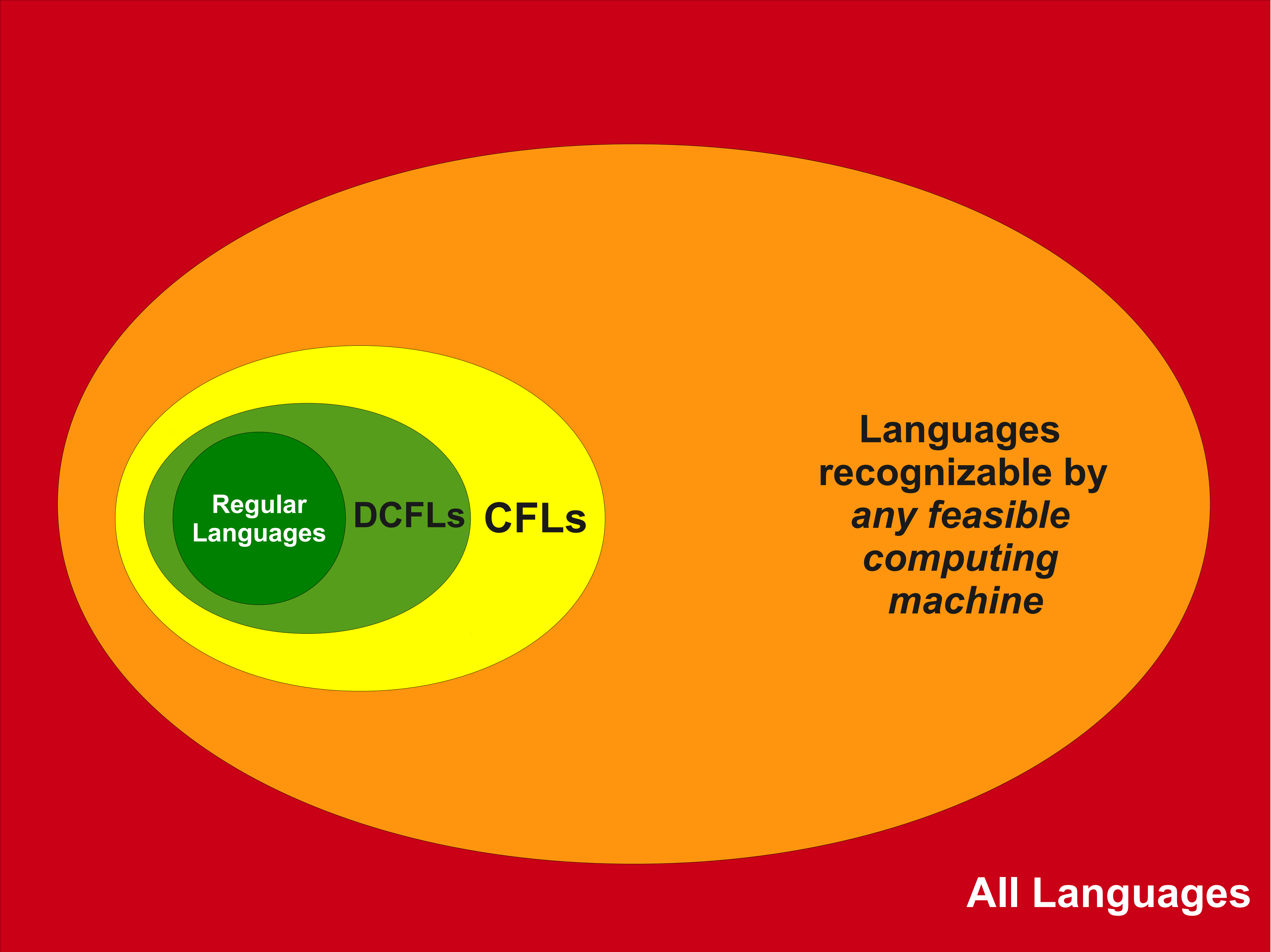
Turing Machines

Announcements

- Problem Set 6 due next Monday, February 25, at 12:50PM.
- Midterm graded, will be returned at end of lecture.

Are some problems inherently harder than others?


Regular Languages
DCFLs
CFLs
Languages recognizable by *any feasible computing machine*
All Languages

That same drawing, to scale.

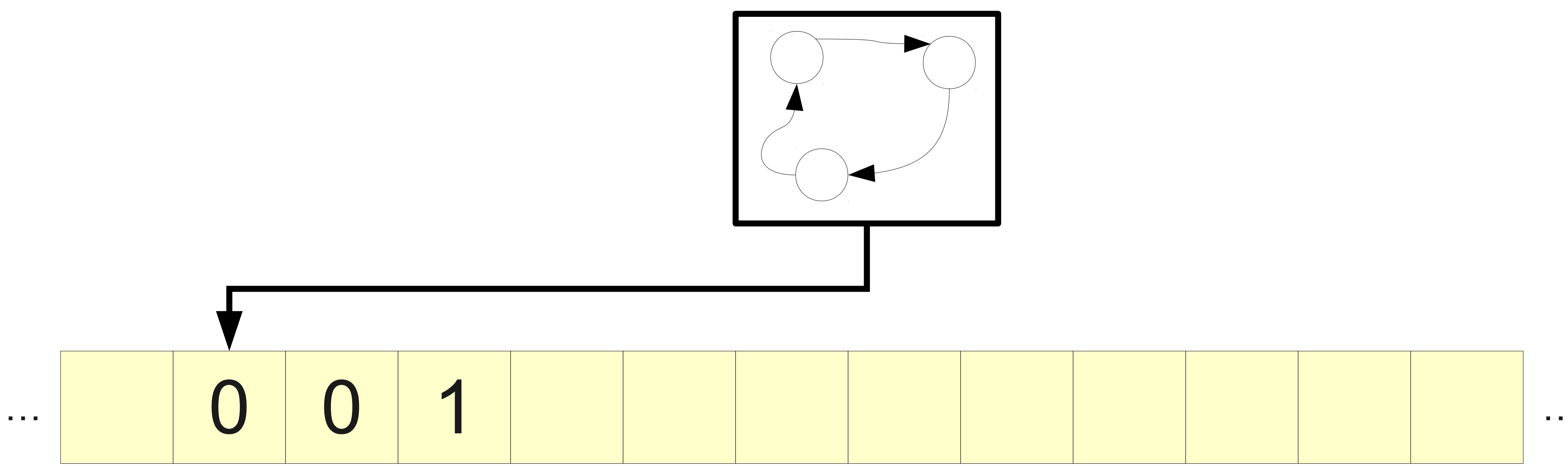
All Languages

Defining Computability

- To talk about “any feasible computing machine,” we'll need a formal model of computation.
- The standard automaton for this job is the **Turing machine**, named after Alan Turing, the “Father of Computer Science.”

A Better Memory Device

- A **Turing machine** is a finite automaton equipped with an **infinite tape** as its memory.
- The tape begins with the input to the machine written on it, surrounded by infinitely many blank cells.
- The machine has a **tape head** that can read and write a single memory cell at a time.



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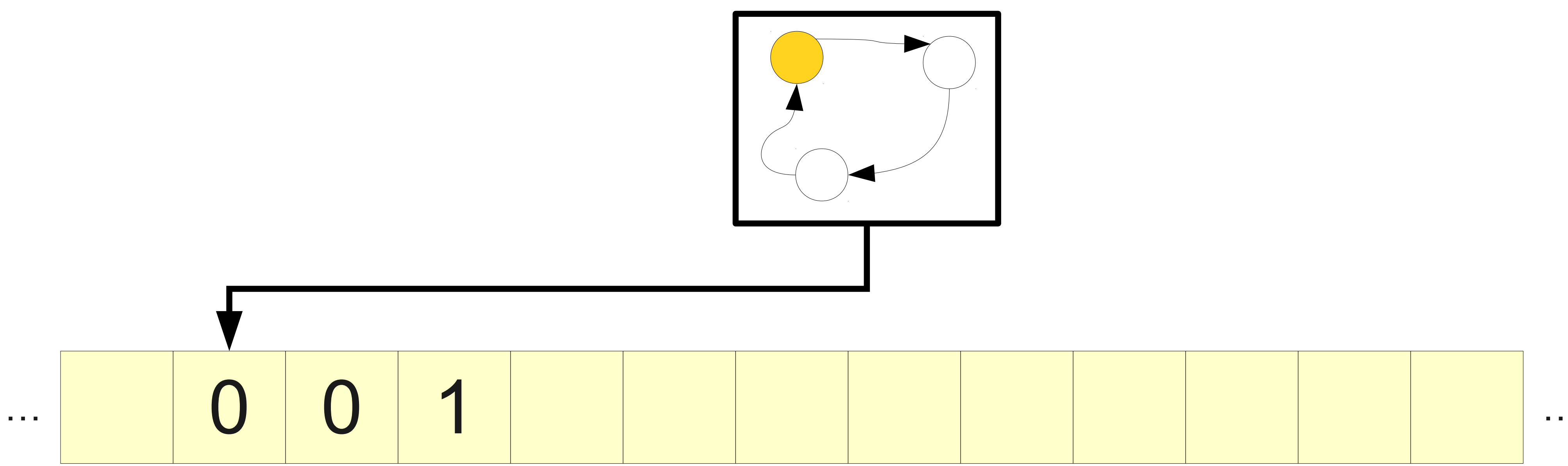

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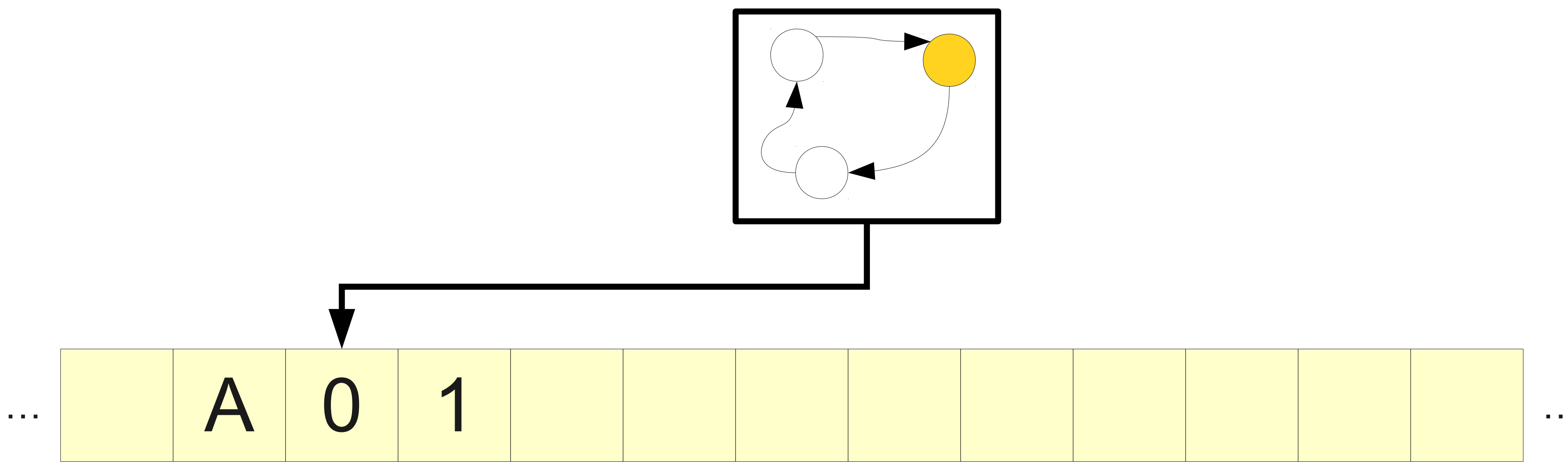

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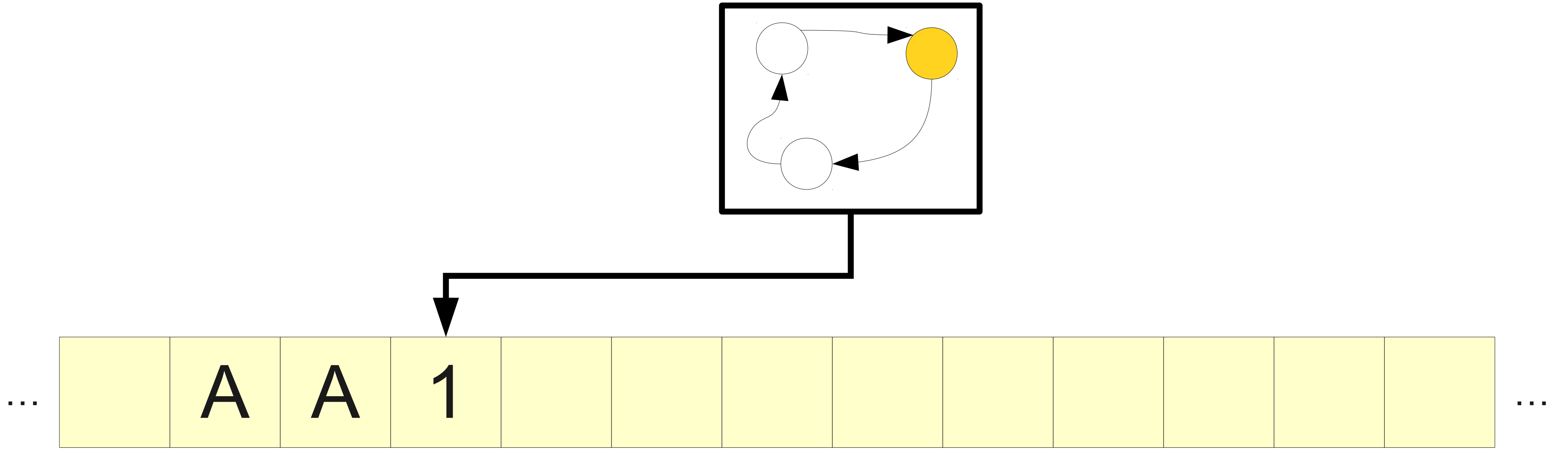

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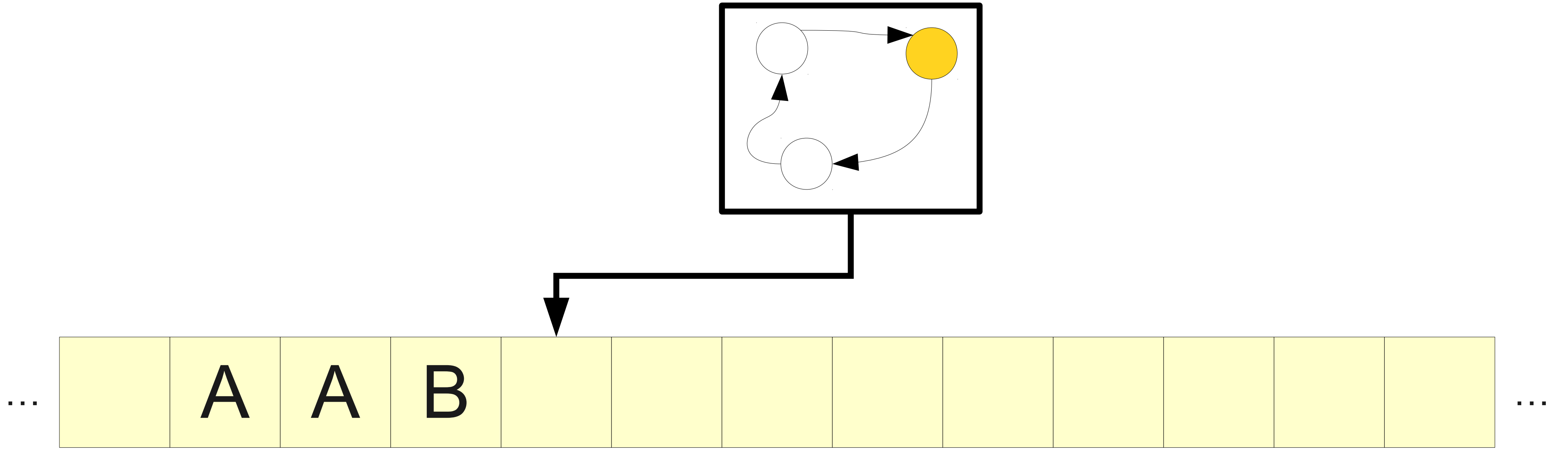

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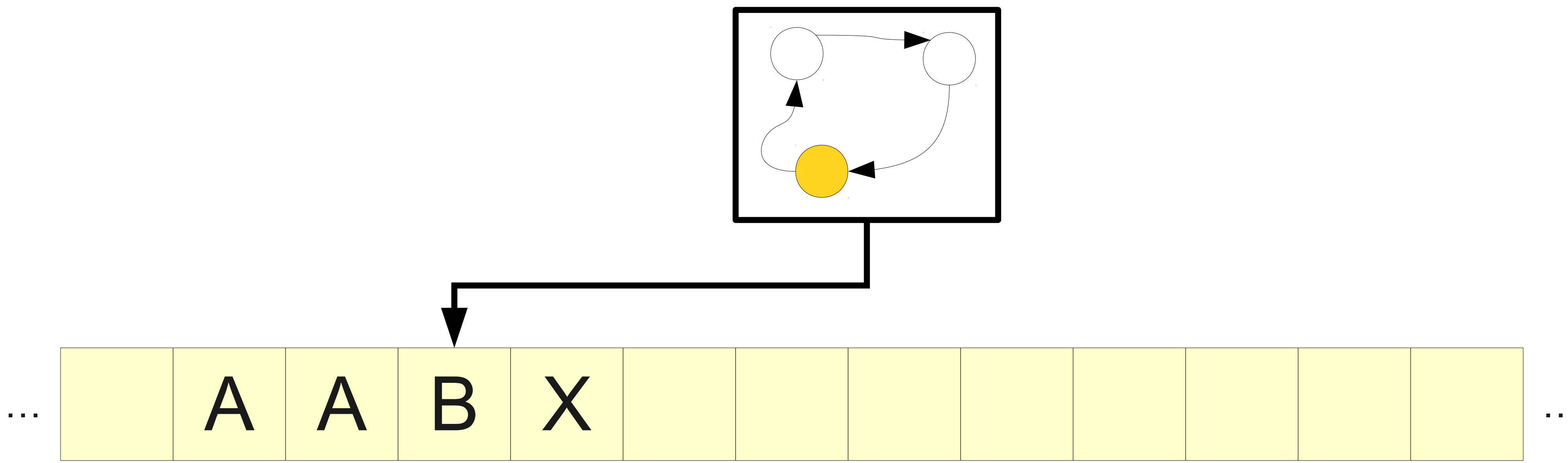

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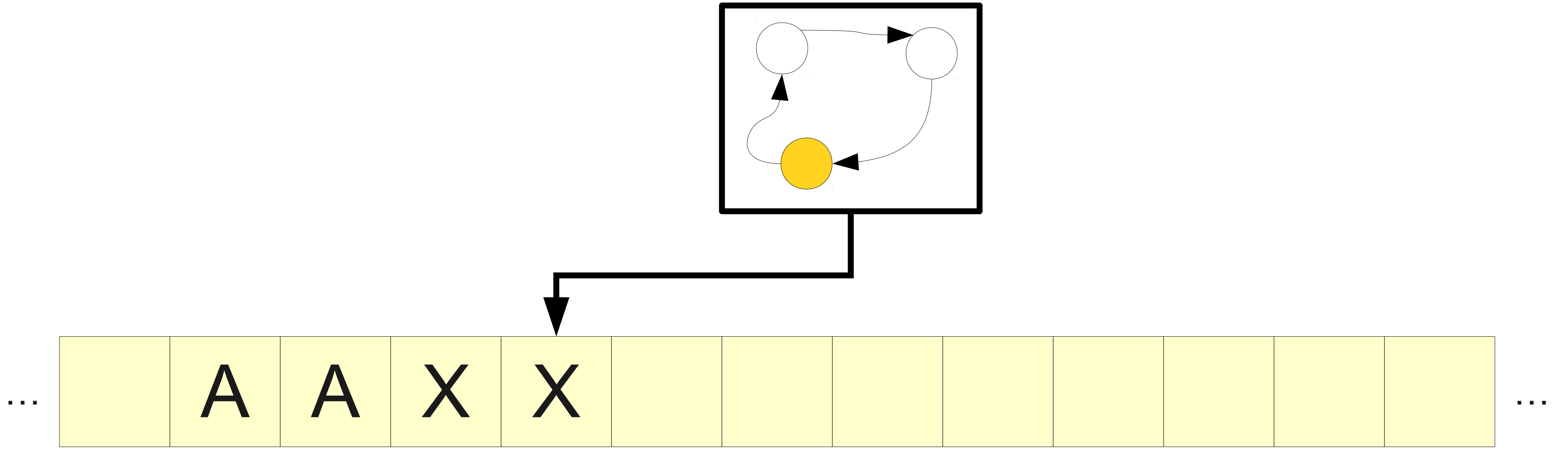

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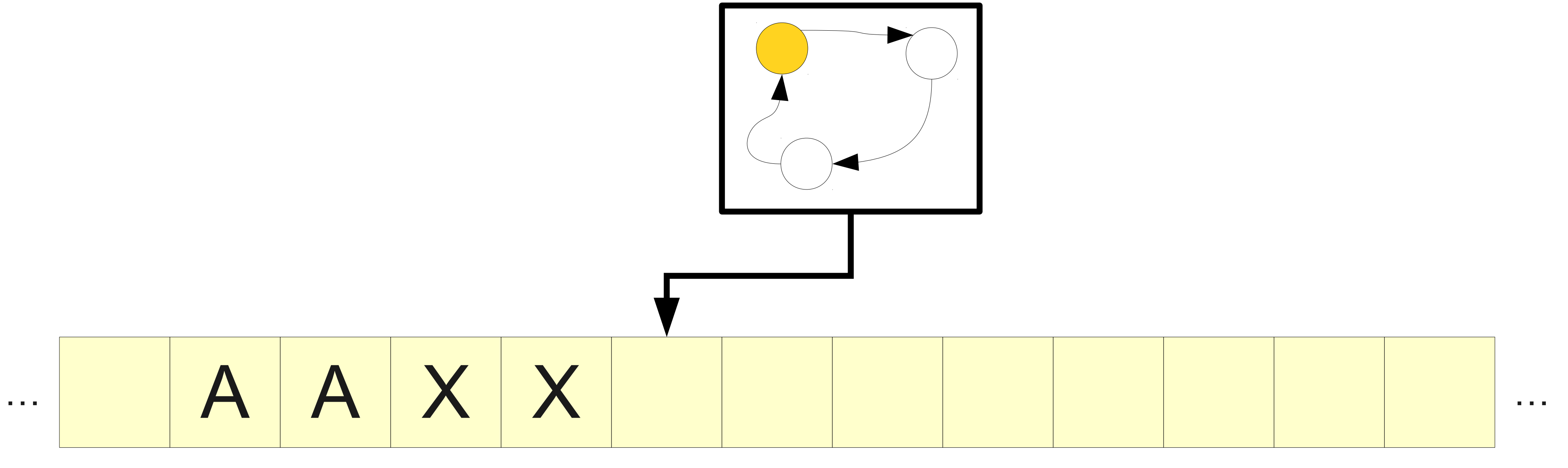

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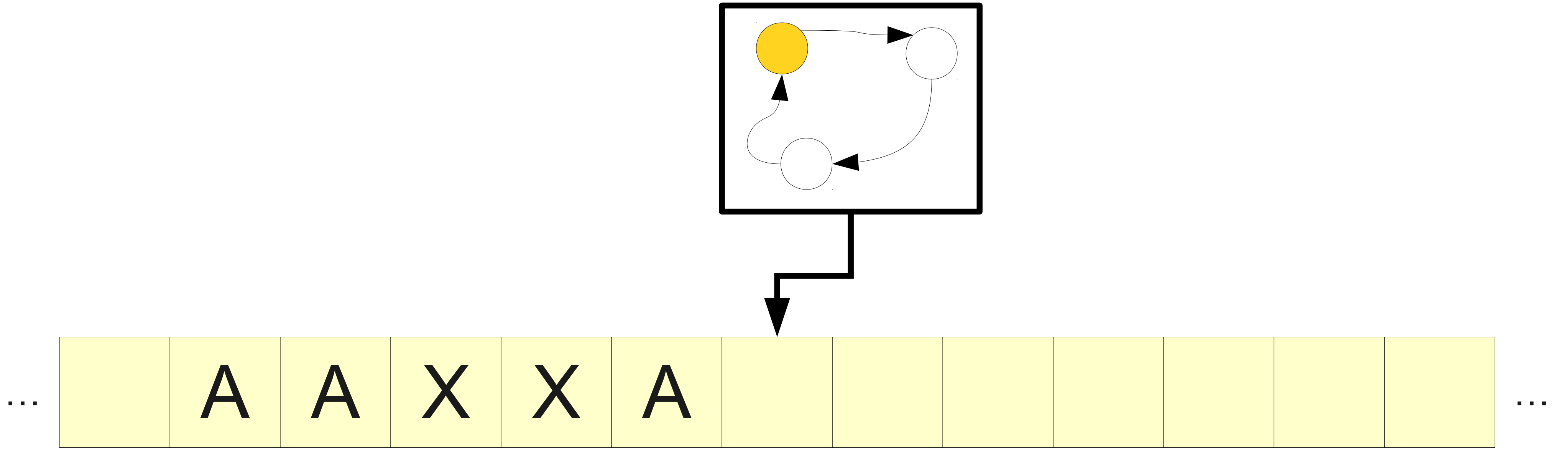

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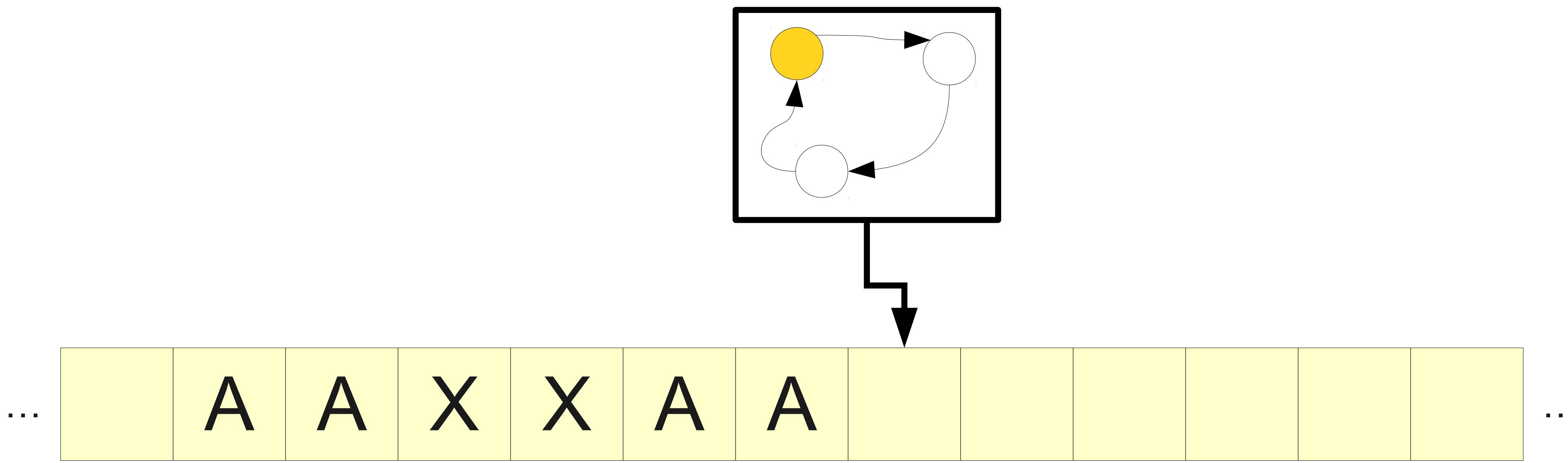

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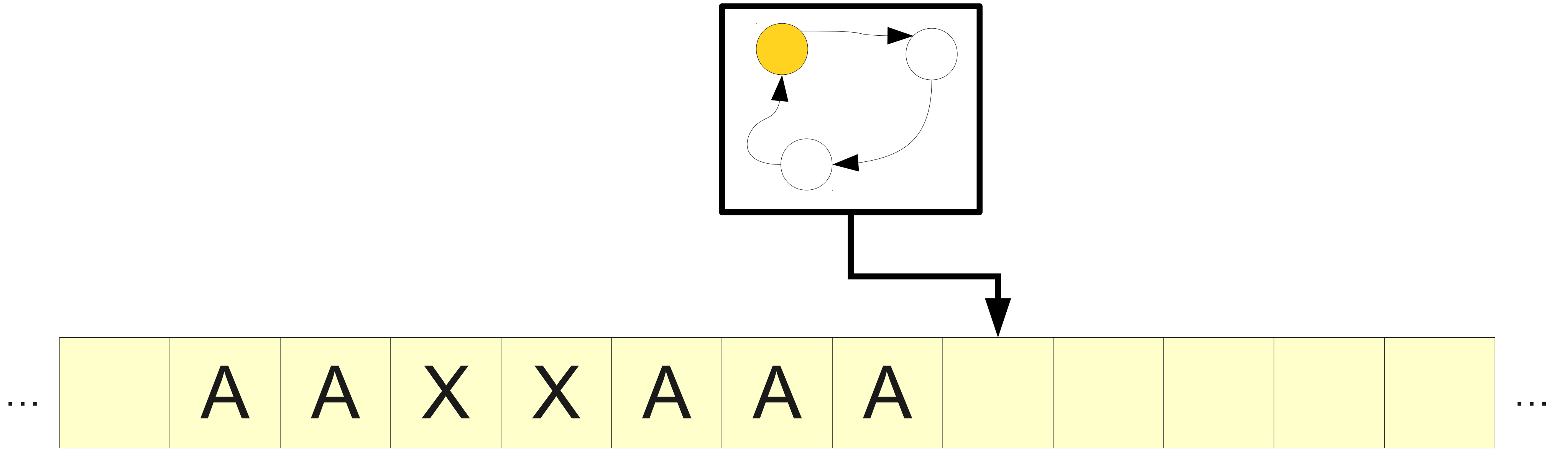


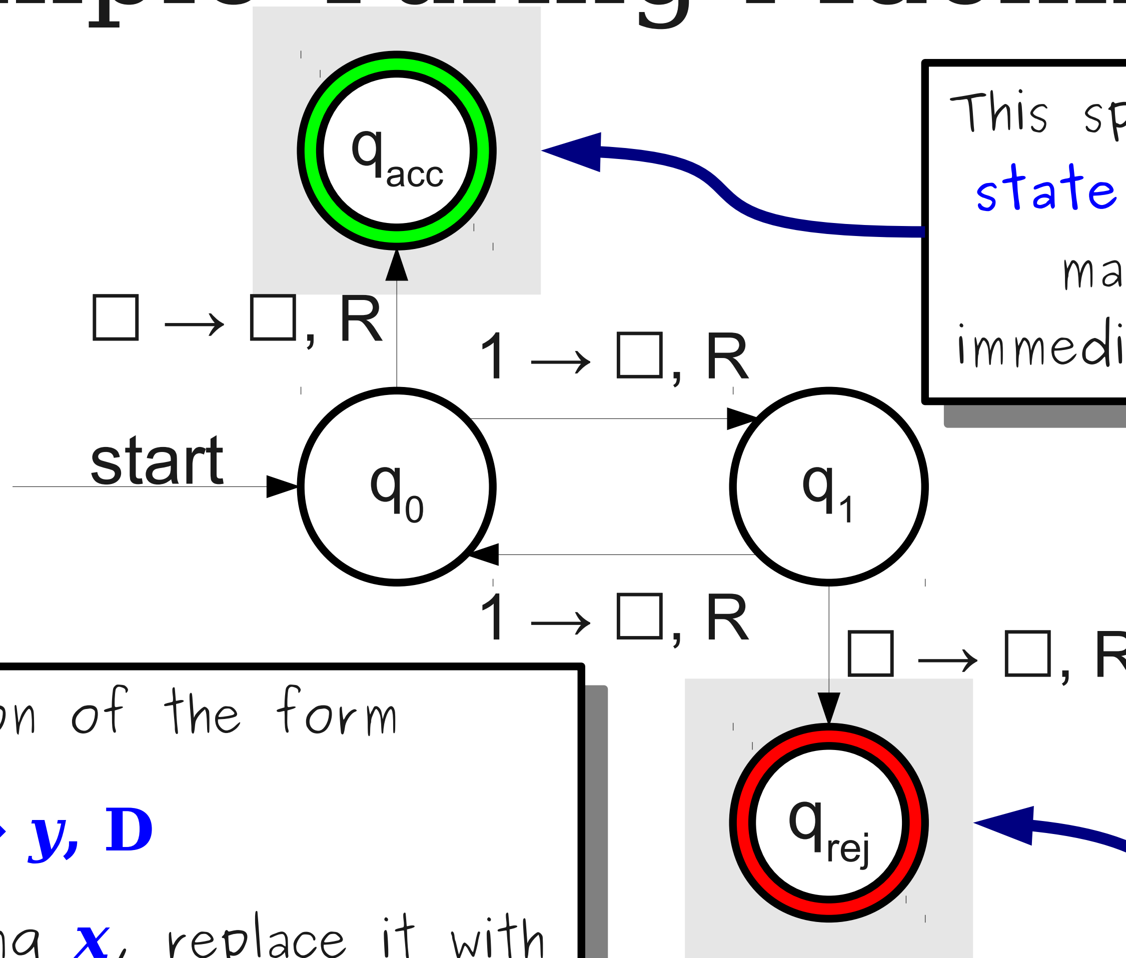
The Turing Machine

- A Turing machine consists of three parts:
  - A **finite-state control** that issues commands,
  - an **infinite tape** for input and scratch space, and
  - a **tape head** that can read and write a single tape cell.
- At each step, the Turing machine
  - Writes a symbol to the tape cell under the tape head,
  - changes state, and
  - moves the tape head to the left or to the right.

Input and Tape Alphabets

- A Turing machine has two alphabets:
  - An **input alphabet** Σ. All input strings are written in the input alphabet.
  - A **tape alphabet** Γ, where $\Sigma \subseteq \Gamma$. The tape alphabet contains all symbols that can be written onto the tape.
- The tape alphabet Γ can contain any number of symbols, but always contains at least one **blank symbol**, denoted $\square$. You are guaranteed $\square \notin \Sigma$.
- At startup, the Turing machine begins with an infinite tape of $\square$ symbols with the input written at some location. The tape head is at the start of the input.

A Simple Turing Machine



This special **accept state** causes the machine to immediately accept.

This special **reject state** causes the machine to immediately reject.

Each transition of the form

$$\boldsymbol{x \to y, D}$$

means "upon reading $\boldsymbol{x}$, replace it with symbol $\boldsymbol{y}$ and move the tape head in direction $\mathbf{D}$ (which is either $\mathbf{L}$ or $\mathbf{R}$). The symbol $\square$ represents the **blank symbol**.

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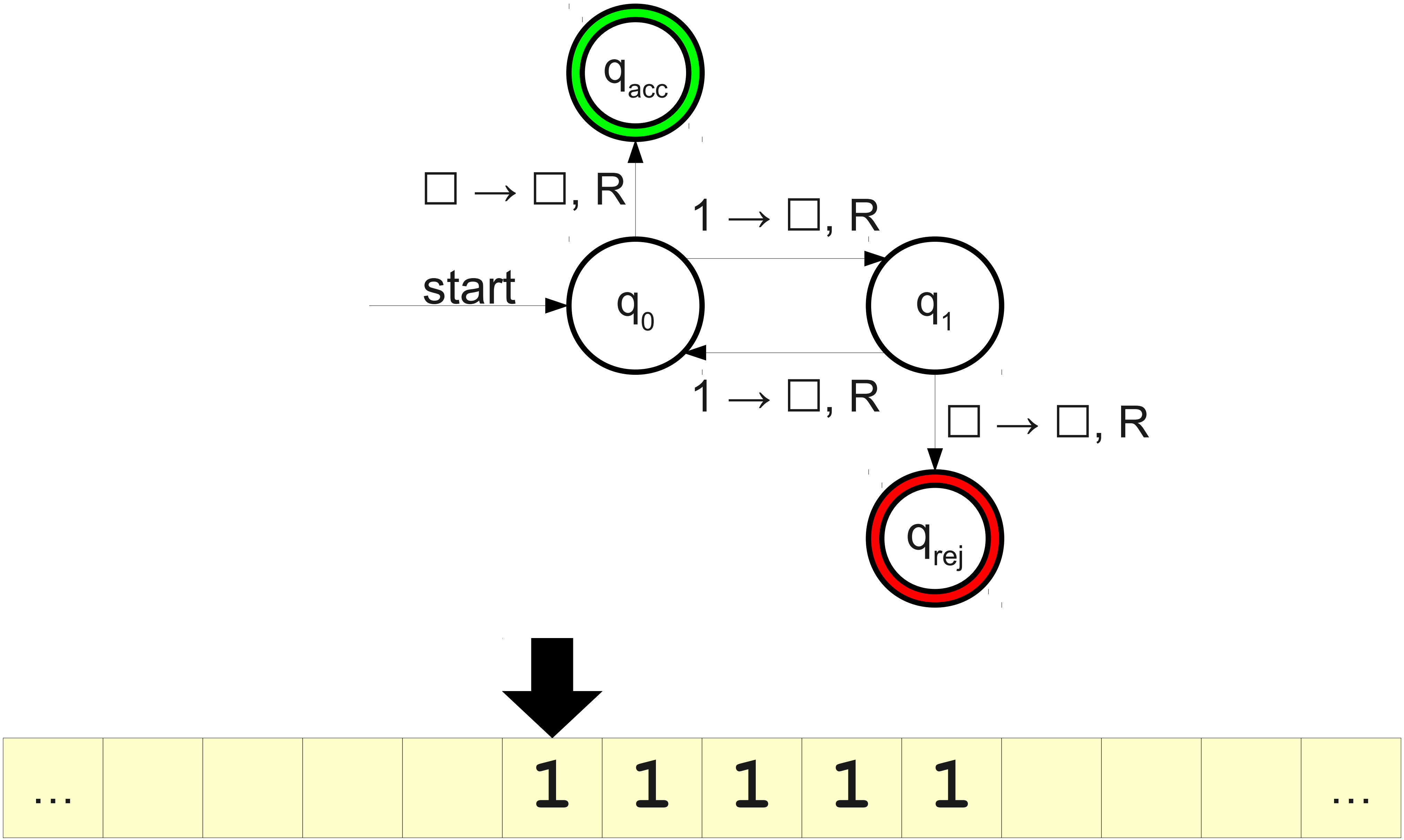

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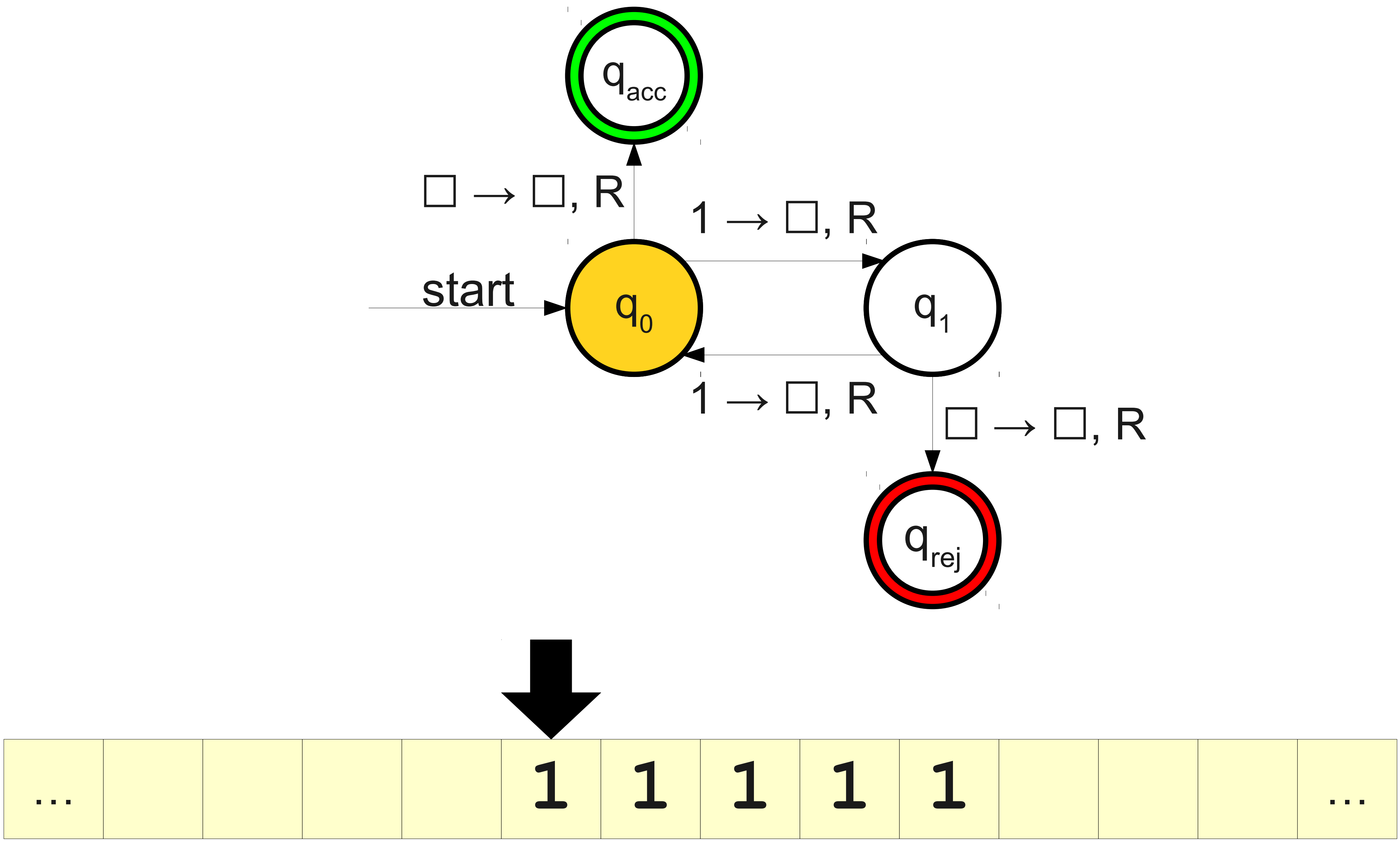

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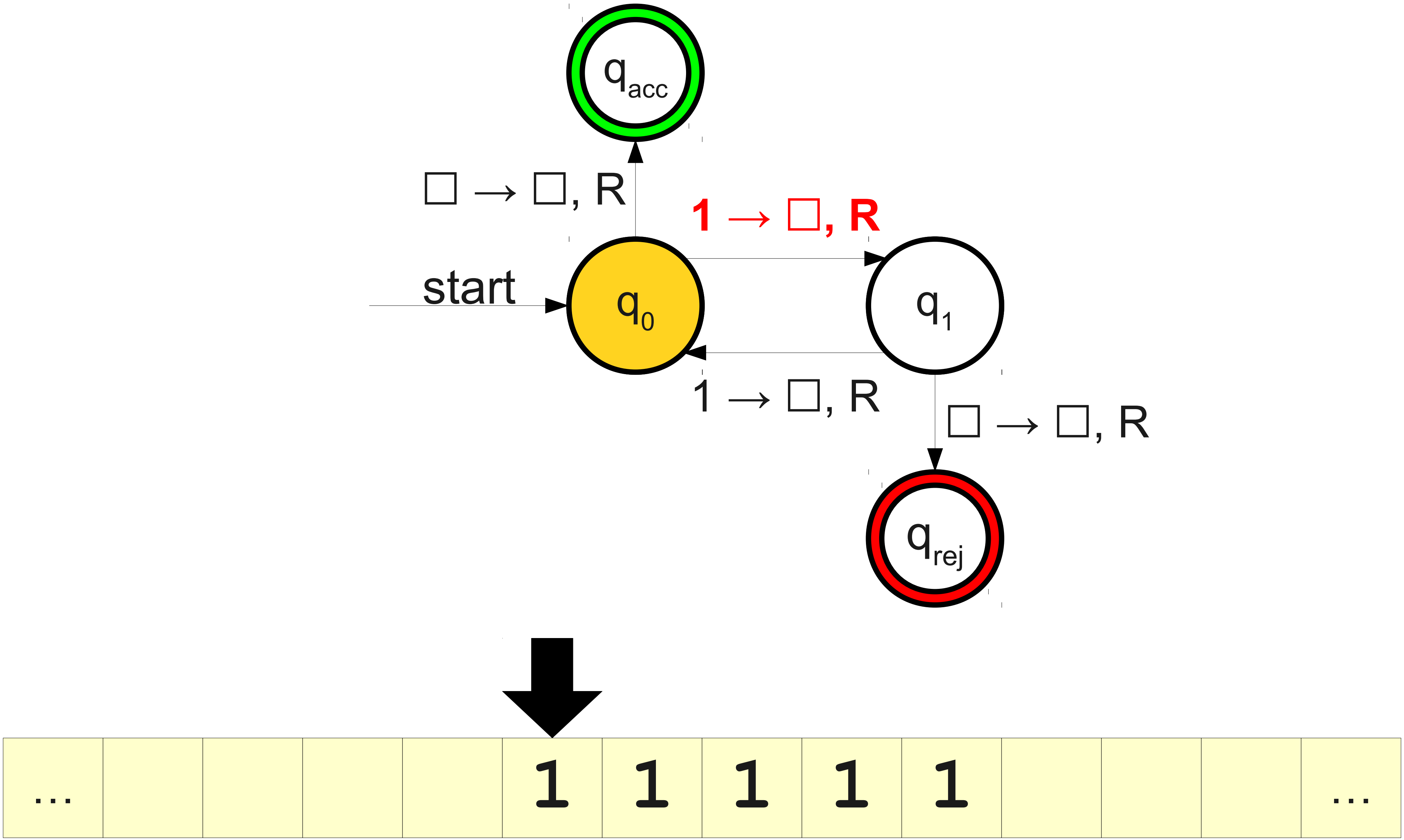

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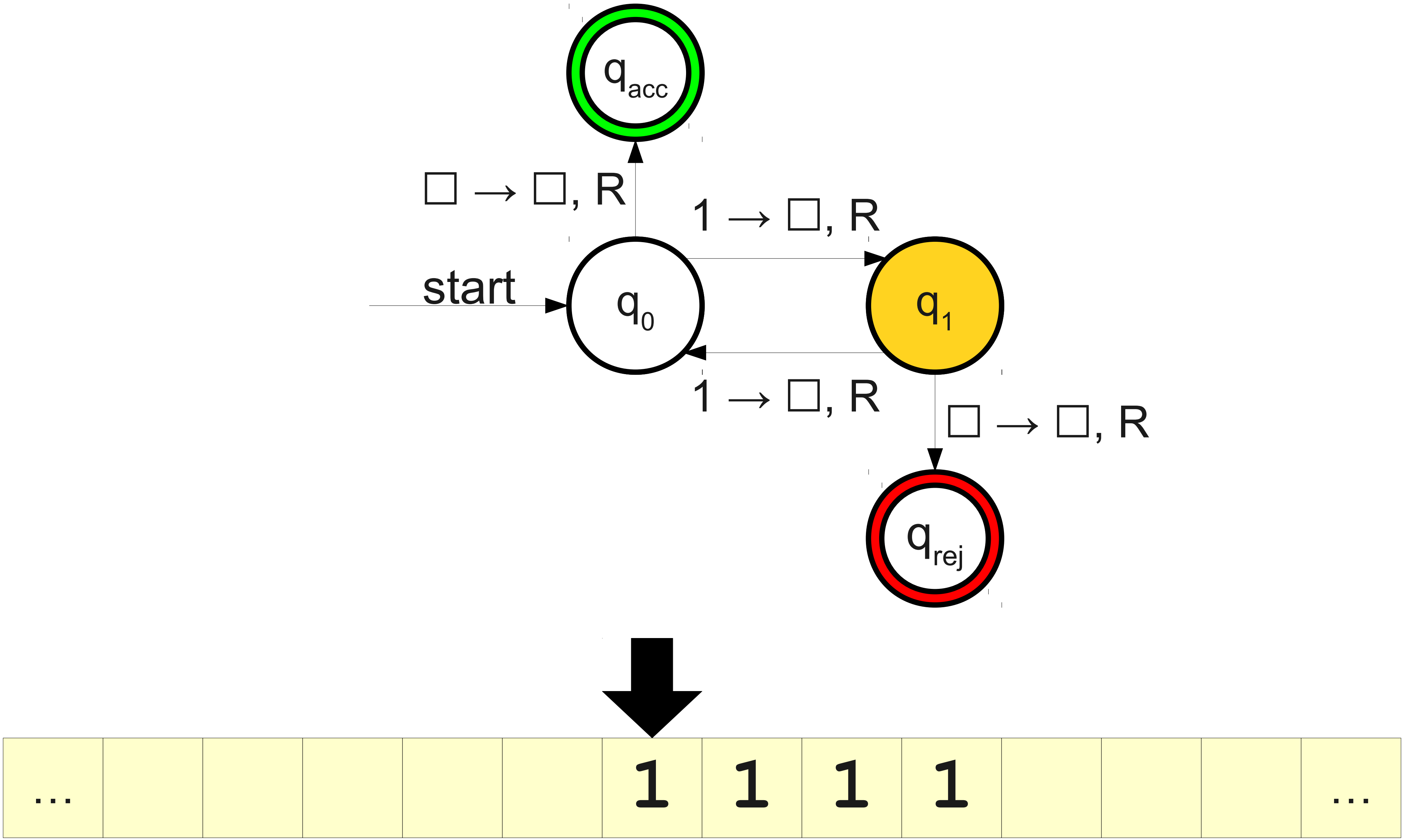

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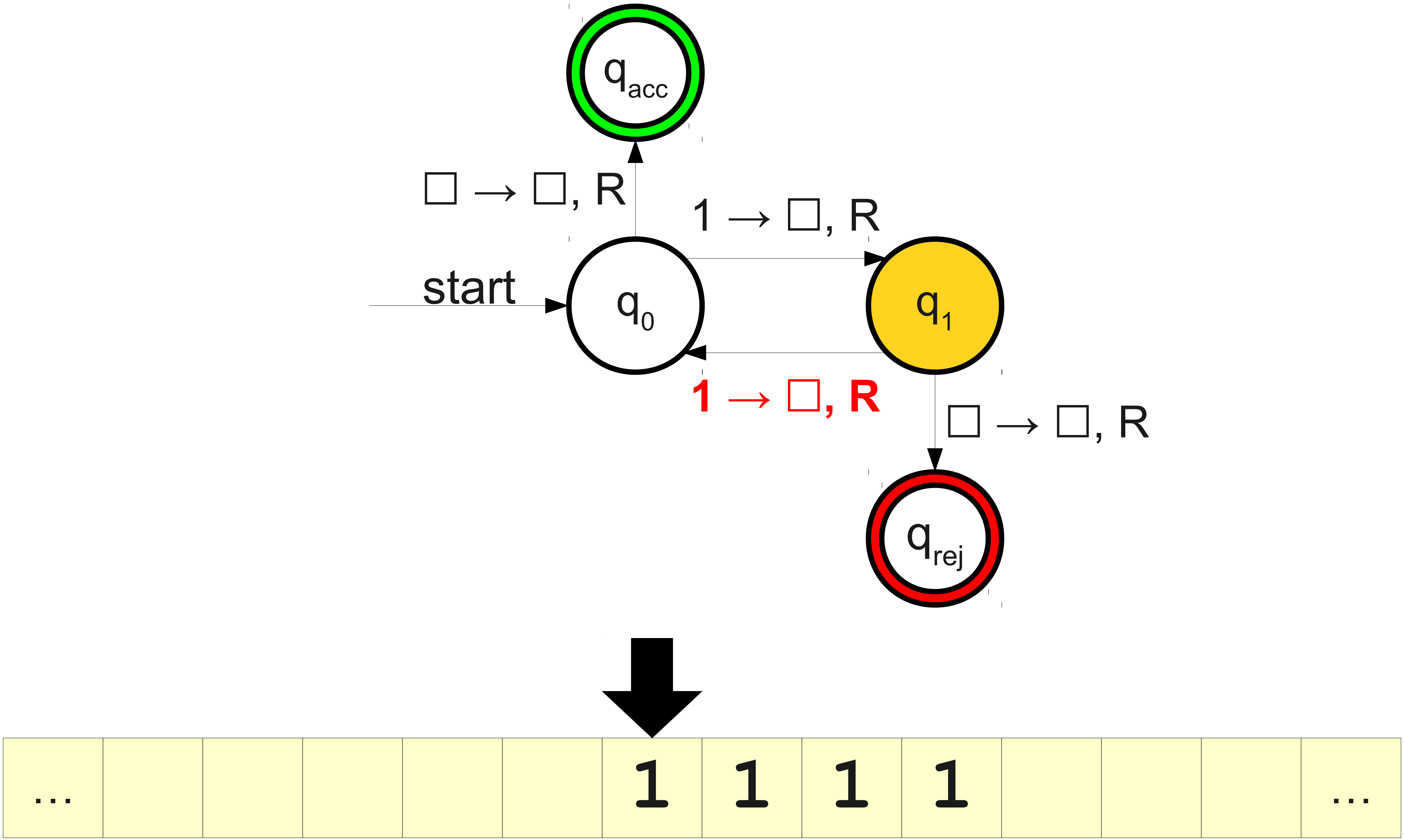

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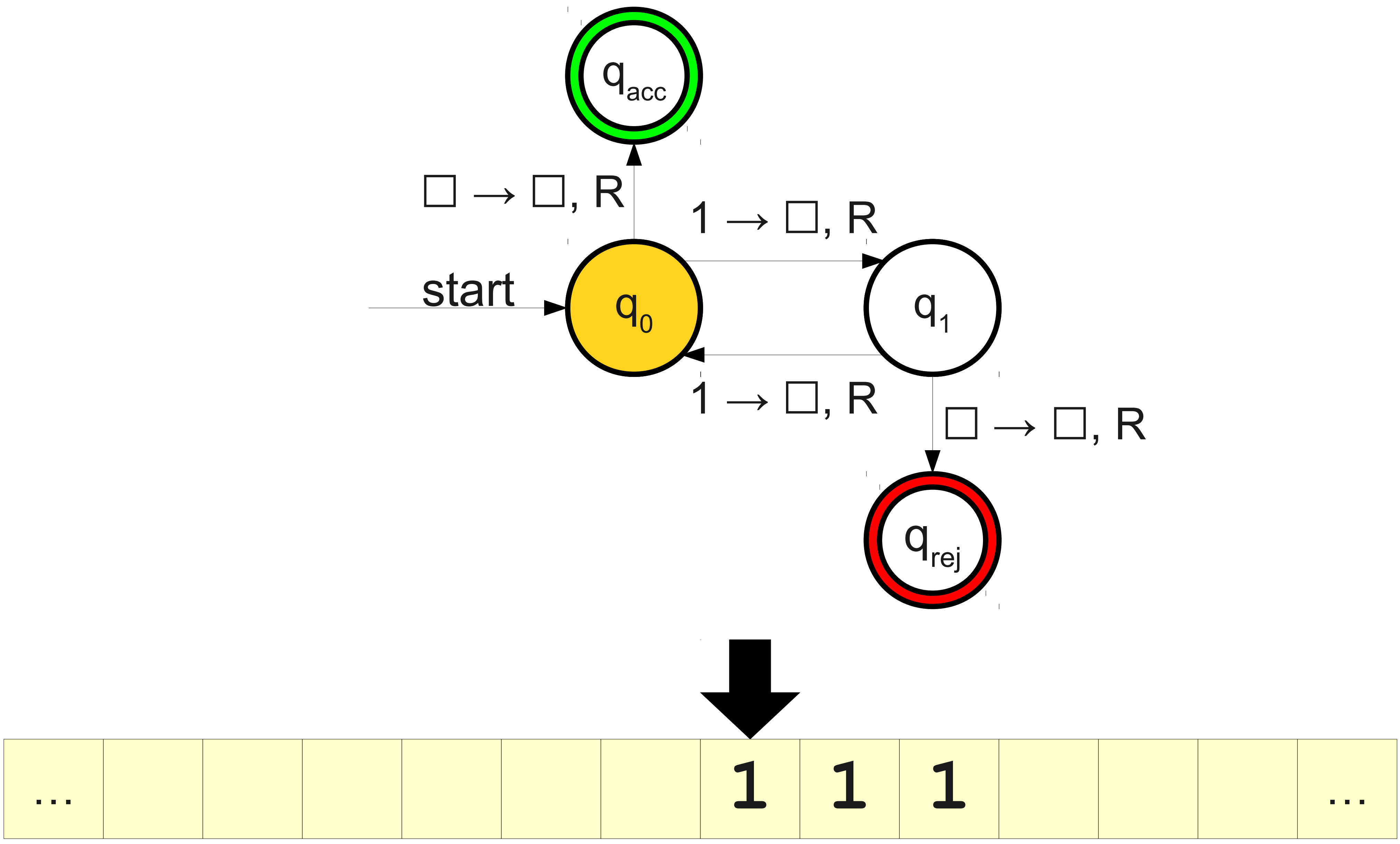

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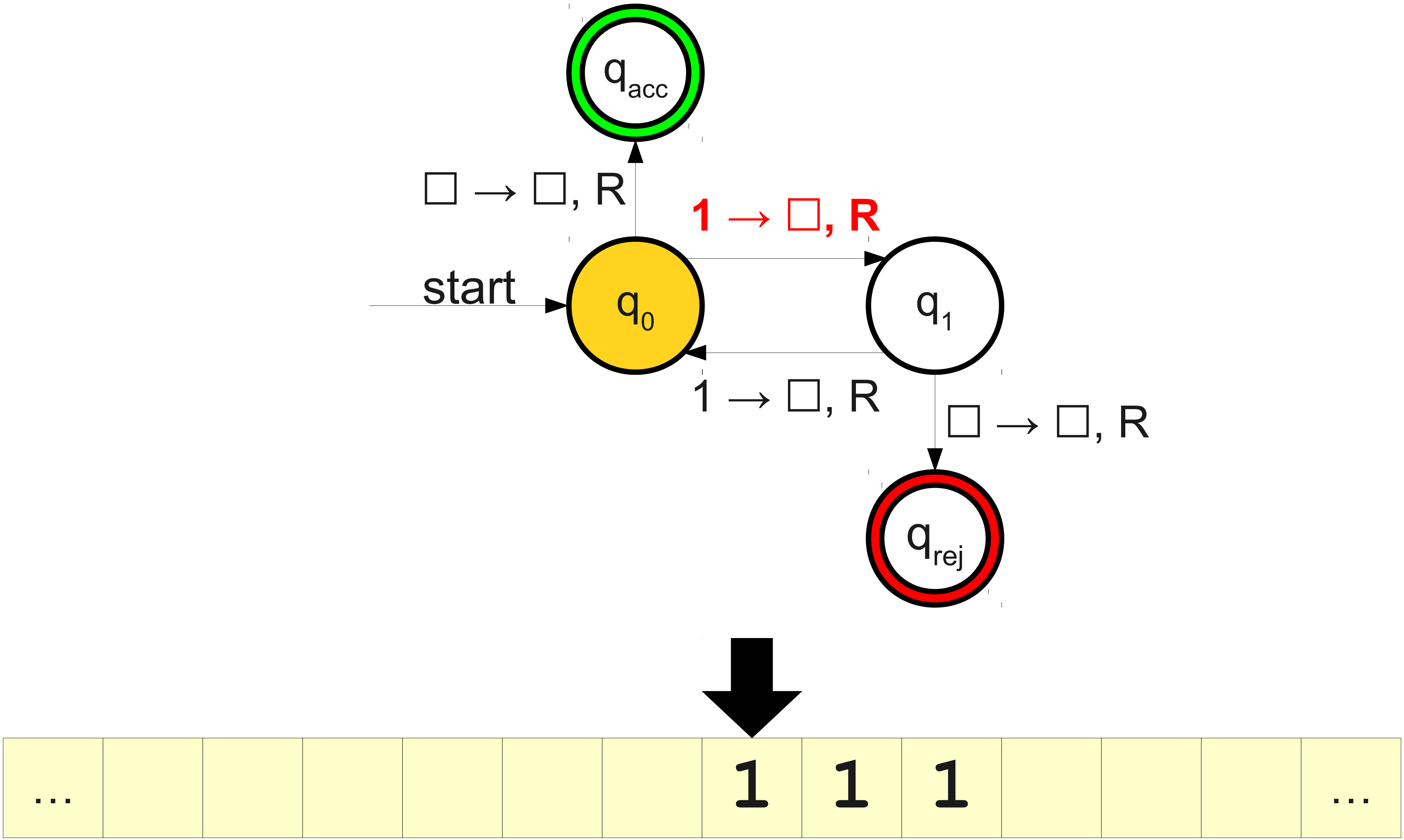

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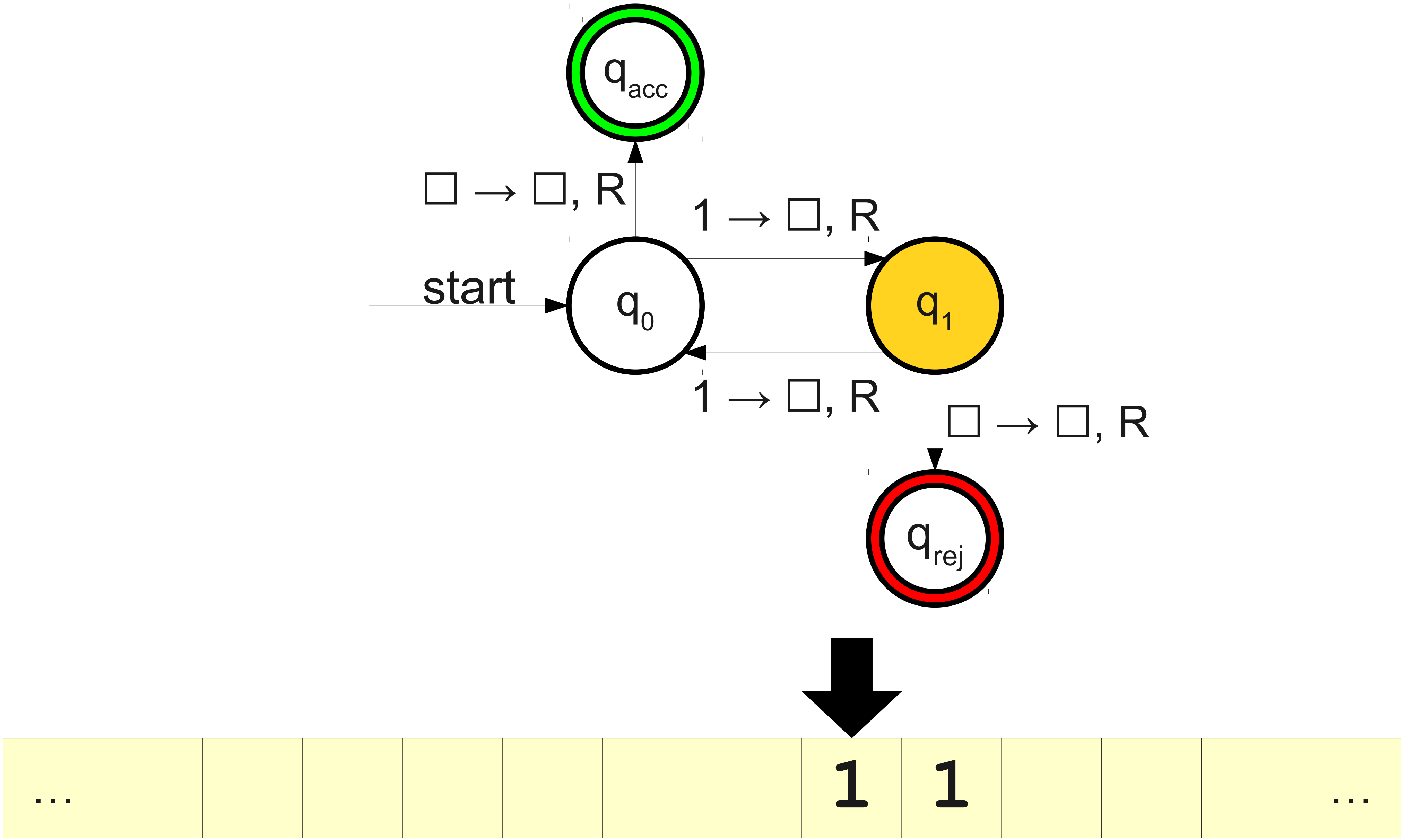

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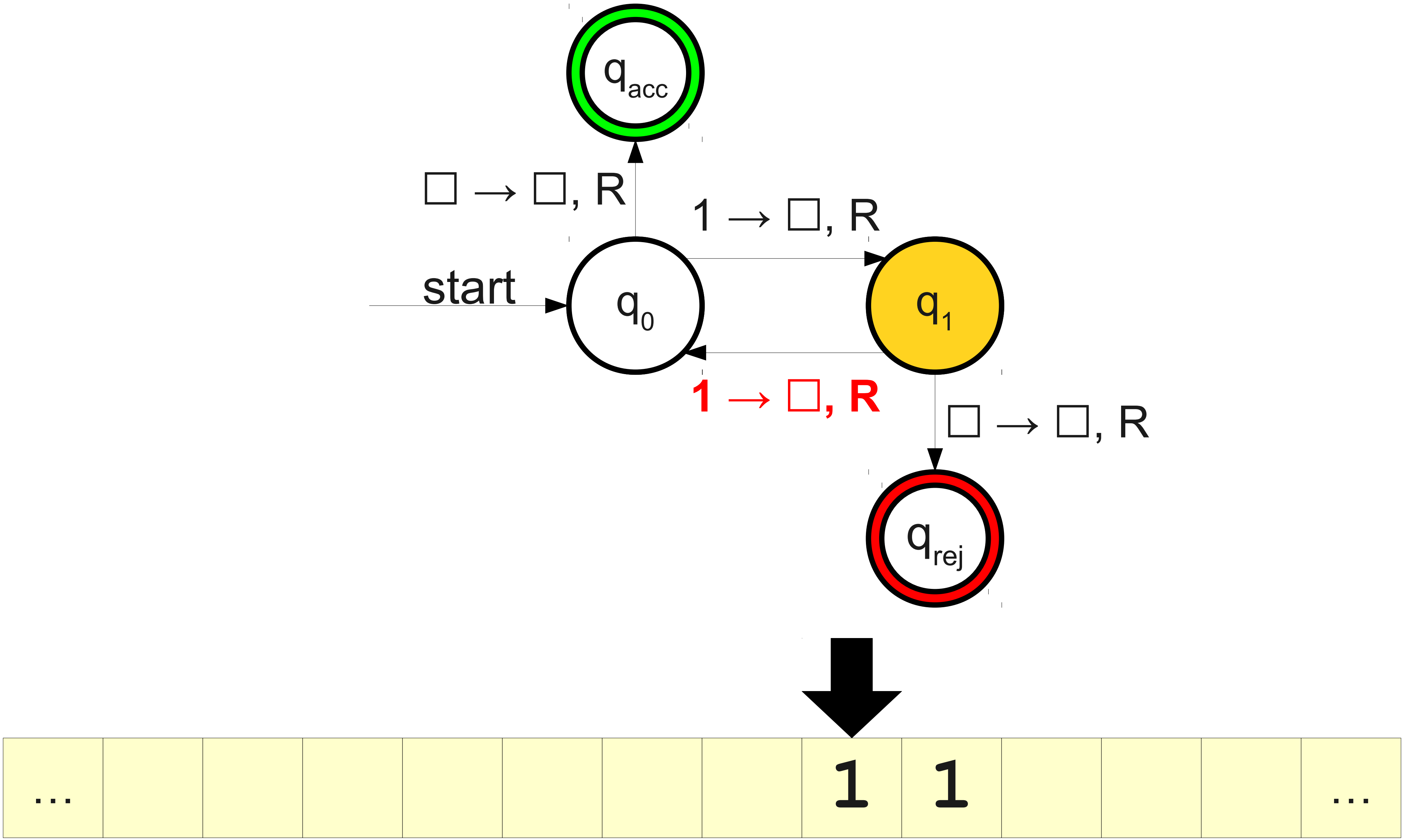

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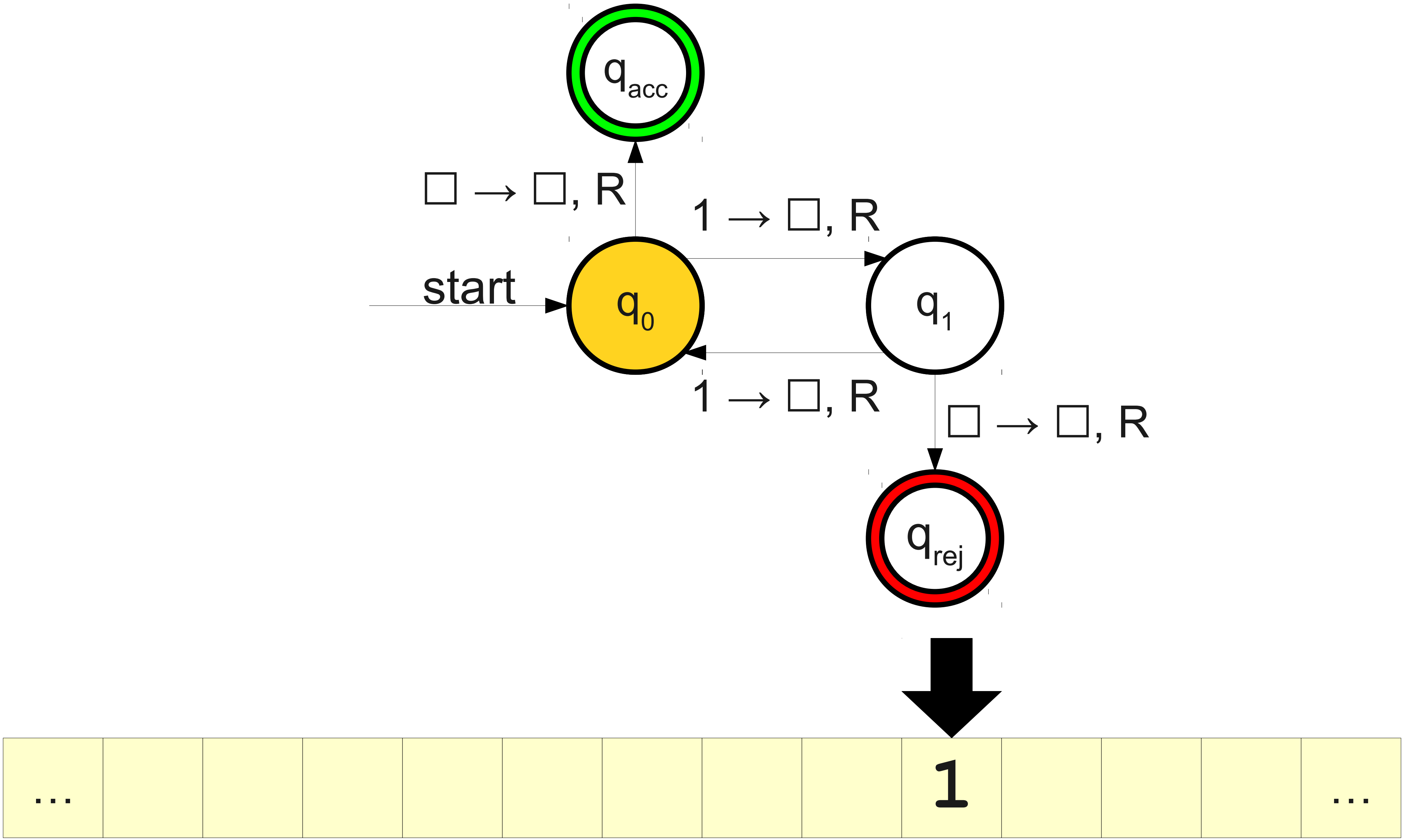

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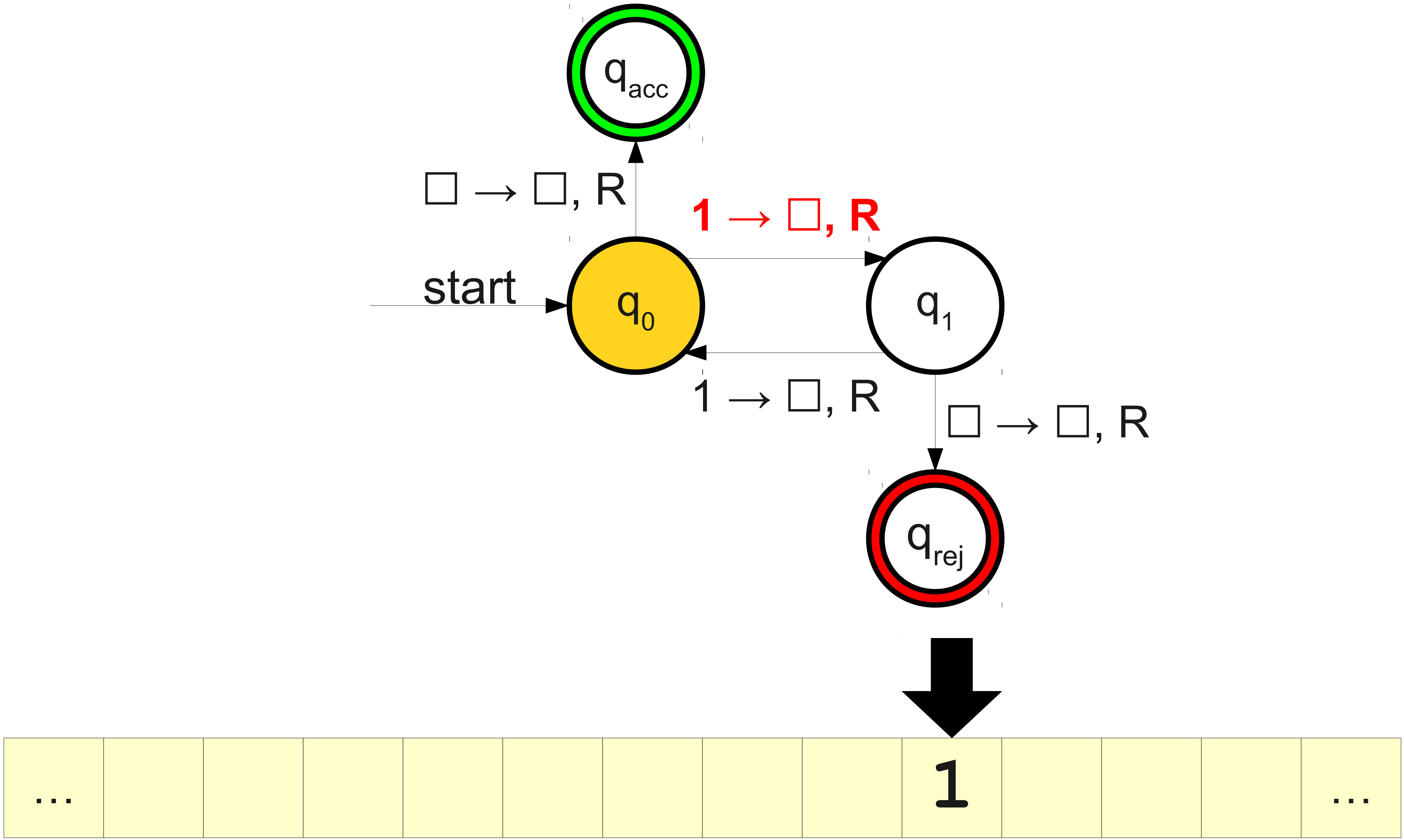

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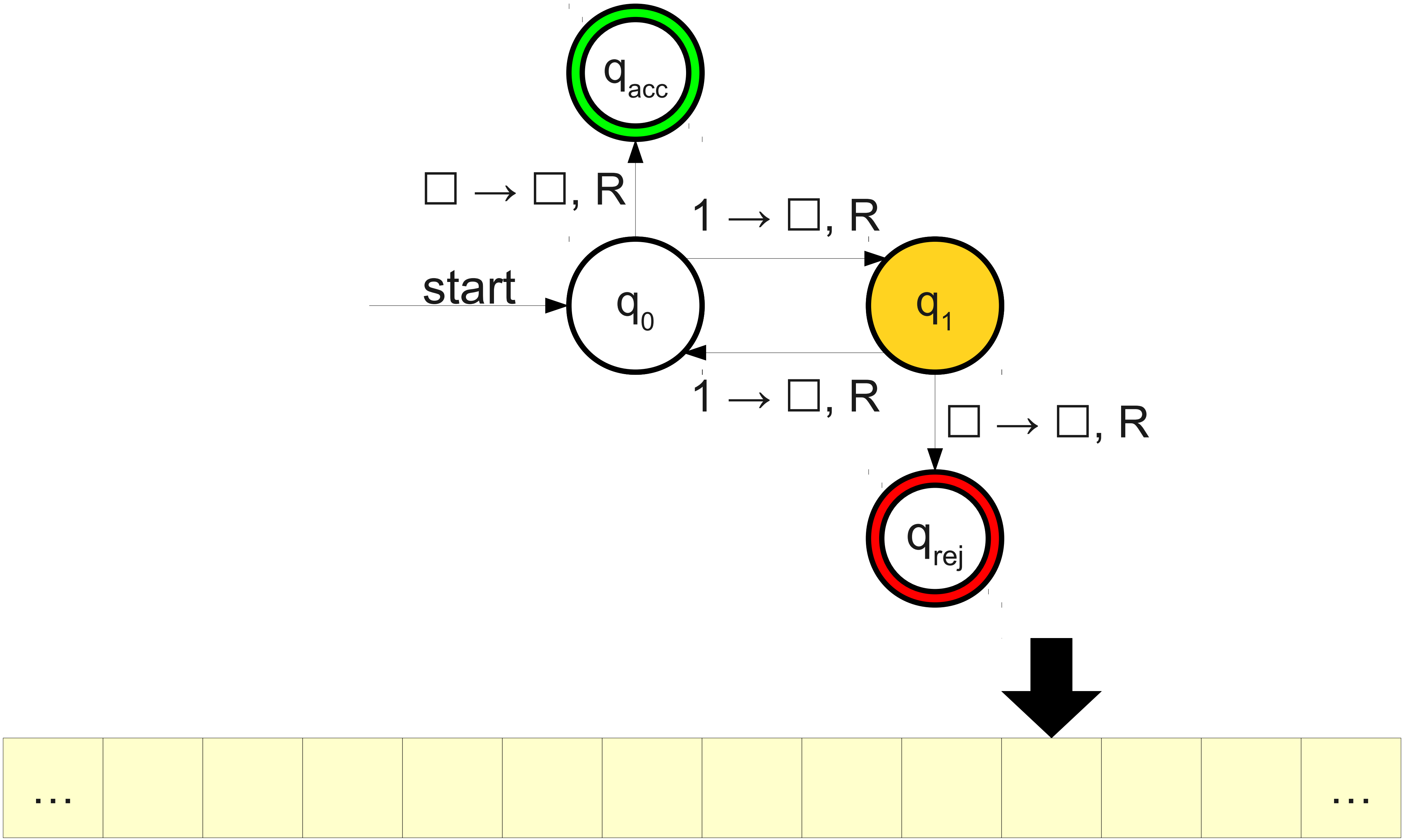

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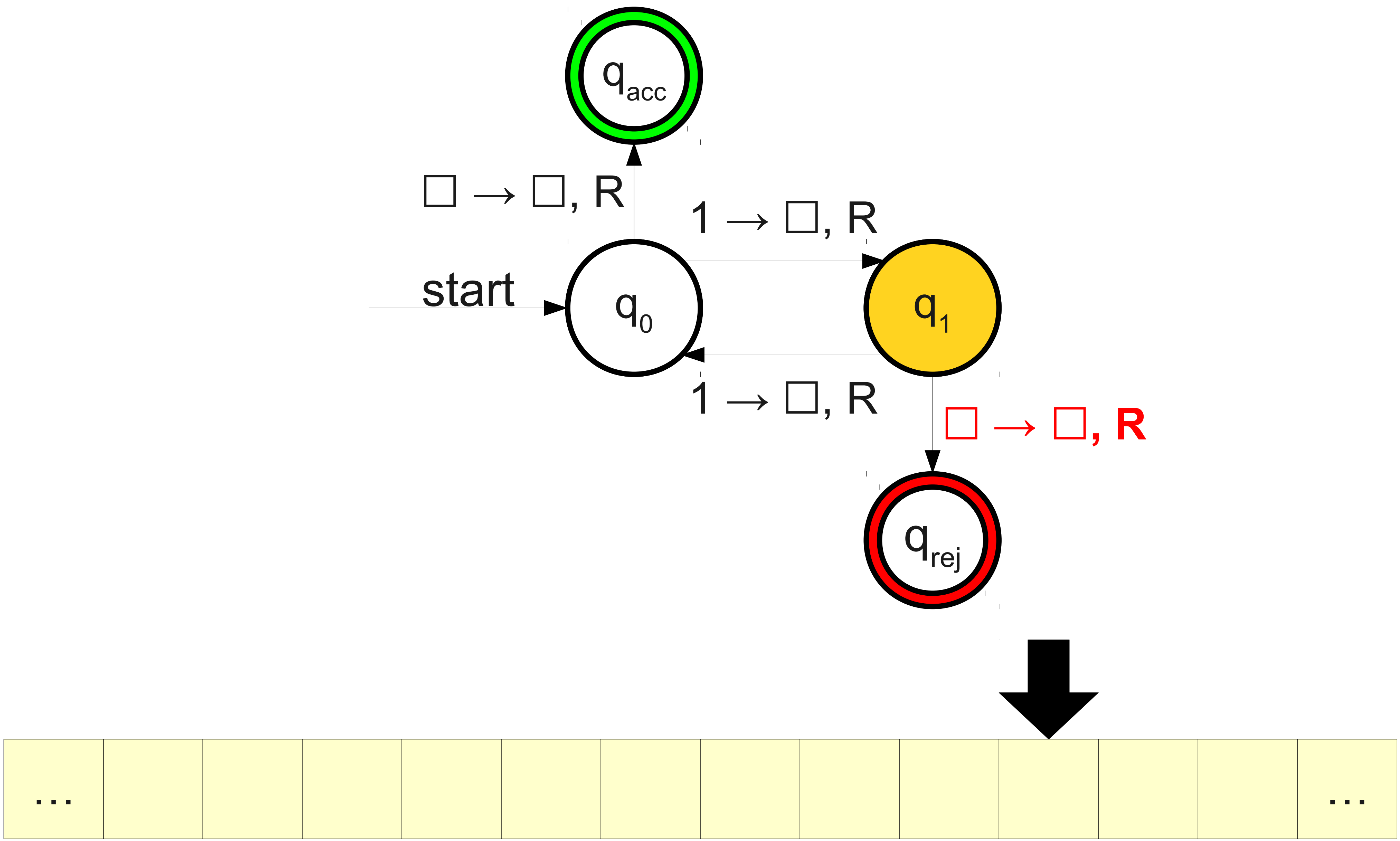

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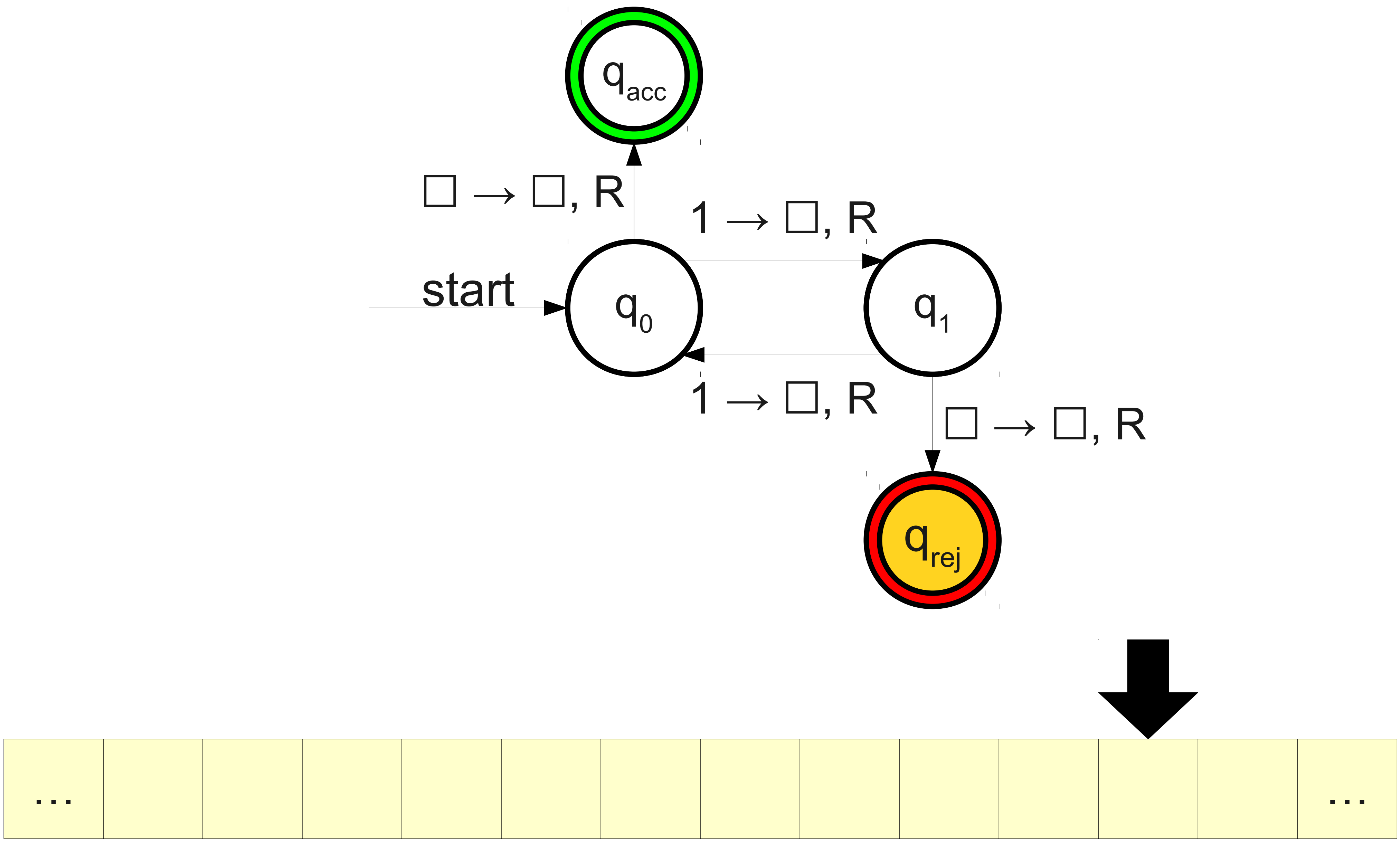

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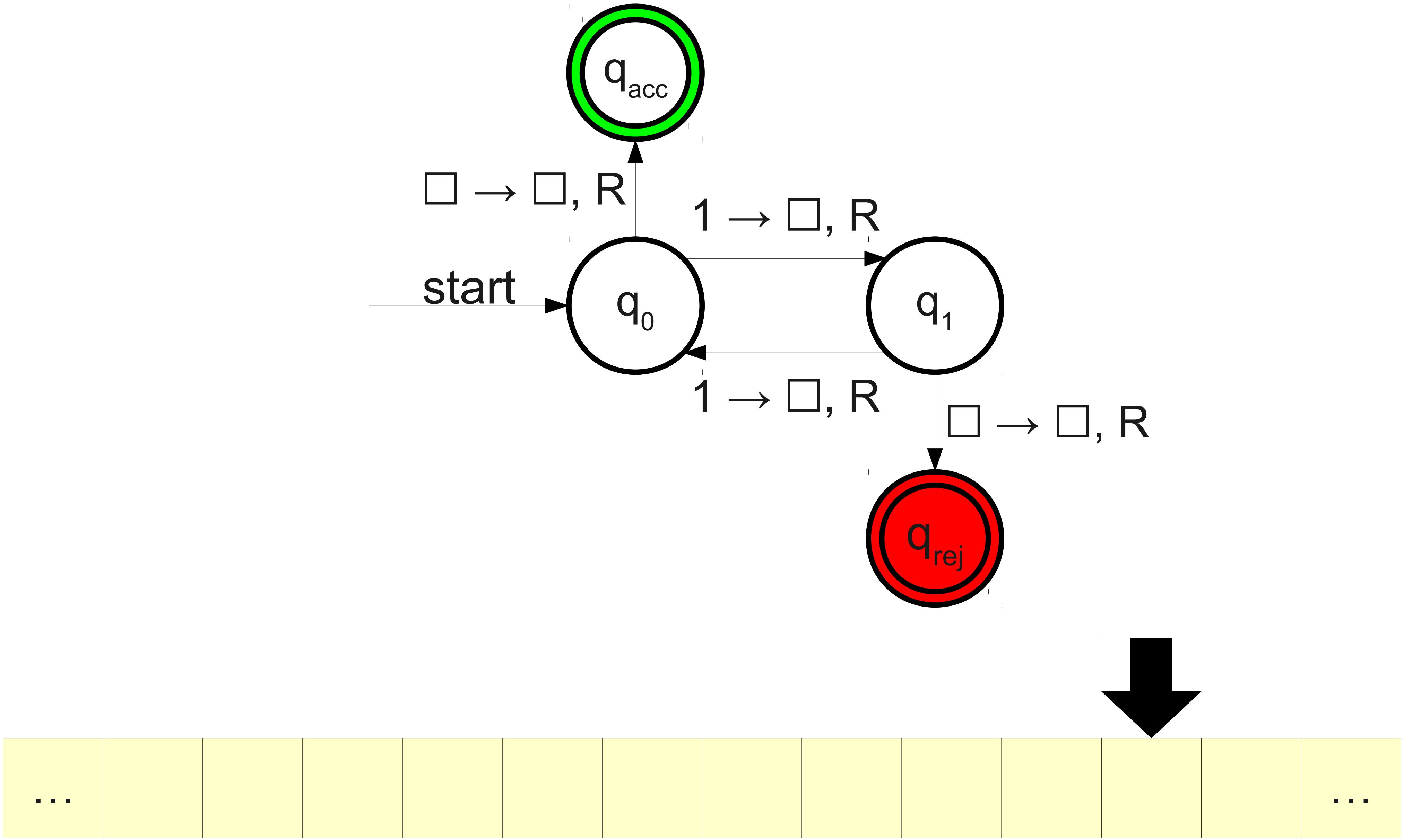

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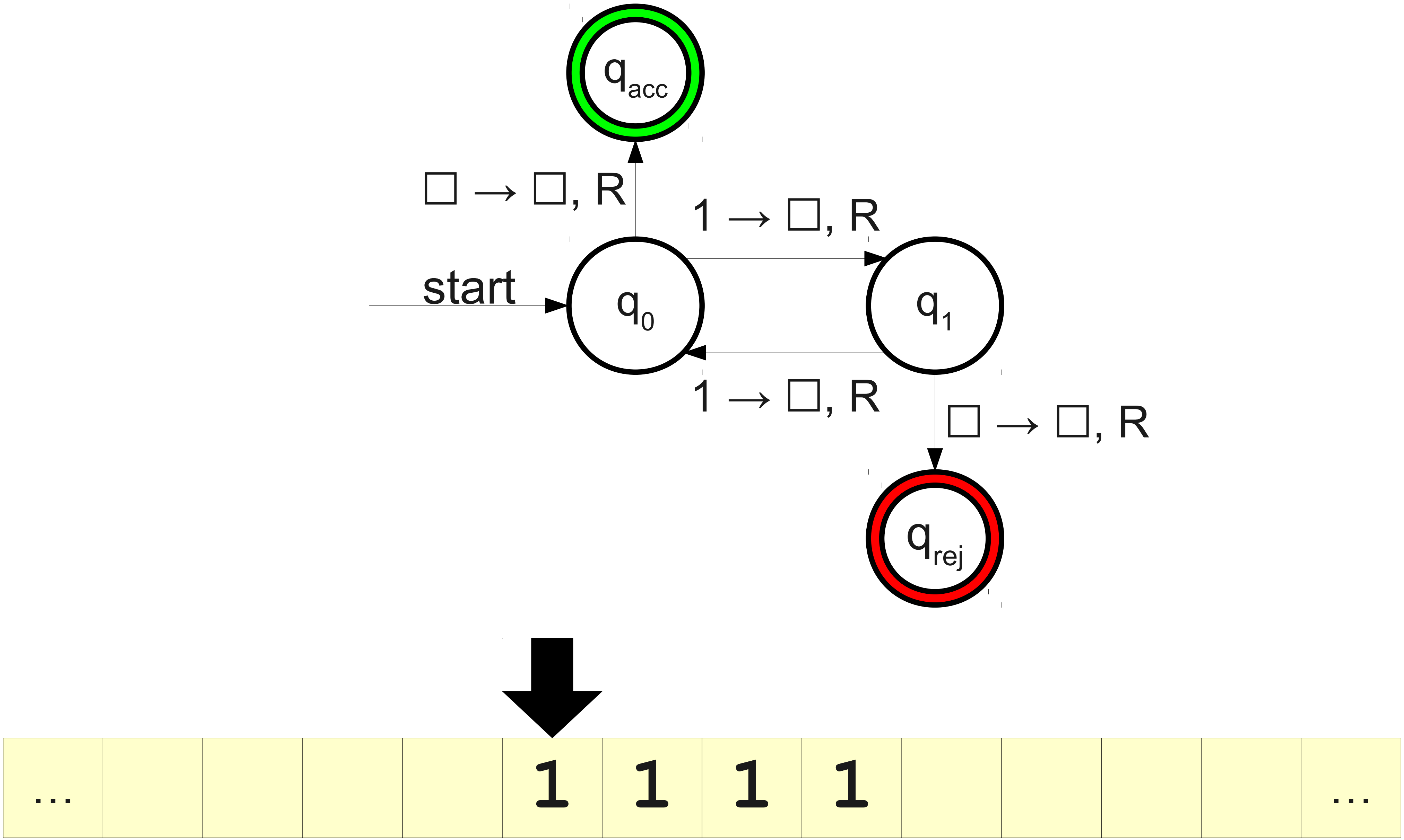

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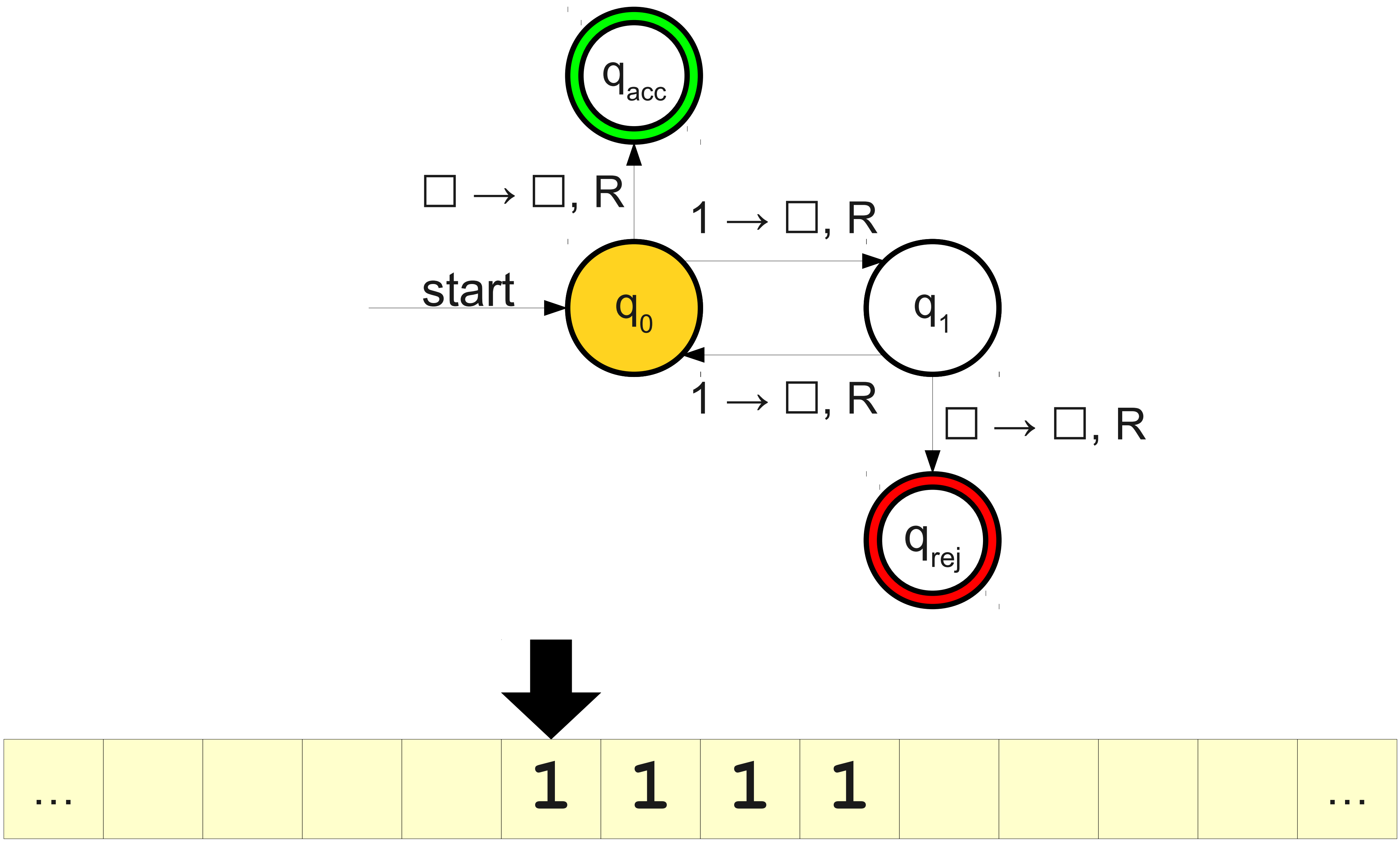

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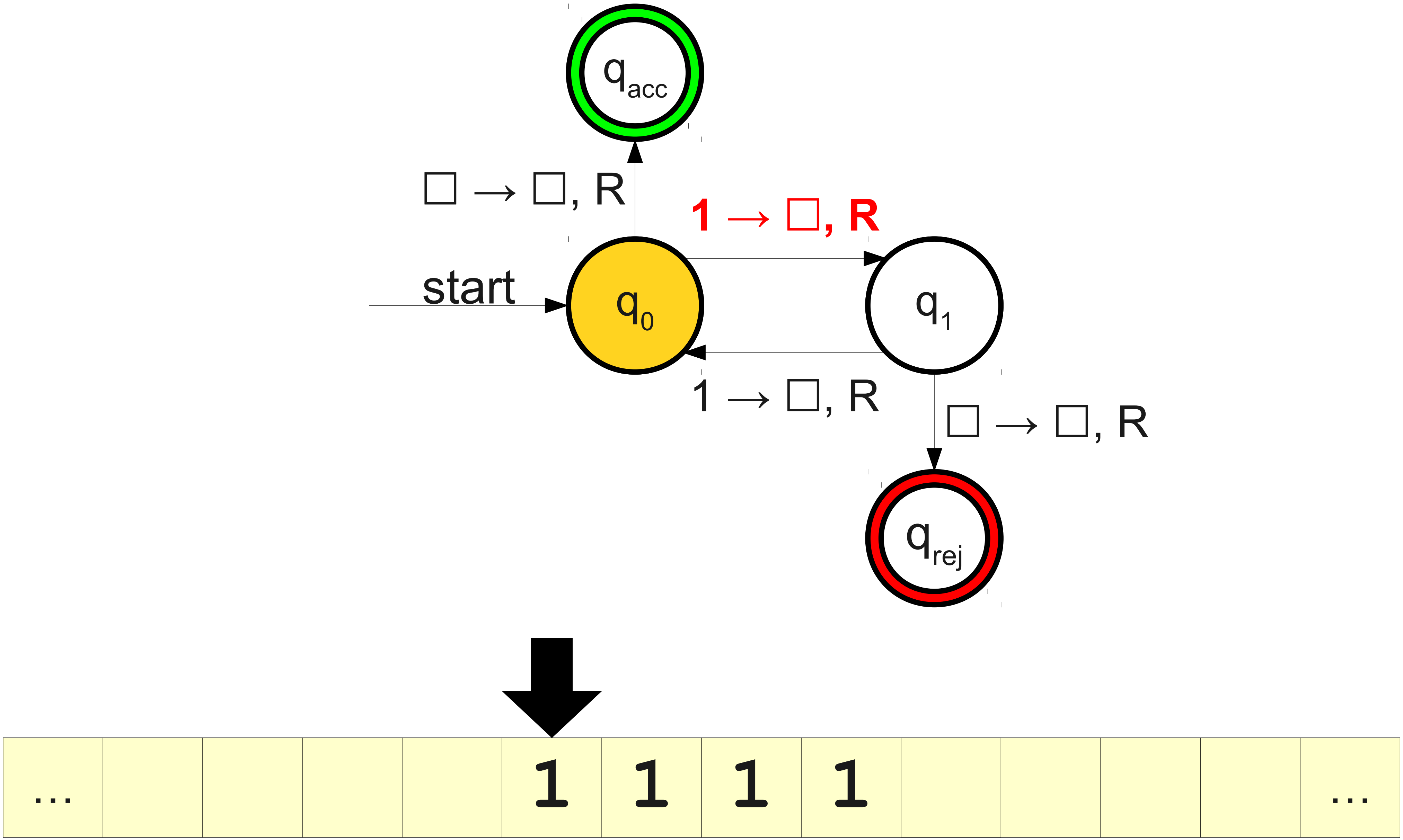

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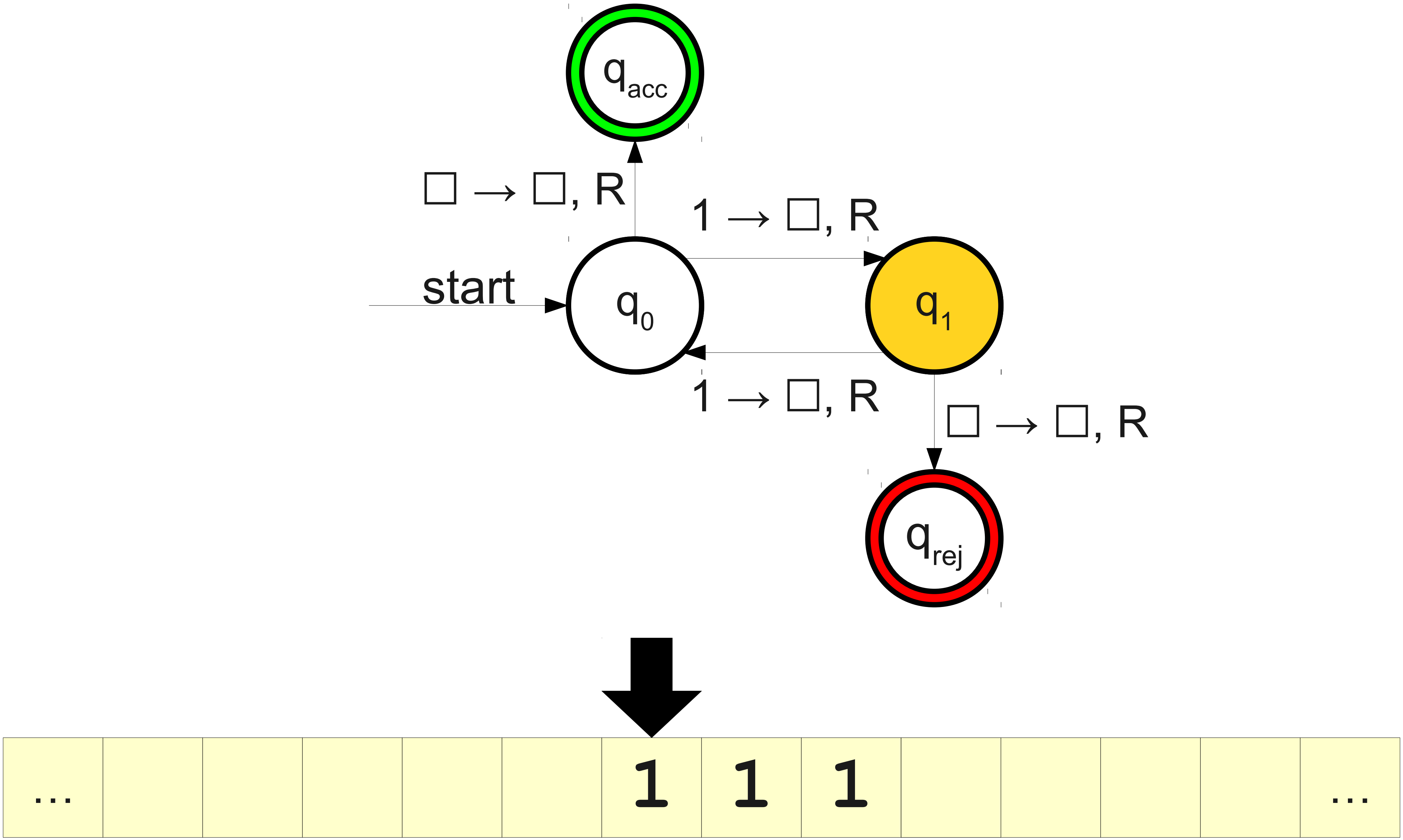

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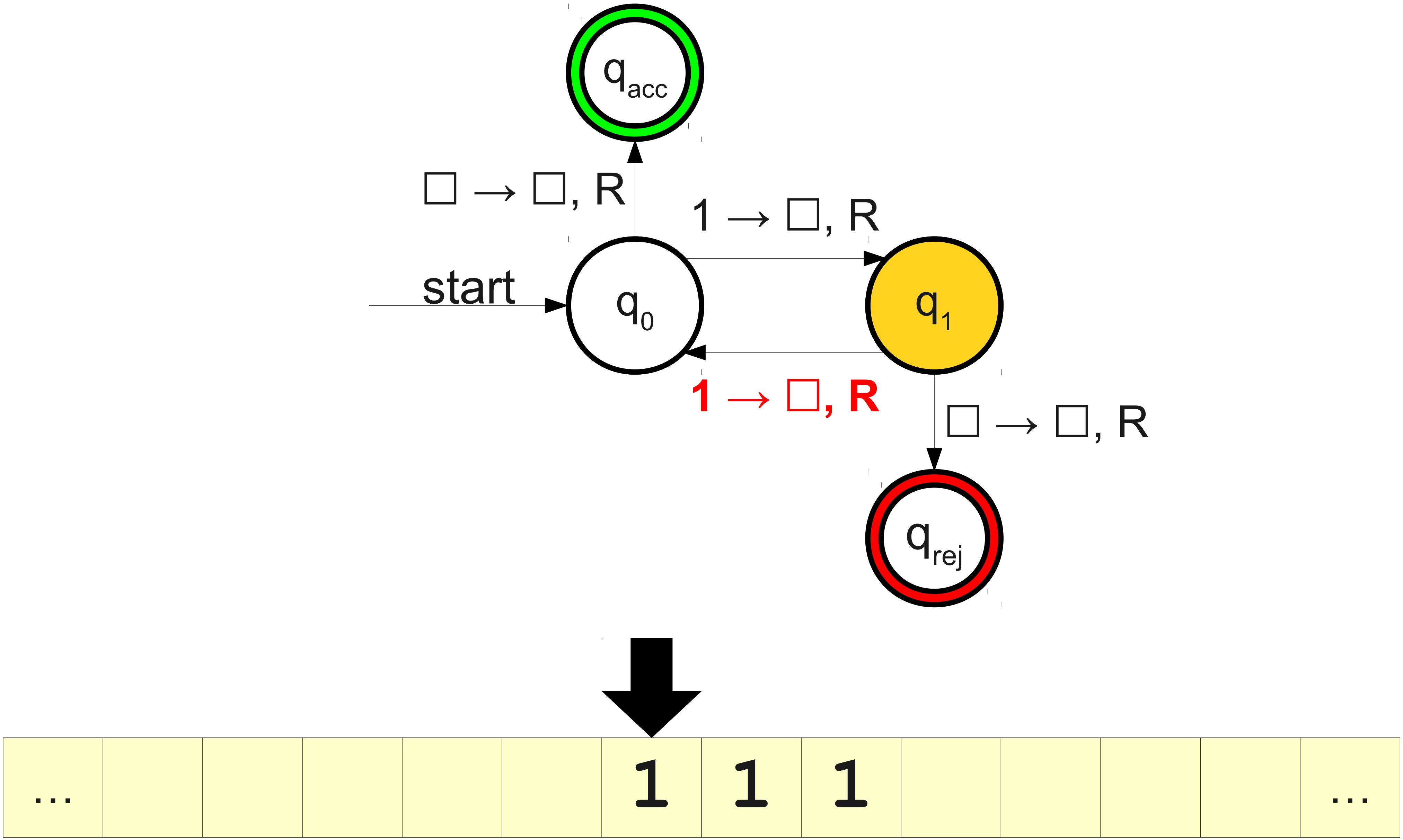

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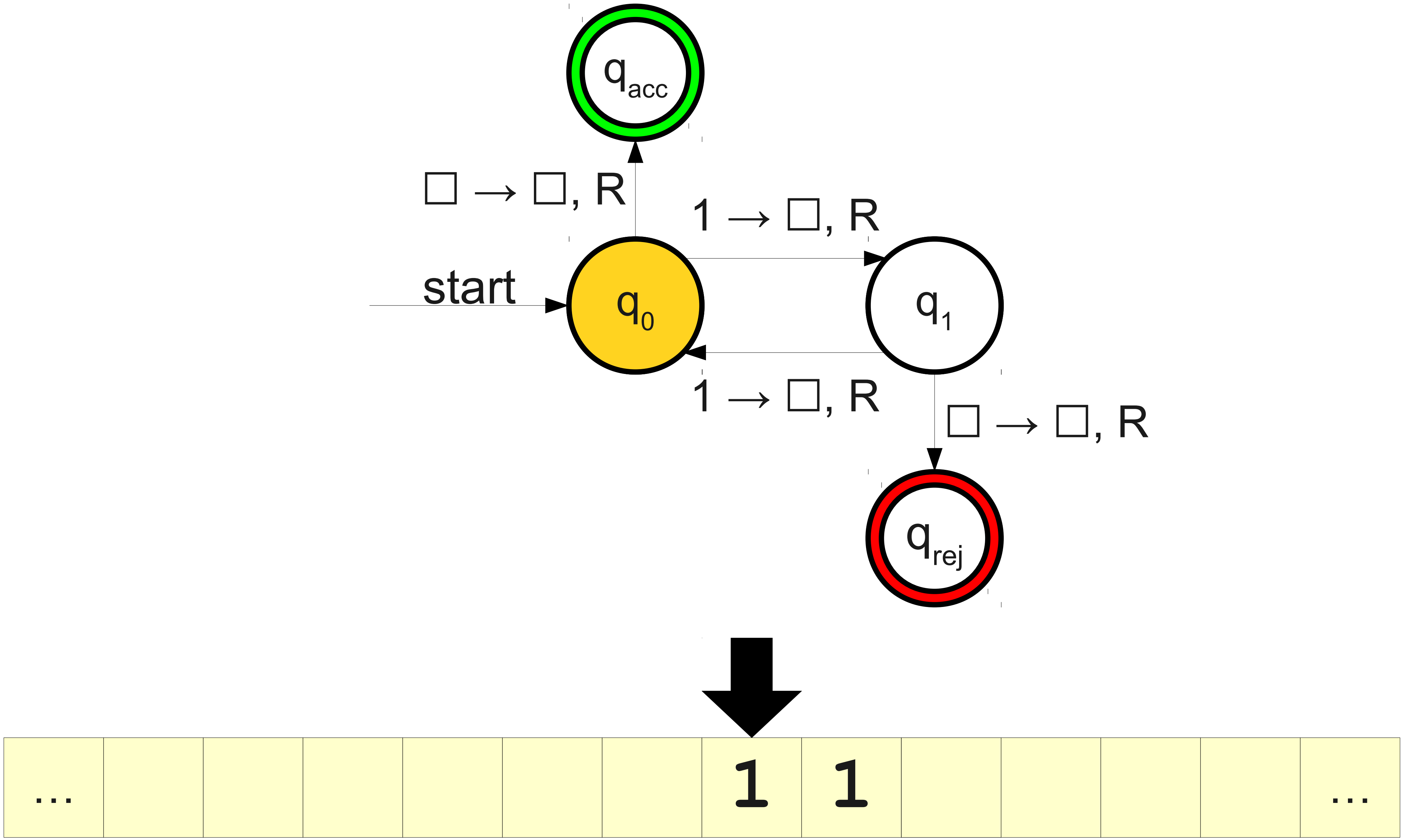

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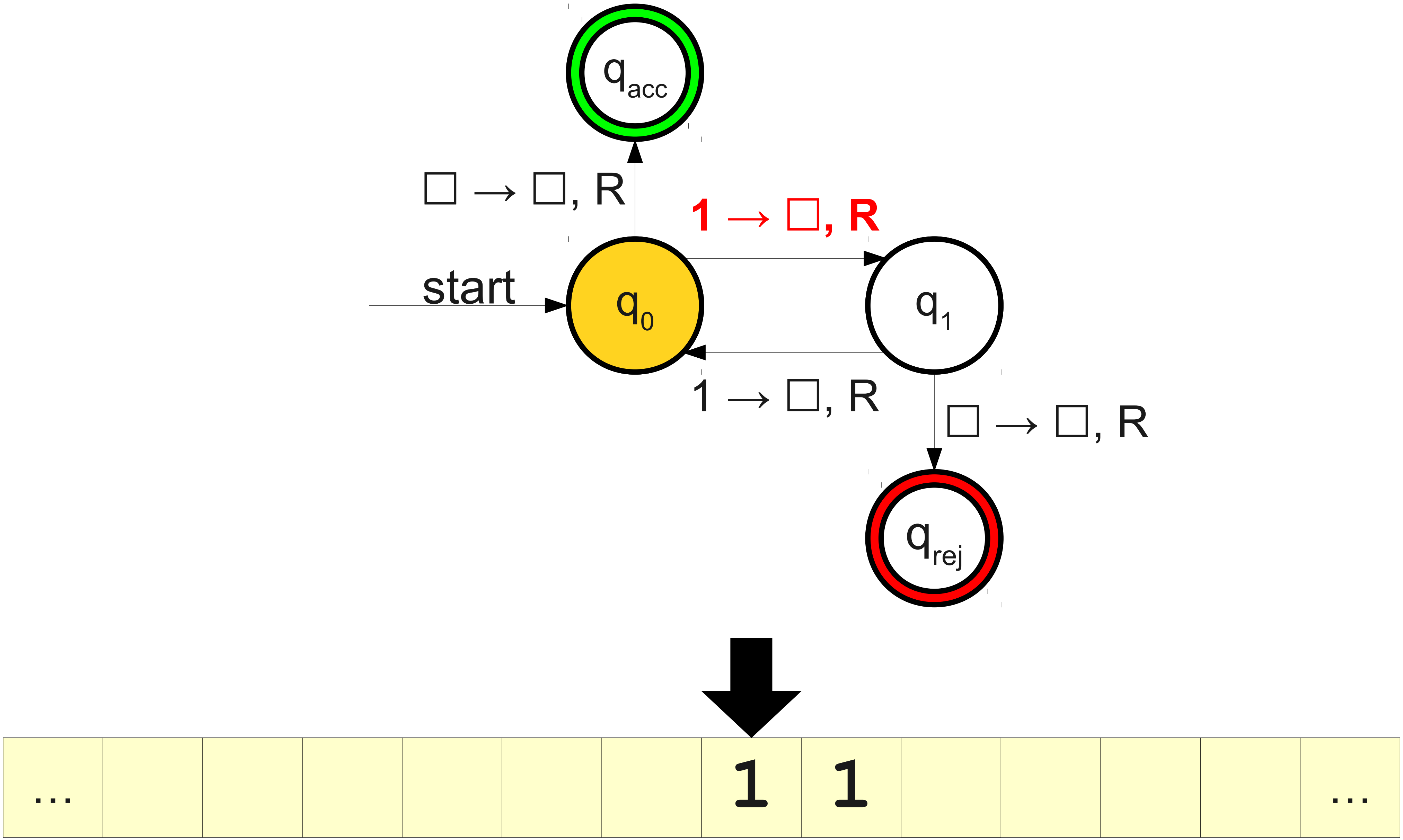

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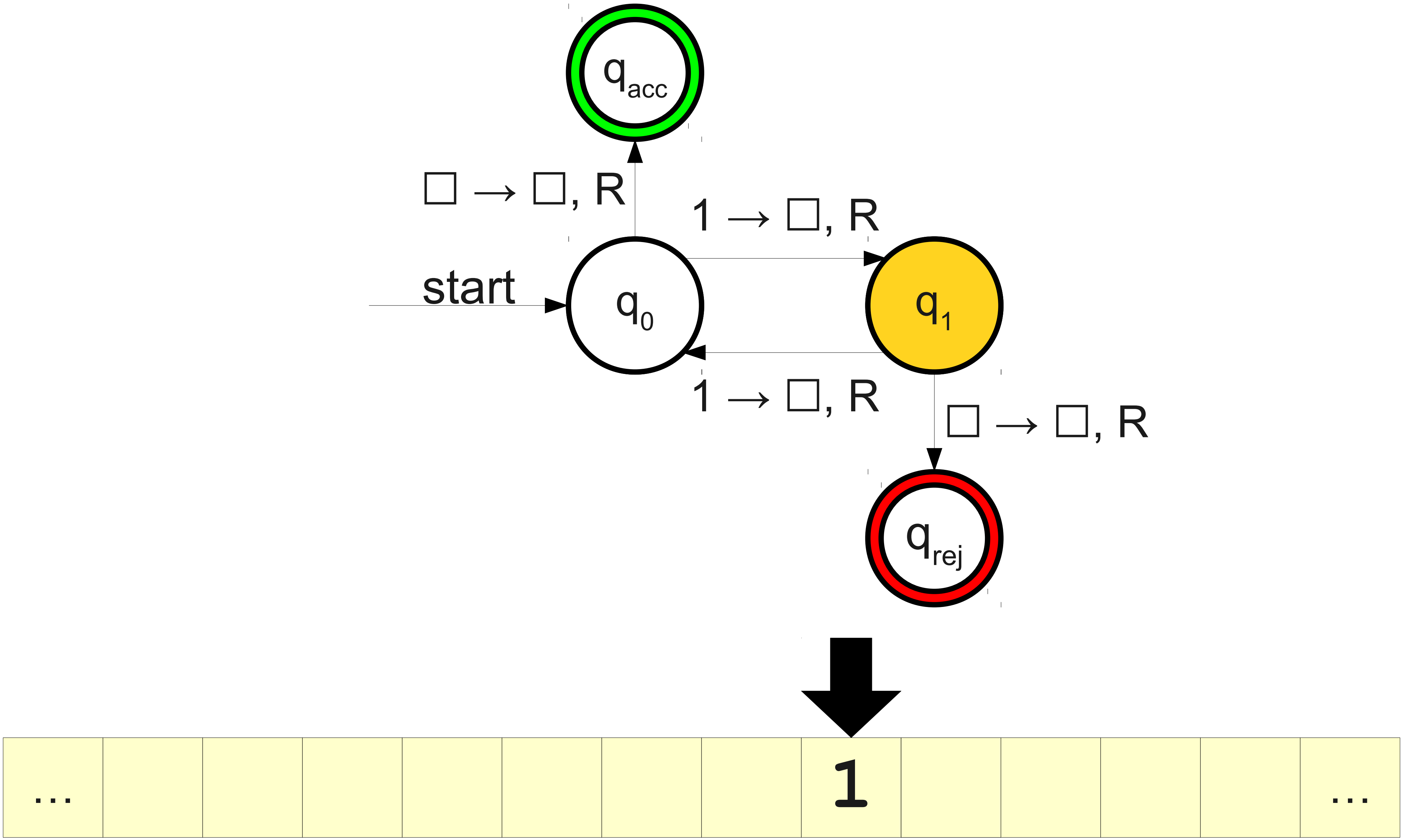

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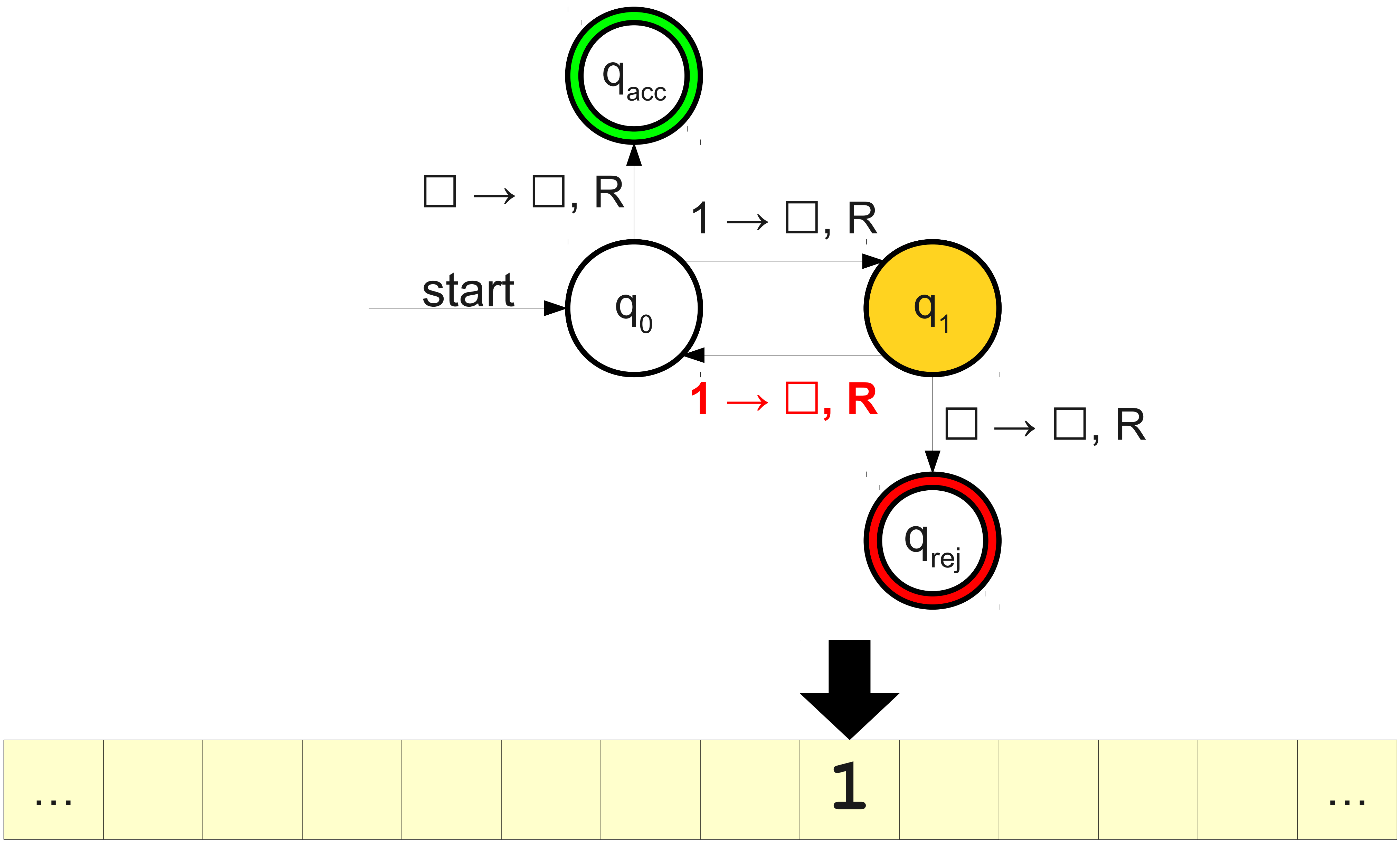

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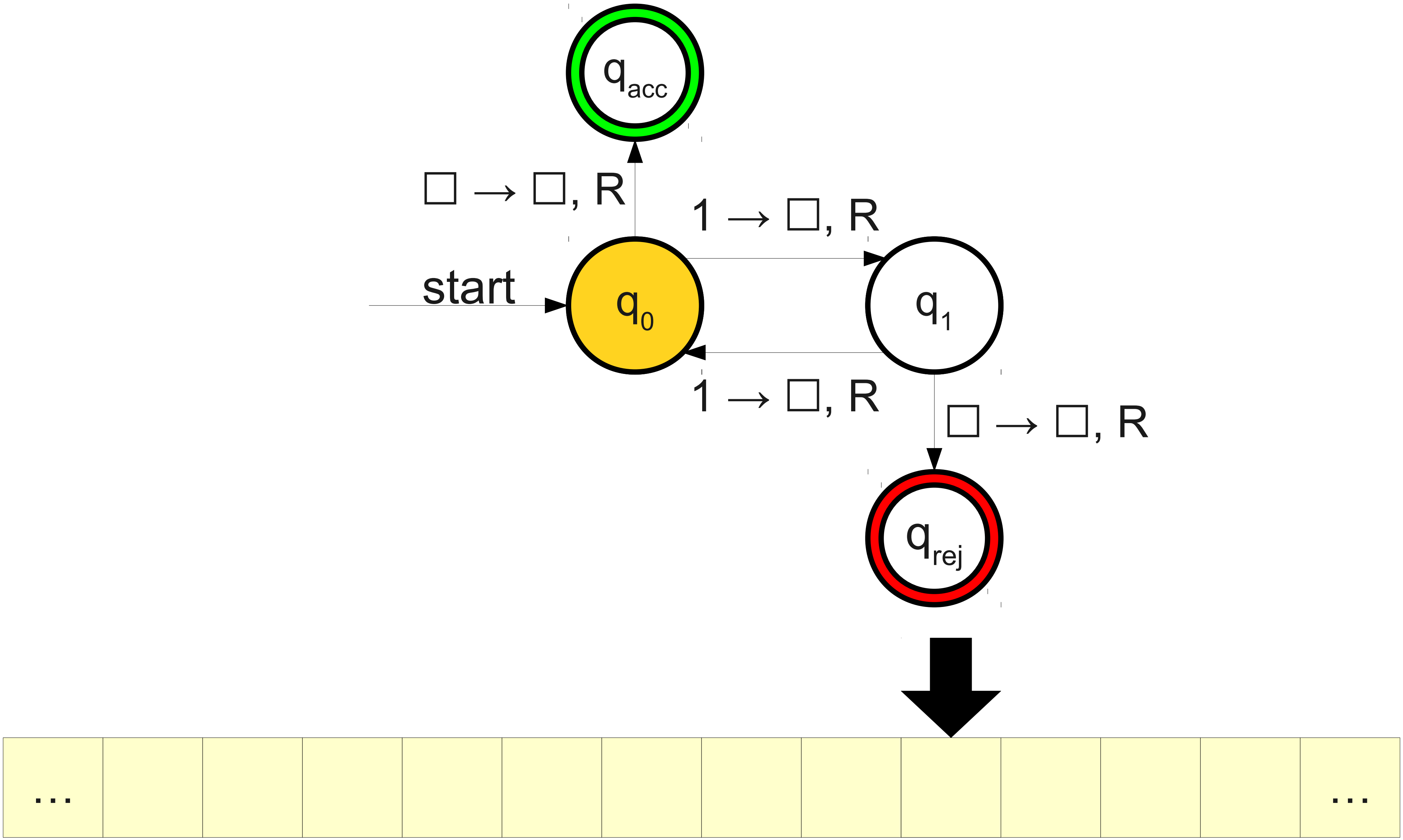

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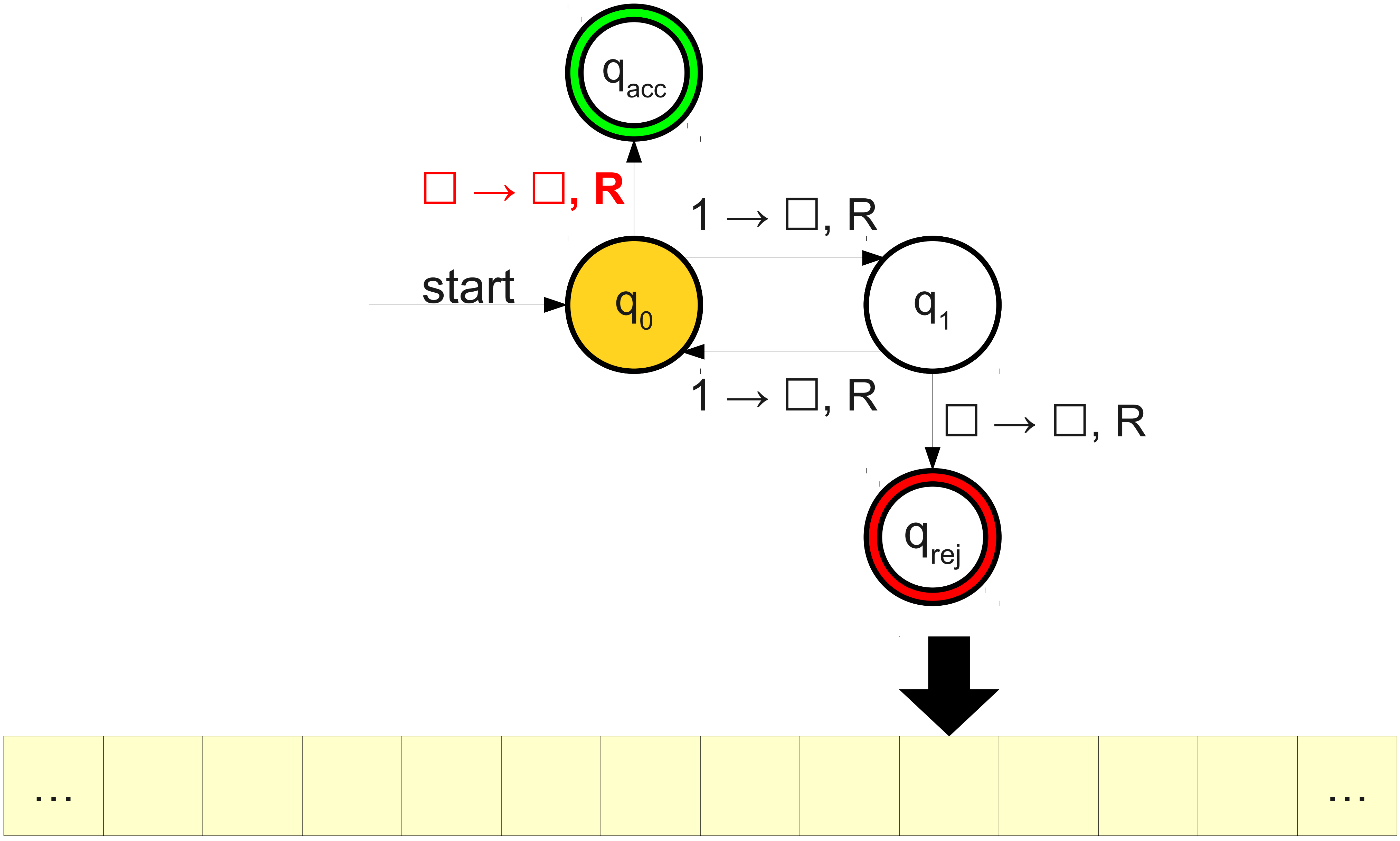

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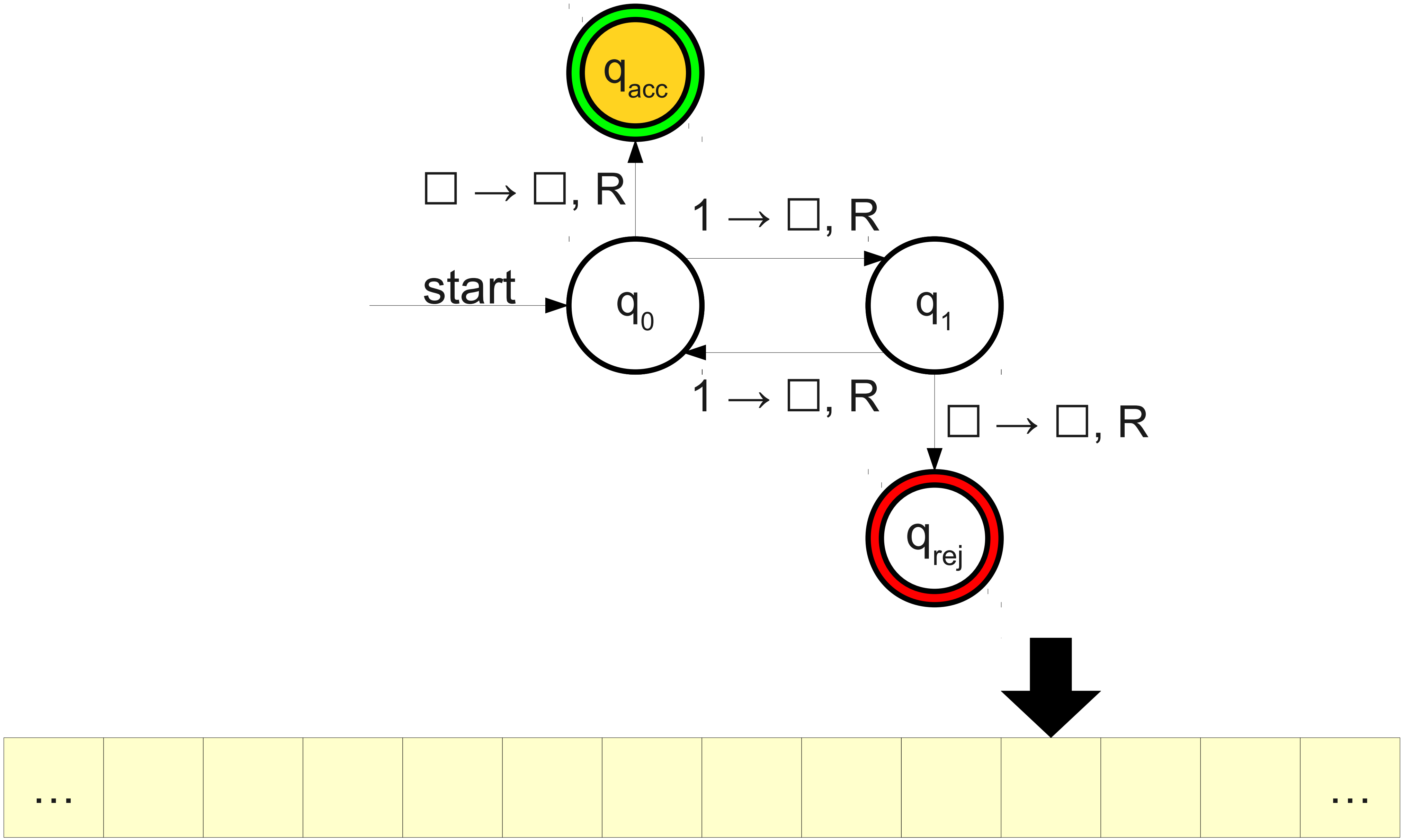

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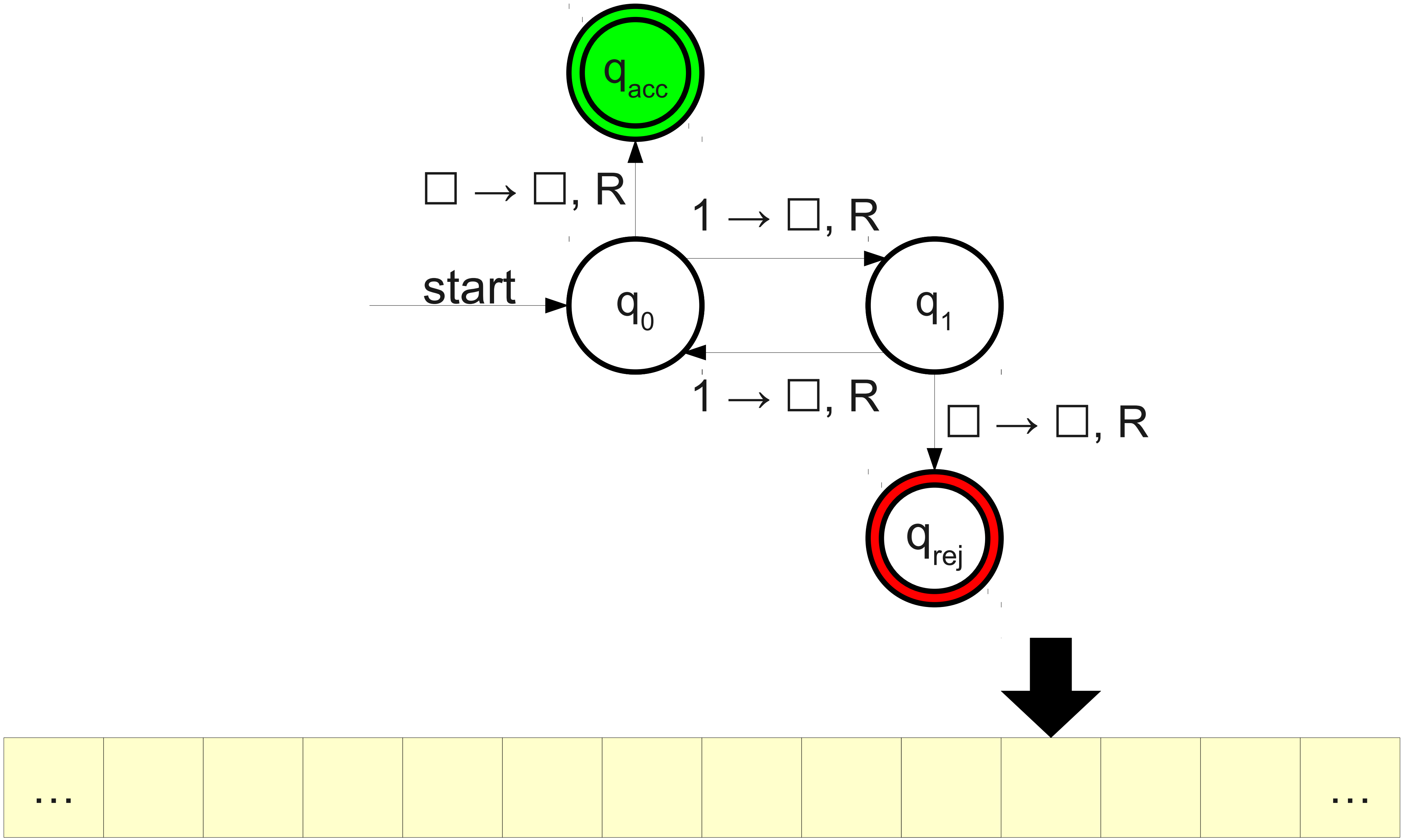


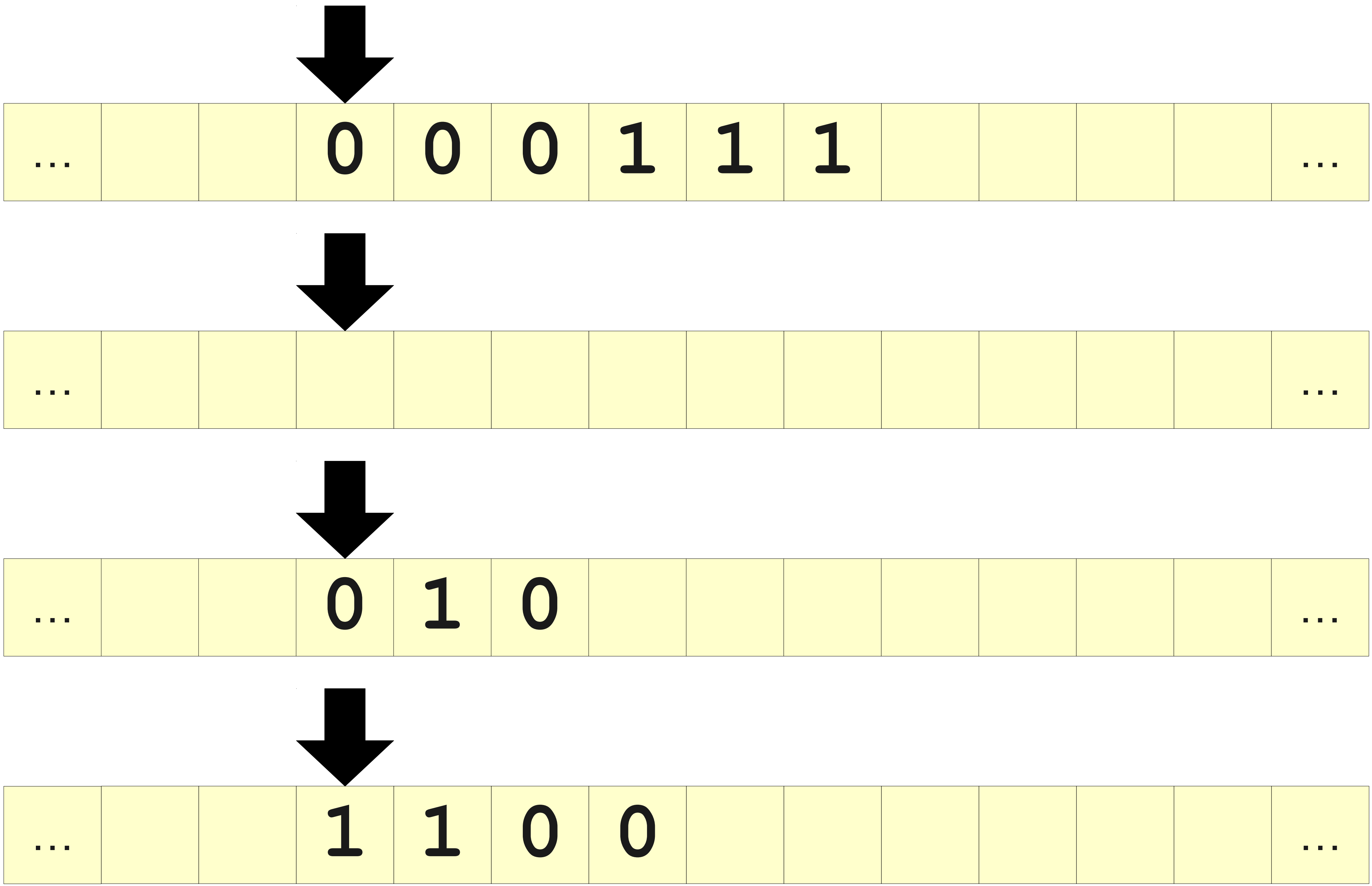
Designing Turing Machines

- Despite their simplicity, Turing machines are very powerful computing devices.
- Today's lecture explores how to design Turing machines for various languages.

Designing Turing Machines

- Let $\Sigma = \{\texttt{0}, \texttt{1}\}$ and consider the language $L = \{\texttt{0}^n\texttt{1}^n \mid n \in \mathbb{N}\}$.
- We know that L is context-free.
- How might we build a Turing machine for it?

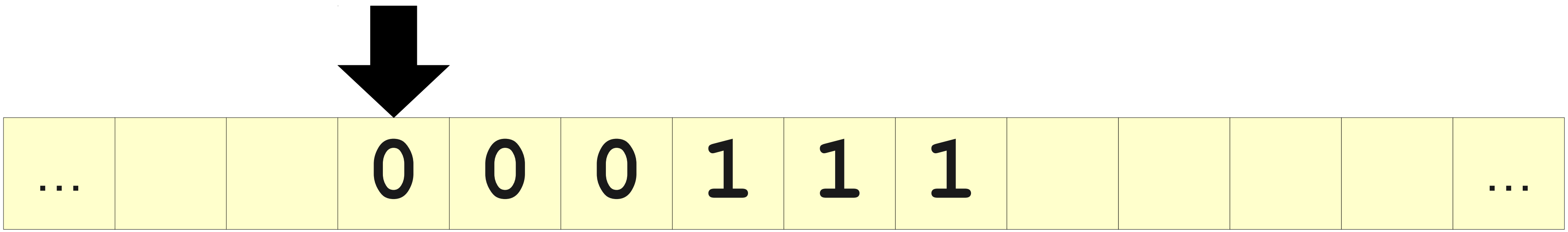
$$L = \{ \mathbf{0}^n \mathbf{1}^n \mid n \in \mathbb{N} \}$$



A Recursive Approach

- The string ε is in L.
- The string `0`w`1` is in L iff w is in L.
- Any string starting with `1` is not in L.
- Any string ending with `0` is not in L.

A Sketch of the TM



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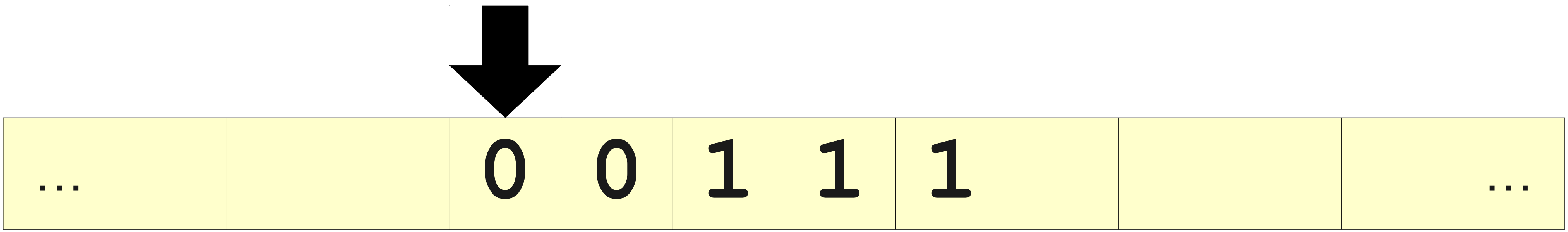

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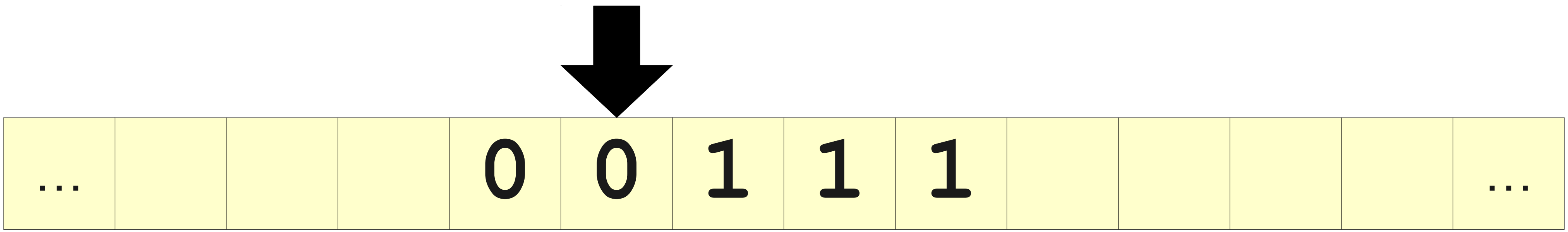

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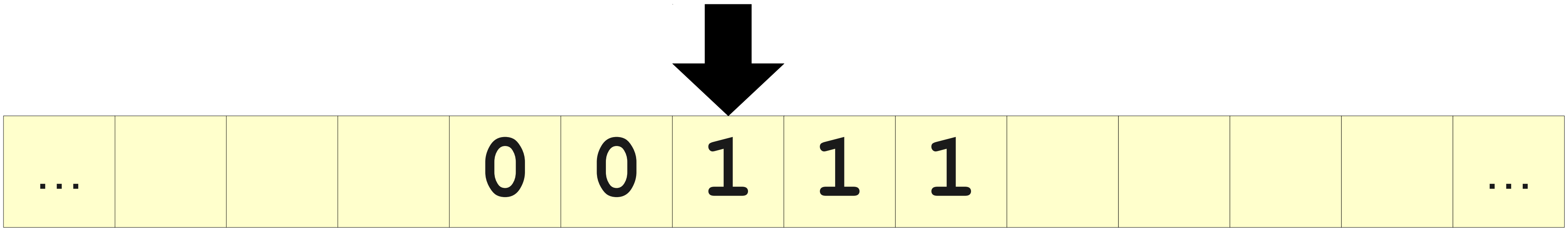

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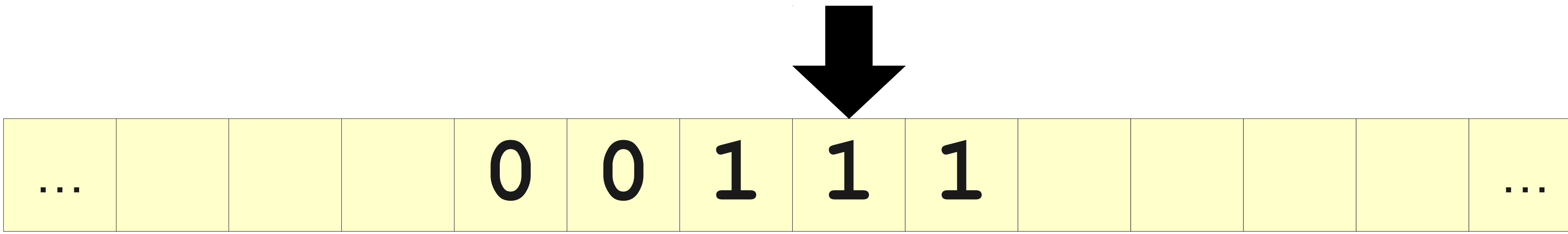

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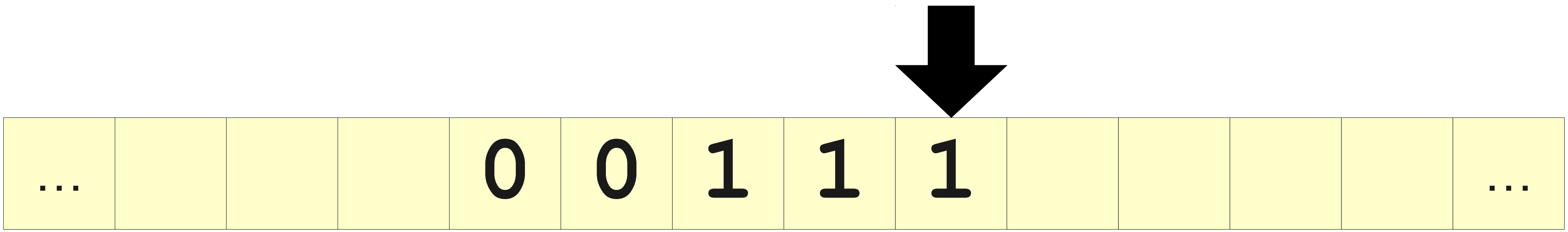

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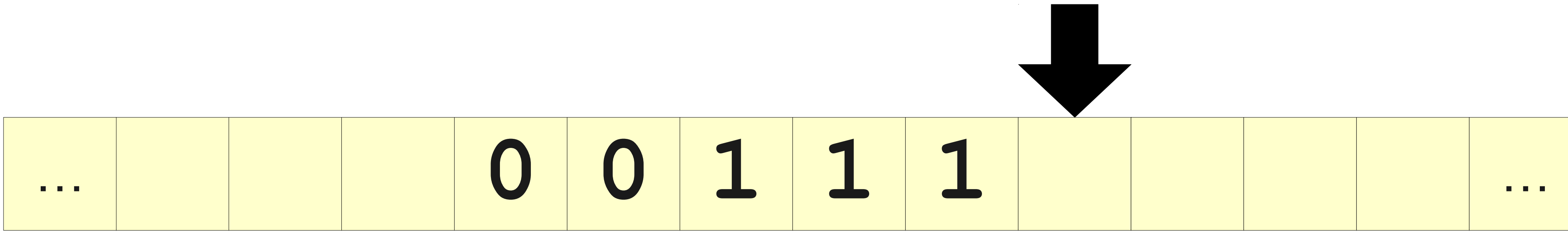

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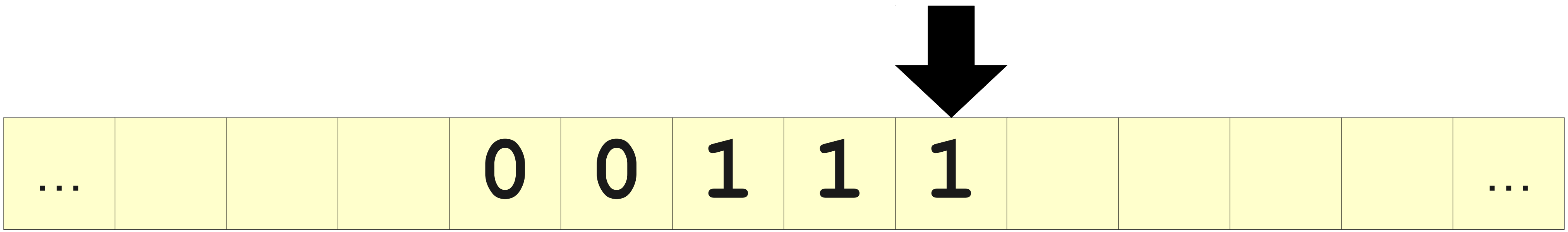

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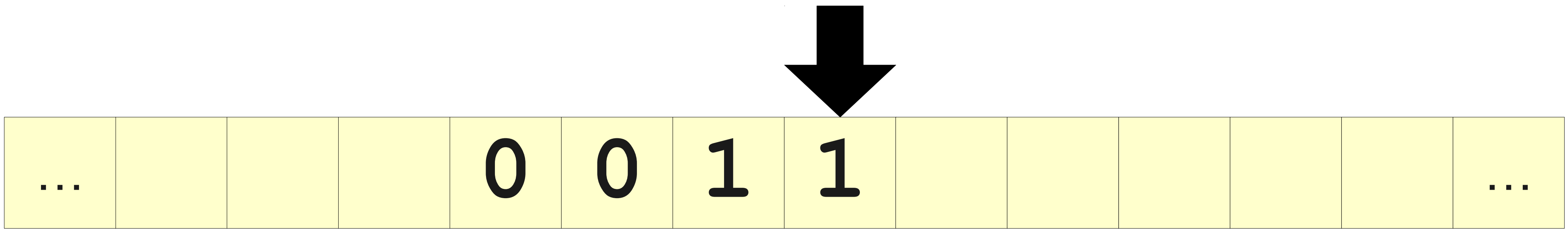

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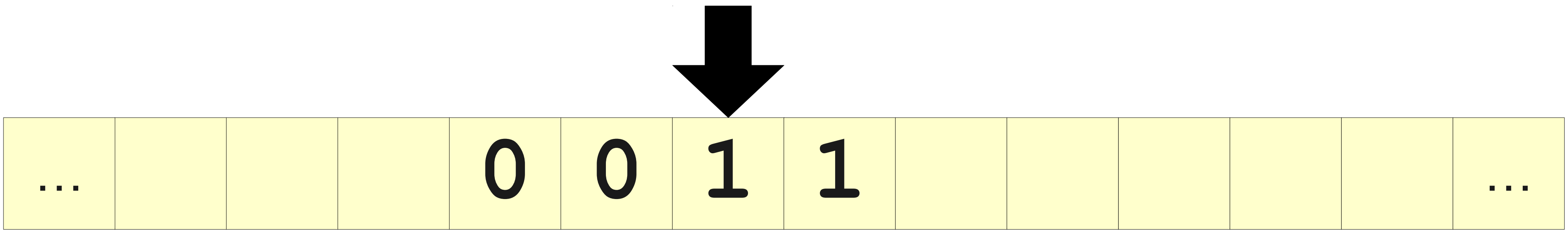

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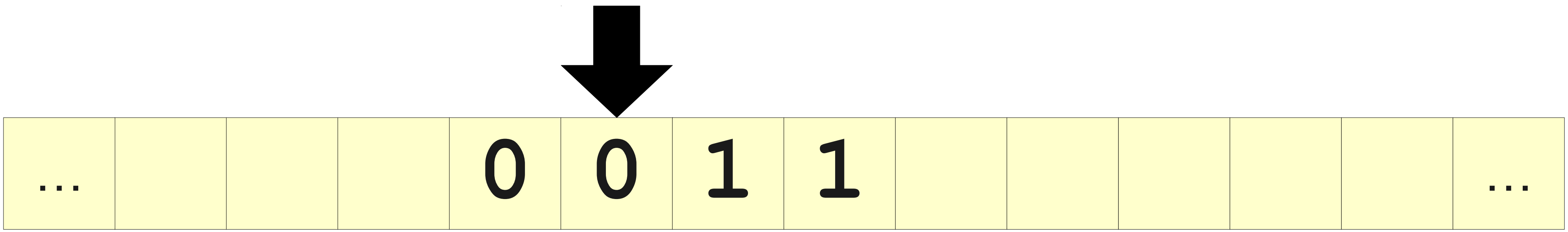

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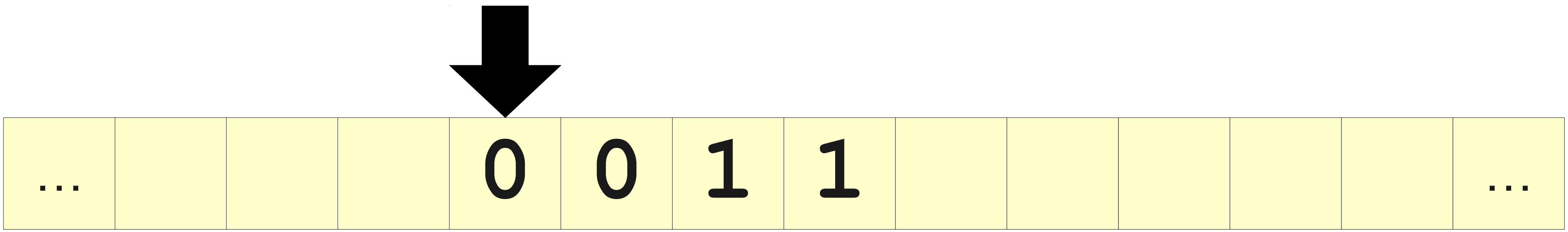

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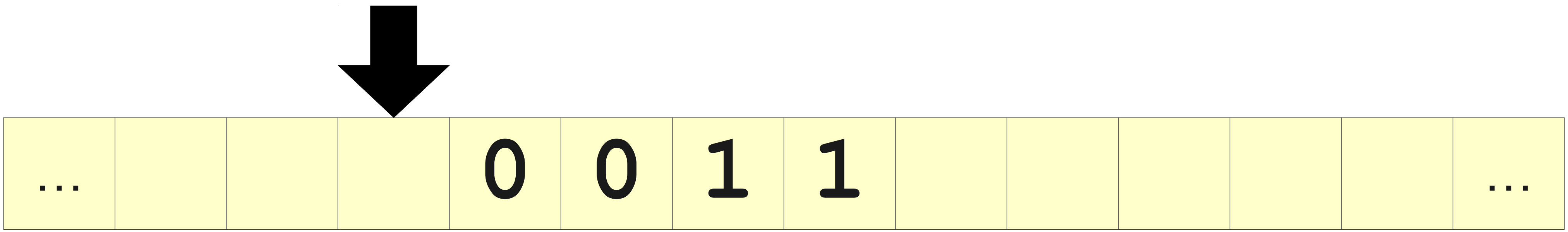

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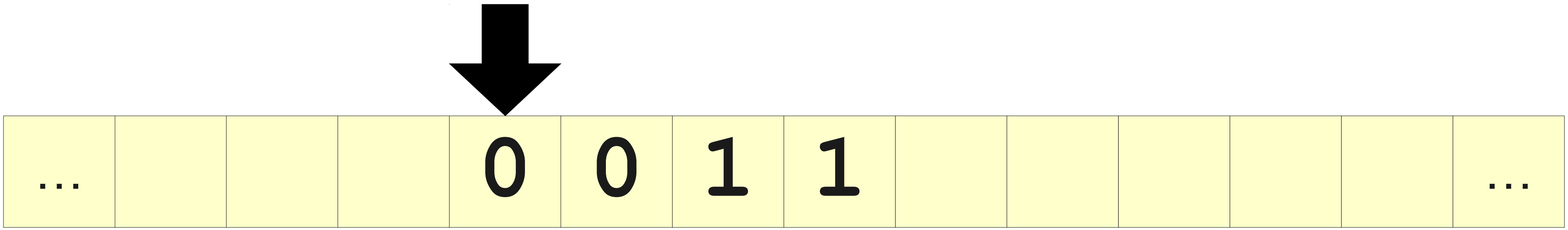

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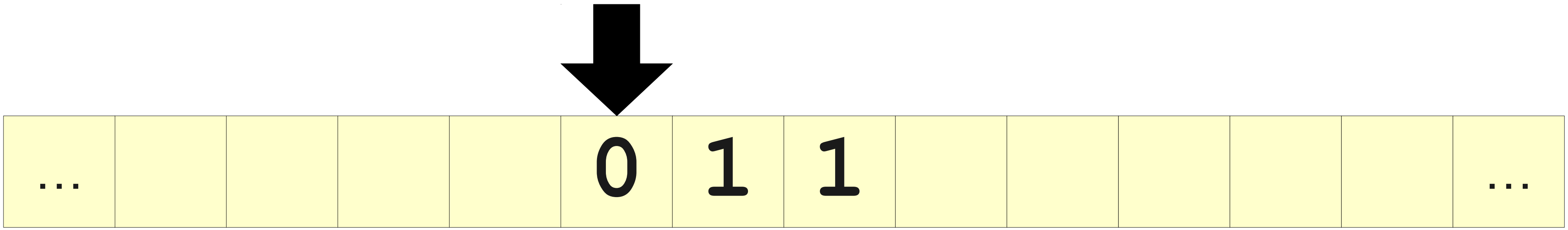

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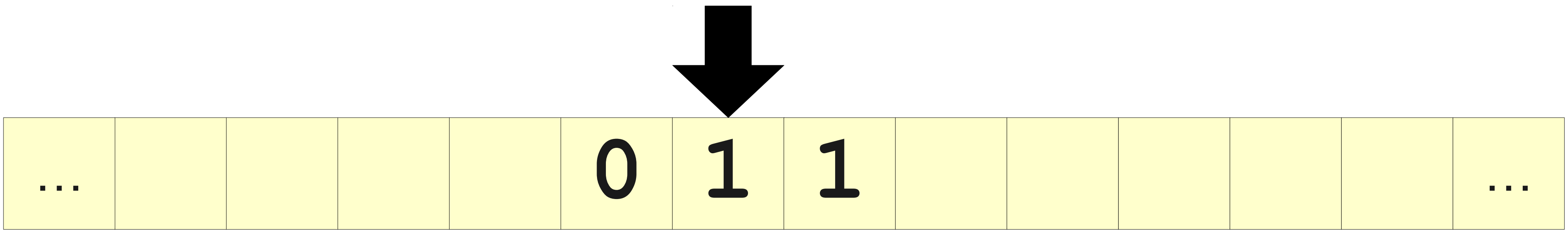
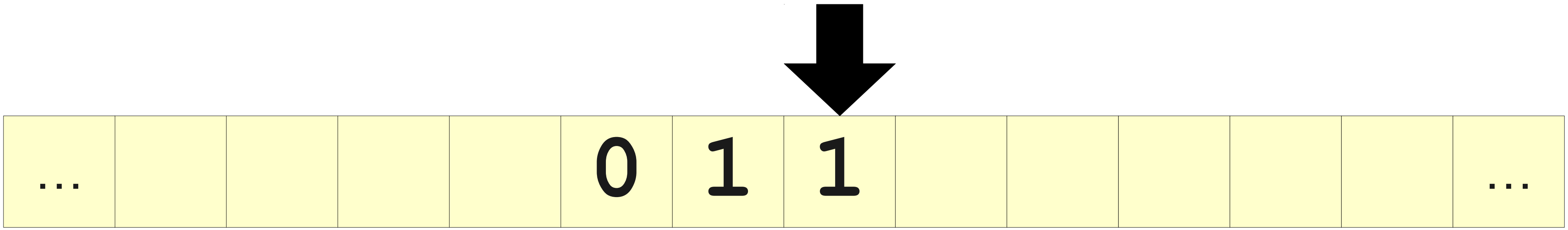


A Sketch of the TM



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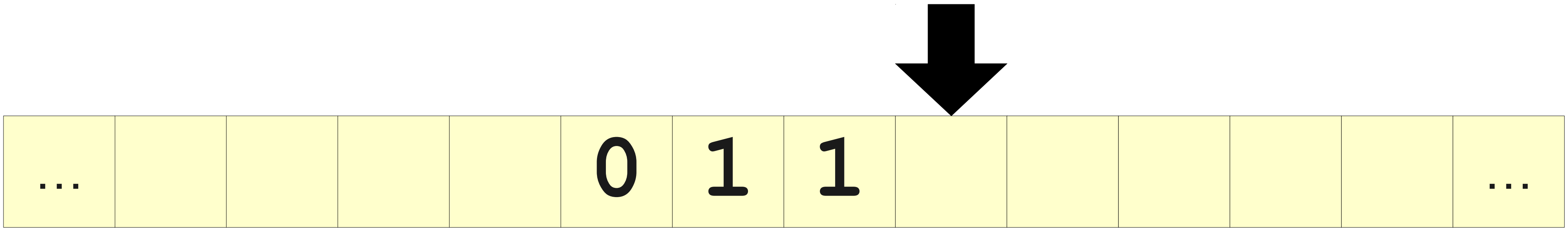

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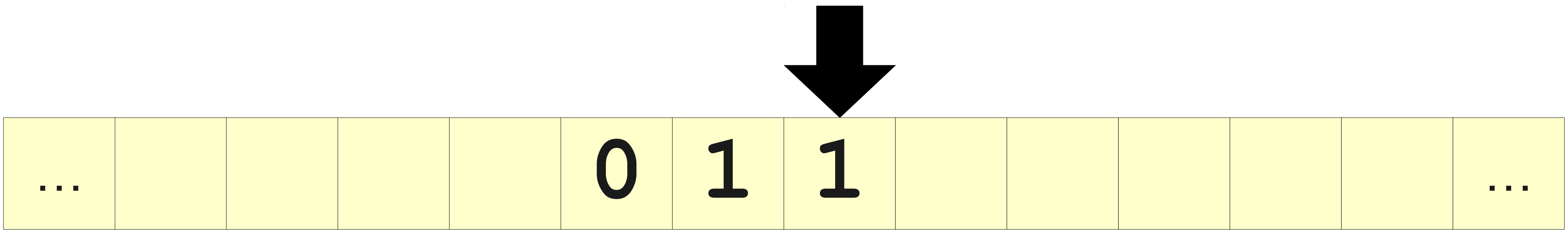

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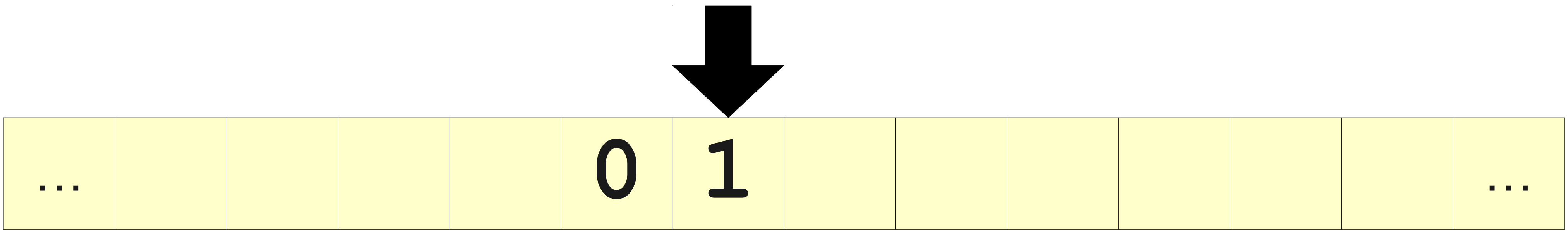

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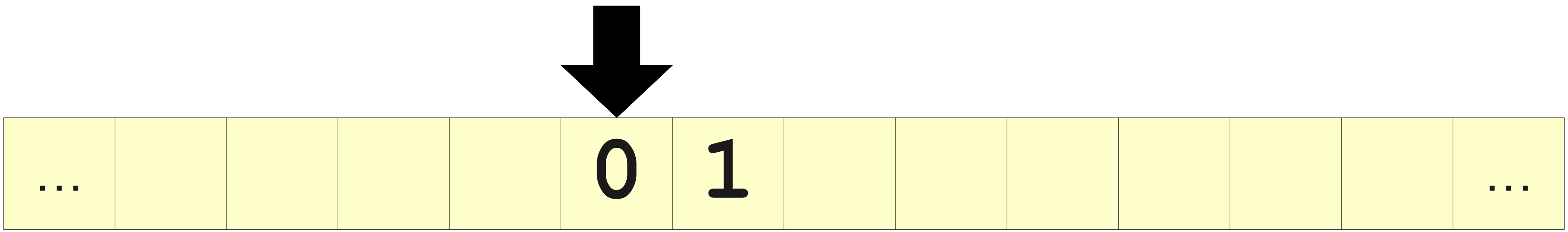

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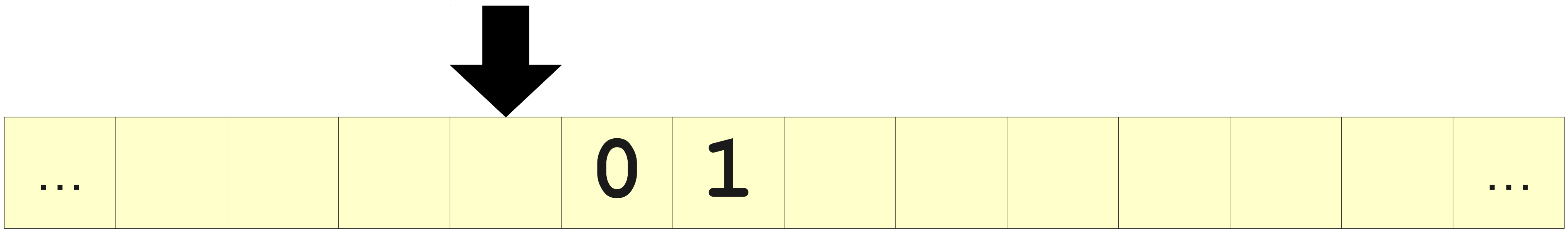

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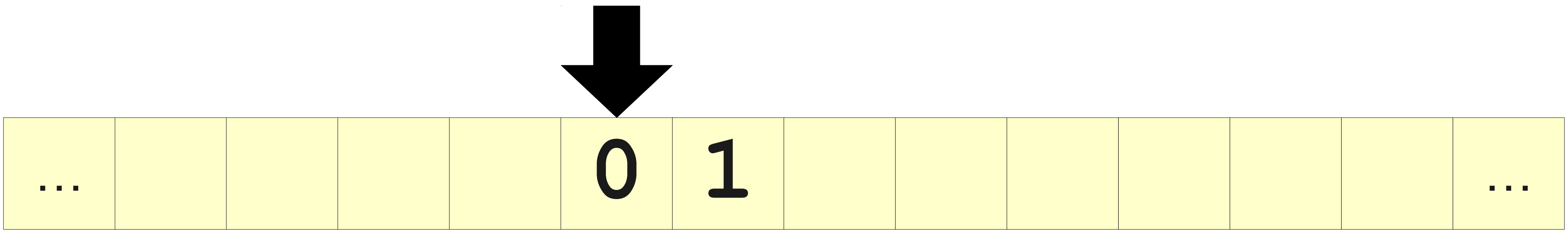

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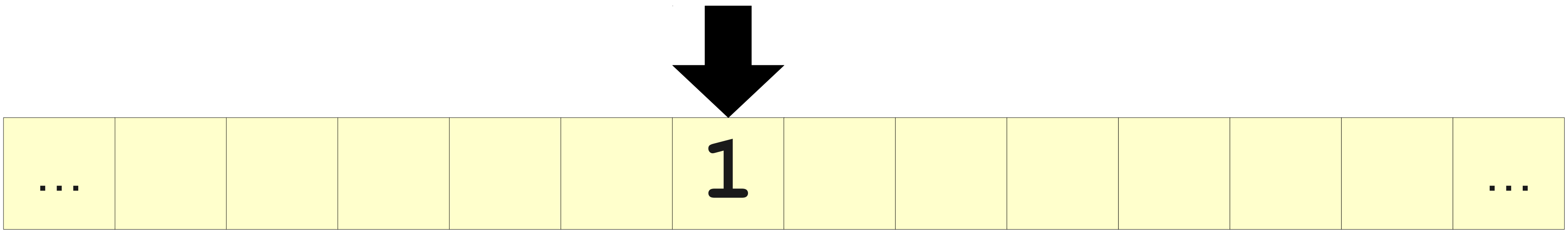

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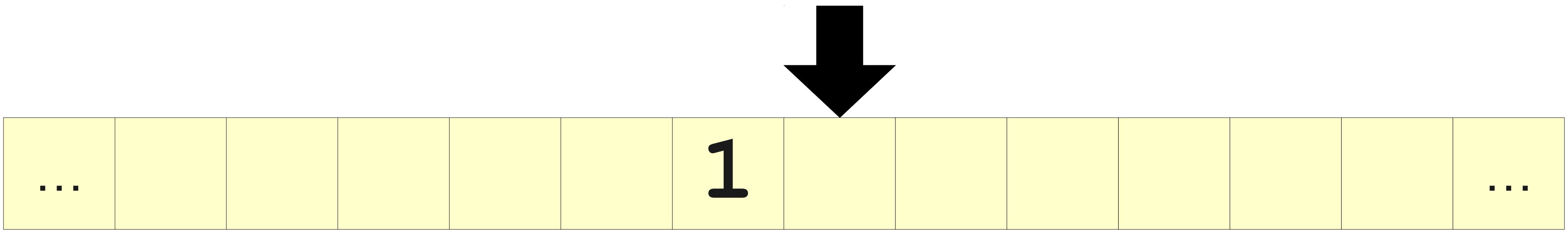

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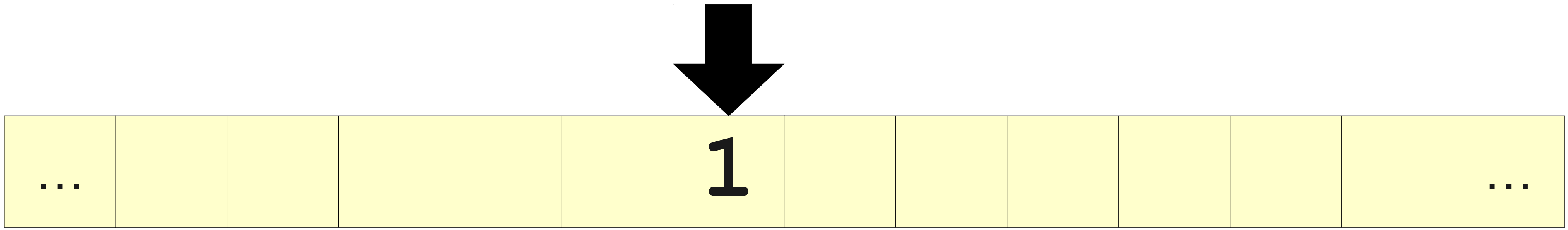

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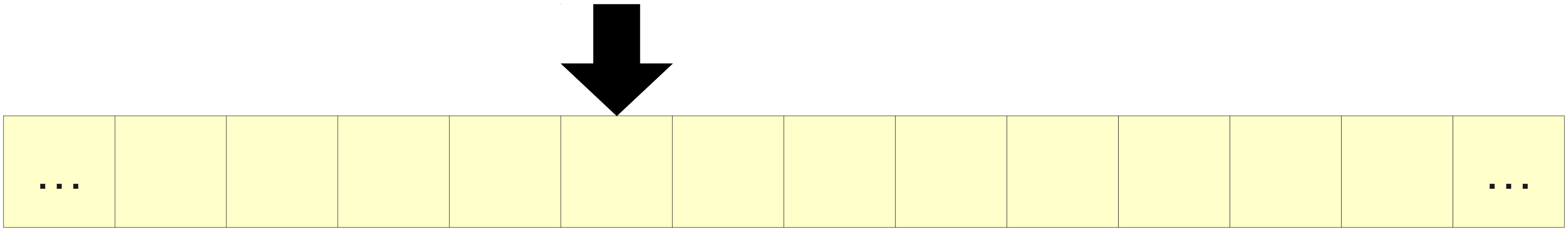

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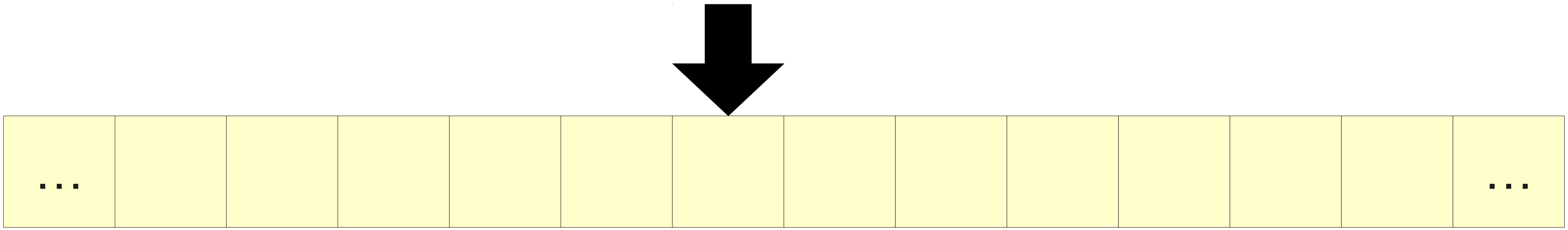

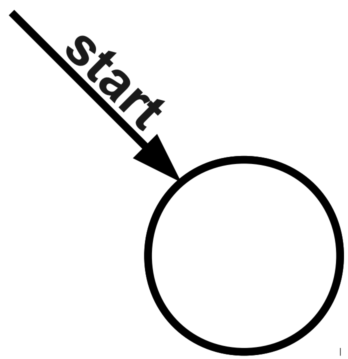
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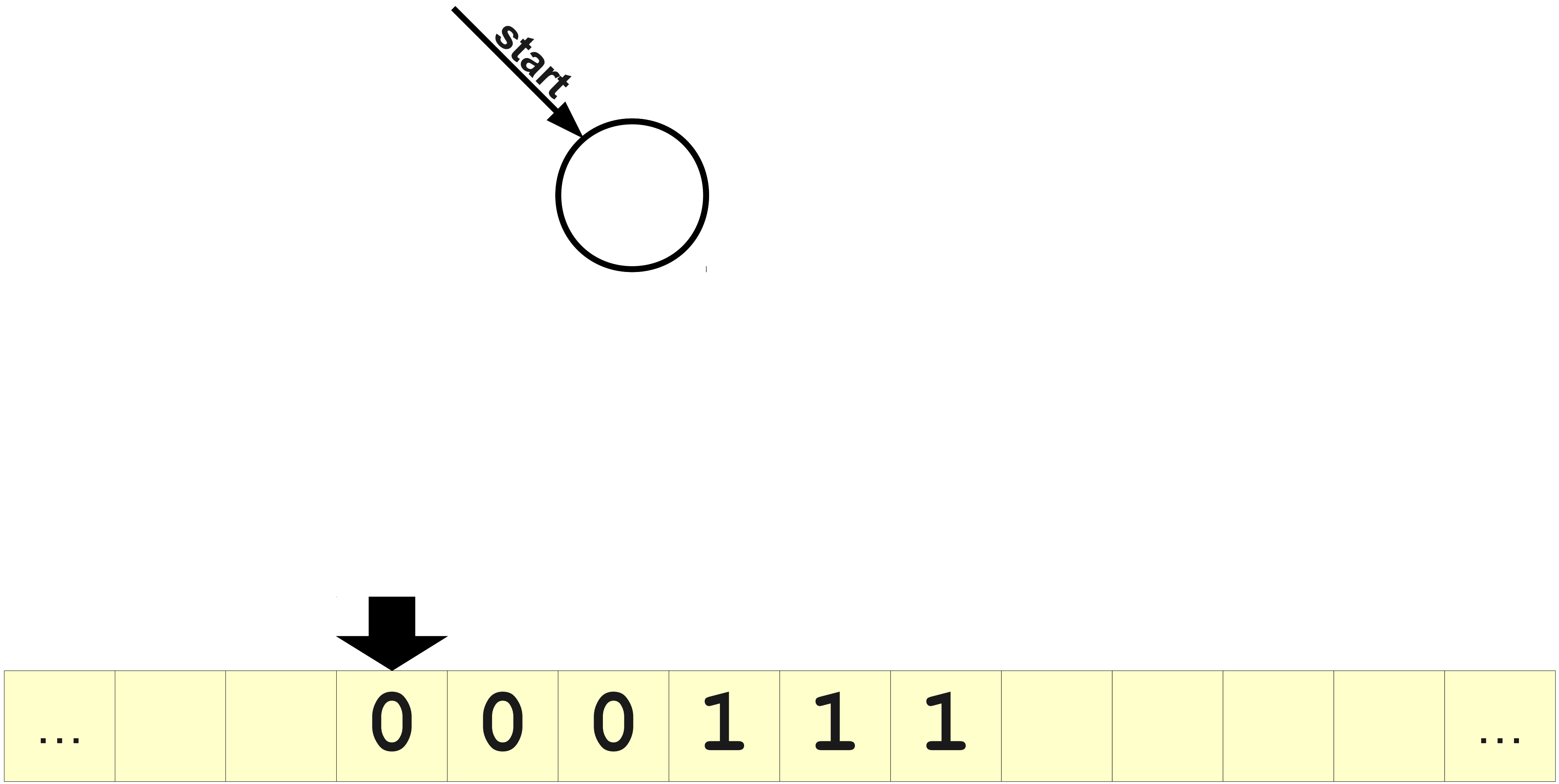
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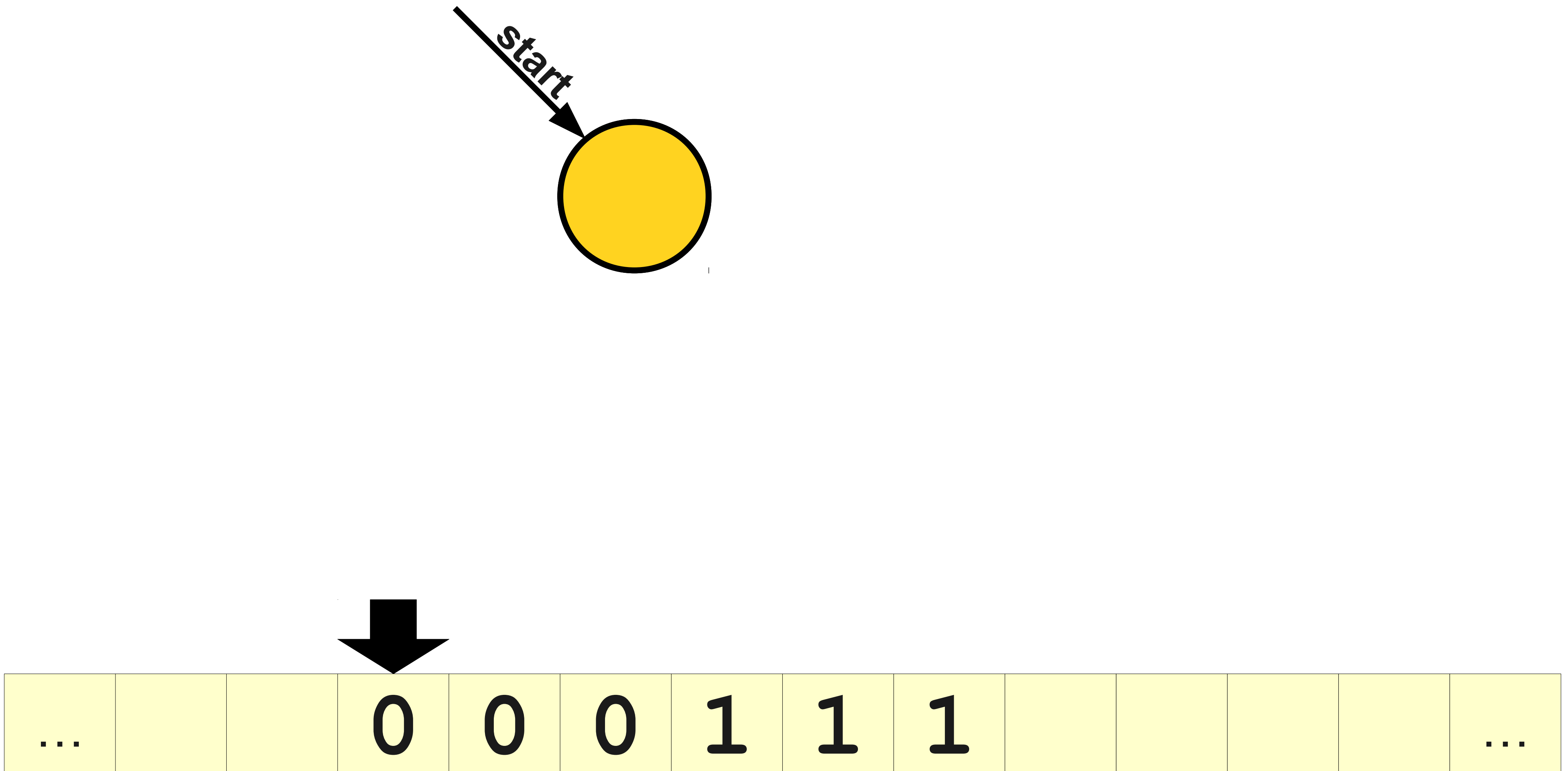
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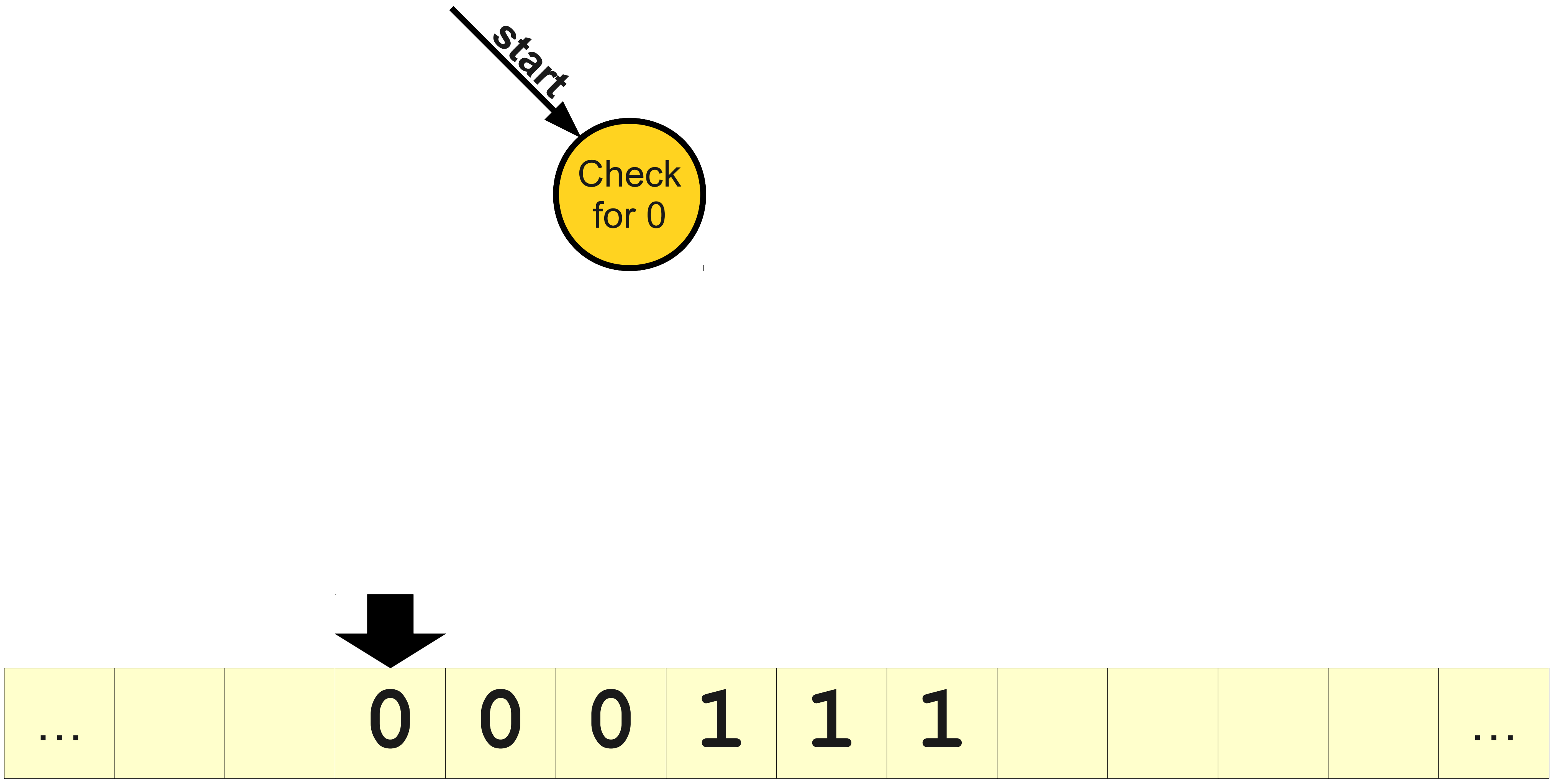

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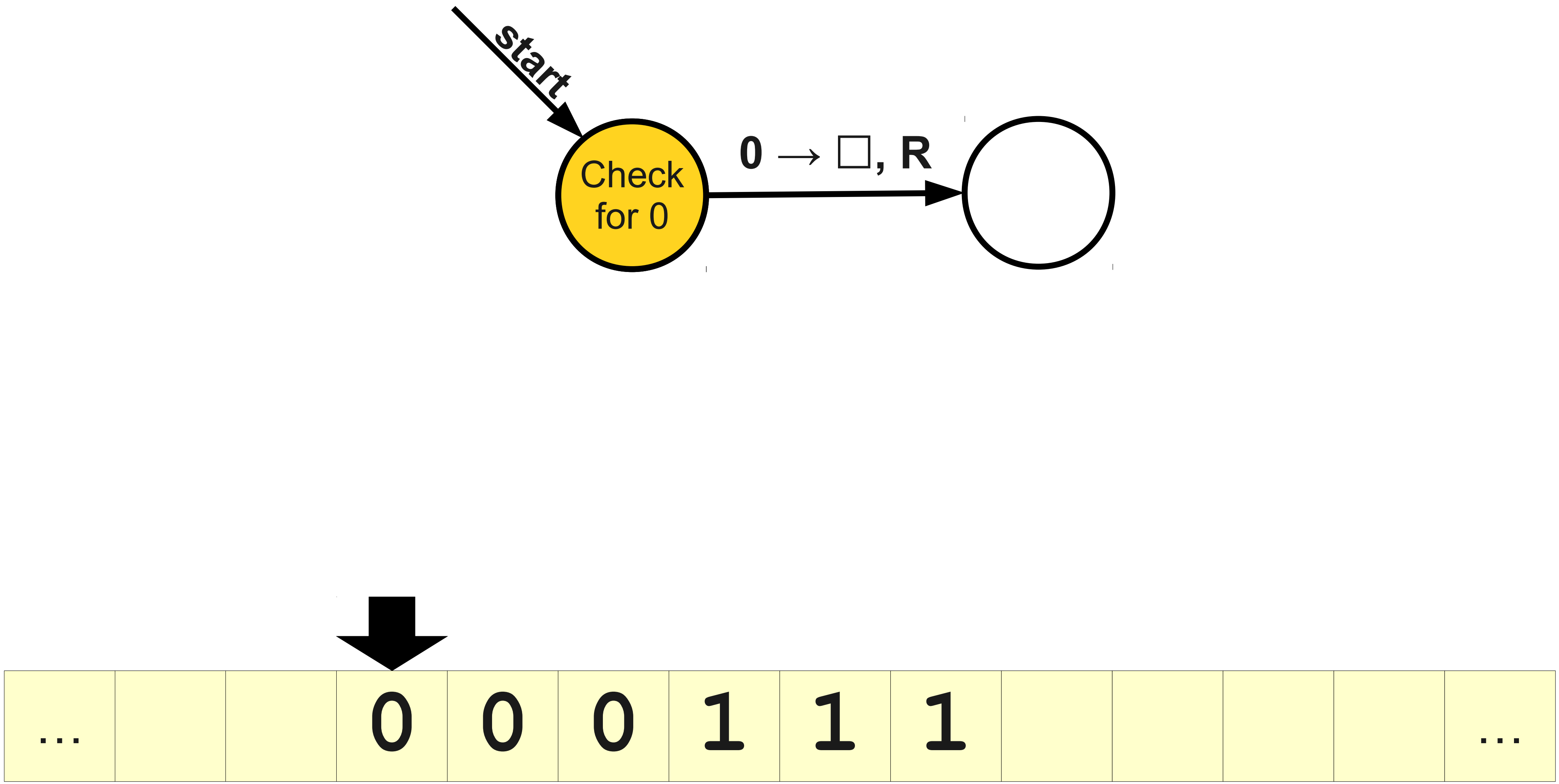

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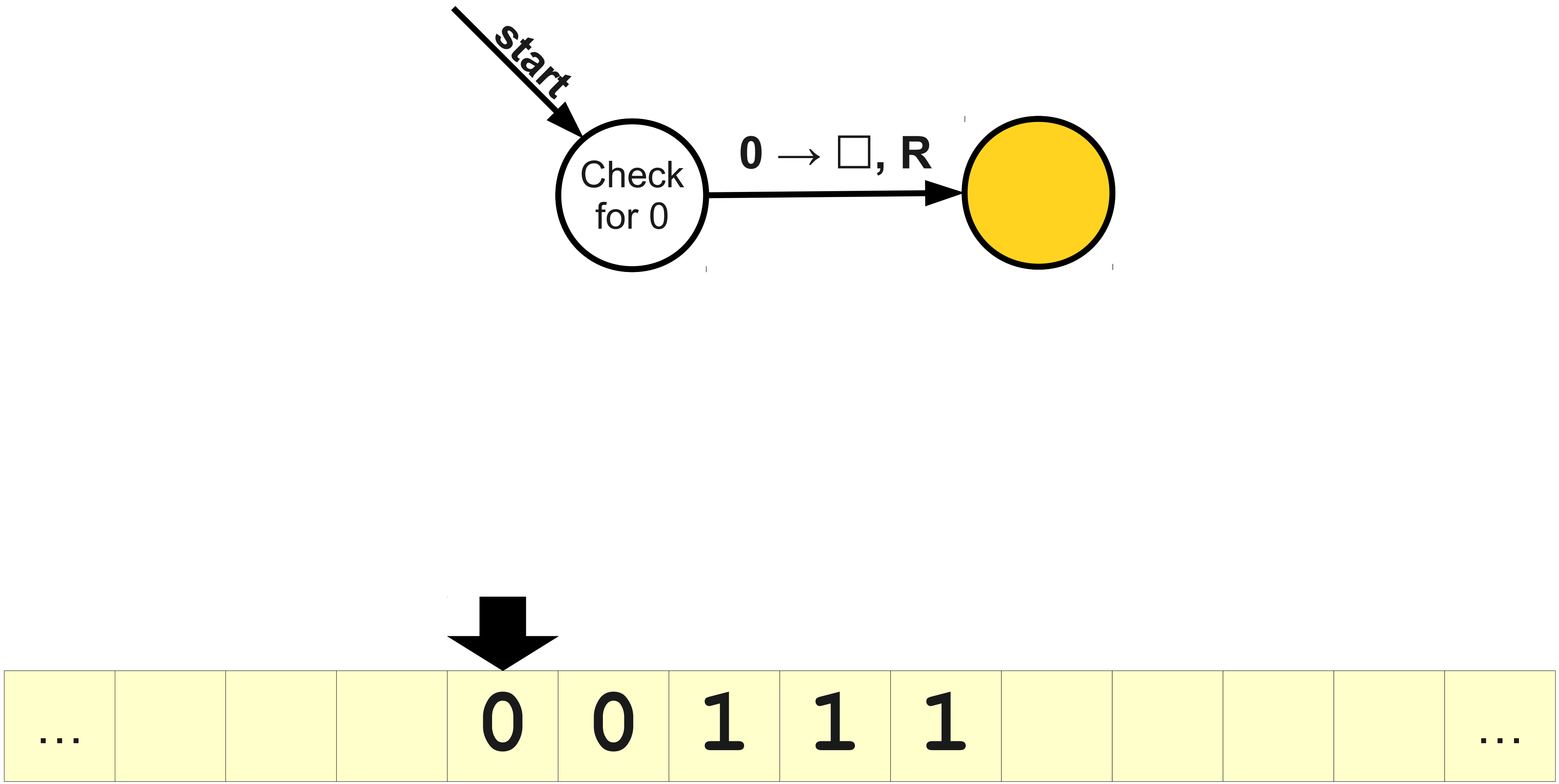
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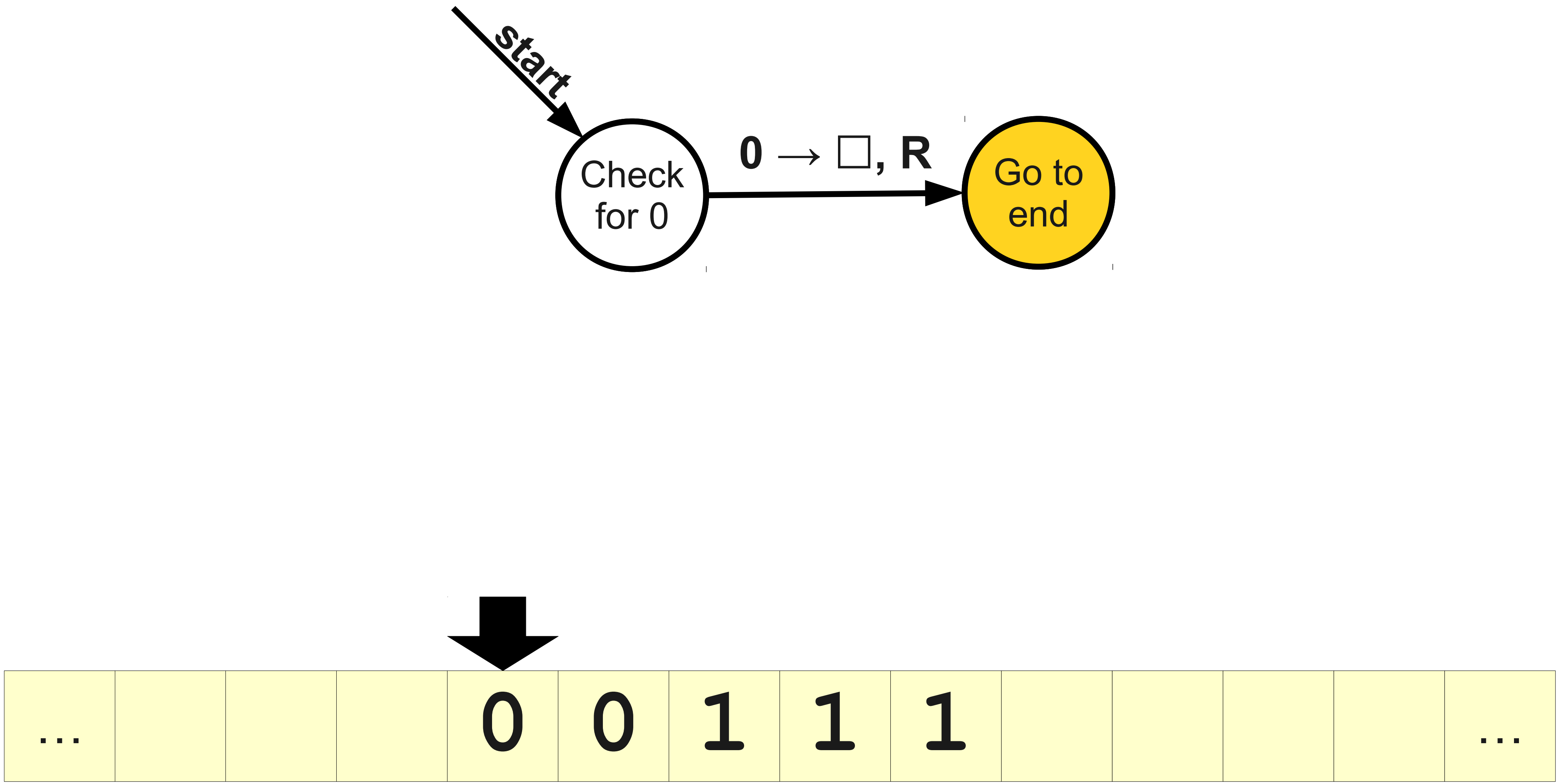
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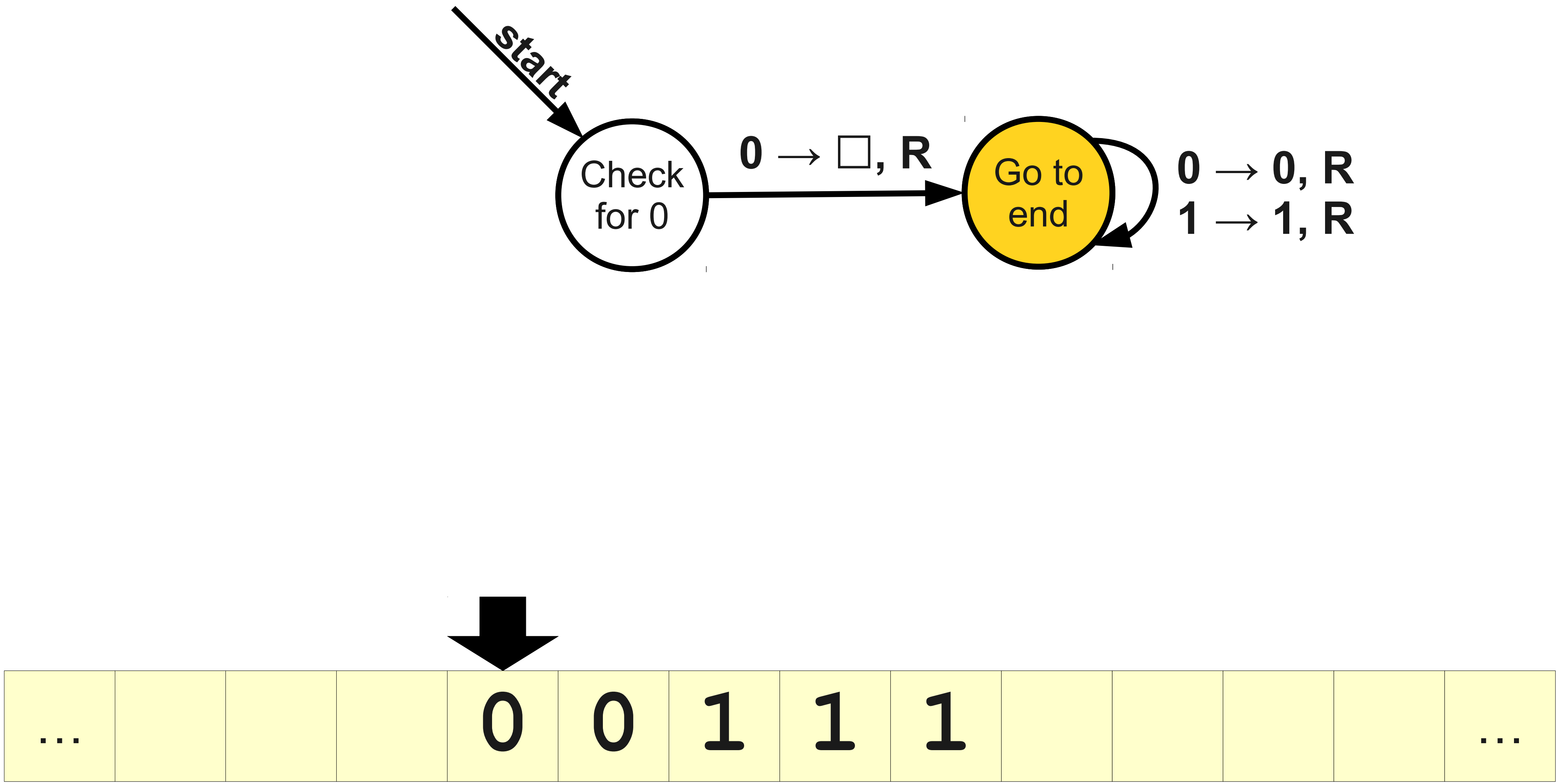
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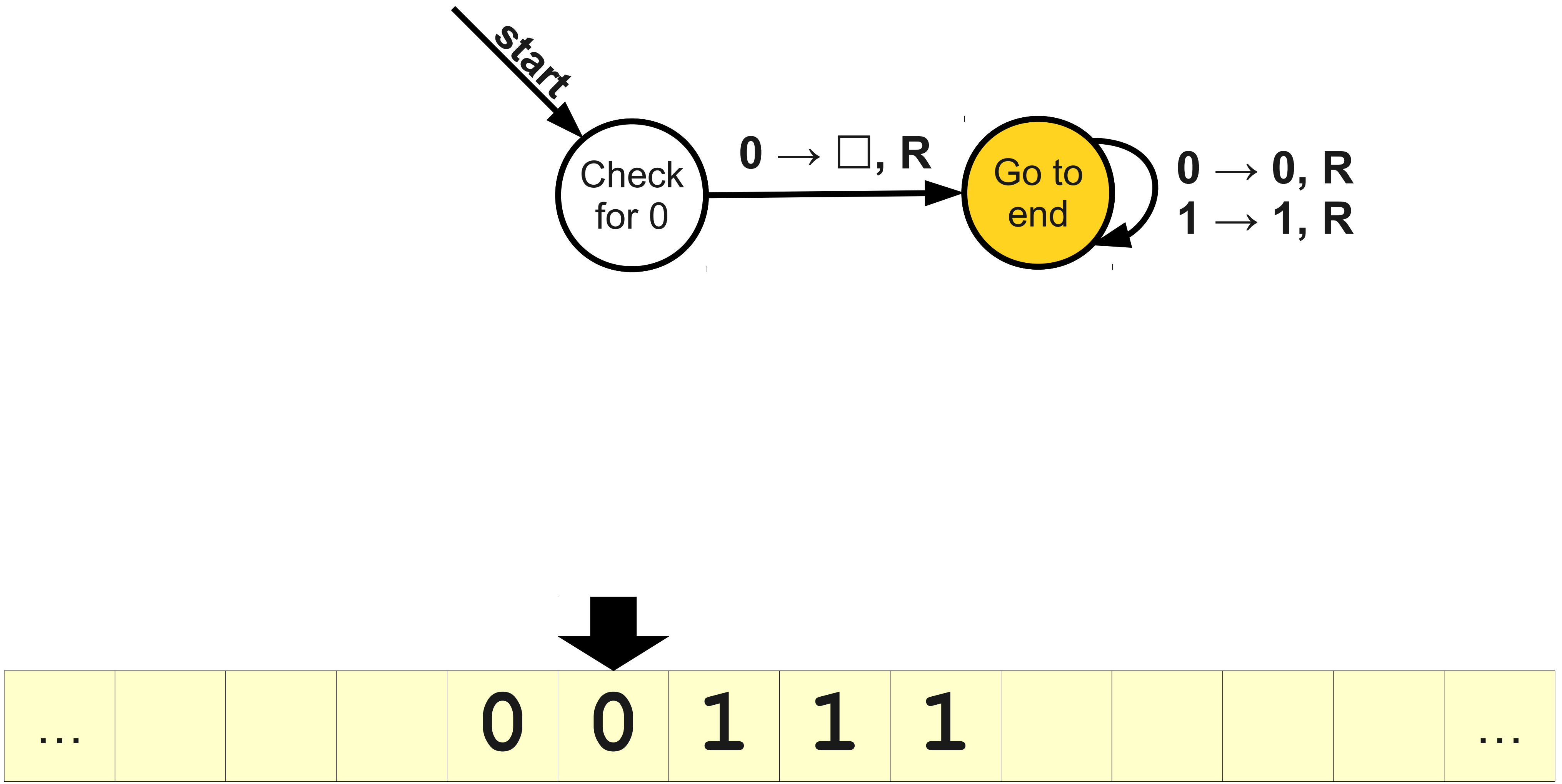
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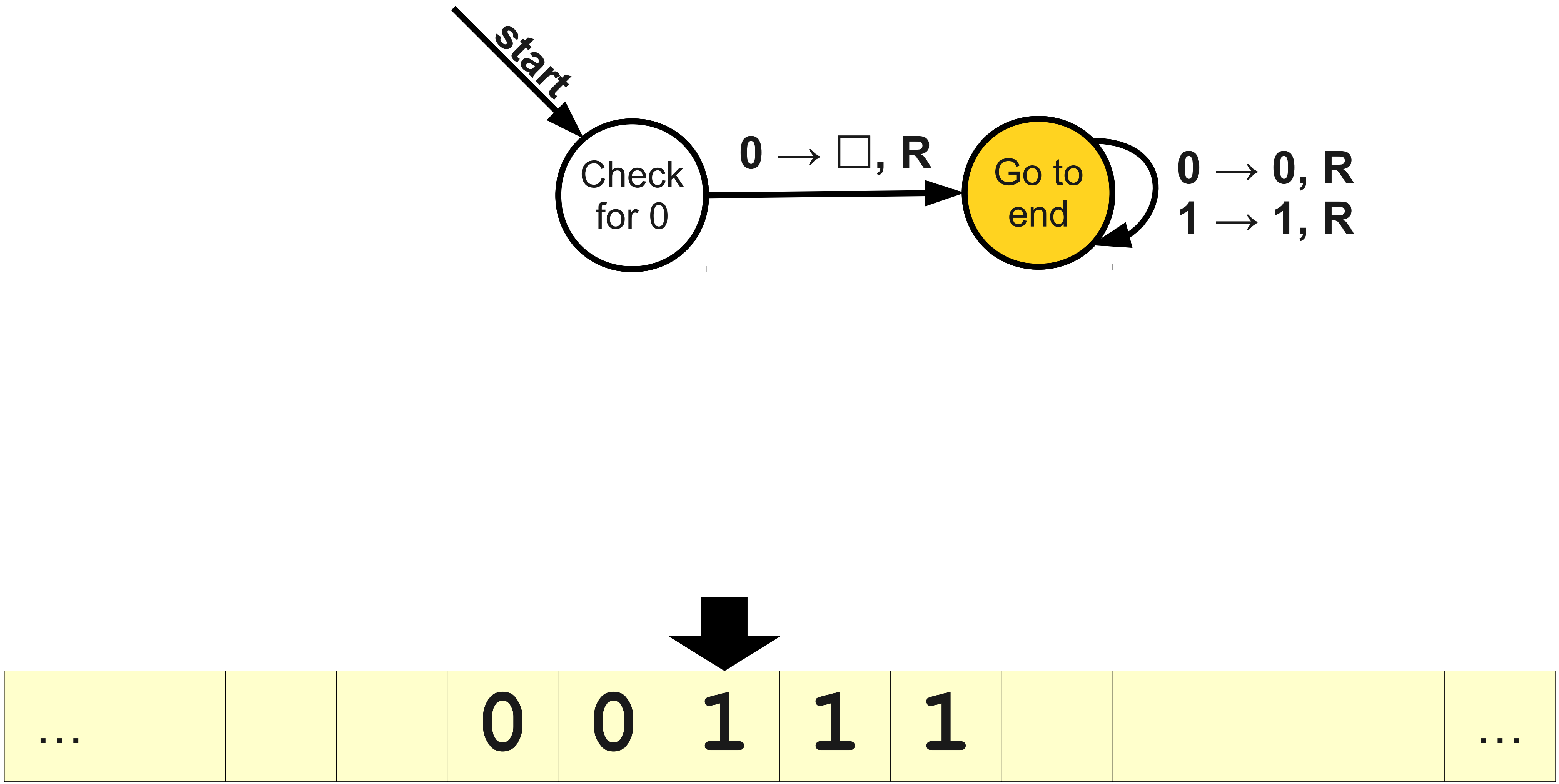

