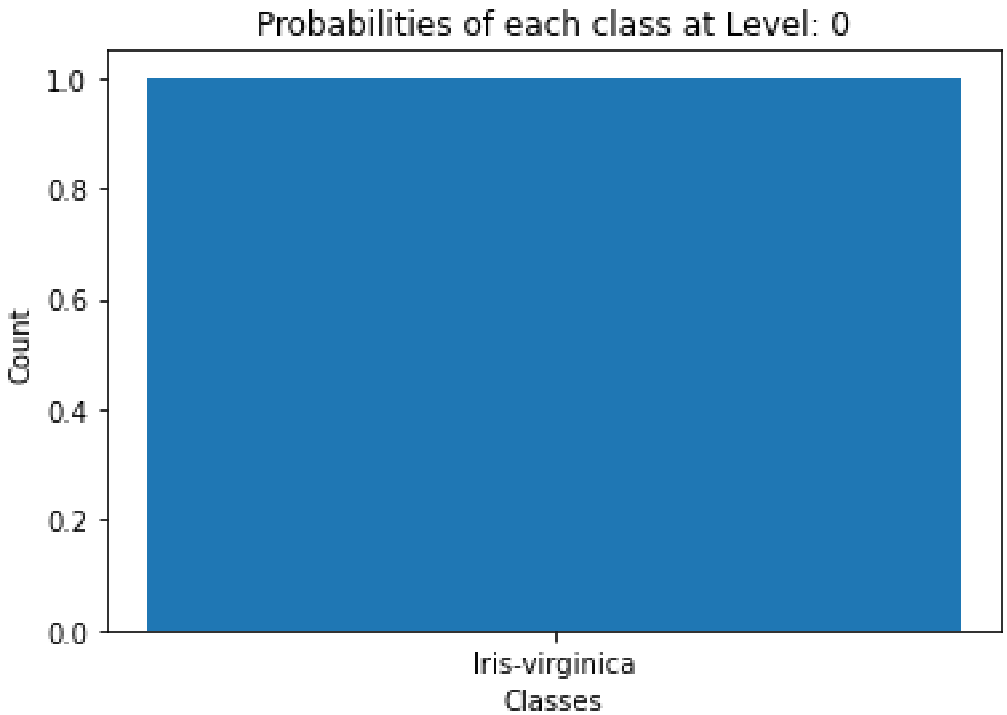

Probabilities of each class at Level: 0
Count
1.0
0.8
0.6
0.4
0.2
0.0
Iris-virginica
Classes

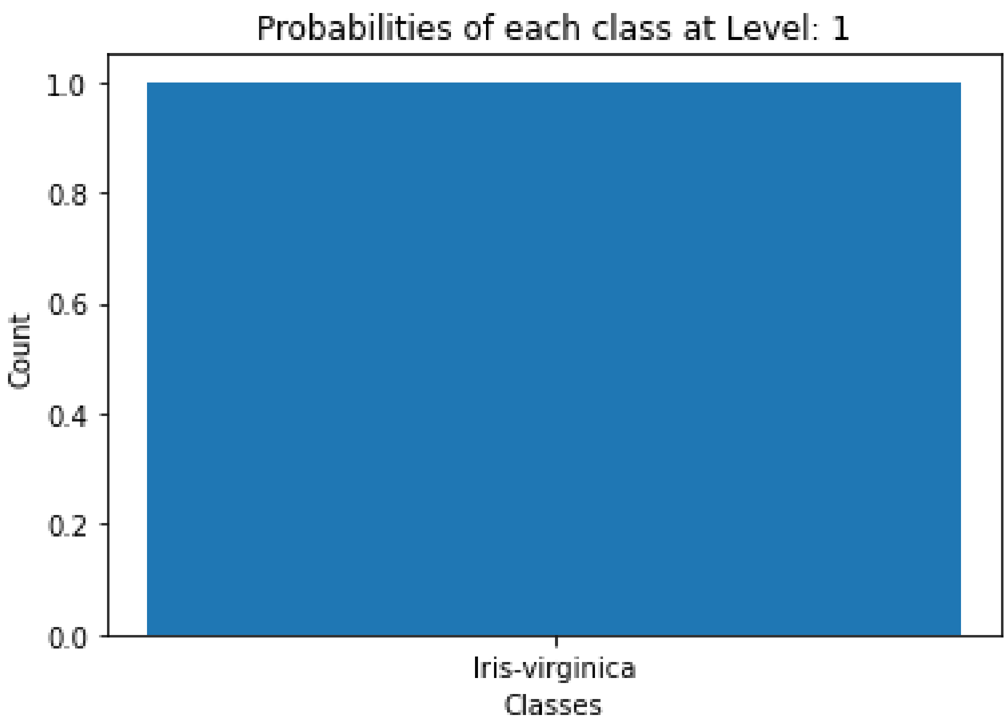

Probabilities of each class at Level: 0
1.0
0.8
0.6
0.4
0.2
0.0
Count
Iris-virginica
Classes


Probabilities of each class at Level: 2
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Count
Iris-virginica
Classes


Probabilities of each class at Level: 1
Count
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
Iris-virginica
Classes


Probabilities of each class at Level: 0
Count
1.0
0.8
0.6
0.4
0.2
0.0
Iris-virginica
Classes


Probabilities of each class at Level: 0
1.0
0.8
0.6
0.4
0.2
0.0
Count
Iris-virginica
Classes


Probabilities of each class at Level: 1
1.0
0.8
0.6
0.4
0.2
0.0
Count
Iris-virginica
Classes

```
Printing the Tree using a breath first tree traversal:

check for petal_width <= 0.75
True:
  check for petal_width <= 0.45
  True:
    check for petal_width <= 0.35
    True:
      check for petal_width <= 0.25
      True:
        check for petal_width <= 0.15000000000000002
        True:
          Leaf Node: Iris-setosa
        False:
          Leaf Node: Iris-setosa
      False:
        check for petal_length <= 1.45
        True:
          Leaf Node: Iris-setosa
        False:
          Leaf Node: Iris-setosa
    False:
      check for petal_length <= 1.65
      True:
        check for petal_length <= 1.4500000000000002
        True:
          Leaf Node: Iris-setosa
        False:
          Leaf Node: Iris-setosa
      False:
        Leaf Node: Iris-setosa
  False:
    Leaf Node: Iris-setosa
False:
  check for petal_width <= 1.75
  True:
    check for petal_length <= 5.35
    True:
      check for sepal_length <= 4.95
      True:
        Leaf Node: Iris-virginica
      False:
        check for petal_width <= 1.65
        True:
          Leaf Node: Iris-versicolor
        False:
          Leaf Node: Iris-versicolor
    False:
```

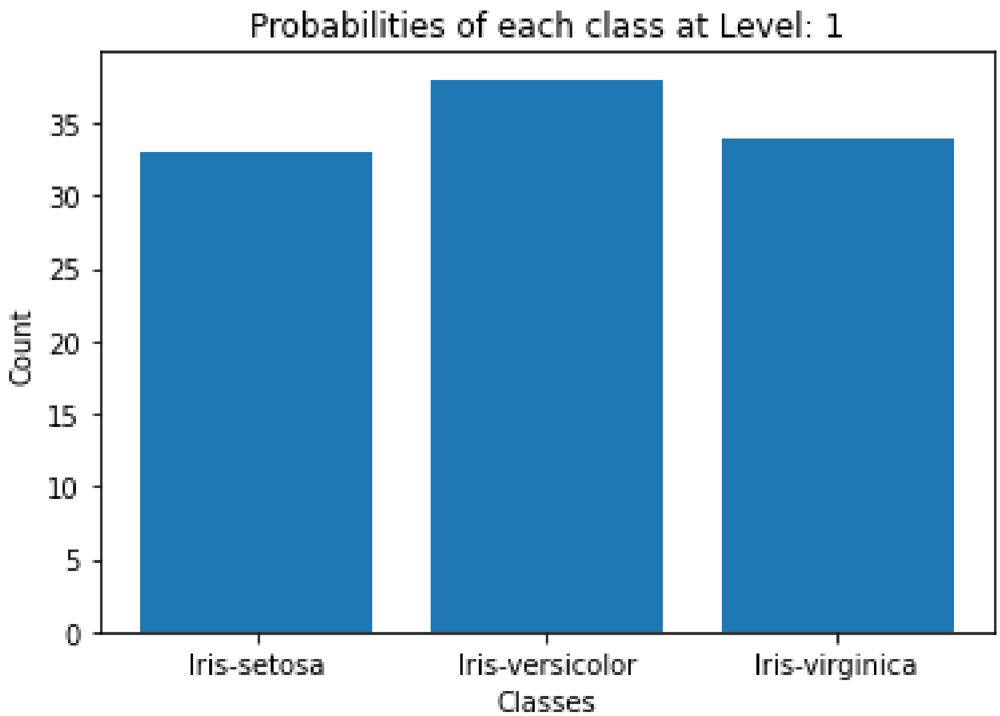
```
      Leaf Node: Iris-virginica
  False:
    check for petal_width <= 2.45
    True:
      check for petal_width <= 2.3499999999999996
      True:
        check for petal_width <= 2.25
        True:
          Leaf Node: Iris-virginica
        False:
          Leaf Node: Iris-virginica
      False:
        check for sepal_width <= 3.25
        True:
          Leaf Node: Iris-virginica
        False:
          Leaf Node: Iris-virginica
    False:
      check for petal_length <= 6.05
      True:
        check for petal_length <= 5.85
        True:
          Leaf Node: Iris-virginica
        False:
          Leaf Node: Iris-virginica
      False:
        Leaf Node: Iris-virginica

Accuracy: 0.9545454545454546

Cross Entropy Loss: 0.04652001563489282
```

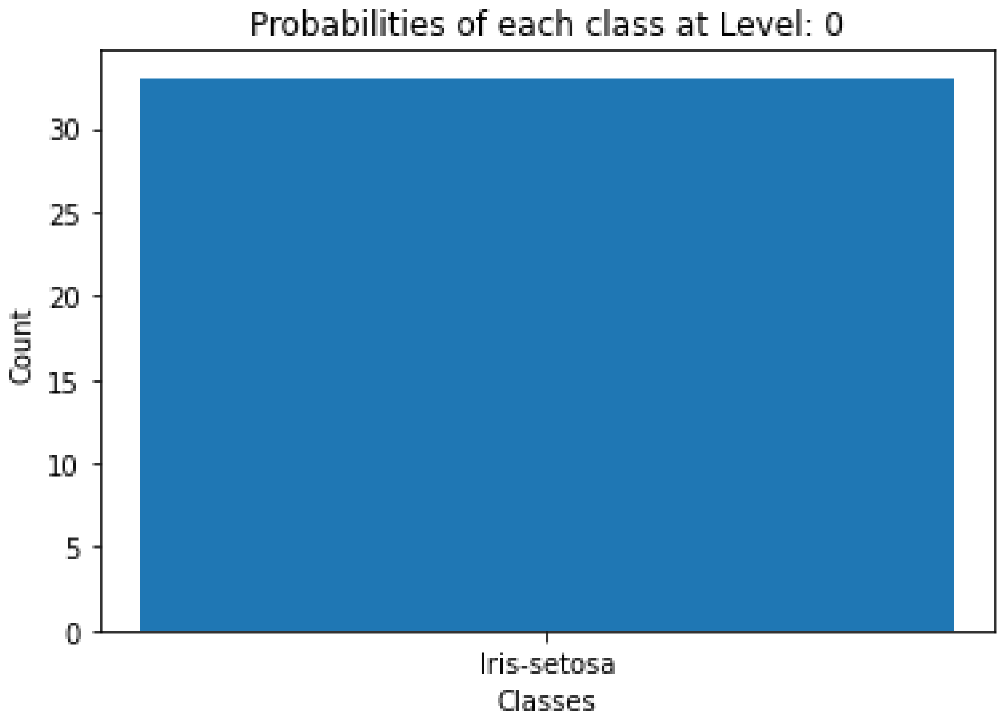
Part B: Experimenting with other Quality-criterion: Information Gain

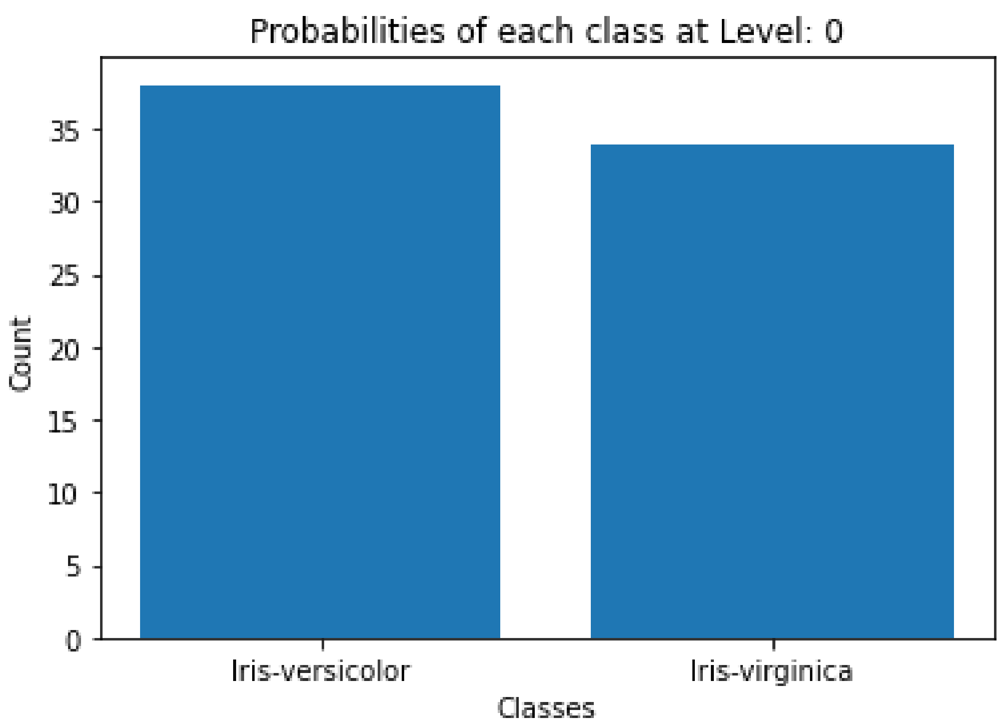
Applying Grid Search on Max-Depth, Printing the Resultant Tree and Displaying the Accuarcy and Cross Entropy Loss