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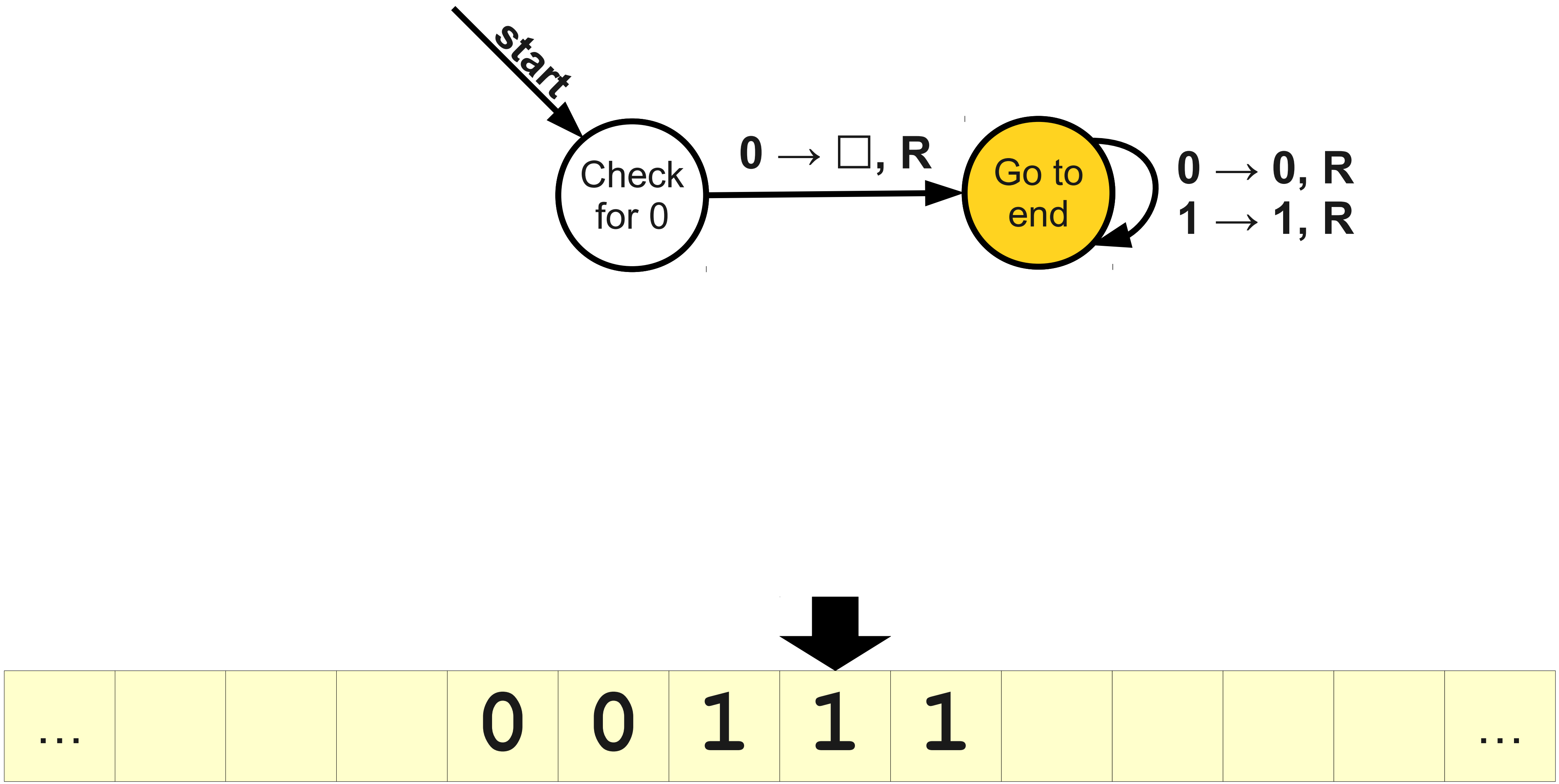
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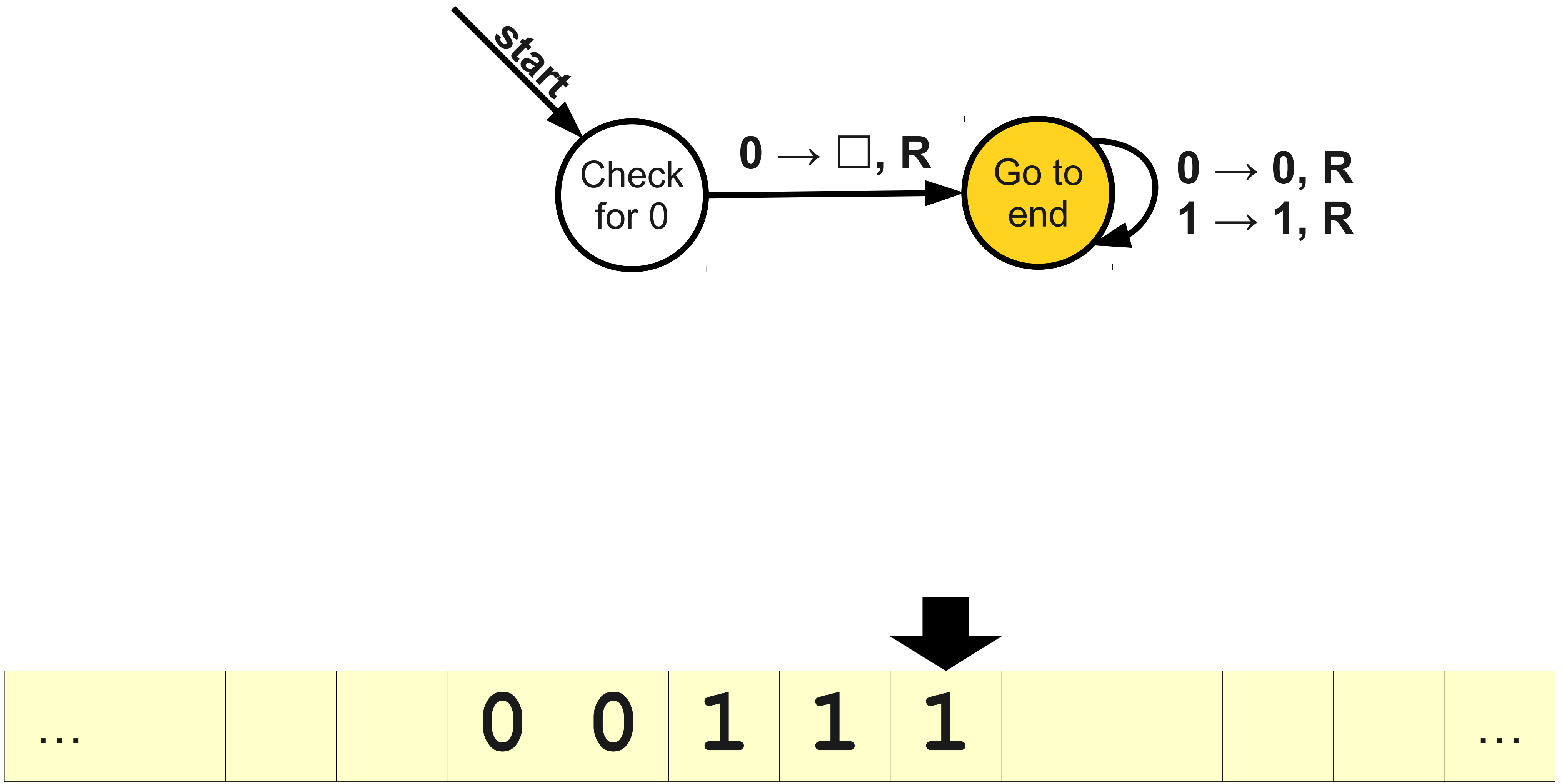
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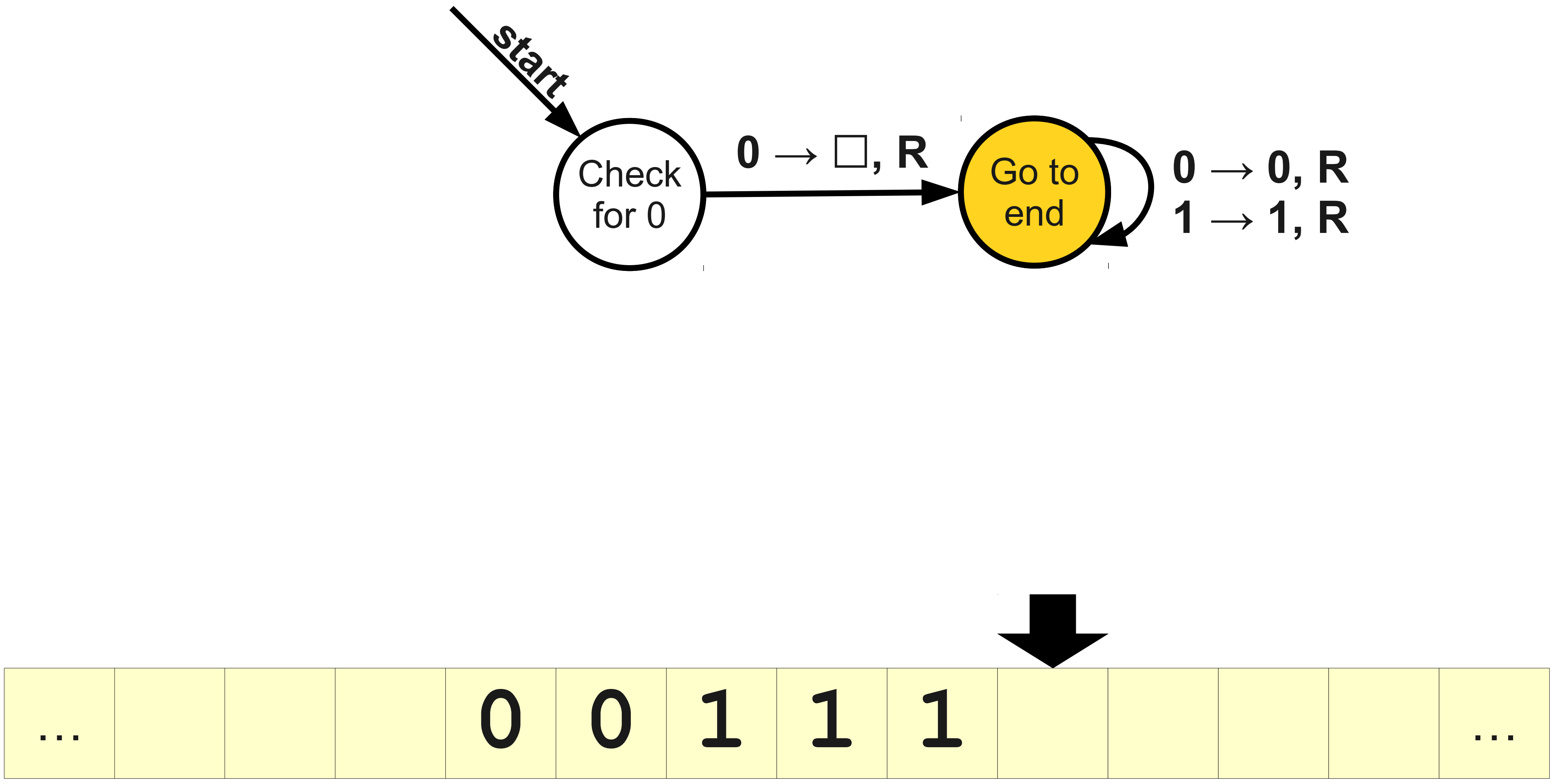

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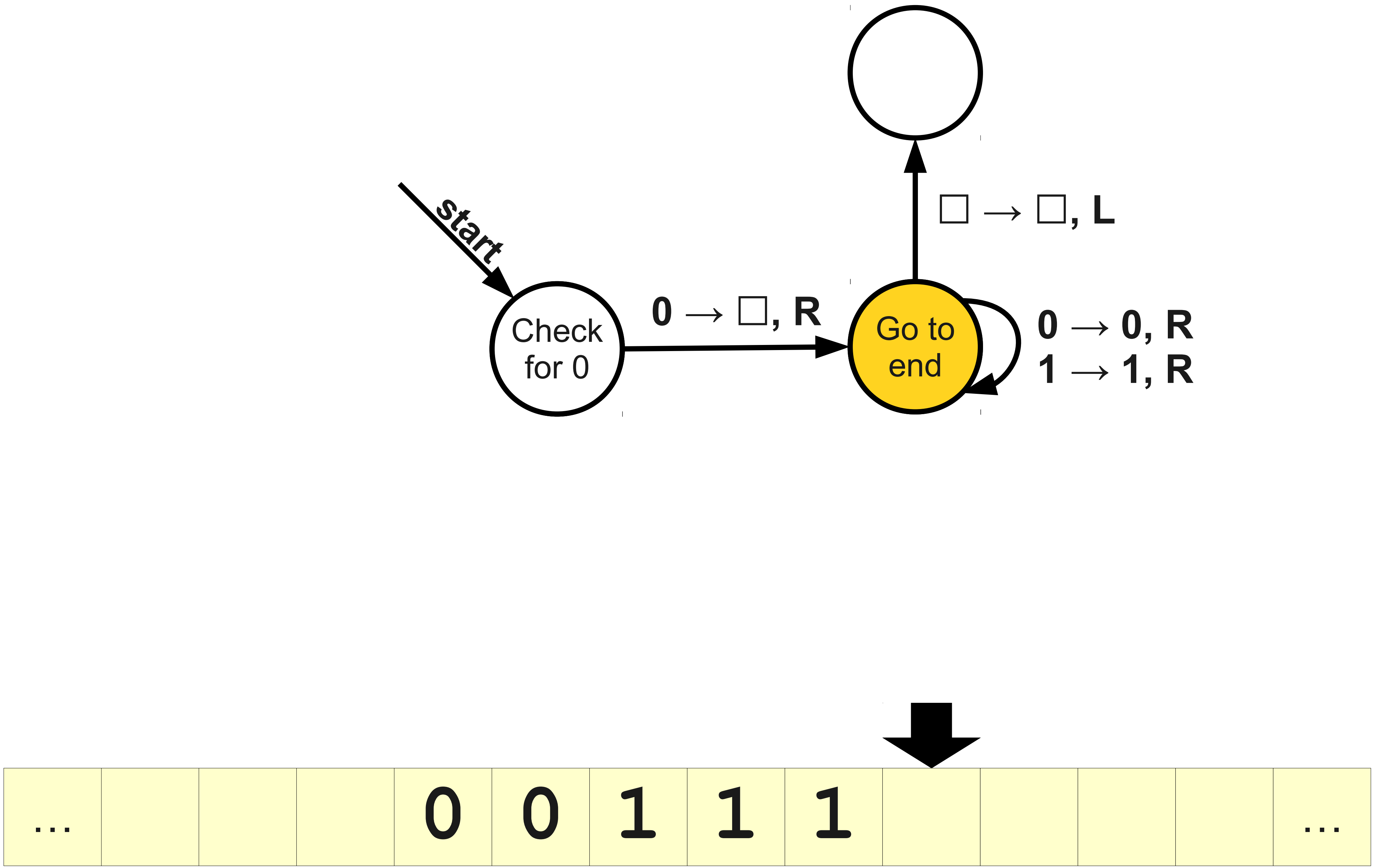
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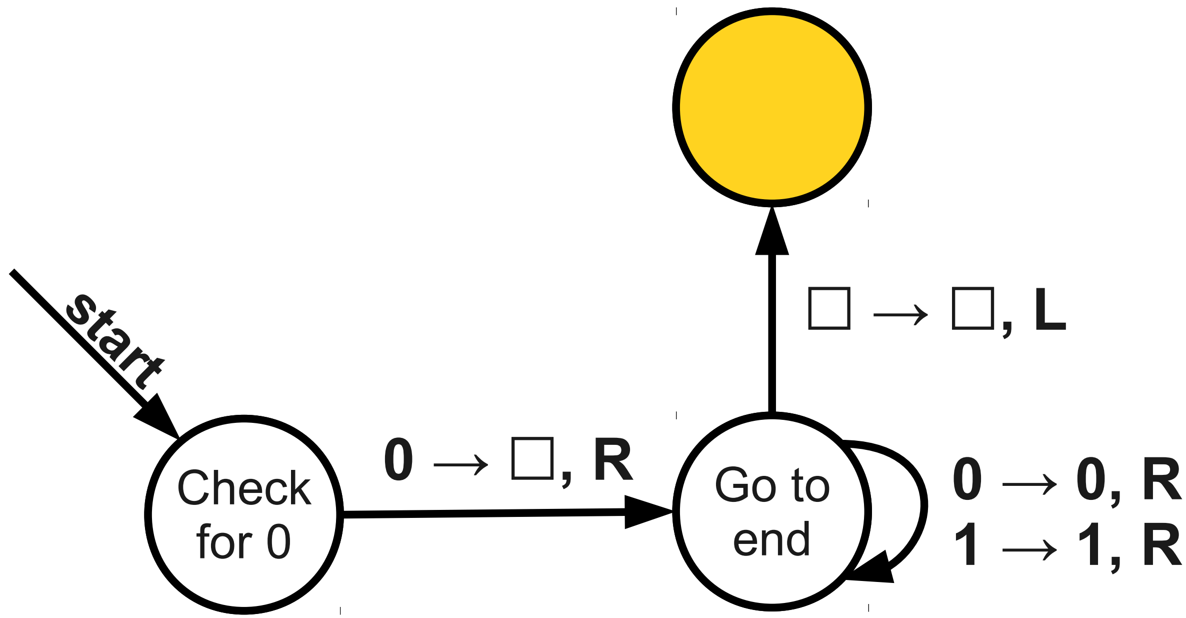

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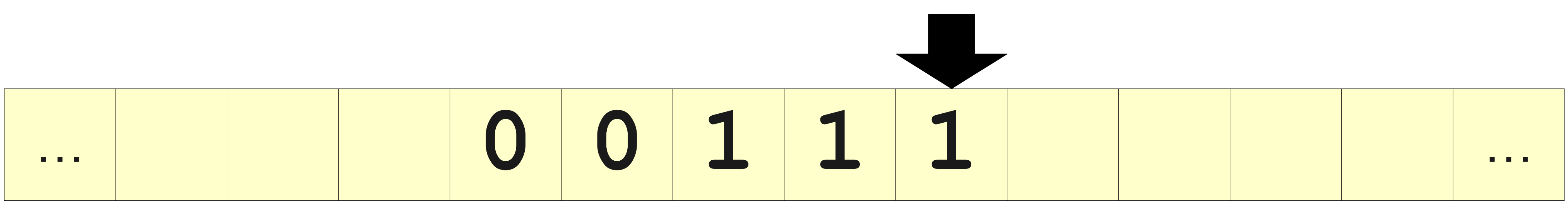

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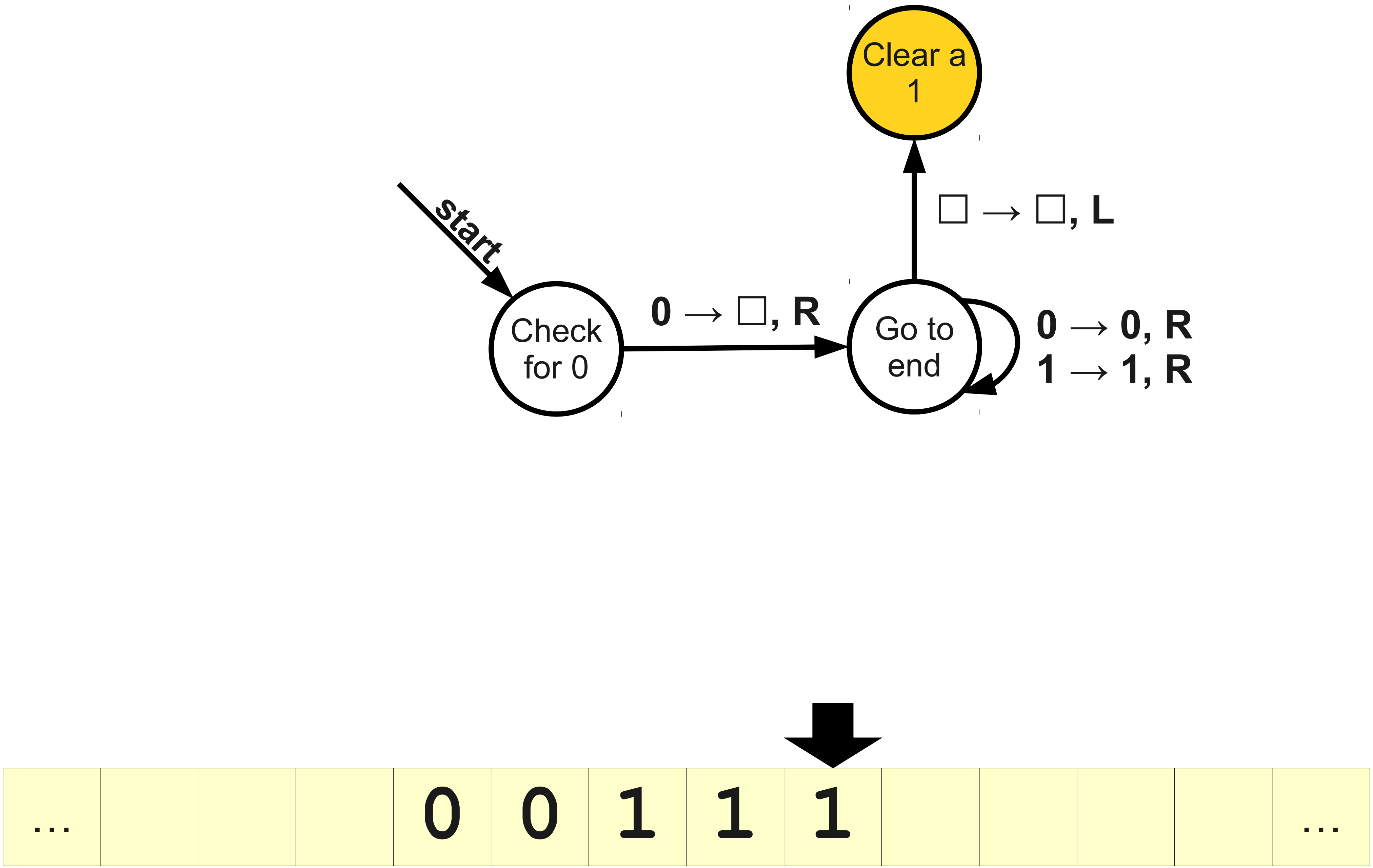

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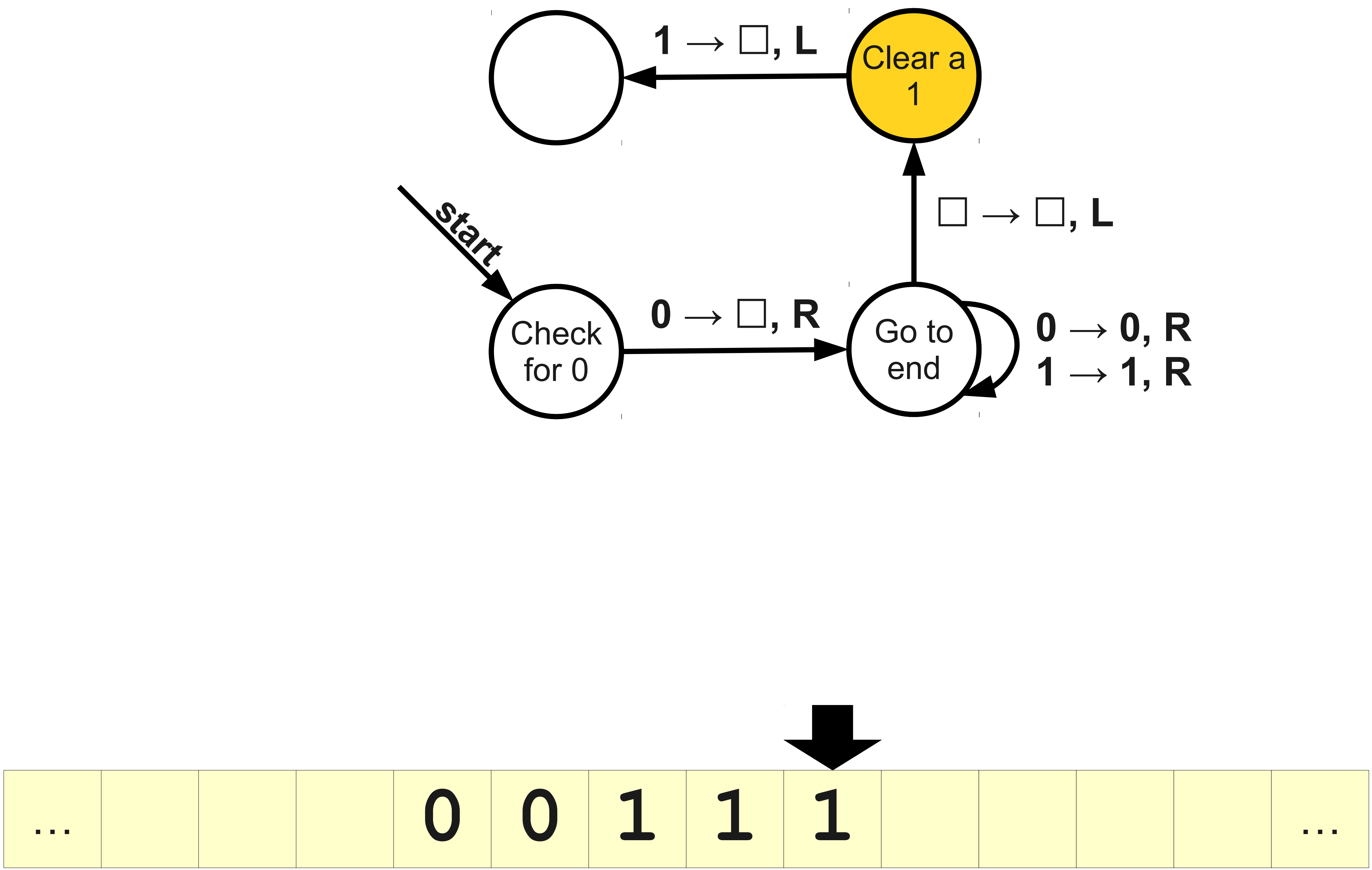

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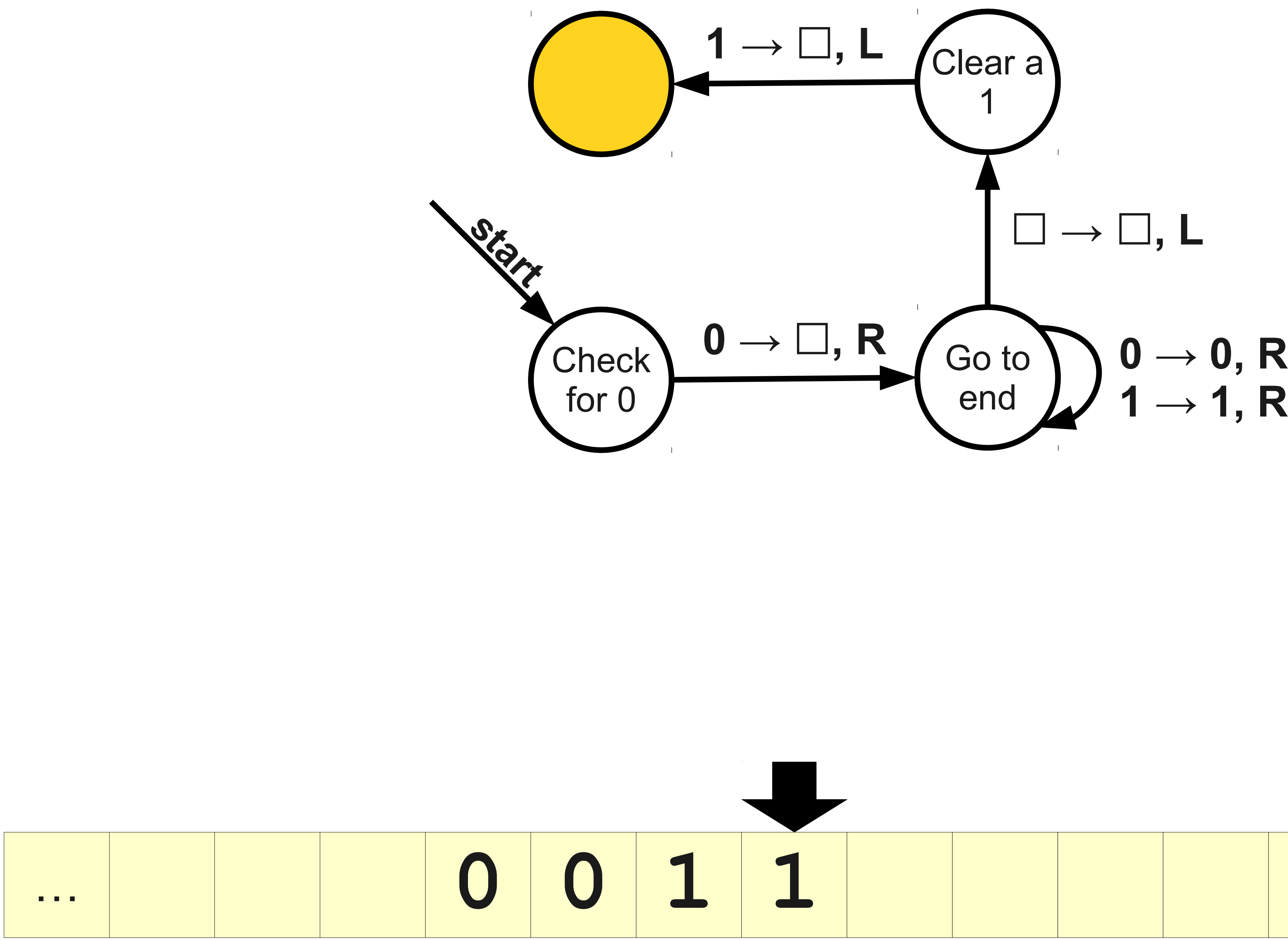

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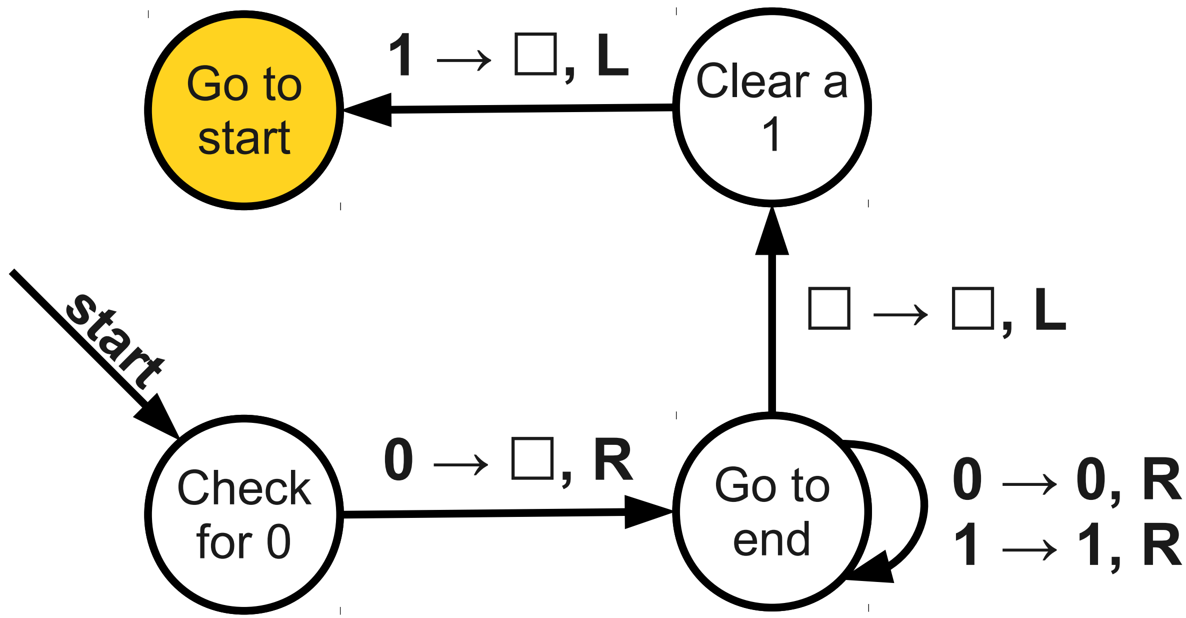
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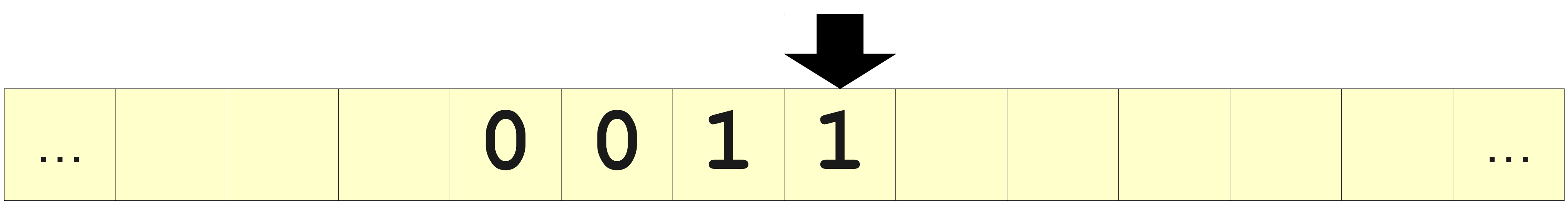
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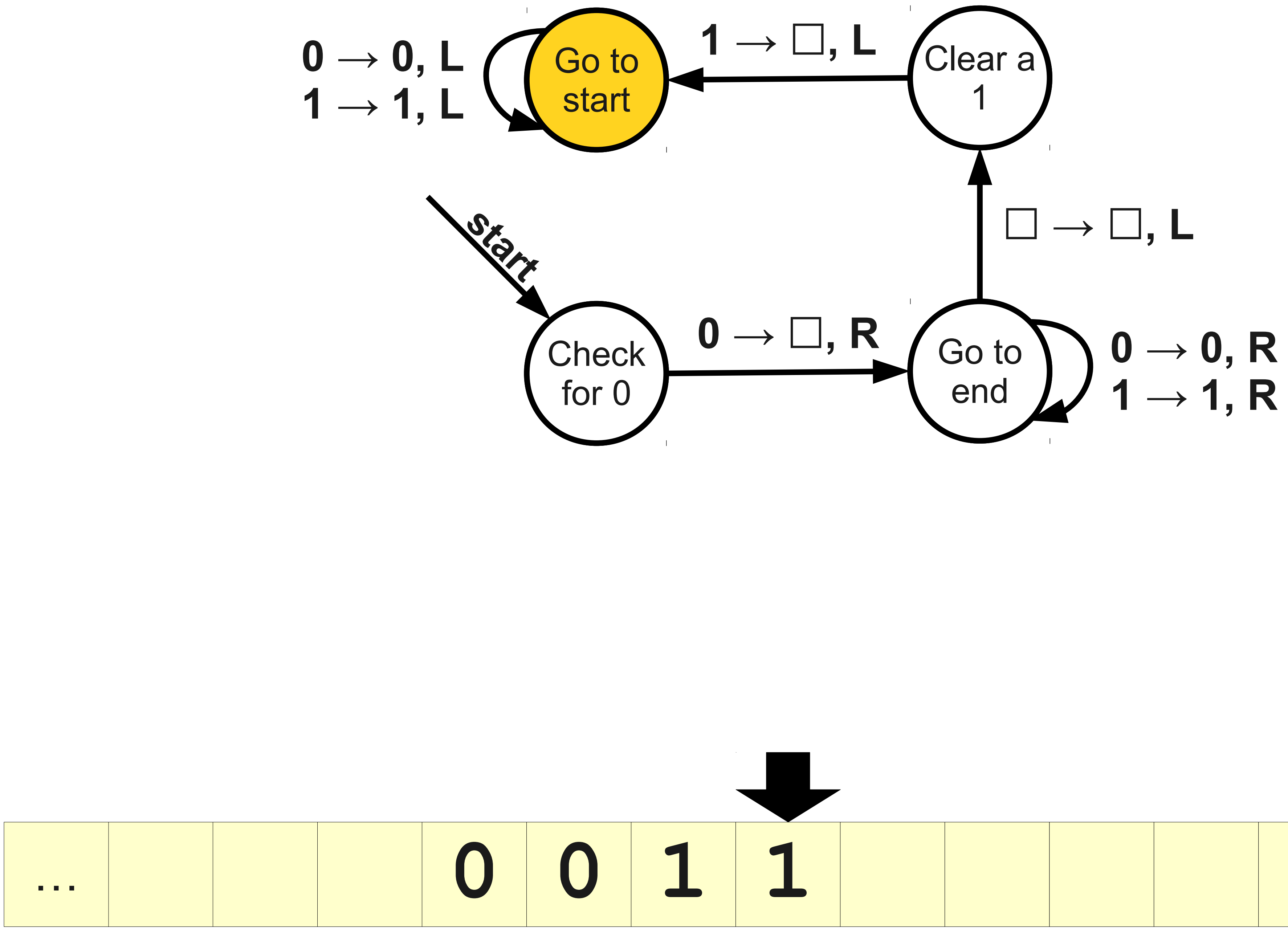

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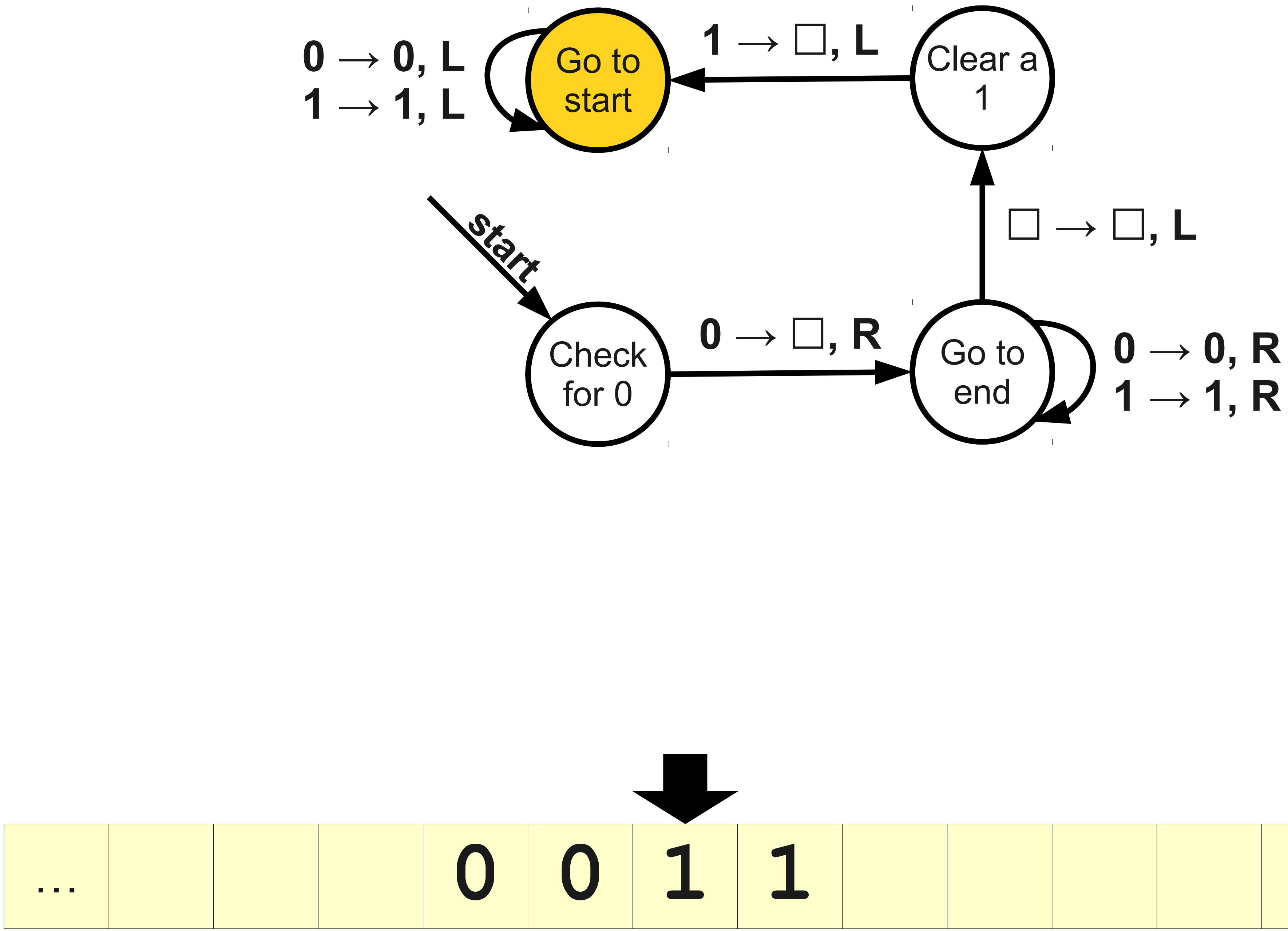

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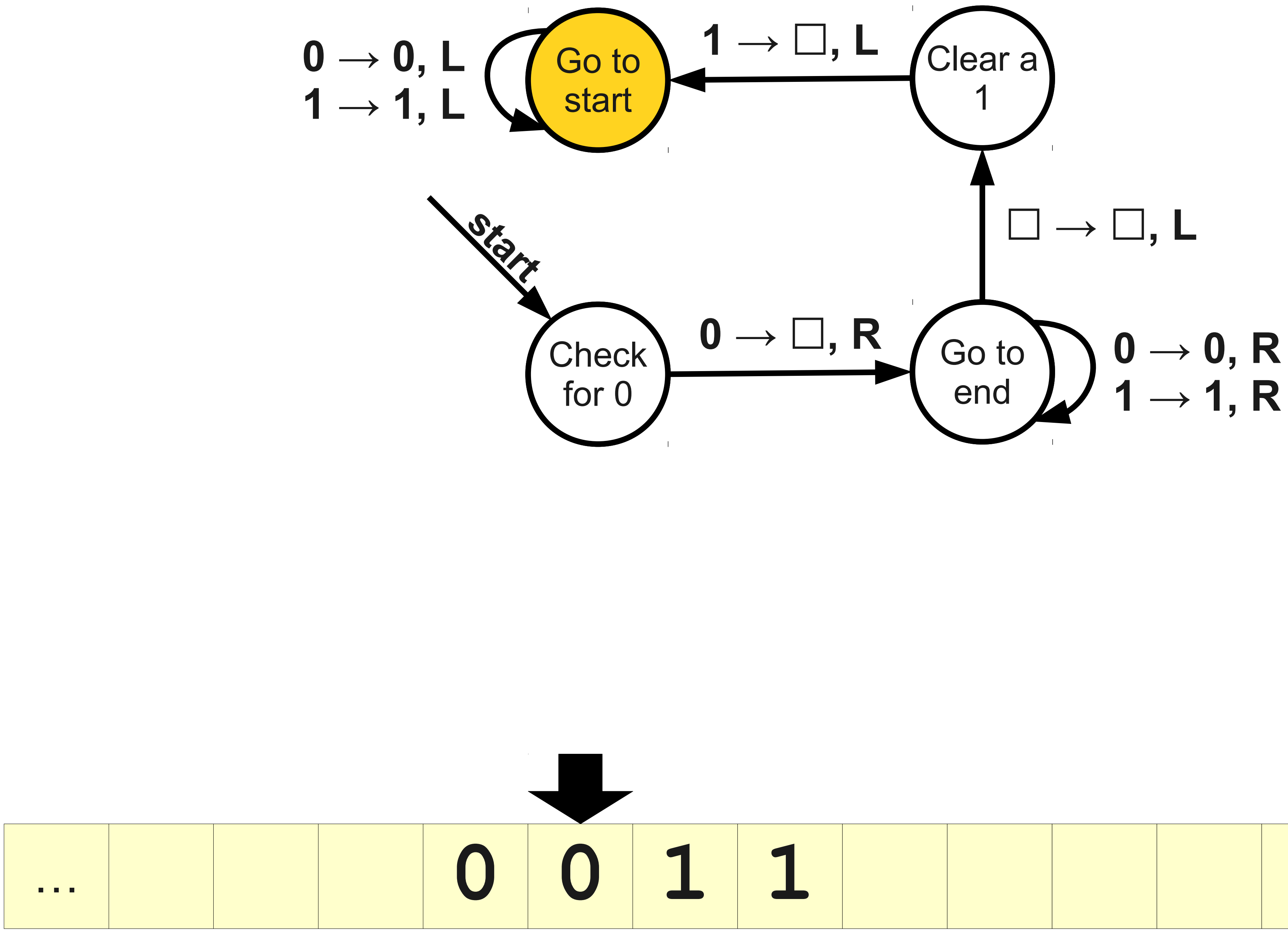
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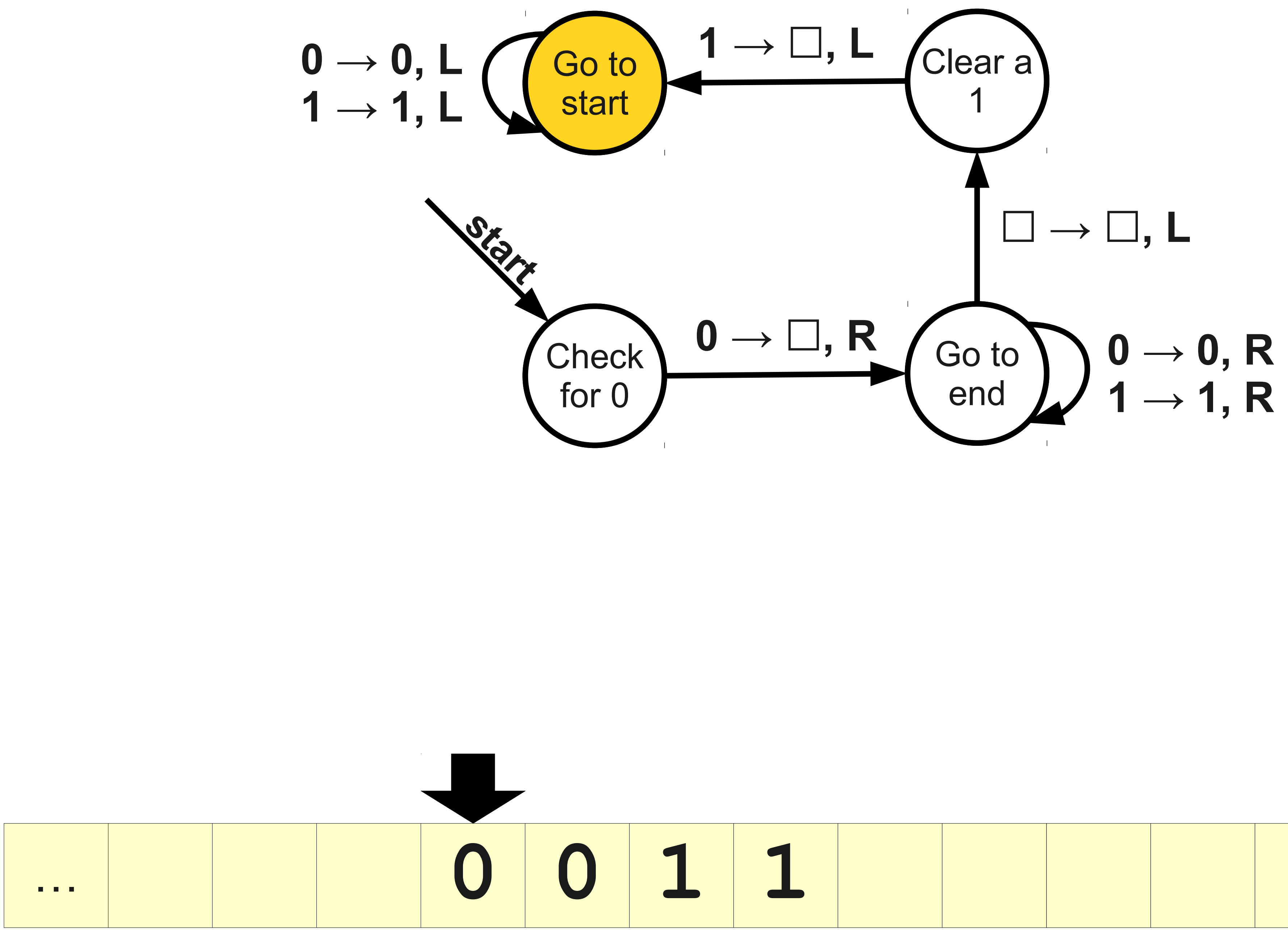

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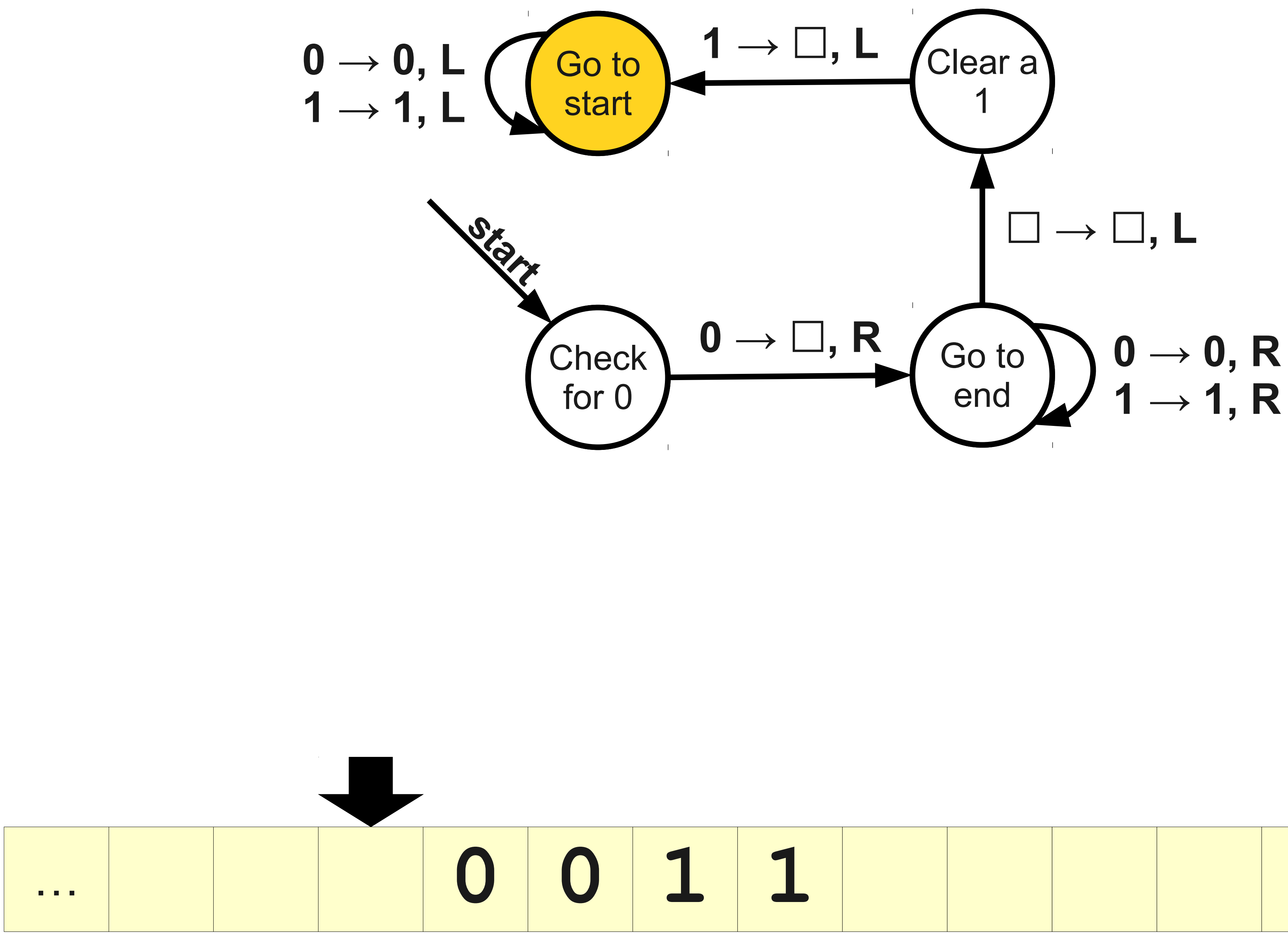
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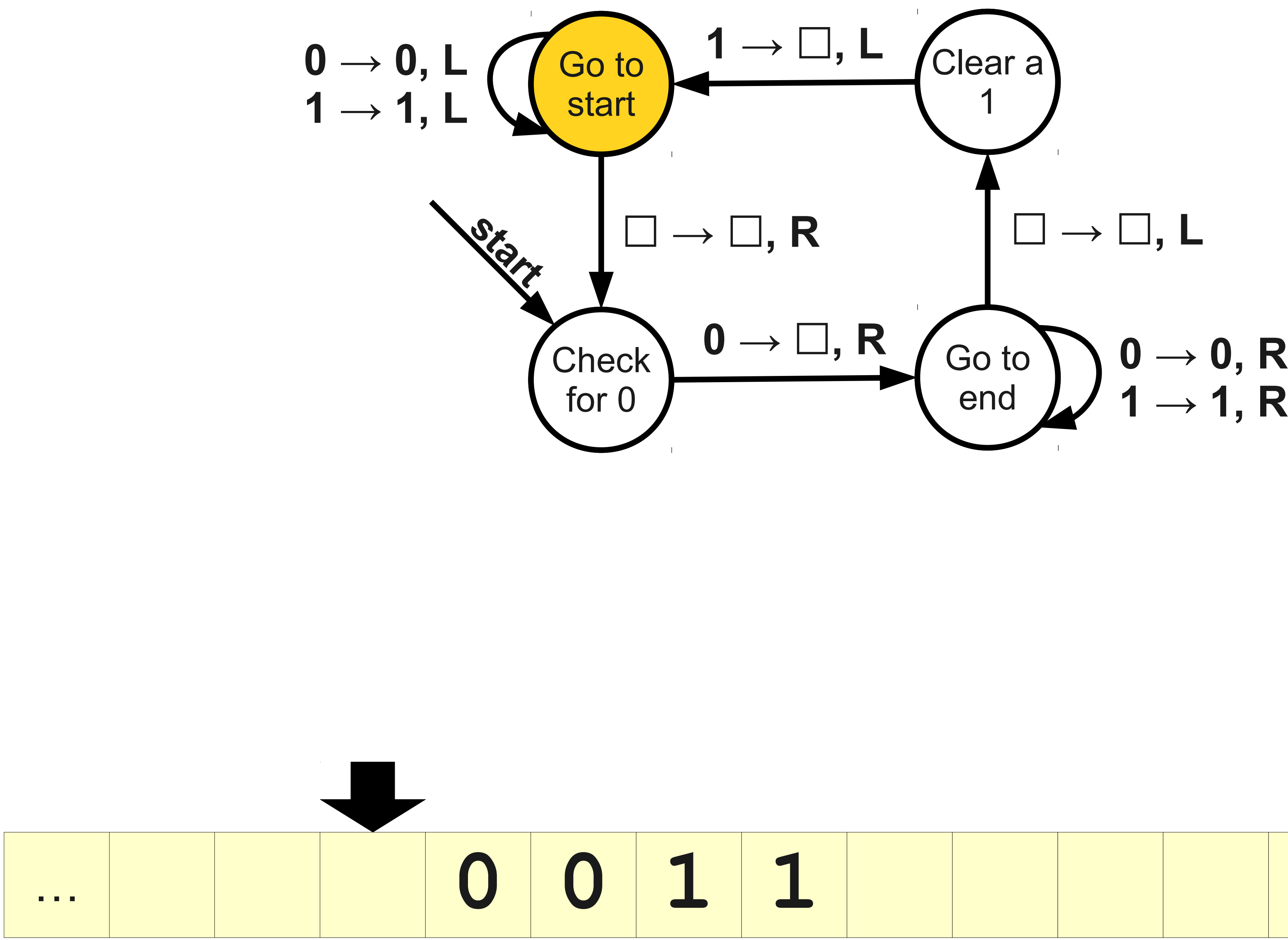

0 → 0, L
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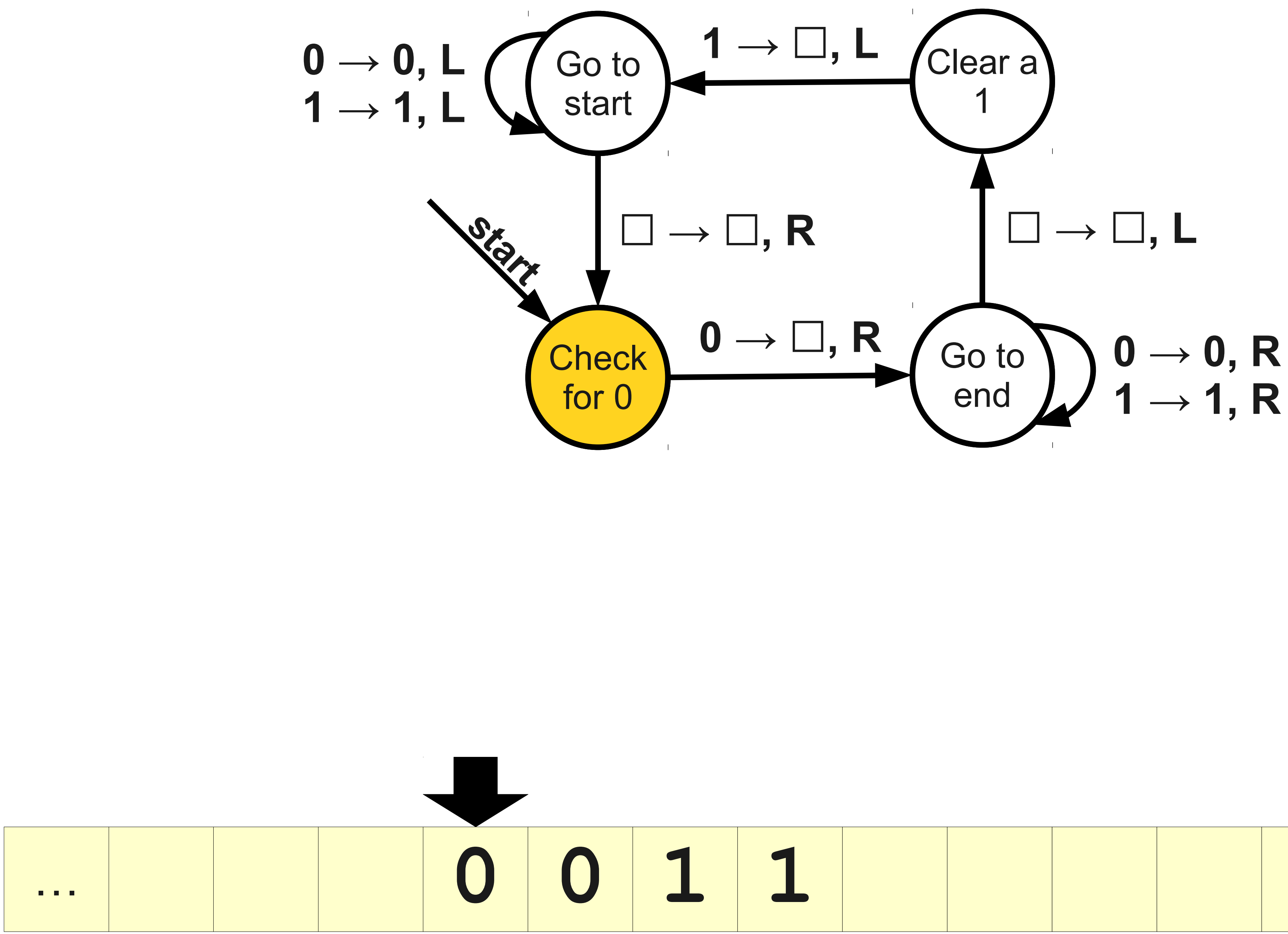

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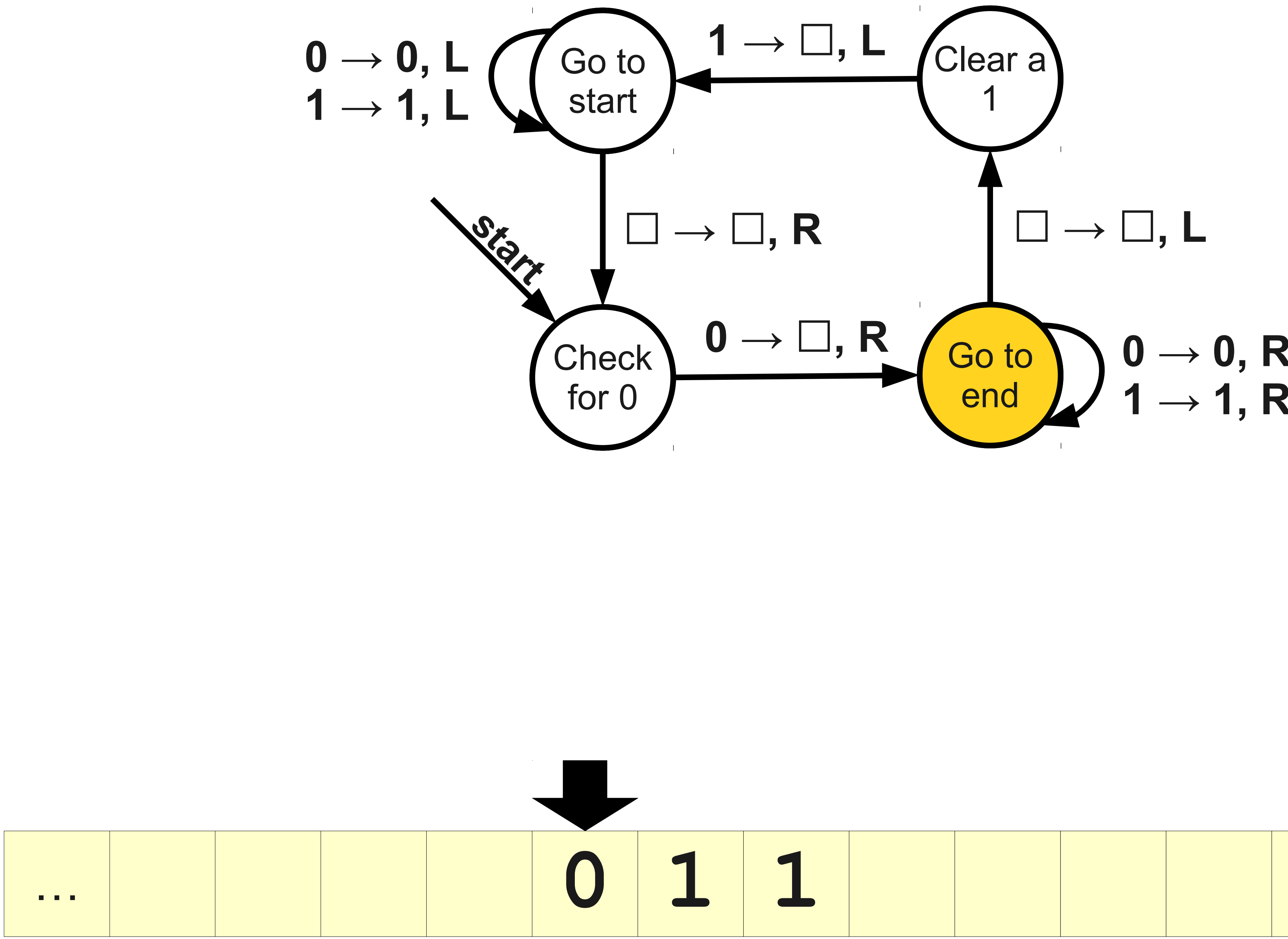
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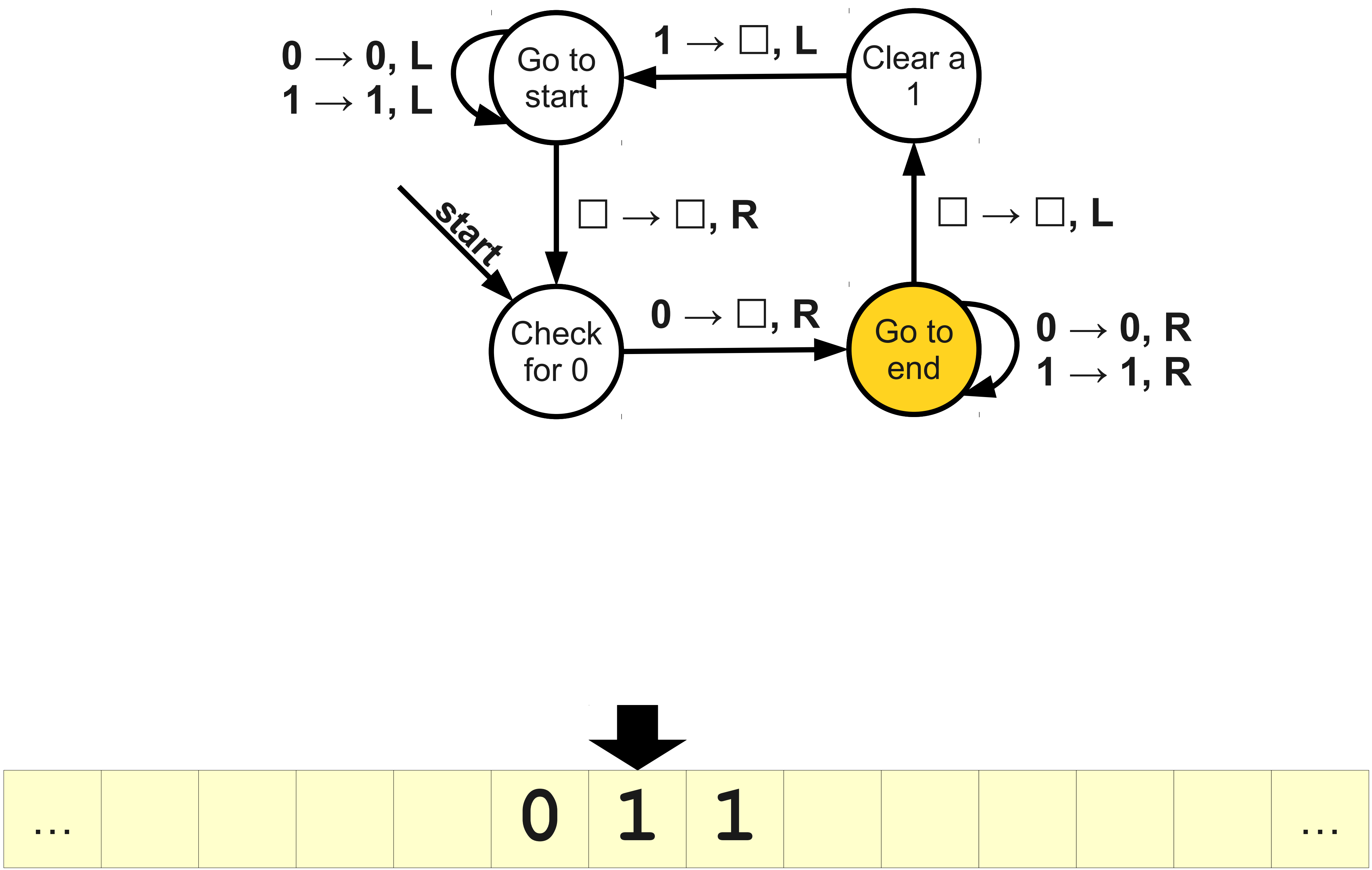

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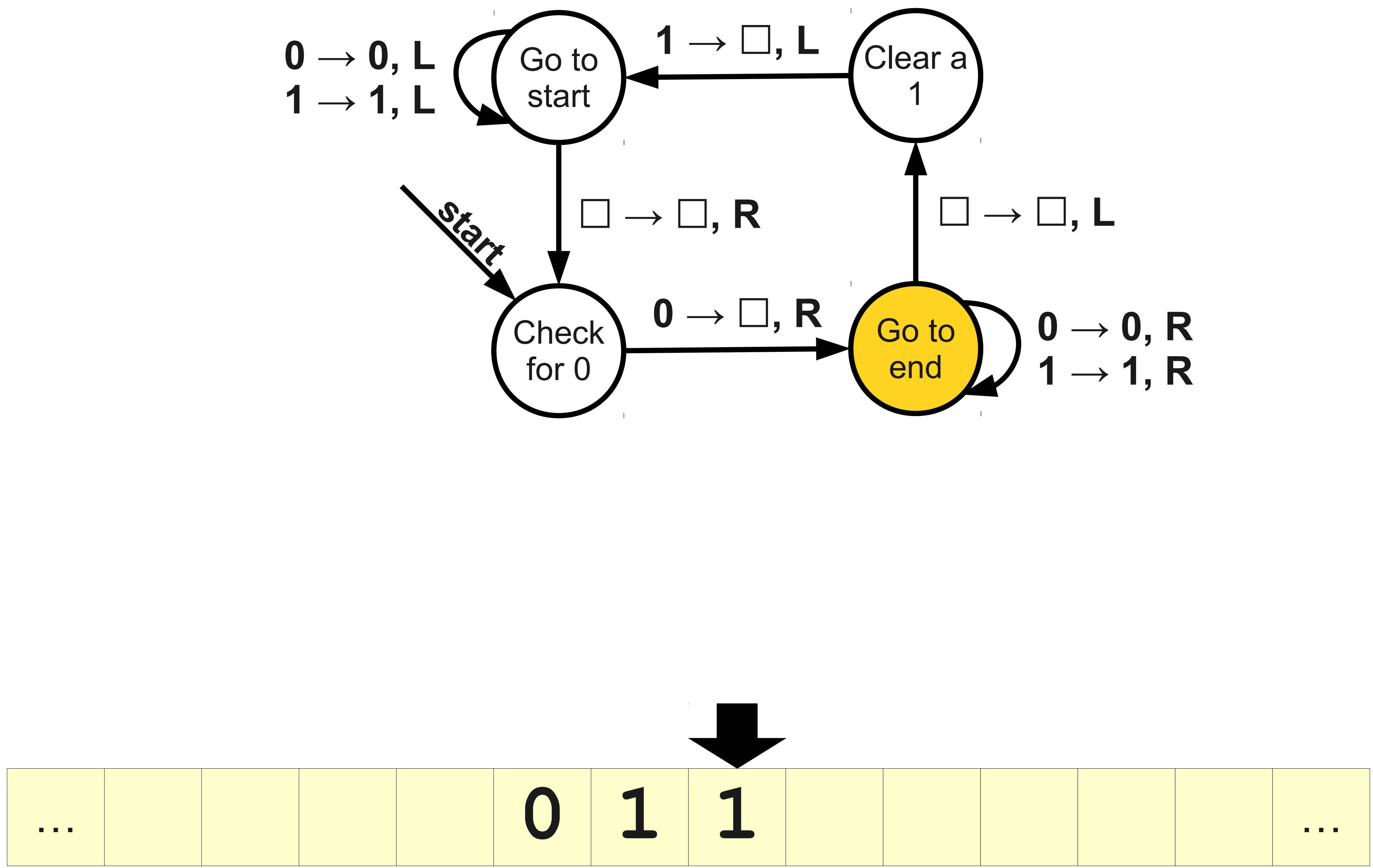

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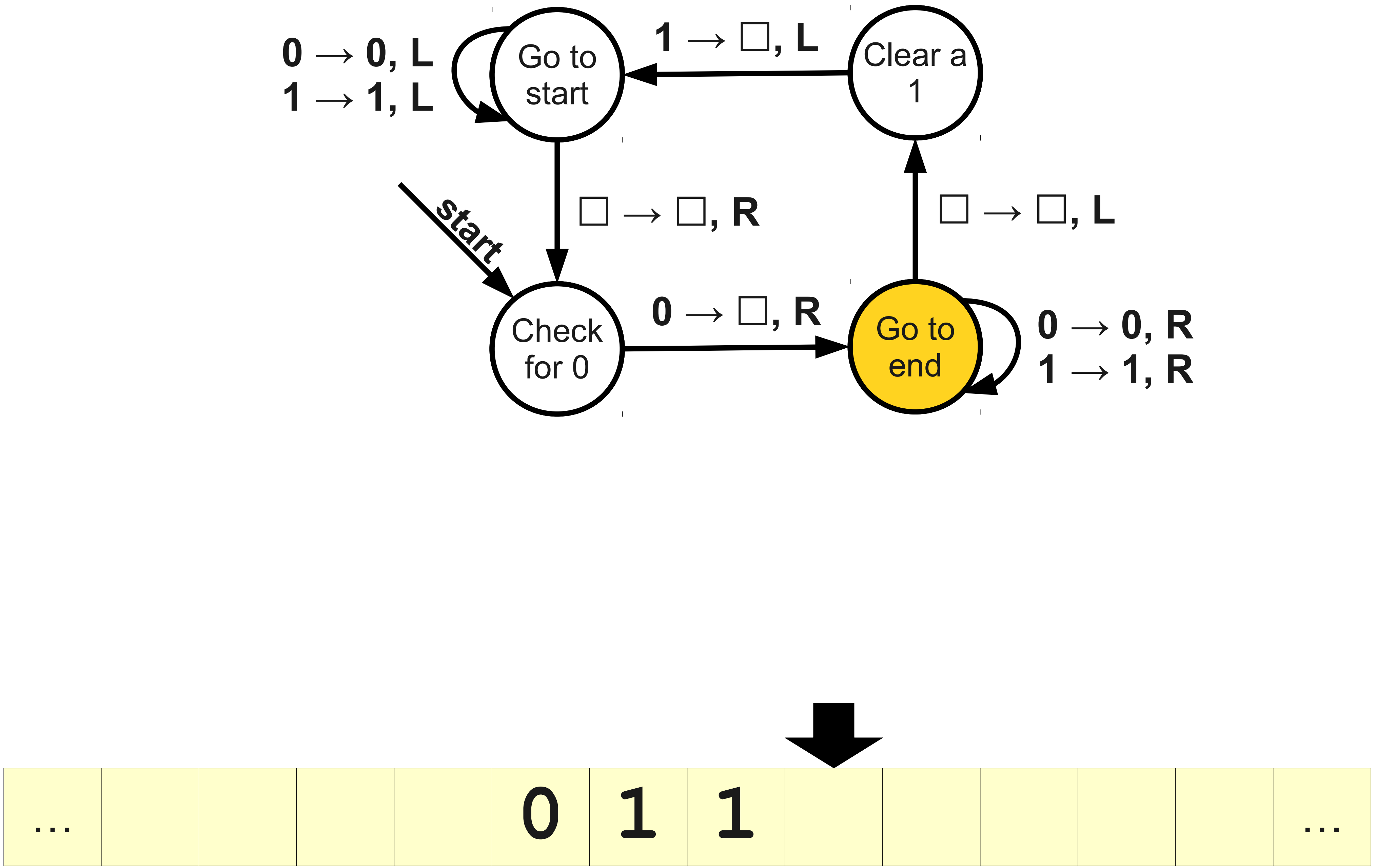

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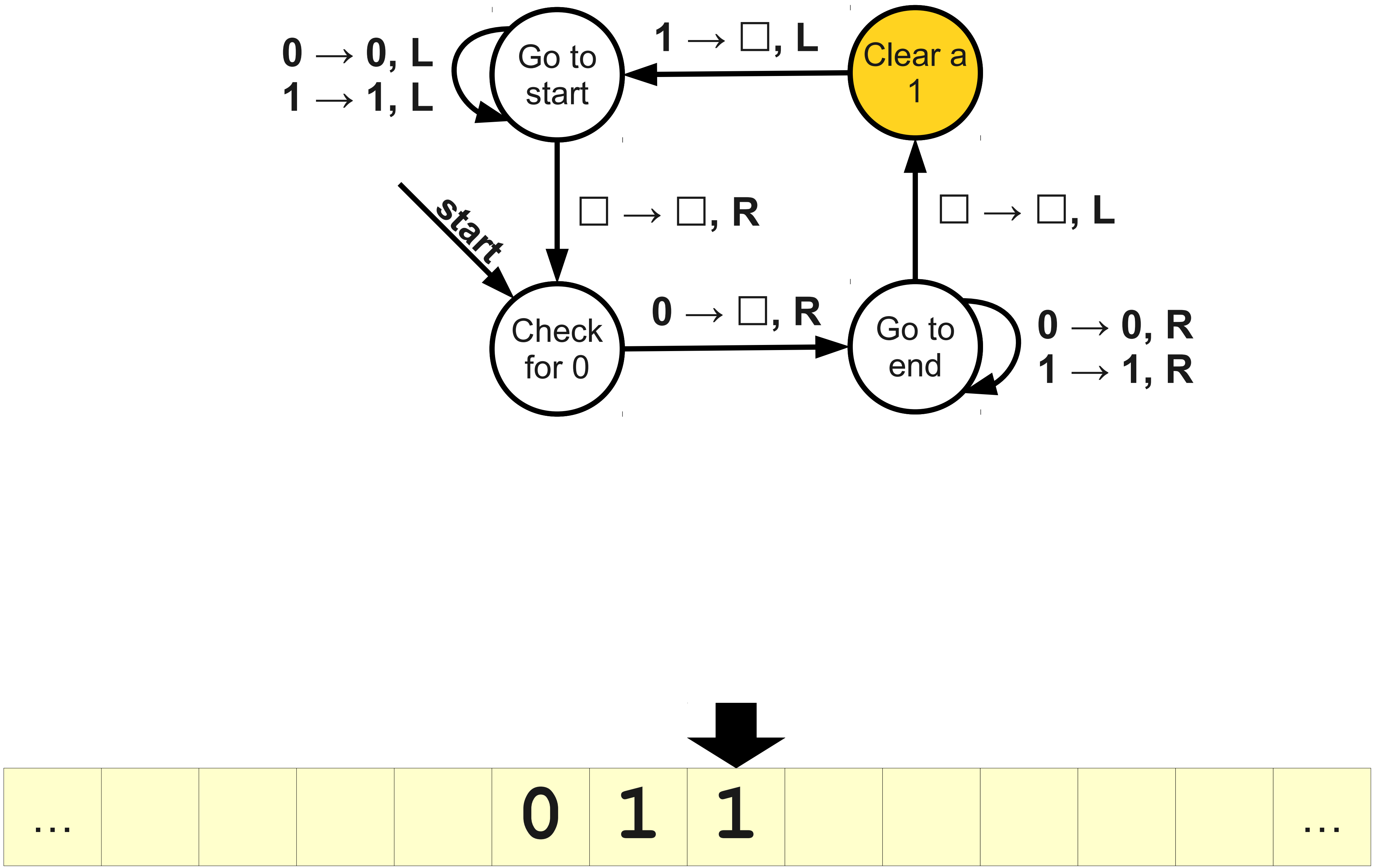

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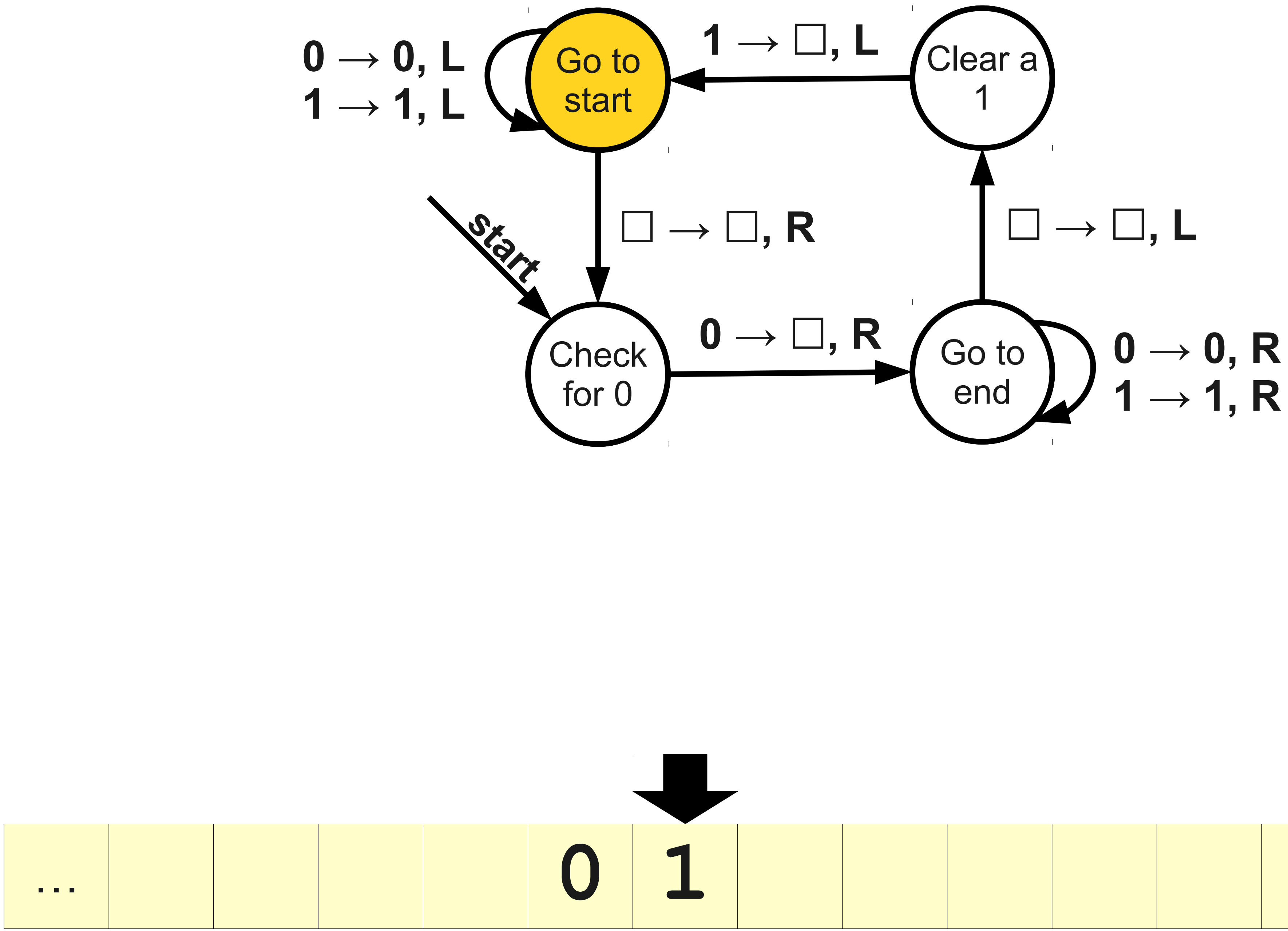

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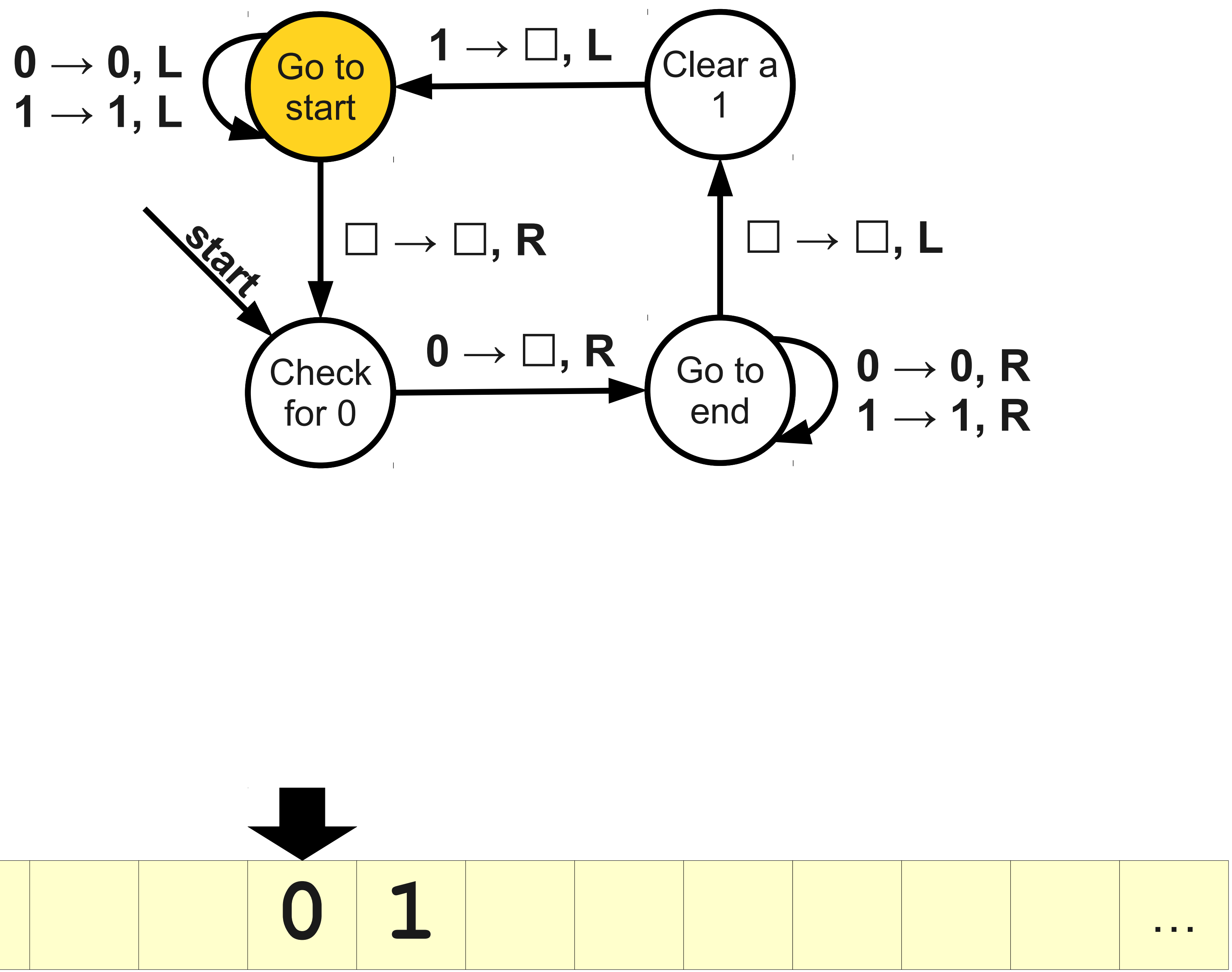
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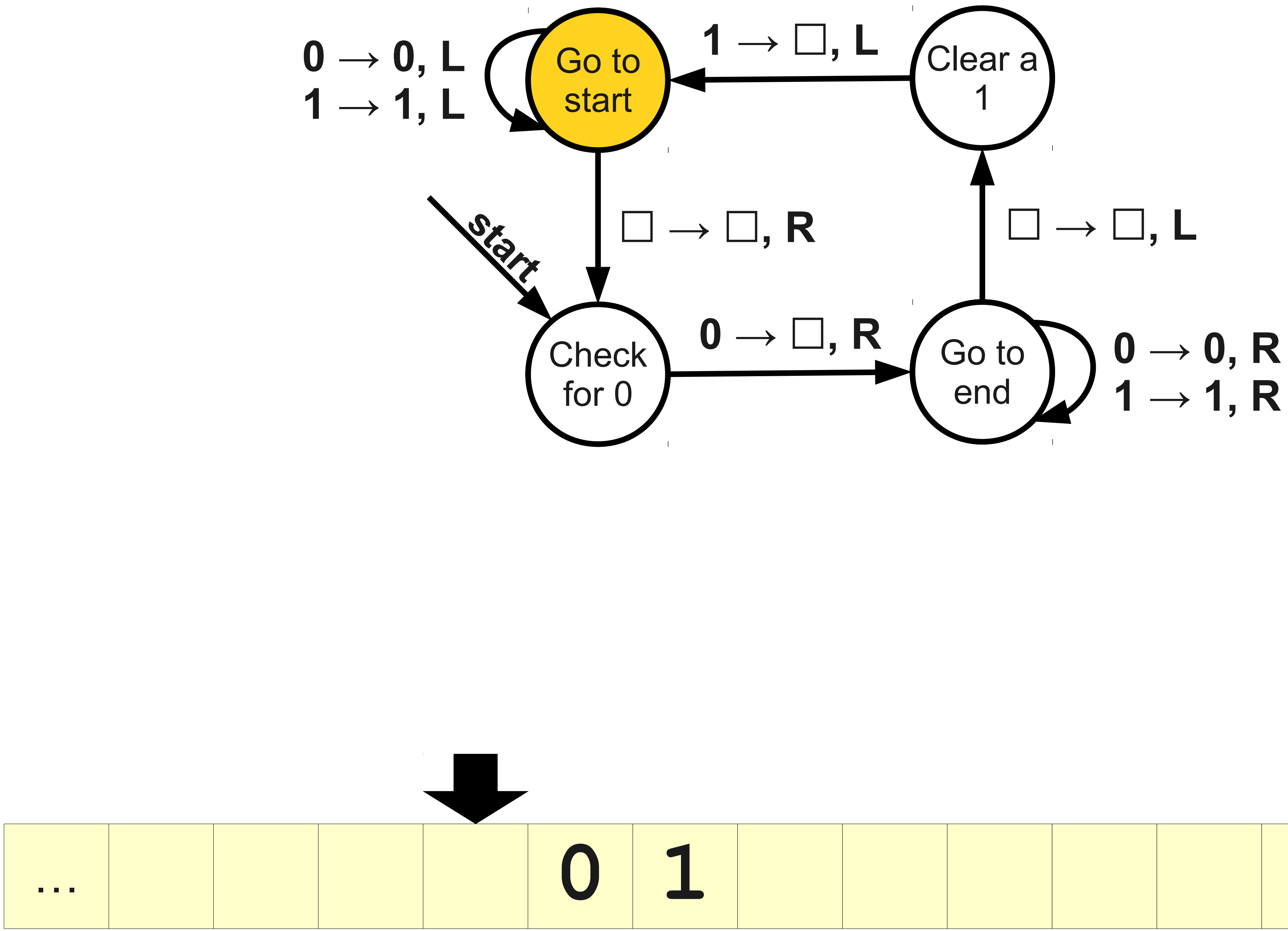
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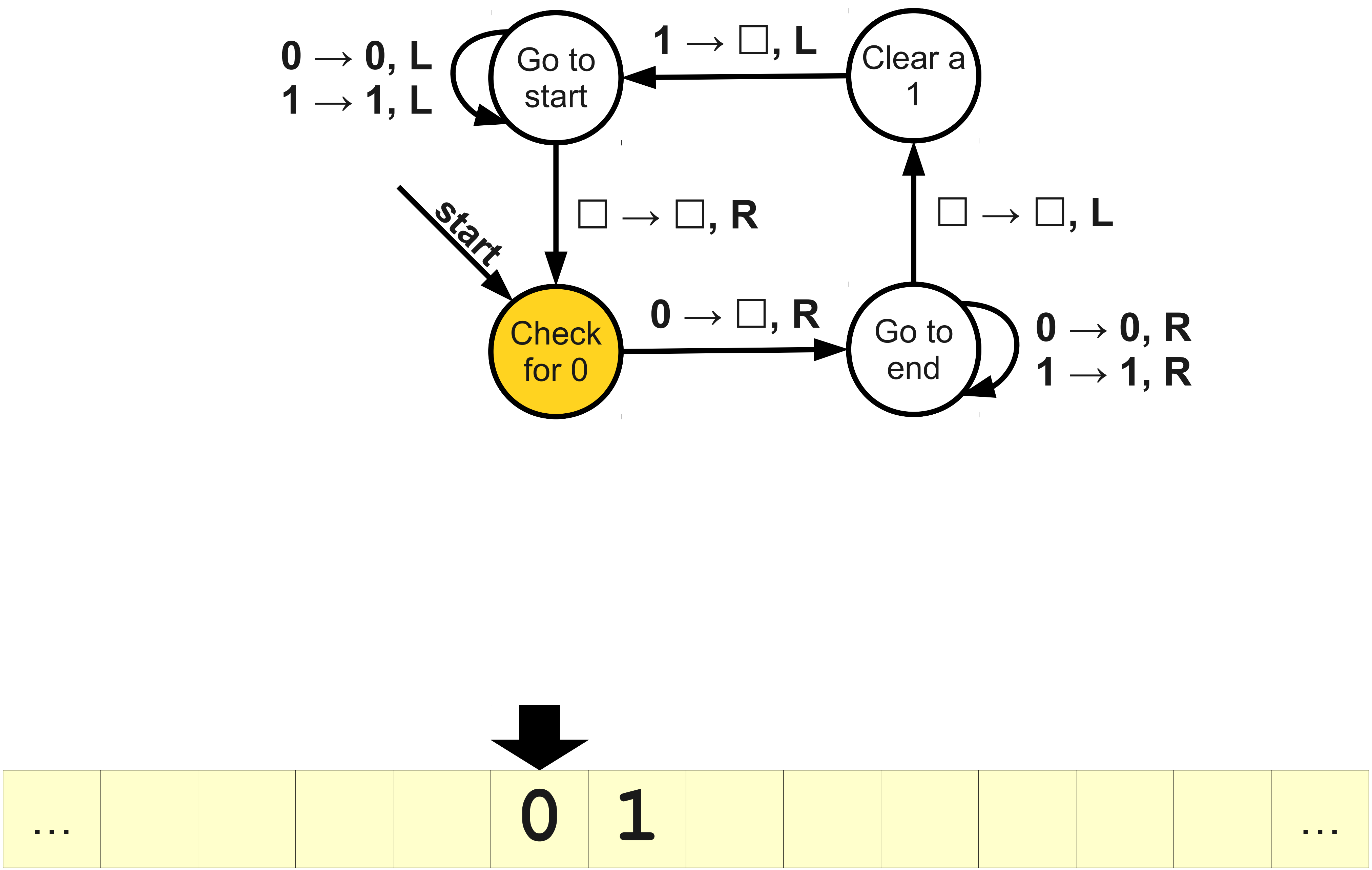
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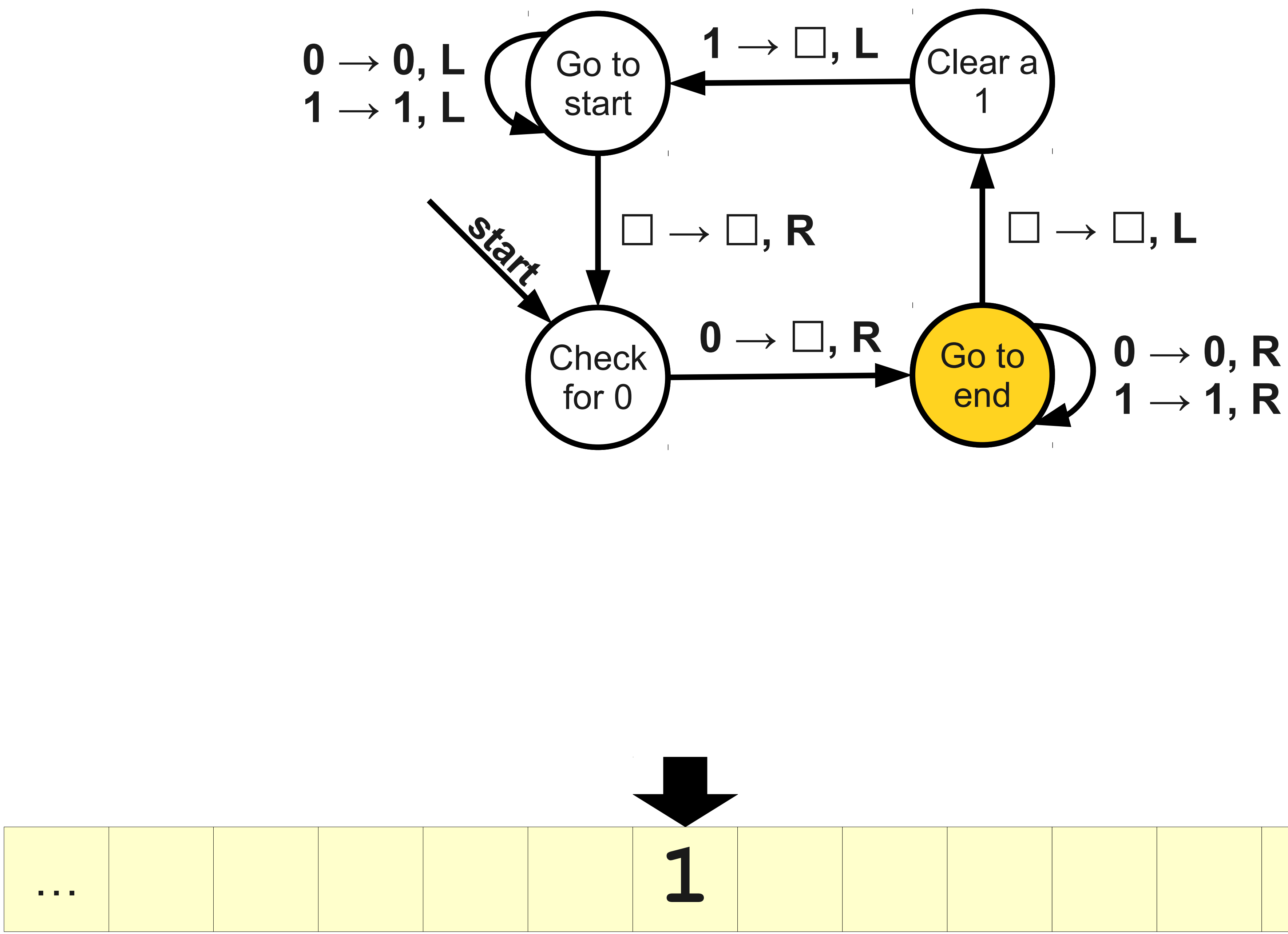

0 → 0, L
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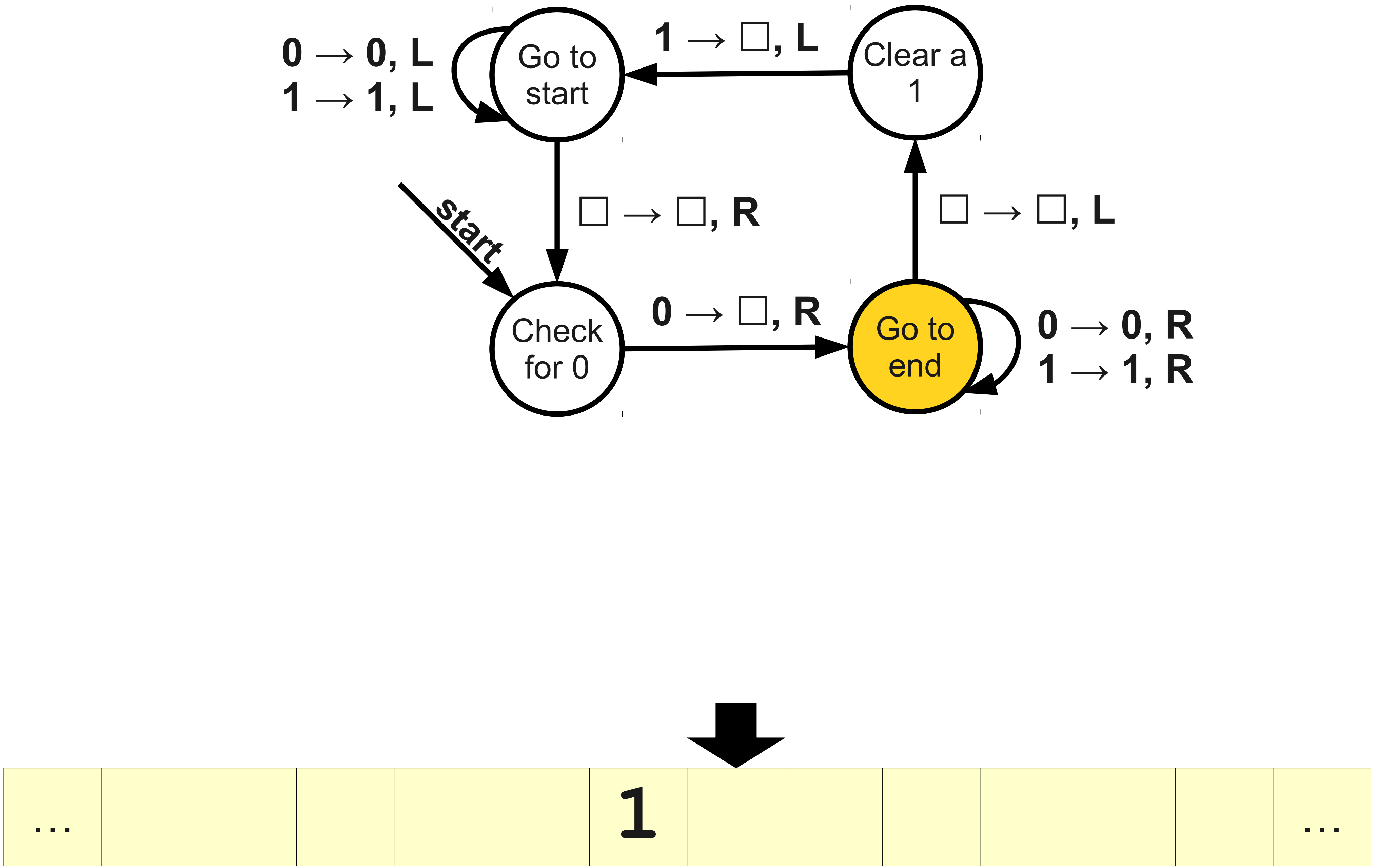
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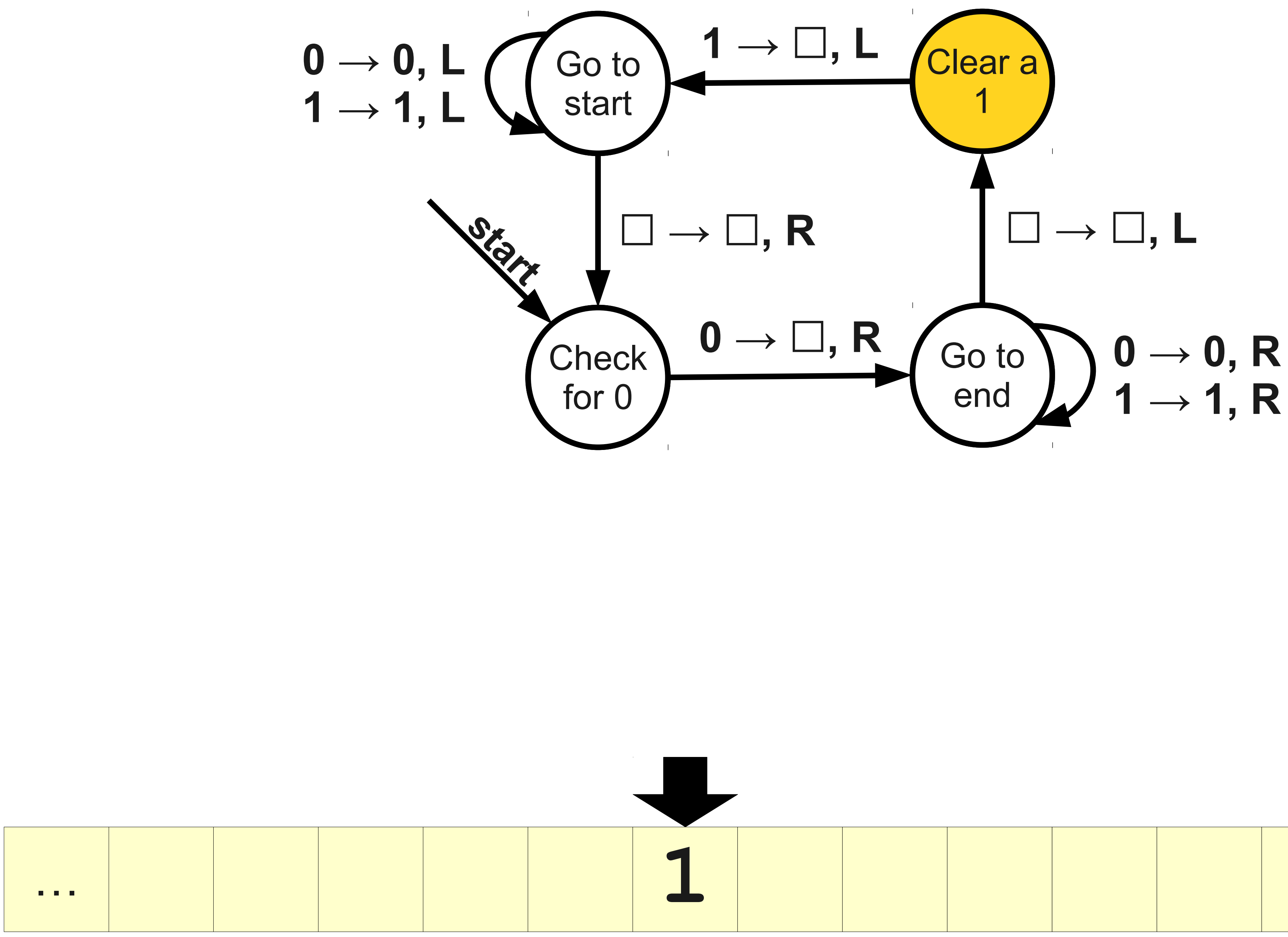
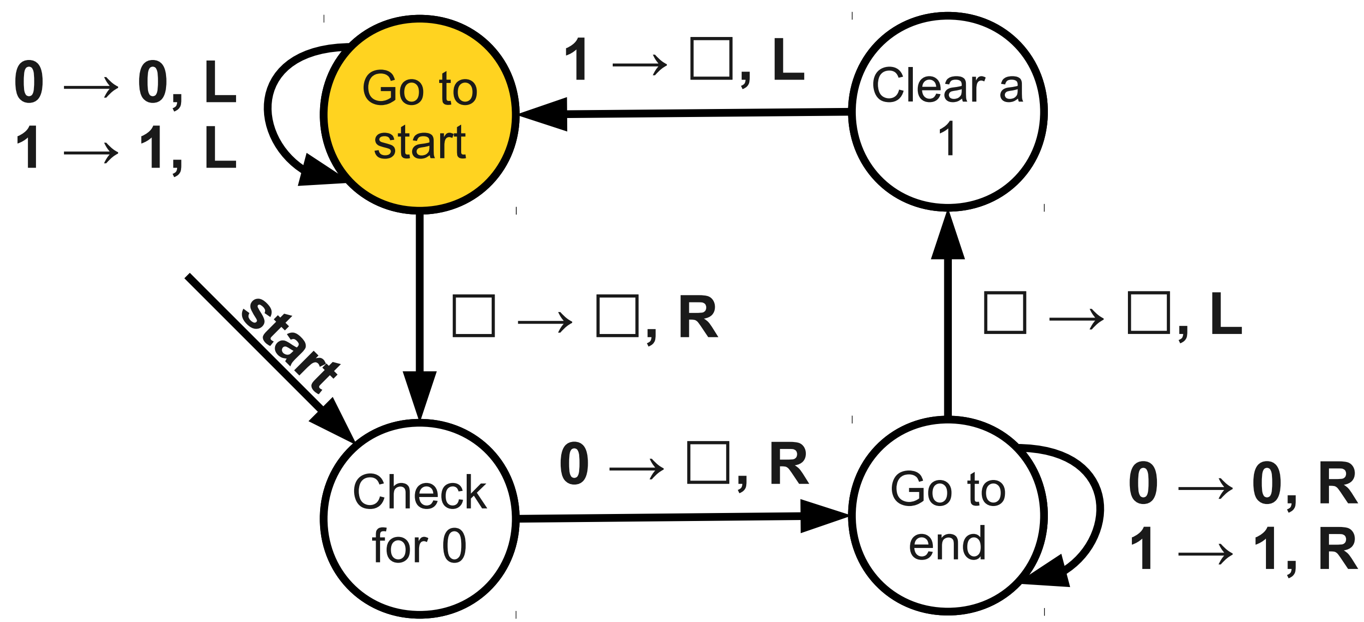
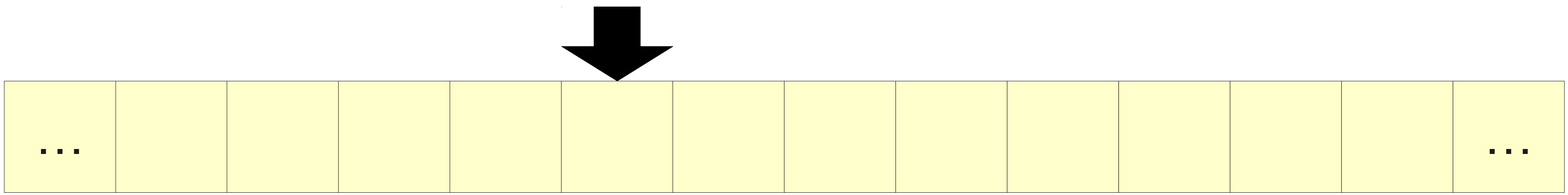


Go to start
Clear a 1
Check for 0
Go to end
start
0 → 0, L
1 → 1, L
1 → □, L
□ → □, R
□ → □, L
0 → □, R
0 → 0, R
1 → 1, R
…
1
…



0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
start
□ → □, R
□ → □, L
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R



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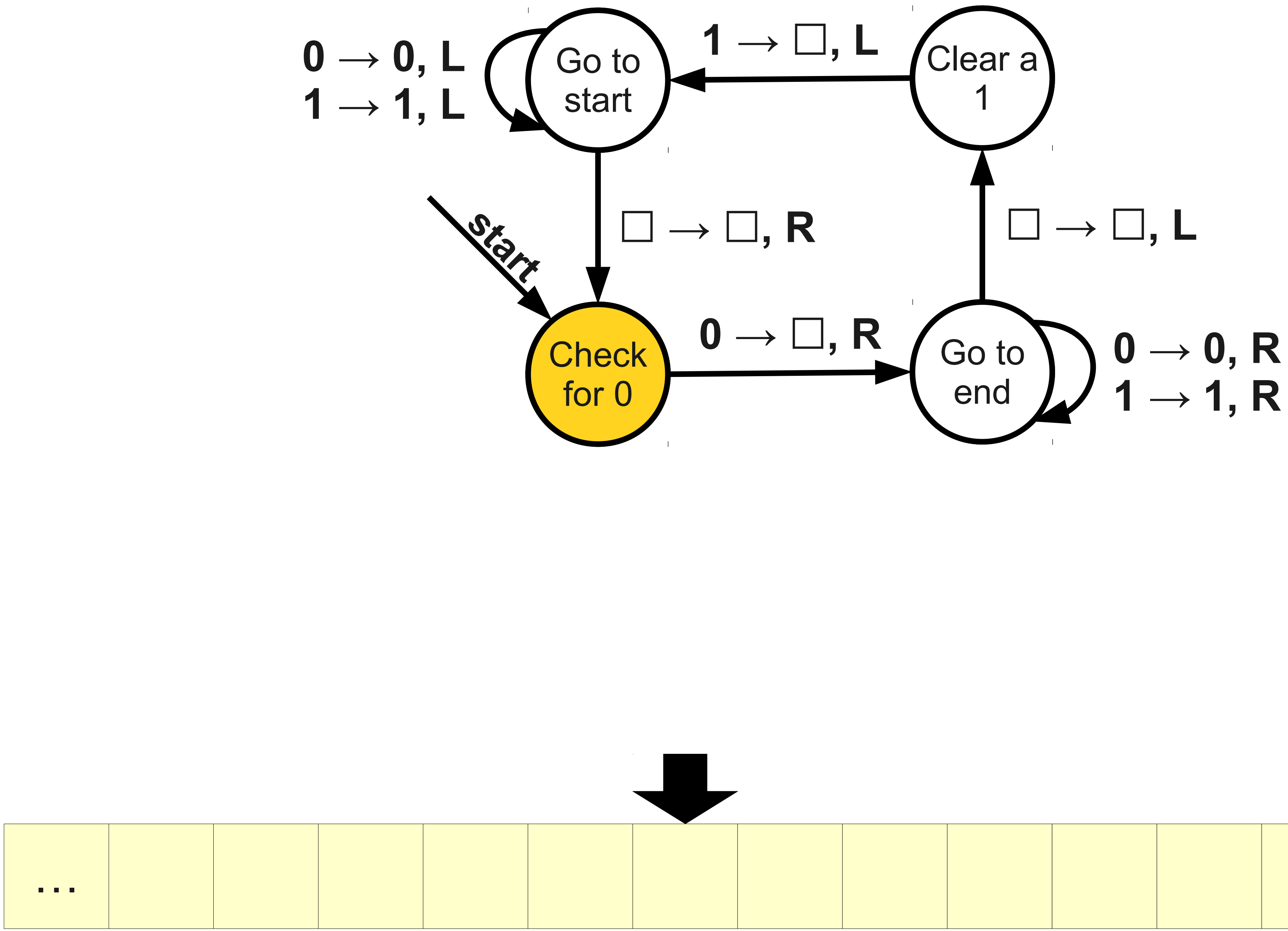
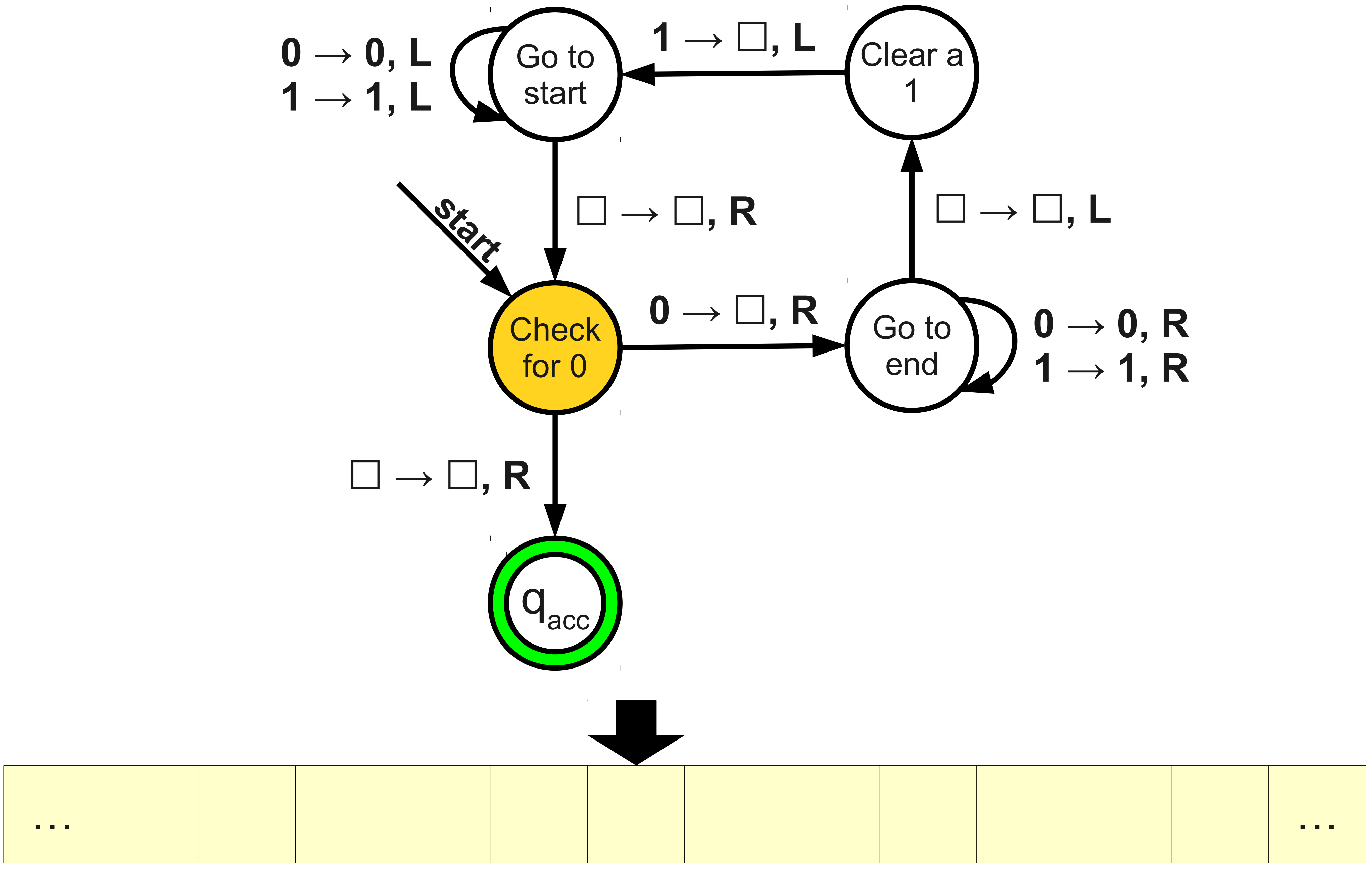
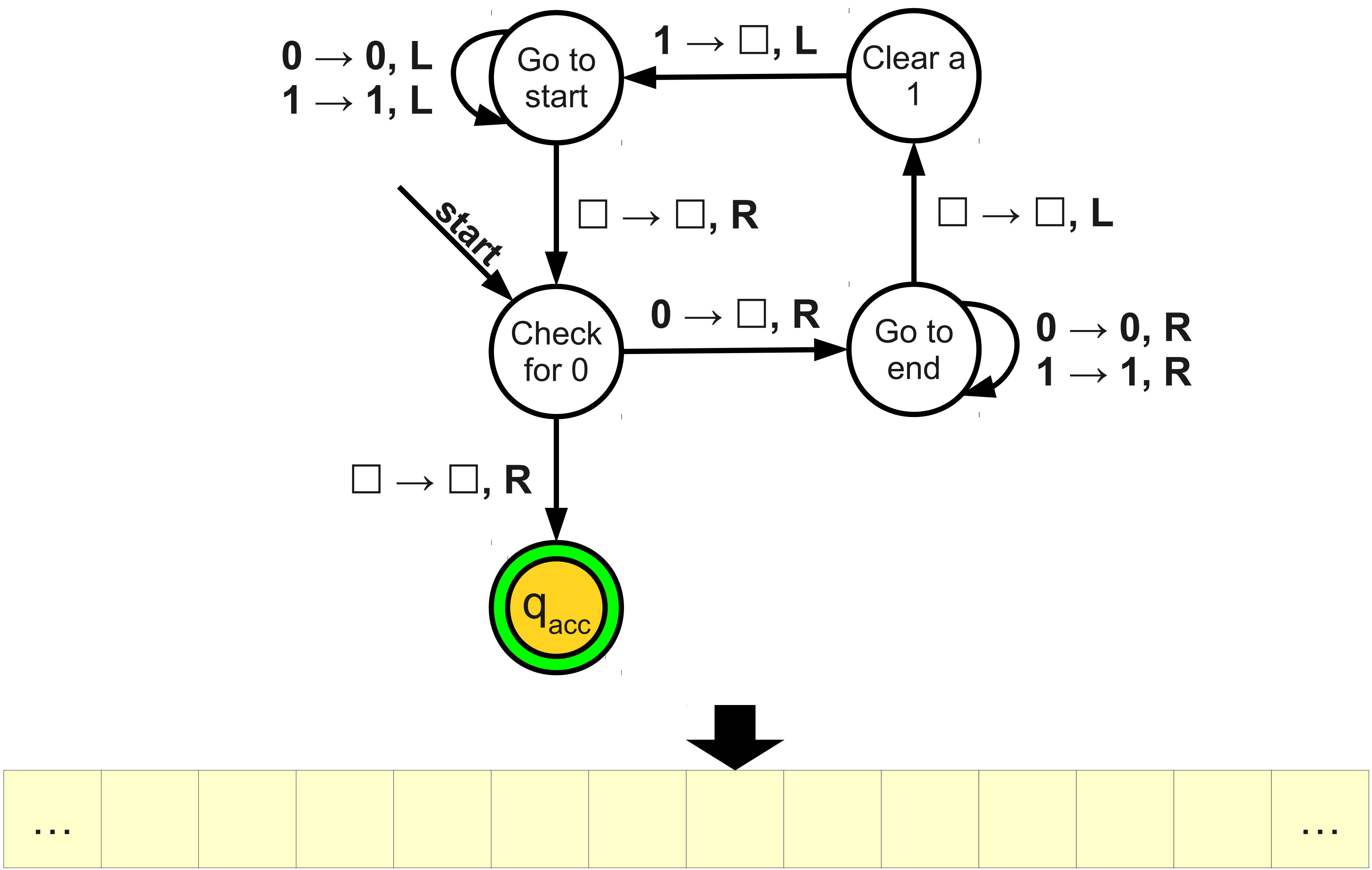


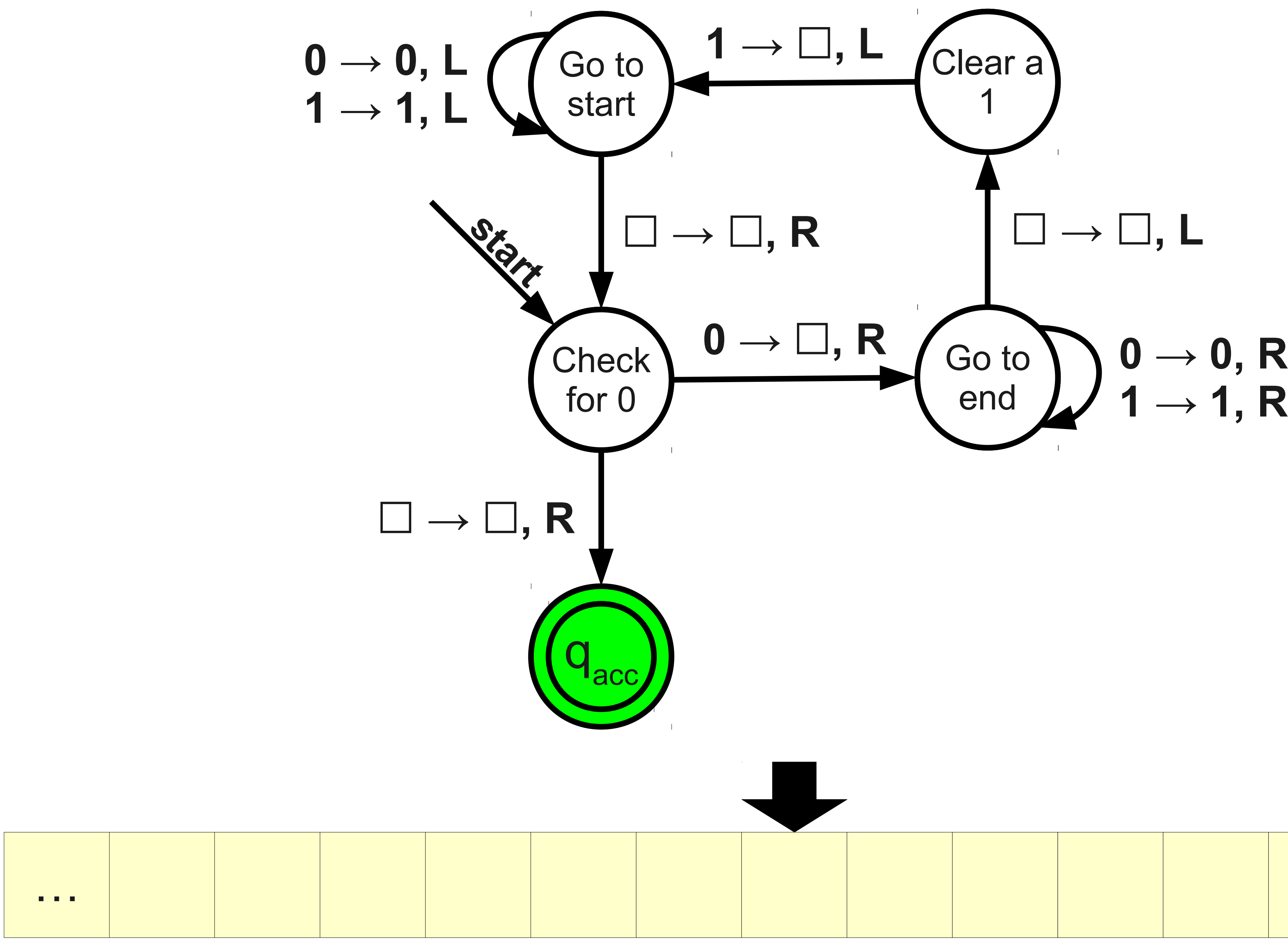
0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
start
□ → □, R
□ → □, L
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
...
...



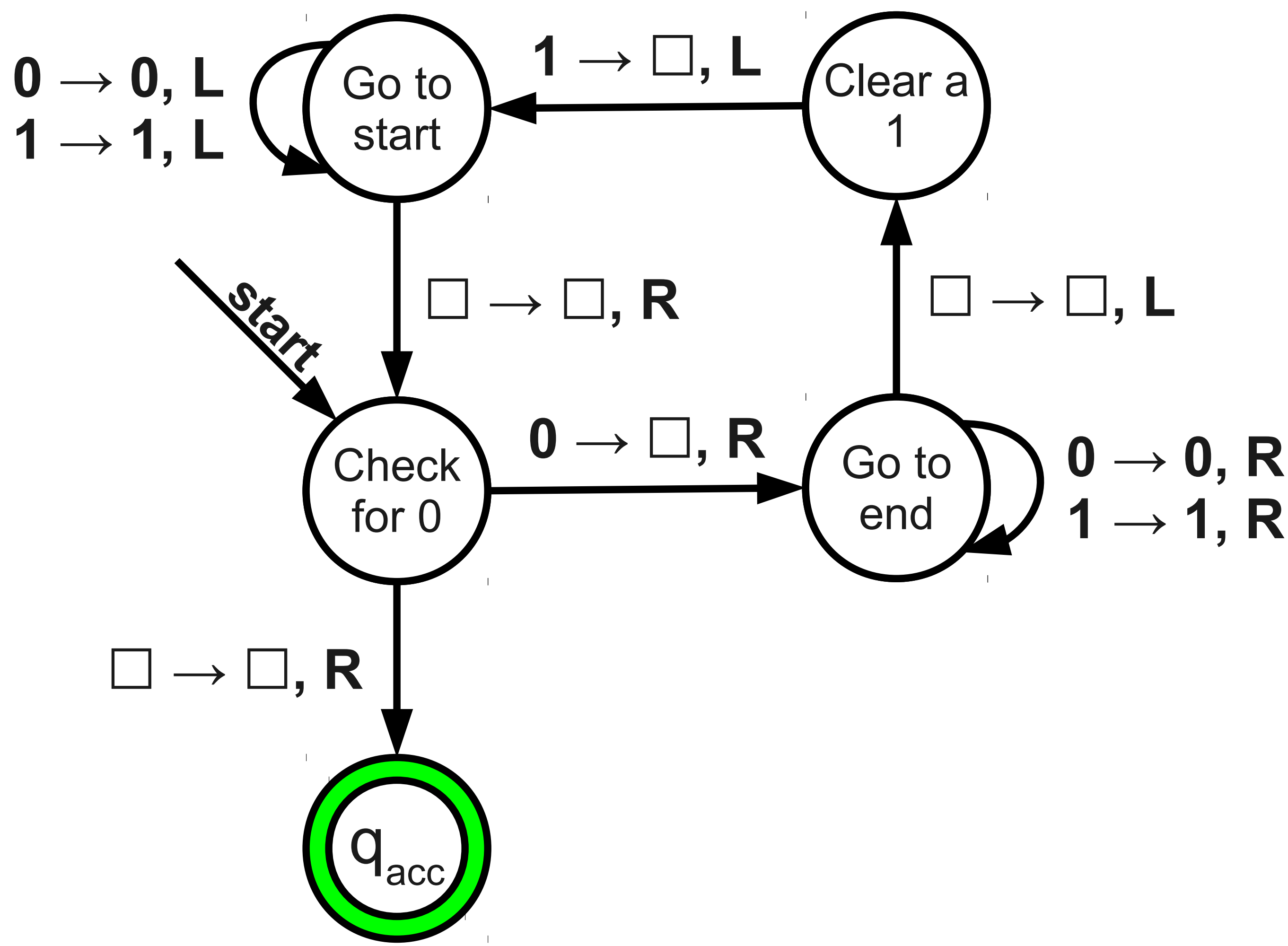
0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
start
□ → □, R
□ → □, L
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
□ → □, R
q_{acc}
...
...



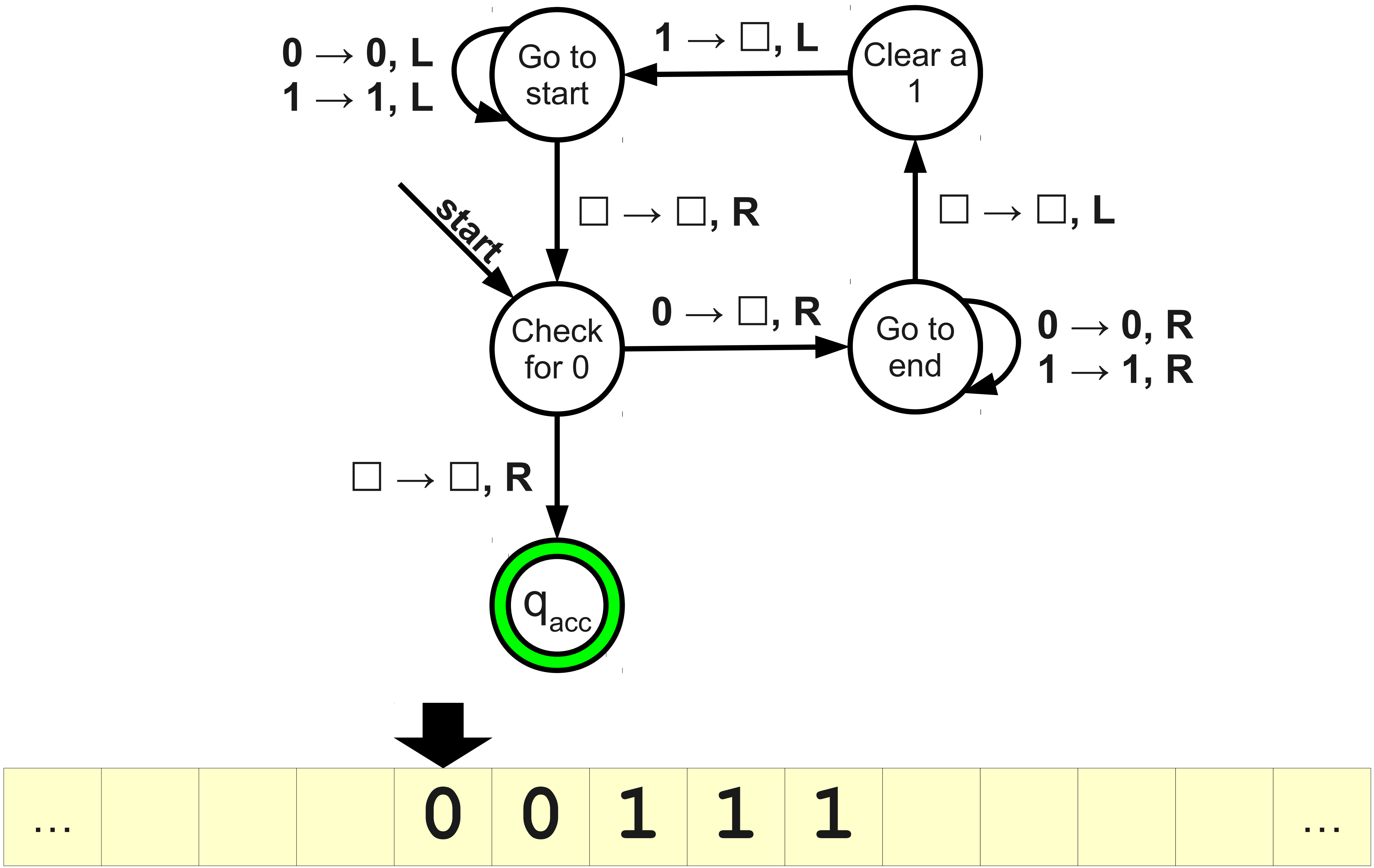
Go to start
Clear a 1
Check for 0
Go to end
q_acc
start
0 → 0, L
1 → 1, L
1 → □, L
□ → □, R
□ → □, L
0 → □, R
0 → 0, R
1 → 1, R
□ → □, R
…
…

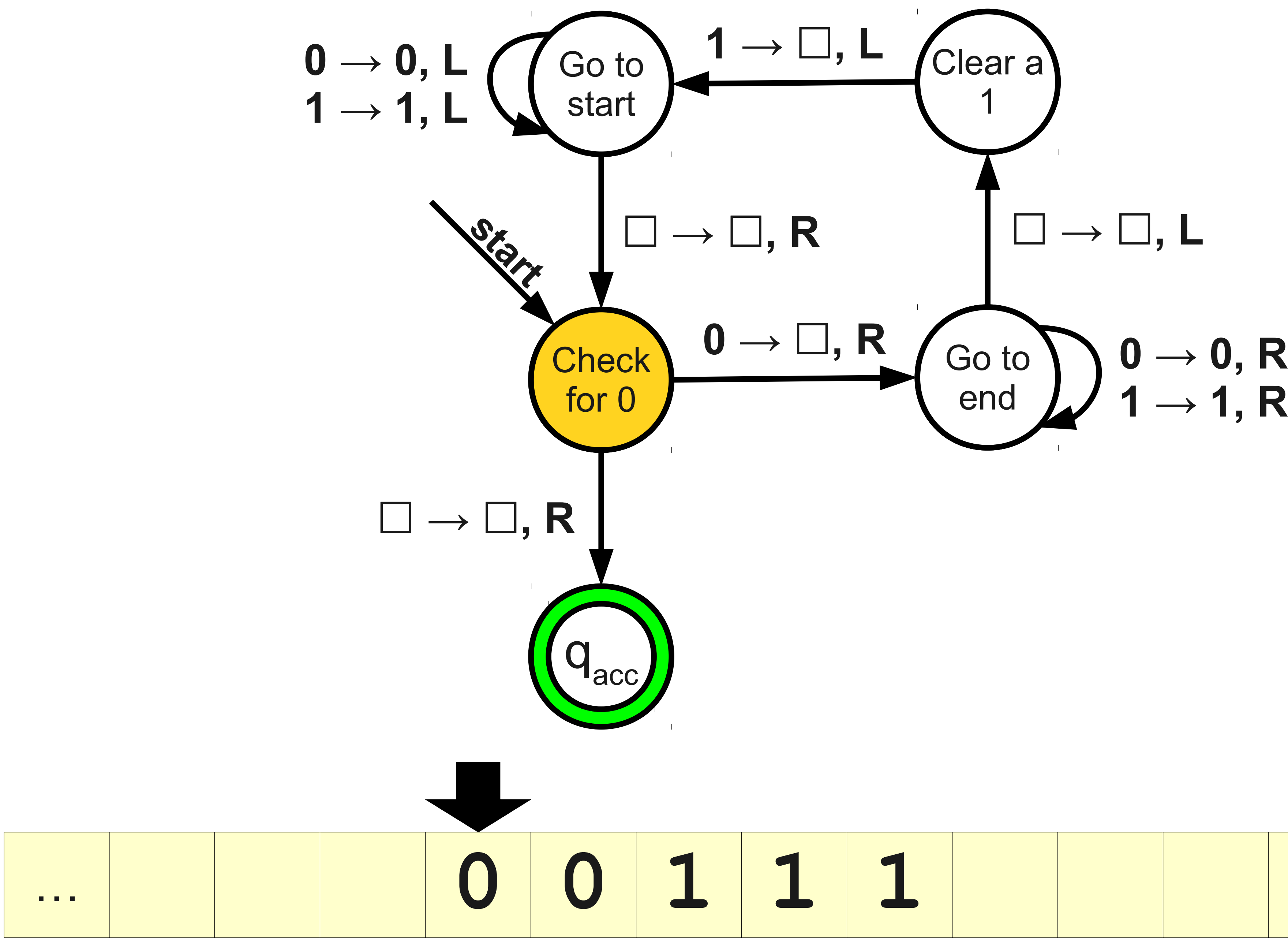


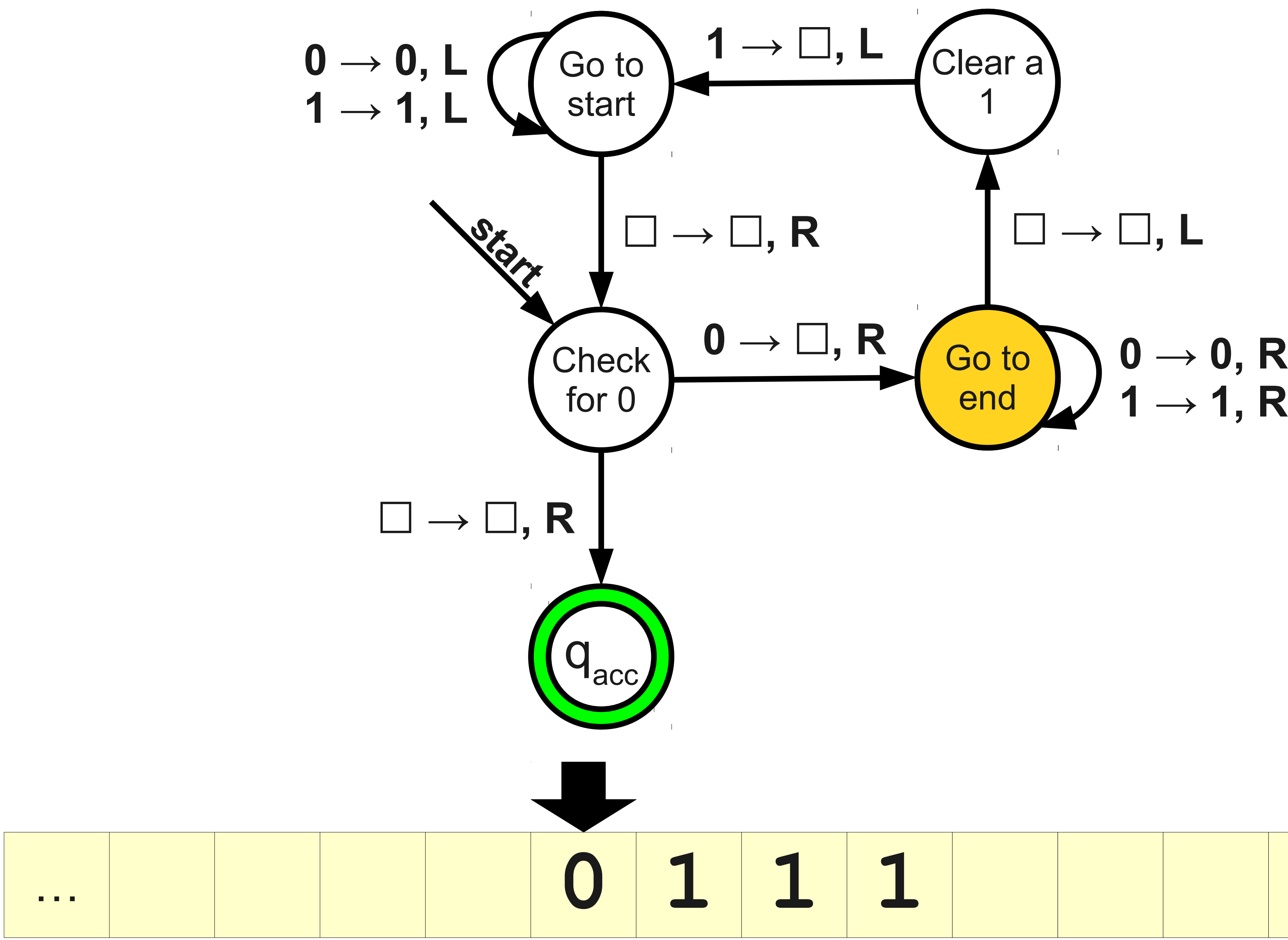
Go to start
Clear a 1
Check for 0
Go to end
q_acc
start
0 → 0, L
1 → 1, L
1 → □, L
□ → □, R
□ → □, L
0 → □, R
0 → 0, R
1 → 1, R
□ → □, R
…
…



start
Go to start
Clear a 1
Check for 0
Go to end
q_{acc}
0 → 0, L
1 → 1, L
1 → □, L
□ → □, R
□ → □, L
0 → □, R
0 → 0, R
1 → 1, R
□ → □, R


0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
□ → □, R
start
□ → □, L
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
□ → □, R
q_acc
…
0 0 1 1 1
…


0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
start
□ → □, R
□ → □, L
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
□ → □, R
q_acc
…
0
0
1
1
1
…



0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
start
□ → □, R
□ → □, L
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
□ → □, R
q_acc
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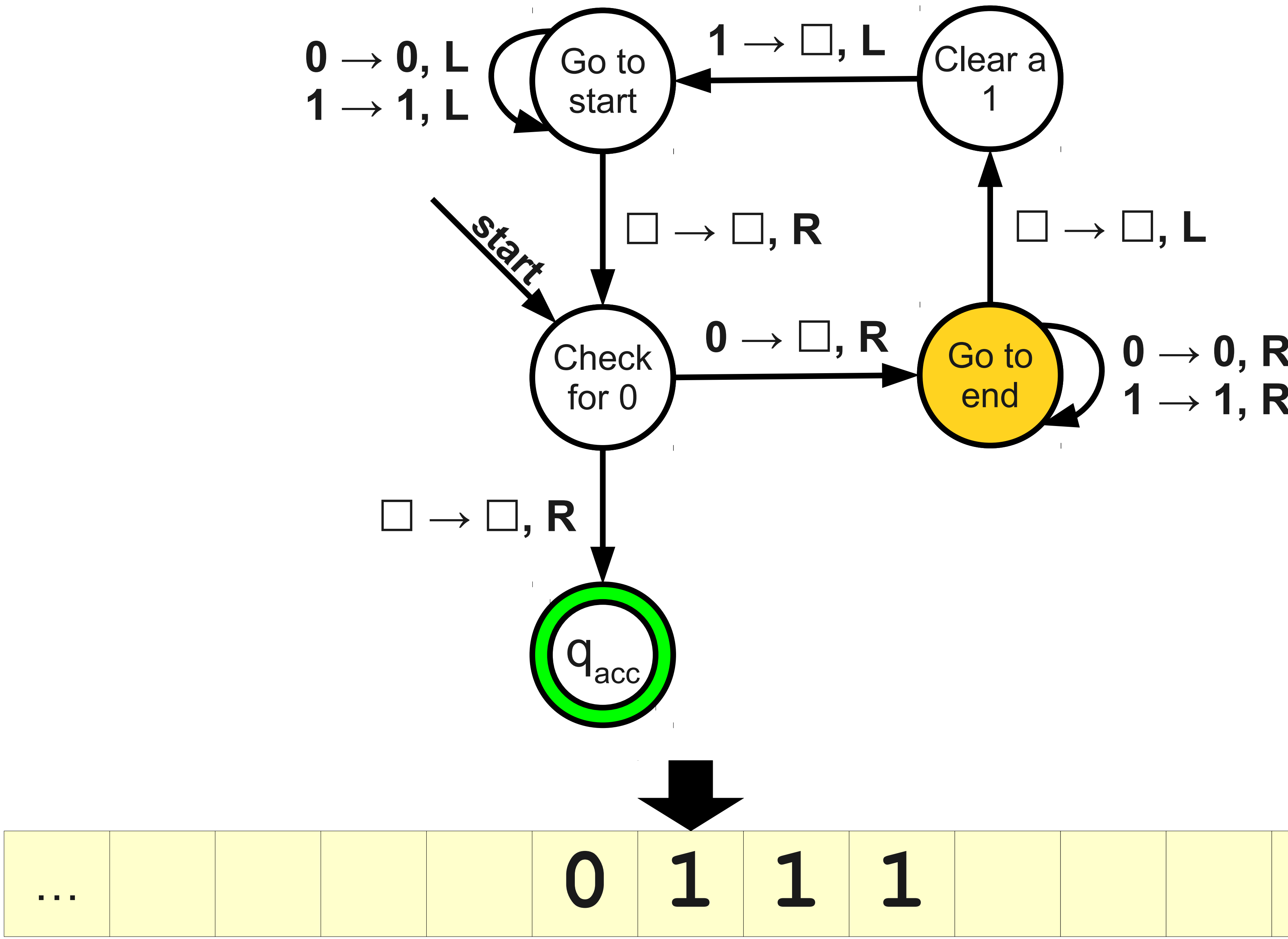
0
1
1
1
…



0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
start
□ → □, R
□ → □, L
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
□ → □, R
q_{acc}
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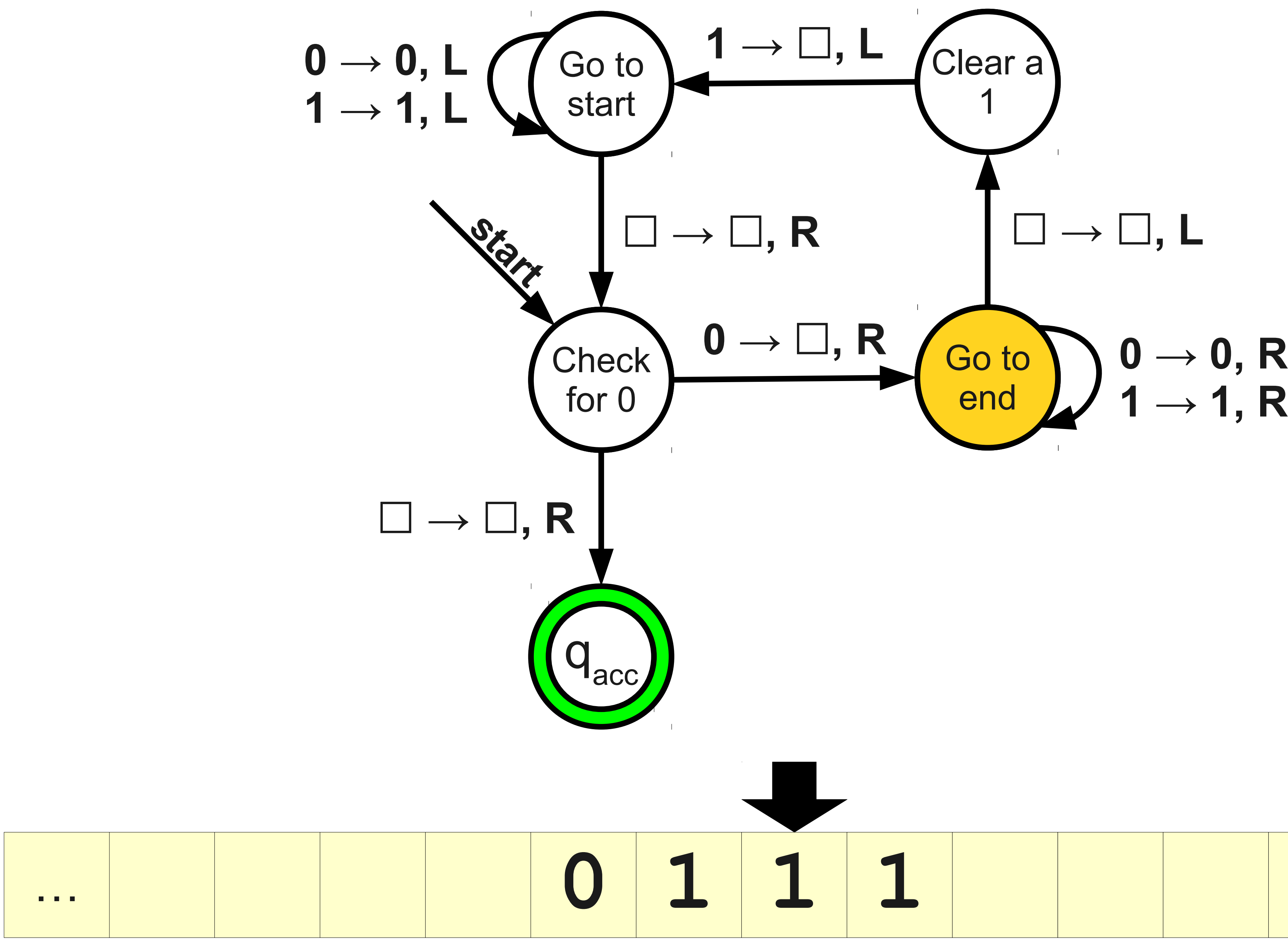
0
1
1
1
…


0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
start
□ → □, R
□ → □, L
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
□ → □, R
q_{acc}
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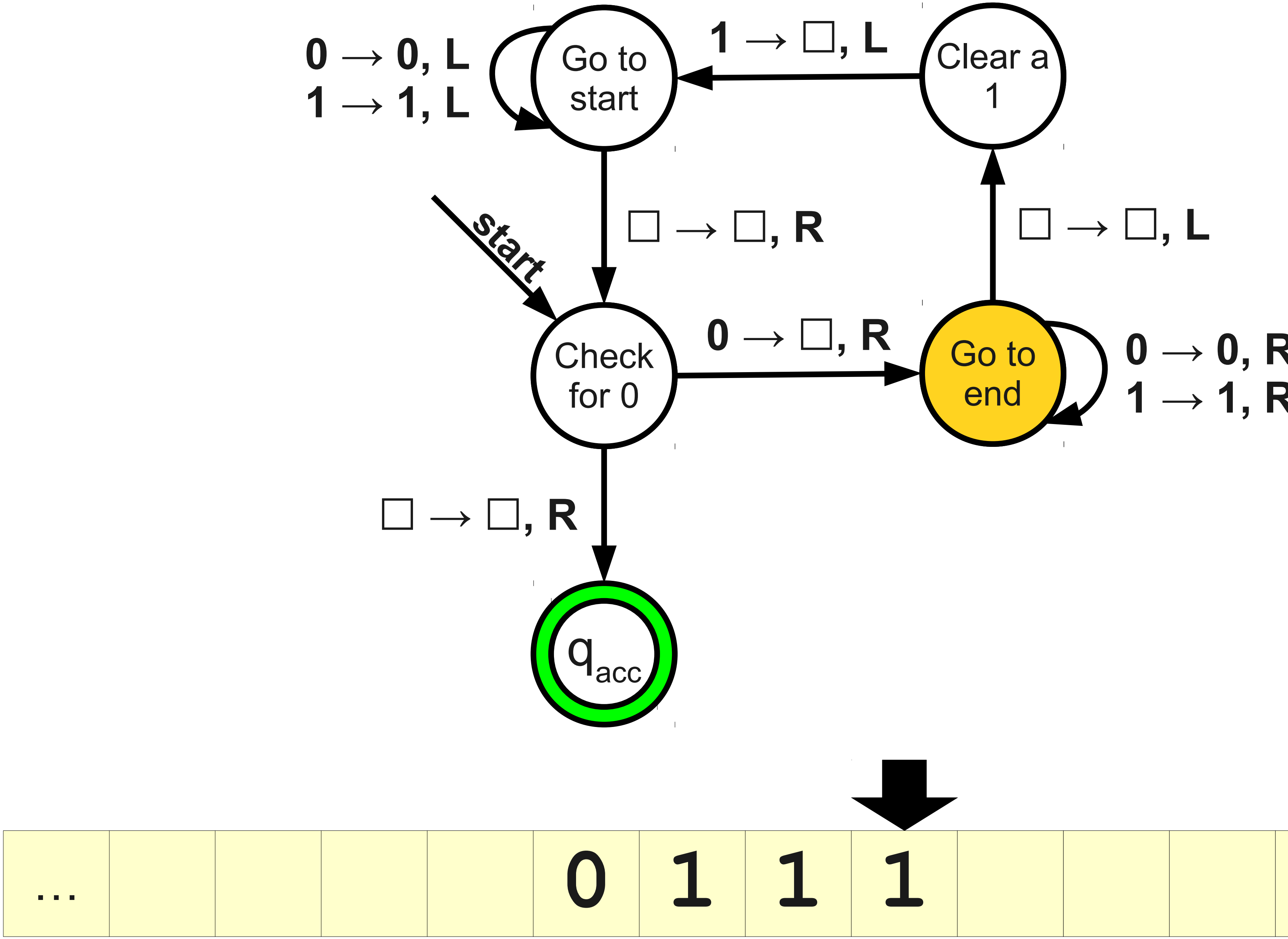
0
1
1
1
…



0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
□ → □, R
start
□ → □, L
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
□ → □, R
q_acc
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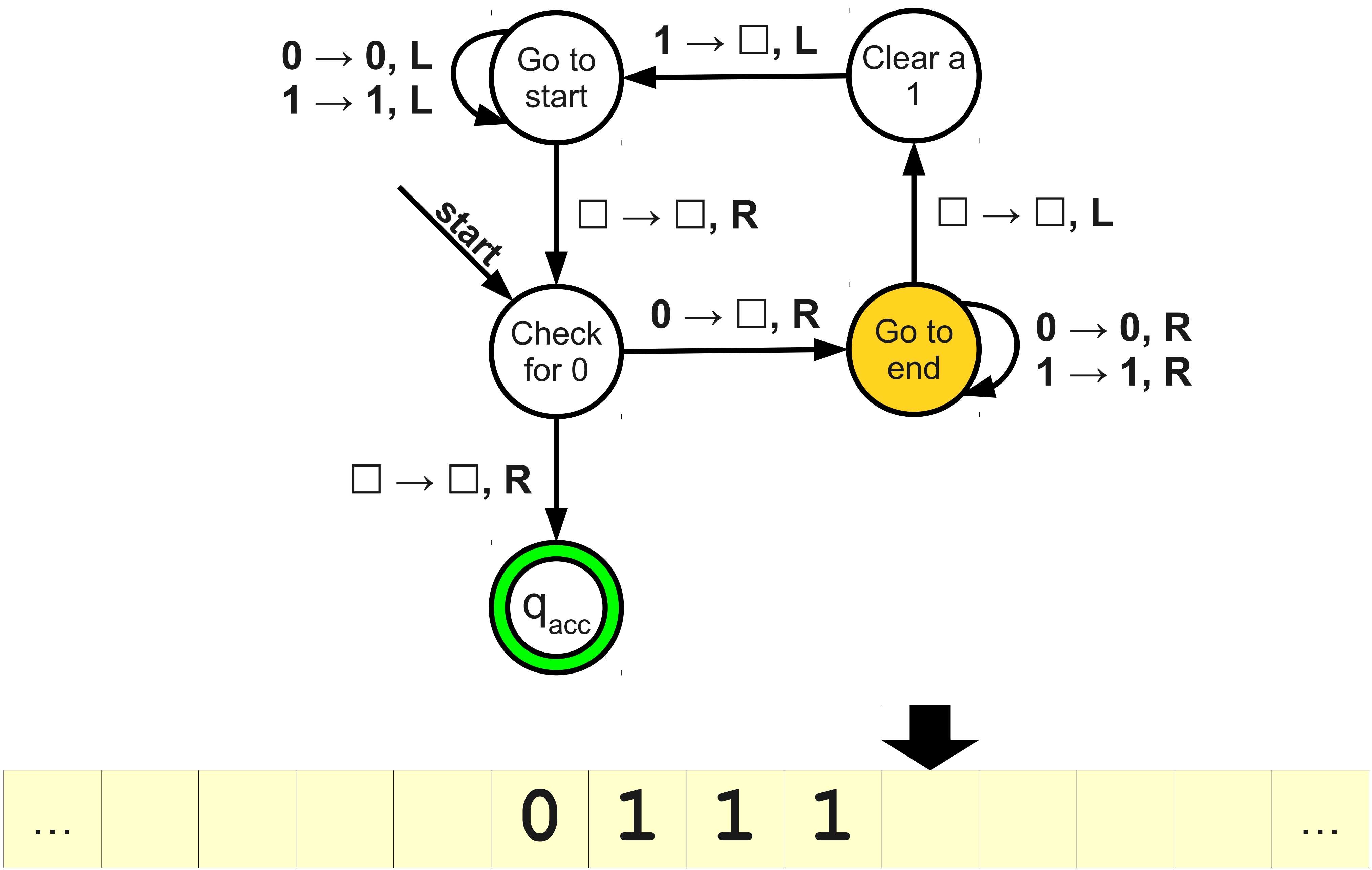
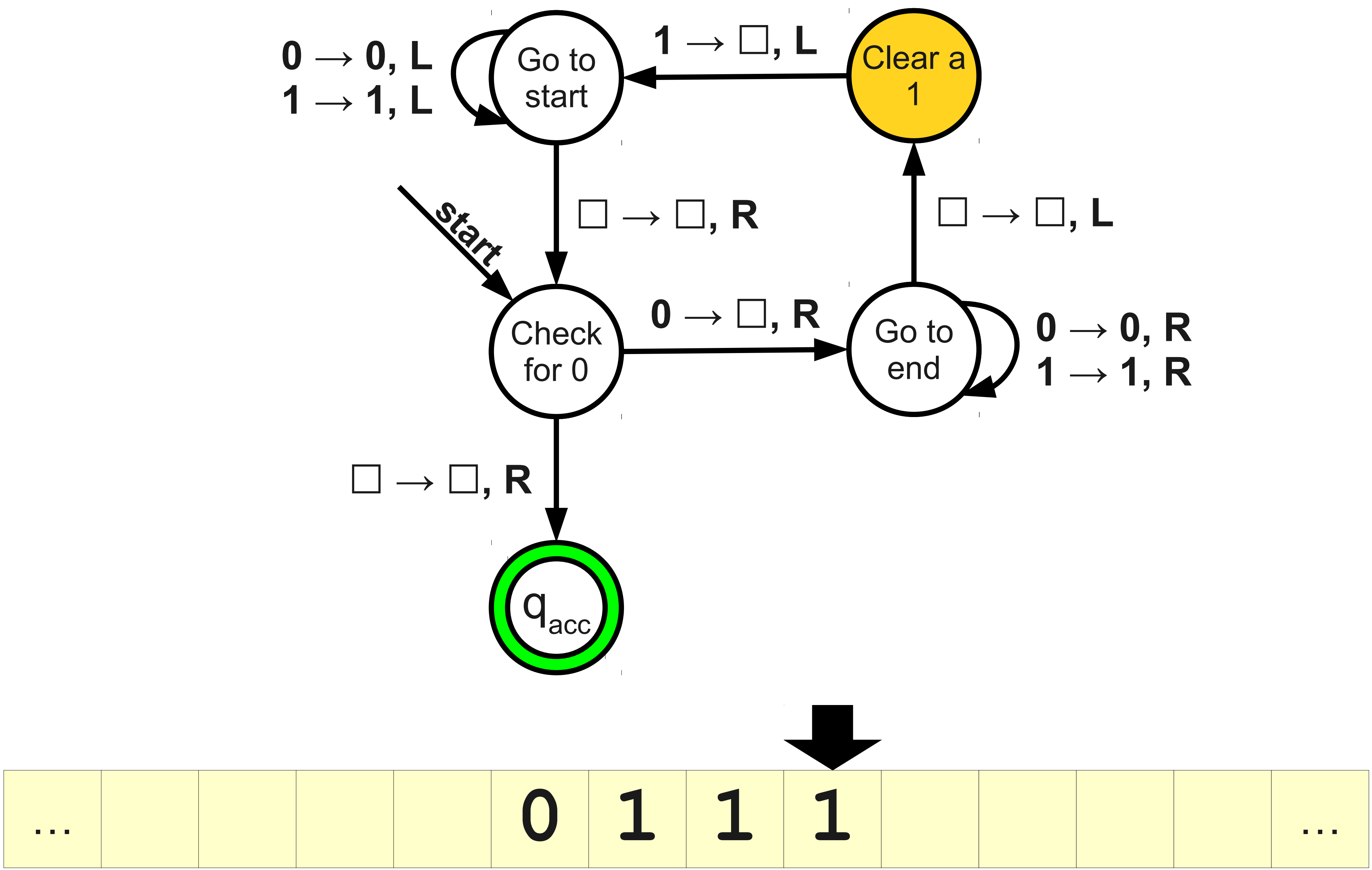
0
1
1
1
…



Go to start
Clear a 1
Check for 0
Go to end
q_{acc}
start
$0 \to 0, L$
$1 \to 1, L$
$1 \to \square, L$
$\square \to \square, R$
$\square \to \square, L$
$0 \to \square, R$
$0 \to 0, R$
$1 \to 1, R$
$\square \to \square, R$
… 0 1 1 1 …



0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
start
□ → □, R
□ → □, L
0 → □, R
Check for 0
Go to end
0 → 0, R
1 → 1, R
□ → □, R
q_{acc}
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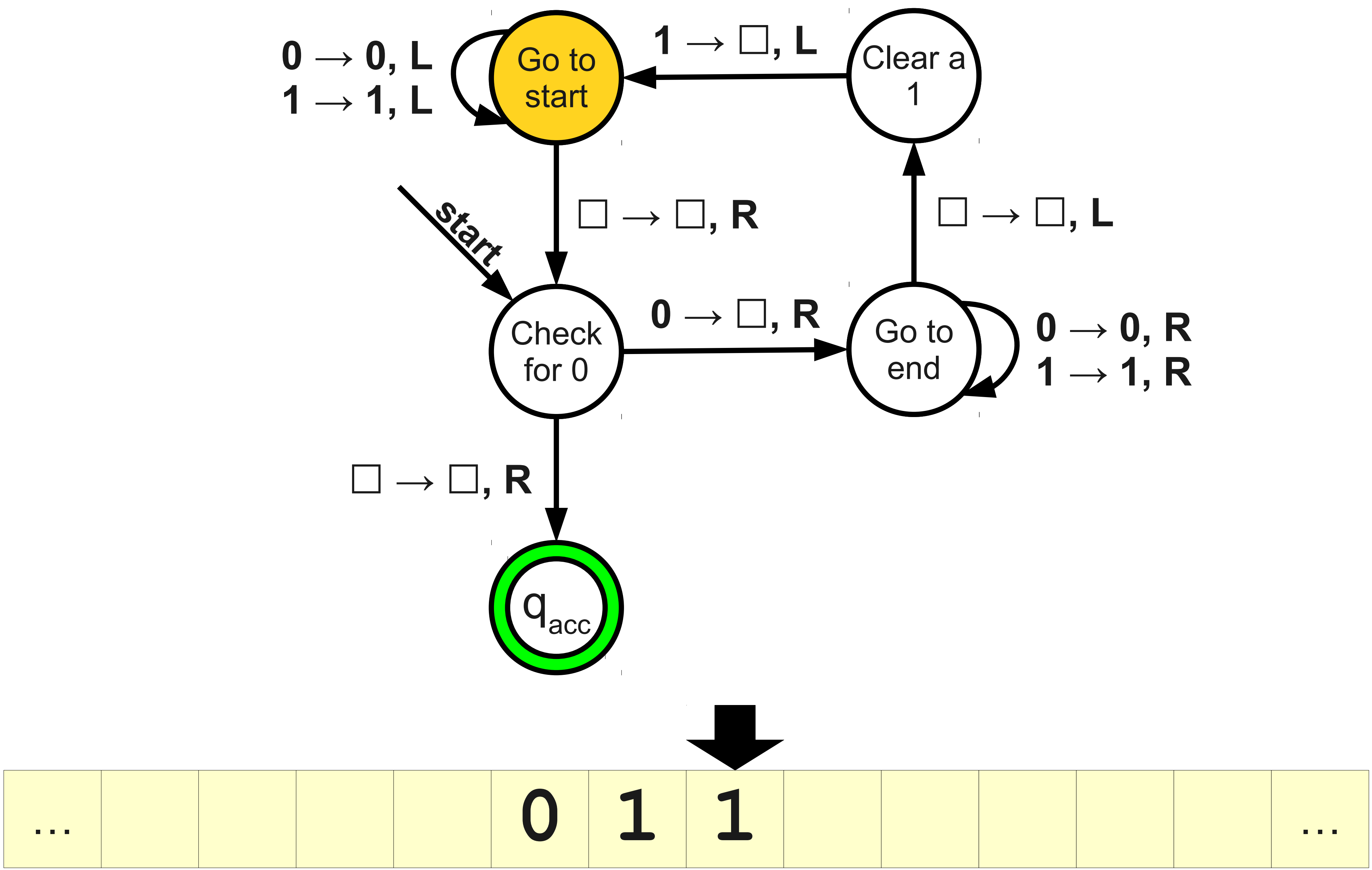
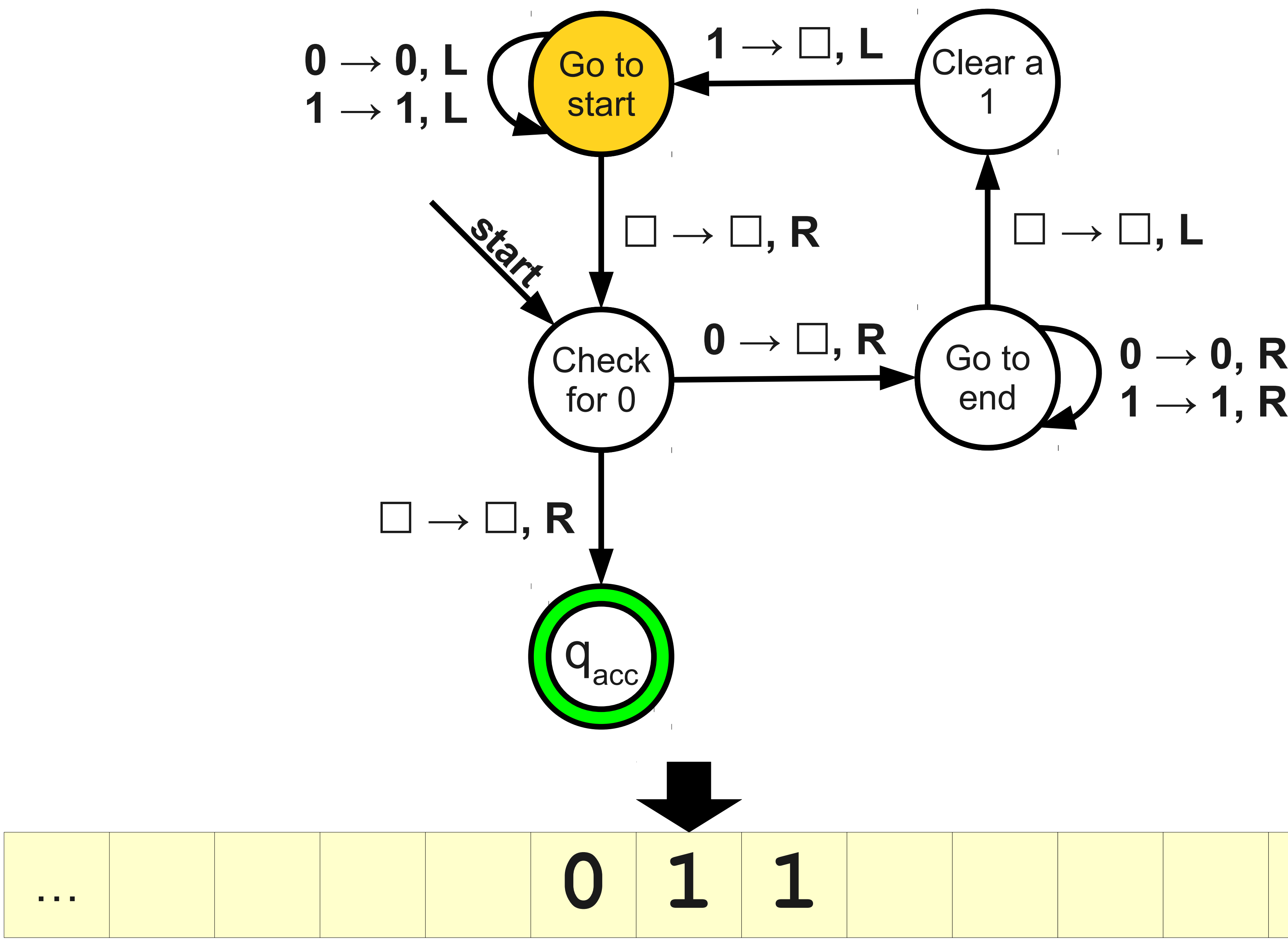
0
1
1
1
…



Go to start
Clear a 1
Check for 0
Go to end
q_{acc}
start
0 → 0, L
1 → 1, L
1 → □, L
□ → □, R
□ → □, L
0 → □, R
0 → 0, R
1 → 1, R
□ → □, R
…
0
1
1
…


0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
start
□ → □, R
□ → □, L
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
□ → □, R
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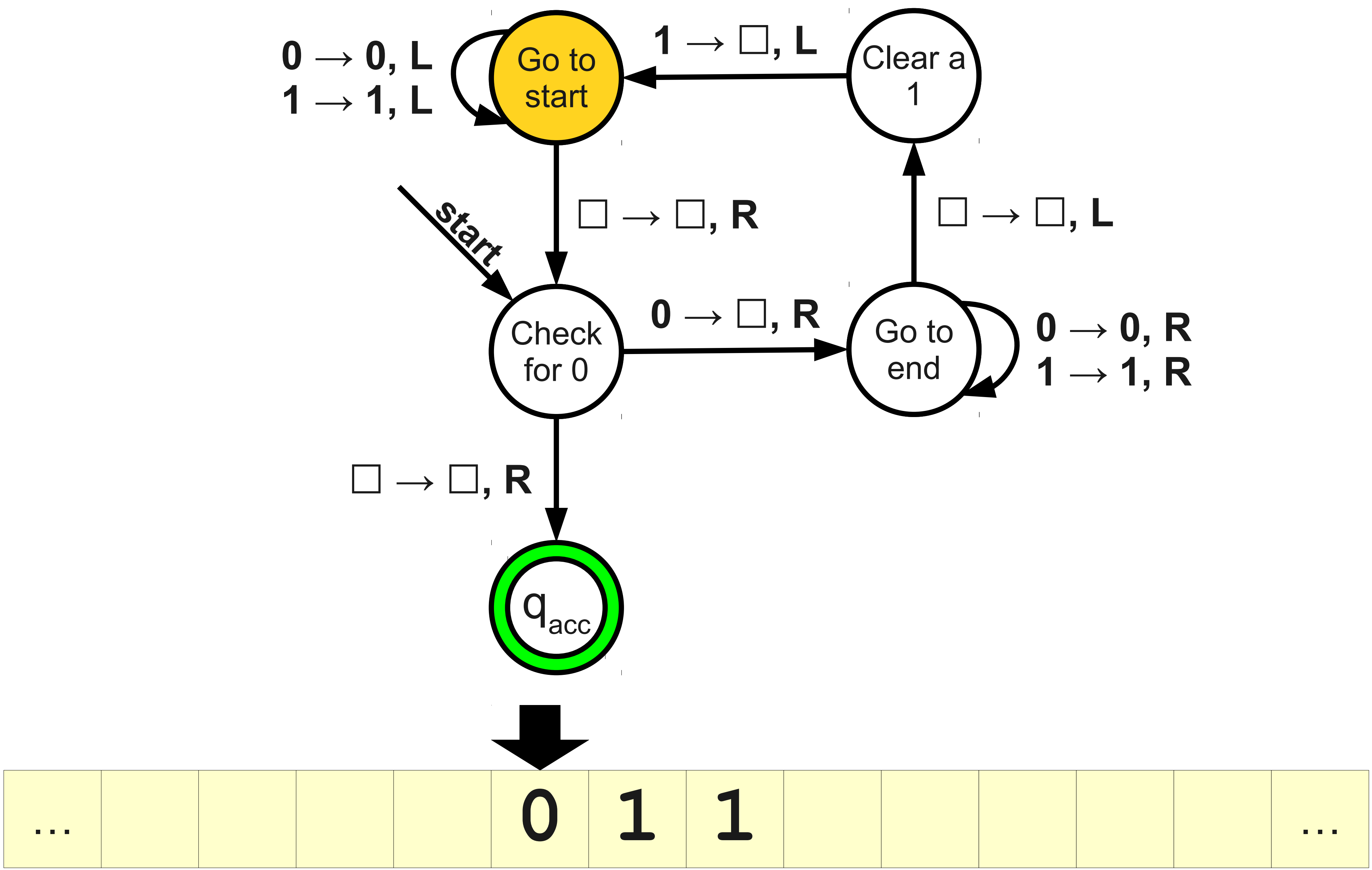
…
0
1
1
…



0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
start
□ → □, R
□ → □, L
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
□ → □, R
q_acc
…
0
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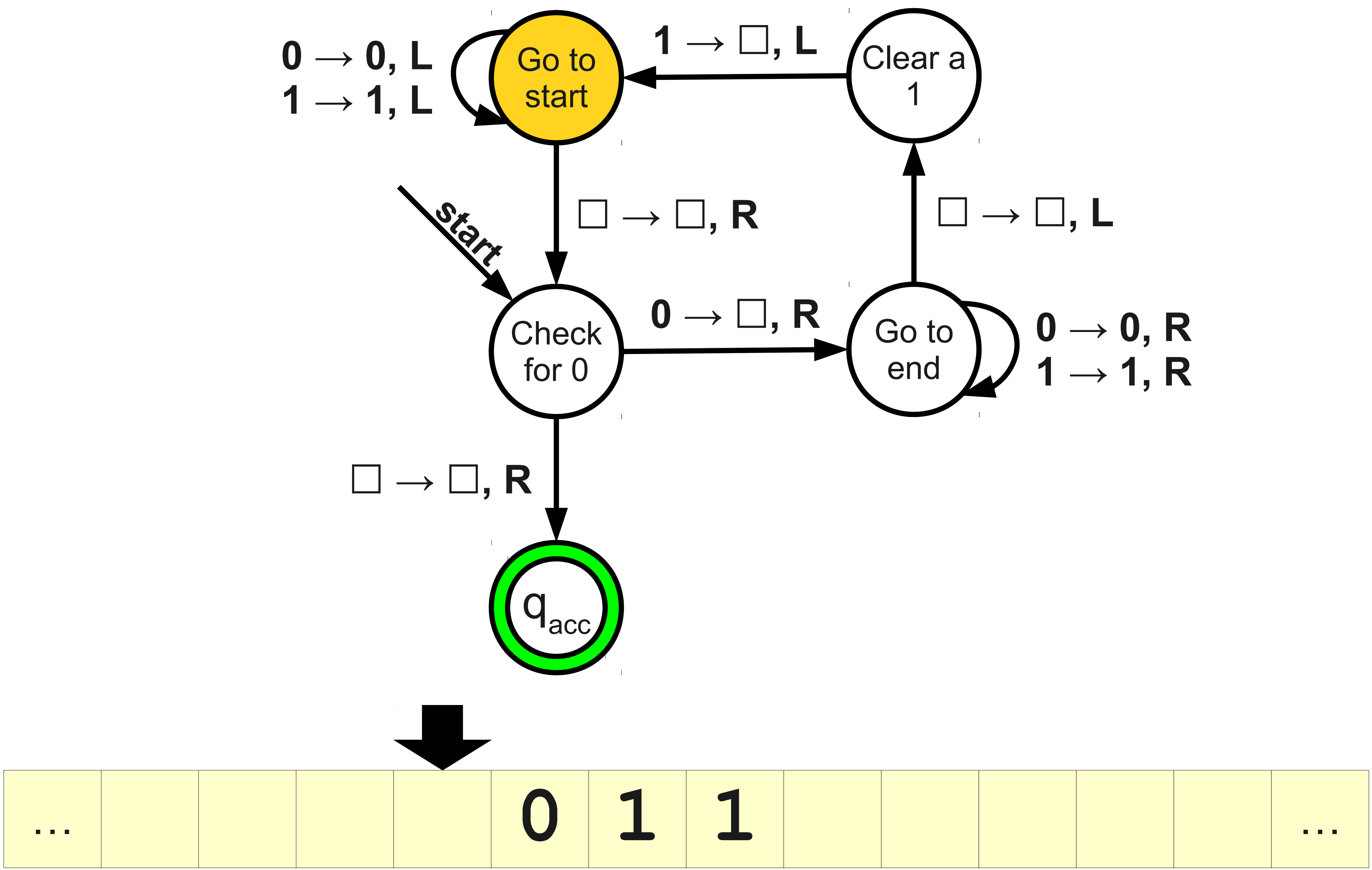
1
…



0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
start
□ → □, R
□ → □, L
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
□ → □, R
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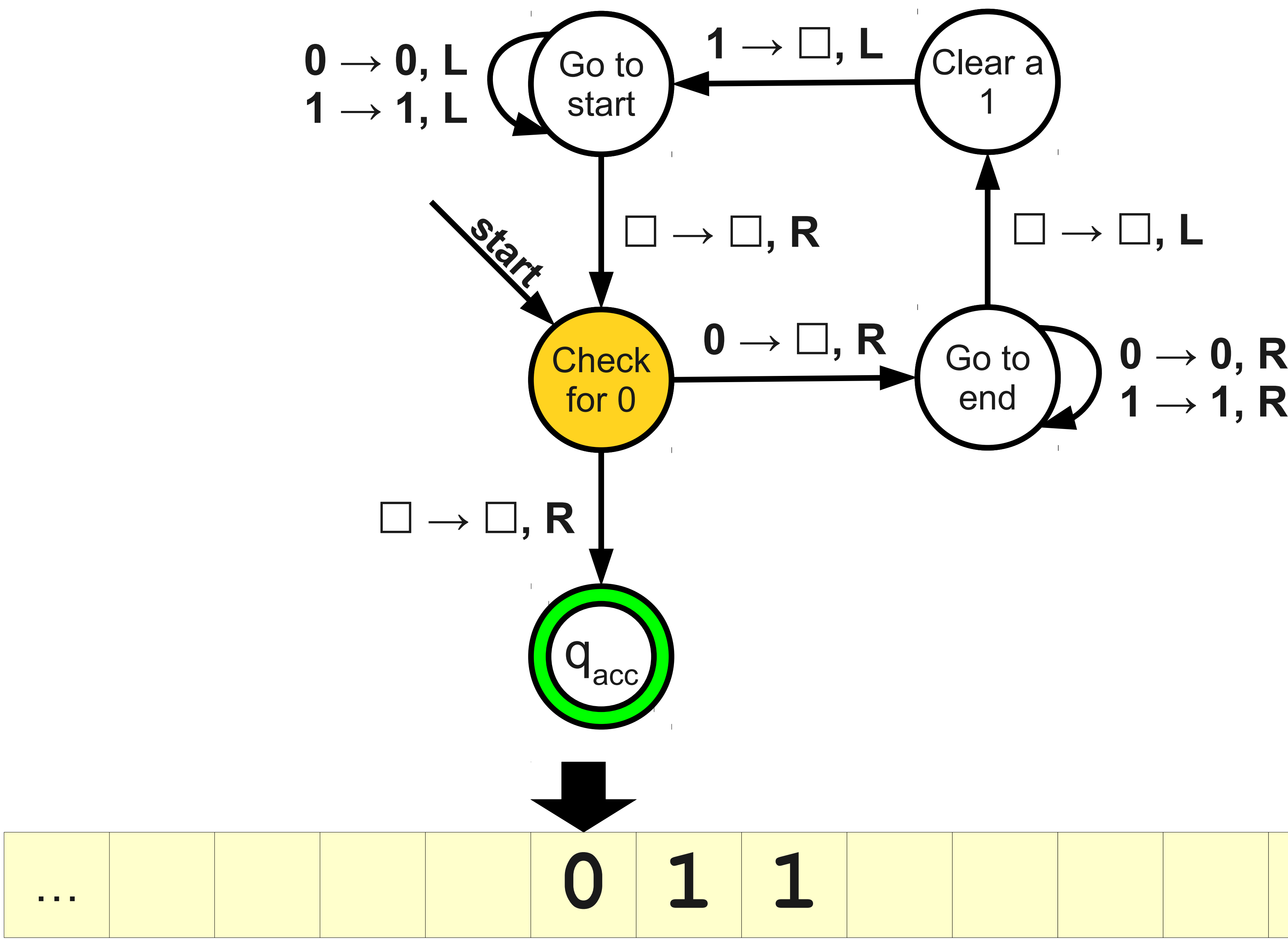
…
0
1
1
…



0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
start
□ → □, R
□ → □, L
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
□ → □, R
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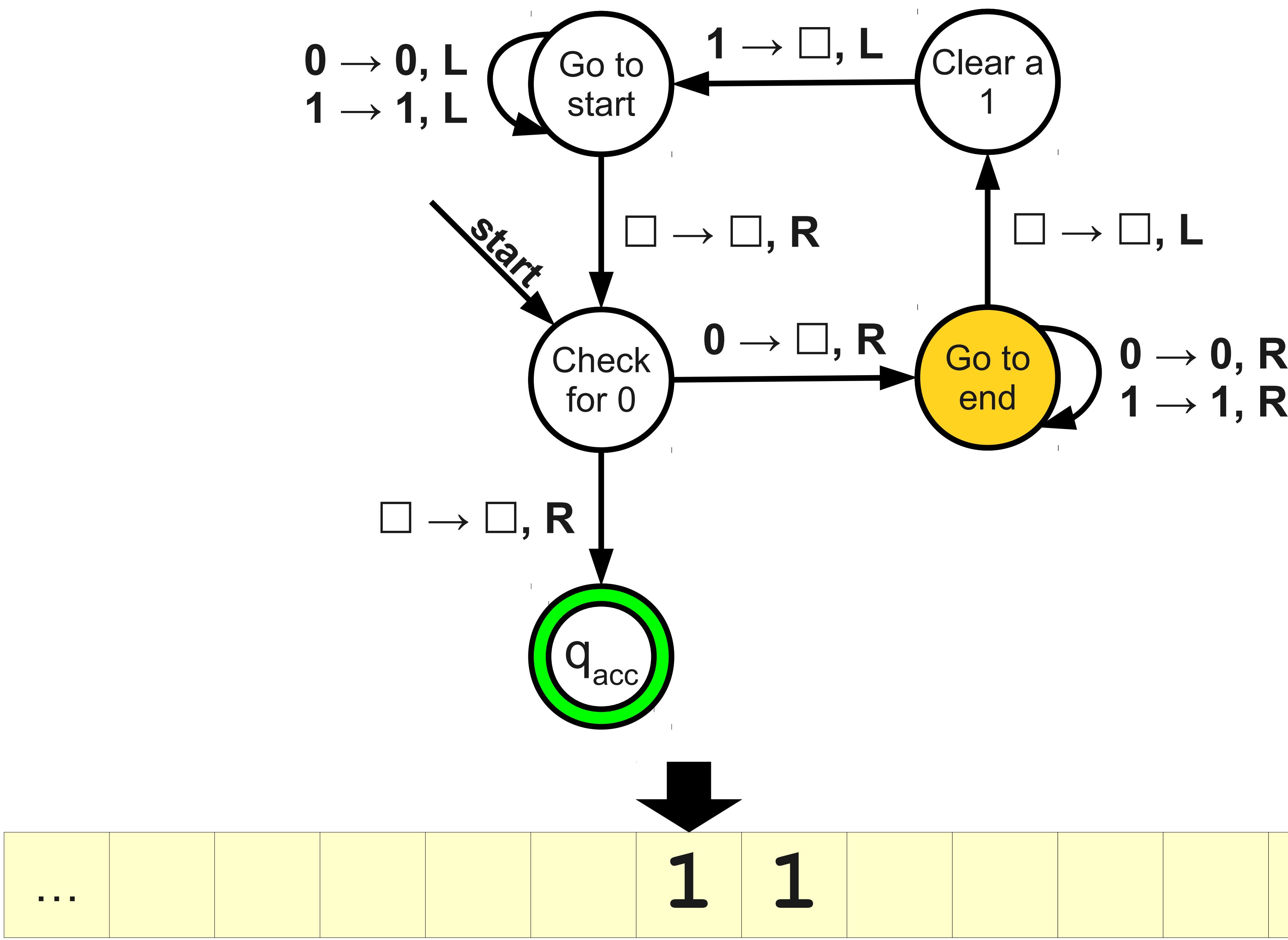
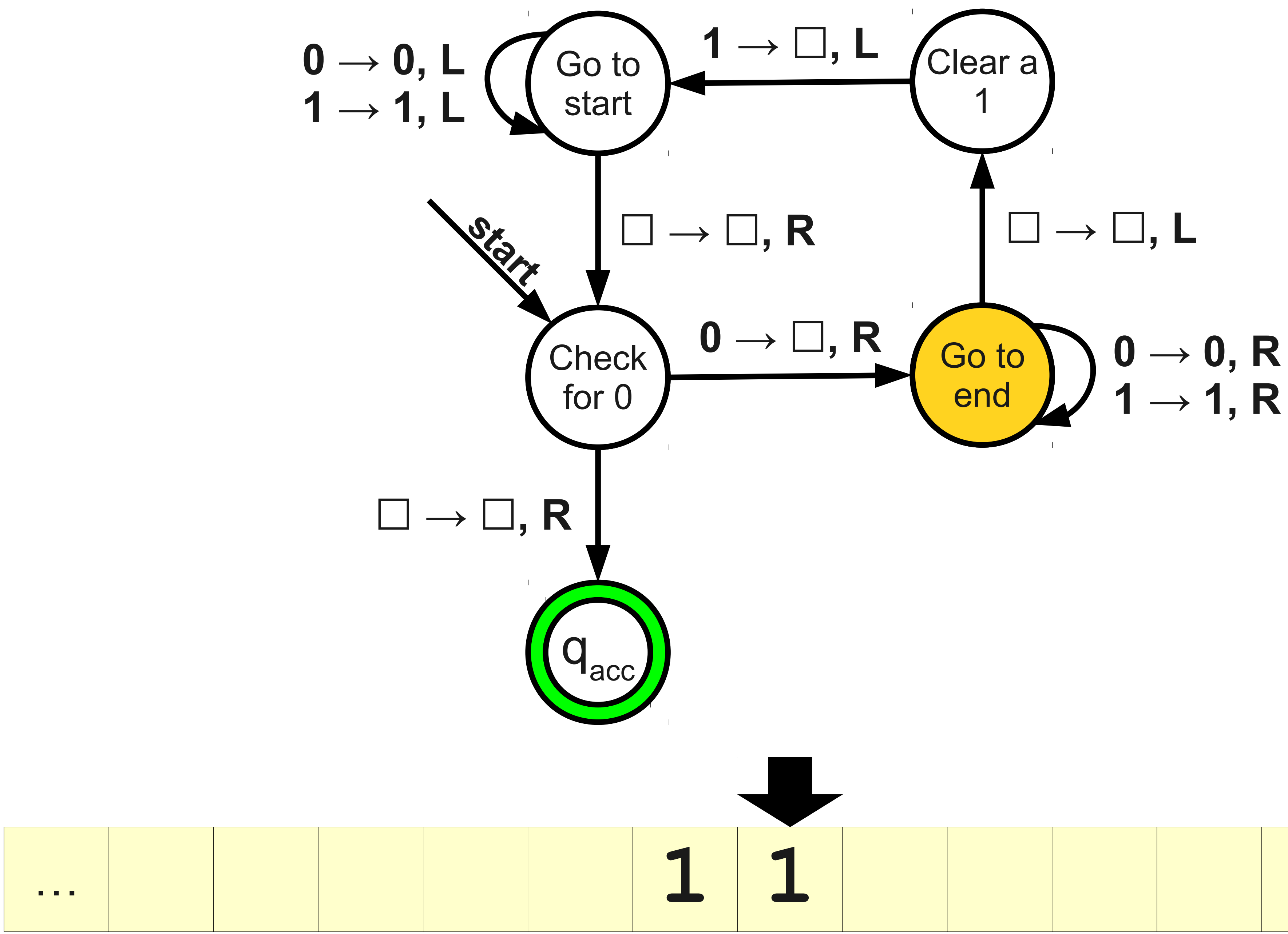
…
0
1
1
…


0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
start
□ → □, R
□ → □, L
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
□ → □, R
q_acc
…
1
1
…



0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
start
□ → □, R
□ → □, L
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
□ → □, R
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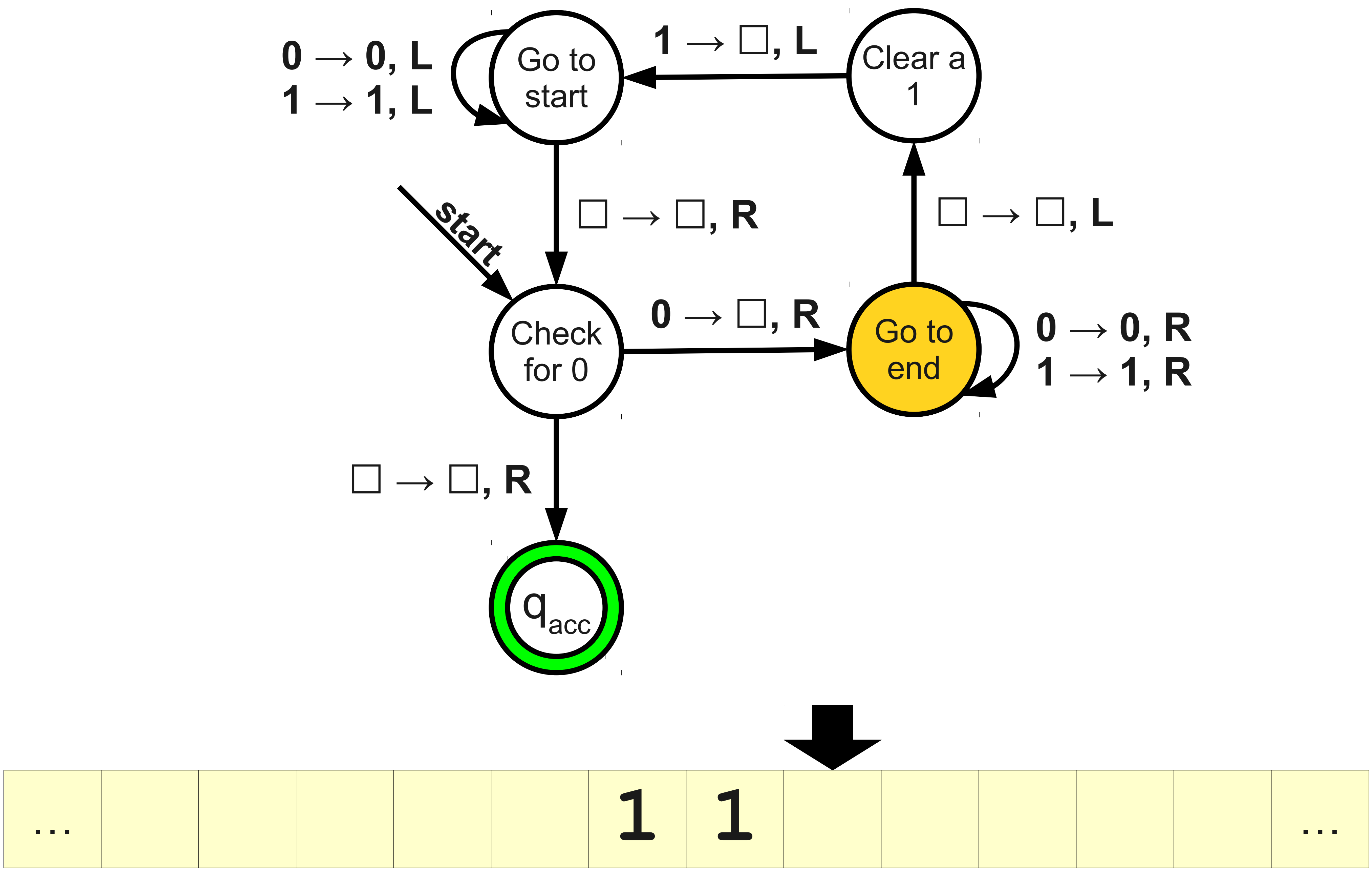
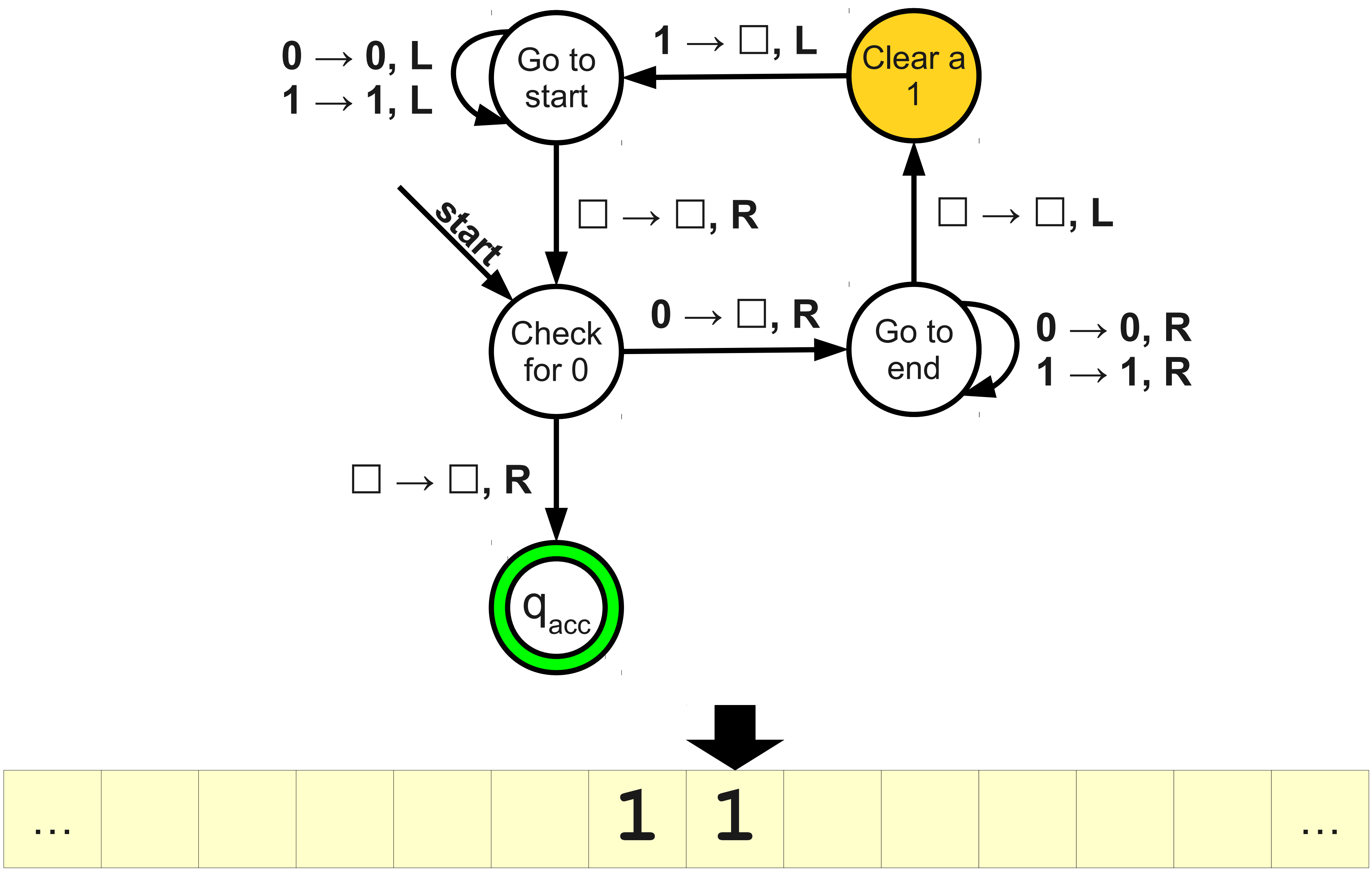
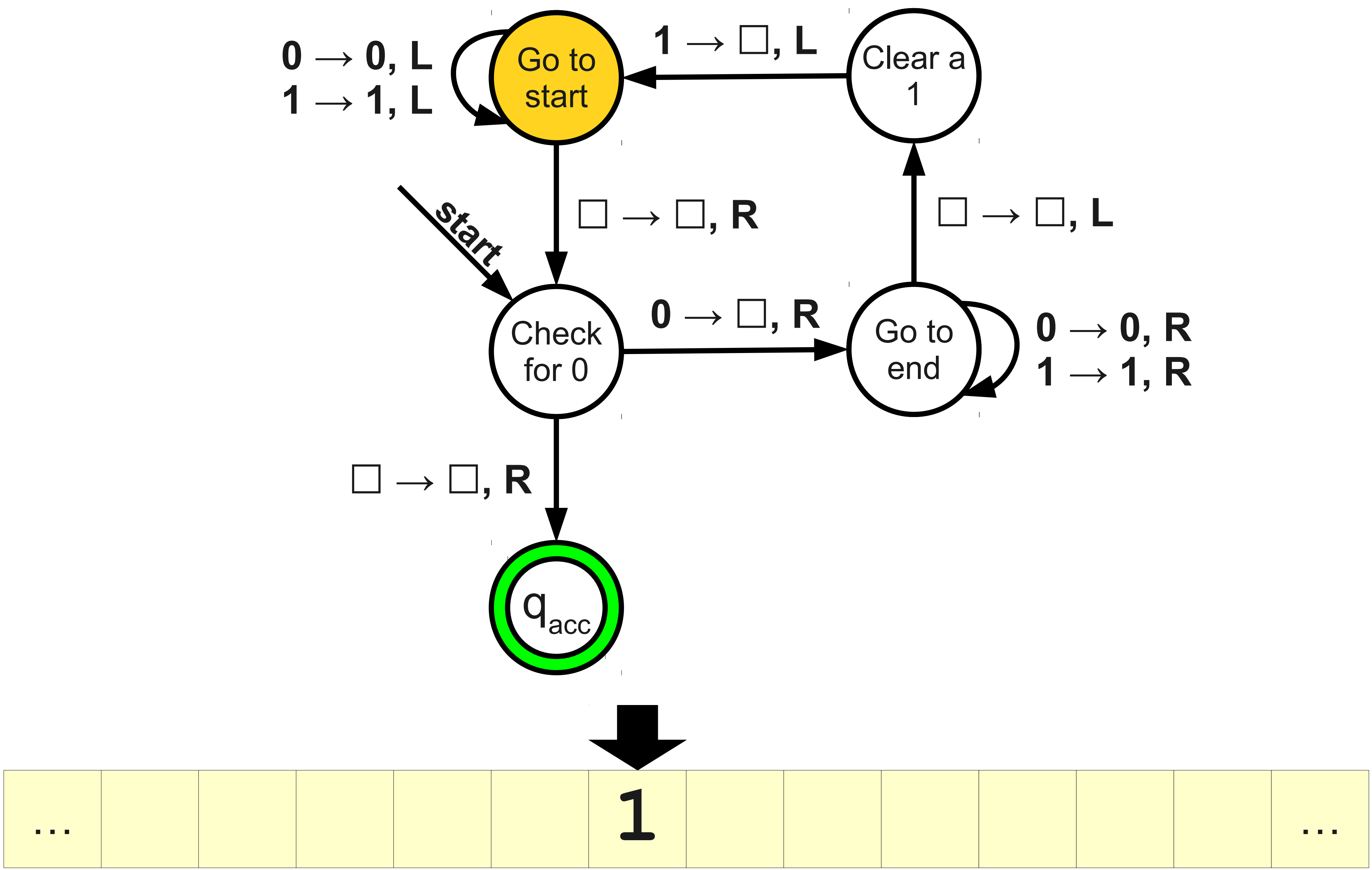
…
1
1
…

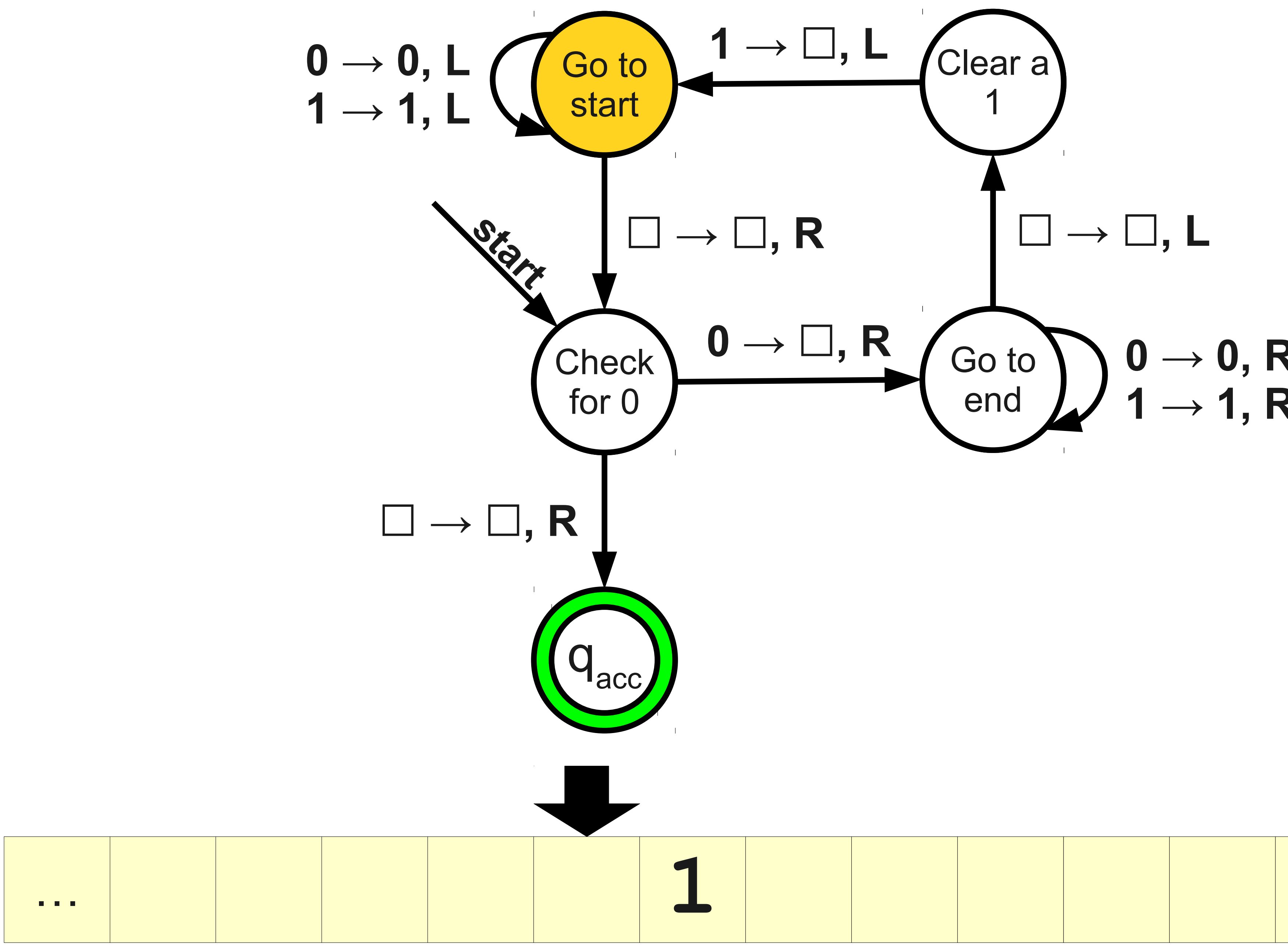


Go to start
Clear a 1
Check for 0
Go to end
q_{acc}
start
0 → 0, L
1 → 1, L
1 → □, L
□ → □, R
□ → □, L
0 → □, R
0 → 0, R
1 → 1, R
□ → □, R
…
1
1
…

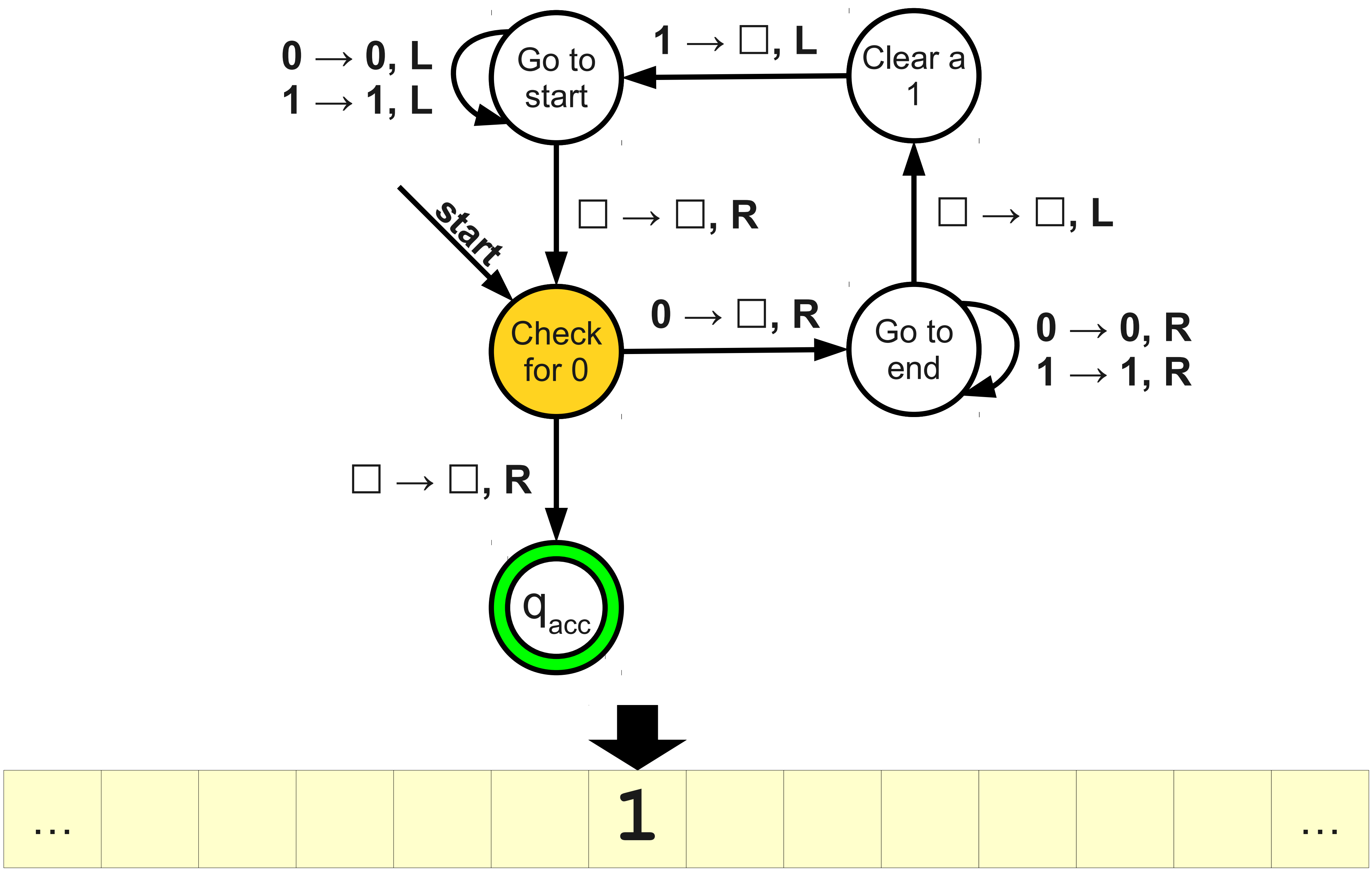


0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
start
□ → □, R
□ → □, L
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
□ → □, R
q_{acc}
…
1
1
…

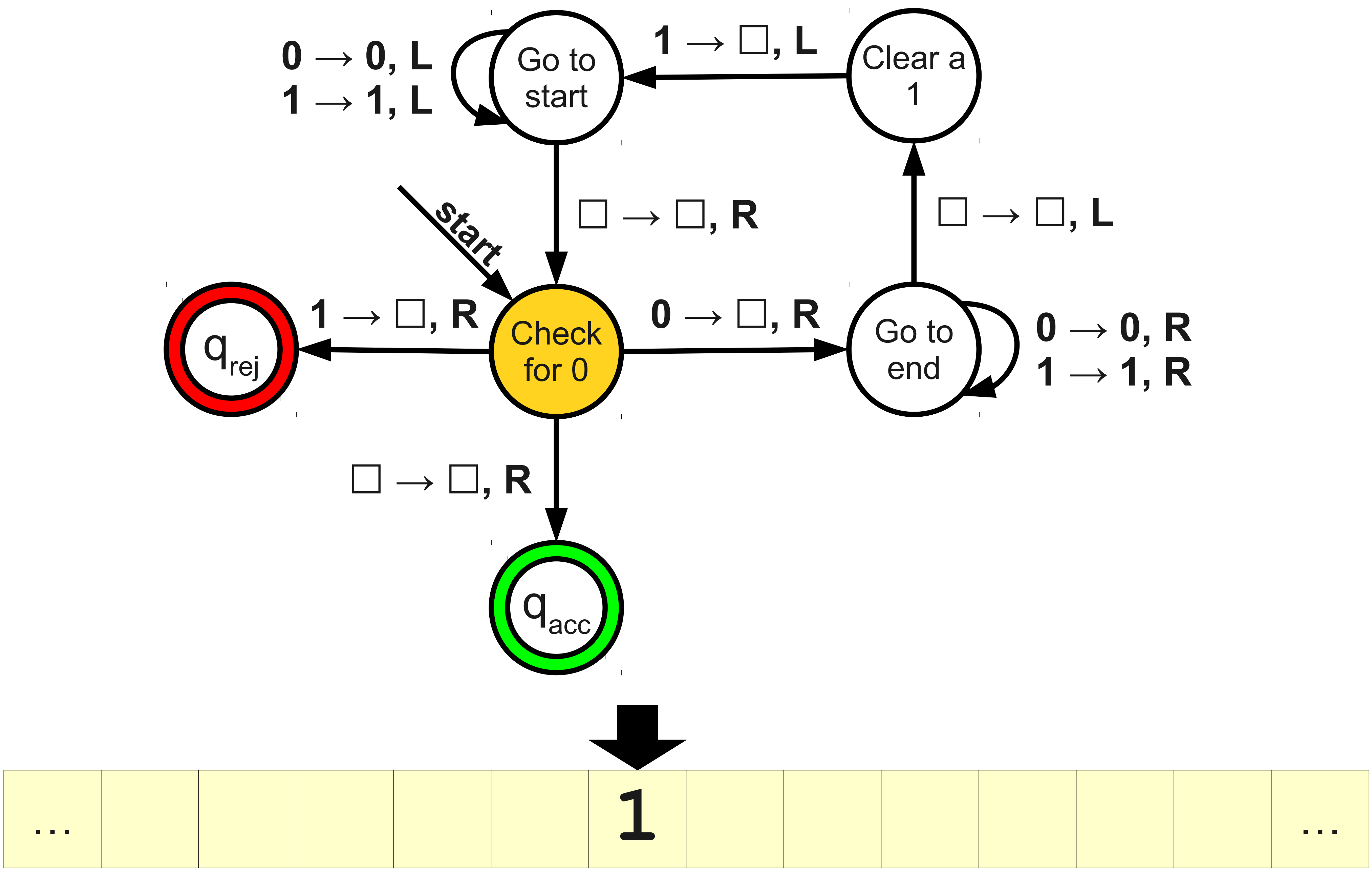

0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
start
□ → □, R
□ → □, L
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
□ → □, R
q_{acc}
…
1
…



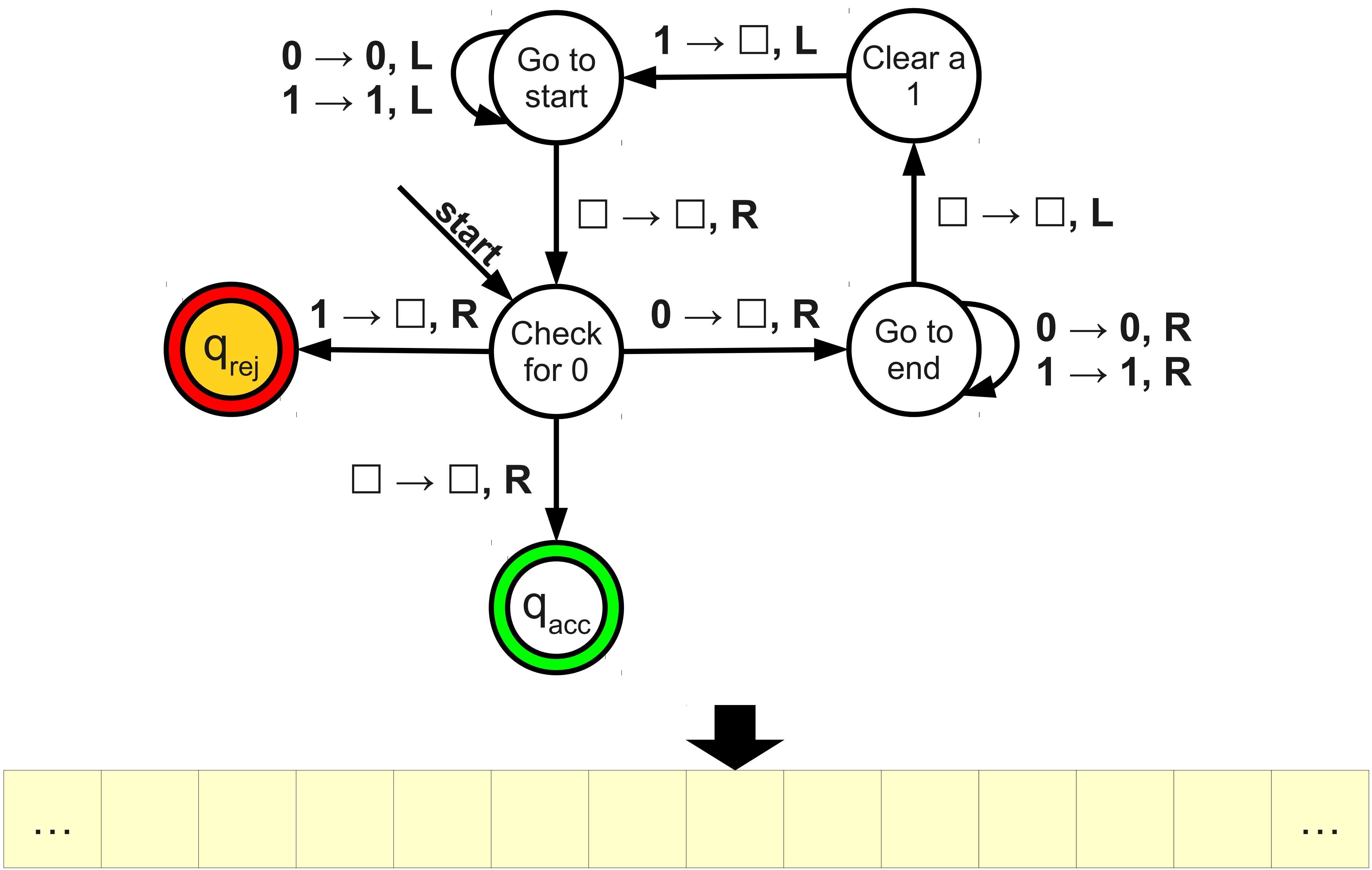
0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
start
□ → □, R
□ → □, L
0 → □, R
Check for 0
Go to end
0 → 0, R
1 → 1, R
□ → □, R
q_acc
…
1
…

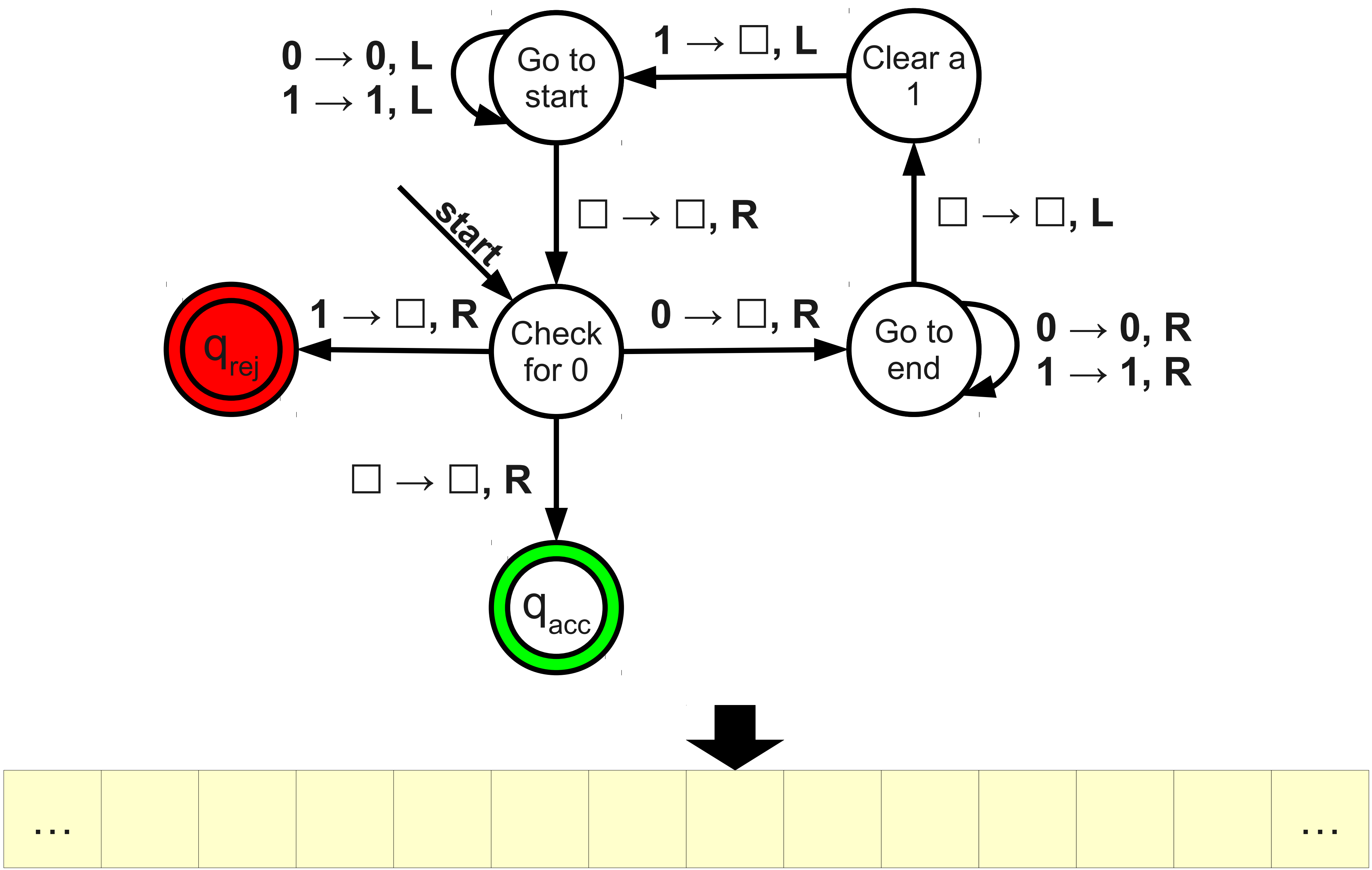


Go to start
Clear a 1
Check for 0
Go to end
q_{acc}
start
$0 \to 0, L$
$1 \to 1, L$
$1 \to \square, L$
$\square \to \square, R$
$\square \to \square, L$
$0 \to \square, R$
$0 \to 0, R$
$1 \to 1, R$
$\square \to \square, R$
…
1
…



0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
□ → □, R
□ → □, L
start
1 → □, R
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
q_{rej}
□ → □, R
q_{acc}
…
1
…


0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
start
□ → □, R
□ → □, L
1 → □, R
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
q_rej
□ → □, R
q_acc
...
...


0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
start
□ → □, R
□ → □, L
1 → □, R
q_{rej}
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
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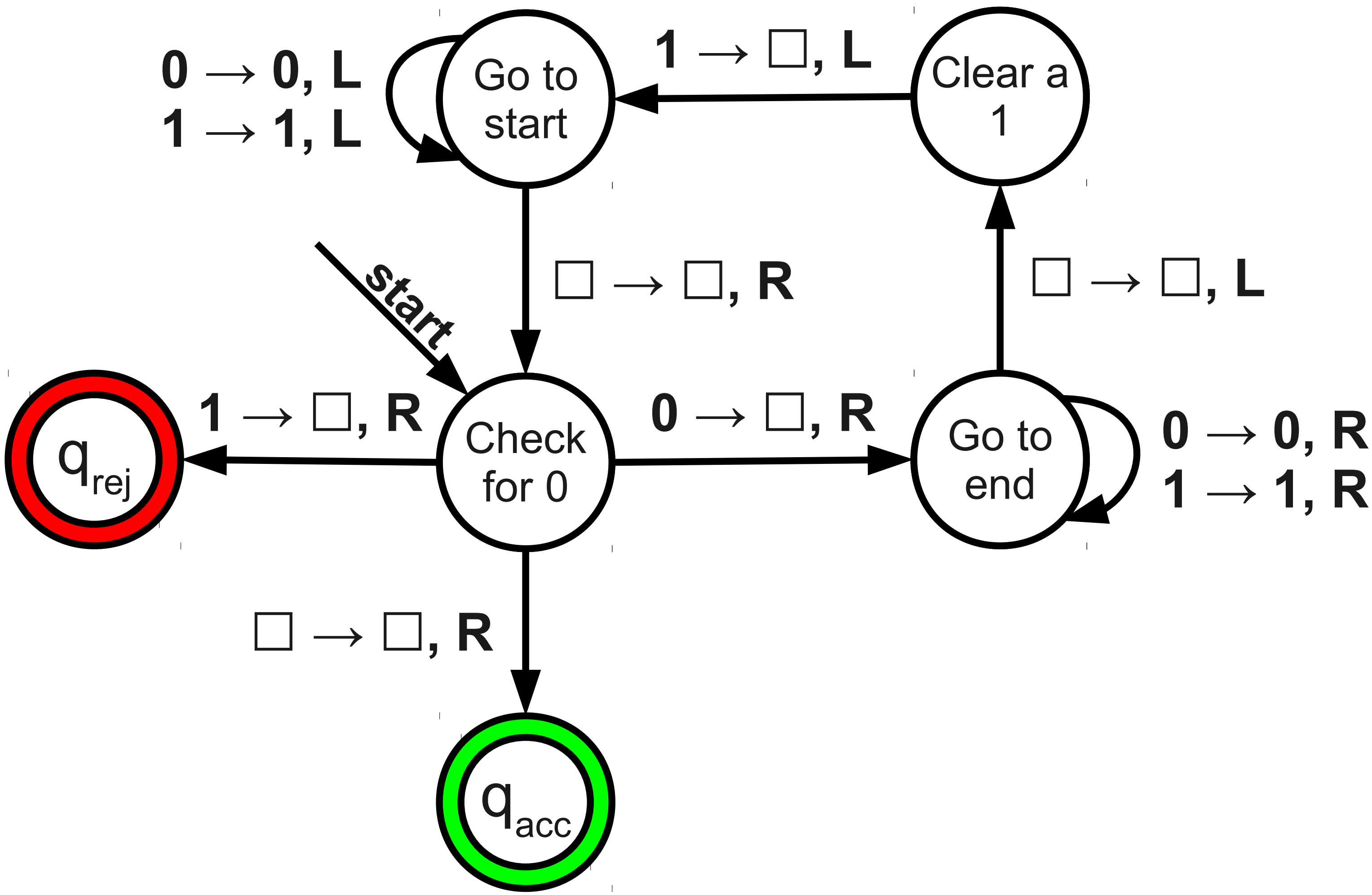
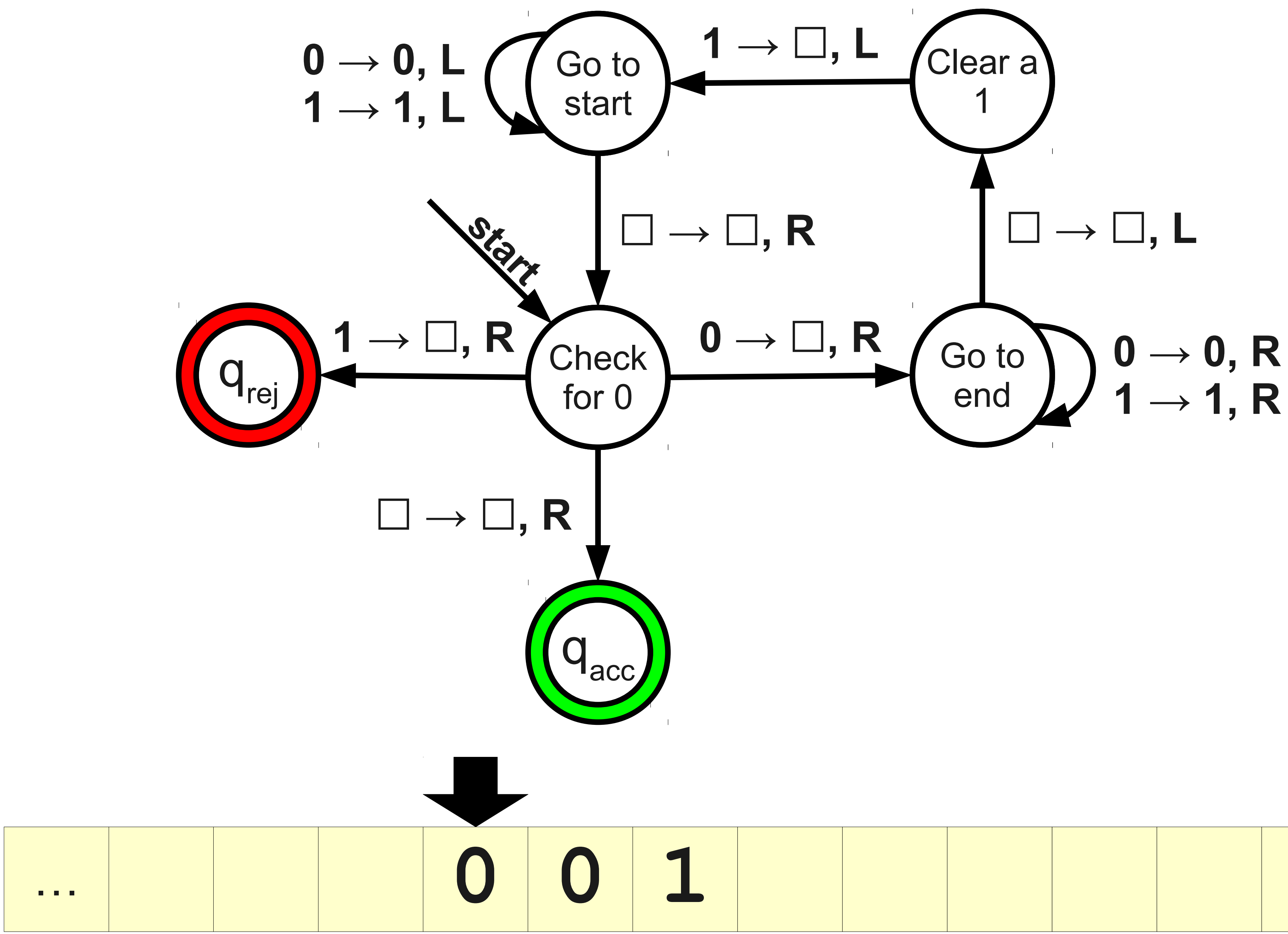
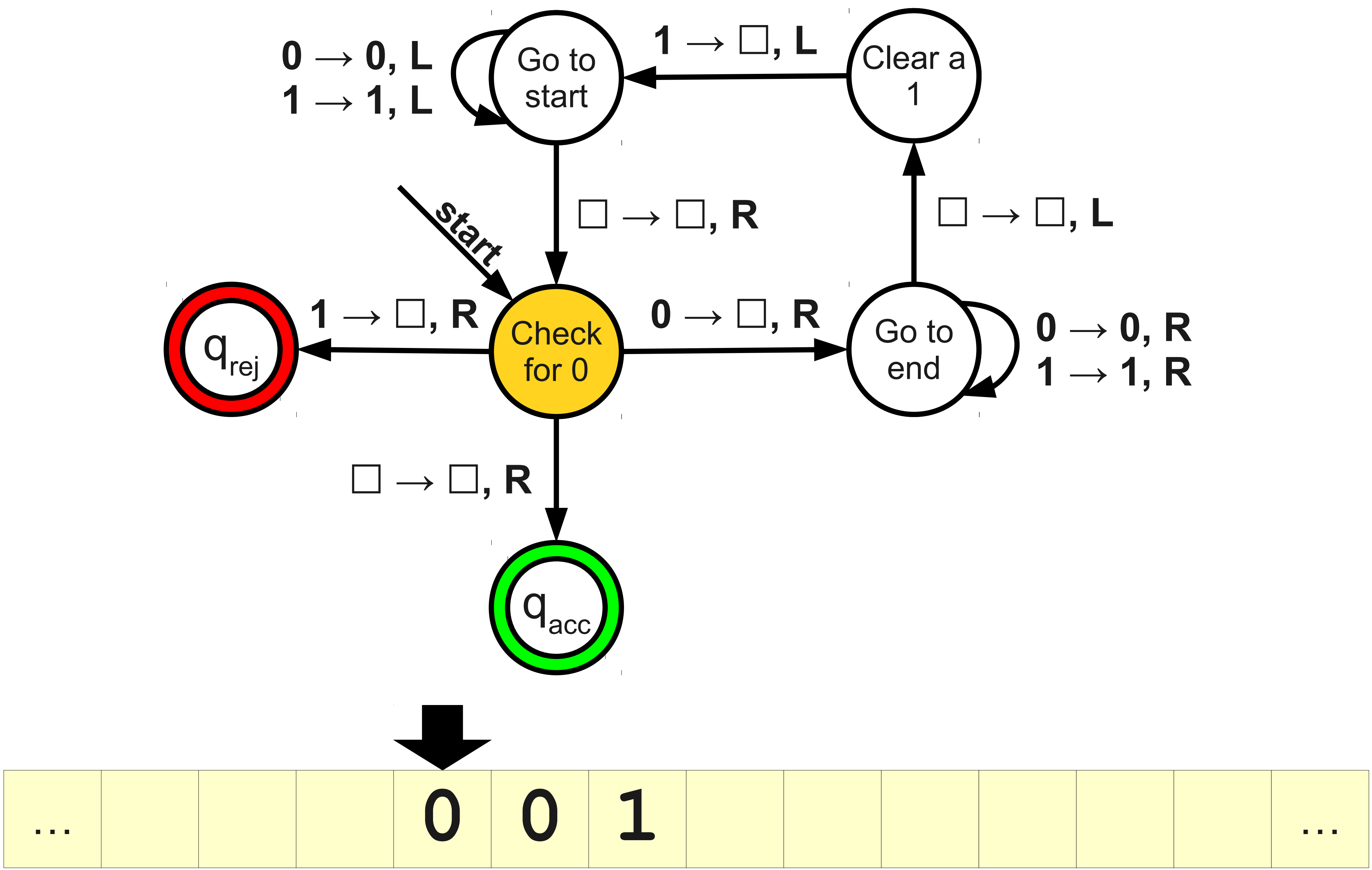
q_{acc}
…
…


Go to start
Clear a 1
Check for 0
Go to end
q_{rej}
q_{acc}
start
0 → 0, L
1 → 1, L
1 → ☐, L
☐ → ☐, R
☐ → ☐, L
1 → ☐, R
0 → ☐, R
0 → 0, R
1 → 1, R
☐ → ☐, R



0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
□ → □, R
□ → □, L
start
1 → □, R
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
q_rej
□ → □, R
q_acc
...
0
0
1
...


Go to start
Clear a 1
Check for 0
Go to end
q_rej
q_acc
start
0 → 0, L
1 → 1, L
1 → □, L
□ → □, R
□ → □, L
1 → □, R
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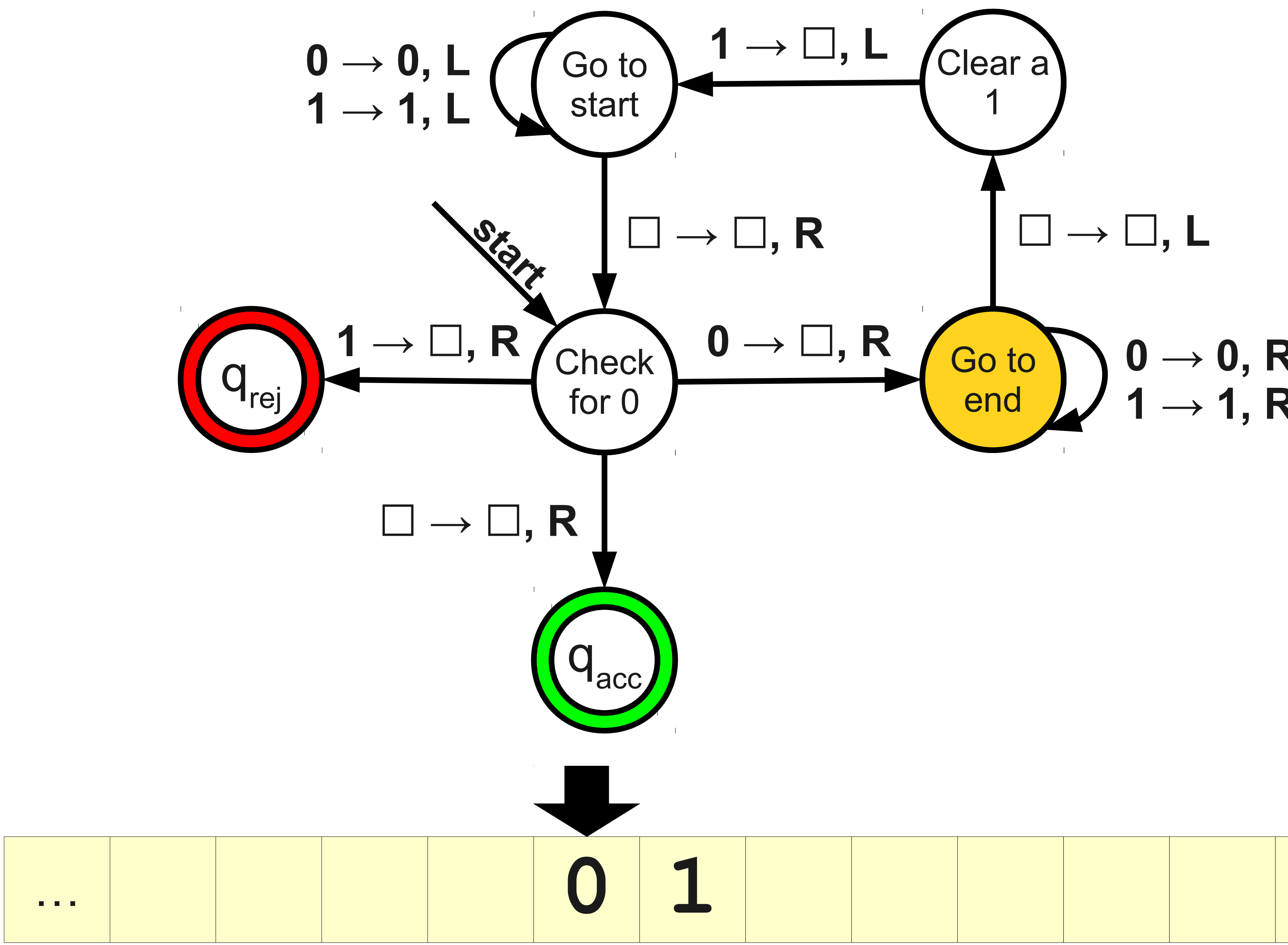
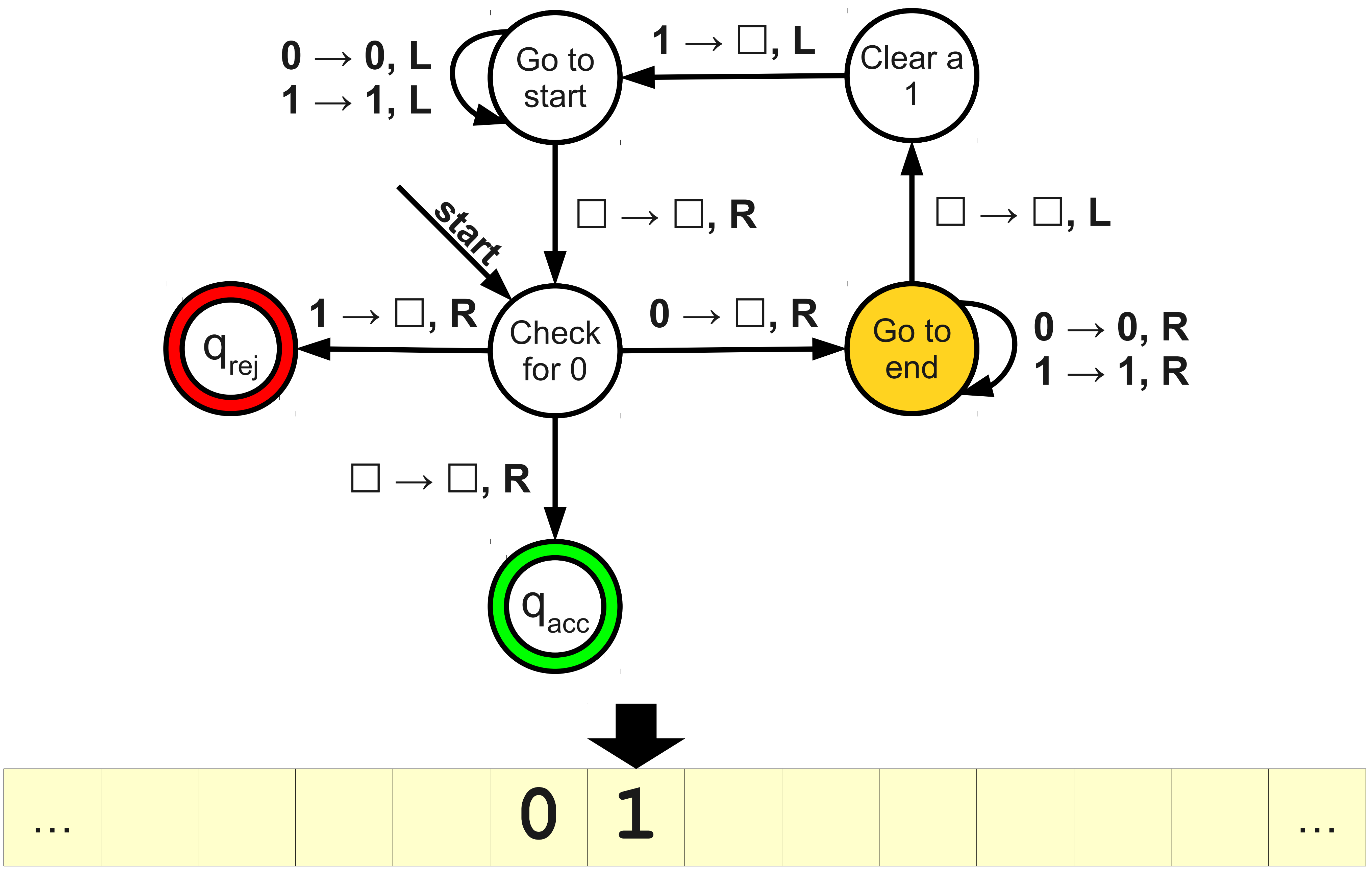
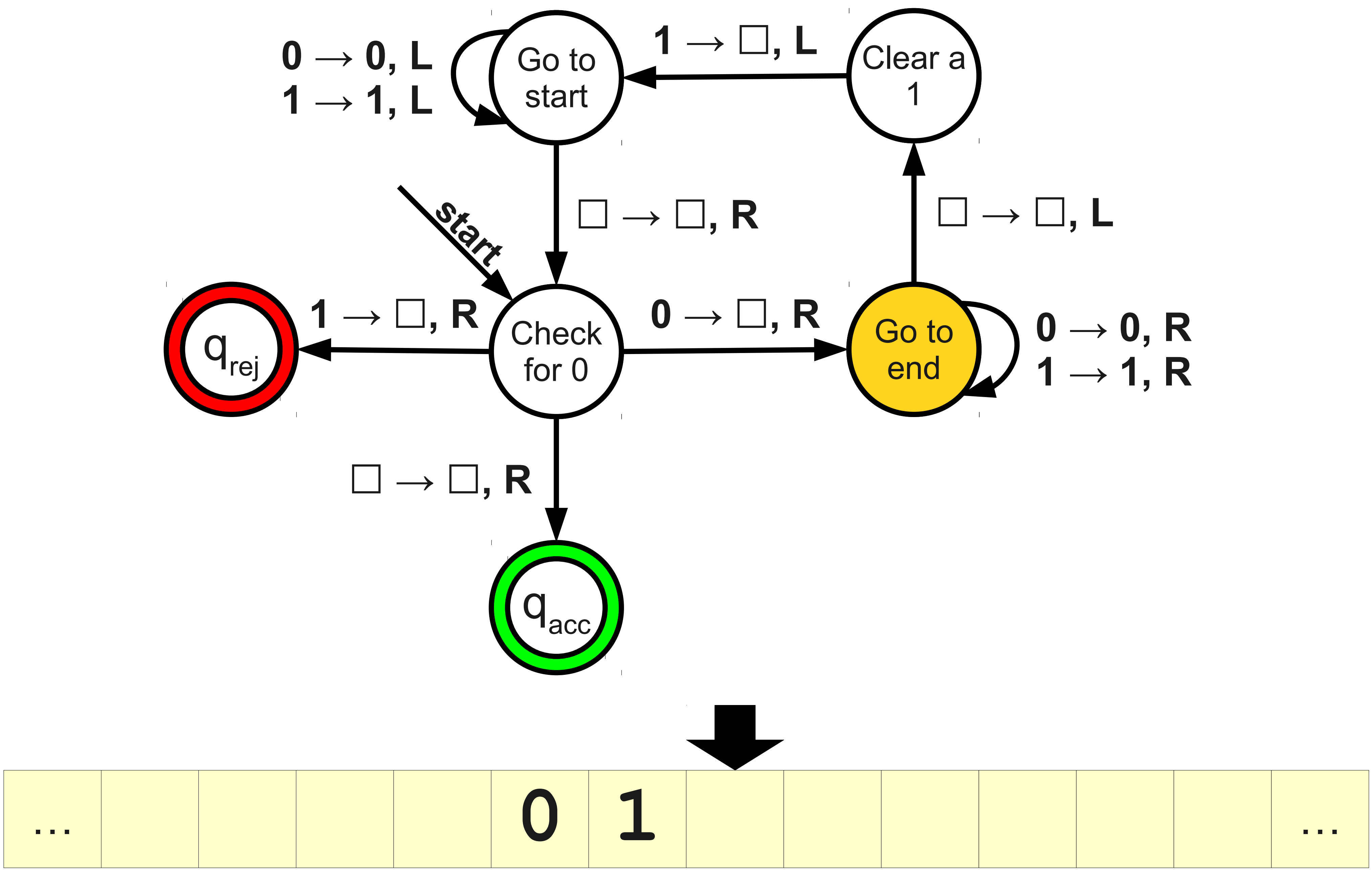
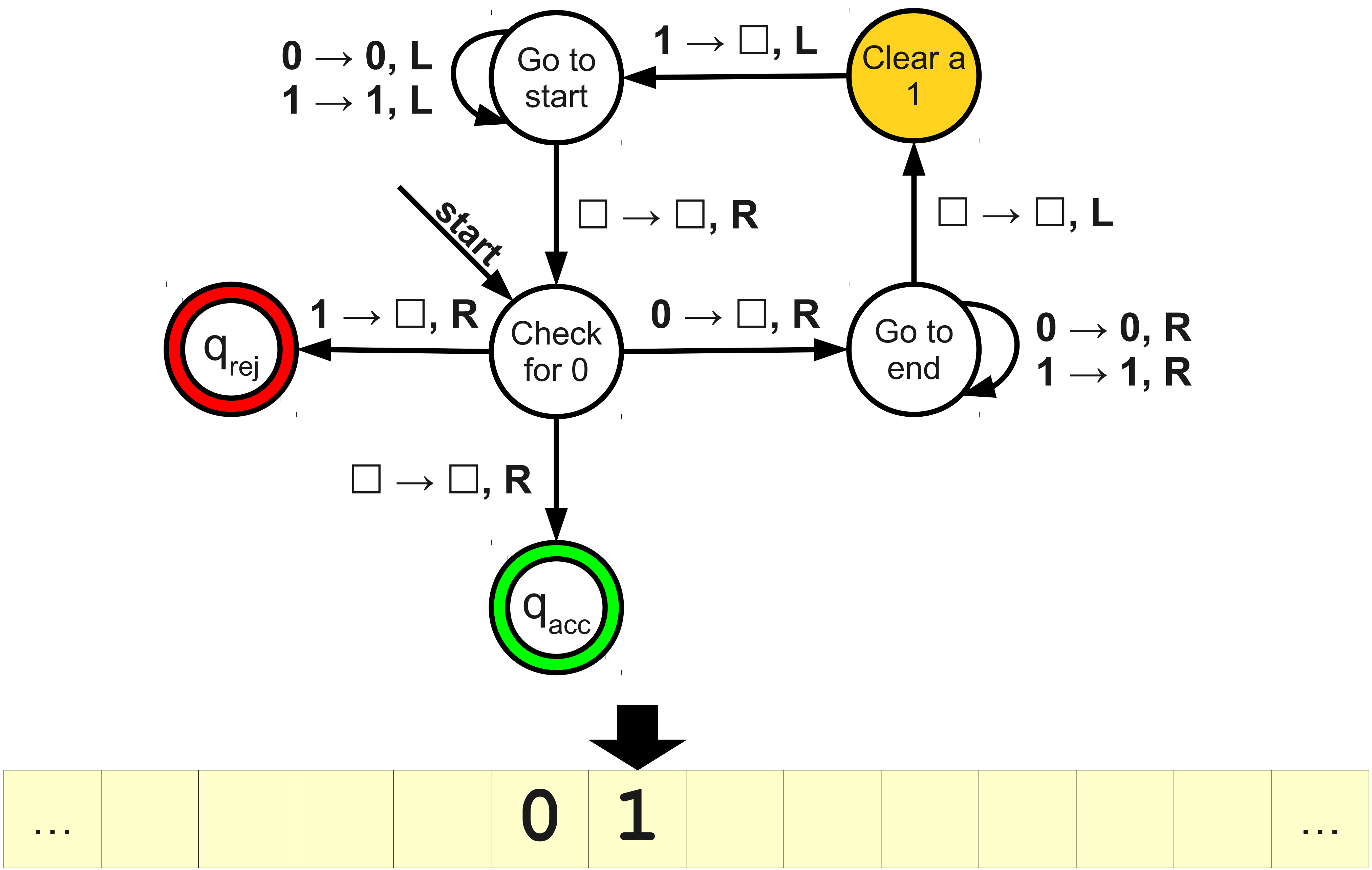
0 → 0, R
1 → 1, R
□ → □, R
…
0
0
1
…


0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
□ → □, R
□ → □, L
start
1 → □, R
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
q_{rej}
□ → □, R
q_{acc}
…
0
1
…

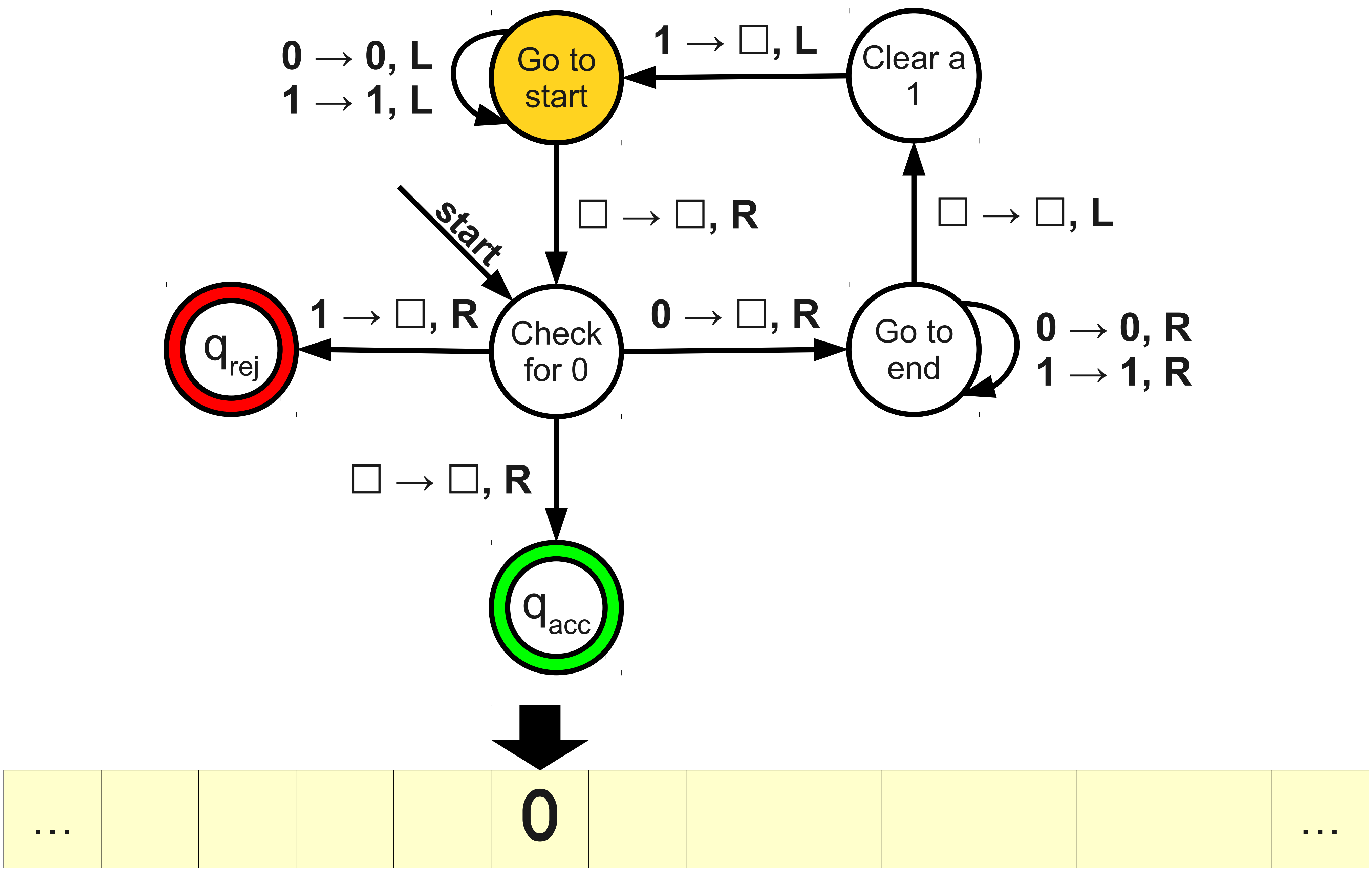


0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
start
□ → □, R
□ → □, L
1 → □, R
q_rej
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
□ → □, R
q_acc
…
0
1
…


0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
start
□ → □, R
□ → □, L
1 → □, R
q_rej
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
□ → □, R
q_acc
…
0
1
…



0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
□ → □, R
□ → □, L
start
1 → □, R
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
q_{rej}
□ → □, R
q_{acc}
…
0
1
…



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1 → 1, L
Go to start
1 → □, L
Clear a 1
start
□ → □, R
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1 → □, R
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Go to end
0 → 0, R
1 → 1, R
□ → □, R
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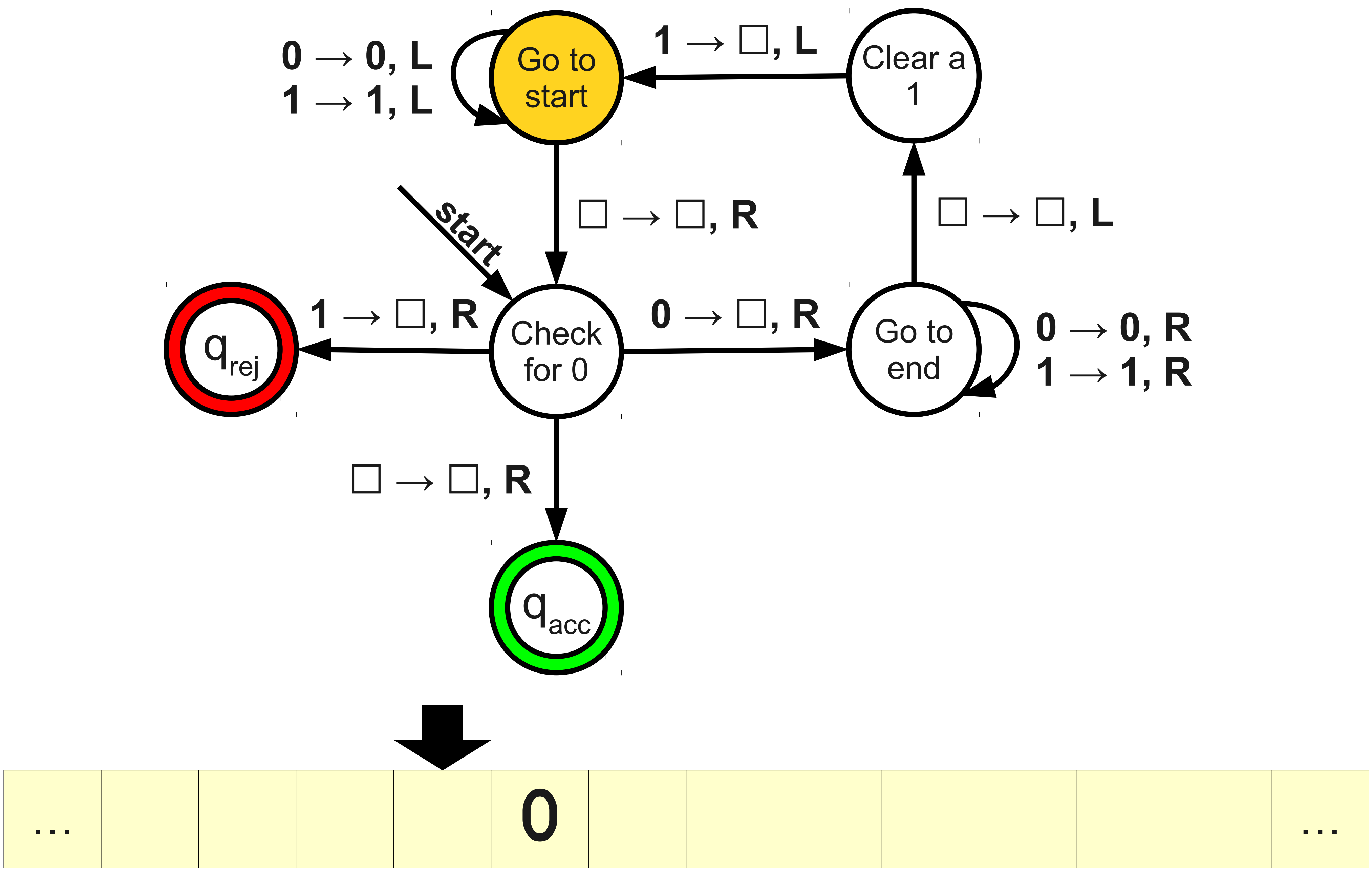

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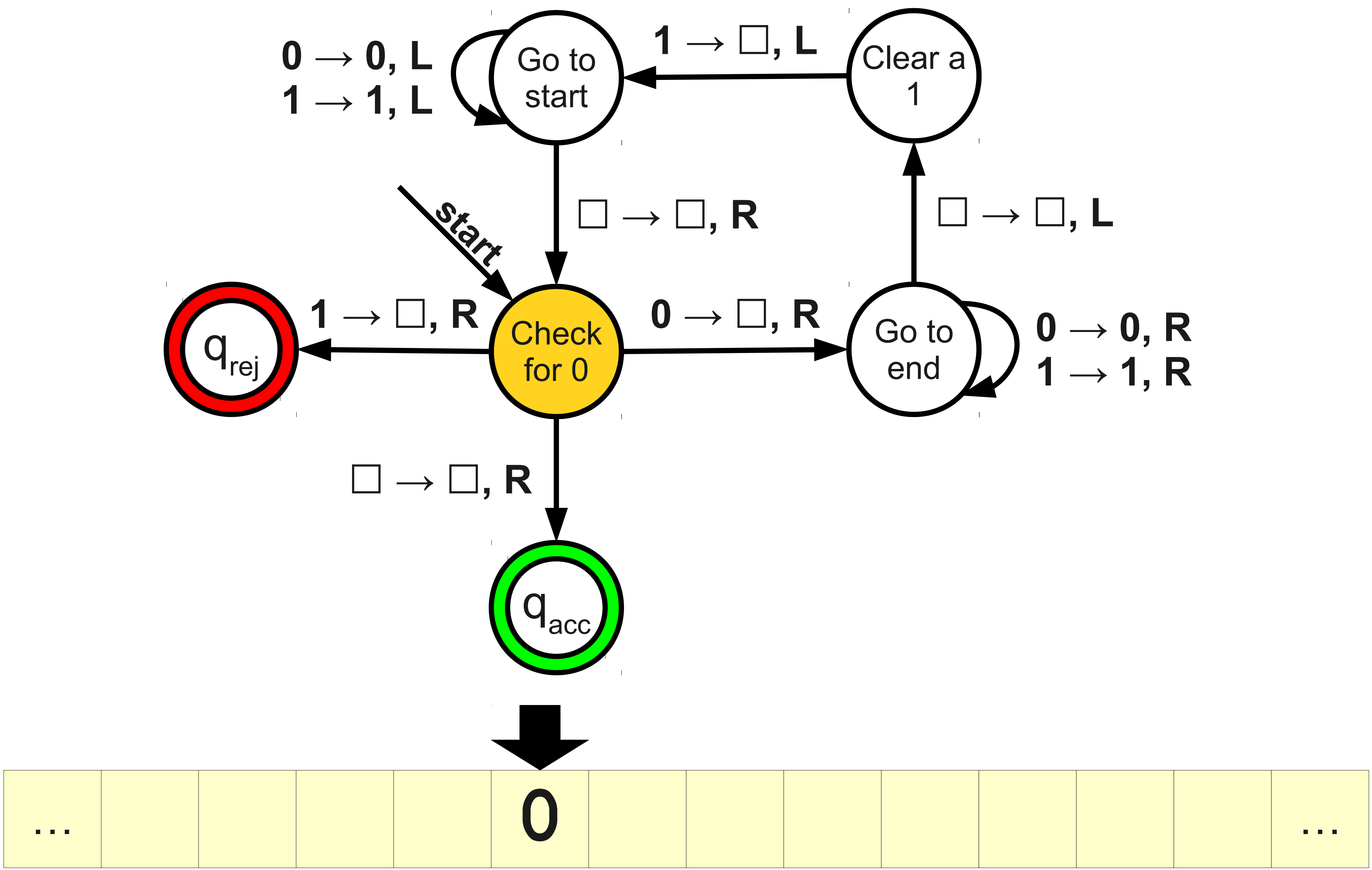
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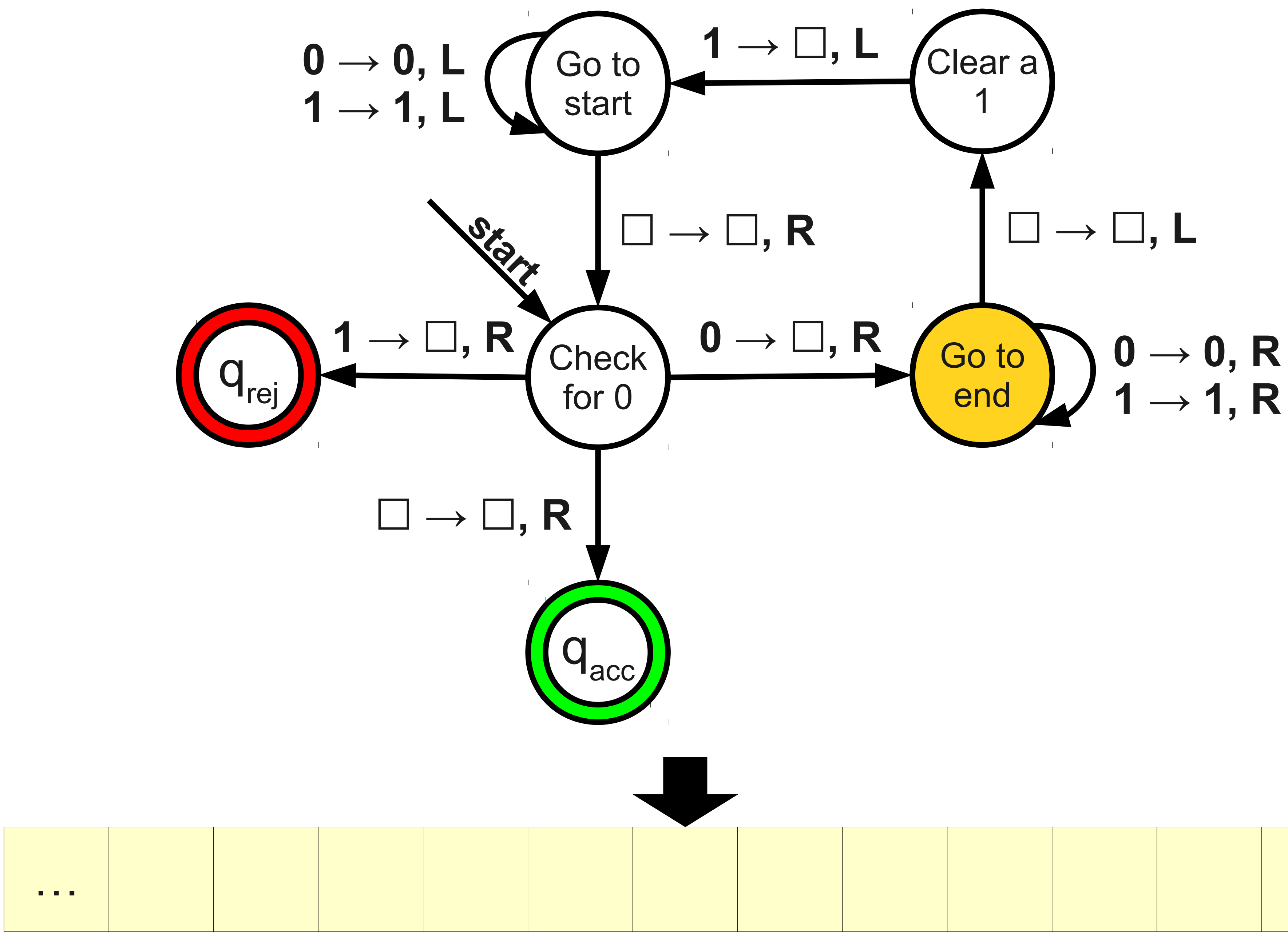

Go to start
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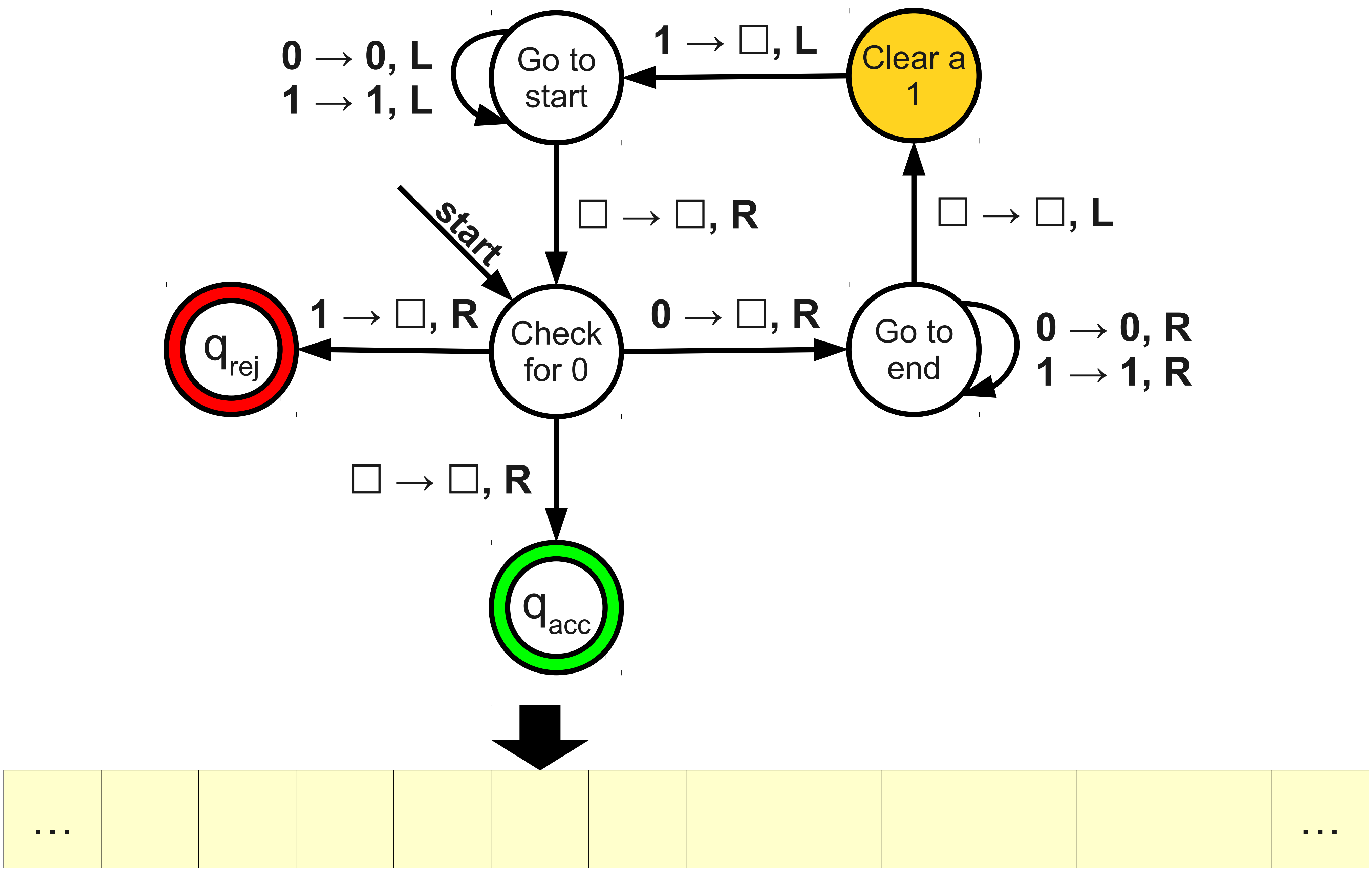

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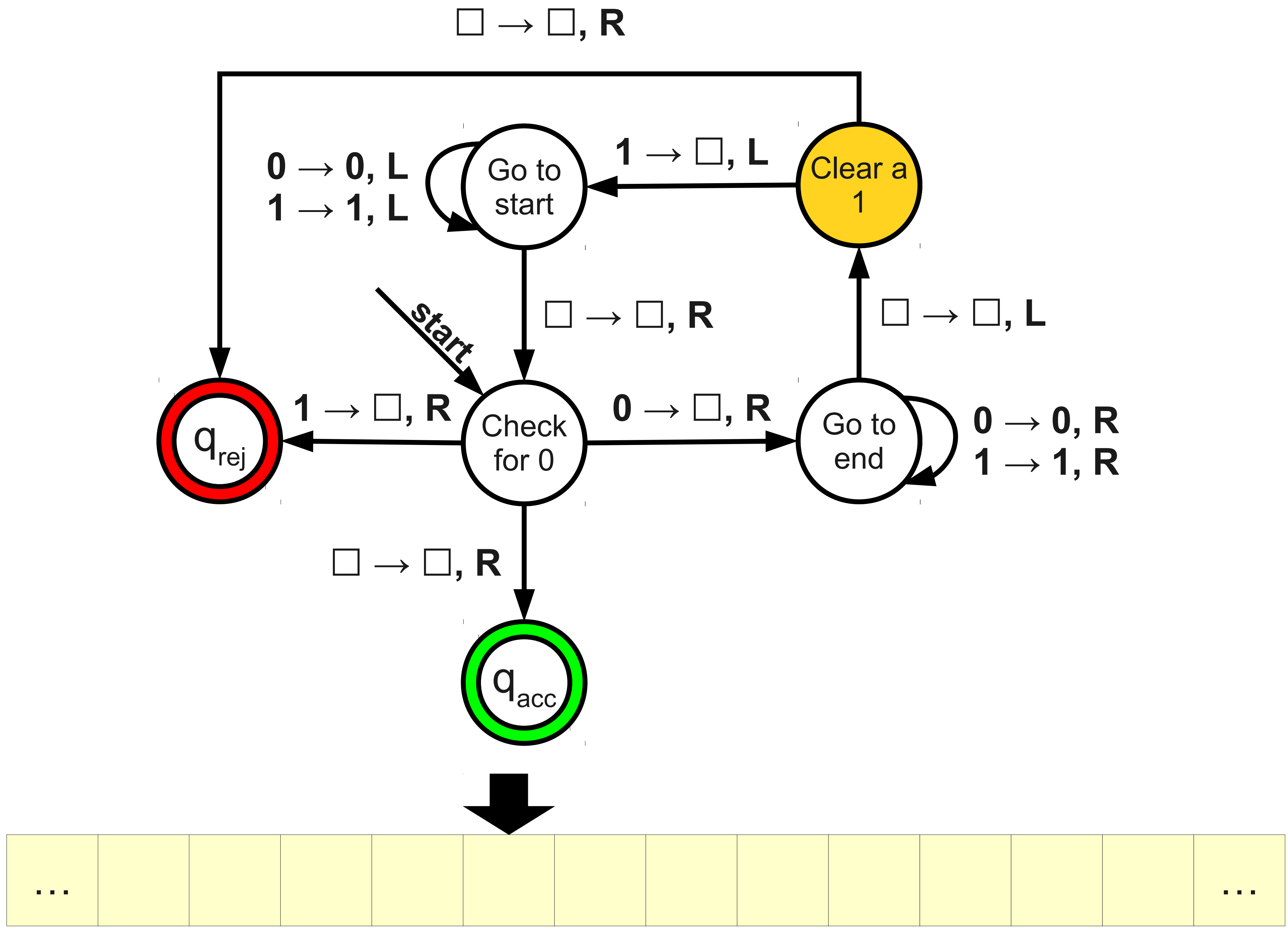

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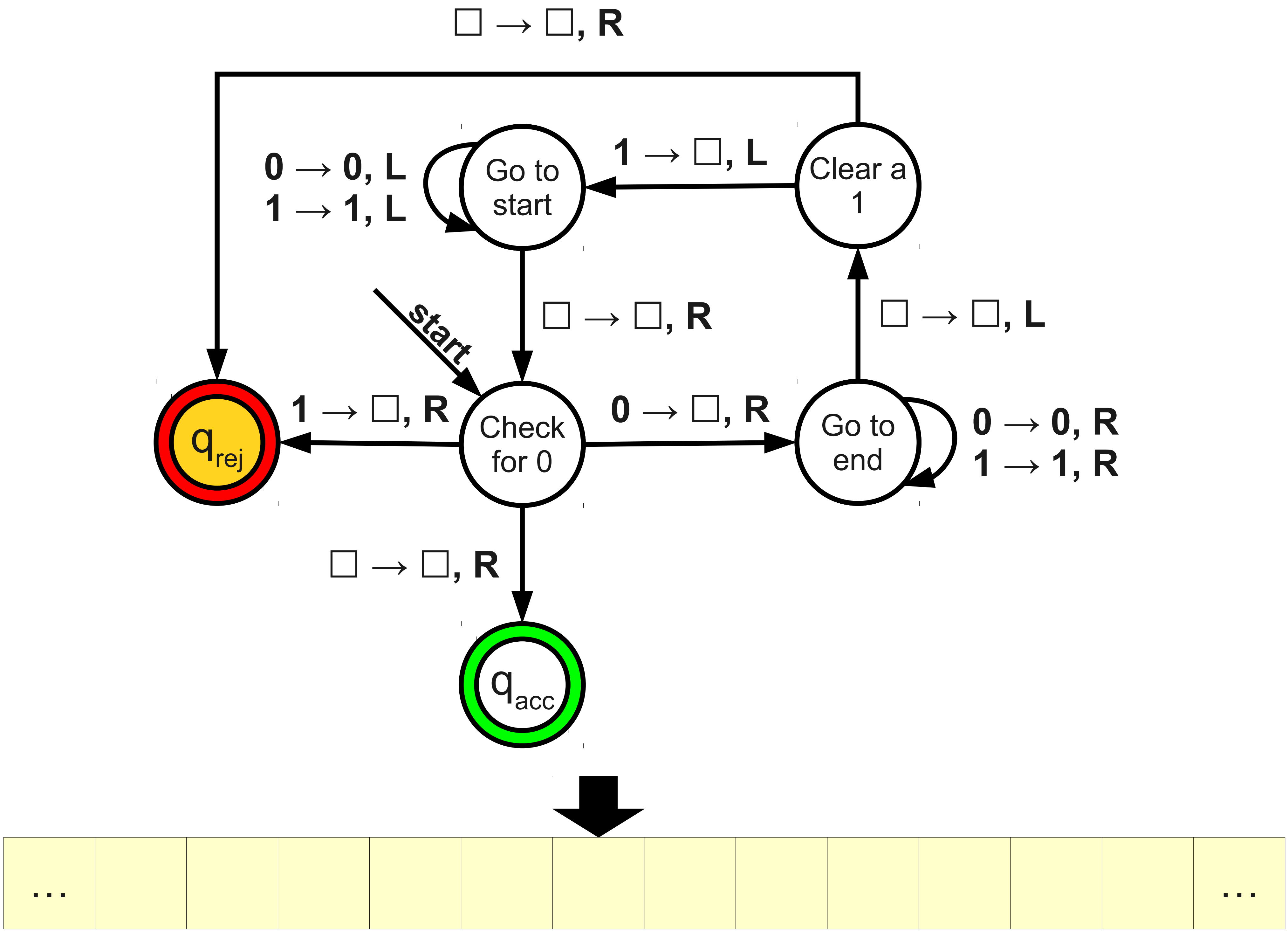

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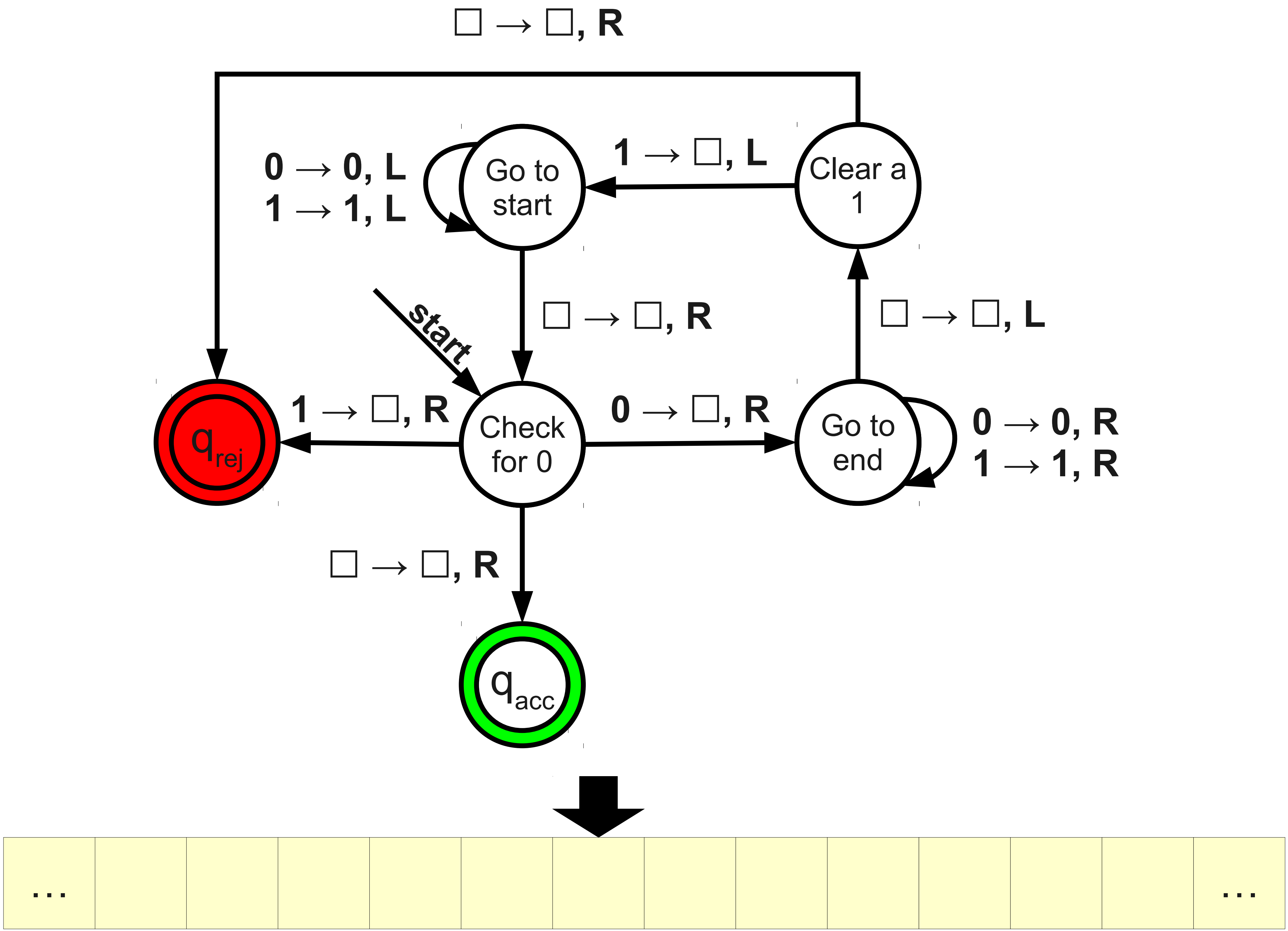

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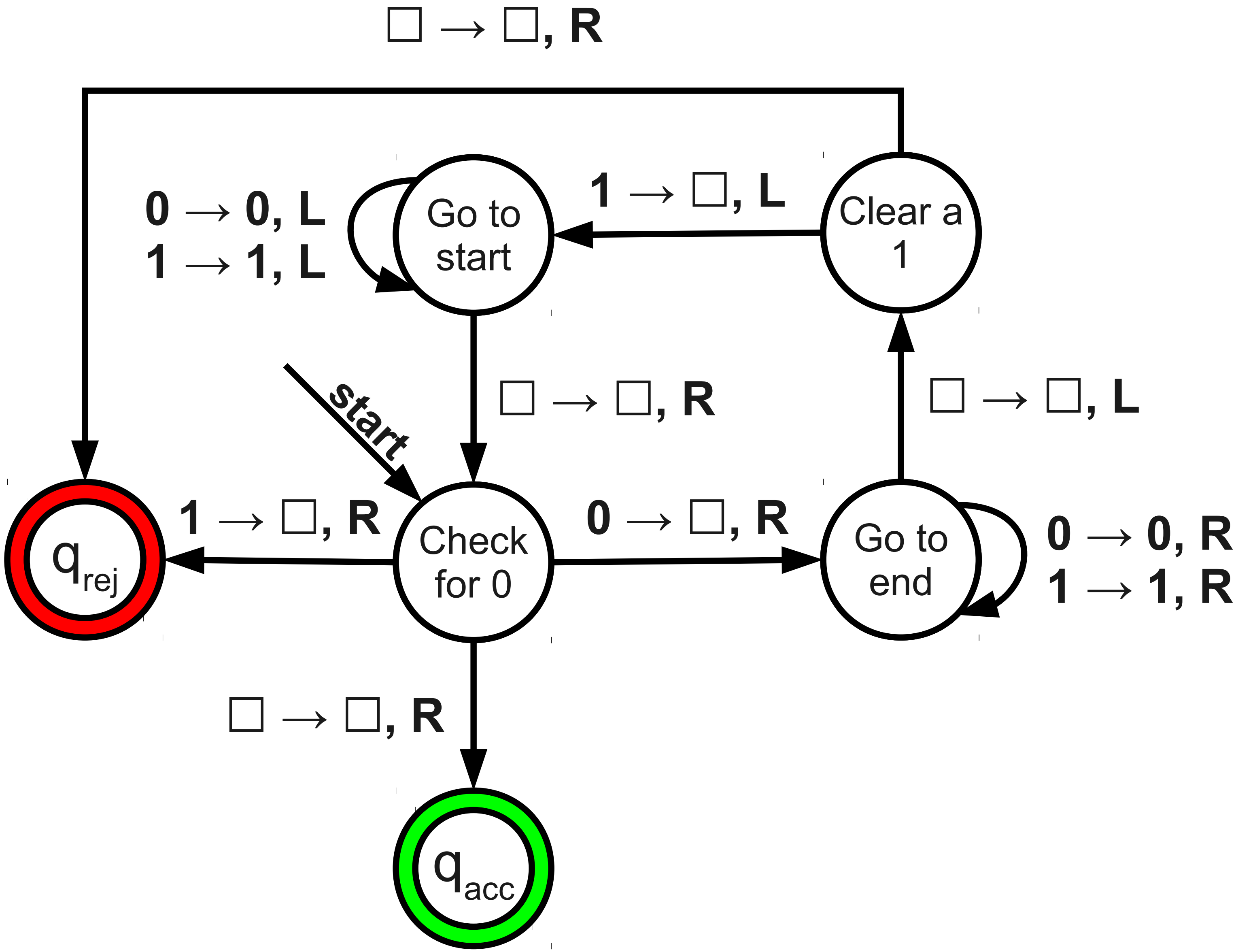

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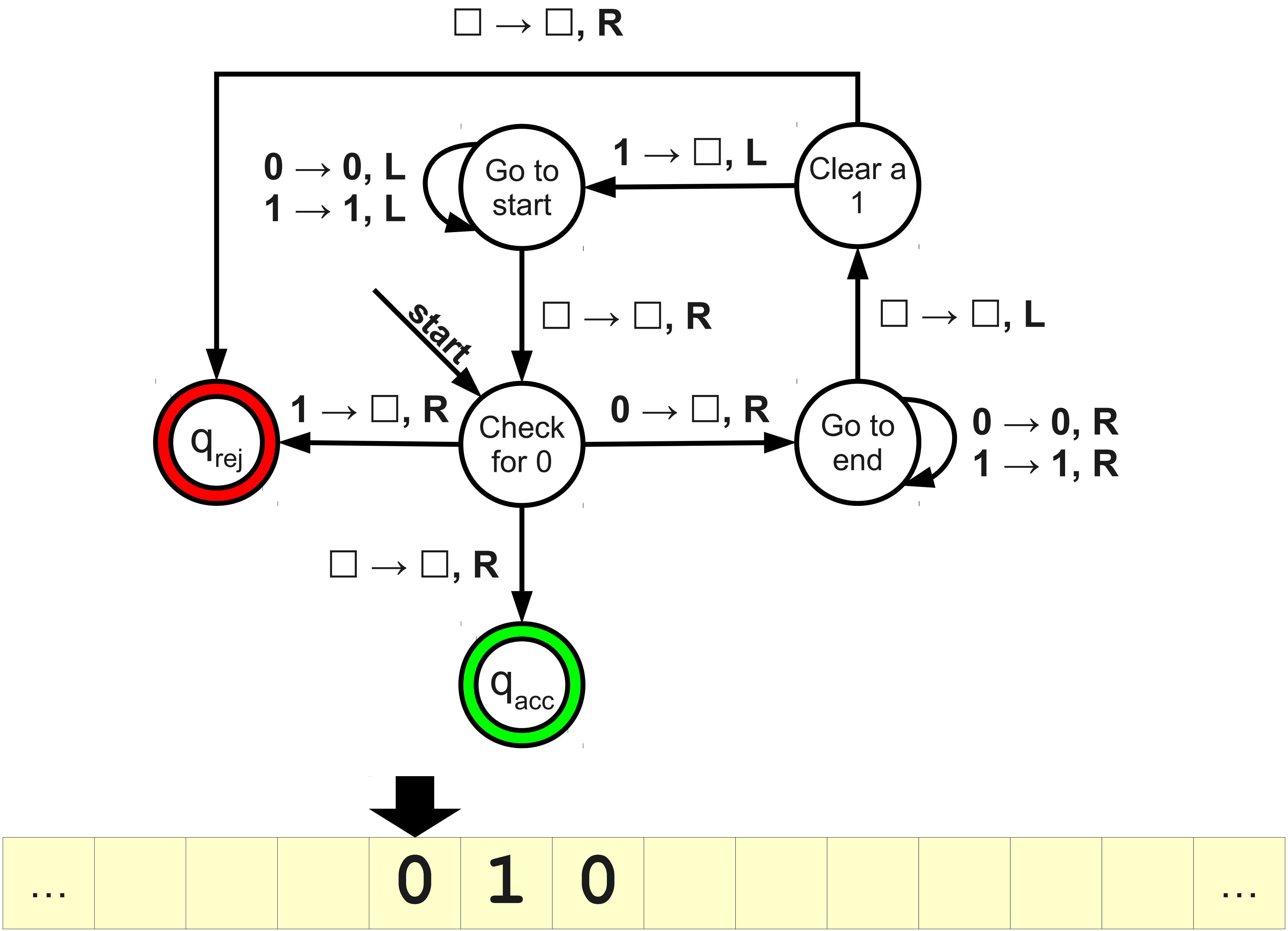
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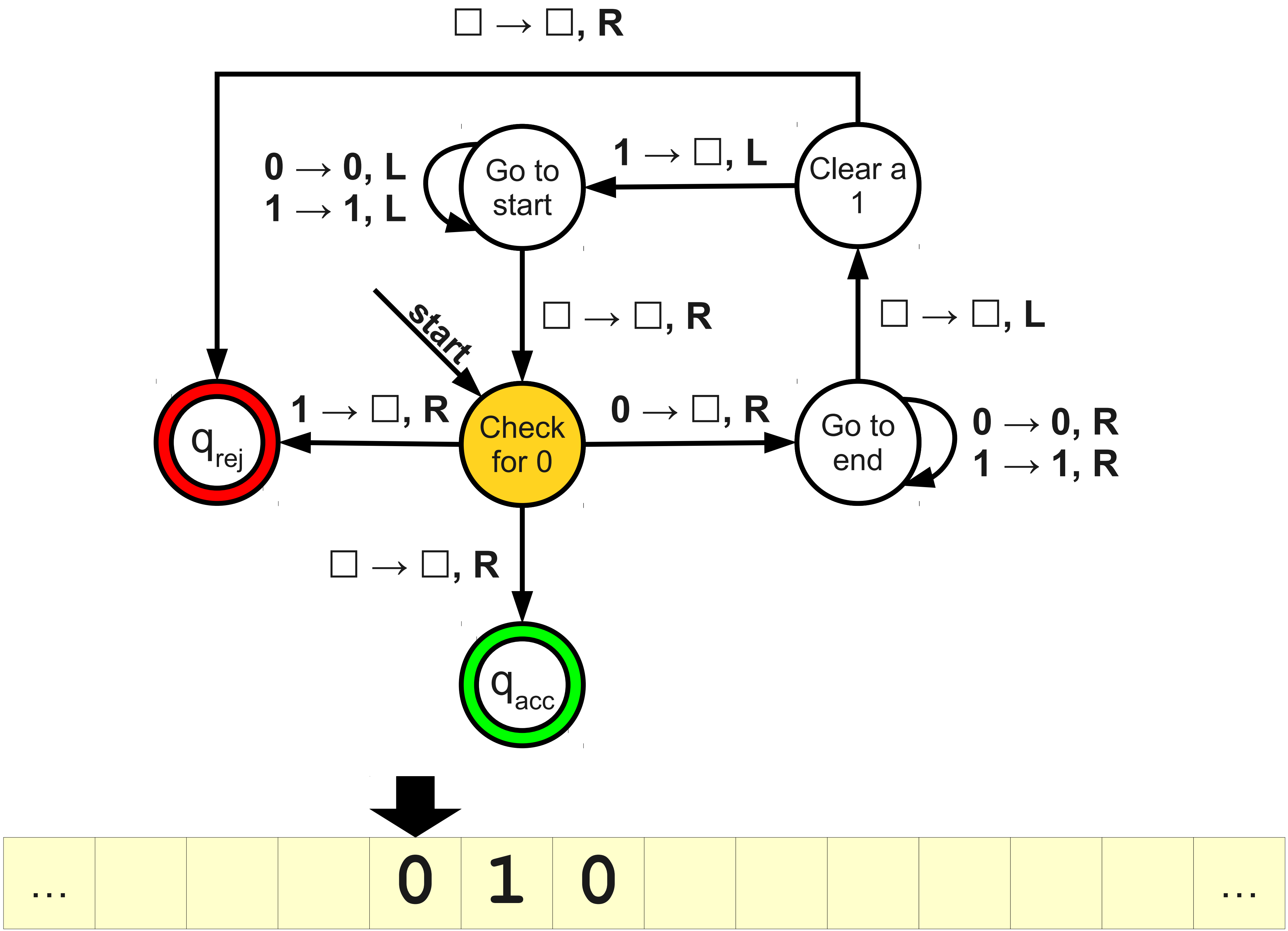

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q_{rej}
□ → □, R
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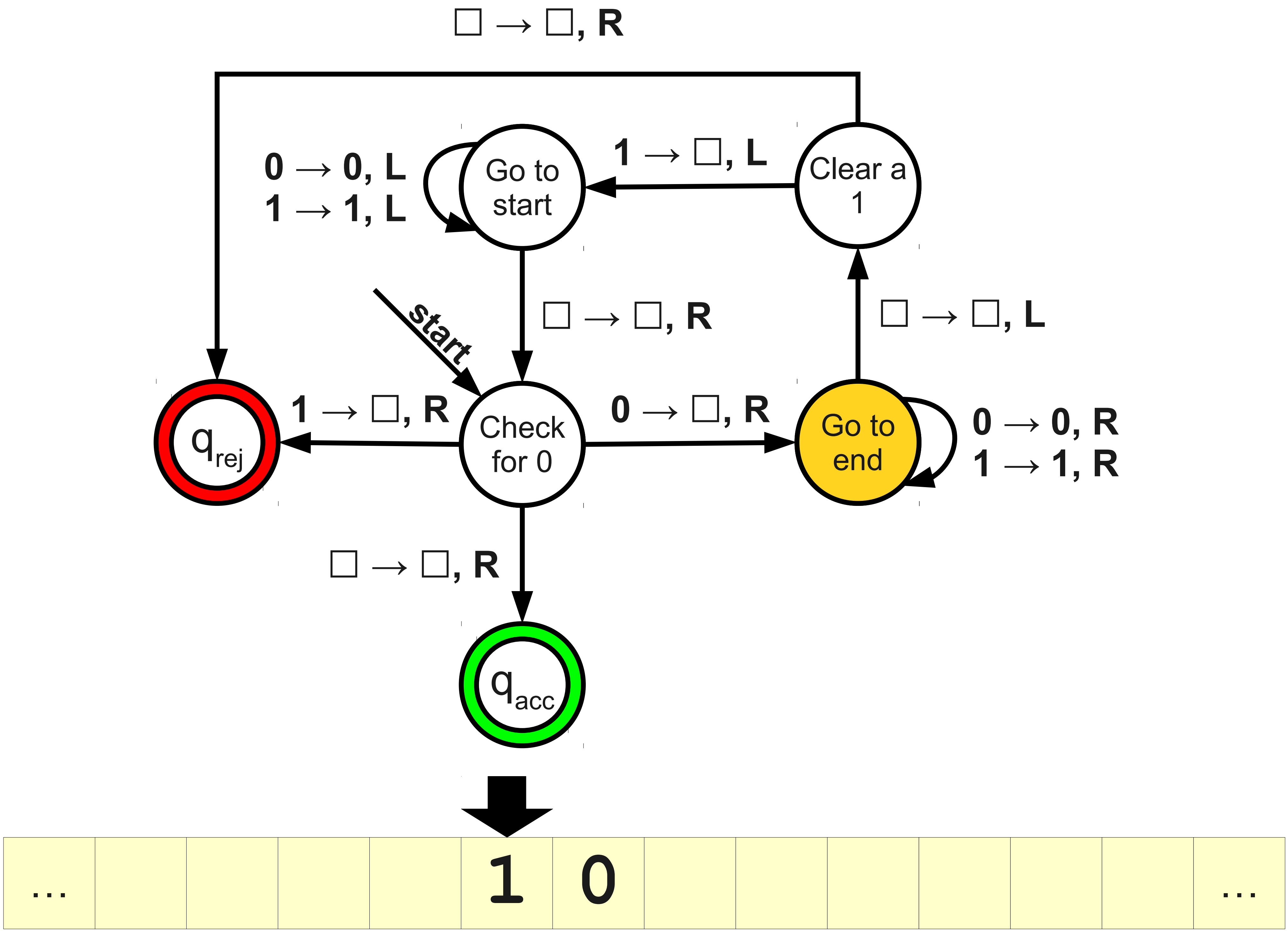

□ → □, R
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Check for 0
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Go to end
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1 → 1, R
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q_acc
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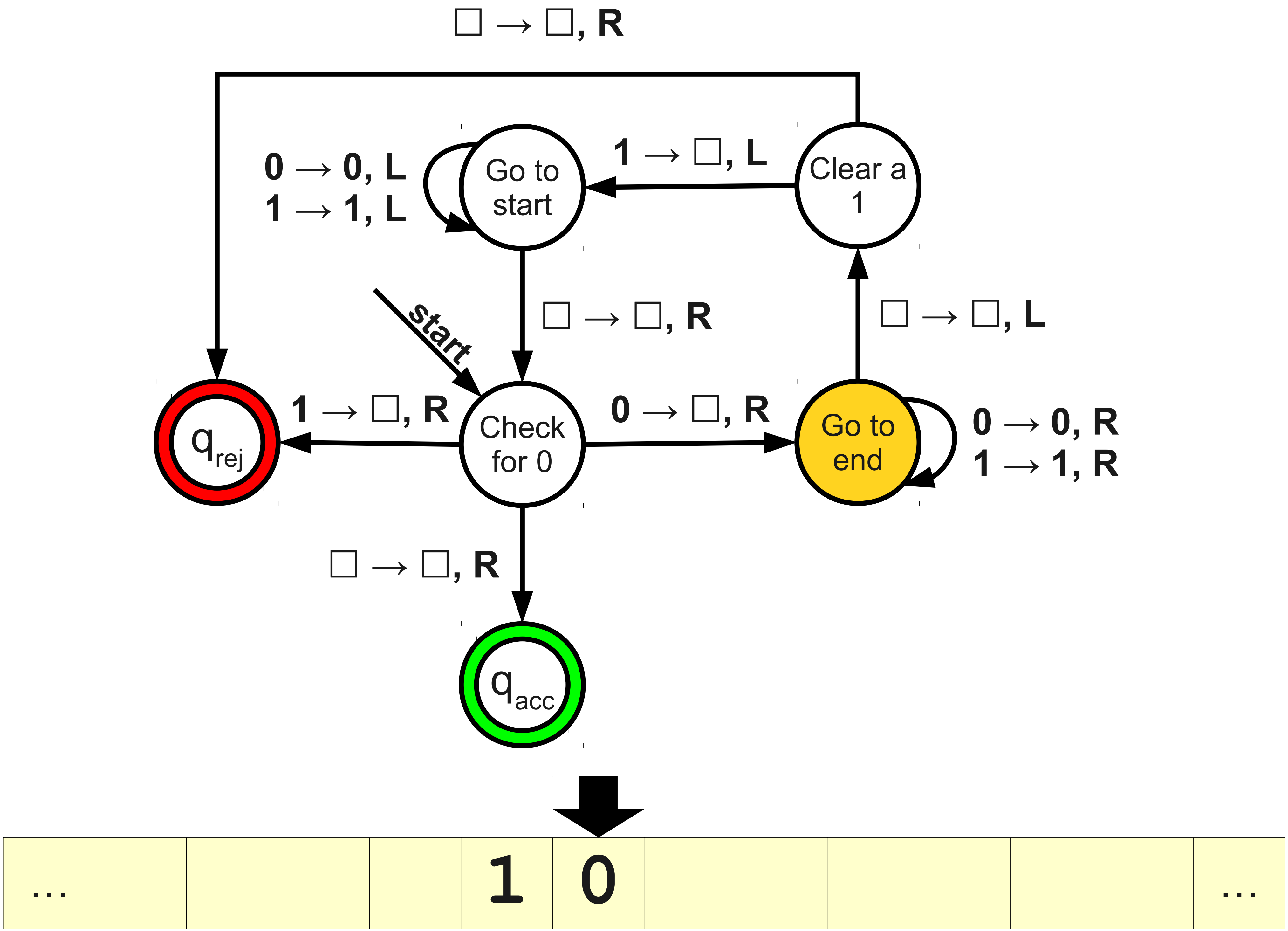

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1 → □, R
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Go to end
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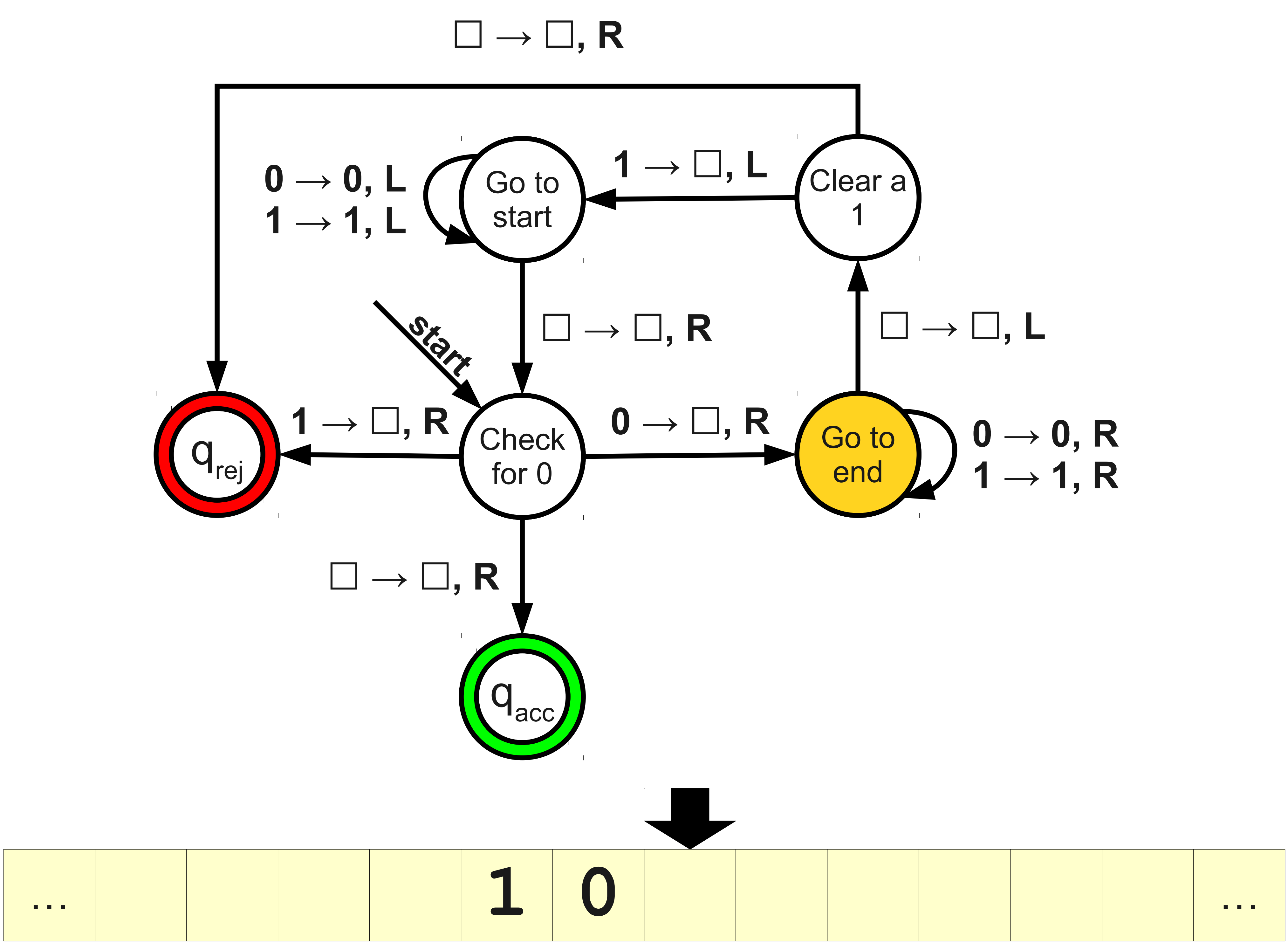
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□ → □, L
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1 → □, R
Check for 0
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Go to end
0 → 0, R
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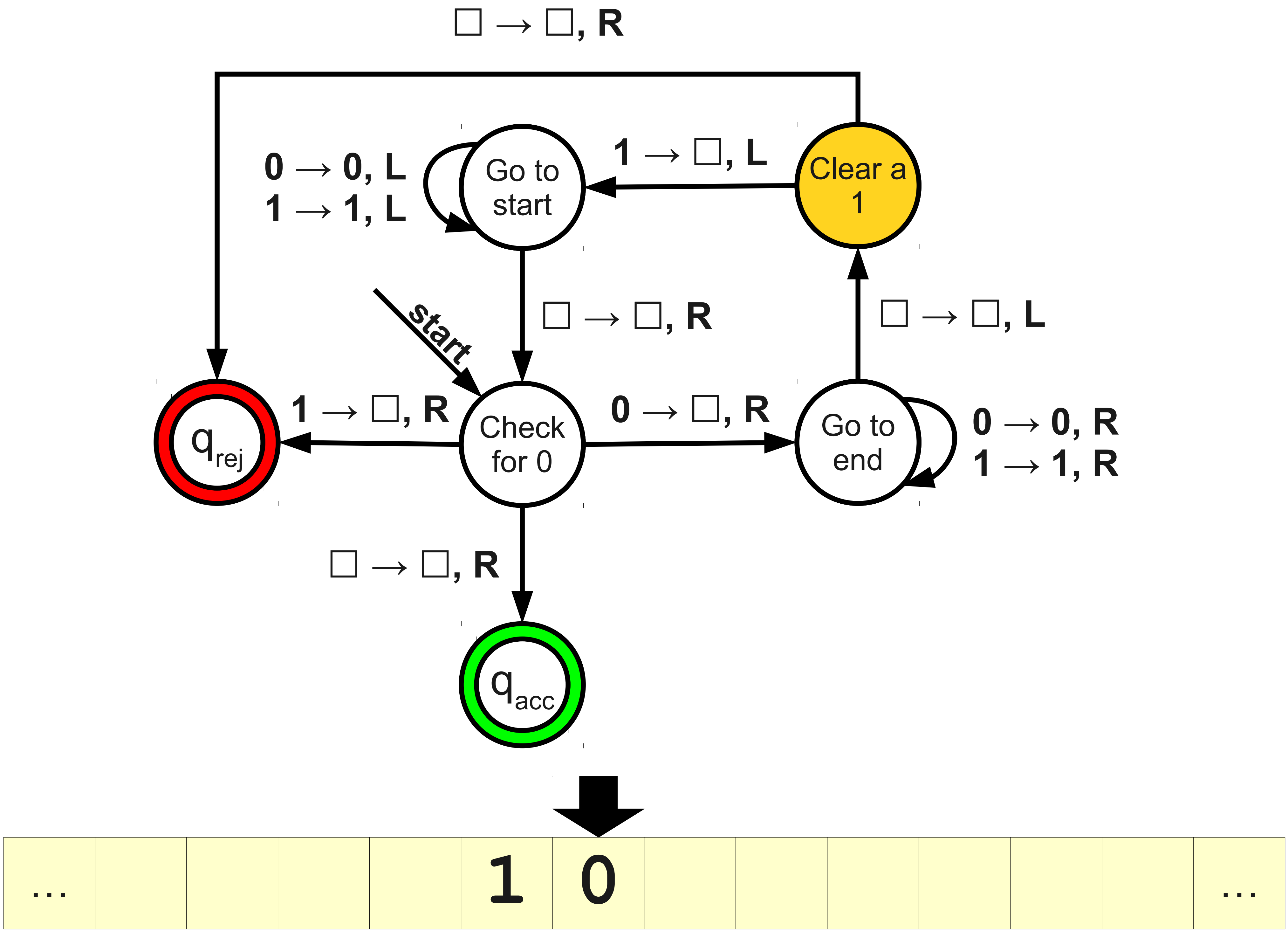

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1 → □, L
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□ → □, L
q_rej
1 → □, R
Check for 0
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Go to end
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1 → 1, R
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q_acc
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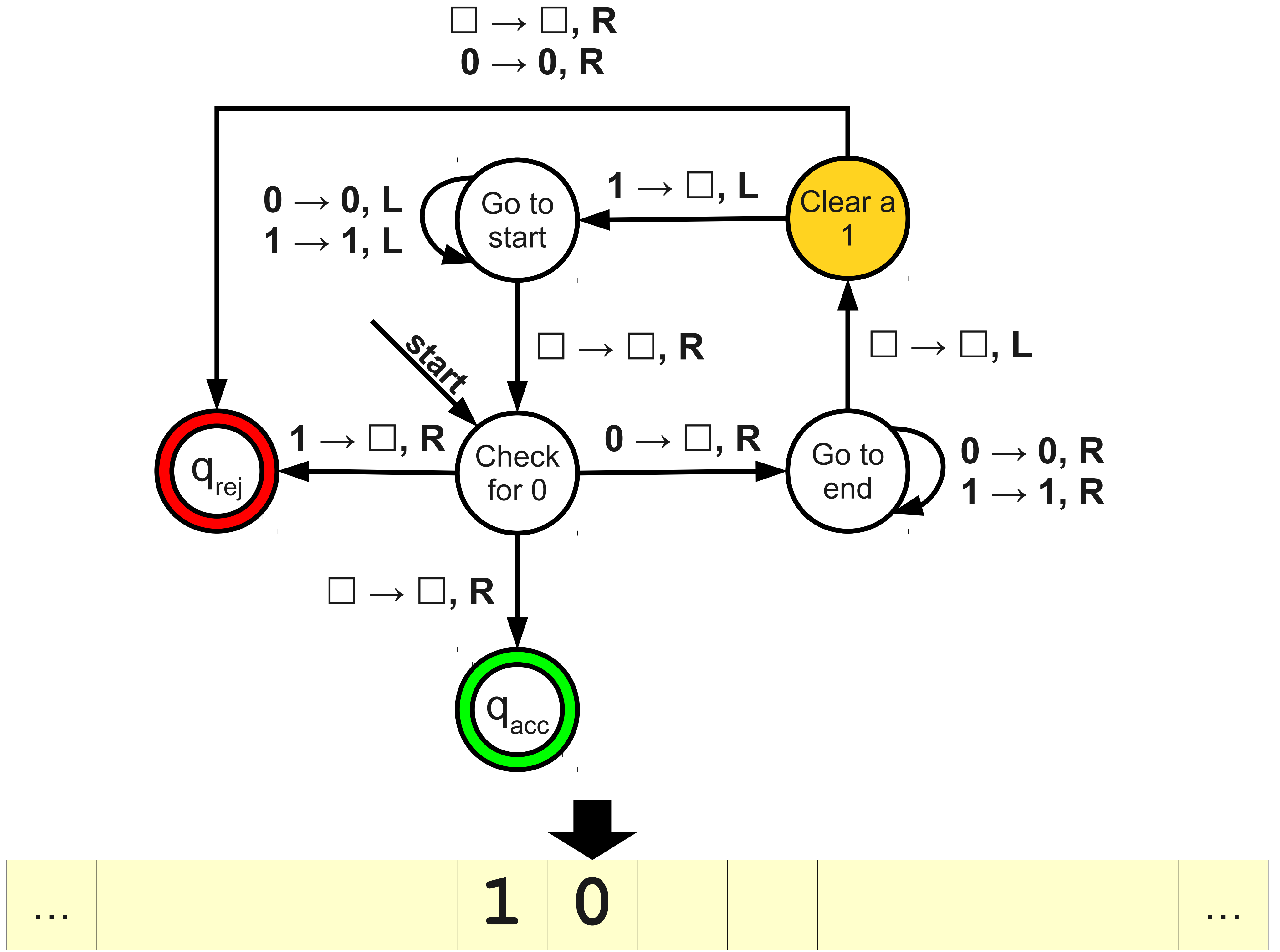

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Go to start
1 → □, L
Clear a 1
start
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q_{rej}
1 → □, R
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
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…
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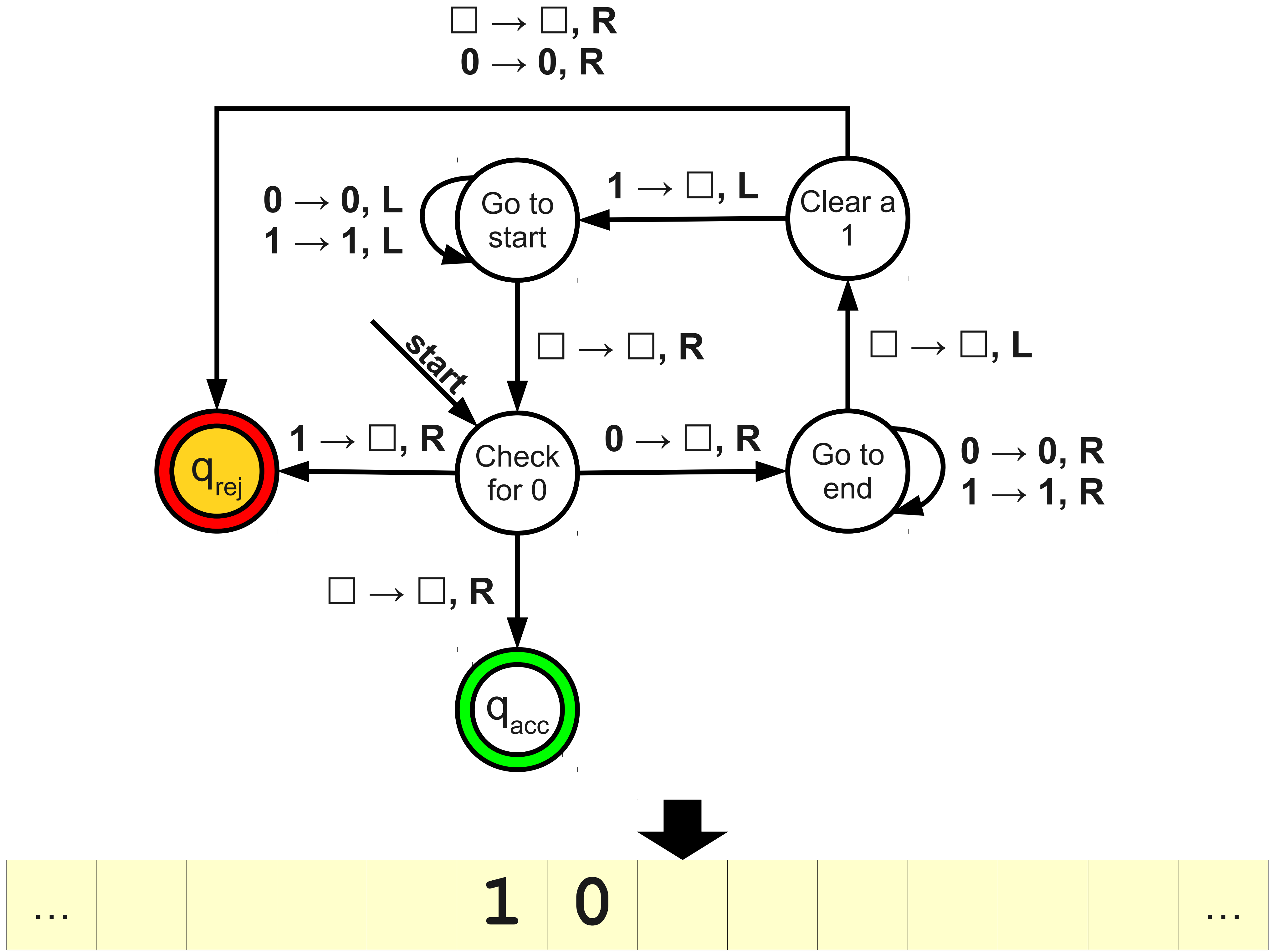

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0 → 0, R
0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
start
□ → □, R
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q_rej
1 → □, R
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
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q_acc
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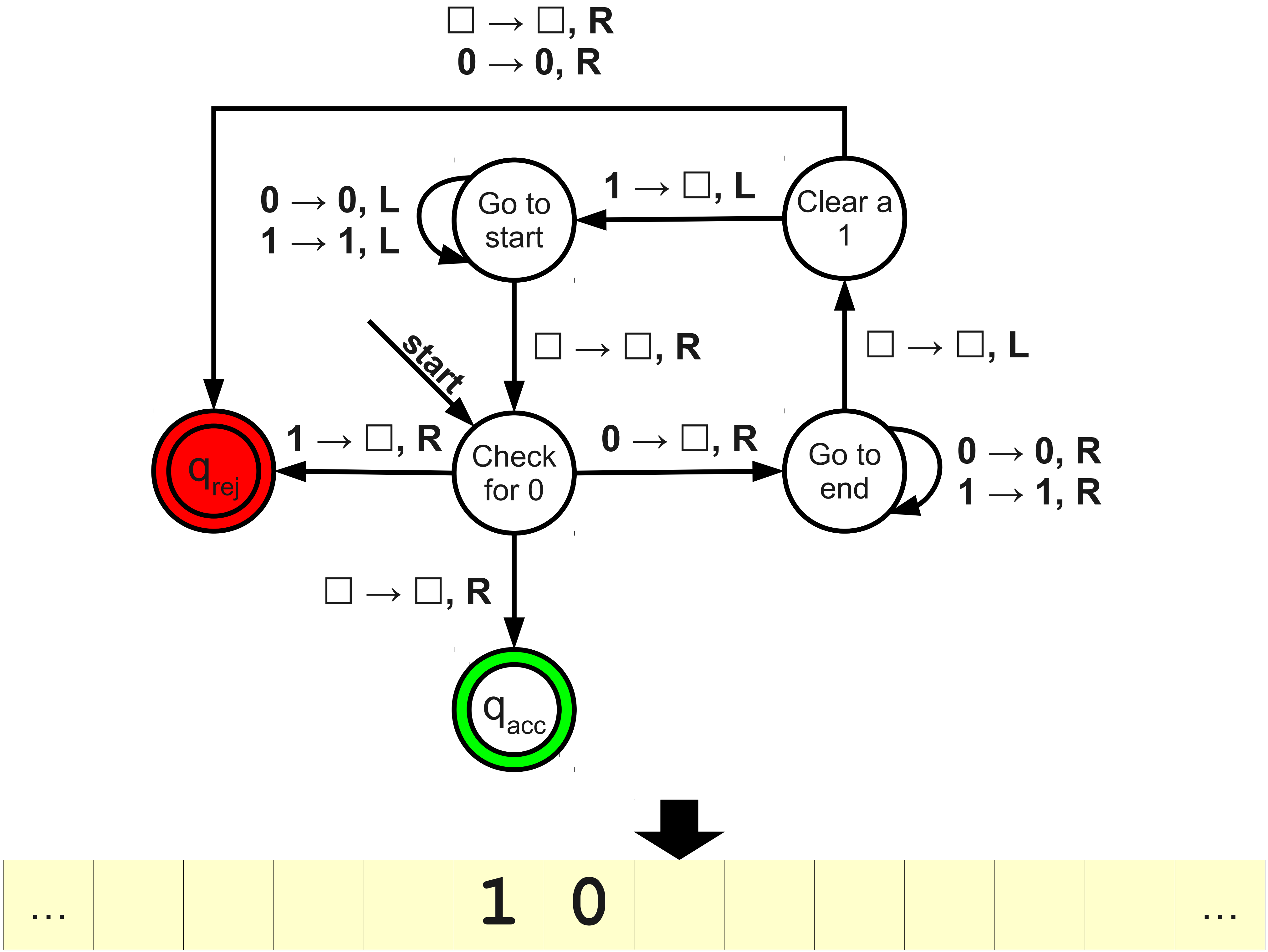
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0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
start
□ → □, R
□ → □, L
q_rej
1 → □, R
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
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q_acc
…
1
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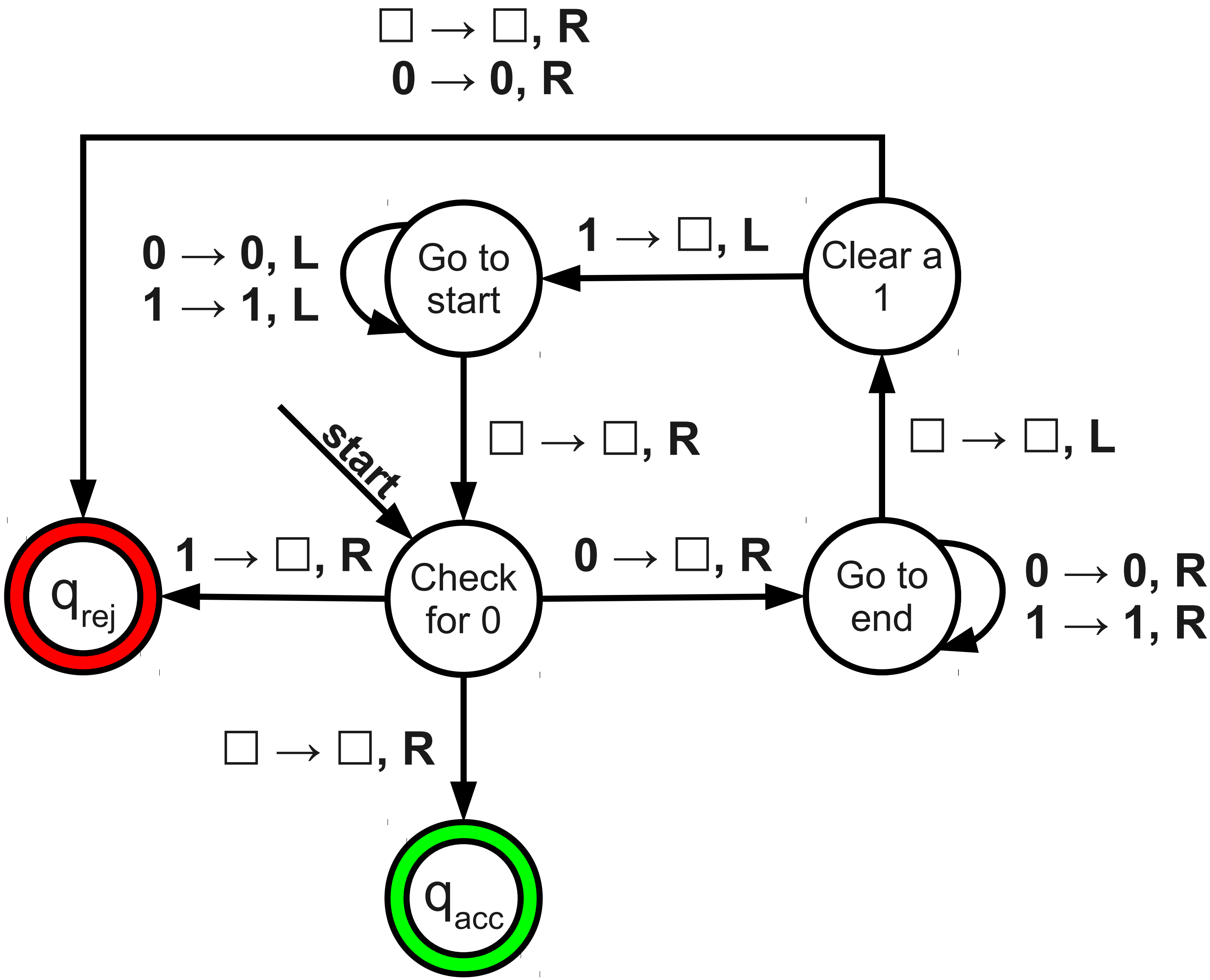


☐ → ☐, R
0 → 0, R
0 → 0, L
1 → 1, L
Go to start
1 → ☐, L
Clear a 1
start
☐ → ☐, R
☐ → ☐, L
q_{rej}
1 → ☐, R
Check for 0
0 → ☐, R
Go to end
0 → 0, R
1 → 1, R
☐ → ☐, R
q_{acc}

A Second Example

Multiplication

- Let $\Sigma = \{\mathtt{1}, \mathtt{\times}, \mathtt{=}\}$ and consider the language $L = \{ \mathtt{1}^m\mathtt{\times}\mathtt{1}^n\mathtt{=}\mathtt{1}^{mn} \mid m, n \in \mathbb{N} \}$
- This language is ***not*** regular (use the pumping lemma).
- This language is ***not*** context-free (use the pumping lemma).
- Can we build a TM for it?

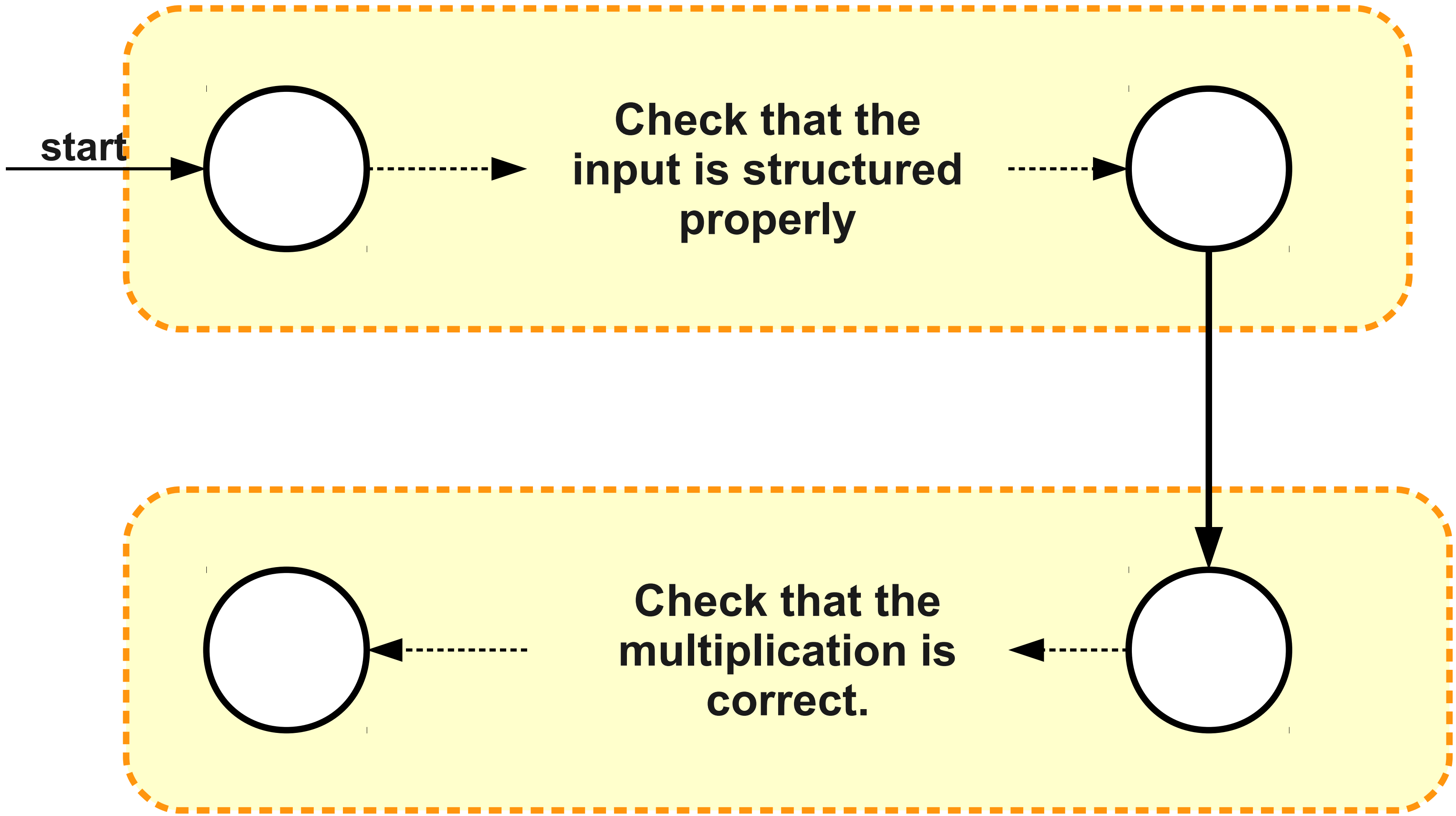
Things To Watch For

- The input has to have the right format.
  - Don't allow `11==×11×`, etc.
- The input must do the multiplication correctly.
  - Don't allow `11×11=11111`, for example.
- How do we handle this?

Key Idea: Subroutines

- A **subroutine** of a Turing machine is a small set of states in the TM such that performs a small computation.
- Usually, a single entry state and a single exit state.
- Many very complicated tasks can be performed by TMs by breaking those tasks into smaller subroutines.

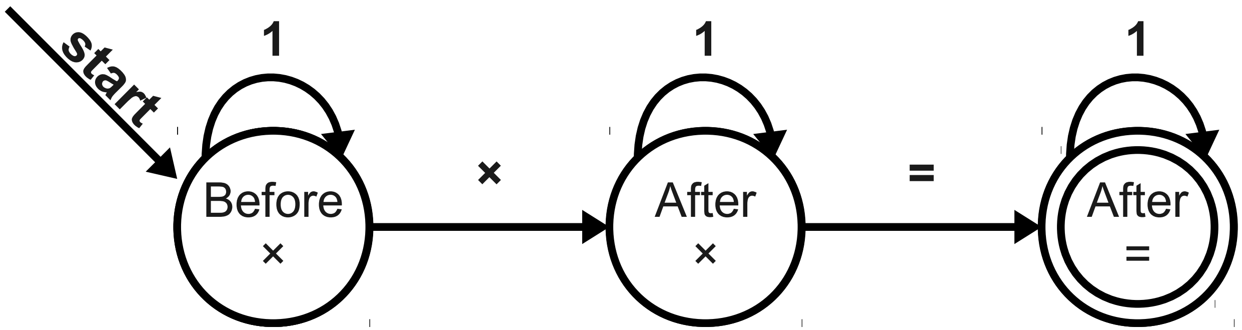
$$L = \{ 1^m \times 1^n = 1^{mn} \mid m, n \in \mathbb{N} \}$$



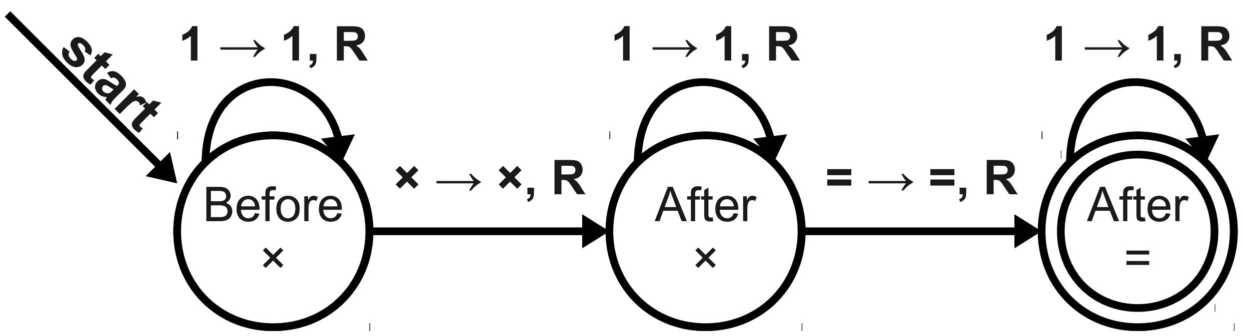
Validating the Input

- We'll check that the input has the form **1*****×1*****=1***.
- Just checking relative ordering of symbols, not the quantity of the symbols.
- How might we do this?

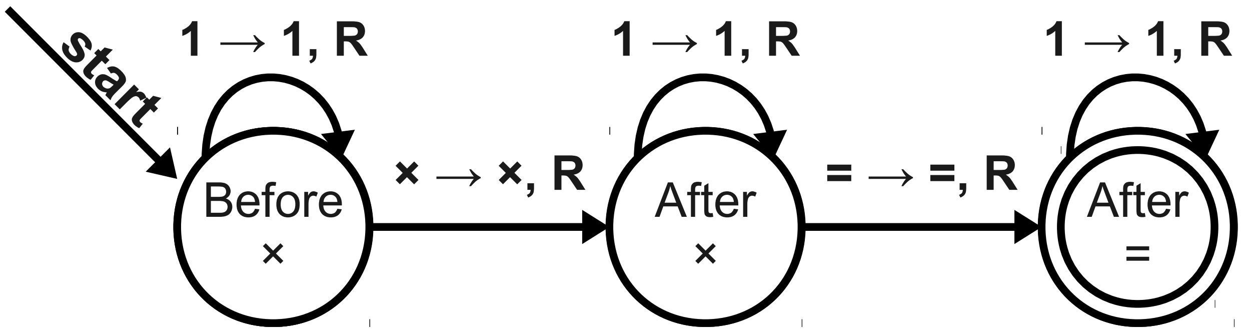
Checking for $\mathtt{1}^*\mathtt{\times 1}^*\mathtt{=1}^*$

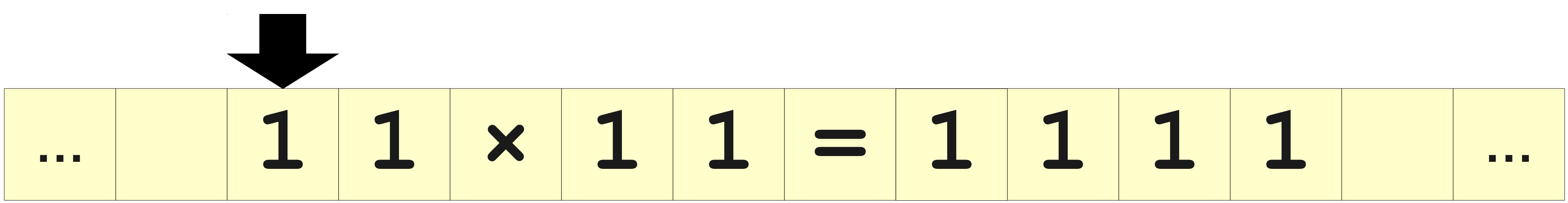


Checking for $\mathtt{1}^*\mathtt{×1}^*\mathtt{=1}^*$

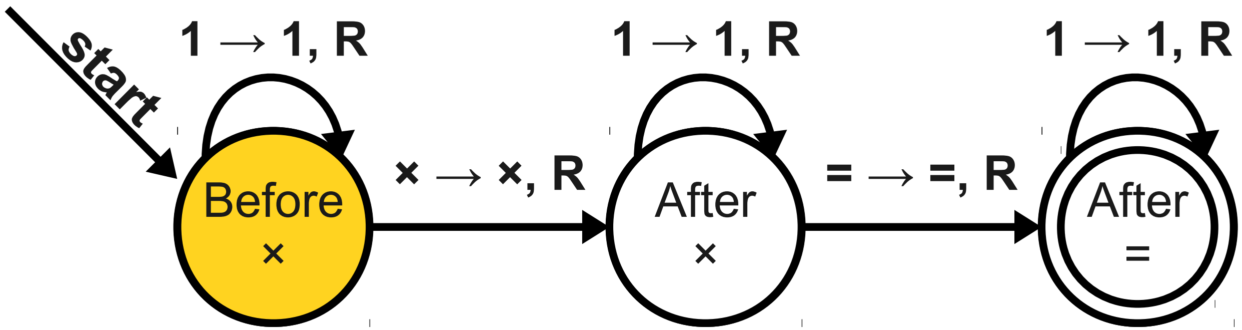


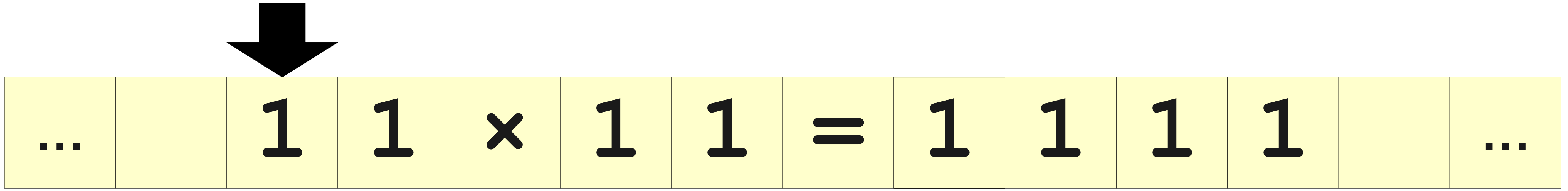
Checking for $1^*\times1^*=1^*$



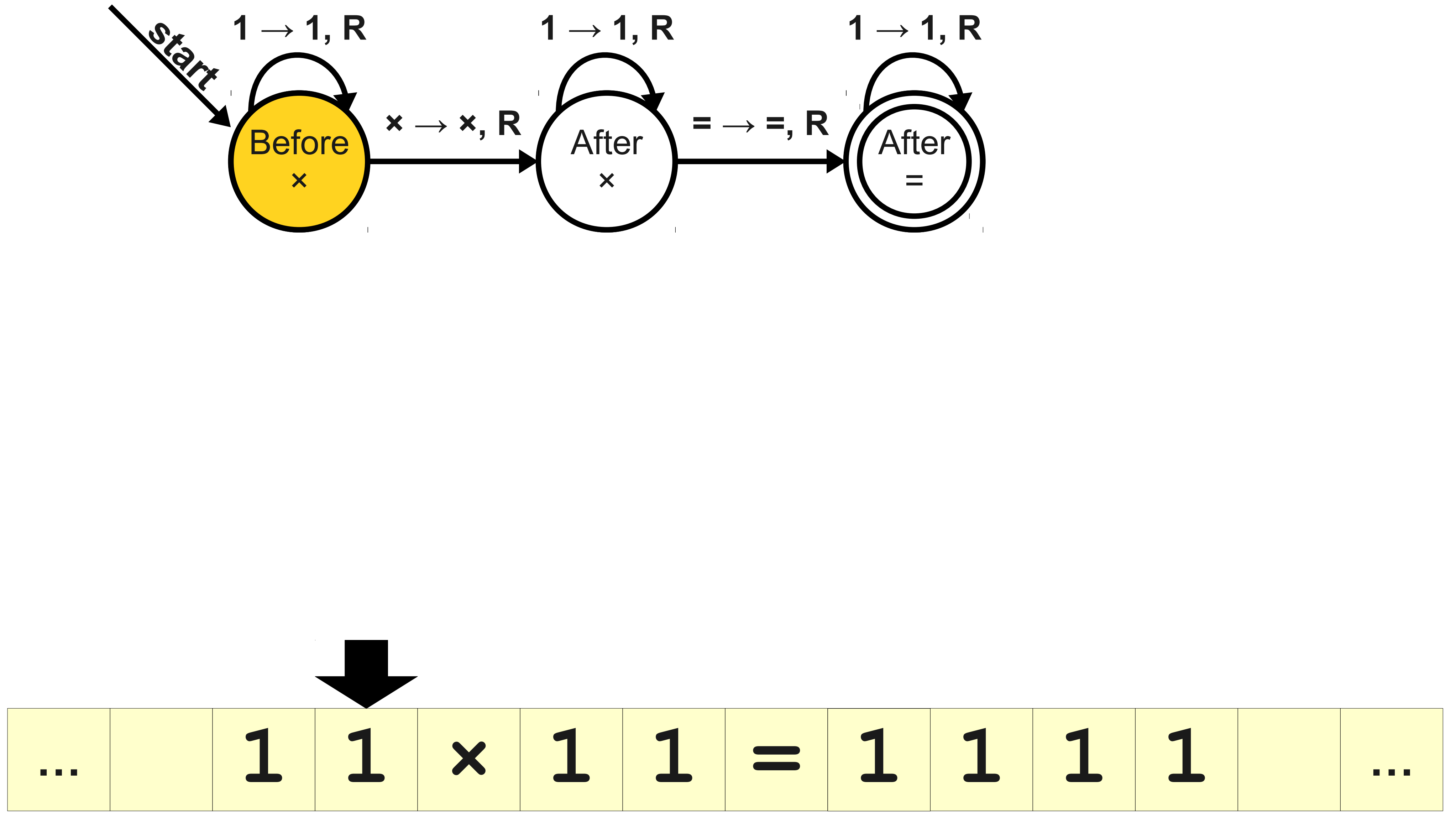


Checking for $\mathbf{1^*{\times}1^*{=}1^*}$

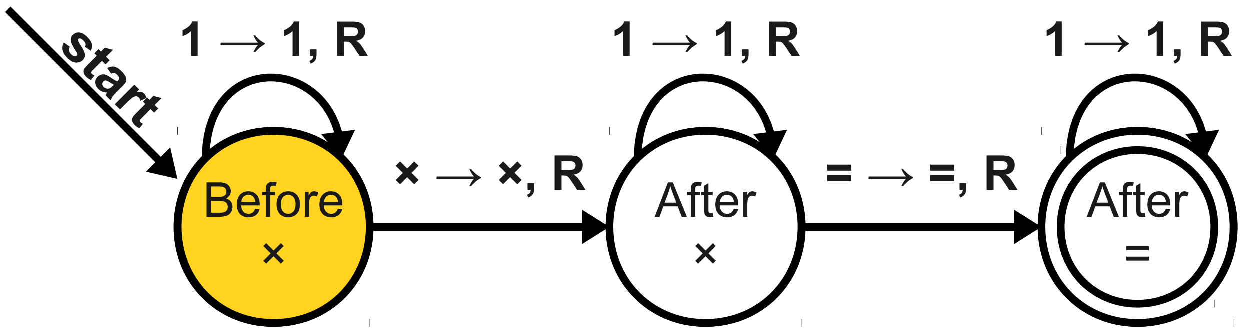


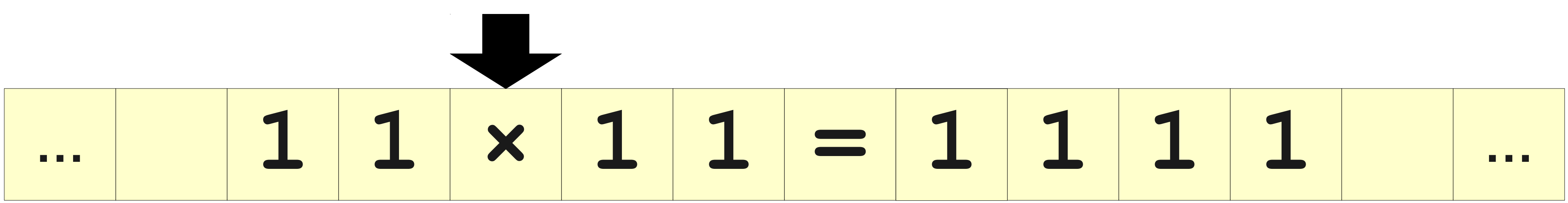


Checking for $1^*\times 1^*=1^*$

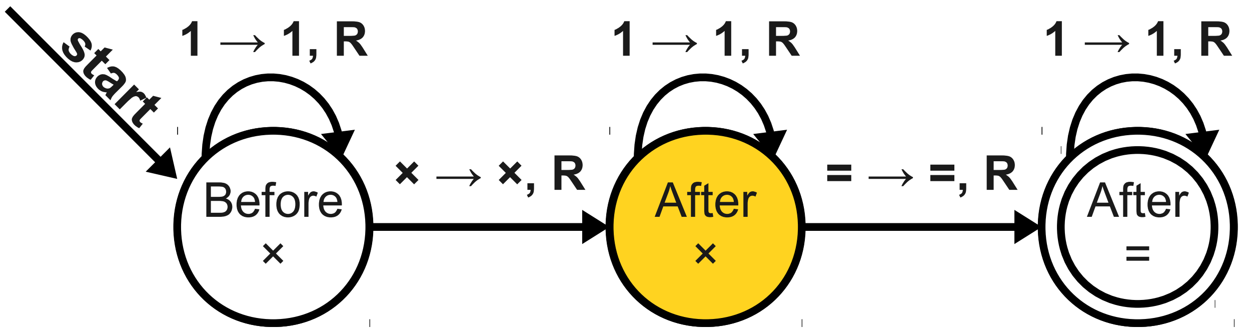


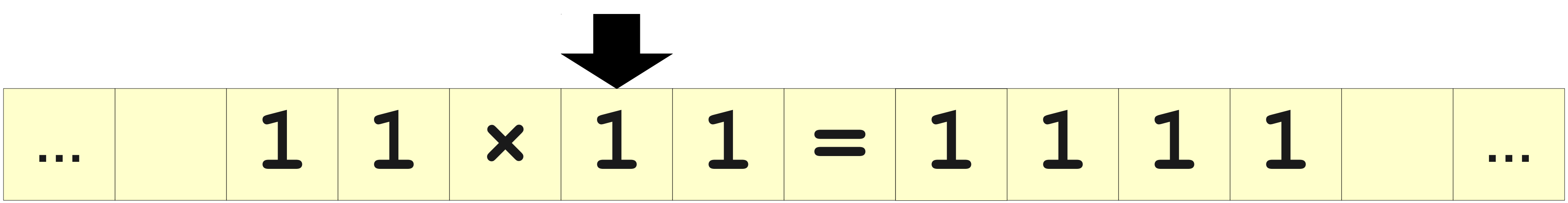
Checking for 1*×1*=1*



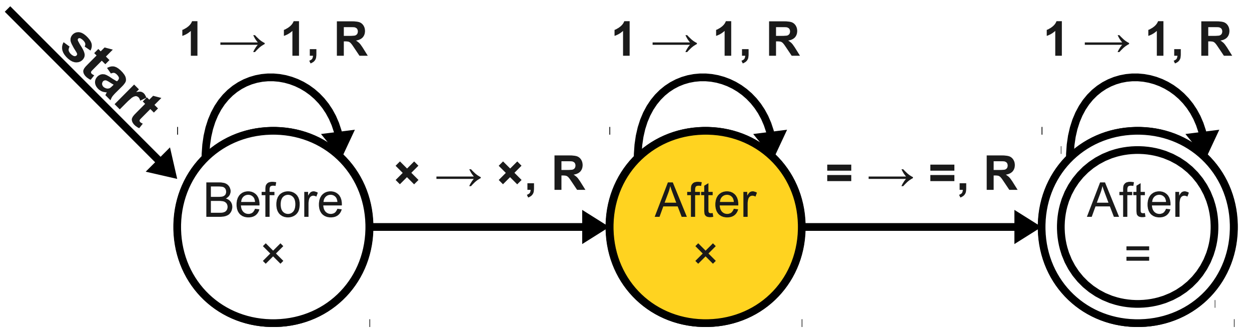


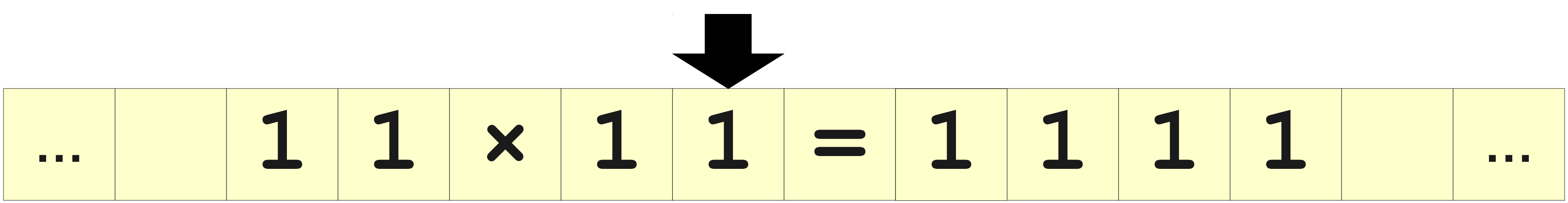
Checking for $1^*\times1^*=1^*$



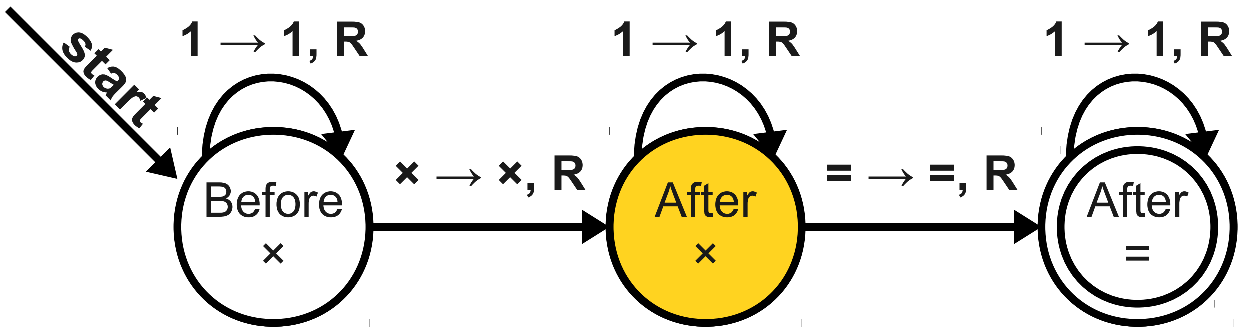


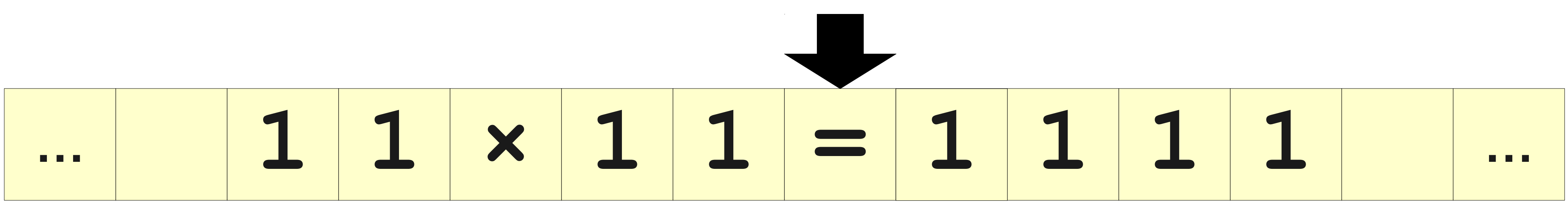
Checking for $1^{*}\times 1^{*}=1^{*}$



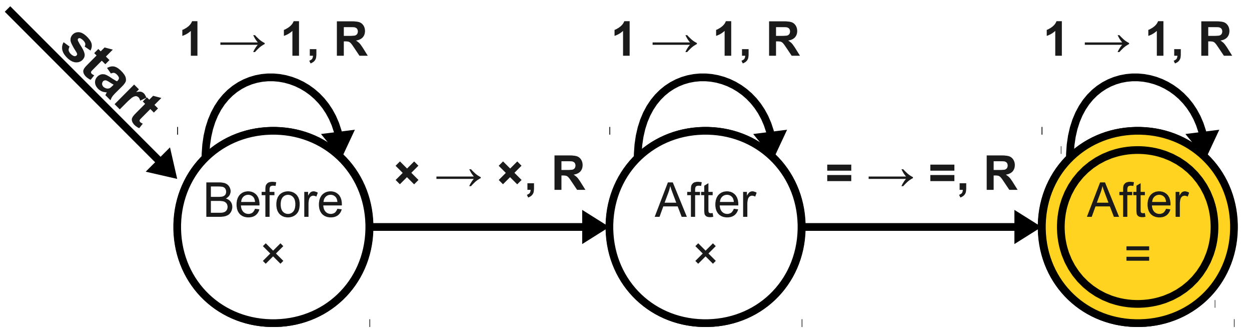


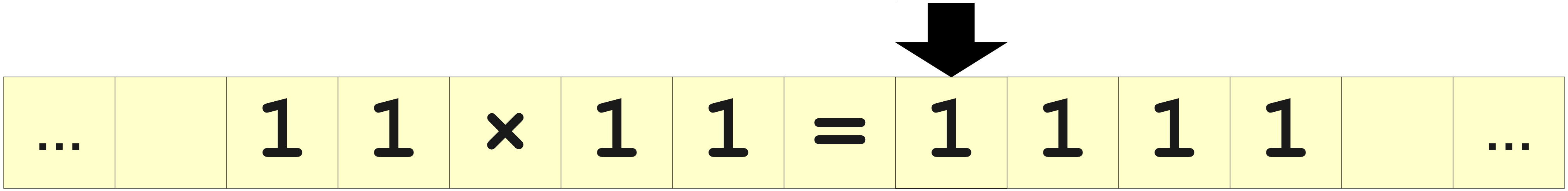
Checking for $1^*\times 1^*=1^*$



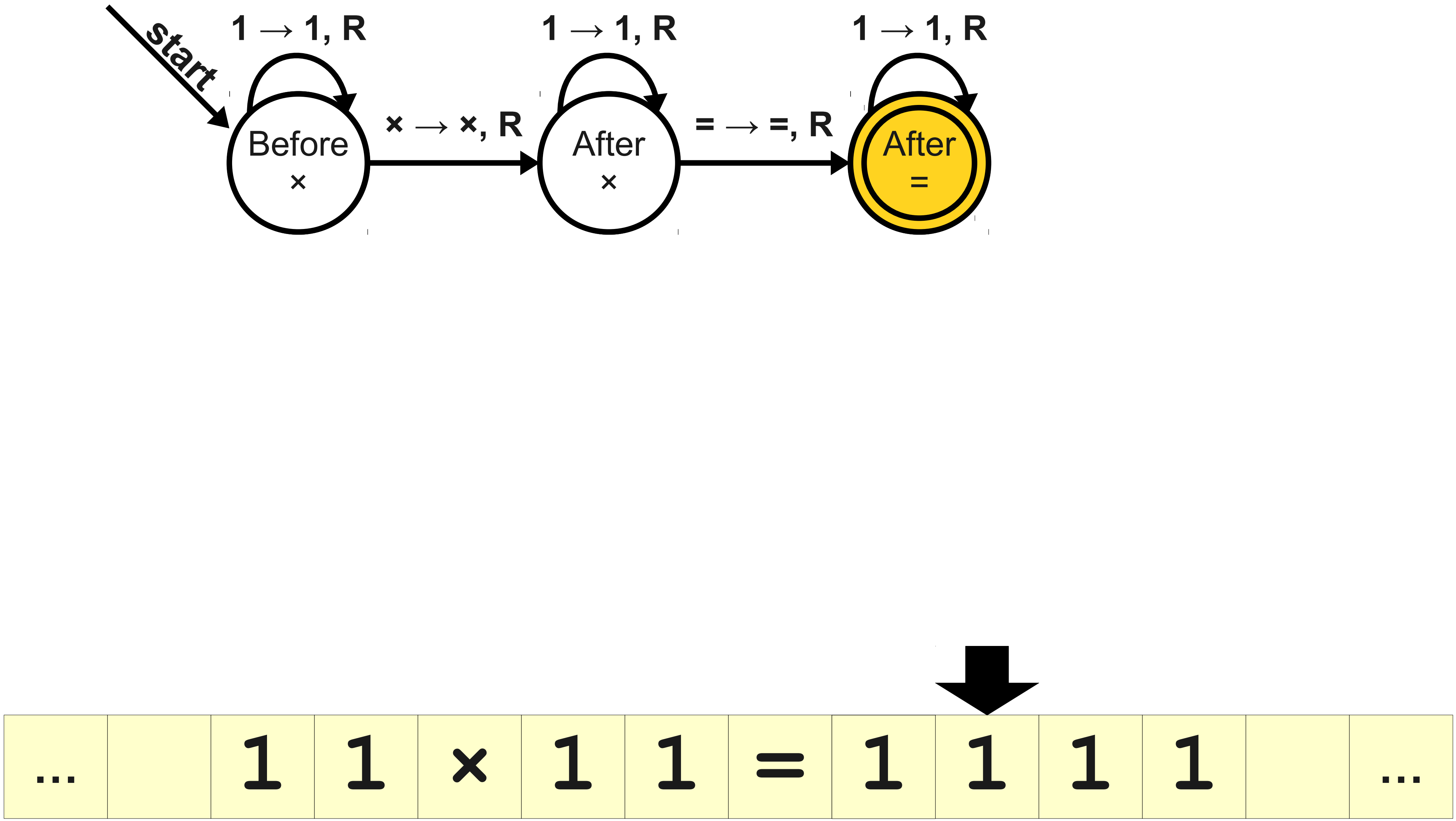


Checking for $1^*\times1^*=1^*$

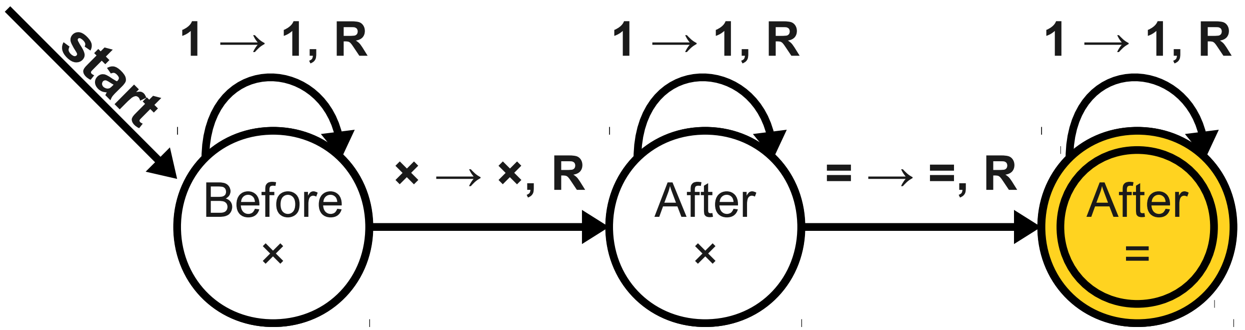


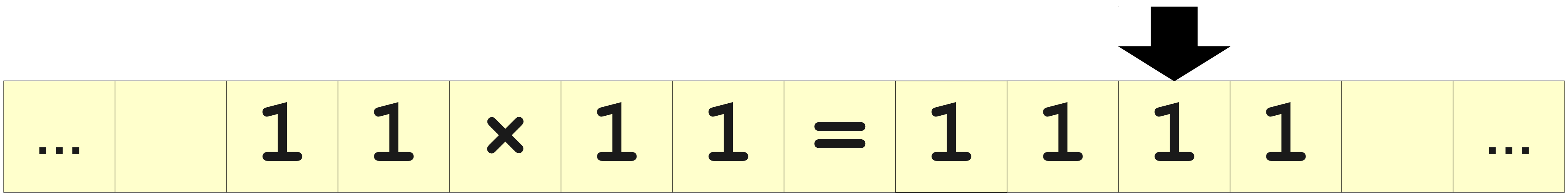


Checking for $1^*\times1^*=1^*$

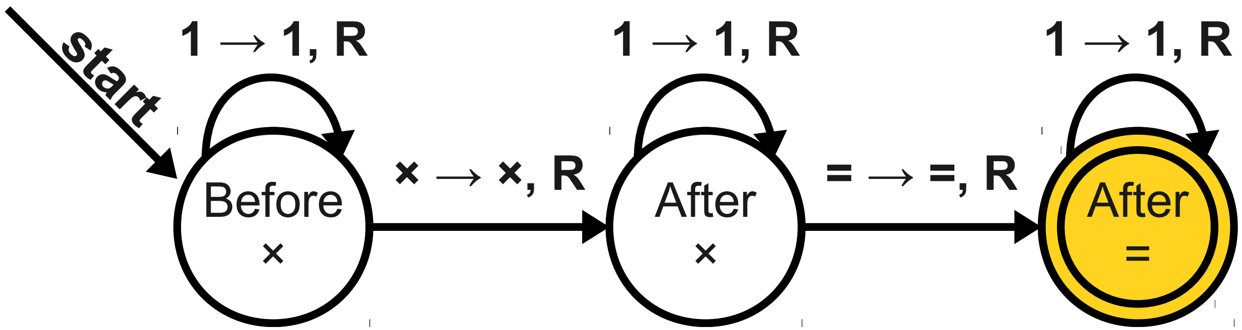


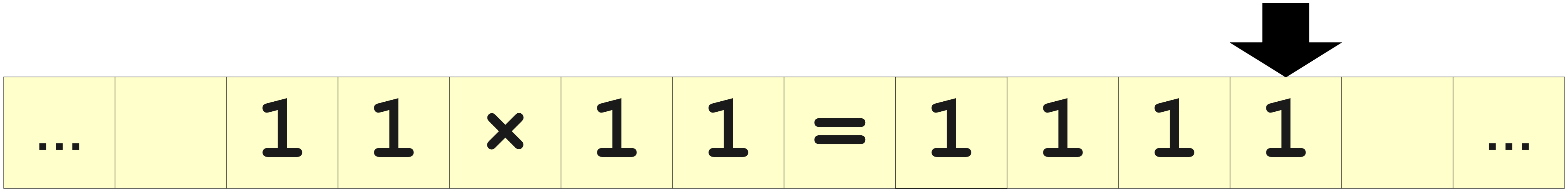
Checking for $\mathtt{1^*\times 1^*=1^*}$



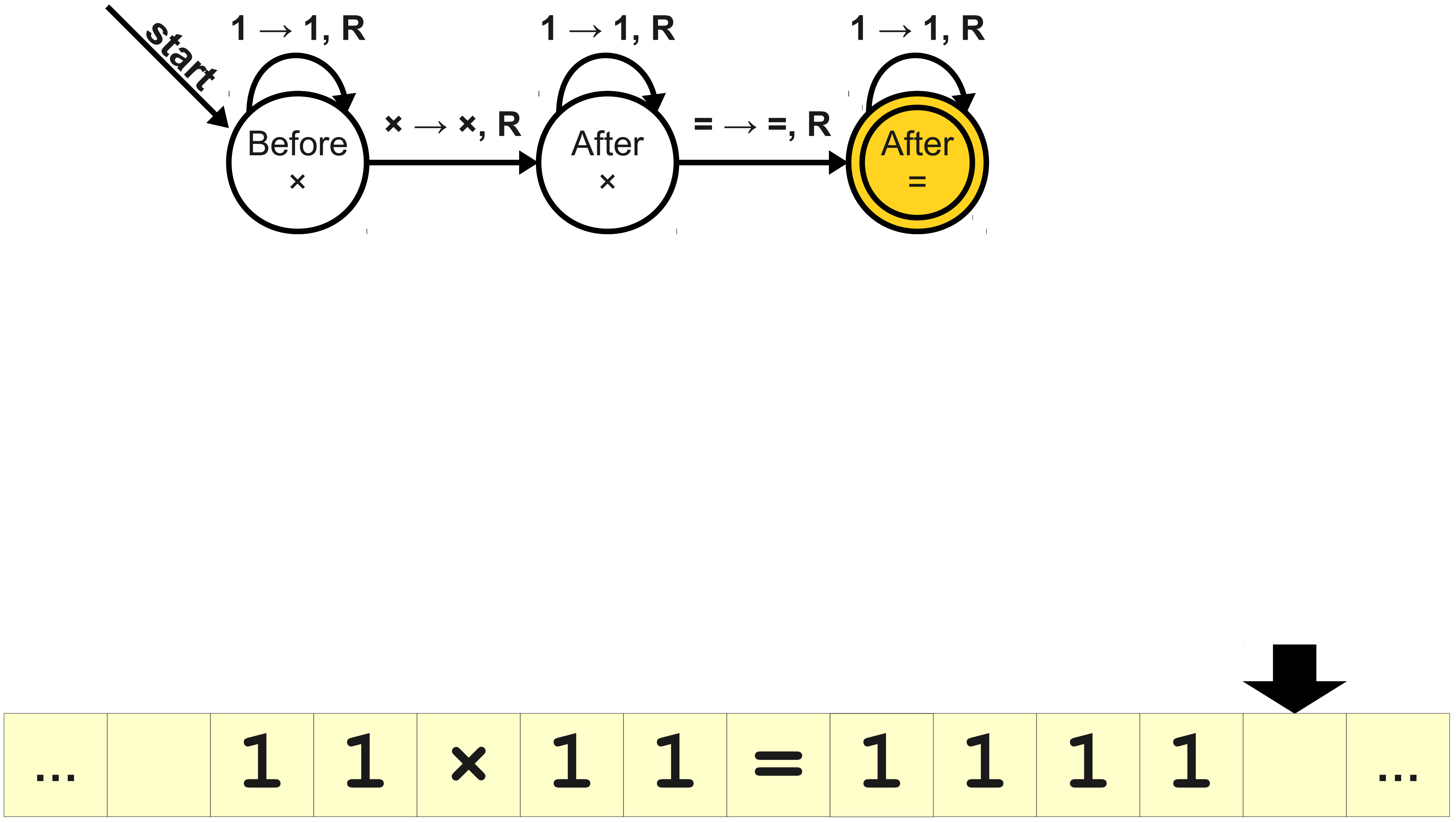


Checking for $\mathtt{1}^*\mathtt{\times 1}^*\mathtt{=1}^*$

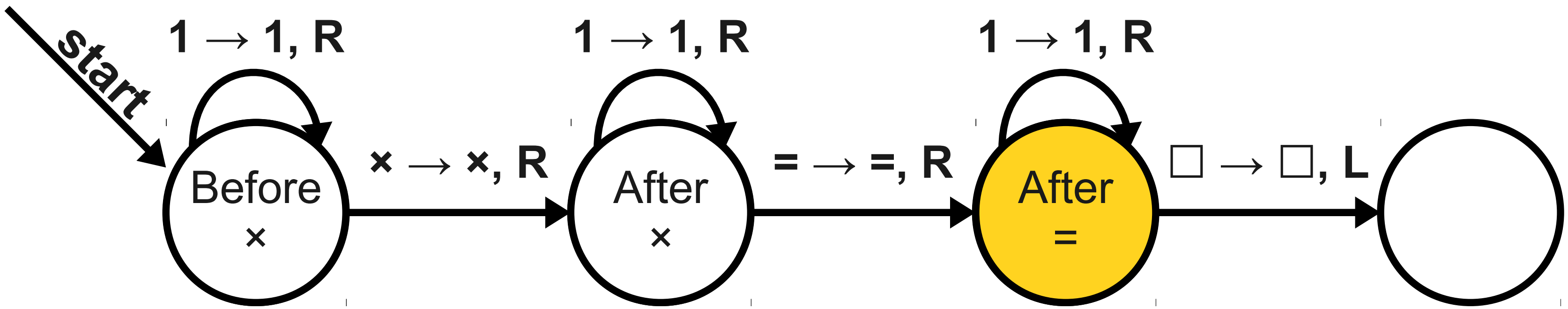


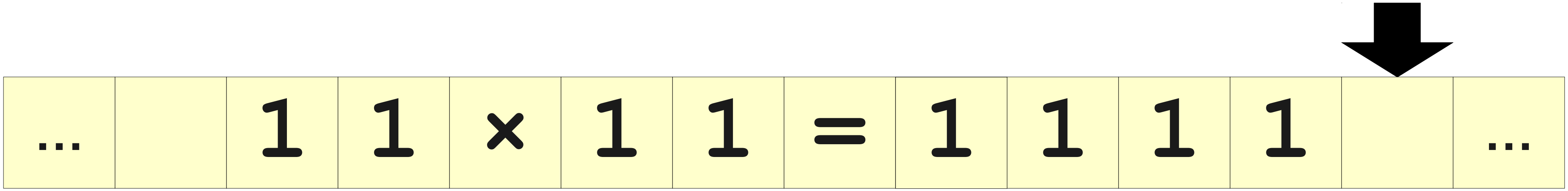


Checking for $1^*\times1^*=1^*$

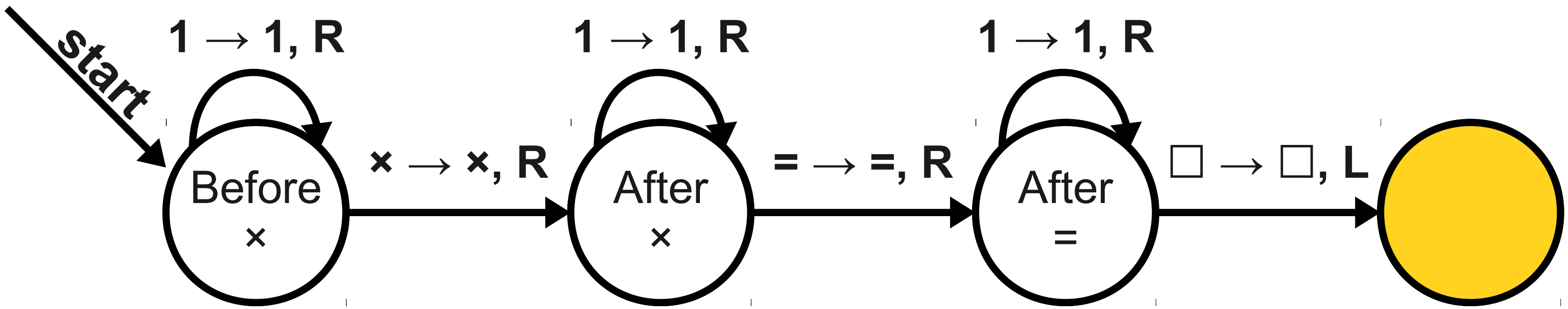


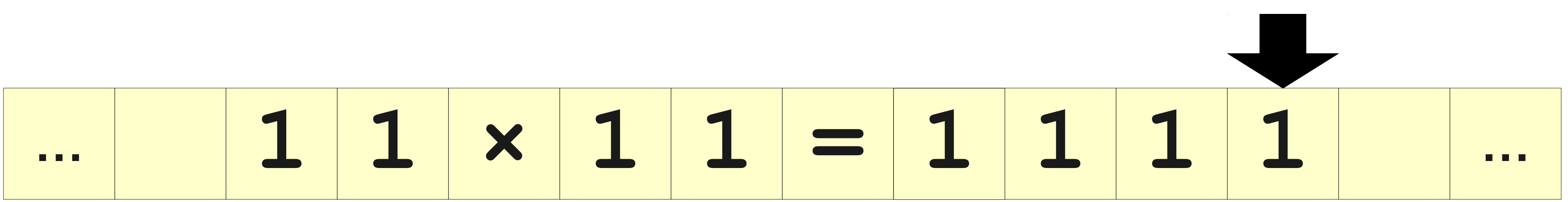
Checking for $1^*\times1^*=1^*$



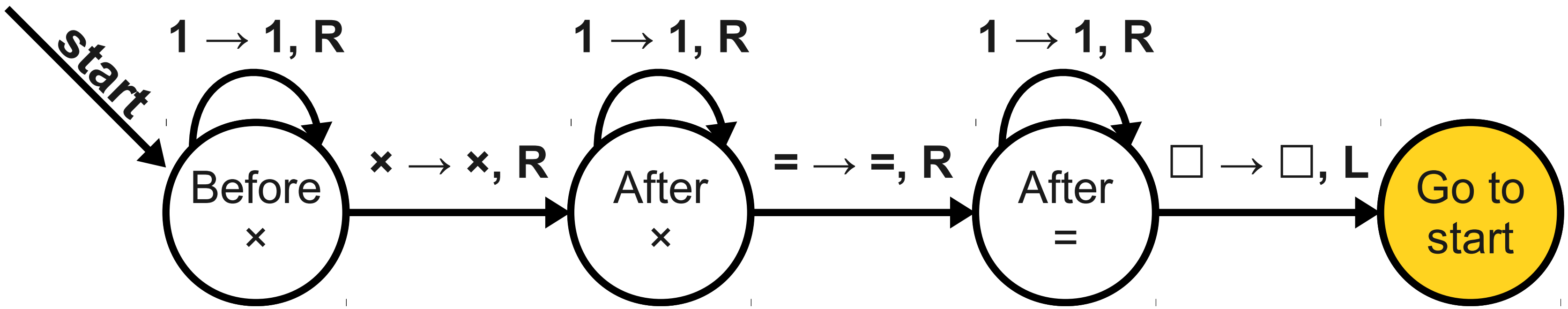


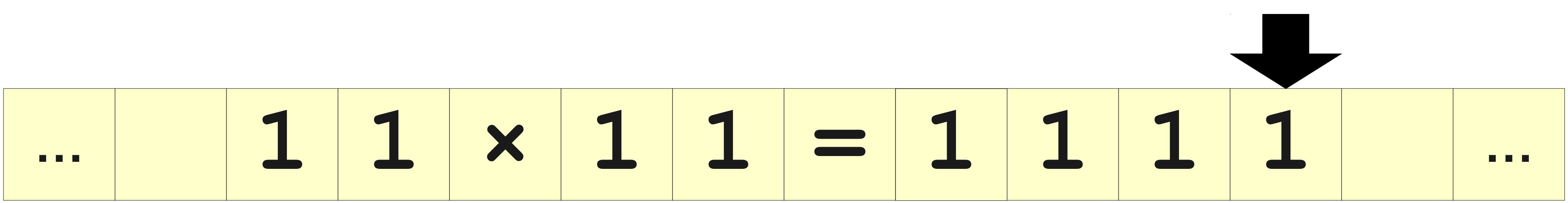
Checking for $1^*\times 1^* = 1^*$





Checking for $1^*\times 1^*=1^*$





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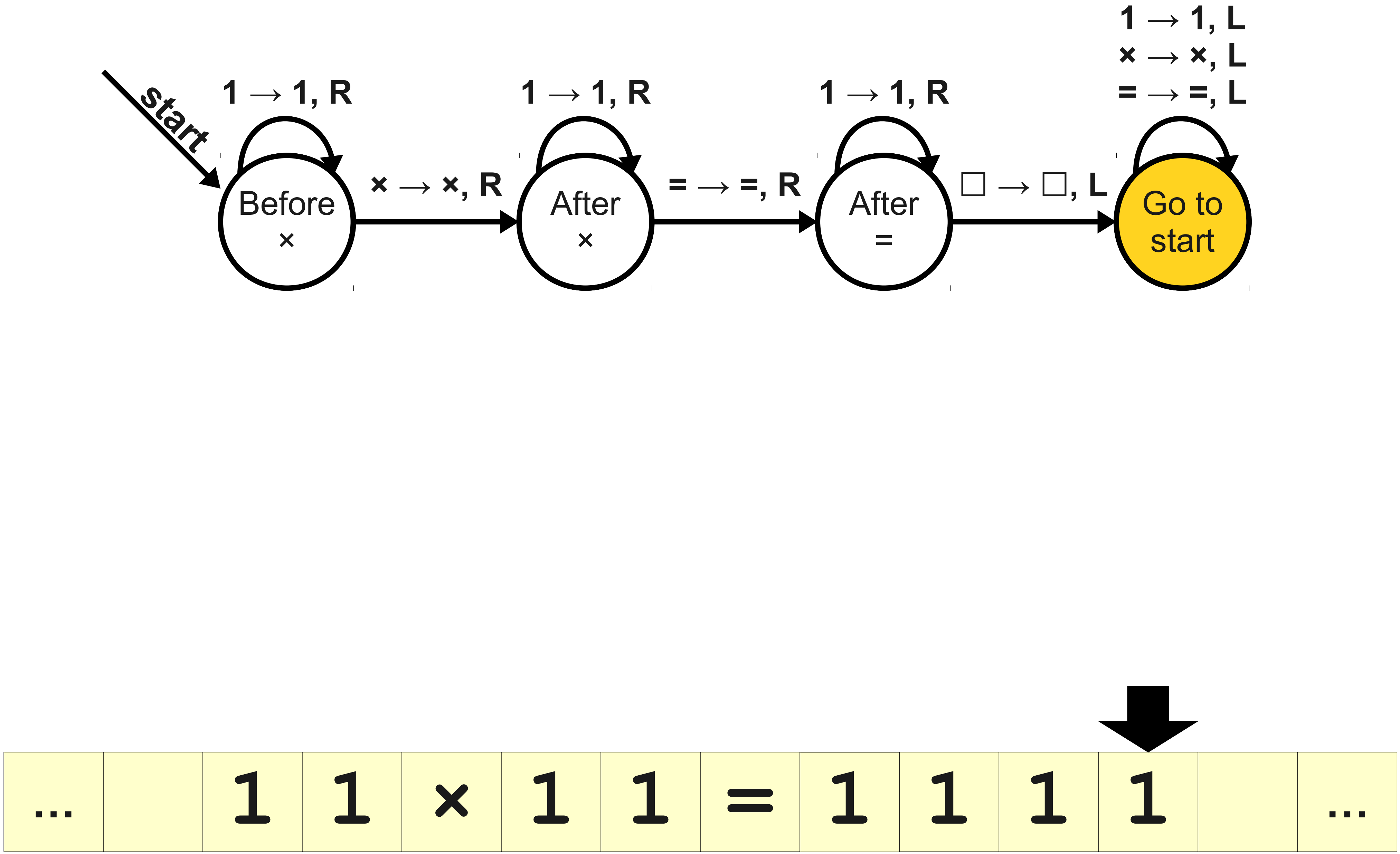

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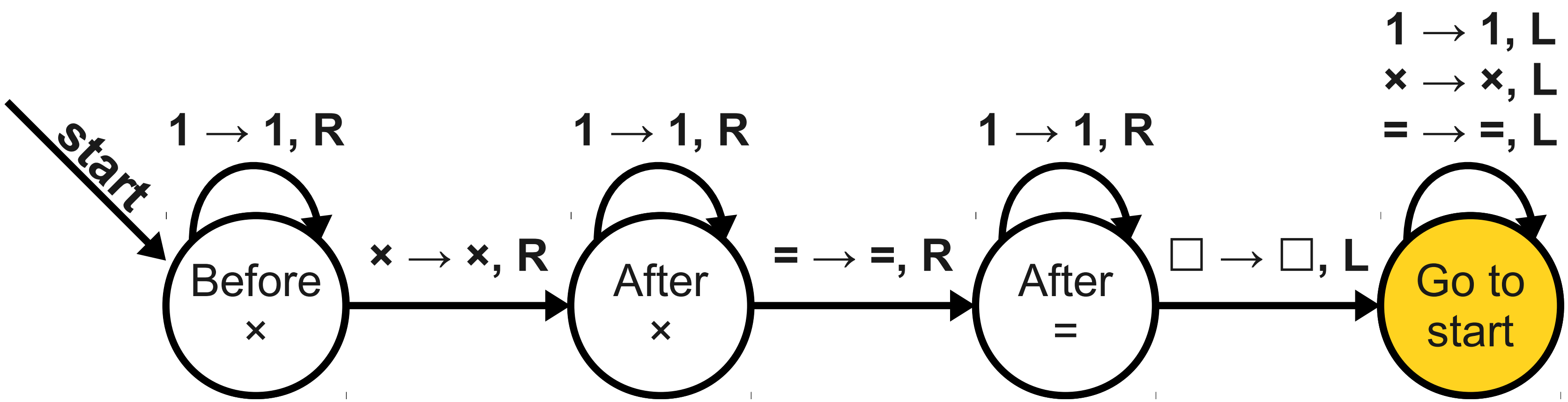

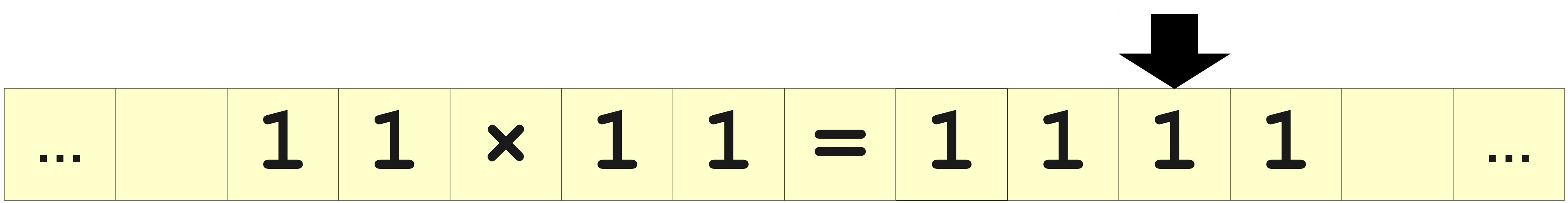

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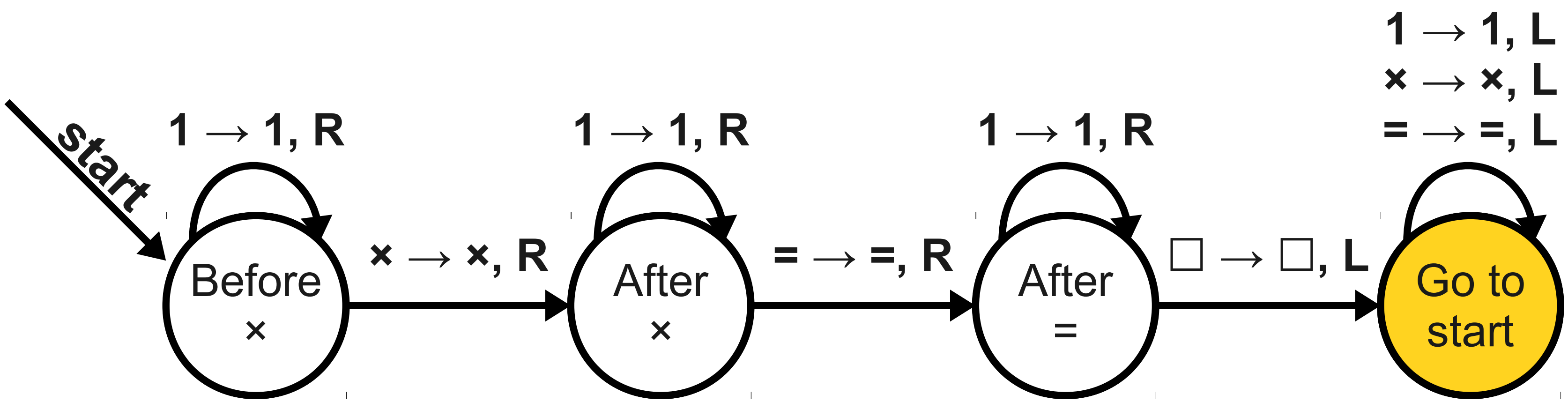

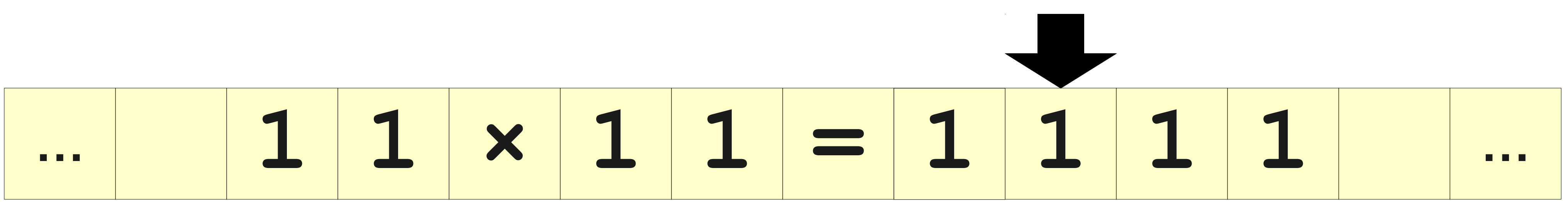

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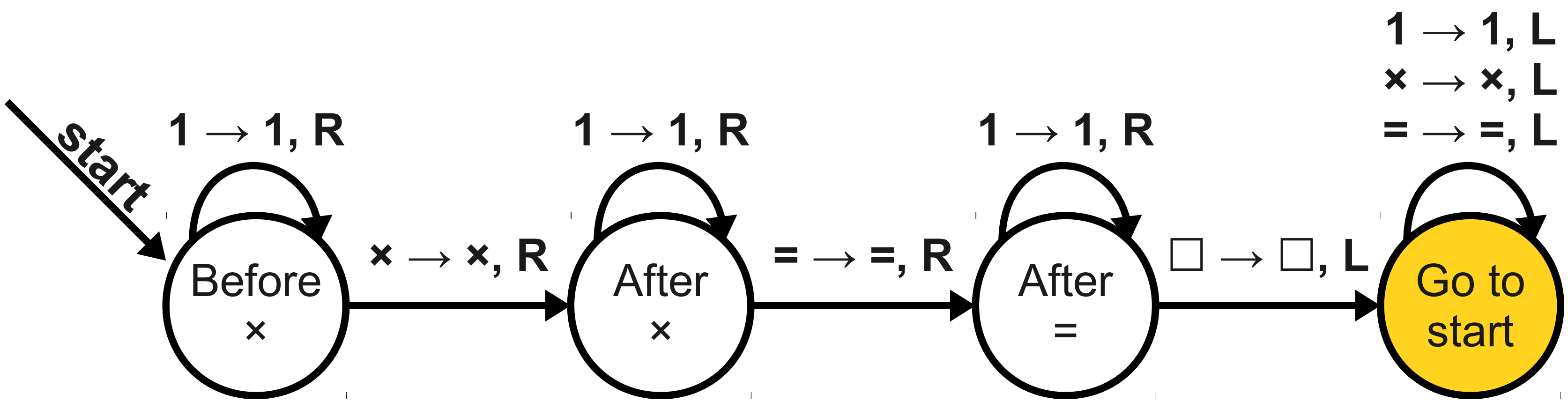

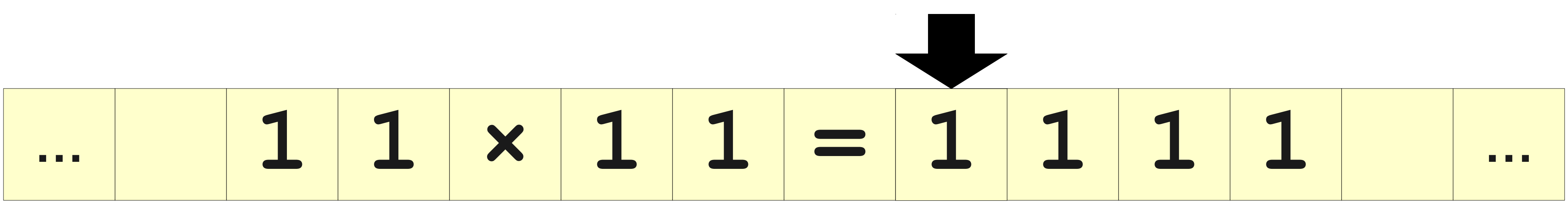

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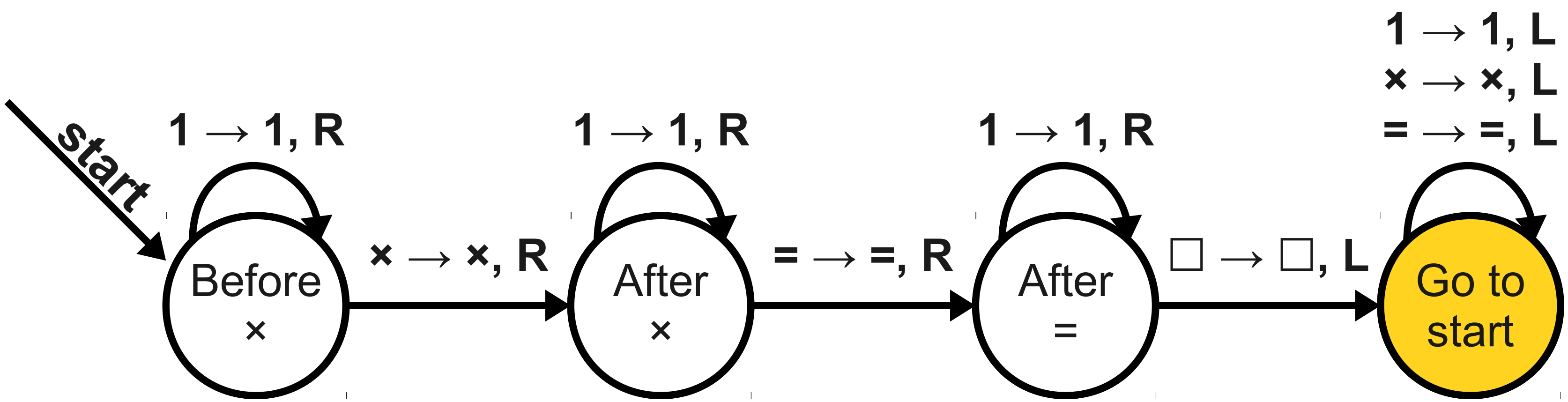

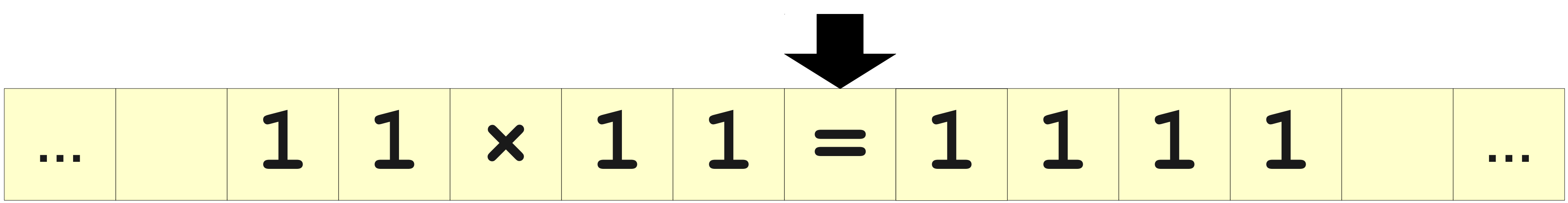

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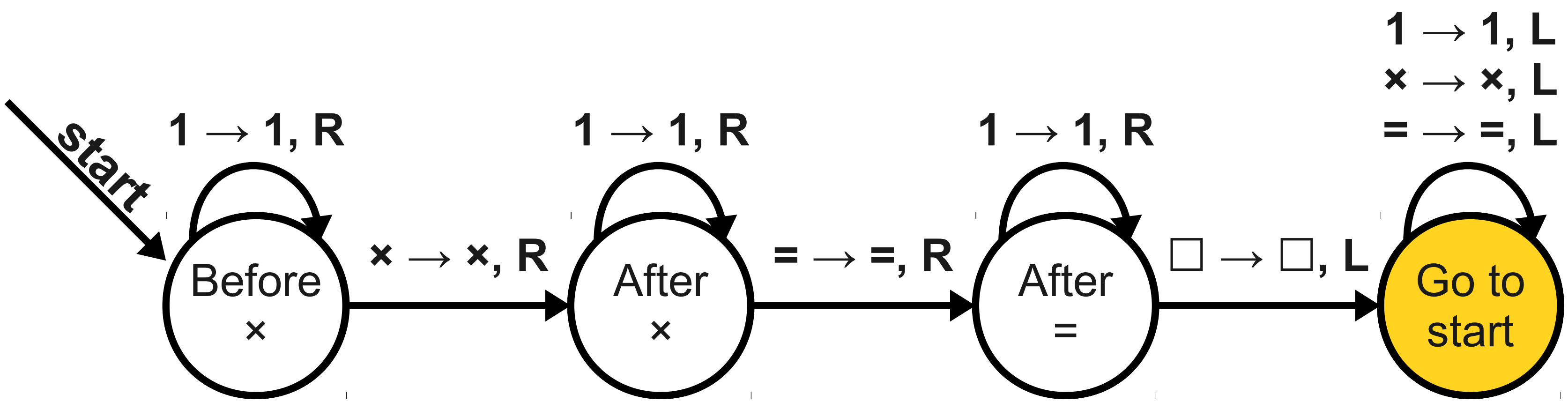

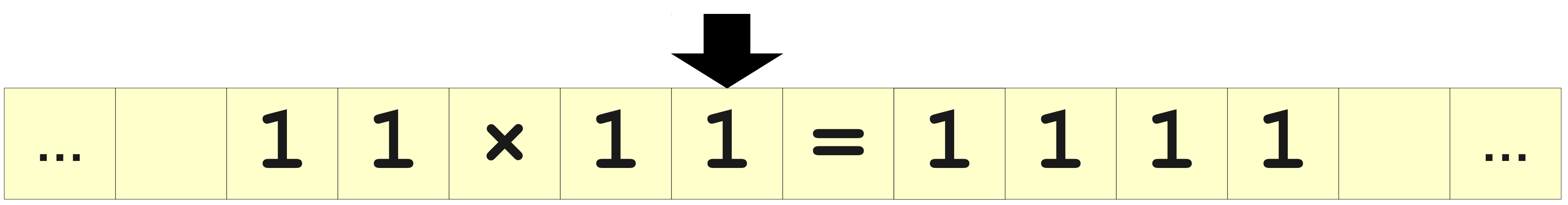

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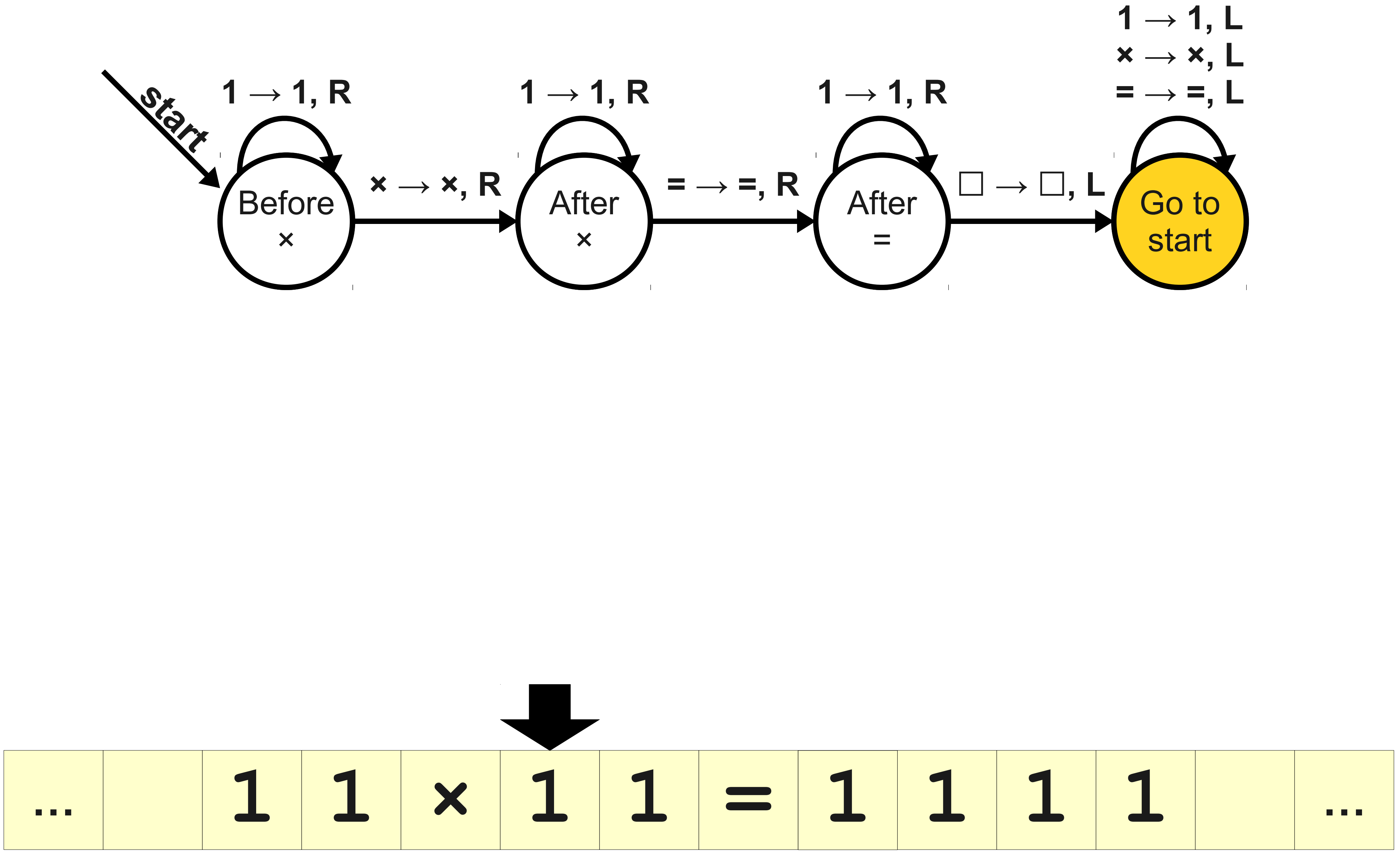

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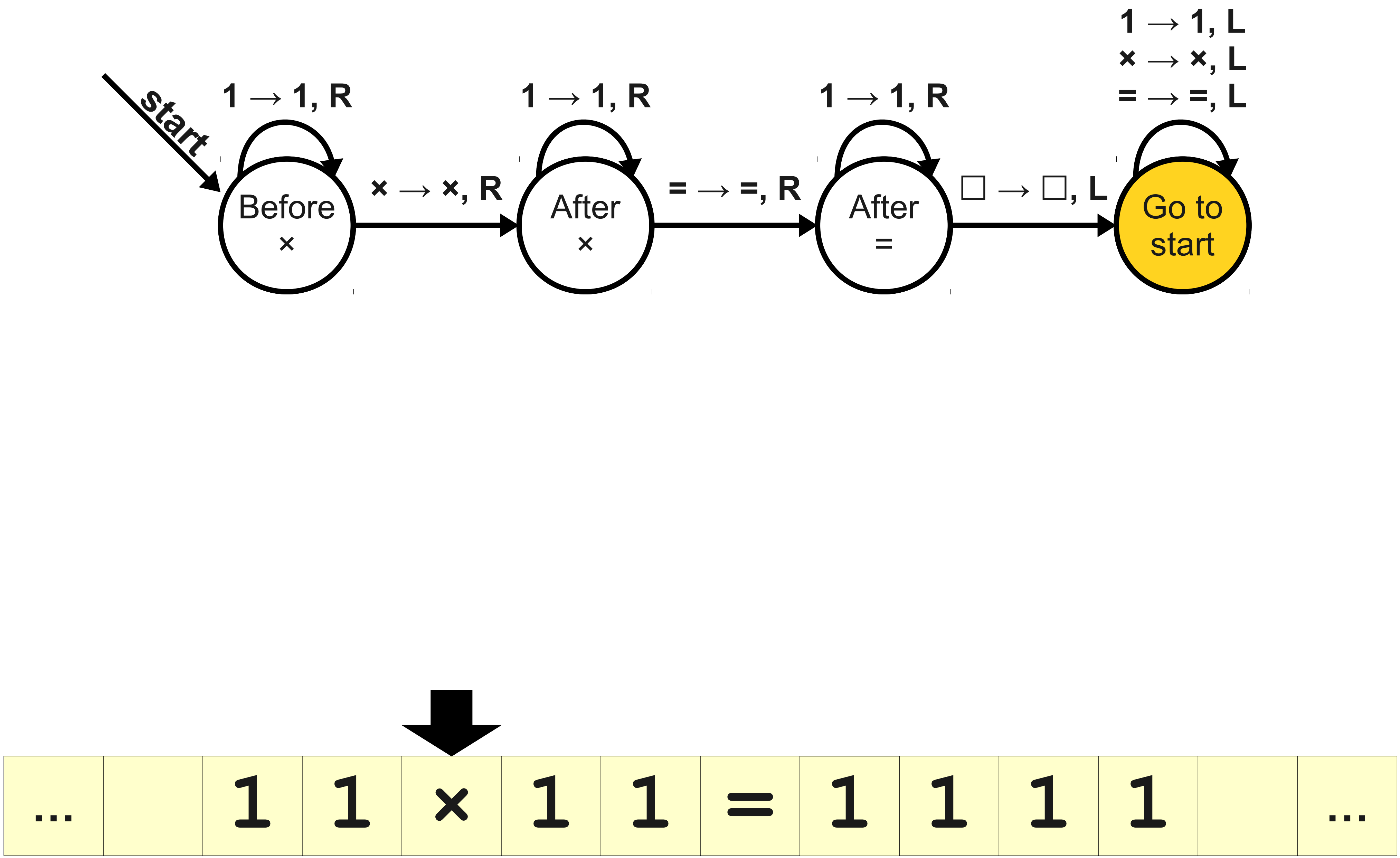

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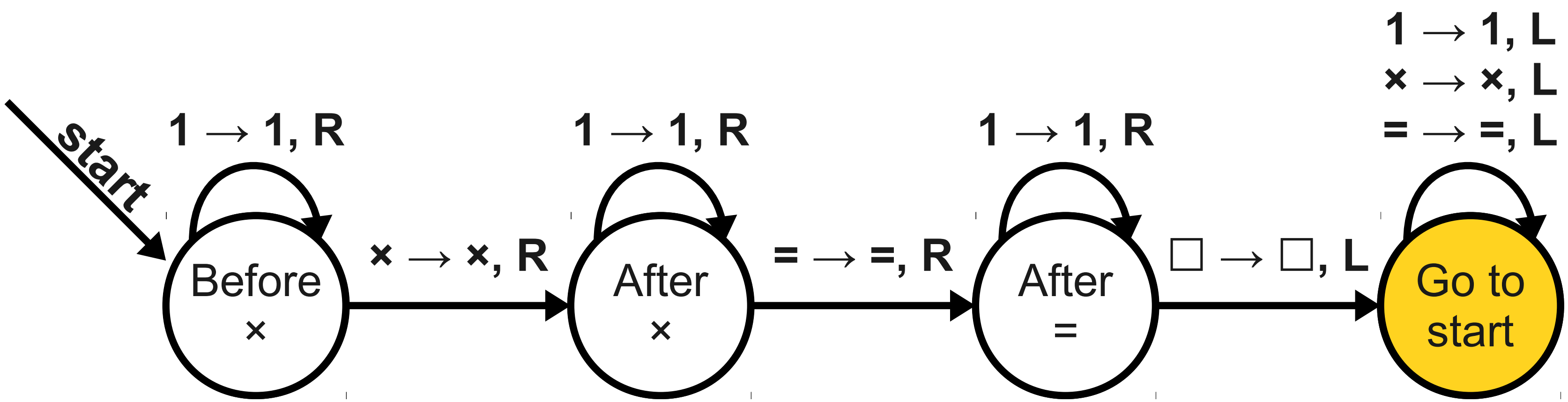

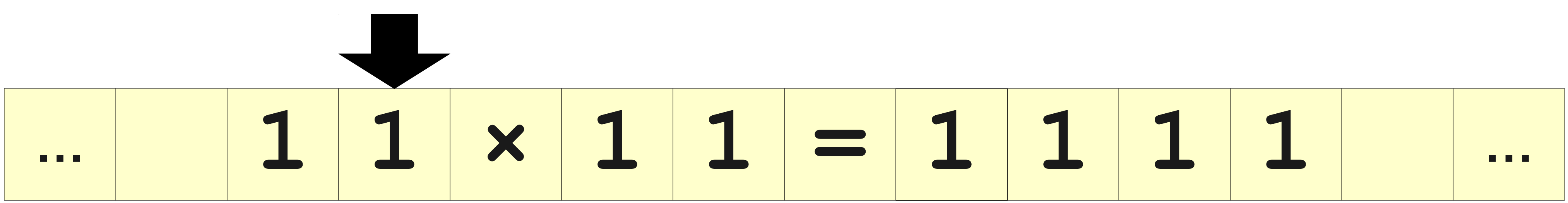

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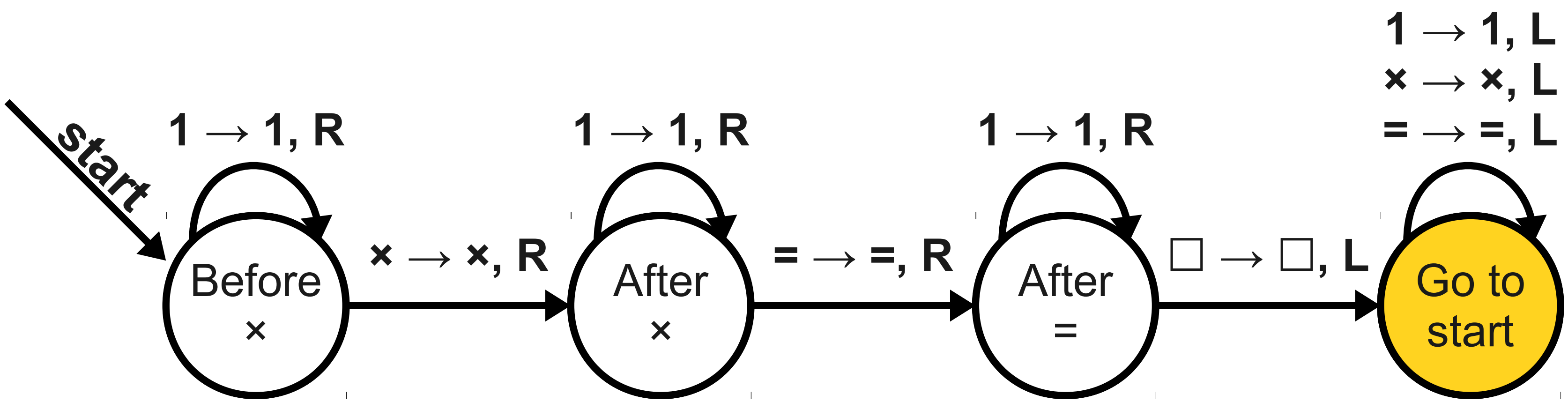

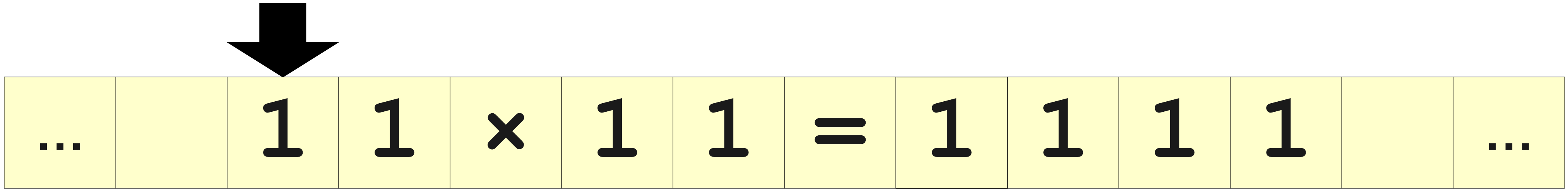

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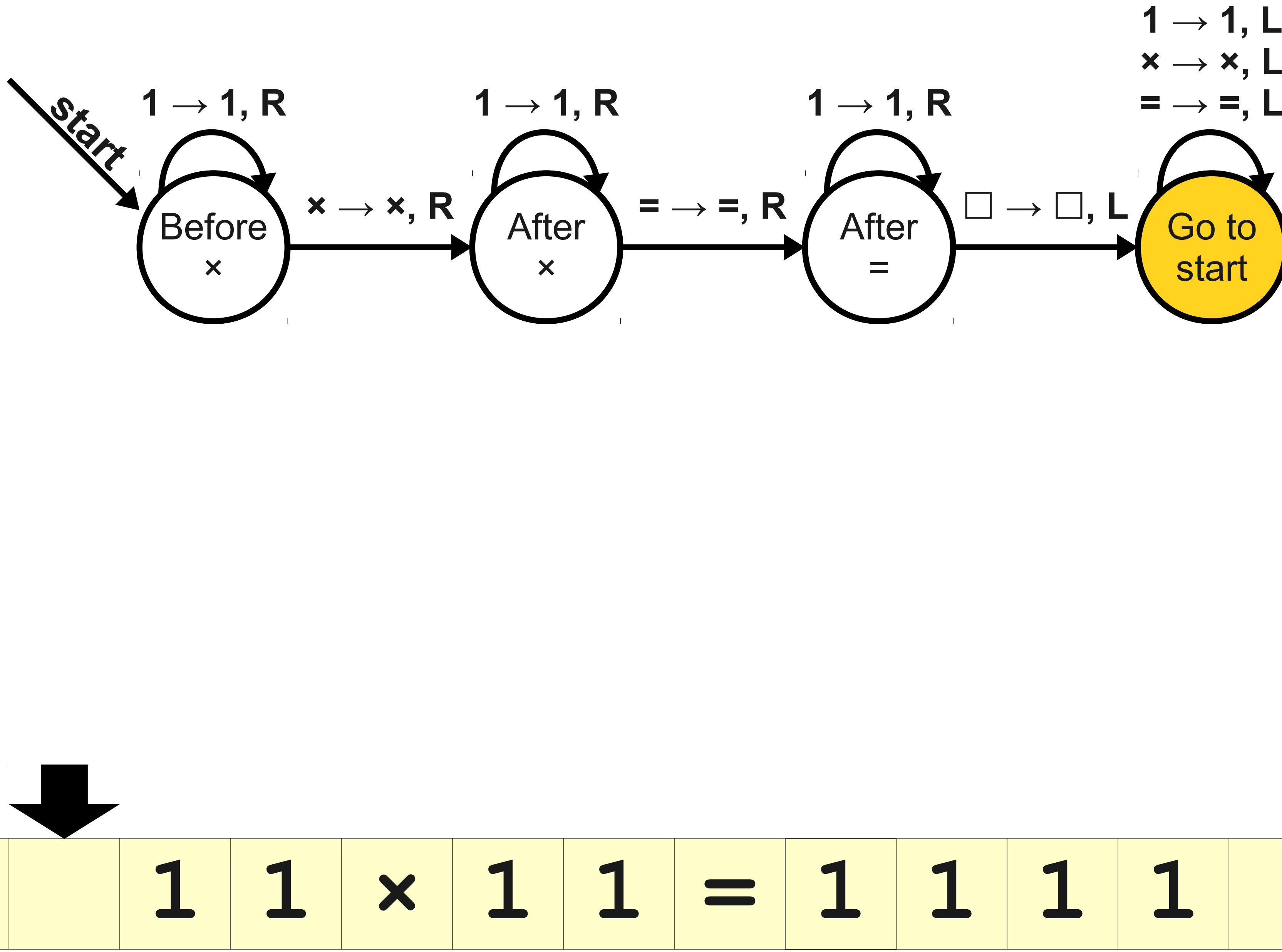

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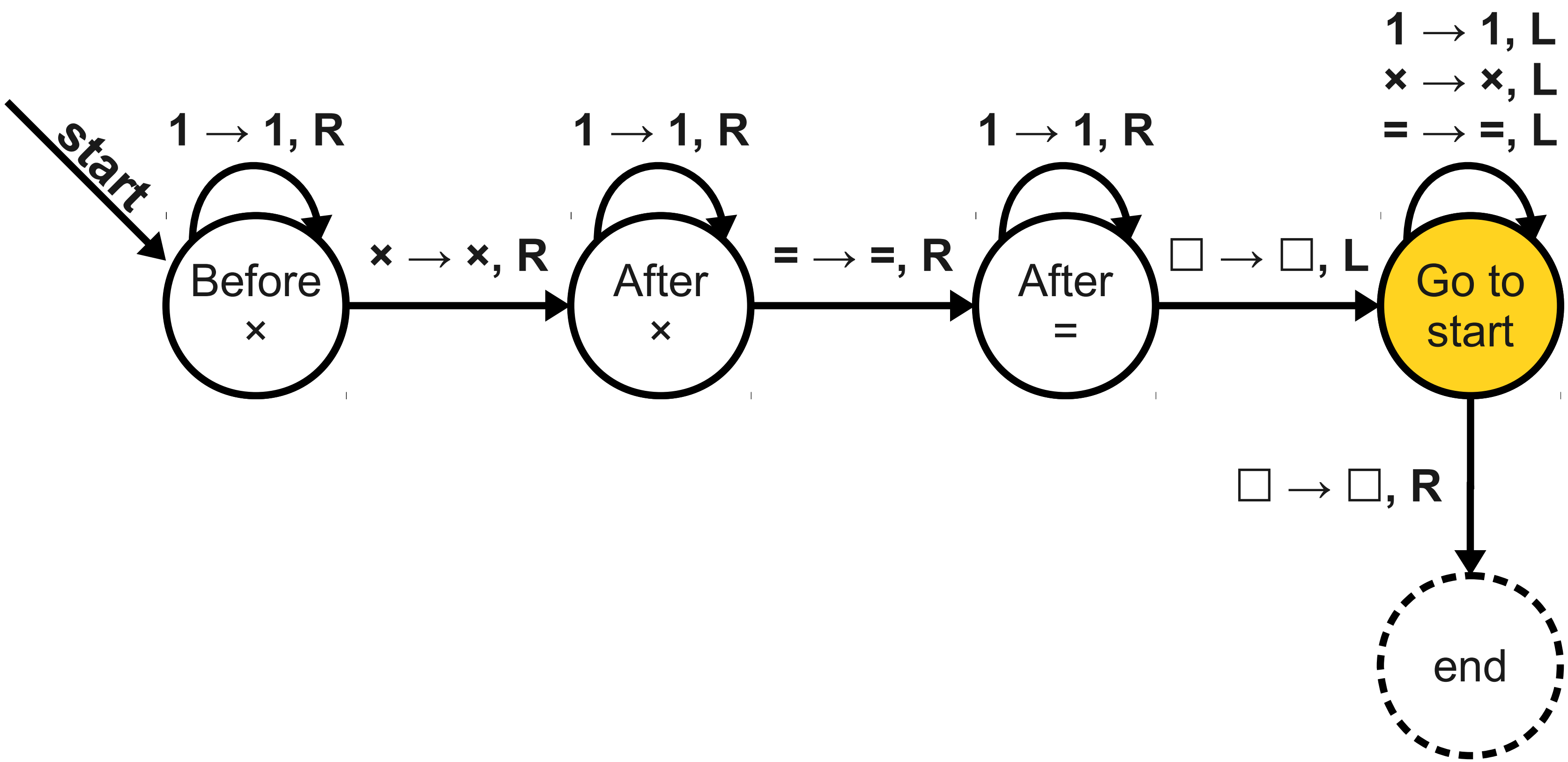

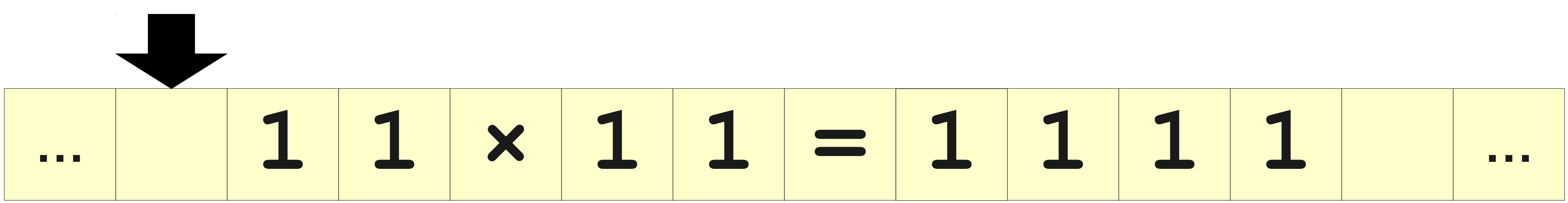

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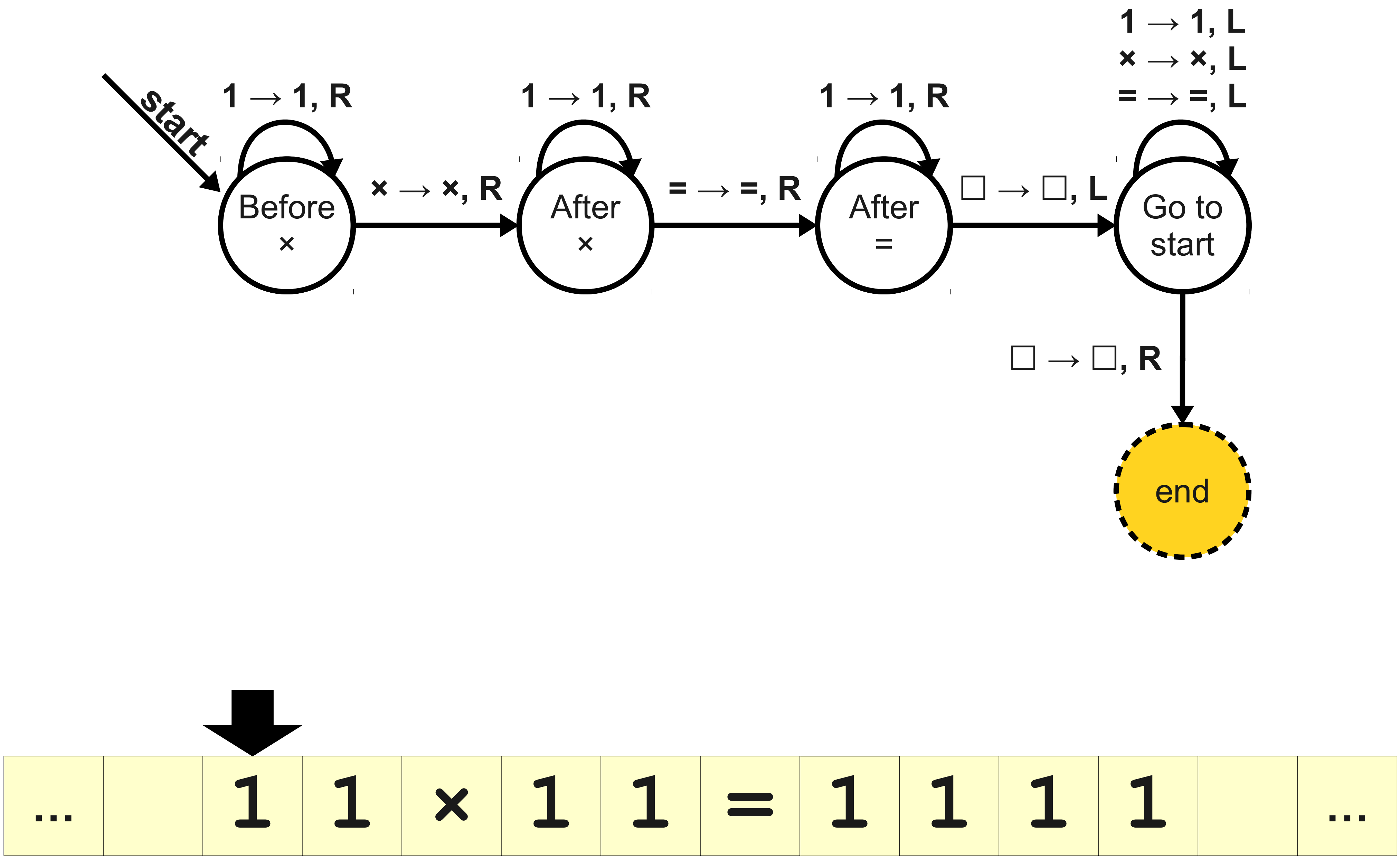

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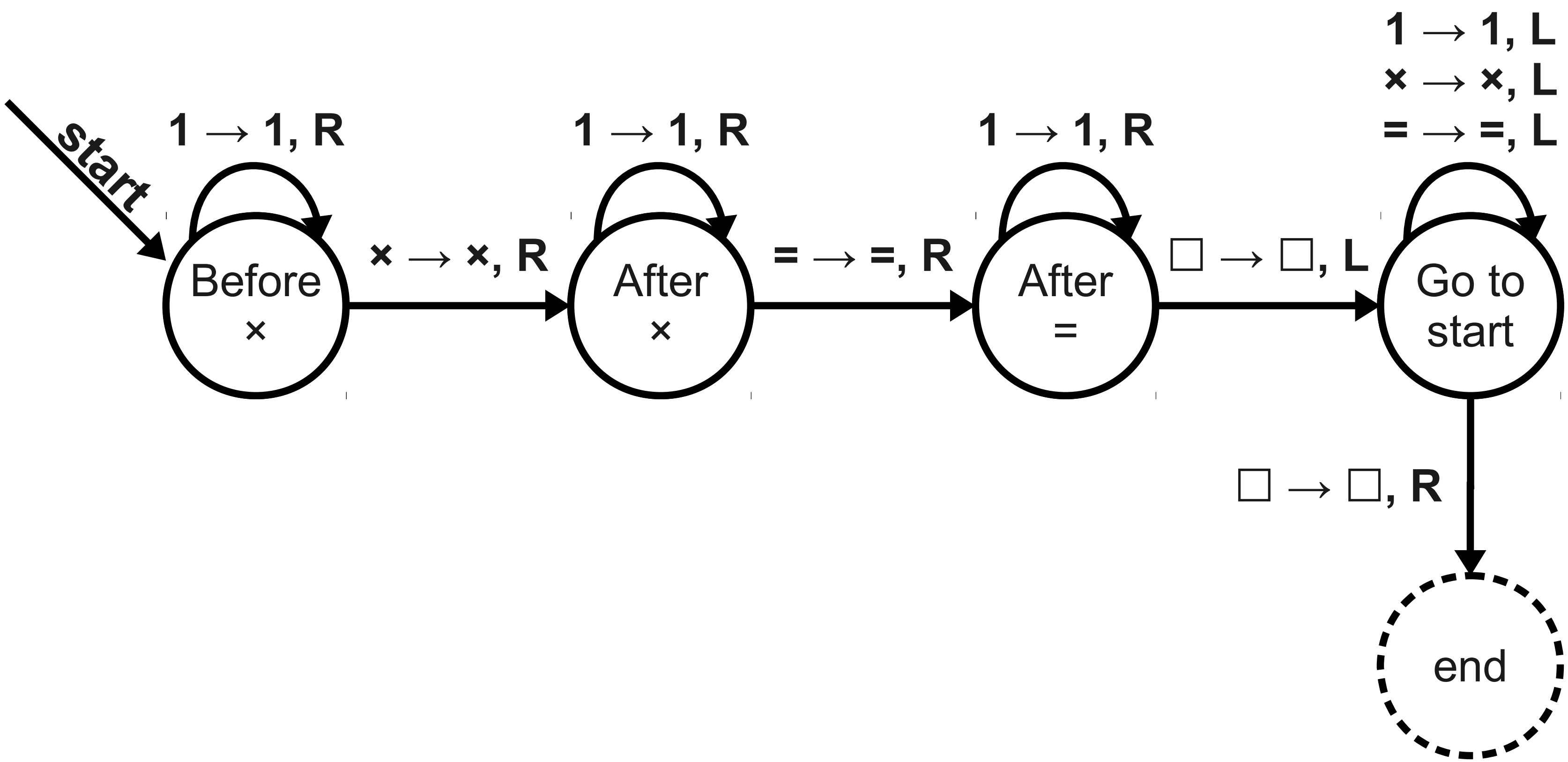
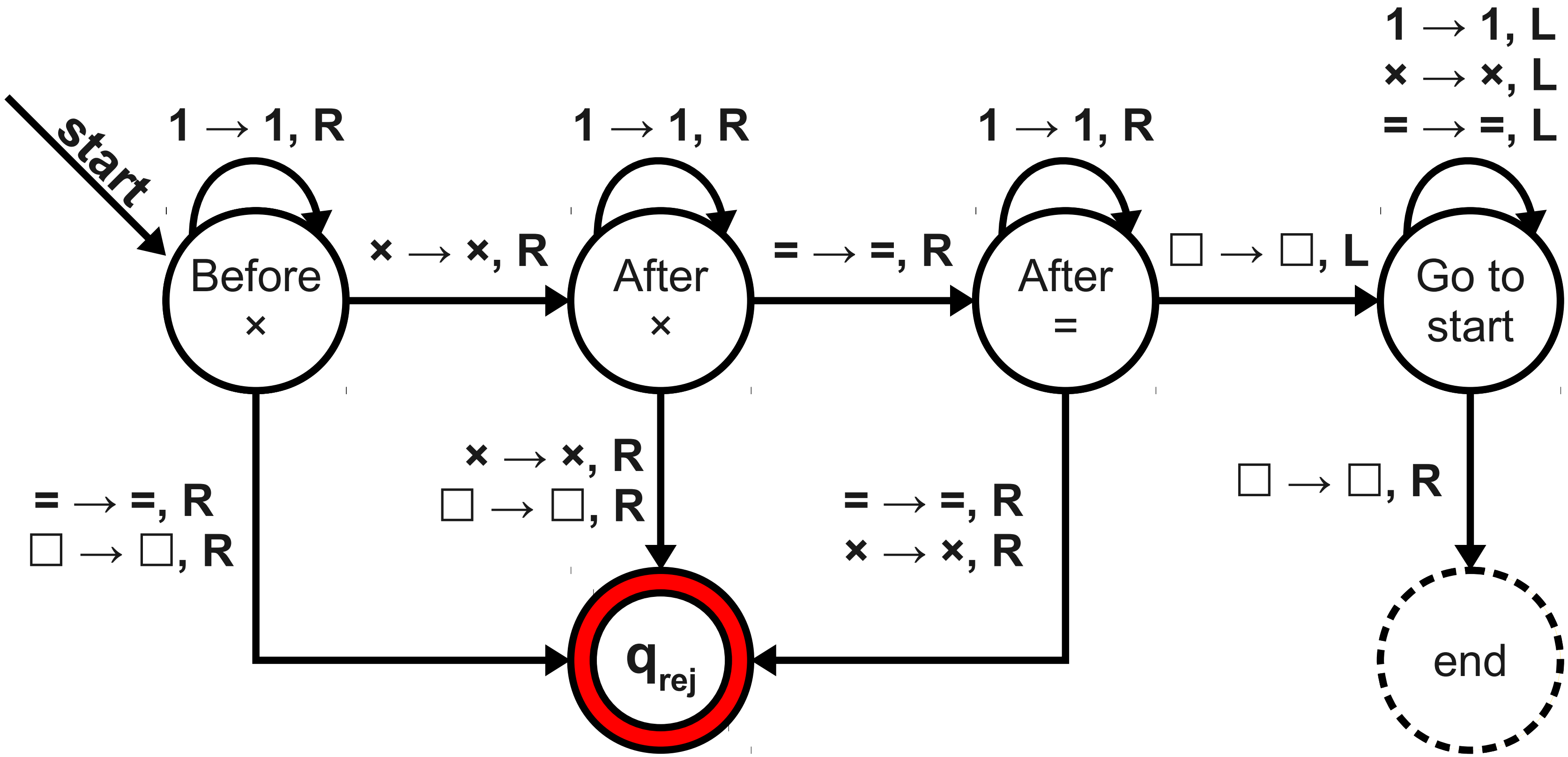


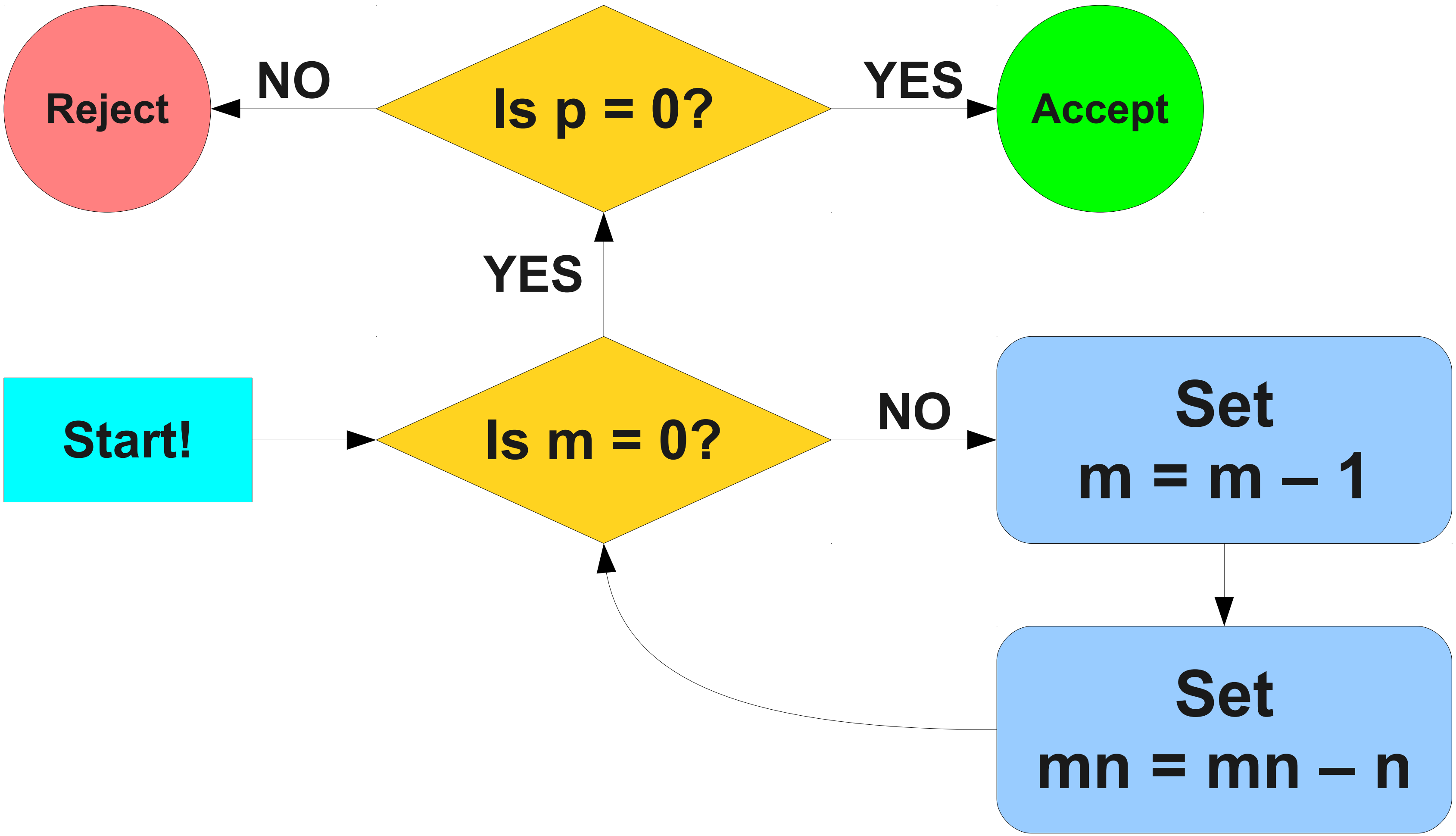
Checking for $\mathbf{1^* \times 1^* = 1^*}$



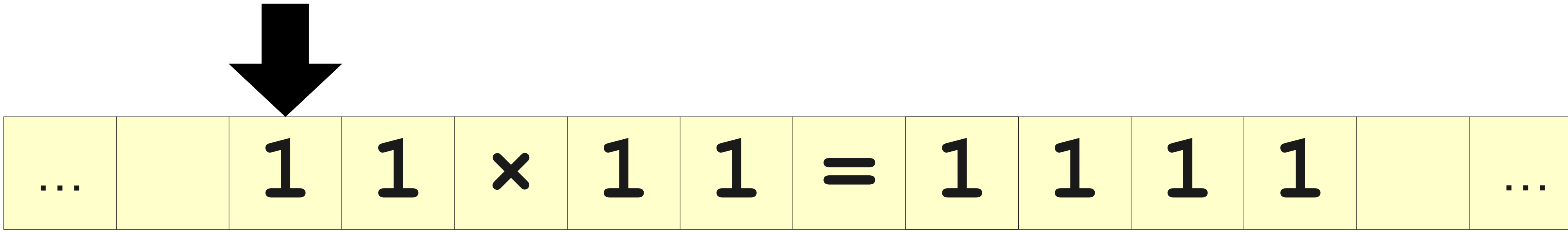
Multiplication Via TMs

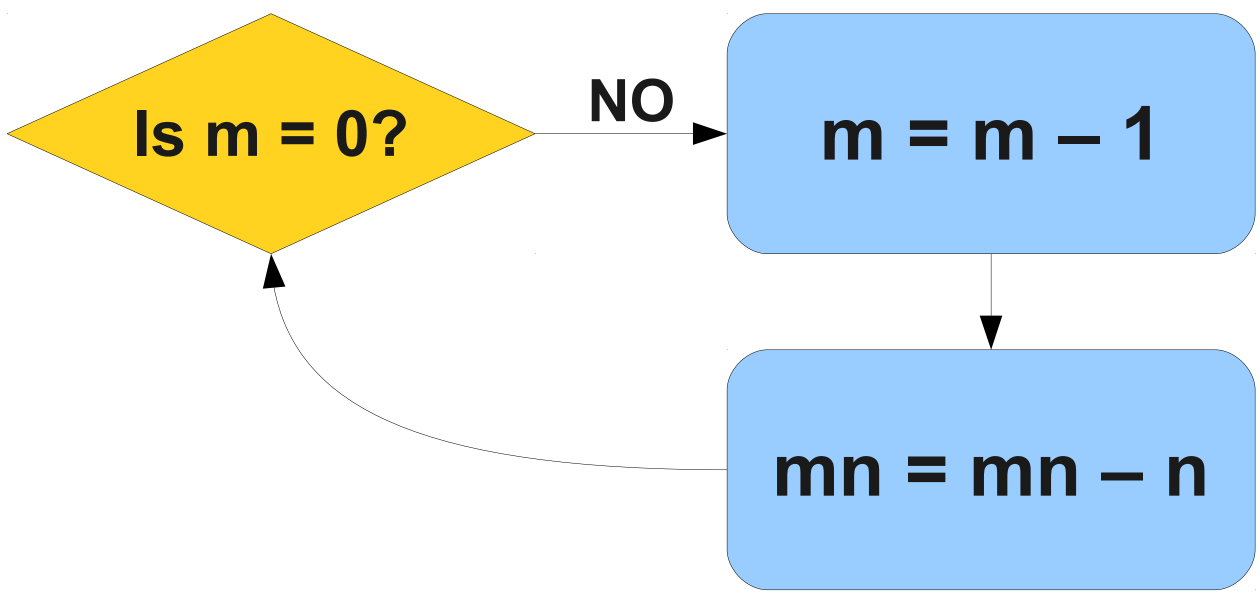
- Now that we can check that the input is valid, how do we confirm the math is right?
- **Idea**: Use a recursive formulation of multiplication!
- If $m = 0$, then $m \times n = 0$.
- If $m > 0$, then $m \times n = n + (m - 1) \times n$.
- Our algorithm: Given $\mathtt{1}^m\mathtt{\times}\mathtt{1}^n\mathtt{=}\mathtt{1}^p$:
  - If $m = 0$, accept iff $p = 0$.
  - Otherwise, accept iff $\mathtt{1}^{m-1}\mathtt{\times}\mathtt{1}^n\mathtt{=}\mathtt{1}^{p-n}$ is accepted.