[ ]:
```
for max_depth in max_depth_list:
    print('-------------------------------------------------------------\n')
    print('Learning Decision Tree with Information Gain and Max Depth: {}\n'.
 ↪format(max_depth))
    root = learn_decision_tree(iris_train, iris_headers, 4, 'gain' , max_depth)
    print('Printing the Tree using a breath first tree traversal:\n')
    print_tree(root)
    print('\nAccuracy: {}\n'.format(accuracy(root, iris_validation, 4)))
    print('Cross Entropy Loss: {}\n'.format(-1 * np.log(accuracy(root,␣
 ↪iris_validation, 4))))
```

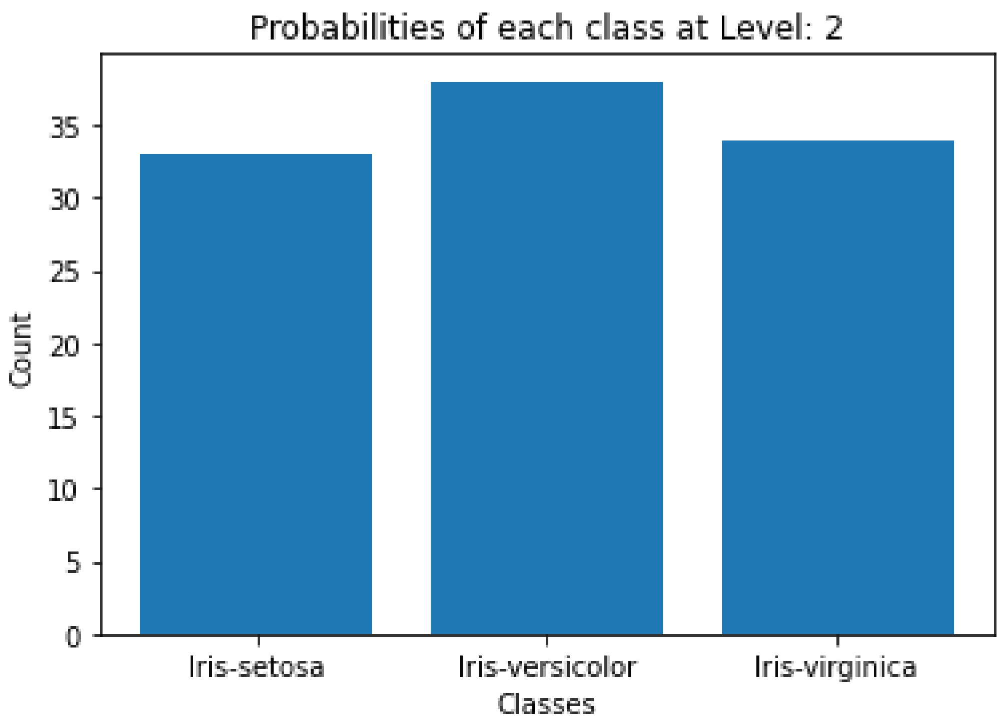
Learning Decision Tree with Information Gain and Max Depth: 1

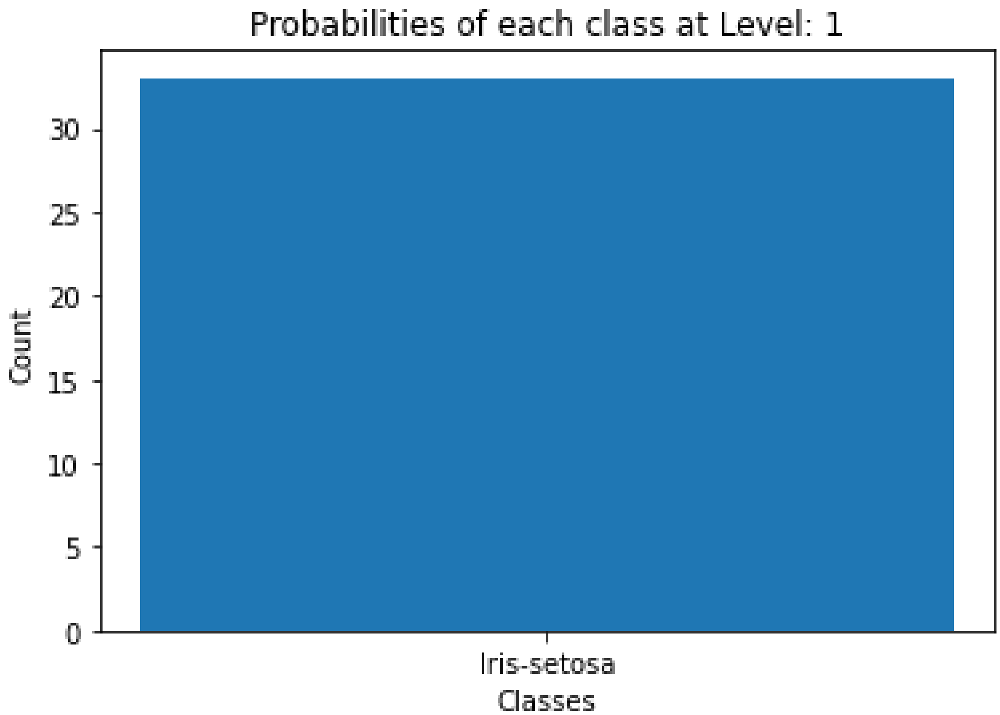


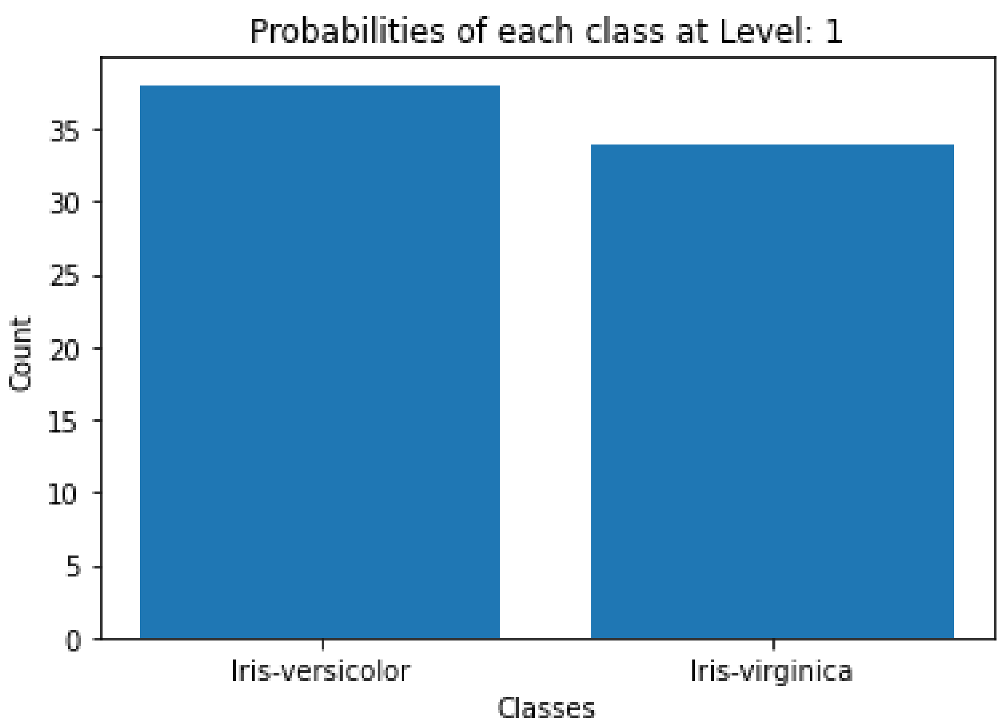

Probabilities of each class at Level: 0
Count
0
5
10
15
20
25
30
Iris-setosa
Classes

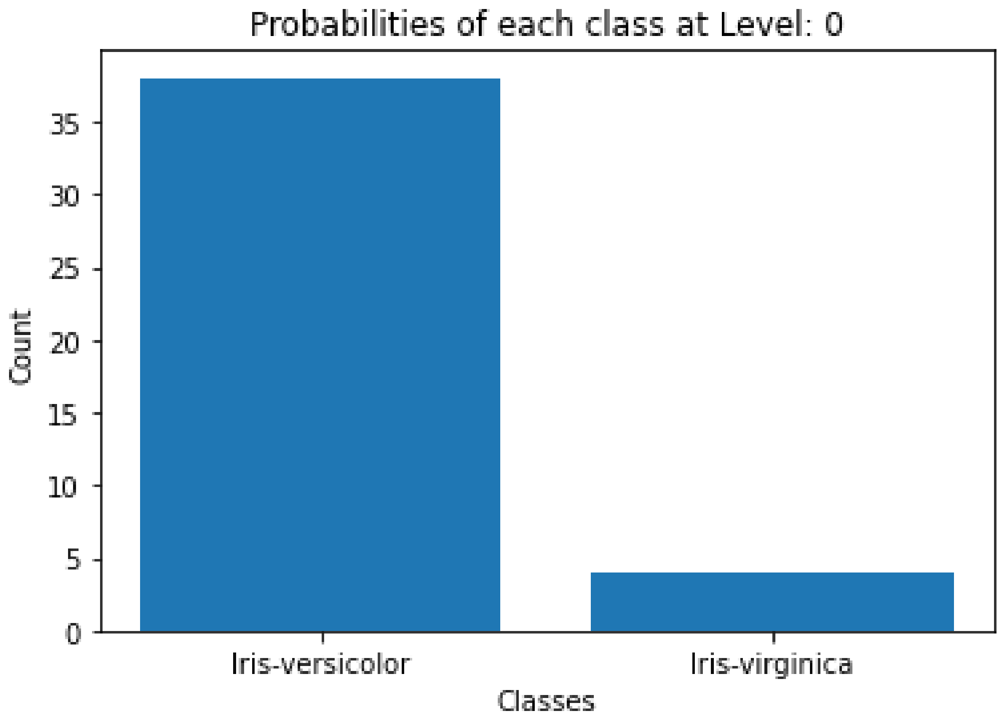

Probabilities of each class at Level: 0
Count
0
5
10
15
20
25
30
35
Iris-versicolor
Iris-virginica
Classes

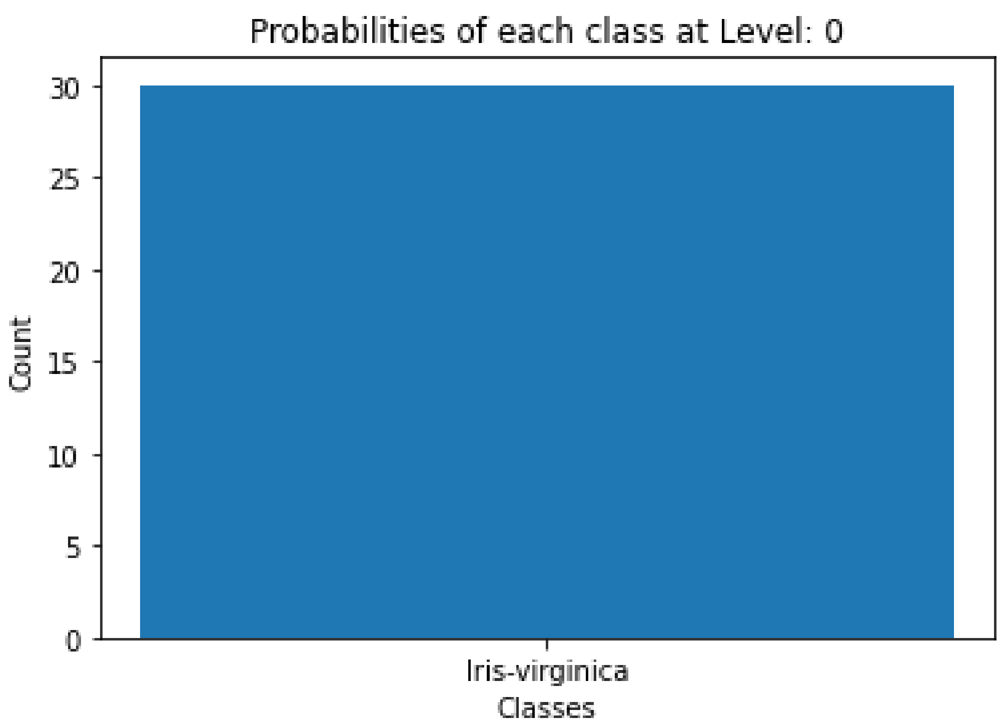
```
Printing the Tree using a breath first tree traversal:

check for petal_width <= 0.75
True:
  Leaf Node: Iris-setosa
False:
  Leaf Node: Iris-versicolor

Accuracy: 0.6363636363636364

Cross Entropy Loss: 0.45198512374305727

--------------------------------------------------------------

Learning Decision Tree with Information Gain and Max Depth: 2
```



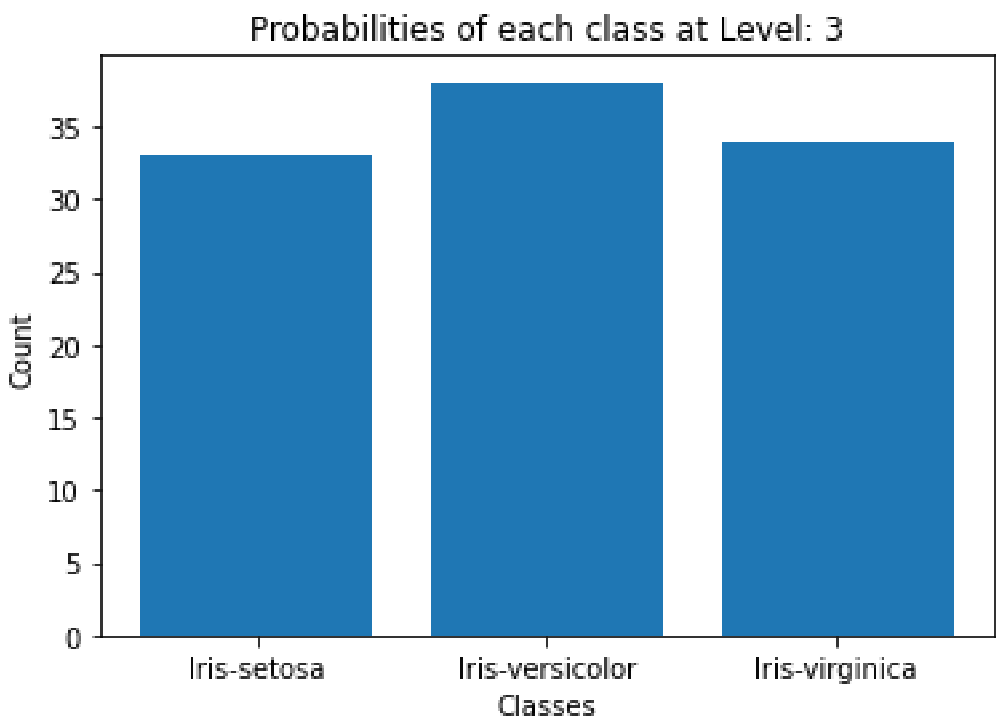

Probabilities of each class at Level: 1
Count
0
5
10
15
20
25
30
Iris-setosa
Classes