Schematically



A Sketch of the Algorithm





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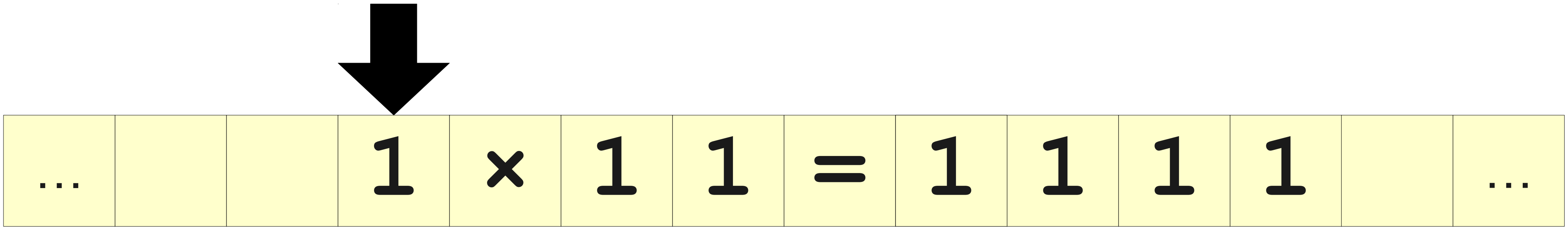

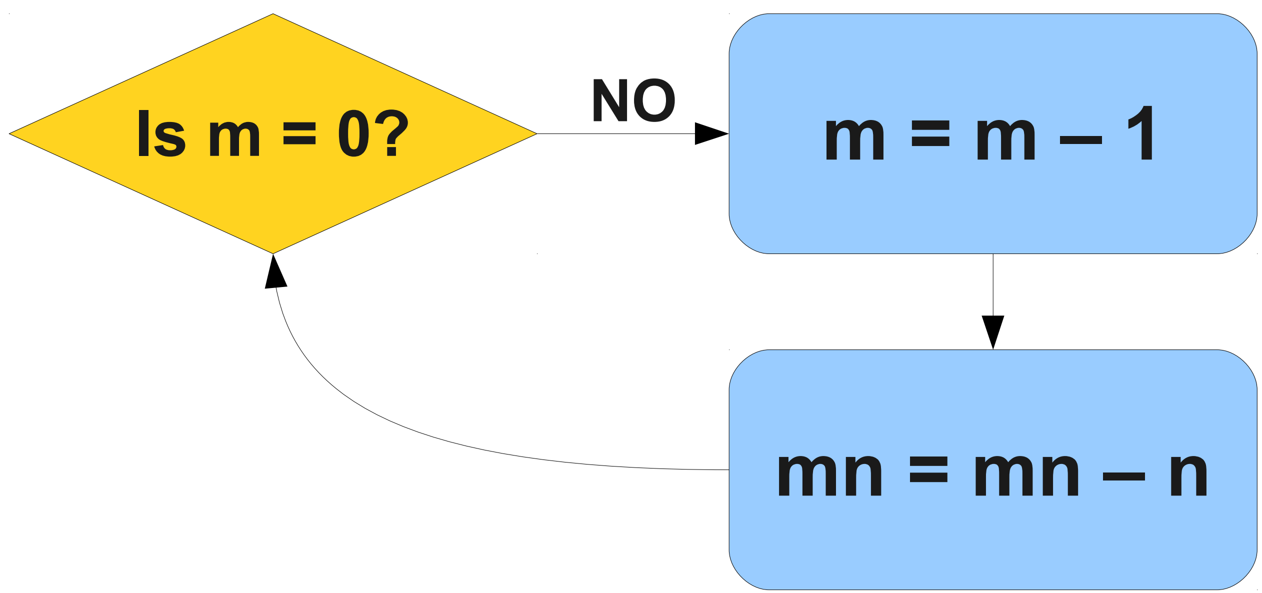

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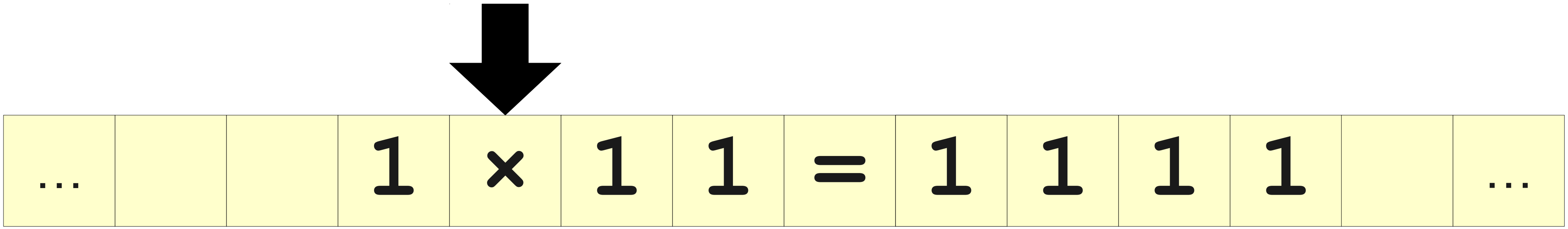

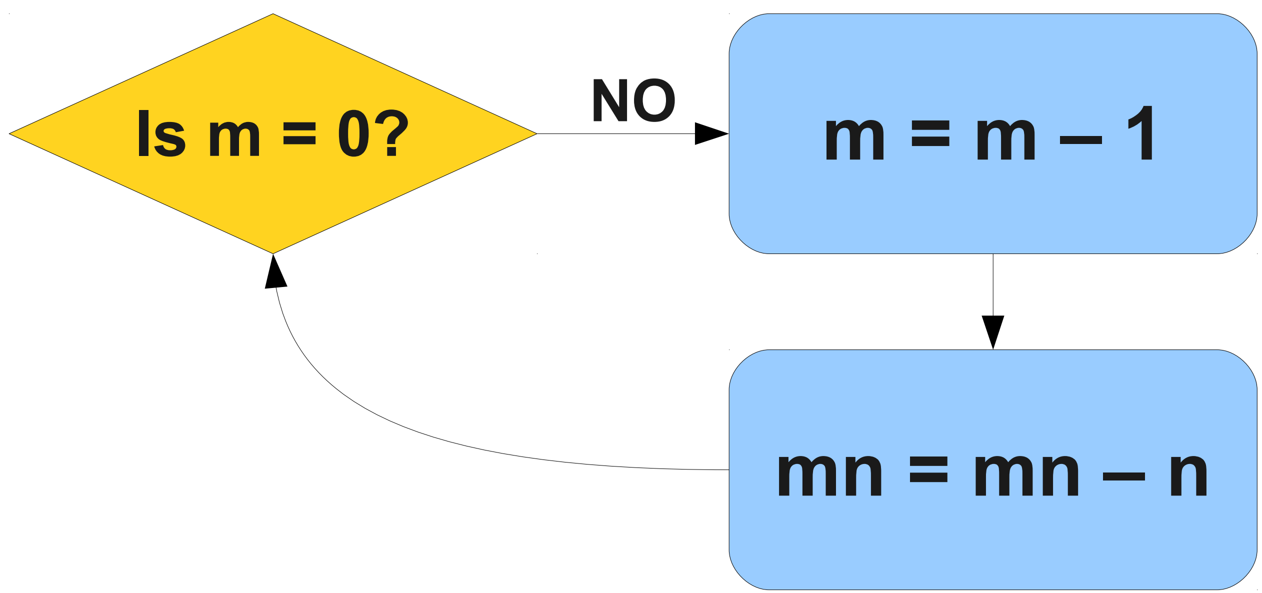

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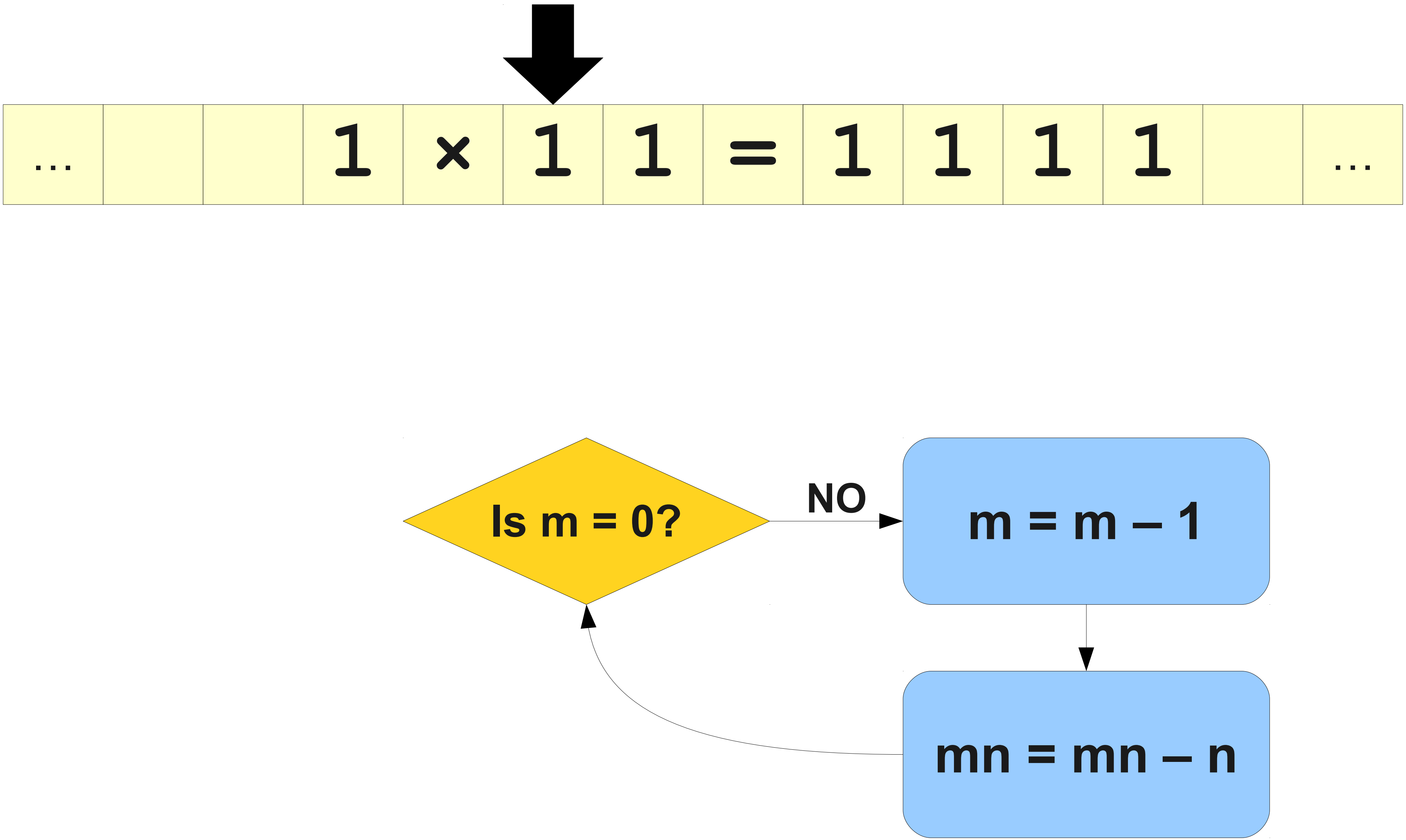

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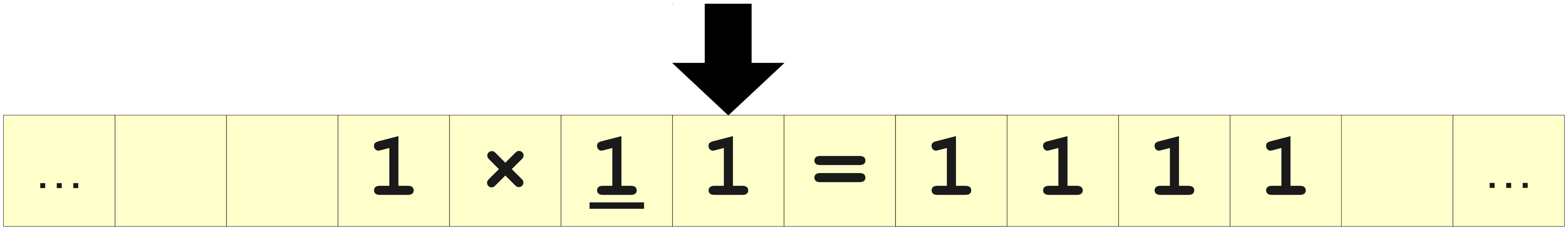

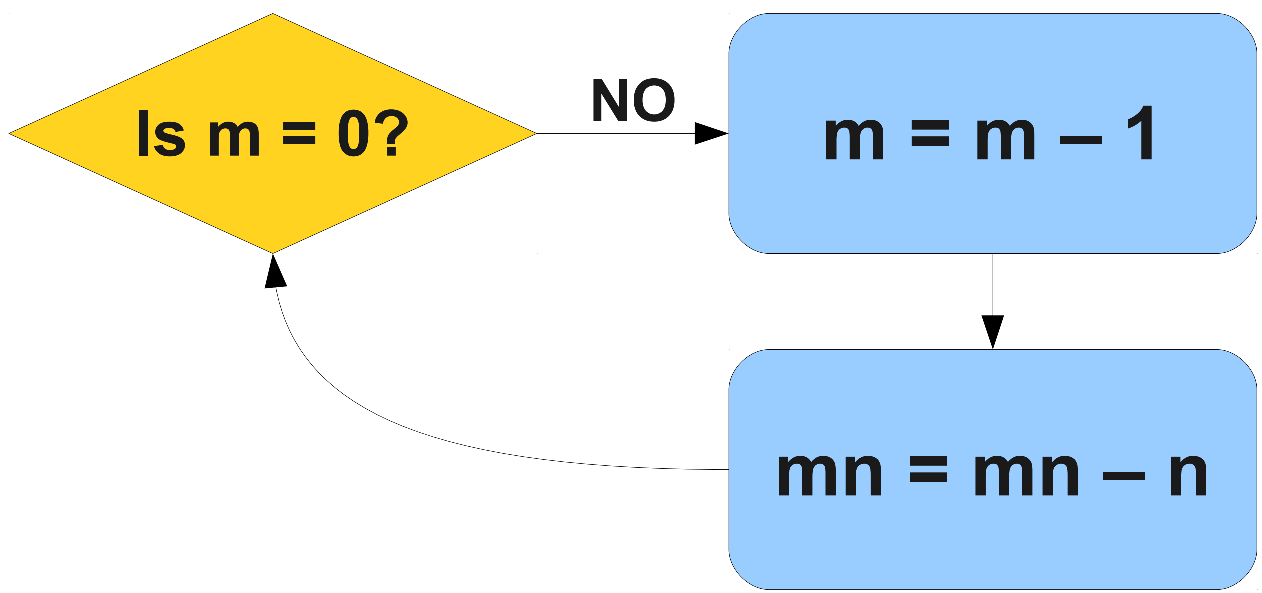

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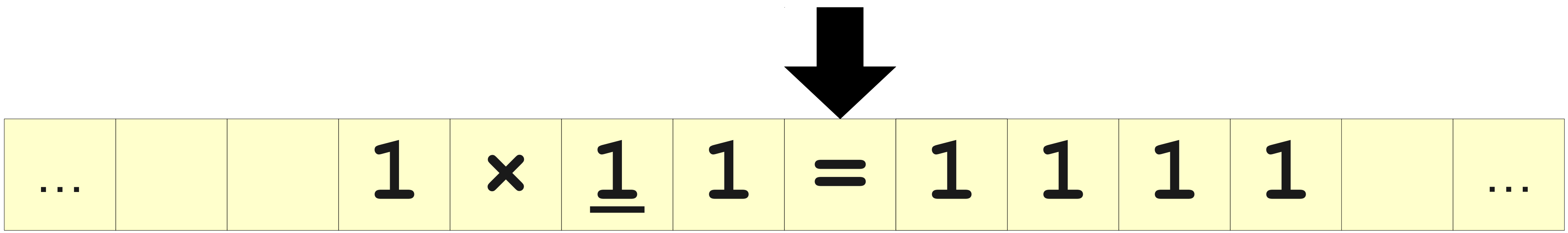

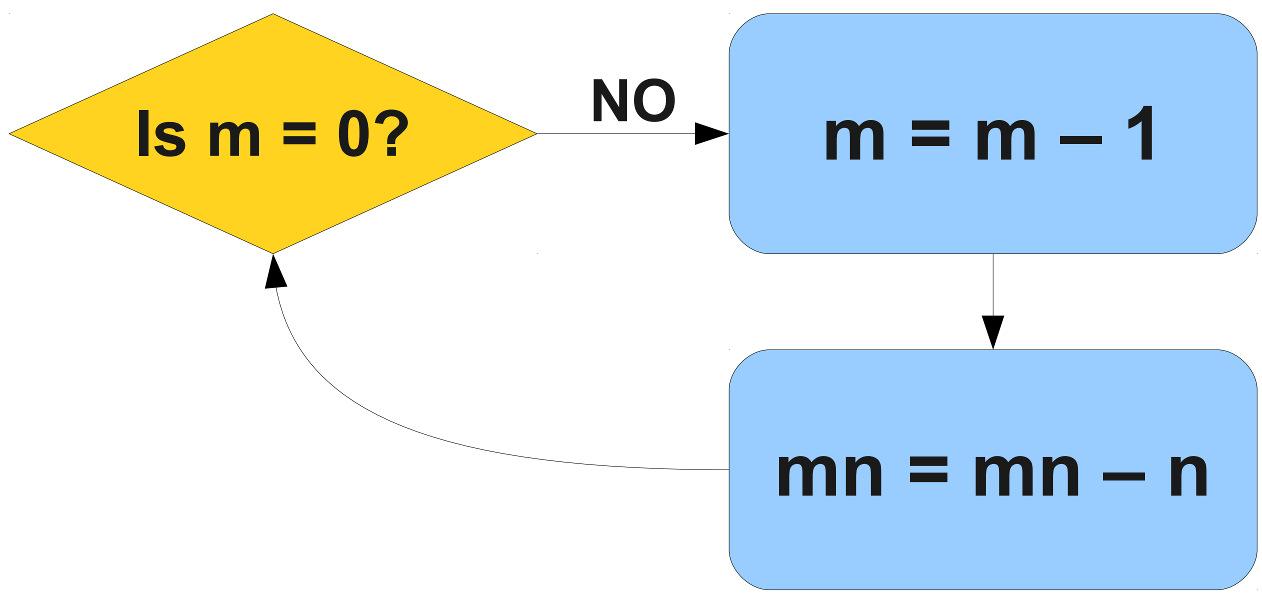

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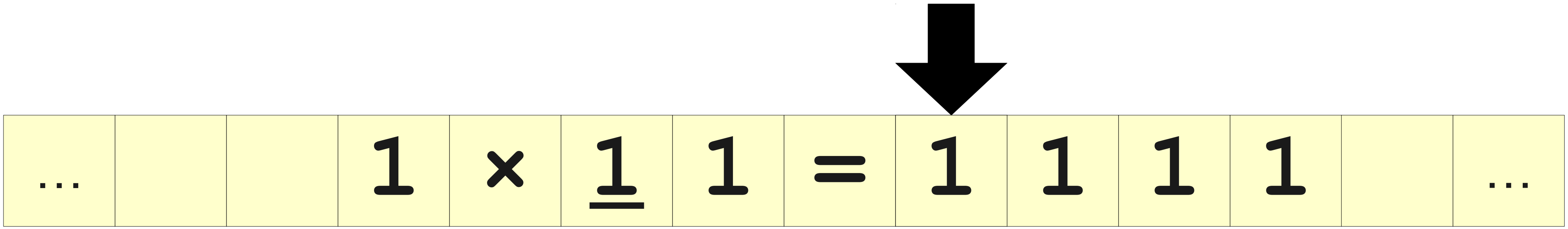

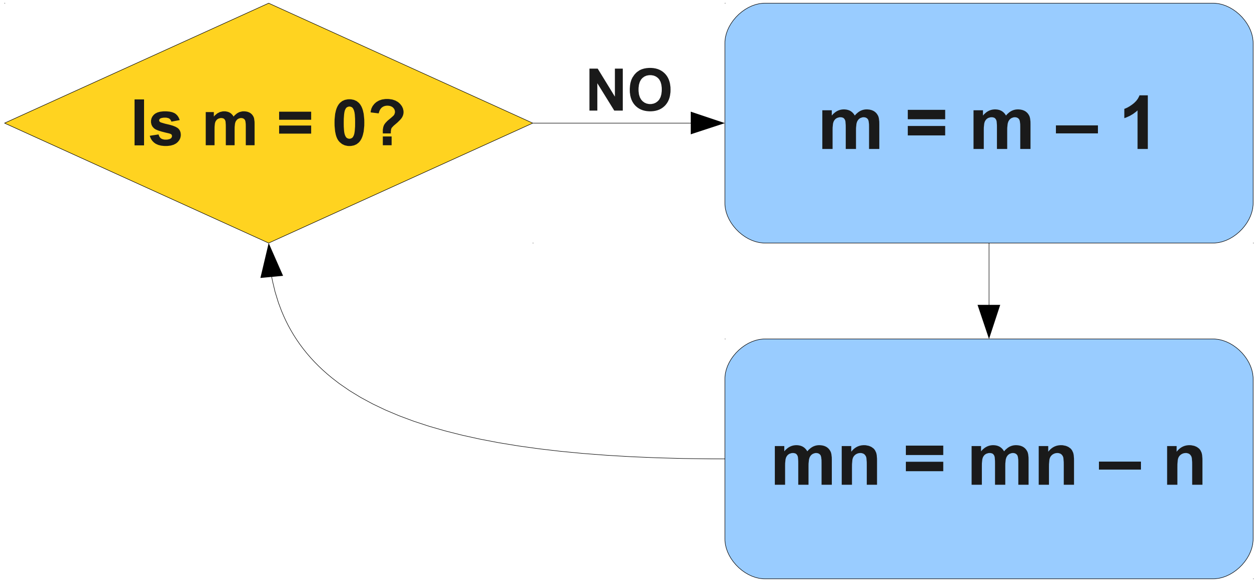

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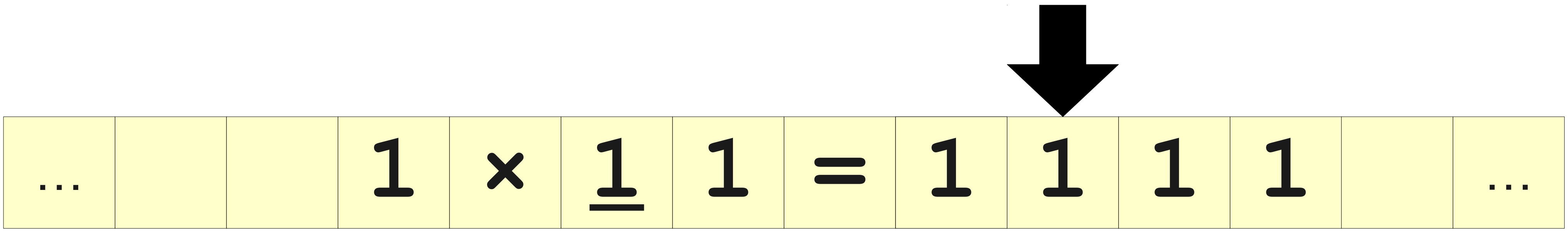

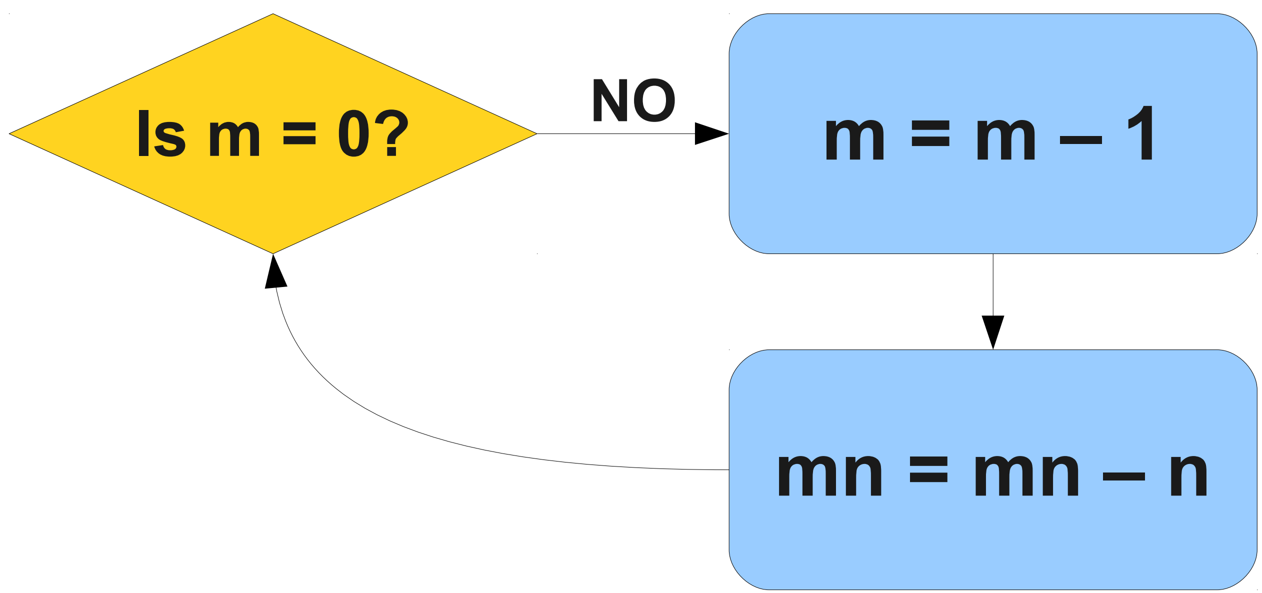

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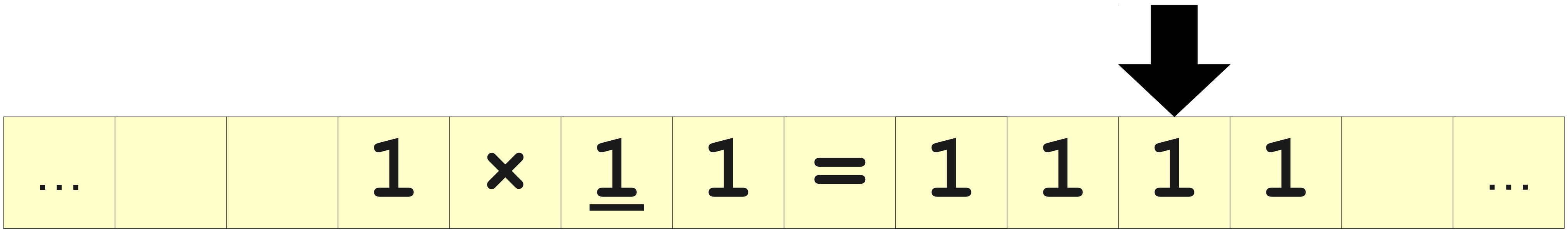

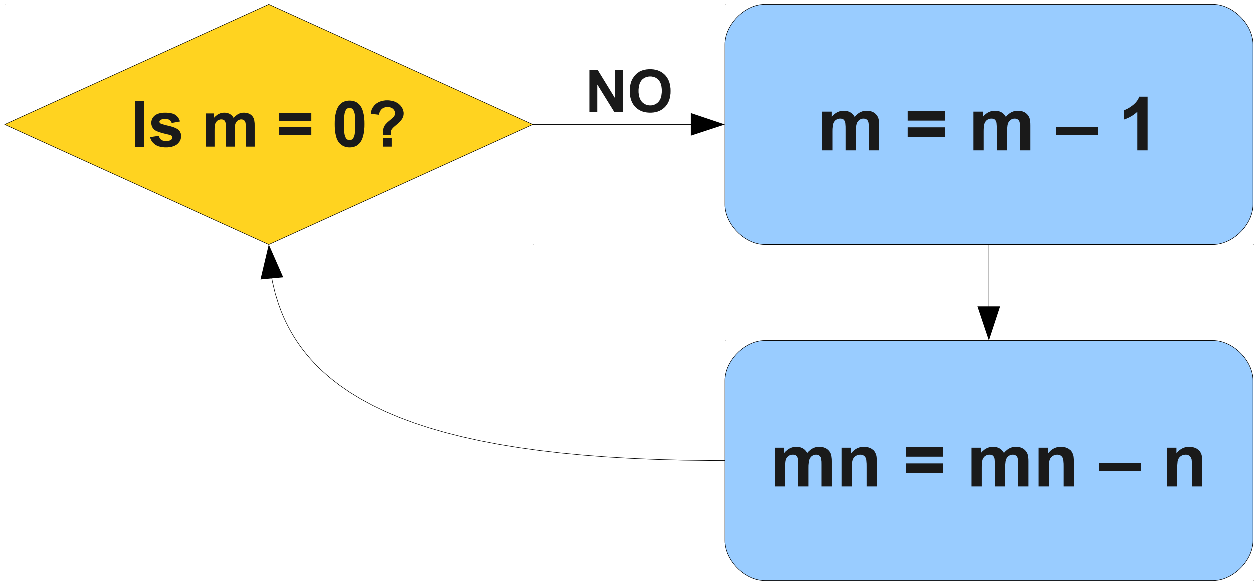

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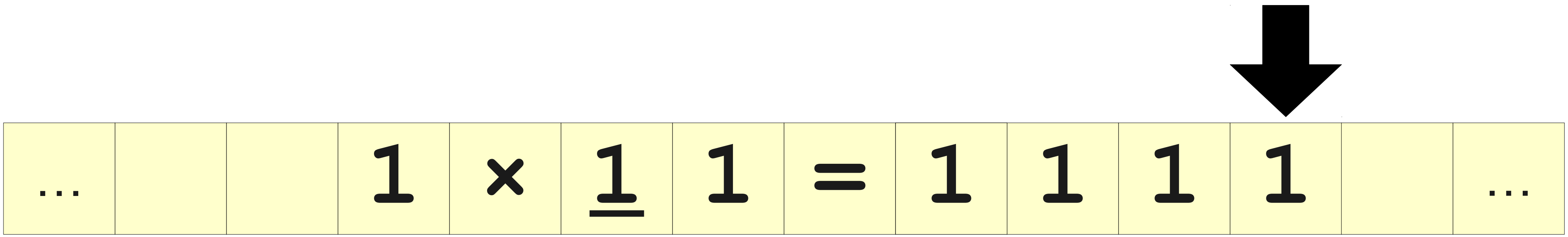

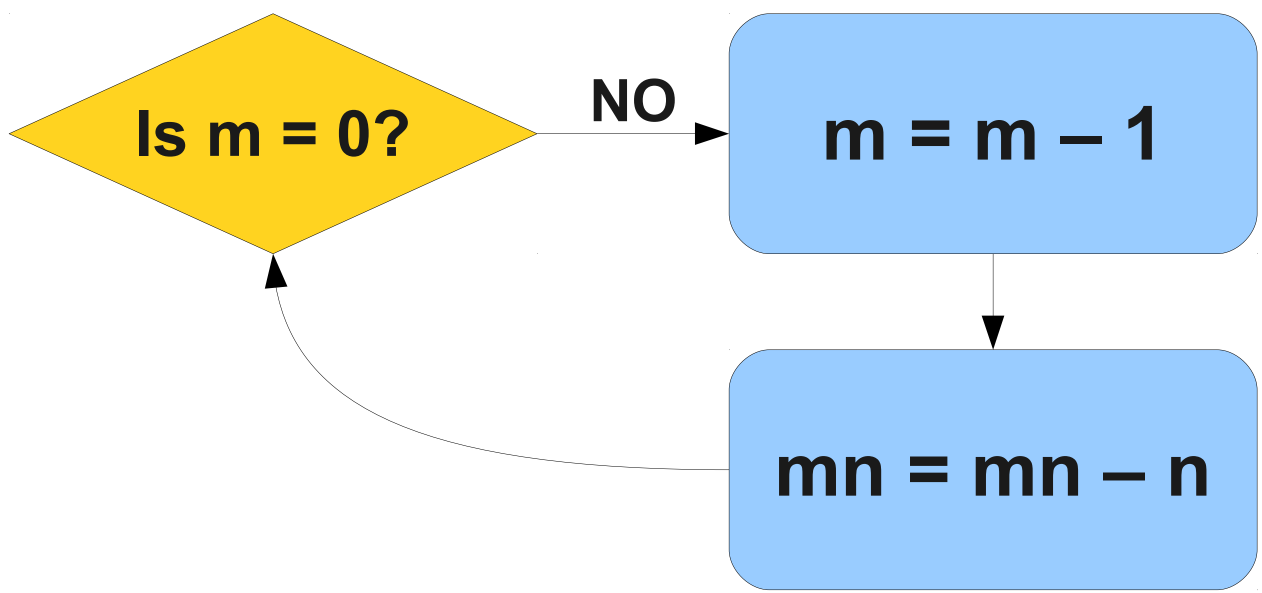

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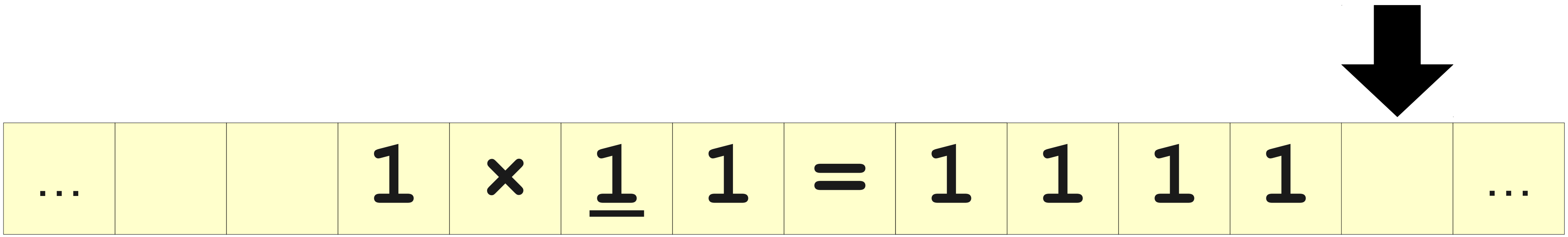

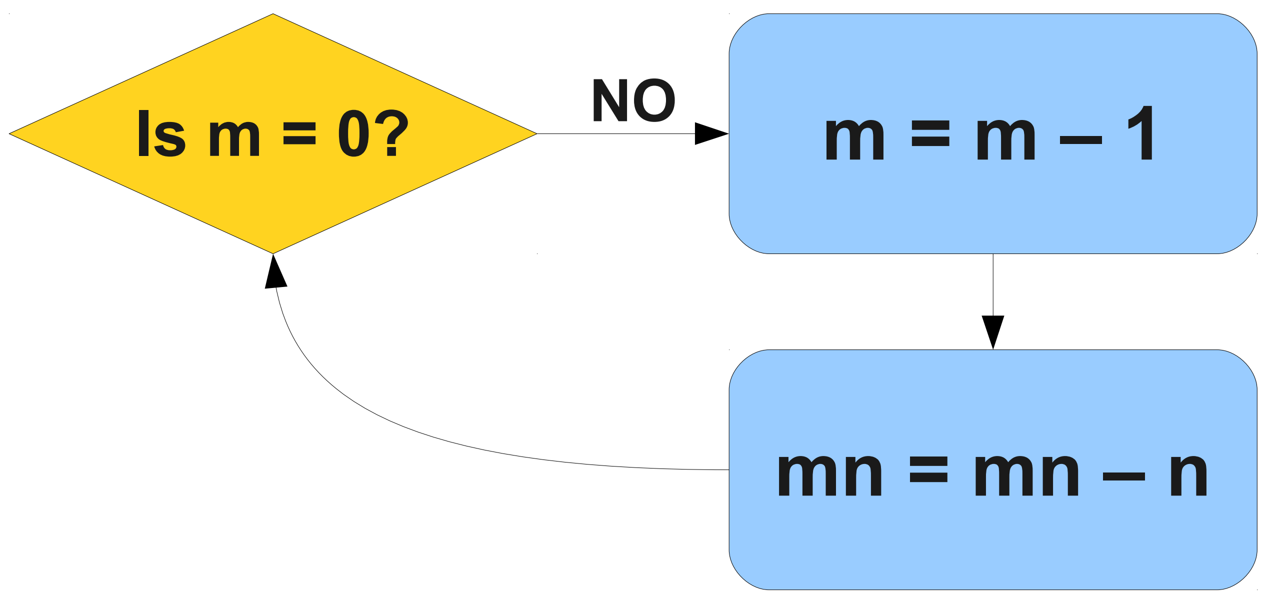

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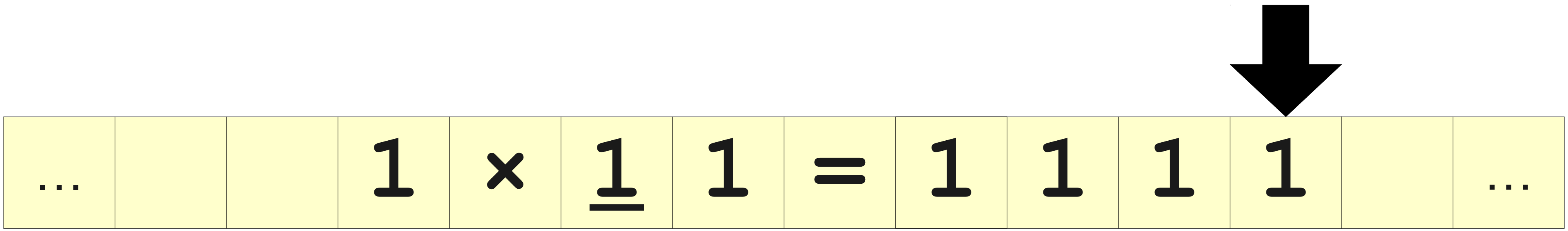

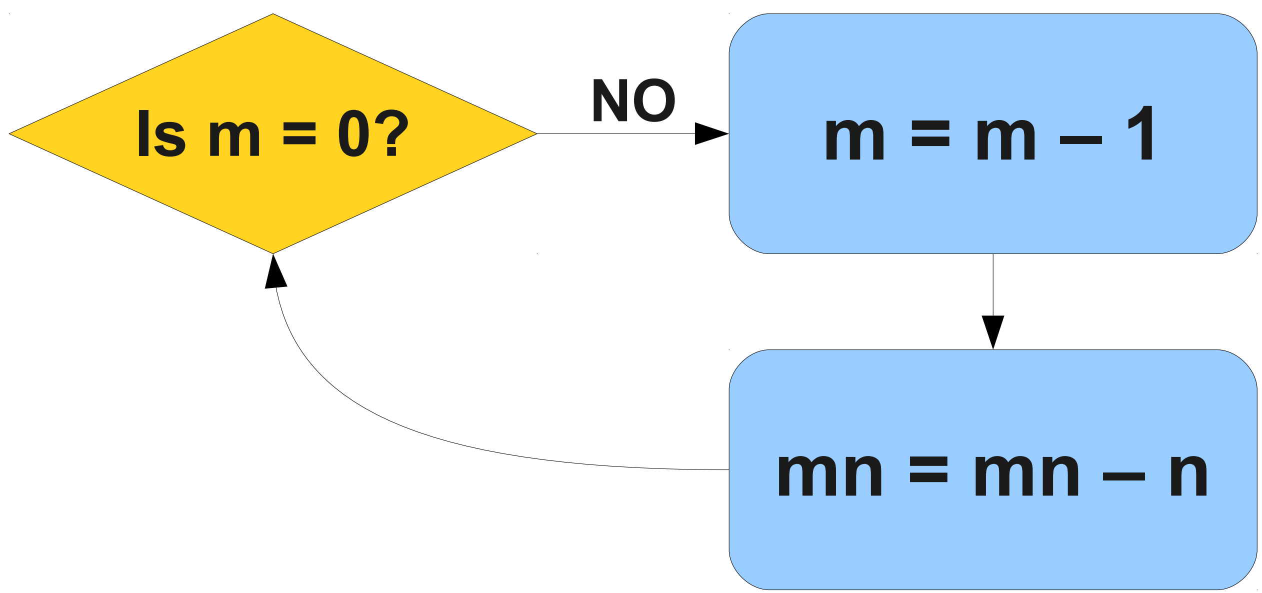

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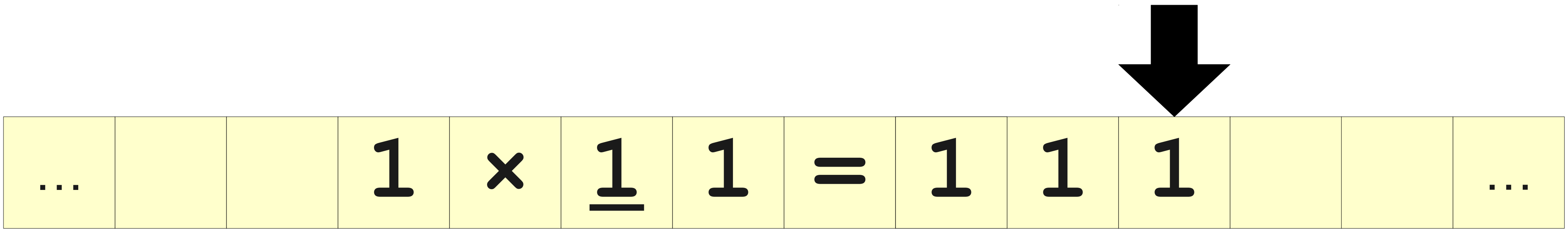

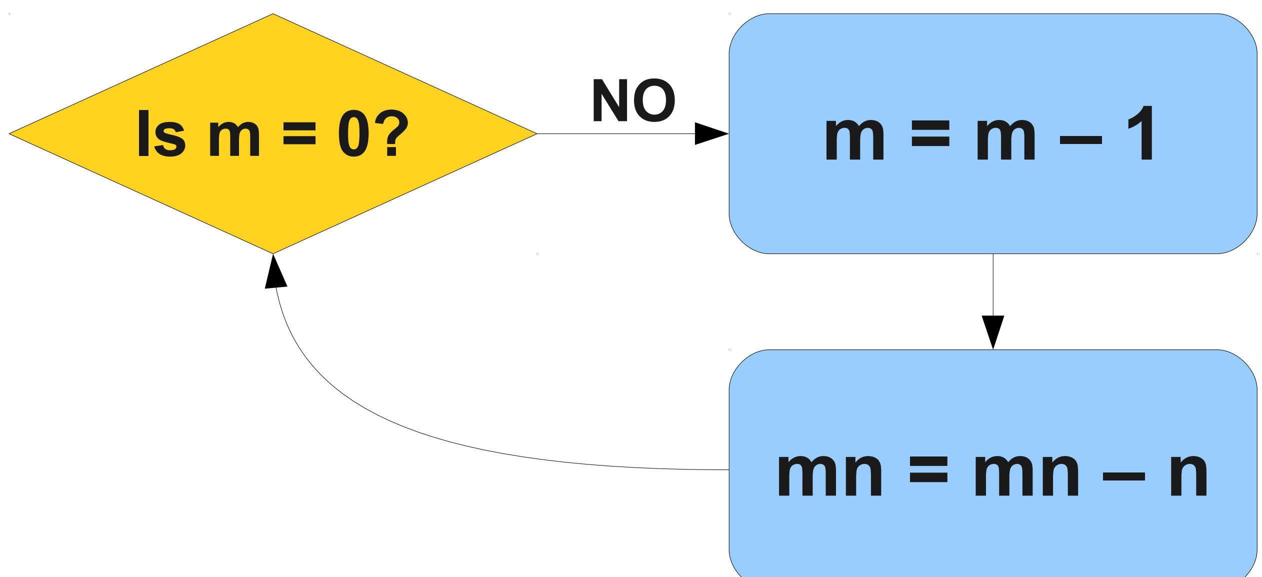

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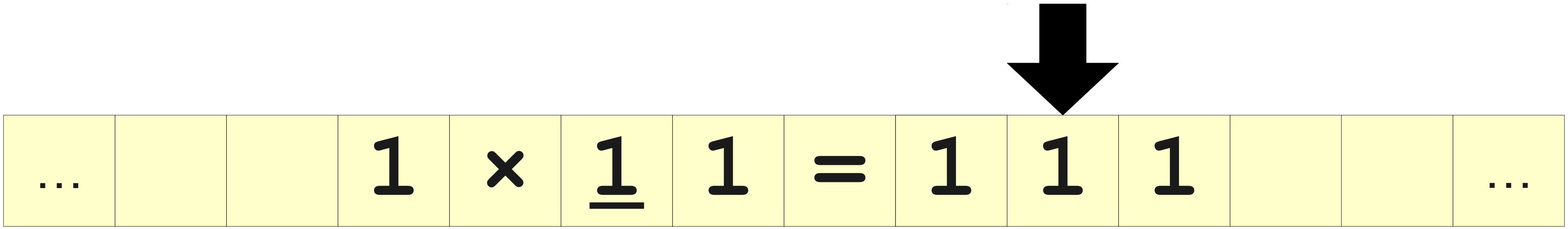

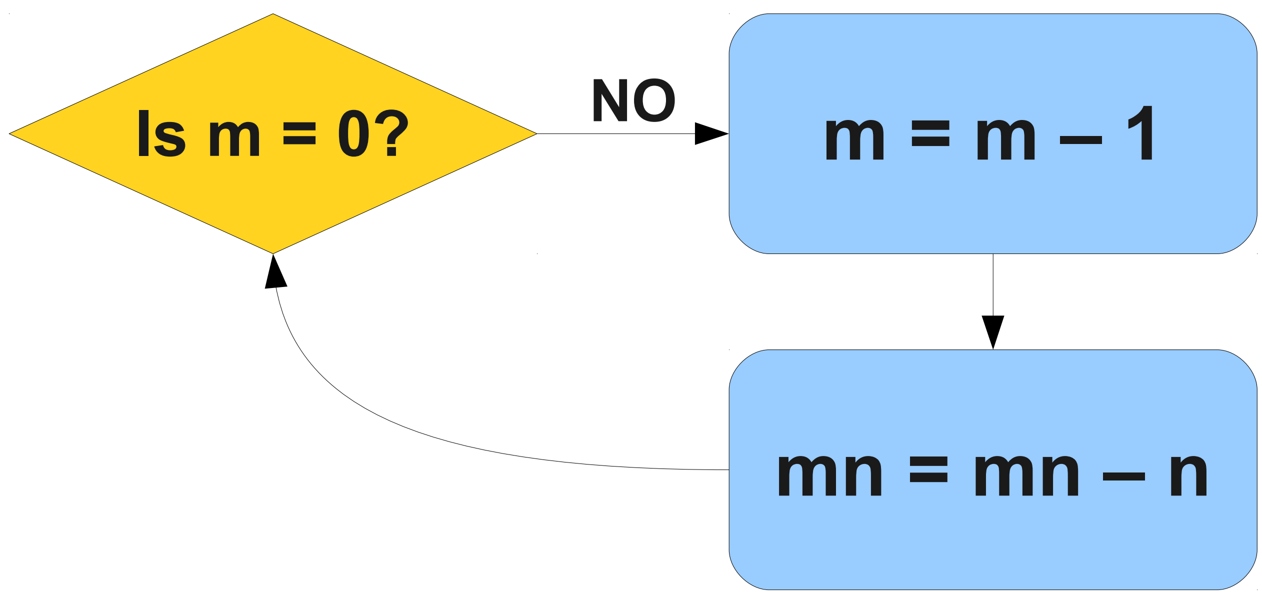

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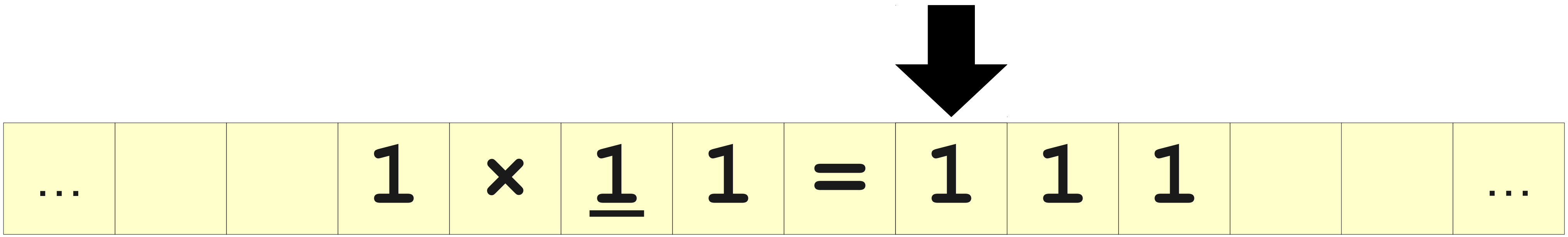

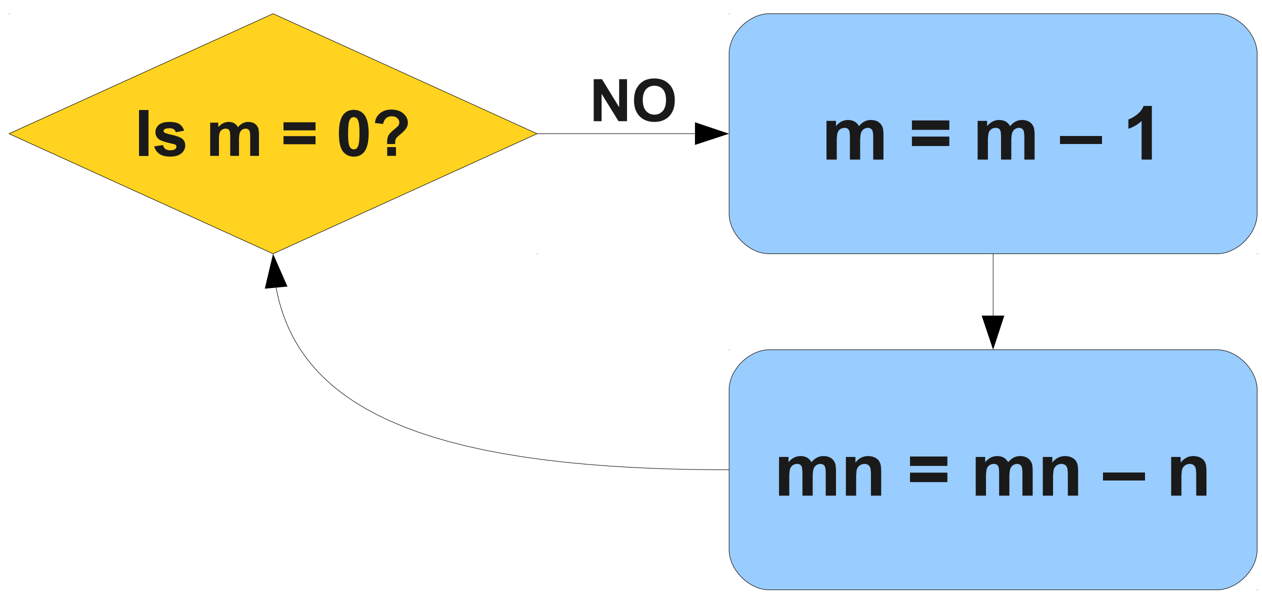

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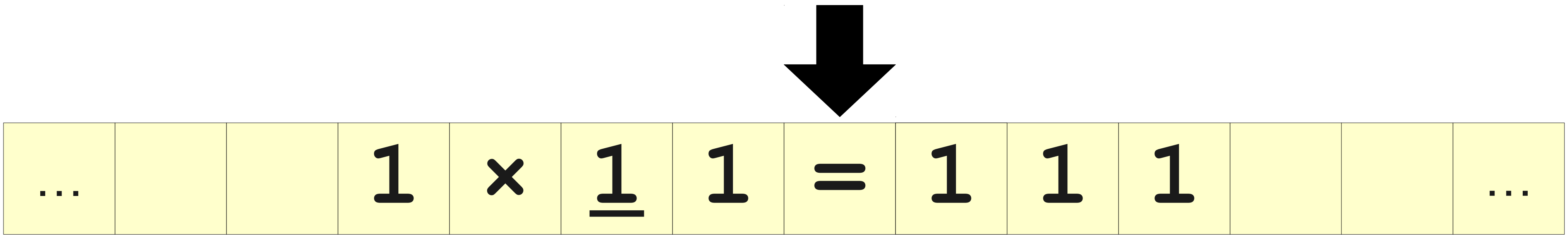

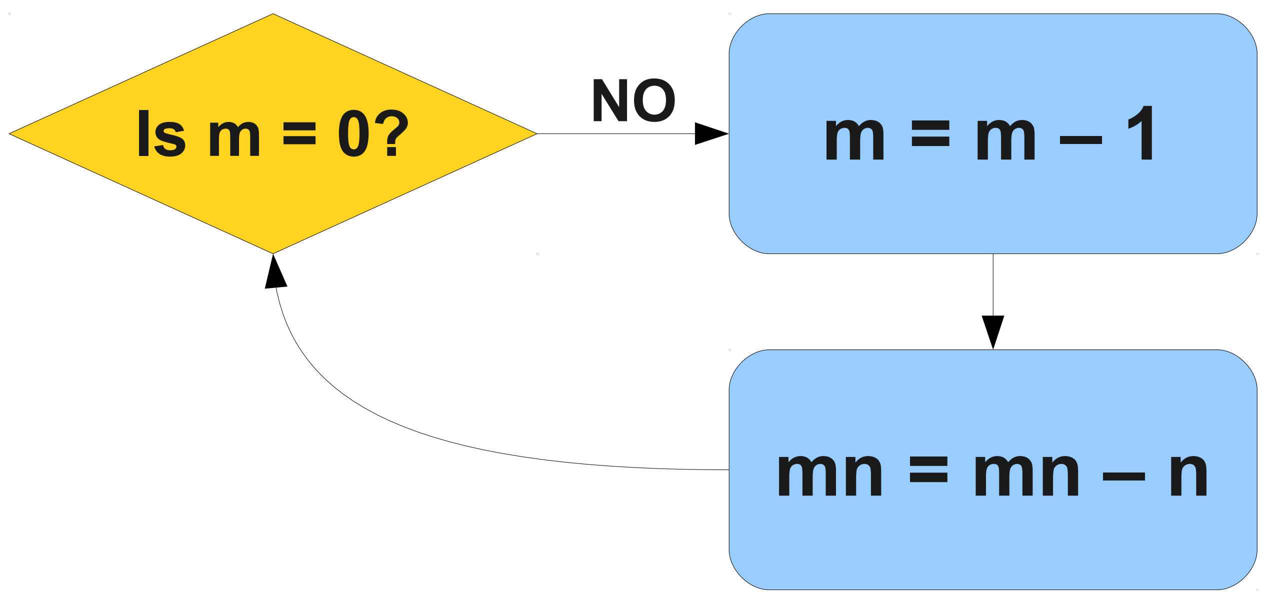

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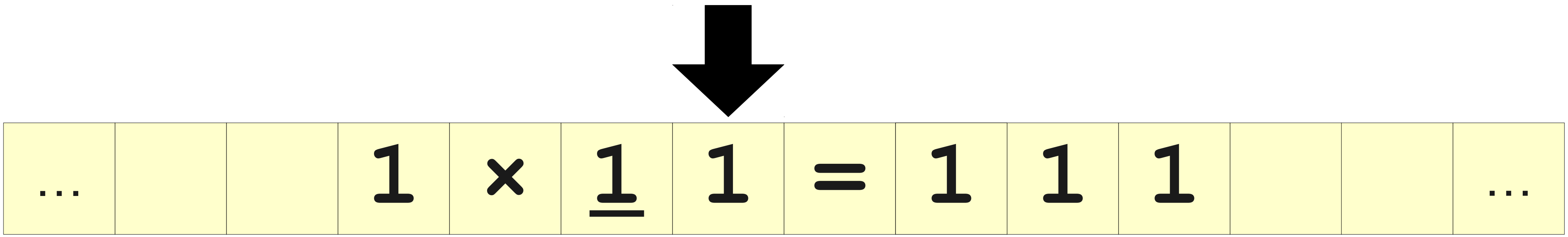

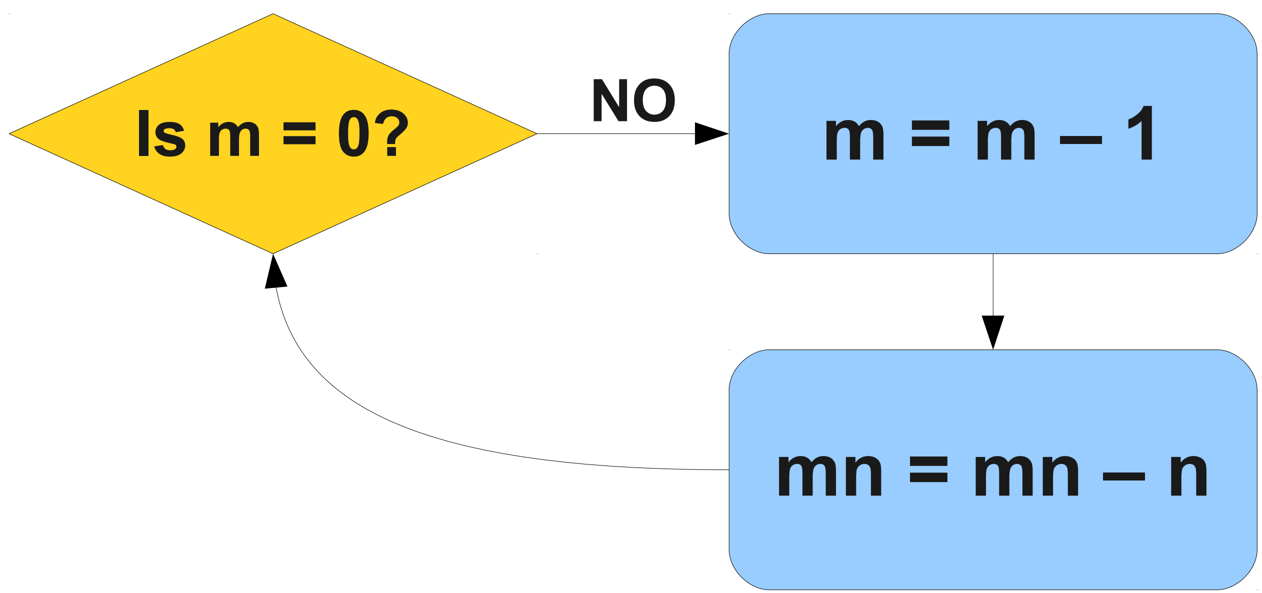

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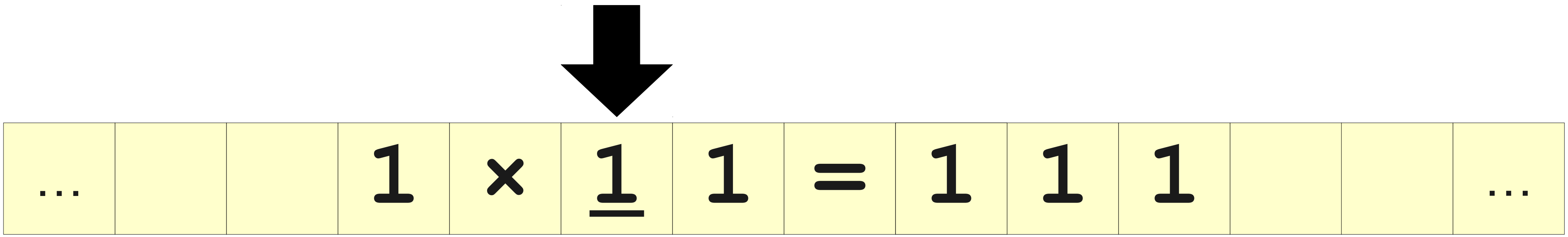

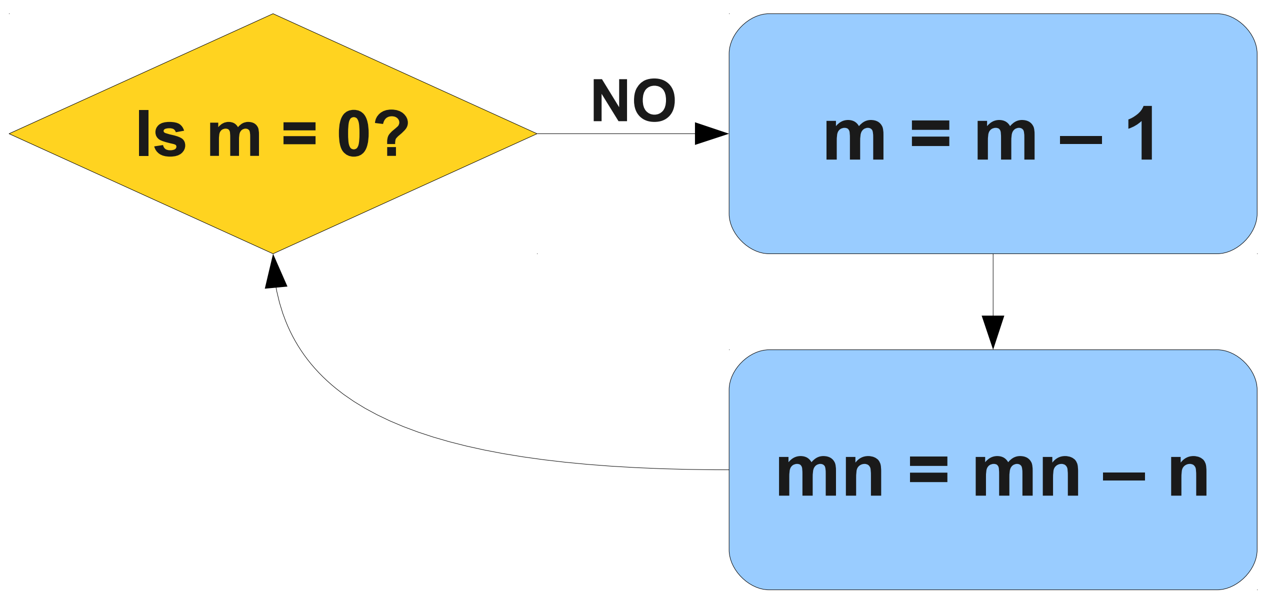

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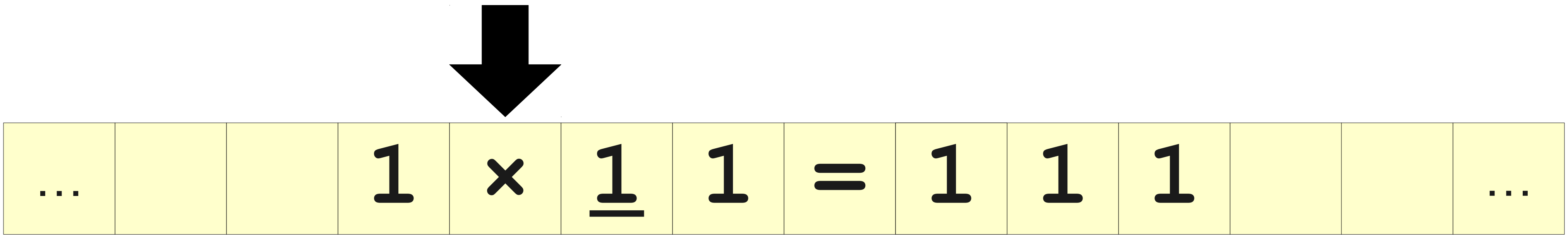

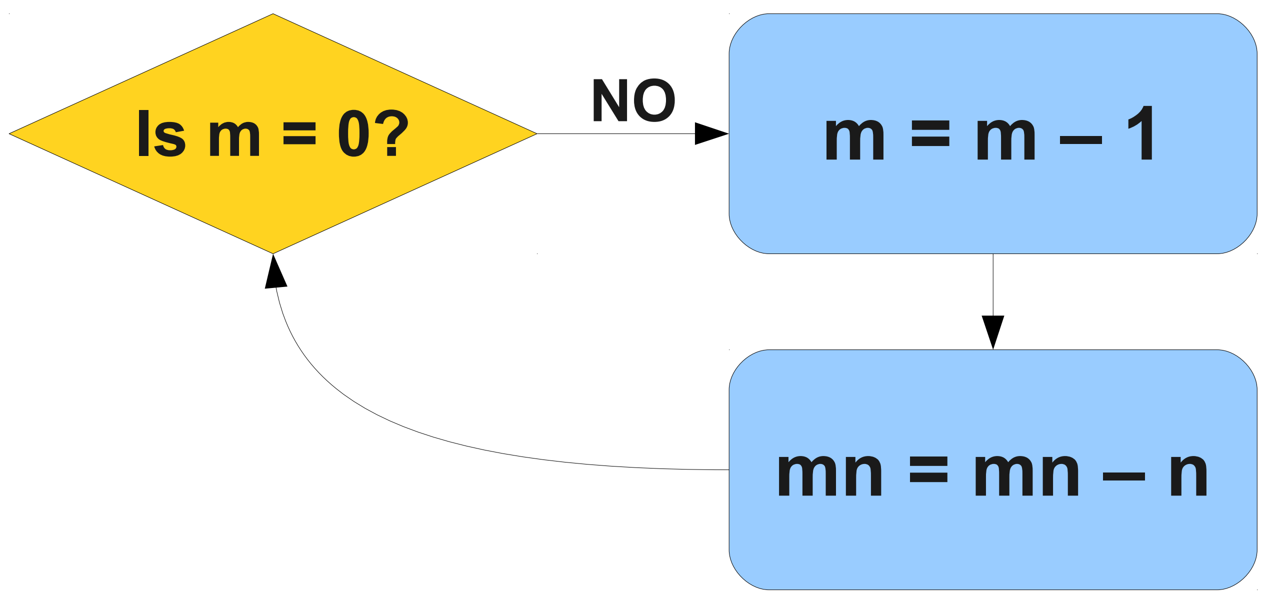

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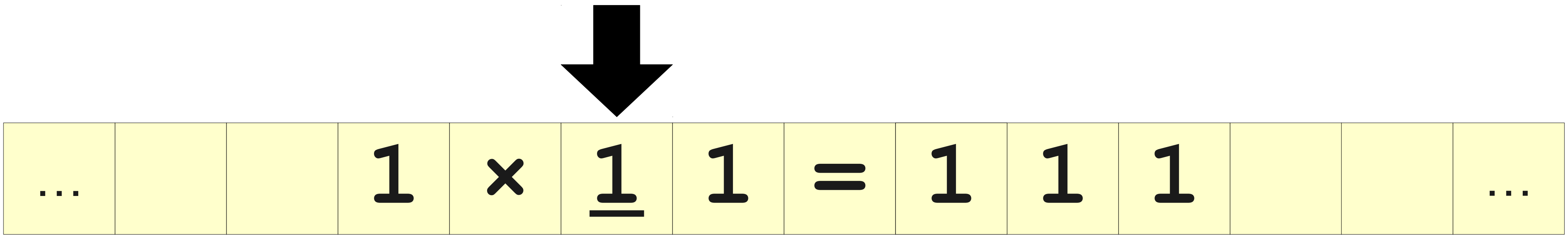

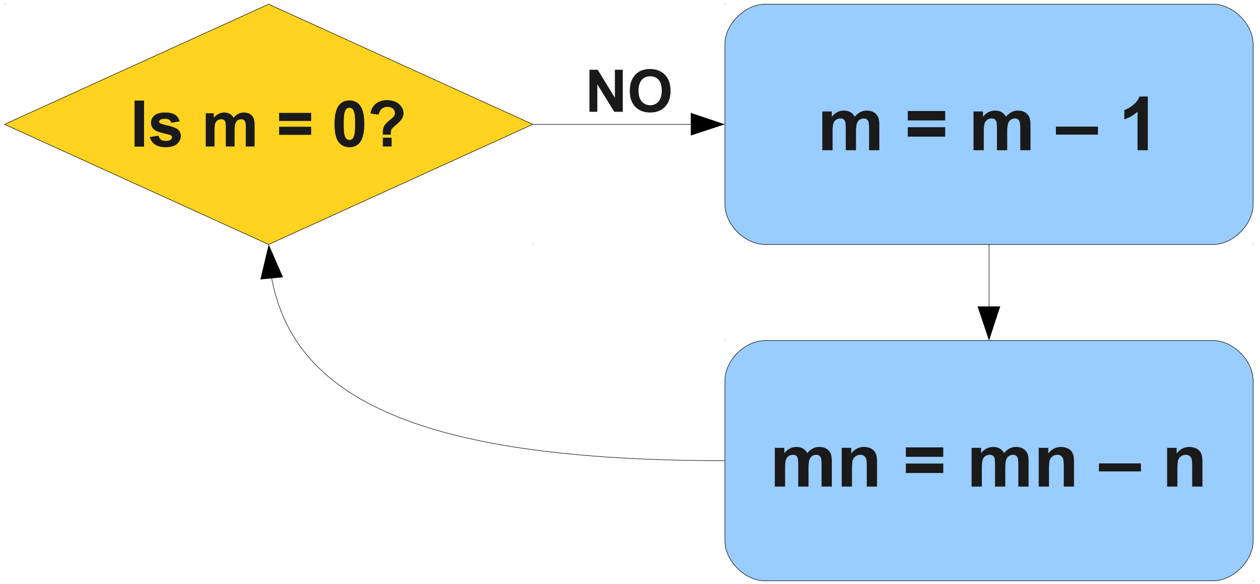

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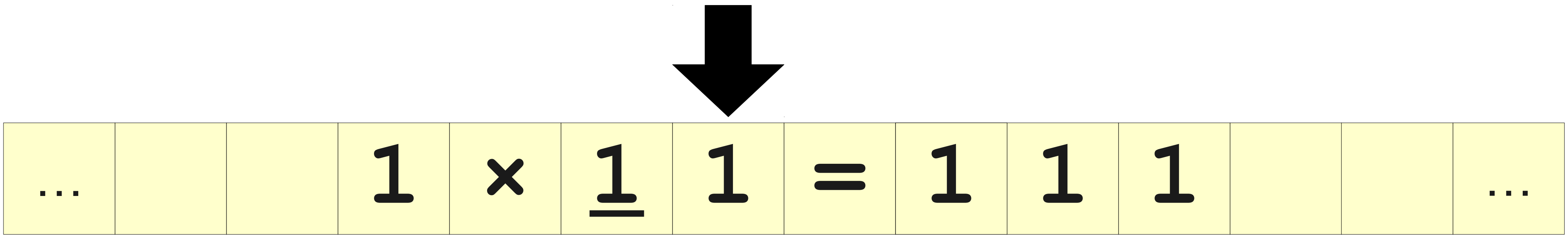

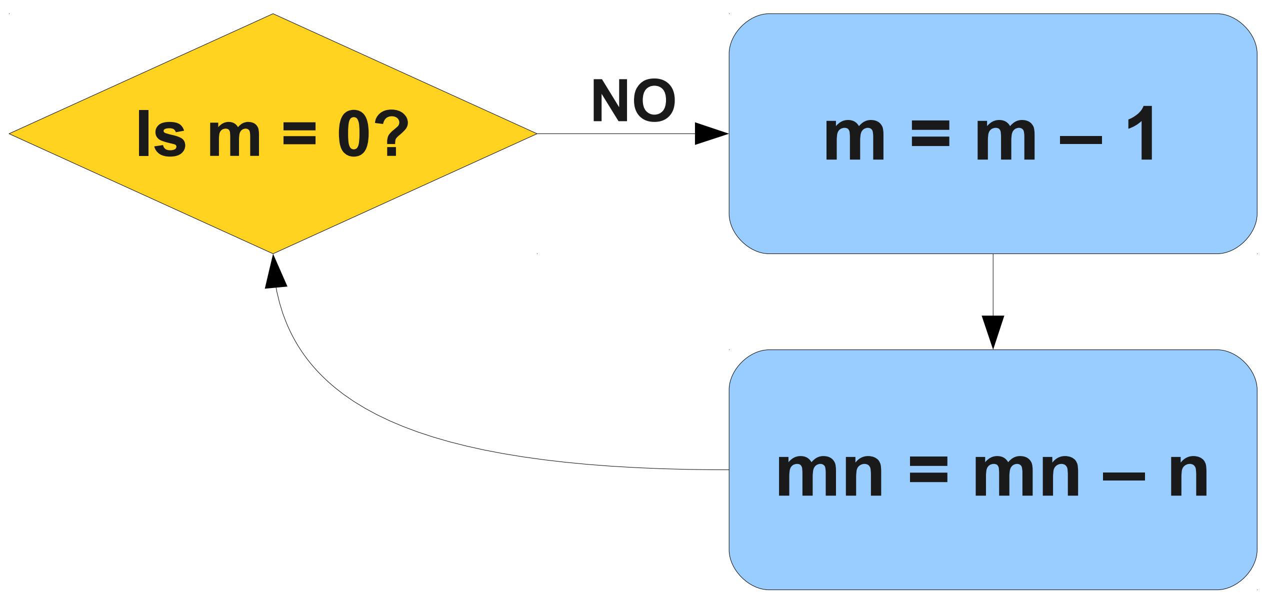

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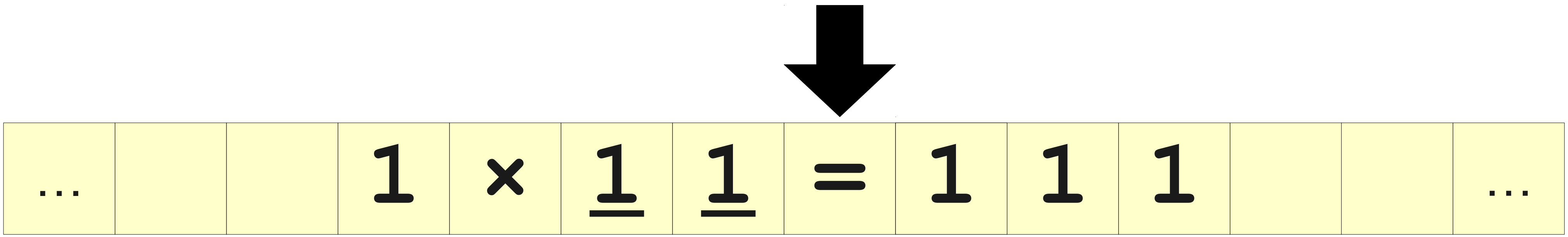

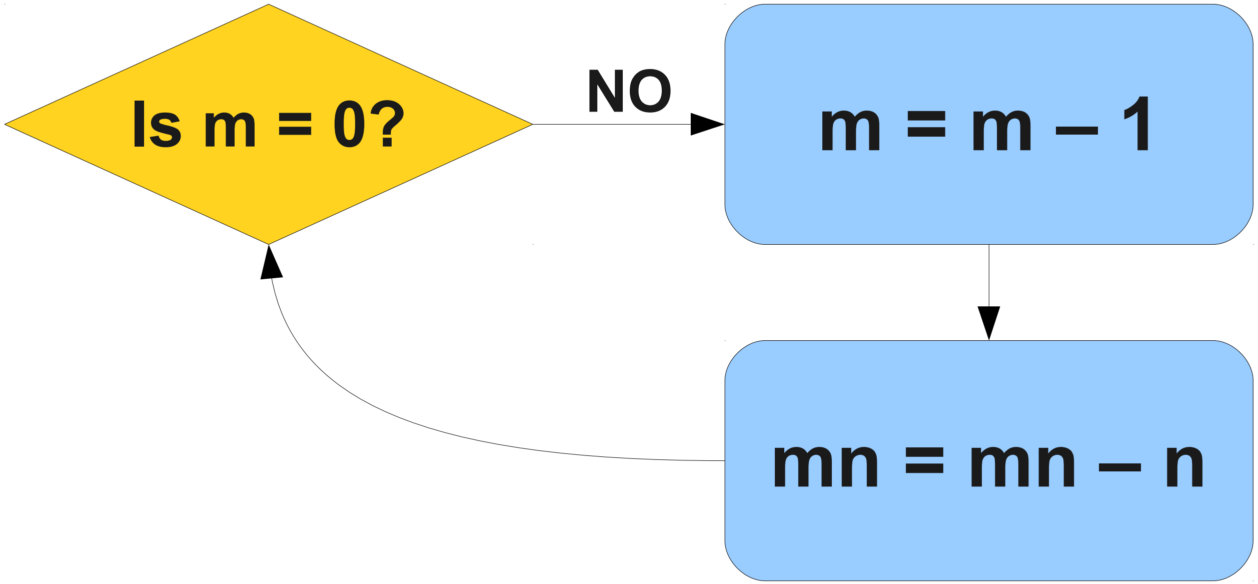

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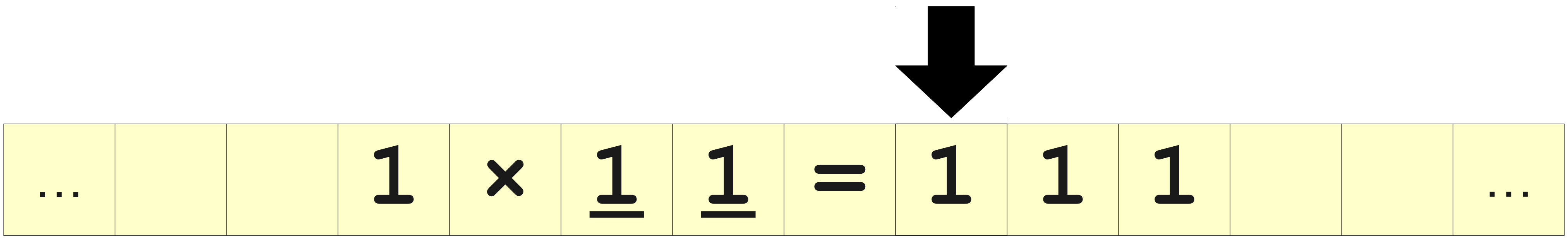
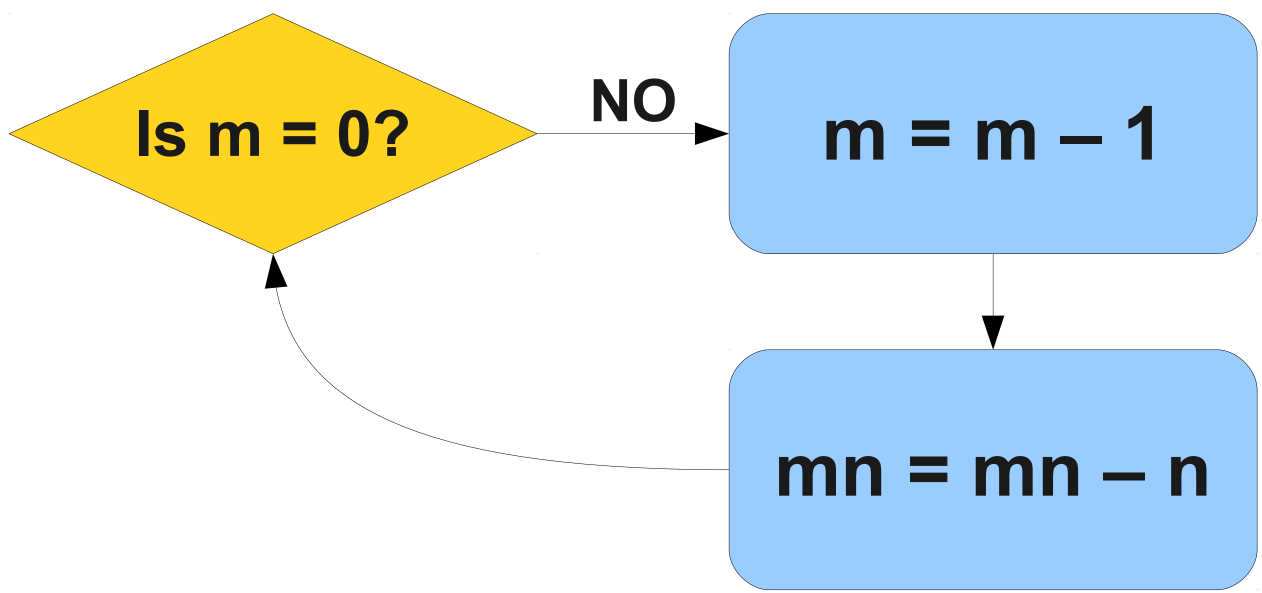
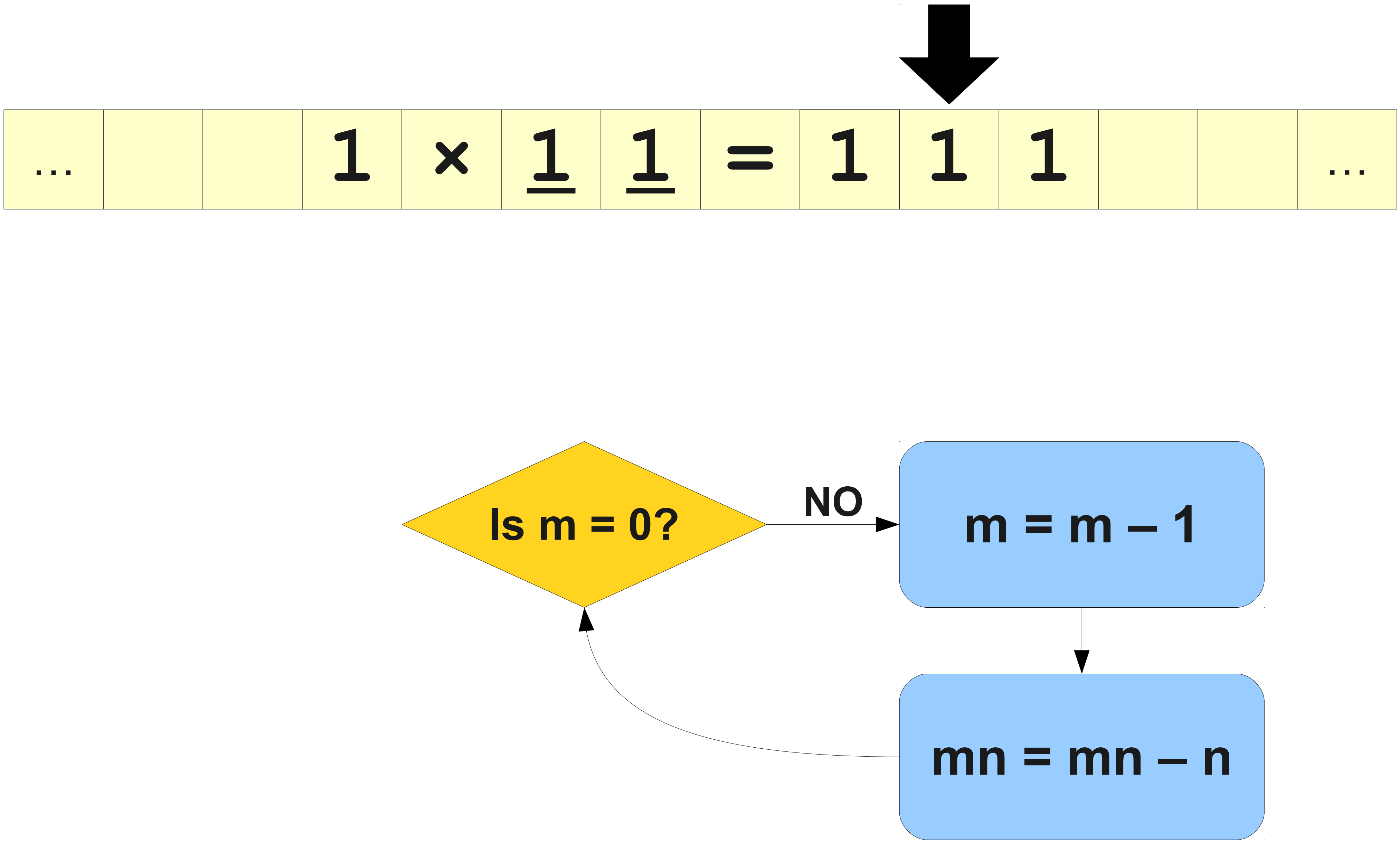




A Sketch of the Algorithm



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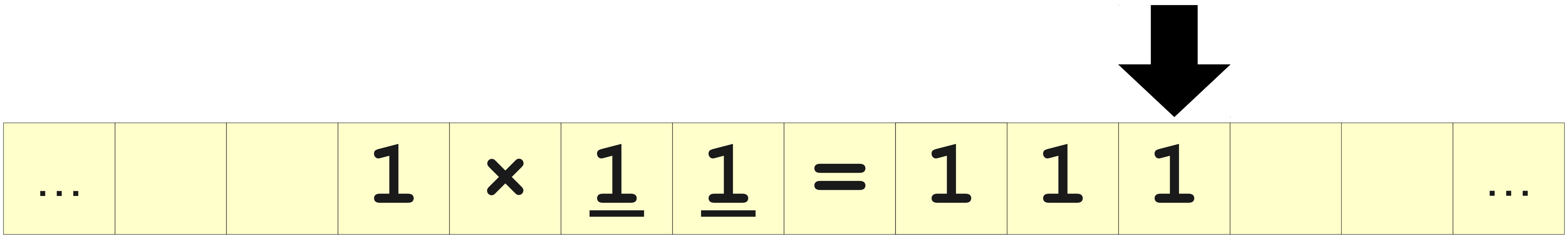

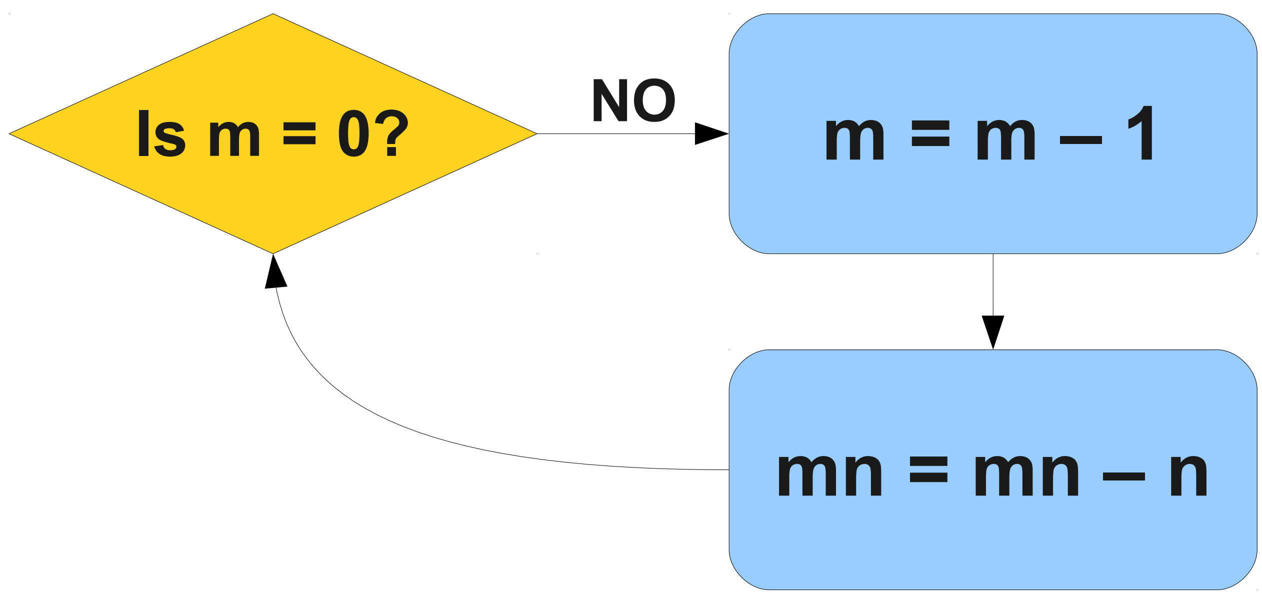

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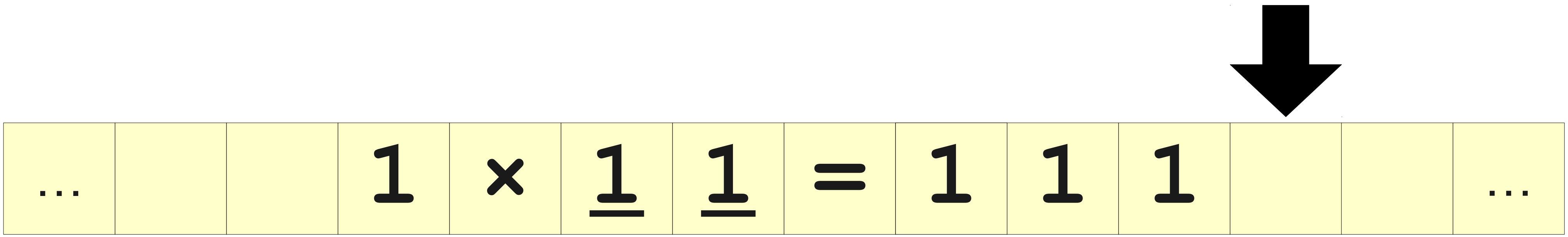

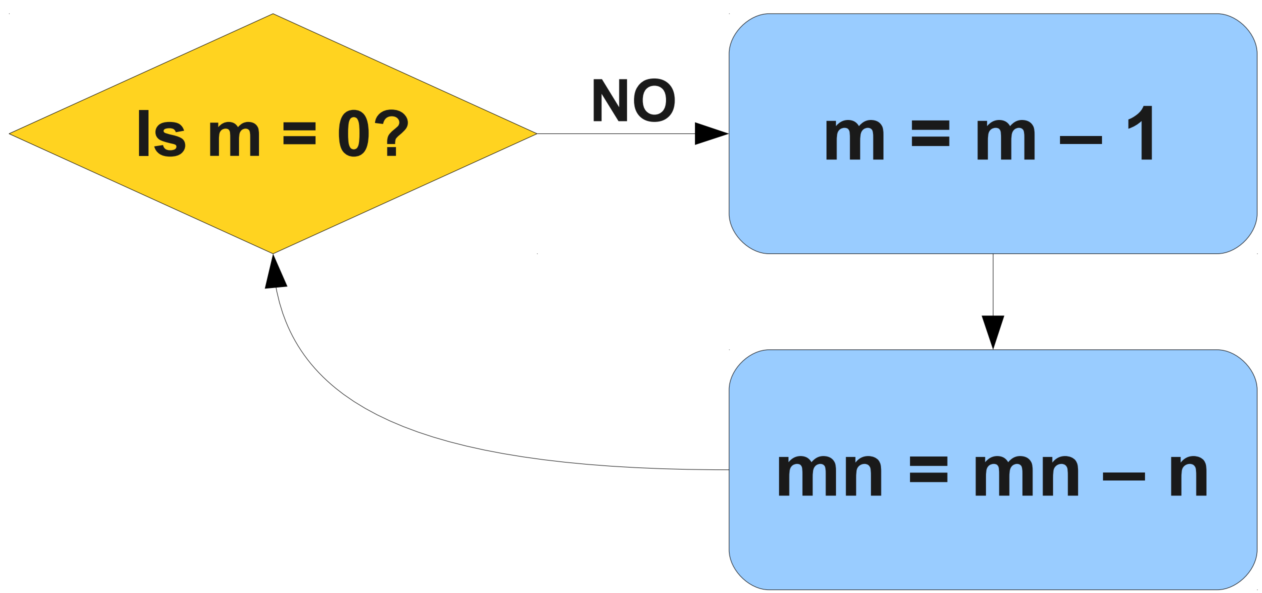

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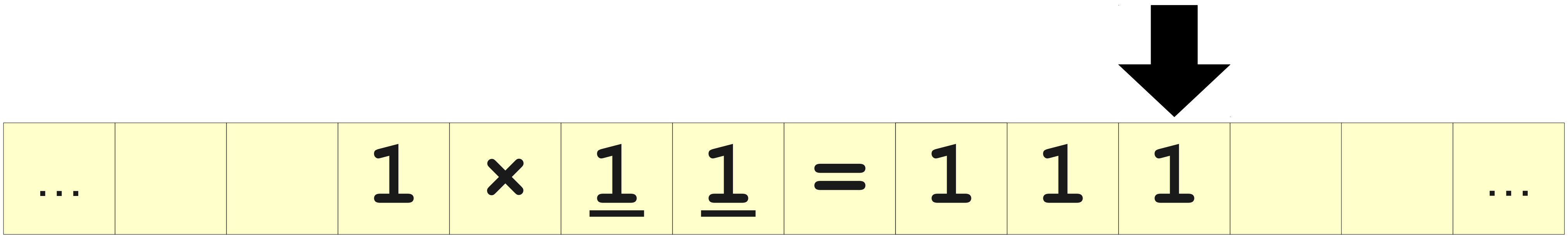

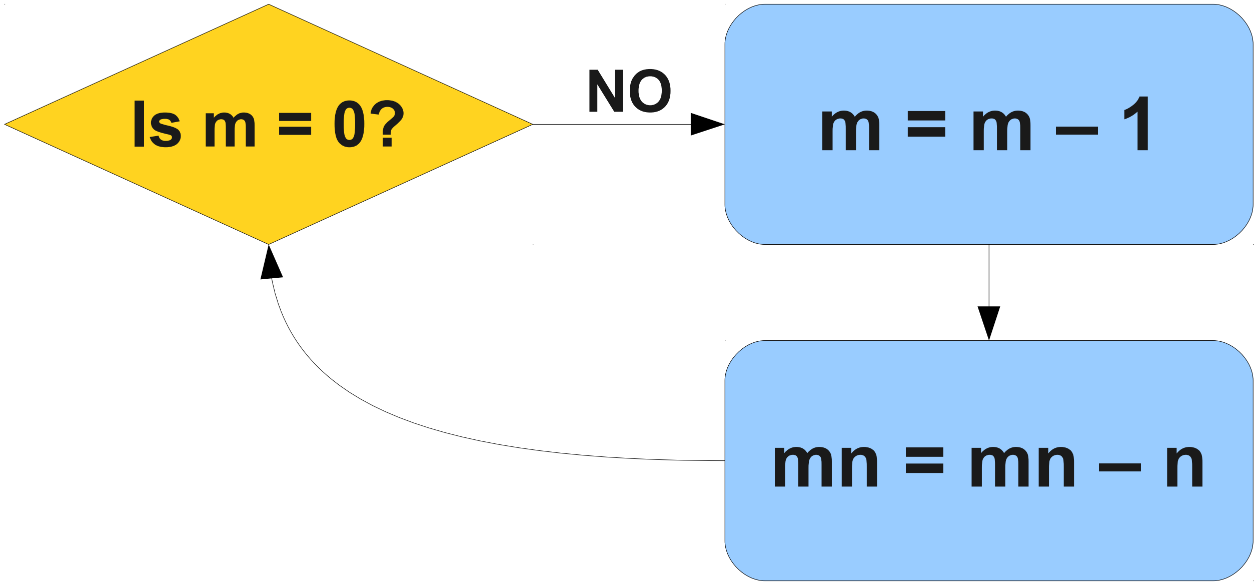

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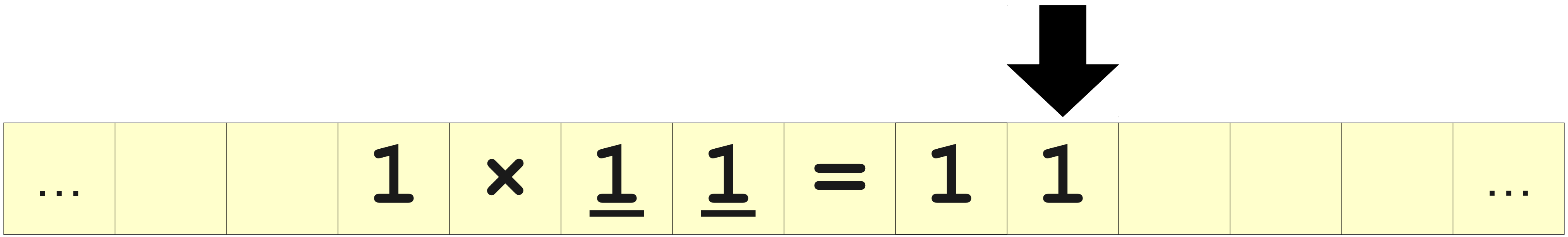

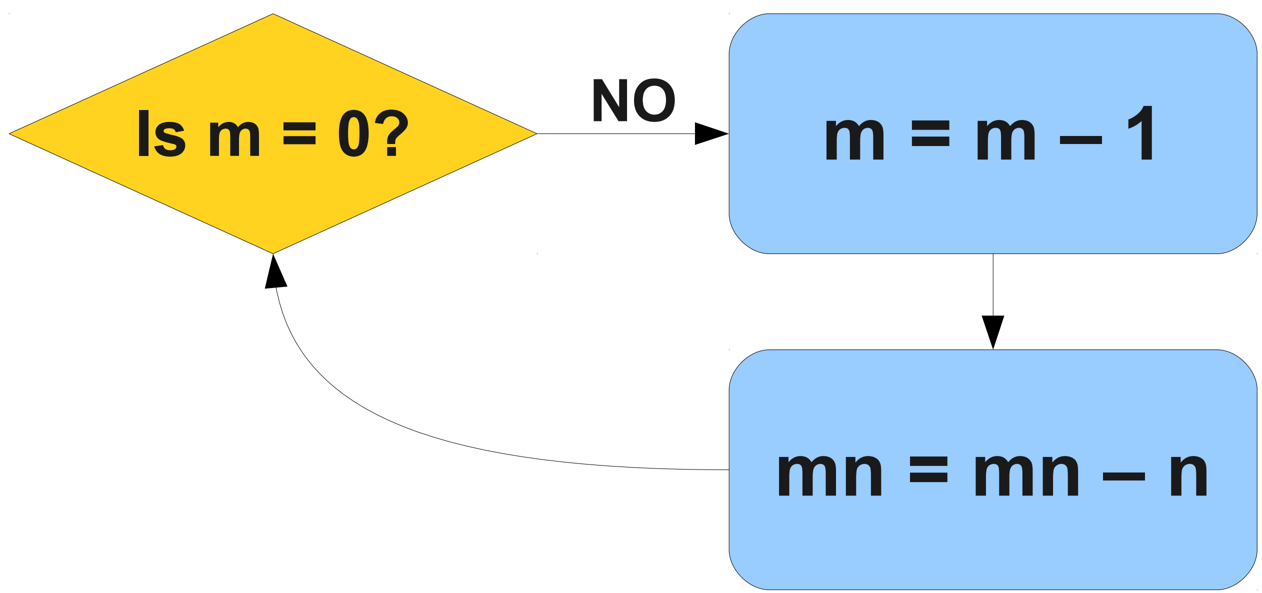

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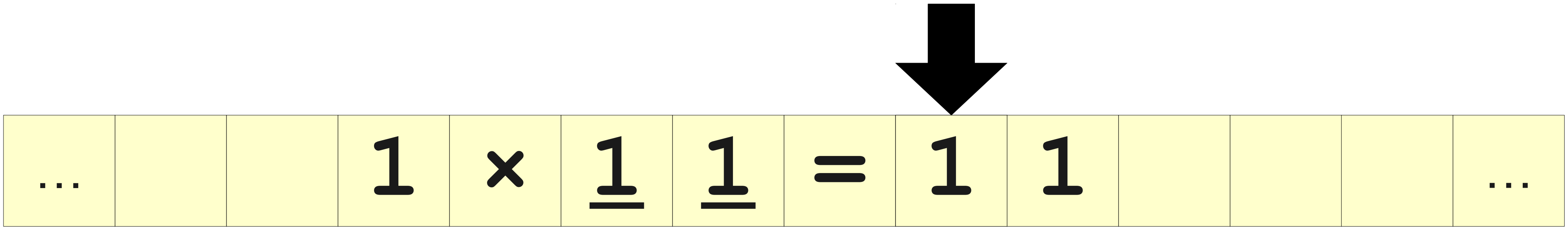

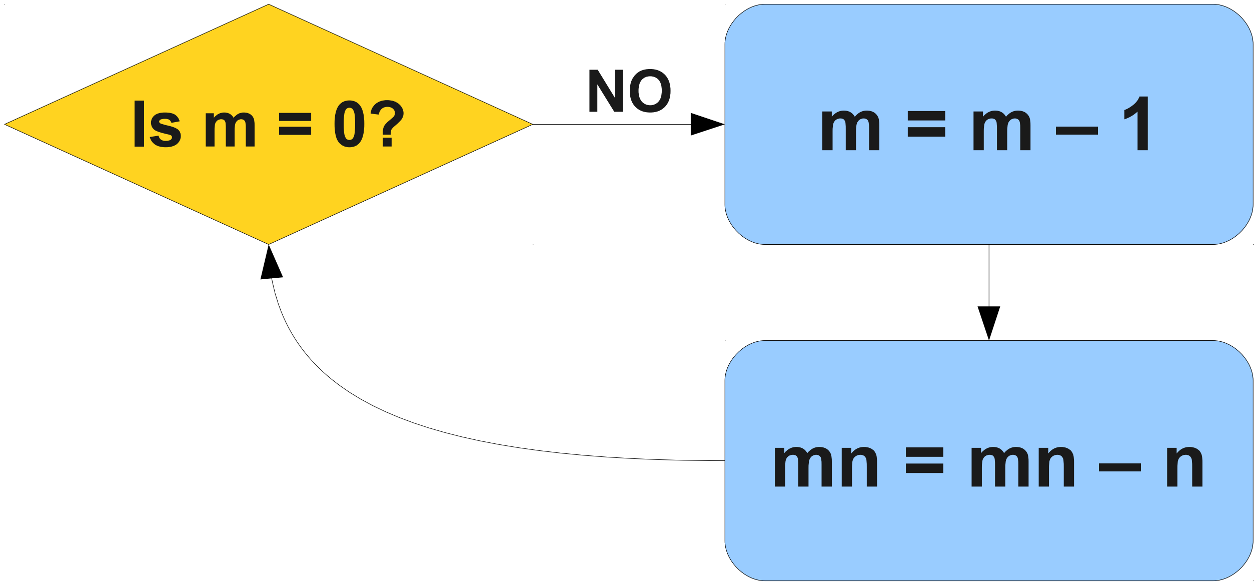

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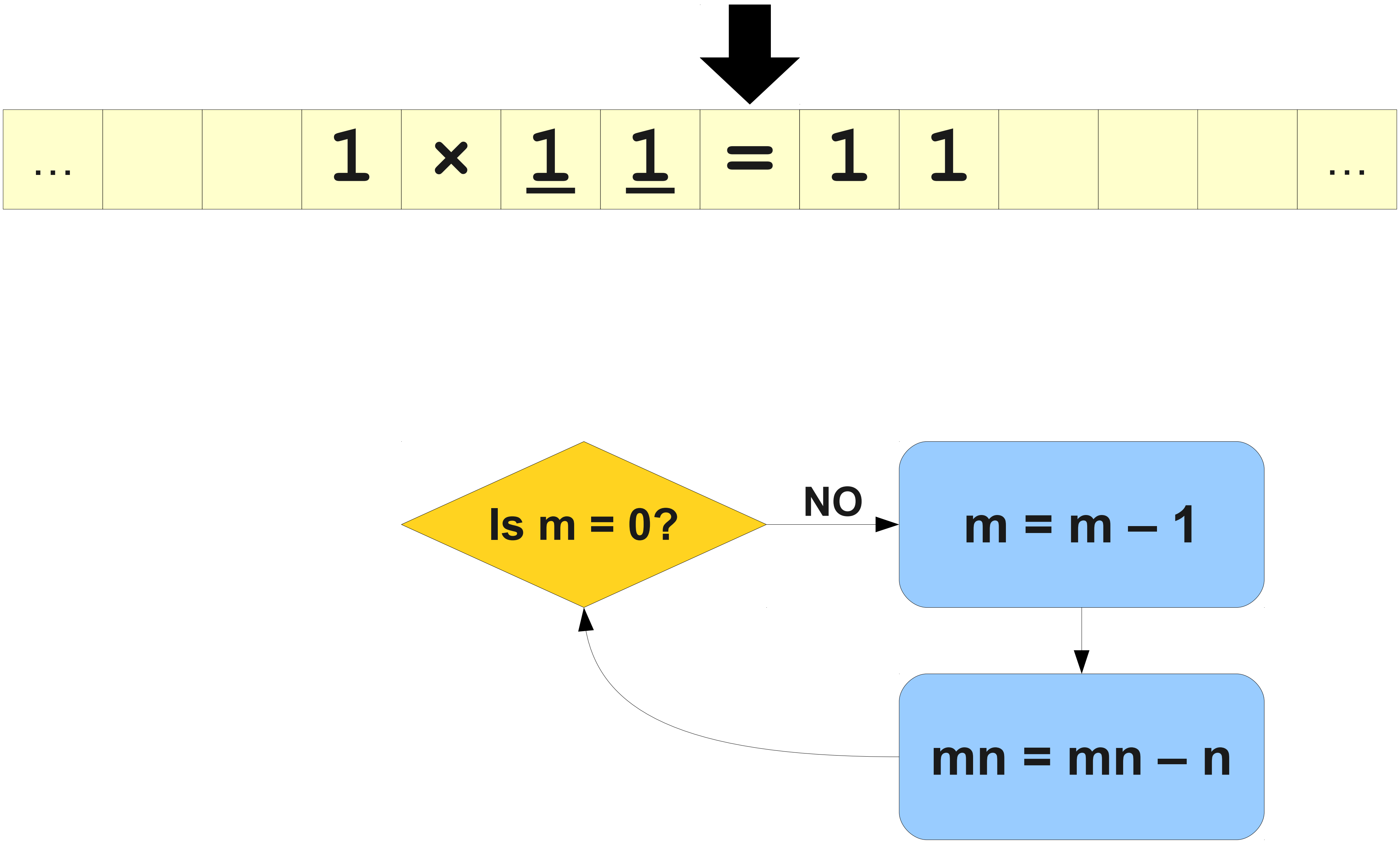

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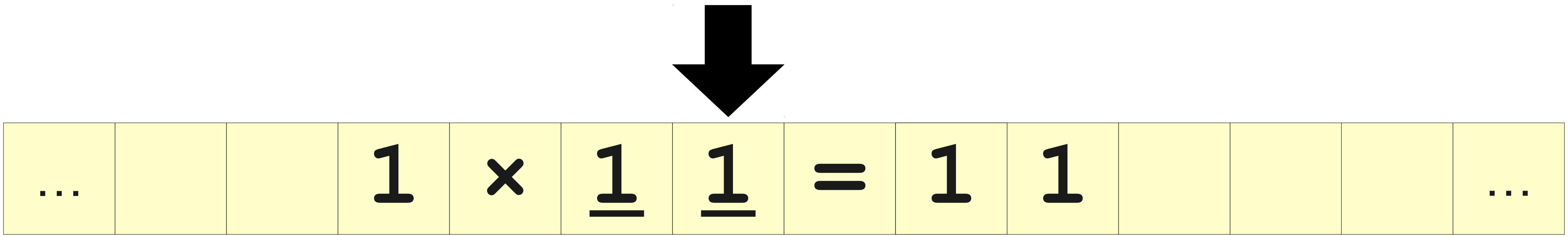

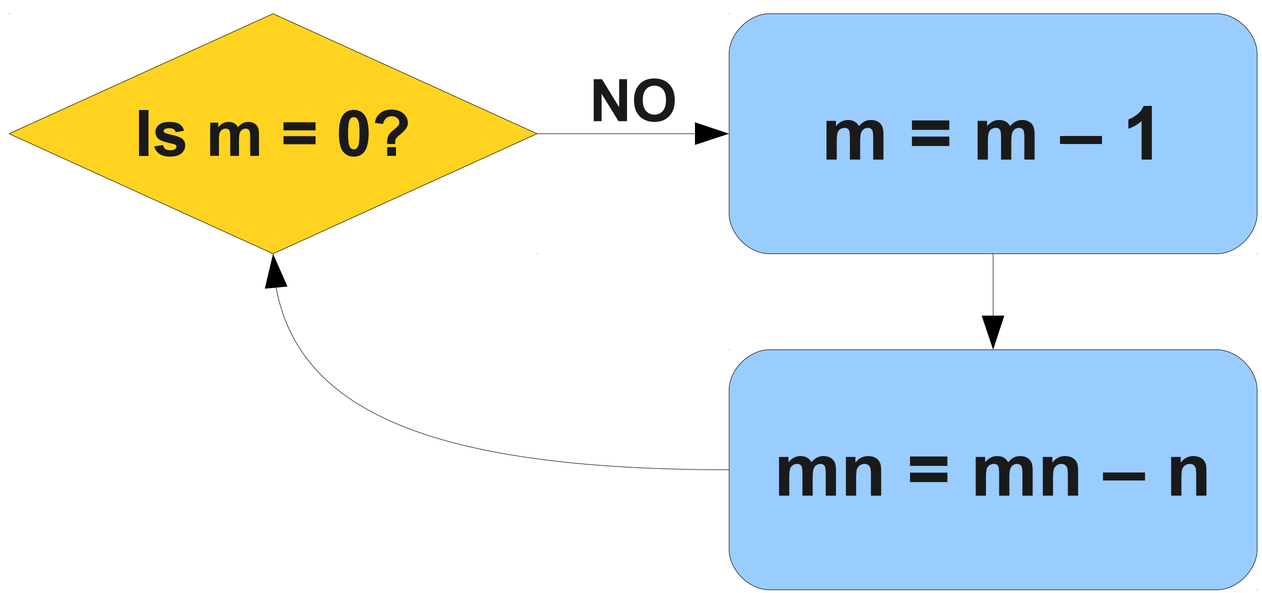

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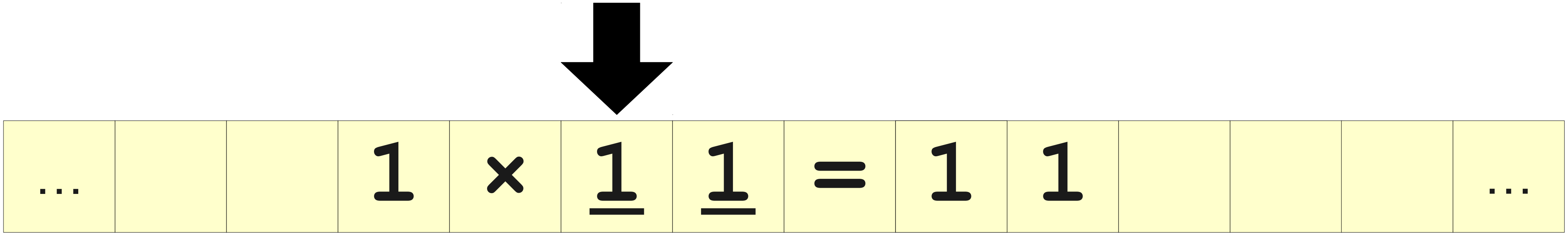

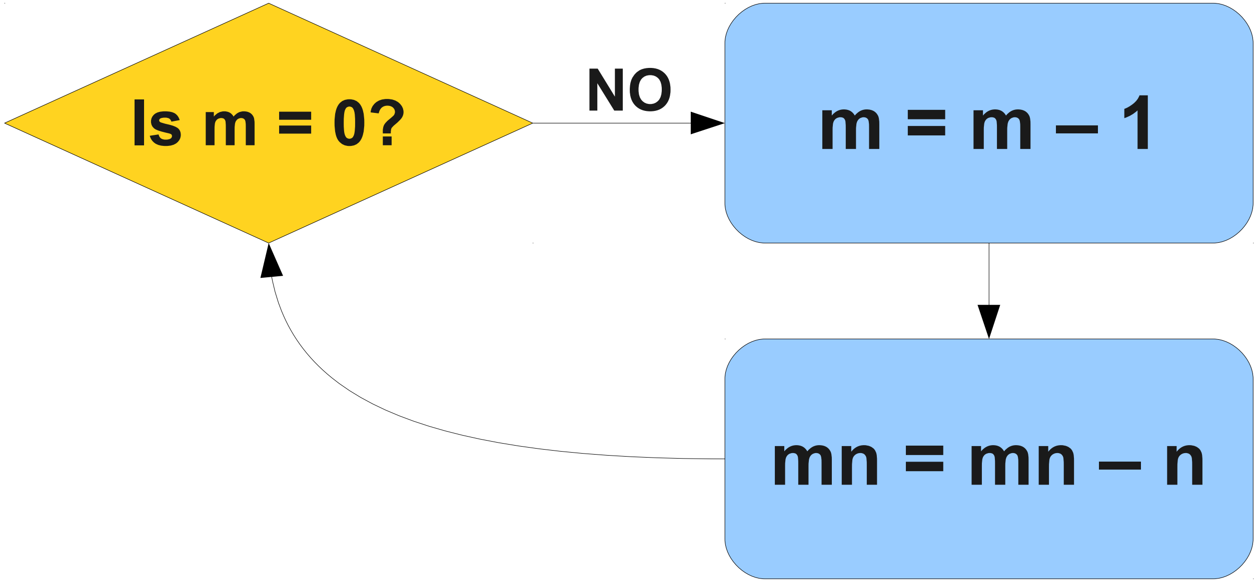

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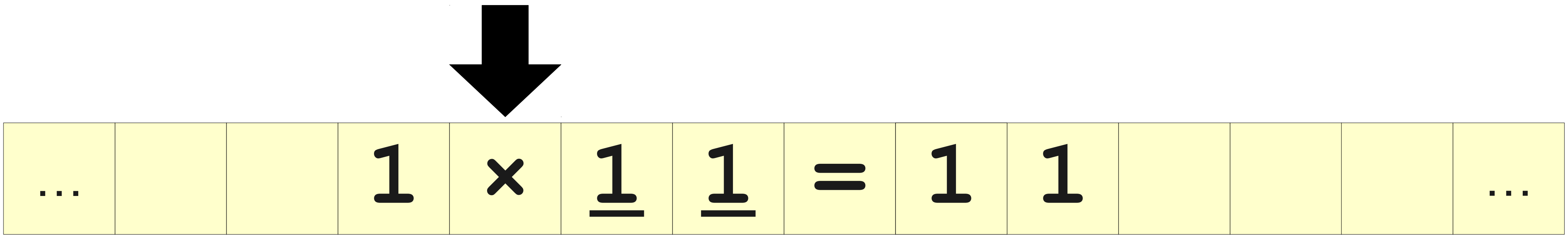

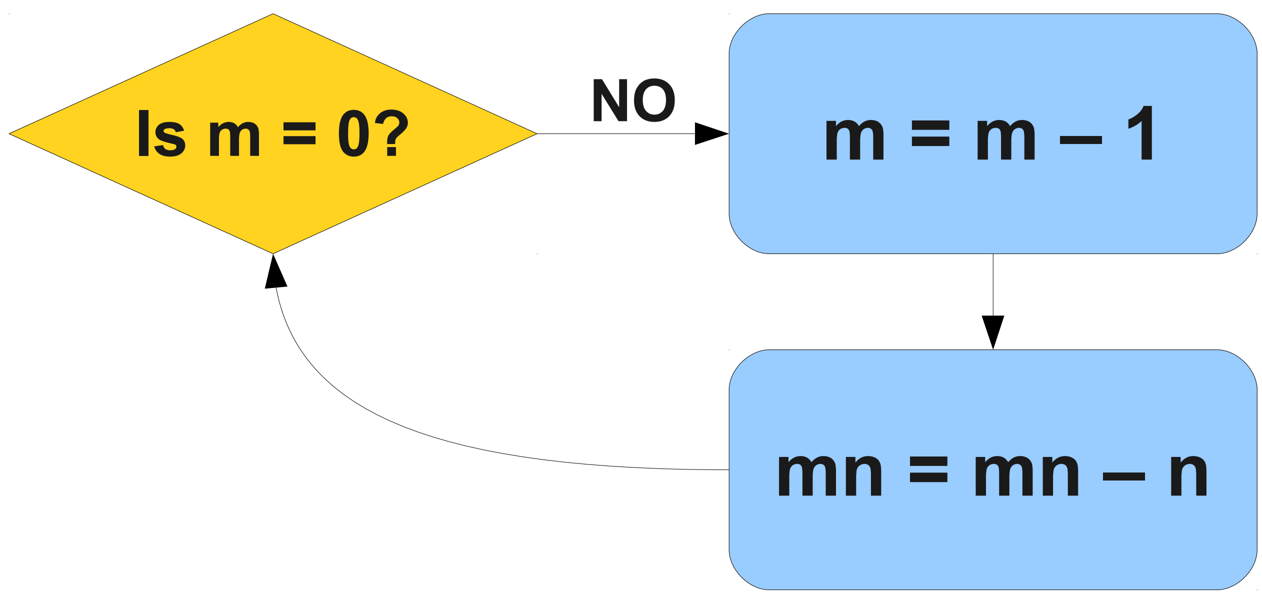

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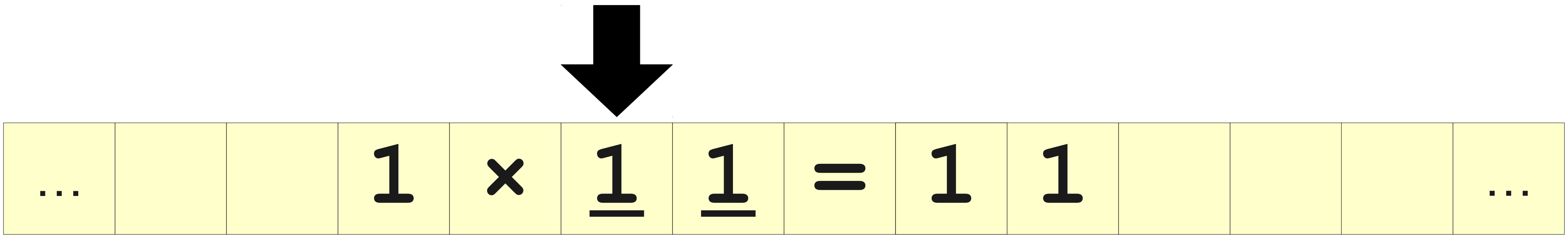

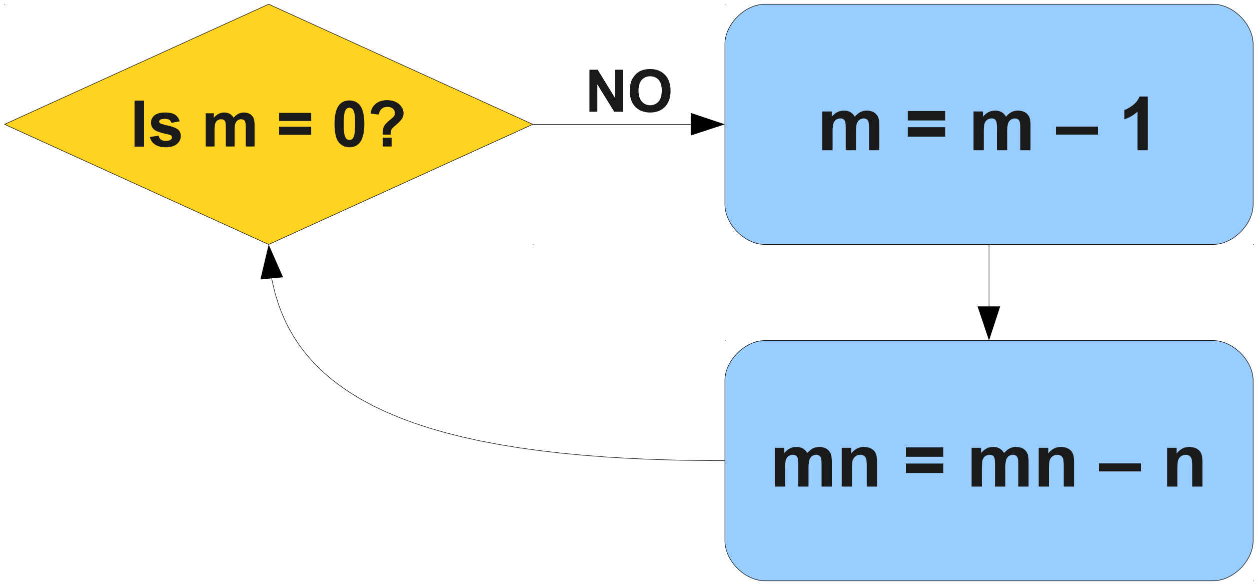

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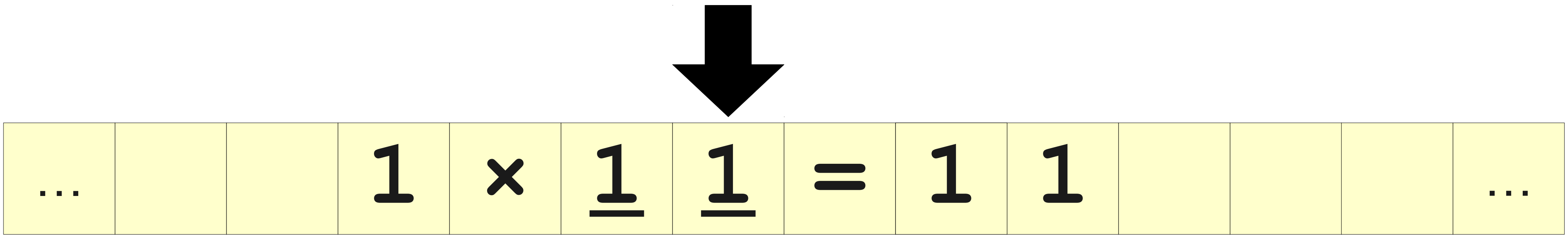

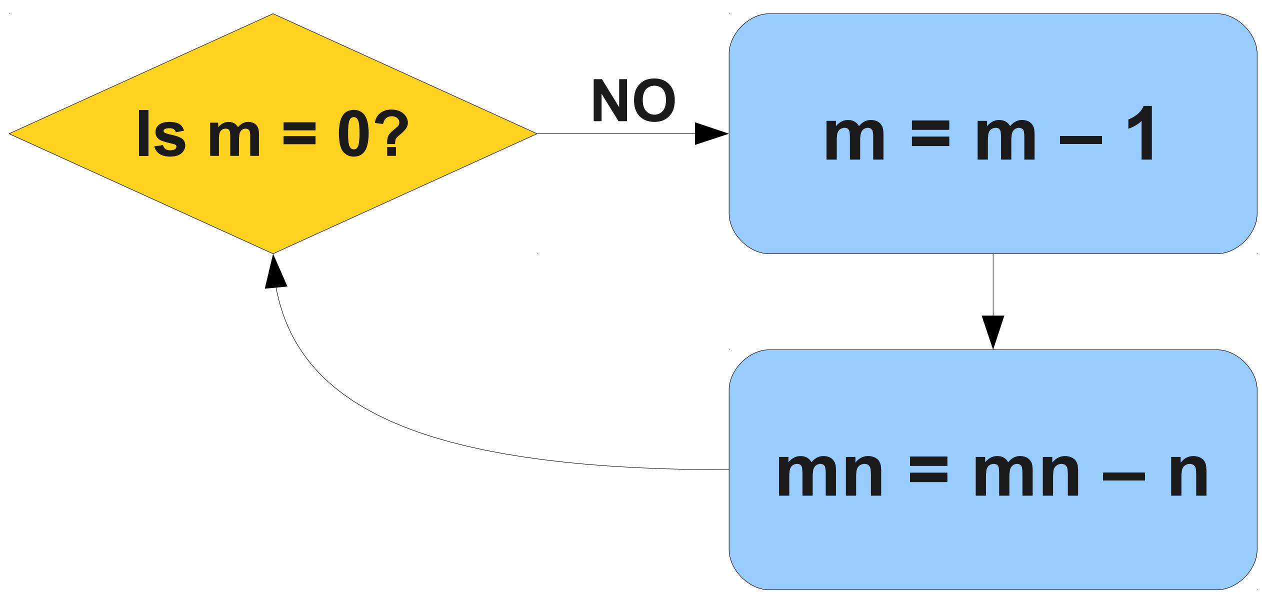

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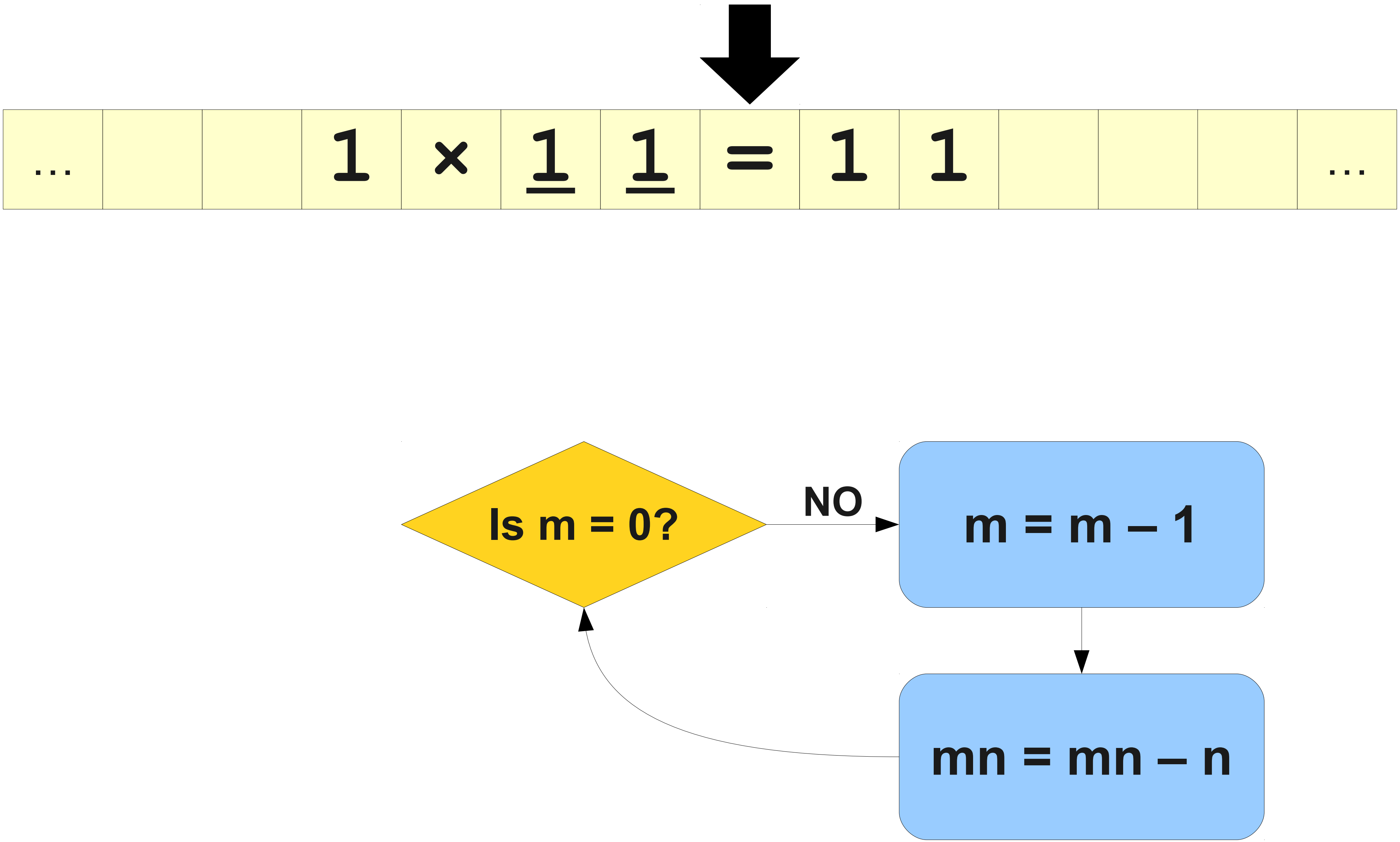

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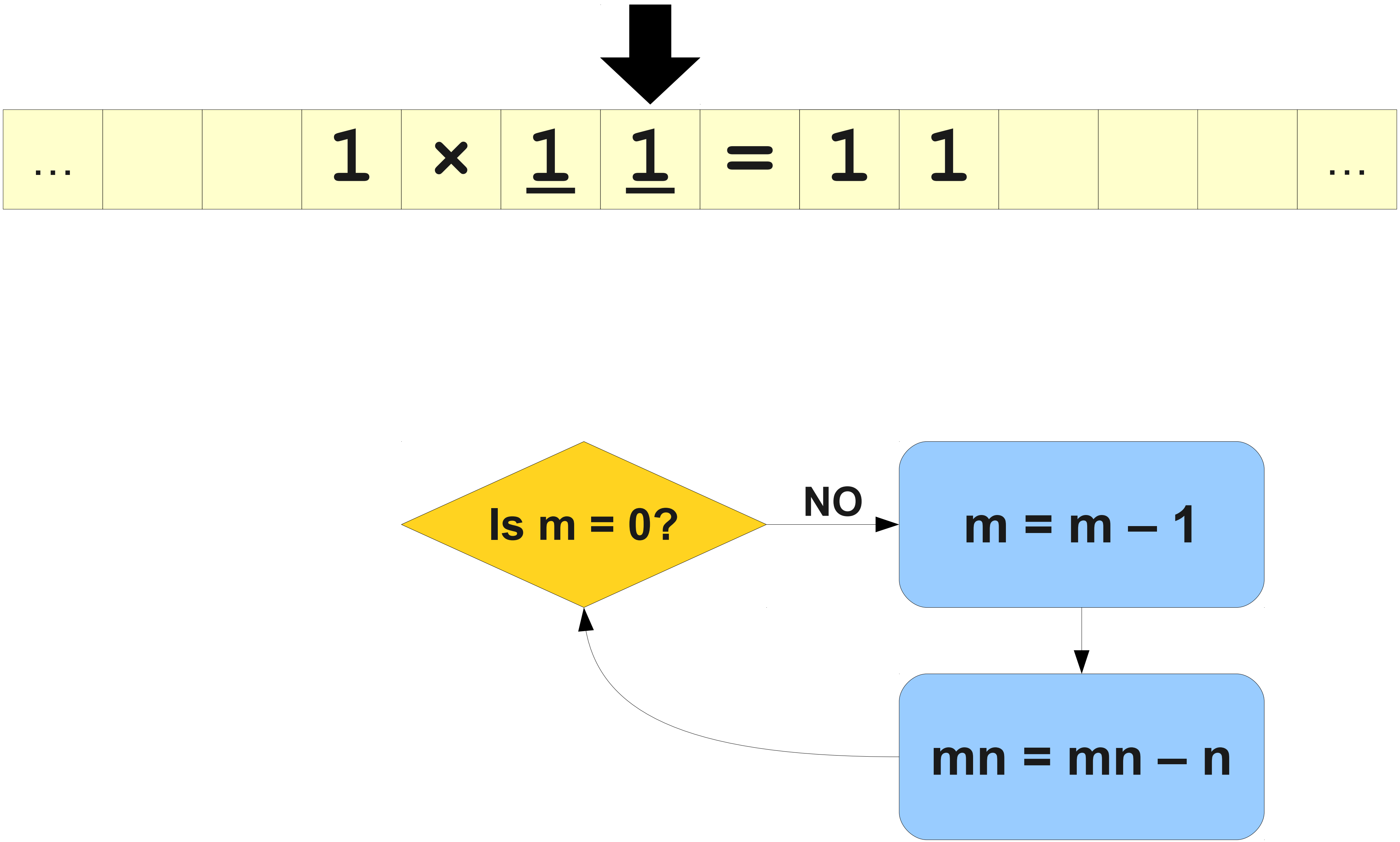

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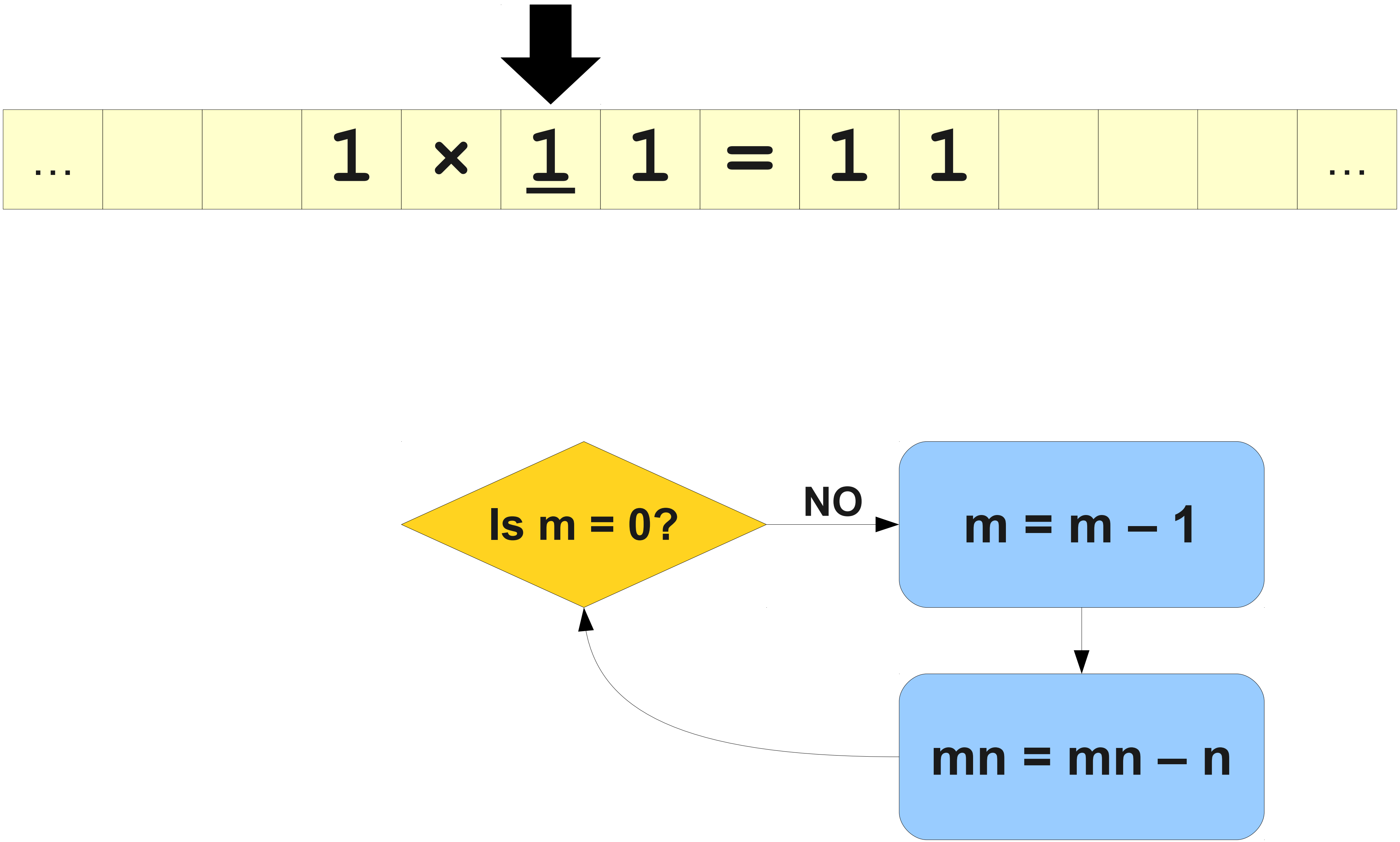

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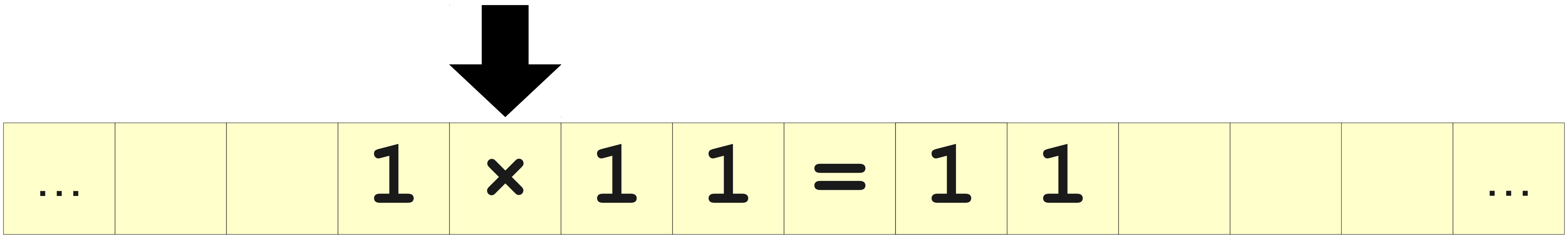

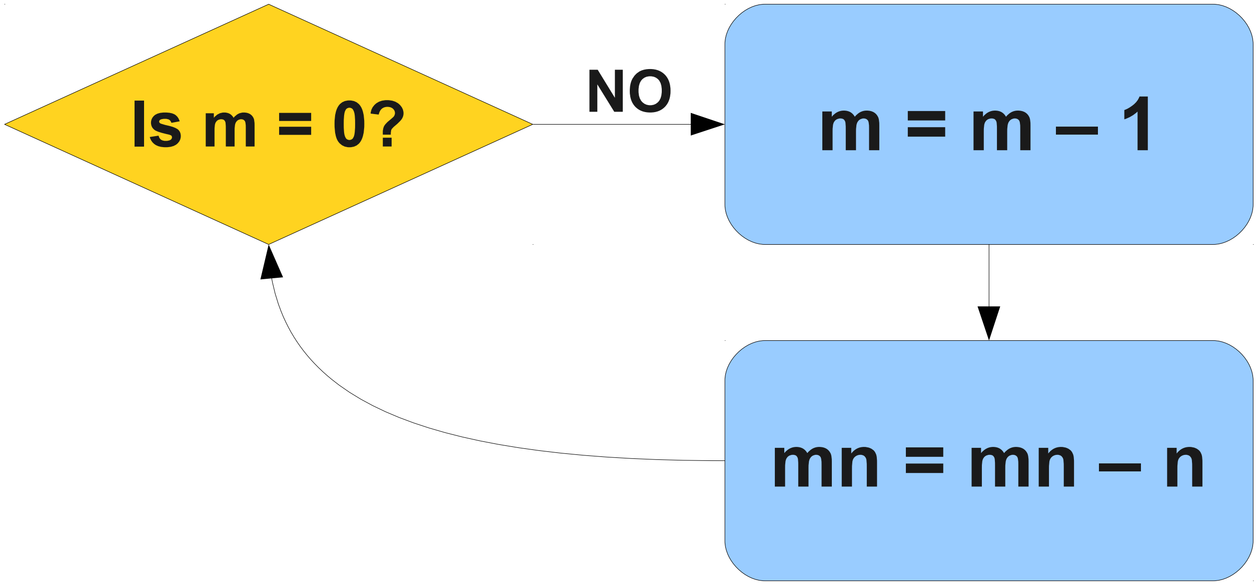

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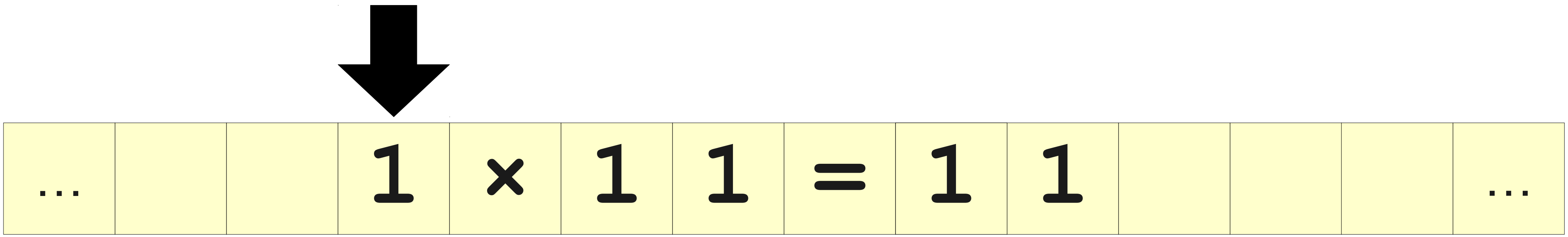

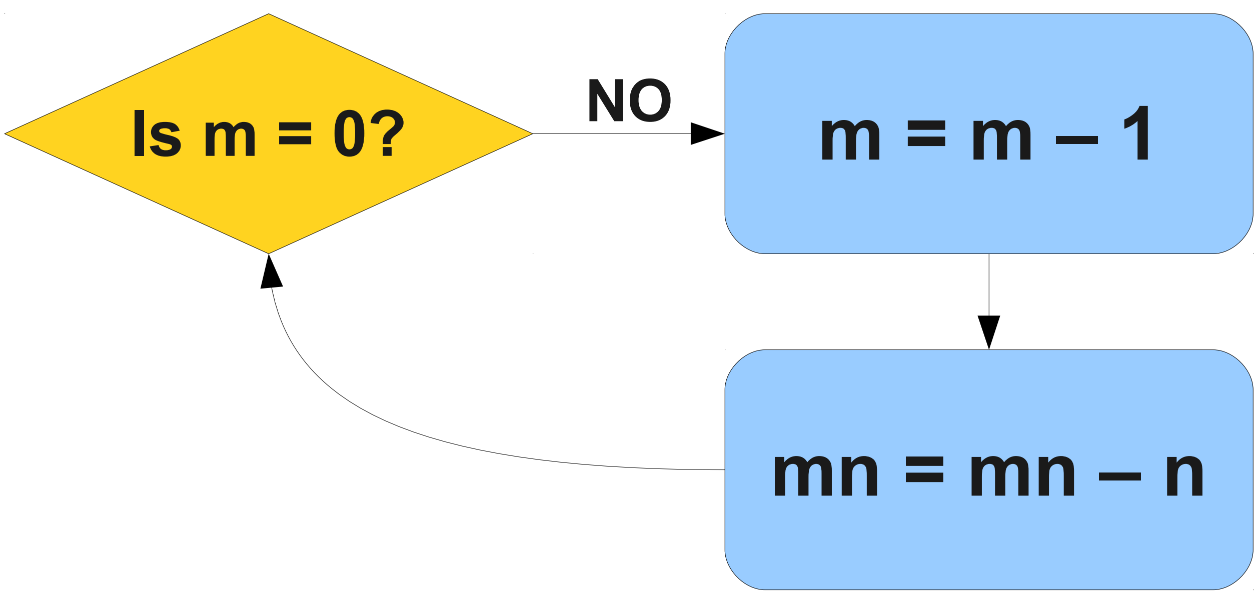

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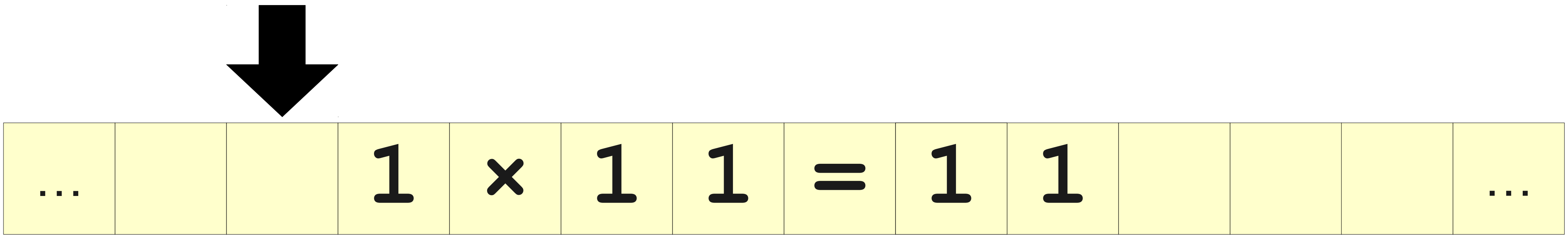

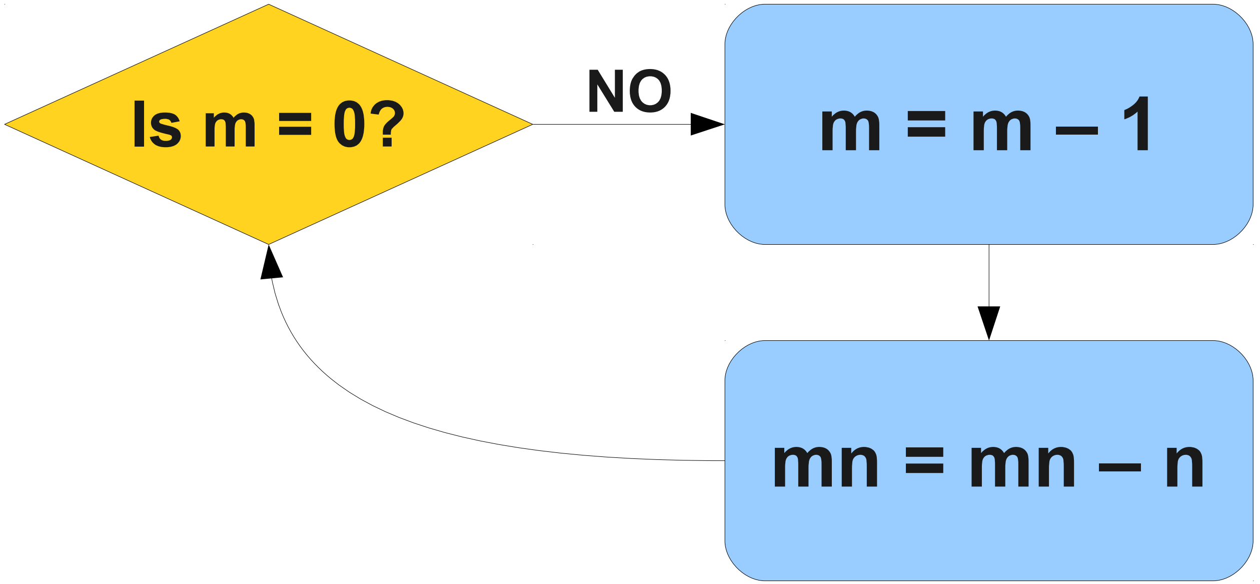

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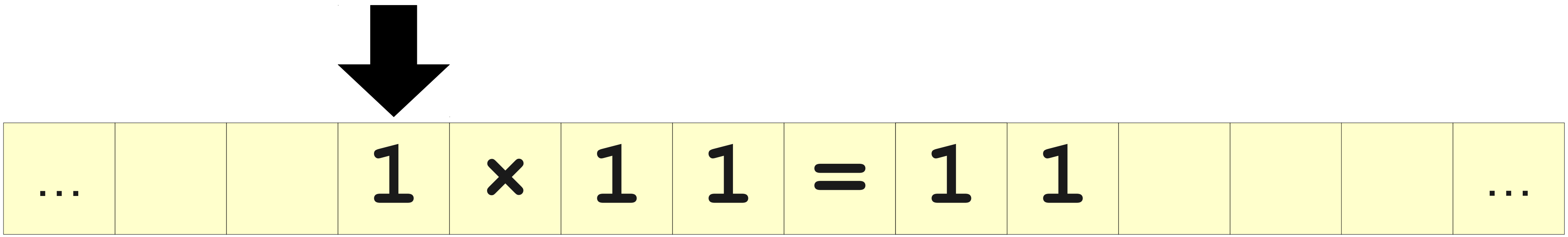

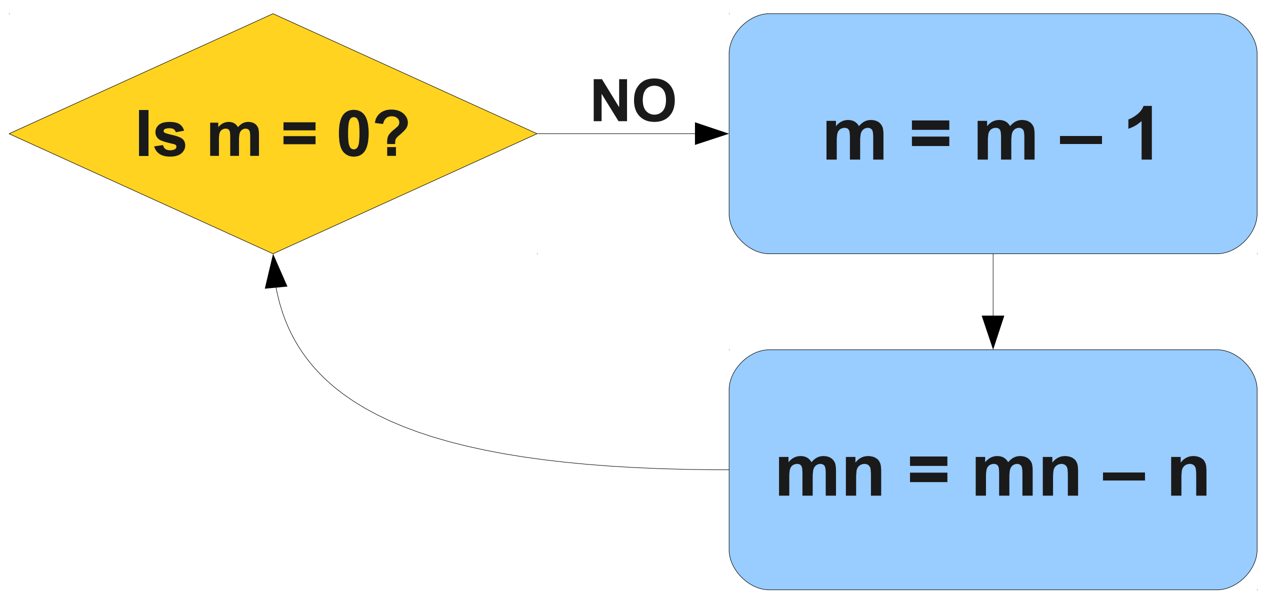

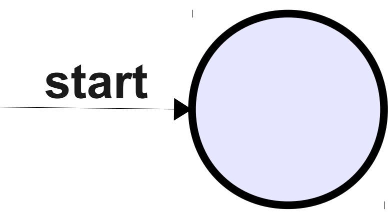
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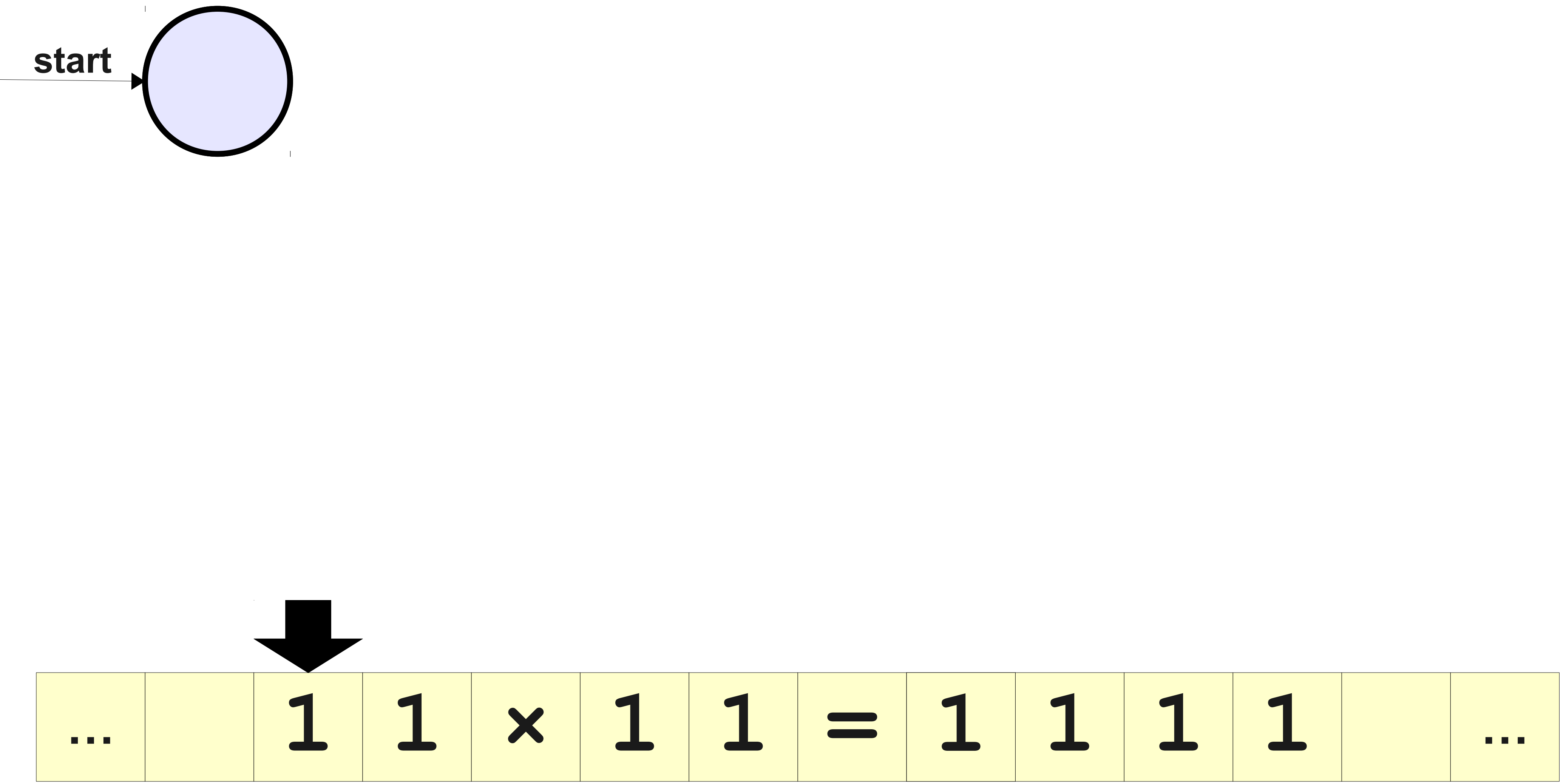

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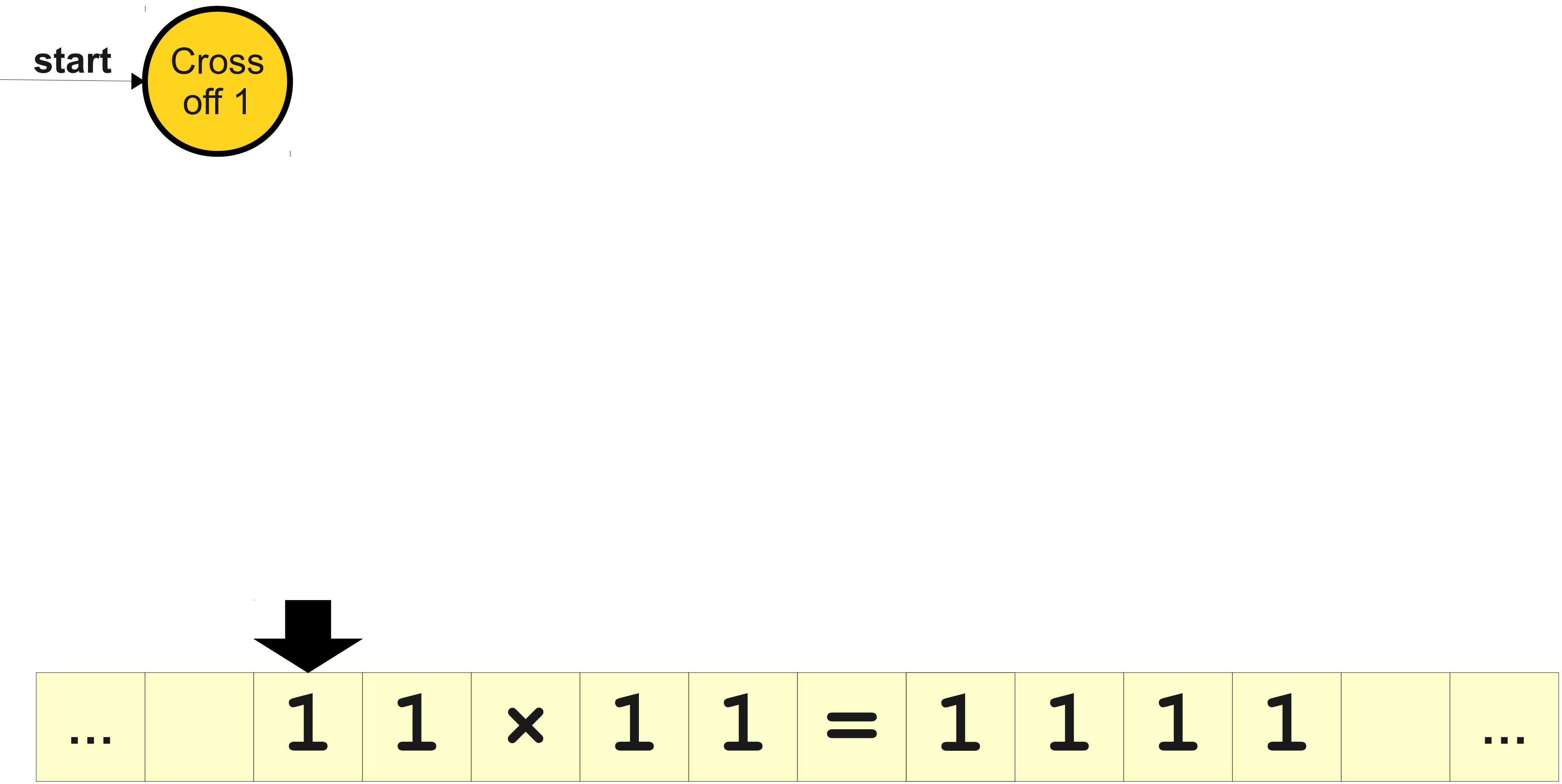
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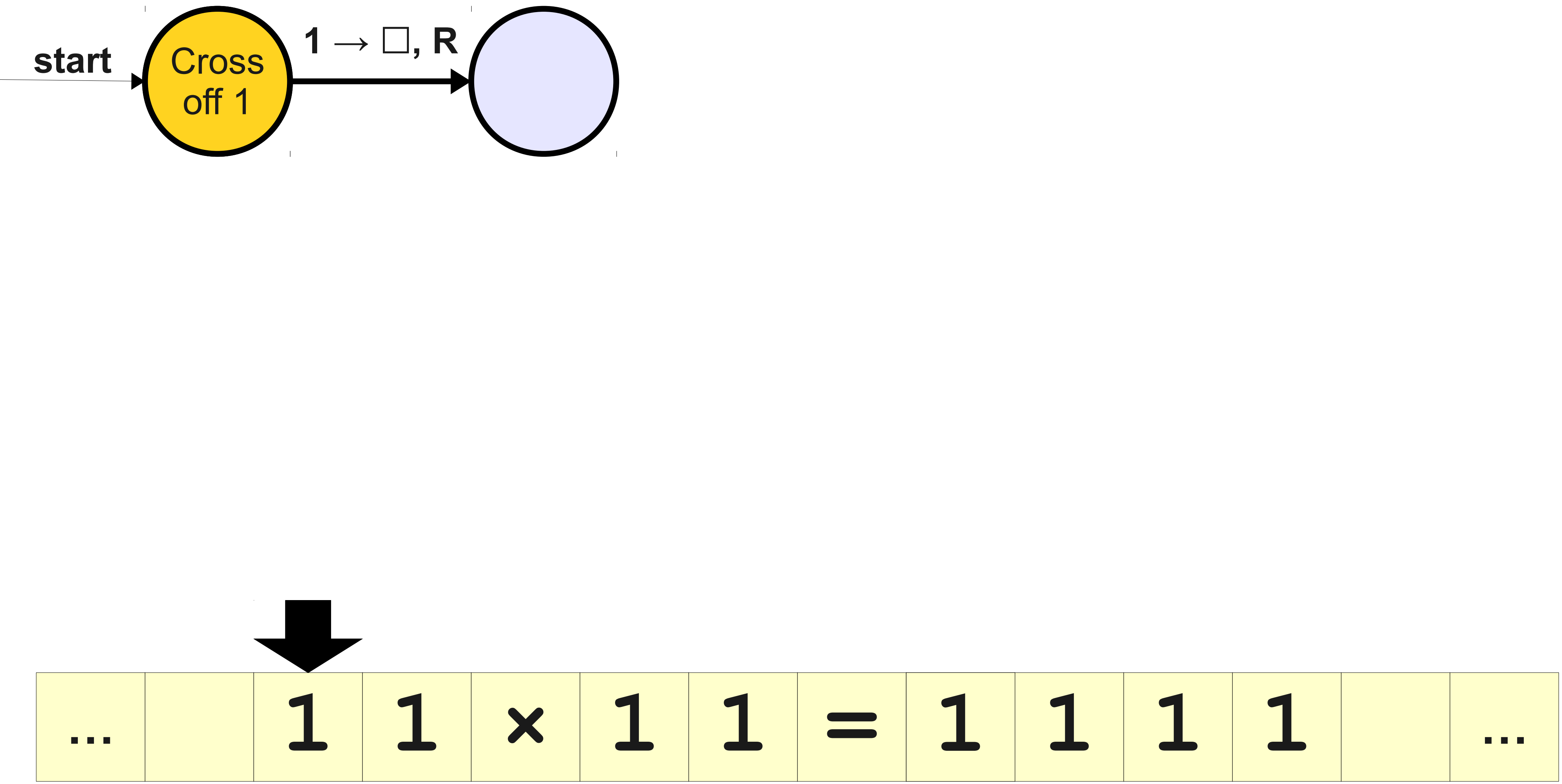
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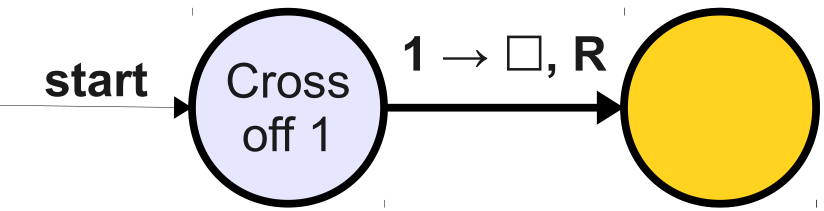
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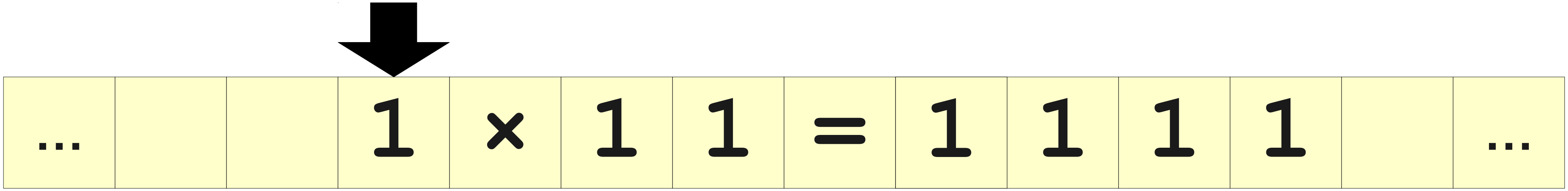
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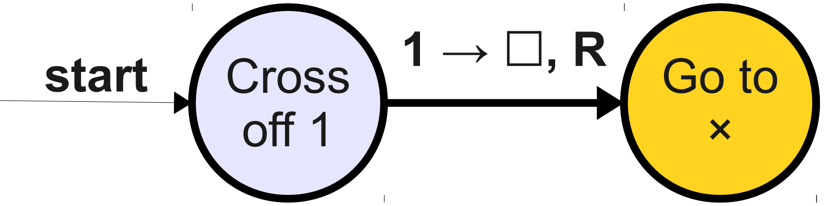
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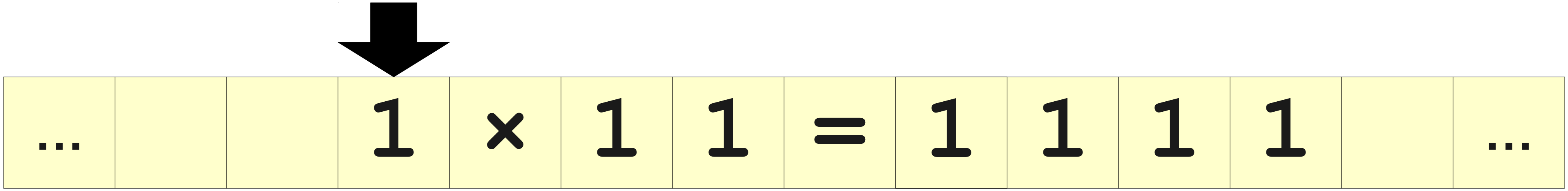
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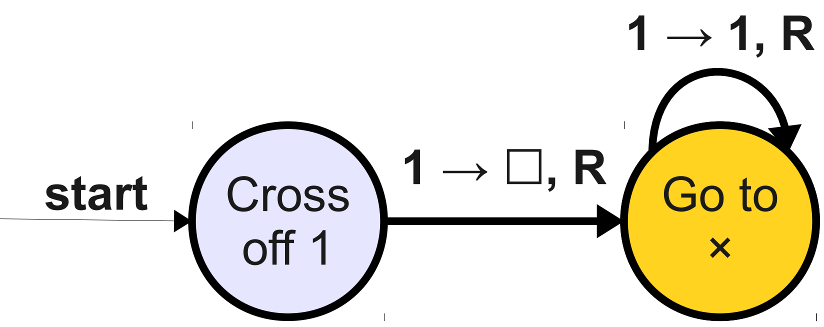
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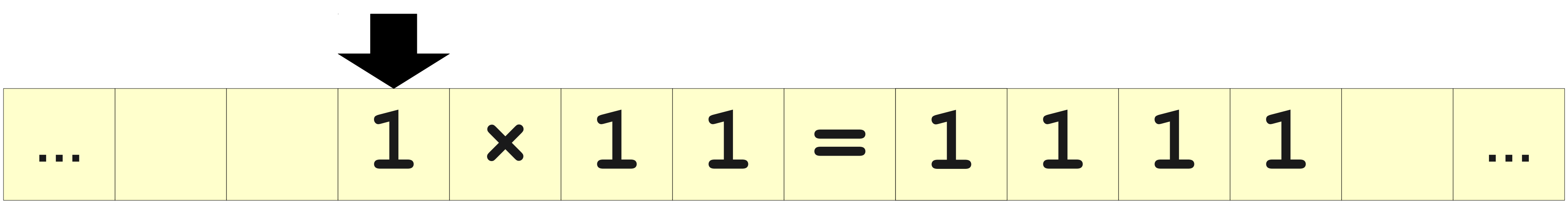
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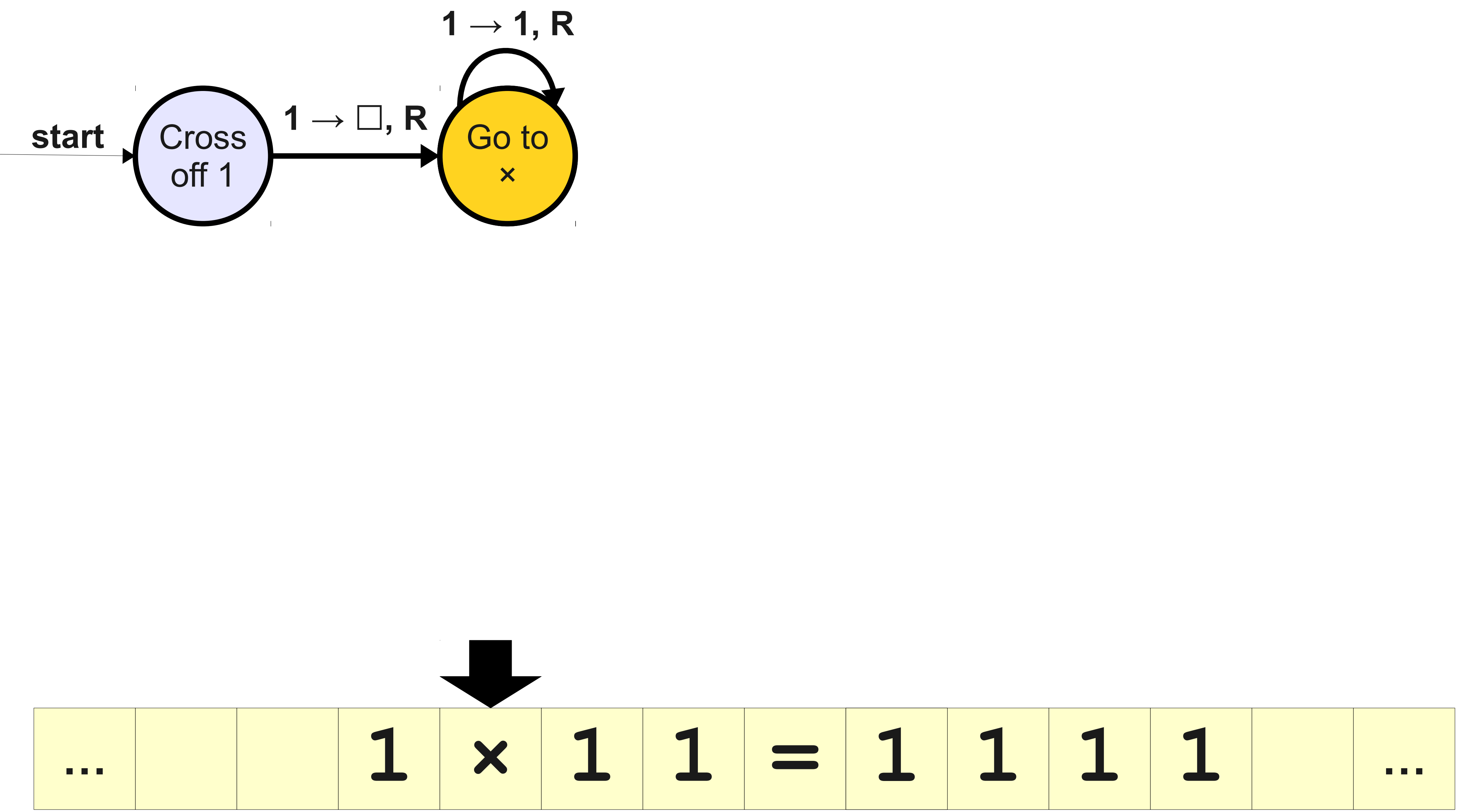
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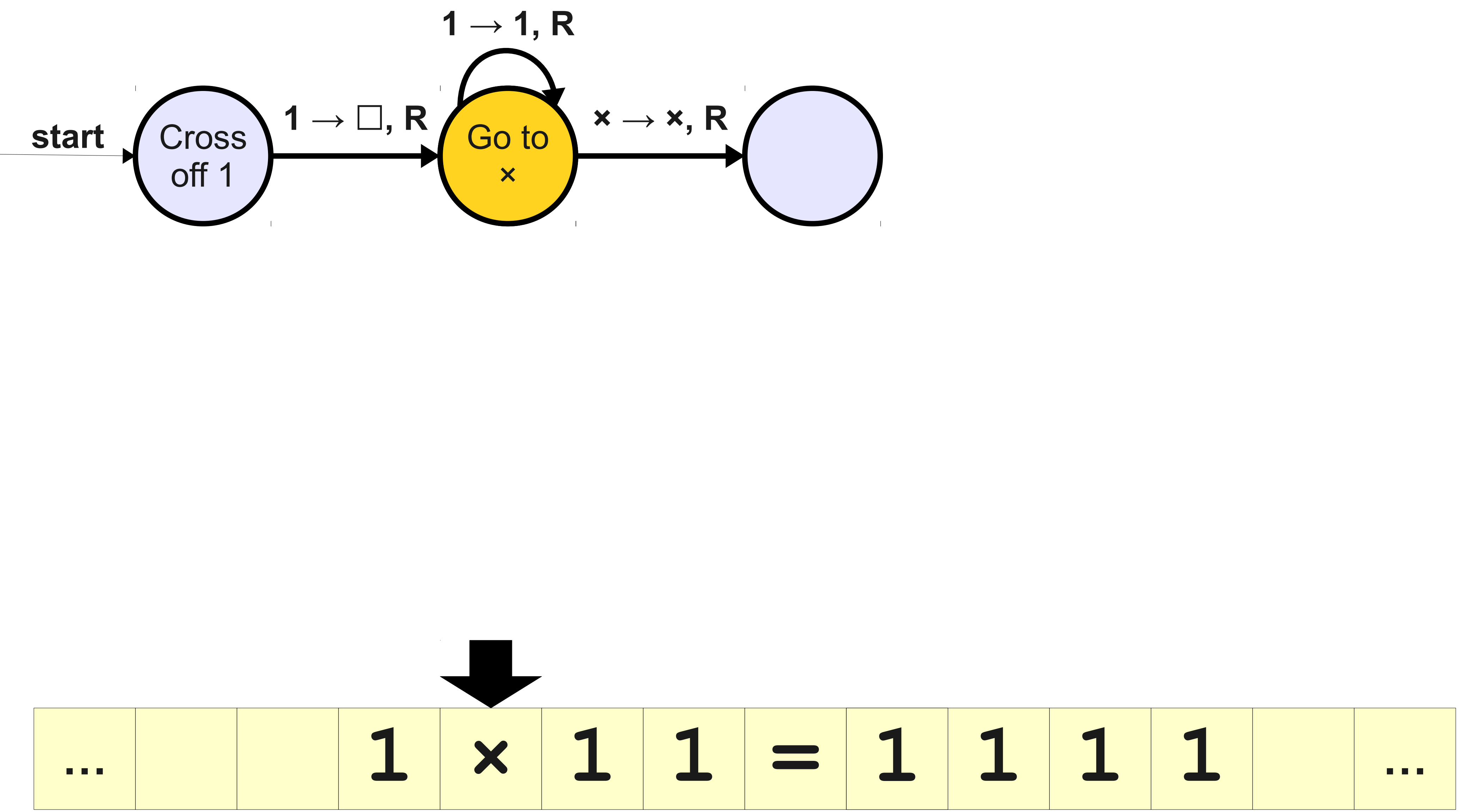
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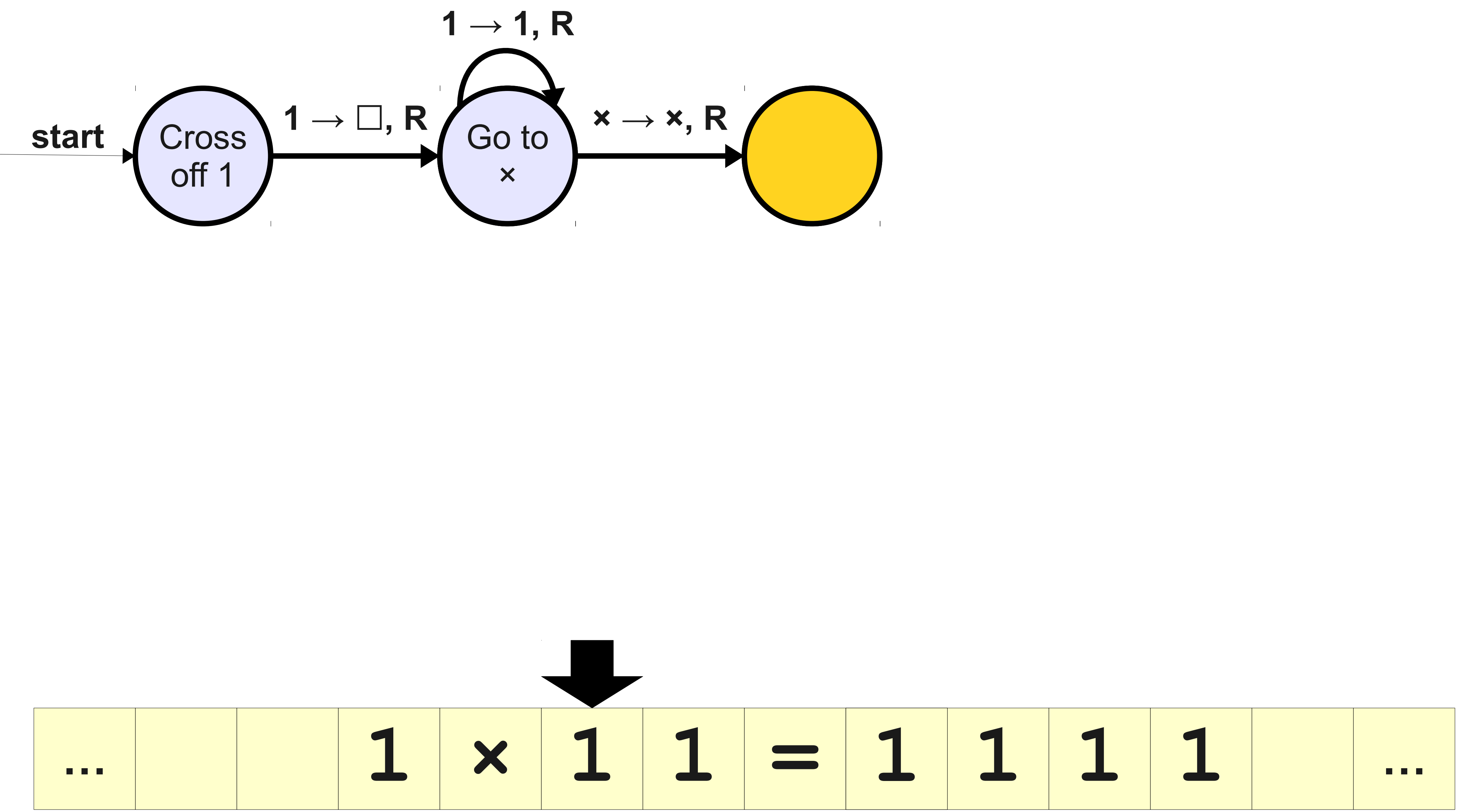
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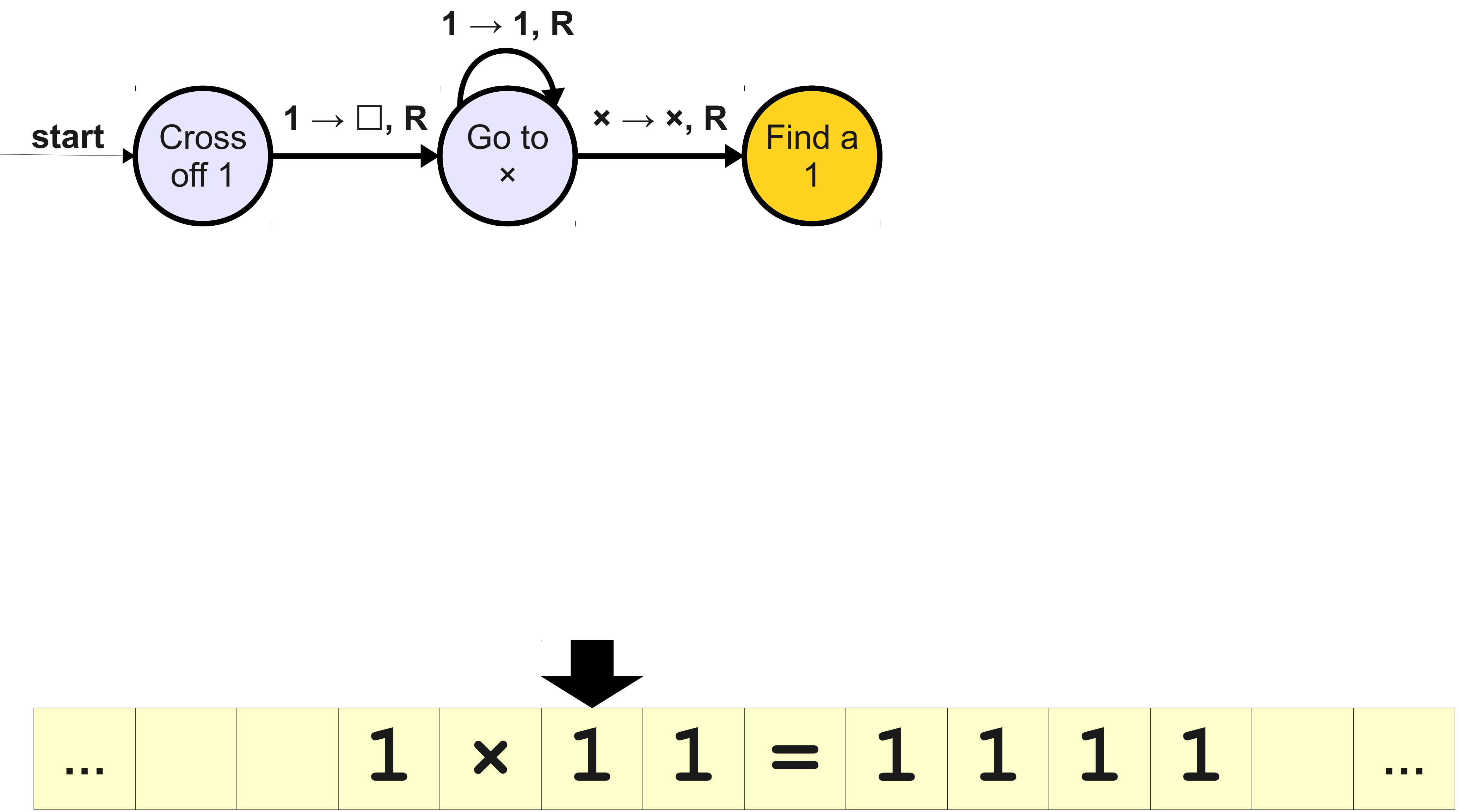
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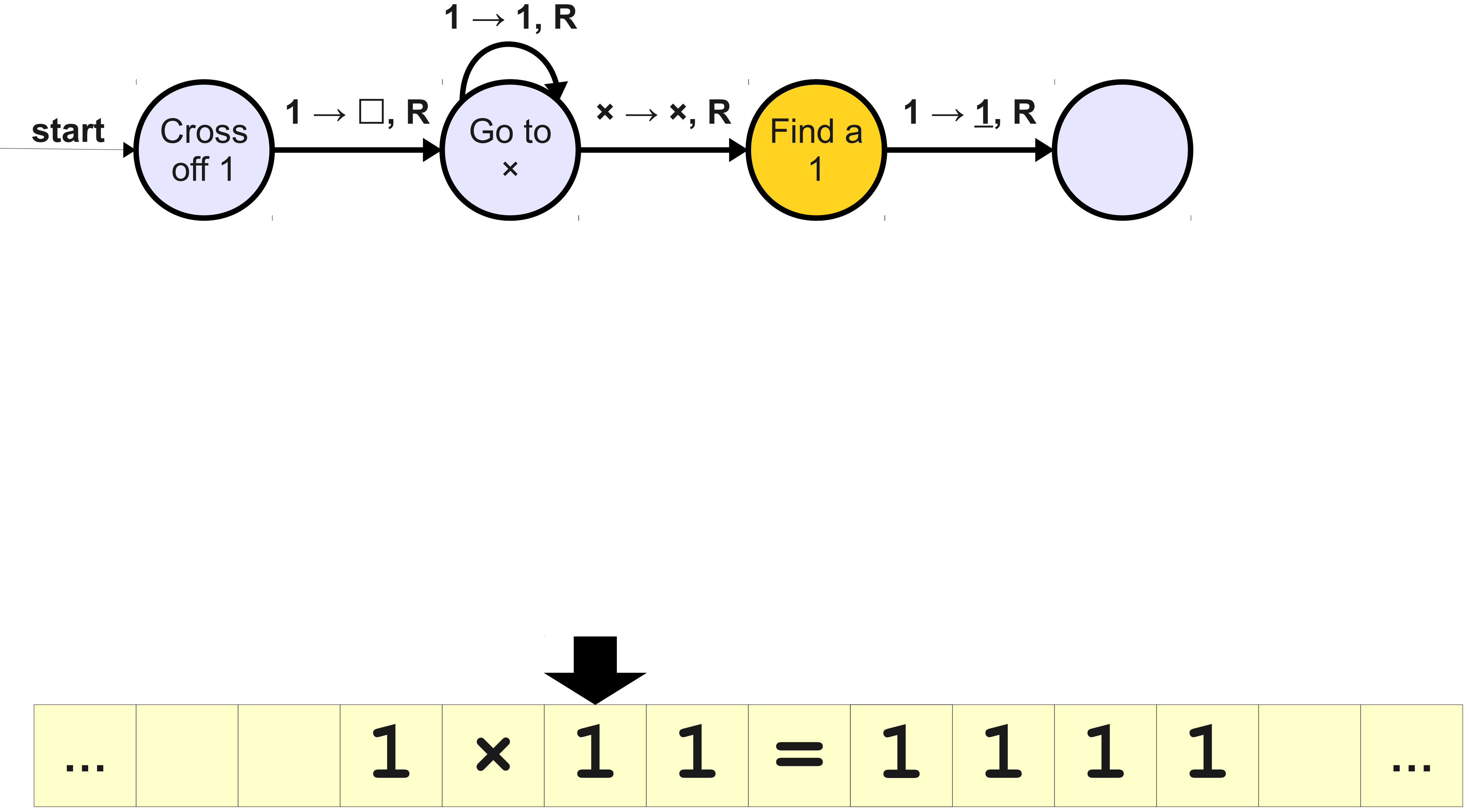

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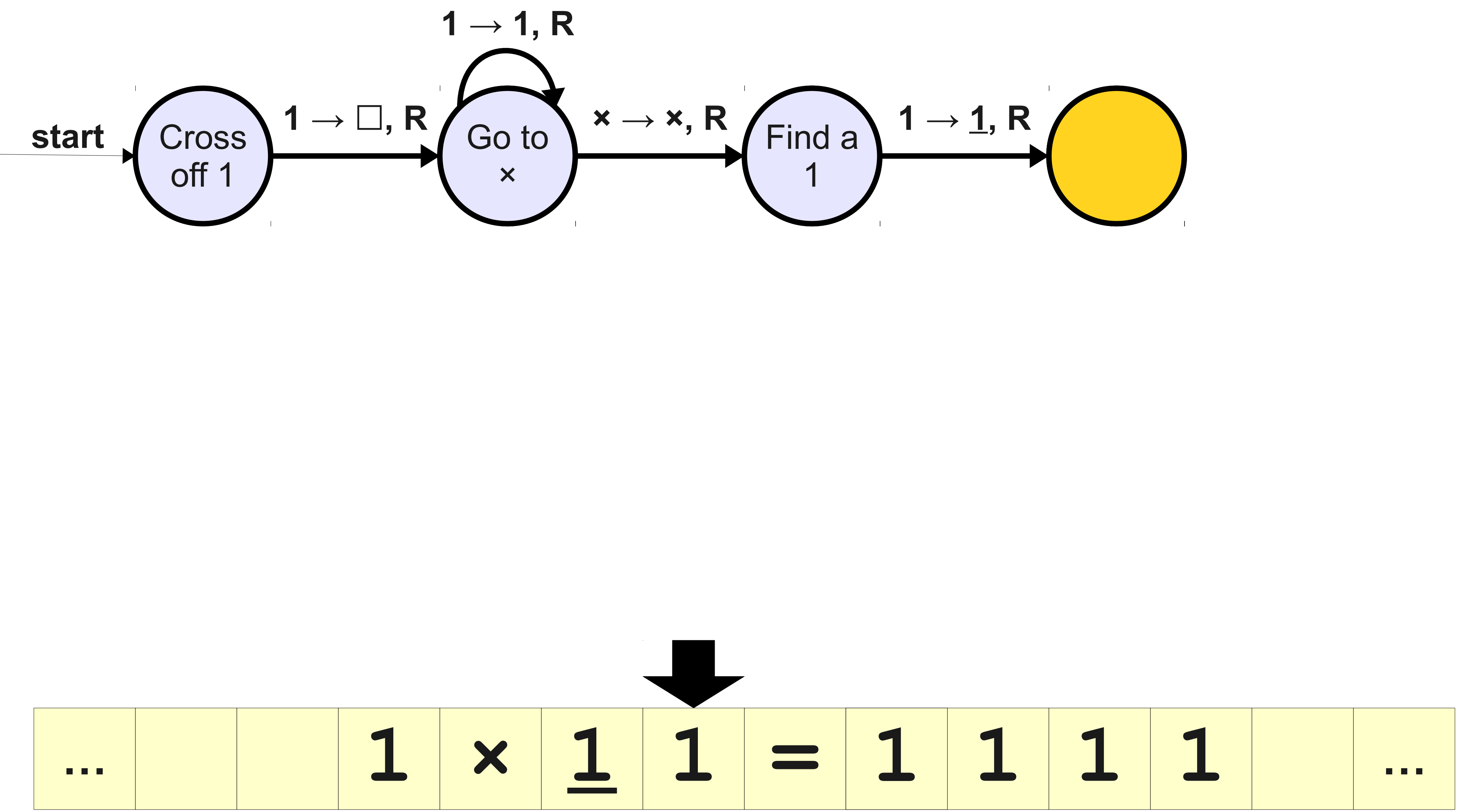