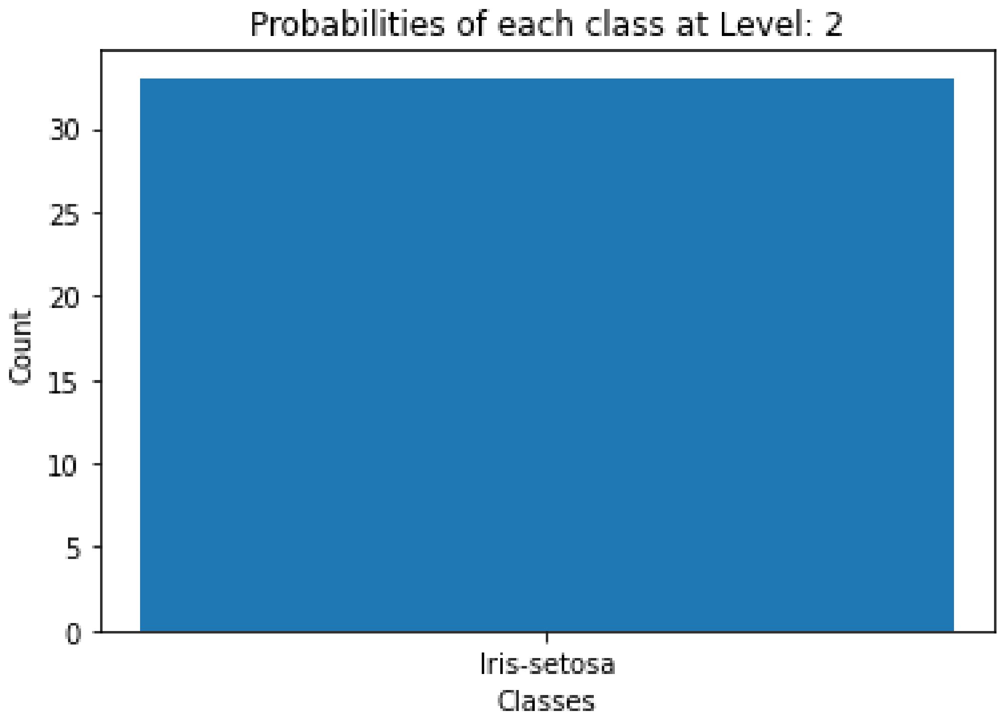

Probabilities of each class at Level: 1
Count
0
5
10
15
20
25
30
35
Iris-versicolor
Iris-virginica
Classes

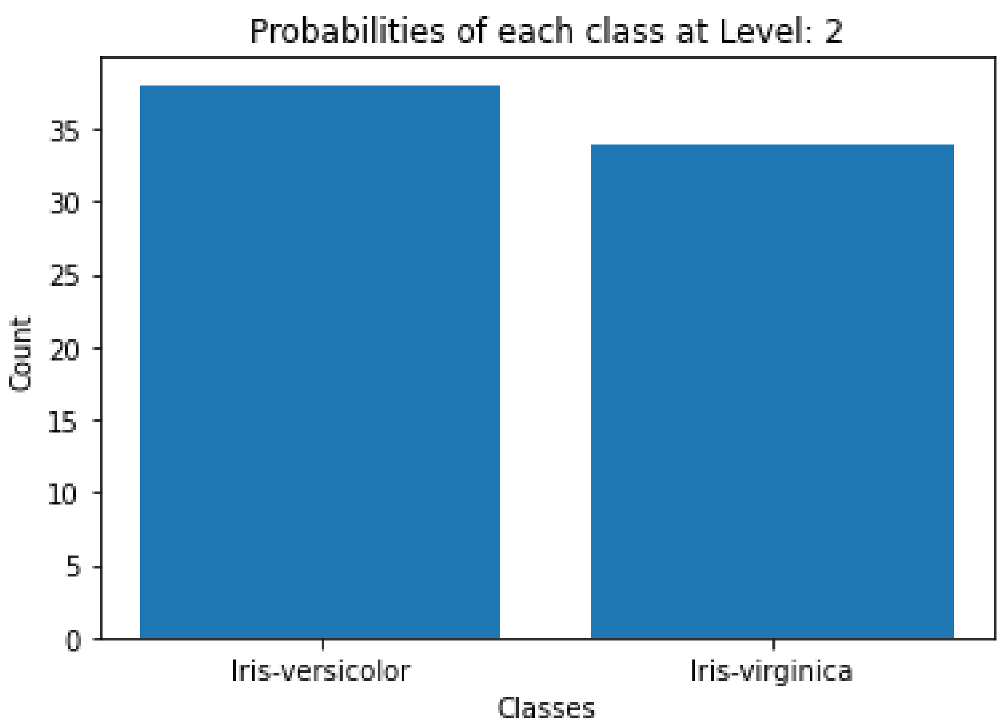

Probabilities of each class at Level: 0
Count
0
5
10
15
20
25
30
35
Iris-versicolor
Iris-virginica
Classes

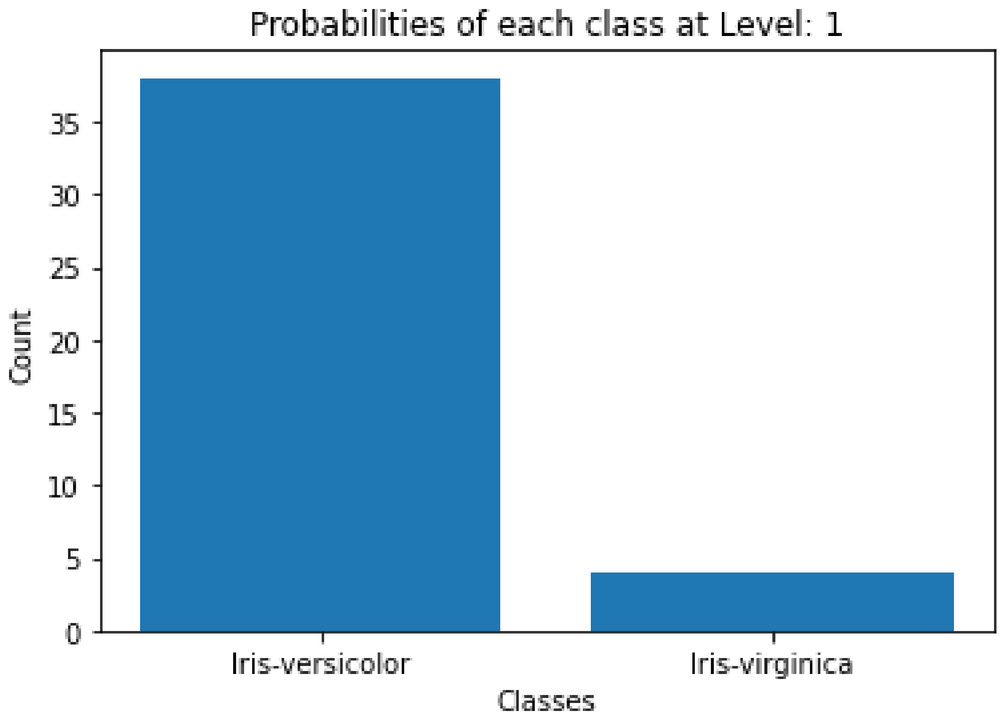

Probabilities of each class at Level: 0
Count
0
5
10
15
20
25
30
Iris-virginica
Classes

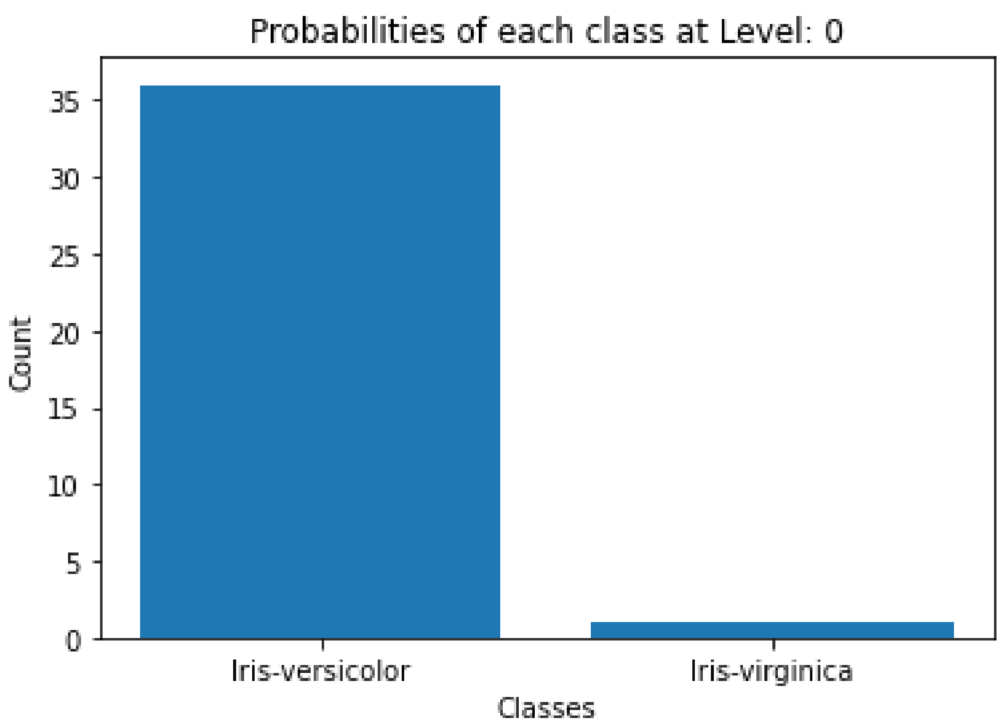
```
Printing the Tree using a breath first tree traversal:

check for petal_width <= 0.75
True:
  Leaf Node: Iris-setosa
False:
  check for petal_width <= 1.75
  True:
    Leaf Node: Iris-versicolor
  False:
    Leaf Node: Iris-virginica

Accuracy: 0.9545454545454546

Cross Entropy Loss: 0.04652001563489282

--------------------------------------------------------------

Learning Decision Tree with Information Gain and Max Depth: 3
```

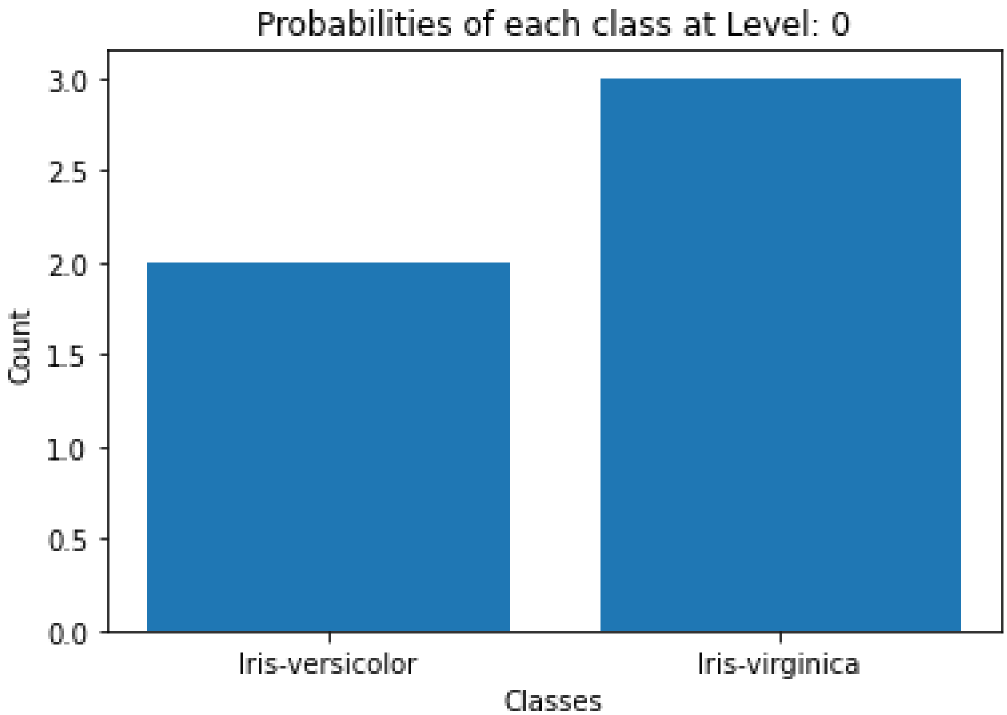


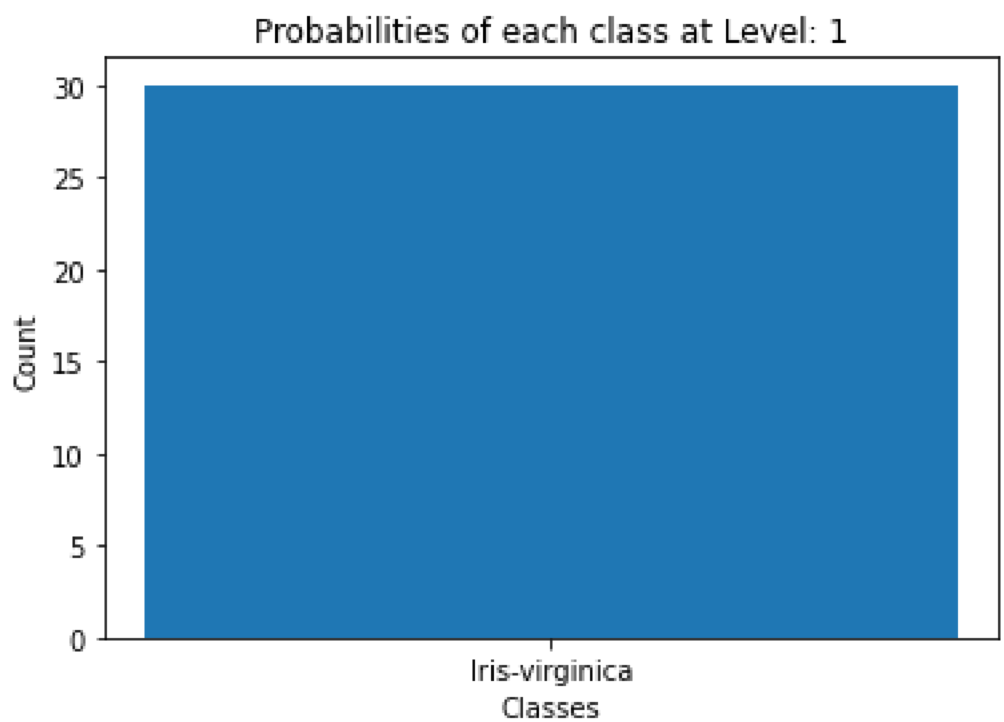

Probabilities of each class at Level: 2
Count
0
5
10
15
20
25
30
Iris-setosa
Classes