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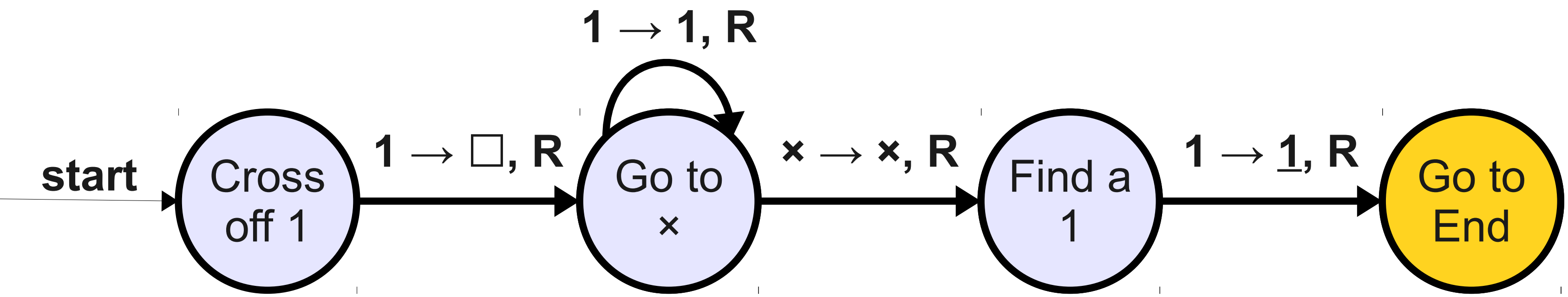

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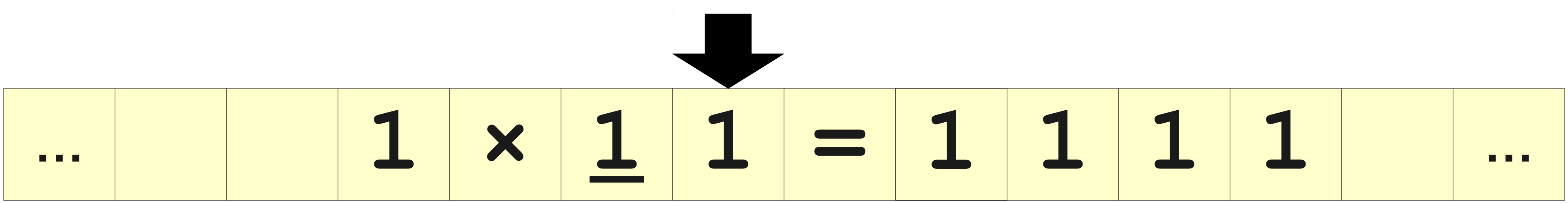

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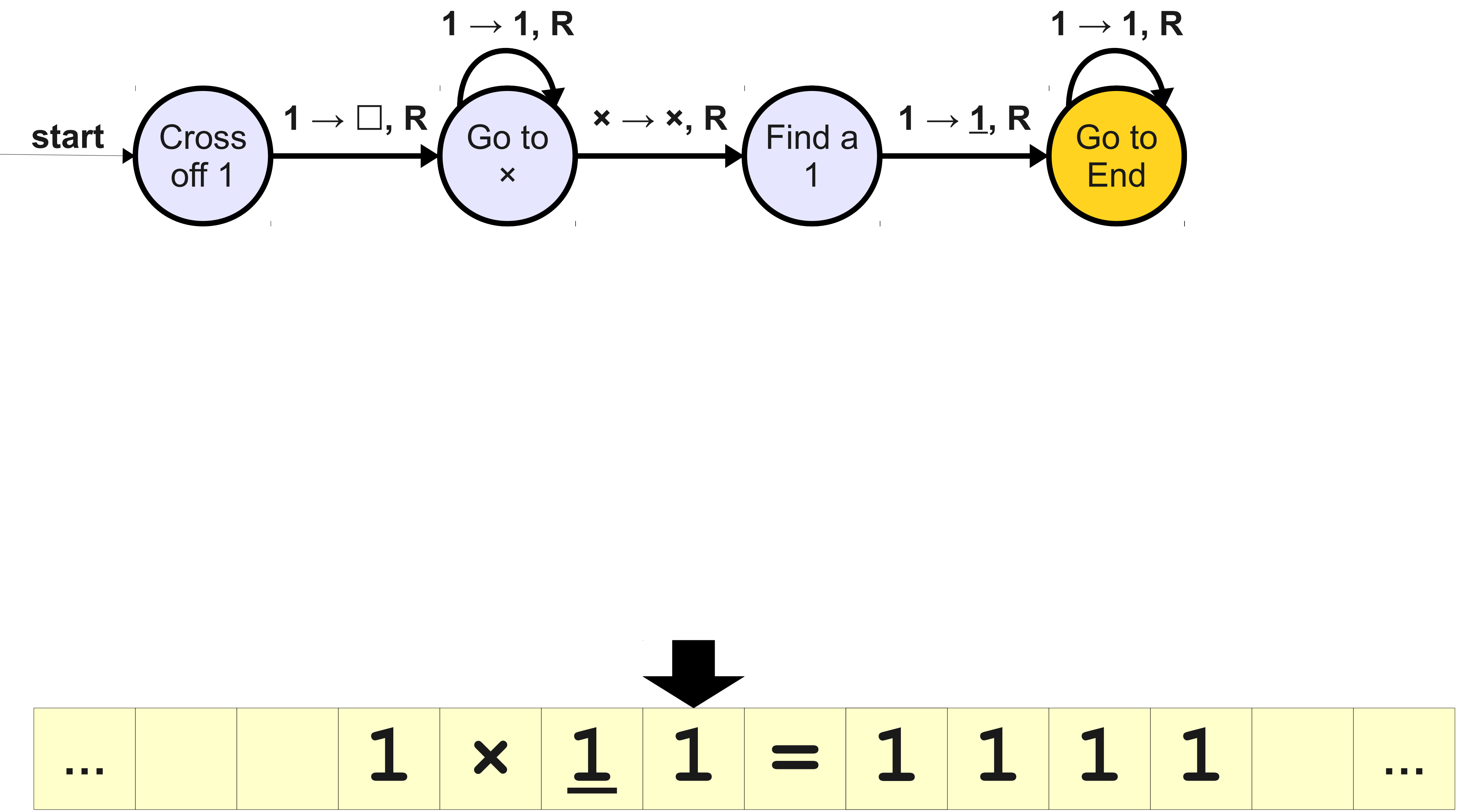

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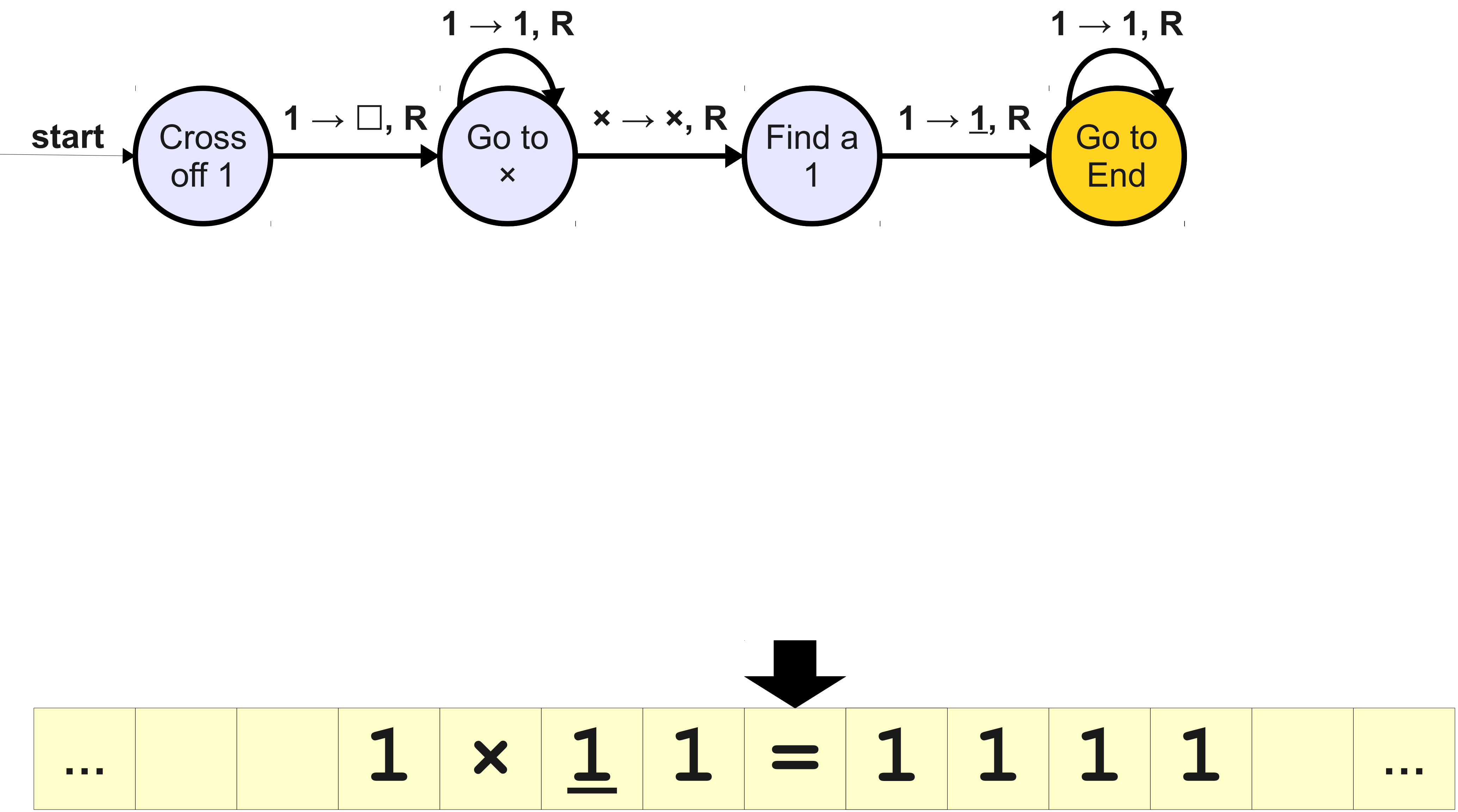

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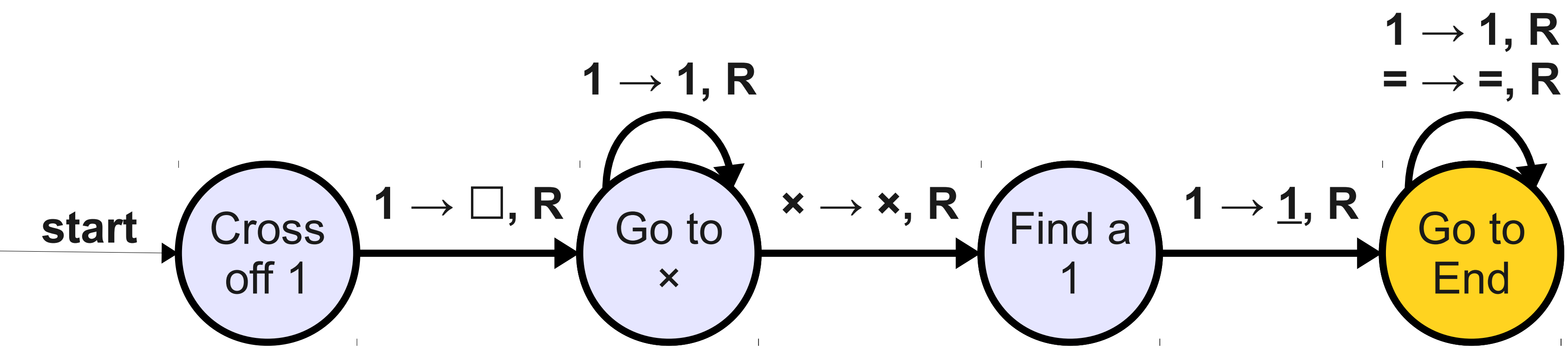
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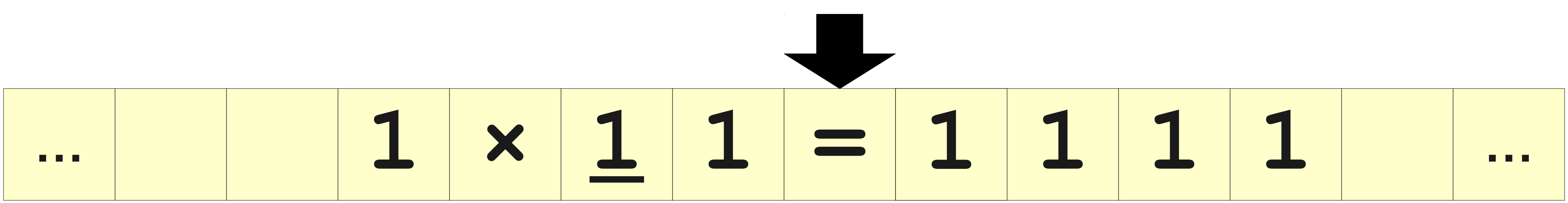
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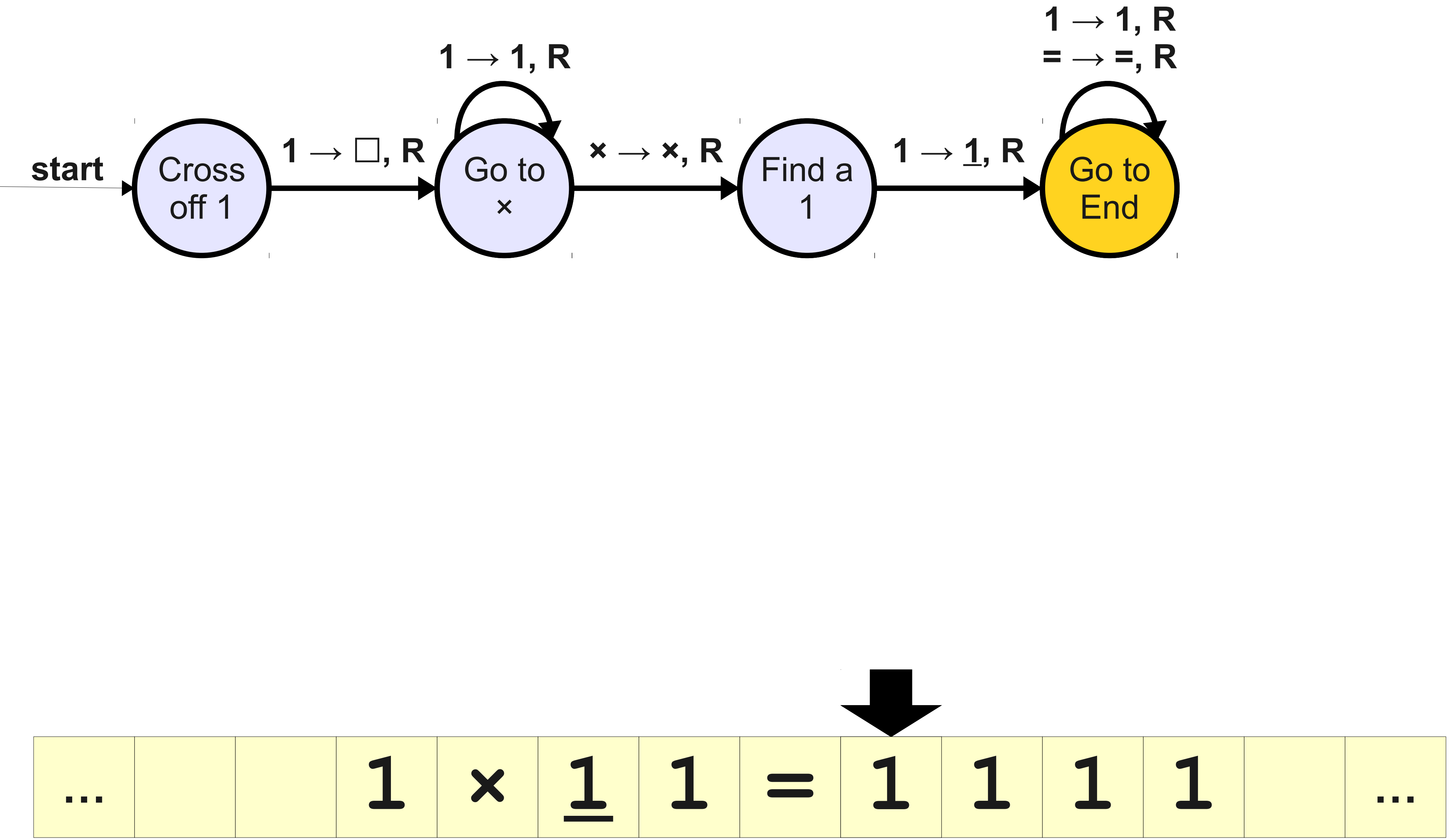
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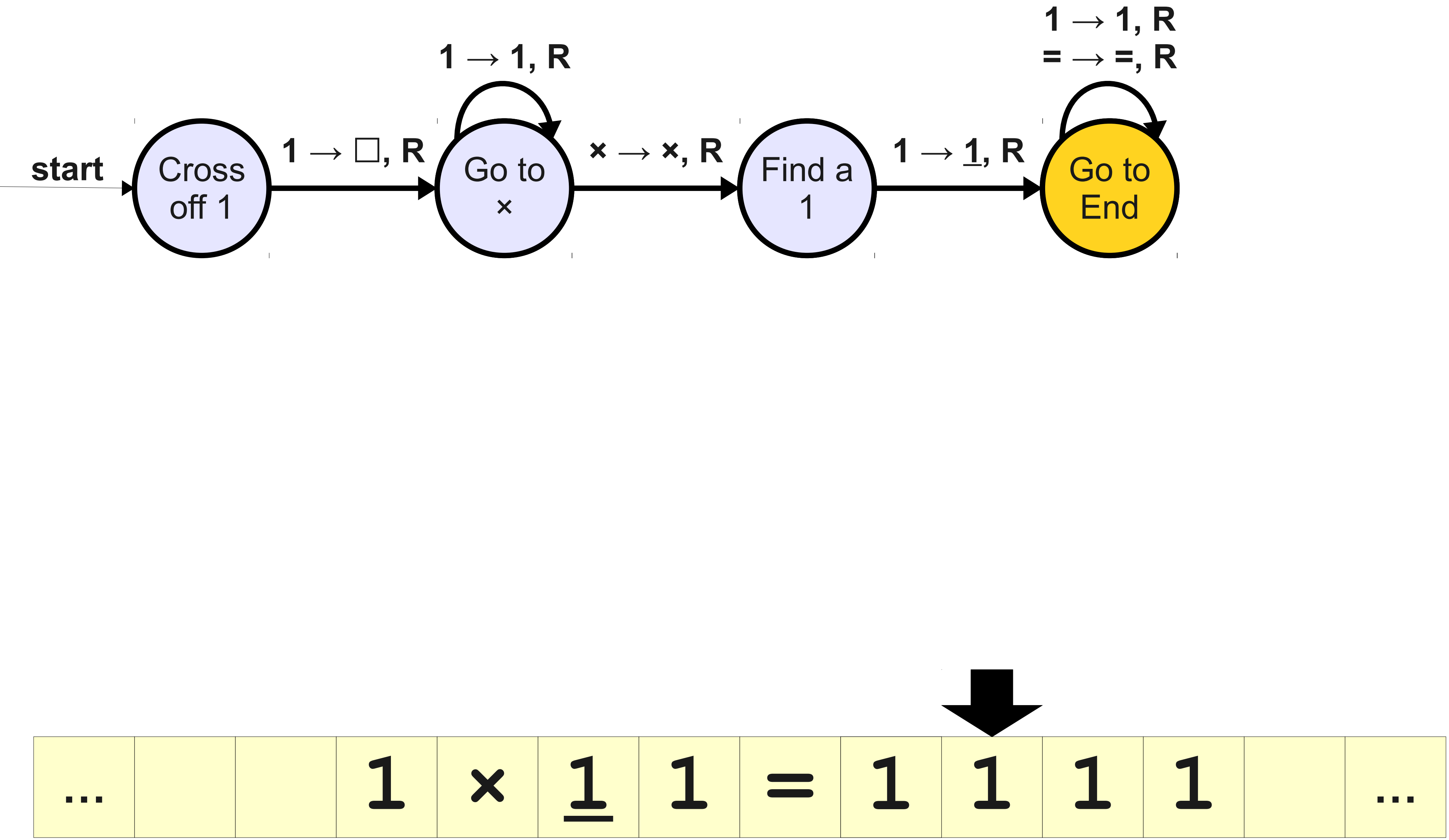

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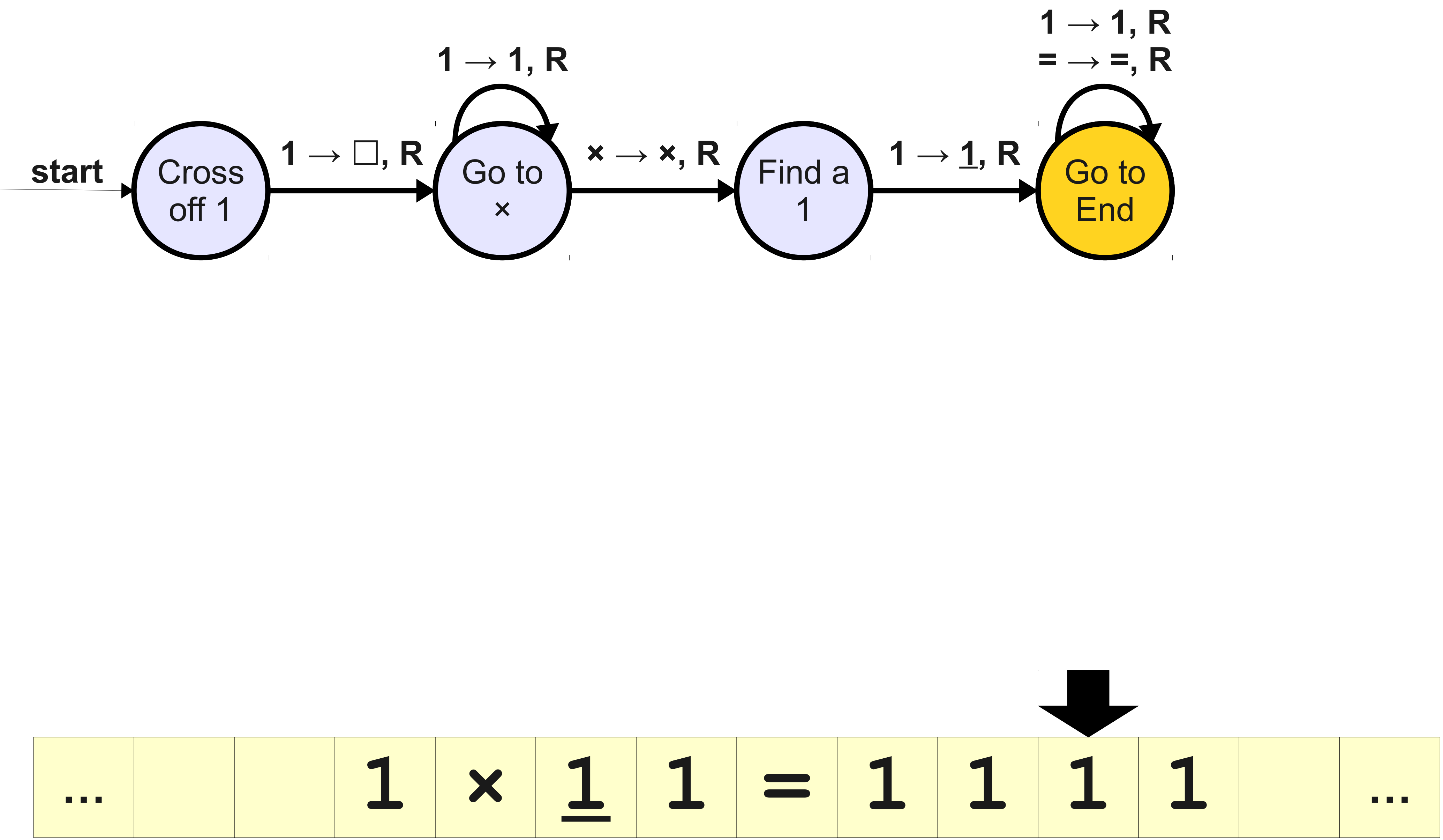
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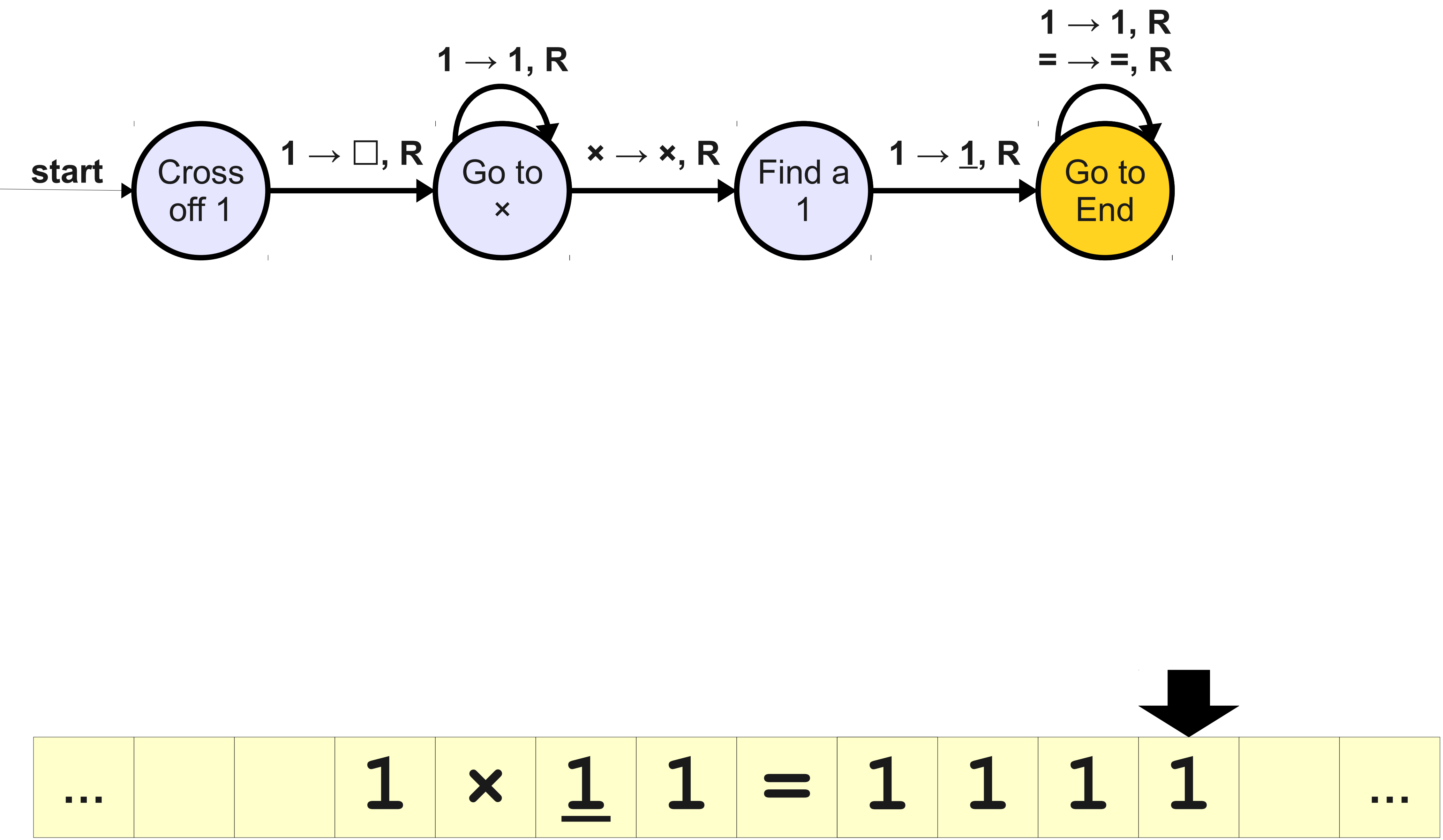
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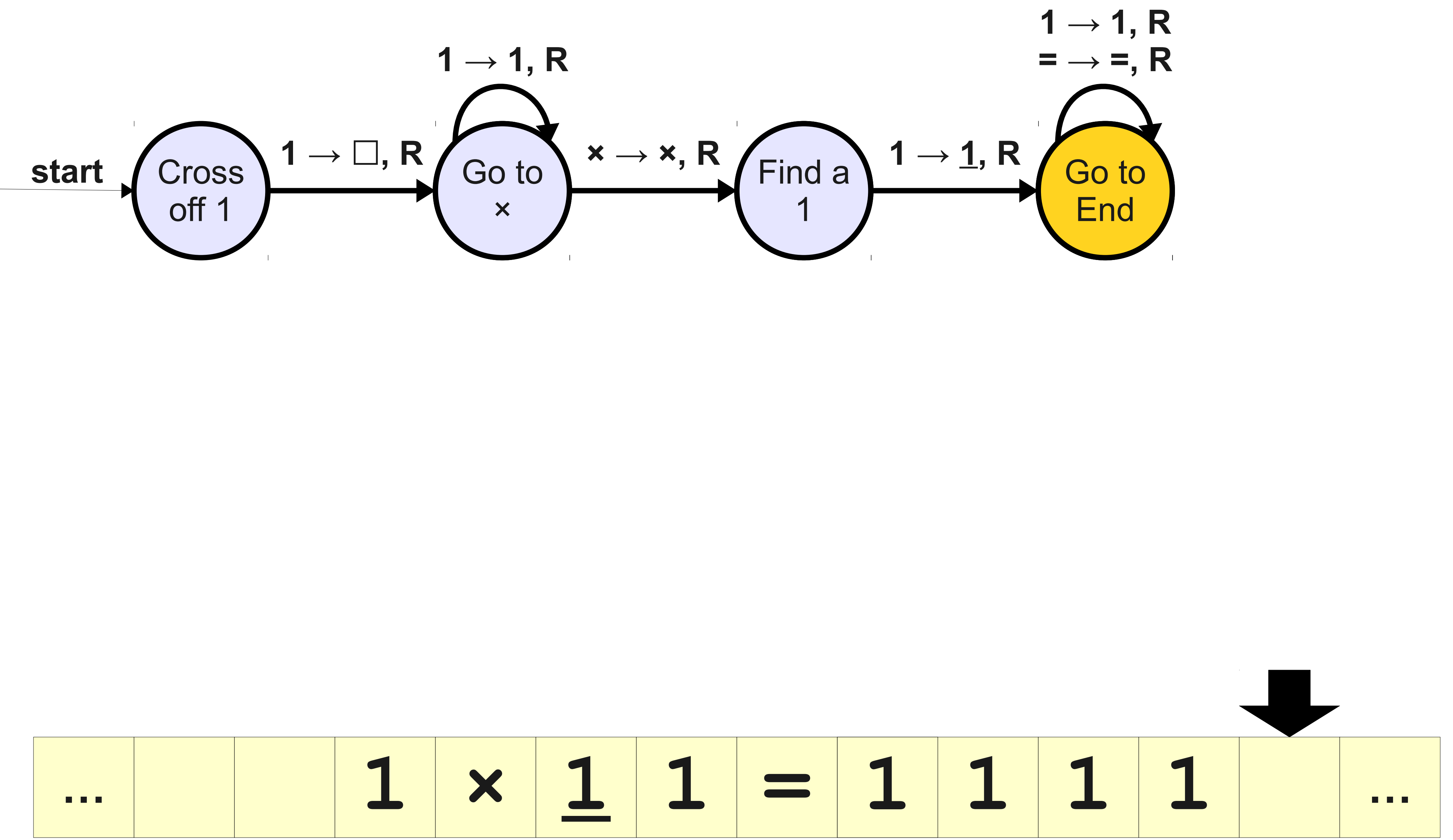

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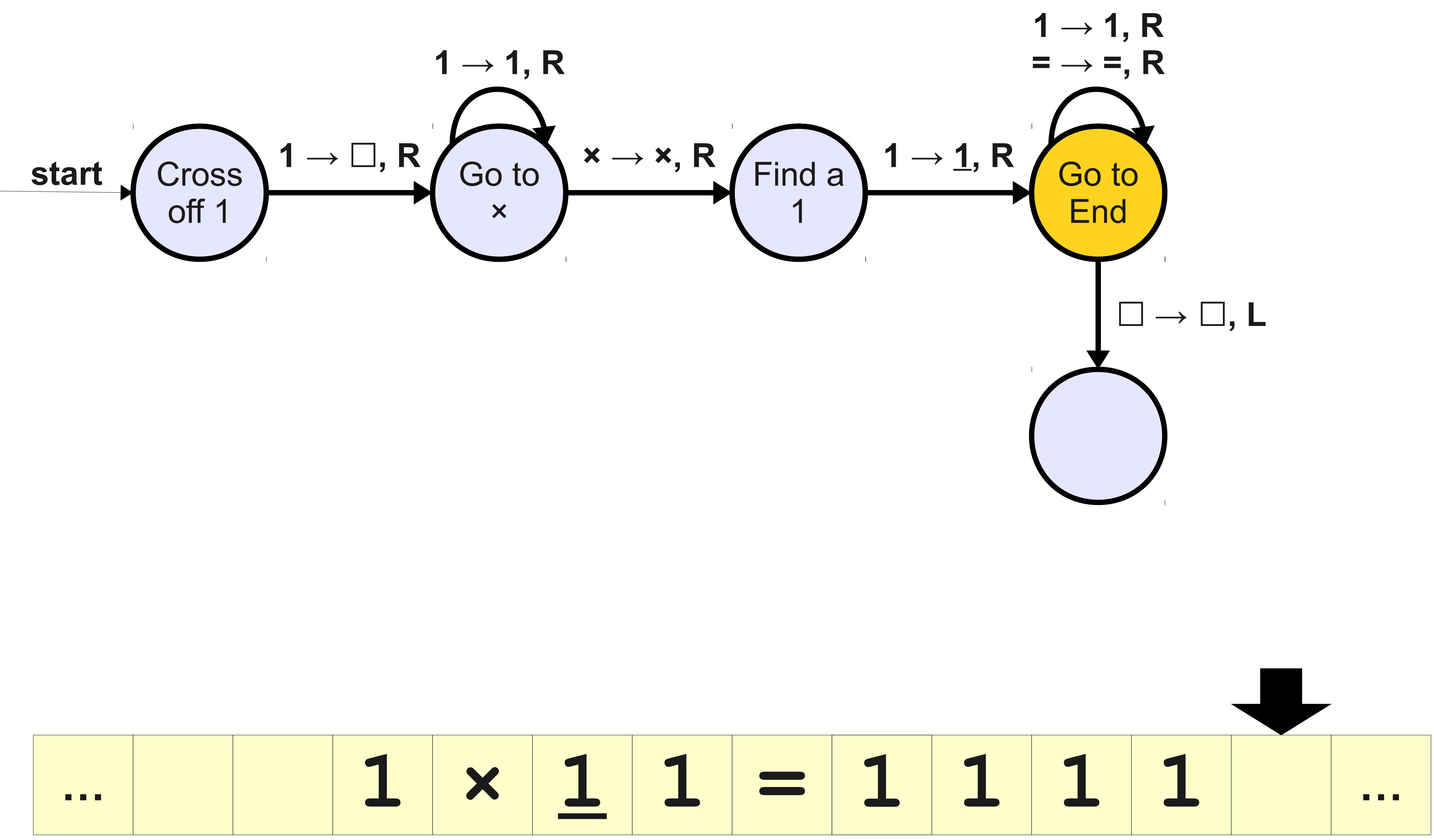

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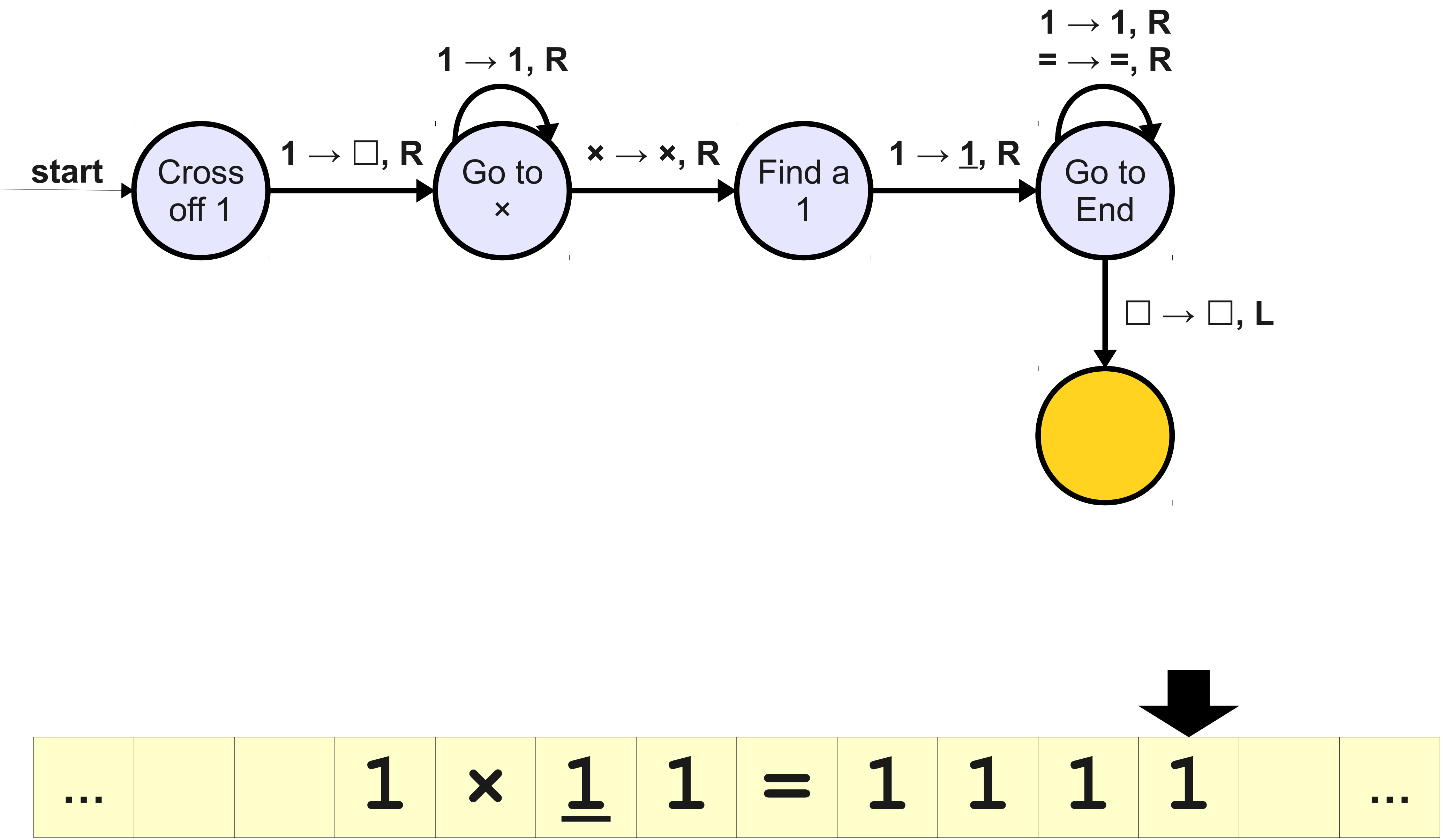
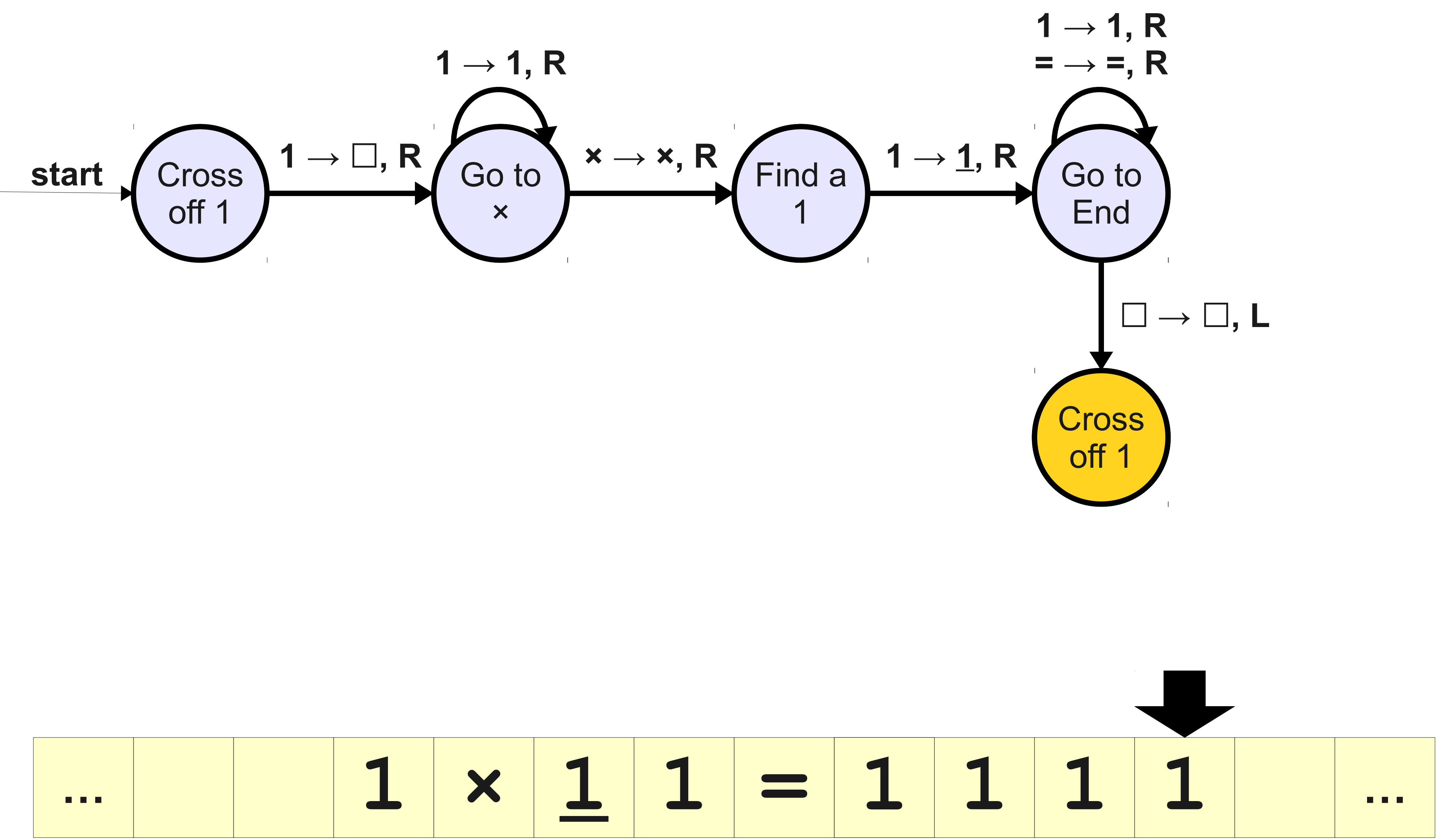
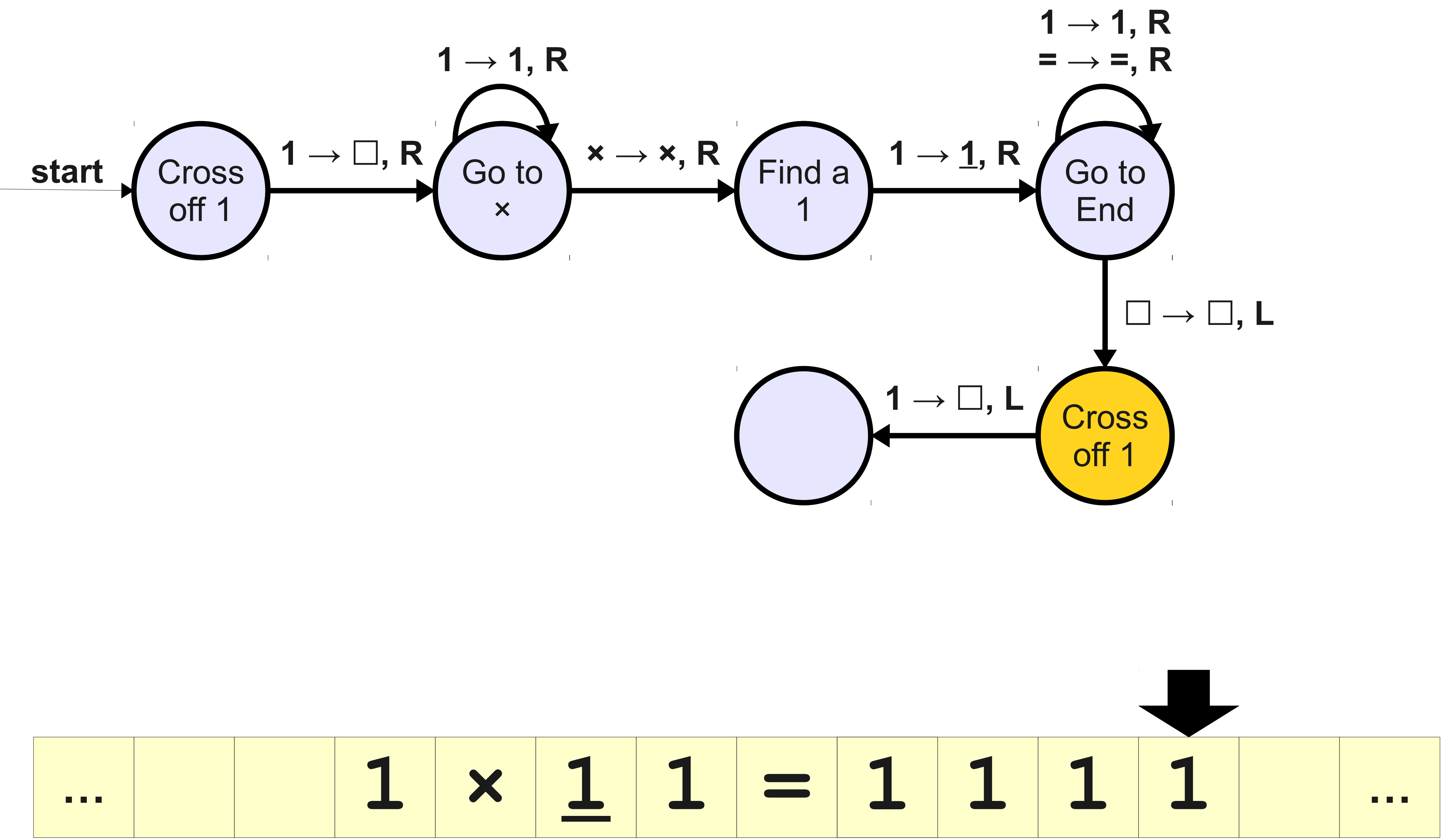
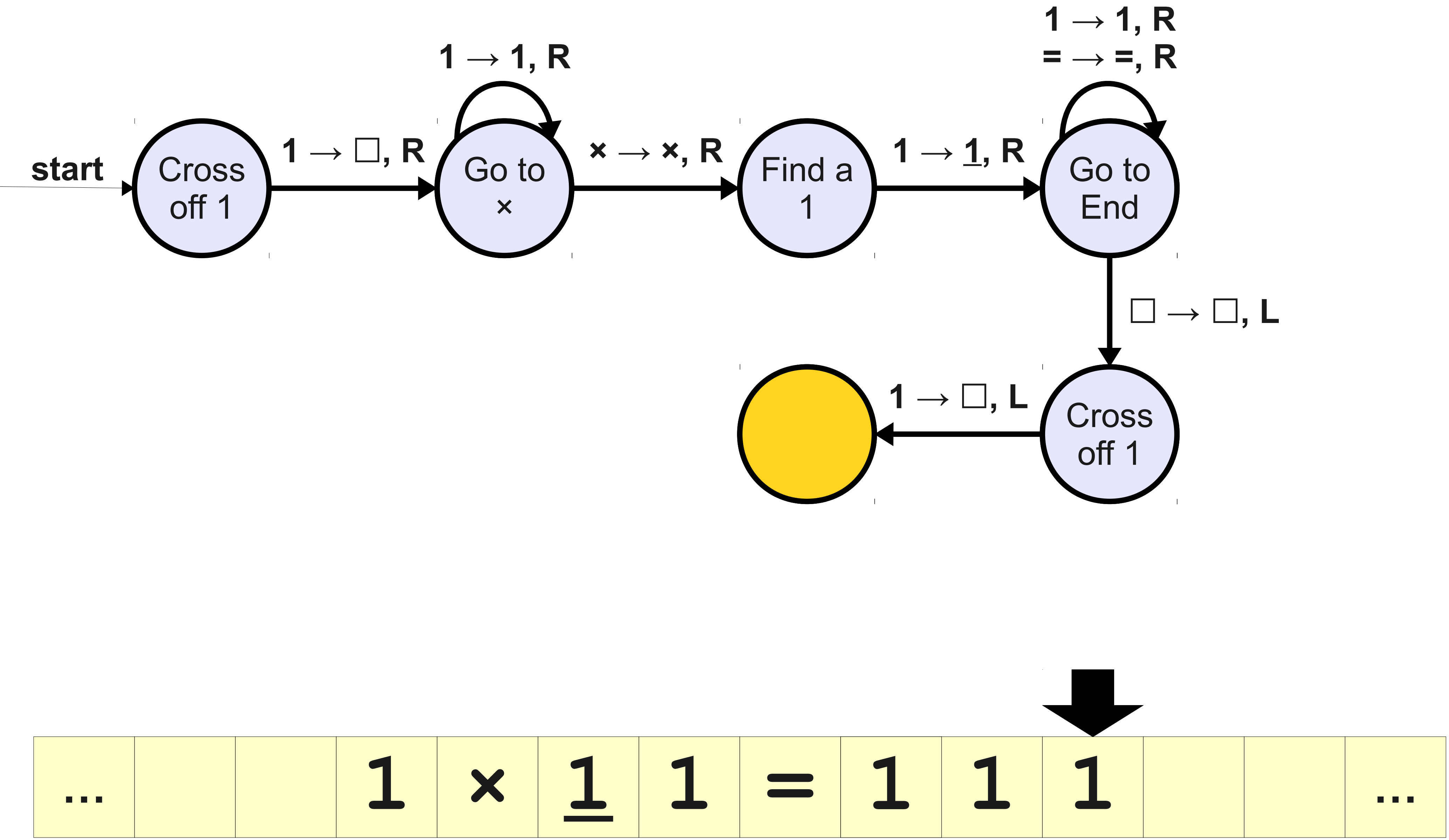
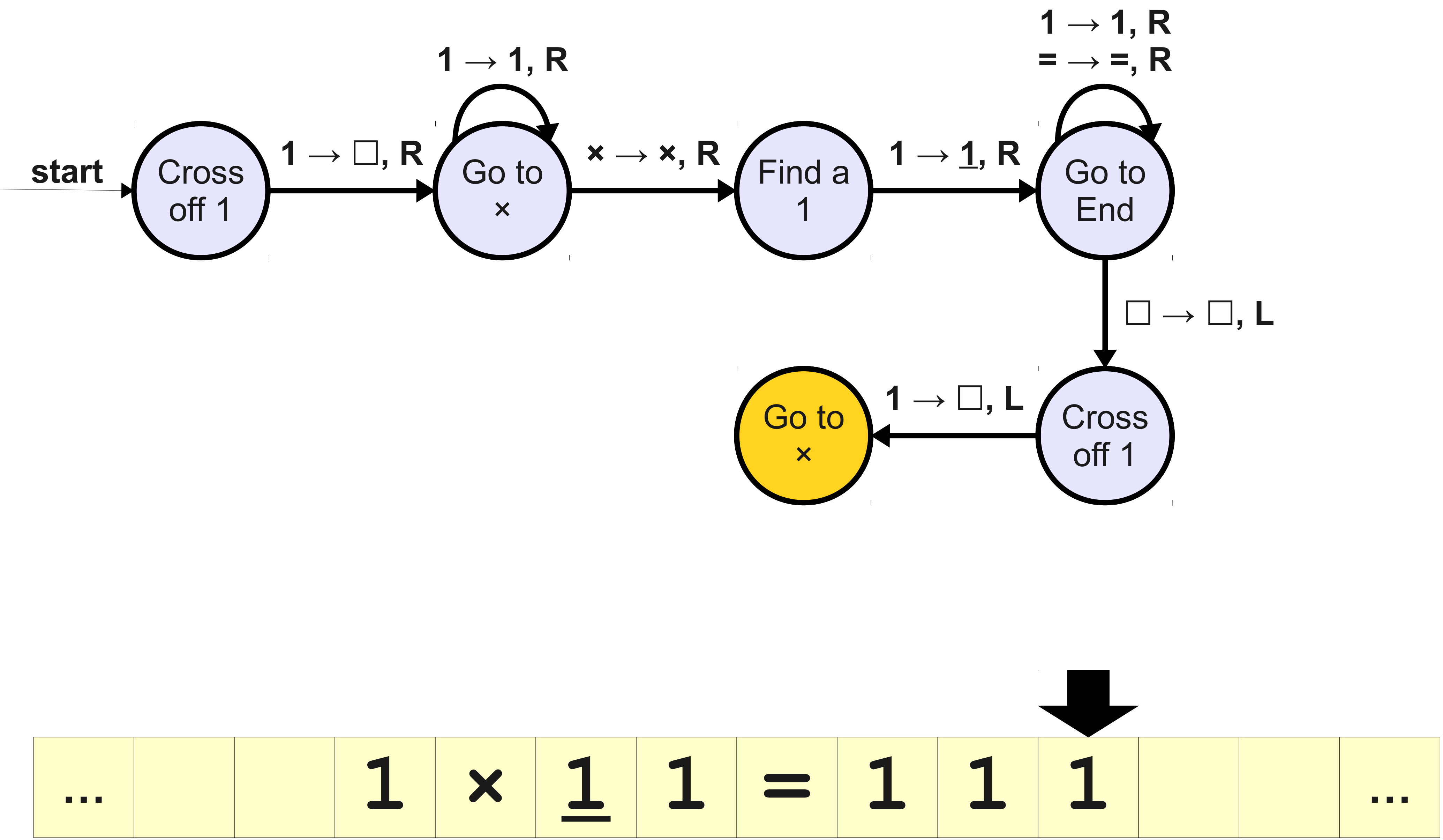


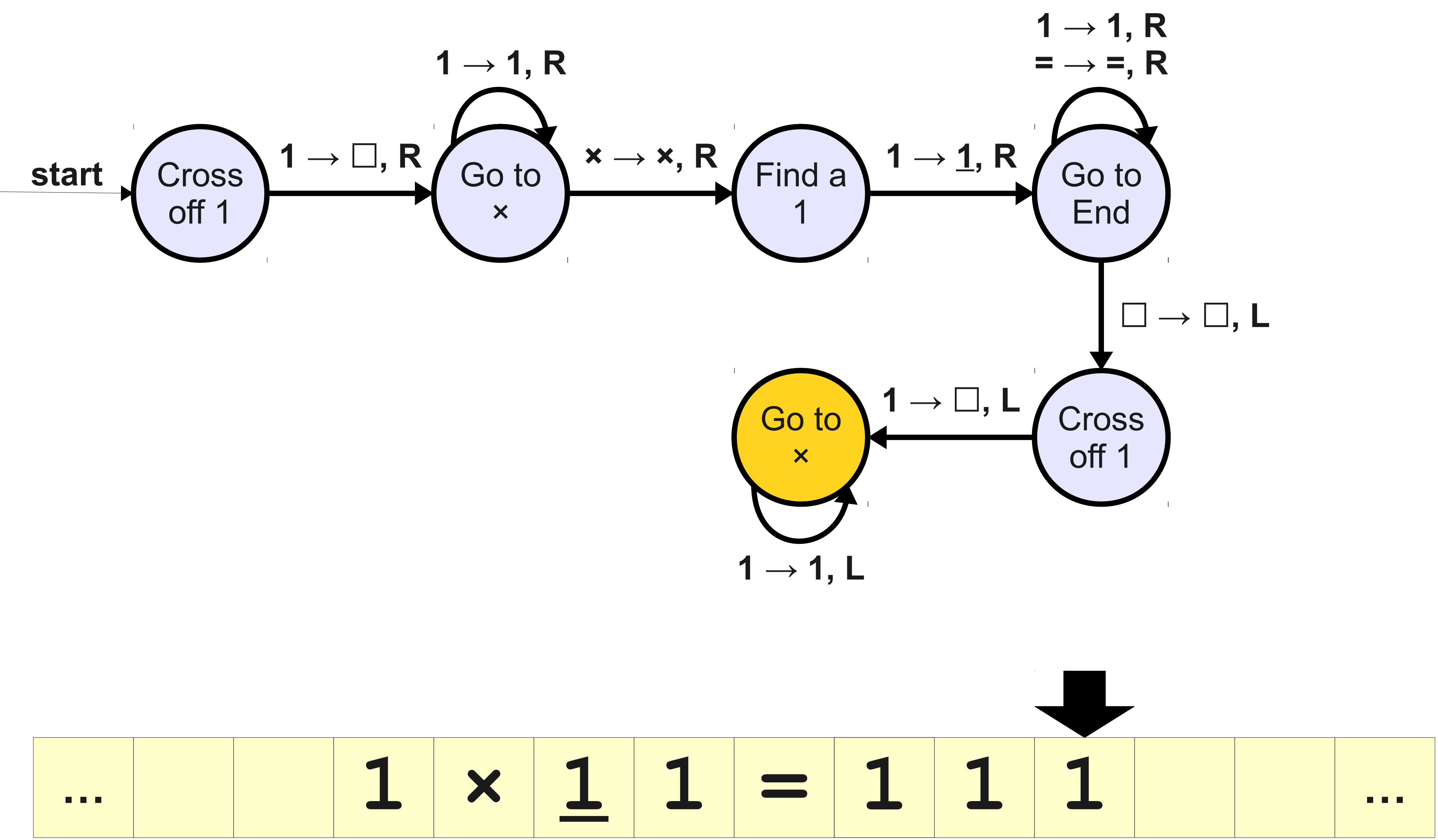
start
Cross off 1
1 → □, R
Go to ×
1 → 1, R
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
= → =, R
□ → □, L
…
1
×
1
1
=
1
1
1
1
…

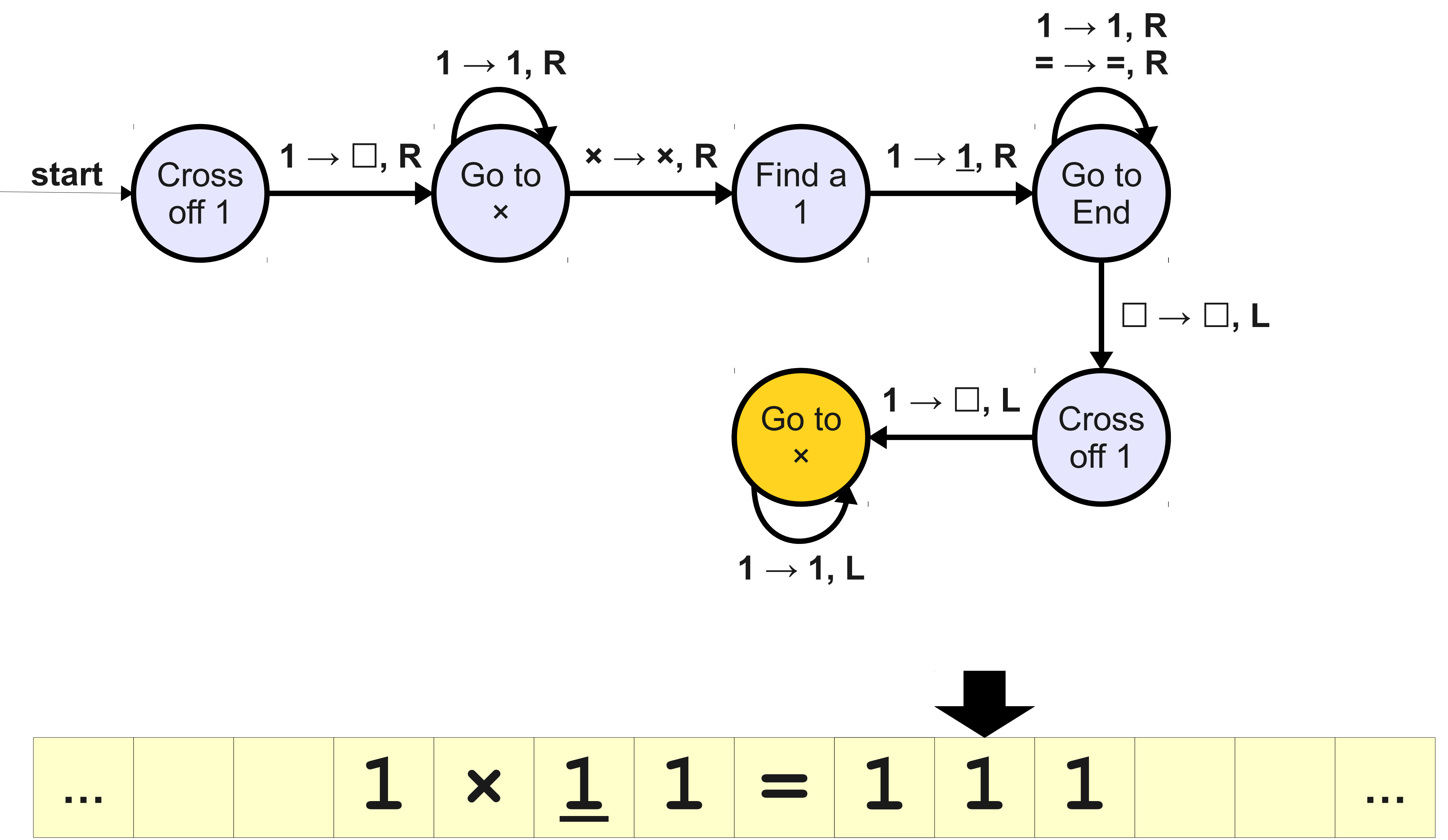

1 → 1, R
1 → 1, R
= → =, R
start
Cross off 1
1 → □, R
Go to ×
× → ×, R
Find a 1
1 → 1, R
Go to End
□ → □, L
Cross off 1
...
1
×
1
1
=
1
1
1
1
...


start
Cross off 1
1 → □, R
Go to ×
1 → 1, R
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
= → =, R
□ → □, L
Cross off 1
1 → □, L
...
1
×
1
1
=
1
1
1
1
...

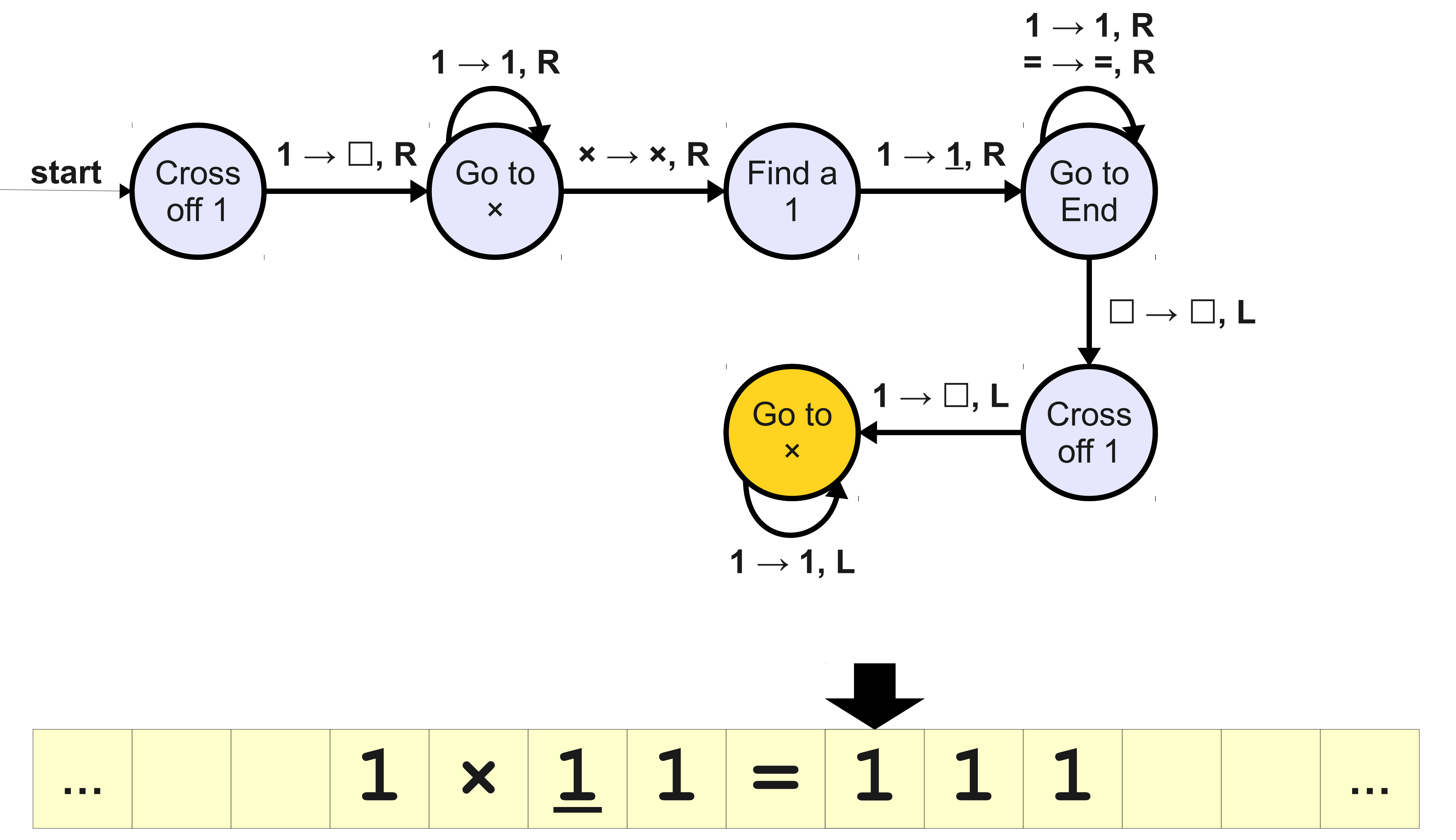

start
Cross off 1
1 → □, R
Go to ×
1 → 1, R
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
= → =, R
□ → □, L
Cross off 1
1 → □, L
...
1
×
1
1
=
1
1
1
...

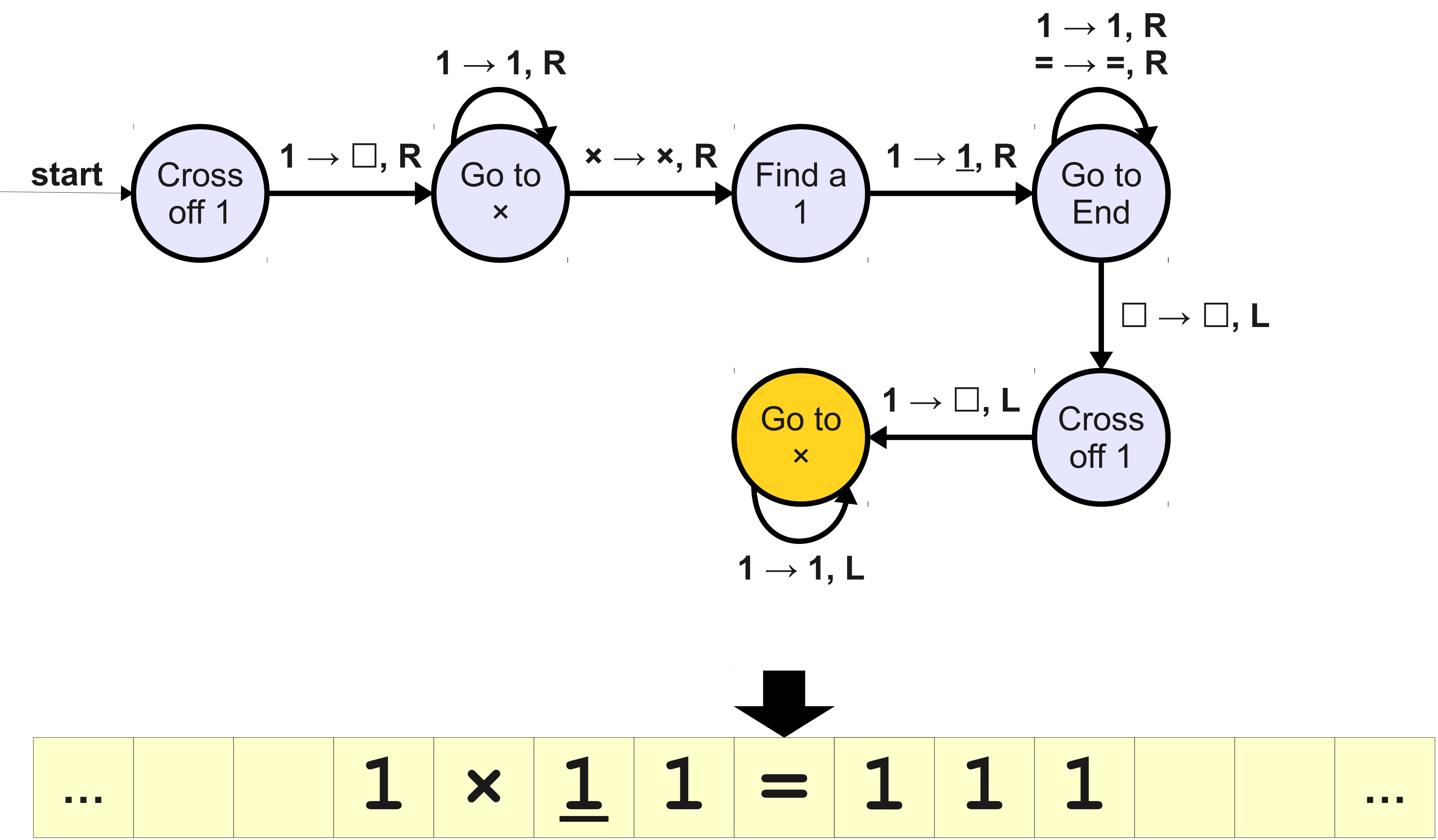

start
Cross off 1
1 → □, R
Go to ×
1 → 1, R
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
= → =, R
□ → □, L
Cross off 1
1 → □, L
Go to ×
… 1 × 1 1 = 1 1 1 …


start
Cross off 1
1 → □, R
Go to ×
1 → 1, R
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
= → =, R
□ → □, L
Cross off 1
1 → □, L
Go to ×
1 → 1, L
...
1
×
1
1
=
1
1
1
...



start
Cross off 1
1 → □, R
Go to ×
1 → 1, R
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
= → =, R
□ → □, L
Cross off 1
1 → □, L
Go to ×
1 → 1, L
… 1 × 1 1 = 1 1 1 …


start
Cross off 1
1 → □, R
Go to ×
1 → 1, R
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
= → =, R
□ → □, L
Cross off 1
1 → □, L
Go to ×
1 → 1, L
…
1
×
1
1
=
1
1
1
…


start
Cross off 1
1 → □, R
Go to ×
1 → 1, R
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
= → =, R
□ → □, L
Cross off 1
1 → □, L
Go to ×
1 → 1, L
…
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×
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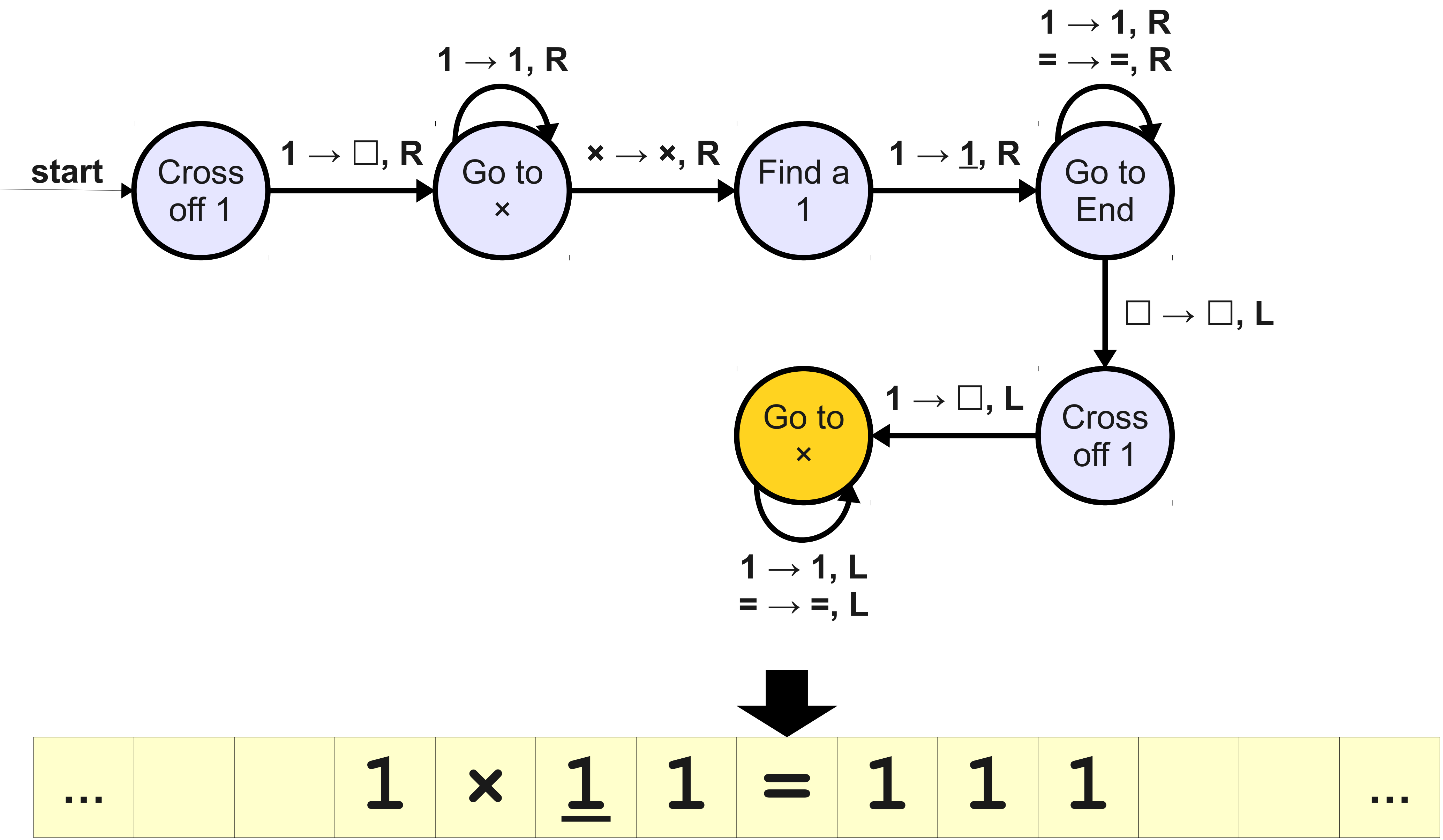
1
=
1
1
1
…



start
Cross off 1
1 → □, R
Go to ×
1 → 1, R
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
= → =, R
□ → □, L
Cross off 1
1 → □, L
Go to ×
1 → 1, L
= → =, L
…
1
×
1
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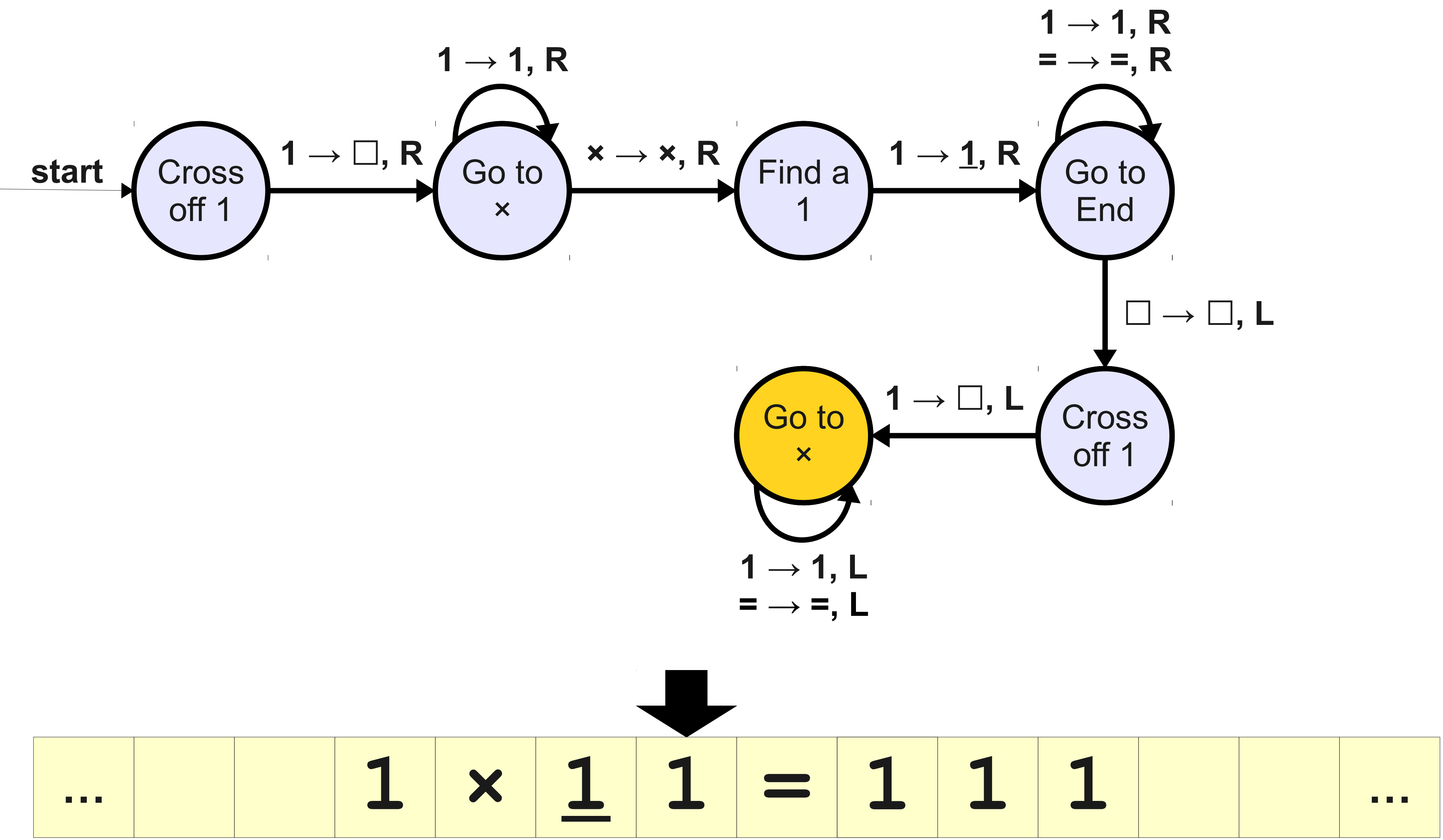
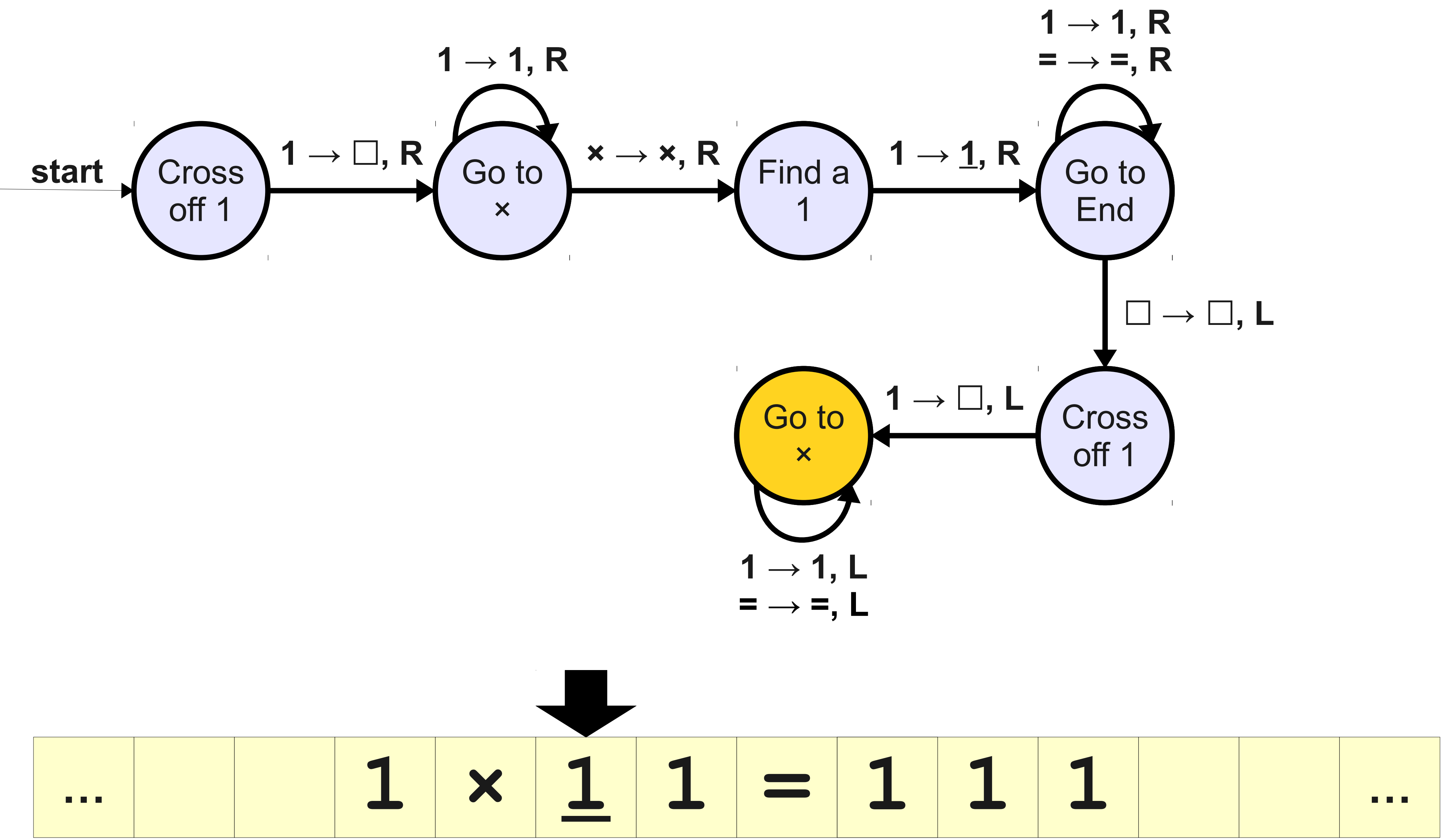
=
1
1
1
…


start
Cross off 1
1 → □, R
Go to ×
1 → 1, R
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
= → =, R
□ → □, L
Cross off 1
1 → □, L
Go to ×
1 → 1, L
= → =, L
... 1 × 1 1 = 1 1 1 ...


start
Cross off 1
1 → □, R
Go to ×
1 → 1, R
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
= → =, R
□ → □, L
Cross off 1
1 → □, L
Go to ×
1 → 1, L
= → =, L
…
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×
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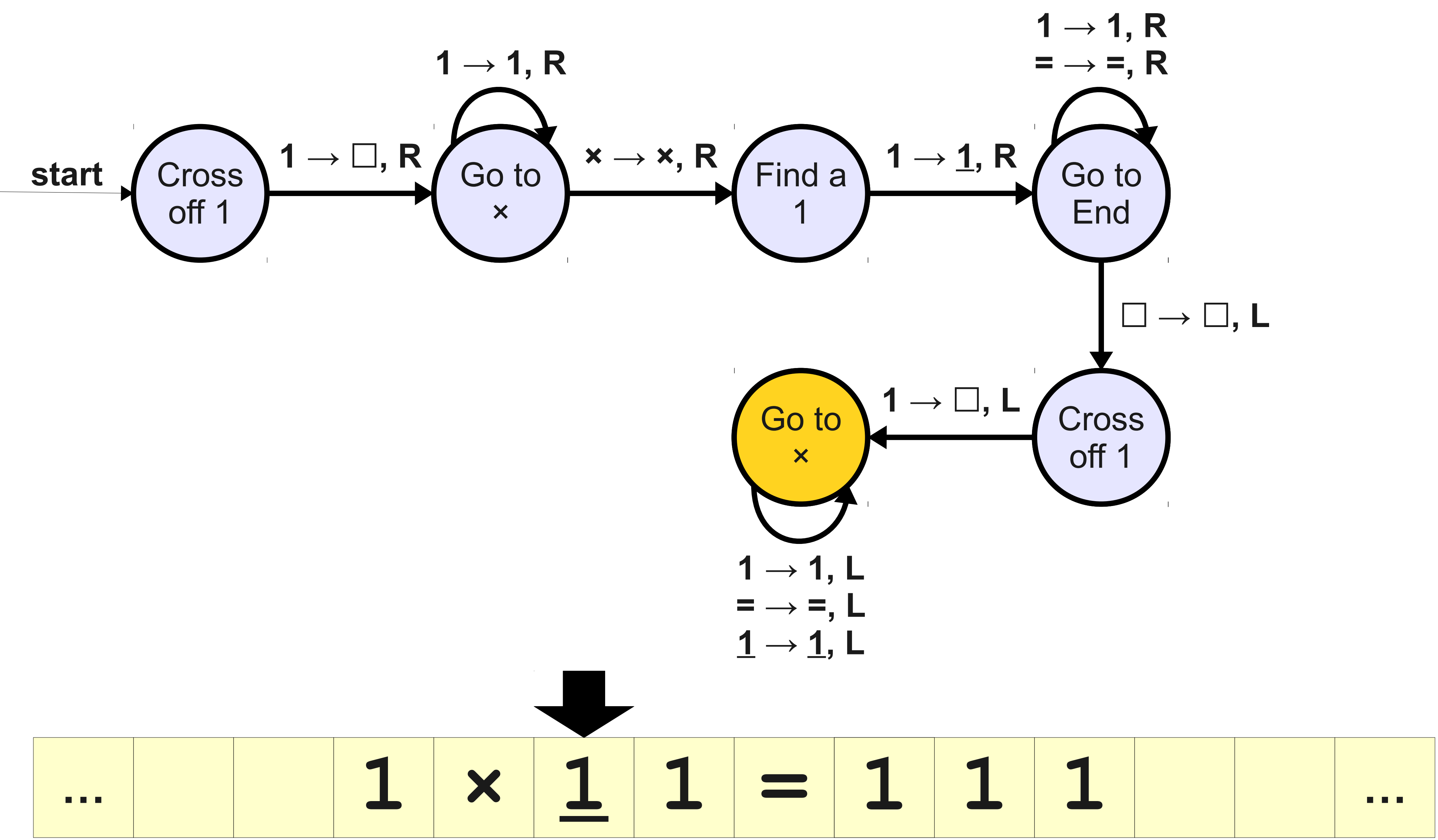
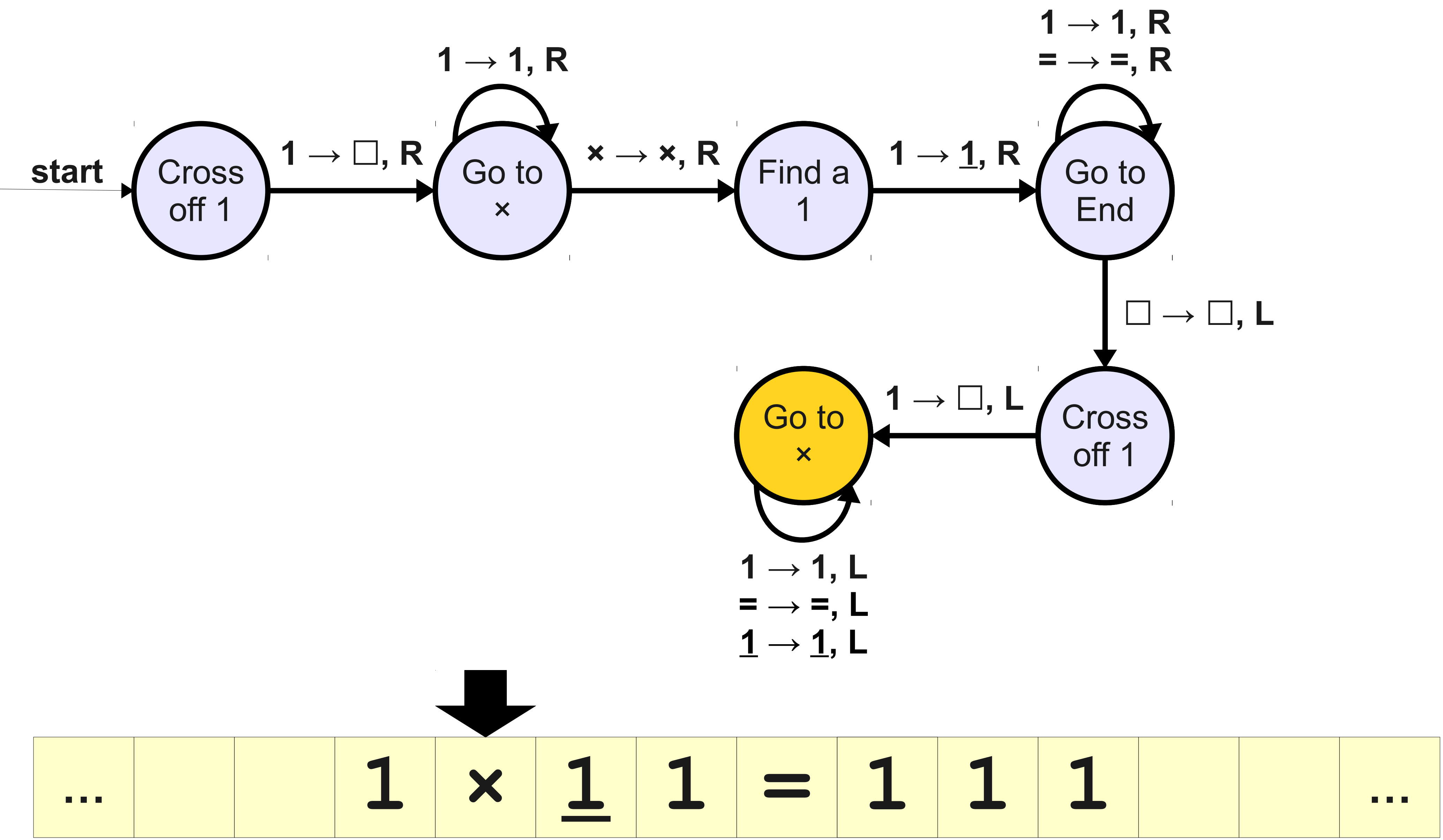
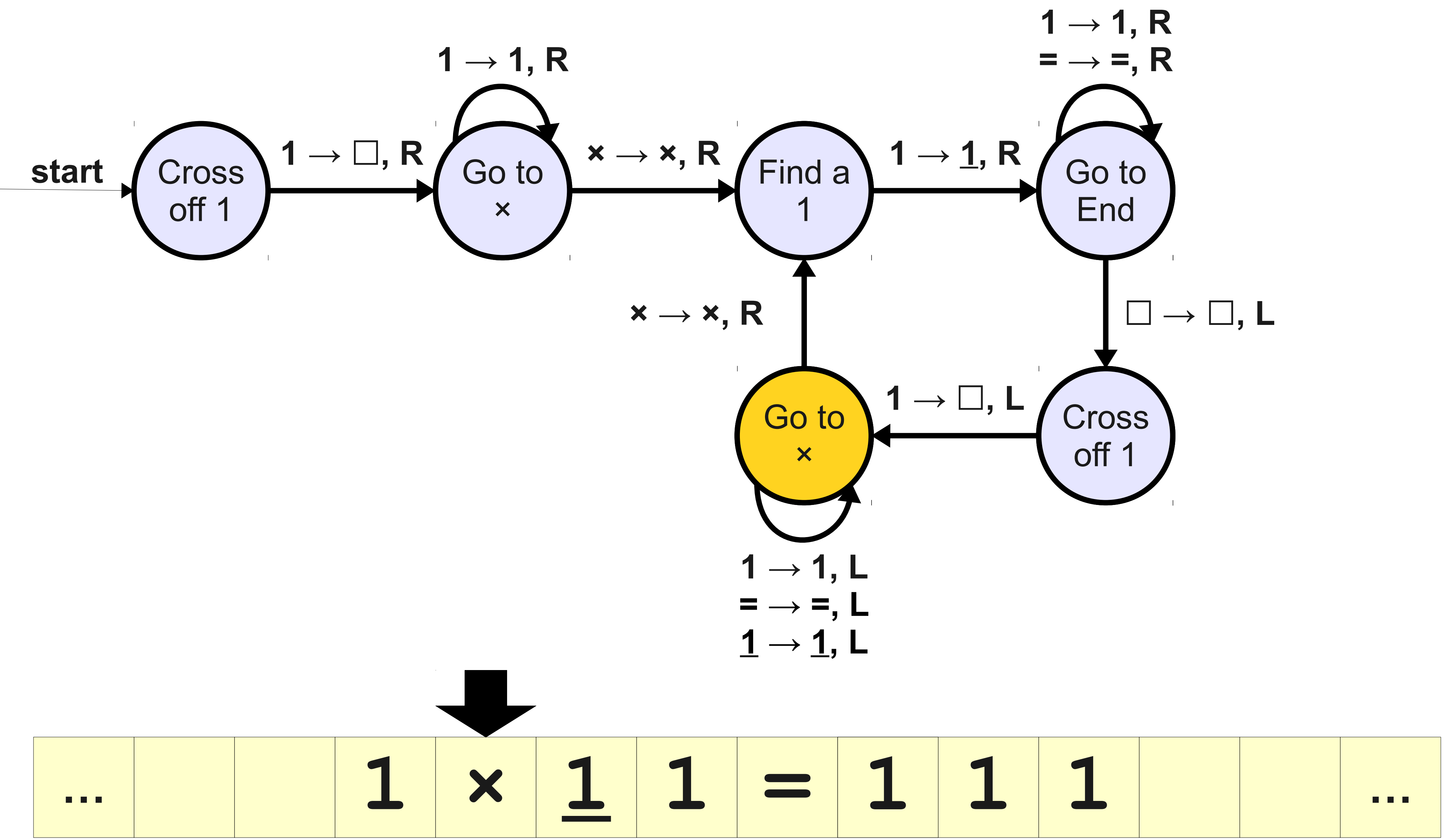
=
1
1
1
…

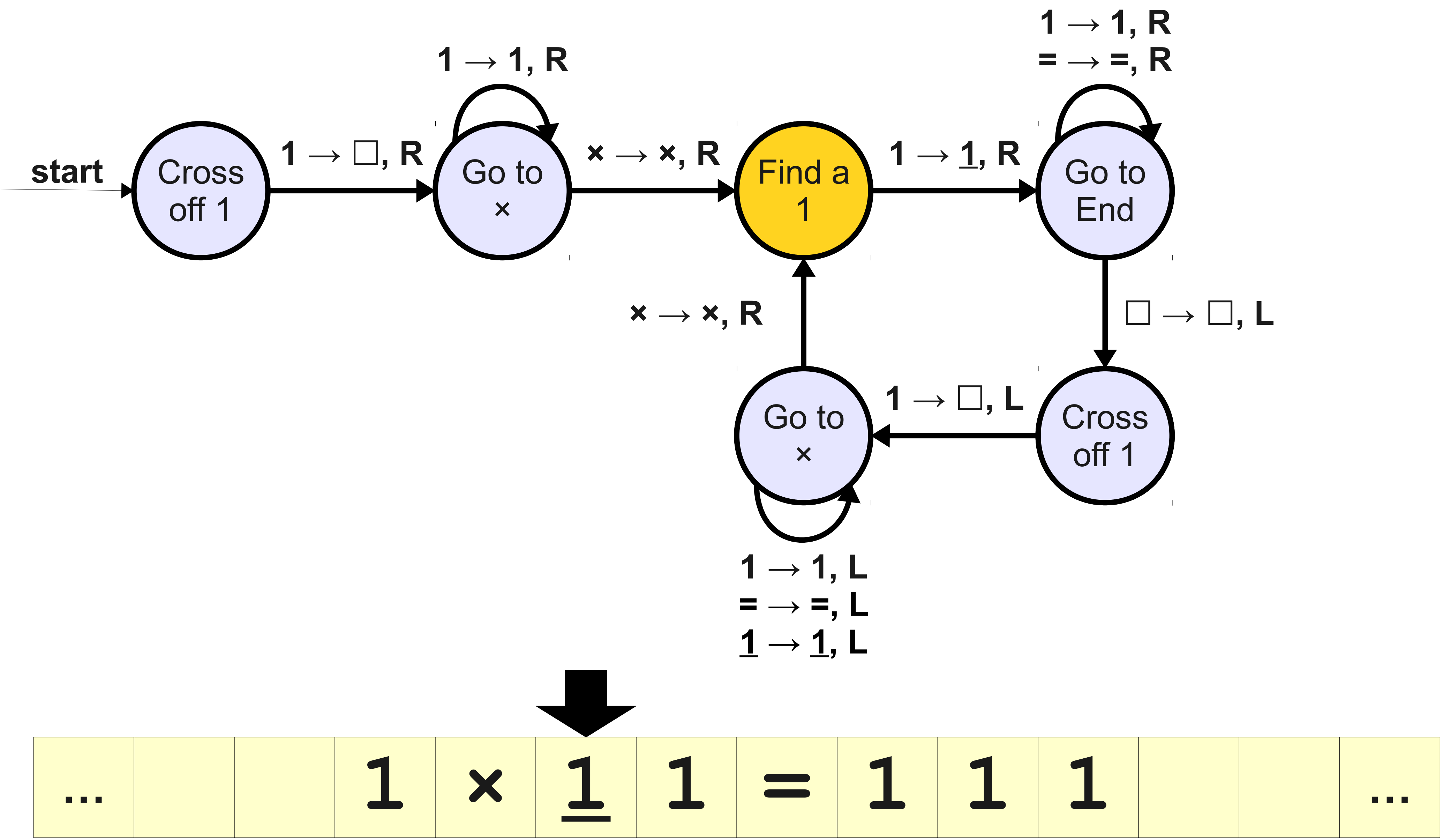


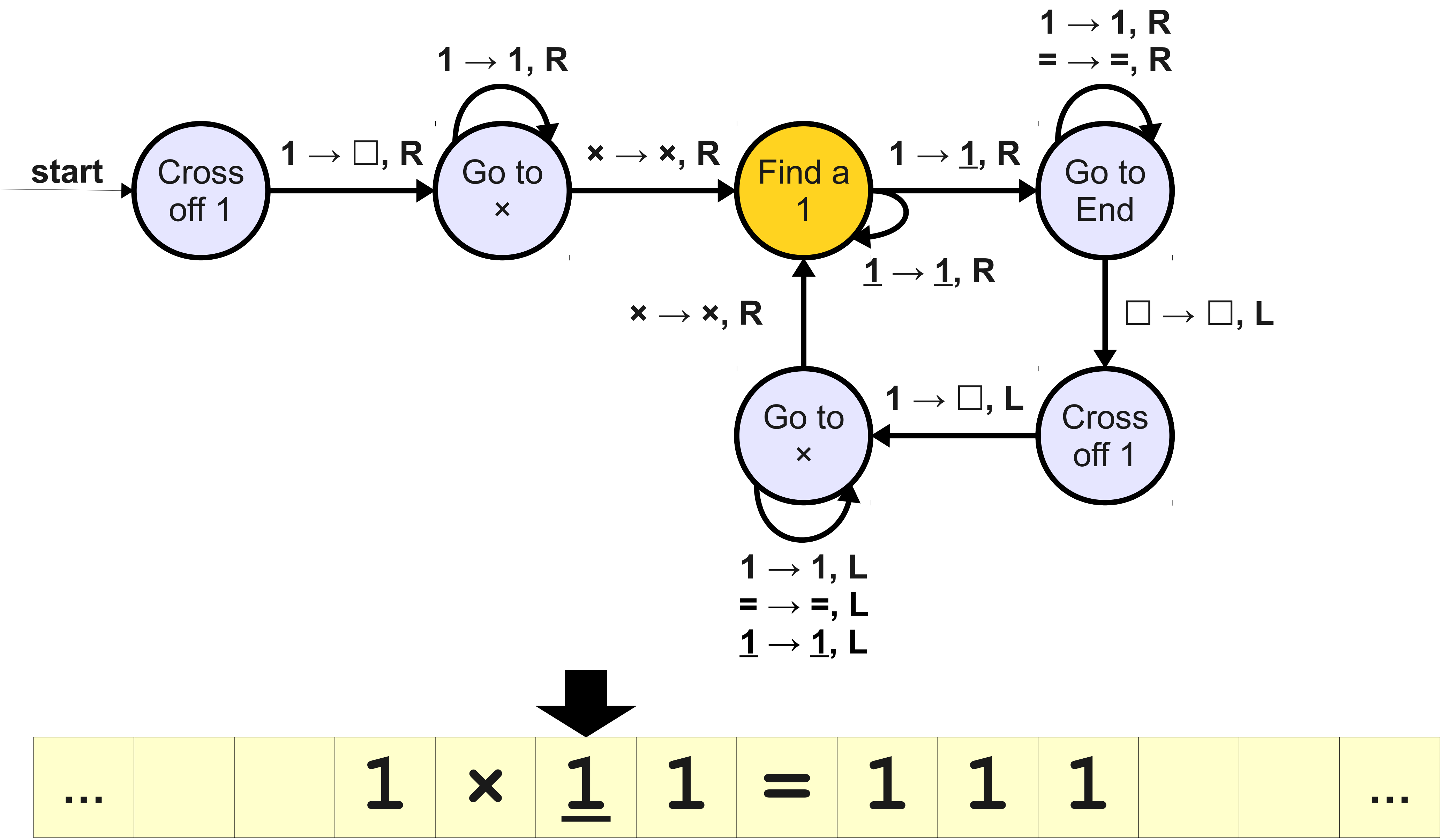
start
Cross off 1
1 → □, R
Go to ×
1 → 1, R
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
= → =, R
□ → □, L
Cross off 1
1 → □, L
Go to ×
1 → 1, L
= → =, L
1 → 1, L
… 1 × 1 1 = 1 1 1 …

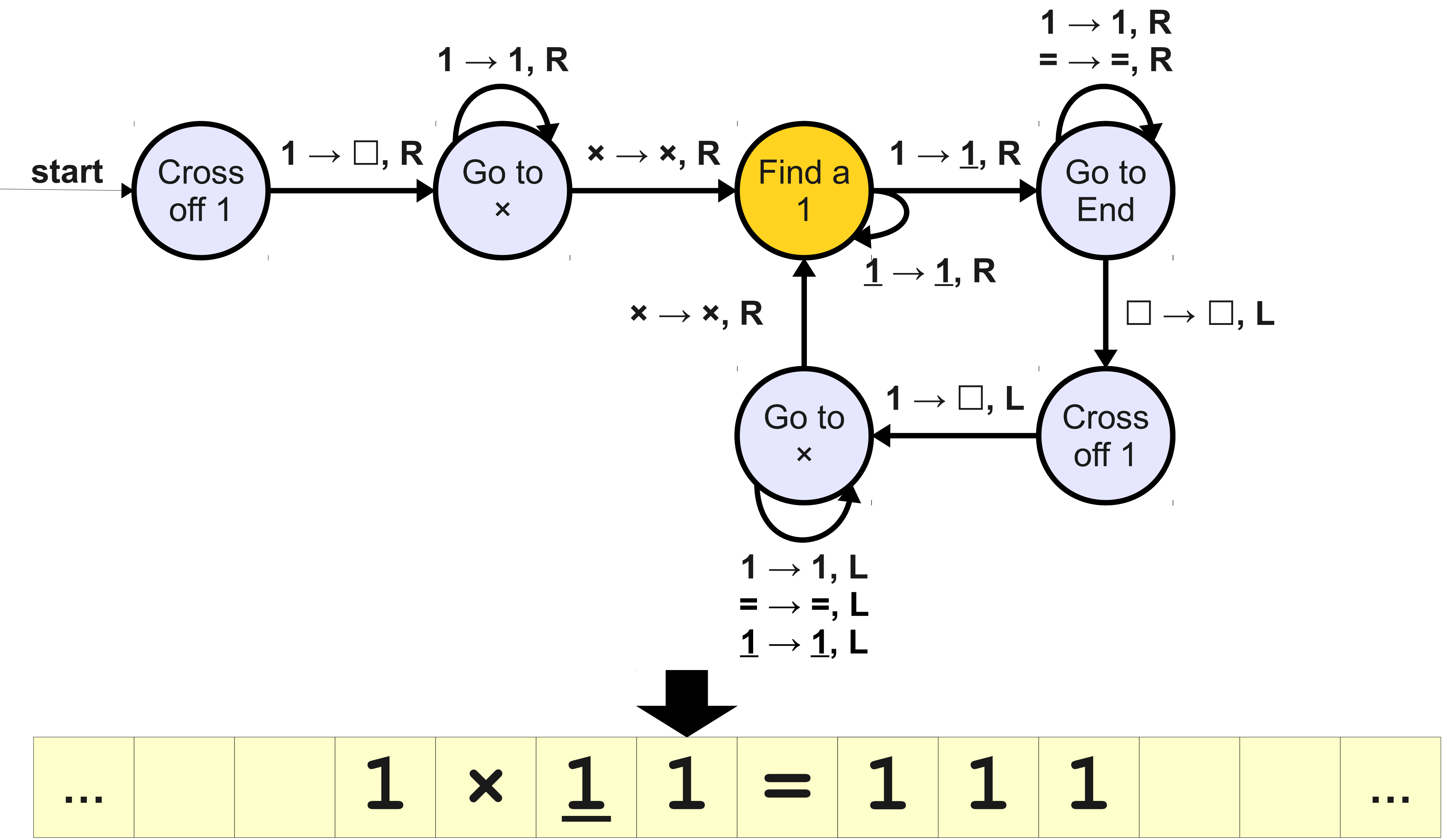


start
Cross off 1
1 → ☐, R
Go to ×
1 → 1, R
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
= → =, R
☐ → ☐, L
Cross off 1
1 → ☐, L
Go to ×
1 → 1, L
= → =, L
1 → 1, L
...
1
×
1
1
=
1
1
1
...

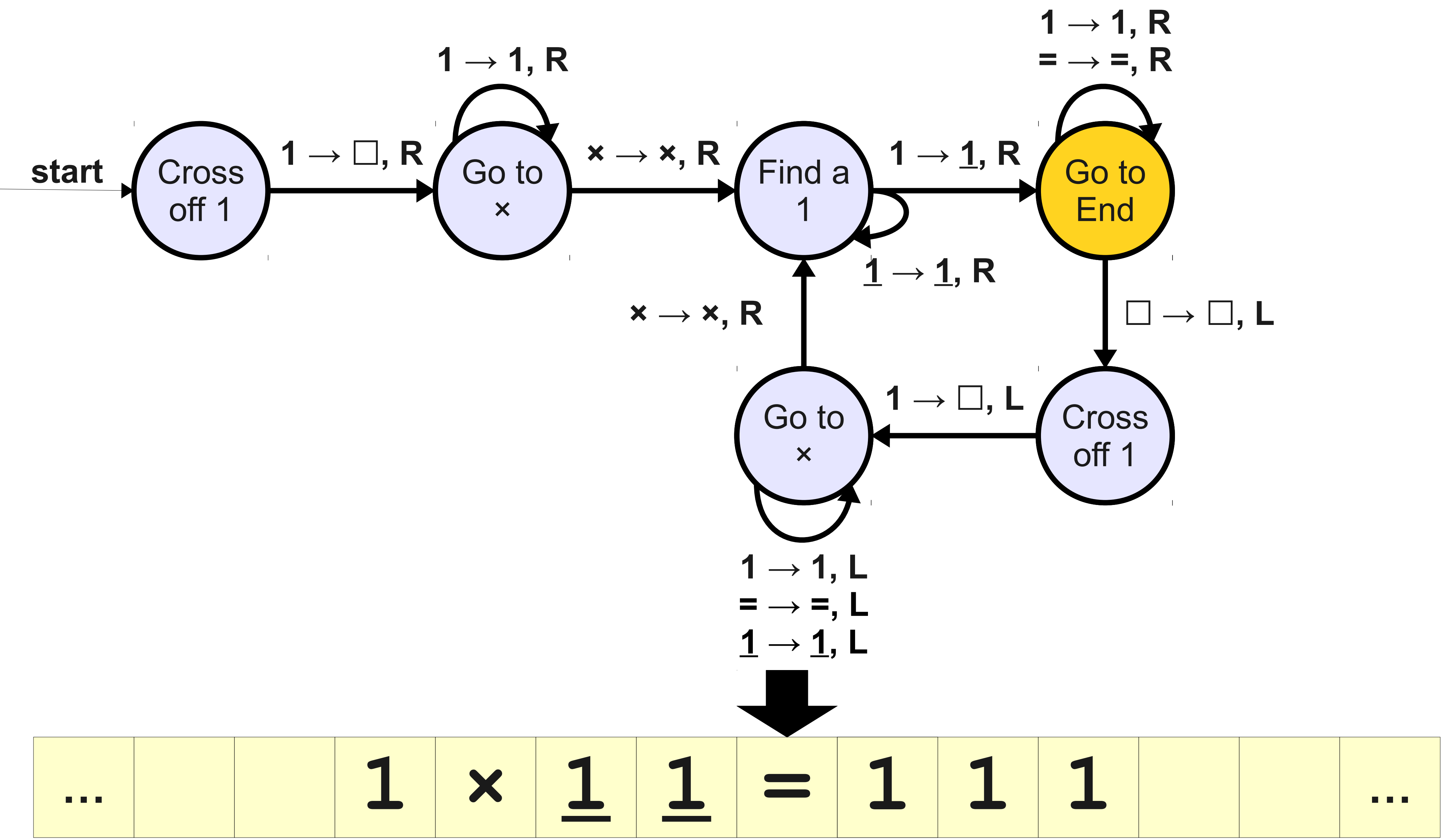

1 → 1, R
1 → 1, R
= → =, R
start
Cross off 1
1 → □, R
Go to ×
× → ×, R
Find a 1
1 → 1, R
Go to End
× → ×, R
□ → □, L
Go to ×
1 → □, L
Cross off 1
1 → 1, L
= → =, L
1 → 1, L
… 1 × 1 1 = 1 1 1 …


start
Cross off 1
1 → □, R
Go to ×
1 → 1, R
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
= → =, R
□ → □, L
Cross off 1
1 → □, L
Go to ×
× → ×, R
1 → 1, L
= → =, L
1 → 1, L
...
1
×
1
1
=
1
1
1
...

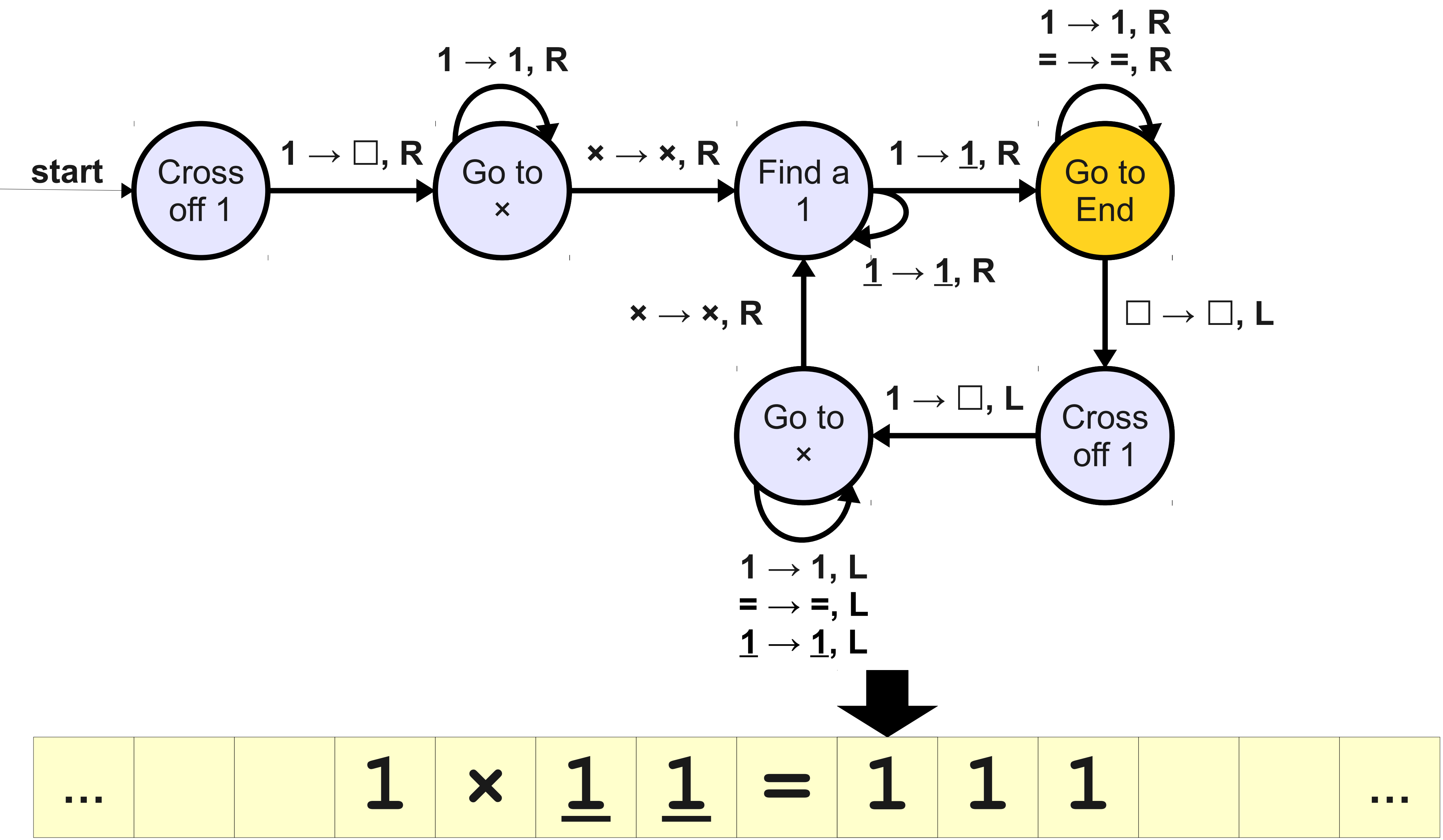

start
Cross off 1
1 → □, R
Go to ×
1 → 1, R
× → ×, R
Find a 1
1 → 1, R
1 → 1, R
Go to End
1 → 1, R
= → =, R
□ → □, L
Cross off 1
1 → □, L
Go to ×
× → ×, R
1 → 1, L
= → =, L
1 → 1, L
… 1 × 1 1 = 1 1 1 …



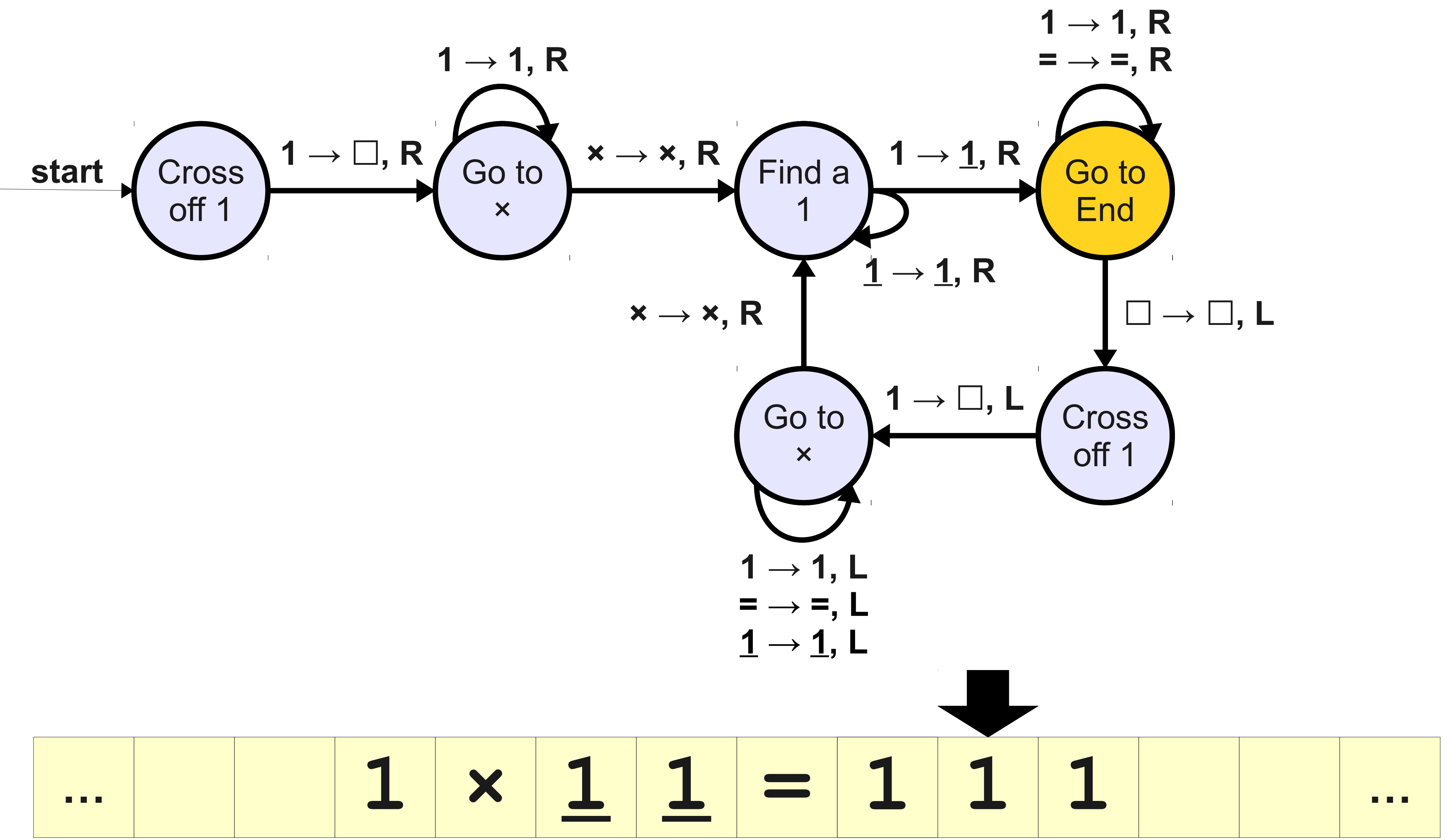
start
Cross off 1
1 → □, R
Go to ×
1 → 1, R
× → ×, R
Find a 1
1 → 1, R
1 → 1, R
= → =, R
Go to End
1 → 1, R
□ → □, L
× → ×, R
Go to ×
1 → □, L
Cross off 1
1 → 1, L
= → =, L
1 → 1, L
…
1
×
1
1
=
1
1
1
…



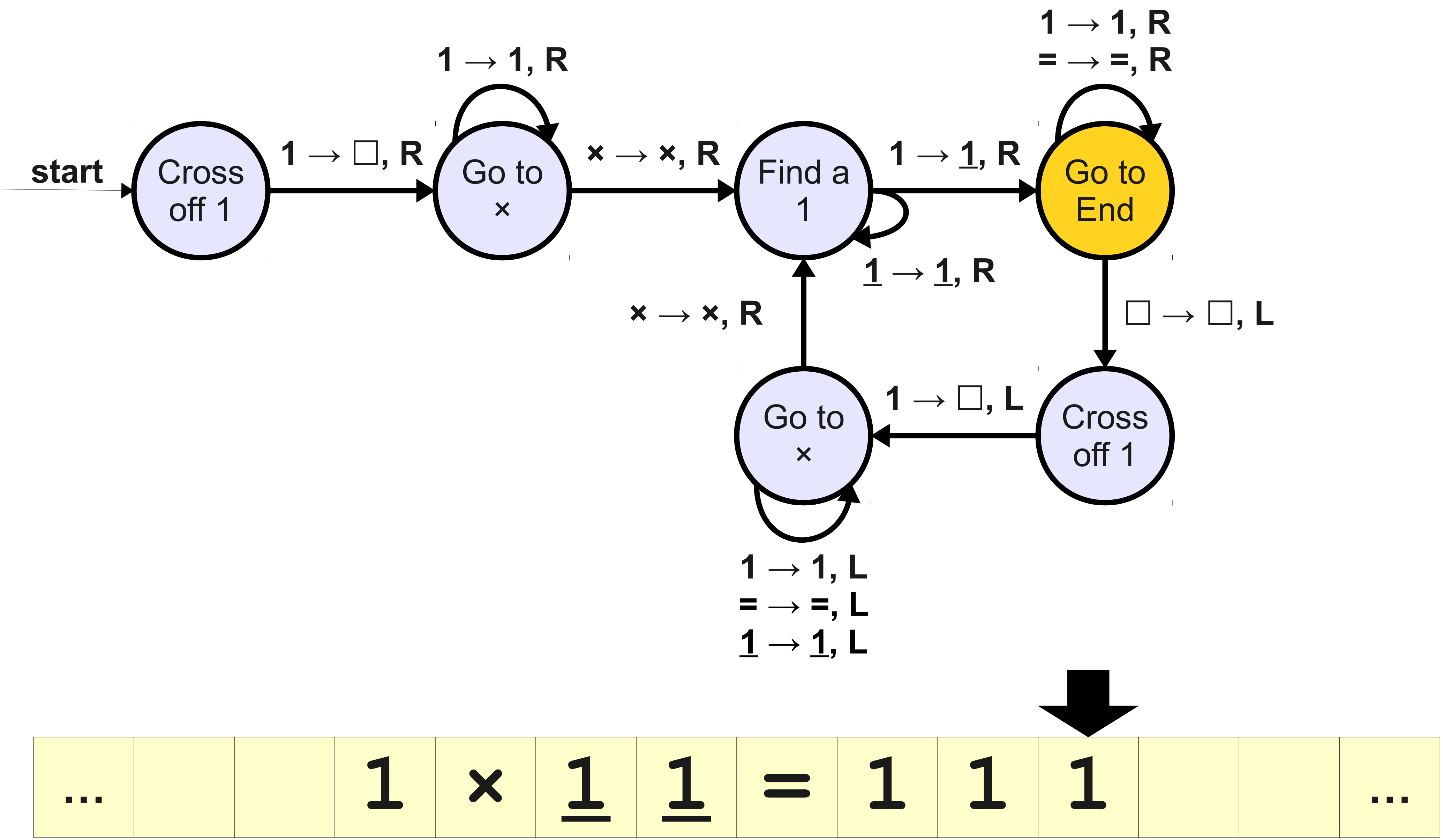
start
Cross off 1
1 → □, R
Go to ×
1 → 1, R
× → ×, R
Find a 1
1 → 1, R
1 → 1, R
Go to End
1 → 1, R
= → =, R
□ → □, L
Cross off 1
1 → □, L
Go to ×
× → ×, R
1 → 1, L
= → =, L
1 → 1, L
… 1 × 1 1 = 1 1 1 …



start
Cross off 1
1 → □, R
Go to ×
1 → 1, R
× → ×, R
Find a 1
1 → 1, R
1 → 1, R
Go to End
1 → 1, R
= → =, R
□ → □, L
Cross off 1
1 → □, L
Go to ×
× → ×, R
1 → 1, L
= → =, L
1 → 1, L
... 1 × 1 1 = 1 1 1 ...



start
Cross off 1
1 → □, R
Go to ×
1 → 1, R
× → ×, R
Find a 1
1 → 1, R
1 → 1, R
Go to End
1 → 1, R
= → =, R
□ → □, L
Cross off 1
1 → □, L
Go to ×
× → ×, R
1 → 1, L
= → =, L
1 → 1, L
… 1 × 1 1 = 1 1 1 …


start
Cross off 1
1 → □, R
Go to ×
1 → 1, R
× → ×, R
Find a 1
1 → 1, R
1 → 1, R
Go to End
1 → 1, R
= → =, R
□ → □, L
Cross off 1
1 → □, L
Go to ×
× → ×, R
1 → 1, L
= → =, L
1 → 1, L
…
1
×
1
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=
1
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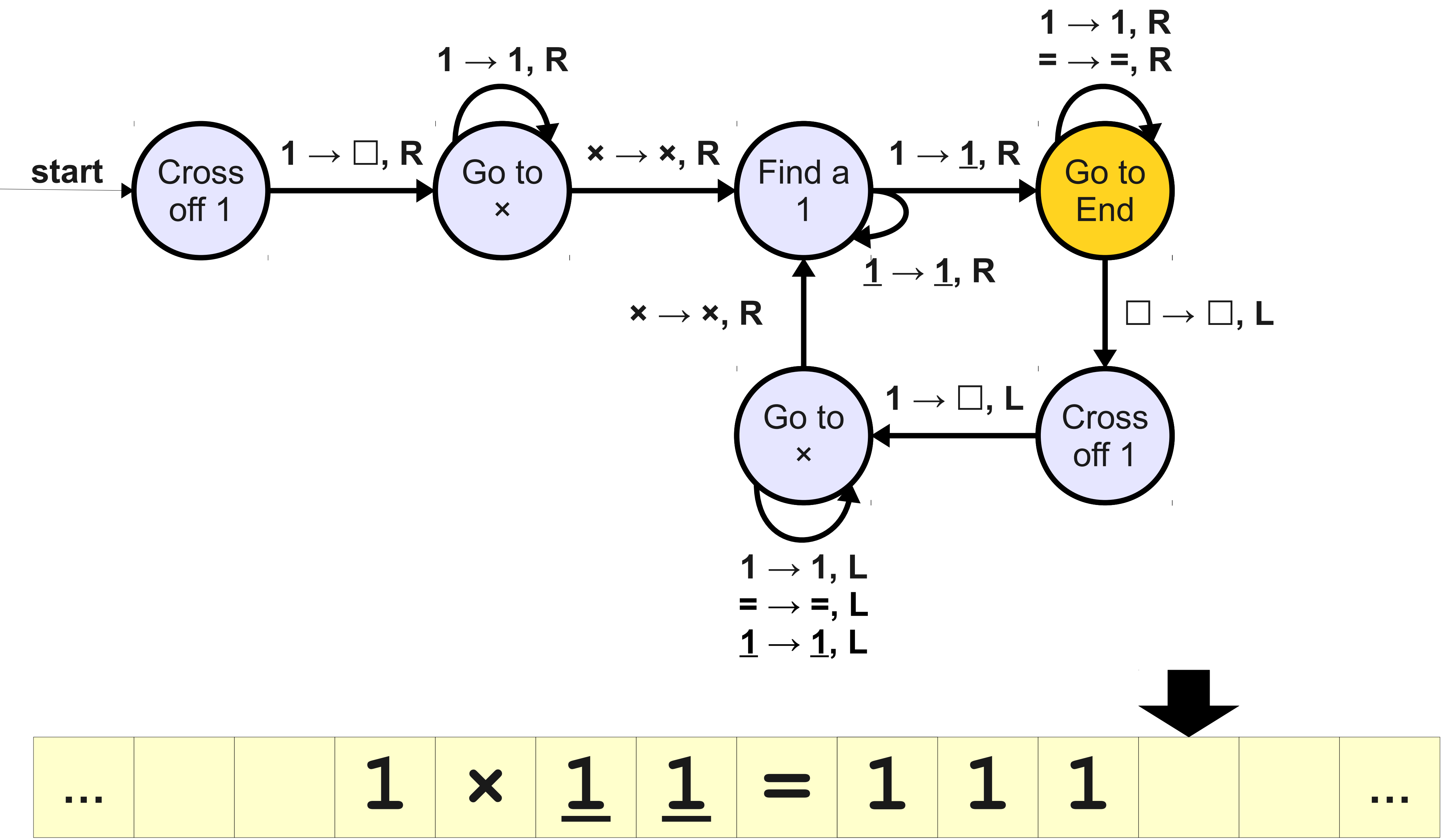
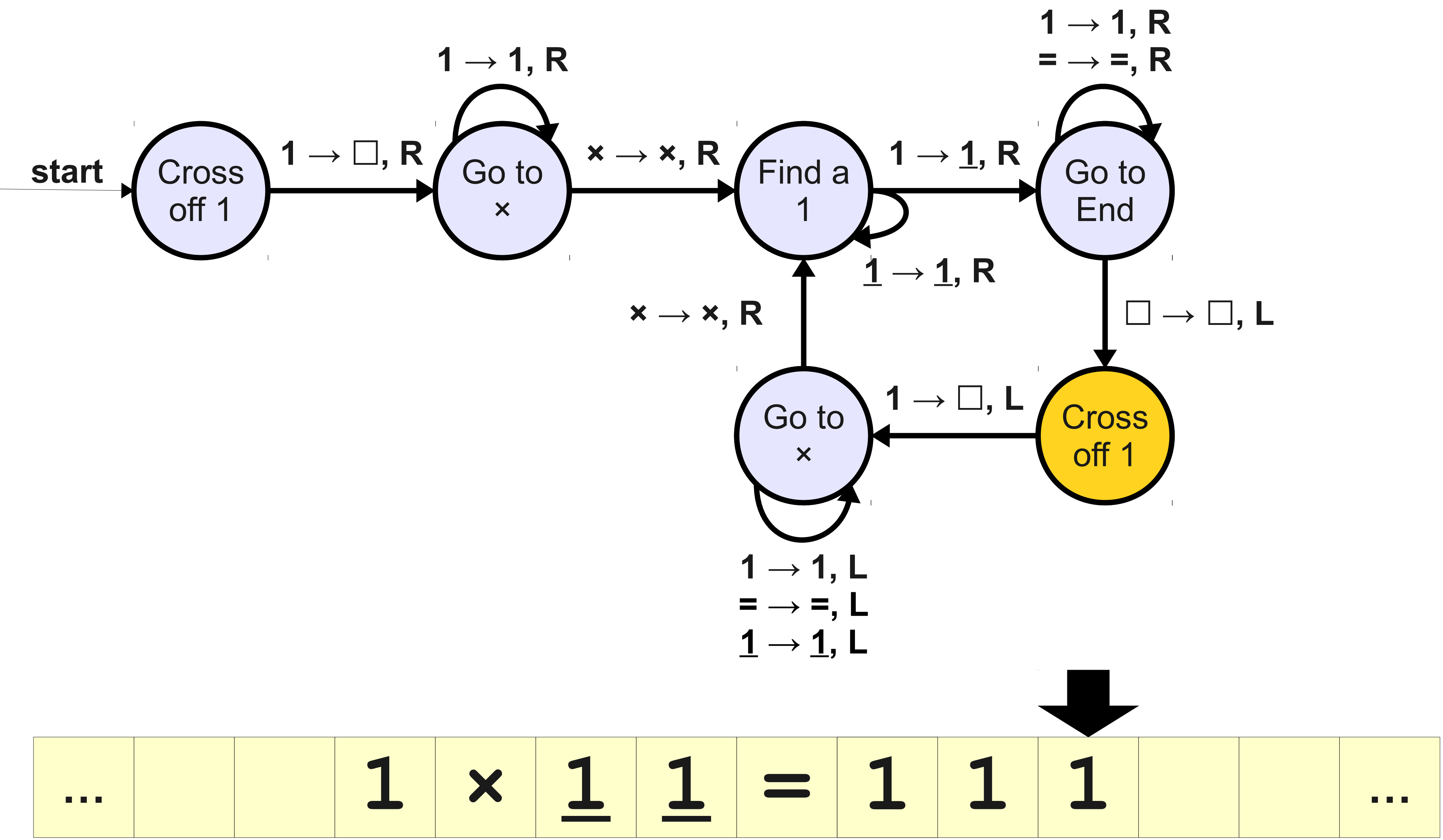
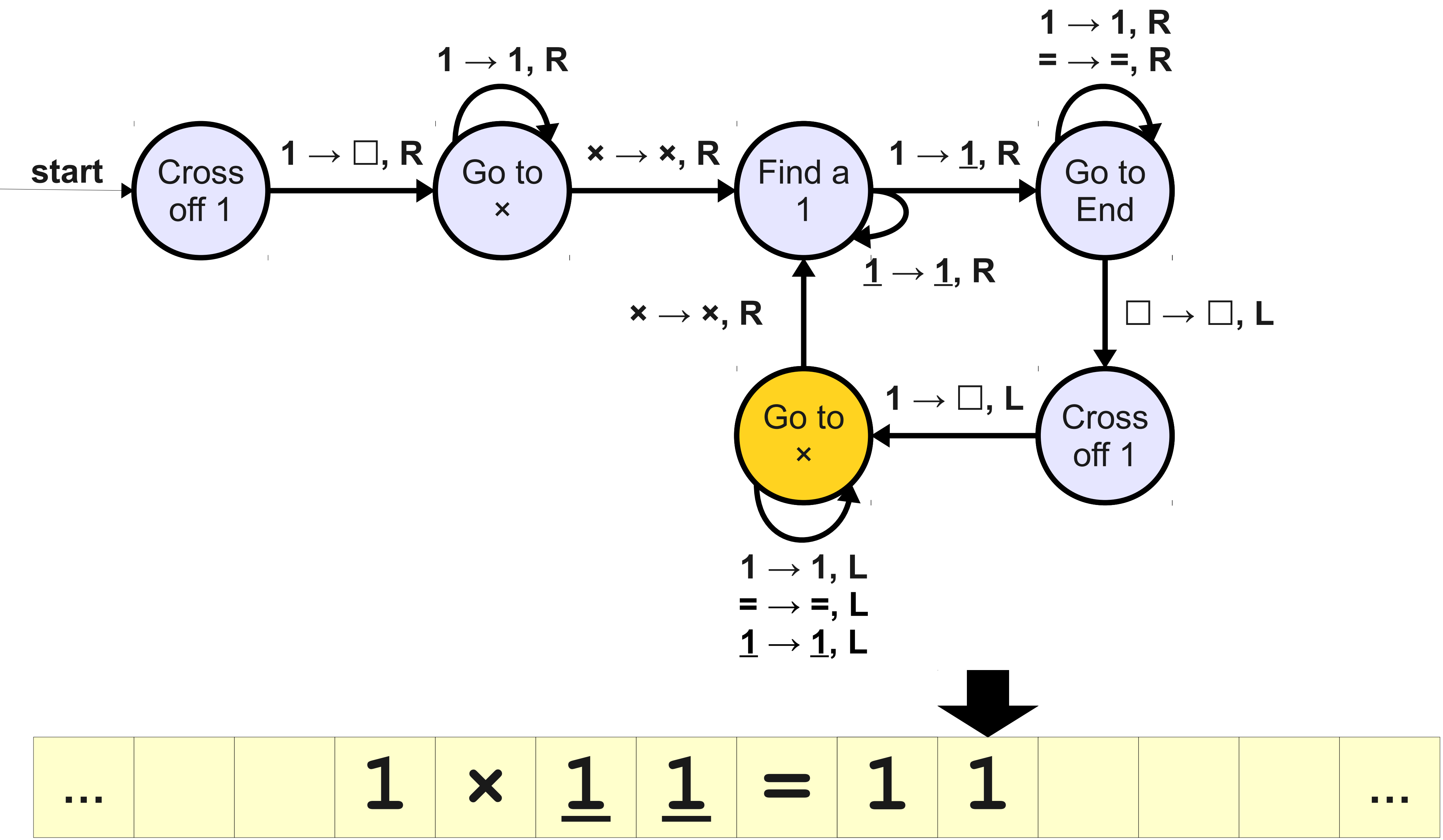
…

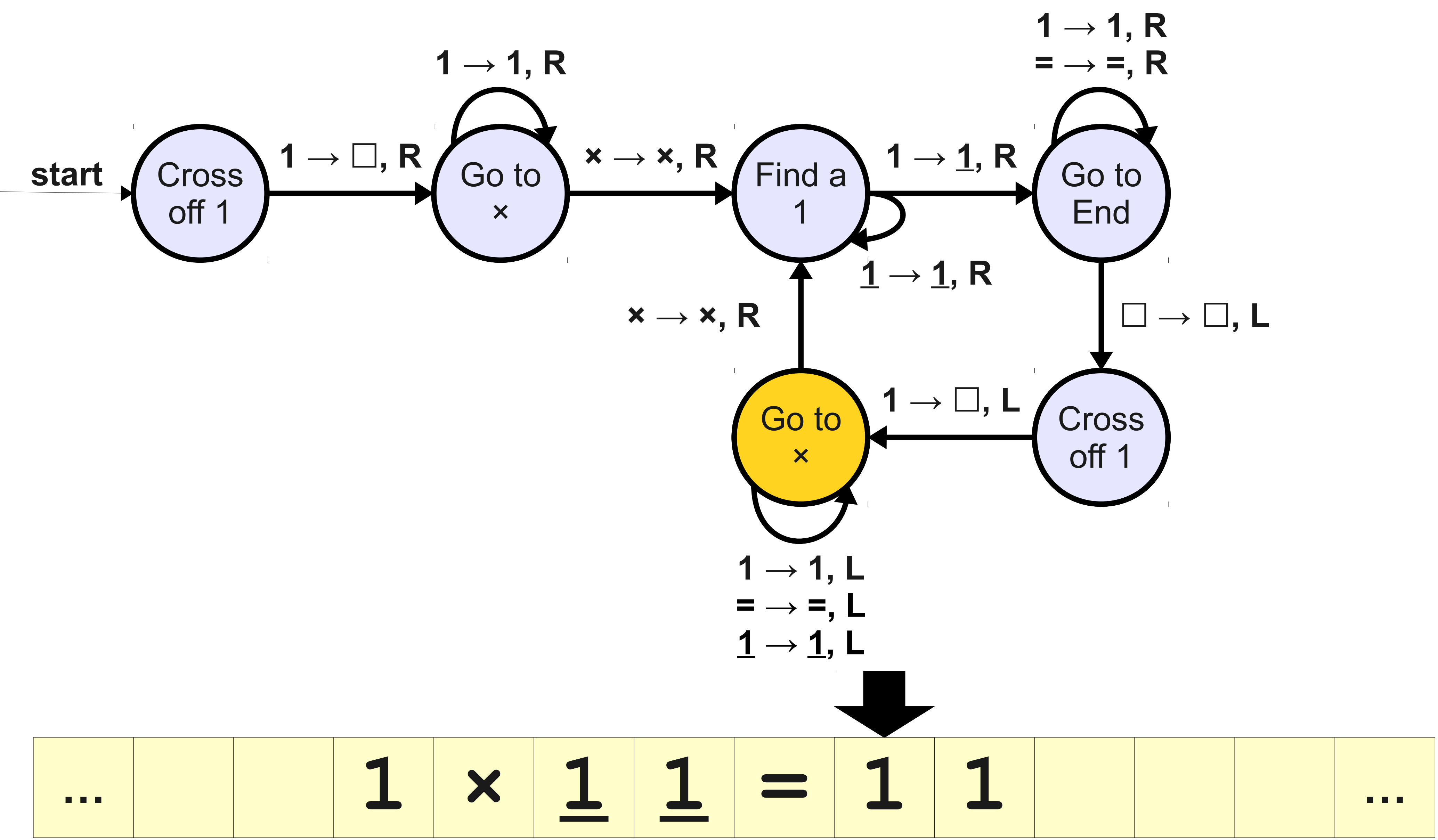


start
Cross off 1
1 → □, R
Go to ×
1 → 1, R
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
= → =, R
1 → 1, R
□ → □, L
× → ×, R
Cross off 1
1 → □, L
Go to ×
1 → 1, L
= → =, L
1 → 1, L
…
1
×
1
1
=
1
1
1
…

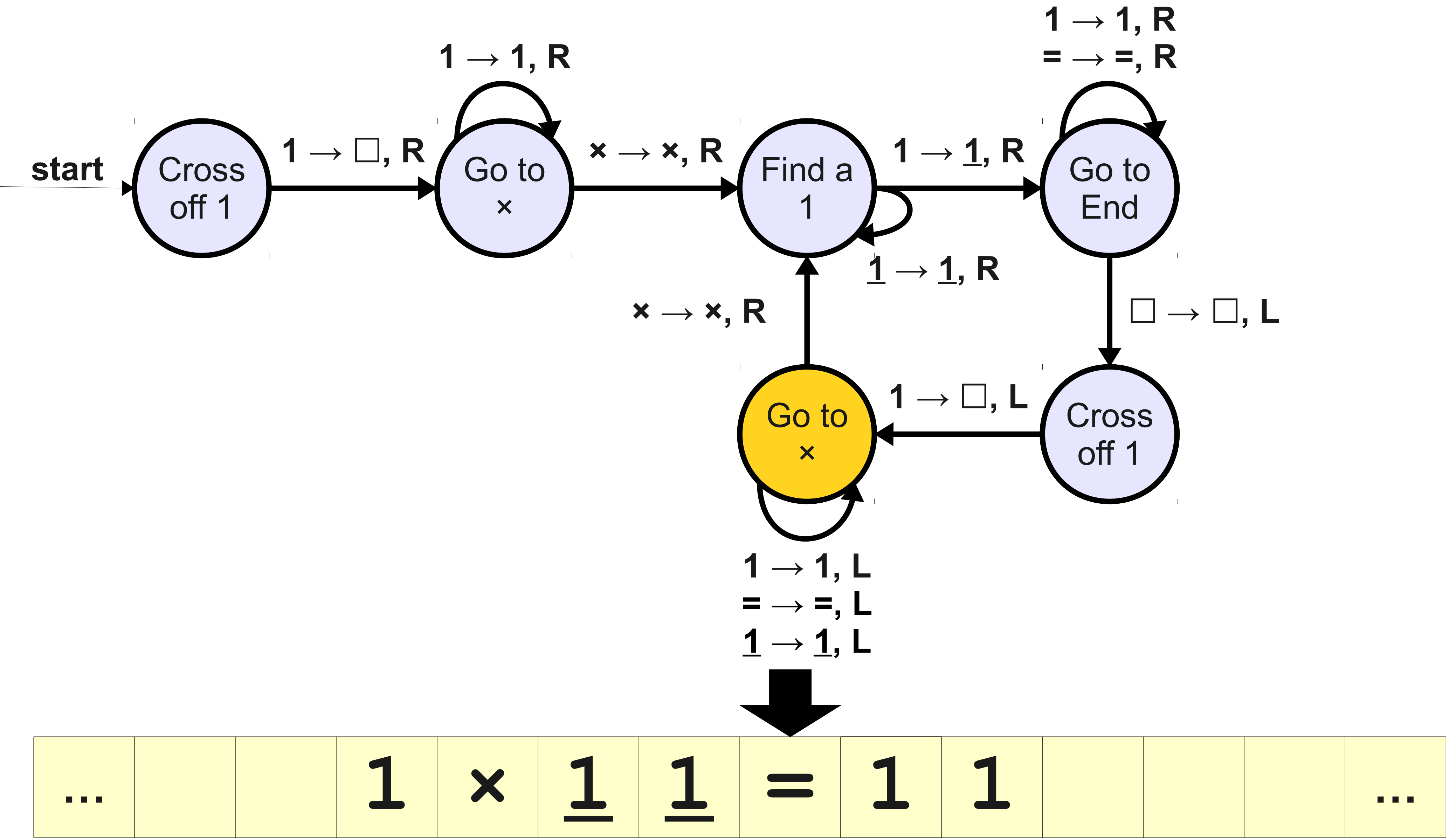


start
Cross off 1
1 → □, R
Go to ×
1 → 1, R
× → ×, R
Find a 1
1 → 1, R
1 → 1, R
Go to End
1 → 1, R
= → =, R
□ → □, L
Cross off 1
1 → □, L
Go to ×
1 → 1, L
= → =, L
1 → 1, L
× → ×, R
… 1 × 1 1 = 1 1 1 …

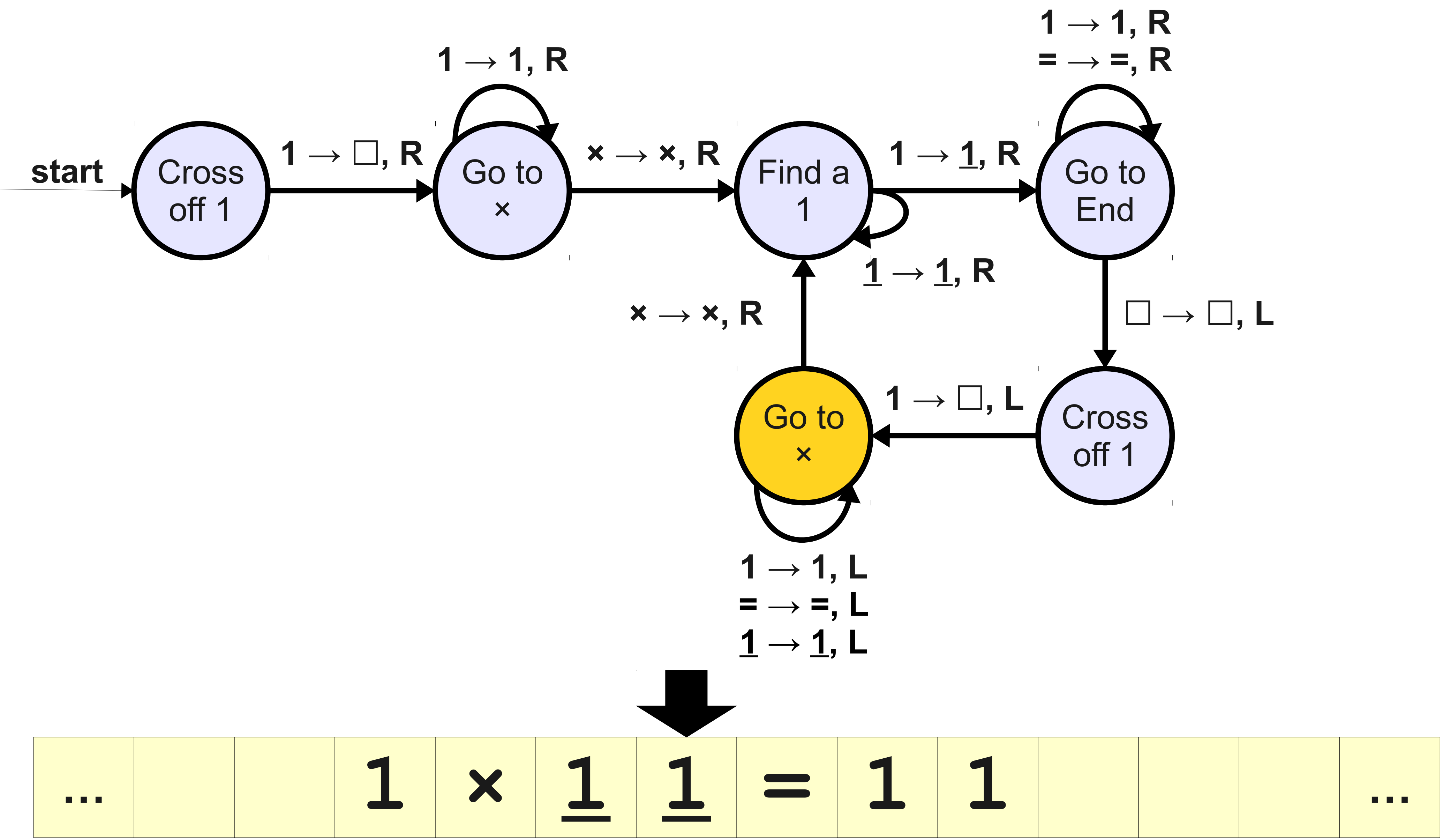

start
Cross off 1
1 → □, R
Go to ×
1 → 1, R
× → ×, R
Find a 1
1 → 1, R
1 → 1, R
Go to End
1 → 1, R
= → =, R
□ → □, L
Cross off 1
1 → □, L
Go to ×
× → ×, R
1 → 1, L
= → =, L
1 → 1, L
…
1
×
1
1
=
1
1
…



start
Cross off 1
1 → □, R
Go to ×
1 → 1, R
× → ×, R
Find a 1
1 → 1, R
1 → 1, R
Go to End
1 → 1, R
= → =, R
□ → □, L
Cross off 1
1 → □, L
Go to ×
× → ×, R
1 → 1, L
= → =, L
1 → 1, L
… 1 × 1 1 = 1 1 …



start
Cross off 1
1 → □, R
Go to ×
1 → 1, R
× → ×, R
Find a 1
1 → 1, R
1 → 1, R
Go to End
1 → 1, R
= → =, R
□ → □, L
Cross off 1
1 → □, L
Go to ×
× → ×, R
1 → 1, L
= → =, L
1 → 1, L
... 1 × 1 1 = 1 1 ...


start
Cross off 1
1 → □, R
Go to ×
1 → 1, R
× → ×, R
Find a 1
1 → 1, R
1 → 1, R
Go to End
1 → 1, R
= → =, R
□ → □, L
Cross off 1
1 → □, L
Go to ×
× → ×, R
1 → 1, L
= → =, L
1 → 1, L
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×
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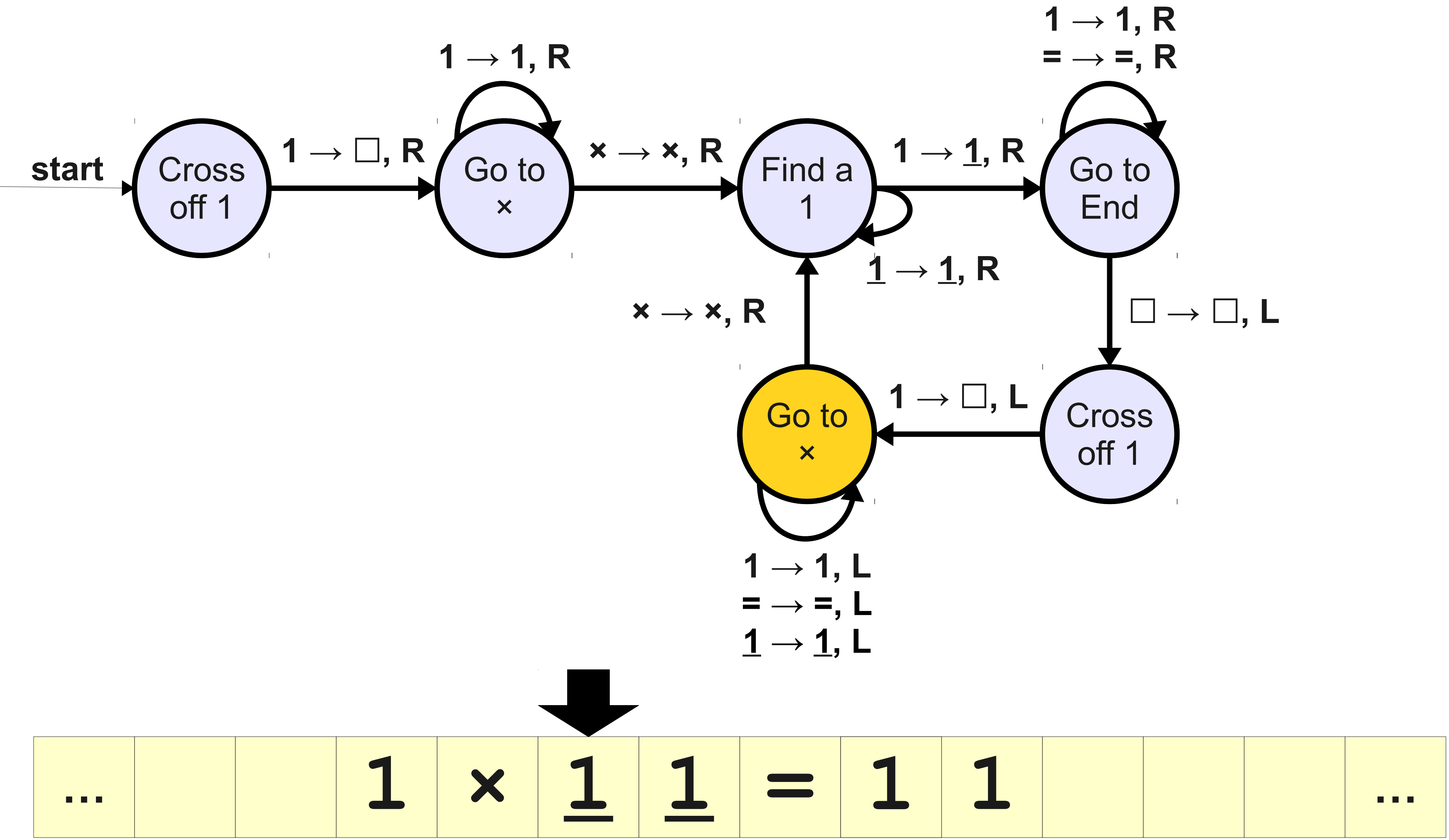
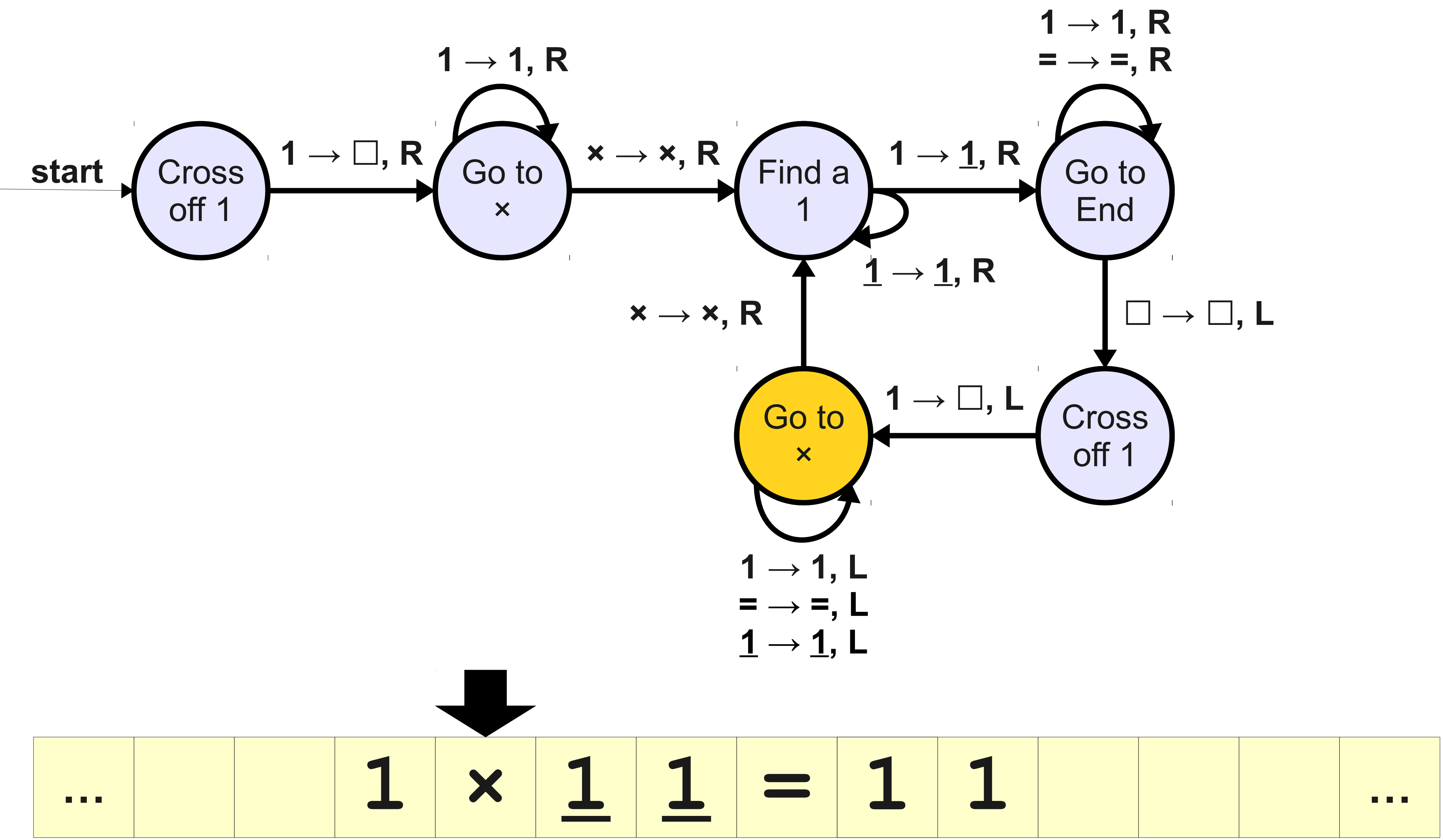
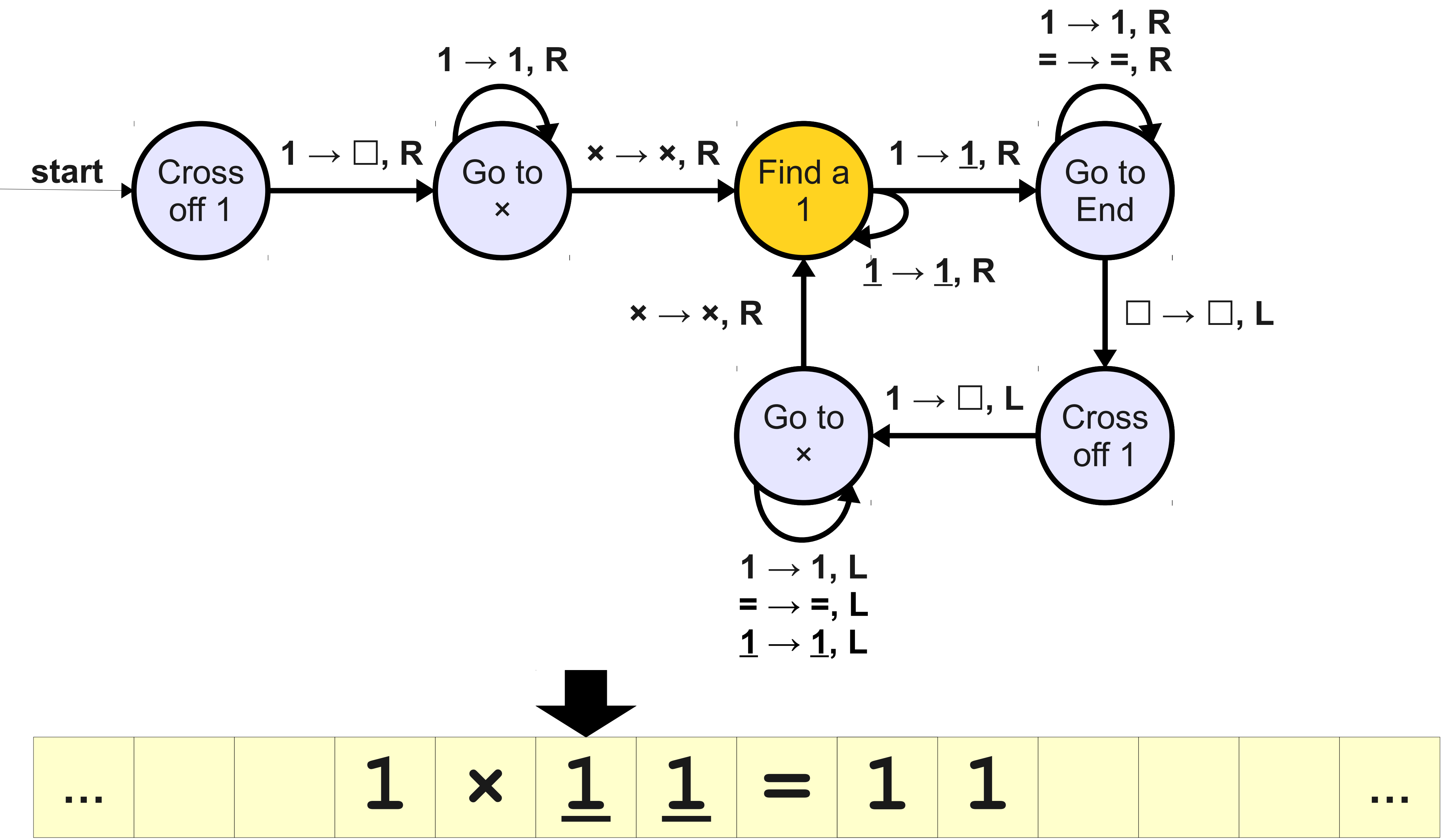
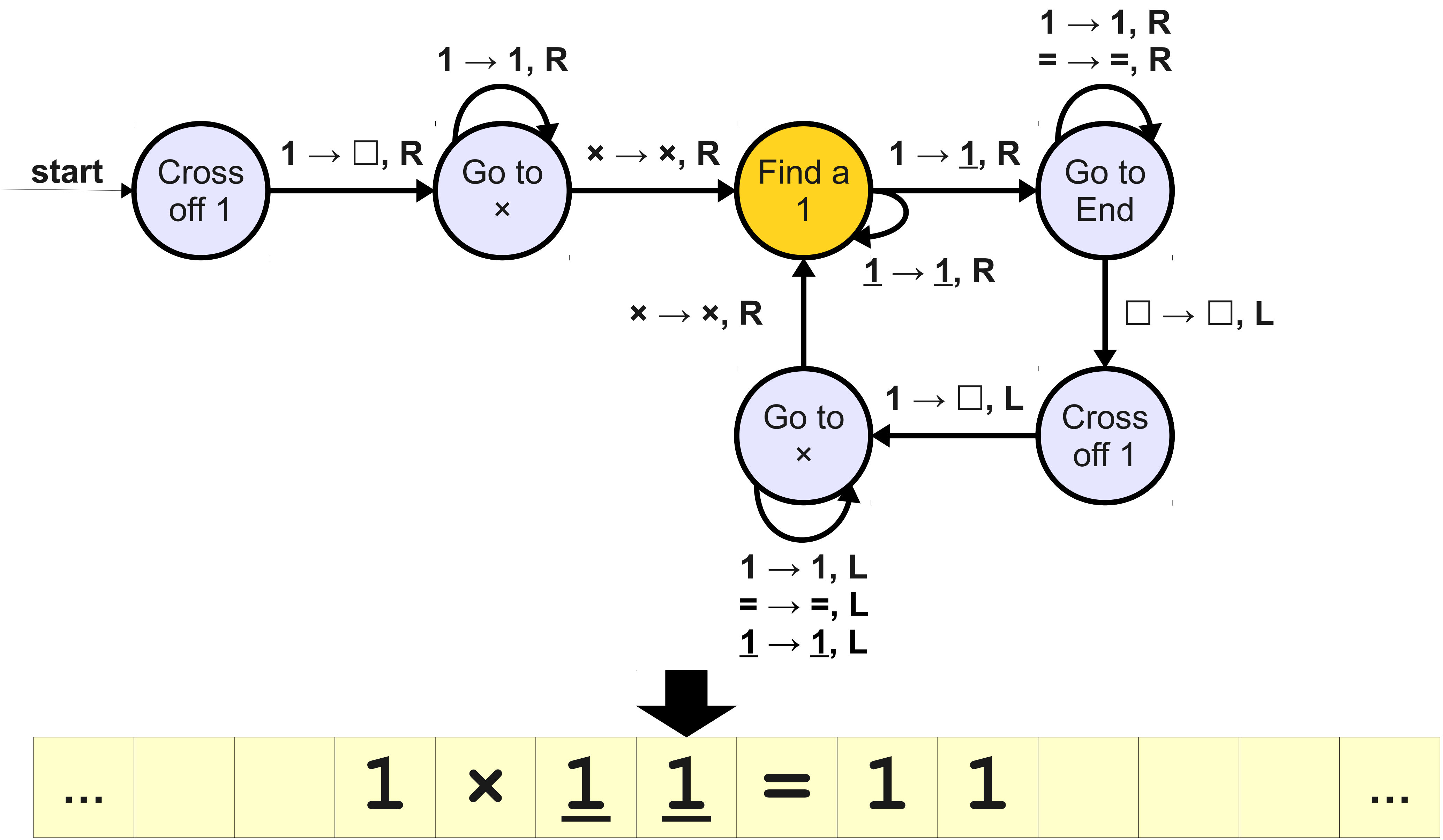
1
=
1
1
…

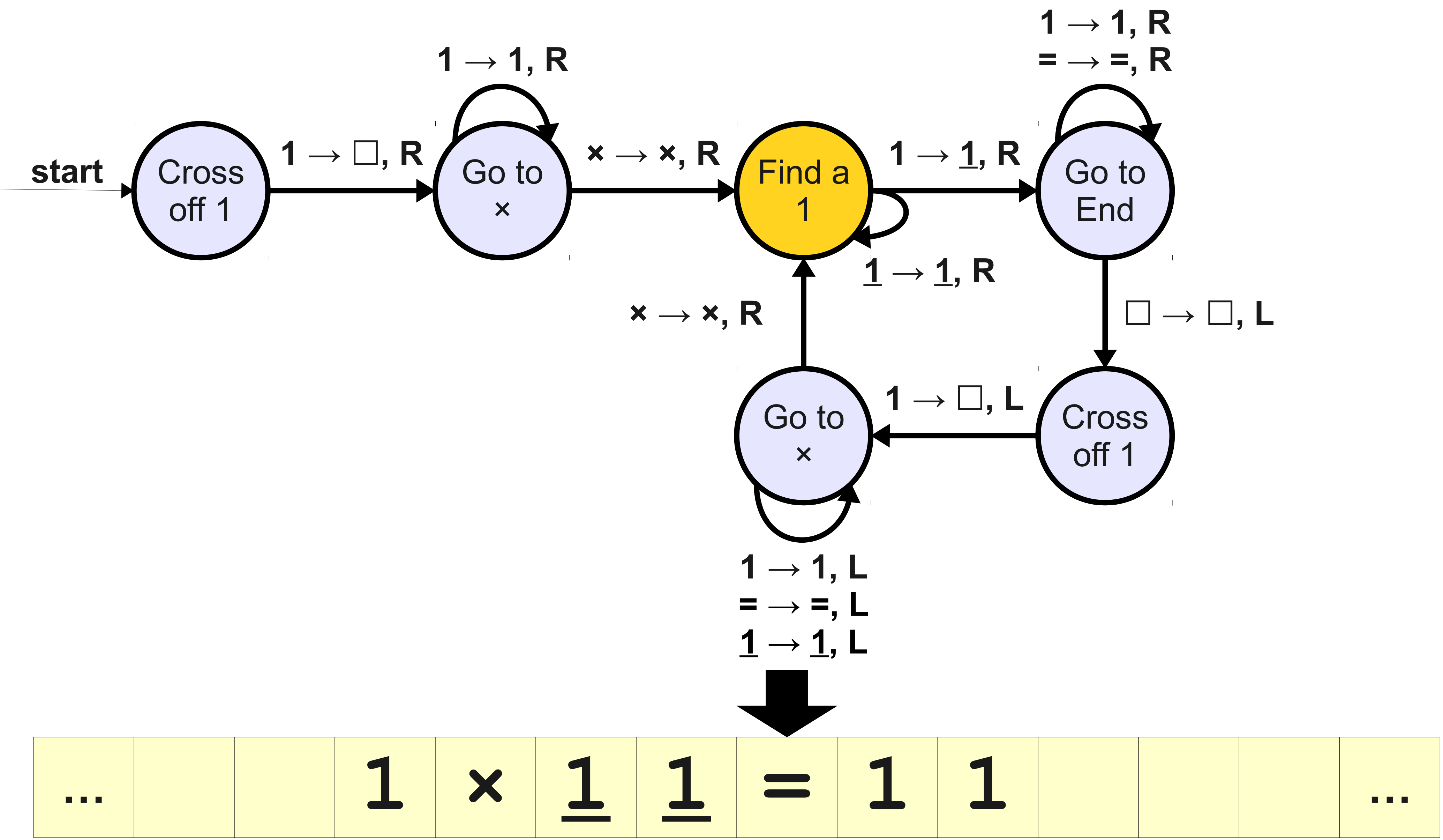

start
Cross off 1
1 → □, R
Go to ×
1 → 1, R
× → ×, R
Find a 1
1 → 1, R
1 → 1, R
Go to End
1 → 1, R
= → =, R
□ → □, L
Cross off 1
1 → □, L
Go to ×
× → ×, R
1 → 1, L
= → =, L
1 → 1, L
... 1 × 1 1 = 1 1 ...

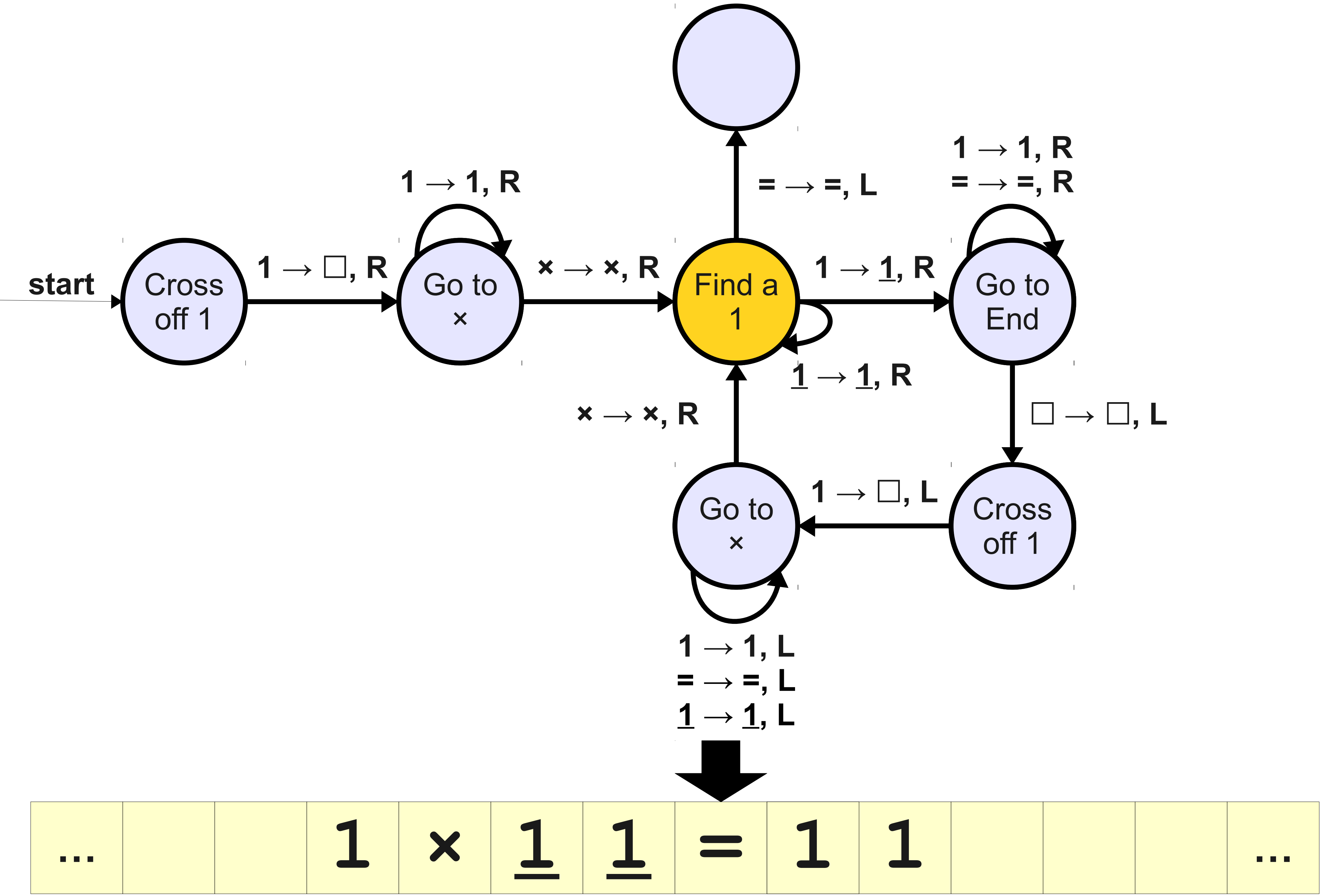

start
Cross off 1
1 → □, R
Go to ×
1 → 1, R
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
= → =, R
1 → 1, R
□ → □, L
× → ×, R
Cross off 1
1 → □, L
Go to ×
1 → 1, L
= → =, L
1 → 1, L
…
1
×
1
1
=
1
1
…

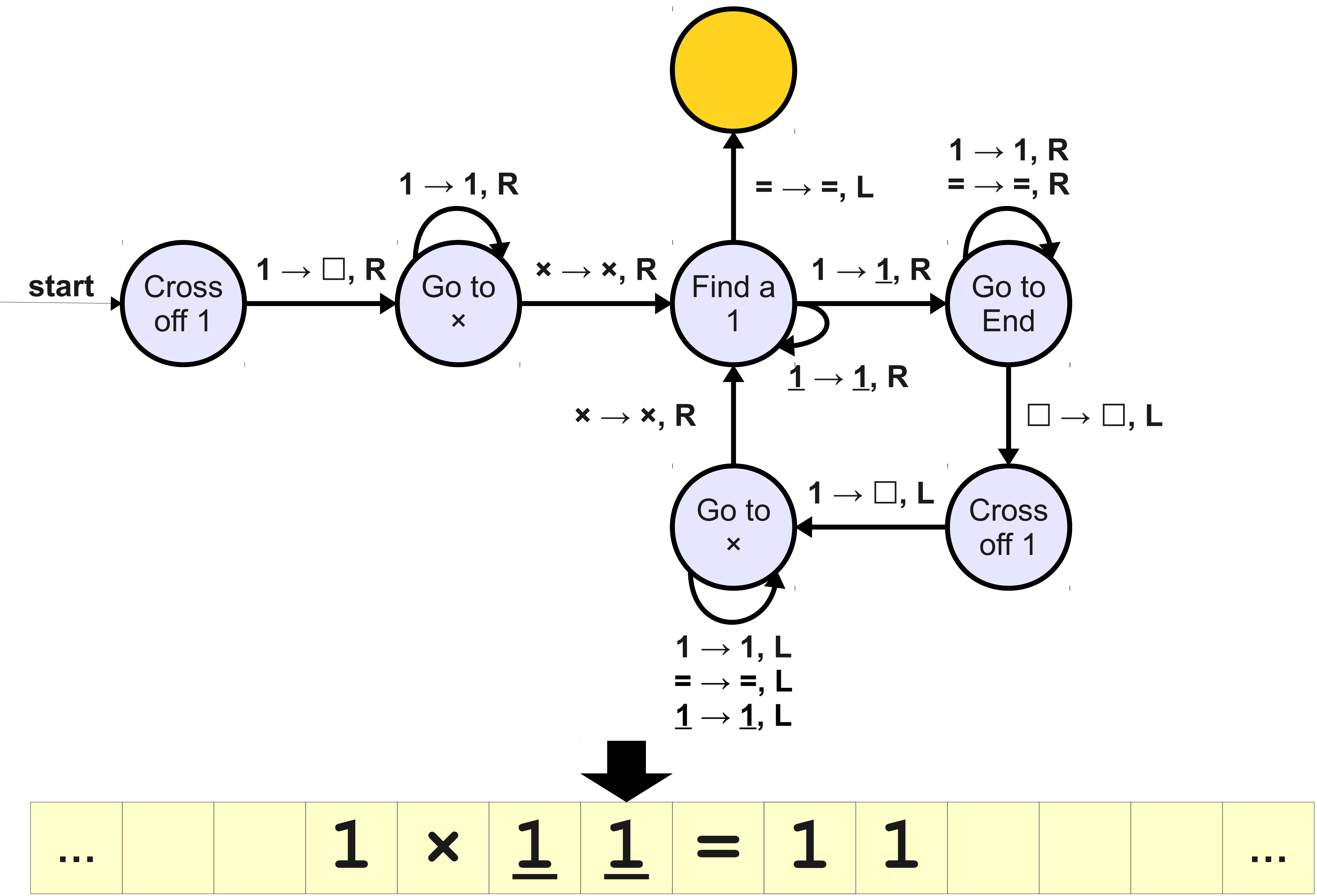


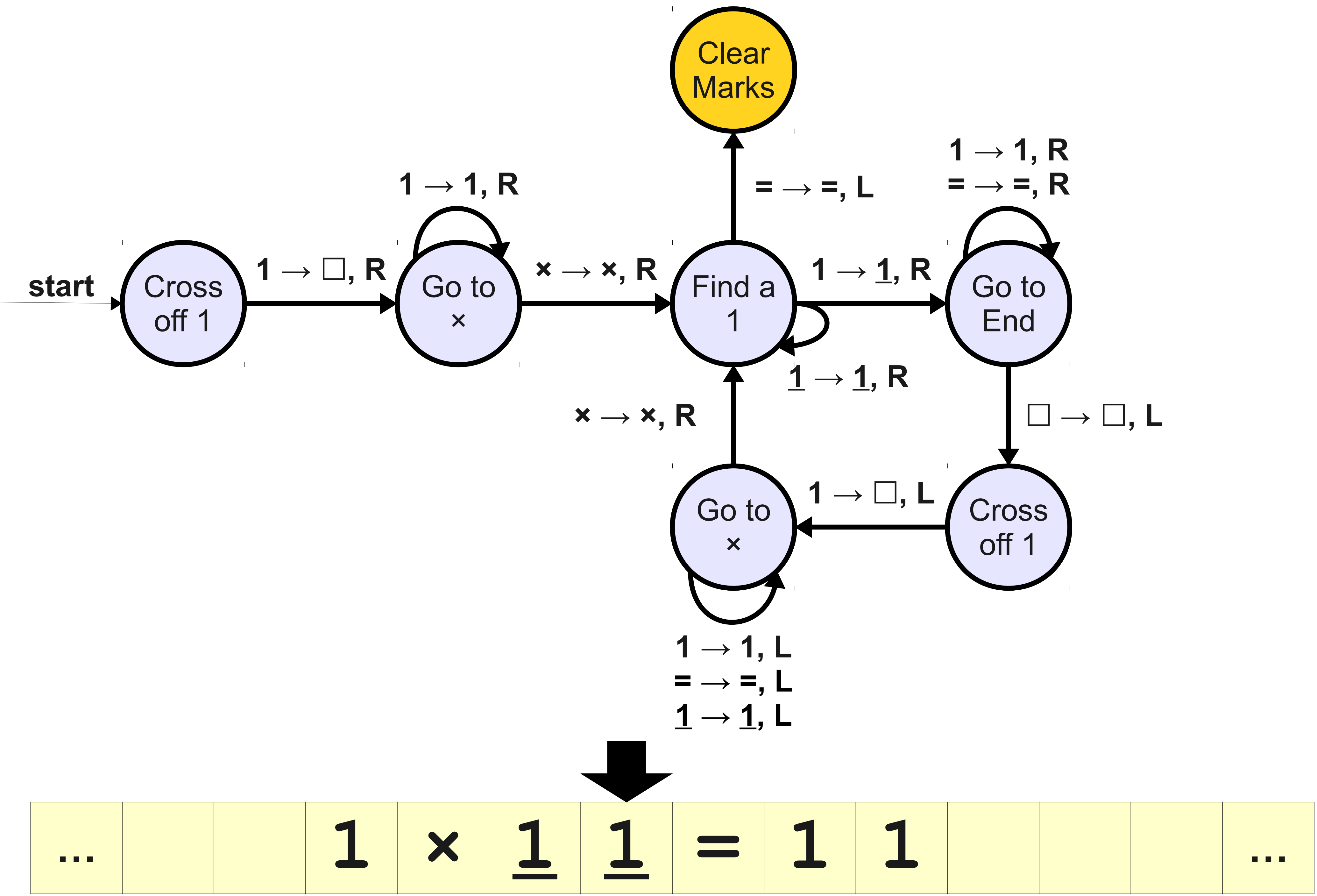
start
Cross off 1
1 → □, R
Go to ×
1 → 1, R
× → ×, R
Find a 1
1 → 1, R
1 → 1, R
Go to End
1 → 1, R
= → =, R
□ → □, L
Cross off 1
1 → □, L
Go to ×
1 → 1, L
= → =, L
1 → 1, L
× → ×, R
… 1 × 1 1 = 1 1 …

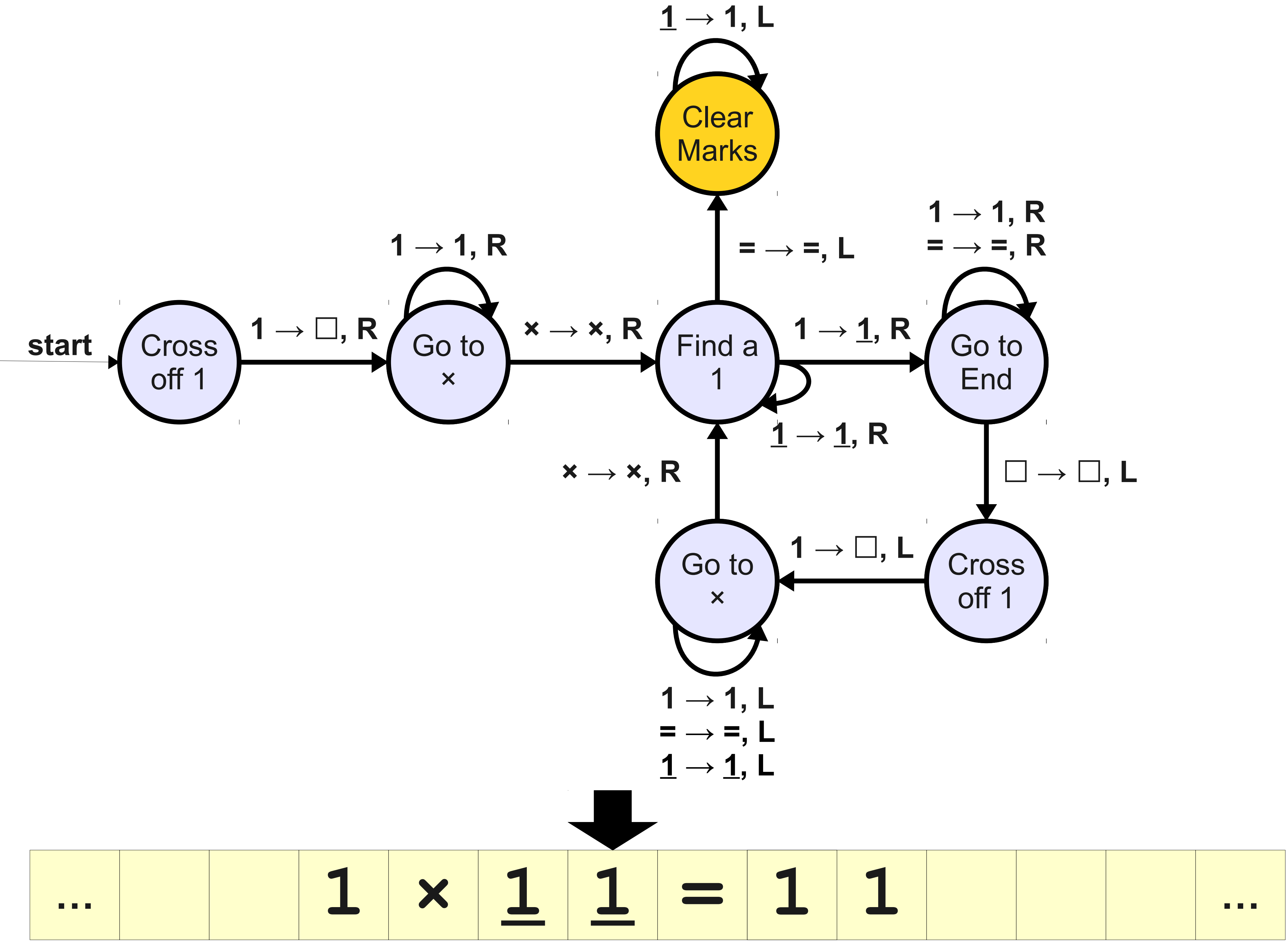

start
Cross off 1
1 → □, R
Go to ×
1 → 1, R
× → ×, R
Find a 1
1 → 1, R
1 → 1, R
Go to End
1 → 1, R
= → =, R
□ → □, L
Cross off 1
1 → □, L
Go to ×
× → ×, R
1 → 1, L
= → =, L
1 → 1, L
… 1 × 1 1 = 1 1 …


start
Cross off 1
1 → □, R
Go to ×
1 → 1, R
× → ×, R
Find a 1
1 → 1, R
1 → 1, R
Go to End
1 → 1, R
= → =, R
□ → □, L
Cross off 1
1 → □, L
Go to ×
× → ×, R
1 → 1, L
= → =, L
1 → 1, L
… 1 × 1 1 = 1 1 …

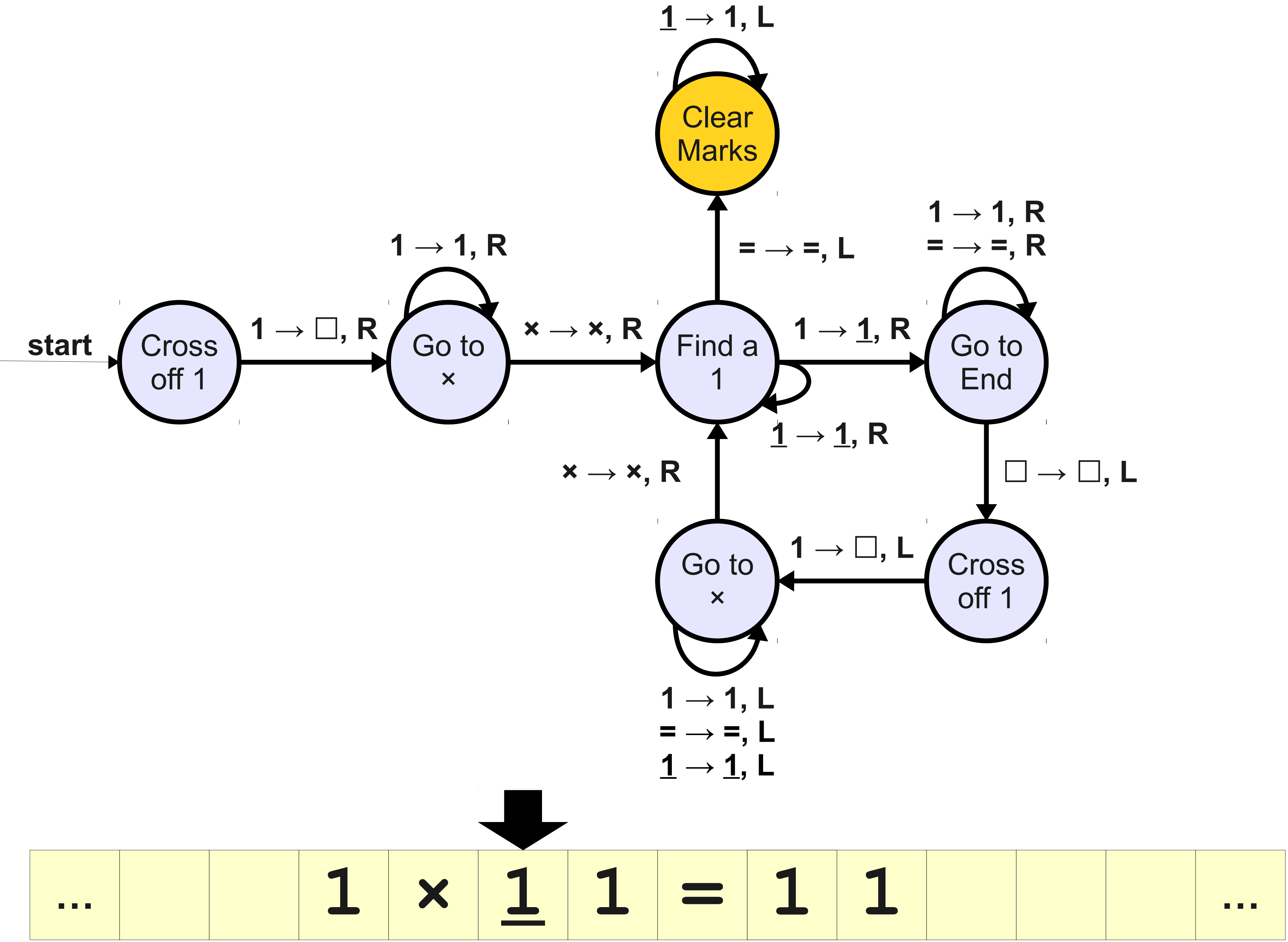

start
Cross off 1
1 → □, R
Go to ×
1 → 1, R
× → ×, R
Find a 1
= → =, L
1 → 1, R
1 → 1, R
1 → 1, R
= → =, R
Go to End
□ → □, L
Cross off 1
1 → □, L
Go to ×
× → ×, R
1 → 1, L
= → =, L
1 → 1, L
... 1 × 1 1 = 1 1 ...


start
Cross off 1
1 → □, R
Go to ×
1 → 1, R
× → ×, R
Find a 1
= → =, L
1 → 1̲, R
Go to End
1 → 1, R
= → =, R
1̲ → 1̲, R
□ → □, L
Cross off 1
1 → □, L
Go to ×
× → ×, R
1 → 1, L
= → =, L
1̲ → 1̲, L
… 1 × 1̲ 1̲ = 1 1 …

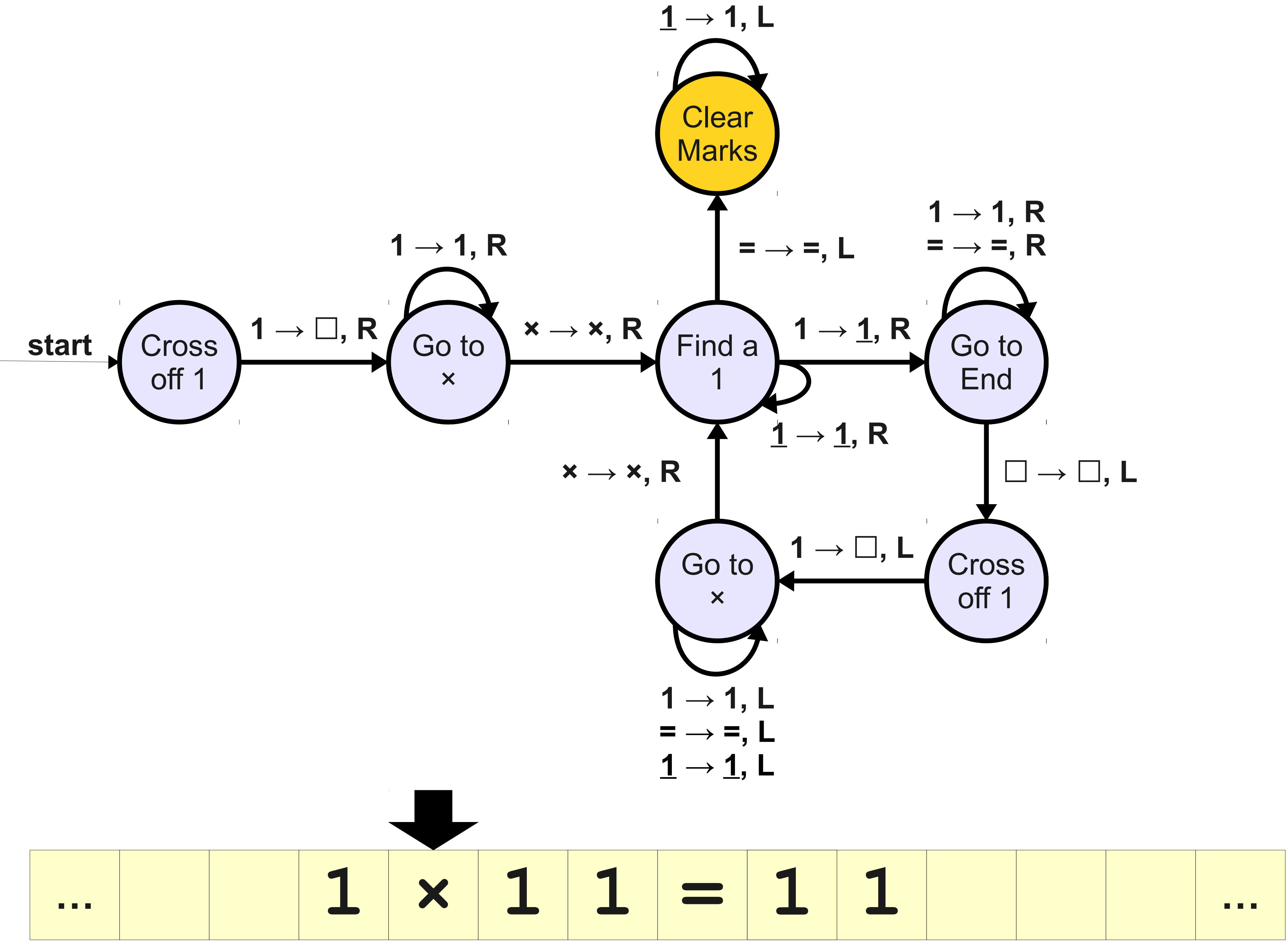

start
Cross off 1
1 → □, R
Go to ×
1 → 1, R
× → ×, R
Find a 1
Clear Marks
= → =, L
1 → 1, R
Go to End
1 → 1, R
= → =, R
1 → 1, R
× → ×, R
□ → □, L
1 → □, L
Cross off 1
Go to ×
1 → 1, L
= → =, L
1 → 1, L
...
1
×
1
1
=
1
1
...



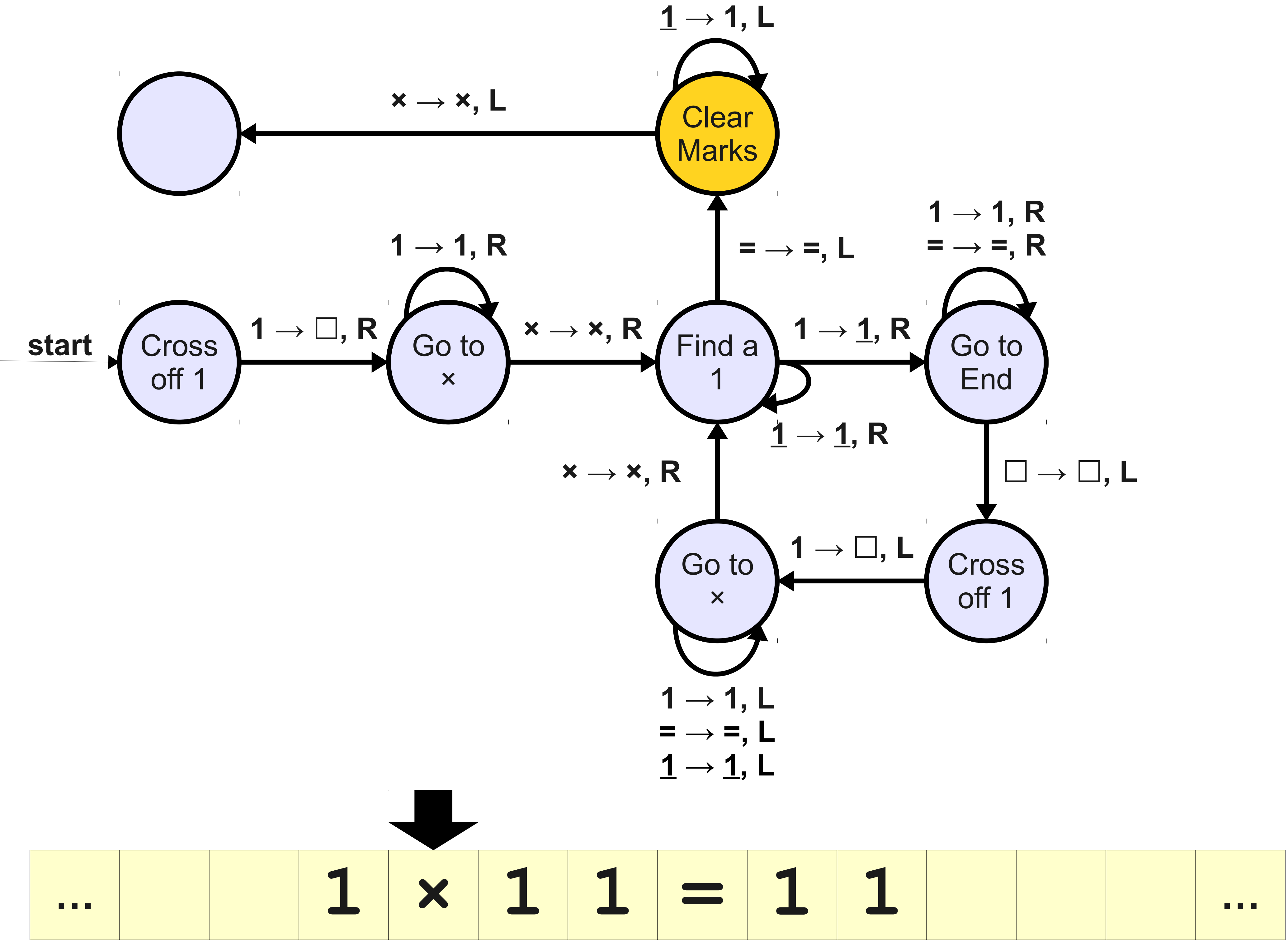
Clear Marks
1 → 1, L
start
Cross off 1
1 → □, R
Go to ×
1 → 1, R
× → ×, R
Find a 1
= → =, L
1 → 1, R
1 → 1, R
Go to End
1 → 1, R
= → =, R
□ → □, L
Cross off 1
1 → □, L
Go to ×
× → ×, R
1 → 1, L
= → =, L
1 → 1, L
… 1 × 1 1 = 1 1 …



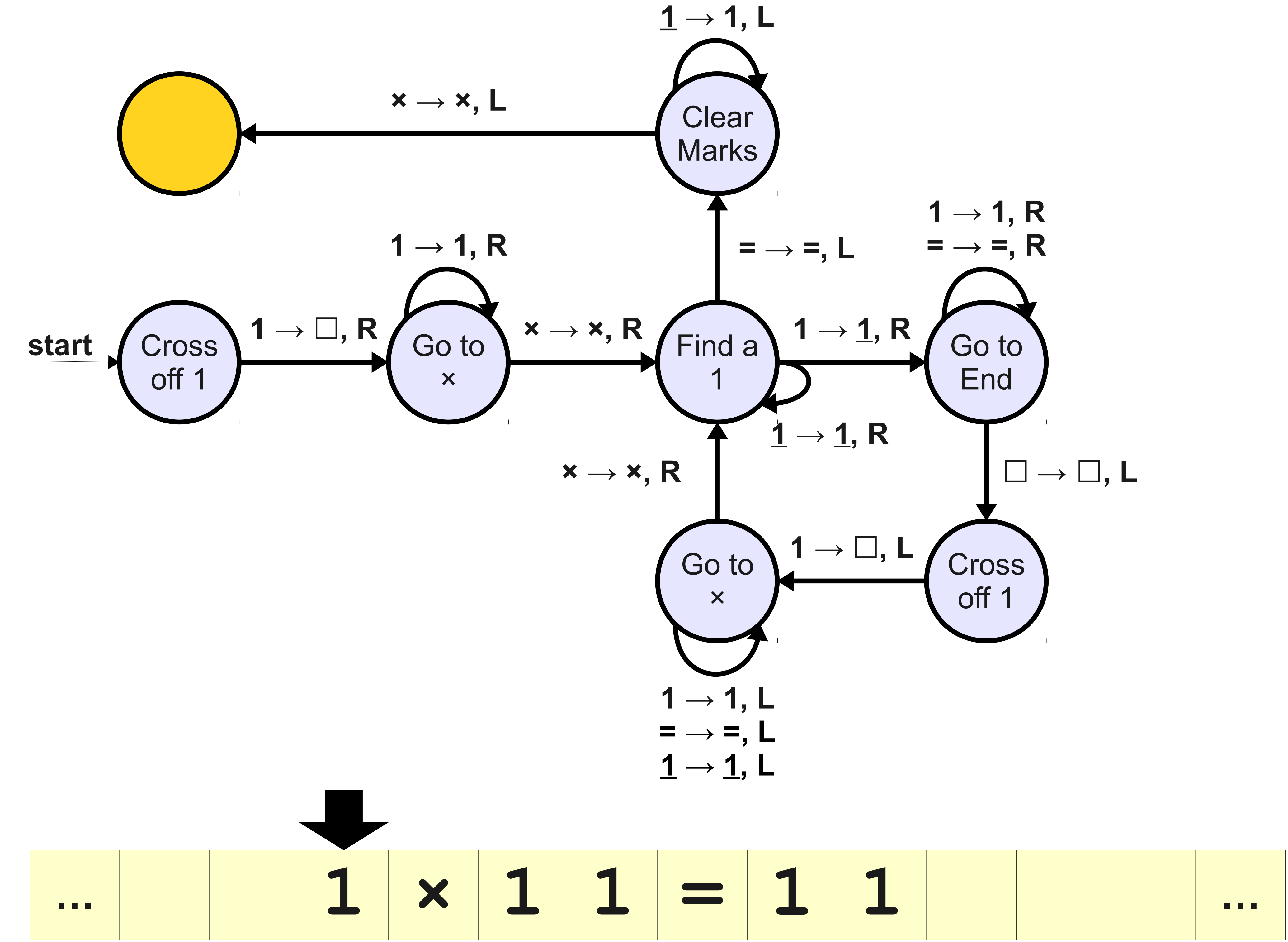
start
Cross off 1
1 → □, R
Go to ×
1 → 1, R
× → ×, R
Find a 1
1 → 1, R
1 → 1, R
= → =, L
Clear Marks
1 → 1, L
Go to End
1 → 1, R
= → =, R
□ → □, L
Cross off 1
1 → □, L
Go to ×
× → ×, R
1 → 1, L
= → =, L
1 → 1, L
… 1 × 1 1 = 1 1 …

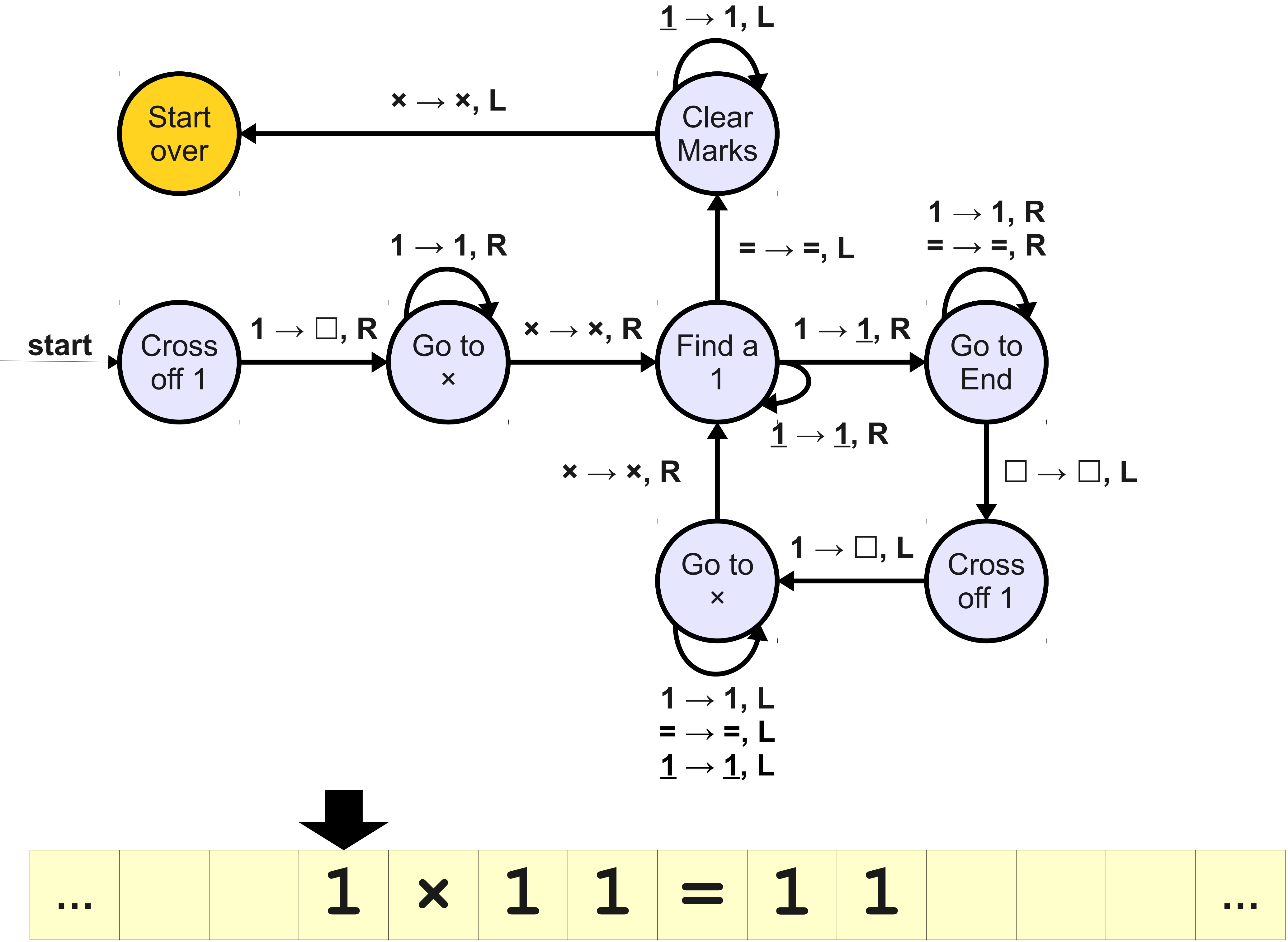


start
Cross off 1
1 → □, R
Go to ×
1 → 1, R
× → ×, R
Find a 1
= → =, L
Clear Marks
1 → 1, L
1 → 1, R
Go to End
1 → 1, R
= → =, R
1 → 1, R
□ → □, L
Cross off 1
1 → □, L
Go to ×
× → ×, R
1 → 1, L
= → =, L
1 → 1, L
...
1
×
1
1
=
1
1
...

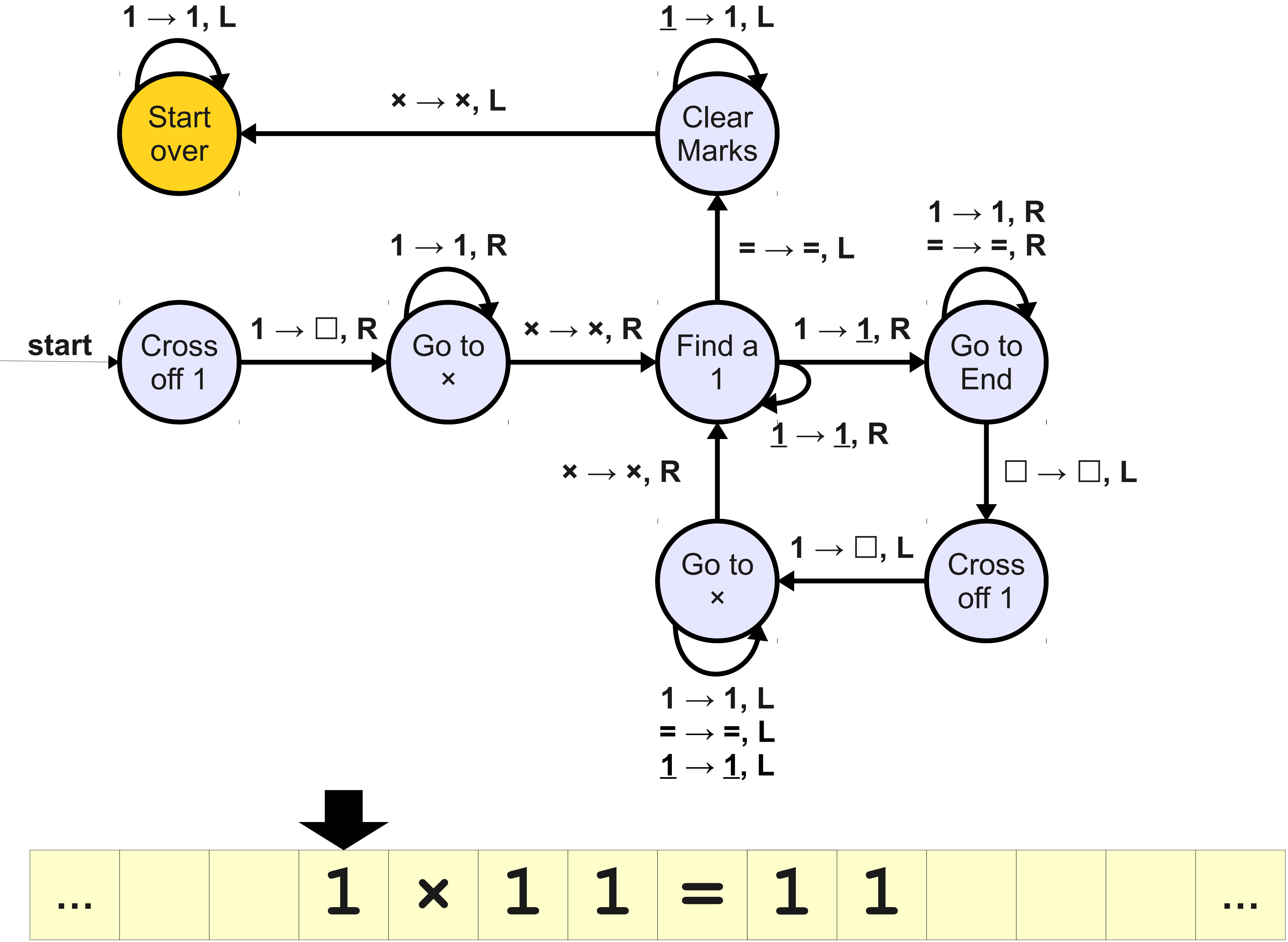


1 → 1, L
× → ×, L
Clear Marks
1 → 1, R
= → =, L
1 → 1, R
= → =, R
start
Cross off 1
1 → □, R
Go to ×
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
× → ×, R
□ → □, L
Go to ×
1 → □, L
Cross off 1
1 → 1, L
= → =, L
1 → 1, L
… 1 × 1 1 = 1 1 …

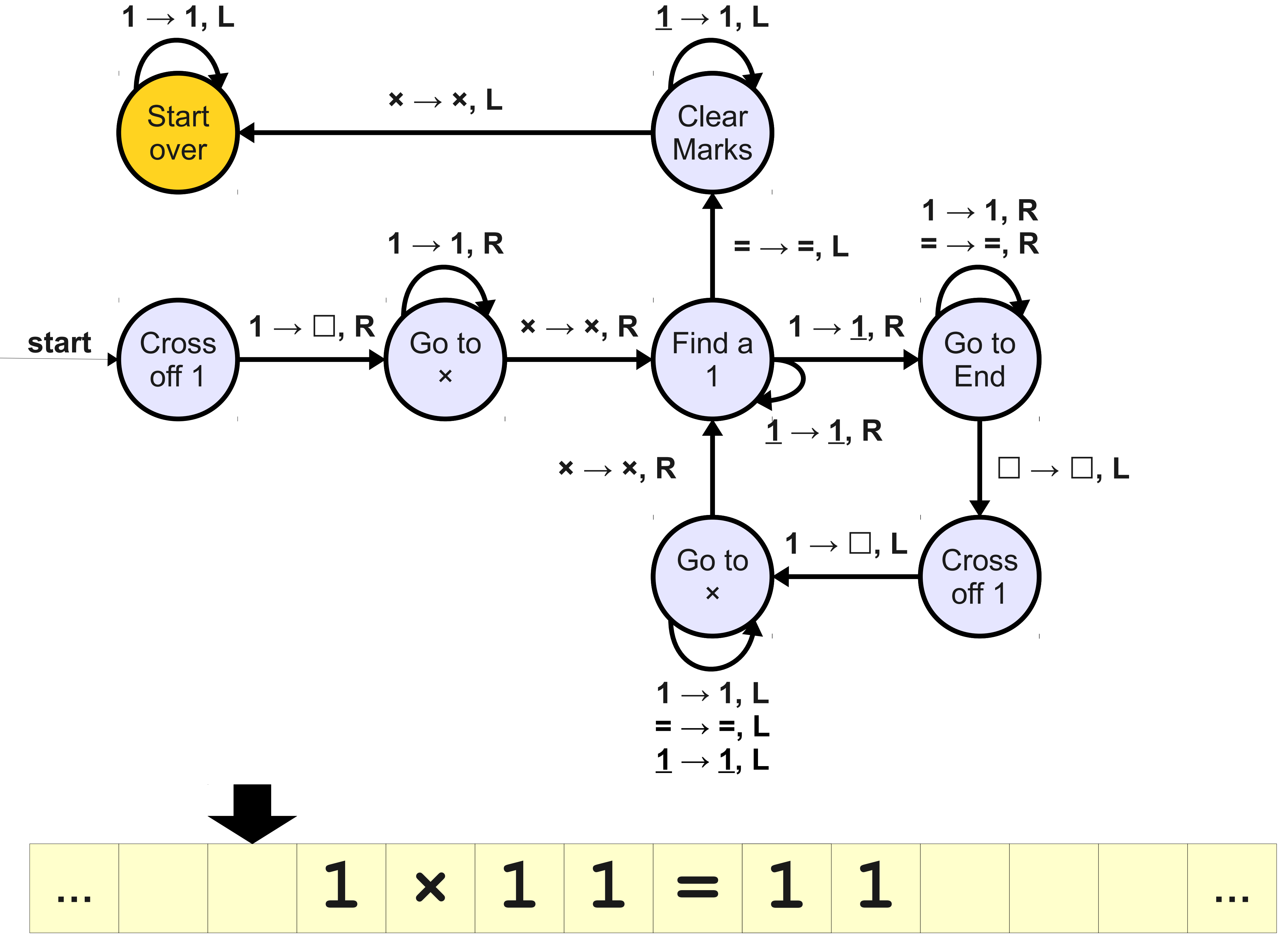

start
Cross off 1
1 → □, R
Go to ×
1 → 1, R
× → ×, R
Find a 1
1 → 1, R
1 → 1, R
Go to End
1 → 1, R
= → =, R
□ → □, L
Cross off 1
1 → □, L
Go to ×
1 → 1, L
= → =, L
1 → 1, L
× → ×, R
= → =, L
Clear Marks
1 → 1, L
× → ×, L
…
1
×
1
1
=
1
1
…



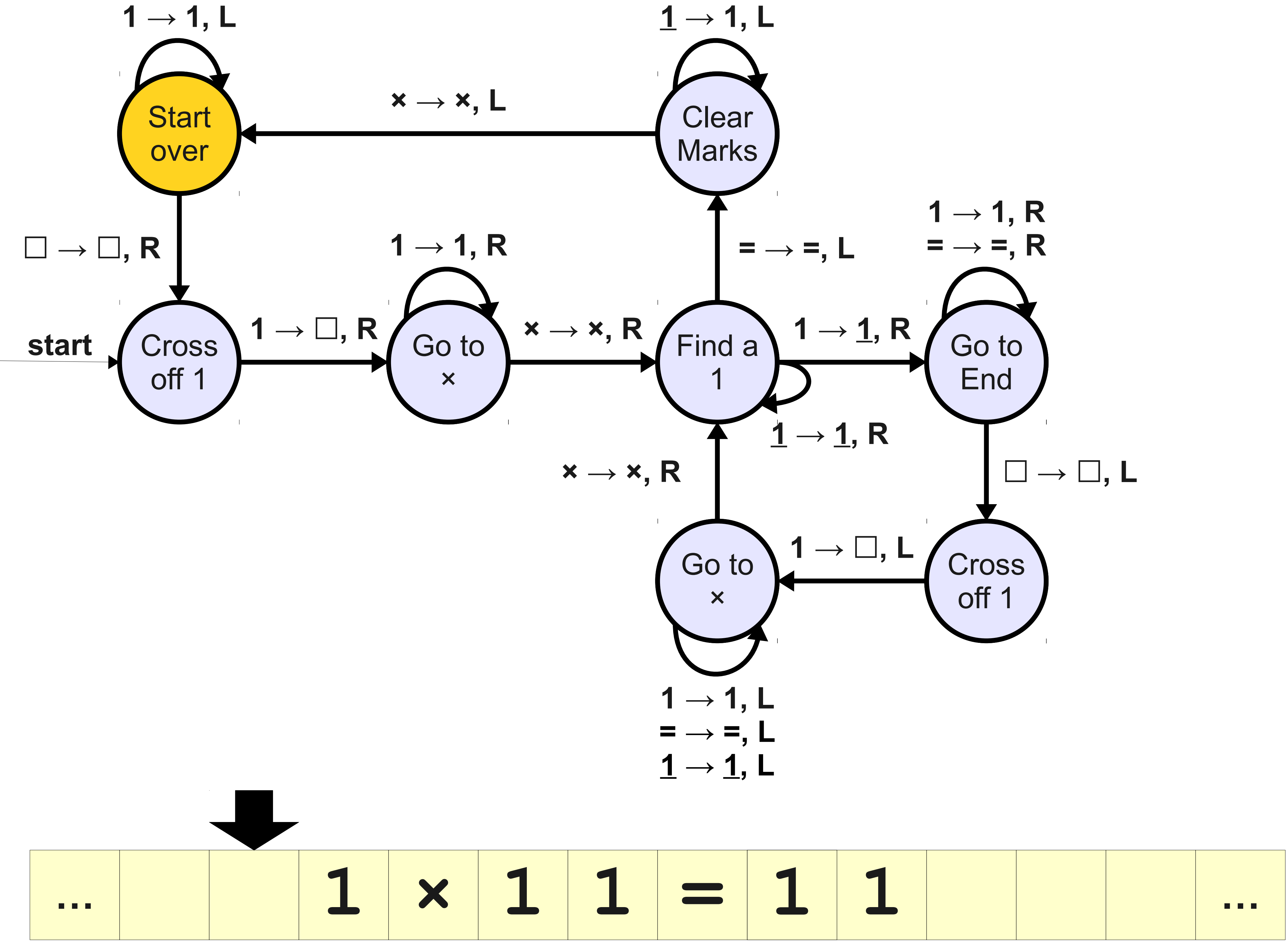
1 → 1, L
Start over
× → ×, L
Clear Marks
1 → 1, R
= → =, L
1 → 1, R
= → =, R
start
Cross off 1
1 → □, R
Go to ×
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
× → ×, R
□ → □, L
Go to ×
1 → □, L
Cross off 1
1 → 1, L
= → =, L
1 → 1, L
...
1
×
1
1
=
1
1
...

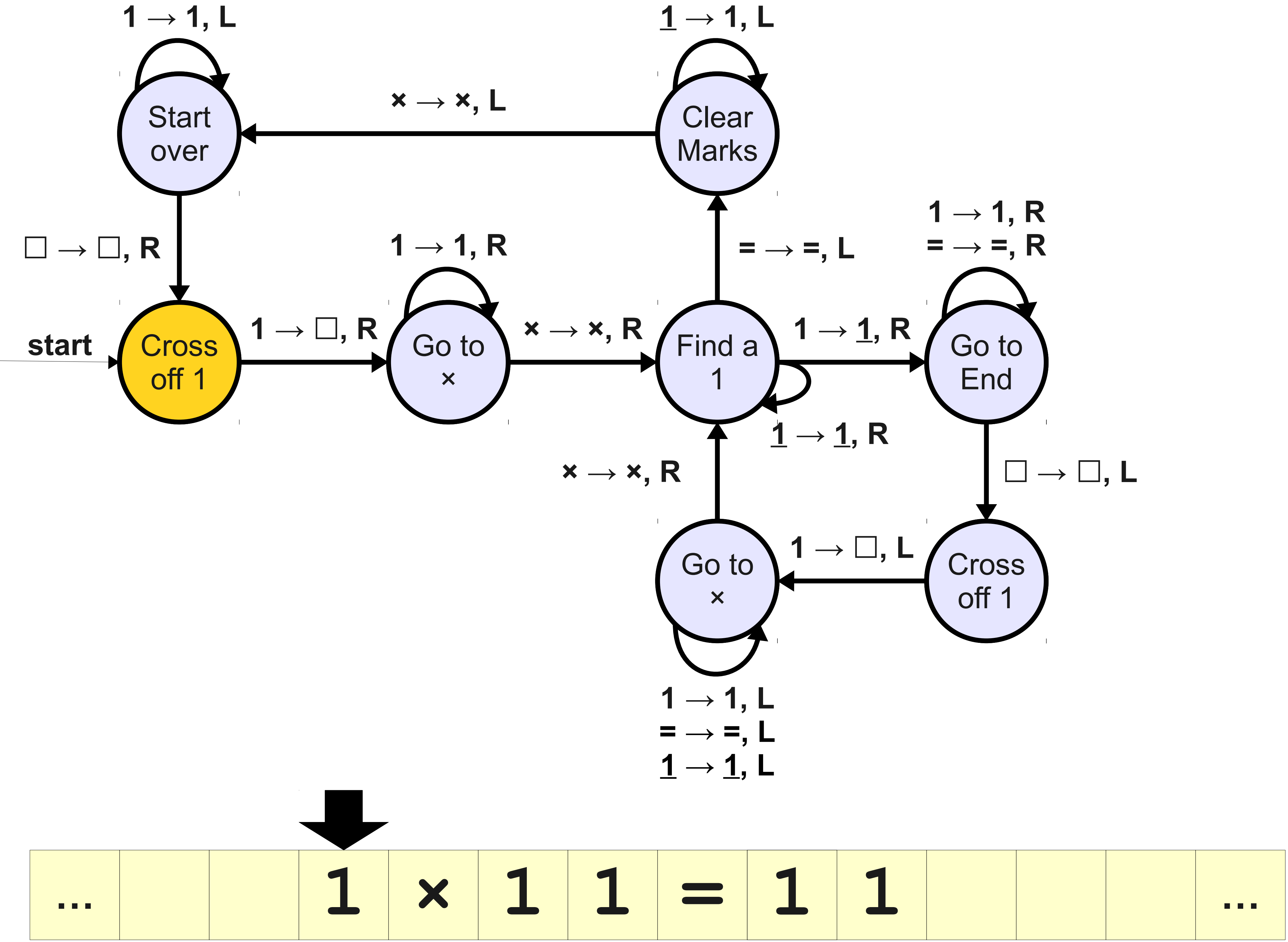


1 → 1, L
1 → 1, L
Start over
× → ×, L
Clear Marks
= → =, L
1 → 1, R
= → =, R
1 → 1, R
start
Cross off 1
1 → □, R
Go to ×
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
× → ×, R
□ → □, L
Go to ×
1 → □, L
Cross off 1
1 → 1, L
= → =, L
1 → 1, L
… 1 × 1 1 = 1 1 …

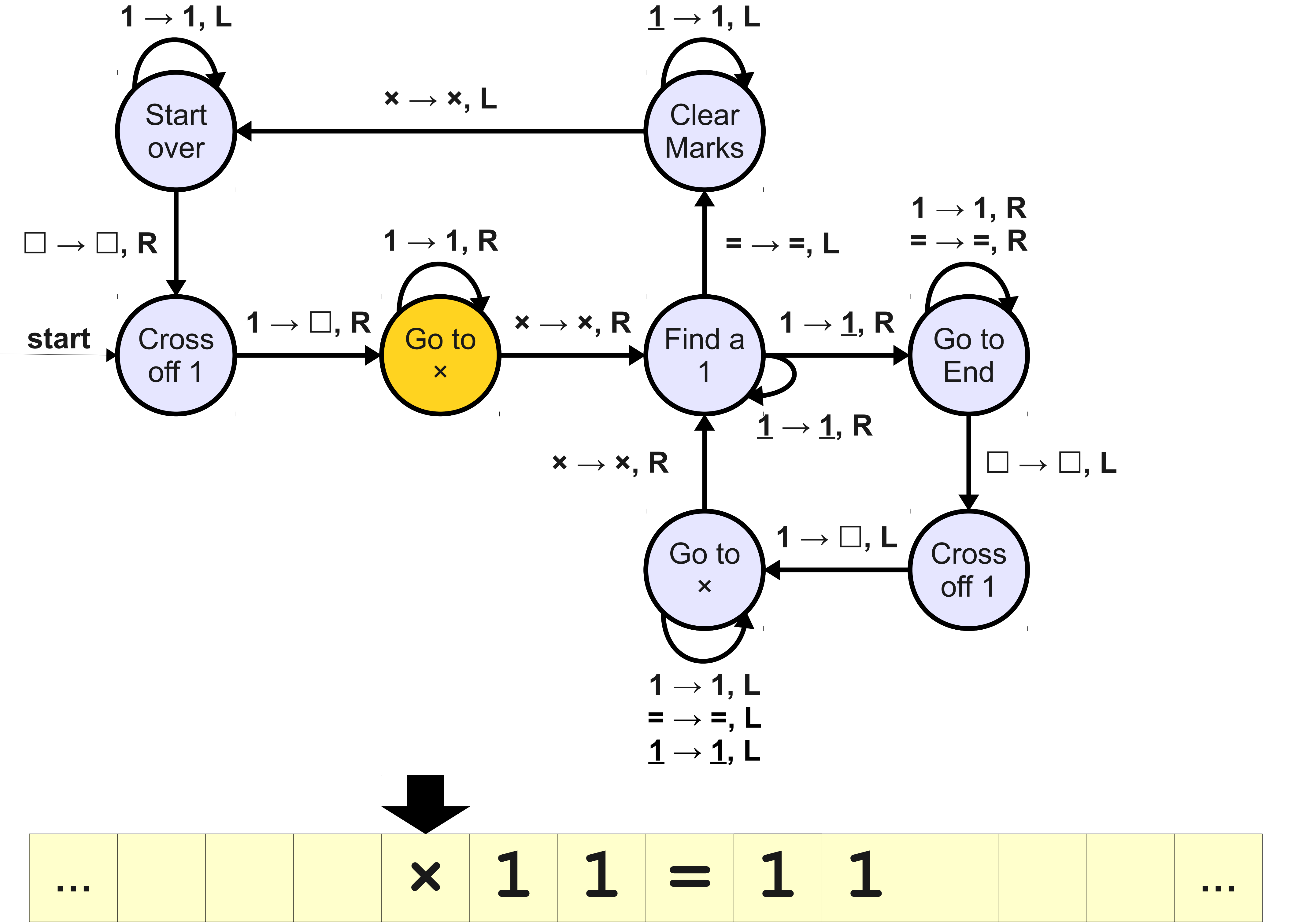


1 → 1, L
Start over
× → ×, L
1 → 1, L
Clear Marks
= → =, L
1 → 1, R
= → =, R
1 → 1, R
start
Cross off 1
1 → □, R
Go to ×
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
× → ×, R
□ → □, L
Go to ×
1 → □, L
Cross off 1
1 → 1, L
= → =, L
1 → 1, L
… 1 × 1 1 = 1 1 …

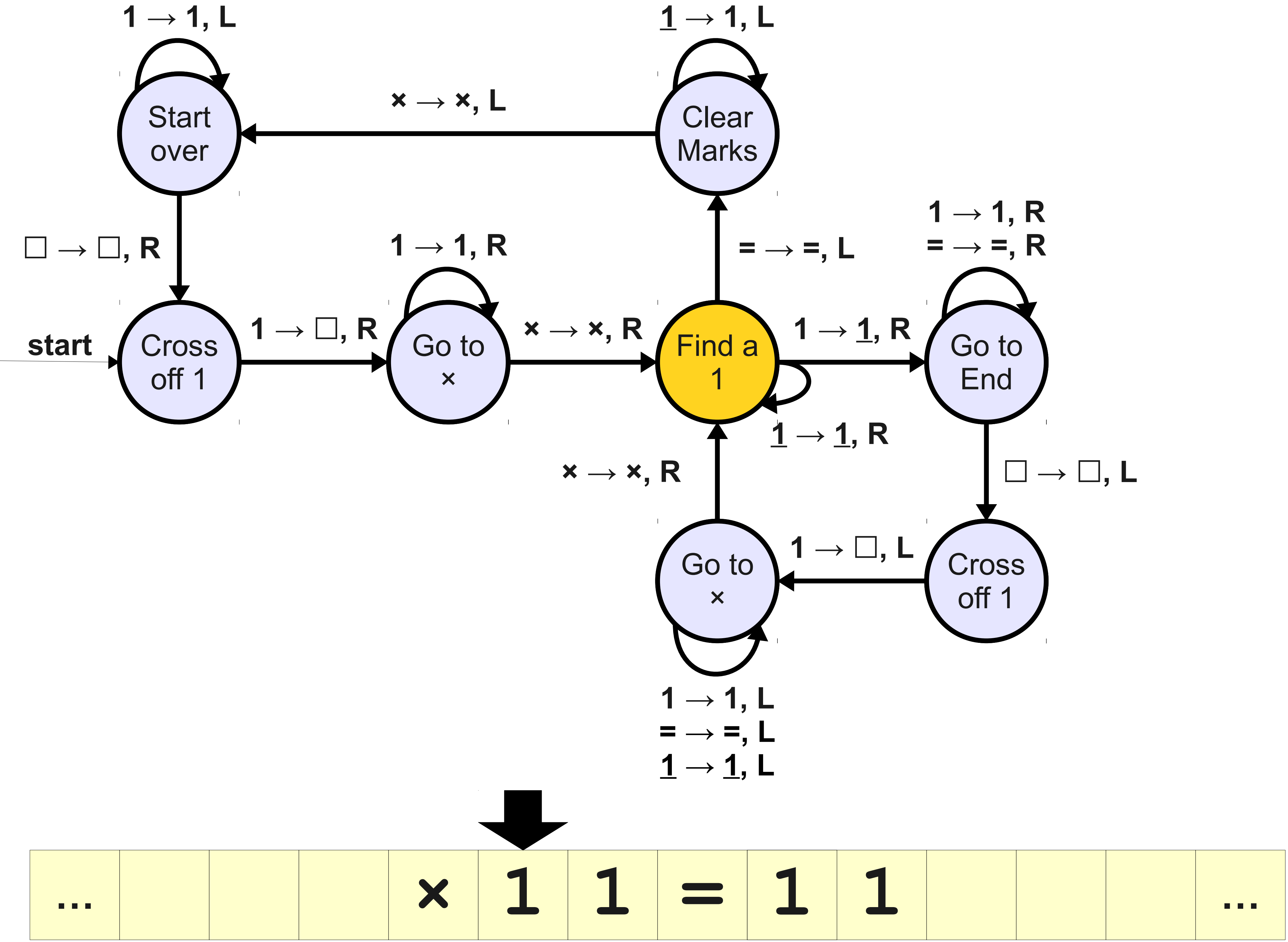

1 → 1, L
Start over
× → ×, L
1 → 1, L
Clear Marks
□ → □, R
1 → 1, R
1 → 1, R
= → =, R
= → =, L
start
Cross off 1
1 → □, R
Go to ×
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
× → ×, R
□ → □, L
Go to ×
1 → □, L
Cross off 1
1 → 1, L
= → =, L
1 → 1, L
… 1 × 1 1 = 1 1 …



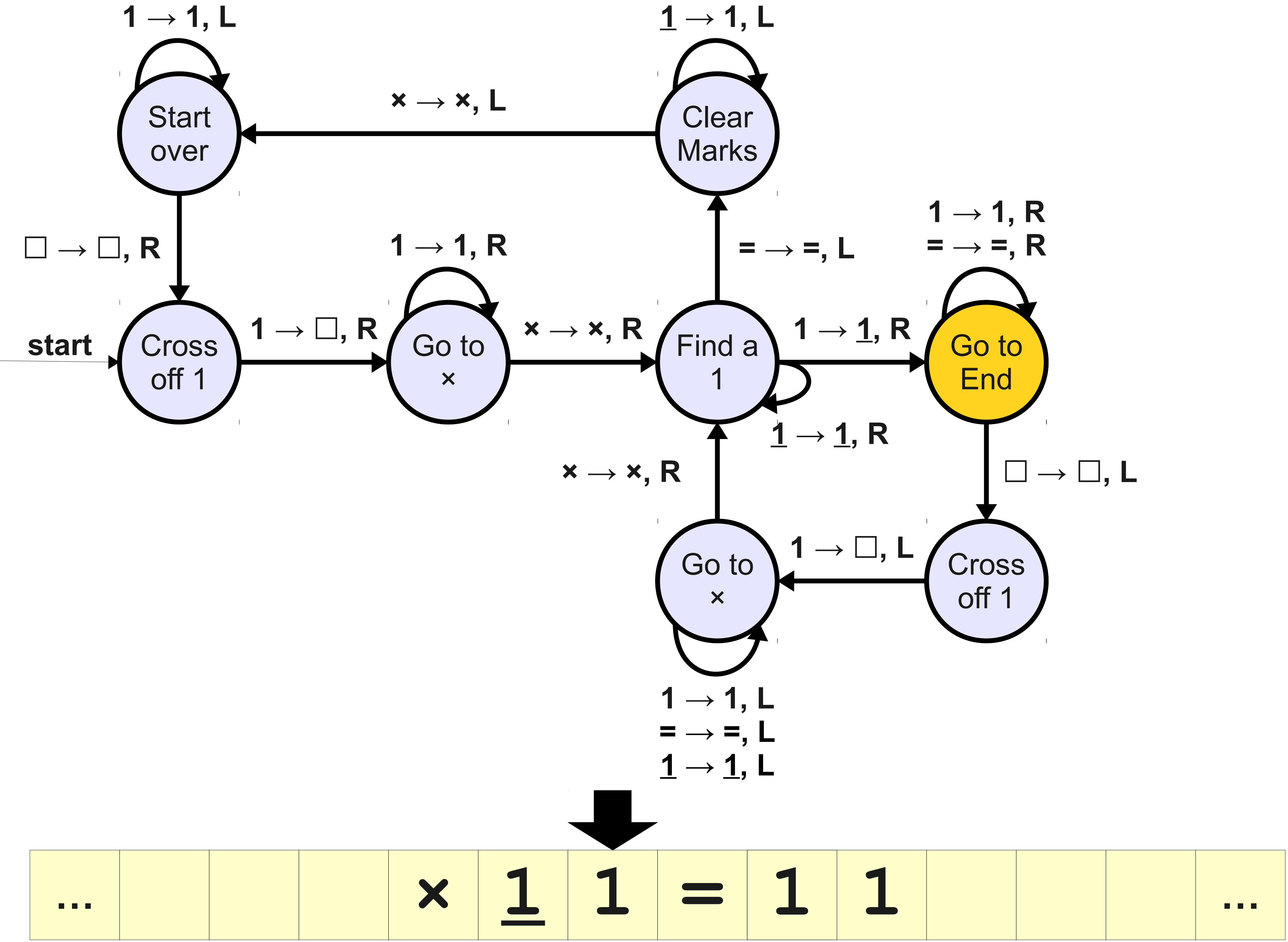
1 → 1, L
Start over
× → ×, L
1 → 1, L
Clear Marks
□ → □, R
1 → 1, R
= → =, L
1 → 1, R
= → =, R
start
Cross off 1
1 → □, R
Go to ×
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
× → ×, R
□ → □, L
Go to ×
1 → □, L
Cross off 1
1 → 1, L
= → =, L
1 → 1, L
… 1 × 1 1 = 1 1 …



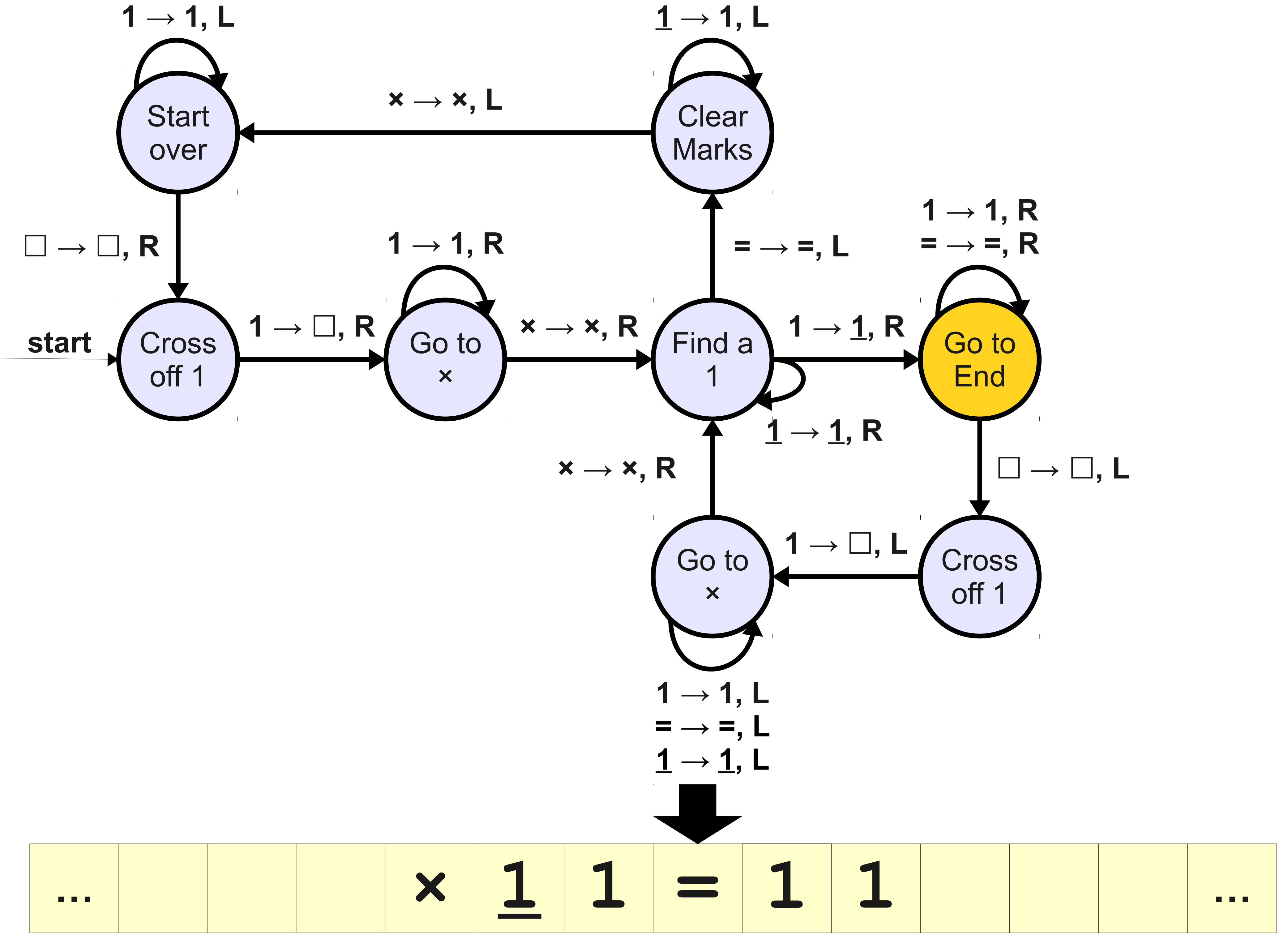
Start over
Clear Marks
Cross off 1
Go to ×
Find a 1
Go to End
Go to ×
Cross off 1
start
1 → 1, L
1 → 1, L
× → ×, L
□ → □, R
1 → 1, R
= → =, L
1 → 1, R
= → =, R
1 → □, R
× → ×, R
1 → 1, R
1 → 1, R
× → ×, R
□ → □, L
1 → □, L
1 → 1, L
= → =, L
1 → 1, L
…
×
1
1
=
1
1
…



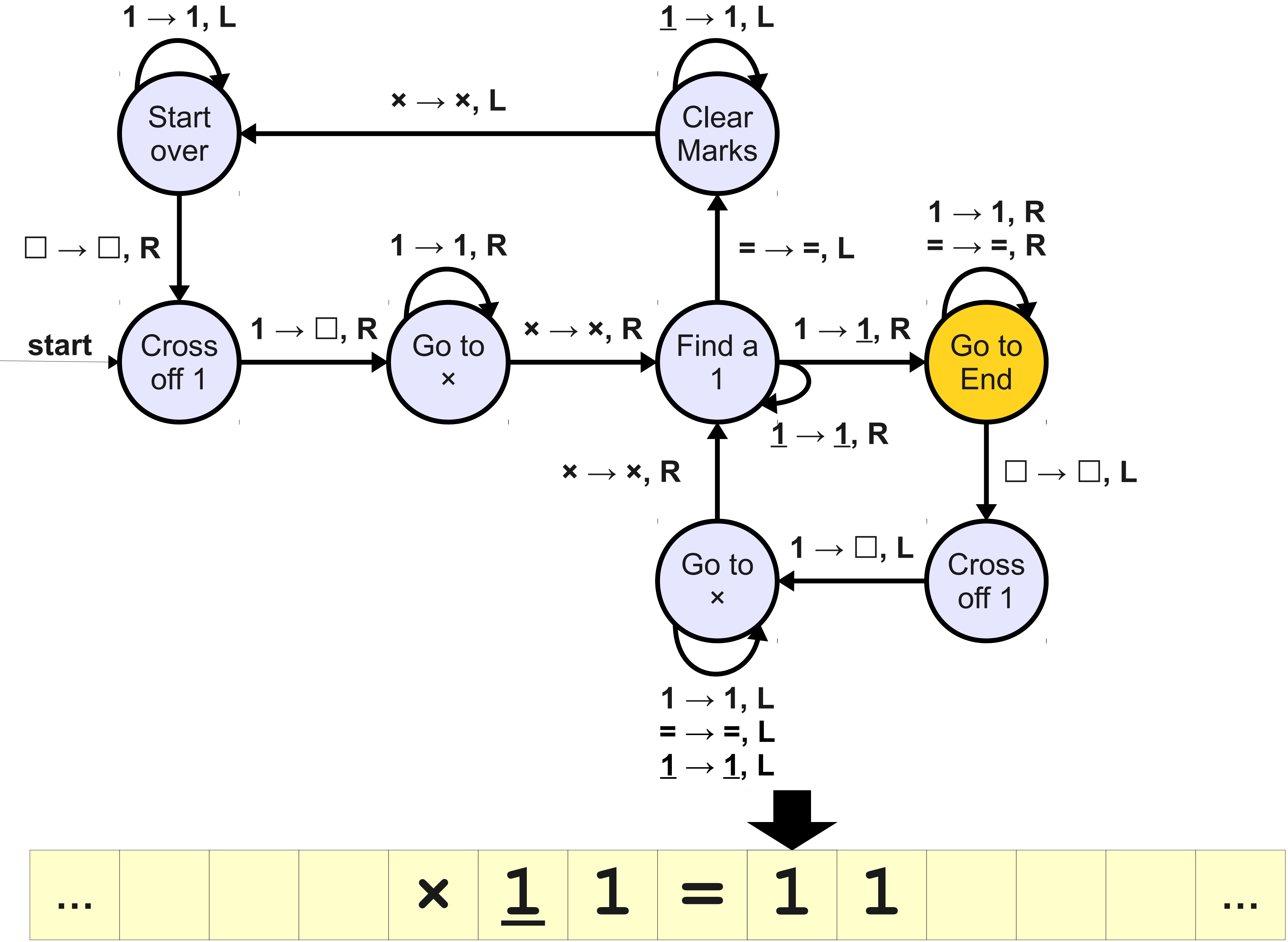
1 → 1, L
Start over
× → ×, L
1 → 1, L
Clear Marks
□ → □, R
1 → 1, R
= → =, L
1 → 1, R
= → =, R
start
Cross off 1
1 → □, R
Go to ×
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
× → ×, R
□ → □, L
Go to ×
1 → □, L
Cross off 1
1 → 1, L
= → =, L
1 → 1, L
…
×
1
1
=
1
1
…

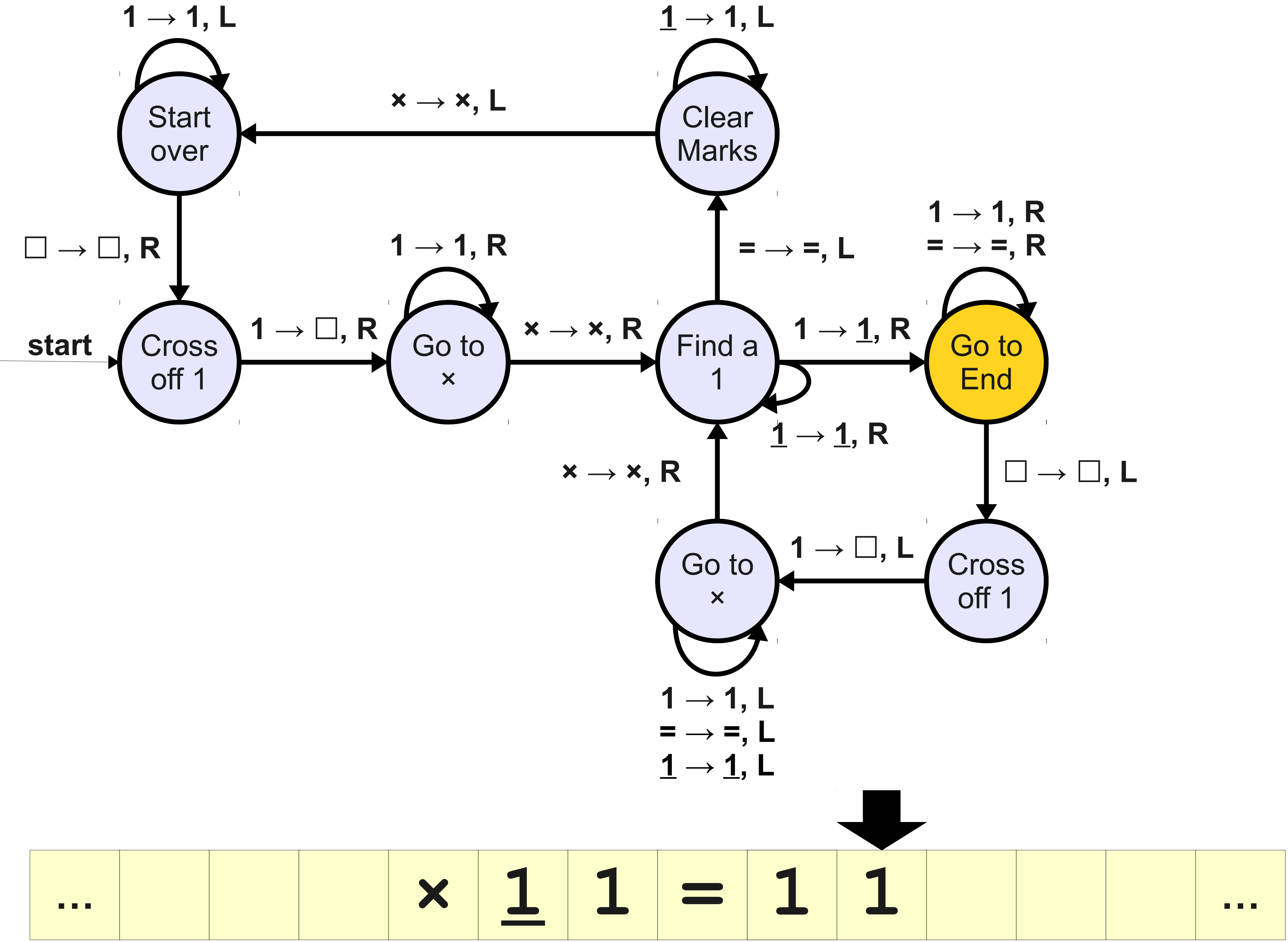


start
Start over
Clear Marks
Cross off 1
Go to ×
Find a 1
Go to End
Go to ×
Cross off 1
1 → 1, L
1 → 1, L
× → ×, L
□ → □, R
1 → 1, R
1 → □, R
× → ×, R
= → =, L
1 → 1, R
= → =, R
1 → 1, R
1 → 1, R
× → ×, R
□ → □, L
1 → □, L
1 → 1, L
= → =, L
1 → 1, L
...
×
1
1
=
1
1
...

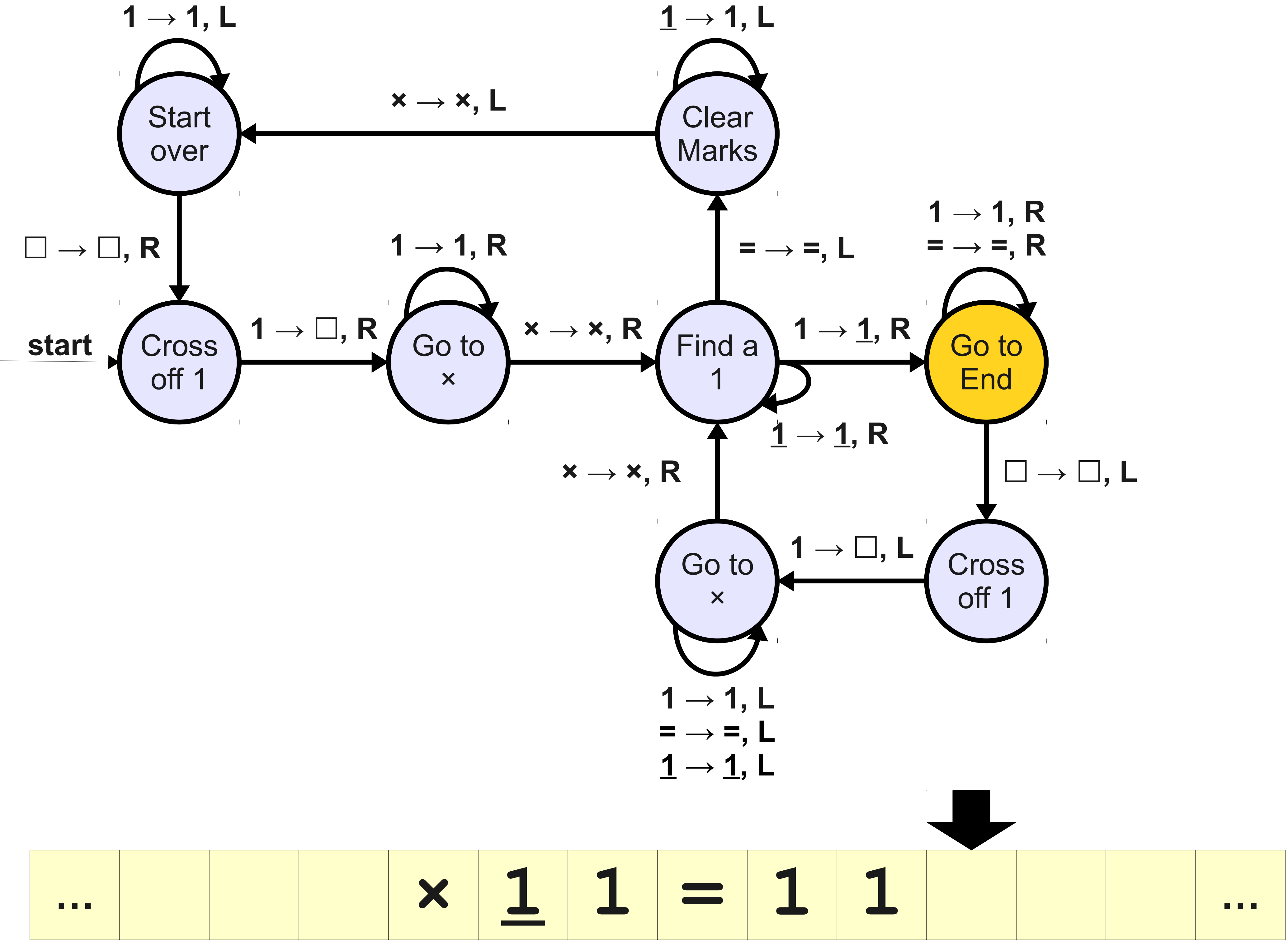


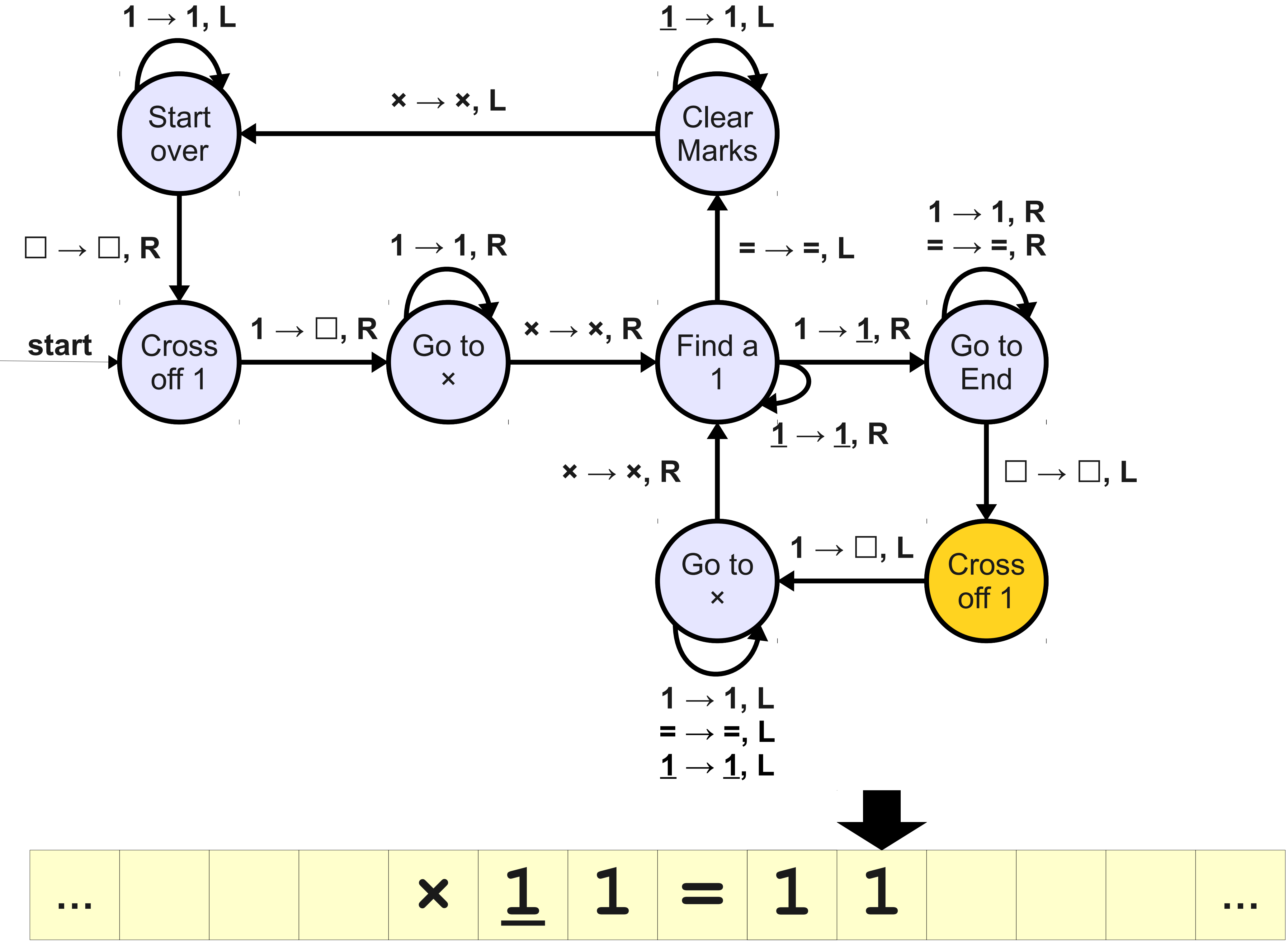
1 → 1, L
Start over
1 → 1, L
Clear Marks
× → ×, L
□ → □, R
1 → 1, R
= → =, L
1 → 1, R
= → =, R
start
Cross off 1
1 → □, R
Go to ×
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
× → ×, R
□ → □, L
Go to ×
1 → □, L
Cross off 1
1 → 1, L
= → =, L
1 → 1, L
… × 1 1 = 1 1 …


1 → 1, L
Start over
× → ×, L
1 → 1, L
Clear Marks
☐ → ☐, R
1 → 1, R
= → =, L
1 → 1, R
= → =, R
start
Cross off 1
1 → ☐, R
Go to ×
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
× → ×, R
☐ → ☐, L
Go to ×
1 → ☐, L
Cross off 1
1 → 1, L
= → =, L
1 → 1, L
…
×
1
1
=
1
1
…

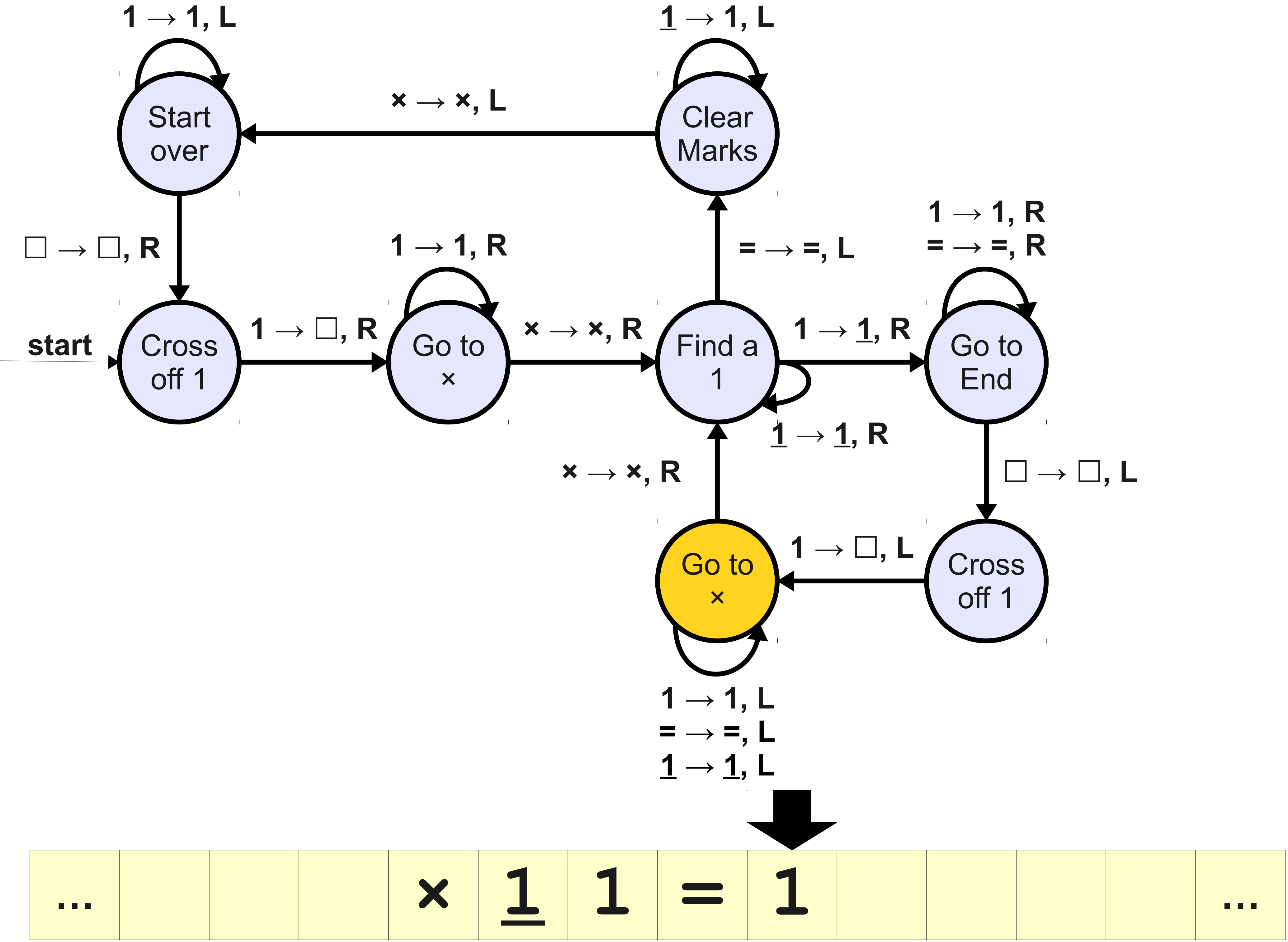


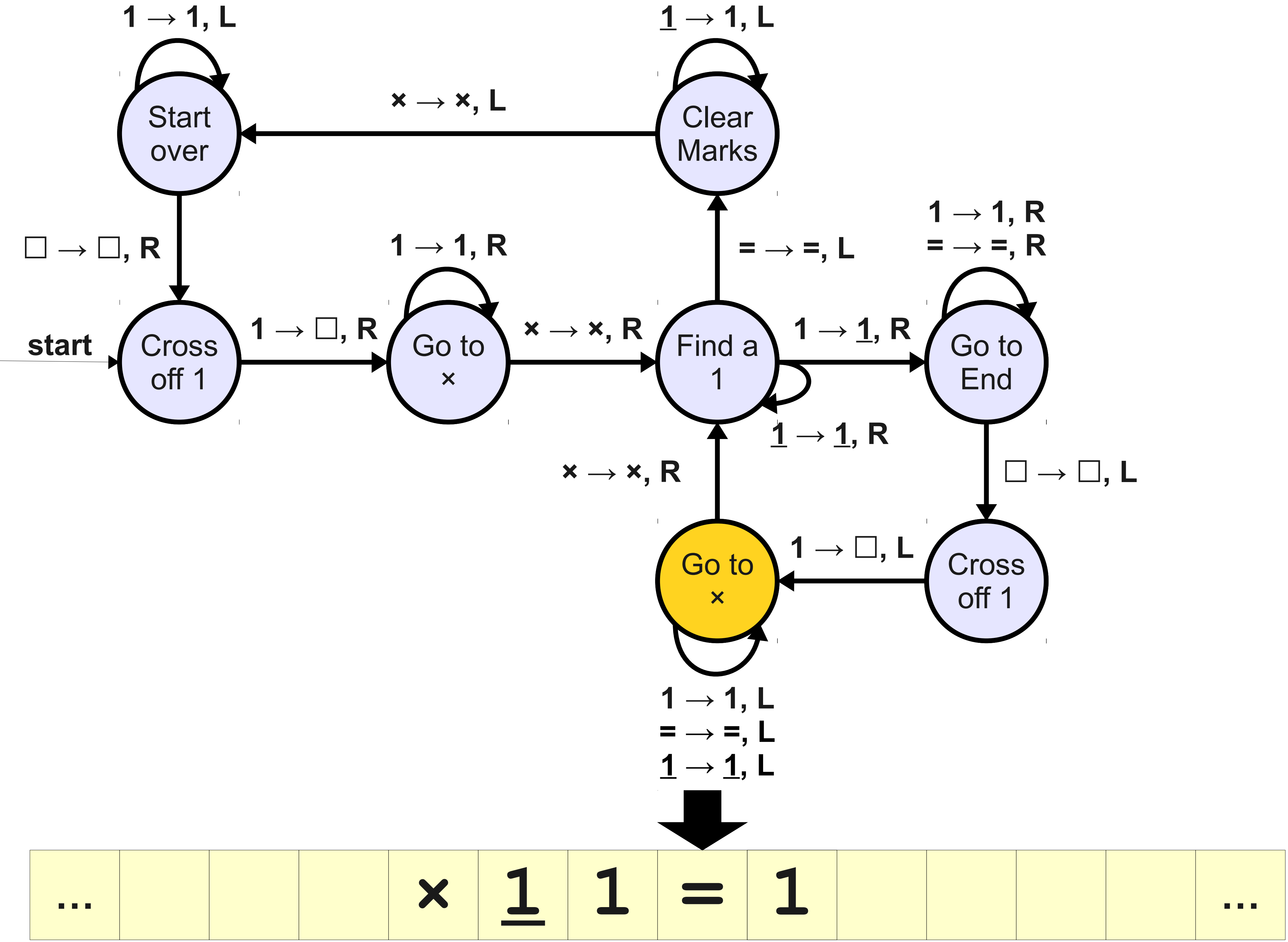
Start over
Clear Marks
Cross off 1
Go to ×
Find a 1
Go to End
Go to ×
Cross off 1
start
1 → 1, L
1 → 1, L
× → ×, L
□ → □, R
1 → 1, R
= → =, L
1 → 1, R
= → =, R
1 → □, R
× → ×, R
1 → 1, R
1 → 1, R
× → ×, R
□ → □, L
1 → □, L
1 → 1, L
= → =, L
1 → 1, L
…
×
1
1
=
1
1
…

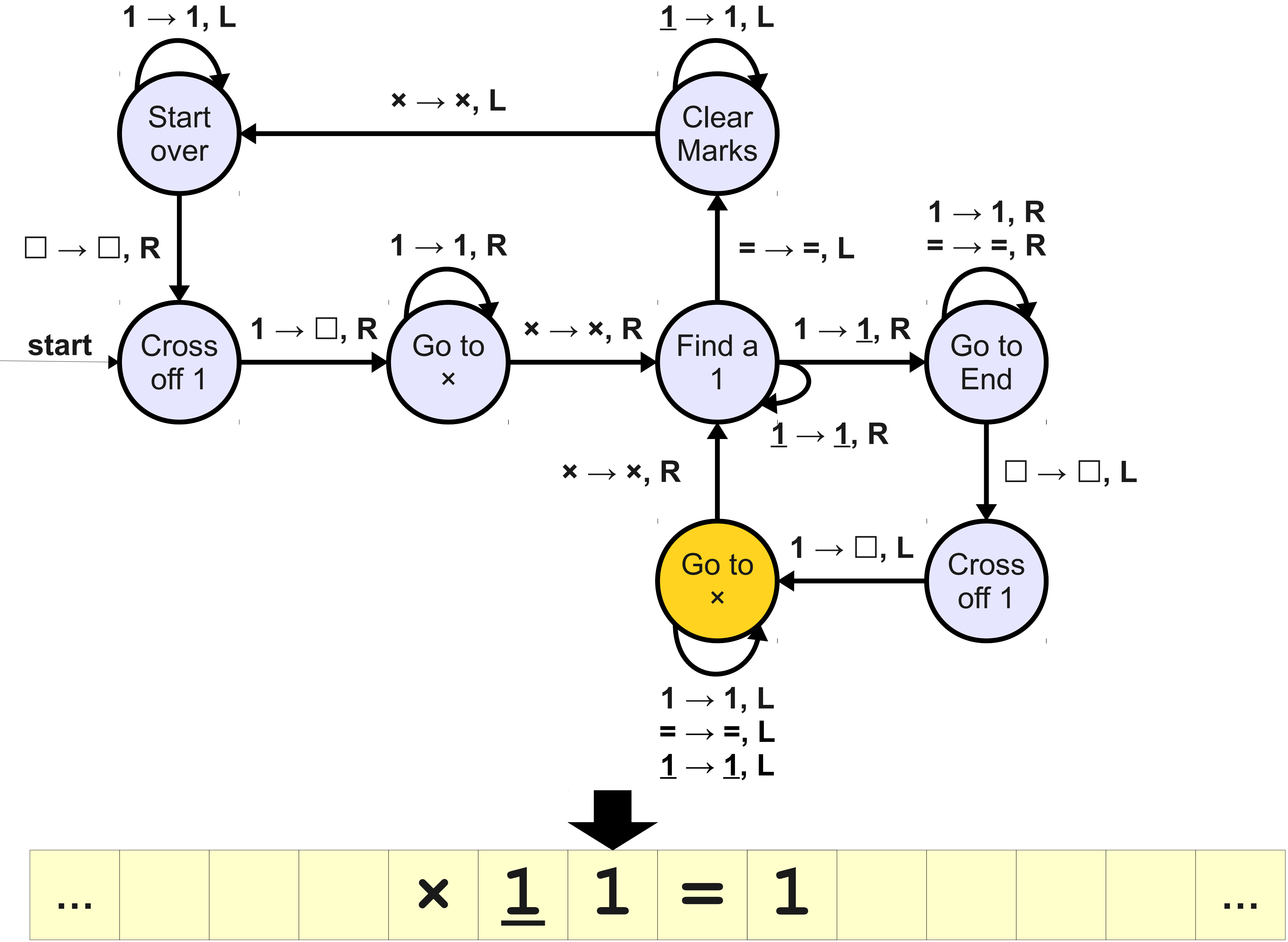

1 → 1, L
$\underline{1}$ → 1, L
× → ×, L
Start over
Clear Marks
□ → □, R
1 → 1, R
= → =, L
1 → 1, R
= → =, R
start
Cross off 1
1 → □, R
Go to ×
× → ×, R
Find a 1
1 → $\underline{1}$, R
Go to End
$\underline{1}$ → $\underline{1}$, R
× → ×, R
□ → □, L
Go to ×
1 → □, L
Cross off 1
1 → 1, L
= → =, L
$\underline{1}$ → $\underline{1}$, L
…
×
$\underline{1}$
1
=
1
1
…

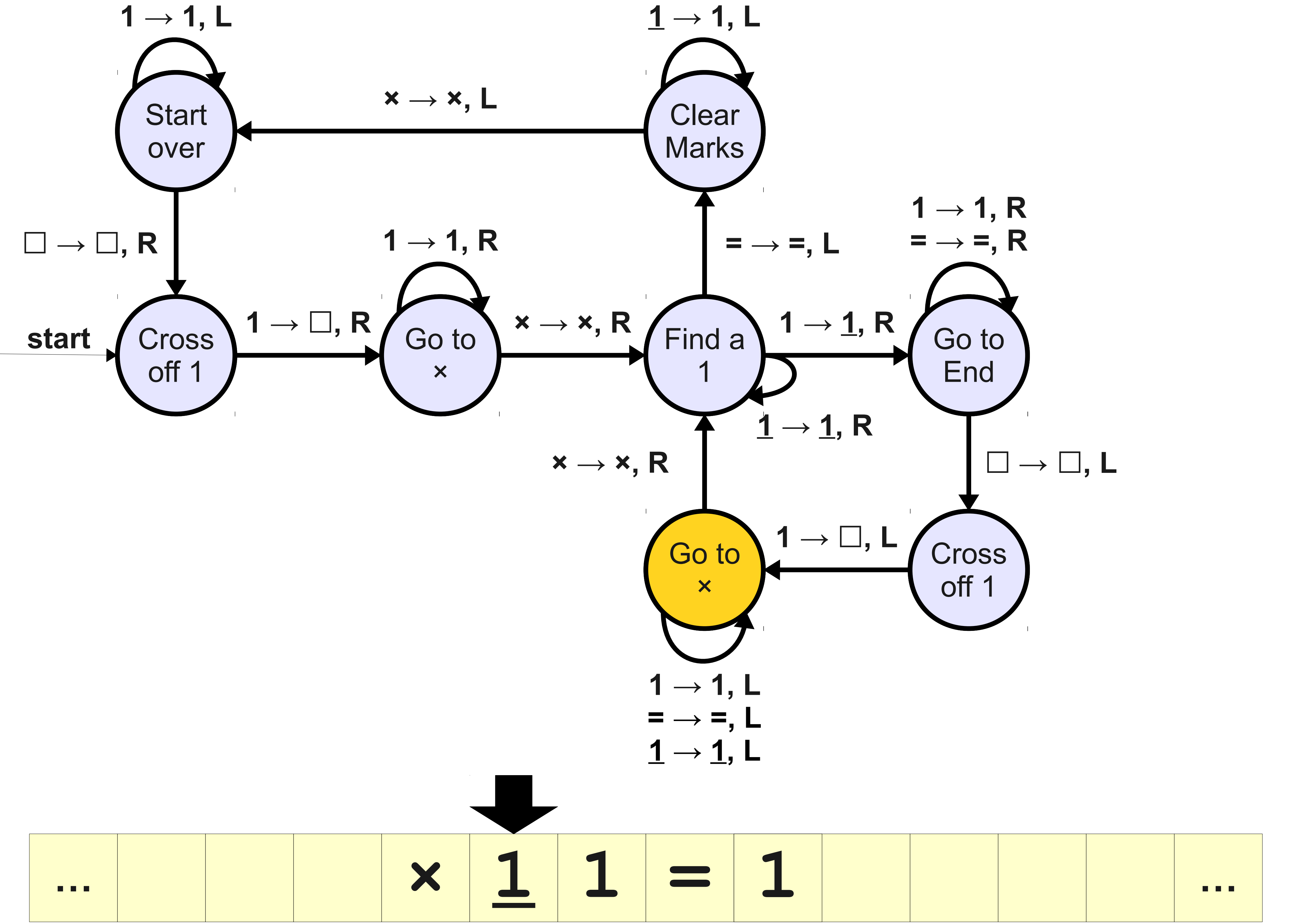


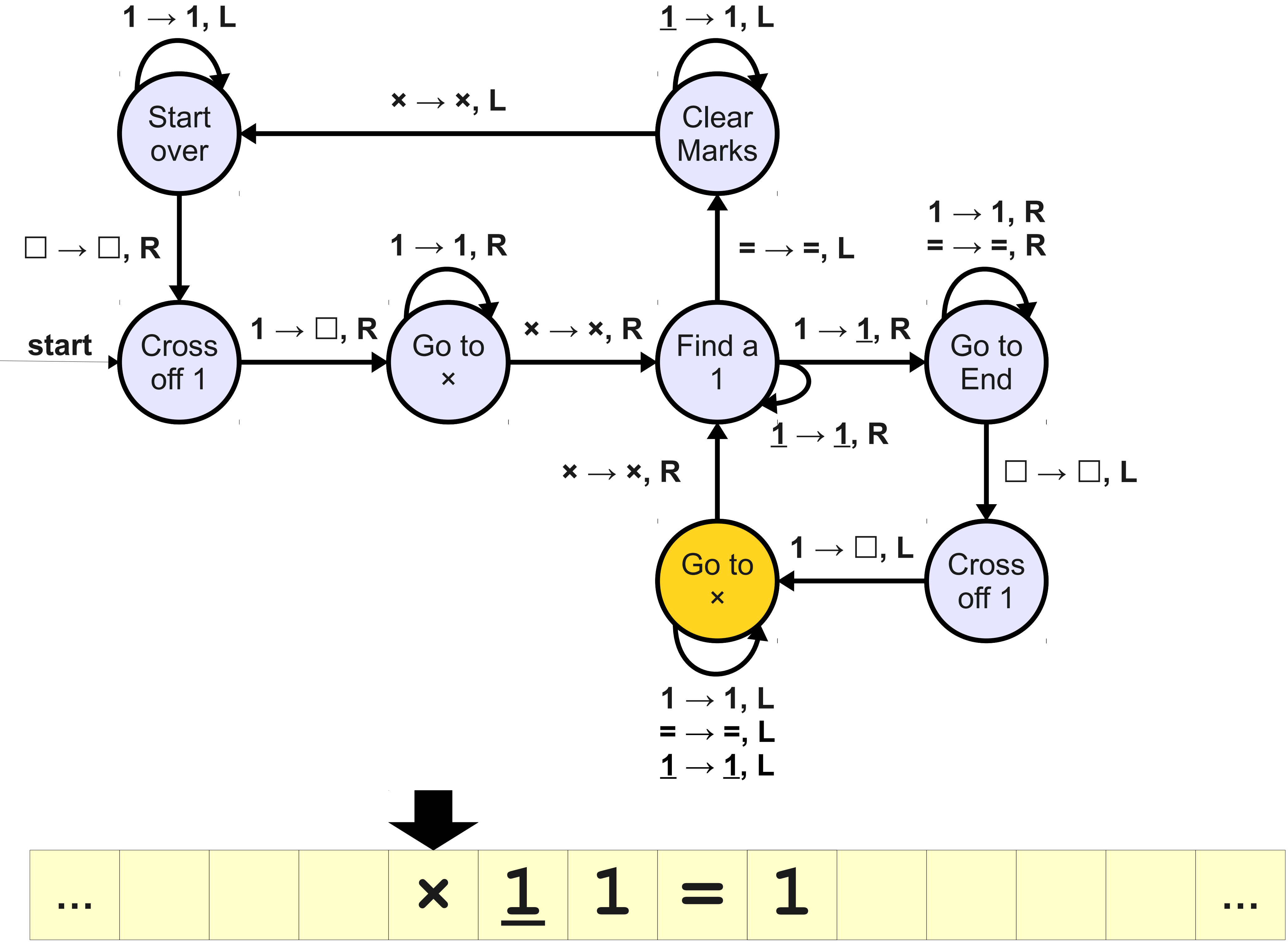
1 → 1, L
1 → 1, L
× → ×, L
Start over
Clear Marks
□ → □, R
1 → 1, R
= → =, L
1 → 1, R
= → =, R
start
Cross off 1
1 → □, R
Go to ×
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
× → ×, R
□ → □, L
Go to ×
1 → □, L
Cross off 1
1 → 1, L
= → =, L
1 → 1, L
…
×
1
1
=
1
1
…


start
Start over
Clear Marks
Cross off 1
Go to ×
Find a 1
Go to End
Go to ×
Cross off 1
1 → 1, L
1 → 1, L
× → ×, L
□ → □, R
1 → 1, R
= → =, L
1 → 1, R
= → =, R
1 → □, R
× → ×, R
1 → 1, R
1 → 1, R
× → ×, R
□ → □, L
1 → □, L
1 → 1, L
= → =, L
1 → 1, L
…
×
1
1
=
1
…

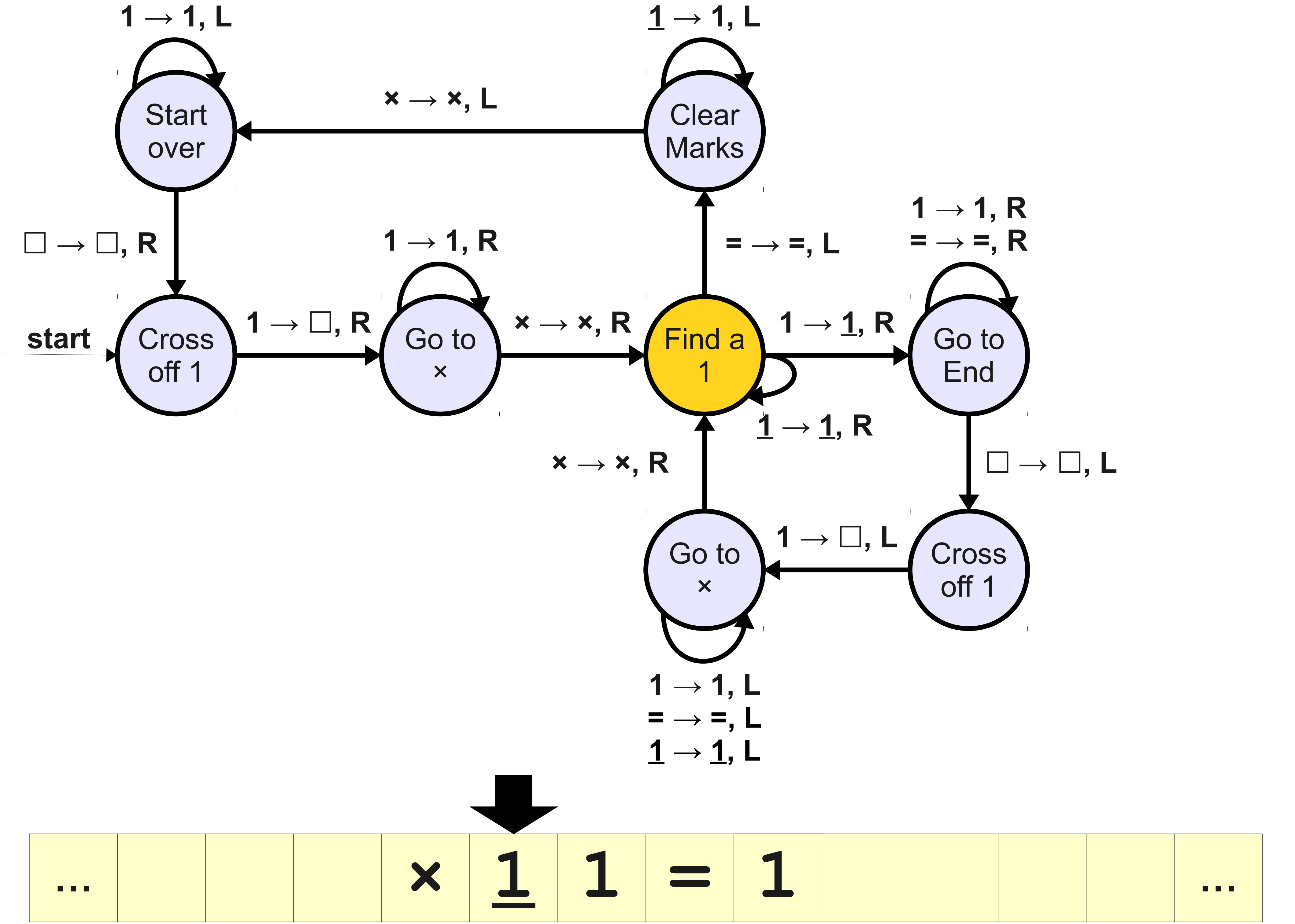

1 → 1, L
Start over
× → ×, L
1 → 1, L
Clear Marks
□ → □, R
1 → 1, R
= → =, L
1 → 1, R
= → =, R
start
Cross off 1
1 → □, R
Go to ×
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
× → ×, R
□ → □, L
Go to ×
1 → □, L
Cross off 1
1 → 1, L
= → =, L
1 → 1, L
...
×
1
1
=
1
...


1 → 1, L
Start over
1 → 1, L
Clear Marks
× → ×, L
□ → □, R
start
Cross off 1
1 → □, R
Go to ×
1 → 1, R
× → ×, R
Find a 1
= → =, L
1 → 1, R
Go to End
1 → 1, R
= → =, R
1 → 1, R
□ → □, L
× → ×, R
Go to ×
1 → □, L
Cross off 1
1 → 1, L
= → =, L
1 → 1, L
...
×
1
1
=
1
...

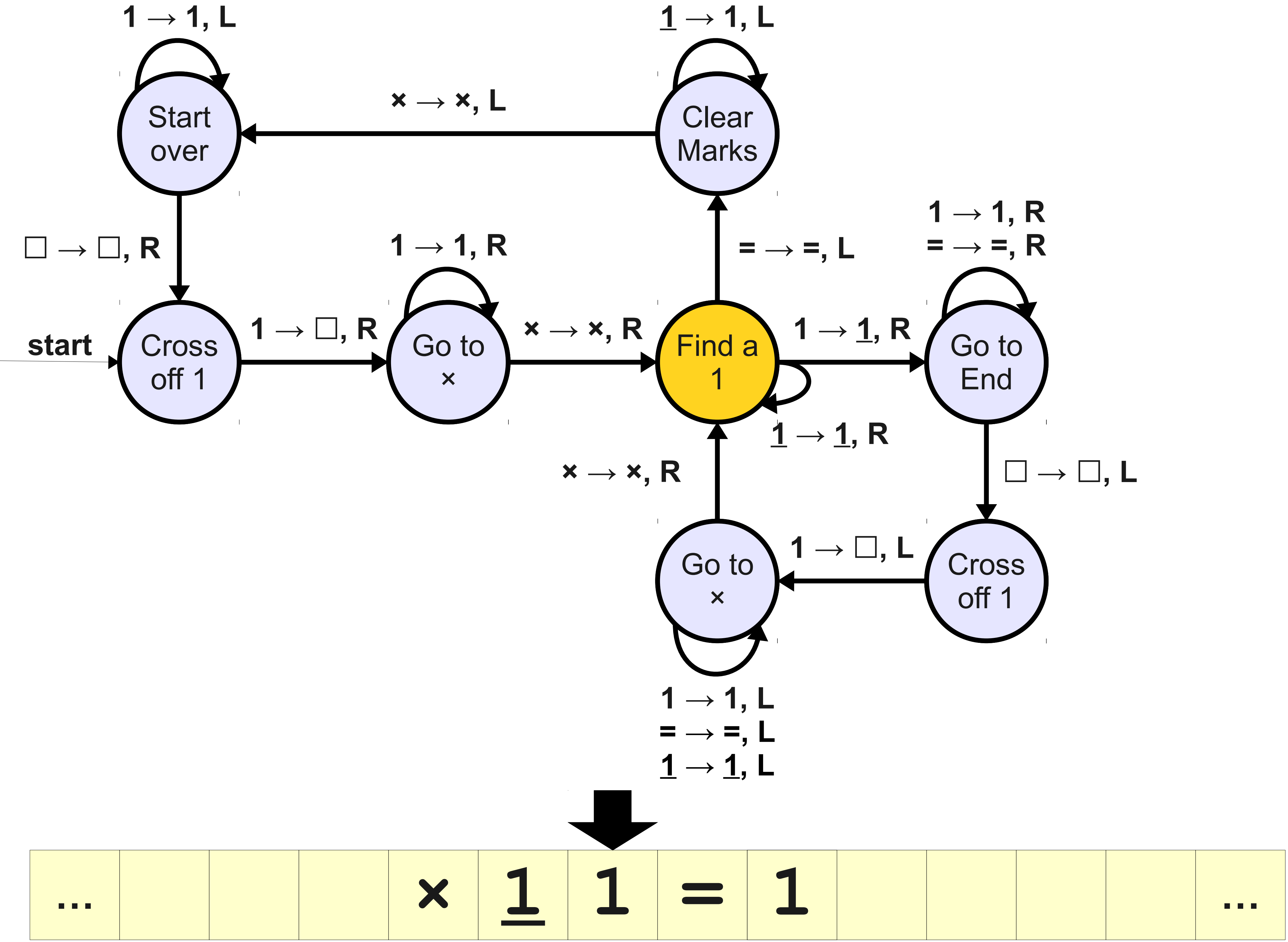

start
Start over
Clear Marks
Cross off 1
Go to ×
Find a 1
Go to End
Go to ×
Cross off 1
1 → 1, L
1 → 1, L
× → ×, L
□ → □, R
1 → 1, R
1 → □, R
× → ×, R
= → =, L
1 → 1, R
= → =, R
1 → 1, R
1 → 1, R
× → ×, R
□ → □, L
1 → □, L
1 → 1, L
= → =, L
1 → 1, L
...
×
1
1
=
1
...



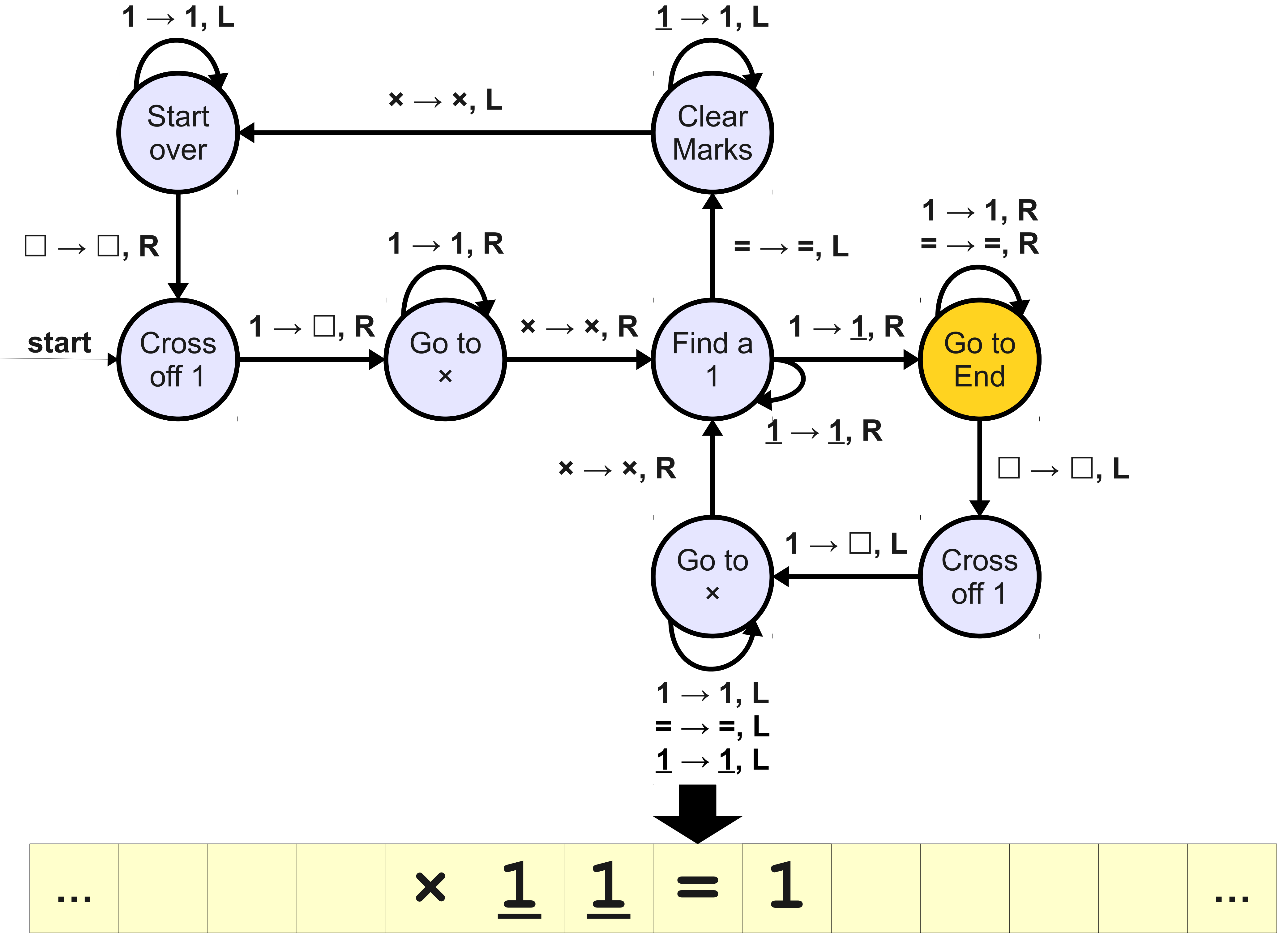
1 → 1, L
Start over
1 → 1, L
Clear Marks
× → ×, L
□ → □, R
start
Cross off 1
1 → □, R
1 → 1, R
Go to ×
× → ×, R
= → =, L
Find a 1
1 → 1, R
1 → 1, R
1 → 1, R
= → =, R
Go to End
× → ×, R
□ → □, L
Go to ×
1 → □, L
Cross off 1
1 → 1, L
= → =, L
1 → 1, L
… × 1 1 = 1 …

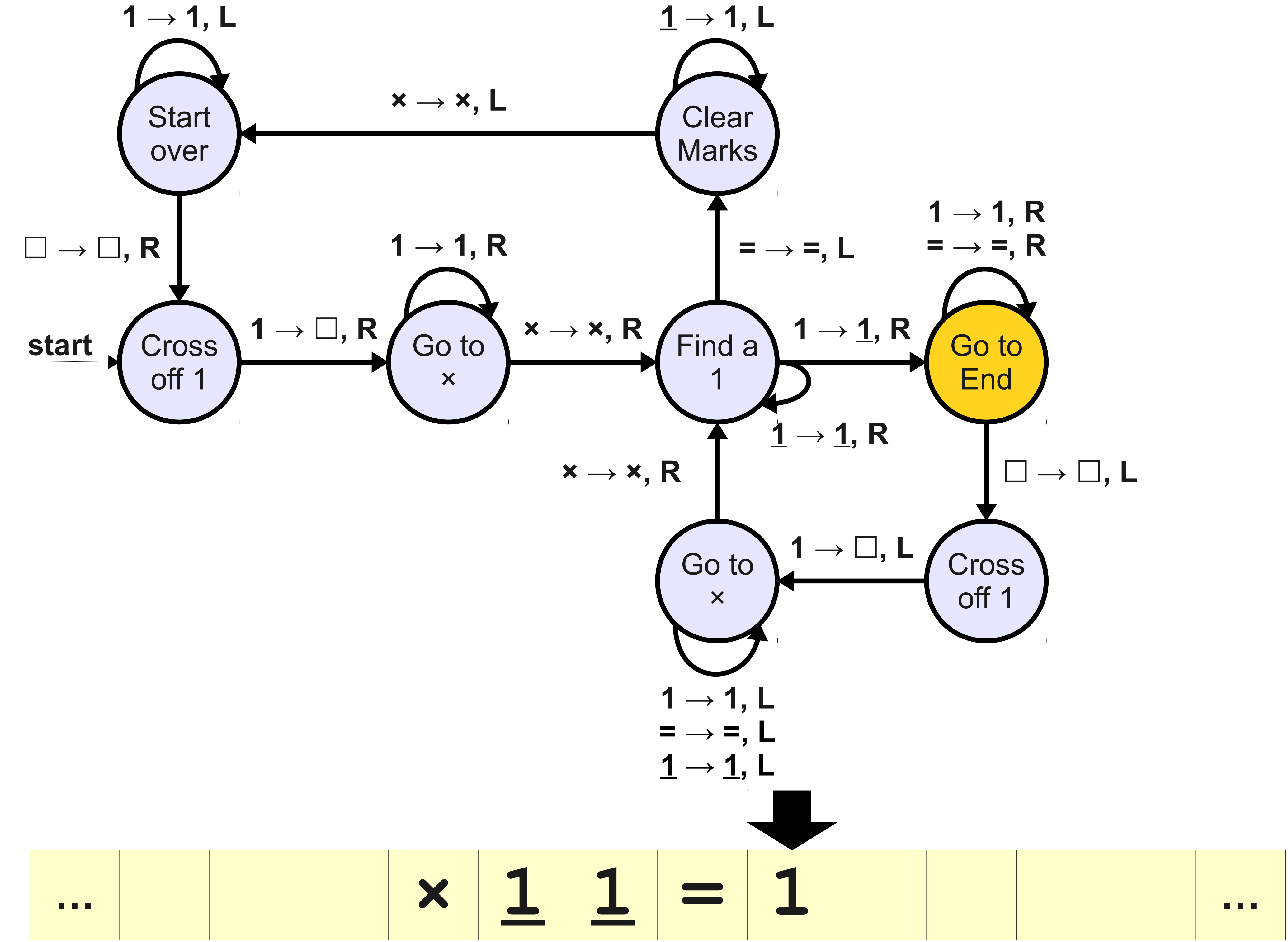


1 → 1, L
Start over
1 → 1, L
Clear Marks
× → ×, L
□ → □, R
1 → 1, R
1 → 1, R
= → =, R
= → =, L
start
Cross off 1
1 → □, R
Go to ×
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
× → ×, R
□ → □, L
Go to ×
1 → □, L
Cross off 1
1 → 1, L
= → =, L
1 → 1, L
...
×
1
1
=
1
...

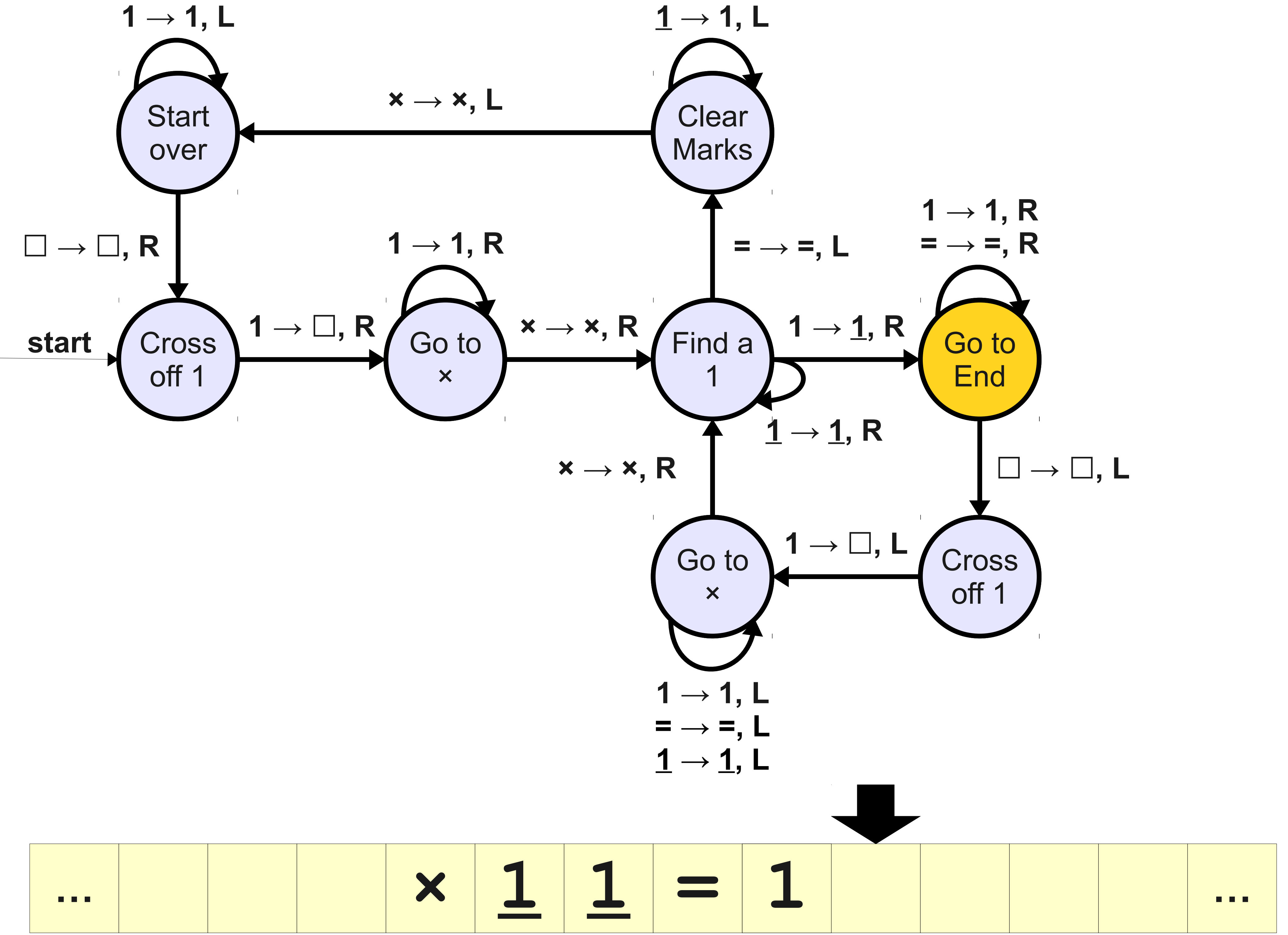


1 → 1, L
Start over
× → ×, L
1 → 1, L
Clear Marks
□ → □, R
start
Cross off 1
1 → □, R
1 → 1, R
Go to ×
× → ×, R
= → =, L
Find a 1
1 → 1, R
1 → 1, R
= → =, R
Go to End
1 → 1, R
× → ×, R
□ → □, L
1 → □, L
Go to ×
Cross off 1
1 → 1, L
= → =, L
1 → 1, L
… × 1 1 = 1 …

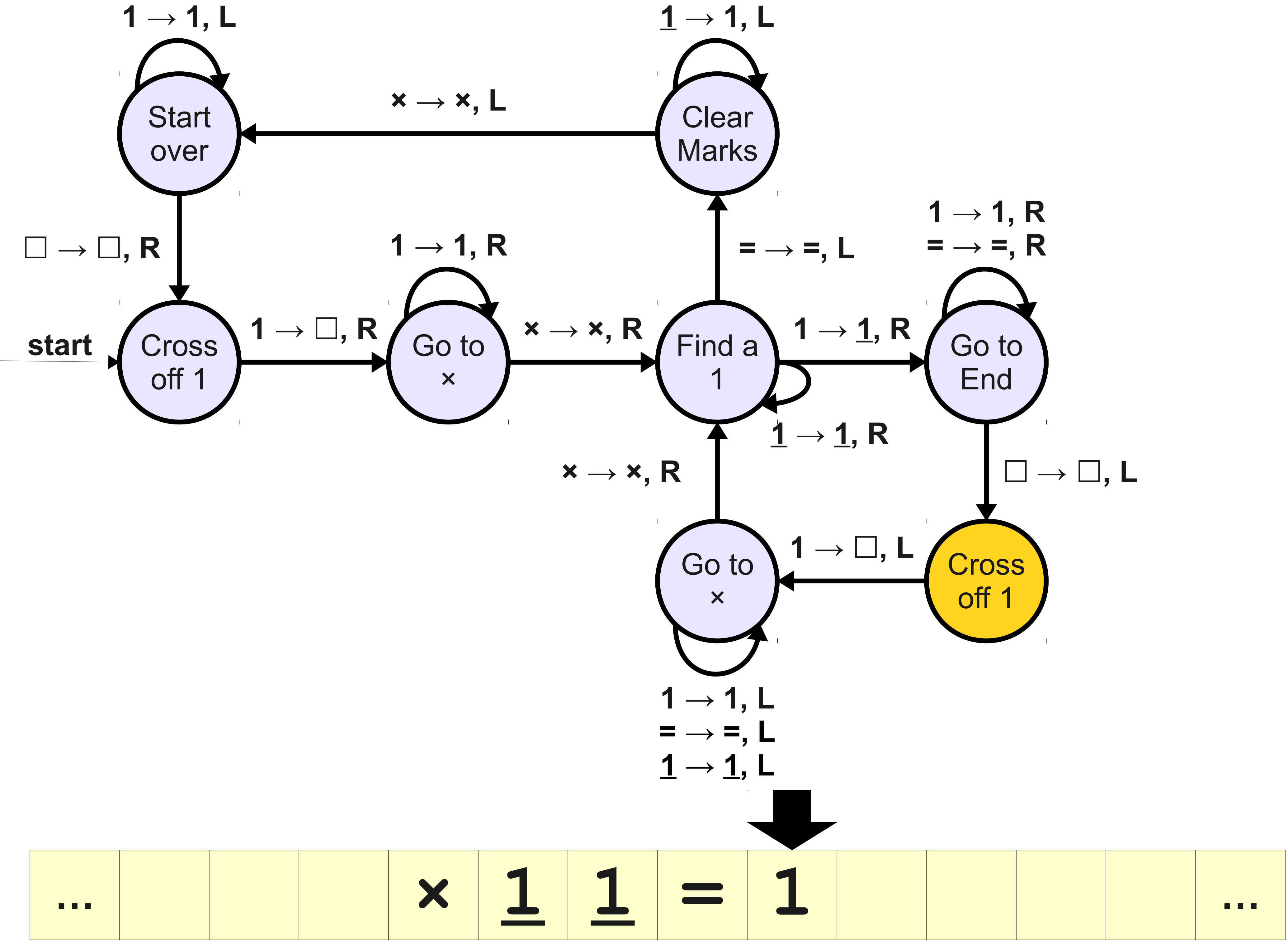

1 → 1, L
Start over
□ → □, R
1 → 1, L
Clear Marks
× → ×, L
= → =, L
1 → 1, R
= → =, R
start
Cross off 1
1 → □, R
1 → 1, R
Go to ×
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
□ → □, L
× → ×, R
Go to ×
1 → □, L
Cross off 1
1 → 1, L
= → =, L
1 → 1, L
… × 1 1 = 1 …



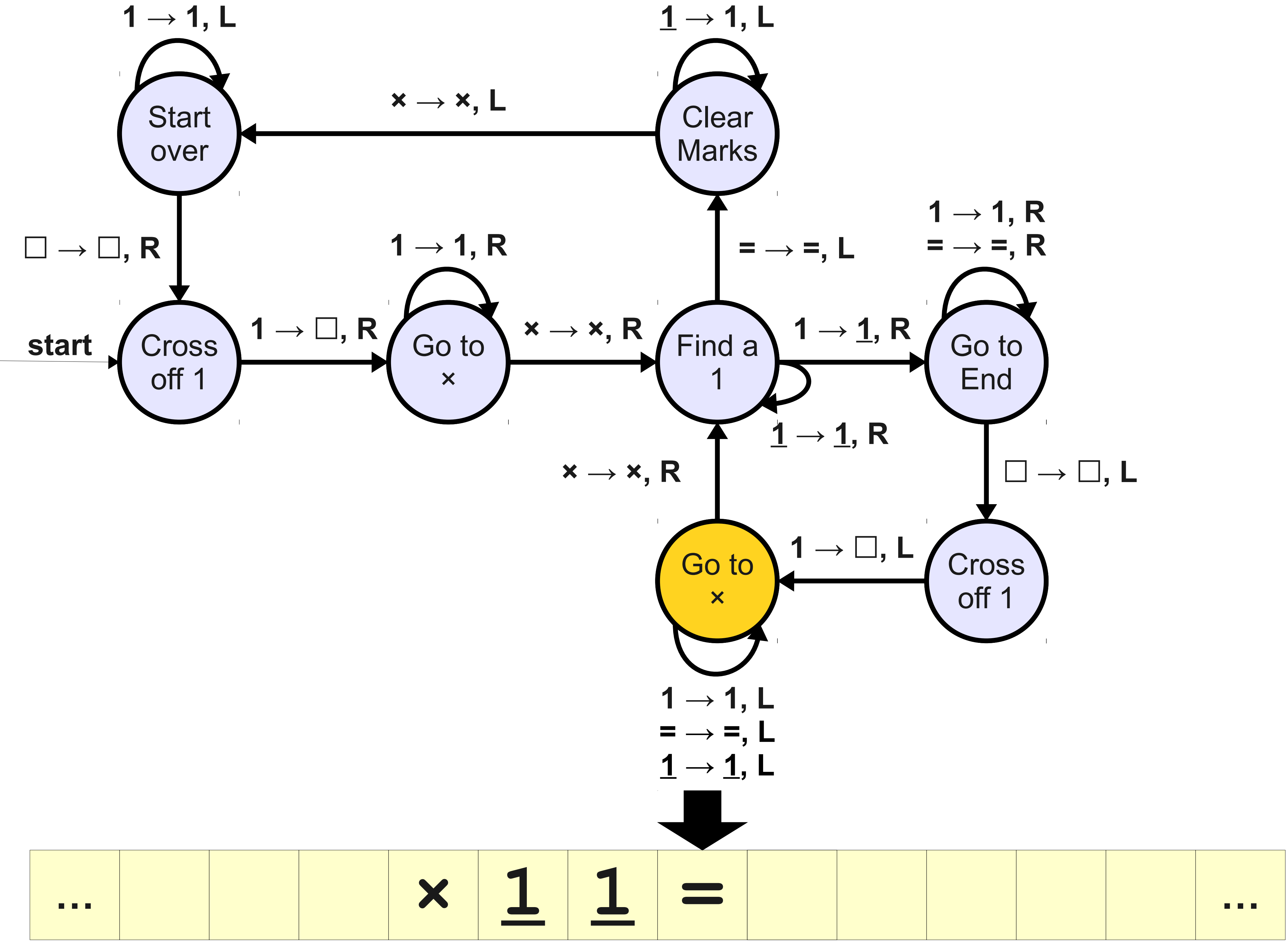
1 → 1, L
Start over
1 → 1, L
Clear Marks
× → ×, L
□ → □, R
1 → 1, R
1 → 1, R
= → =, R
= → =, L
start
Cross off 1
1 → □, R
Go to ×
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
× → ×, R
□ → □, L
Go to ×
1 → □, L
Cross off 1
1 → 1, L
= → =, L
1 → 1, L
…
×
1
1
=
1
…



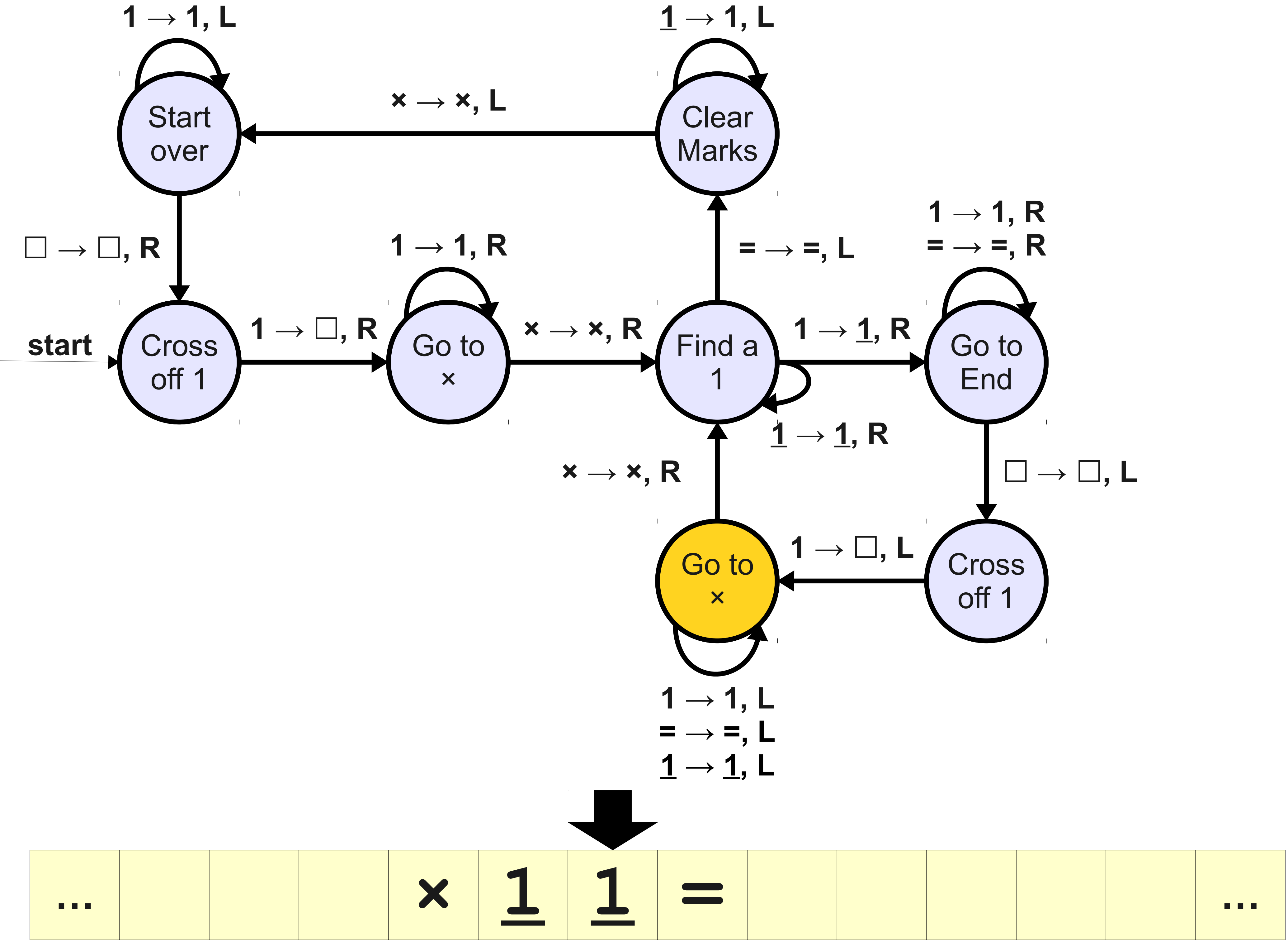
1 → 1, L
Start over
1 → 1, L
Clear Marks
× → ×, L
□ → □, R
1 → 1, R
= → =, L
1 → 1, R
= → =, R
start
Cross off 1
1 → □, R
Go to ×
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
× → ×, R
□ → □, L
Go to ×
1 → □, L
Cross off 1
1 → 1, L
= → =, L
1 → 1, L
…
×
1
1
=
1
…



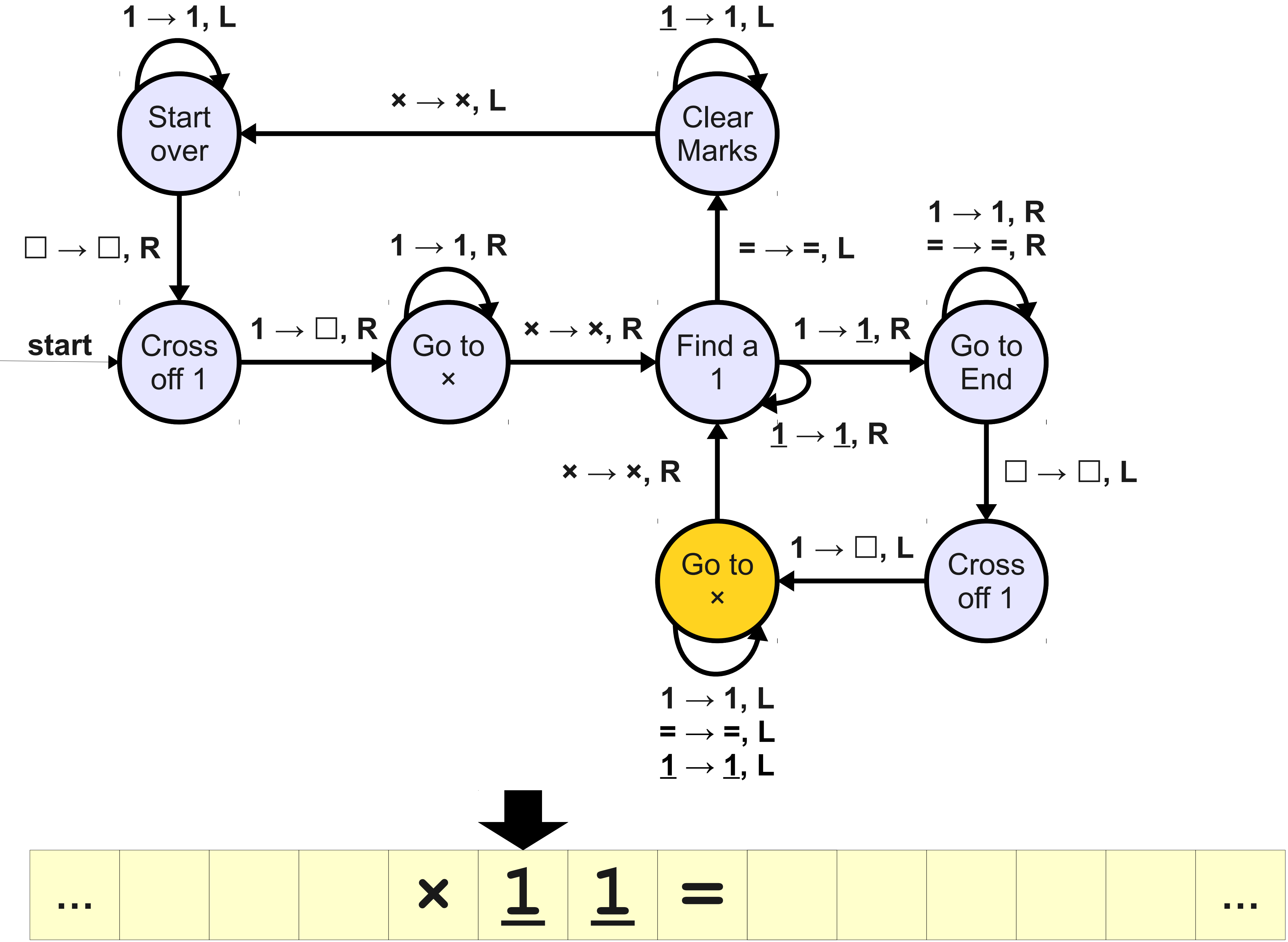
1 → 1, L
Start over
× → ×, L
1 → 1, L
Clear Marks
□ → □, R
1 → 1, R
= → =, L
1 → 1, R
= → =, R
start
Cross off 1
1 → □, R
Go to ×
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
× → ×, R
□ → □, L
Go to ×
1 → □, L
Cross off 1
1 → 1, L
= → =, L
1 → 1, L
... × 1 1 = 1 ...



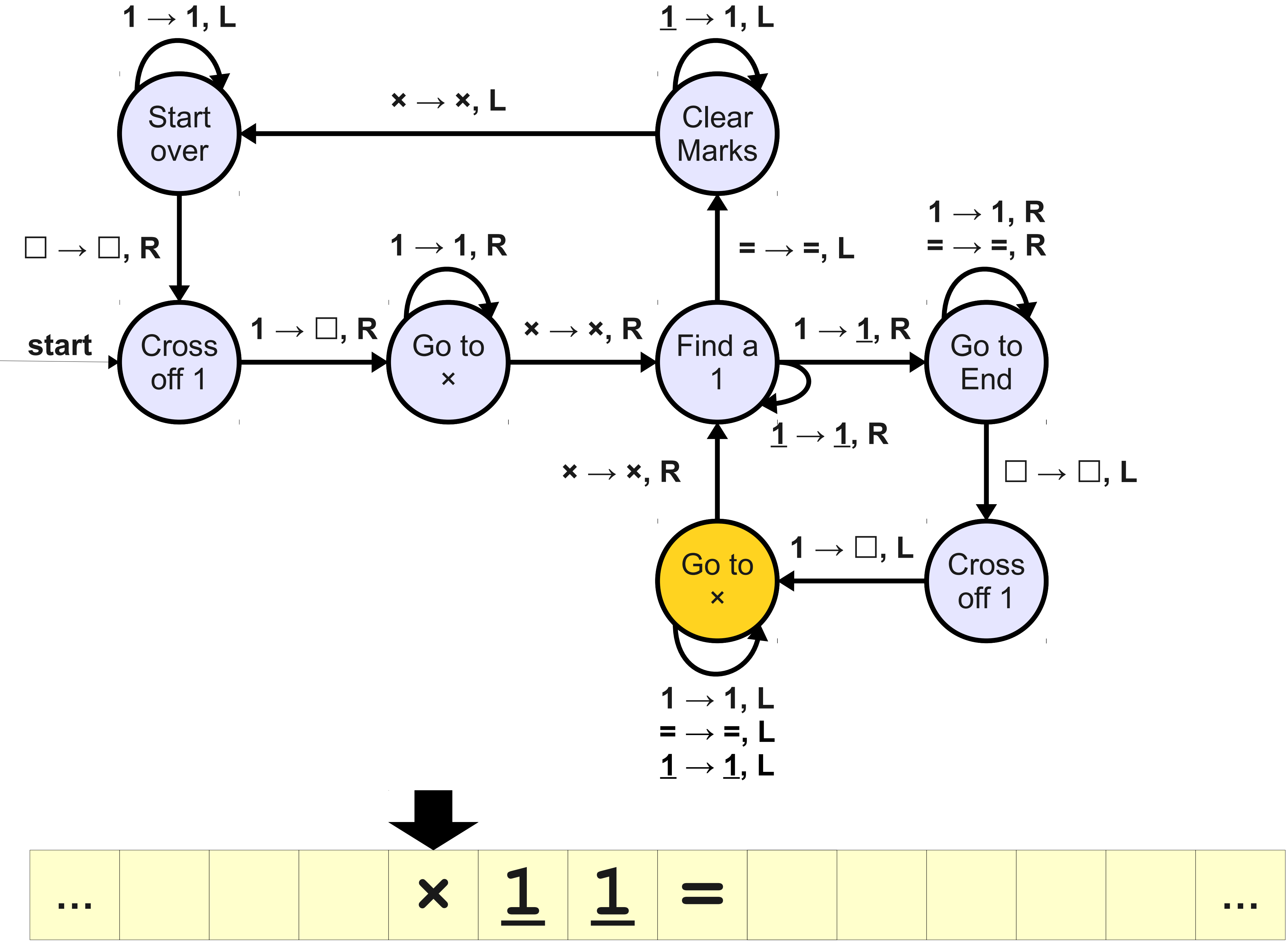
1 → 1, L
Start over
× → ×, L
1 → 1, L
Clear Marks
□ → □, R
1 → 1, R
= → =, L
1 → 1, R
= → =, R
start
Cross off 1
1 → □, R
Go to ×
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
× → ×, R
□ → □, L
1 → □, L
Go to ×
Cross off 1
1 → 1, L
= → =, L
1 → 1, L
... × 1 1 = ...

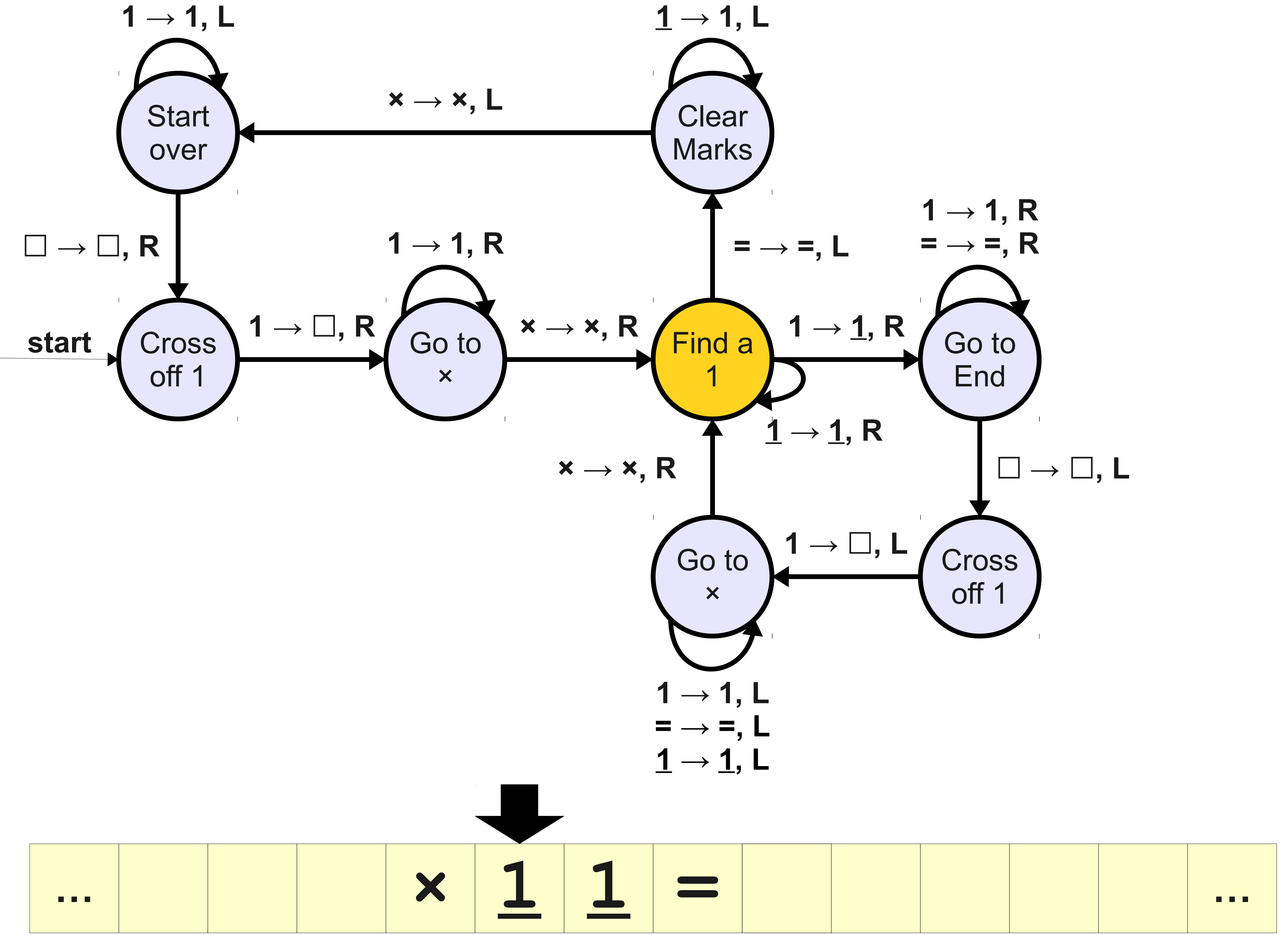


1 → 1, L
Start over
× → ×, L
1 → 1, L
Clear Marks
□ → □, R
1 → 1, R
= → =, L
1 → 1, R
= → =, R
start
Cross off 1
1 → □, R
Go to ×
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
× → ×, R
□ → □, L
Go to ×
1 → □, L
Cross off 1
1 → 1, L
= → =, L
1 → 1, L
...
×
1
1
=
...

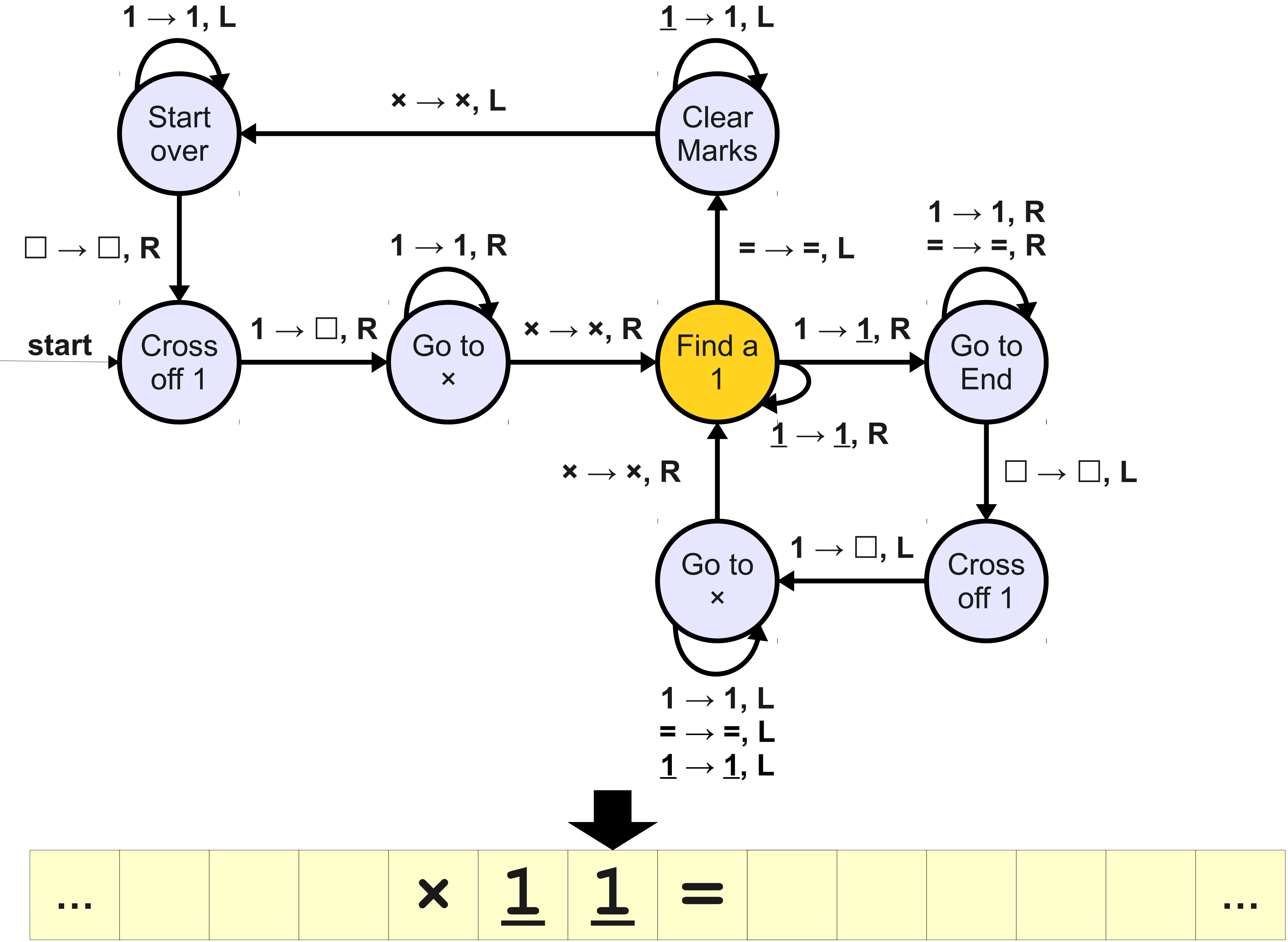


Start over
Clear Marks
Cross off 1
Go to ×
Find a 1
Go to End
Go to ×
Cross off 1
start
1 → 1, L
1 → 1, L
× → ×, L
□ → □, R
1 → 1, R
1 → 1, R
= → =, R
= → =, L
1 → □, R
× → ×, R
1 → 1, R
1 → 1, R
× → ×, R
□ → □, L
1 → □, L
1 → 1, L
= → =, L
1 → 1, L
… × 1 1 = …

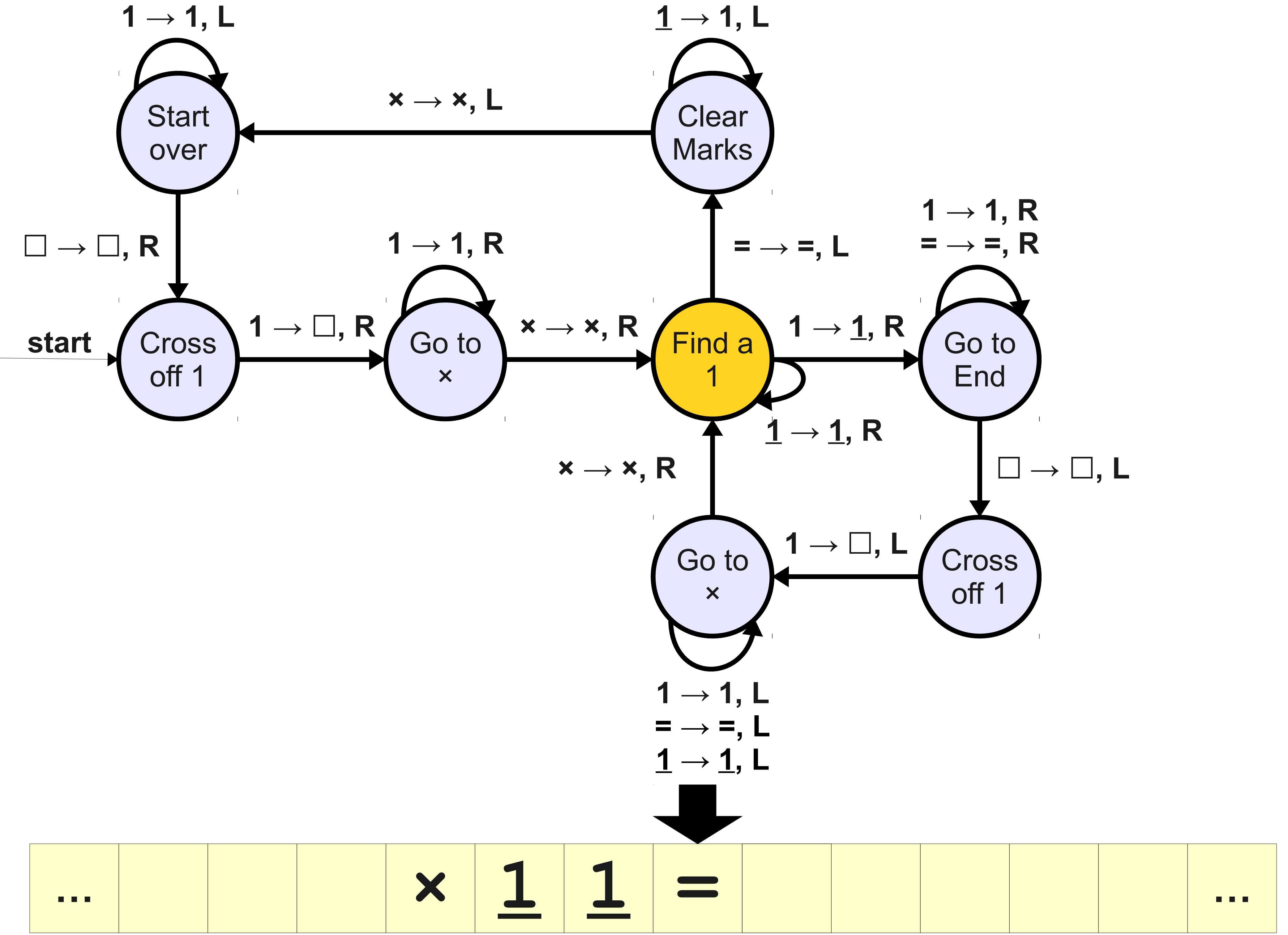

1 → 1, L
1 → 1, L
× → ×, L
Start over
Clear Marks
□ → □, R
1 → 1, R
= → =, L
1 → 1, R
= → =, R
start
Cross off 1
1 → □, R
Go to ×
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
× → ×, R
□ → □, L
Go to ×
1 → □, L
Cross off 1
1 → 1, L
= → =, L
1 → 1, L
…
×
1
1
=
…



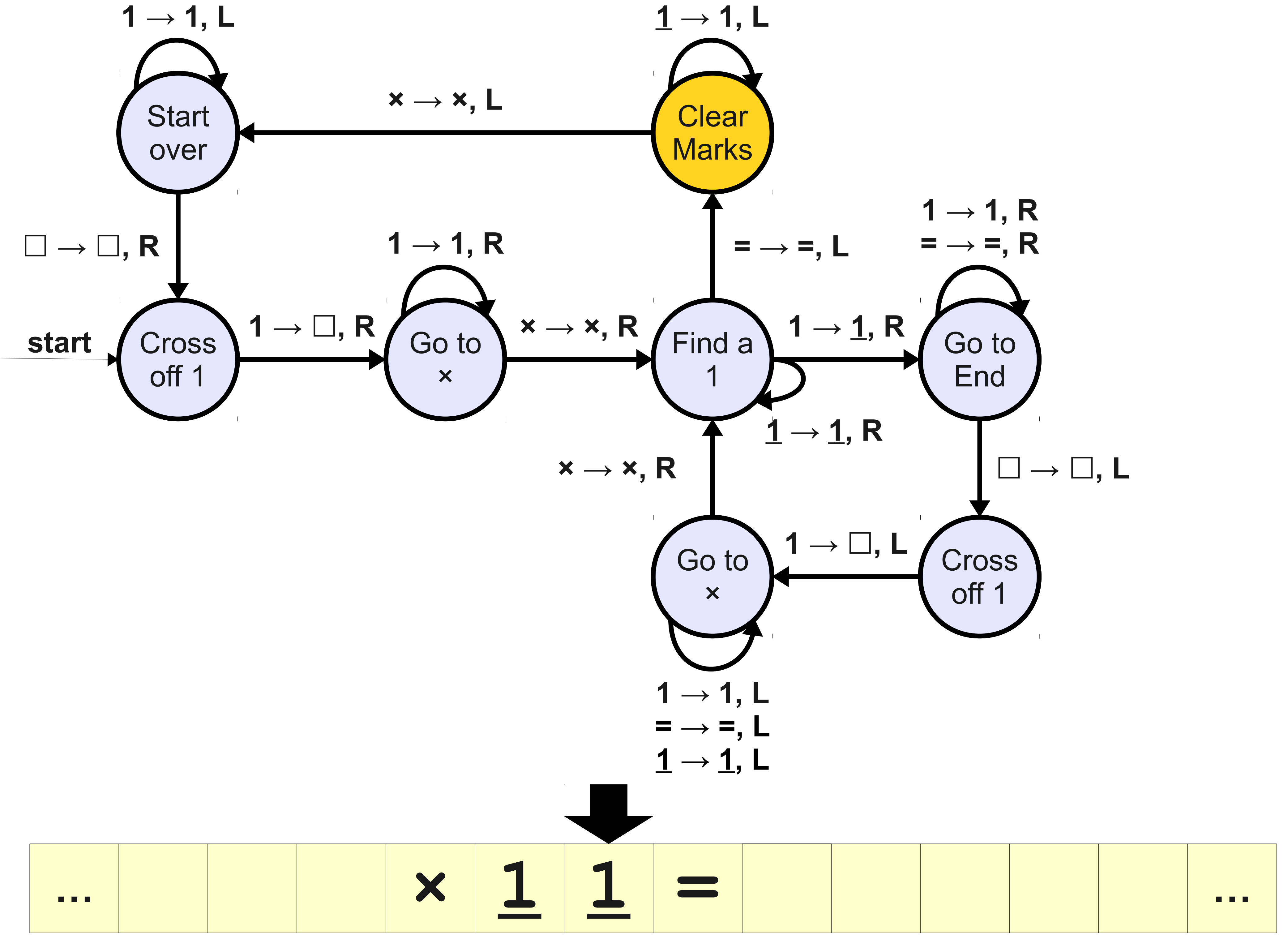
Start over
Clear Marks
Cross off 1
Go to ×
Find a 1
Go to End
Cross off 1
Go to ×
start
1 → 1, L
1 → 1, L
× → ×, L
□ → □, R
1 → 1, R
= → =, L
1 → 1, R
= → =, R
1 → □, R
× → ×, R
1 → 1, R
1 → 1, R
× → ×, R
□ → □, L
1 → □, L
1 → 1, L
= → =, L
1 → 1, L
… × 1 1 = …



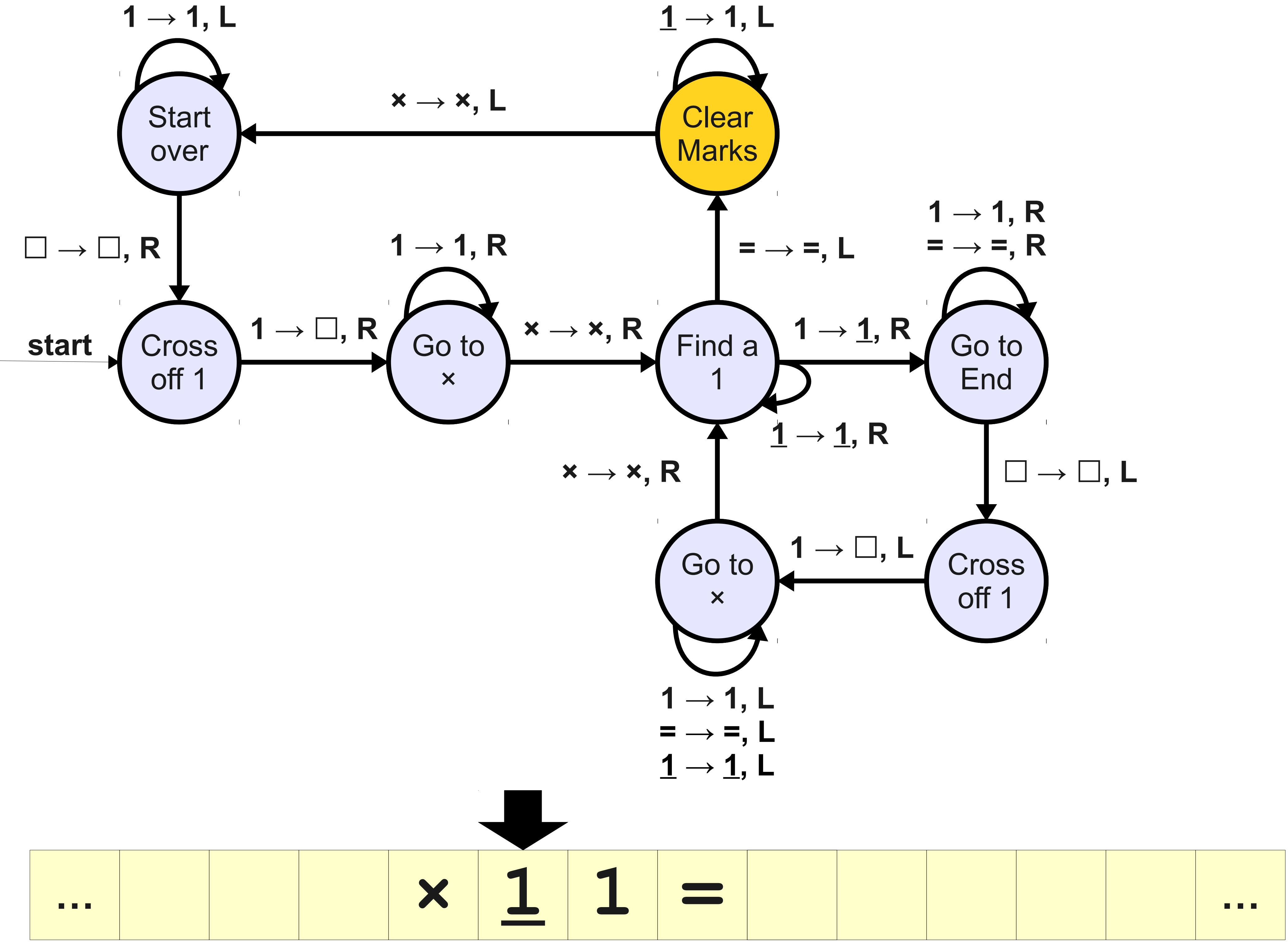
1 → 1, L
Start over
× → ×, L
1 → 1, L
Clear Marks
□ → □, R
1 → 1, R
= → =, L
1 → 1, R
= → =, R
start
Cross off 1
1 → □, R
Go to ×
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
× → ×, R
□ → □, L
1 → □, L
Go to ×
Cross off 1
1 → 1, L
= → =, L
1 → 1, L
...
×
1
1
=
...

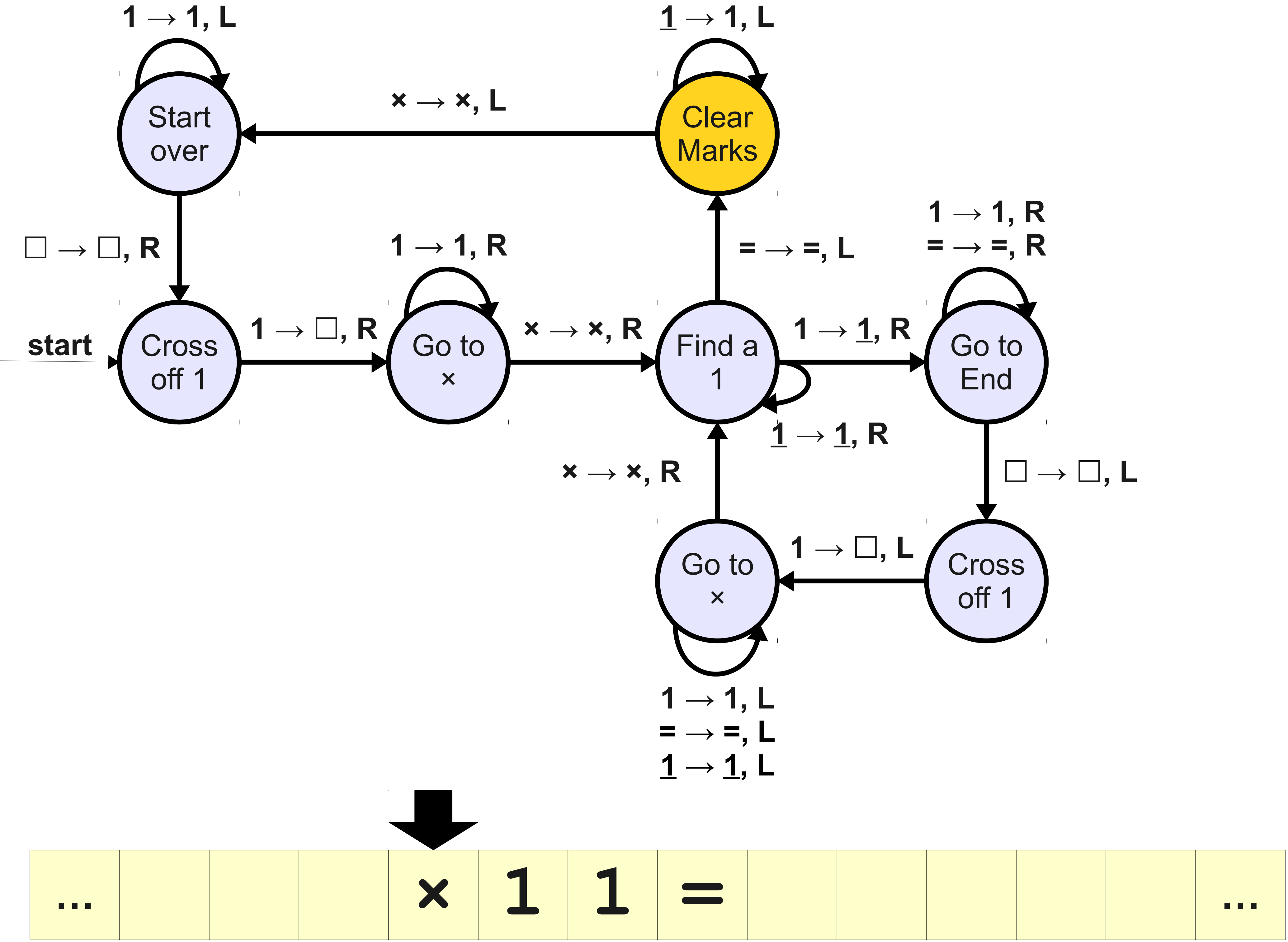


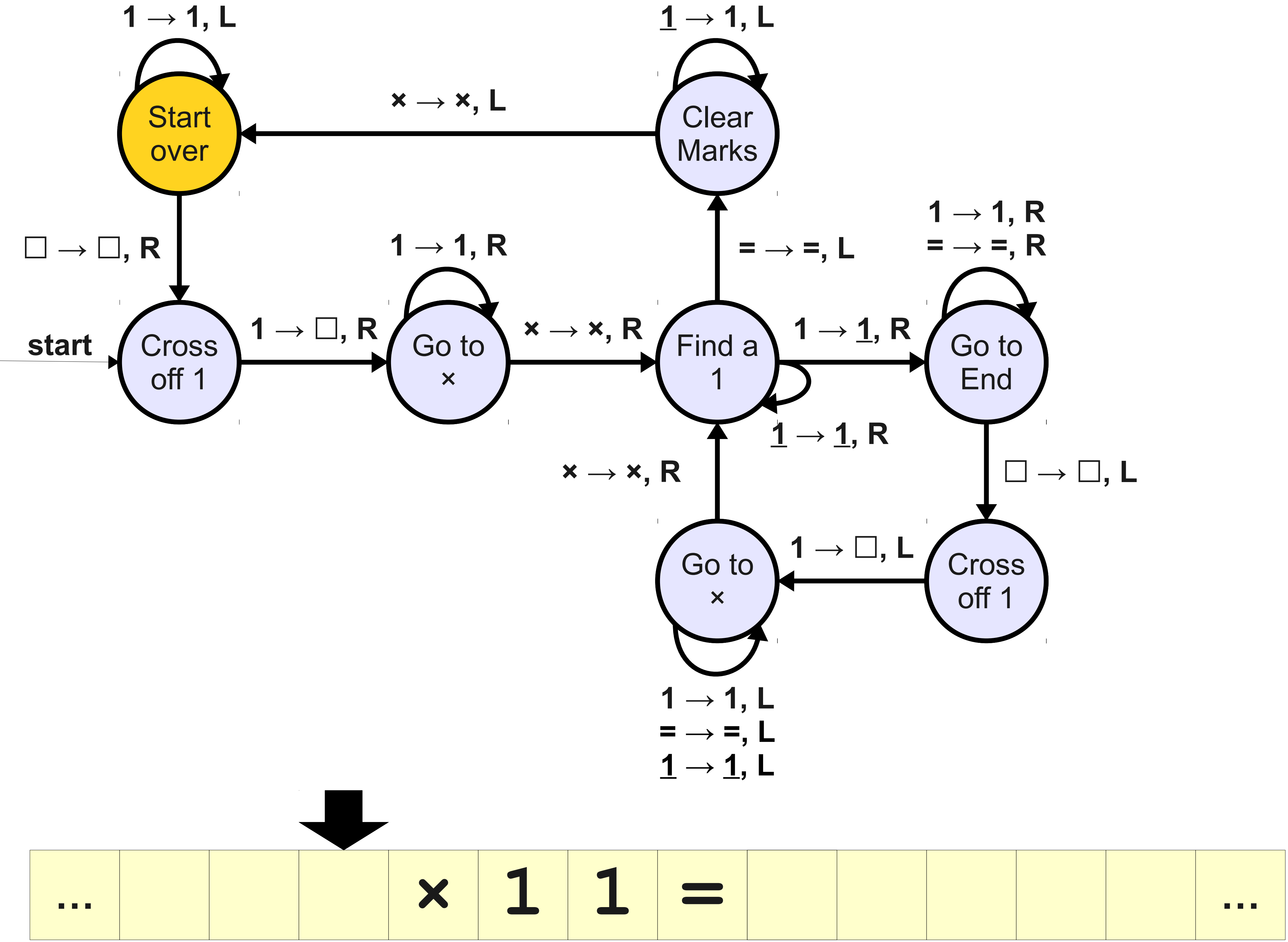
1 → 1, L
Start over
× → ×, L
1 → 1, L
Clear Marks
□ → □, R
1 → 1, R
= → =, L
1 → 1, R
= → =, R
start
Cross off 1
1 → □, R
Go to ×
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
× → ×, R
□ → □, L
Go to ×
1 → □, L
Cross off 1
1 → 1, L
= → =, L
1 → 1, L
…
×
1
1
=
…



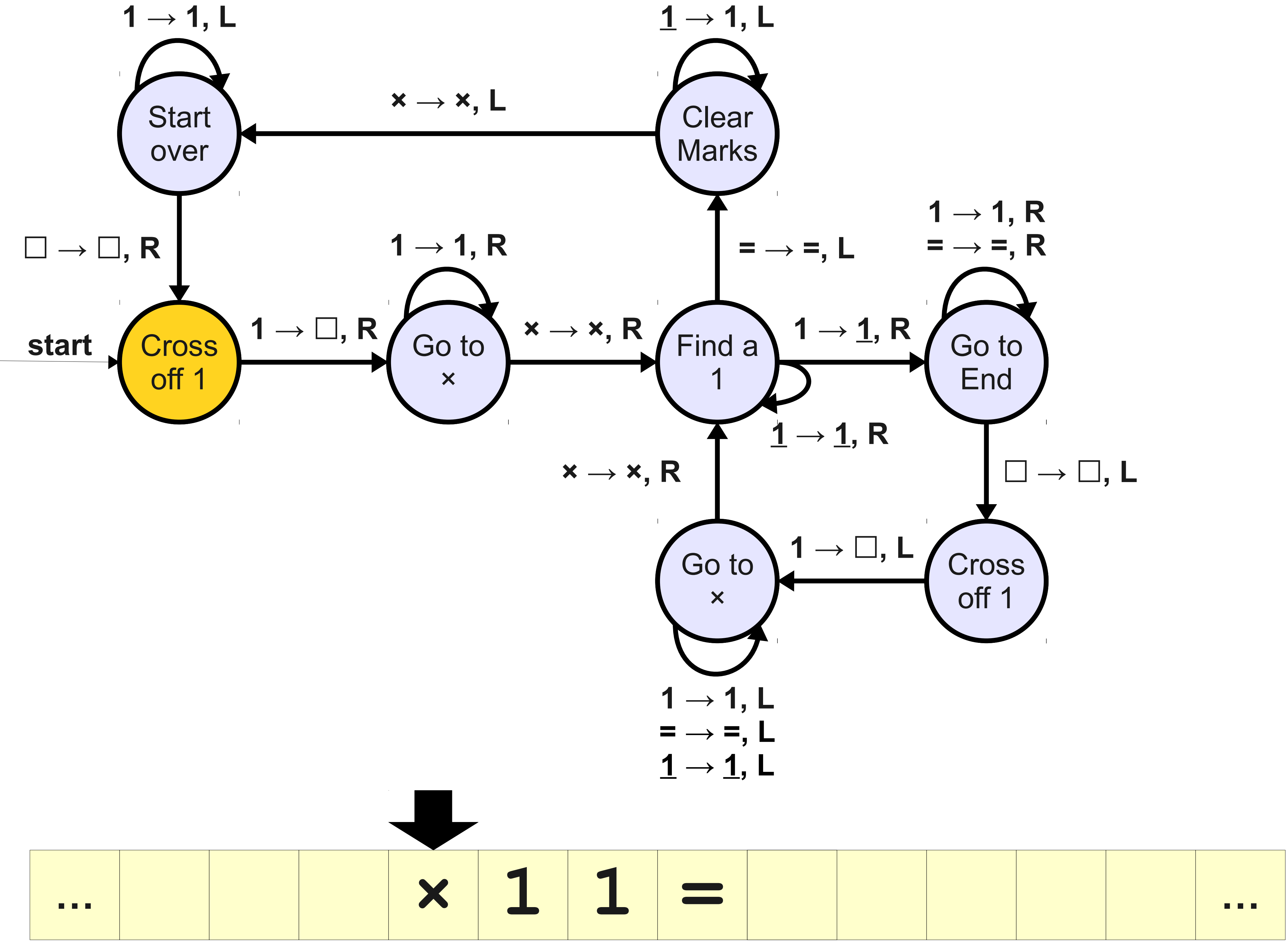
Start over
Clear Marks
Cross off 1
Go to ×
Find a 1
Go to End
Go to ×
Cross off 1
start
1 → 1, L
1 → 1, L
× → ×, L
□ → □, R
1 → 1, R
= → =, L
1 → 1, R
= → =, R
1 → □, R
× → ×, R
1 → 1, R
1 → 1, R
× → ×, R
□ → □, L
1 → □, L
1 → 1, L
= → =, L
1 → 1, L
…
×
1
1
=
…


1 → 1, L
1 → 1, L
× → ×, L
Start over
Clear Marks
□ → □, R
1 → 1, R
= → =, L
1 → 1, R
= → =, R
start
Cross off 1
1 → □, R
Go to ×
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
× → ×, R
□ → □, L
Go to ×
1 → □, L
Cross off 1
1 → 1, L
= → =, L
1 → 1, L
...
×
1
1
=
...

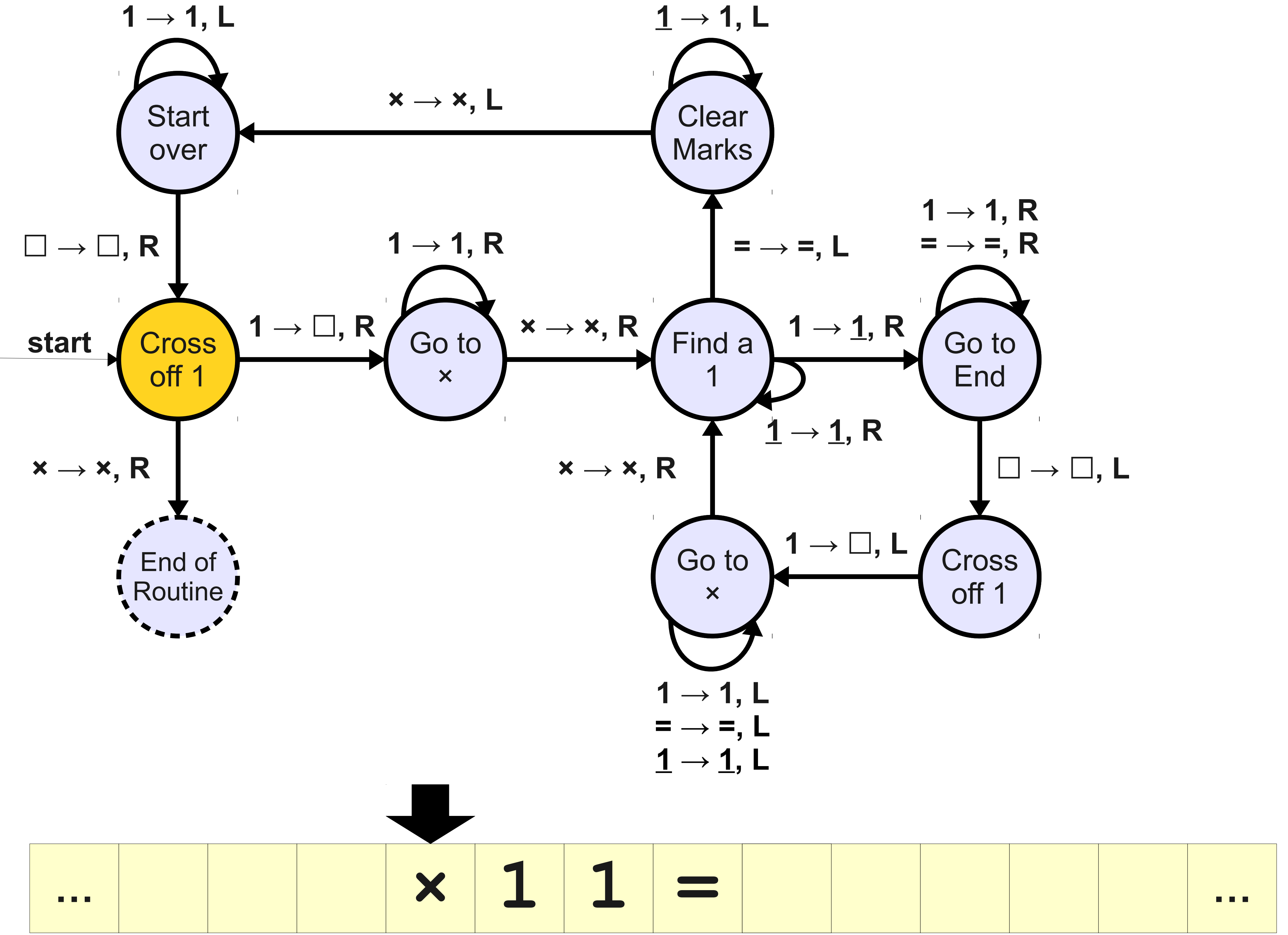

1 → 1, L
1 → 1, L
× → ×, L
Start over
Clear Marks
□ → □, R
1 → 1, R
= → =, L
1 → 1, R
= → =, R
start
Cross off 1
1 → □, R
Go to ×
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
× → ×, R
□ → □, L
Go to ×
1 → □, L
Cross off 1
1 → 1, L
= → =, L
1 → 1, L
…
×
1
1
=
…



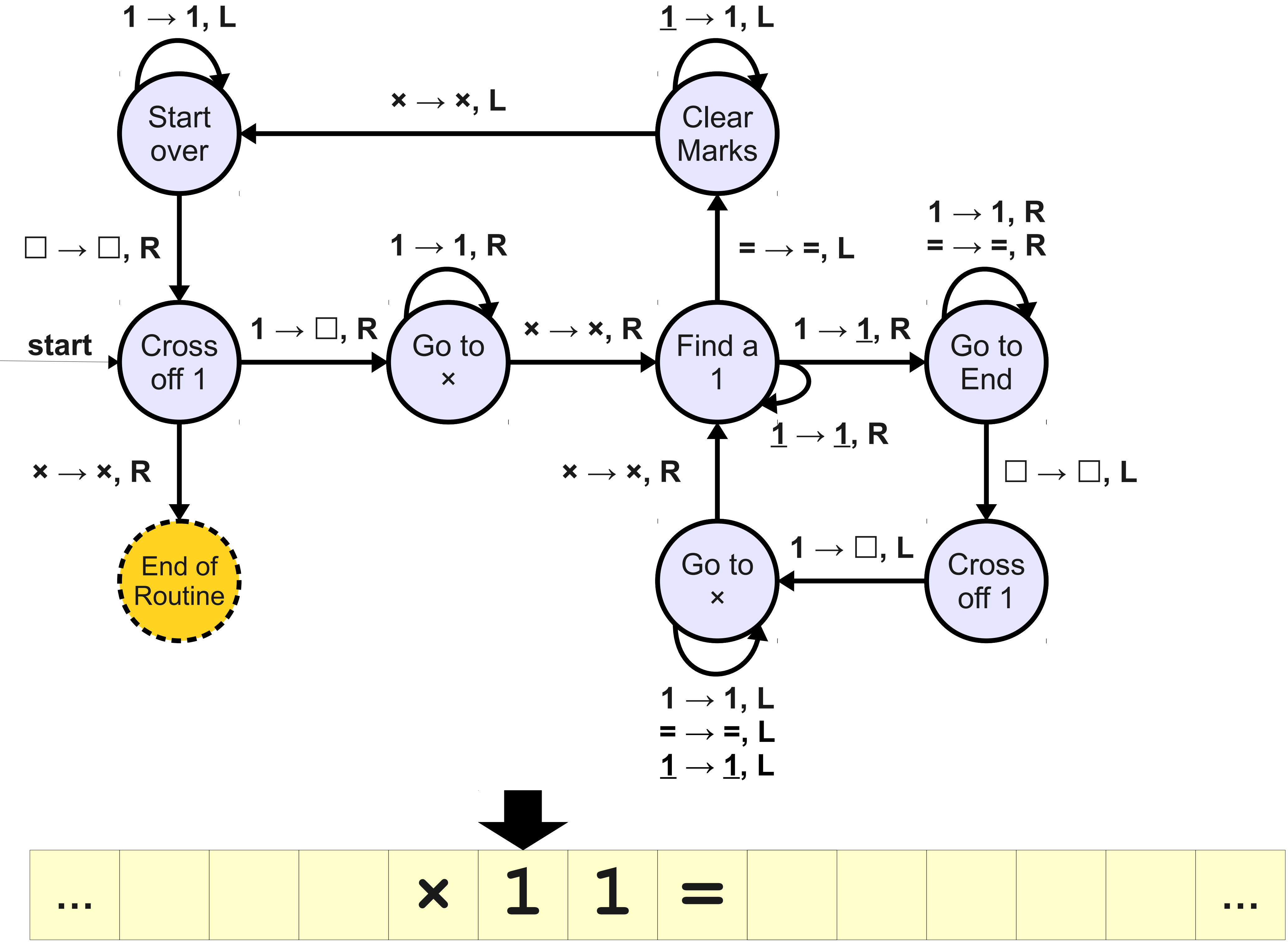
1 → 1, L
Start over
× → ×, L
1 → 1, L
Clear Marks
□ → □, R
1 → 1, R
= → =, L
1 → 1, R
= → =, R
start
Cross off 1
1 → □, R
Go to ×
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
× → ×, R
□ → □, L
Go to ×
1 → □, L
Cross off 1
1 → 1, L
= → =, L
1 → 1, L
…
×
1
1
=
…



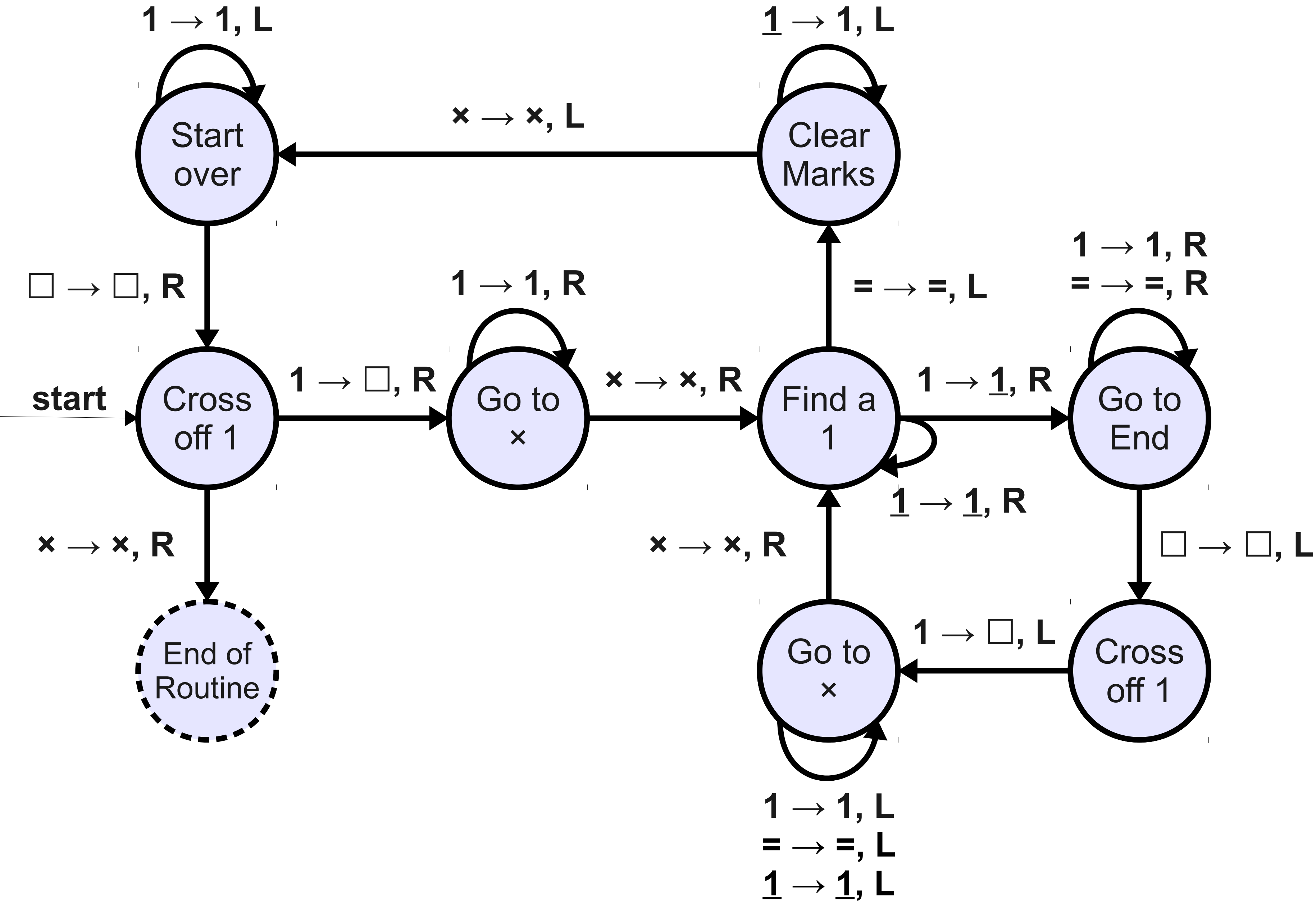
1 → 1, L
1 → 1, L
× → ×, L
Start over
Clear Marks
□ → □, R
1 → 1, R
= → =, L
1 → 1, R
= → =, R
start
Cross off 1
1 → □, R
Go to ×
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
× → ×, R
□ → □, L
Go to ×
1 → □, L
Cross off 1
1 → 1, L
= → =, L
1 → 1, L
…
× 1 1 =
…



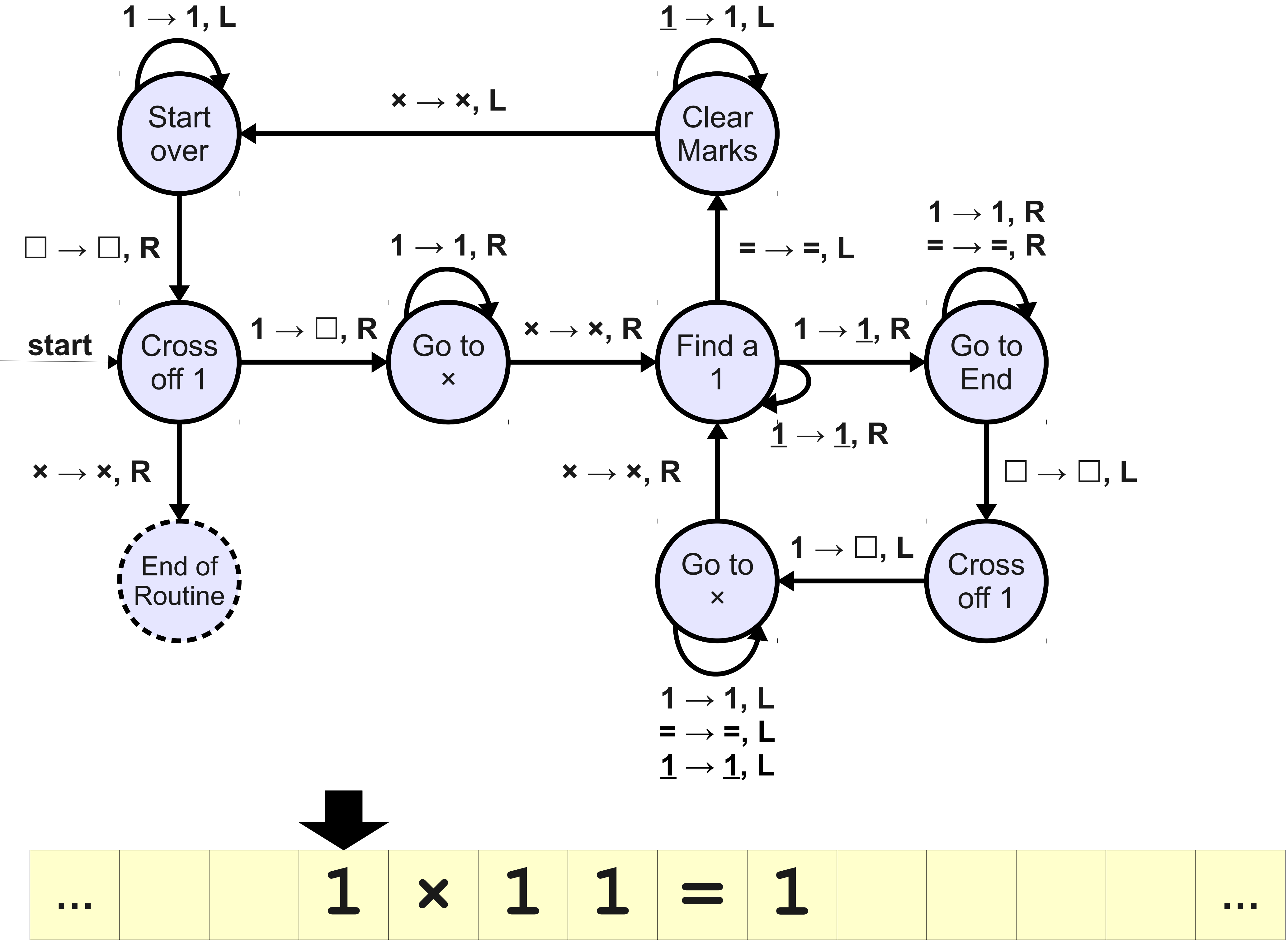
Start over
Clear Marks
Cross off 1
Go to ×
Find a 1
Go to End
End of Routine
Go to ×
Cross off 1
start
1 → 1, L
1 → 1, L
× → ×, L
□ → □, R
1 → 1, R
= → =, L
1 → 1, R
= → =, R
1 → □, R
× → ×, R
1 → 1, R
1 → 1, R
× → ×, R
× → ×, R
□ → □, L
1 → □, L
1 → 1, L
= → =, L
1 → 1, L
... × 1 1 = ...



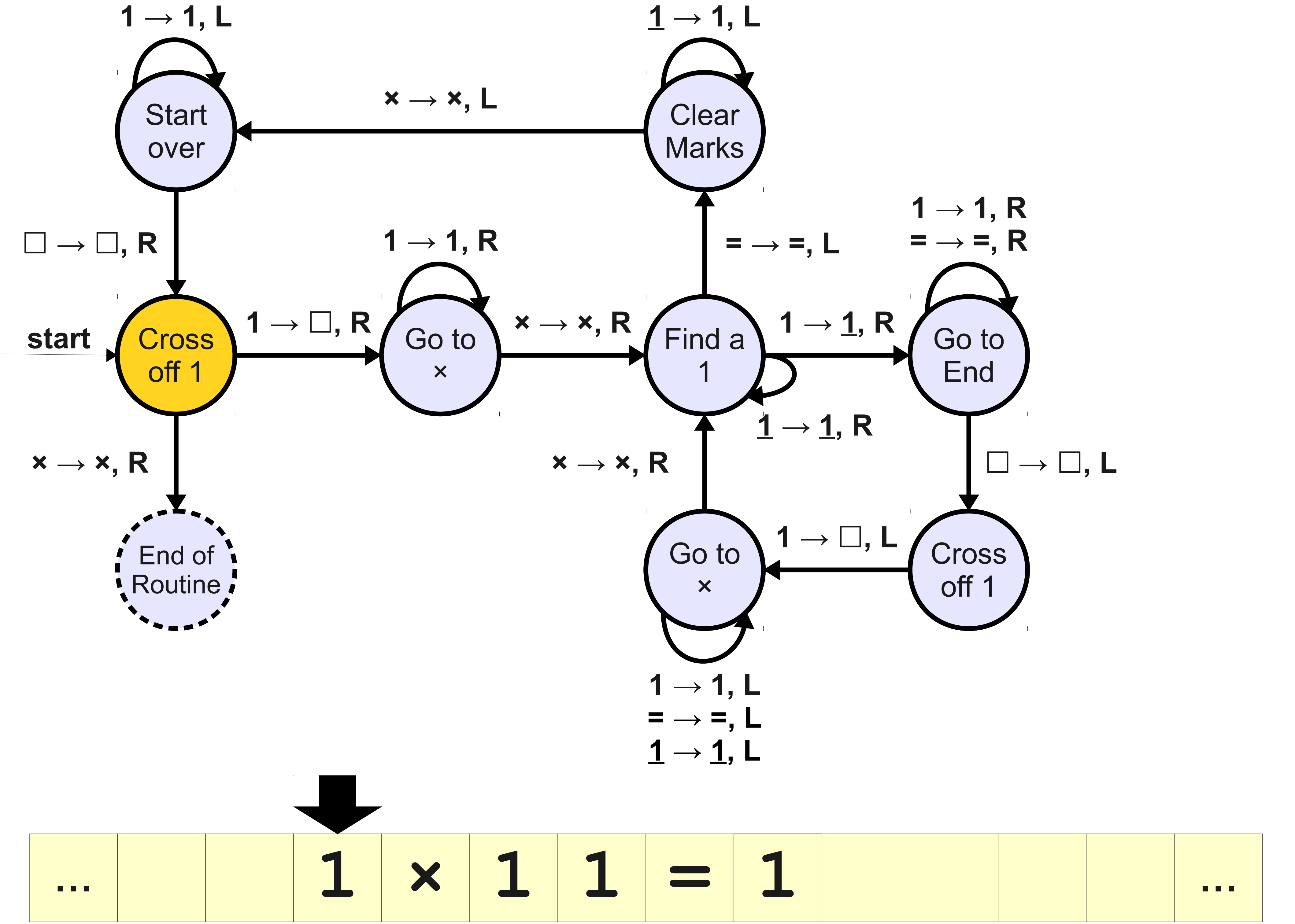
1 → 1, L
Start over
× → ×, L
1 → 1, L
Clear Marks
□ → □, R
1 → 1, R
= → =, L
1 → 1, R
= → =, R
start
Cross off 1
1 → □, R
Go to ×
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
× → ×, R
× → ×, R
□ → □, L
End of Routine
Go to ×
1 → □, L
Cross off 1
1 → 1, L
= → =, L
1 → 1, L
...
×
1
1
=
...



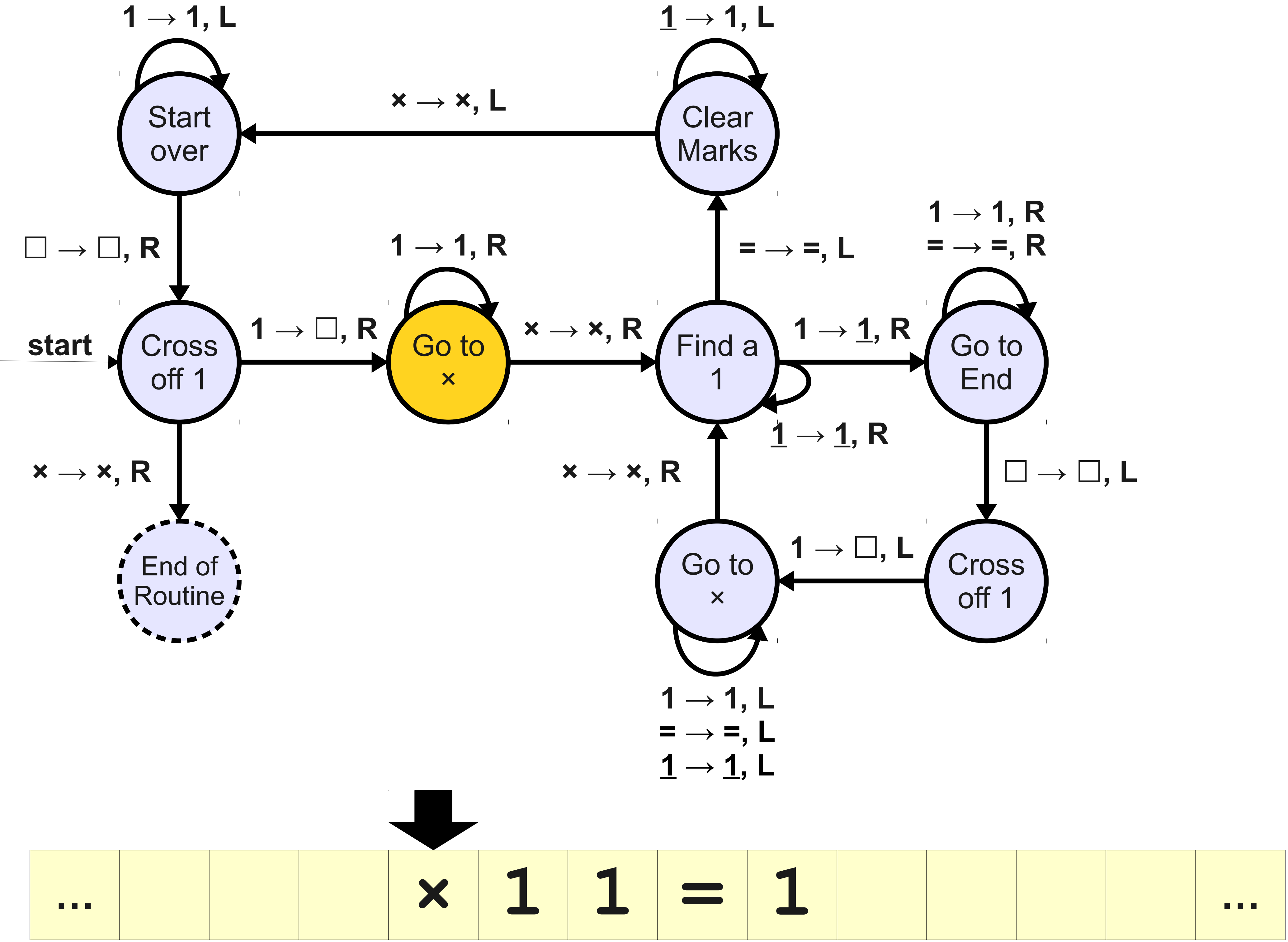
1 → 1, L
Start over
× → ×, L
1 → 1, L
Clear Marks
□ → □, R
1 → 1, R
= → =, L
1 → 1, R
= → =, R
start
Cross off 1
1 → □, R
Go to ×
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
× → ×, R
× → ×, R
□ → □, L
End of Routine
Go to ×
1 → □, L
Cross off 1
1 → 1, L
= → =, L
1 → 1, L



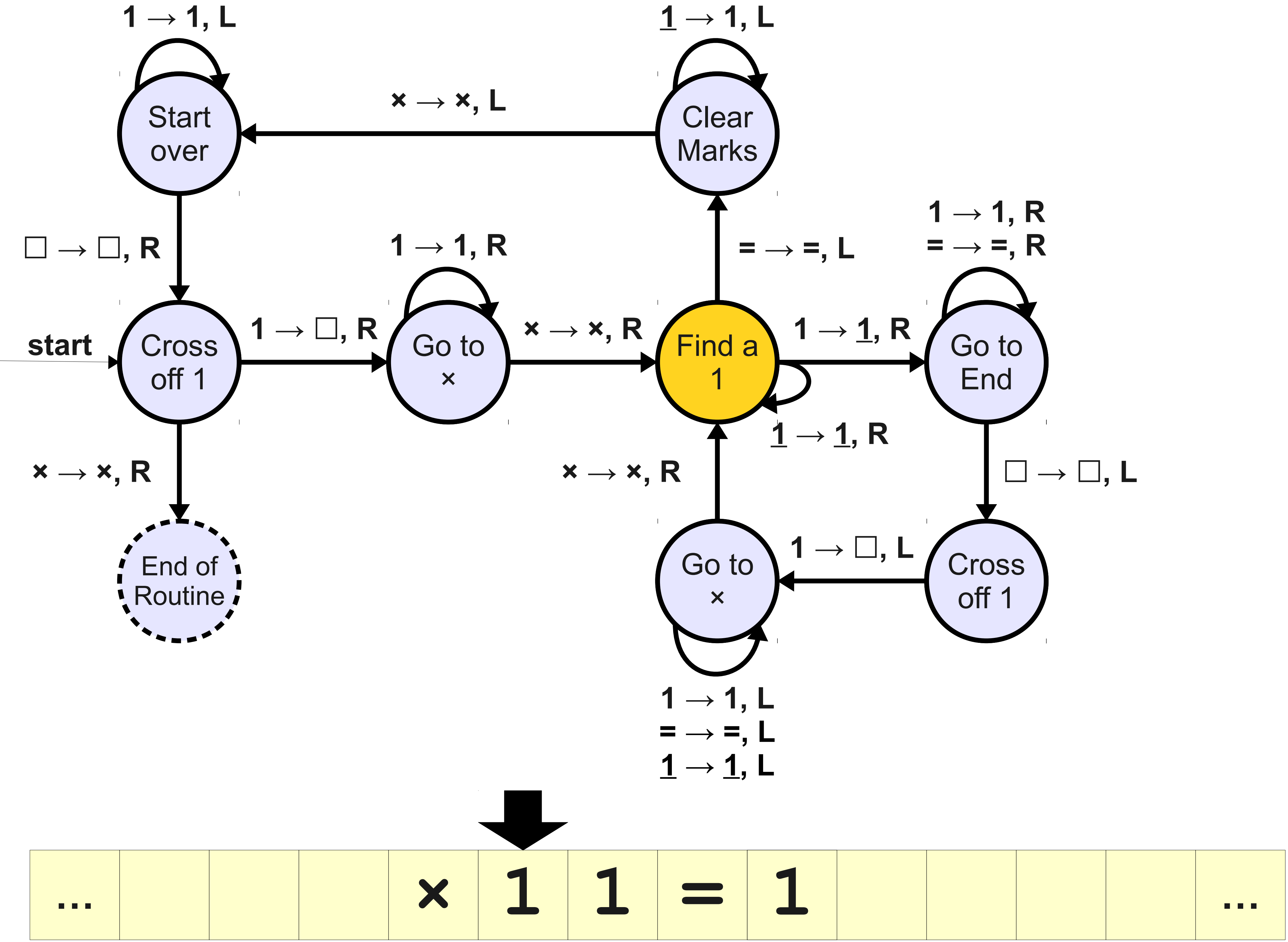
1 → 1, L
Start over
× → ×, L
1 → 1, L
Clear Marks
□ → □, R
1 → 1, R
= → =, L
1 → 1, R
= → =, R
start
Cross off 1
1 → □, R
Go to ×
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
× → ×, R
× → ×, R
□ → □, L
End of Routine
Go to ×
1 → □, L
Cross off 1
1 → 1, L
= → =, L
1 → 1, L
...
1
×
1
1
=
1
...



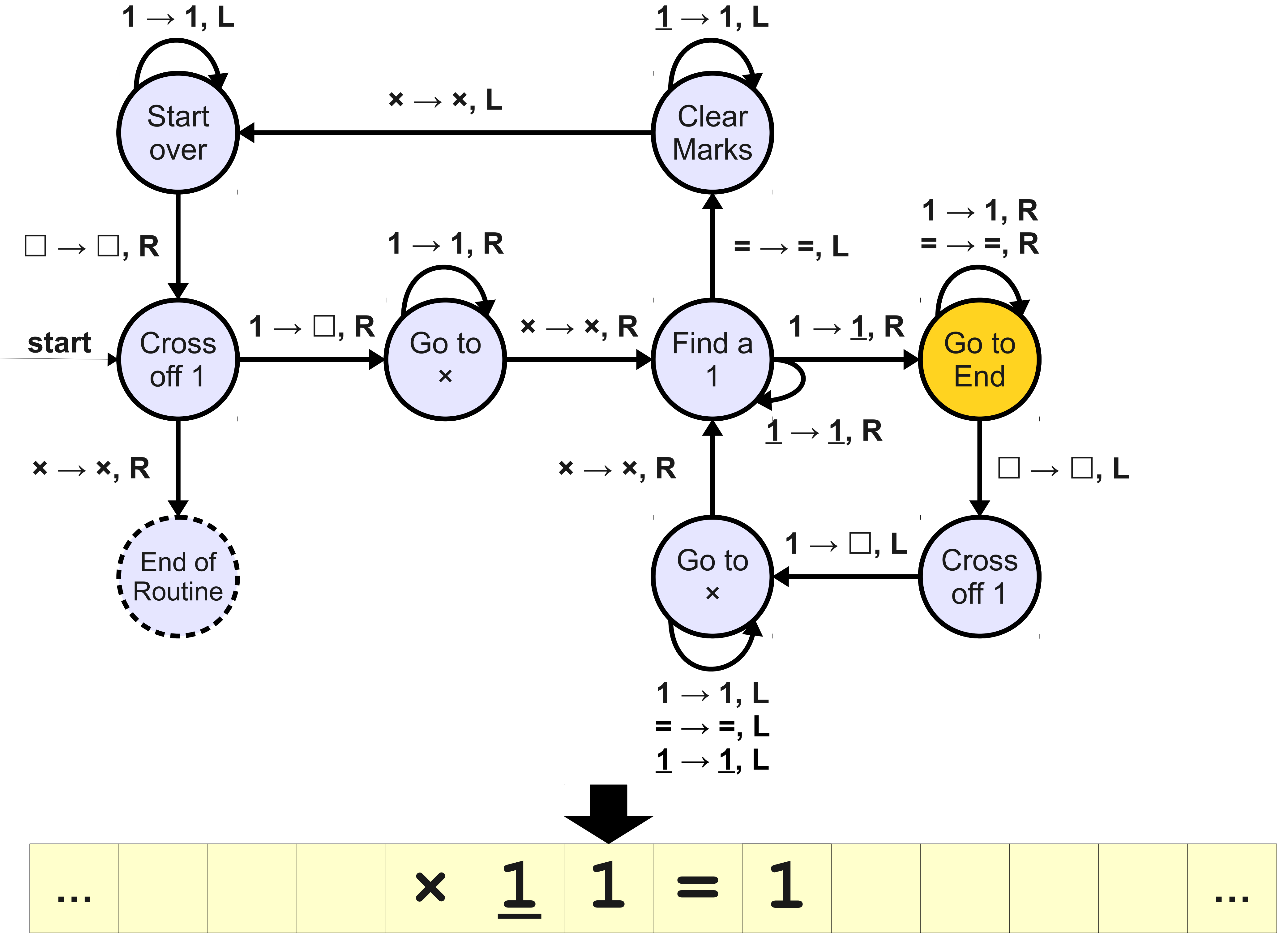
Start over
Clear Marks
Cross off 1
Go to ×
Find a 1
Go to End
End of Routine
Go to ×
Cross off 1
start
1 → 1, L
1 → 1, L
× → ×, L
□ → □, R
1 → 1, R
= → =, L
1 → 1, R
= → =, R
1 → □, R
× → ×, R
1 → 1, R
1 → 1, R
× → ×, R
× → ×, R
□ → □, L
1 → □, L
1 → 1, L
= → =, L
1 → 1, L
… 1 × 1 1 = 1 …



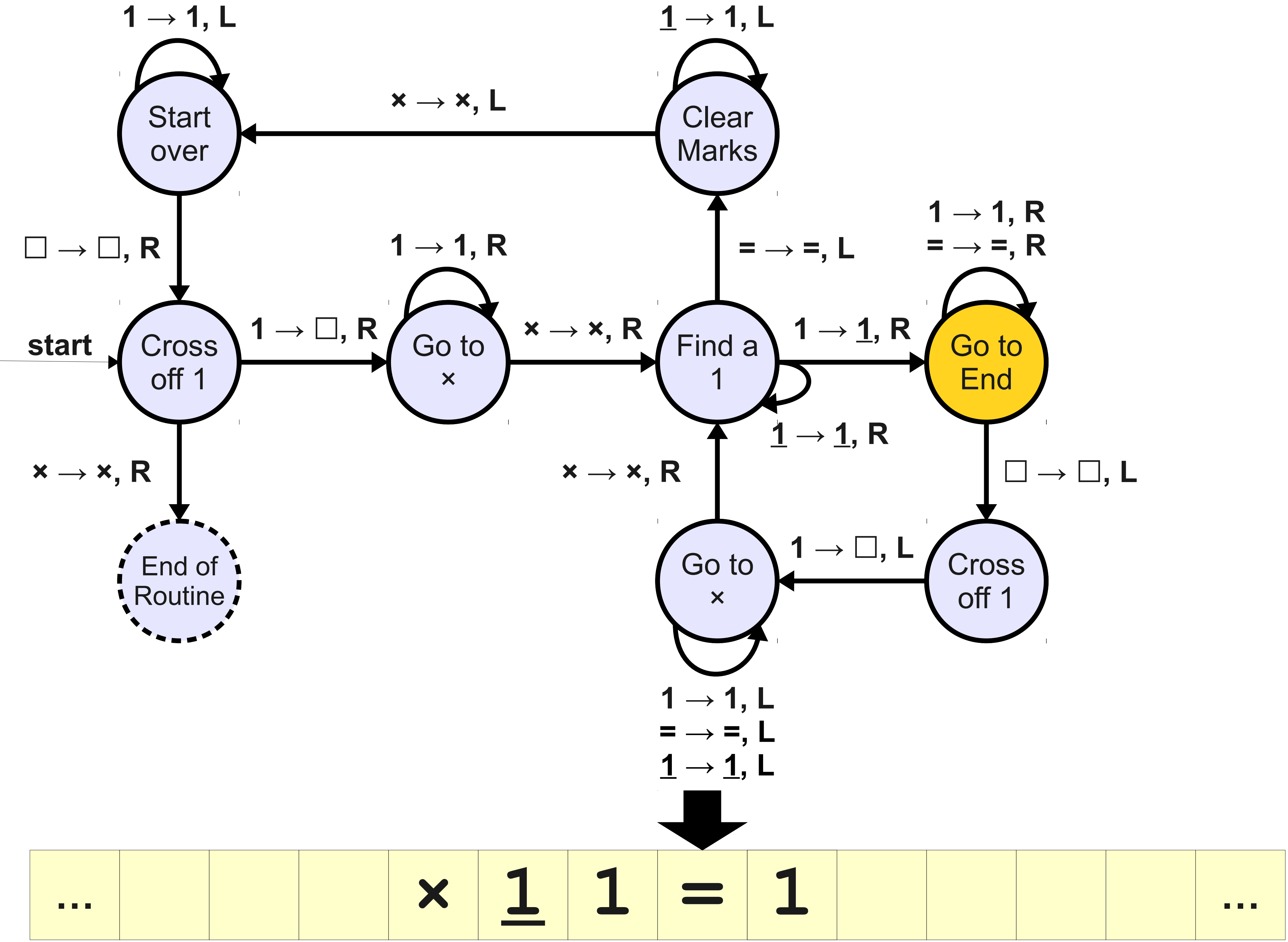
1 → 1, L
Start over
1 → 1, L
Clear Marks
× → ×, L
□ → □, R
1 → 1, R
1 → 1, R
= → =, R
= → =, L
start
Cross off 1
1 → □, R
Go to ×
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
× → ×, R
× → ×, R
□ → □, L
End of Routine
1 → □, L
Go to ×
Cross off 1
1 → 1, L
= → =, L
1 → 1, L
× 1 1 = 1
…
…

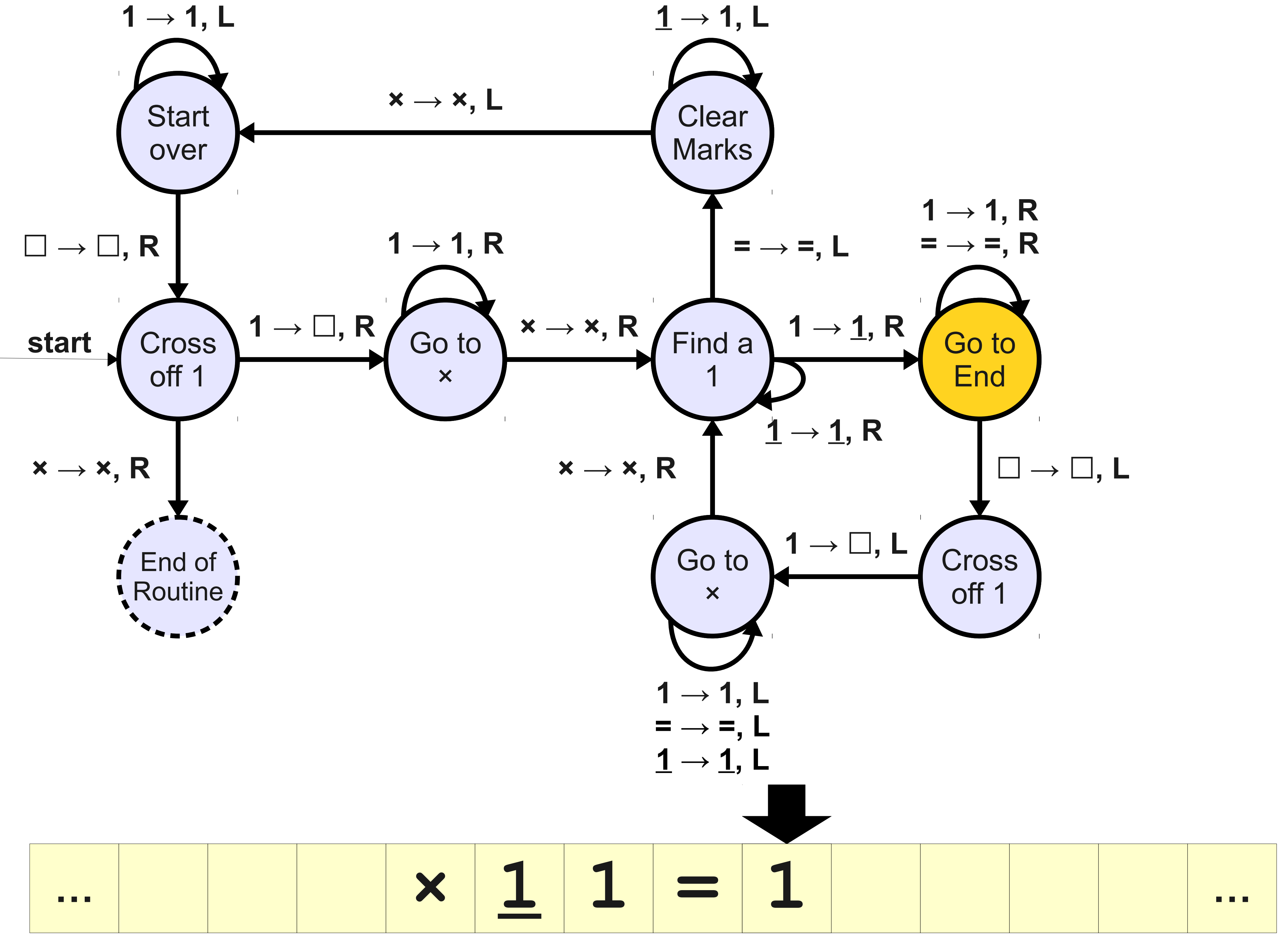


1 → 1, L
Start over
× → ×, L
1 → 1, L
Clear Marks
□ → □, R
1 → 1, R
= → =, L
1 → 1, R
= → =, R
start
Cross off 1
1 → □, R
Go to ×
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
× → ×, R
× → ×, R
□ → □, L
End of Routine
Go to ×
1 → □, L
Cross off 1
1 → 1, L
= → =, L
1 → 1, L
… × 1 1 = 1 …

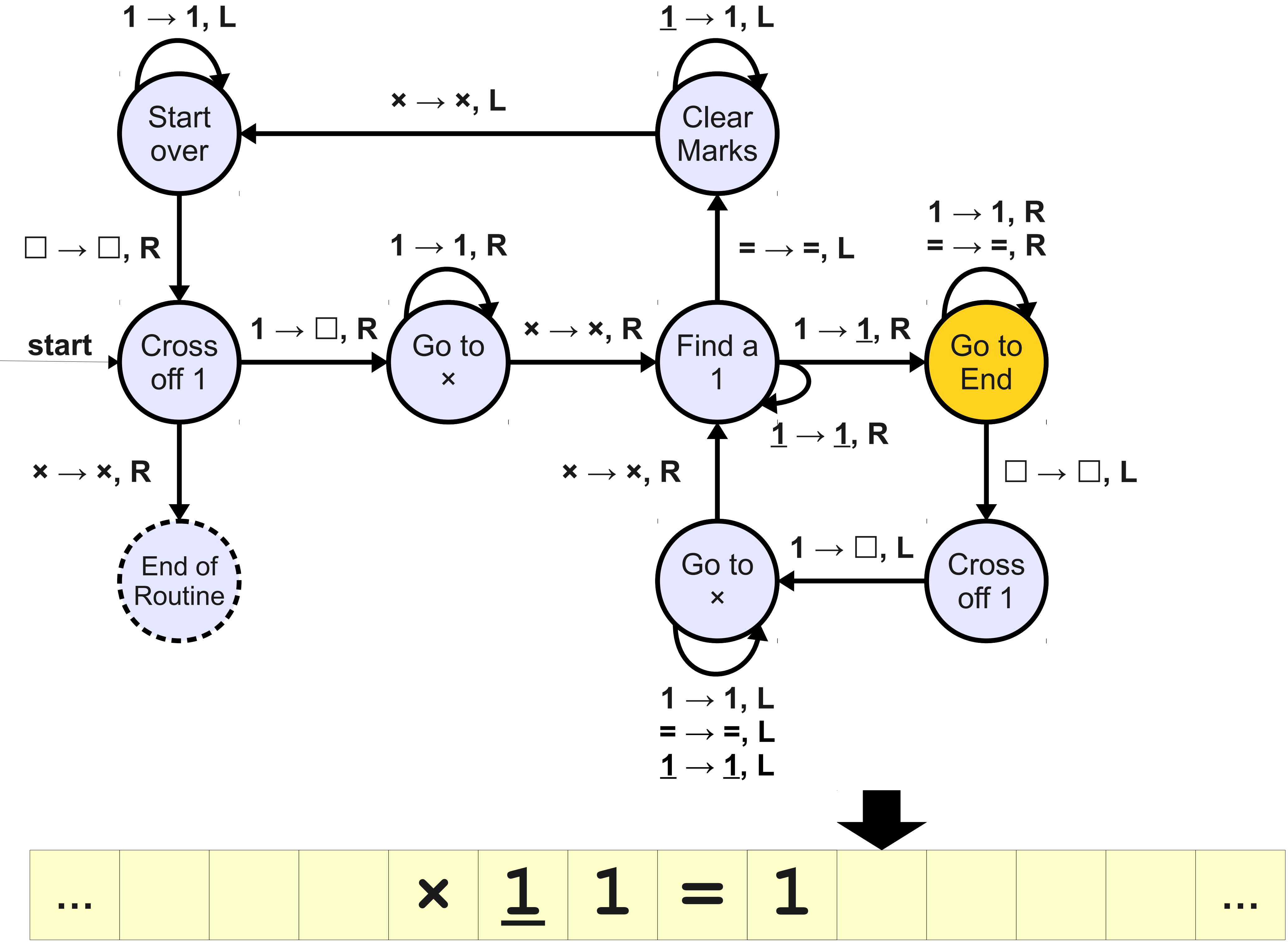


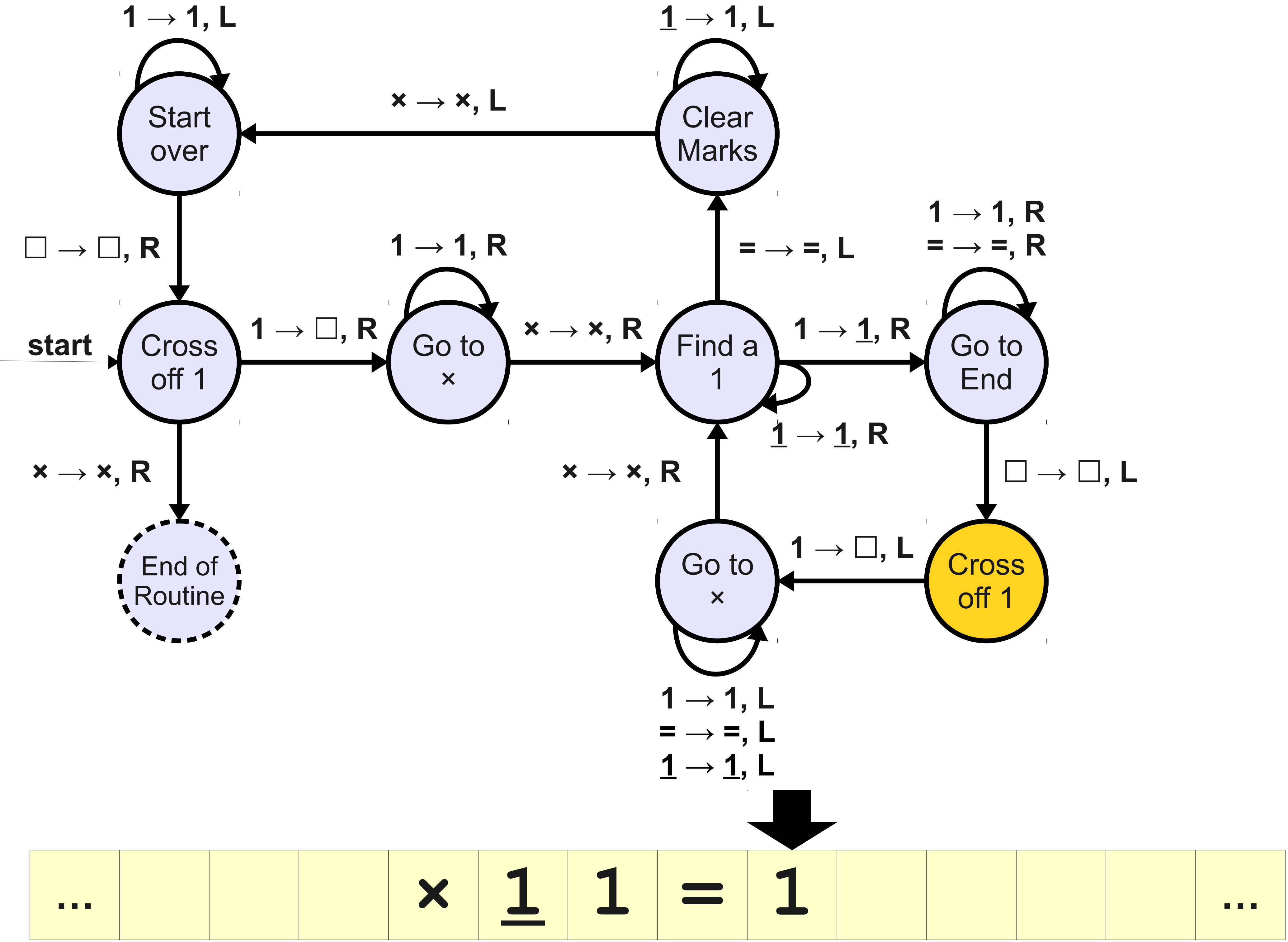
Start over
Clear Marks
Cross off 1
Go to ×
Find a 1
Go to End
End of Routine
Go to ×
Cross off 1
start
1 → 1, L
1 → 1, L
× → ×, L
□ → □, R
1 → 1, R
1 → 1, R
= → =, R
= → =, L
1 → □, R
× → ×, R
1 → 1, R
1 → 1, R
× → ×, R
× → ×, R
□ → □, L
1 → □, L
1 → 1, L
= → =, L
1 → 1, L
…
×
1
1
=
1
…


Start over
Clear Marks
Cross off 1
Go to ×
Find a 1
Go to End
End of Routine
Go to ×
Cross off 1
start
1 → 1, L
1 → 1, L
× → ×, L
□ → □, R
1 → 1, R
1 → 1, R
= → =, R
= → =, L
1 → □, R
× → ×, R
1 → 1, R
1 → 1, R
× → ×, R
× → ×, R
□ → □, L
1 → □, L
1 → 1, L
= → =, L
1 → 1, L
...
×
1
1
=
1
...

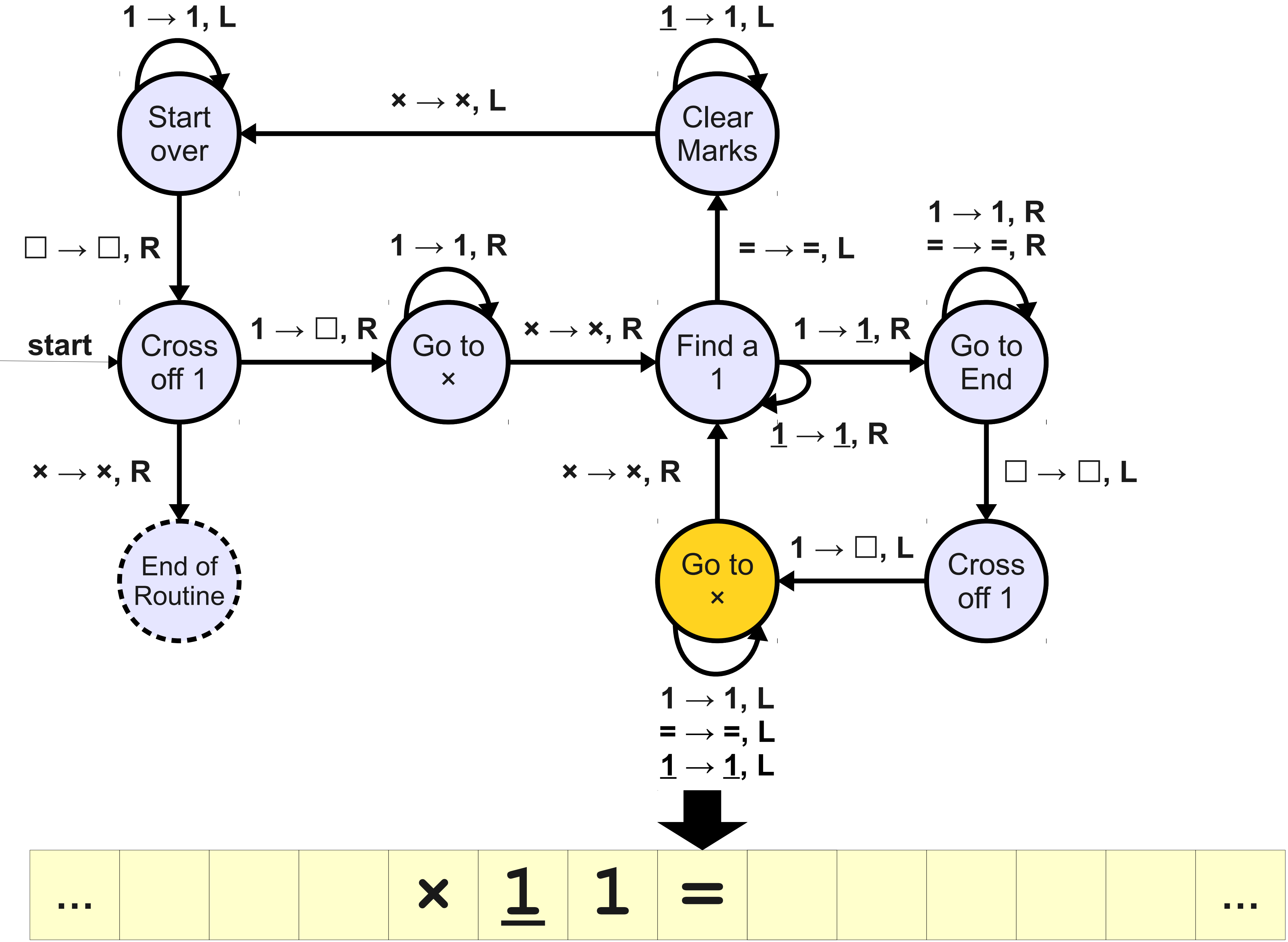


1 → 1, L
Start over
× → ×, L
1 → 1, L
Clear Marks
□ → □, R
1 → 1, R
= → =, L
1 → 1, R
= → =, R
start
Cross off 1
1 → □, R
Go to ×
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
× → ×, R
× → ×, R
□ → □, L
End of Routine
Go to ×
1 → □, L
Cross off 1
1 → 1, L
= → =, L
1 → 1, L
… × 1 1 = 1 …

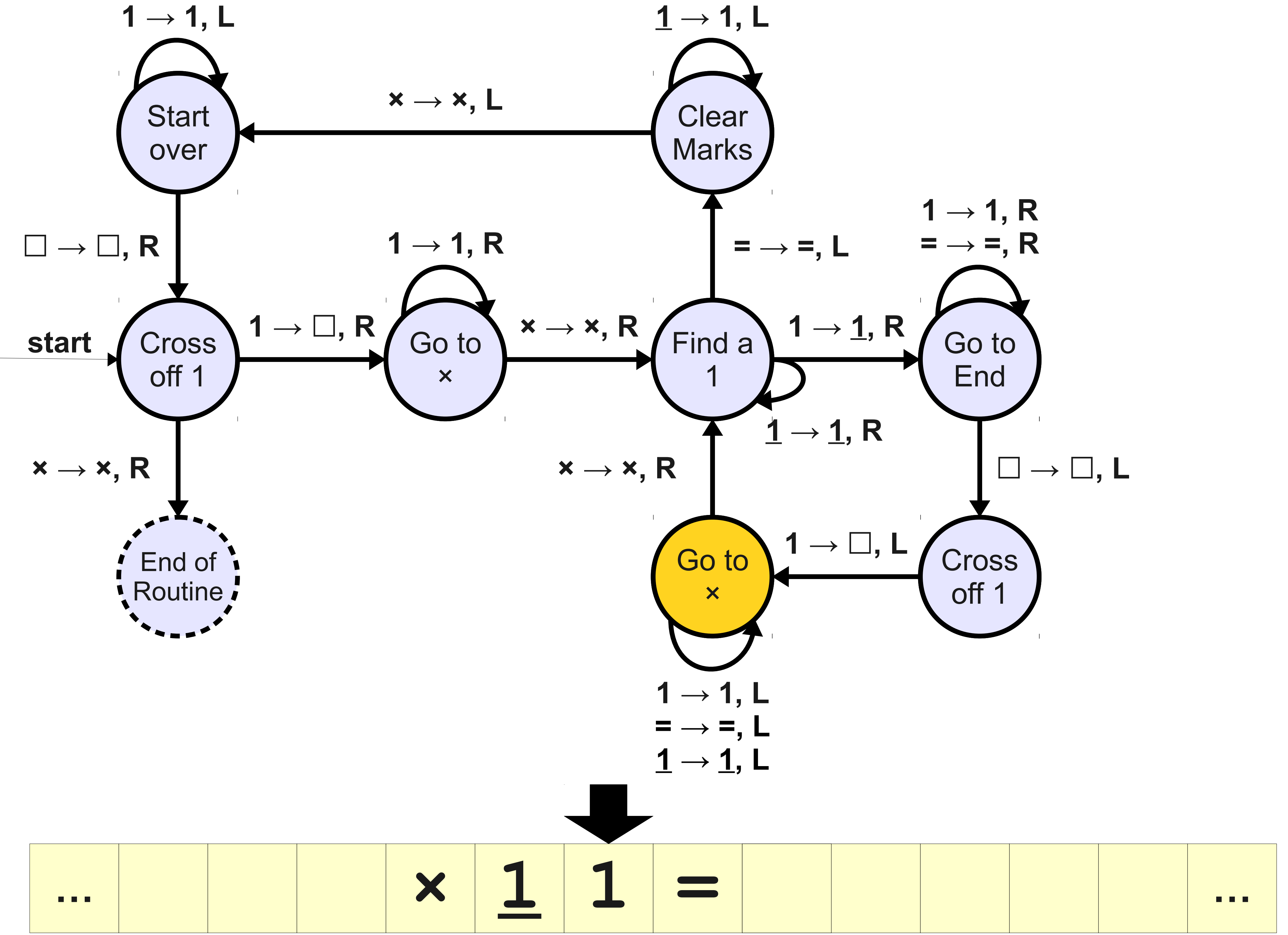

1 → 1, L
Start over
□ → □, R
× → ×, L
1 → 1, L
Clear Marks
= → =, L
1 → 1, R
= → =, R
start
Cross off 1
1 → □, R
1 → 1, R
Go to ×
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
× → ×, R
× → ×, R
□ → □, L
End of Routine
Go to ×
1 → □, L
Cross off 1
1 → 1, L
= → =, L
1 → 1, L
...
×
1
1
=
1
...