Probabilities of each class at Level: 2
Count
0
5
10
15
20
25
30
35
Iris-versicolor
Iris-virginica
Classes

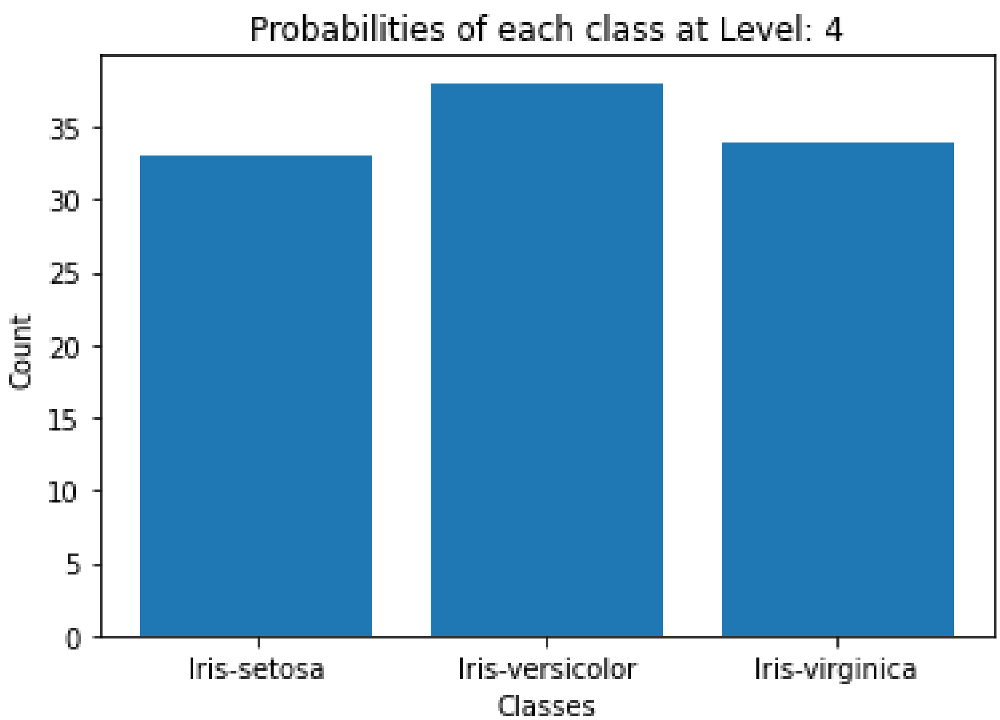

Probabilities of each class at Level: 1
Count
0
5
10
15
20
25
30
35
Iris-versicolor
Iris-virginica
Classes

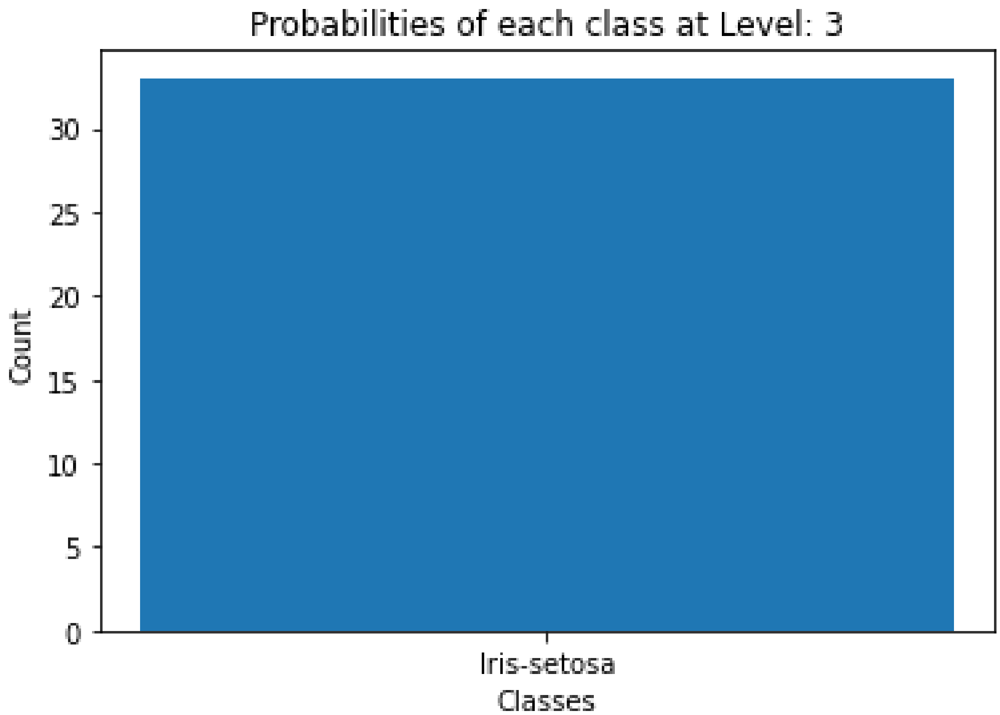

Probabilities of each class at Level: 0
Count
0
5
10
15
20
25
30
35
Iris-versicolor
Iris-virginica
Classes

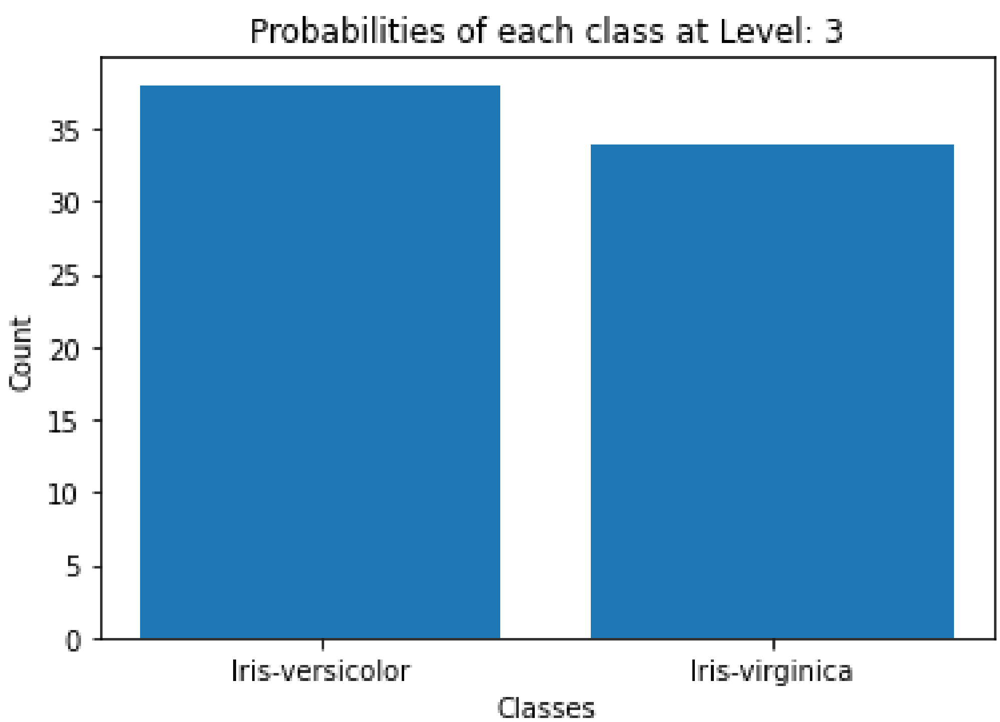

Probabilities of each class at Level: 0
Count
Iris-versicolor
Iris-virginica
Classes

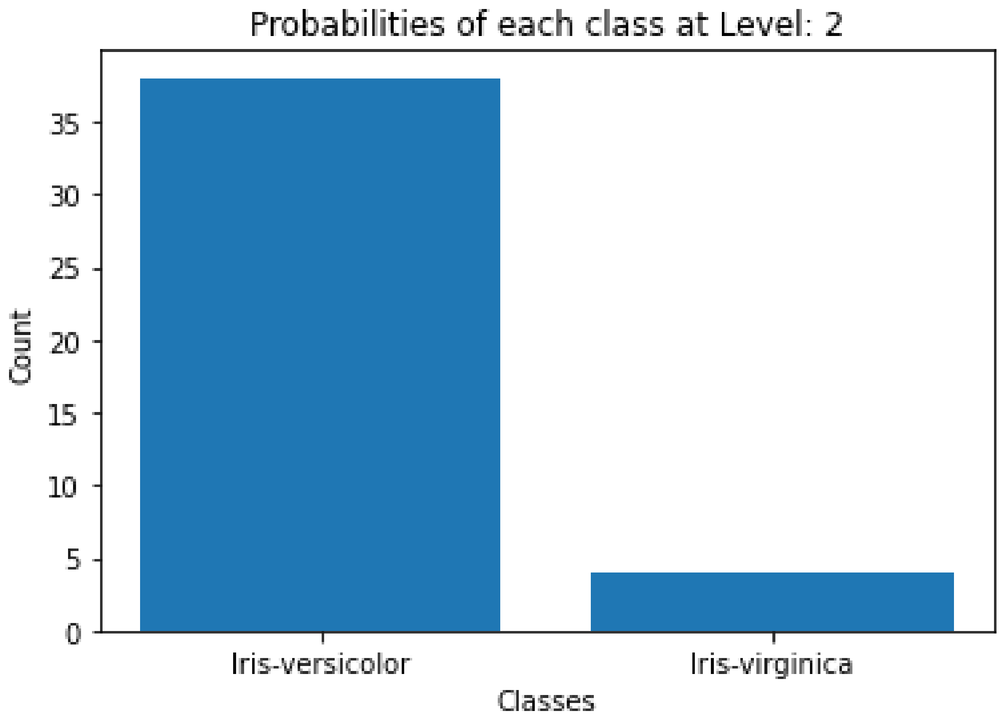

Probabilities of each class at Level: 1
Count
Iris-virginica
Classes

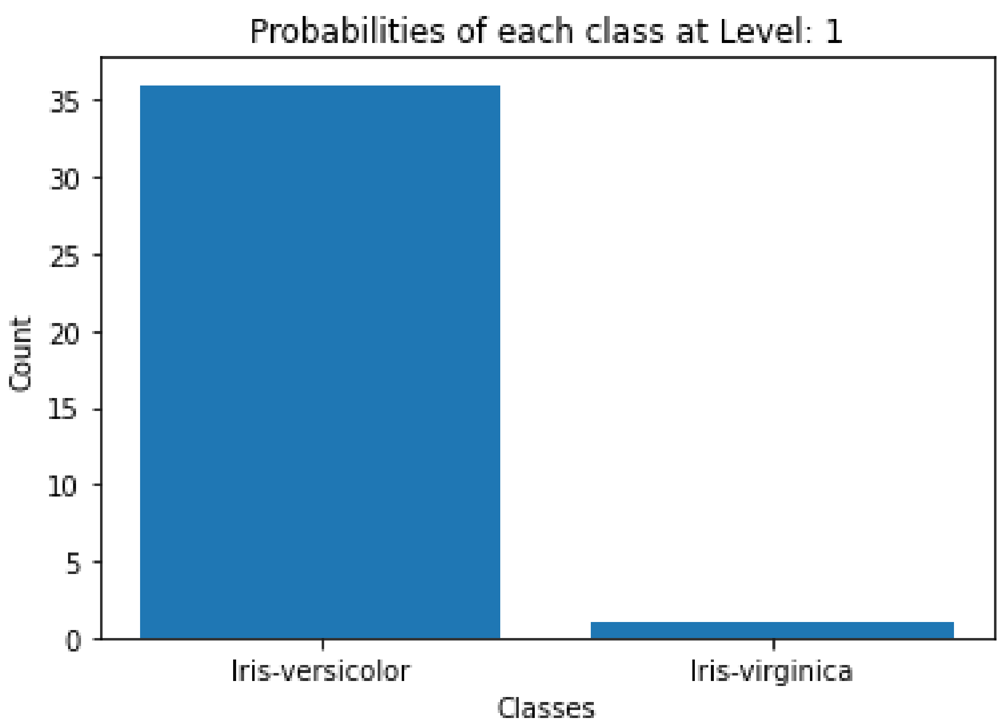
```
Printing the Tree using a breath first tree traversal:

check for petal_width <= 0.75
True:
  Leaf Node: Iris-setosa
False:
  check for petal_width <= 1.75
  True:
    check for petal_length <= 4.95
    True:
      Leaf Node: Iris-versicolor
    False:
      Leaf Node: Iris-virginica
  False:
    Leaf Node: Iris-virginica

Accuracy: 1.0

Cross Entropy Loss: -0.0

--------------------------------------------------------------

Learning Decision Tree with Information Gain and Max Depth: 4
```