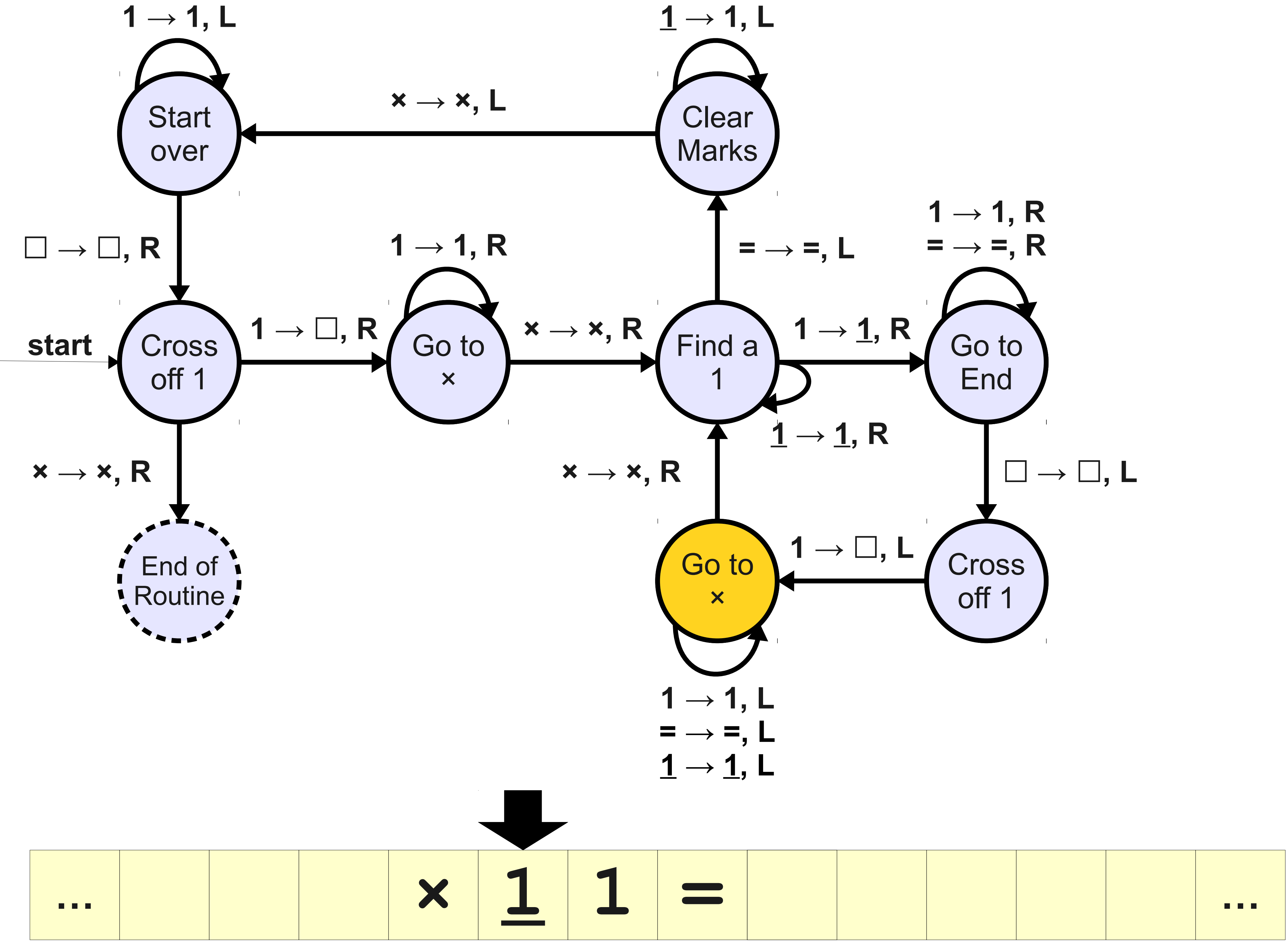
Start over
Clear Marks
Cross off 1
Go to ×
Find a 1
Go to End
End of Routine
Go to ×
Cross off 1
start
1 → 1, L
1 → 1, L
× → ×, L
□ → □, R
1 → 1, R
1 → □, R
× → ×, R
= → =, L
1 → 1, R
= → =, R
1 → 1, R
1 → 1, R
× → ×, R
× → ×, R
□ → □, L
1 → □, L
1 → 1, L
= → =, L
1 → 1, L
…
×
1
1
=
1
…



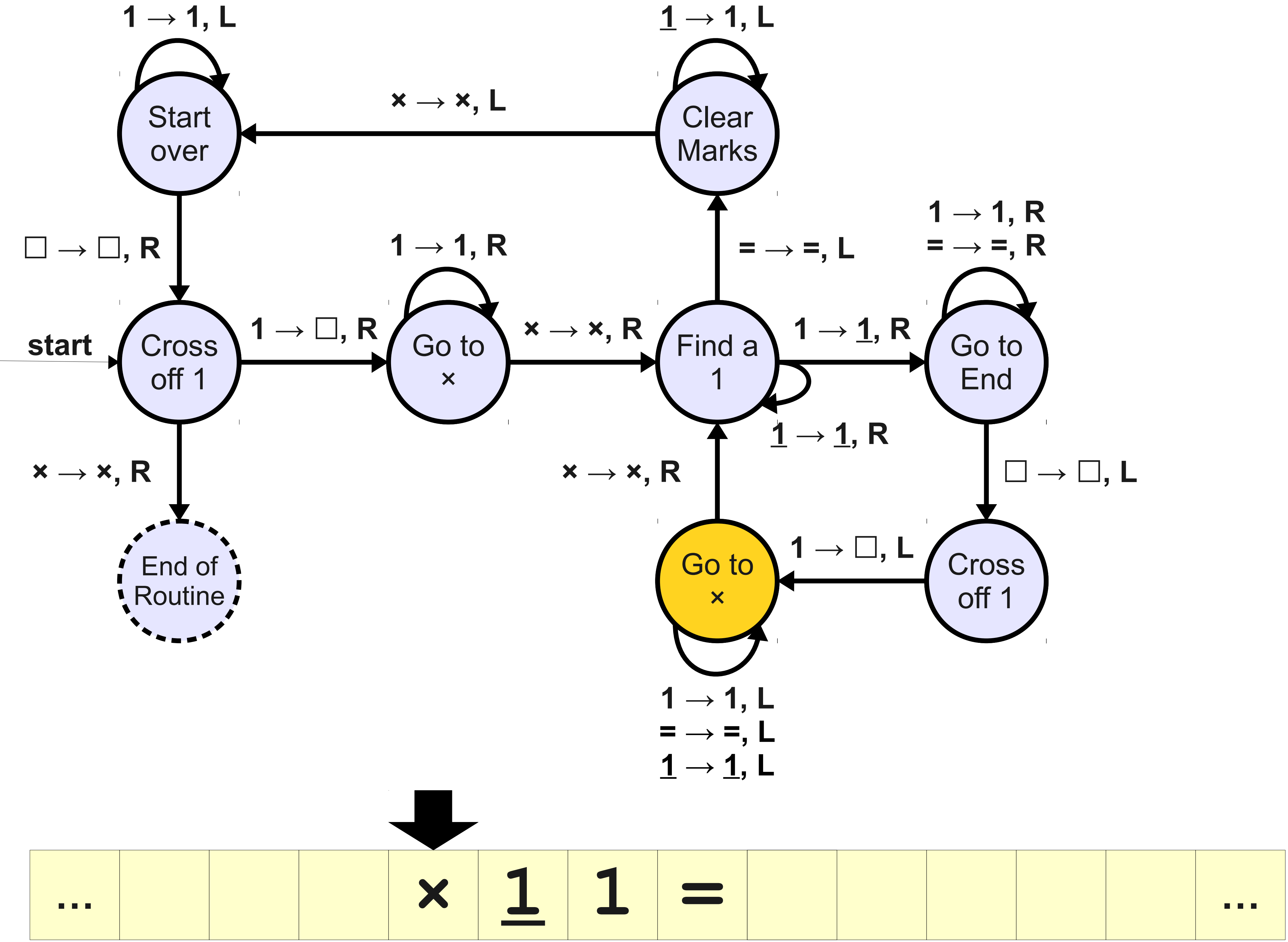
1 → 1, L
Start over
1 → 1, L
Clear Marks
× → ×, L
□ → □, R
1 → 1, R
= → =, L
1 → 1, R
= → =, R
start
Cross off 1
1 → □, R
Go to ×
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
× → ×, R
× → ×, R
□ → □, L
End of Routine
1 → □, L
Go to ×
Cross off 1
1 → 1, L
= → =, L
1 → 1, L
… × 1 1 = …

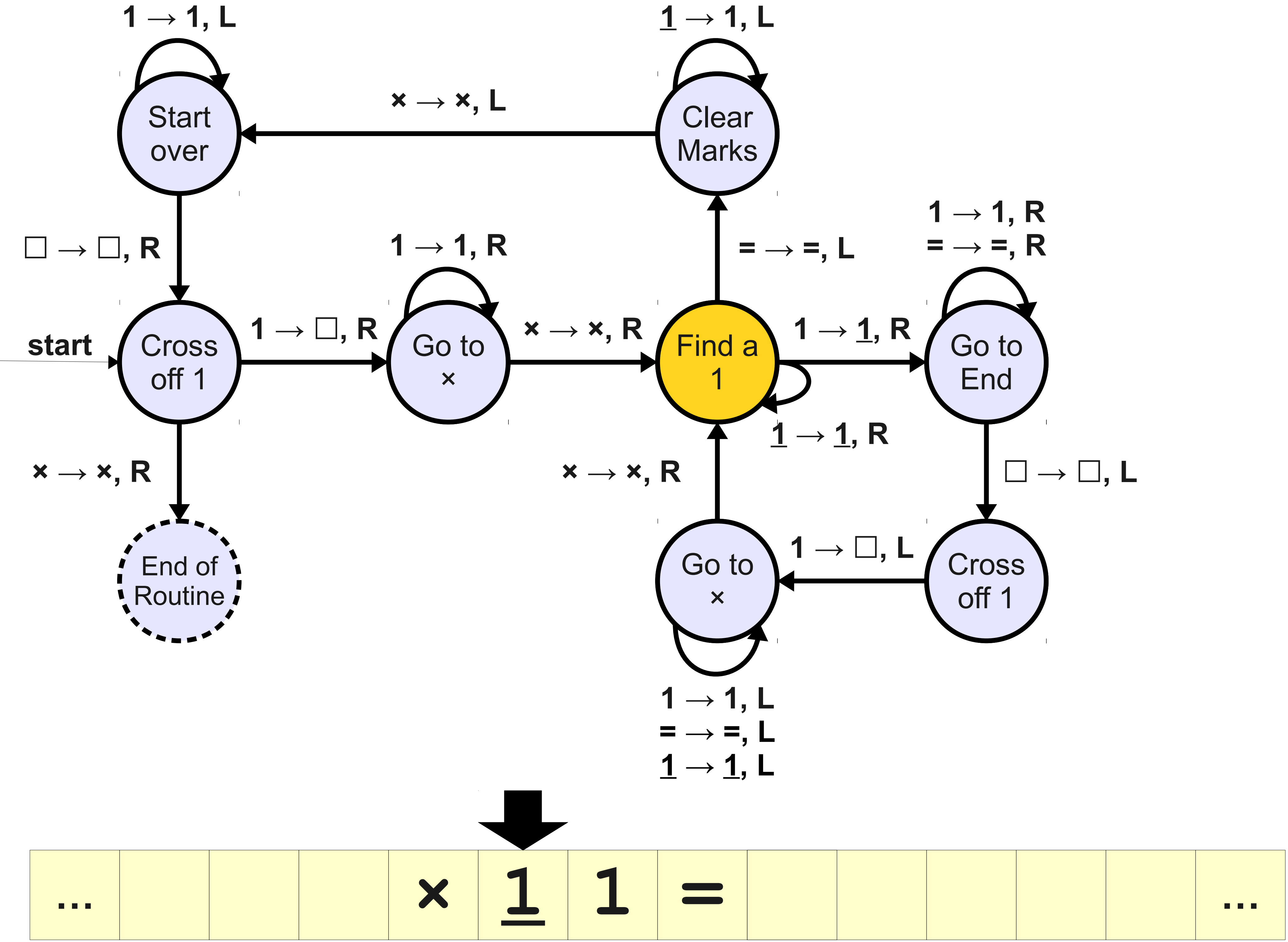


Start over
Clear Marks
Cross off 1
Go to ×
Find a 1
Go to End
End of Routine
Go to ×
Cross off 1
start
1 → 1, L
1 → 1, L
× → ×, L
□ → □, R
1 → 1, R
1 → □, R
× → ×, R
= → =, L
1 → 1, R
= → =, R
1 → 1, R
1 → 1, R
× → ×, R
× → ×, R
□ → □, L
1 → □, L
1 → 1, L
= → =, L
1 → 1, L
… × 1 1 = …

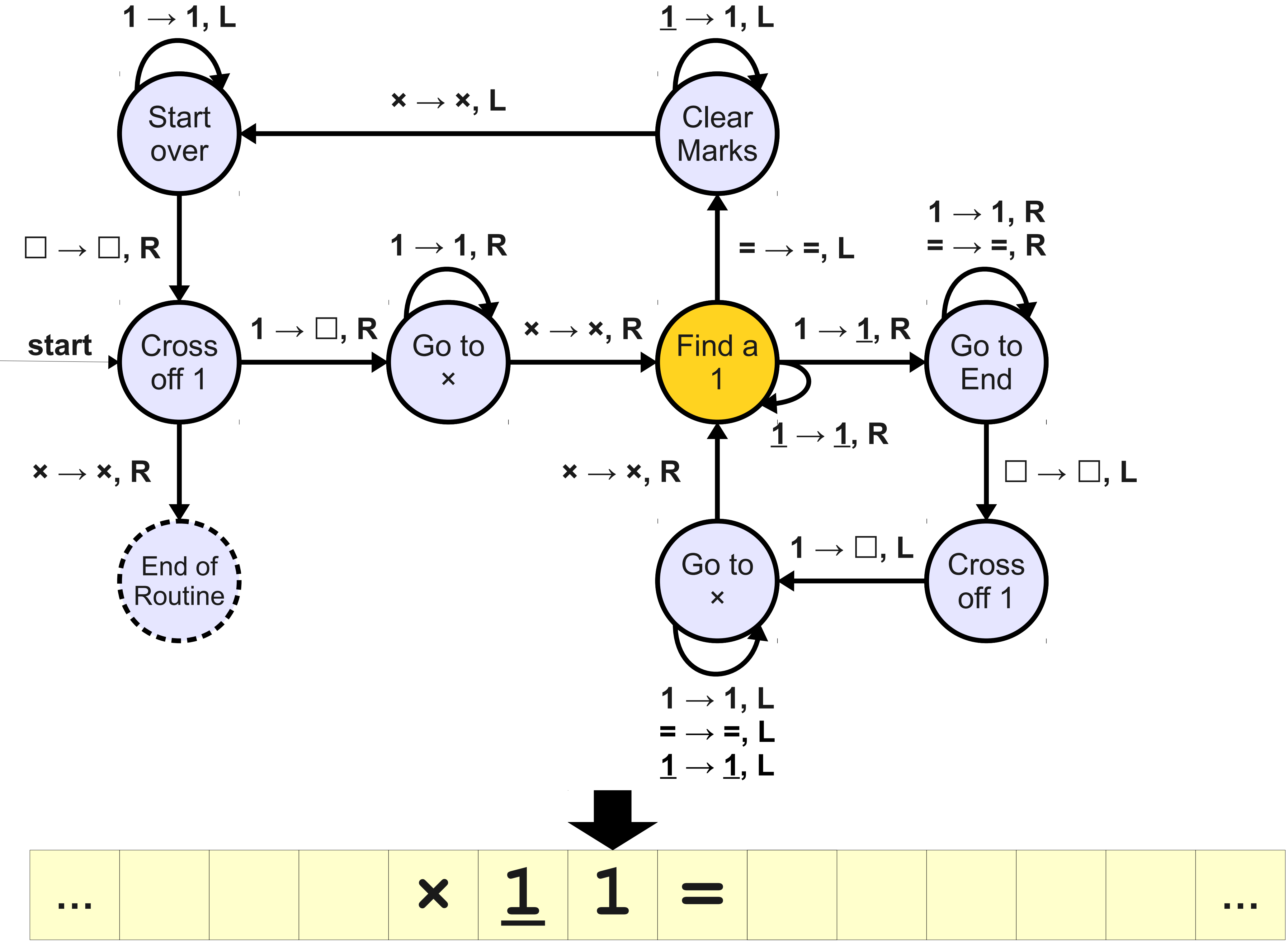


Start over
Clear Marks
Cross off 1
Go to ×
Find a 1
Go to End
End of Routine
Go to ×
Cross off 1
start
1 → 1, L
1 → 1, L
× → ×, L
□ → □, R
1 → 1, R
1 → □, R
× → ×, R
= → =, L
1 → 1, R
= → =, R
1 → 1, R
1 → 1, R
× → ×, R
× → ×, R
□ → □, L
1 → □, L
1 → 1, L
= → =, L
1 → 1, L
...
×
1
1
=
...

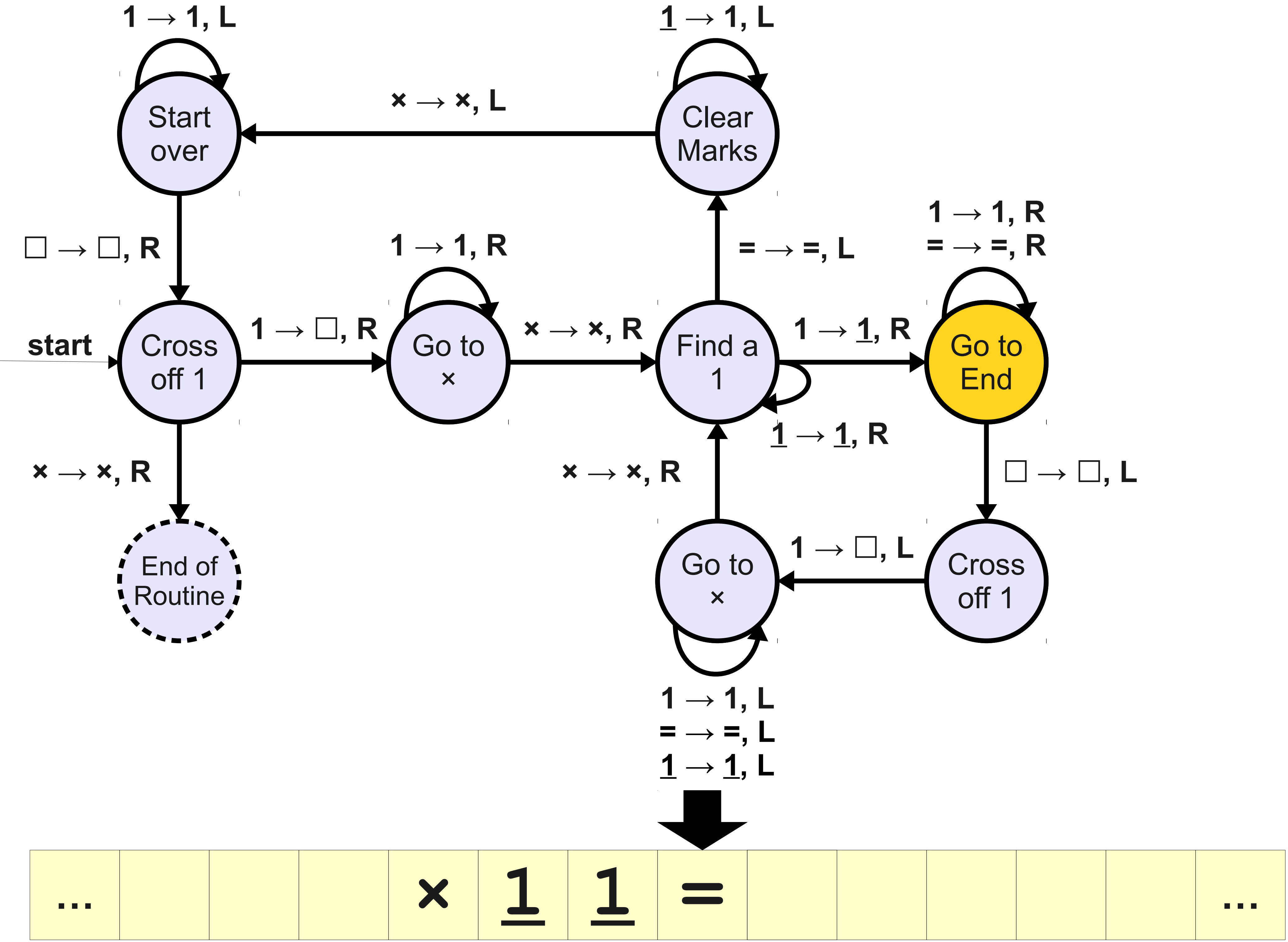

1 → 1, L
Start over
× → ×, L
1 → 1, L
Clear Marks
□ → □, R
1 → 1, R
= → =, L
1 → 1, R
= → =, R
start
Cross off 1
1 → □, R
Go to ×
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
× → ×, R
× → ×, R
□ → □, L
End of Routine
Go to ×
1 → □, L
Cross off 1
1 → 1, L
= → =, L
1 → 1, L
...
×
1
1
=
...



start
Start over
Clear Marks
Cross off 1
Go to ×
Find a 1
Go to End
End of Routine
Go to ×
Cross off 1
1 → 1, L
1 → 1, L
× → ×, L
□ → □, R
1 → □, R
1 → 1, R
× → ×, R
= → =, L
1 → 1, R
1 → 1, R
= → =, R
1 → 1, R
× → ×, R
× → ×, R
□ → □, L
1 → □, L
1 → 1, L
= → =, L
1 → 1, L
… × 1 1 = …



Start over
1 → 1, L
Clear Marks
1 → 1, L
× → ×, L
□ → □, R
1 → 1, R
= → =, L
1 → 1, R
= → =, R
start
Cross off 1
1 → □, R
Go to ×
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
× → ×, R
× → ×, R
□ → □, L
End of Routine
Go to ×
1 → □, L
Cross off 1
1 → 1, L
= → =, L
1 → 1, L
...
×
1
1
=
...



1 → 1, L
1 → 1, L
× → ×, L
Start over
Clear Marks
□ → □, R
1 → 1, R
= → =, L
1 → 1, R
= → =, R
start
Cross off 1
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Go to ×
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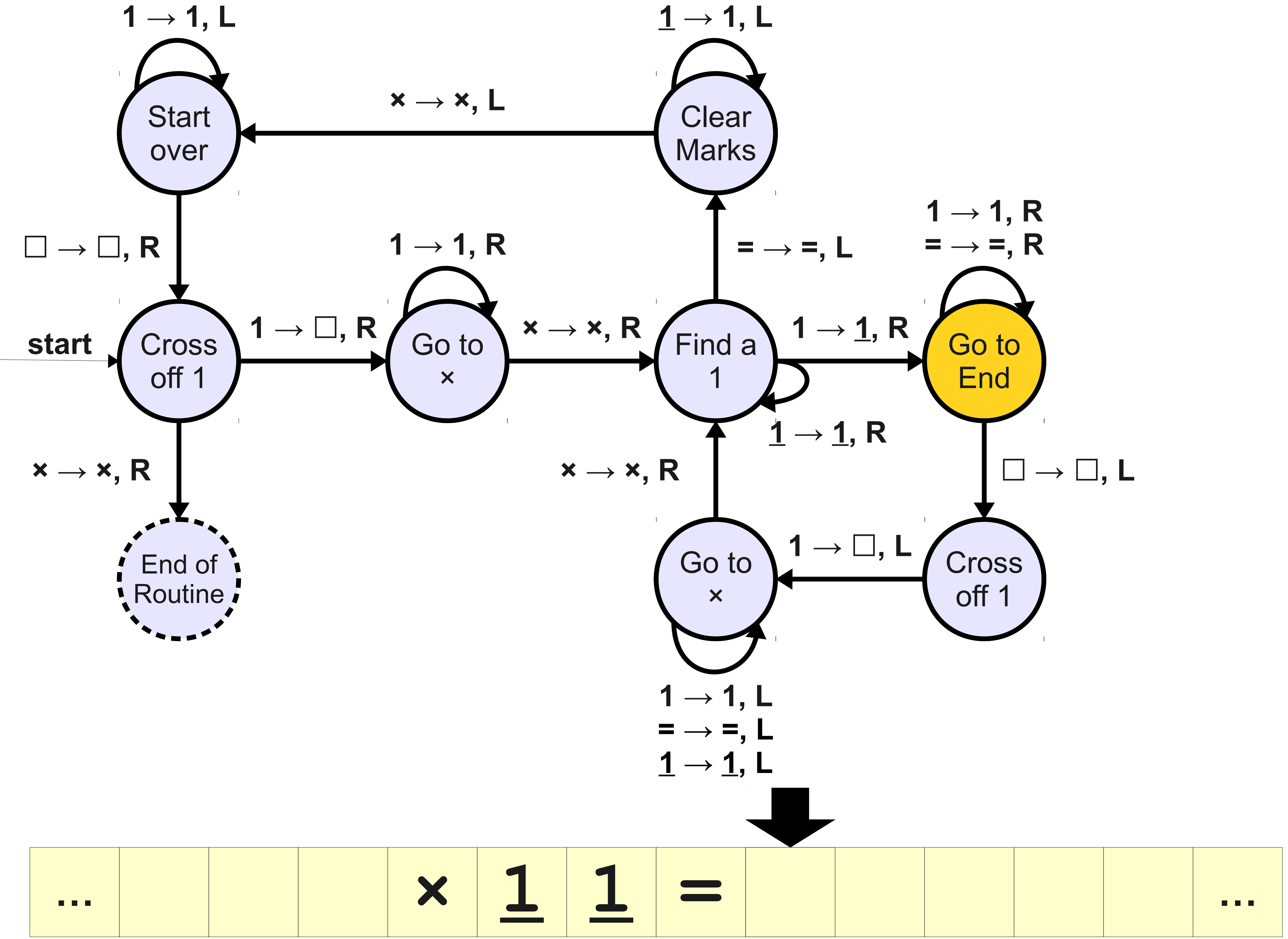
Find a 1
1 → 1, R
Go to End
1 → 1, R
× → ×, R
× → ×, R
□ → □, L
End of Routine
Go to ×
1 → □, L
Cross off 1
1 → 1, L
= → =, L
1 → 1, L
…
×
1
1
=
…



1 → 1, L
Start over
× → ×, L
1 → 1, L
Clear Marks
□ → □, R
1 → 1, R
= → =, L
1 → 1, R
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Cross off 1
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Go to ×
× → ×, R
Find a 1
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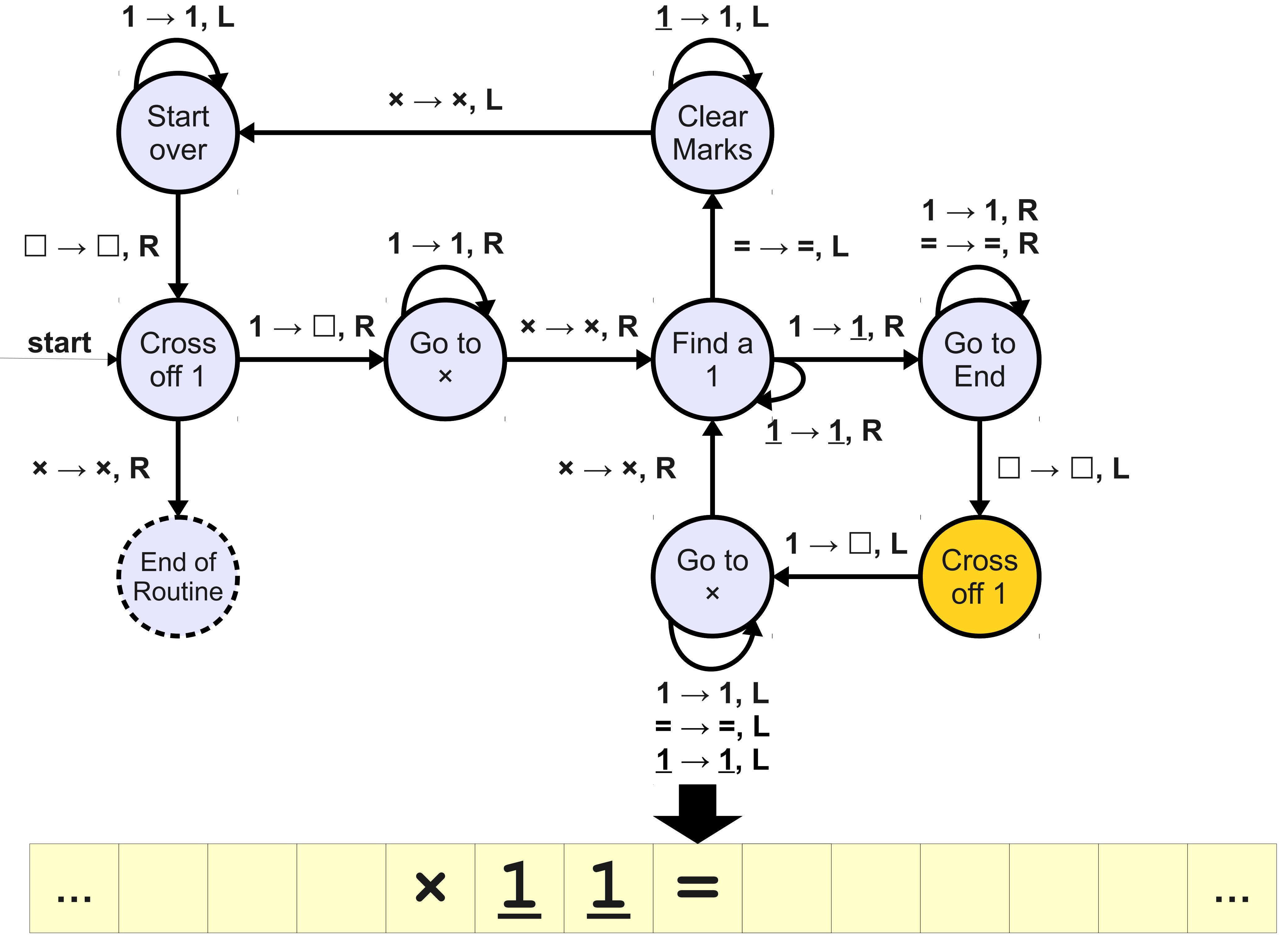
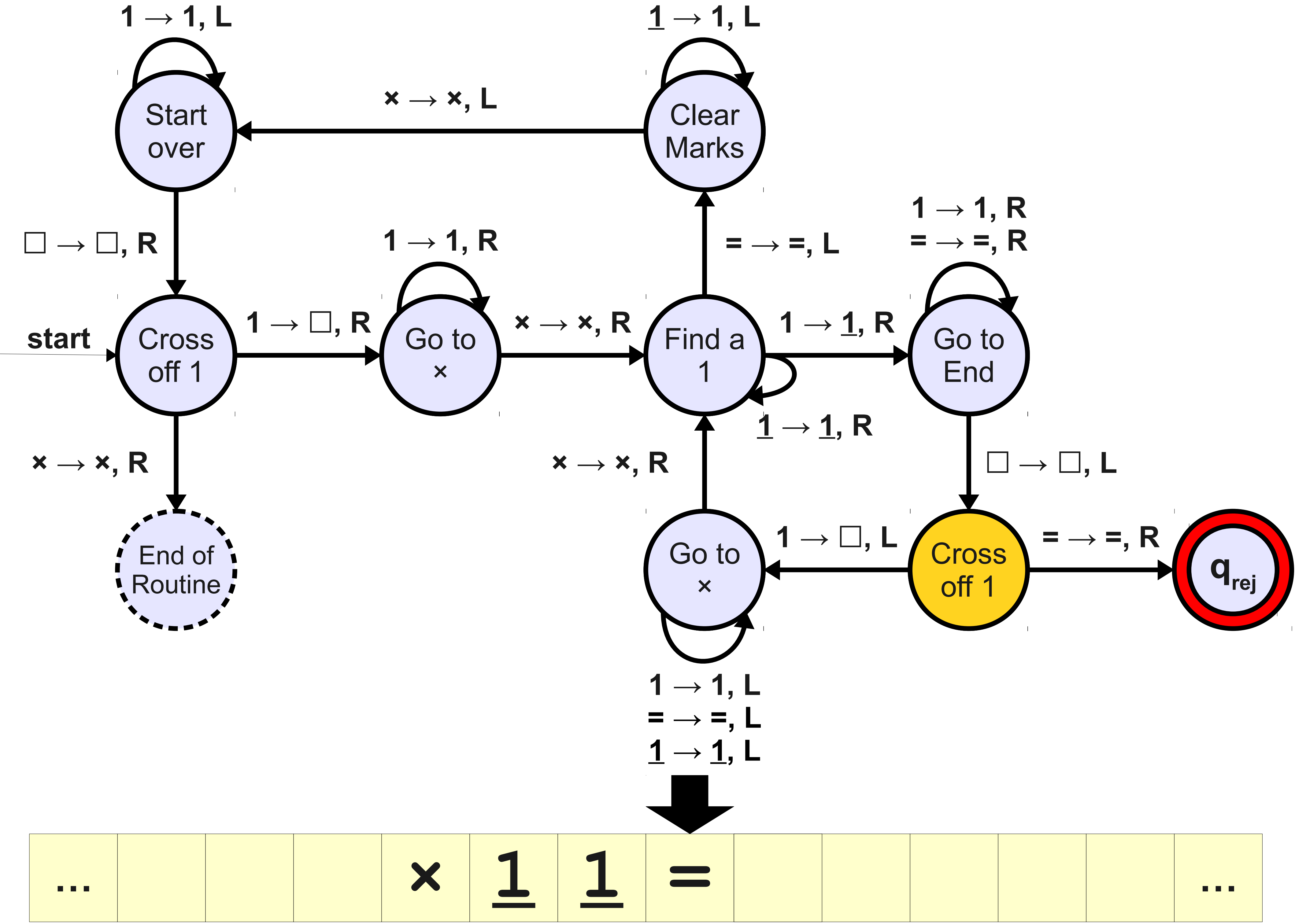
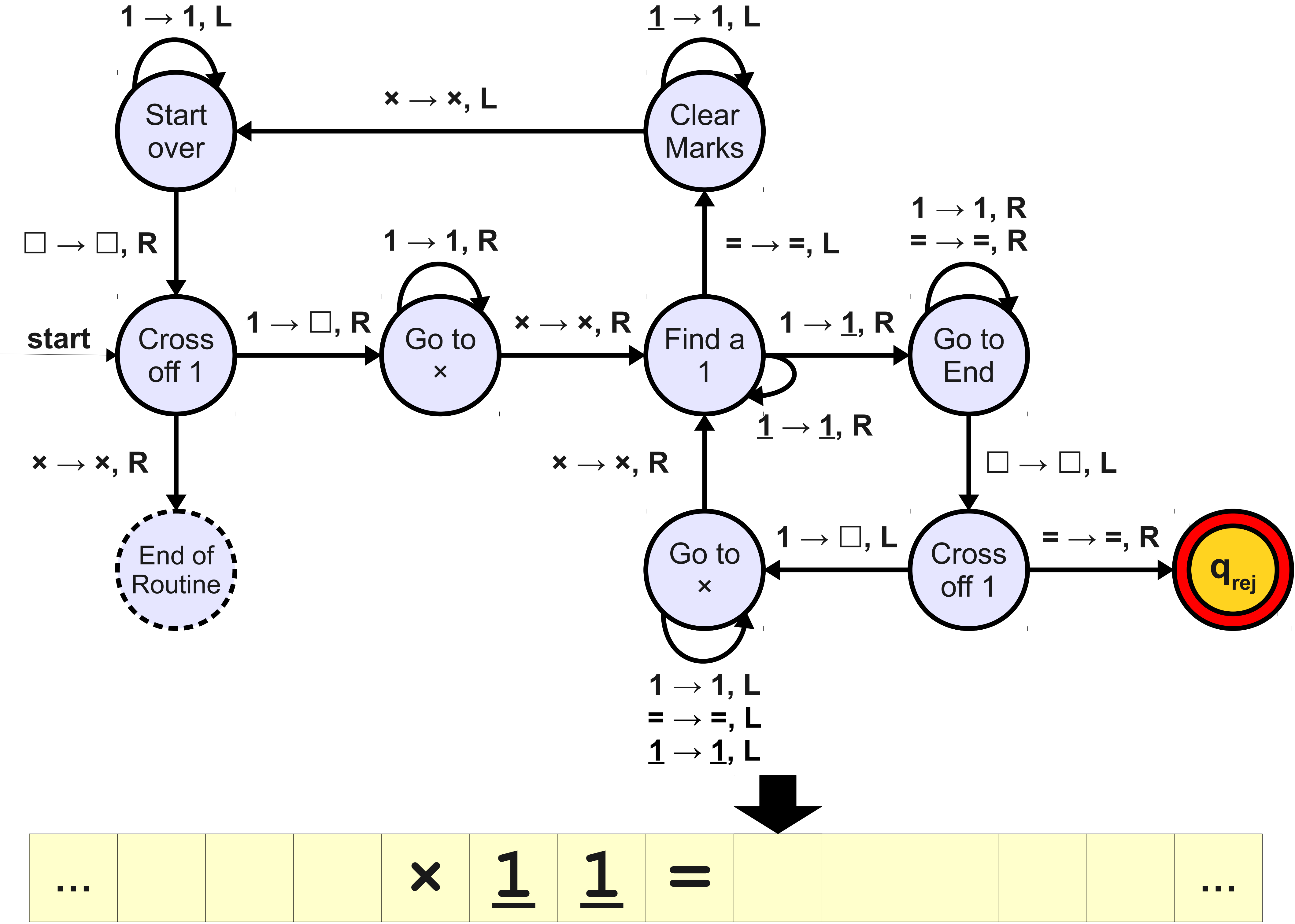
Go to End
1 → 1, R
× → ×, R
× → ×, R
□ → □, L
End of Routine
Go to ×
1 → □, L
Cross off 1
1 → 1, L
= → =, L
1 → 1, L
…
×
1
1
=
…



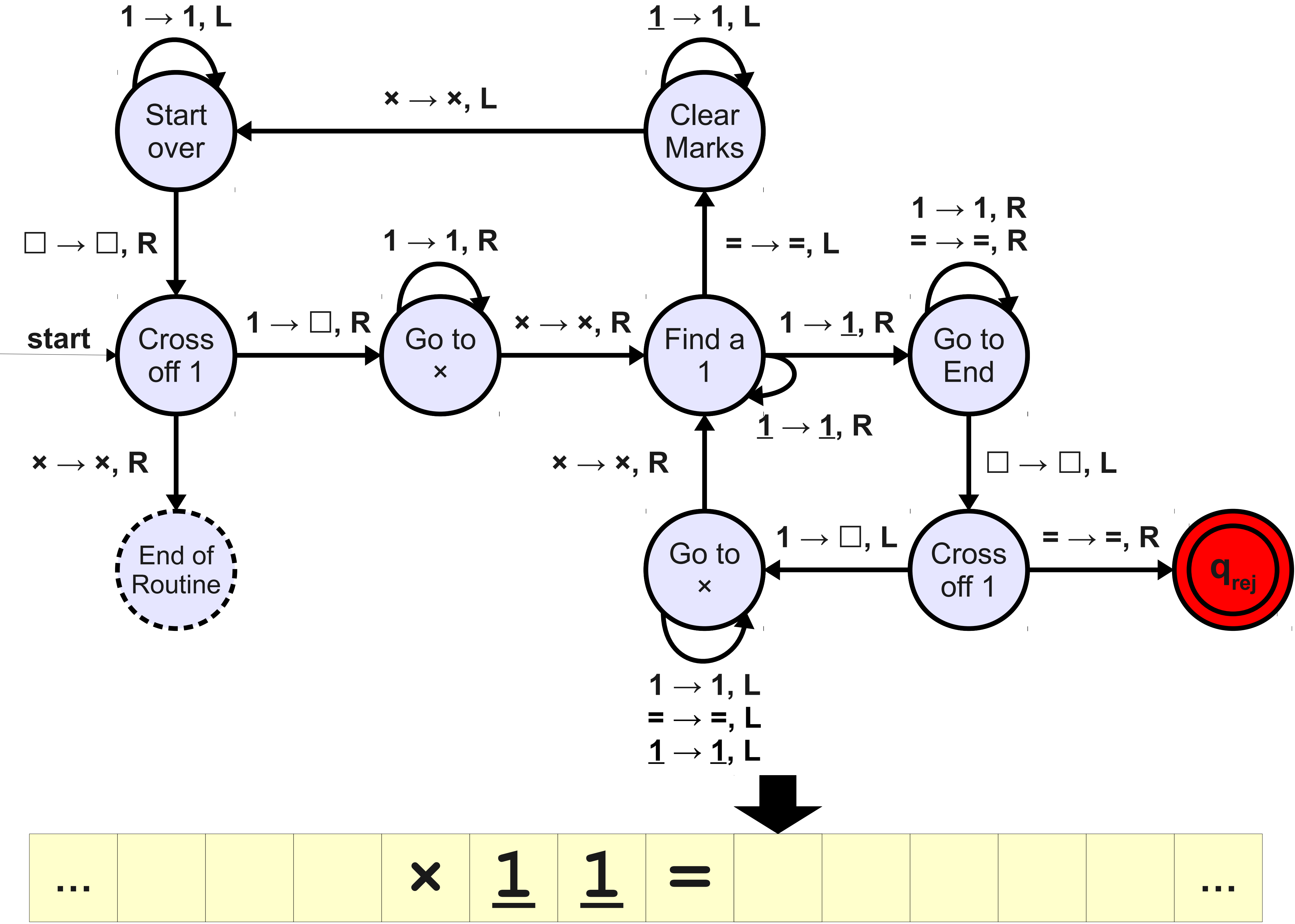
1 → 1, L
Start over
1 → 1, L
Clear Marks
× → ×, L
□ → □, R
1 → 1, R
= → =, L
1 → 1, R
= → =, R
start
Cross off 1
1 → □, R
Go to ×
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
× → ×, R
× → ×, R
□ → □, L
End of Routine
Go to ×
1 → □, L
Cross off 1
1 → 1, L
= → =, L
1 → 1, L
…
×
1
1
=
…



1 → 1, L
Start over
$\underline{1}$ → 1, L
Clear Marks
× → ×, L
□ → □, R
1 → 1, R
1 → 1, R
= → =, R
= → =, L
start
Cross off 1
1 → □, R
Go to ×
× → ×, R
Find a 1
1 → $\underline{1}$, R
Go to End
$\underline{1}$ → $\underline{1}$, R
× → ×, R
× → ×, R
□ → □, L
End of Routine
Go to ×
1 → □, L
Cross off 1
= → =, R
q_{rej}
1 → 1, L
= → =, L
$\underline{1}$ → $\underline{1}$, L
…
×
$\underline{1}$
$\underline{1}$
=
…



1 → 1, L
Start over
× → ×, L
1 → 1, L
Clear Marks
□ → □, R
1 → 1, R
= → =, L
1 → 1, R
= → =, R
start
Cross off 1
1 → □, R
Go to ×
× → ×, R
Find a 1
1 → 1, R
Go to End
1 → 1, R
× → ×, R
× → ×, R
□ → □, L
End of Routine
Go to ×
1 → □, L
Cross off 1
= → =, R
q_rej
1 → 1, L
= → =, L
1 → 1, L
…
×
1
1
=
…



1 → 1, L
Start over
1 → 1, L
Clear Marks
× → ×, L
□ → □, R
start
Cross off 1
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1 → 1, R
Go to ×
× → ×, R
Find a 1
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= → =, R
Go to End
1 → 1, R
1 → 1, R
× → ×, R
× → ×, R
End of Routine
□ → □, L
Go to ×
1 → □, L
Cross off 1
= → =, R
q_rej
1 → 1, L
= → =, L
1 → 1, L
… × 1 1 = …

The Final Piece

- If $m = 0$, we need to check that $p = 0$.
- Input has form $\texttt{×1}^n\texttt{=1}^p$.
- In other words, accept iff string matches the regular expression $\texttt{×1}^*\texttt{=}$.
- Exercise: Build a TM to check this!

Turing Machines and Math

- Turing machines are capable of performing
  - Addition
  - Subtraction
  - Multiplication
  - Integer division
  - Exponentiation
  - Integer logarithms
  - **Plus a whole lot more…**

List Processing

- Suppose we have a list of strings represented as

$$w_1 \; : \; w_2 \; : \; \dots \; : \; w_n \; :$$

- What sorts of transformations can we perform on this list using a Turing machine?

Example: Take Odds

- Given a list of $2n$ strings

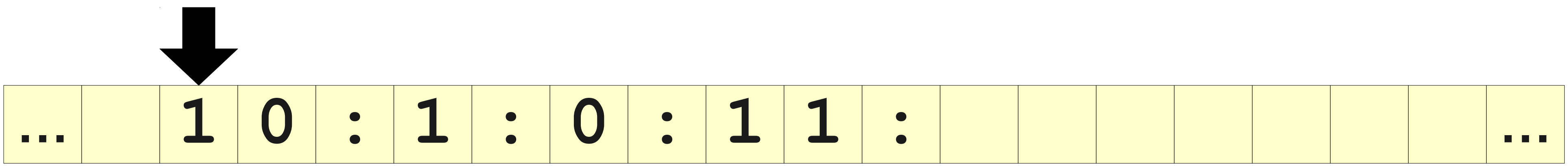
$$w_1 : w_2 : \dots : w_{2n} :$$

  filter the list to get back just the odd-numbered entries:

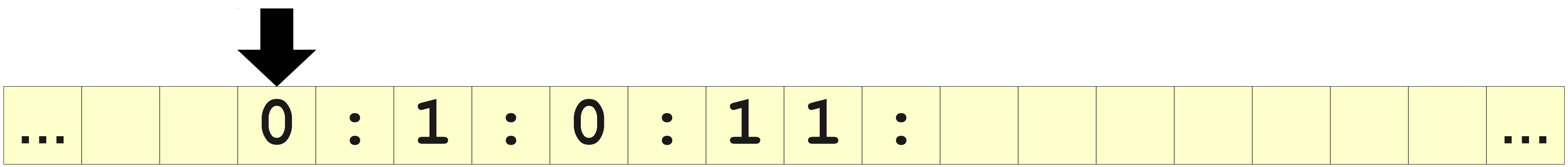
$$w_1 : w_3 : \dots : w_{2n-1} :$$

- How might we do this with a Turing machine?

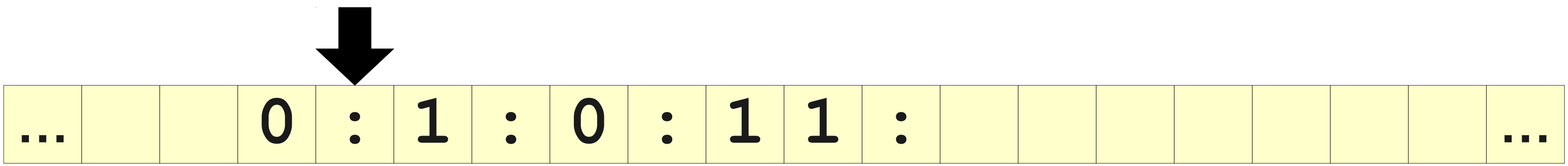
Taking Odds



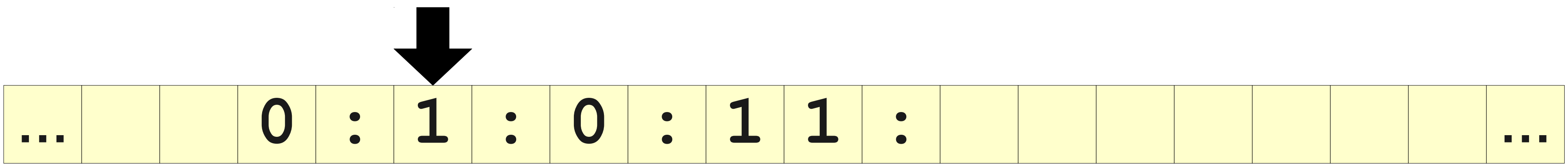
Taking Odds



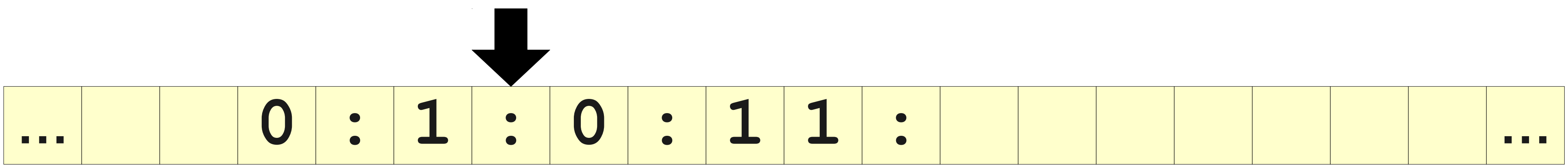
Taking Odds



Taking Odds



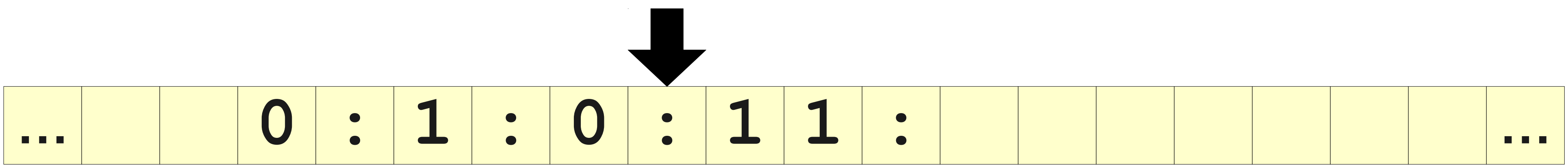
Taking Odds



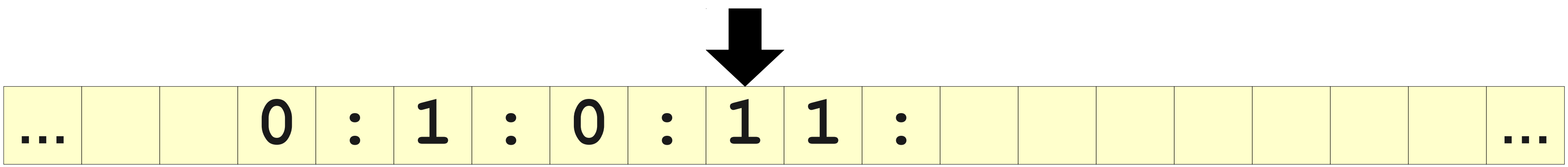
Taking Odds

...			0	:	1	:	0	:	1	1	:								...

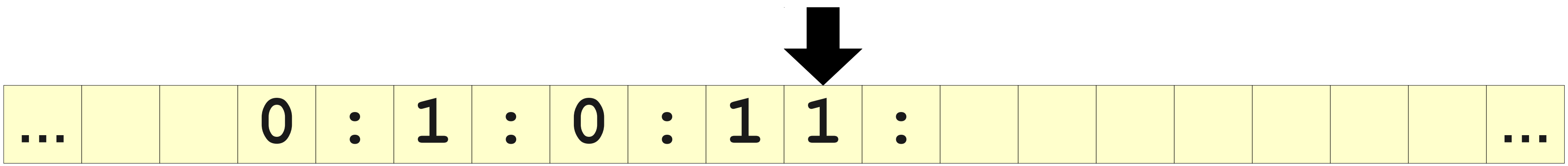
Taking Odds



Taking Odds



Taking Odds



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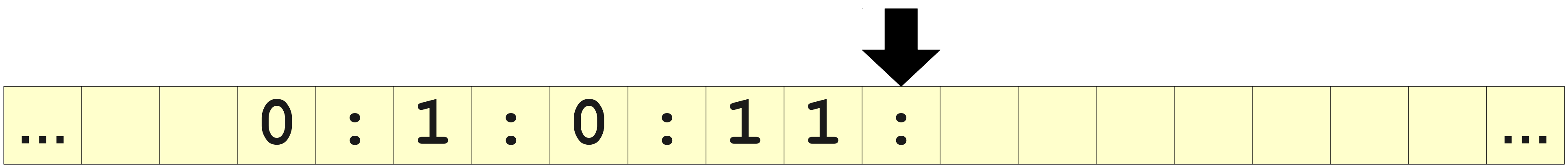
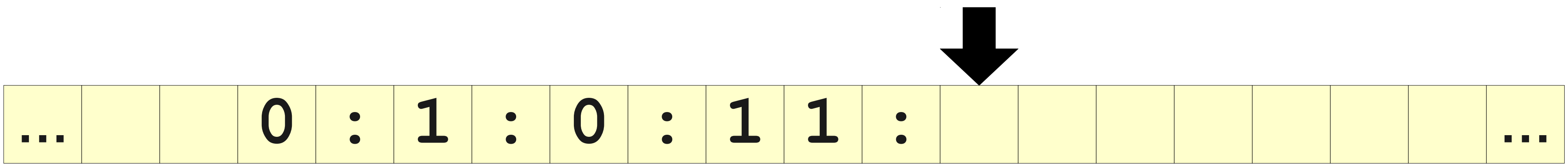
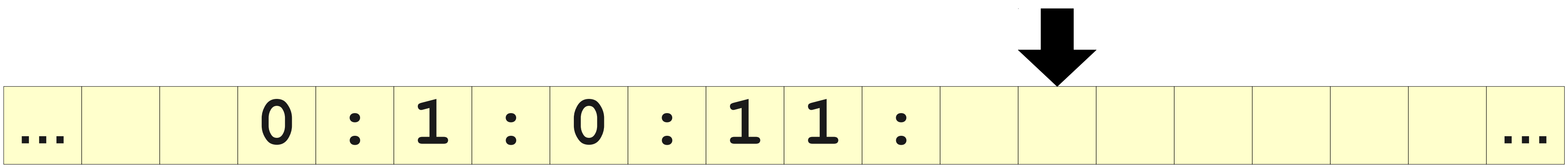


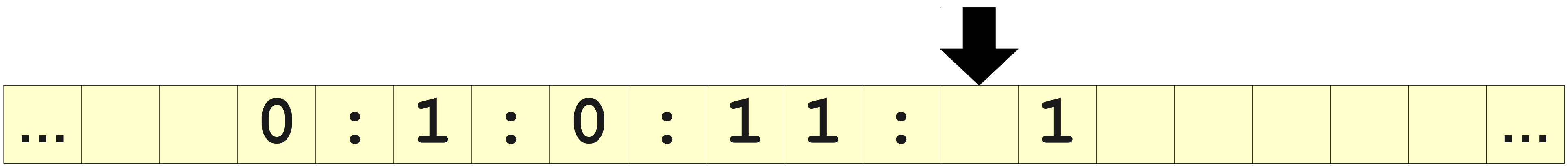
Taking Odds



Taking Odds



Taking Odds



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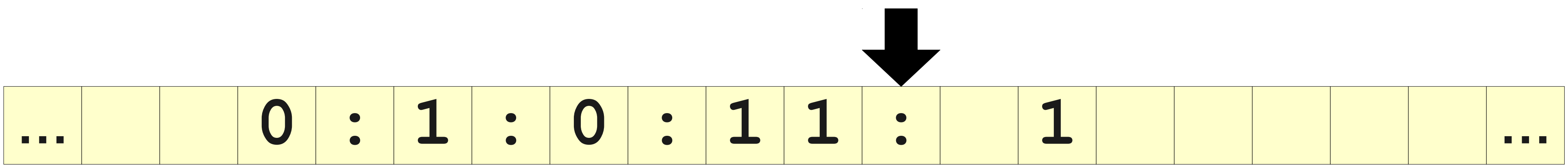

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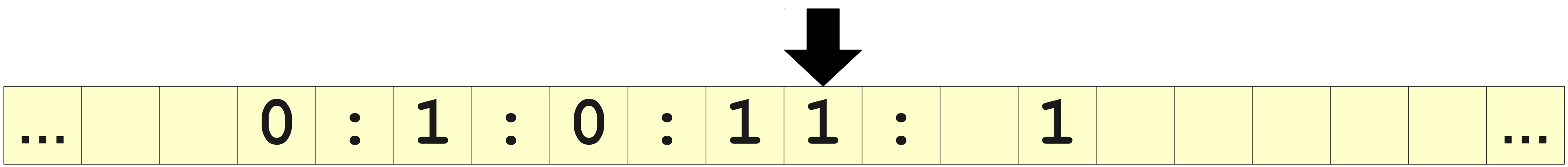

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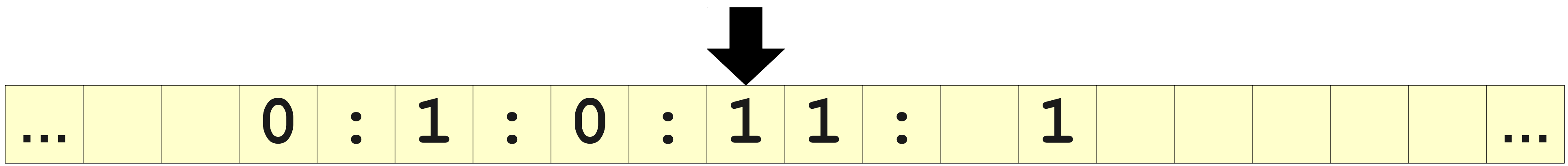
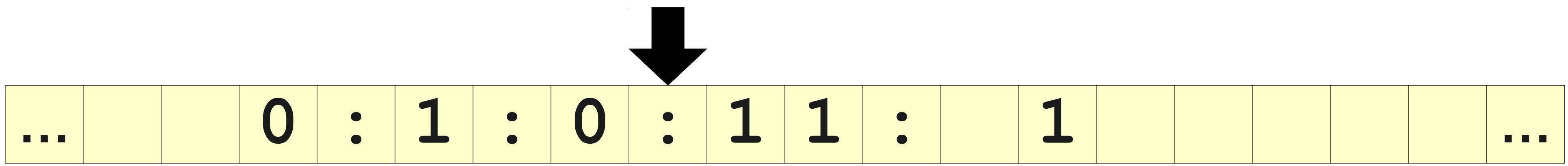


Taking Odds



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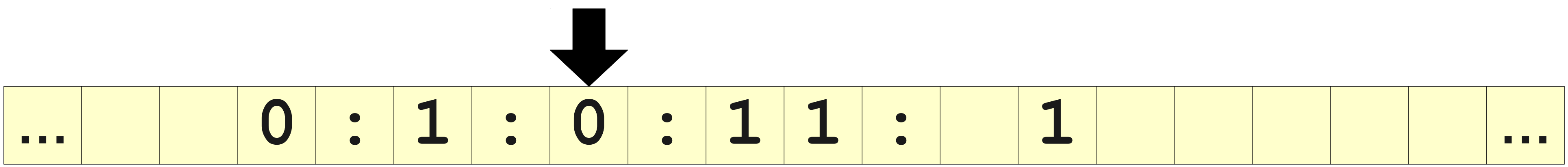
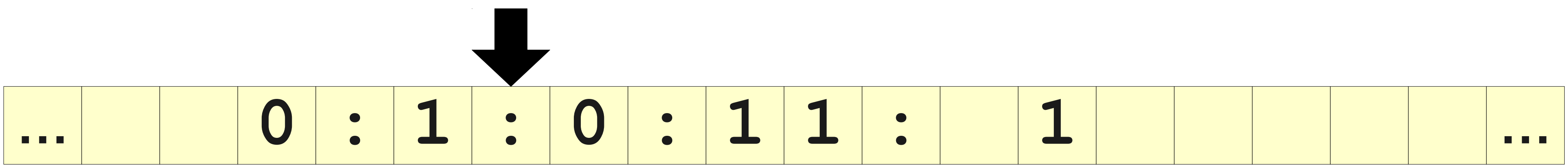
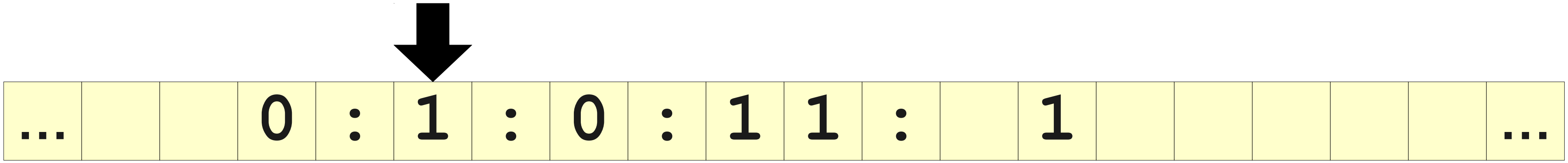


Taking Odds



Taking Odds



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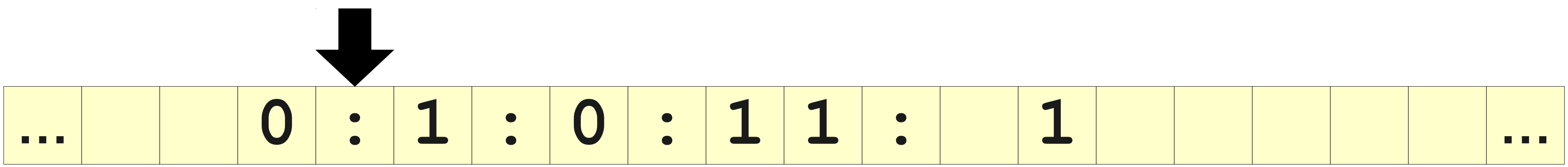

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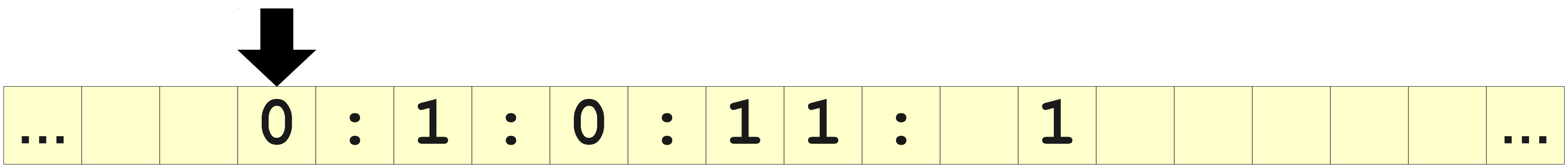

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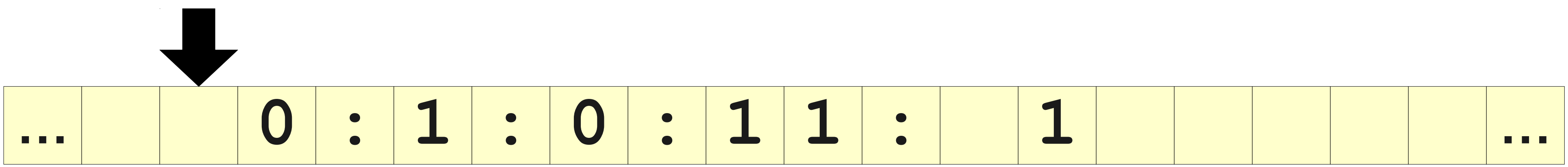

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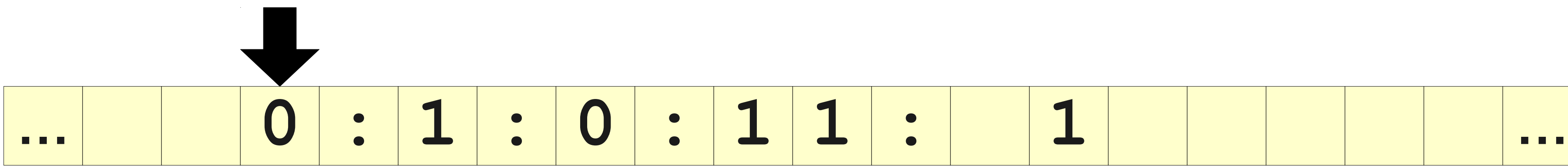

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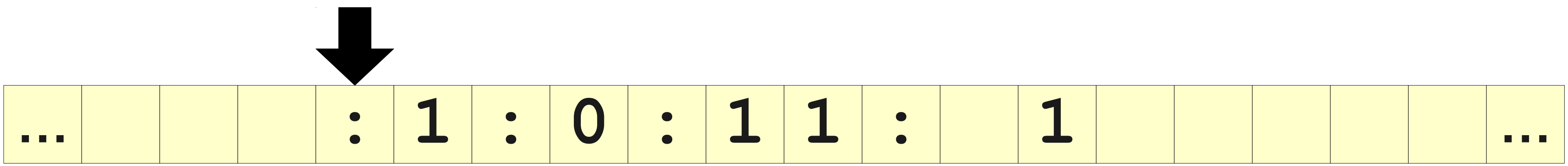

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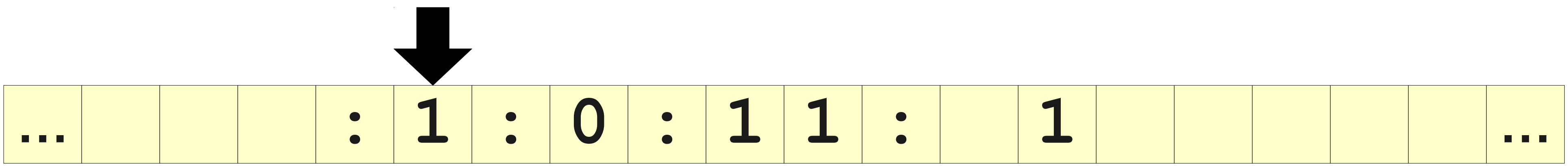

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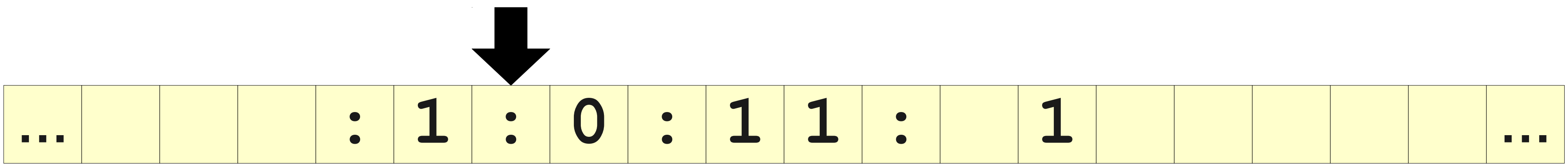
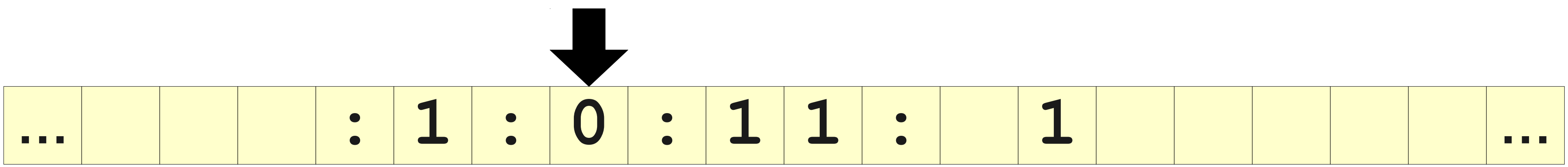


Taking Odds



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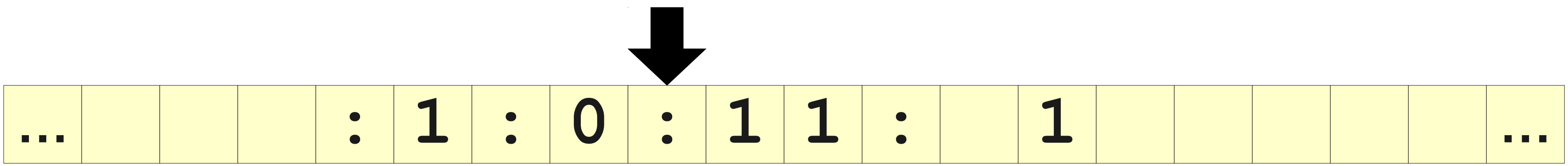

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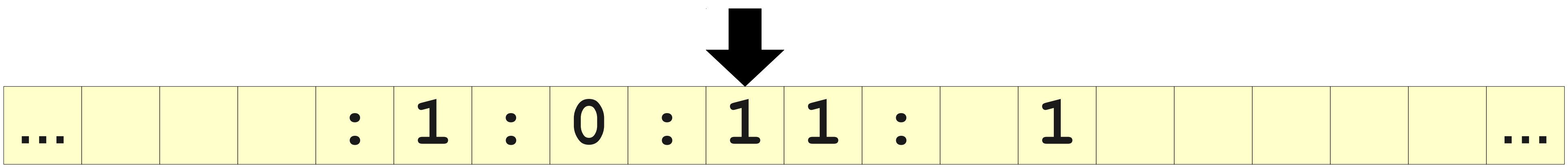

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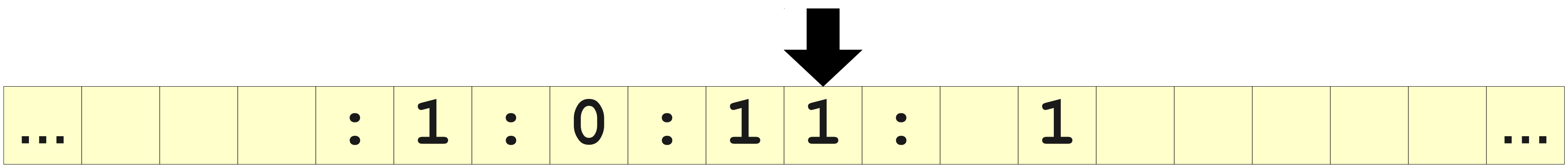

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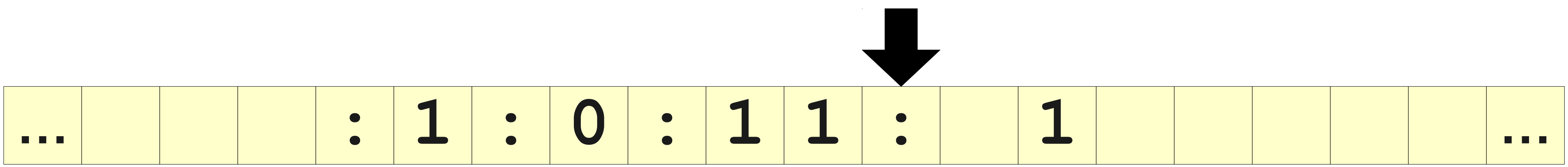

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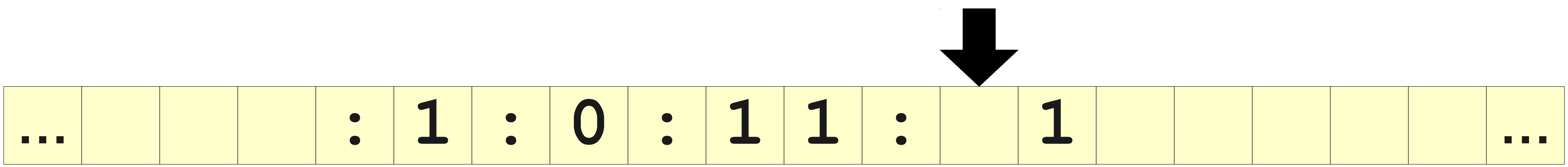

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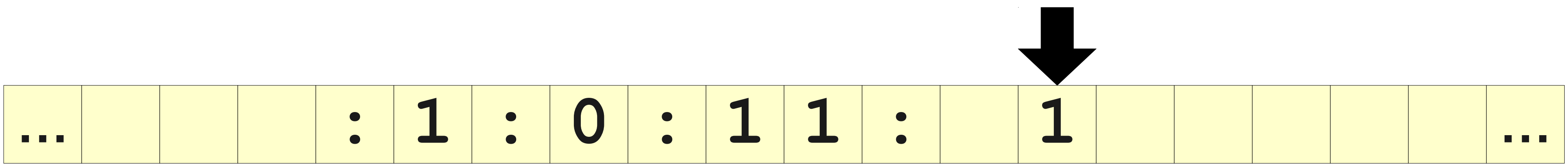
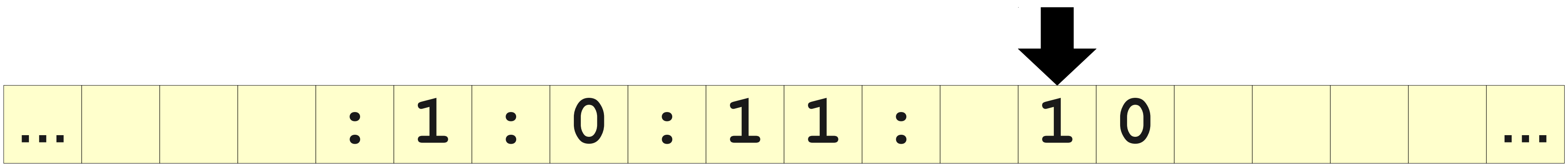


Taking Odds

...				:	1	:	0	:	1	1	:		1						...

Taking Odds



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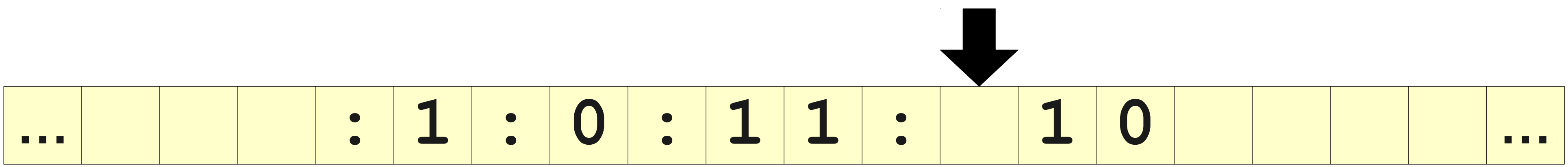

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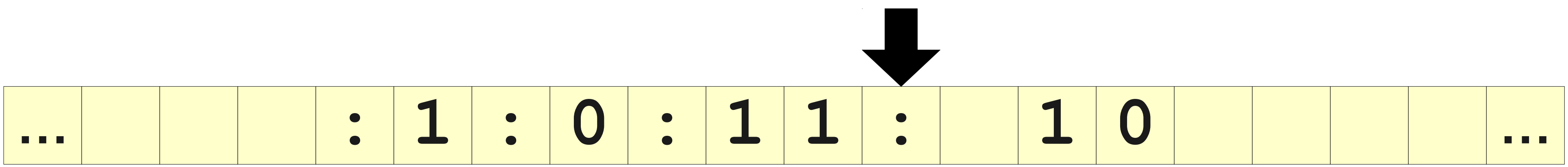

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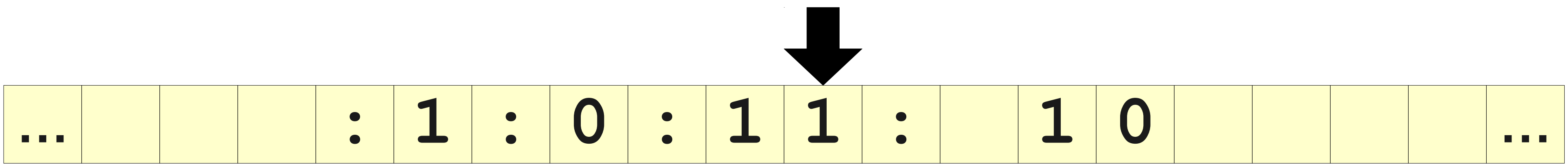

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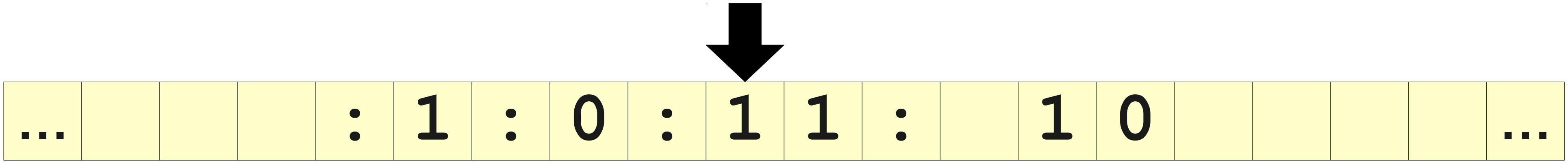
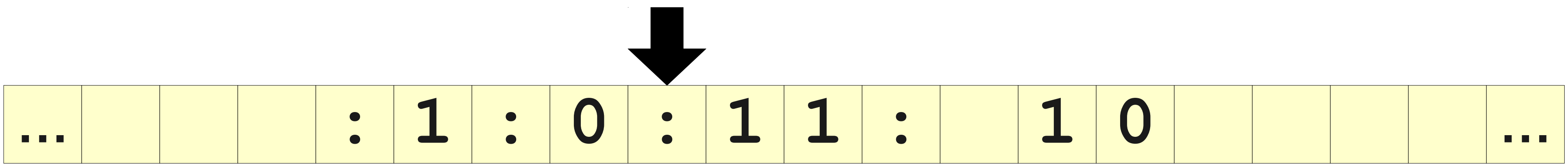


Taking Odds



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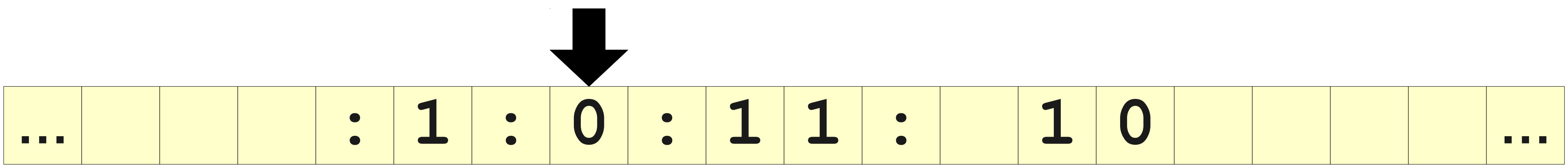

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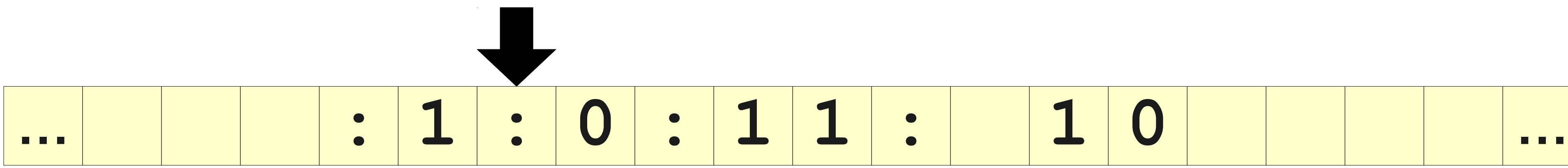

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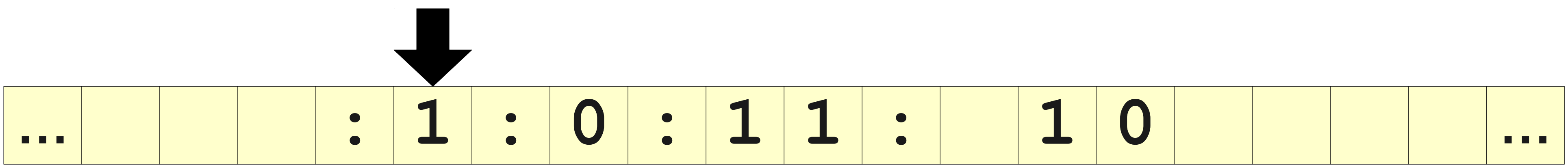

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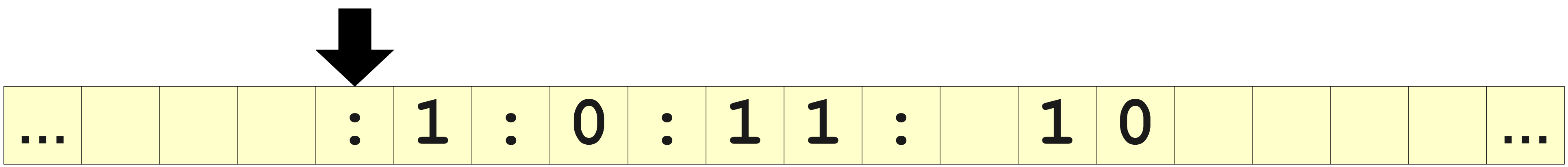

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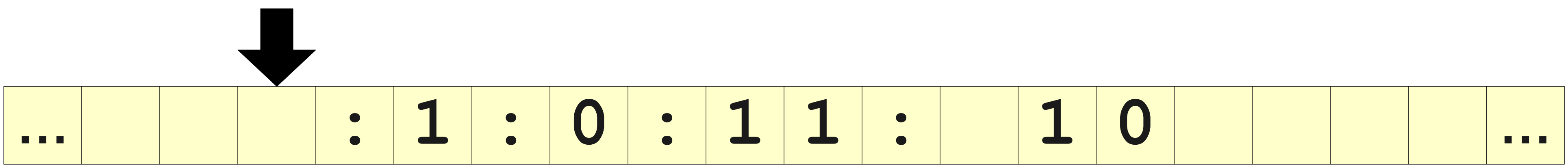

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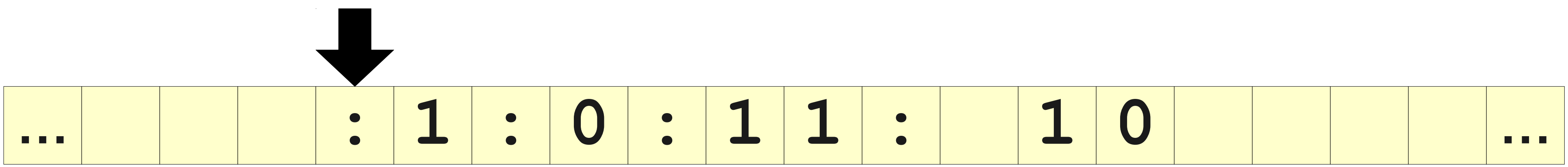

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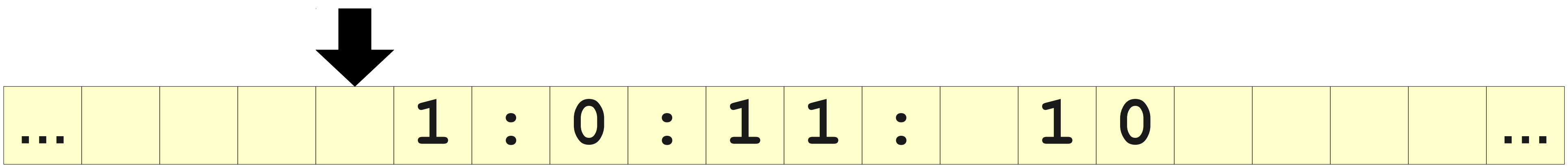

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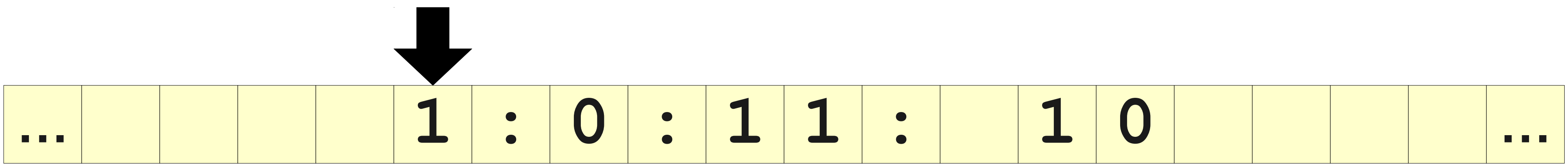

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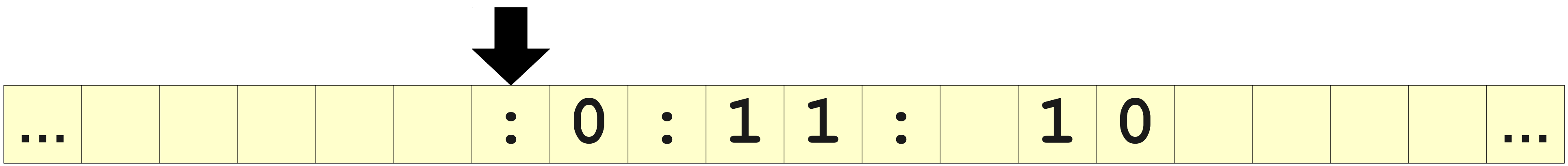

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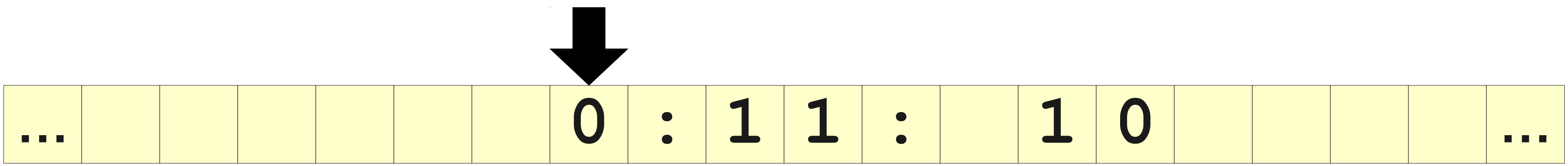

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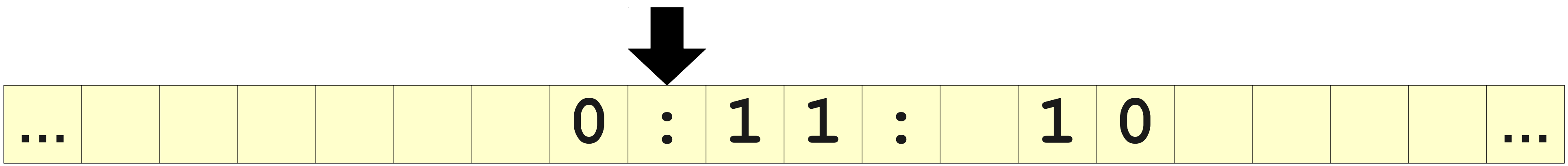

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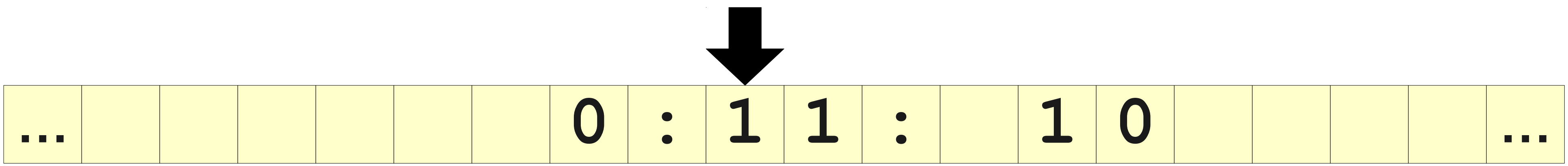

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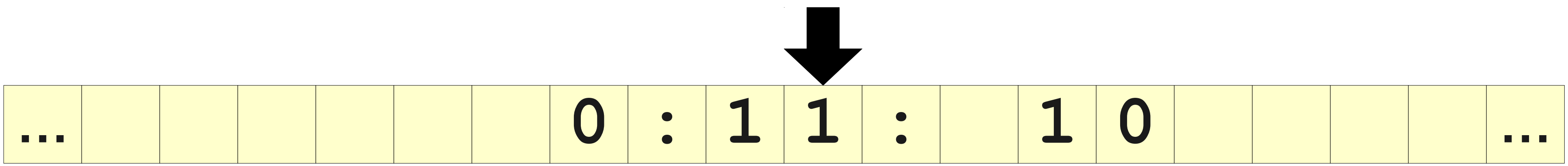

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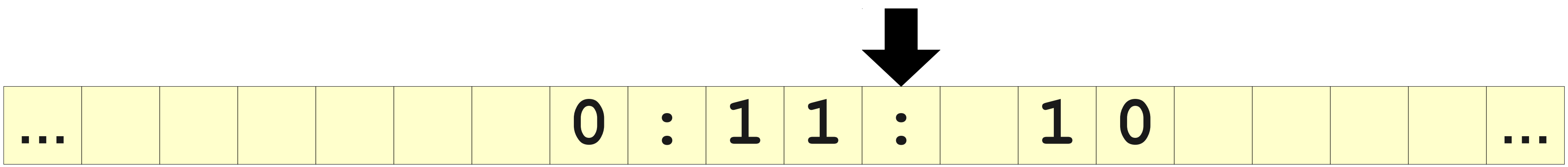

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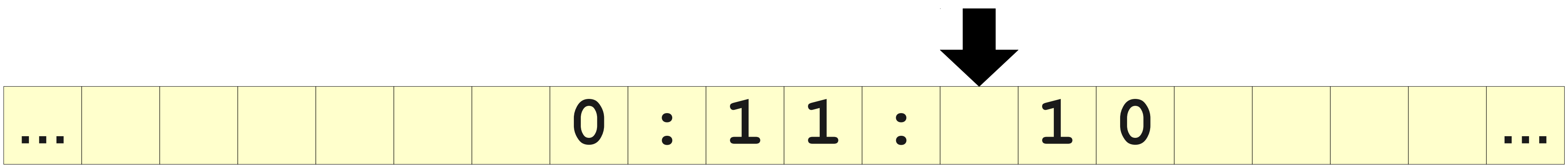

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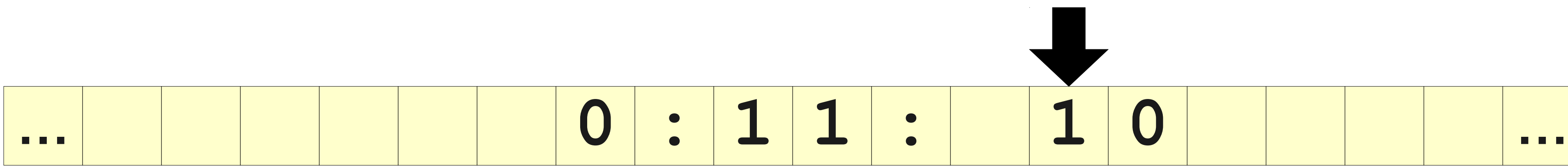

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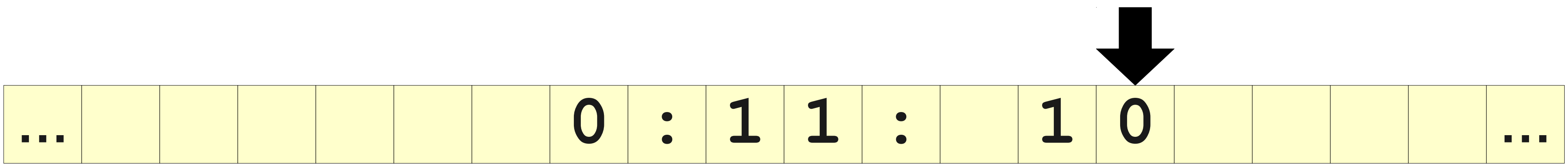

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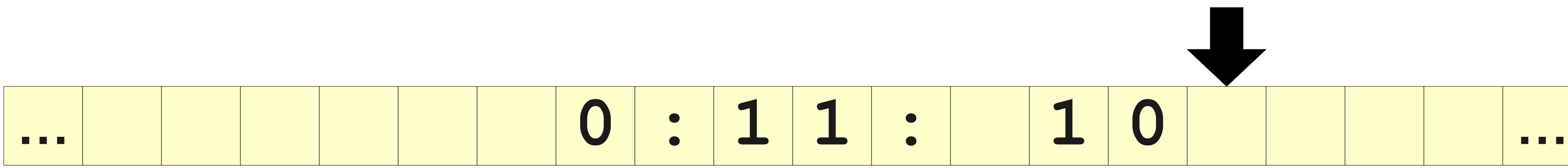

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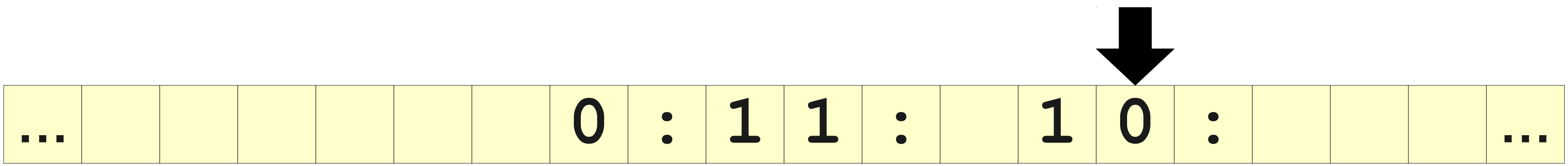

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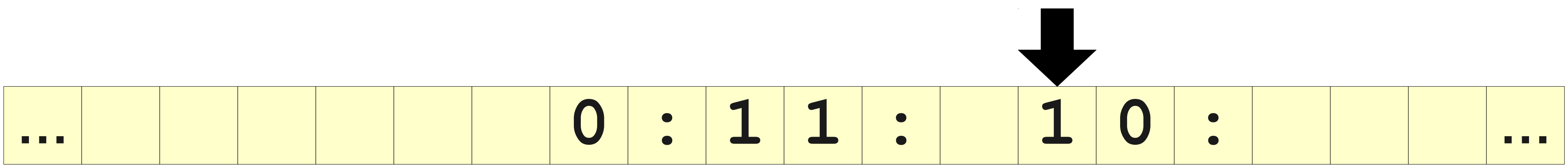

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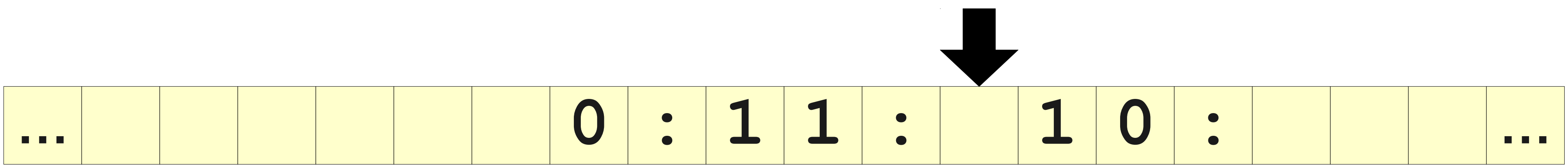

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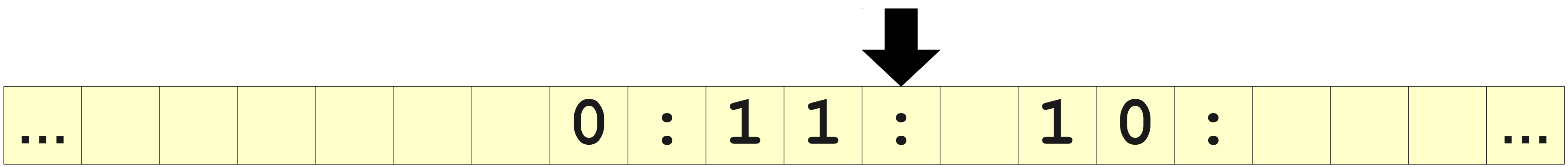

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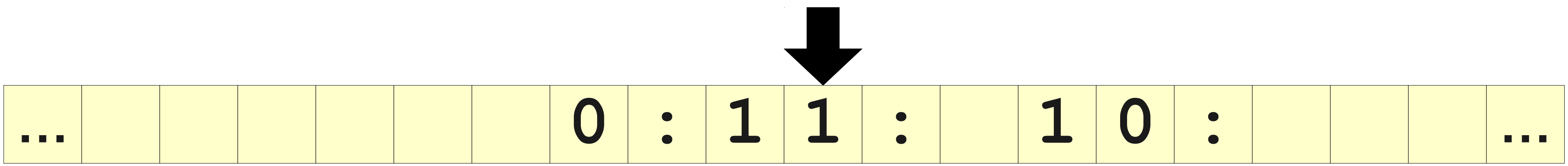

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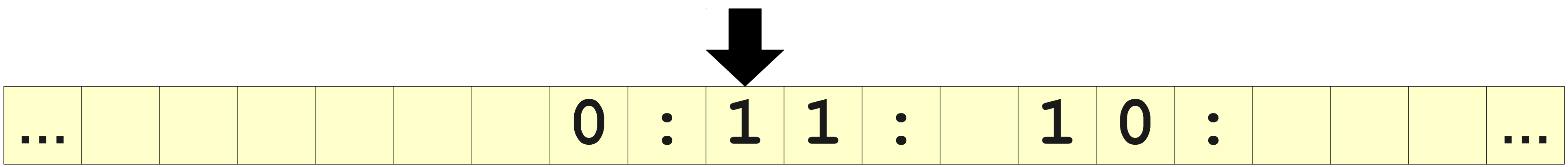

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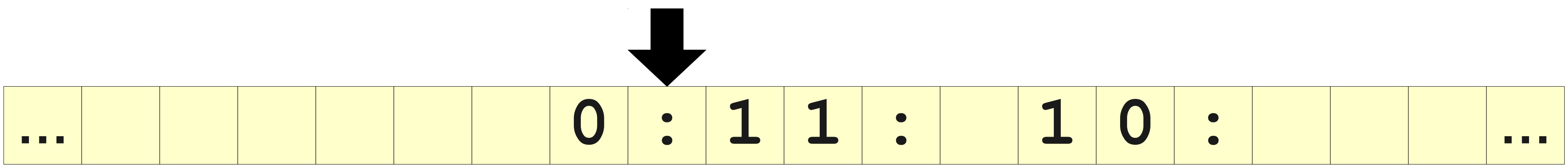

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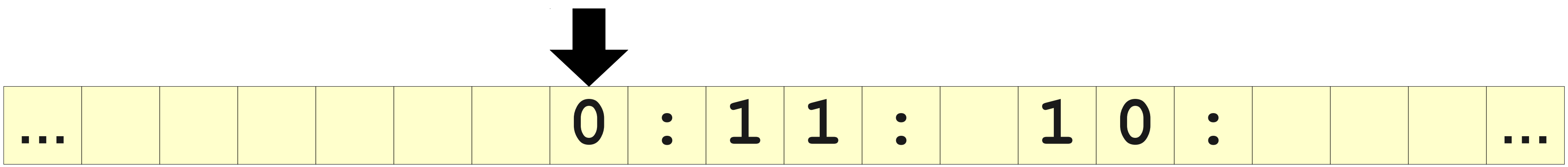

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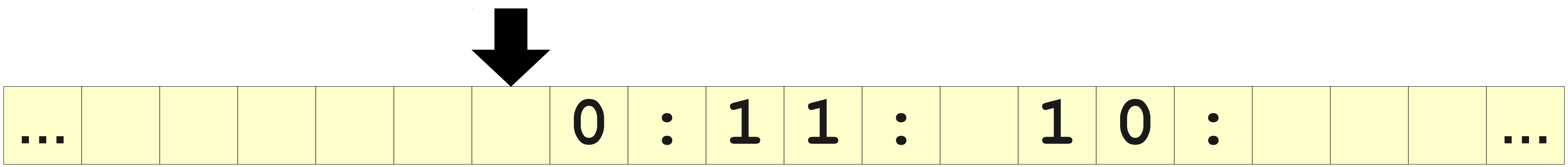

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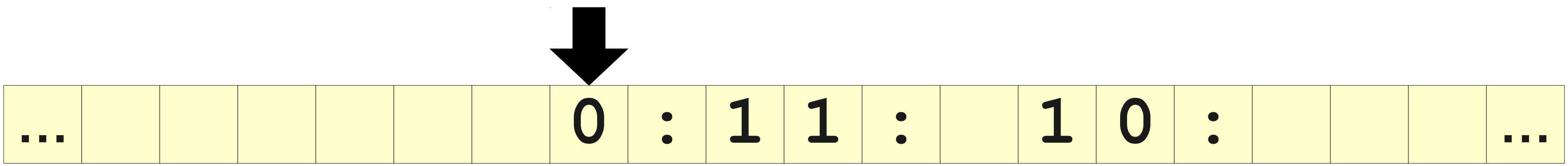

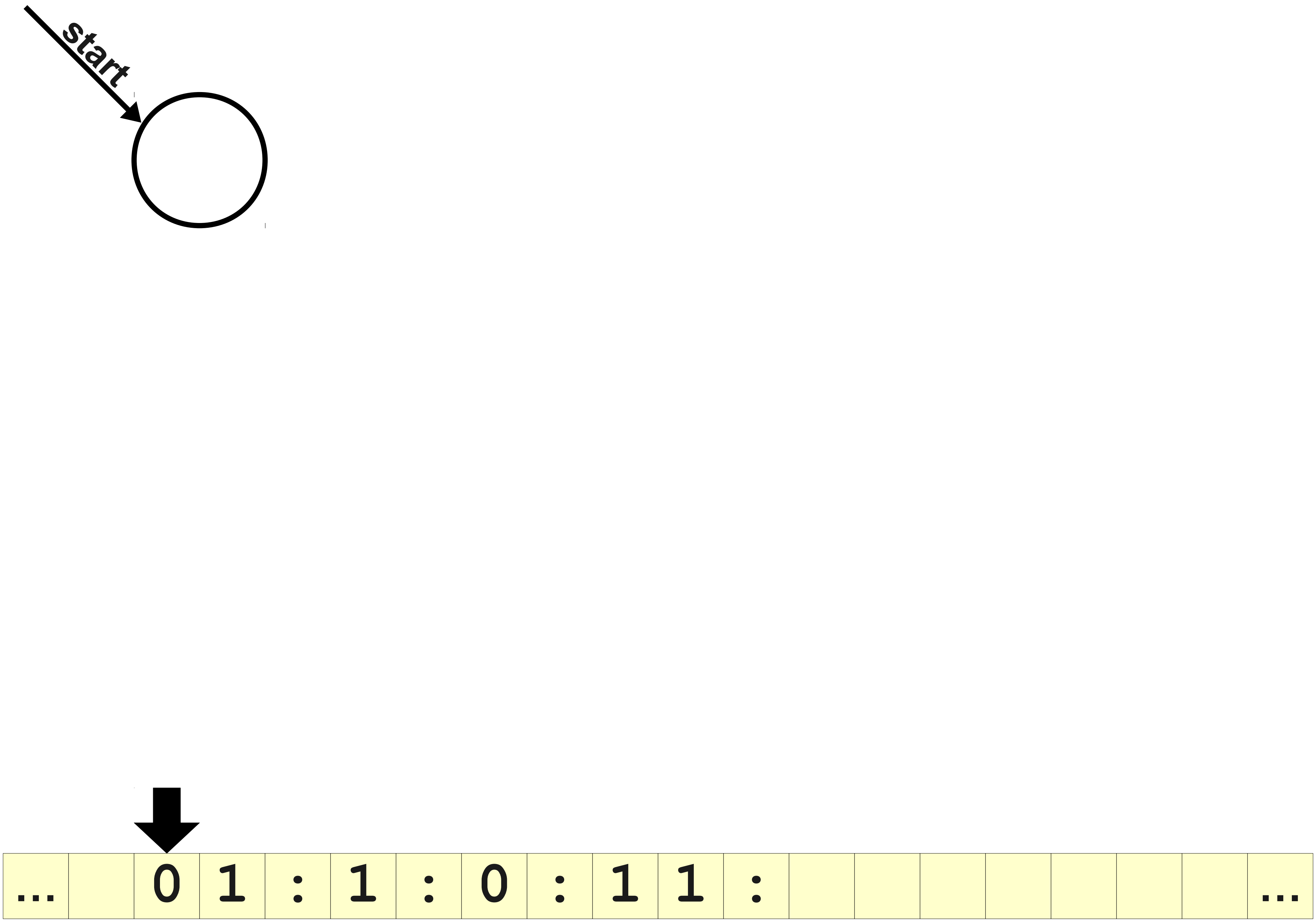
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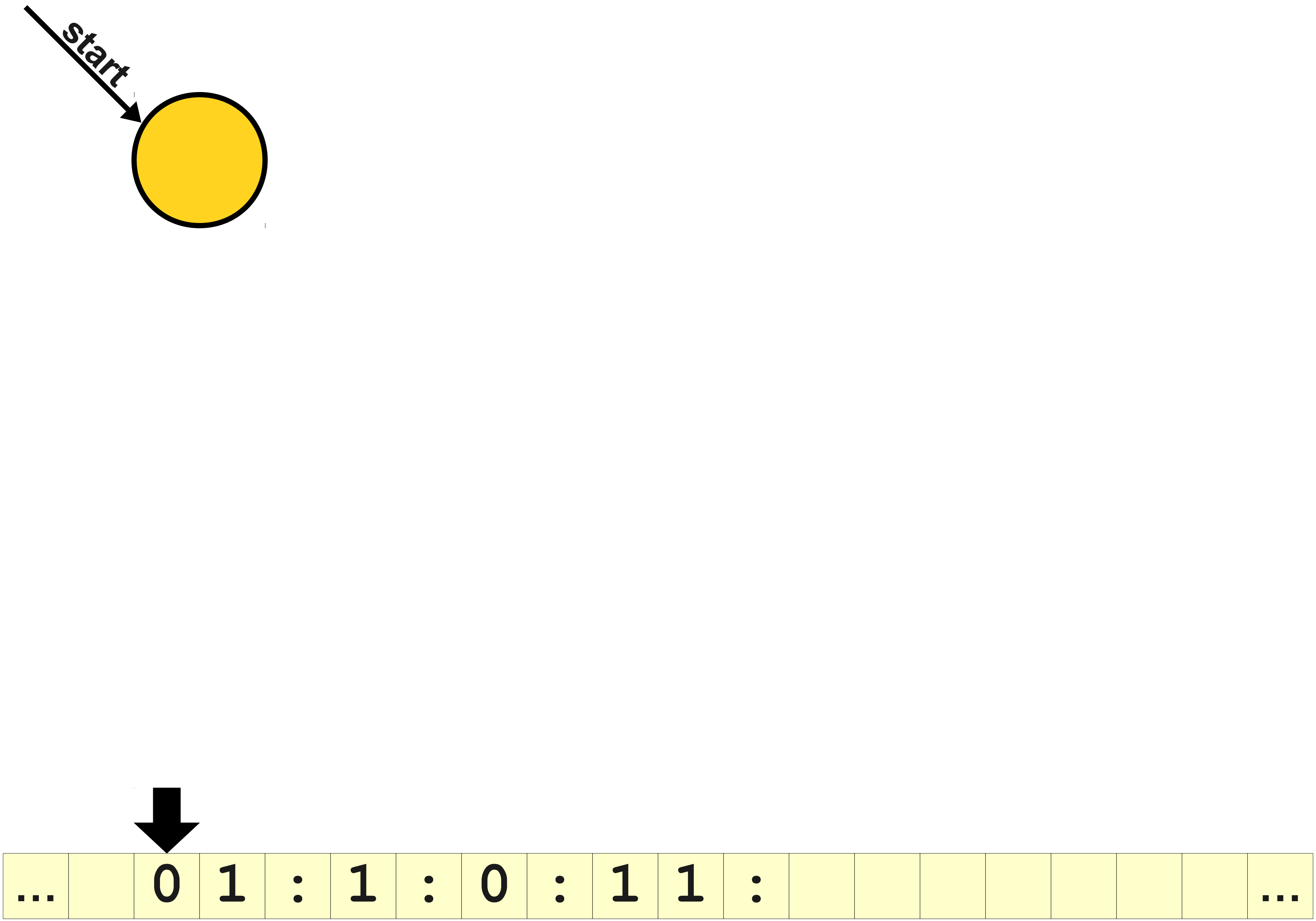
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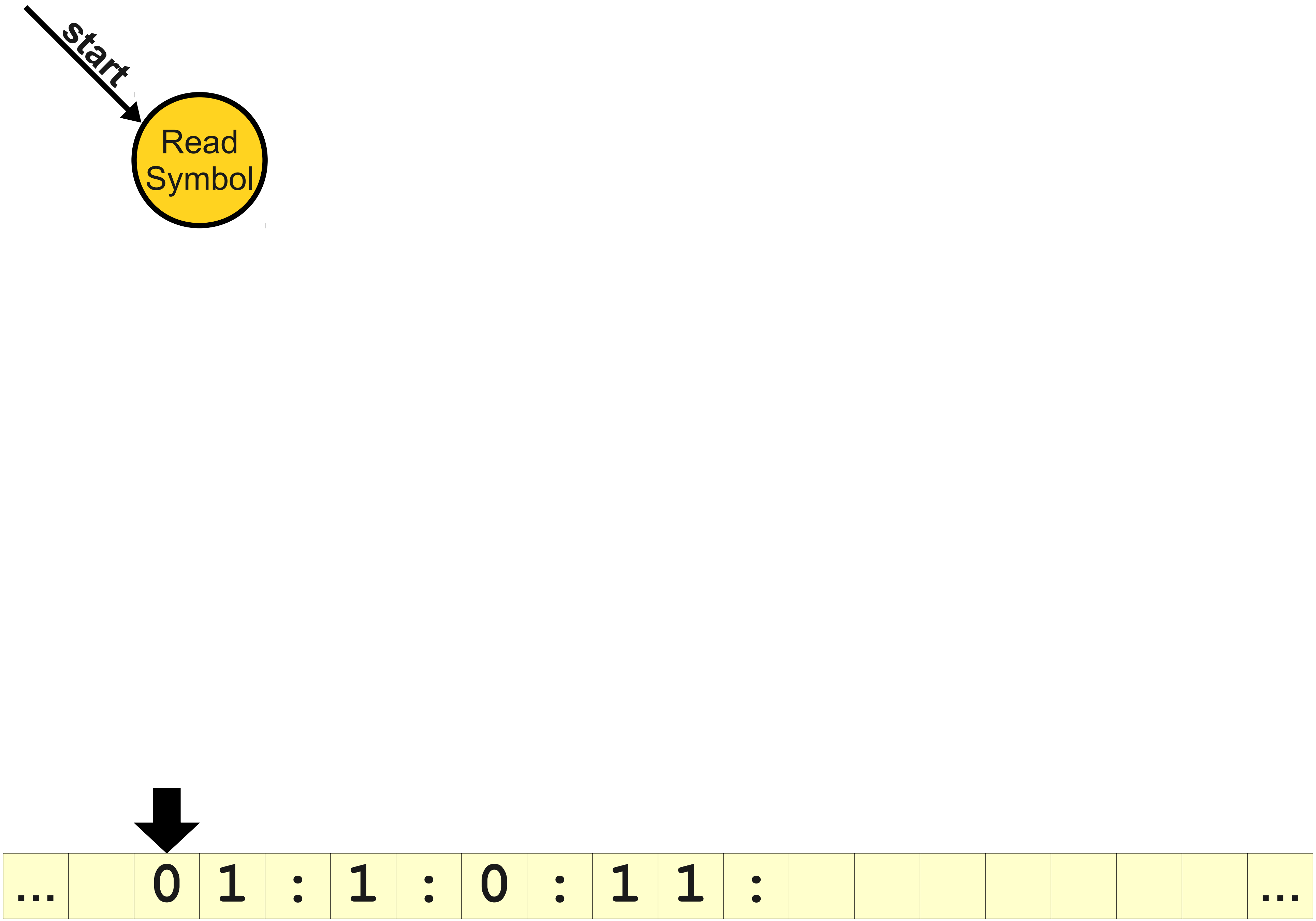
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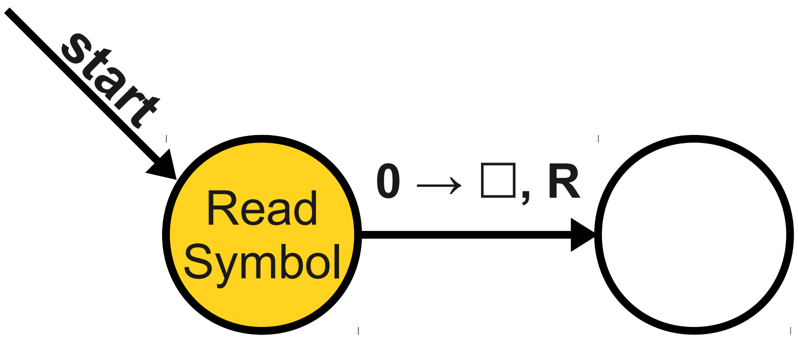
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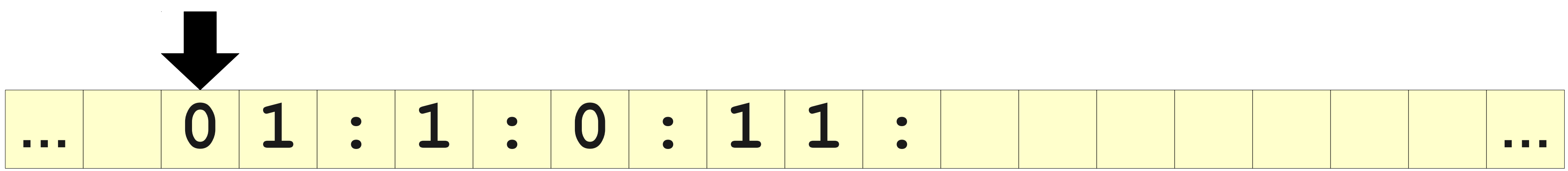
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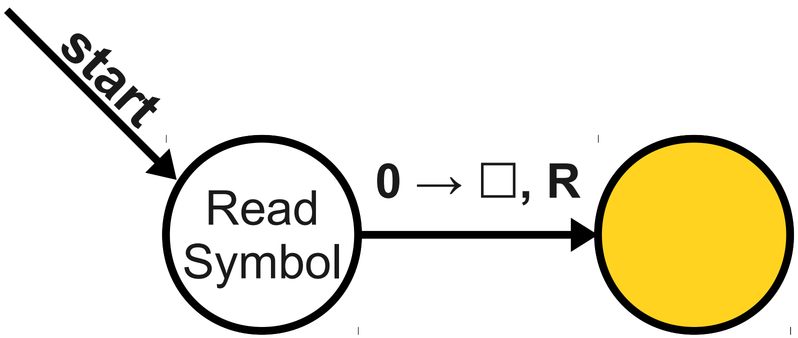
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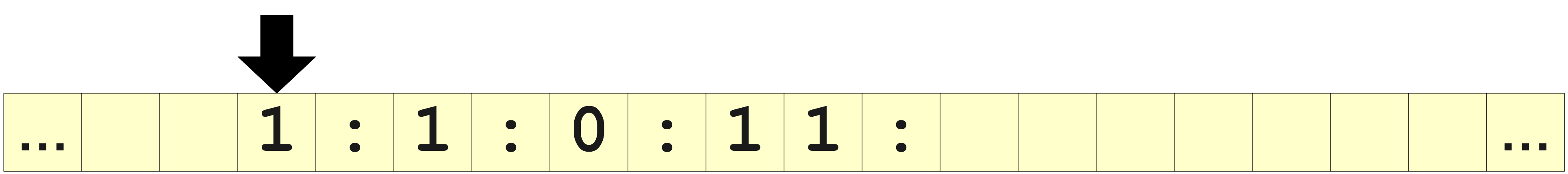
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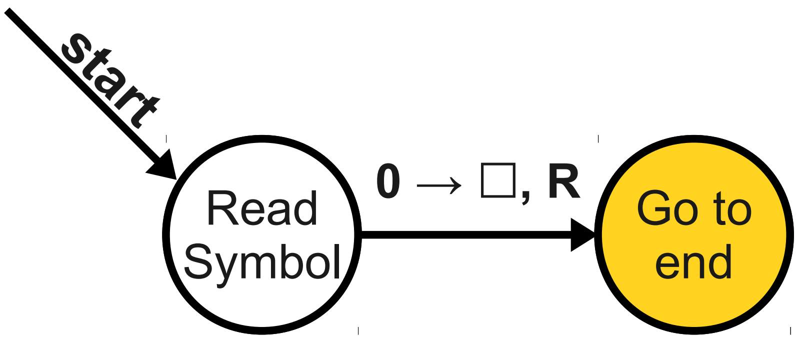
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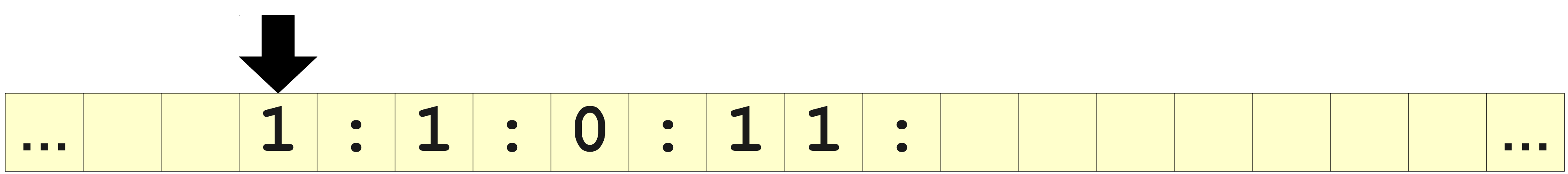
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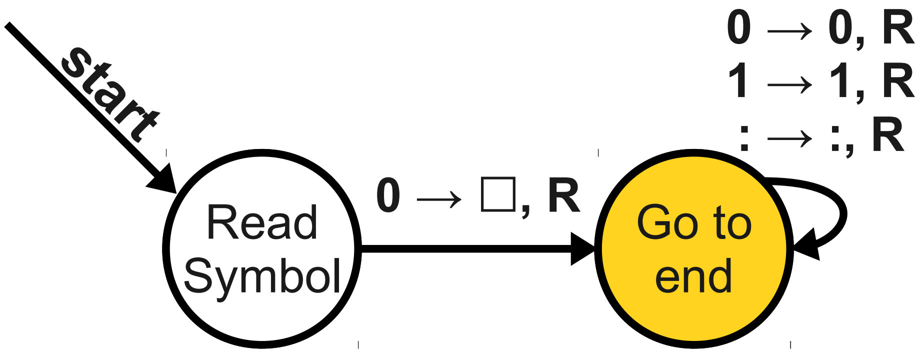

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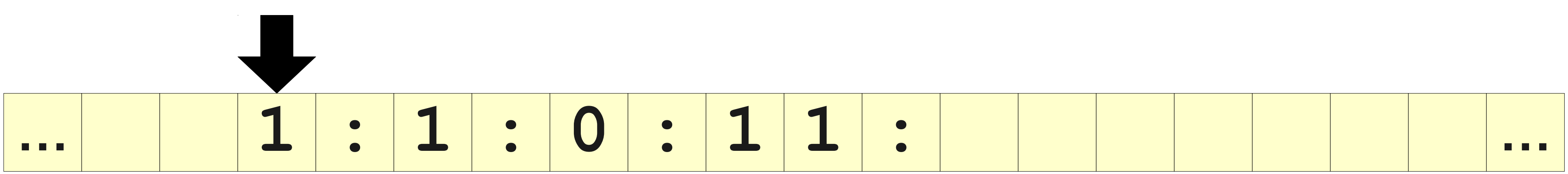

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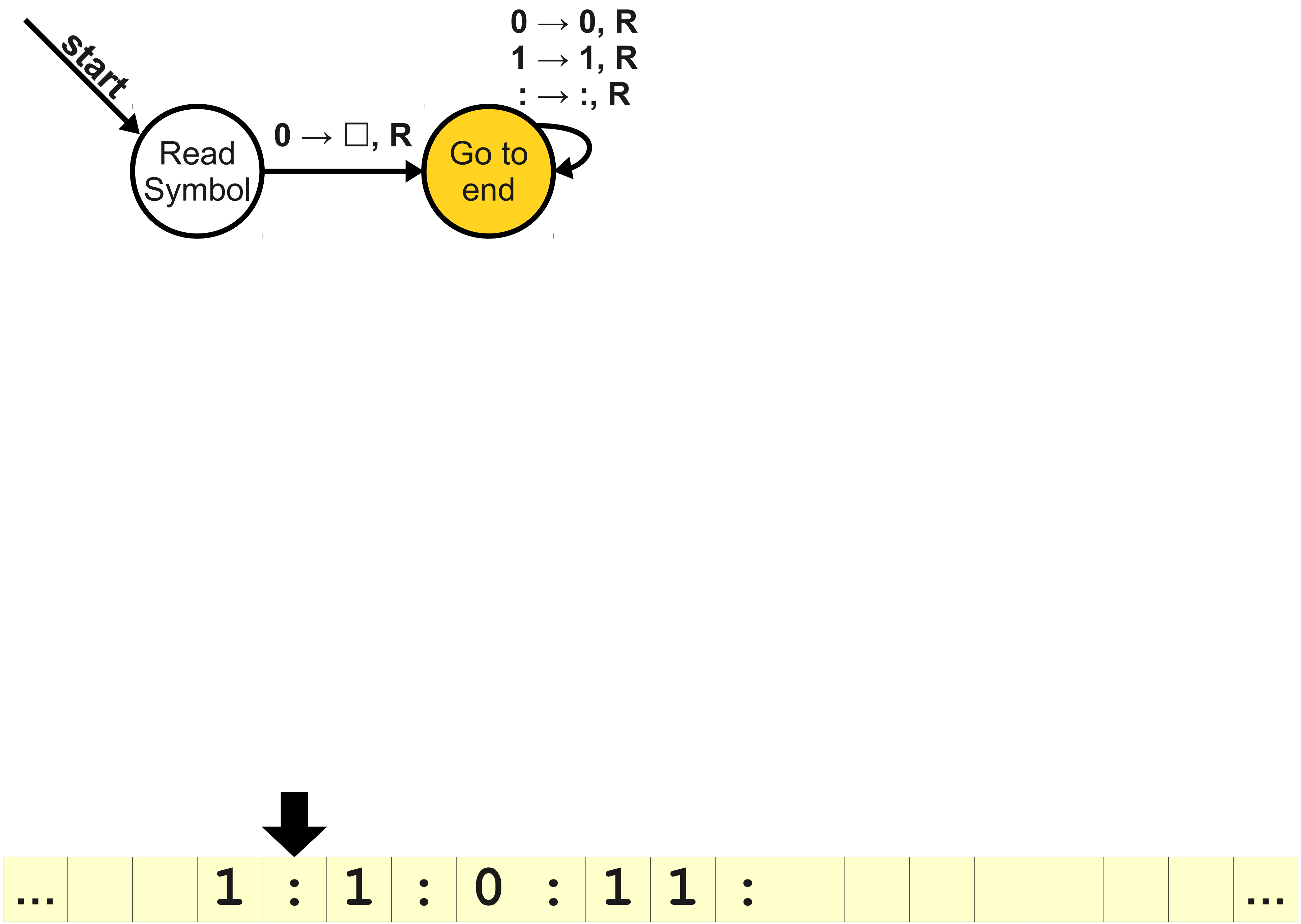
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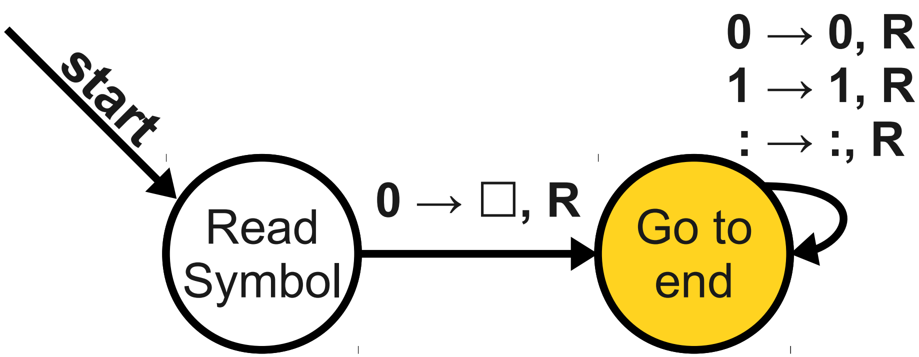
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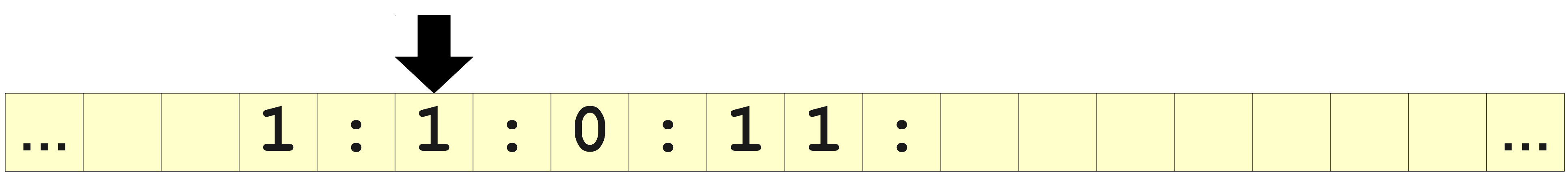
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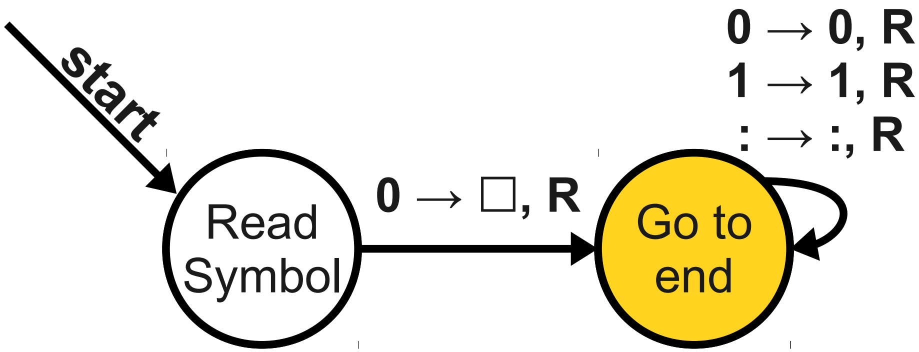
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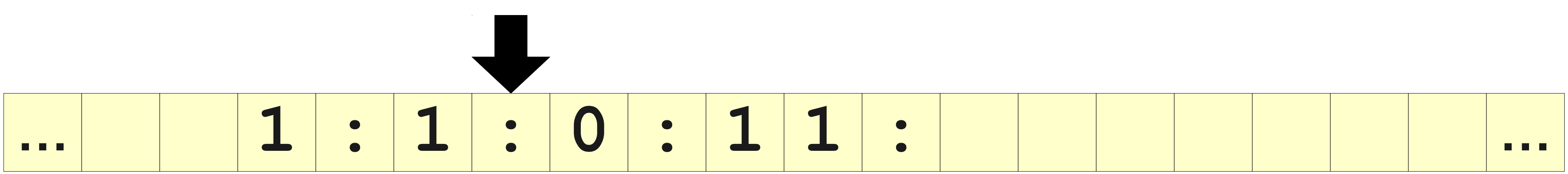
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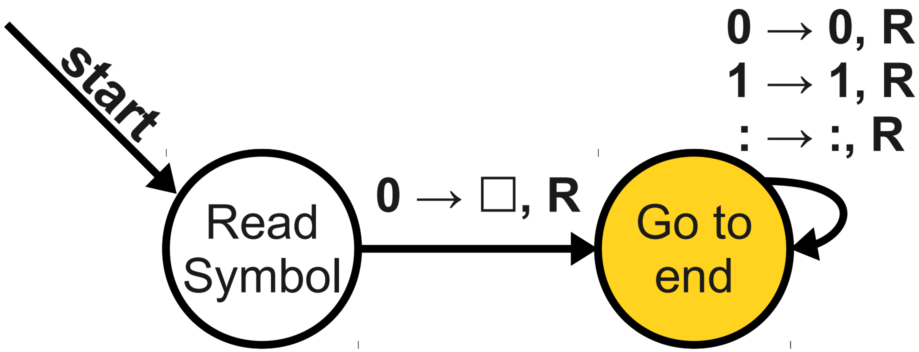
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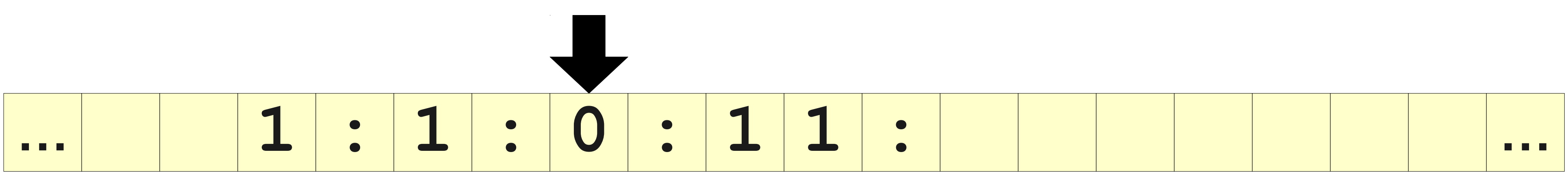
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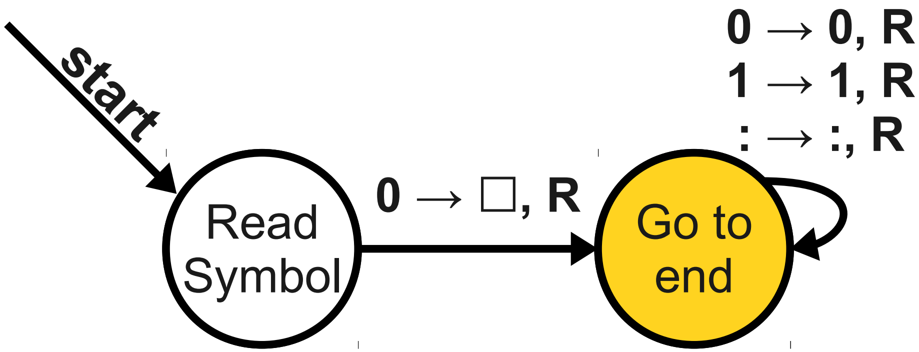
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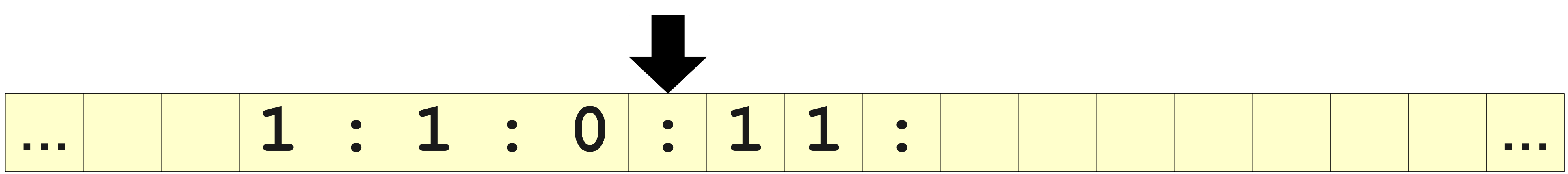
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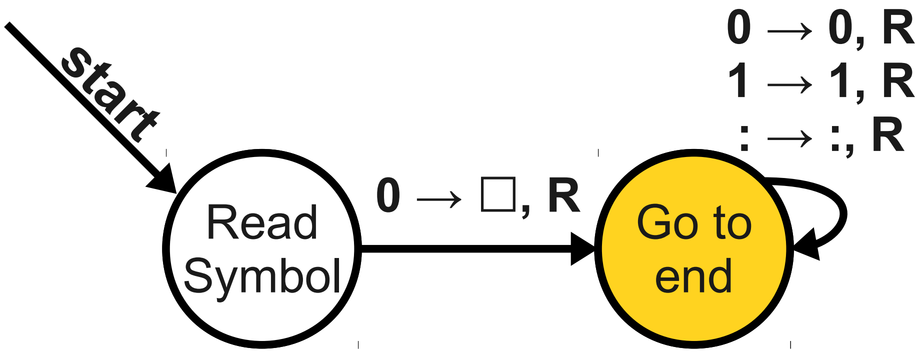
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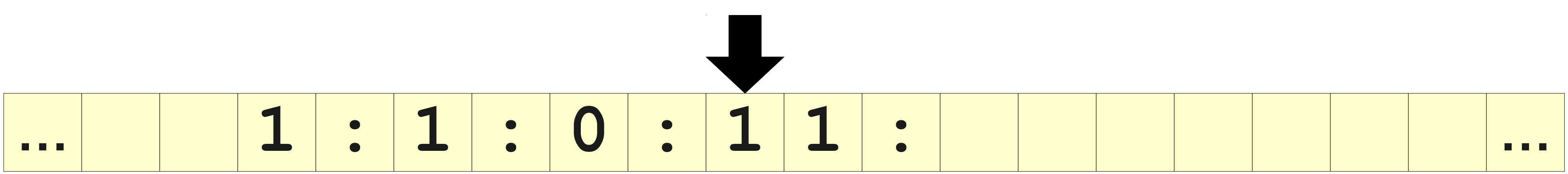
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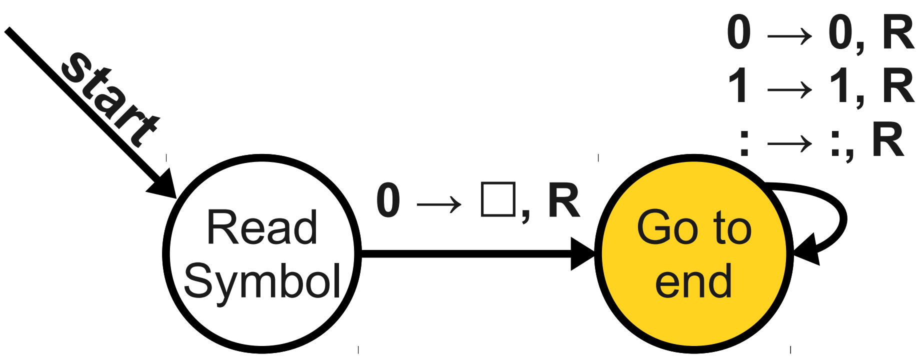
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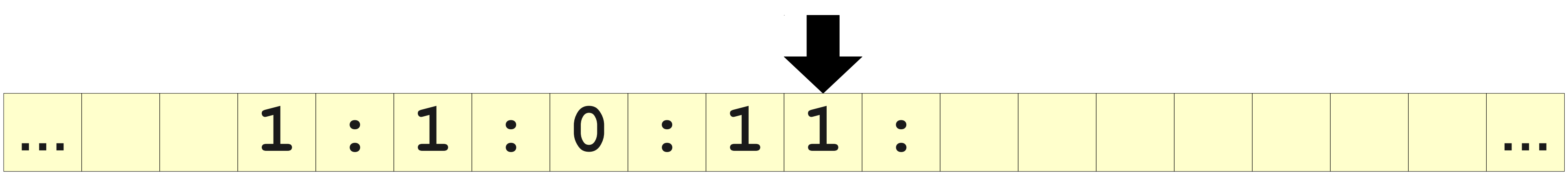
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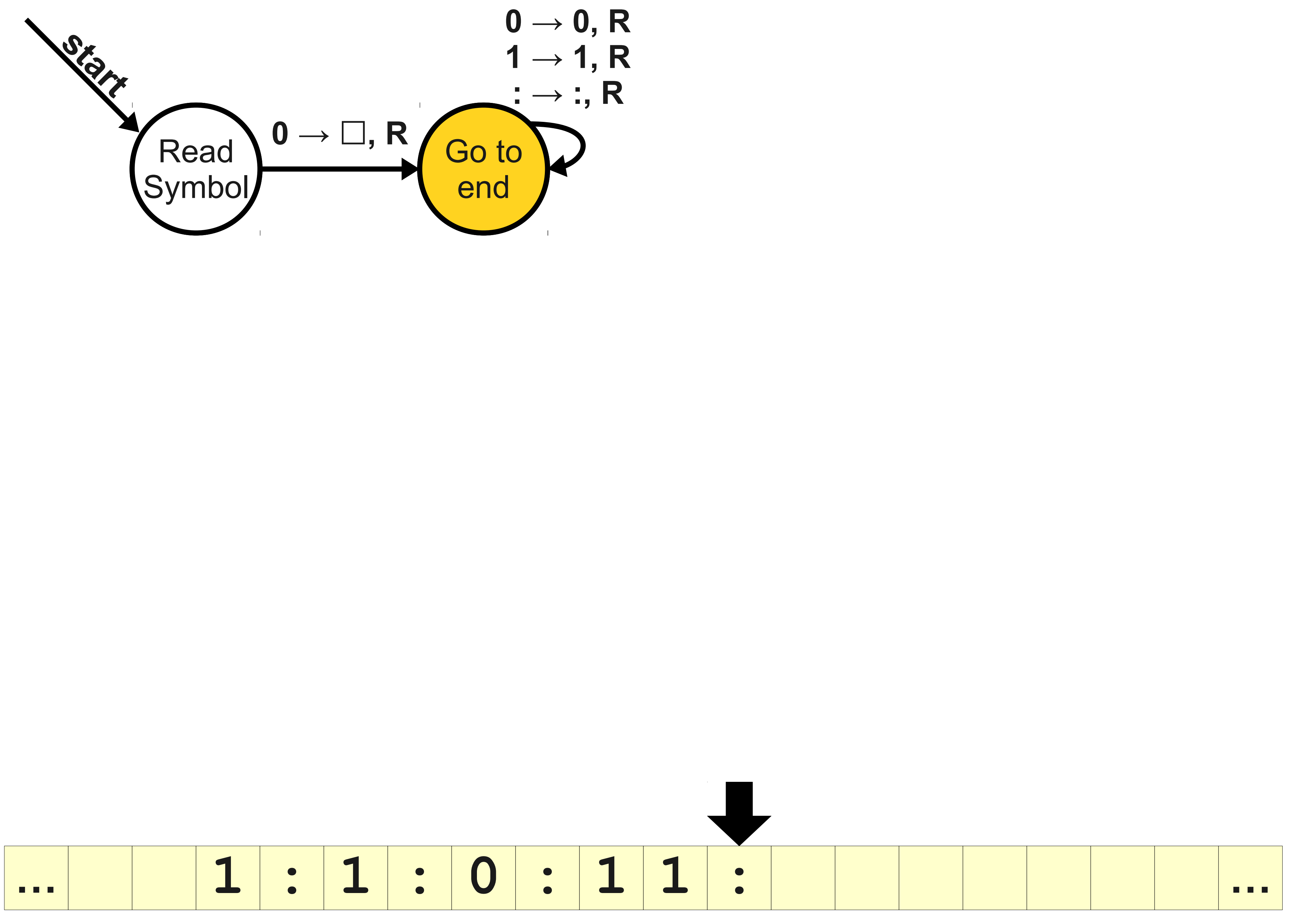

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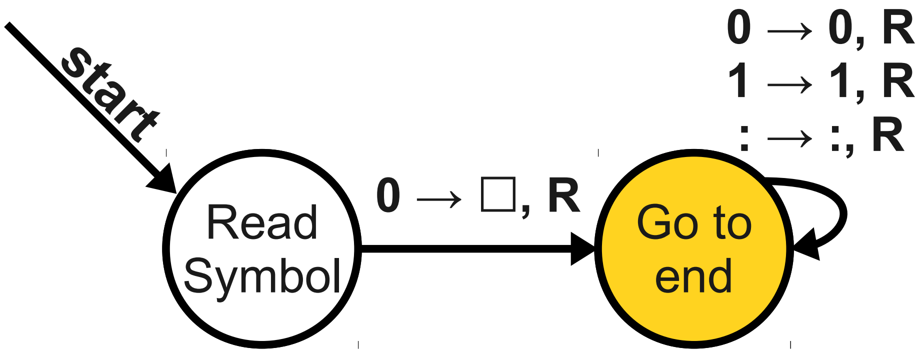
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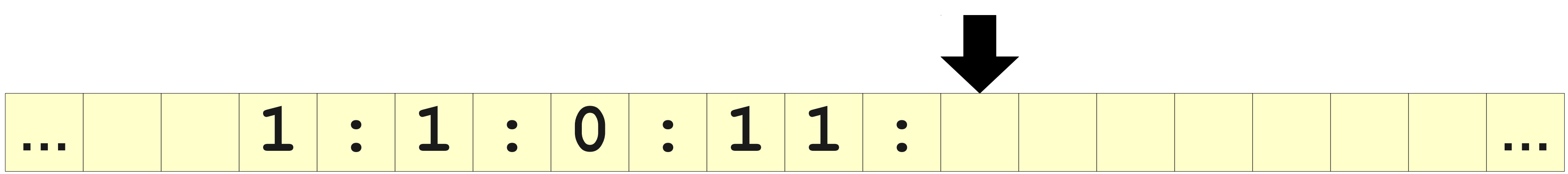
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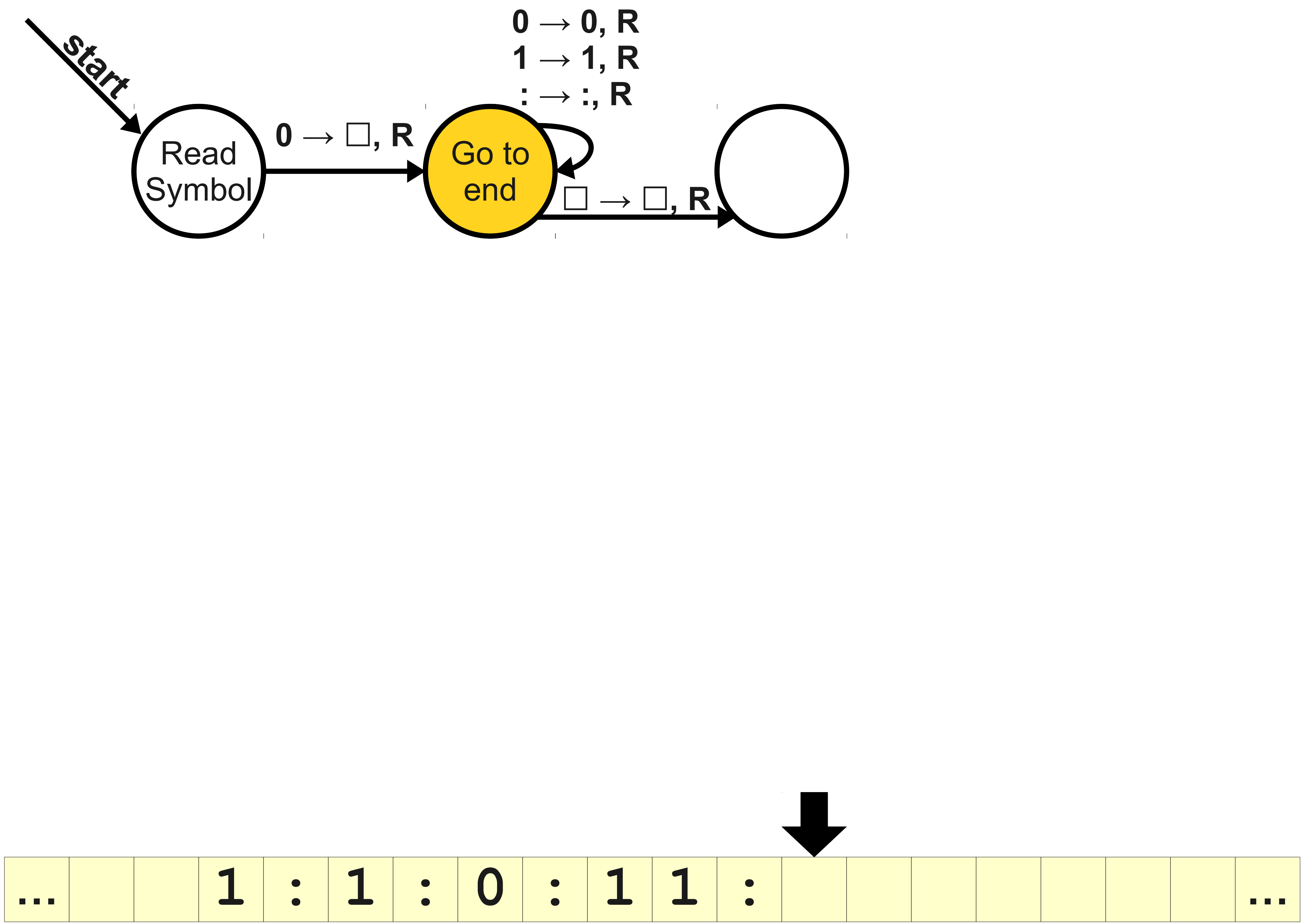
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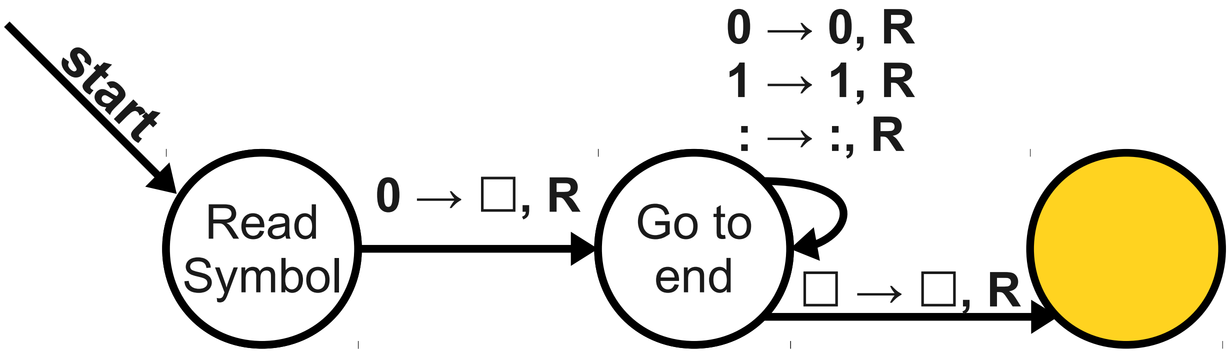
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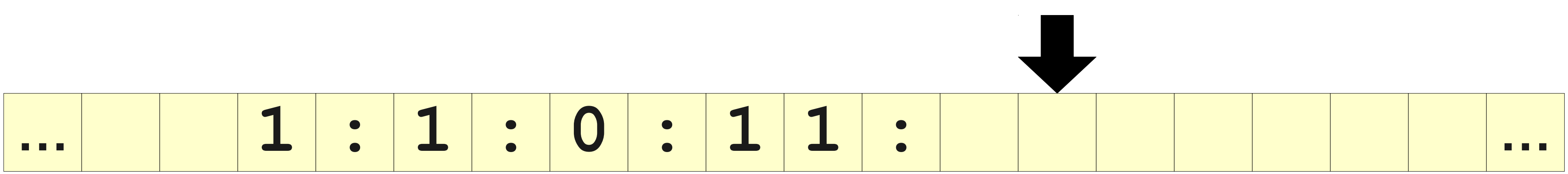
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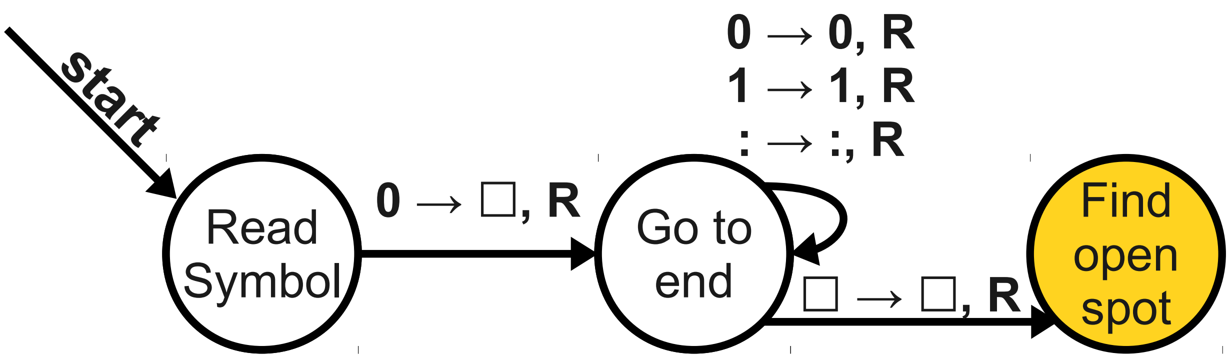
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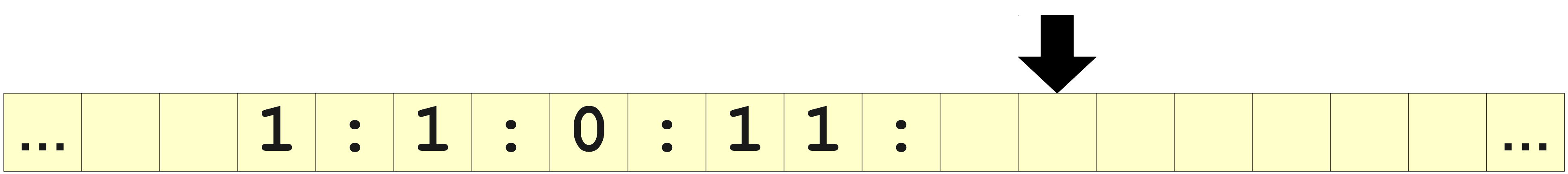
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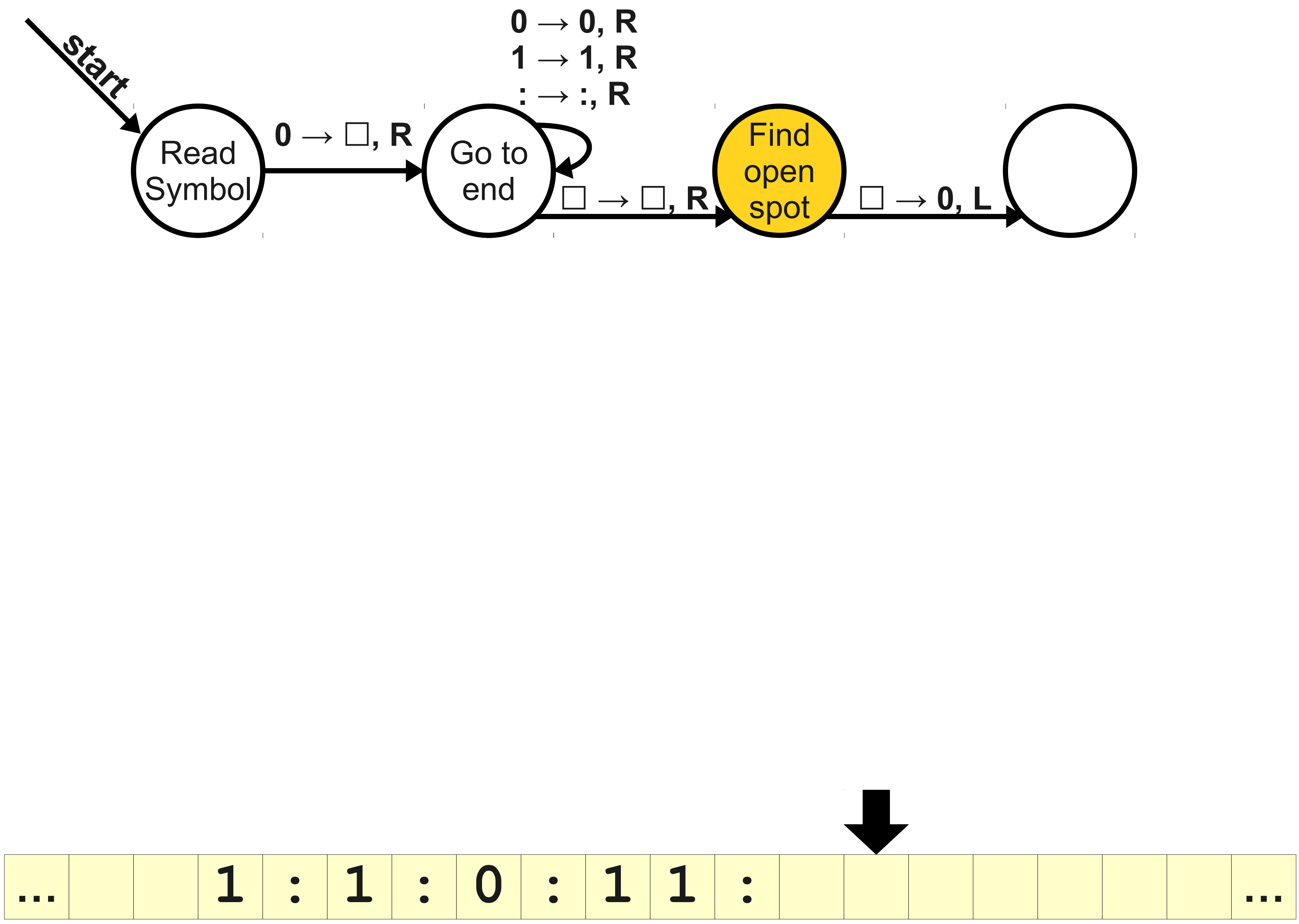
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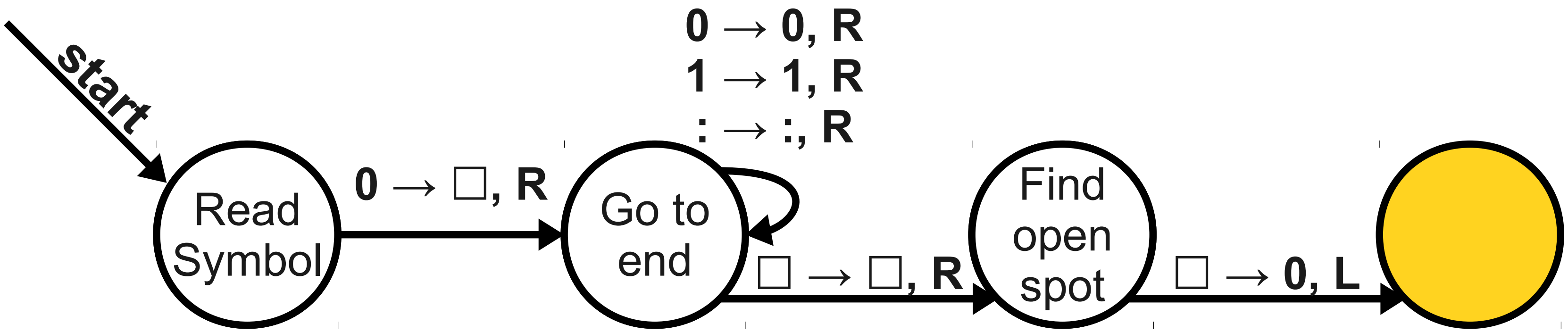
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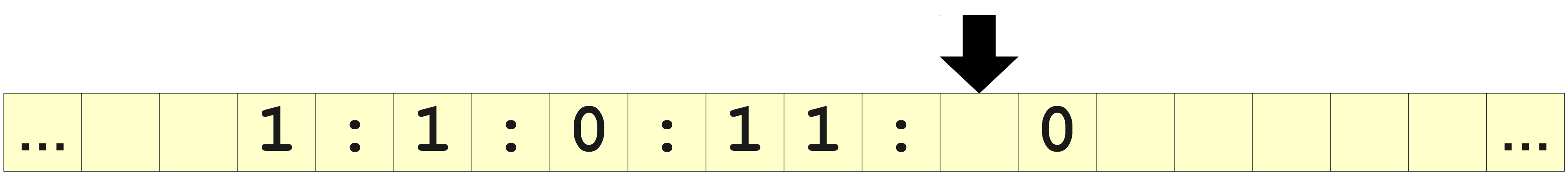
Go to end
Find open spot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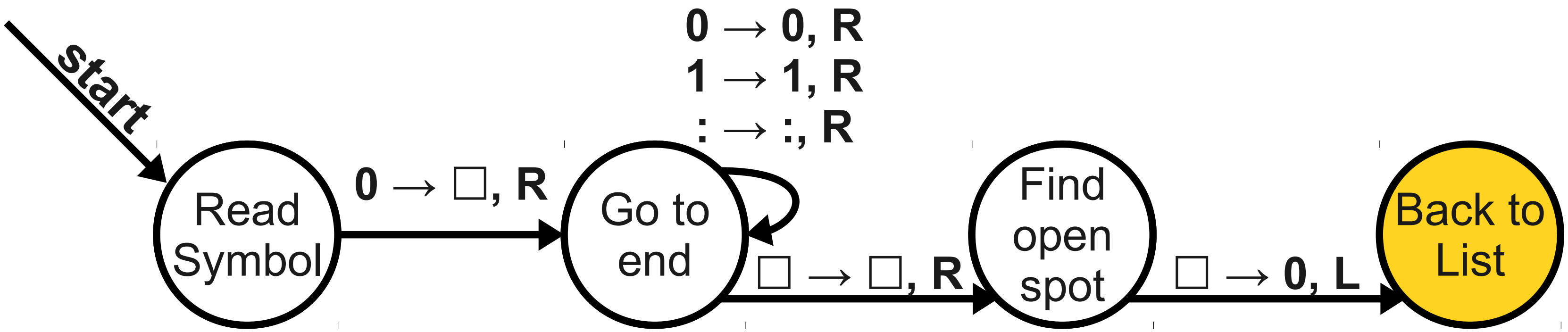
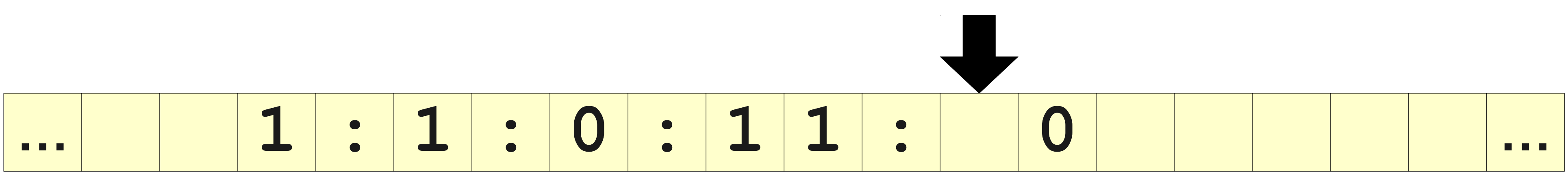

start
Read Symbol
0 → □, R
Go to end
0 → 0, R
1 → 1, R
: → :, R
□ → □, R
Find open spot
□ → 0, L
Back to List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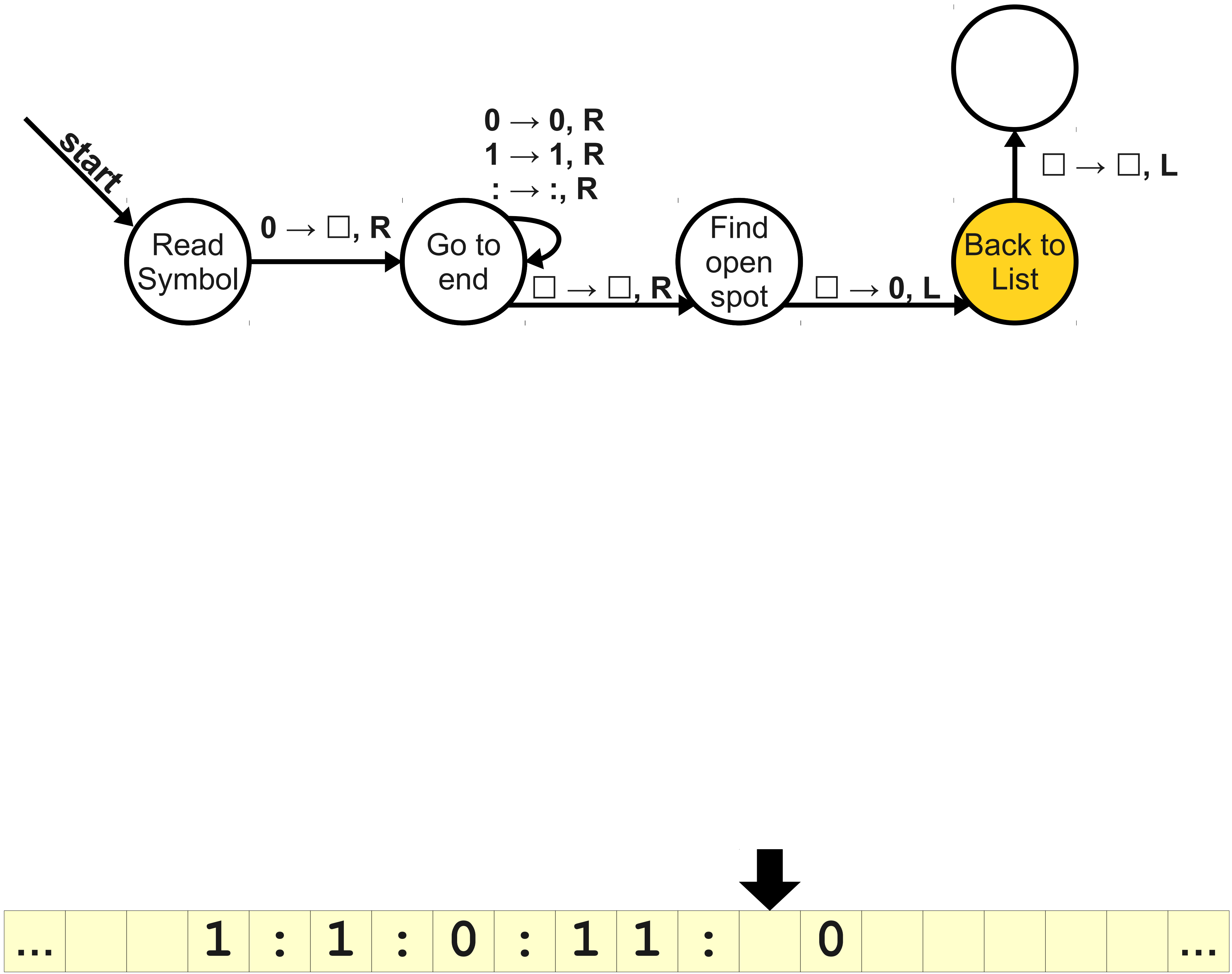
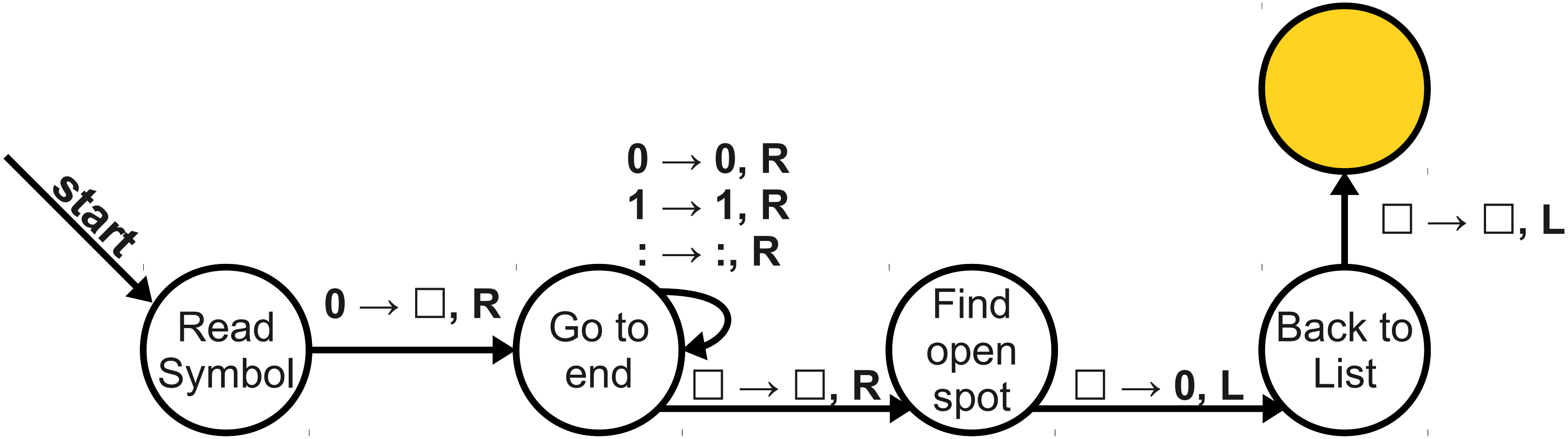


start
Read Symbol
0 → □, R
Go to end
0 → 0, R
1 → 1, R
: → :, R
□ → □, R
Find open spot
□ → 0, L
Back to List
□ → □, L
… 1 : 1 : 0 : 1 1 : 0 …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Go to end
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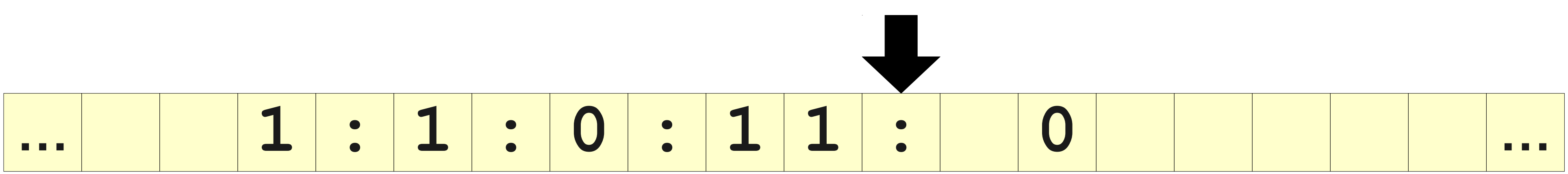
1 → 1, R
: → :, R
□ → □, R
Find open spot
□ → 0, L
Back to List
□ → □, L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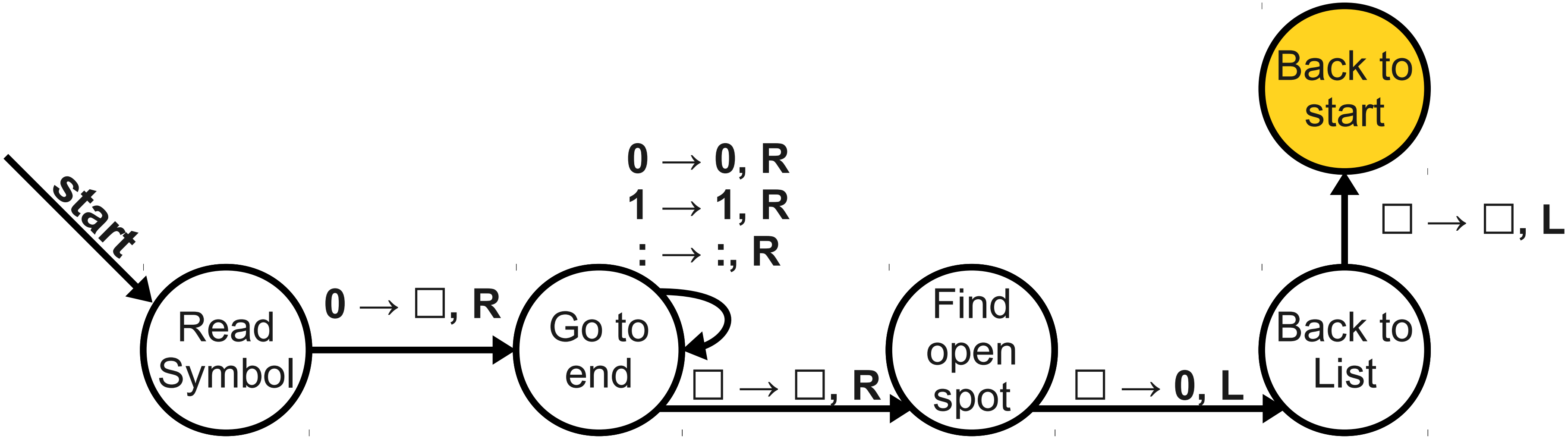
start
Read Symbol
0 → □, R
Go to end
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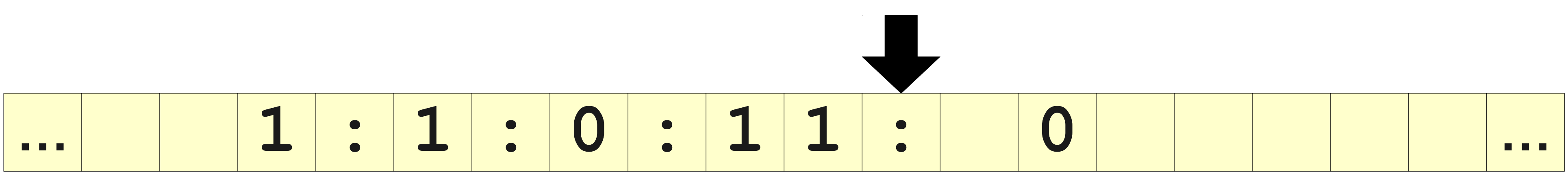
1 → 1, R
: → :, R
□ → □, R
Find open spot
□ → 0, L
Back to List
□ → □, L
Back to start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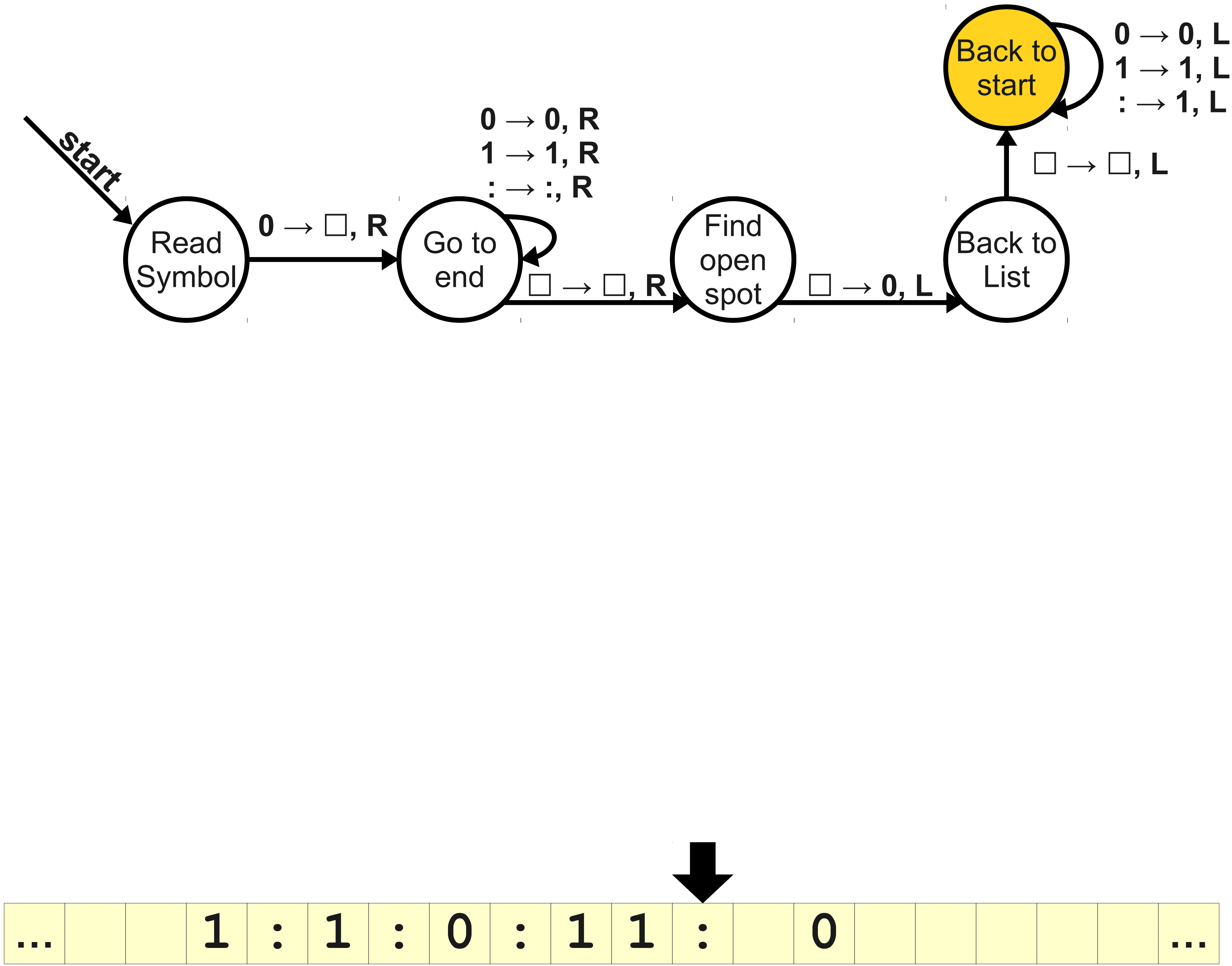

start
Read Symbol
0 → □, R
Go to end
0 → 0, R
1 → 1, R
: → :, R
□ → □, R
Find open spot
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Back to List
□ → □, L
Back to start
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1 → 1, L
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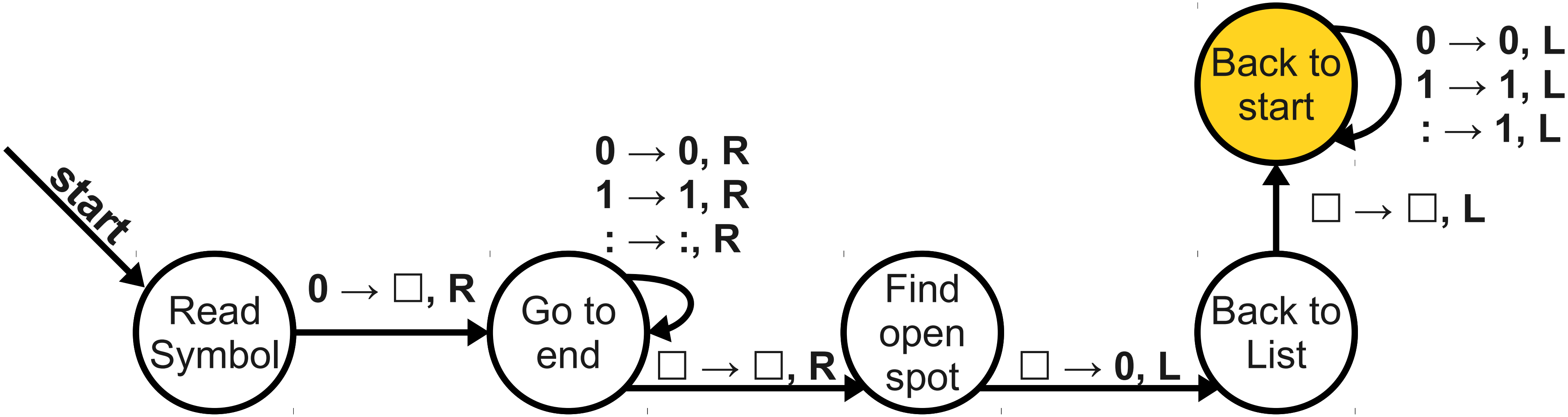
... 1 : 1 : 0 : 1 1 : 0 ...


start
Read Symbol
0 → □, R
Go to end
0 → 0, R
1 → 1, R
: → :, R
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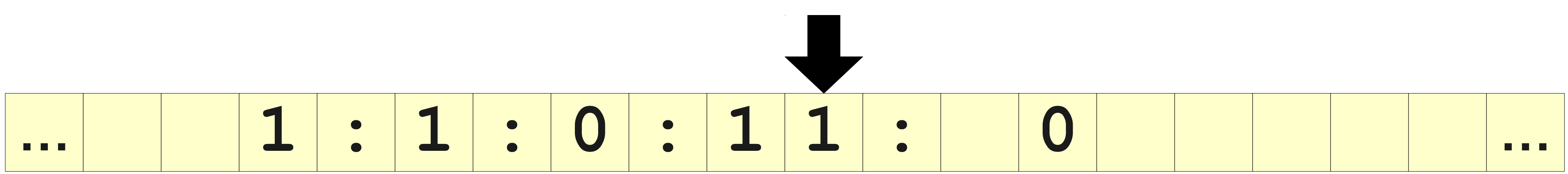
Find open spot
□ → 0, L
Back to List
□ → □, L
Back to start
0 → 0, L
1 → 1, L
: → 1, L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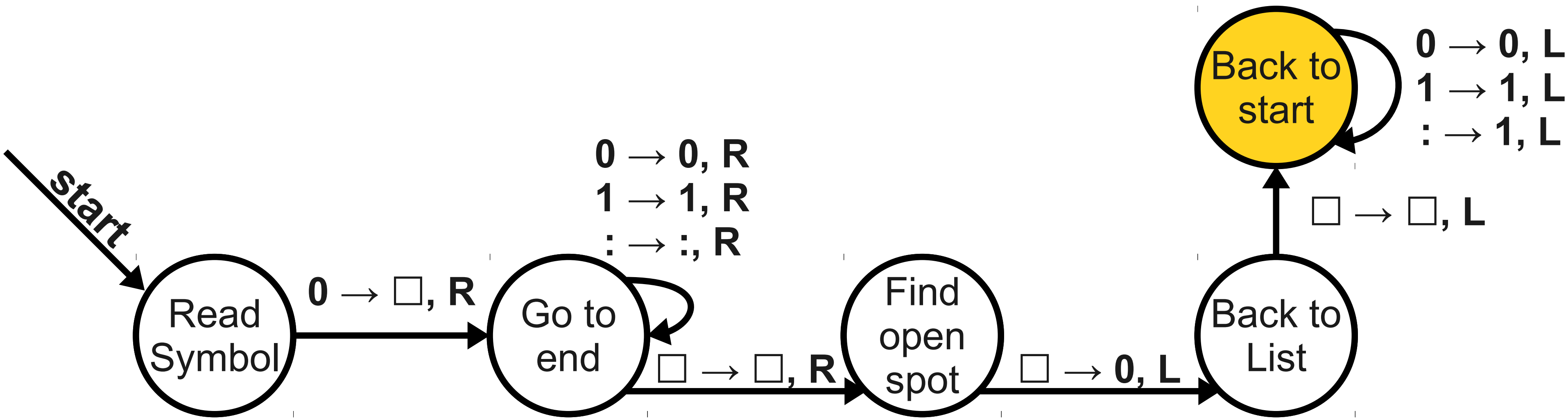
start
Read Symbol
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Go to end
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1 → 1, R
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Find open spot
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Back to List
□ → □, L
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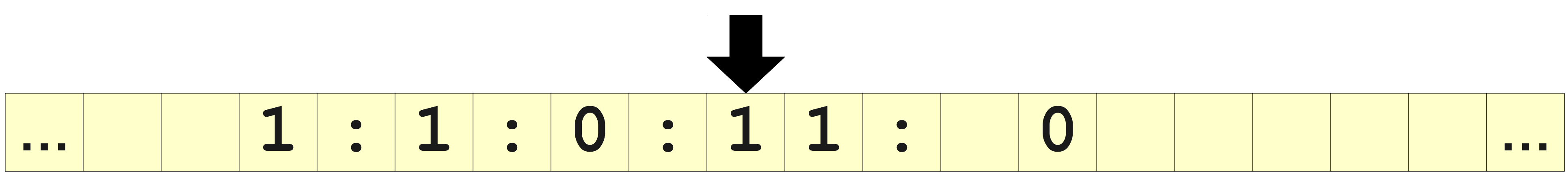
: → 1, L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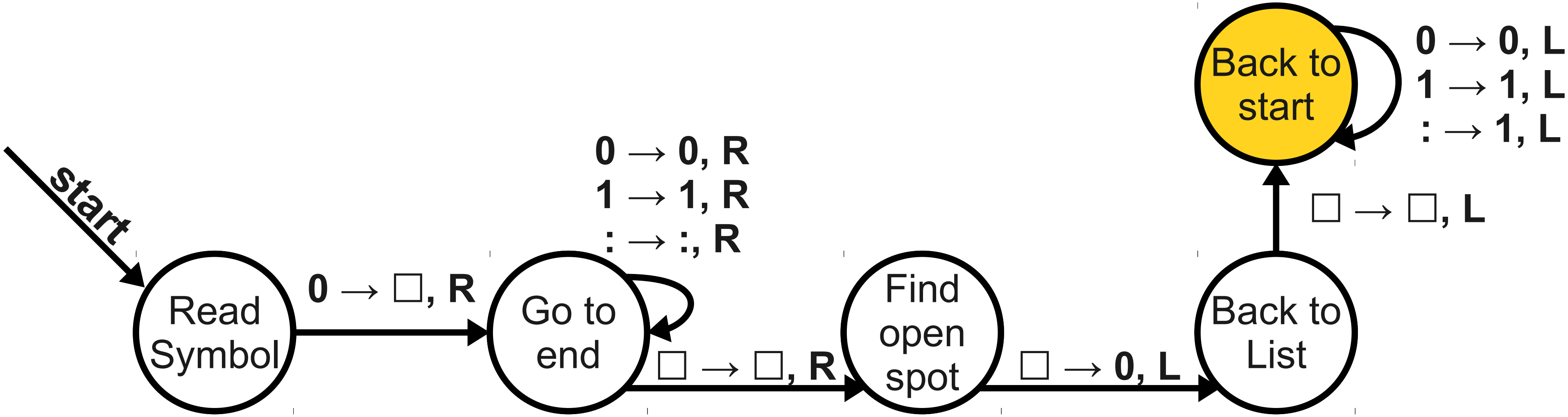
start
Read Symbol
0 → □, R
Go to end
0 → 0, R
1 → 1, R
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Find open spot
□ → 0, L
Back to List
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0 → 0, L
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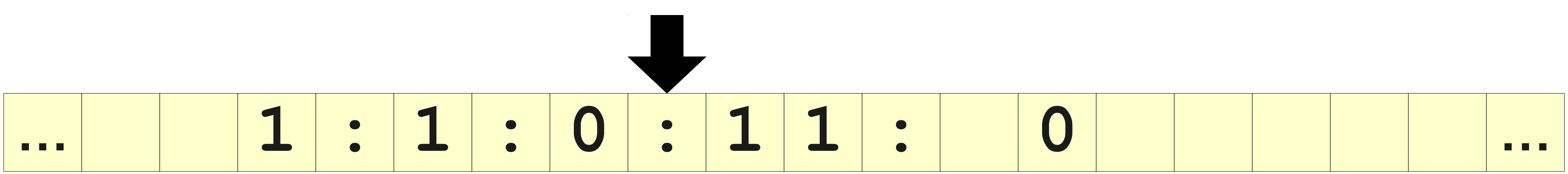
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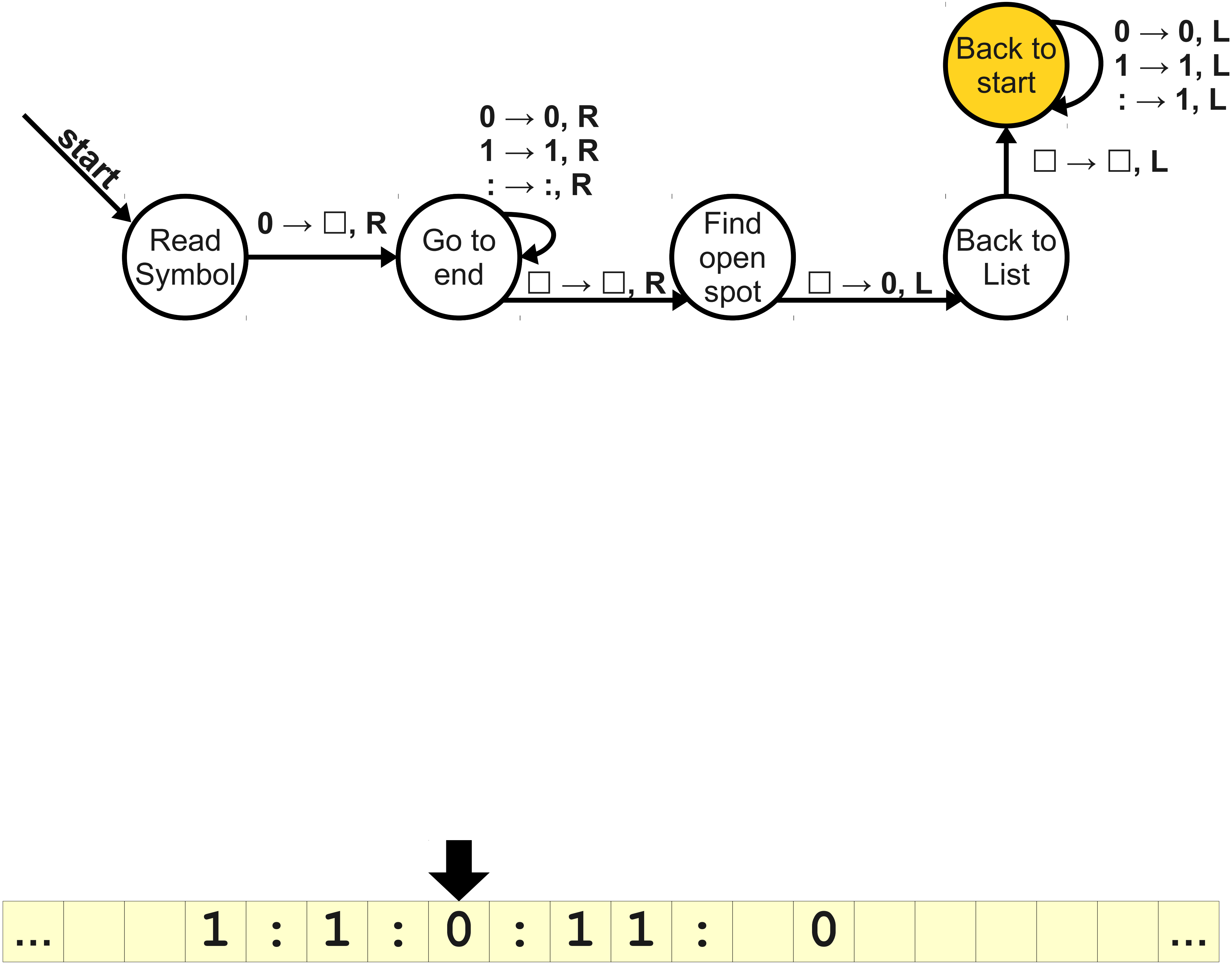

start
Read Symbol
0 → □, R
Go to end
0 → 0, R
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Find open spot
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Back to List
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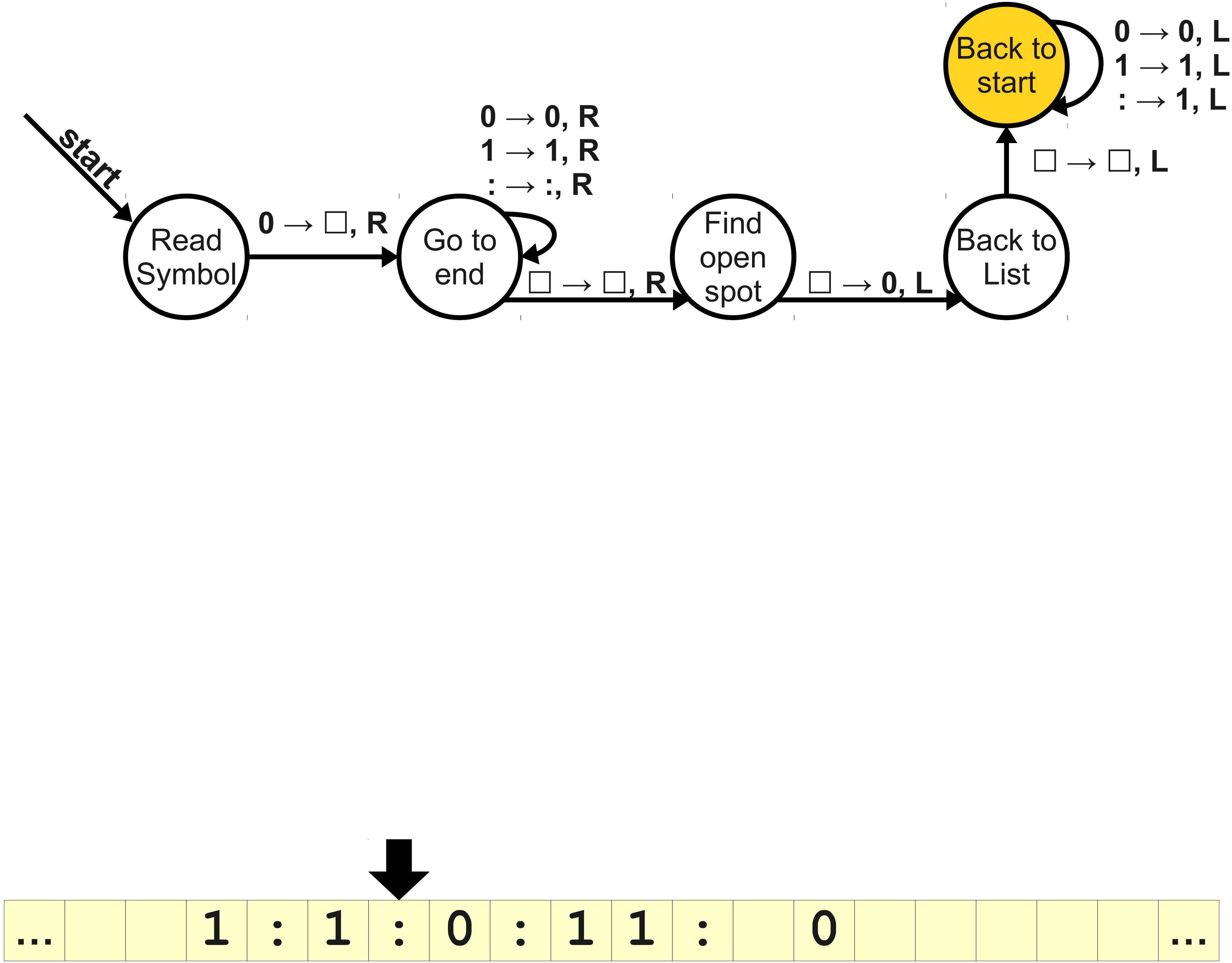
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Go to end
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Find open spot
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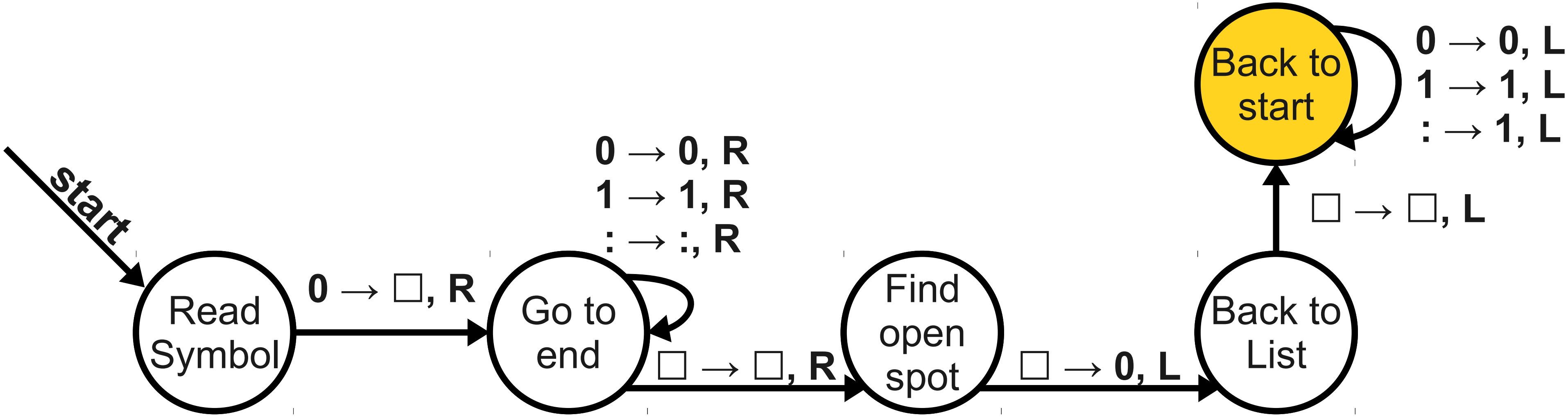
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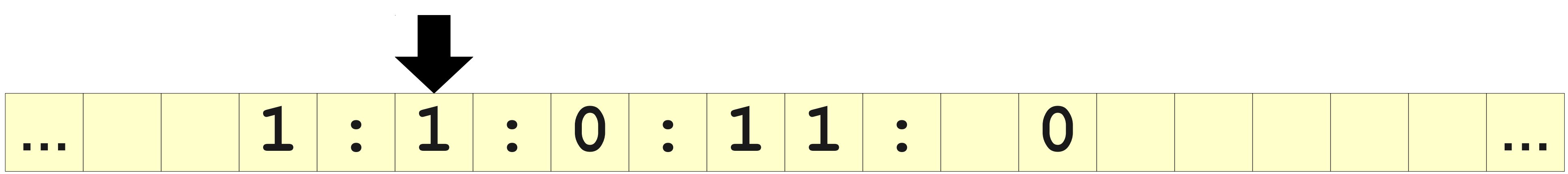
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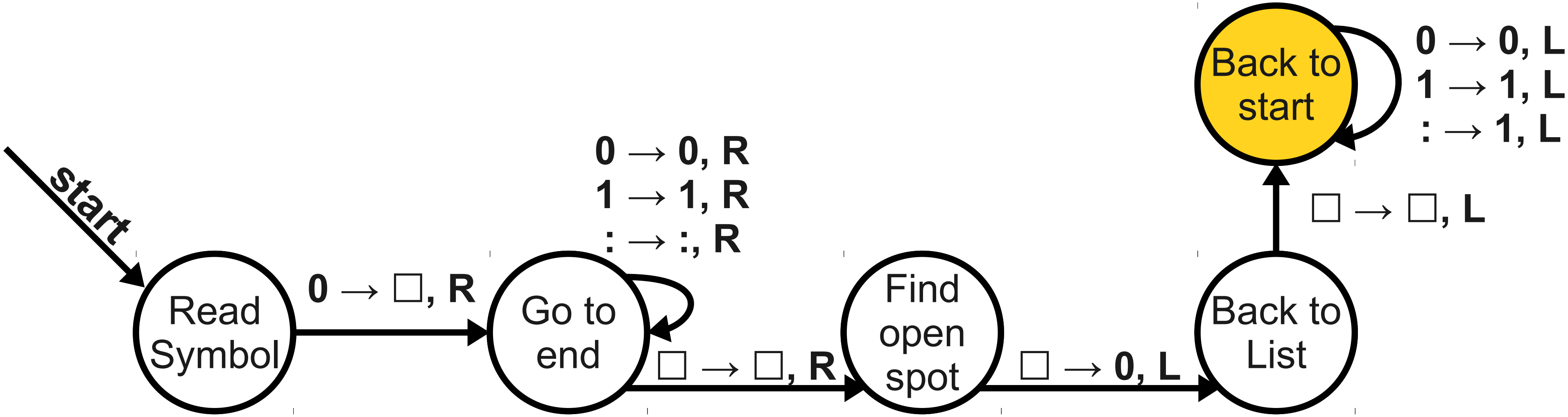
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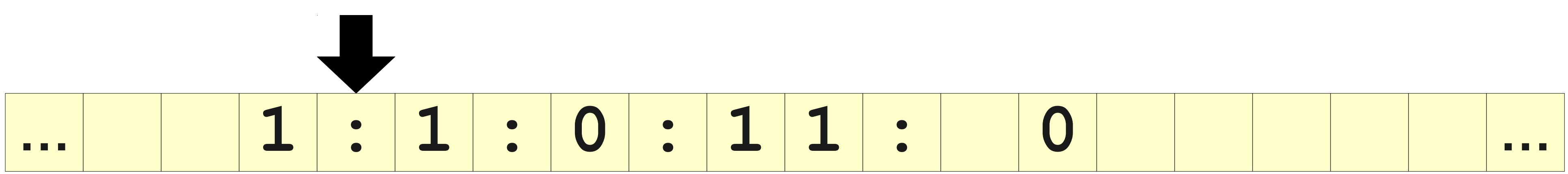
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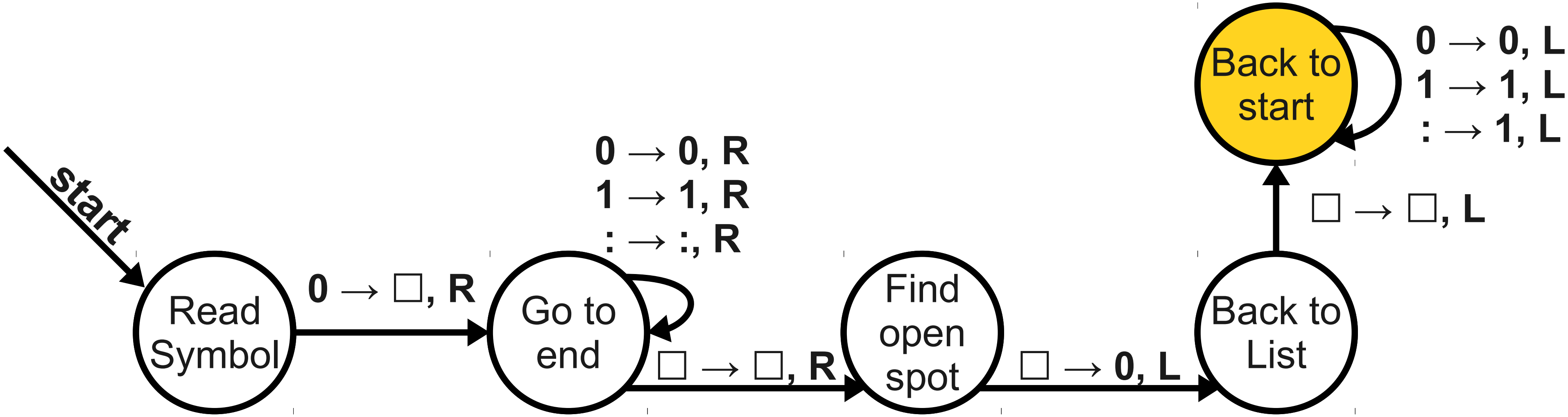
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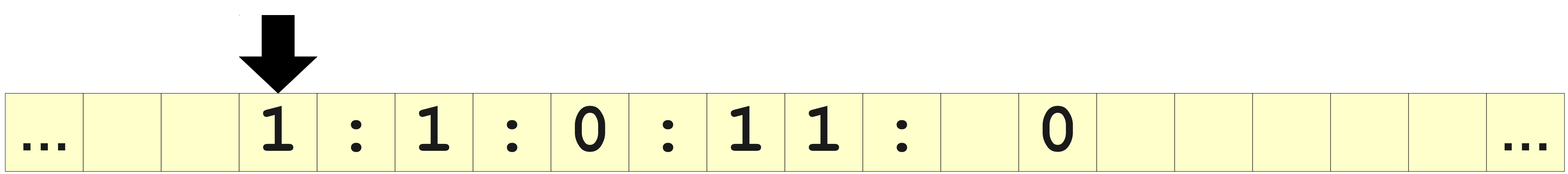
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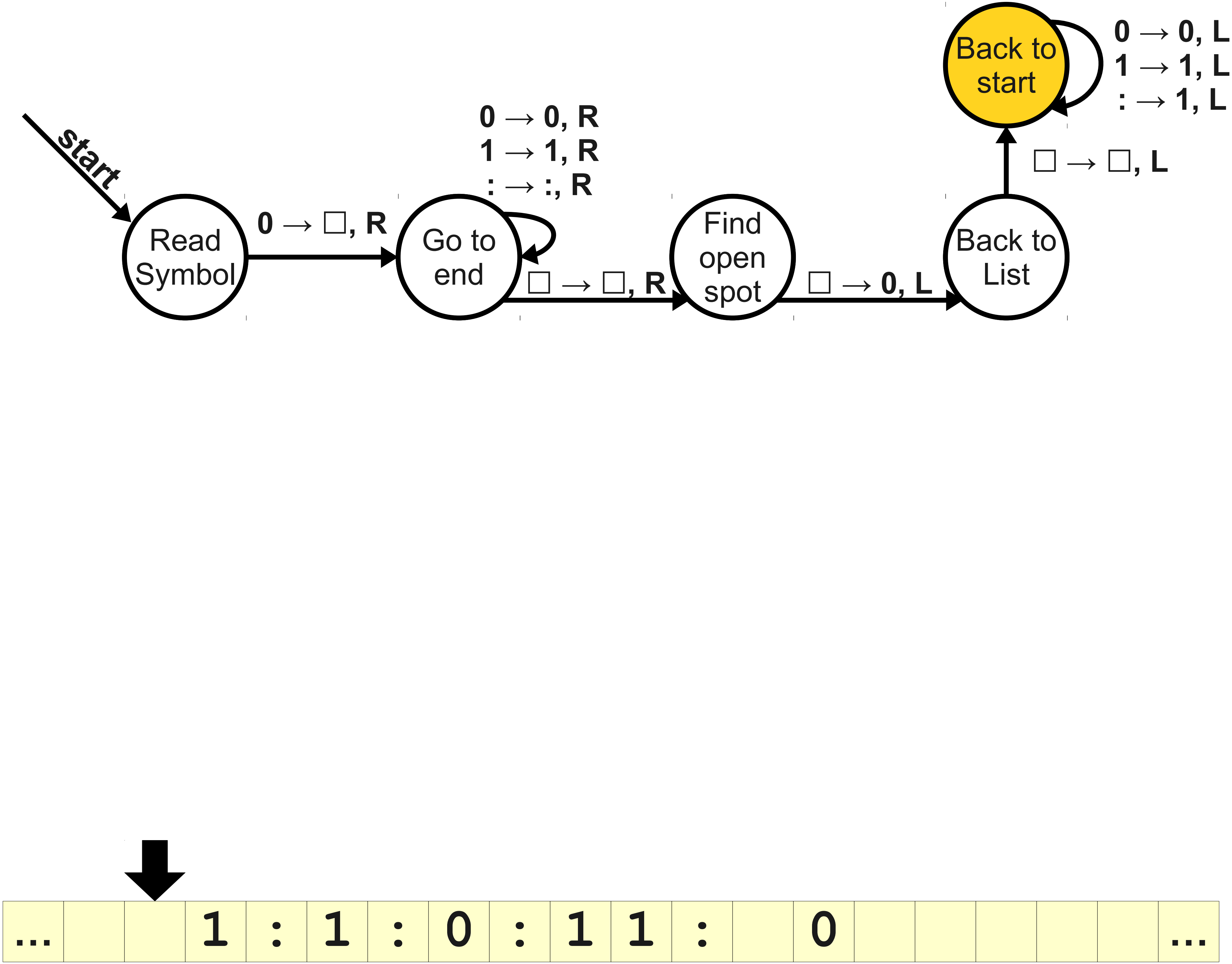
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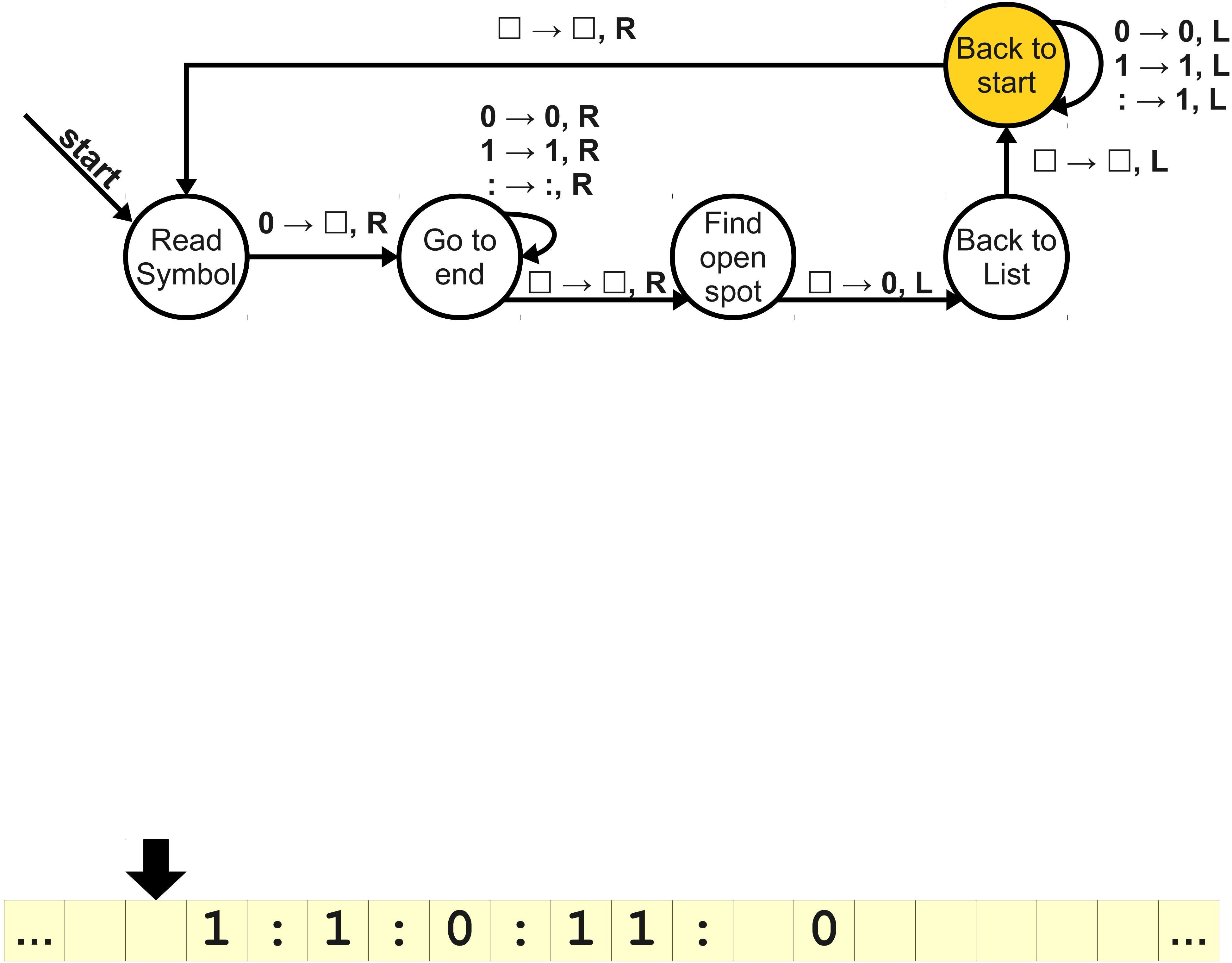

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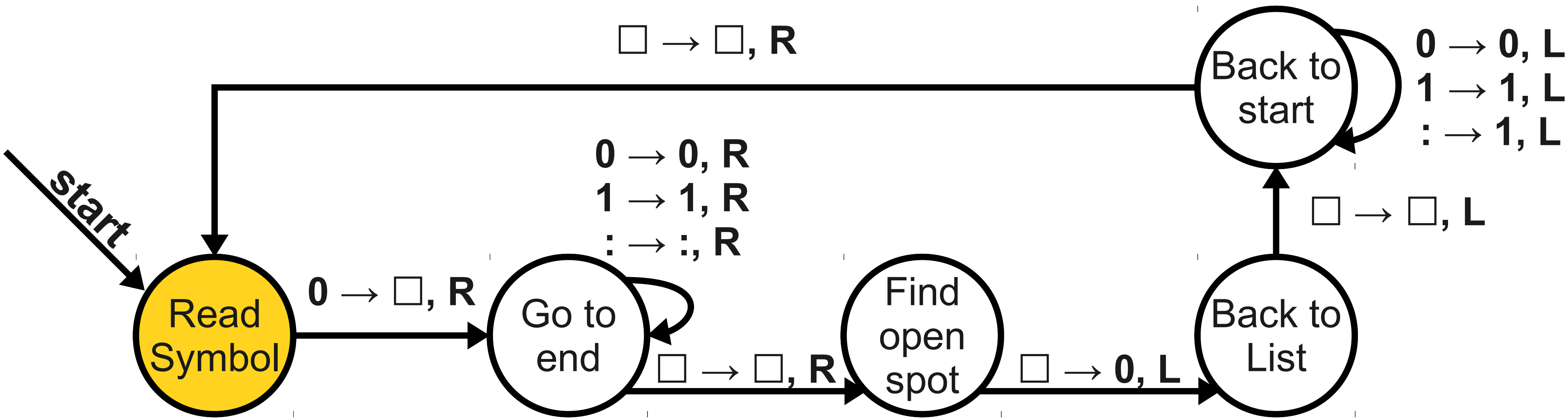