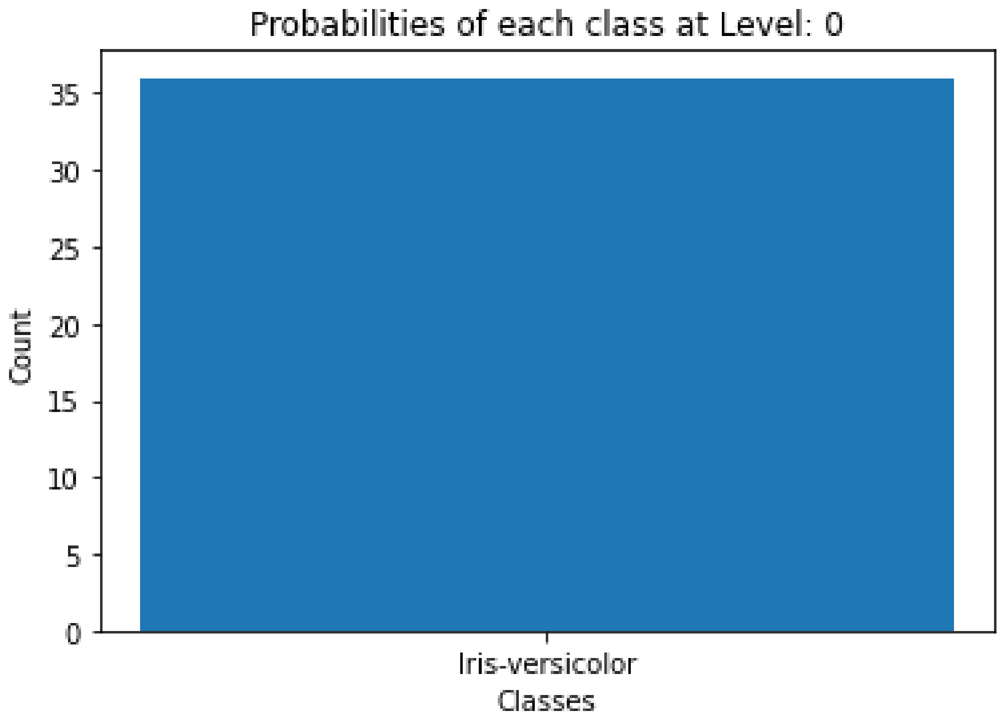


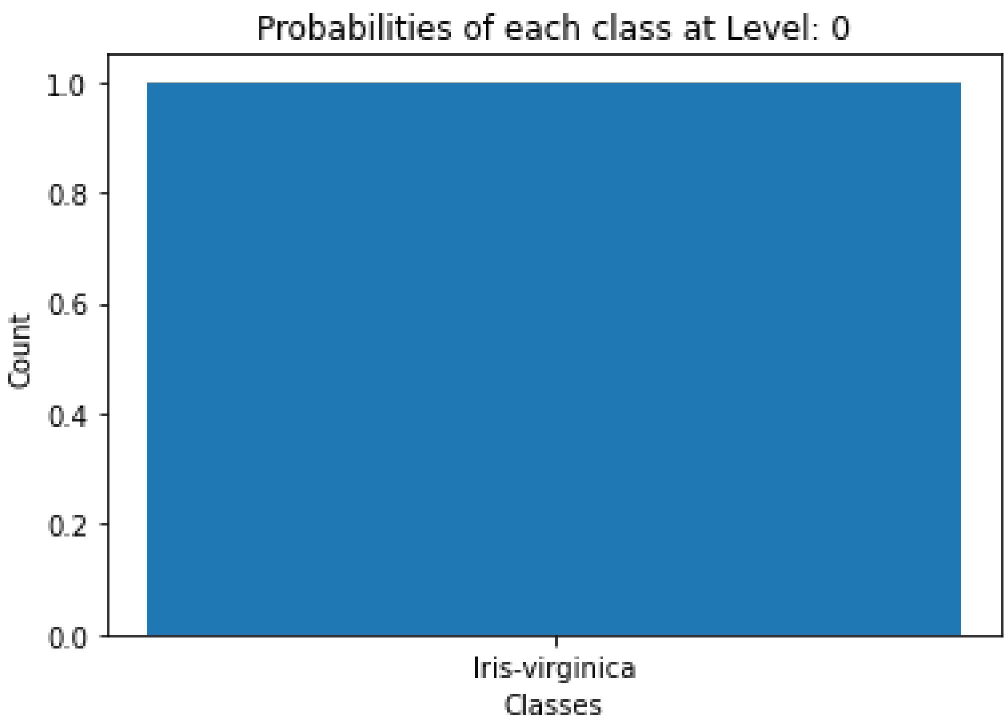

Probabilities of each class at Level: 3
Count
0
5
10
15
20
25
30
Iris-setosa
Classes

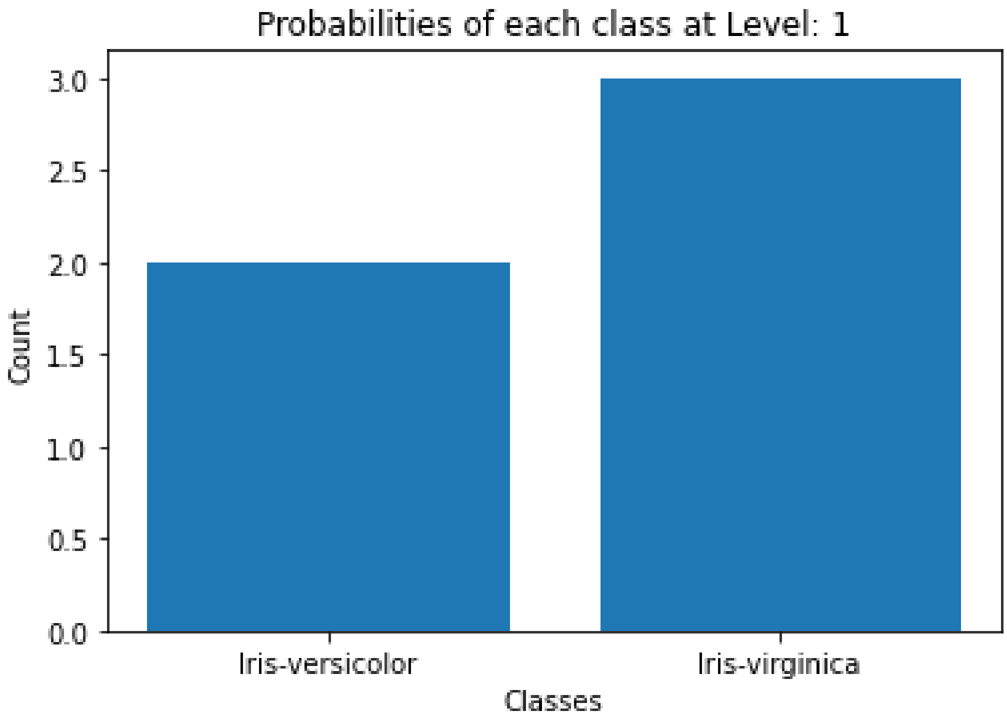

Probabilities of each class at Level: 3
Count
0
5
10
15
20
25
30
35
Iris-versicolor
Iris-virginica
Classes


Probabilities of each class at Level: 2
Count
0
5
10
15
20
25
30
35
Iris-versicolor
Iris-virginica
Classes

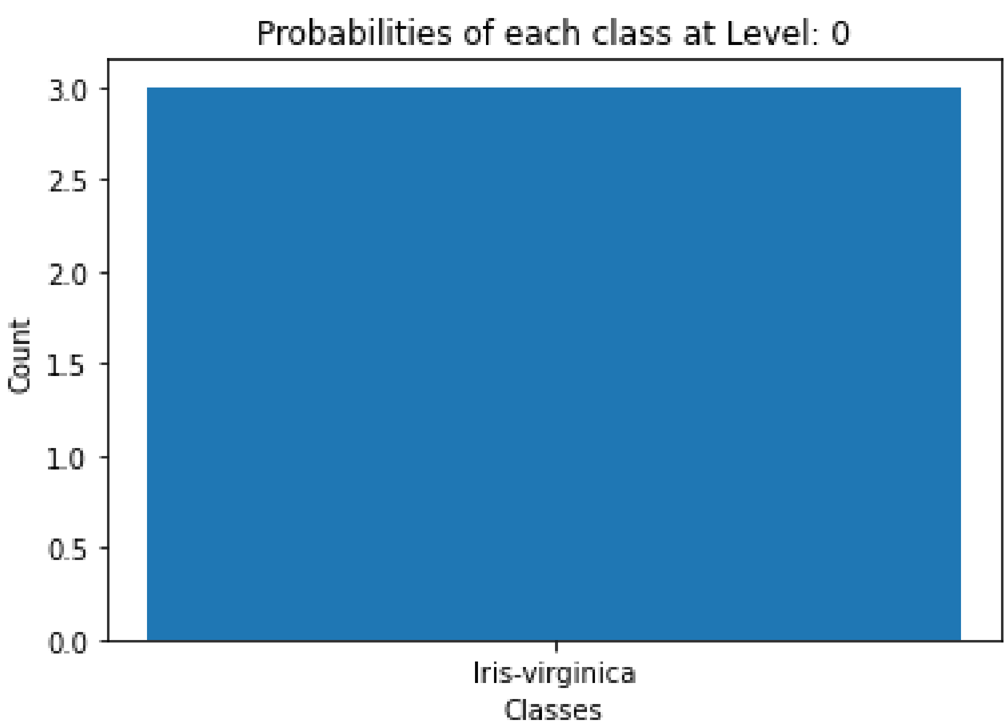

Probabilities of each class at Level: 1
Count
0
5
10
15
20
25
30
35
Iris-versicolor
Iris-virginica
Classes


Probabilities of each class at Level: 0
35
30
25
20
15
10
5
0
Count
Iris-versicolor
Classes

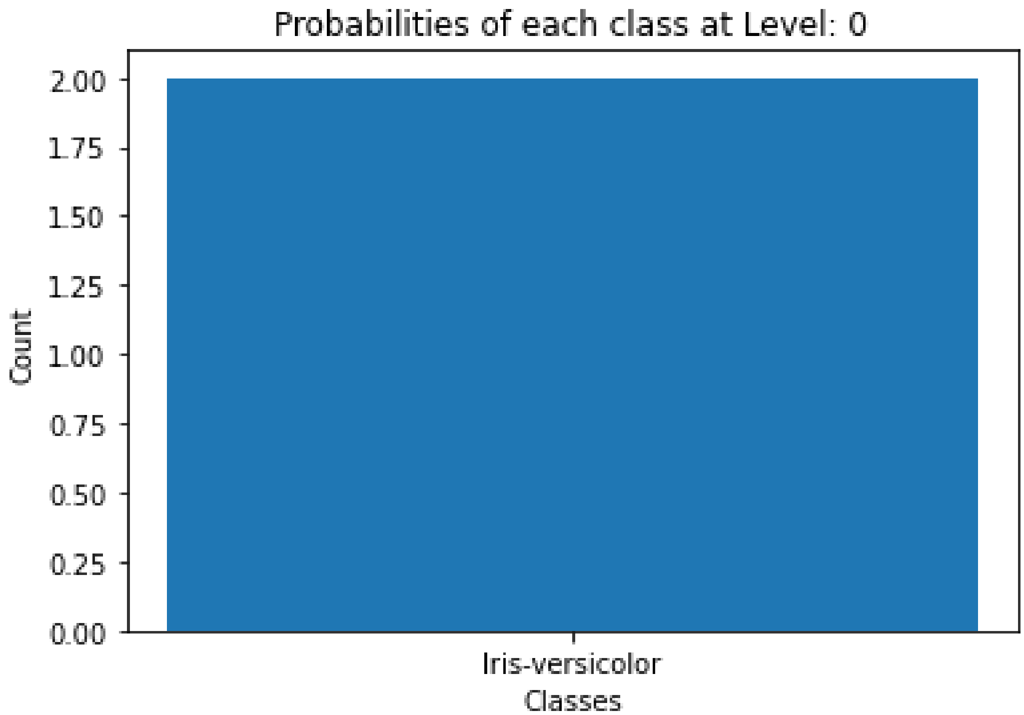

Probabilities of each class at Level: 0
1.0
0.8
0.6
0.4
0.2
0.0
Count
Iris-virginica
Classes

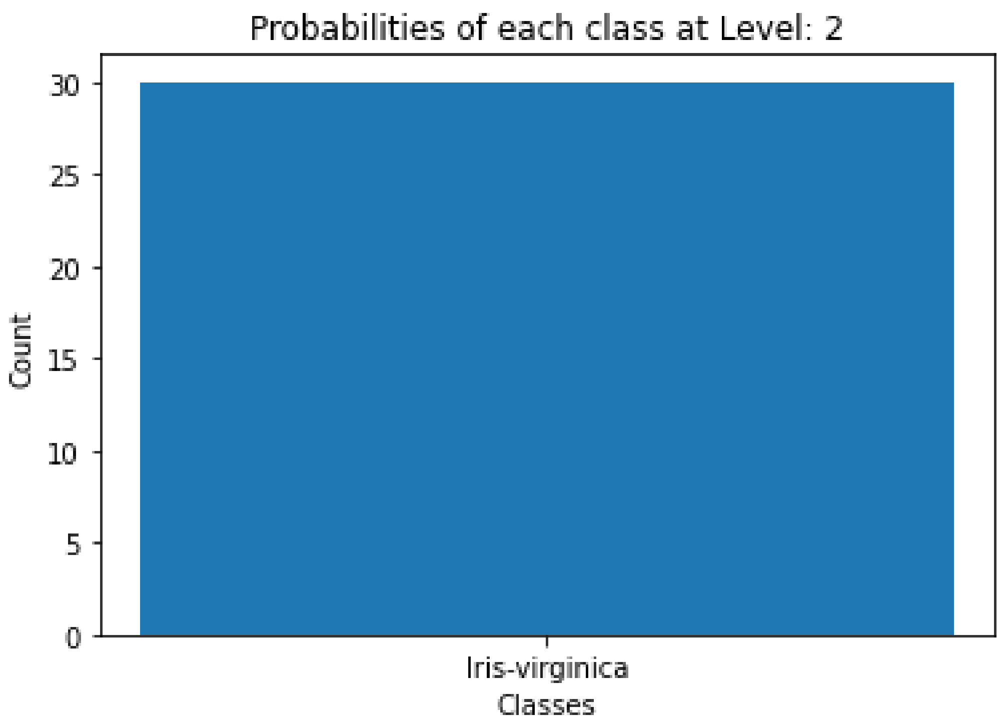


Probabilities of each class at Level: 1
Count
Iris-versicolor
Iris-virginica
Classes

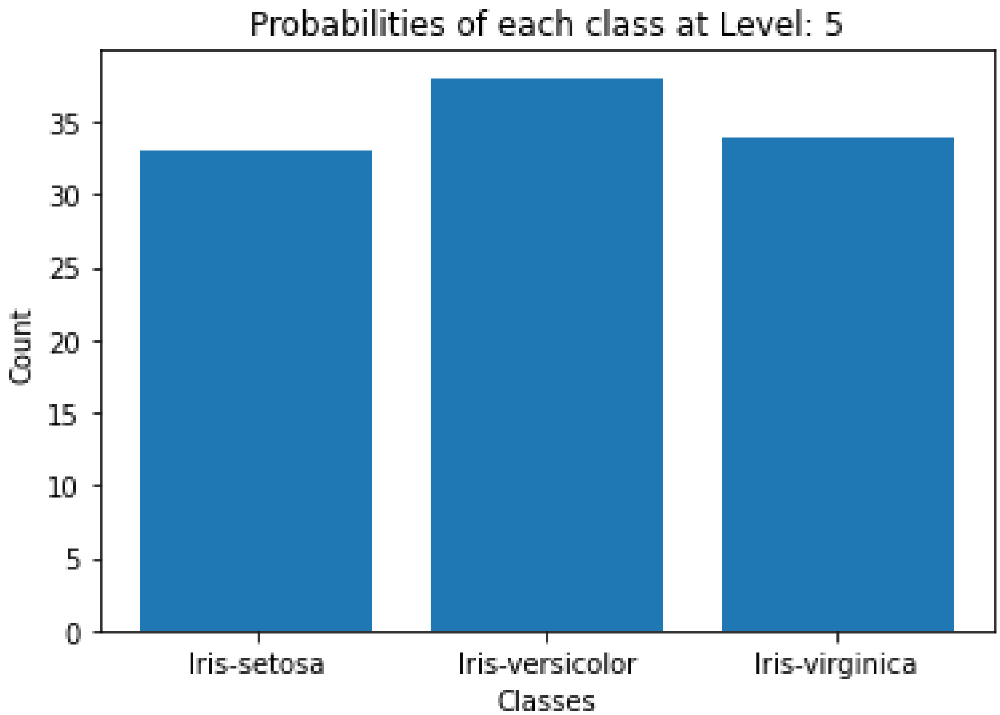


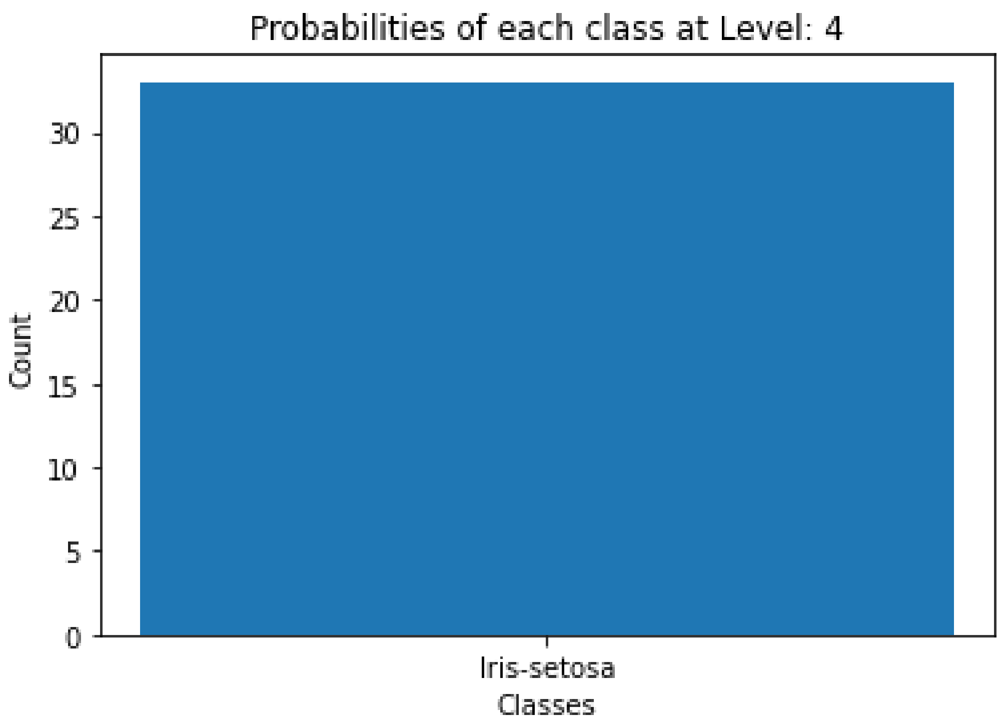
Probabilities of each class at Level: 0
Count
Iris-virginica
Classes

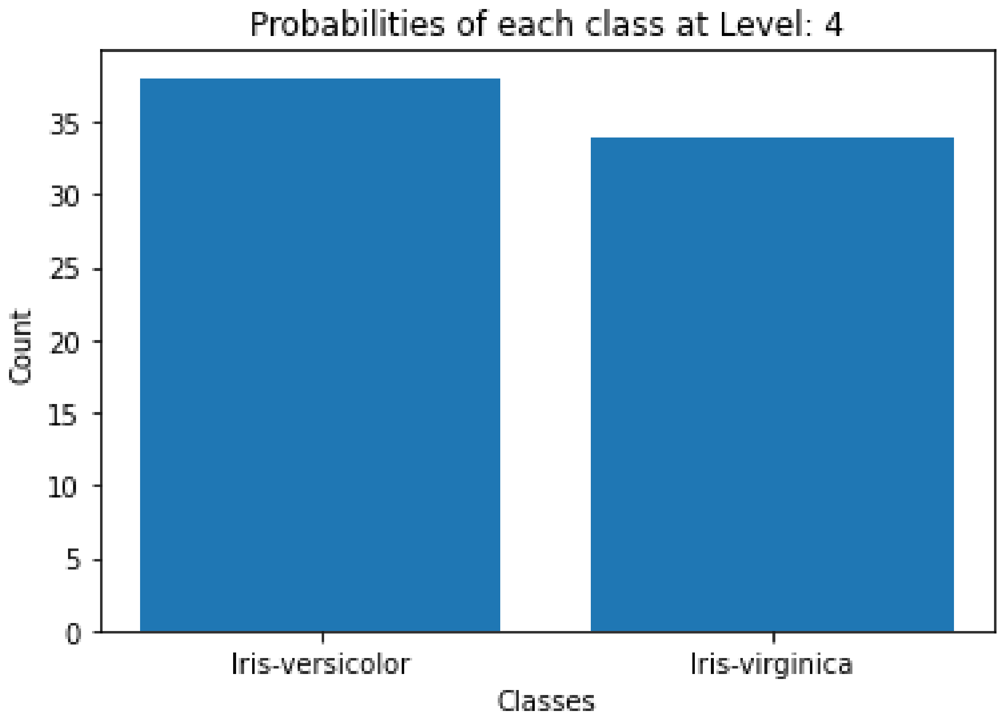

Probabilities of each class at Level: 0
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
Count
Iris-versicolor
Classes

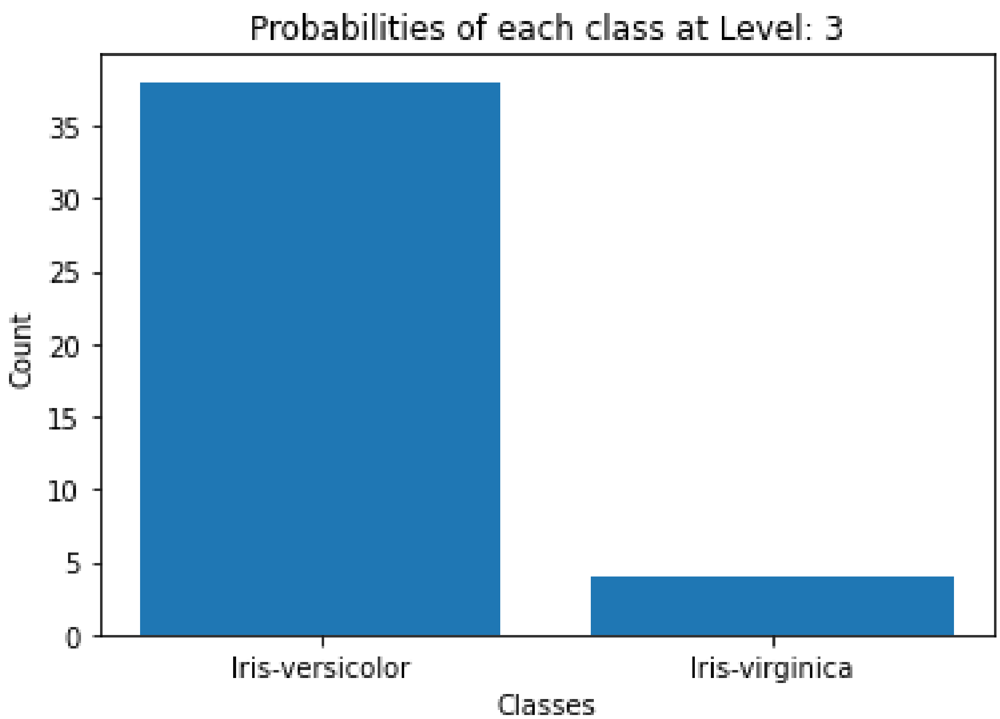

Probabilities of each class at Level: 2
30
25
20
15
10
5
0
Count
Iris-virginica
Classes

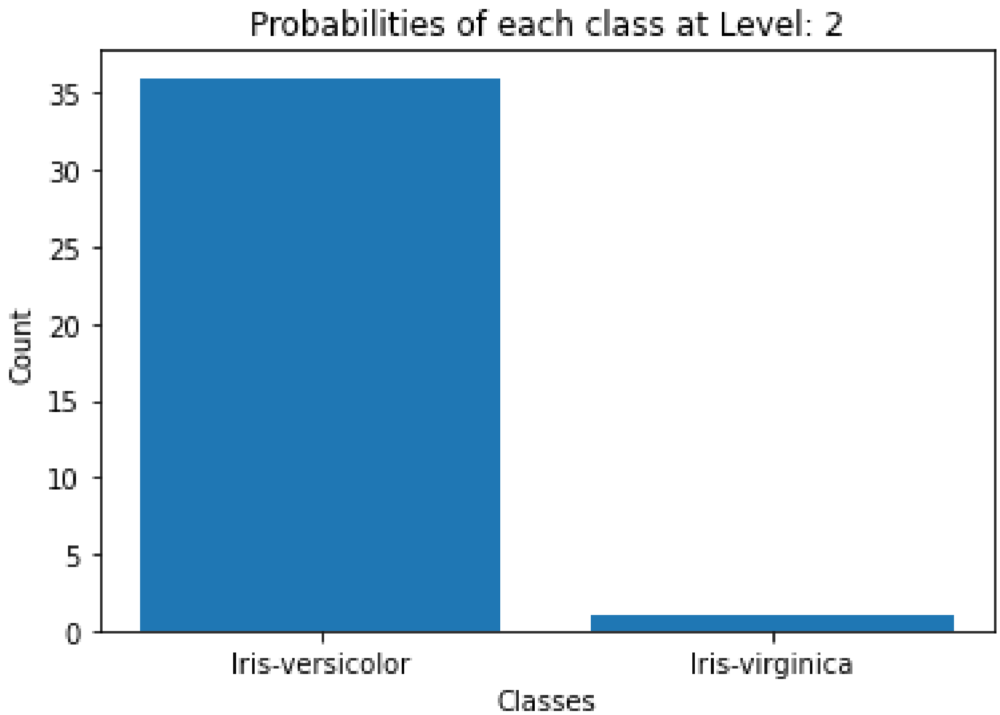
```
Printing the Tree using a breath first tree traversal:

check for petal_width <= 0.75
True:
  Leaf Node: Iris-setosa
False:
  check for petal_width <= 1.75
  True:
    check for petal_length <= 4.95
    True:
      check for petal_width <= 1.65
      True:
        Leaf Node: Iris-versicolor
      False:
        Leaf Node: Iris-virginica
    False:
      check for petal_width <= 1.55
      True:
        Leaf Node: Iris-virginica
      False:
        Leaf Node: Iris-versicolor
  False:
    Leaf Node: Iris-virginica

Accuracy: 0.9545454545454546

Cross Entropy Loss: 0.04652001563489282

--------------------------------------------------------------

Learning Decision Tree with Information Gain and Max Depth: 5
```

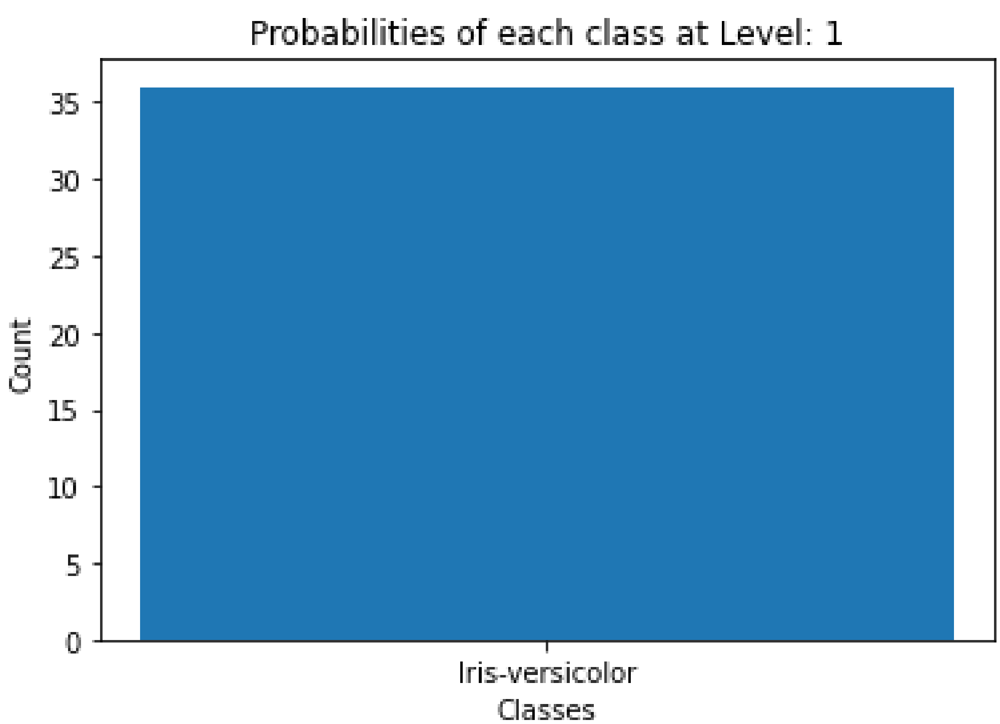

Probabilities of each class at Level: 5
Count
35
30
25
20
15
10
5
0
Iris-setosa
Iris-versicolor
Iris-virginica
Classes

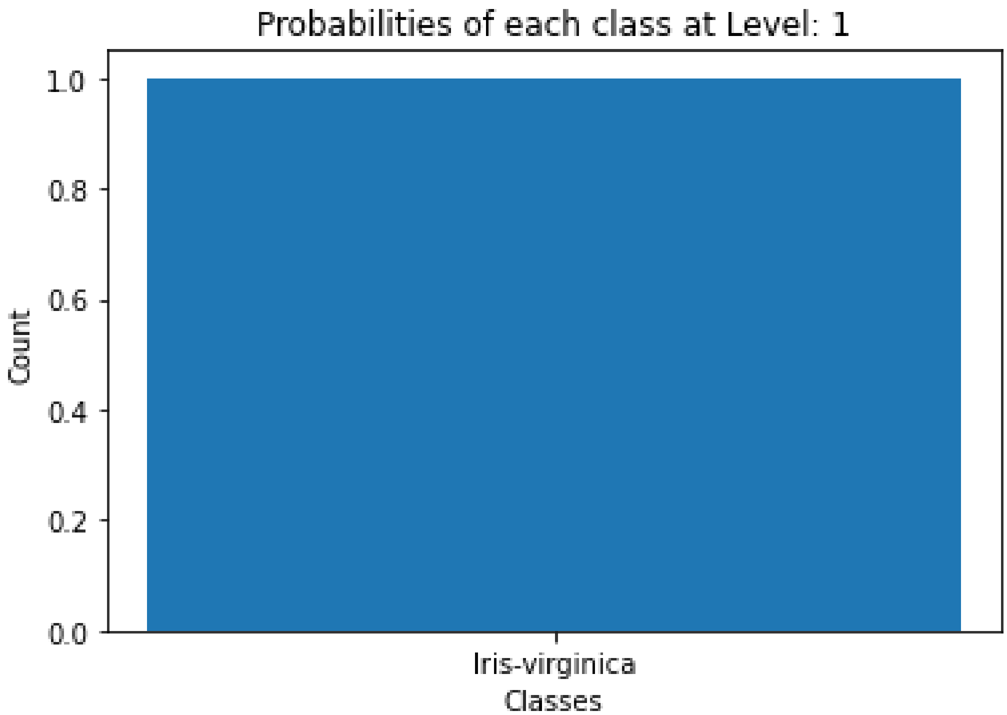

Probabilities of each class at Level: 4
Count
30
25
20
15
10
5
0
Iris-setosa
Classes


Probabilities of each class at Level: 4
35
30
25
20
15
10
5
0
Count
Iris-versicolor
Iris-virginica
Classes

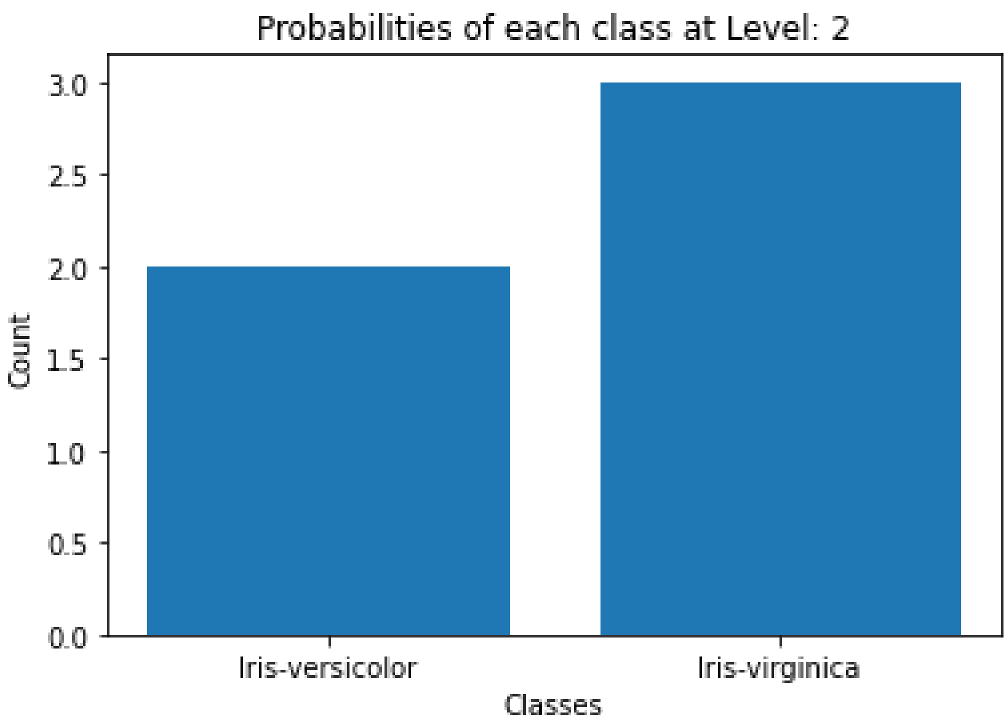

Probabilities of each class at Level: 3
35
30
25
20
15
10
5
0
Count
Iris-versicolor
Iris-virginica
Classes


Probabilities of each class at Level: 2
Count
Iris-versicolor
Iris-virginica
Classes

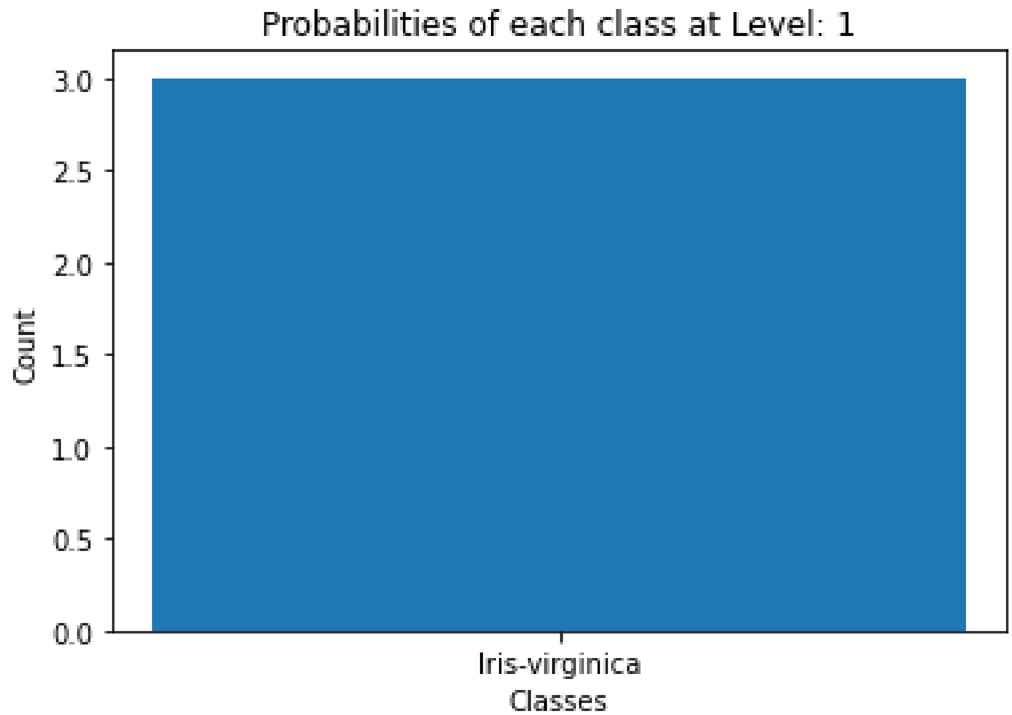

Probabilities of each class at Level: 1
Count
Iris-versicolor
Classes



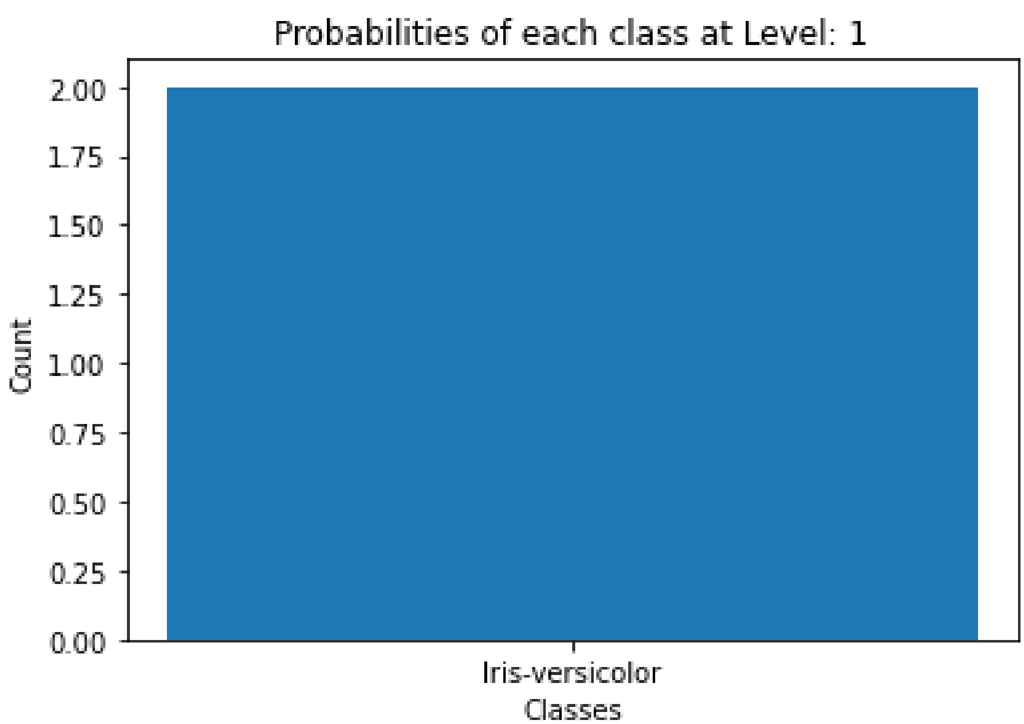
Probabilities of each class at Level: 1
1.0
0.8
0.6
0.4
0.2
0.0
Count
Iris-virginica
Classes



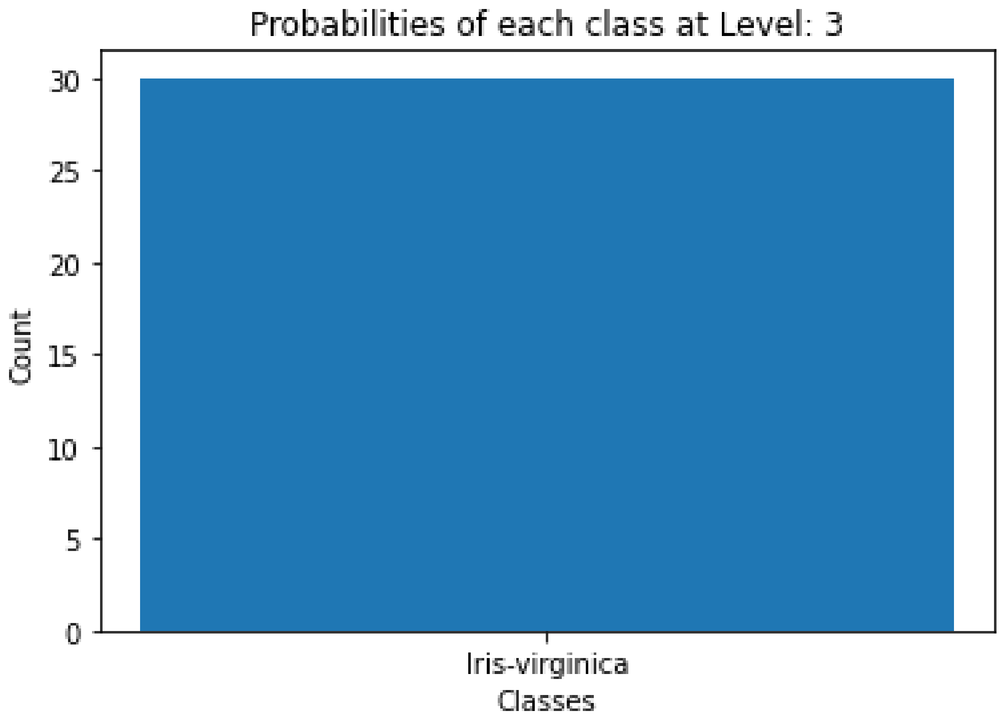
Probabilities of each class at Level: 2
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Count
Iris-versicolor
Iris-virginica
Classes



Probabilities of each class at Level: 1
Count
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Iris-virginica
Classes



Probabilities of each class at Level: 1
Count
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
Iris-versicolor
Classes




```
Printing the Tree using a breath first tree traversal:

check for petal_width <= 0.75
True:
  Leaf Node: Iris-setosa
False:
  check for petal_width <= 1.75
  True:
    check for petal_length <= 4.95
    True:
      check for petal_width <= 1.65
      True:
        Leaf Node: Iris-versicolor
      False:
        Leaf Node: Iris-virginica
    False:
      check for petal_width <= 1.55
      True:
        Leaf Node: Iris-virginica
      False:
        Leaf Node: Iris-versicolor
  False:
    Leaf Node: Iris-virginica
```

```
Accuracy: 0.9545454545454546

Cross Entropy Loss: 0.04652001563489282
```

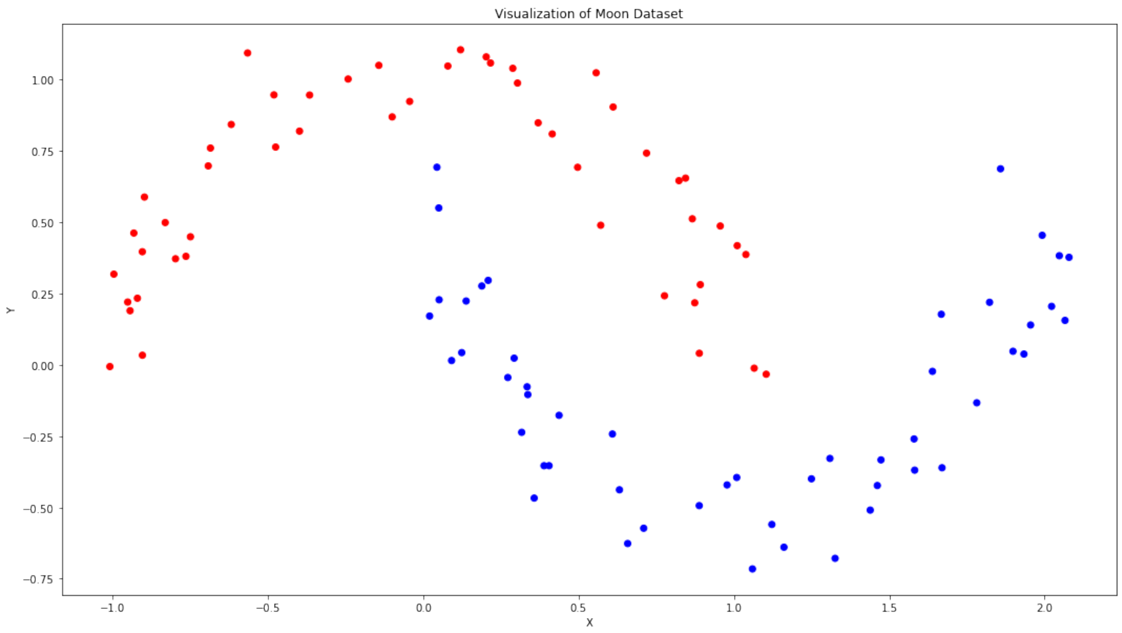
0.0.3 Exercise 2: Gradient Boosted Decision Trees

Generating a binary classification toy dataset from the scikit-learn utility "make-moons" with 100 samples and for 10 different levels of noise

[66]:
```python
#Different Values of noise
noise = np.arange(0.1,0.2,0.01)

#Creating the Binary Classification Dataset
X , y = make_moons(n_samples=100, noise = noise[0])
for n in noise[1:]:
    new_X, new_y = make_moons(n_samples=100, noise=n)
    X = np.append(X, new_X, axis=0)
    y = np.append(y, new_y, axis=0)
```

Visualizing the 100 different pairs of so-called moons

[67]:
```python
#Creating a vector containing the color for each instance based on its class
vec = np.vectorize(lambda x : 'red' if x == 0 else 'blue')

#Plotting the 100 pairs of Moons
fig = plt.figure(figsize=(18,10))
plt.scatter(X[:100,0],X[:100,1], c = vec(y[:100]))
plt.title('Visualization of Moon Dataset')
plt.xlabel('X')
plt.ylabel('Y')
plt.show()
```



Generating train/validation/test splits with the ratios like before

[68]:
```
#Converting the Moon dataset into Dataframe for the purpose of Splitting
moon_df = pd.DataFrame.from_dict({'x':X[:,0], 'y':X[:,1], 'class':y})

#Splitting the Dataset
moon_train, moon_validation, moon_test = split_dataset(moon_df)
```

Keeping the max depth of trees to 2 i.e root node then leaf nodes (also called stumps), and tuning the number of trees in the ensemble on the validation set.

Function for calculating the Sigmoid of any value

[69]:
```
def sigmoid(x):
    #Sigmoid = 1 / (1 + e^-x)
    return 1 / (1 + np.exp(-1 * x))
```

Function to Calculate the L(n) on actual Y and Predicted Y

[70]:
```
def LN(actual_y, predicted_y):
    return -1 * actual_y * np.log(predicted_y) - (1 - actual_y) * np.log(1 -␣
 ↪sigmoid(predicted_y))
```

Function to Calculate the Gradient on actual Y and Predicted Y

[71]:
```
def GN(actual_y, predicted_y):
    return sigmoid(predicted_y) - actual_y
```

Function to Calculate the Hessian on actual Y and Predicted Y

```
[72]: def HN(actual_y, predicted_y):
          return sigmoid(predicted_y) * (1 - sigmoid(predicted_y))
```

Function to Calculate the Gain on the Ensemble Trees

```
[73]: def boosted_gain(dataset, left_dataset, right_dataset, lamda, gn_index,␣
       ↪hn_index):
          return 0.5 * ((sum(left_dataset[:,gn_index])/(lamda + sum(left_dataset[:
       ↪,hn_index]))) +
                        (sum(right_dataset[:,gn_index])/(lamda + sum(right_dataset[:
       ↪,hn_index]))) -
                        (sum(dataset[:,gn_index])/(lamda + sum(dataset[:,hn_index]))))
```

Function to find the Best split based on Ensemble Gain value

```
[74]: def best_fit_boosted_gain(dataset, headers, target_column, lamda, gn_index,␣
       ↪hn_index):
          #Initializing the best Enemble Gain to 0 and best Condition to None
          best_gain = 0
          best_question = None

          #Iterating through all feature columns
          for i in range(len(dataset[0])):
              #We have to skip the class column
              if i == target_column:
                  continue

              #Extracting the unique values from the selected column
              unique_values = np.unique(dataset[:,i])

              #If the column values are numeric than calculating the list containing␣
       ↪the average values between two consecutive numbers
              if isinstance(unique_values[0], int) or isinstance(unique_values[0],␣
       ↪float):
                  unique_values = numerical_splitting_average(unique_values)

              #Iterating through all unique values
              for unique_val in unique_values:

                  #Creating a condition with selected feature and unique value
                  question = Question(headers[i], i, unique_val)

                  #Calculating the Information Gain value for that condition
                  left_data , right_data = question.apply_split(dataset)
                  gain = boosted_gain(dataset, left_data, right_data, lamda,␣
       ↪gn_index, hn_index)
```

```
            #If we find a better Information Gain then swapping it with the␣
↪previous best
            if gain >= best_gain:
                best_gain = gain
                best_question = question

    #Returning the Best Information Gain and Best Condition
    return best_gain, best_question
```

Function to Learn a Gradient Boosted Decision Tree from the Dataset

[75]:
```
def learn_gradient_boosted_decision_tree(dataset, headers, target_column,␣
↪lamda, gn_index, hn_index, max_depth = 2):

    #Calculating the Best Condition based on Either MCR or Information Gain
    eval_val , question = best_fit_boosted_gain(dataset, headers,␣
↪target_column, lamda, gn_index, hn_index)

    #If max depth is achieved than return from Recursion
    if max_depth == 0:
        #Extracting the Dominant Class
        unique_values, counts = np.unique(dataset[:,target_column],␣
↪return_counts=True)
        dominant_class = unique_values[np.argmax(counts)]

        #Returning the Node
        return Node(question, None, None, True, dominant_class)

    #If Ensemble Gain is 0 or the Question is Null, then it is Leaf node
    if eval_val == 0 or question == None:
        #Extracting the Dominant Class
        unique_values, counts = np.unique(dataset[:,target_column],␣
↪return_counts=True)
        dominant_class = unique_values[np.argmax(counts)]

        #Returning the Leaf Node
        return Node(question, None, None, True, dominant_class)

    #Split the Data into Left and Right Dataset based on Best Condition
    left_data, right_data = question.apply_split(dataset)

    #Creating a Left Child Node using Recursive call
    left_node = learn_gradient_boosted_decision_tree(left_data, headers,␣
↪target_column, lamda, gn_index, hn_index, max_depth - 1)

    #Creating a Right Child Node using Recursive call
```

```
    right_node = learn_gradient_boosted_decision_tree(right_data, headers,␣
↪target_column, lamda, gn_index, hn_index, max_depth - 1)

    #Returning the Node which is not Leaf and contains Left and Right Nodes
    return Node(question, left_node, right_node, False, '')
```

Function to update the values of Gradients and Hessians of each instances in the Dataset

[76]:
```
def update_gradient_hessians(dataset, actual_y_index, predicted_y_index,␣
↪sig_index, gn_index, hn_index):
    #Iterating over all rows
    for index, row in enumerate(dataset):
        #Calculating the Sigmoid of predicted value
        dataset[index,sig_index] = sigmoid(dataset[index,predicted_y_index])

        #Calculating the Gradient from the actual and predicted value
        dataset[index,gn_index] =␣
↪GN(dataset[index,actual_y_index],dataset[index,predicted_y_index])

        #Calculating the Hessian from the actual and predicted value
        dataset[index,hn_index] =␣
↪HN(dataset[index,actual_y_index],dataset[index,predicted_y_index])

    #Returning the Dataset
    return dataset
```

Function to apply different Number of Gradient Boosted Decision Trees and Calculating there Accuracies

[77]:
```
total_ensemble_trees = 3
```

[78]:
```
def create_ensemble_trees(train_dataset,validation_dataset,test_dataset,␣
↪headers, target_column, lamda):

    #Appending new Columns in the Training dataset for Gradient, Hessians etc.
    #Here:
    #Column Index 2 denotes: Actual Y
    #Column Index 3 denotes: Predicted Y
    #Column Index 4 denotes: Sigmoid of Predicted Y
    #Column Index 5 denotes: Gradient
    #Column Index 6 denotes: Hessian
    train_dataset = np.append(train_dataset, np.
↪zeros(shape=(len(train_dataset),4)), axis=1)

    #Appending only Predicted Y column in the Validation Dataset
    validation_dataset = np.append(validation_dataset, np.
↪zeros(shape=(len(validation_dataset),1)), axis=1)
```

```
    #Appending only Predicted Y column in the Test Dataset
    test_dataset = np.append(test_dataset, np.
↪zeros(shape=(len(test_dataset),1)), axis=1)

    #Updating the values of Gradient and Hessians for each instance in the␣
↪Training dataset
    train_dataset = update_gradient_hessians(train_dataset, 2,3,4,5,6)

    #List for storing different trees
    trees = []

    #Iterating over different number of Trees in an Ensemble
    for i in range(1,total_ensemble_trees+1):

        ␣
↪print('----------------------------------------------------------------------')
        print('Creating a Ensemble with {} Trees'.format(i))
        ␣
↪print('----------------------------------------------------------------------')

        #Iterating for creating specified number of Trees in the Ensemble
        for j in range(i):

            print('\nTree {}:'.format(j+1))

            #Creating a Tree
            root = learn_gradient_boosted_decision_tree(train_dataset, headers,␣
↪target_column, lamda, 5, 6, max_depth = 2)

            #Calculating the Predicted Y and storing it into the Dataset
            for index, row in enumerate(train_dataset):
                train_dataset[index,3] = train_dataset[index,3] + (lamda *␣
↪predict_class(root, row))

            #Updating the Gradients and Hessians for the next Iteration
            train_dataset = update_gradient_hessians(train_dataset, 2,3,4,5,6)

            #Appending the new Created Tree into the list
            trees.append(root)

            #Printing the Tree
            print_tree(root)

        #Iterating over all Trees with Validation Dataset
        for j in range(i):
```

```
            #Extracting the current Tree
            current_tree = trees[j]

            #Calculating the Predicted Y for the Validation Dataset
            for index, row in enumerate(validation_dataset):
                validation_dataset[index,3] = validation_dataset[index,3] +␣
↪(lamda * predict_class(current_tree, row))

        #Mapping each Predicted Y values in the Validation dataset with the␣
↪Sigmoid Activation function
        validation_dataset[:,3] = list(map(lambda x : 0 if sigmoid(x) <= 0.5␣
↪else 1, validation_dataset[:,3]))

        #Calculating the Total Accuracy of the current Number of Trees in the␣
↪Ensemble
        correct_prediction = 0

        #Foreach validation row checking if the predicted Class is equal to␣
↪Actual Class
        for row in validation_dataset:
            if row[2] == row[3]:
                correct_prediction += 1

        #Printing the Accuracy of the current Number of Trees in the Ensemble
        print('\nAccuracy of Validation with Total Ensemble Trees: {} is: {}\n'.
↪format(i, correct_prediction/len(validation_dataset)))

        #Iterating over all Trees with Test Dataset
        for j in range(i):

            #Extracting the current Tree
            current_tree = trees[j]

            #Calculating the Predicted Y for the Test Dataset
            for index, row in enumerate(test_dataset):
                test_dataset[index,3] = test_dataset[index,3] + (lamda *␣
↪predict_class(current_tree, row))

        #Mapping each Predicted Y values in the Validation dataset with the␣
↪Sigmoid Activation function
        test_dataset[:,3] = list(map(lambda x : 0 if sigmoid(x) <= 0.5 else 1,␣
↪test_dataset[:,3]))

        #Calculating the Total Accuracy of the current Number of Trees in the␣
↪Ensemble
```

```
        correct_prediction = 0

        #Foreach validation row checking if the predicted Class is equal to␣
↪Actual Class
        for row in test_dataset:
            if row[2] == row[3]:
                correct_prediction += 1

        #Printing the Accuracy of the current Number of Trees in the Ensemble
        print('\nAccuracy of Test with Total Ensemble Trees: {} is: {}\n'.
↪format(i, correct_prediction/len(test_dataset)))

        #Emptying list for next iteration
        trees = []

        #Renewing Columns in the Training dataset for Gradient, Hessians etc.
        train_dataset = np.append(train_dataset, np.
↪zeros(shape=(len(train_dataset),4)), axis=1)

        #Renewing Predicted Y column in the Validation Dataset
        validation_dataset = np.append(validation_dataset, np.
↪zeros(shape=(len(validation_dataset),1)), axis=1)

        #Appending only Predicted Y column in the Test Dataset
        test_dataset = np.append(test_dataset, np.
↪zeros(shape=(len(test_dataset),1)), axis=1)

        #Renewing the values of Gradient and Hessians for each instance in the␣
↪Training dataset
        train_dataset = update_gradient_hessians(train_dataset, 2,3,4,5,6)
```

Creating different Ensembles with different Number of Trees

[80]:
```
create_ensemble_trees(moon_train, moon_validation,moon_test,␣
↪['X','Y','Class','Predicted_class', 'Sigmoid', 'GN', 'HN'], 2, 0.1)
```

```
---------------------------------------------------------------------
Creating a Ensemble with 1 Trees
---------------------------------------------------------------------

Tree 1:
check for Y <= 1.1128462219796047
True:
  check for Y <= 1.0533527269806844
  True:
    Leaf Node: 1.0
  False:
    Leaf Node: 0.0
```

```
False:
  check for Y <= 1.1682457059718352
  True:
    Leaf Node: 0.0
  False:
    Leaf Node: 0.0

Accuracy of Validation with Total Ensemble Trees: 1 is: 0.52


Accuracy of Test with Total Ensemble Trees: 1 is: 0.5066666666666667

---------------------------------------------------------------------
Creating a Ensemble with 2 Trees
---------------------------------------------------------------------

Tree 1:
check for X <= -1.069998683844033
True:
  check for Y <= 0.20836093309419113
  True:
    Leaf Node: 0.0
  False:
    Leaf Node: 0.0
False:
  check for X <= -0.9555080724324898
  True:
    Leaf Node: 0.0
  False:
    Leaf Node: 1.0

Tree 2:
check for Y <= 1.1128462219796047
True:
  check for Y <= 1.025473796585373
  True:
    Leaf Node: 1.0
  False:
    Leaf Node: 0.0
False:
  check for Y <= 1.1682457059718352
  True:
    Leaf Node: 0.0
  False:
    Leaf Node: 0.0

Accuracy of Validation with Total Ensemble Trees: 2 is: 0.48
```

```
Accuracy of Test with Total Ensemble Trees: 2 is: 0.43333333333333335

---------------------------------------------------------------------
Creating a Ensemble with 3 Trees
---------------------------------------------------------------------

Tree 1:
check for X <= -1.0546509432272249
True:
  check for Y <= 0.20836093309419113
  True:
    Leaf Node: 0.0
  False:
    Leaf Node: 0.0
False:
  check for X <= -0.9169906934790861
  True:
    Leaf Node: 0.0
  False:
    Leaf Node: 1.0

Tree 2:
check for Y <= 1.106517629684896
True:
  check for Y <= 0.9633966902216651
  True:
    Leaf Node: 1.0
  False:
    Leaf Node: 0.0
False:
  check for Y <= 1.1682457059718352
  True:
    Leaf Node: 0.0
  False:
    Leaf Node: 0.0

Tree 3:
check for X <= -1.0546509432272249
True:
  check for Y <= 0.20836093309419113
  True:
    Leaf Node: 0.0
  False:
    Leaf Node: 0.0
False:
  check for X <= -0.8646704651581854
  True:
```

```
    Leaf Node: 0.0
  False:
    Leaf Node: 1.0

Accuracy of Validation with Total Ensemble Trees: 3 is: 0.48


Accuracy of Test with Total Ensemble Trees: 3 is: 0.43333333333333335
```