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Find open spot
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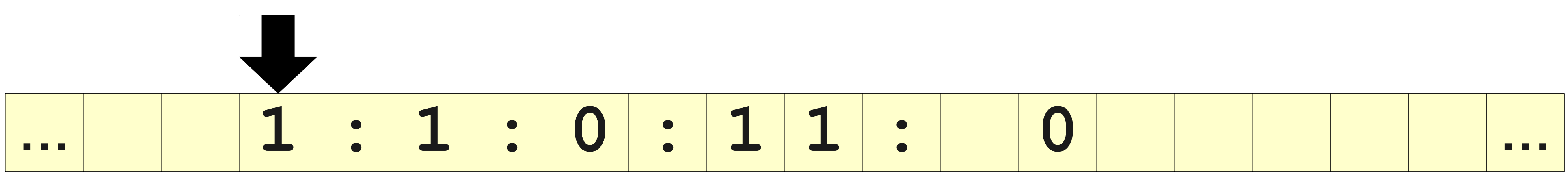

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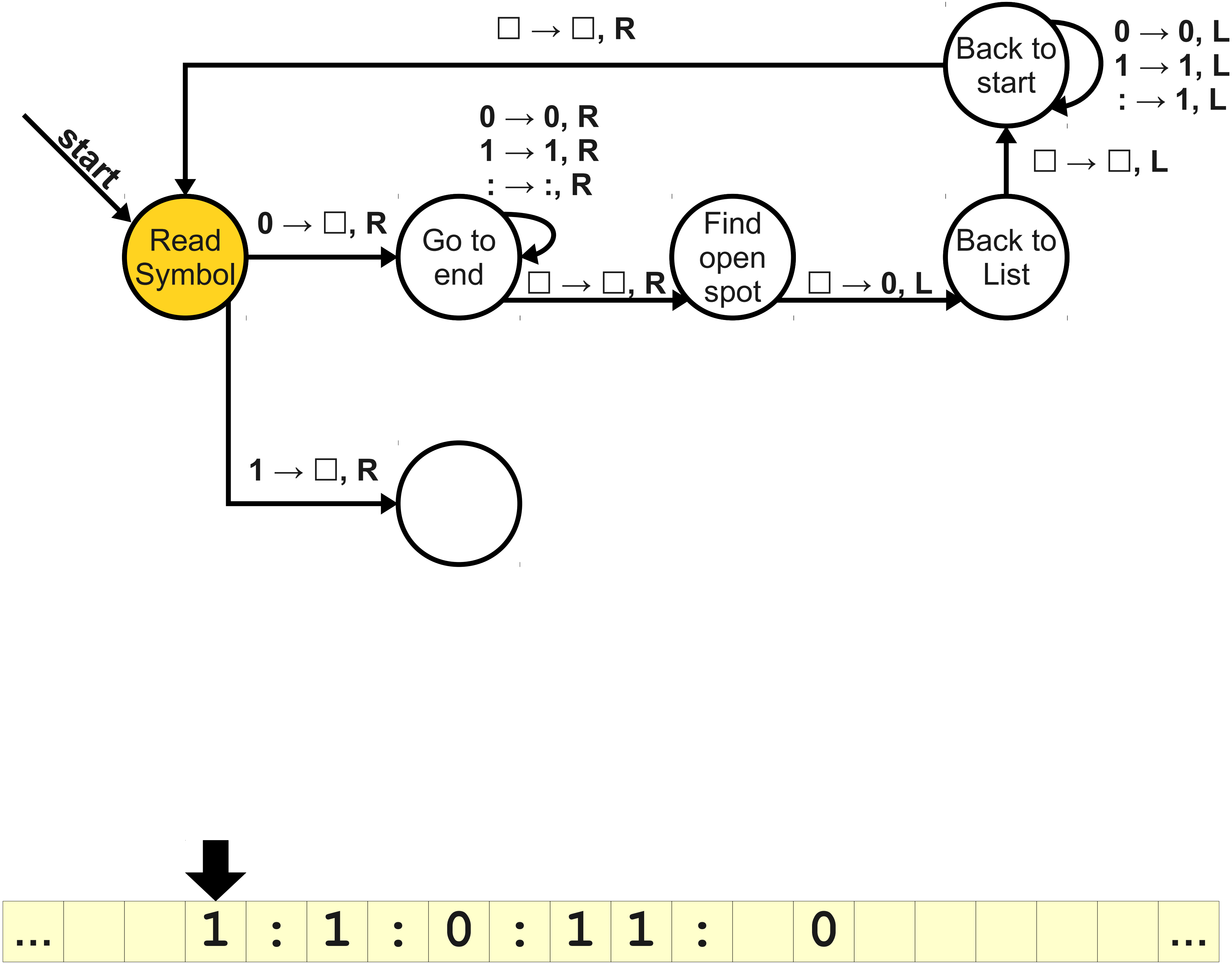
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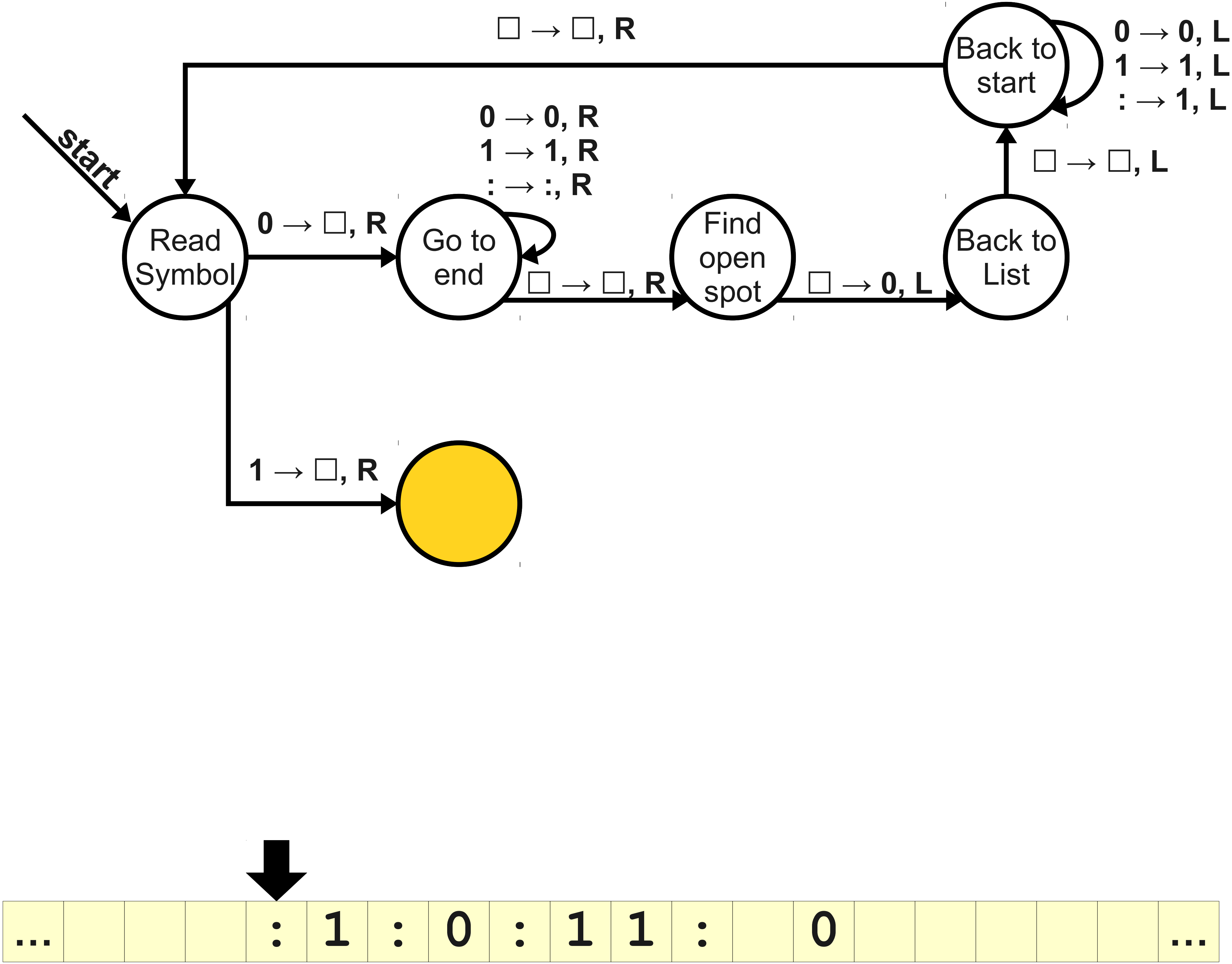

start
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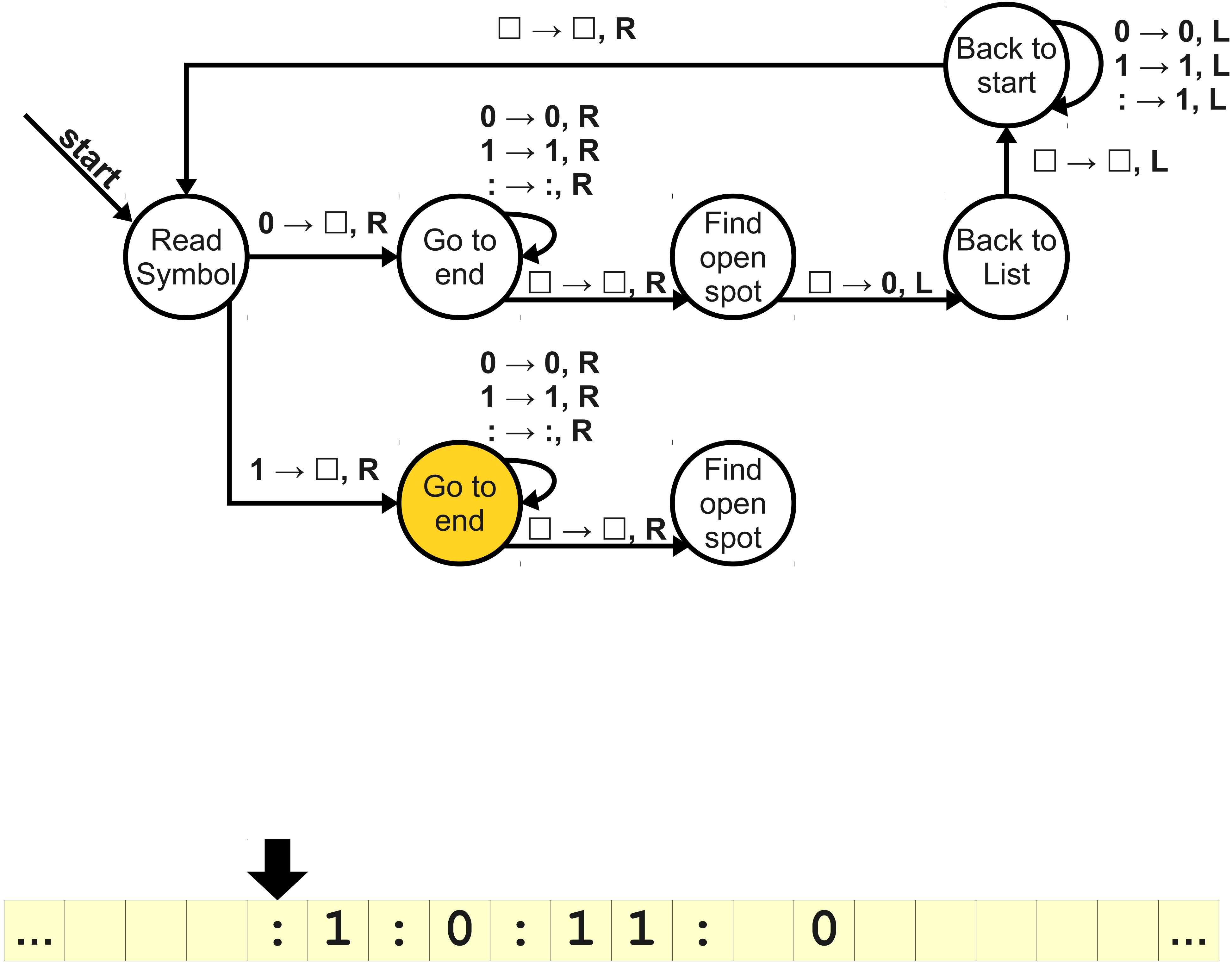

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Go to end
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Back to List
□ → □, R
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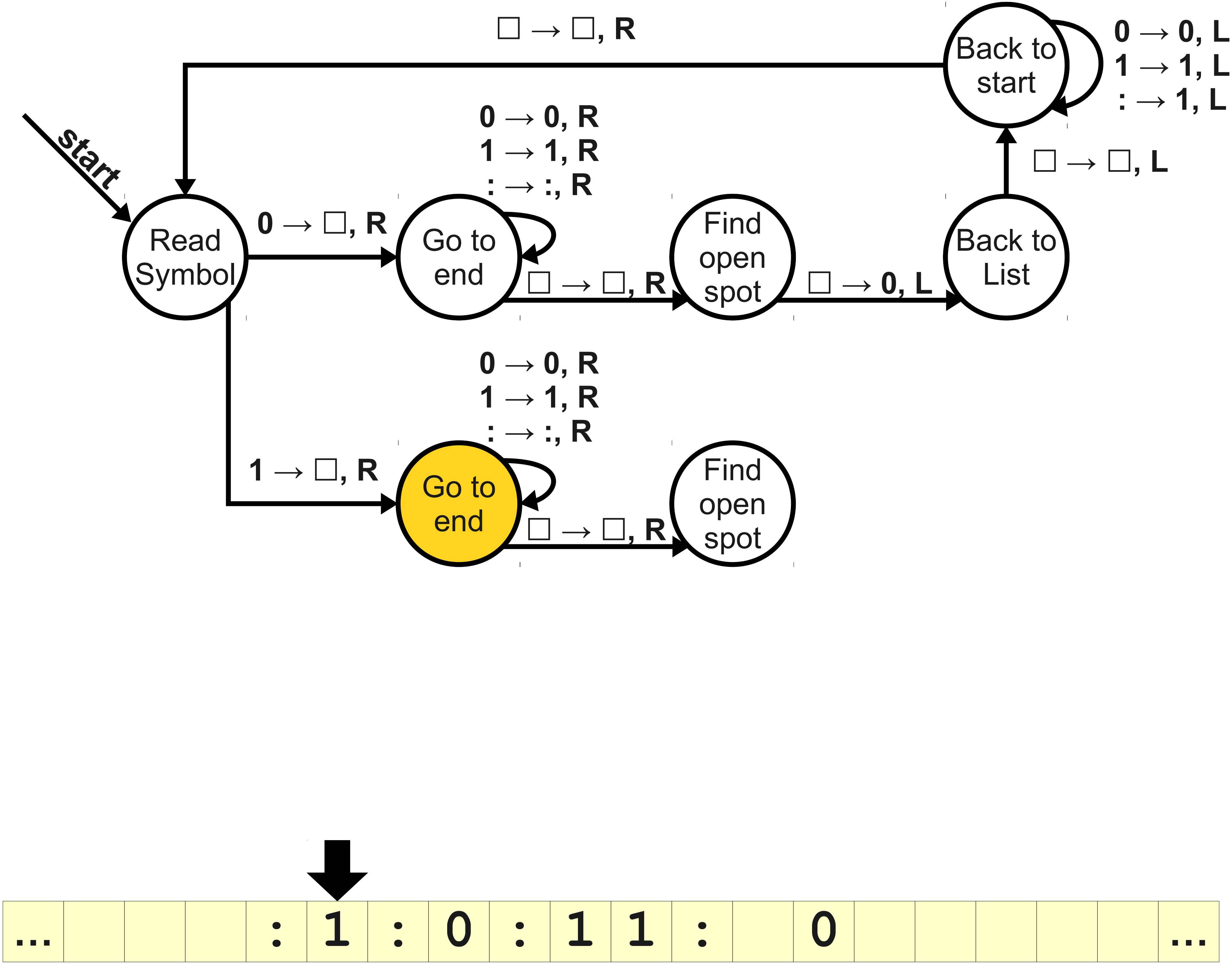
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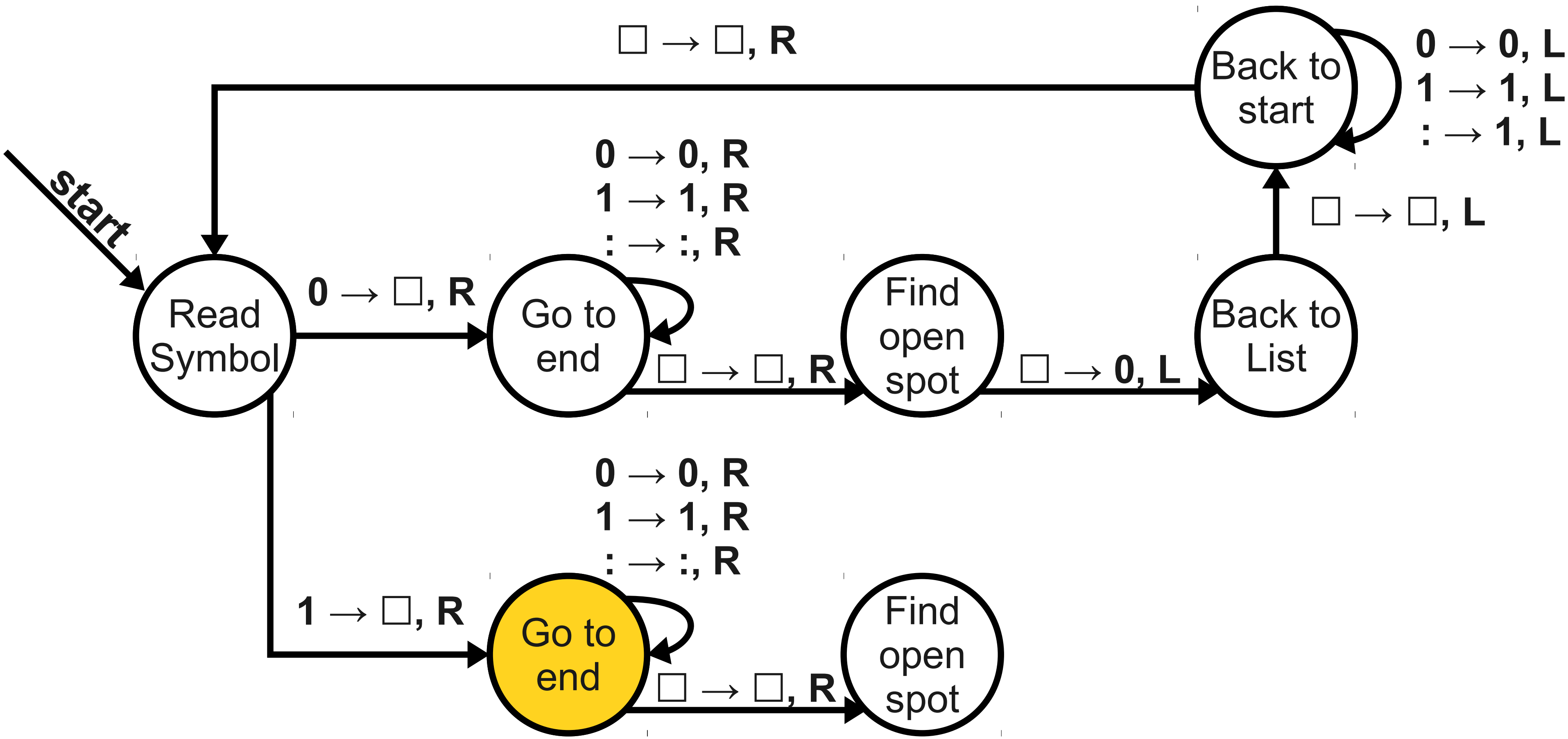

start
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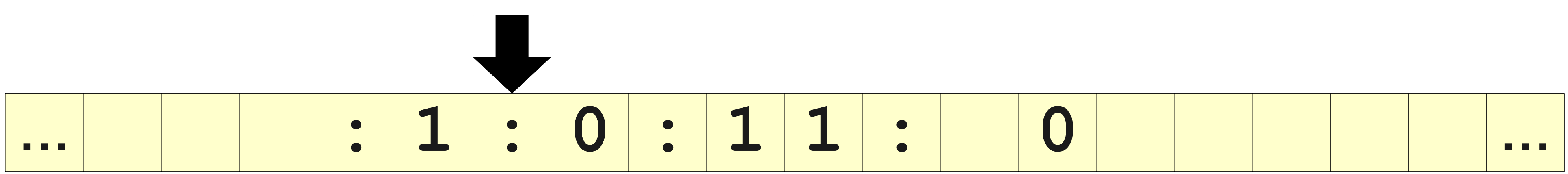
Find open spot



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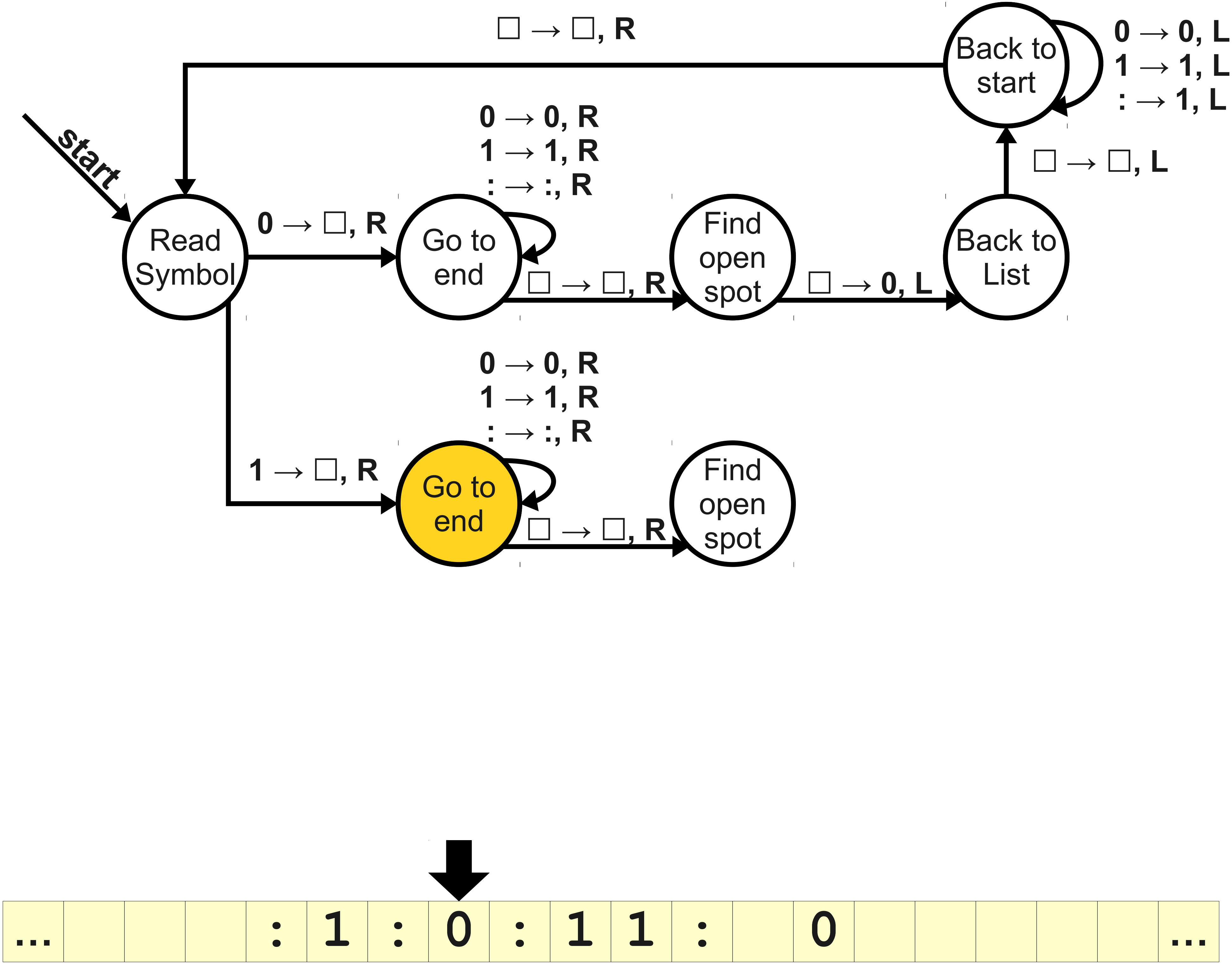
□ → □, R
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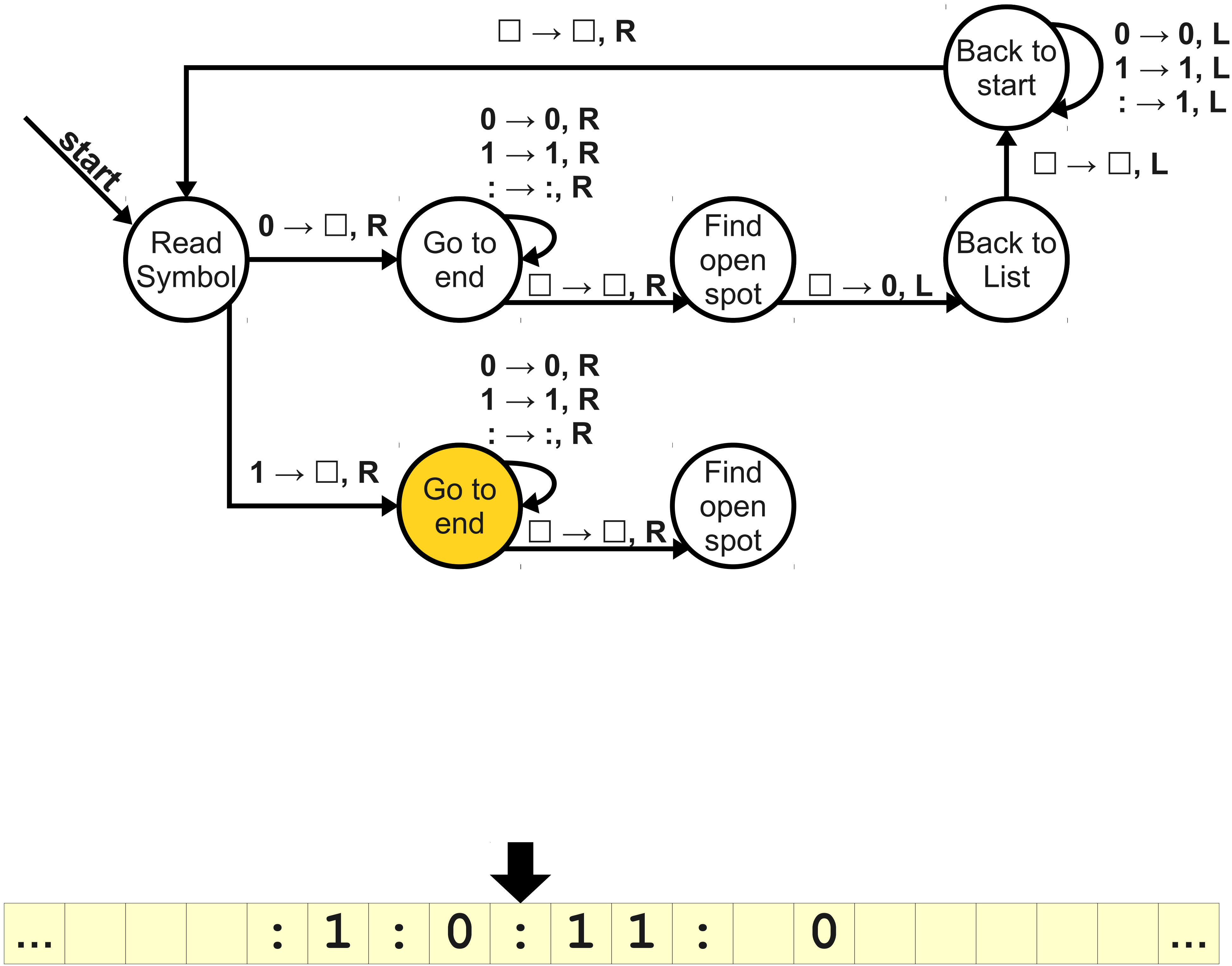
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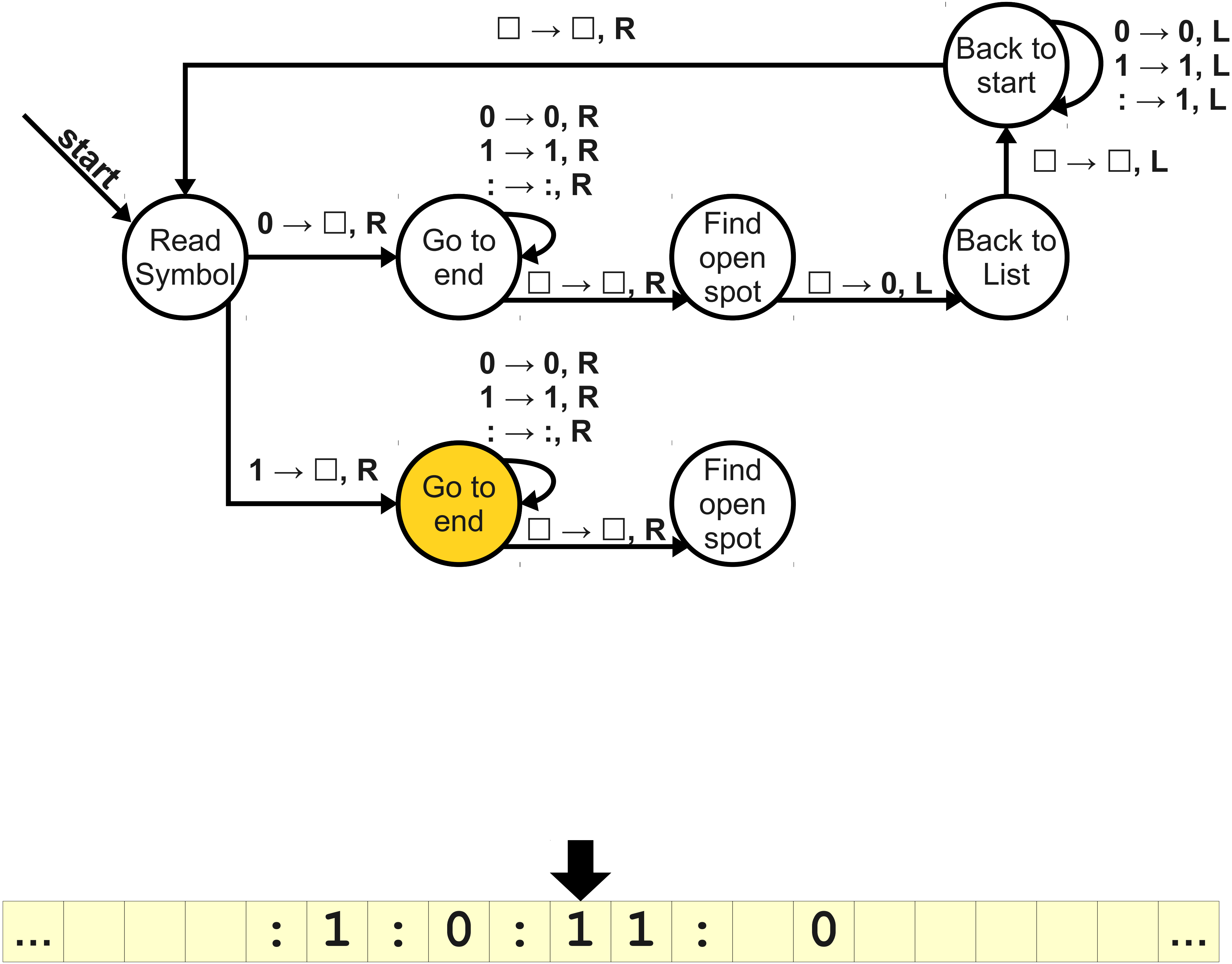
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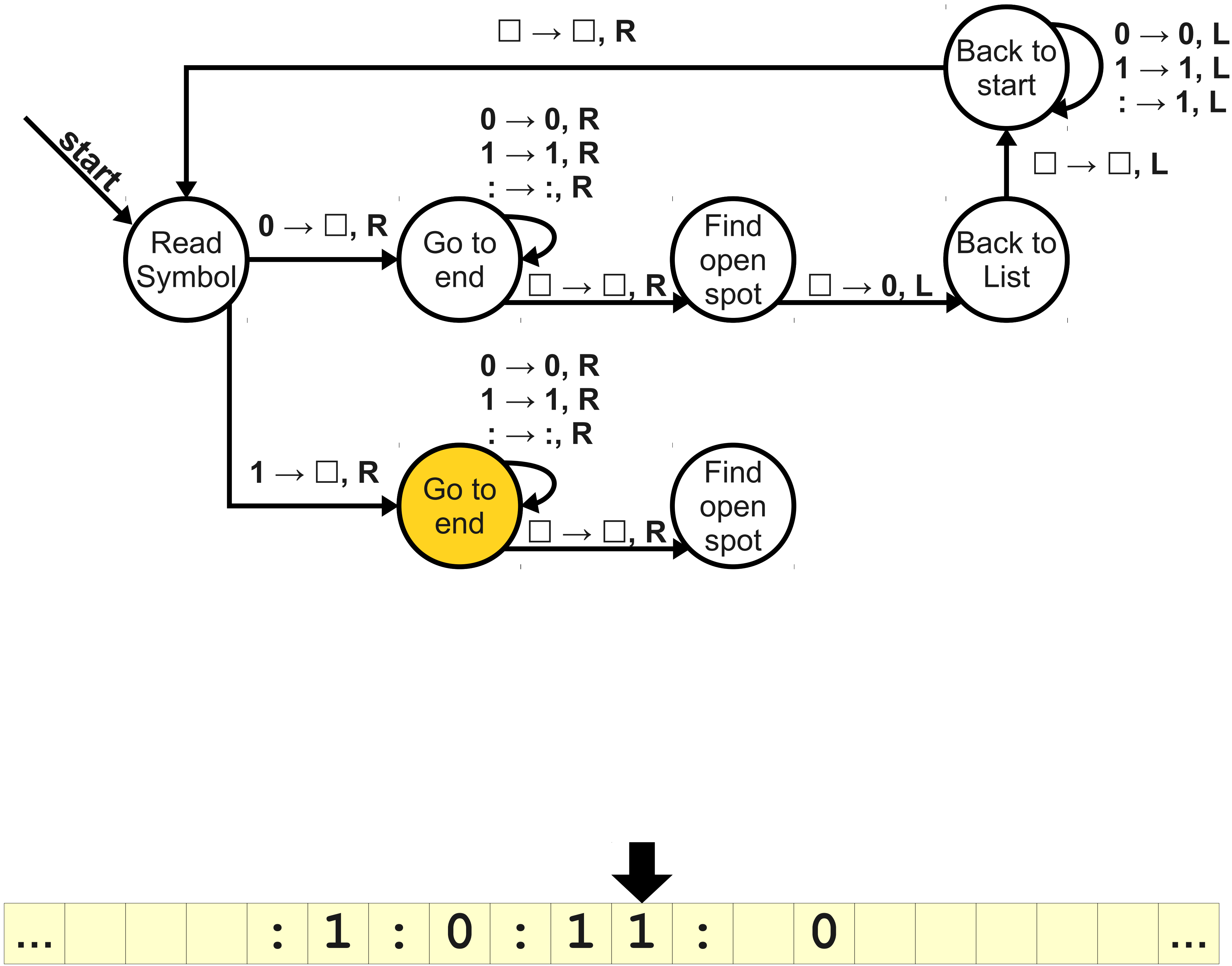

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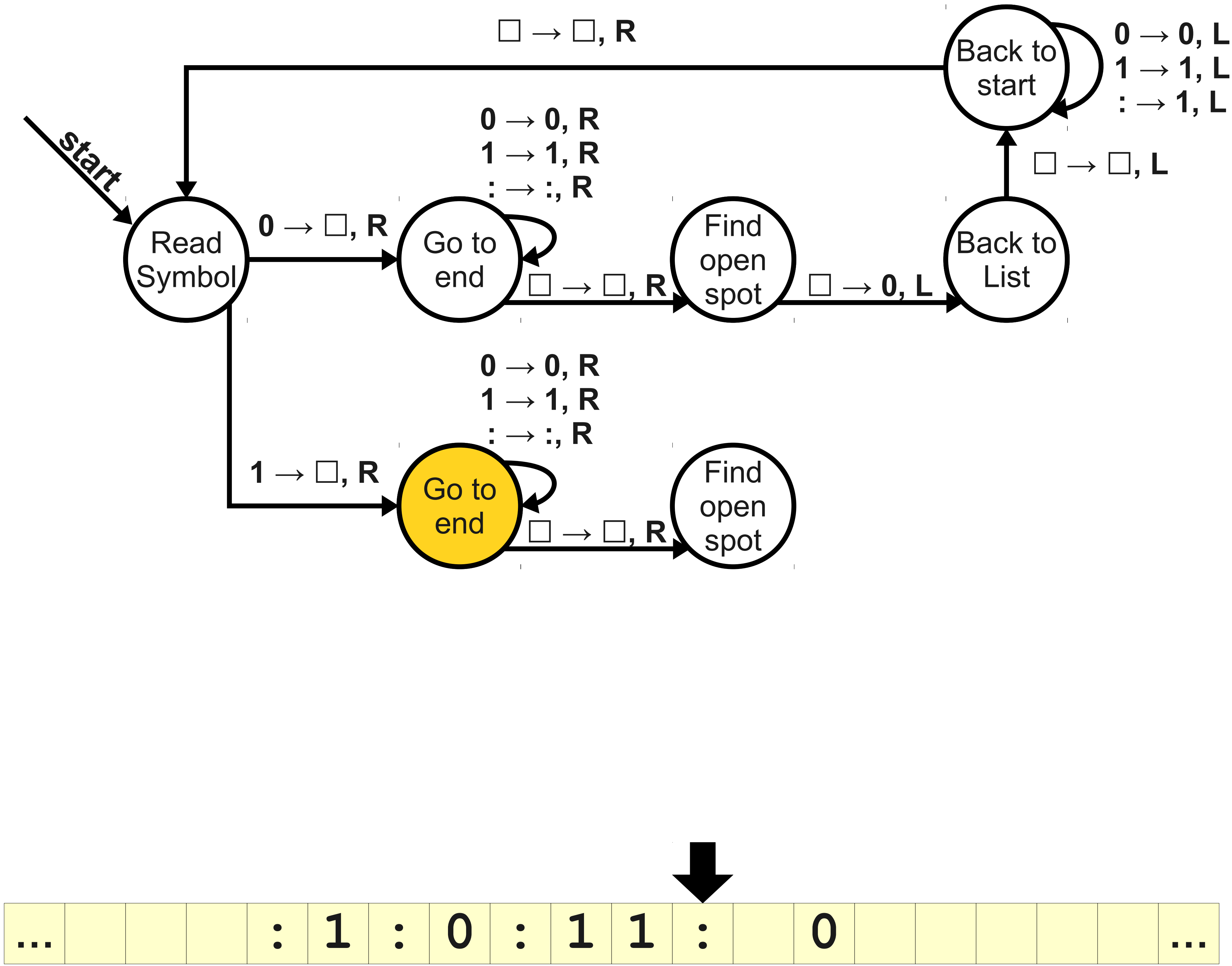

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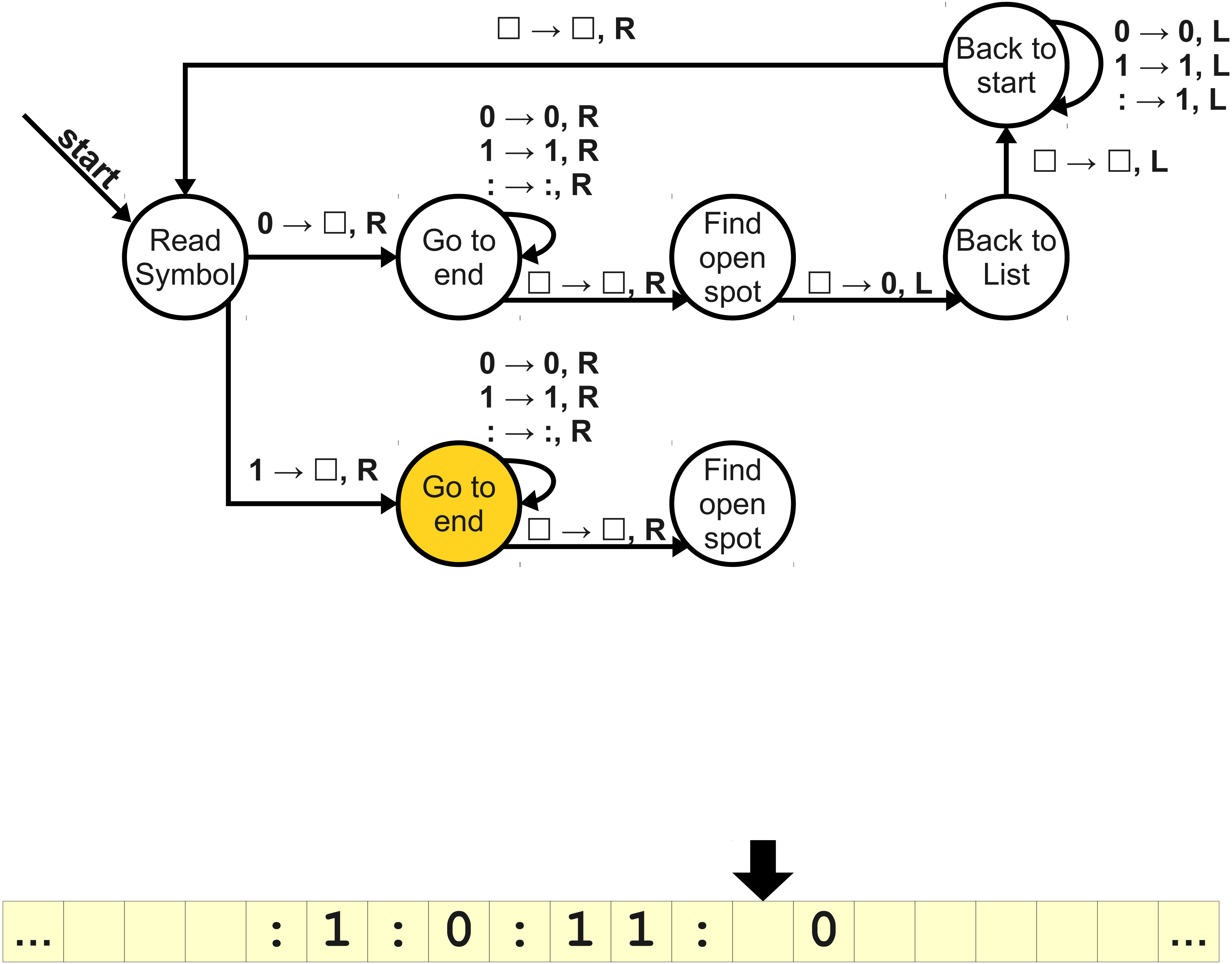
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Go to end
Find open spot
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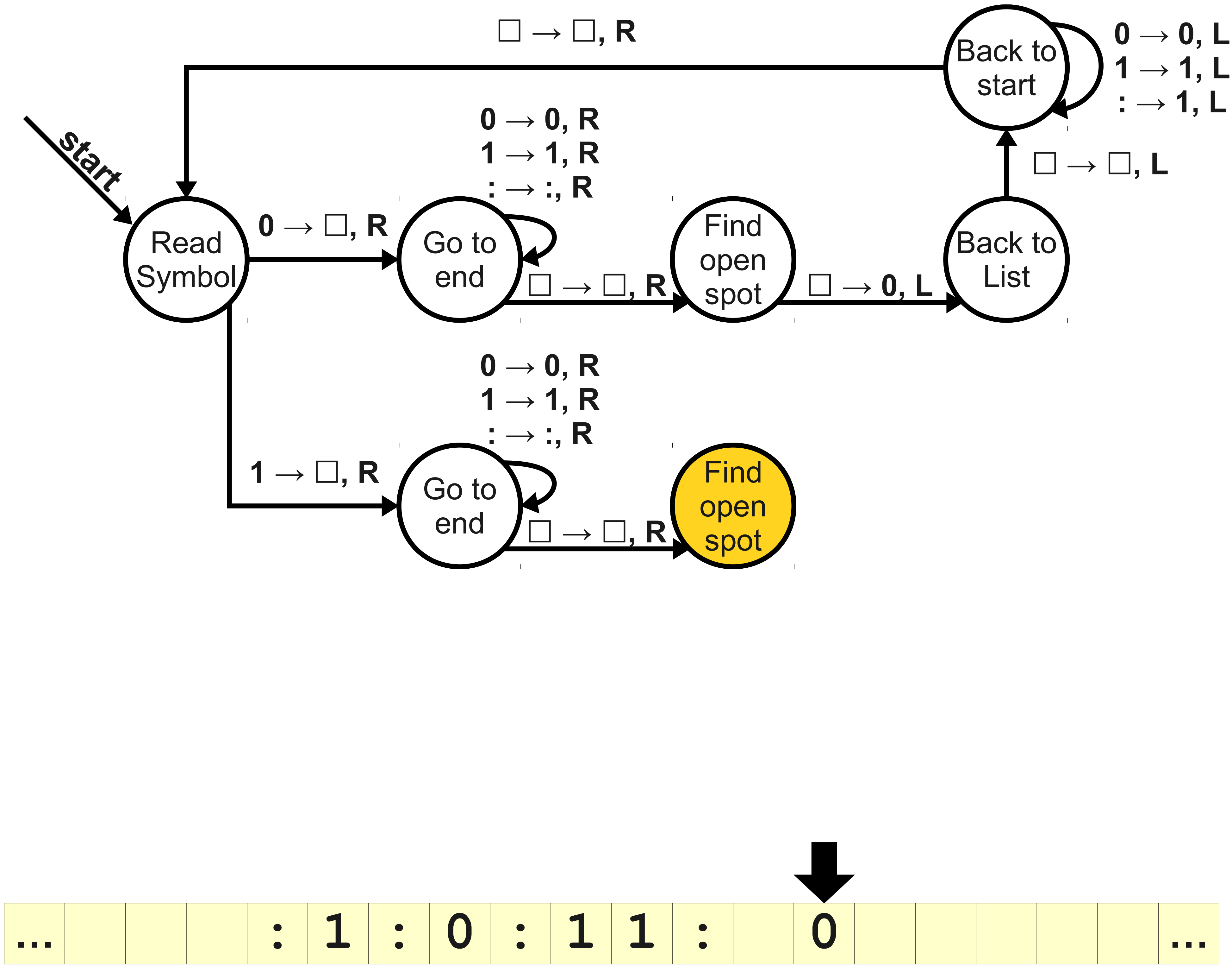

□ → □, R
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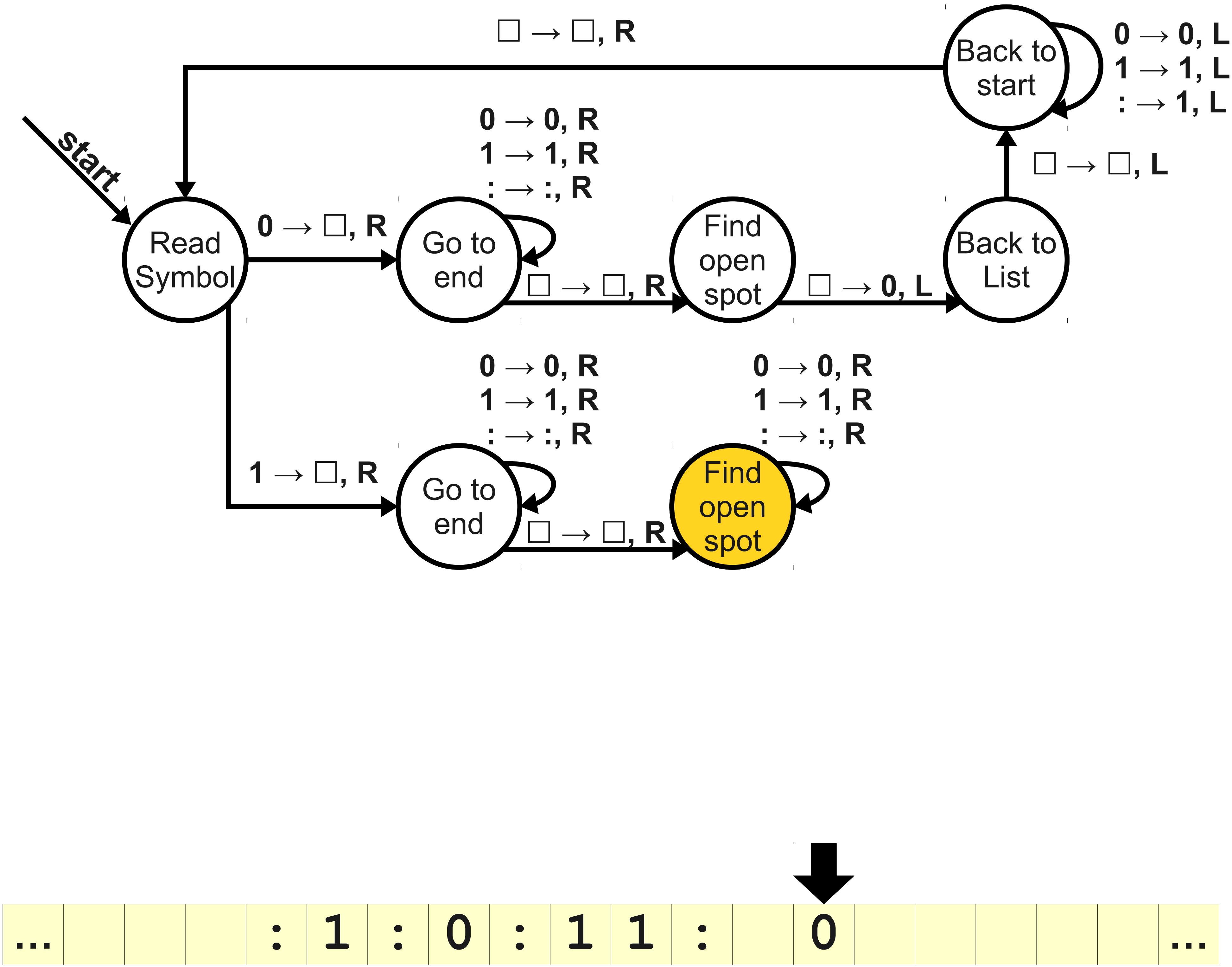
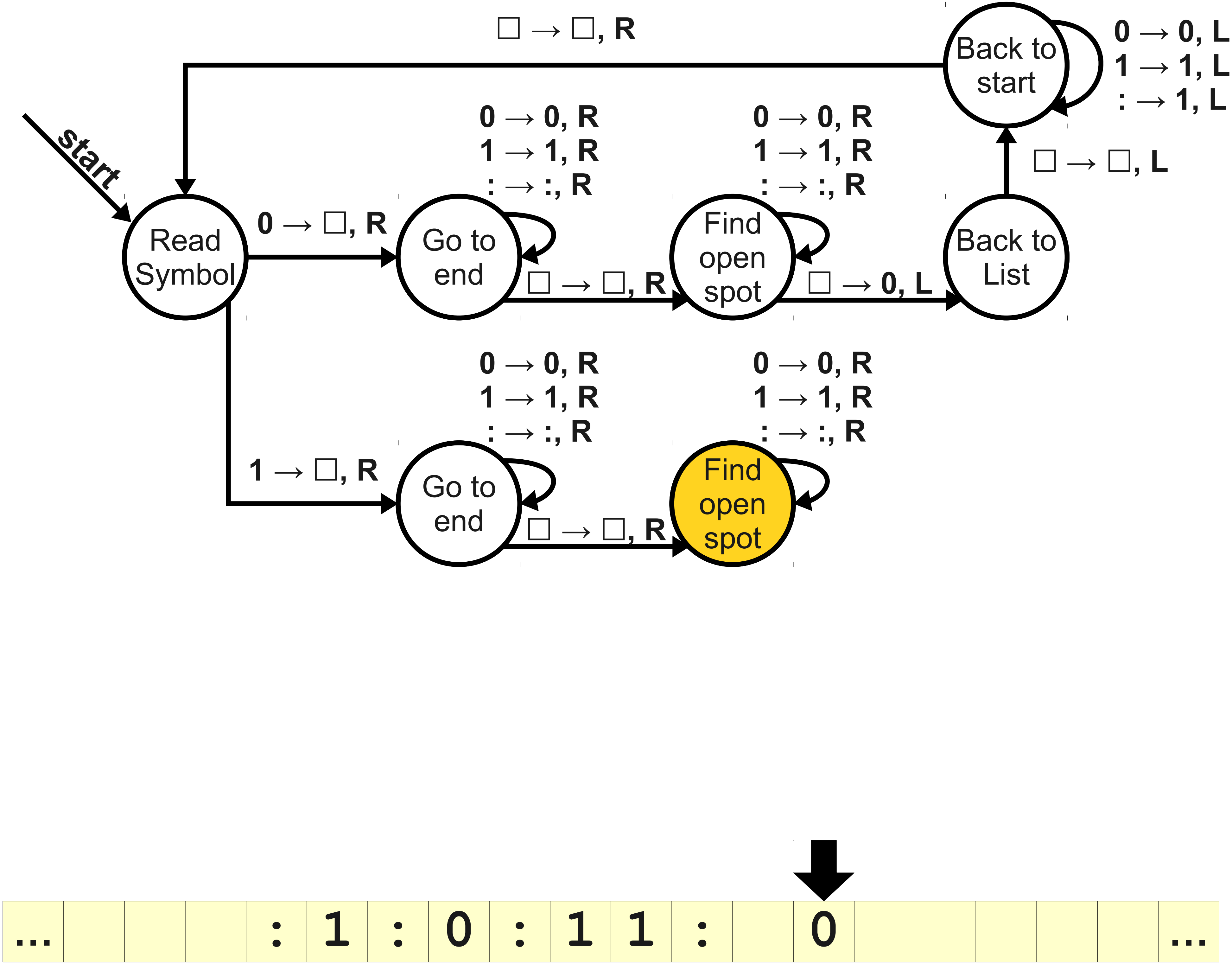
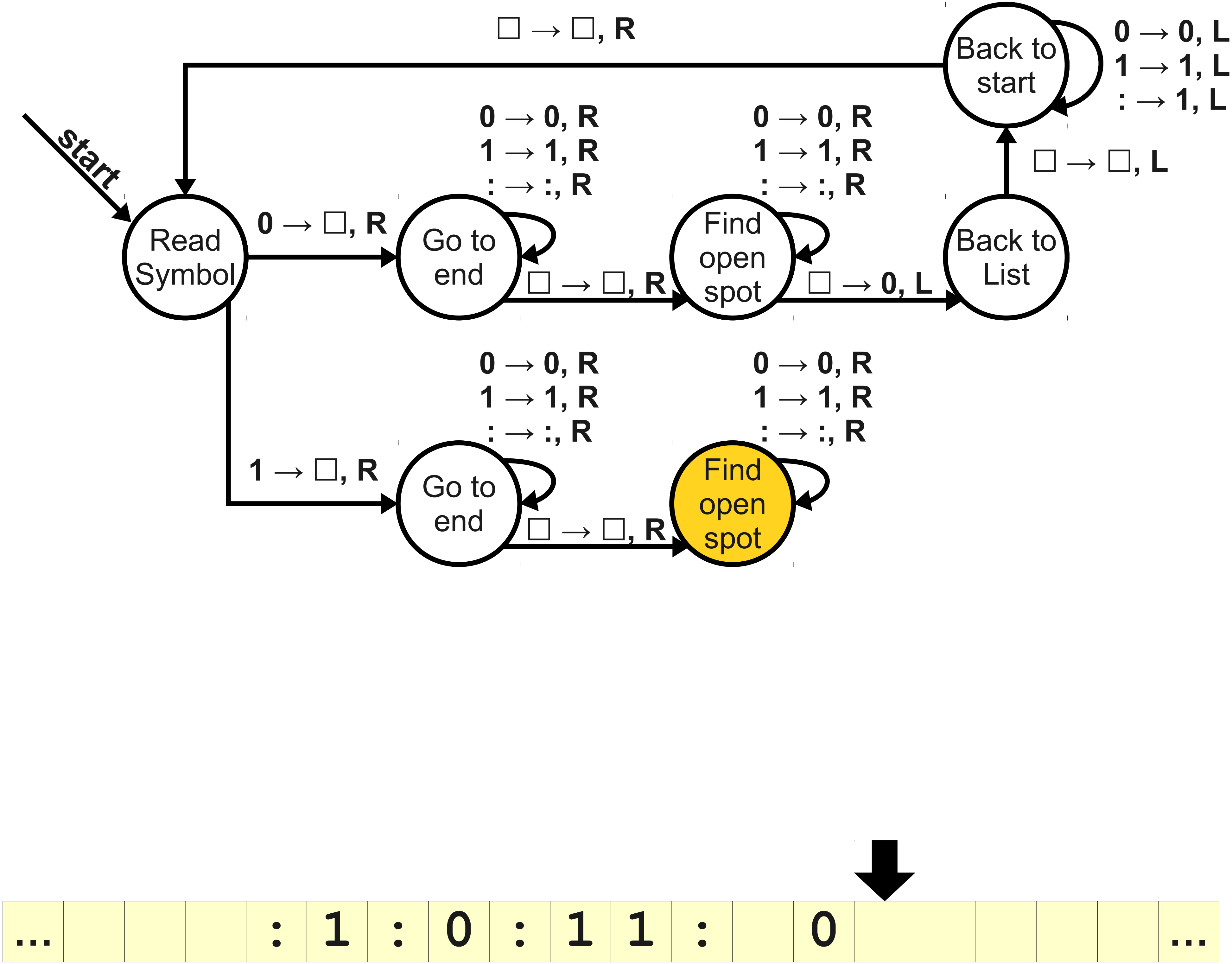
0 → 0, R
1 → 1, R
: → :, R
1 → □, R
Go to end
Find open spot
□ → □, R
...
:
1
:
0
:
1
1
:
0
...



start
Read Symbol
Go to end
Find open spot
Back to List
Back to start
Go to end
Find open spot
0 → □, R
1 → □, R
0 → 0, R
1 → 1, R
: → :, R
□ → □, R
□ → 0, L
□ → □, L
0 → 0, L
1 → 1, L
: → 1, L
□ → □, R
...
:
1
:
0
:
1
1
:
0
...



□ → □, R
Back to start
0 → 0, L
1 → 1, L
: → 1, L
start
0 → 0, R
1 → 1, R
: → :, R
0 → 0, R
1 → 1, R
: → :, R
□ → □, L
Read Symbol
0 → □, R
Go to end
□ → □, R
Find open spot
□ → 0, L
Back to List
0 → 0, R
1 → 1, R
: → :, R
0 → 0, R
1 → 1, R
: → :, R
1 → □, R
Go to end
□ → □, R
Find open spot
... : 1 : 0 : 1 1 : 0 ...



□ → □, R
Back to start
0 → 0, L
1 → 1, L
: → 1, L
start
0 → 0, R
1 → 1, R
: → :, R
0 → 0, R
1 → 1, R
: → :, R
□ → □, L
Read Symbol
0 → □, R
Go to end
□ → □, R
Find open spot
□ → 0, L
Back to List
0 → 0, R
1 → 1, R
: → :, R
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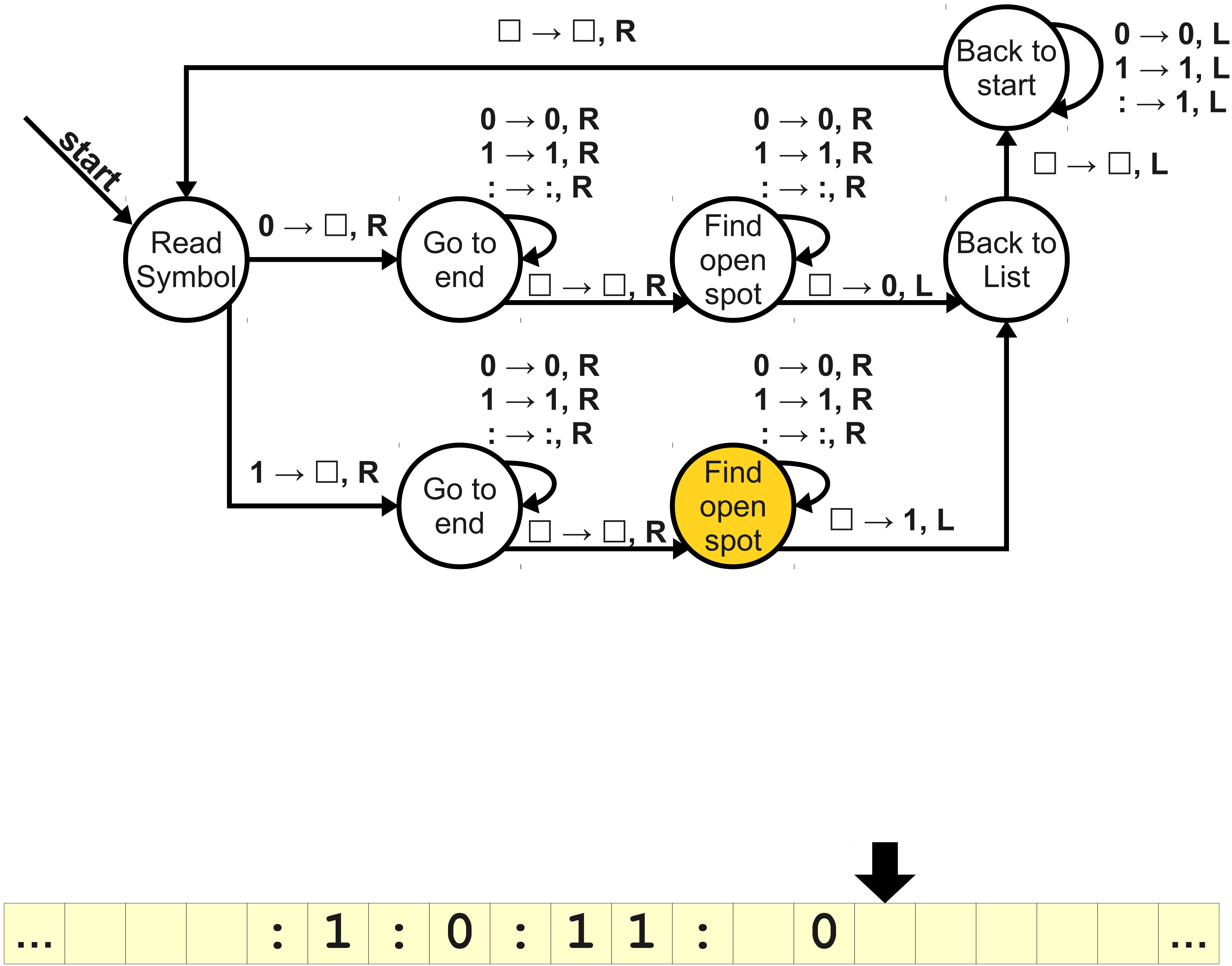
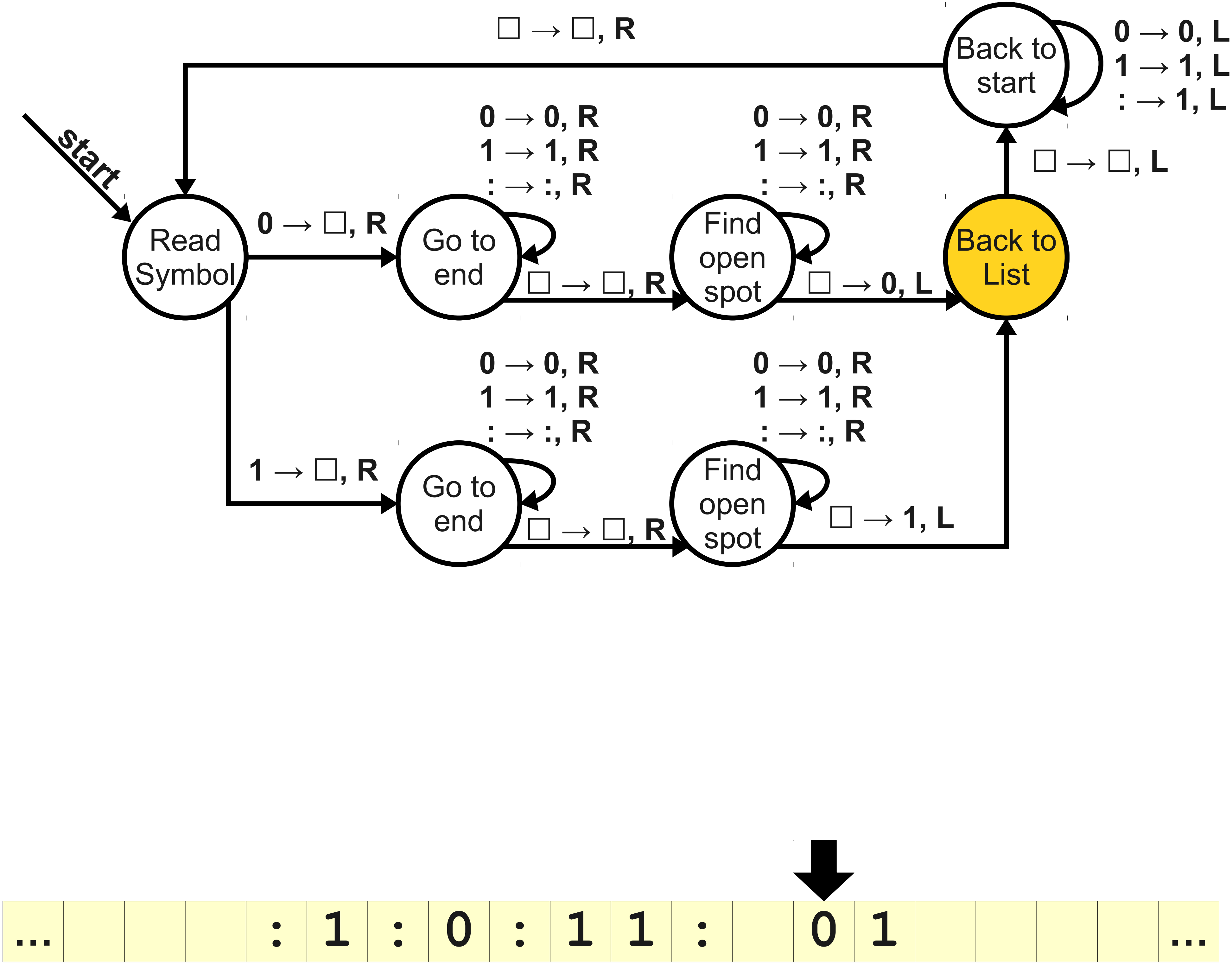
1 → 1, R
: → :, R
1 → □, R
Go to end
□ → □, R
Find open spot
... : 1 : 0 : 1 1 : 0 ...



start
Read Symbol
Go to end
Find open spot
Back to List
Back to start
0 → □, R
1 → □, R
0 → 0, R
1 → 1, R
: → :, R
□ → □, R
□ → 0, L
□ → 1, L
□ → □, L
0 → 0, L
1 → 1, L
: → 1, L
□ → □, R
... : 1 : 0 : 1 1 : 0 ...



start
Read Symbol
Go to end
Find open spot
Back to List
Back to start
Go to end
Find open spot
□ → □, R
0 → □, R
1 → □, R
0 → 0, R
1 → 1, R
: → :, R
□ → □, R
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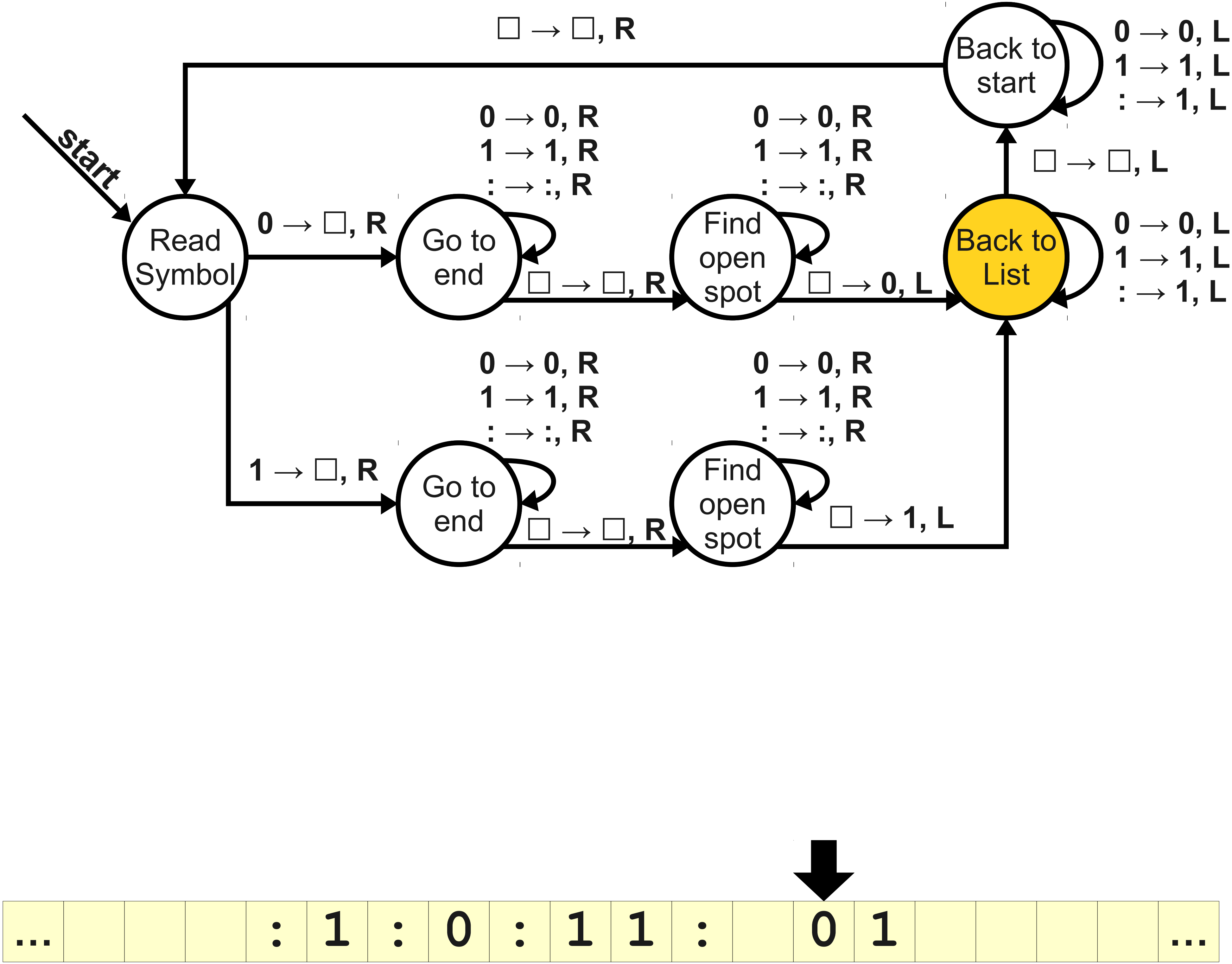
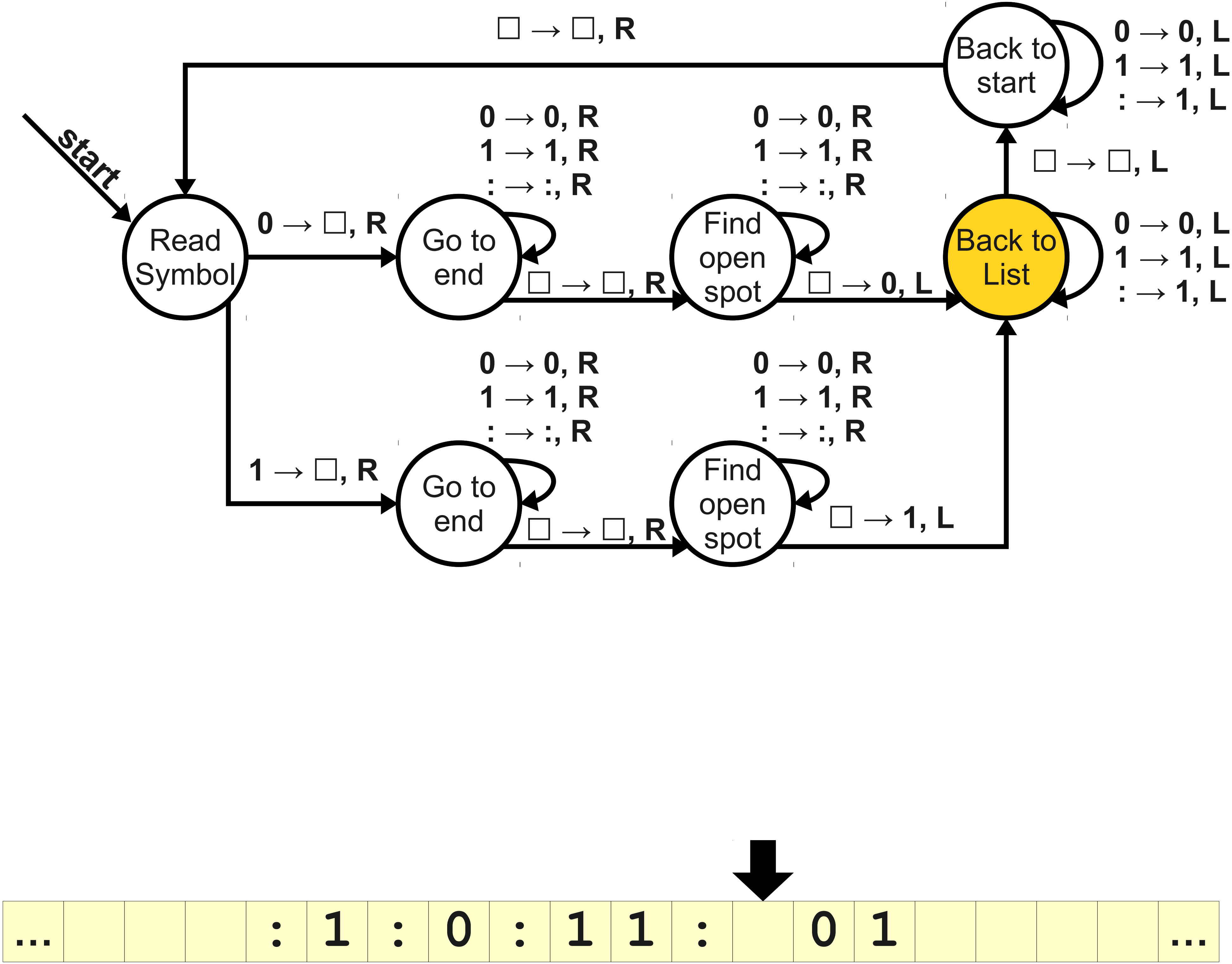
□ → 1, L
□ → □, L
0 → 0, L
1 → 1, L
: → 1, L
… : 1 : 0 : 1 1 : 0 1 …



□ → □, R
Back to start
0 → 0, L
1 → 1, L
: → 1, L
start
0 → 0, R
1 → 1, R
: → :, R
0 → 0, R
1 → 1, R
: → :, R
□ → □, L
Read Symbol
0 → □, R
Go to end
Find open spot
Back to List
0 → 0, L
1 → 1, L
: → 1, L
□ → □, R
□ → 0, L
0 → 0, R
1 → 1, R
: → :, R
0 → 0, R
1 → 1, R
: → :, R
1 → □, R
Go to end
Find open spot
□ → 1, L
□ → □, R
... : 1 : 0 : 1 1 : 0 1 ...



start
Read Symbol
Go to end
Find open spot
Back to List
Back to start
0 → □, R
1 → □, R
0 → 0, R
1 → 1, R
: → :, R
□ → □, R
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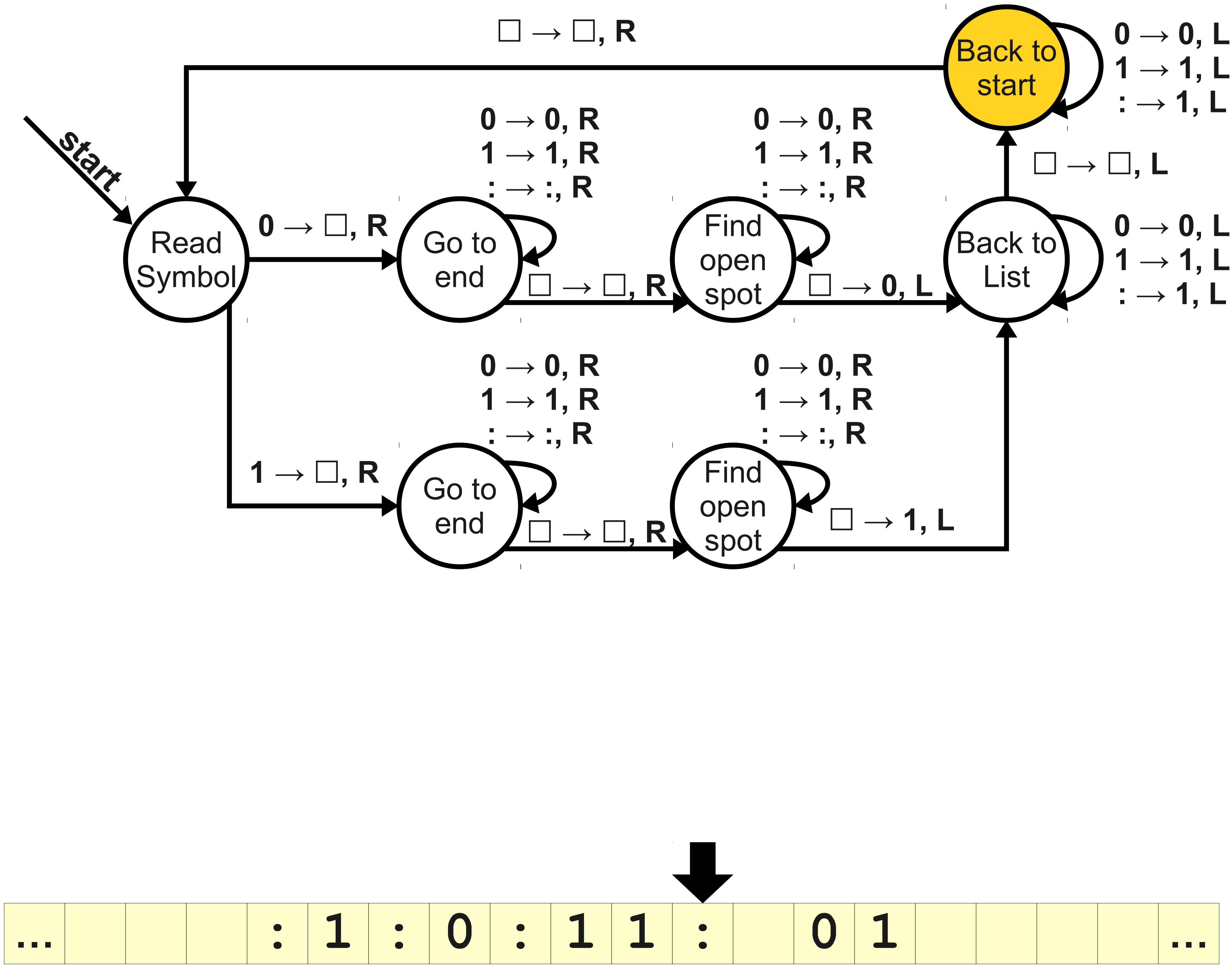
□ → □, L
0 → 0, L
1 → 1, L
: → 1, L
… : 1 : 0 : 1 1 : 0 1 …



start
Read Symbol
Go to end
Find open spot
Back to List
Back to start
0 → □, R
1 → □, R
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1 → 1, R
: → :, R
□ → □, R
□ → 0, L
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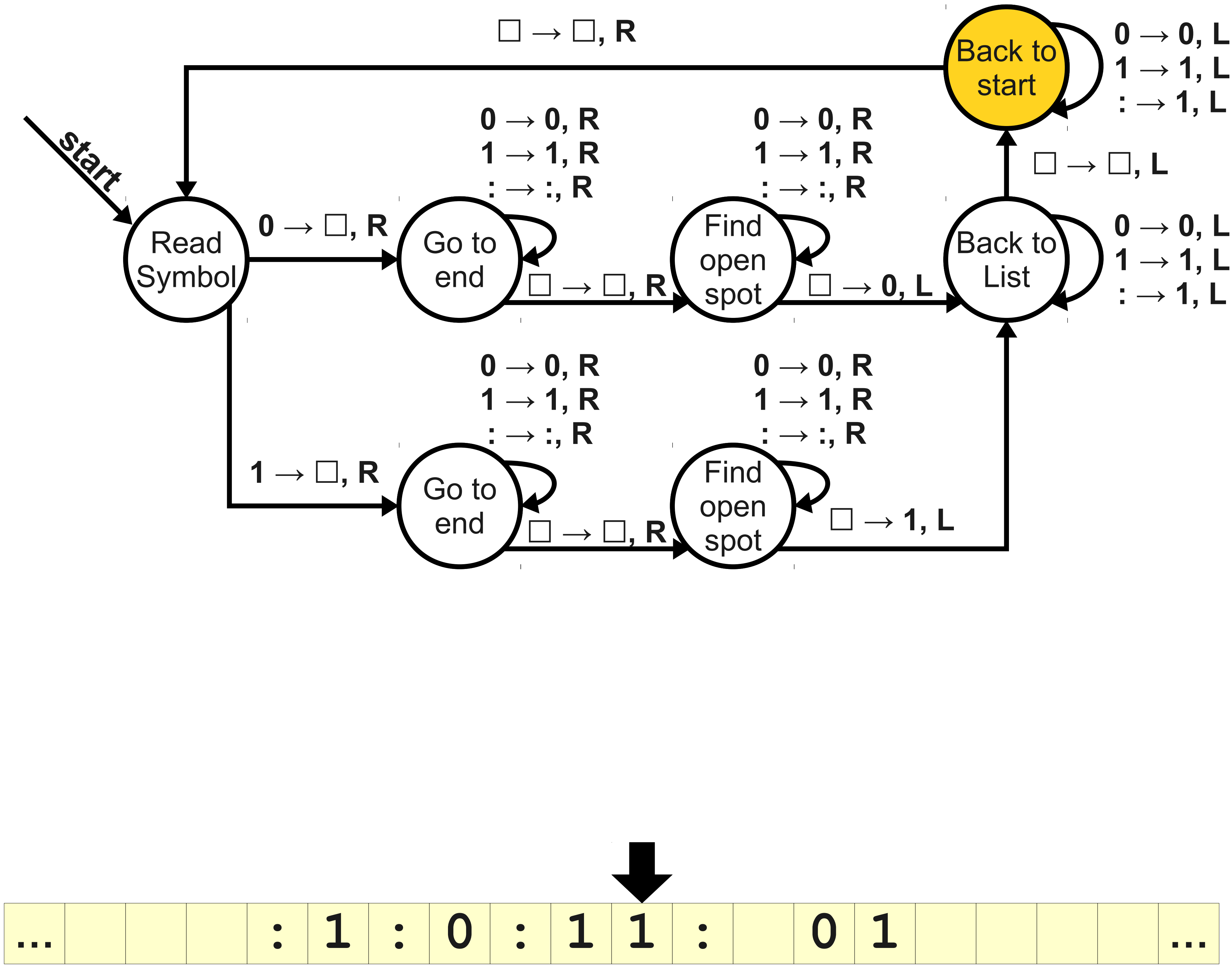
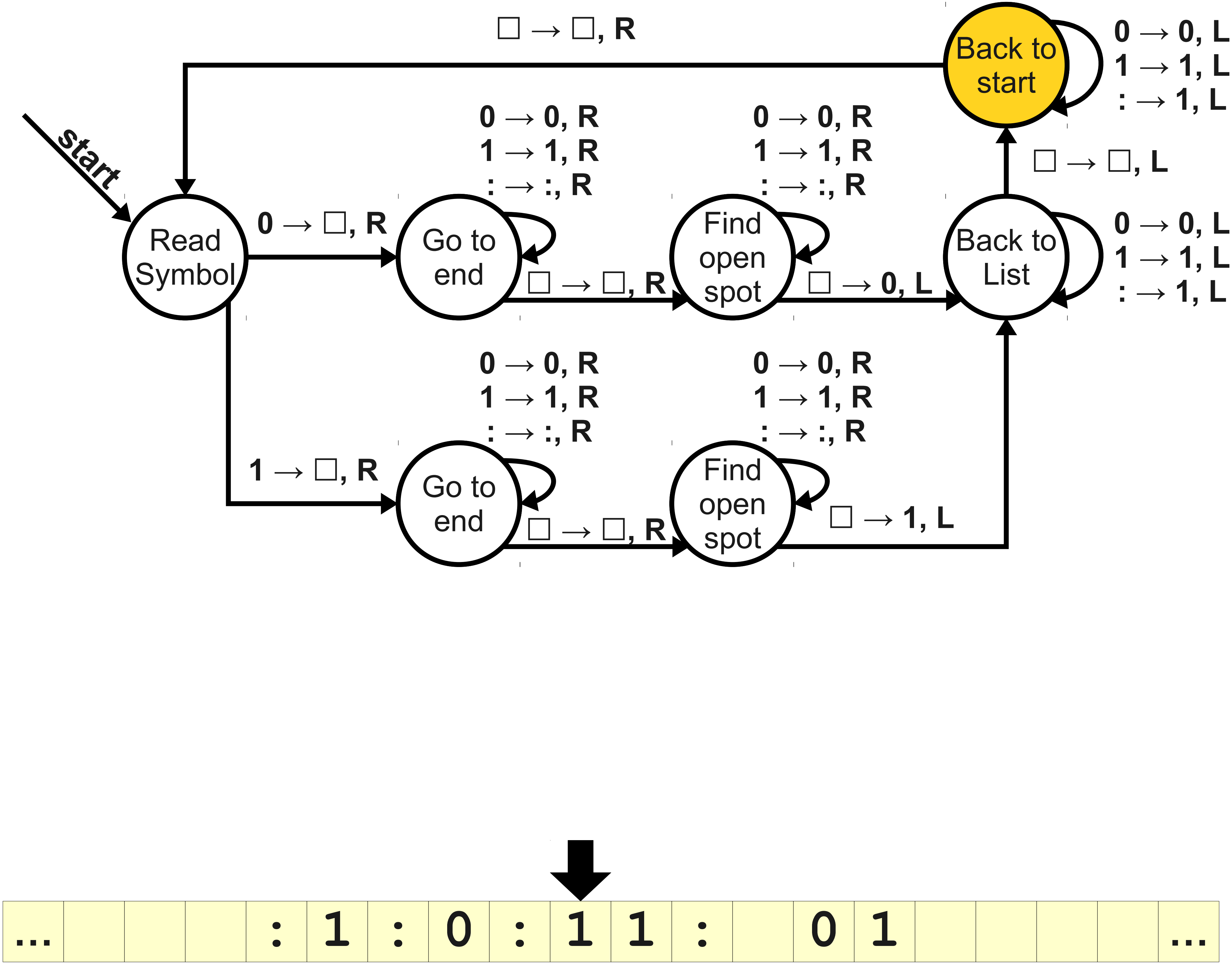
1 → 1, L
: → 1, L
□ → □, L
□ → □, R
... : 1 : 0 : 1 1 : 0 1 ...


□ → □, R
Back to start
0 → 0, L
1 → 1, L
: → 1, L
start
0 → 0, R
1 → 1, R
: → :, R
0 → 0, R
1 → 1, R
: → :, R
□ → □, L
Read Symbol
0 → □, R
Go to end
□ → □, R
Find open spot
□ → 0, L
Back to List
0 → 0, L
1 → 1, L
: → 1, L
0 → 0, R
1 → 1, R
: → :, R
0 → 0, R
1 → 1, R
: → :, R
1 → □, R
Go to end
□ → □, R
Find open spot
□ → 1, L
... : 1 : 0 : 1 1 : 0 1 ...


□ → □, R
Back to start
0 → 0, L
1 → 1, L
: → 1, L
start
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1 → 1, R
: → :, R
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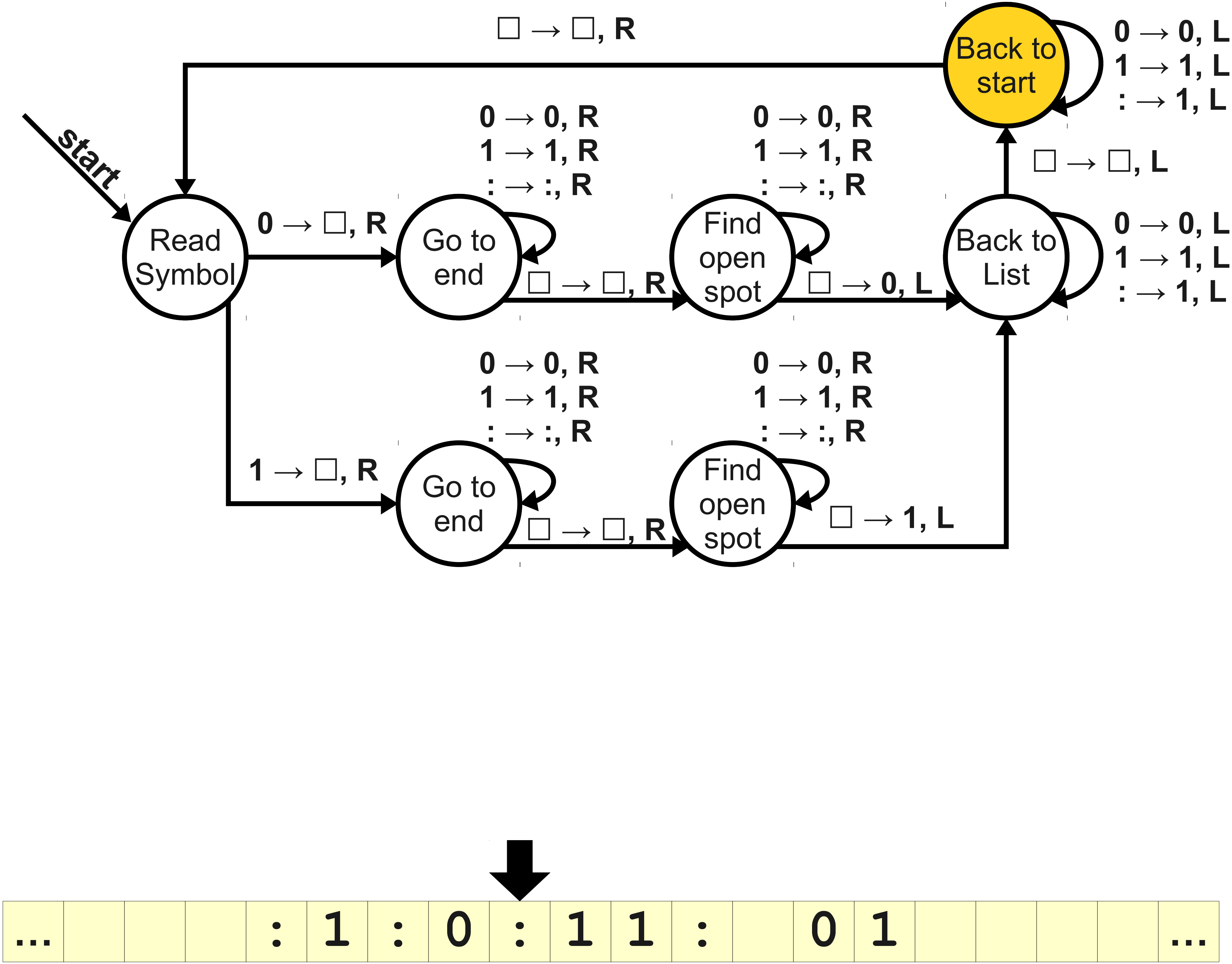
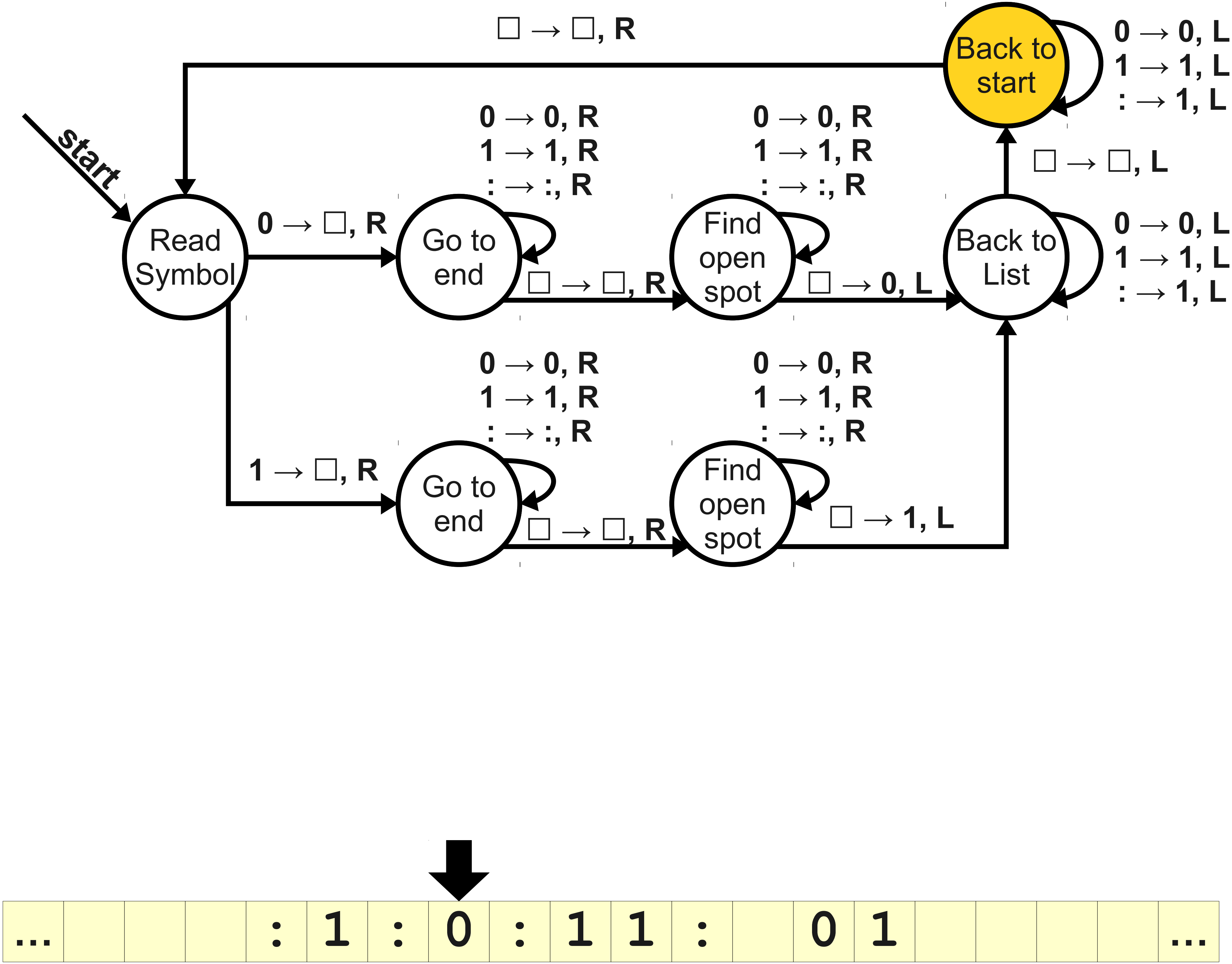
1 → 1, R
: → :, R
□ → □, L
Read Symbol
0 → □, R
Go to end
□ → □, R
Find open spot
□ → 0, L
Back to List
0 → 0, L
1 → 1, L
: → 1, L
0 → 0, R
1 → 1, R
: → :, R
0 → 0, R
1 → 1, R
: → :, R
1 → □, R
Go to end
□ → □, R
Find open spot
□ → 1, L
... : 1 : 0 : 1 1 : 0 1 ...


start
Read Symbol
Go to end
Find open spot
Back to List
Back to start
□ → □, R
0 → 0, L
1 → 1, L
: → 1, L
0 → 0, R
1 → 1, R
: → :, R
0 → □, R
□ → □, R
□ → 0, L
□ → □, L
1 → □, R
□ → 1, L
... : 1 : 0 : 1 1 : 0 1 ...


□ → □, R
Back to start
0 → 0, L
1 → 1, L
: → 1, L
start
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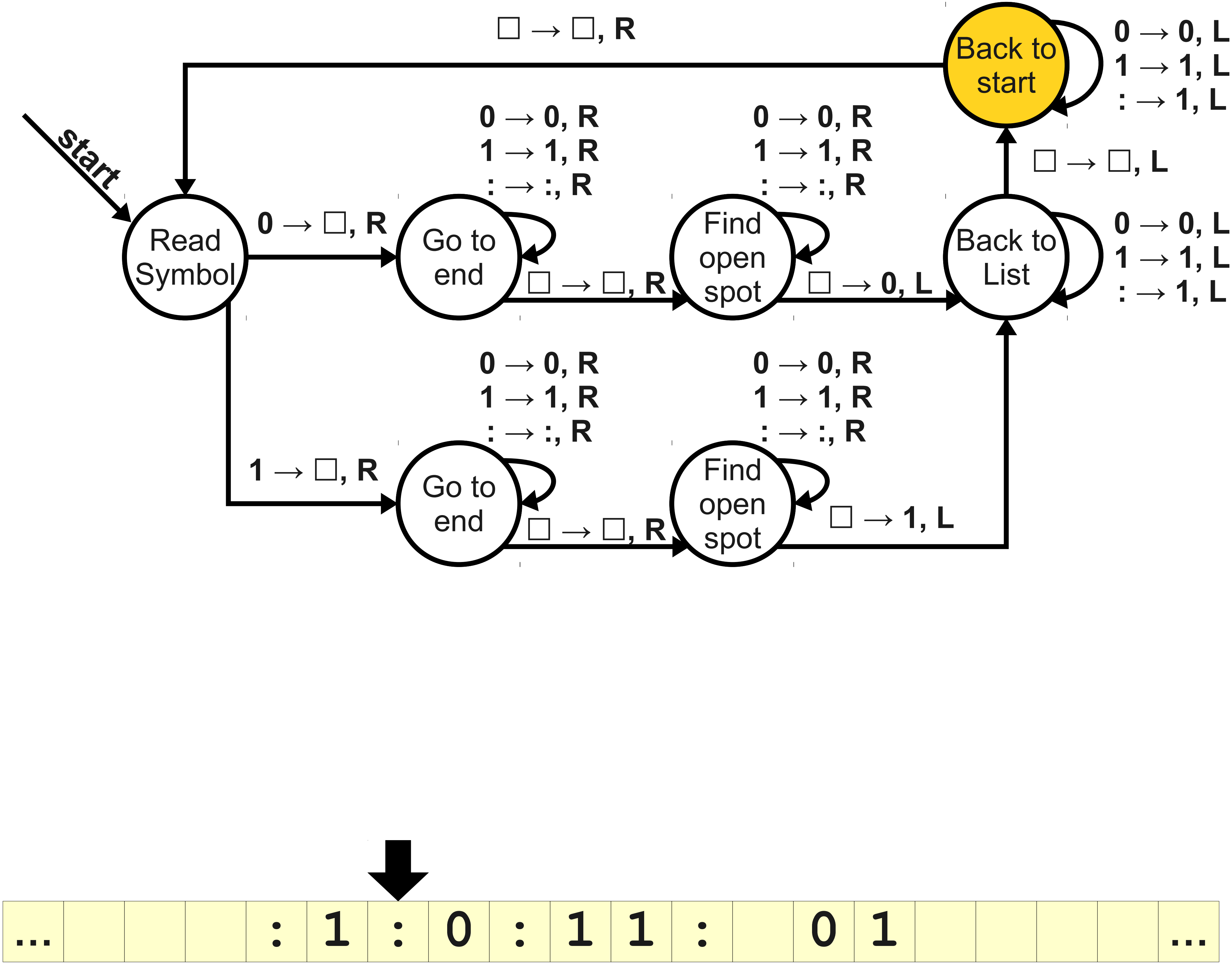
1 → 1, R
: → :, R
0 → 0, R
1 → 1, R
: → :, R
□ → □, L
Read Symbol
0 → □, R
Go to end
□ → □, R
Find open spot
□ → 0, L
Back to List
0 → 0, L
1 → 1, L
: → 1, L
0 → 0, R
1 → 1, R
: → :, R
0 → 0, R
1 → 1, R
: → :, R
1 → □, R
Go to end
□ → □, R
Find open spot
□ → 1, L
... : 1 : 0 : 1 1 : 0 1 ...


start
Read Symbol
Go to end
Find open spot
Back to List
Back to start
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: → :, R
□ → □, R
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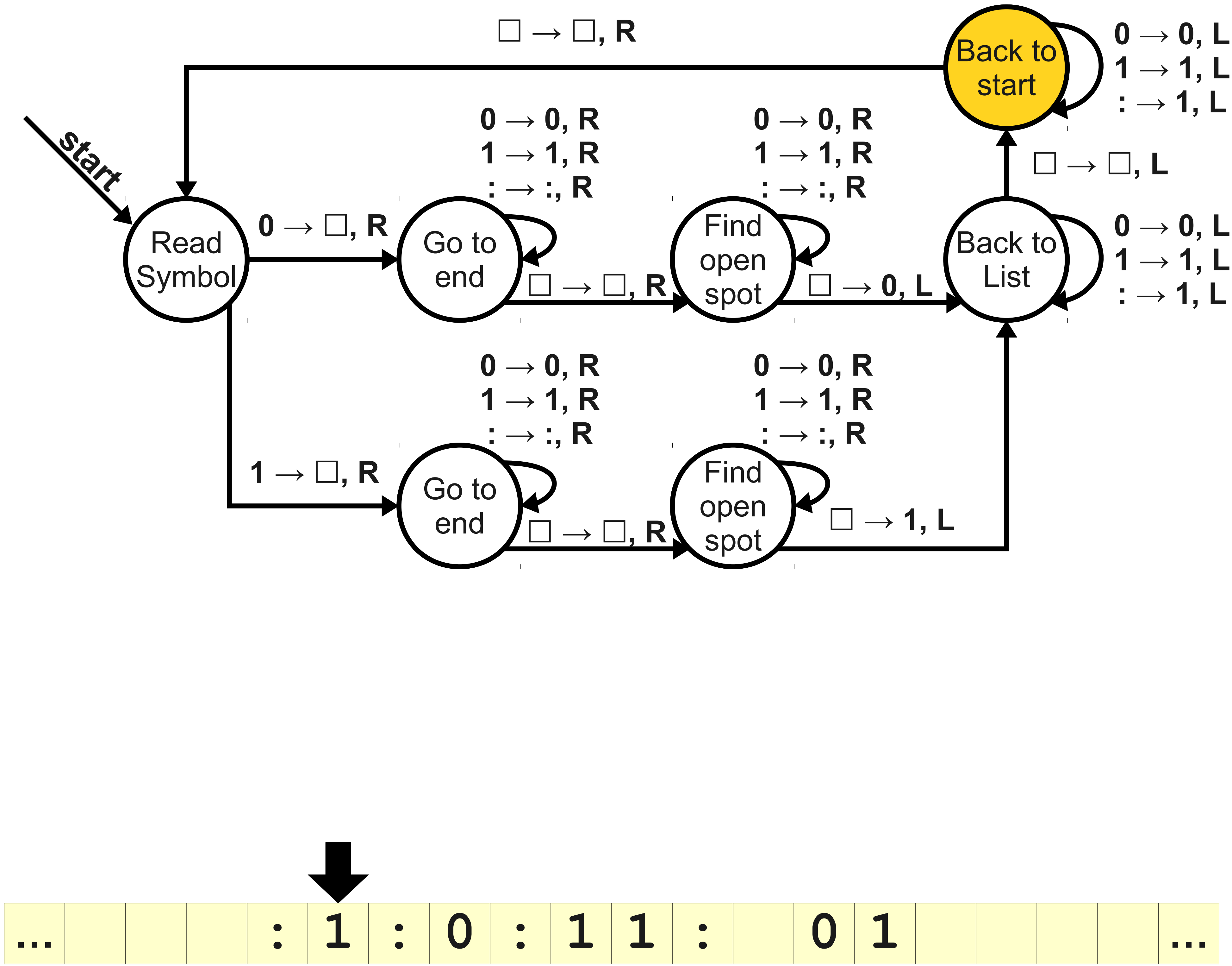
0 → 0, L
1 → 1, L
: → 1, L
□ → □, L
... : 1 : 0 : 1 1 : 0 1 ...


start
Read Symbol
Go to end
Find open spot
Back to List
Back to start
0 → □, R
1 → □, R
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: → :, R
□ → □, R
□ → 0, L
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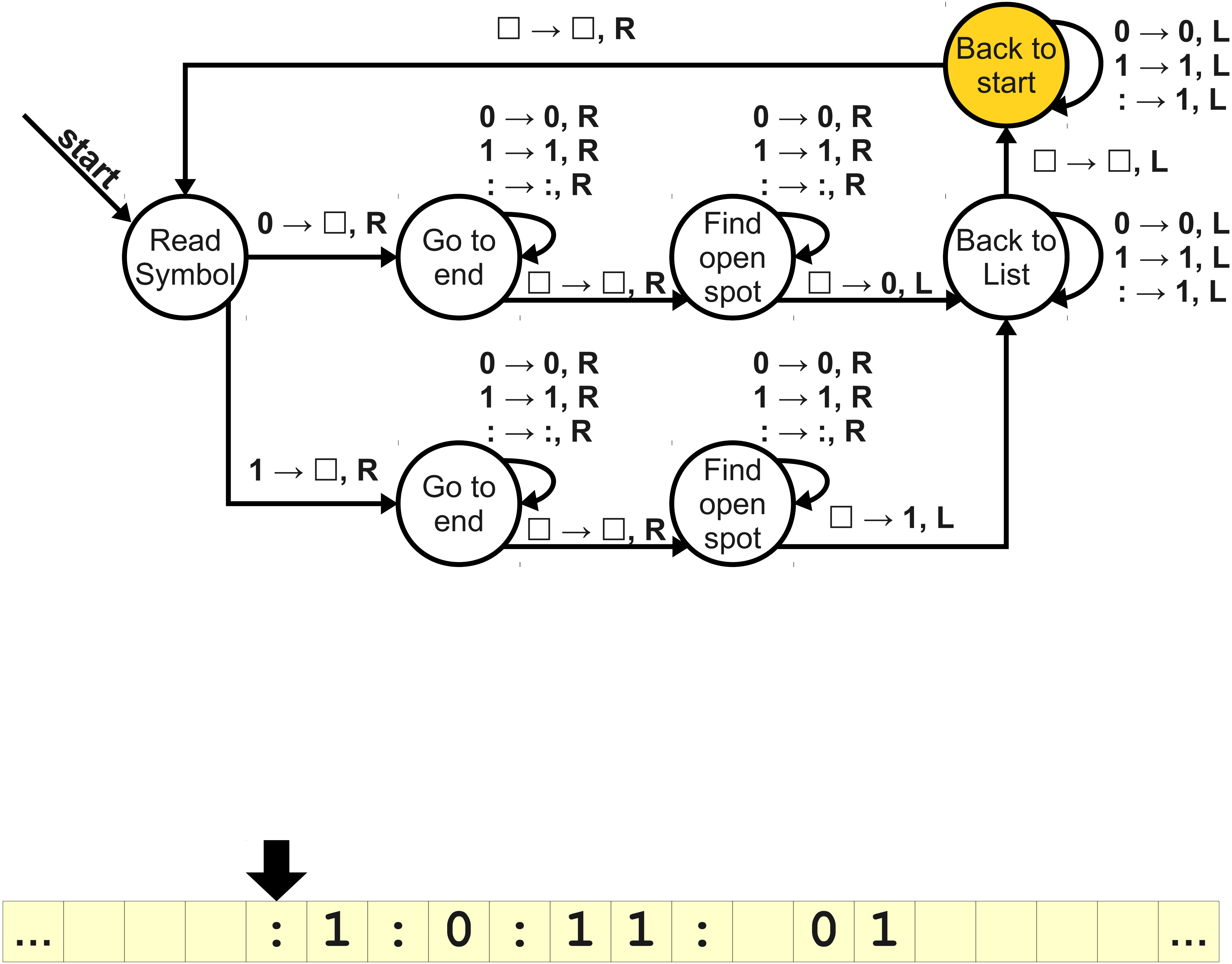
1 → 1, L
: → 1, L
□ → □, L
□ → □, R
… : 1 : 0 : 1 1 : 0 1 …



start
Read Symbol
Go to end
Find open spot
Back to List
Back to start
0 → □, R
1 → □, R
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1 → 1, R
: → :, R
□ → □, R
□ → 0, L
□ → 1, L
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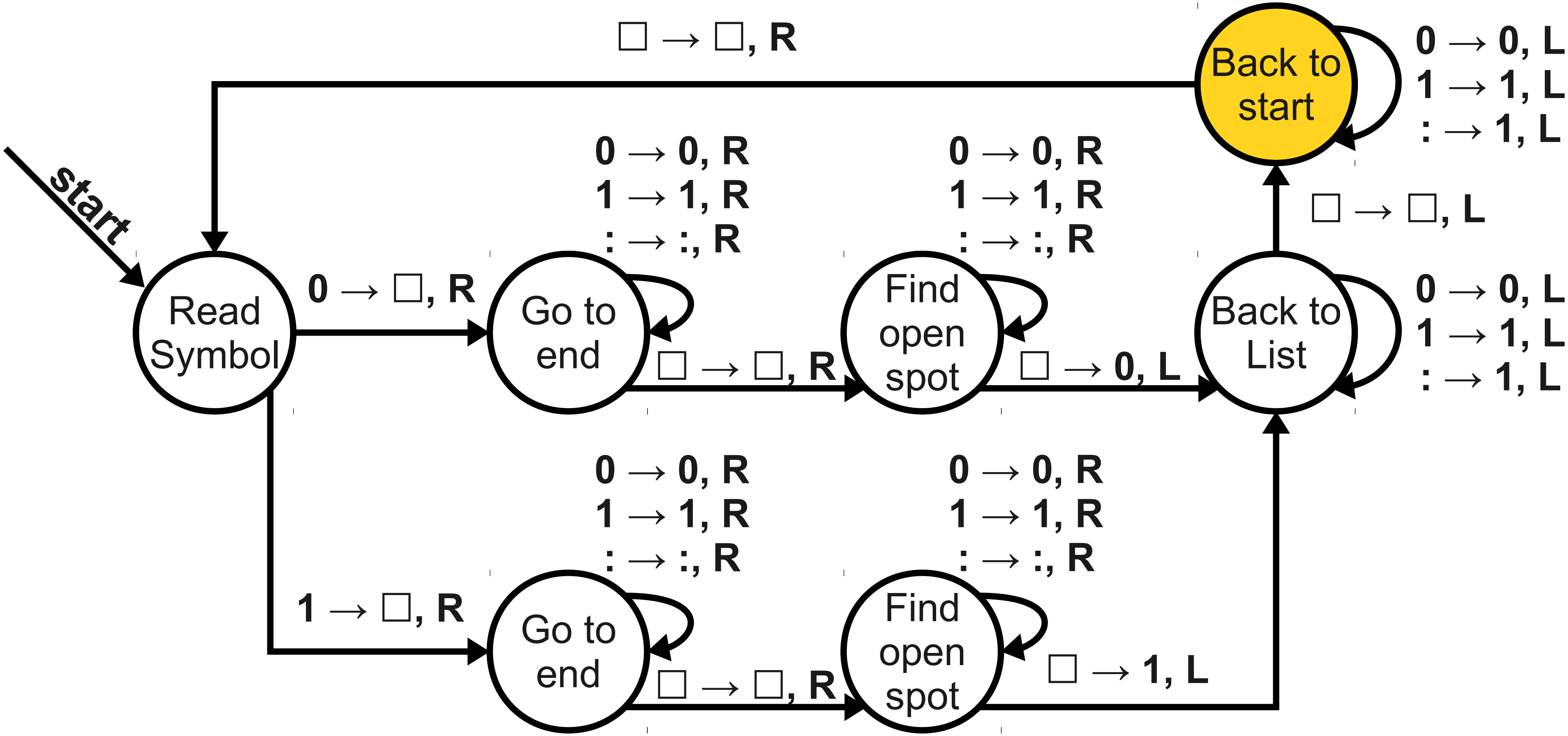
1 → 1, L
: → 1, L
□ → □, L
□ → □, R
... : 1 : 0 : 1 1 : 0 1 ...


start
Read Symbol
Go to end
Find open spot
Back to List
Back to start
Go to end
Find open spot
0 → □, R
1 → □, R
0 → 0, R
1 → 1, R
: → :, R
□ → □, R
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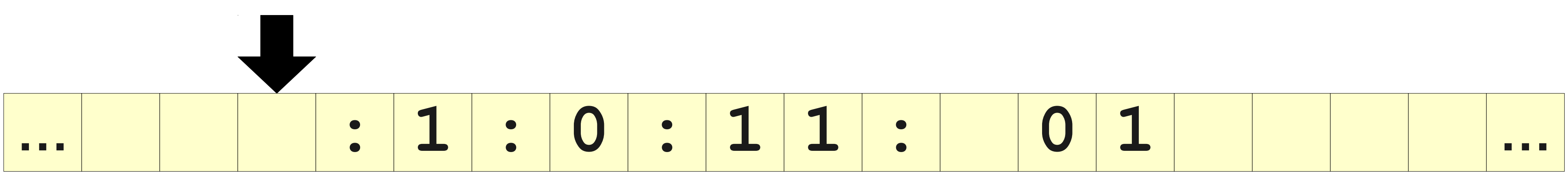
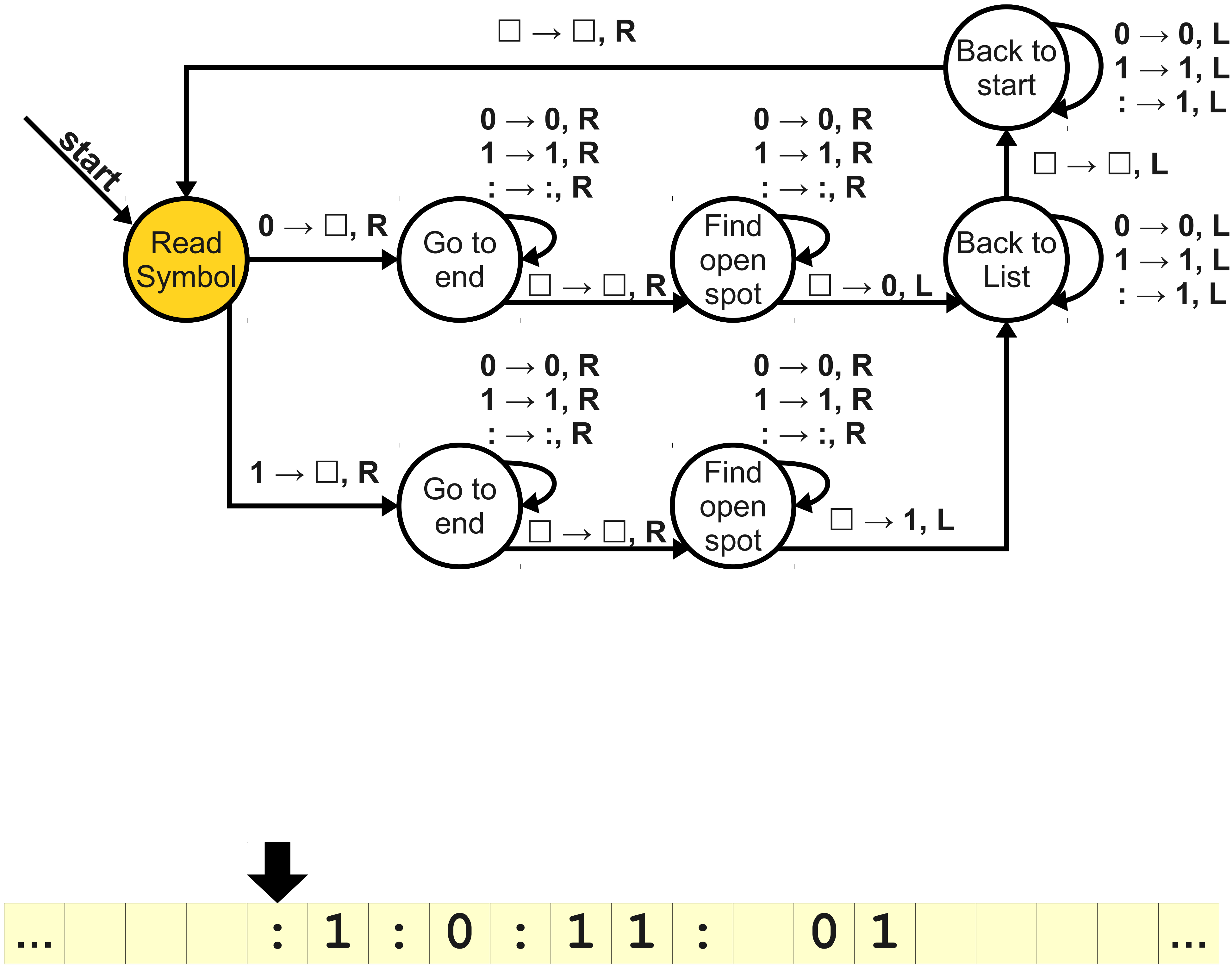
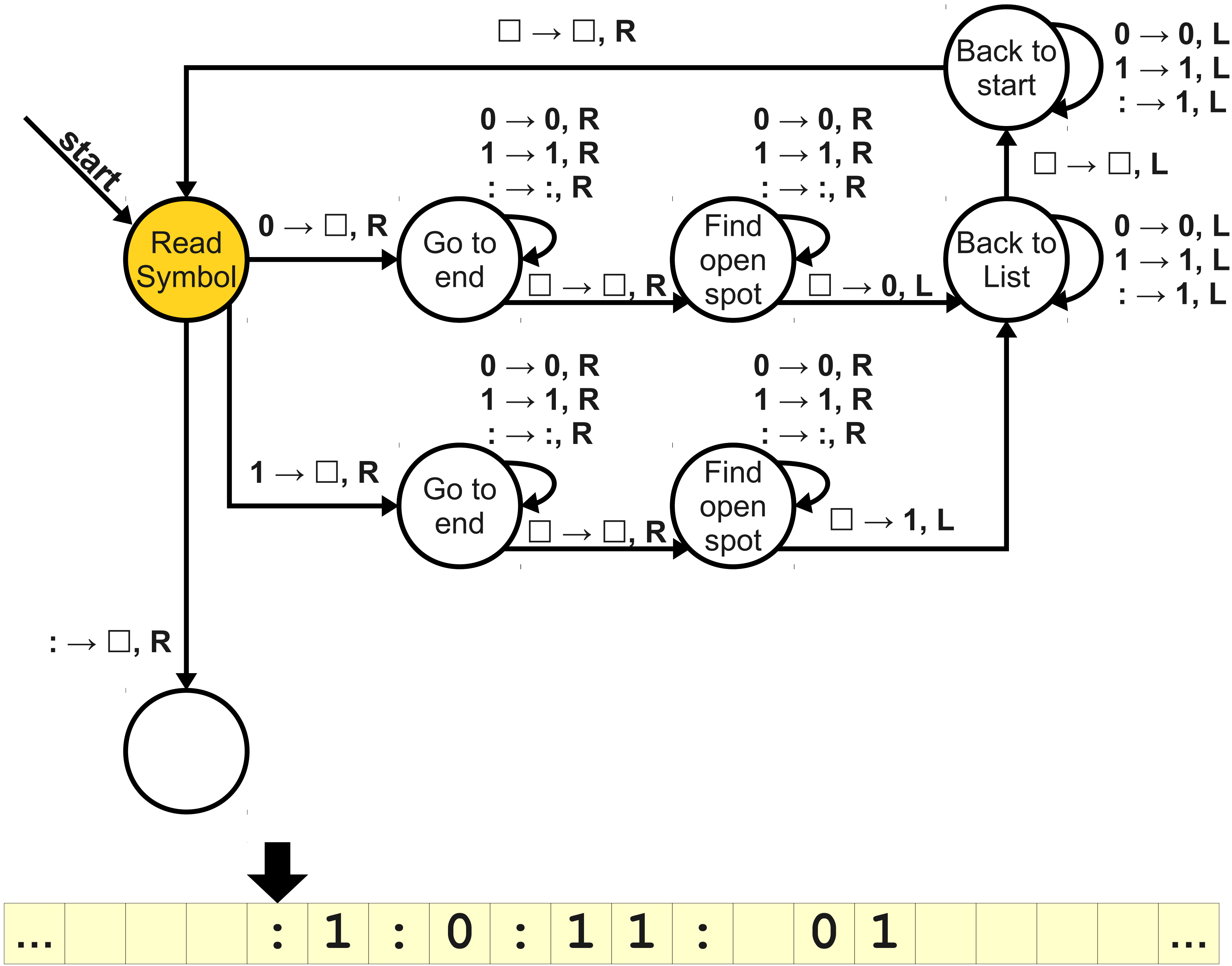
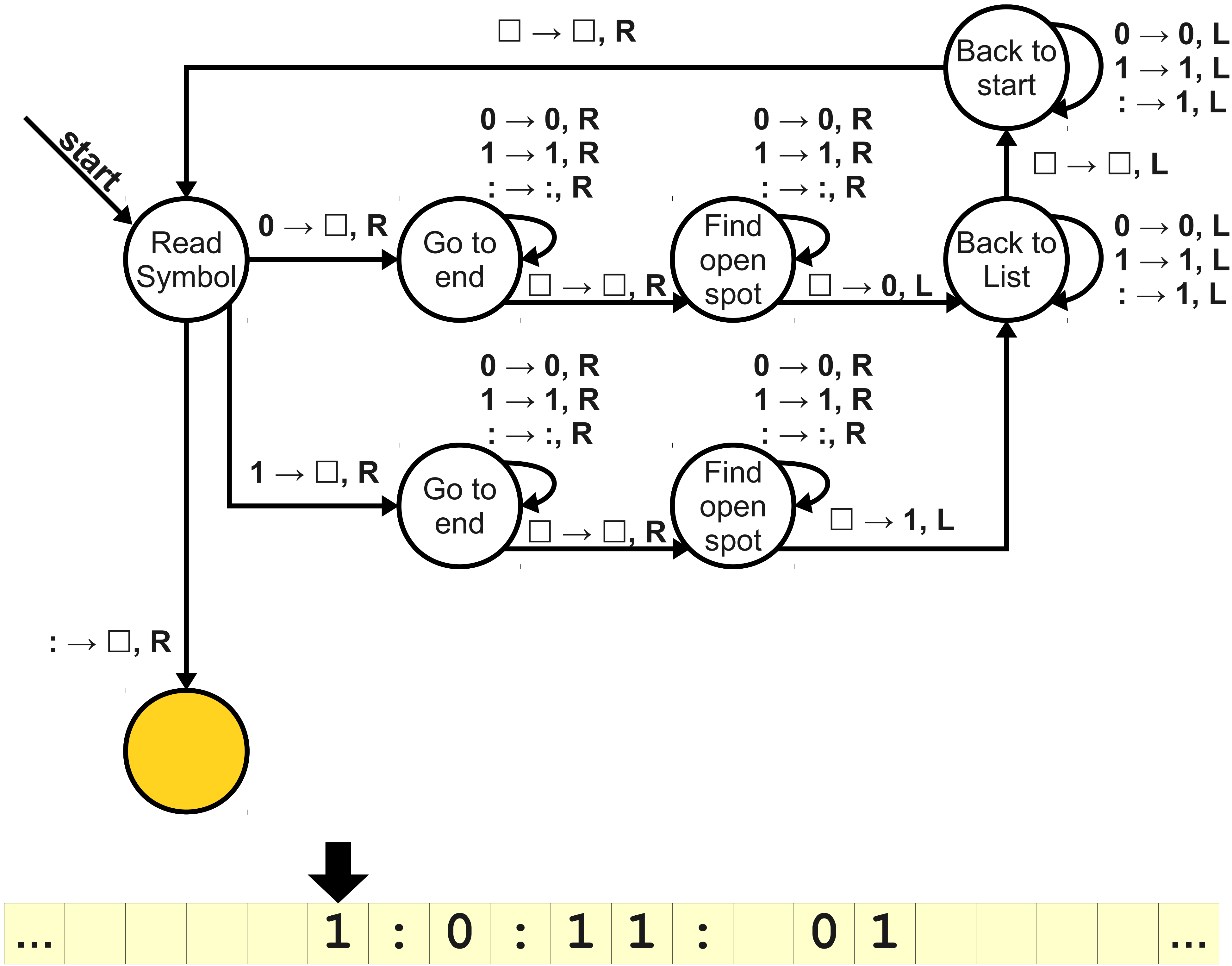
□ → 1, L
0 → 0, L
1 → 1, L
: → 1, L
□ → □, L
□ → □, R


... : 1 : 0 : 1 1 : 0 1 ...


start
Read Symbol
Go to end
Find open spot
Back to List
Back to start
0 → □, R
1 → □, R
0 → 0, R
1 → 1, R
: → :, R
□ → □, R
□ → 0, L
□ → 1, L
□ → □, L
0 → 0, L
1 → 1, L
: → 1, L
□ → □, R
... : 1 : 0 : 1 1 : 0 1 ...


□ → □, R
Back to start
0 → 0, L
1 → 1, L
: → 1, L
start
0 → 0, R
1 → 1, R
: → :, R
0 → 0, R
1 → 1, R
: → :, R
□ → □, L
0 → □, R
Read Symbol
Go to end
□ → □, R
Find open spot
□ → 0, L
Back to List
0 → 0, L
1 → 1, L
: → 1, L
0 → 0, R
1 → 1, R
: → :, R
0 → 0, R
1 → 1, R
: → :, R
1 → □, R
Go to end
□ → □, R
Find open spot
□ → 1, L
: → □, R
... : 1 : 0 : 1 1 : 0 1 ...


□ → □, R
Back to start
0 → 0, L
1 → 1, L
: → 1, L
start
0 → 0, R
1 → 1, R
: → :, R
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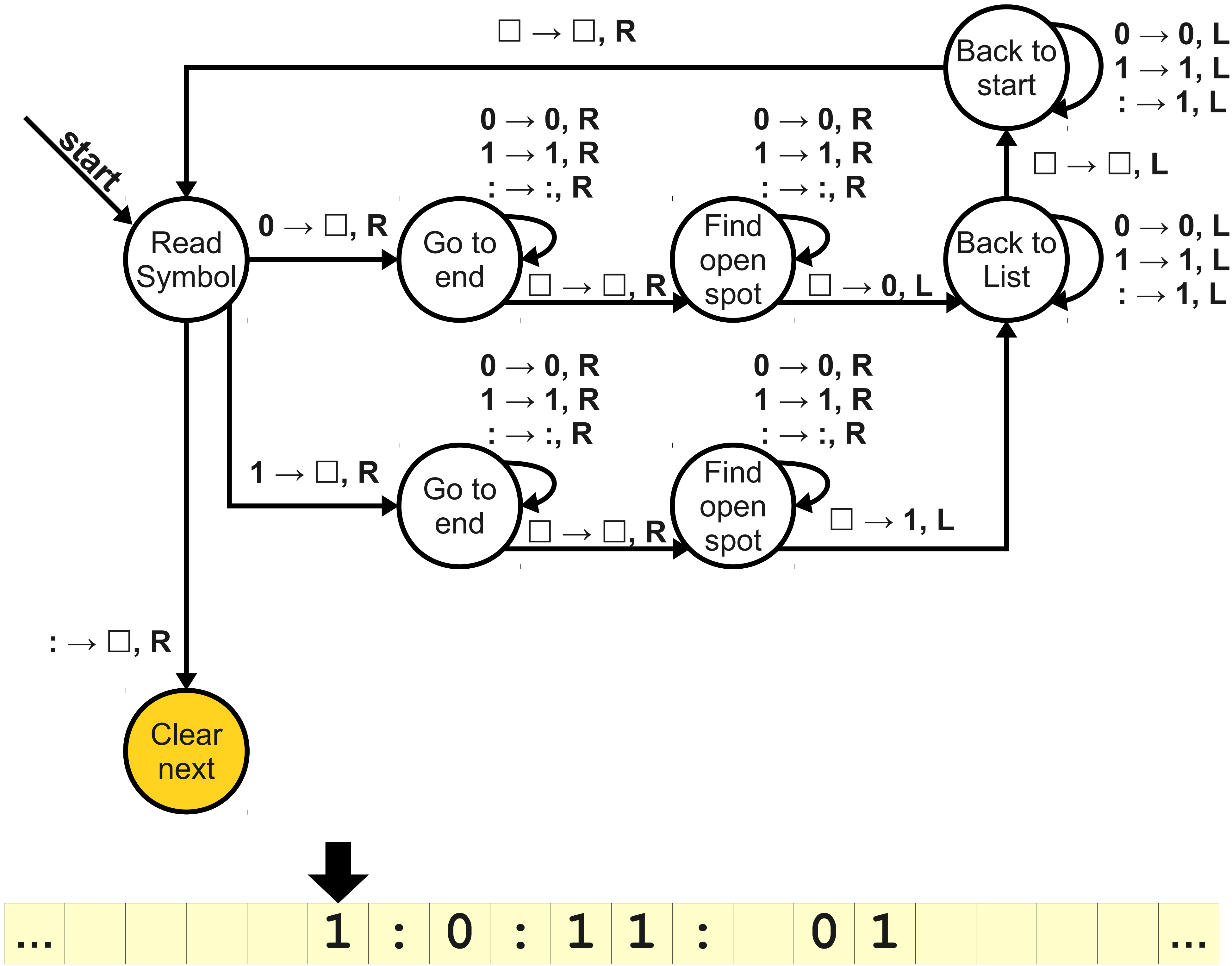
1 → 1, R
: → :, R
□ → □, L
Read Symbol
0 → □, R
Go to end
□ → □, R
Find open spot
□ → 0, L
Back to List
0 → 0, L
1 → 1, L
: → 1, L
0 → 0, R
1 → 1, R
: → :, R
0 → 0, R
1 → 1, R
: → :, R
1 → □, R
Go to end
□ → □, R
Find open spot
□ → 1, L
: → □, R
... 1 : 0 : 1 1 : 0 1 ...



start
Read Symbol
Go to end
Find open spot
Back to List
Back to start
Clear next
□ → □, R
0 → □, R
1 → □, R
: → □, R
0 → 0, R
1 → 1, R
: → :, R
□ → □, R
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□ → 1, L
□ → □, L
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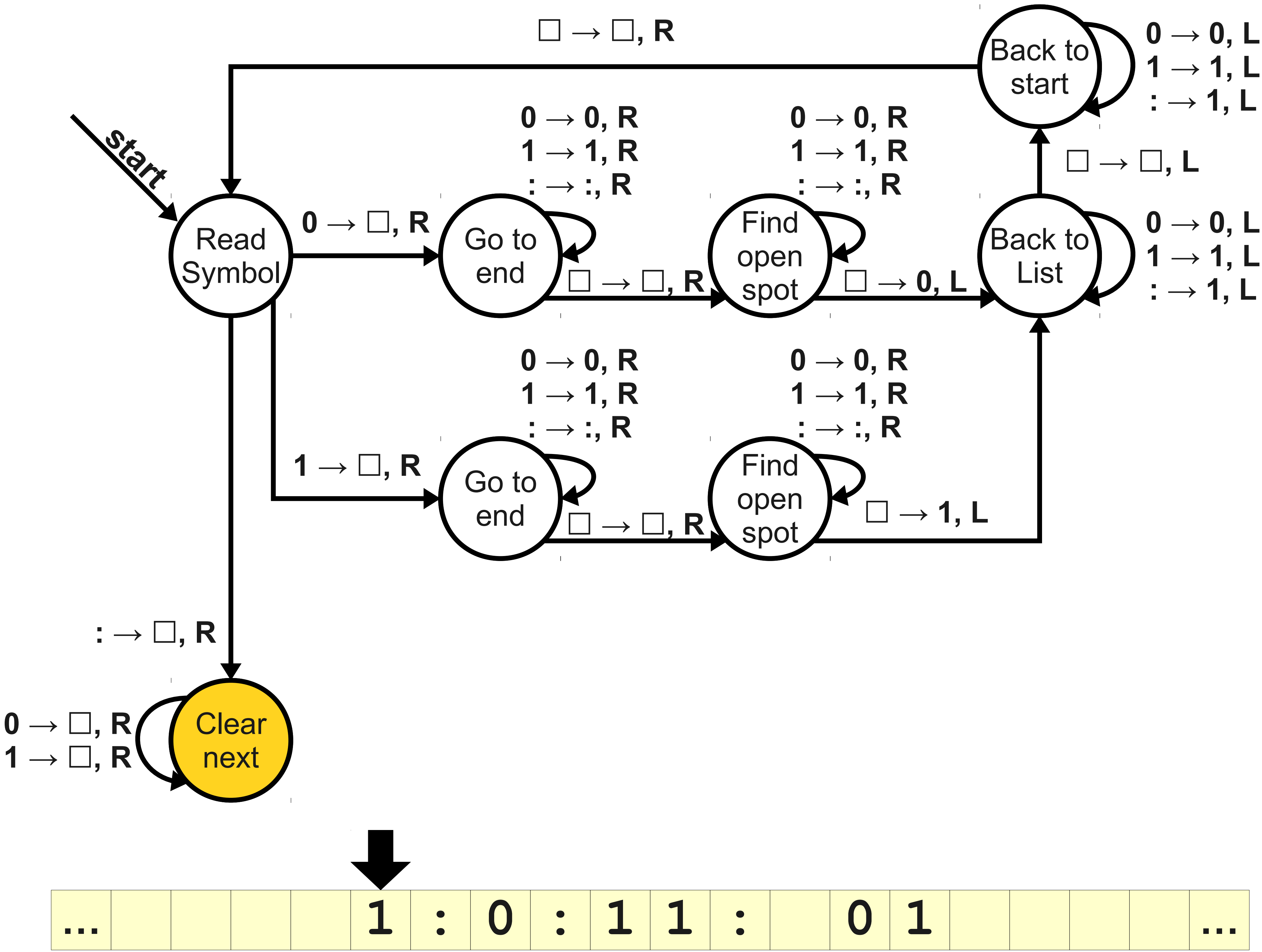
1 → 1, L
: → 1, L
... 1 : 0 : 1 1 : 0 1 ...


start
Read Symbol
Go to end
Find open spot
Back to List
Back to start
Clear next
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: → □, R
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1 → 1, R
: → :, R
□ → □, R
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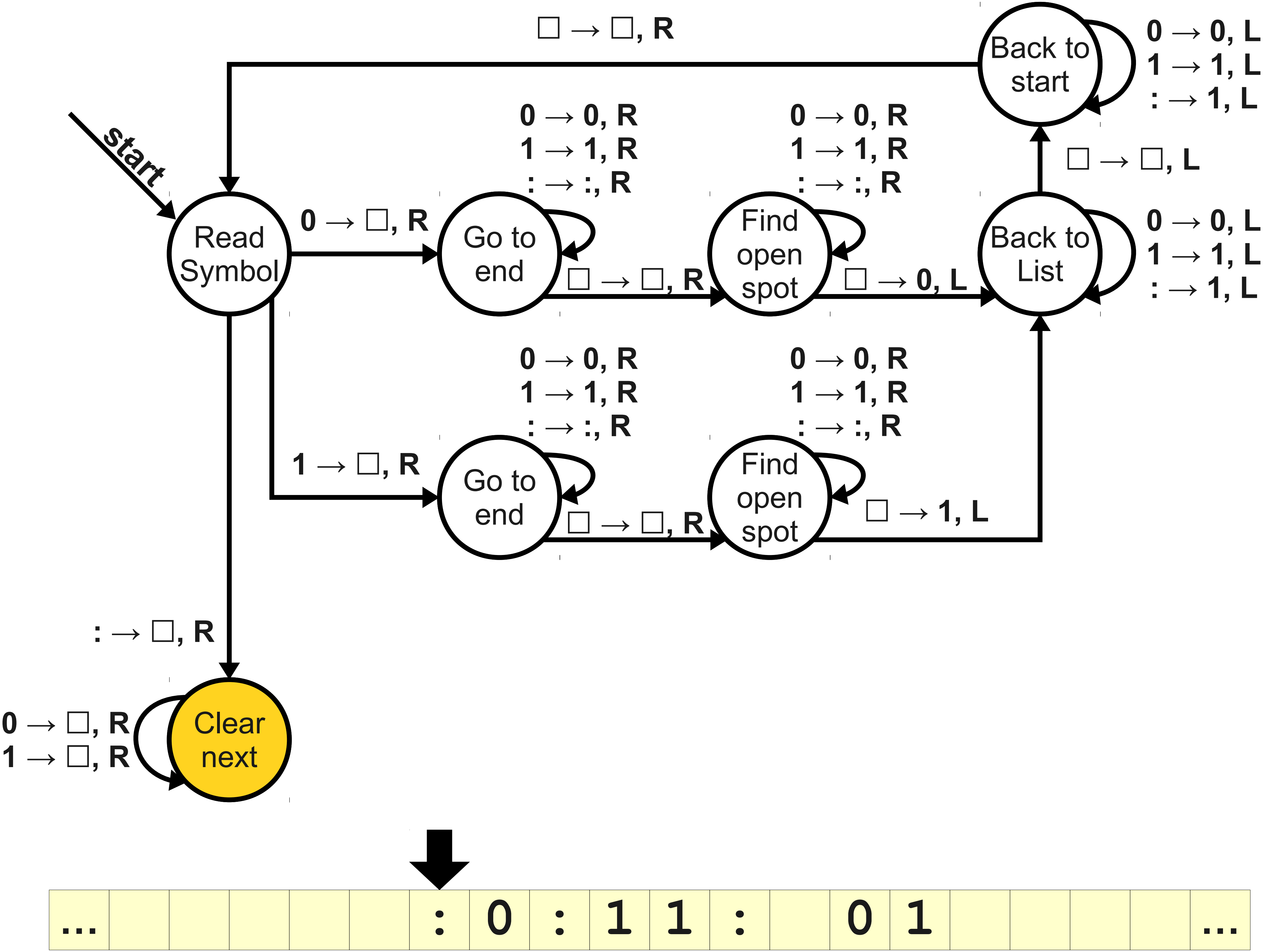
1 → 1, L
: → 1, L
□ → □, L
…
1 : 0 : 1 1 : 0 1
…



□ → □, R
Back to start
0 → 0, L
1 → 1, L
: → 1, L
start
0 → 0, R
1 → 1, R
: → :, R
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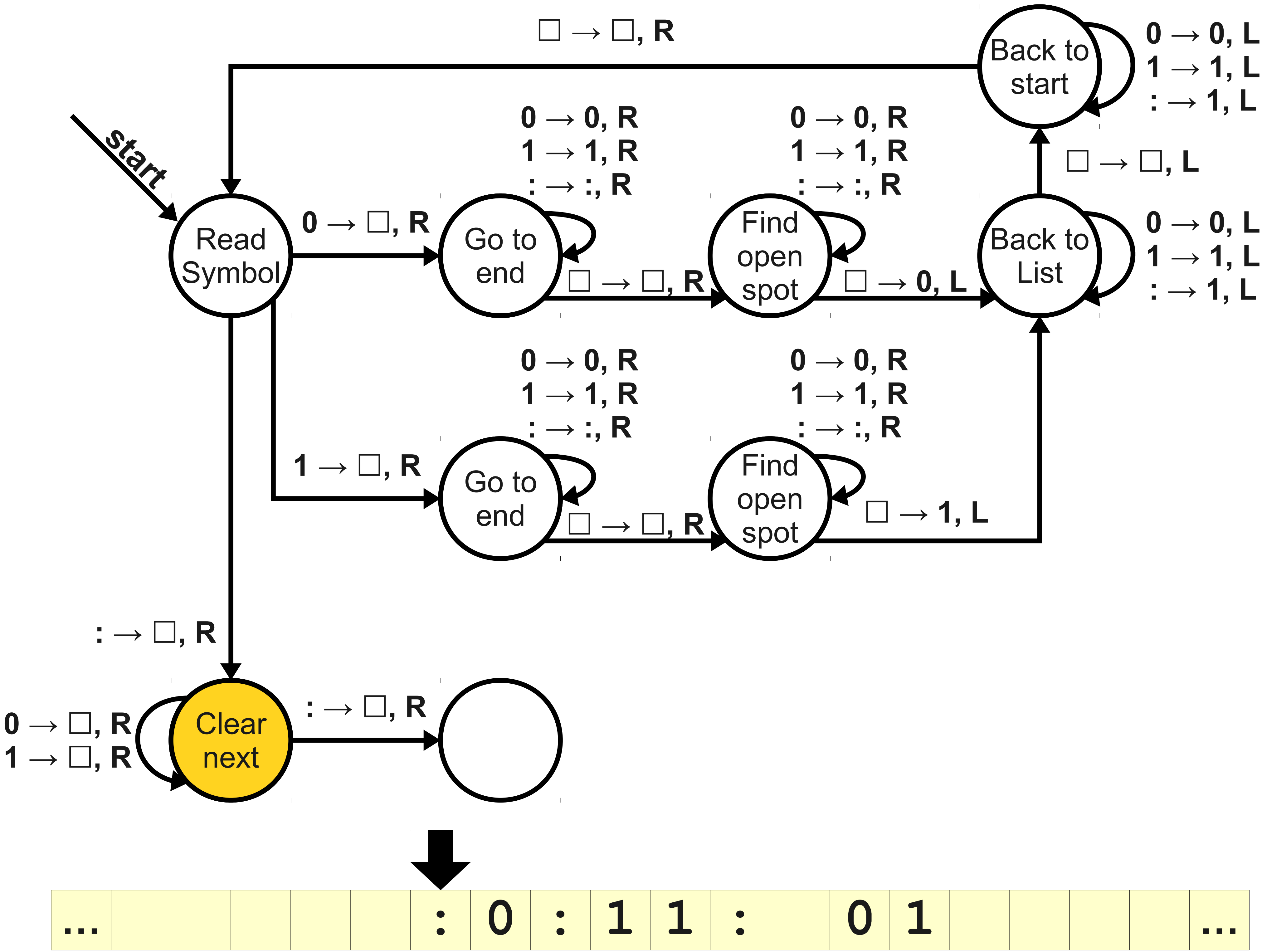
1 → 1, R
: → :, R
□ → □, L
Read Symbol
0 → □, R
Go to end
□ → □, R
Find open spot
□ → 0, L
Back to List
0 → 0, L
1 → 1, L
: → 1, L
0 → 0, R
1 → 1, R
: → :, R
0 → 0, R
1 → 1, R
: → :, R
1 → □, R
Go to end
□ → □, R
Find open spot
□ → 1, L
: → □, R
0 → □, R
1 → □, R
Clear next
... : 0 : 1 1 : 0 1 ...


start
Read Symbol
Go to end
Find open spot
Back to List
Back to start
Clear next
□ → □, R
0 → □, R
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1 → 1, R
: → :, R
□ → □, R
□ → 0, L
□ → 1, L
0 → 0, L
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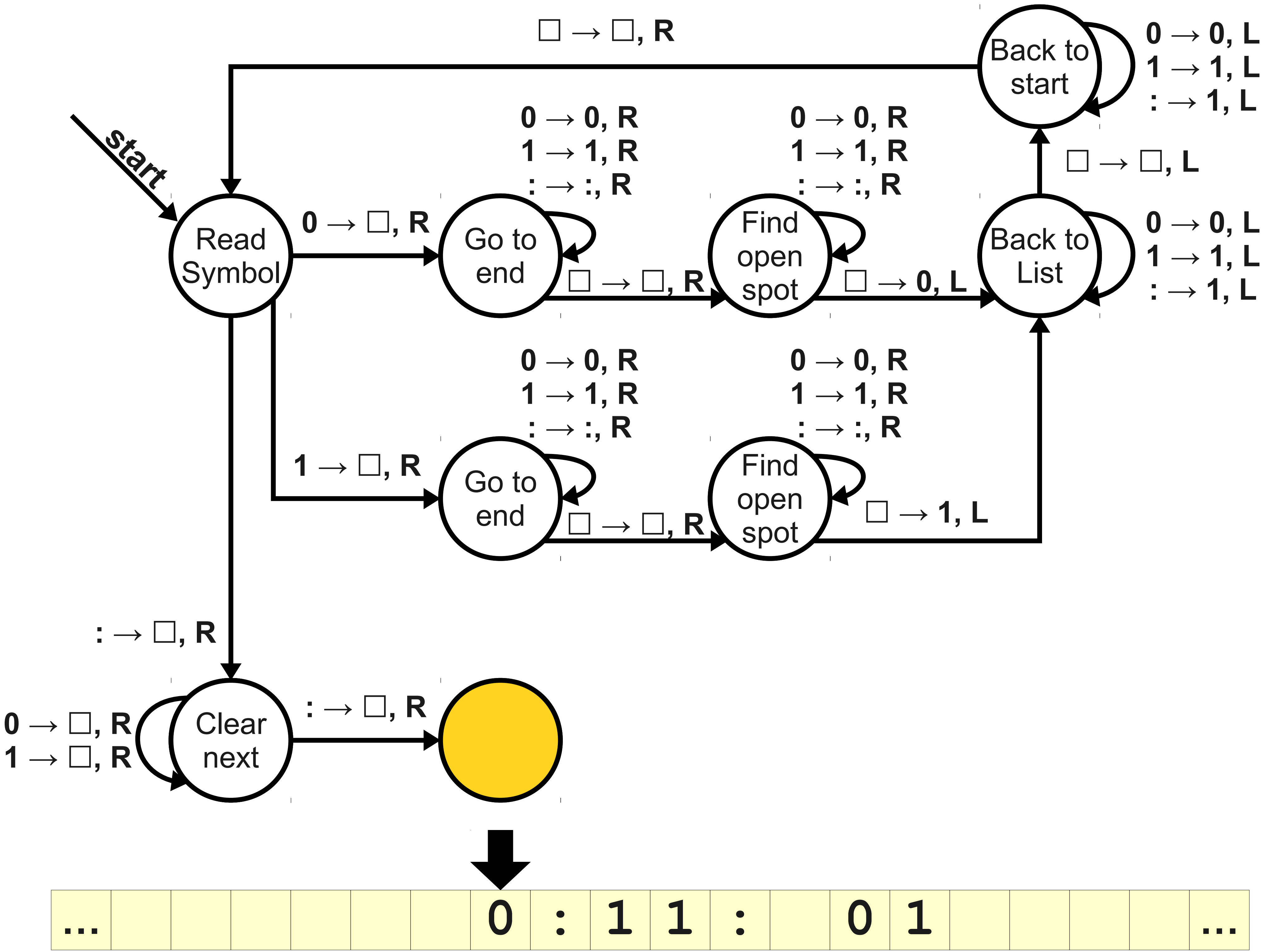
1 → □, R
: → □, R
… : 0 : 1 1 : 0 1 …


start
Read Symbol
Go to end
Find open spot
Back to List
Back to start
Clear next
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1 → 1, R
: → :, R
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□ → □, L
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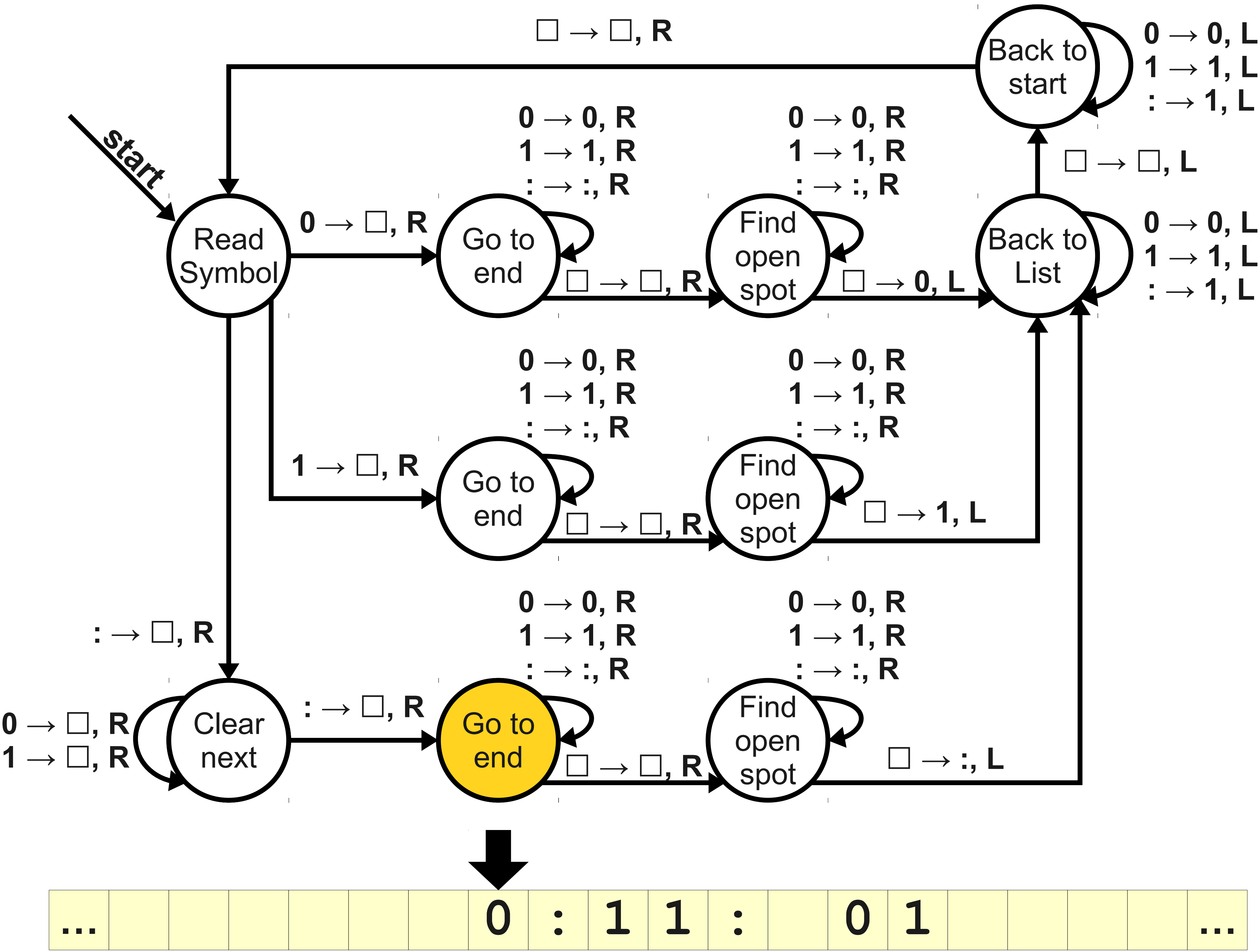
1 → 1, L
: → 1, L
... 0 : 1 1 : 0 1 ...


start
Read Symbol
Go to end
Find open spot
Back to List
Back to start
Clear next
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0 → 0, R
1 → 1, R
: → :, R
□ → □, R
□ → 0, L
□ → 1, L
□ → :, L
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: → 1, L
□ → □, L
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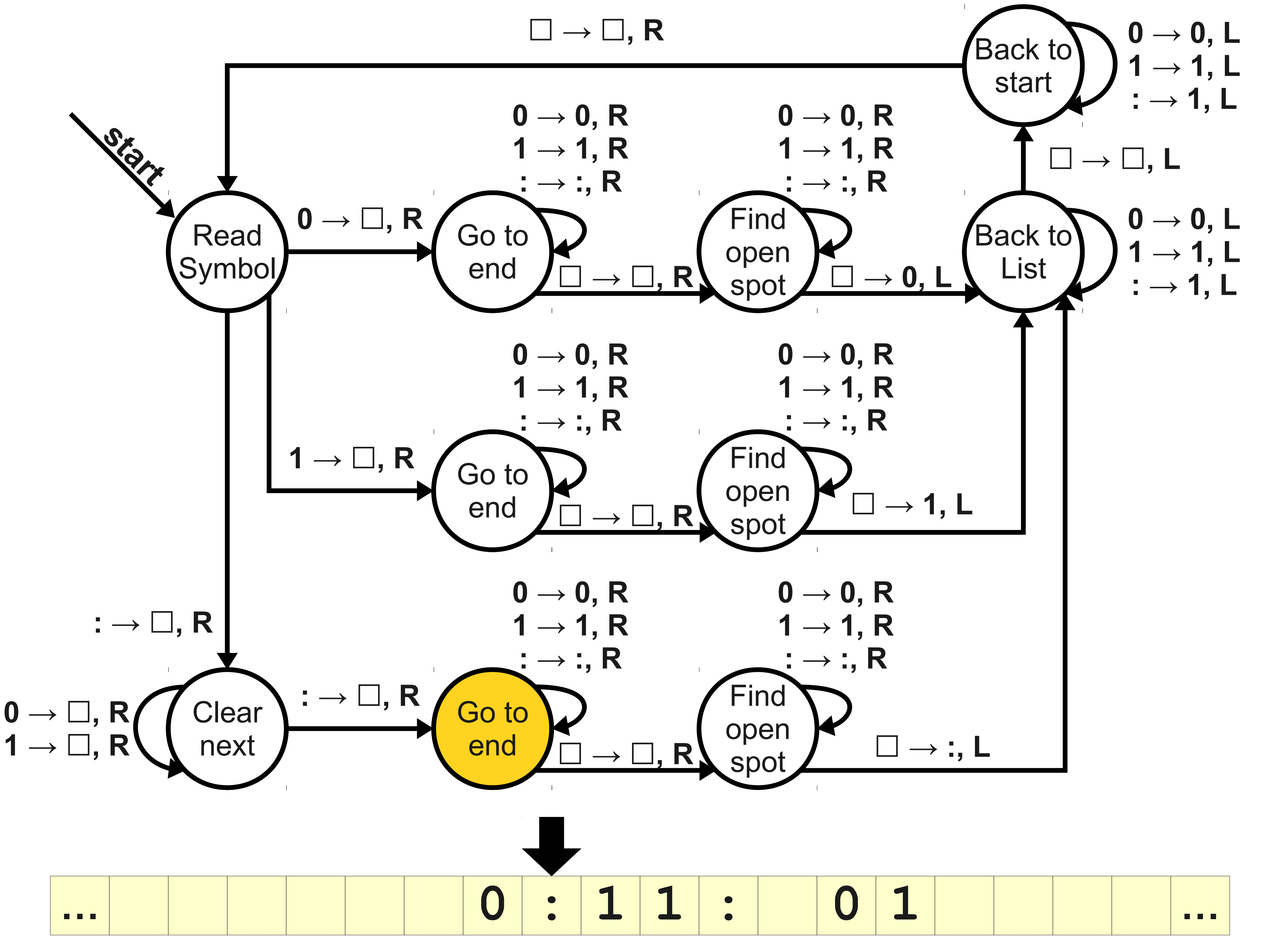
1 → □, R
0 : 1 1 : 0 1
...



start
Read Symbol
Go to end
Find open spot
Back to List
Back to start
Clear next
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1 → □, R
: → □, R
0 → 0, R
1 → 1, R
: → :, R
□ → □, R
□ → 0, L
□ → 1, L
□ → :, L
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: → 1, L
□ → □, L
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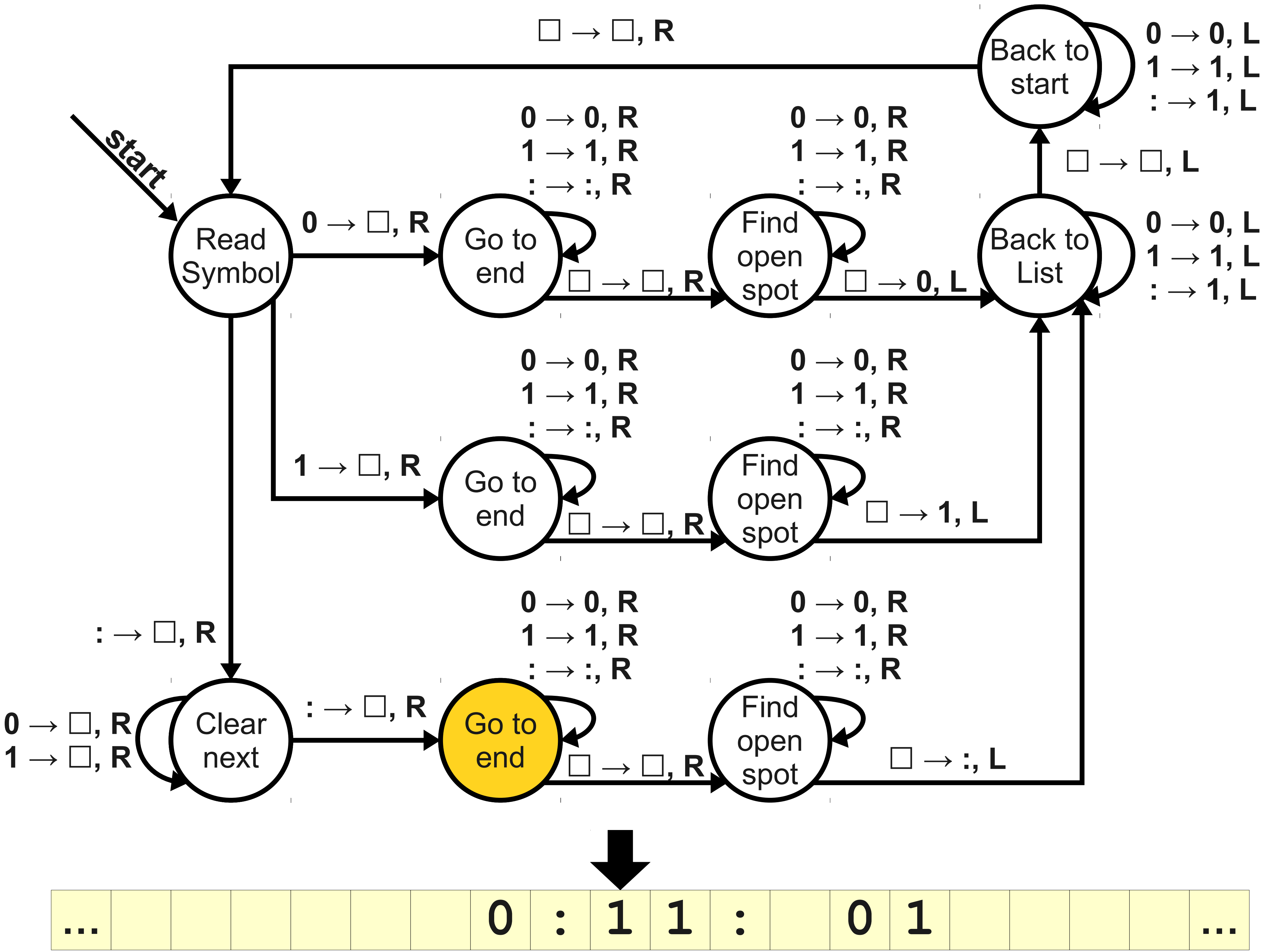
1 → □, R
... 0 : 1 1 : 0 1 ...


start
Read Symbol
Go to end
Find open spot
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Back to start
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□ → 1, L
□ → :, L
□ → □, L
0 → 0, L
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1 → □, R
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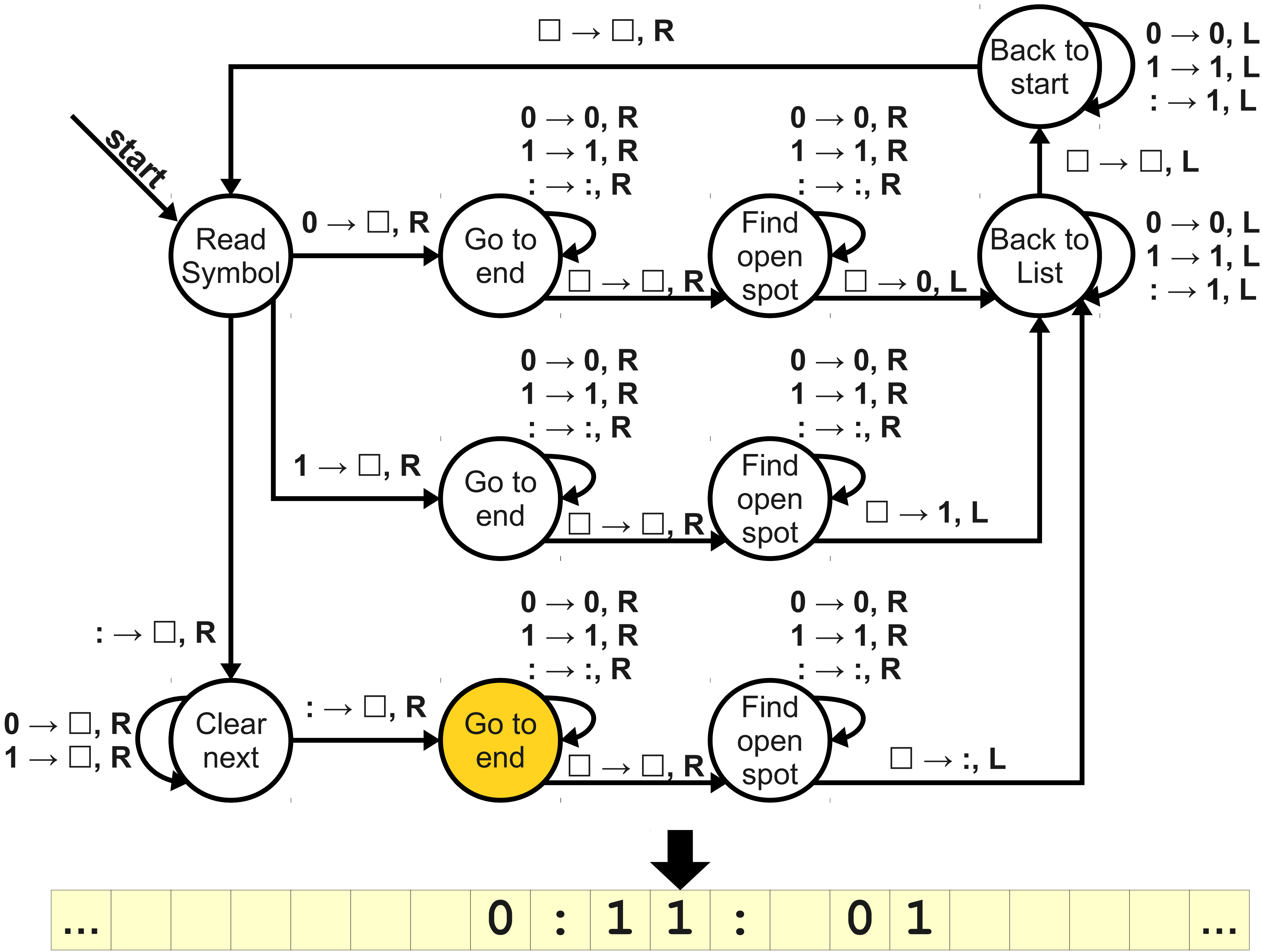
...



□ → □, R
Back to start
0 → 0, L
1 → 1, L
: → 1, L
start
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1 → 1, R
: → :, R
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1 → 1, R
: → :, R
□ → □, L
Read Symbol
0 → □, R
Go to end
Find open spot
Back to List
0 → 0, L
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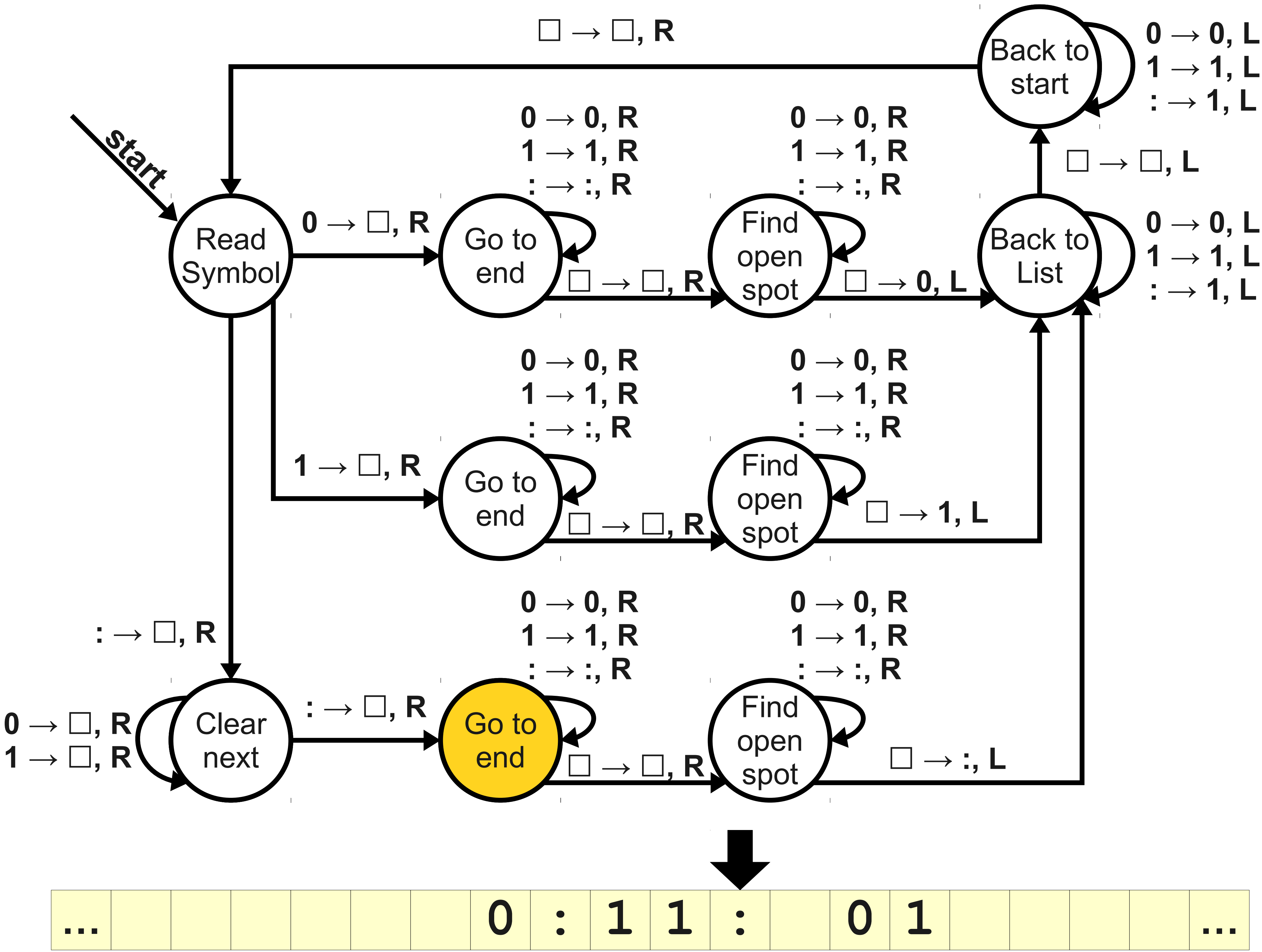
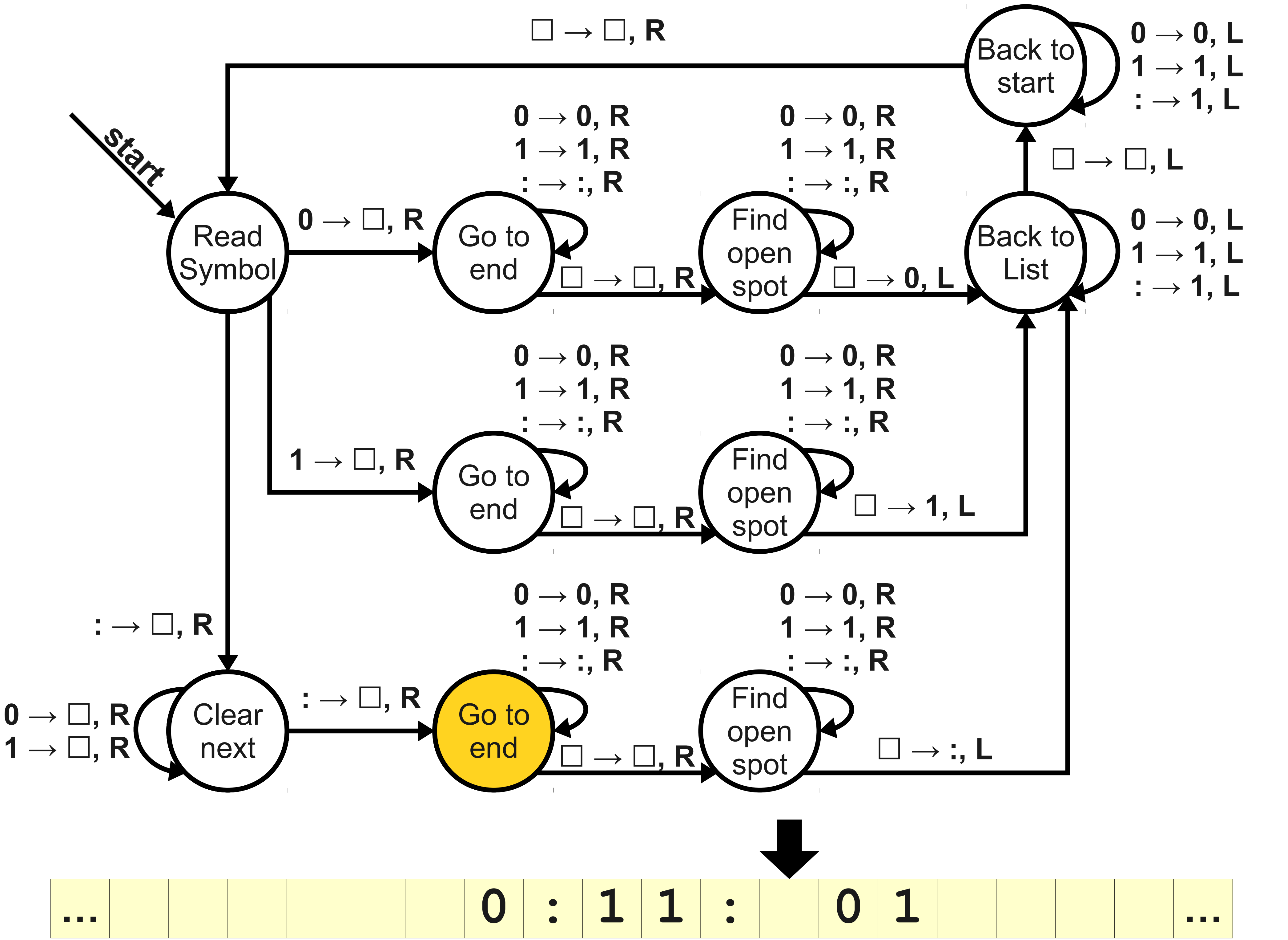
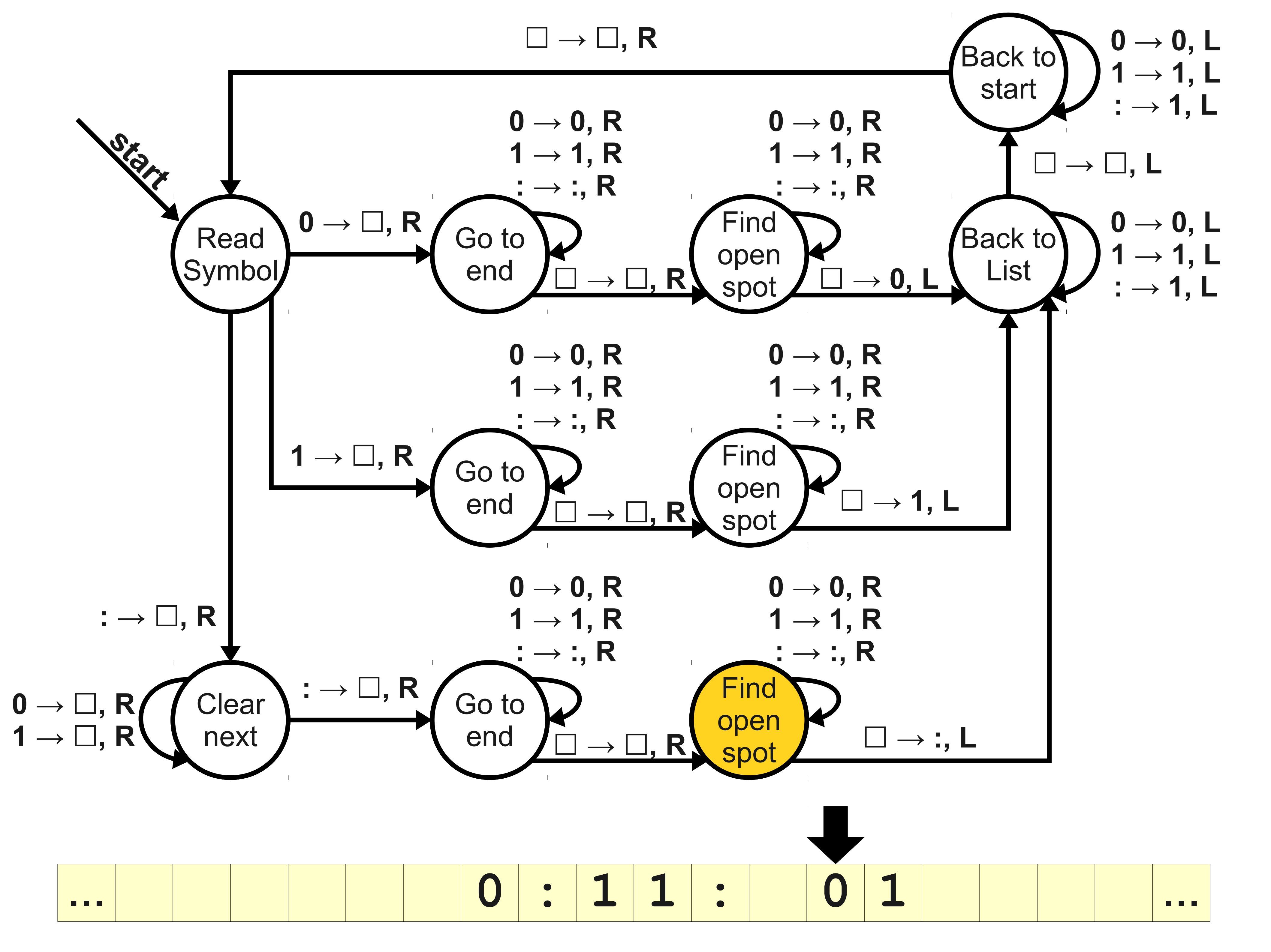
: → 1, L
□ → □, R
□ → 0, L
0 → 0, R
1 → 1, R
: → :, R
0 → 0, R
1 → 1, R
: → :, R
1 → □, R
Go to end
Find open spot
□ → 1, L
□ → □, R
0 → 0, R
1 → 1, R
: → :, R
0 → 0, R
1 → 1, R
: → :, R
: → □, R
Clear next
: → □, R
Go to end
Find open spot
0 → □, R
1 → □, R
□ → □, R
□ → :, L
...
0
:
1
1
:
0
1
...

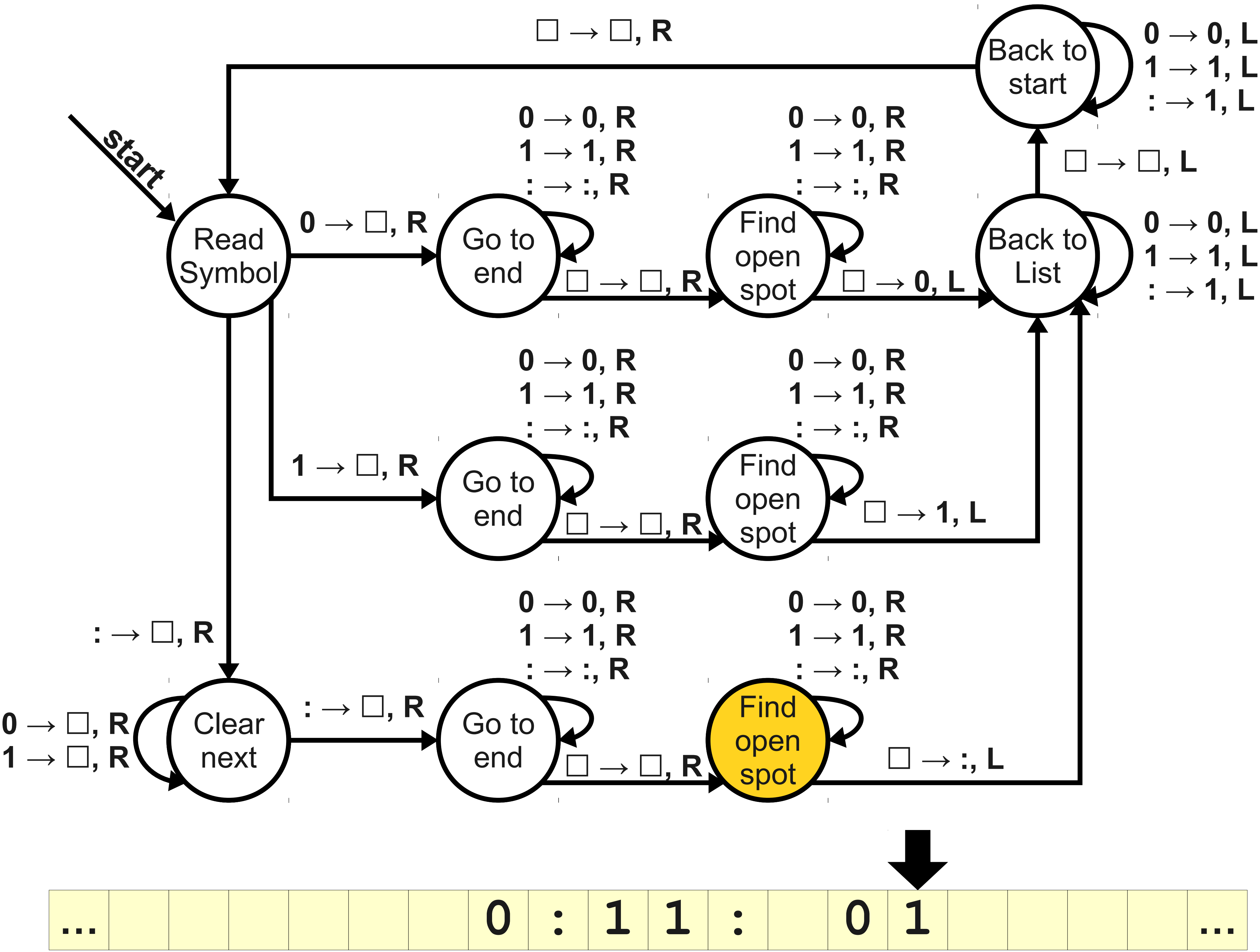


□ → □, R
Back to start
0 → 0, L
1 → 1, L
: → 1, L
start
0 → 0, R
1 → 1, R
: → :, R
□ → □, L
Read Symbol
0 → □, R
Go to end
□ → □, R
Find open spot
□ → 0, L
Back to List
1 → □, R
Go to end
Find open spot
□ → 1, L
: → □, R
Clear next
0 → □, R
1 → □, R
: → □, R
Go to end
Find open spot
□ → :, L
...
0
:
1
1
:
0
1
...



start
Read Symbol
Go to end
Find open spot
Back to List
Back to start
Clear next
□ → □, R
0 → □, R
1 → □, R
: → □, R
0 → 0, R
1 → 1, R
: → :, R
□ → 0, L
□ → 1, L
□ → :, L
0 → 0, L
1 → 1, L
: → 1, L
□ → □, L
0 → □, R
1 → □, R
... 0 : 1 1 : 0 1 ...


start
Read Symbol
Go to end
Find open spot
Back to List
Back to start
Clear next
0 → □, R
1 → □, R
: → □, R
0 → 0, R
1 → 1, R
: → :, R
□ → □, R
□ → 0, L
□ → 1, L
□ → :, L
0 → 0, L
1 → 1, L
: → 1, L
□ → □, L
0 : 1 1 : 0 1
...



□ → □, R
Back to start
0 → 0, L
1 → 1, L
: → 1, L
start
Read Symbol
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Go to end
0 → 0, R
1 → 1, R
: → :, R
□ → □, R
Find open spot
0 → 0, R
1 → 1, R
: → :, R
□ → 0, L
Back to List
0 → 0, L
1 → 1, L
: → 1, L
□ → □, L
1 → □, R
Go to end
0 → 0, R
1 → 1, R
: → :, R
□ → □, R
Find open spot
0 → 0, R
1 → 1, R
: → :, R
□ → 1, L
: → □, R
Clear next
0 → □, R
1 → □, R
: → □, R
Go to end
0 → 0, R
1 → 1, R
: → :, R
□ → □, R
Find open spot
0 → 0, R
1 → 1, R
: → :, R
□ → :, L
...
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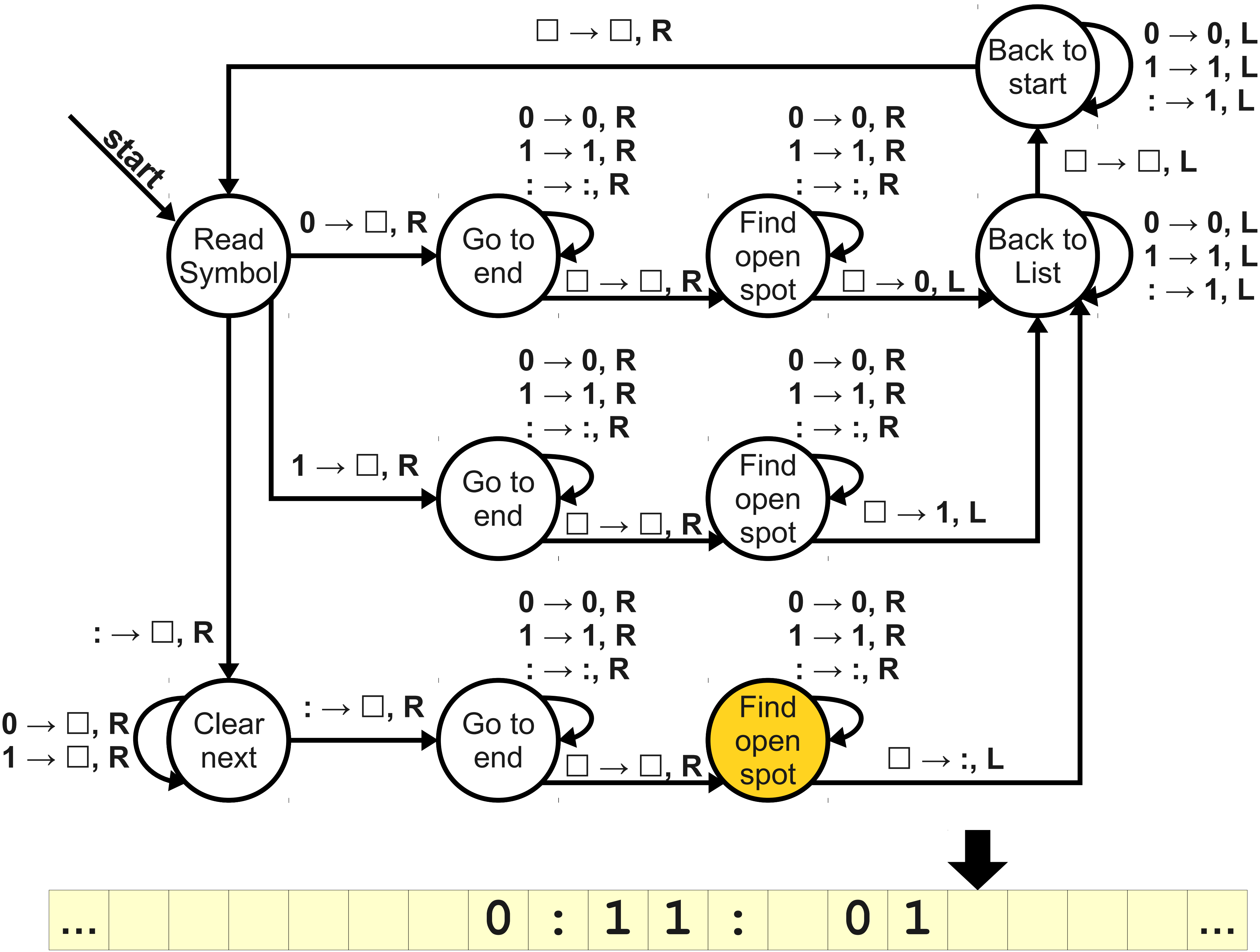
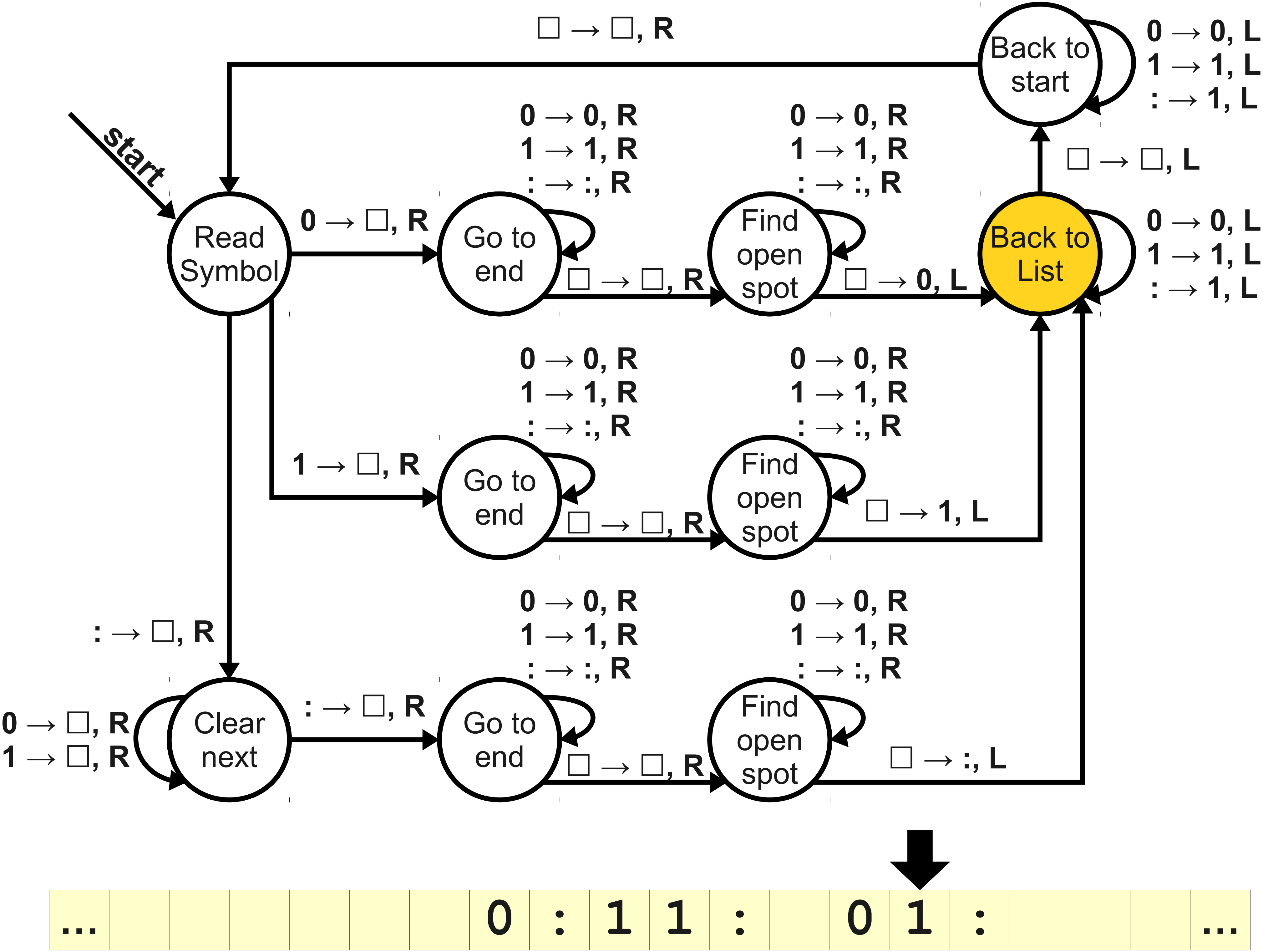
:
0
1
...



□ → □, R
Back to start
0 → 0, L
1 → 1, L
: → 1, L
start
Read Symbol
0 → □, R
Go to end
0 → 0, R
1 → 1, R
: → :, R
□ → □, R
Find open spot
0 → 0, R
1 → 1, R
: → :, R
□ → 0, L
Back to List
□ → □, L
0 → 0, L
1 → 1, L
: → 1, L
1 → □, R
Go to end
0 → 0, R
1 → 1, R
: → :, R
□ → □, R
Find open spot
0 → 0, R
1 → 1, R
: → :, R
□ → 1, L
: → □, R
Clear next
0 → □, R
1 → □, R
: → □, R
Go to end
0 → 0, R
1 → 1, R
: → :, R
□ → □, R
Find open spot
0 → 0, R
1 → 1, R
: → :, R
□ → :, L
... 0 : 1 1 : 0 1 ...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Back to start
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1 → 1, L
: → 1, L
start
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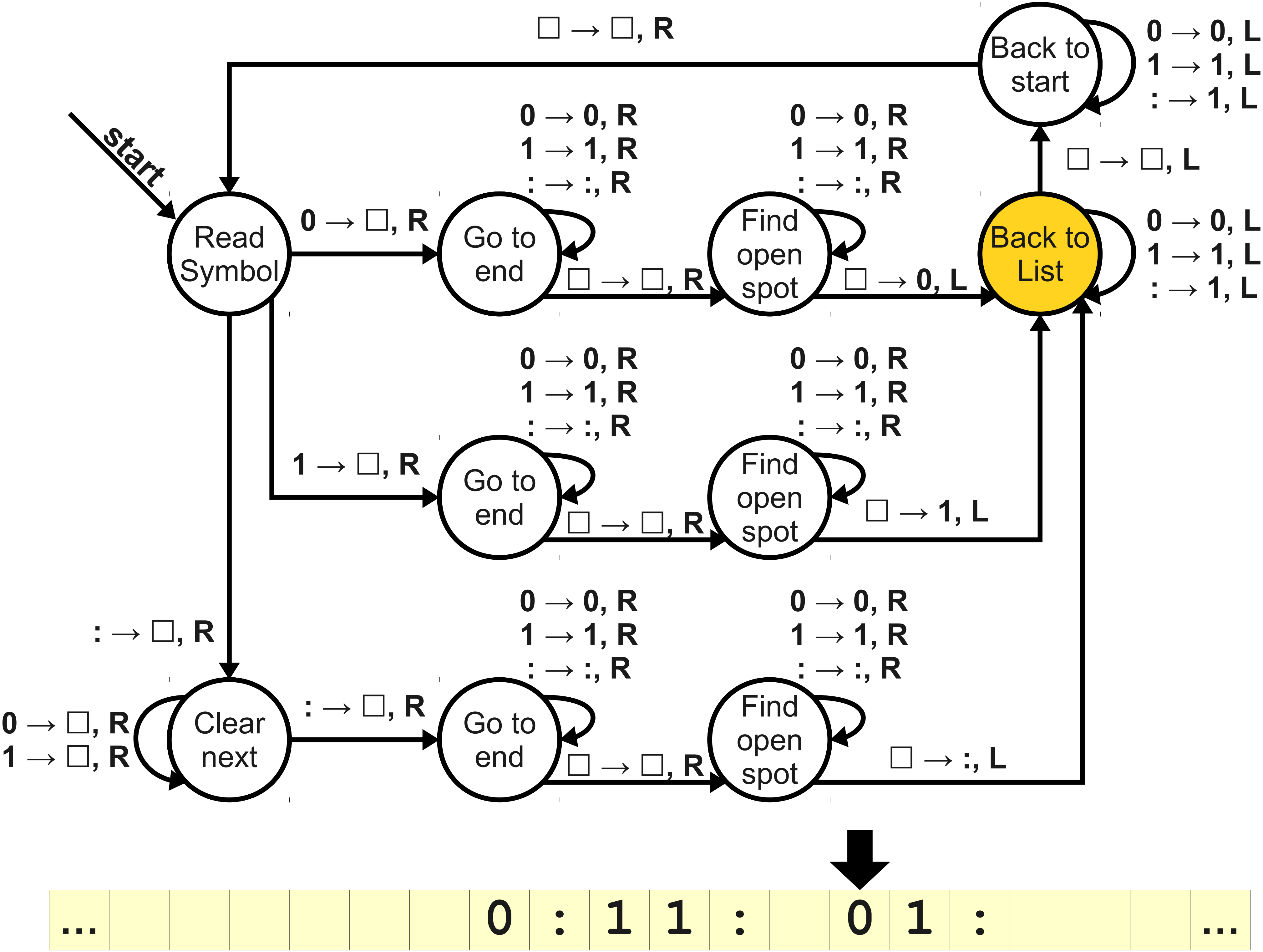
1 → 1, R
: → :, R
□ → □, L
Read Symbol
0 → □, R
Go to end
Find open spot
Back to List
0 → 0, L
1 → 1, L
: → 1, L
□ → □, R
□ → 0, L
1 → □, R
□ → 1, L
: → □, R
Clear next
0 → □, R
1 → □, R
: → □, R
□ → :, L
... 0 : 1 1 : 0 1 : ...



□ → □, R
Back to start
0 → 0, L
1 → 1, L
: → 1, L
start
0 → 0, R
1 → 1, R
: → :, R
□ → □, L
Read Symbol
0 → □, R
Go to end
Find open spot
Back to List
□ → □, R
□ → 0, L
1 → □, R
□ → 1, L
: → □, R
Clear next
0 → □, R
1 → □, R
: → □, R
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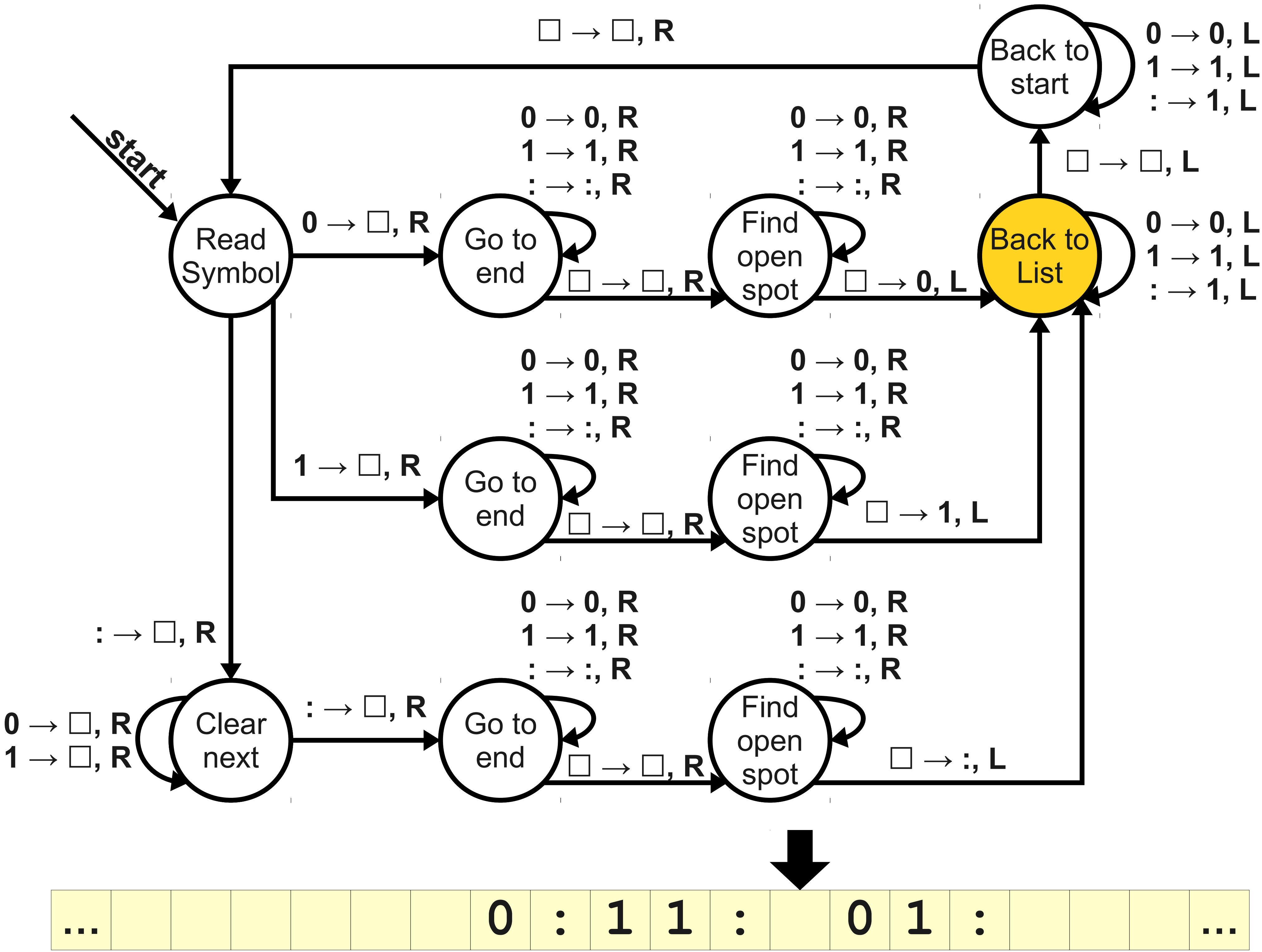
… 0 : 1 1 : 0 1 : …



□ → □, R
Back to start
0 → 0, L
1 → 1, L
: → 1, L
start
Read Symbol
0 → □, R
Go to end
0 → 0, R
1 → 1, R
: → :, R
□ → □, R
Find open spot
0 → 0, R
1 → 1, R
: → :, R
□ → 0, L
□ → □, L
Back to List
0 → 0, L
1 → 1, L
: → 1, L
1 → □, R
Go to end
0 → 0, R
1 → 1, R
: → :, R
□ → □, R
Find open spot
0 → 0, R
1 → 1, R
: → :, R
□ → 1, L
: → □, R
Clear next
0 → □, R
1 → □, R
: → □, R
Go to end
0 → 0, R
1 → 1, R
: → :, R
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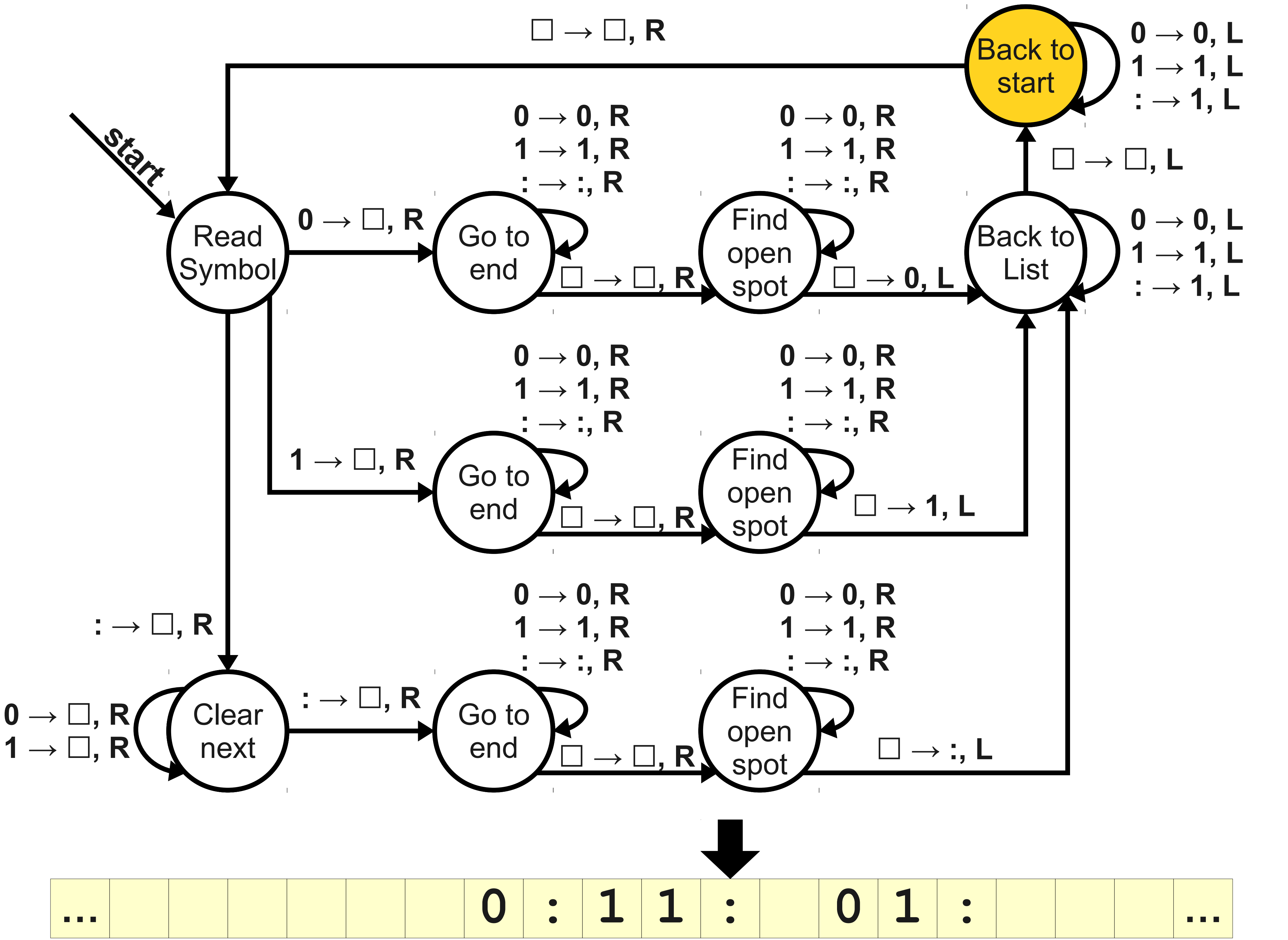
Find open spot
0 → 0, R
1 → 1, R
: → :, R
□ → :, L
... 0 : 1 1 : 0 1 : ...



□ → □, R
Back to start
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: → 1, L
start
Read Symbol
0 → □, R
Go to end
0 → 0, R
1 → 1, R
: → :, R
□ → □, R
Find open spot
0 → 0, R
1 → 1, R
: → :, R
□ → 0, L
Back to List
0 → 0, L
1 → 1, L
: → 1, L
□ → □, L
1 → □, R
Go to end
□ → □, R
Find open spot
□ → 1, L
: → □, R
Clear next
0 → □, R
1 → □, R
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Go to end
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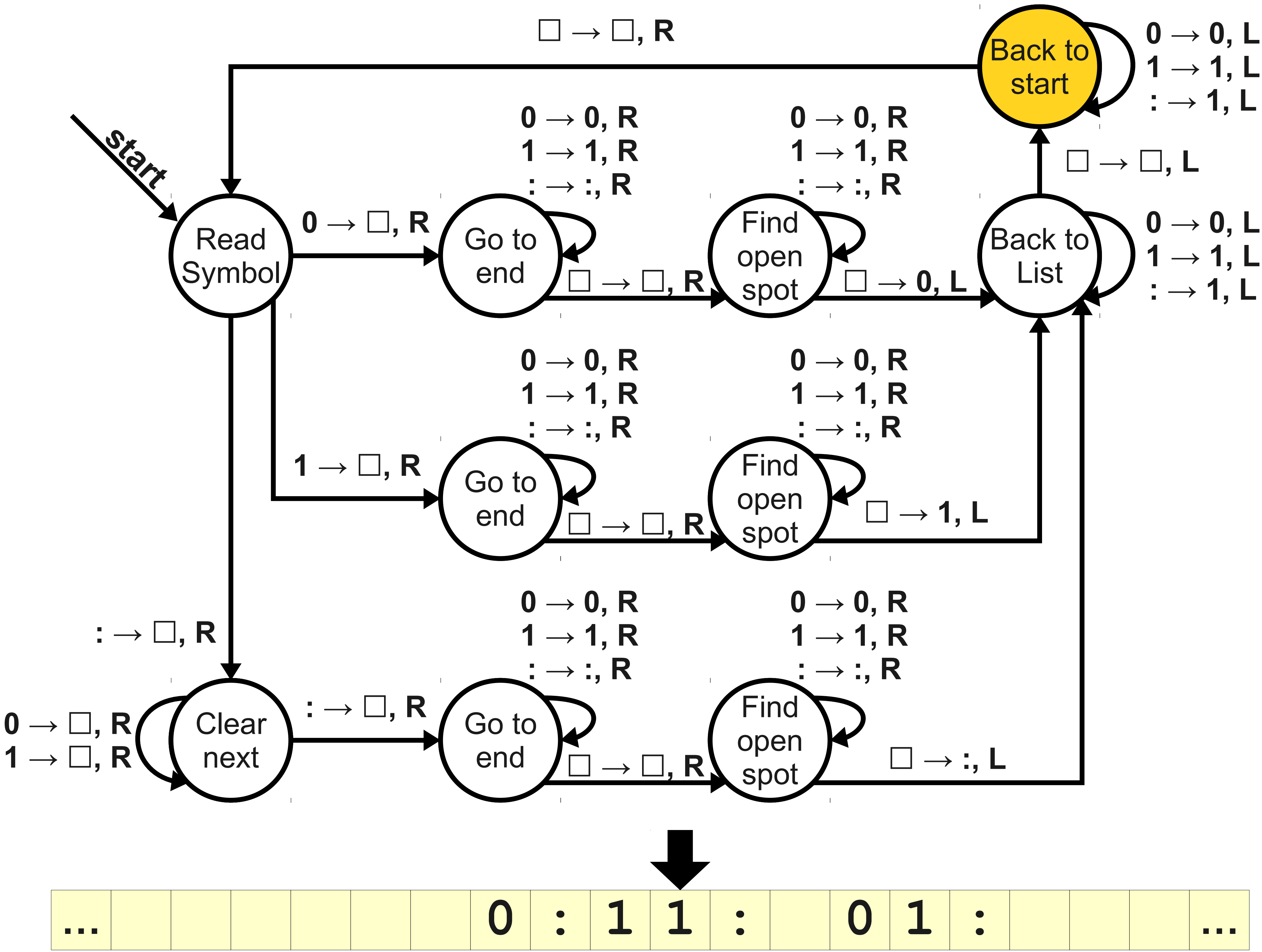
Find open spot
□ → :, L
... 0 : 1 1 : 0 1 : ...


□ → □, R
Back to start
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start
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1 → 1, R
: → :, R
□ → □, L
Read Symbol
0 → □, R
Go to end
□ → □, R
Find open spot
□ → 0, L
Back to List
0 → 0, L
1 → 1, L
: → 1, L
0 → 0, R
1 → 1, R
: → :, R
0 → 0, R
1 → 1, R
: → :, R
1 → □, R
Go to end
□ → □, R
Find open spot
□ → 1, L
: → □, R
0 → 0, R
1 → 1, R
: → :, R
0 → 0, R
1 → 1, R
: → :, R
0 → □, R
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: → □, R
Go to end
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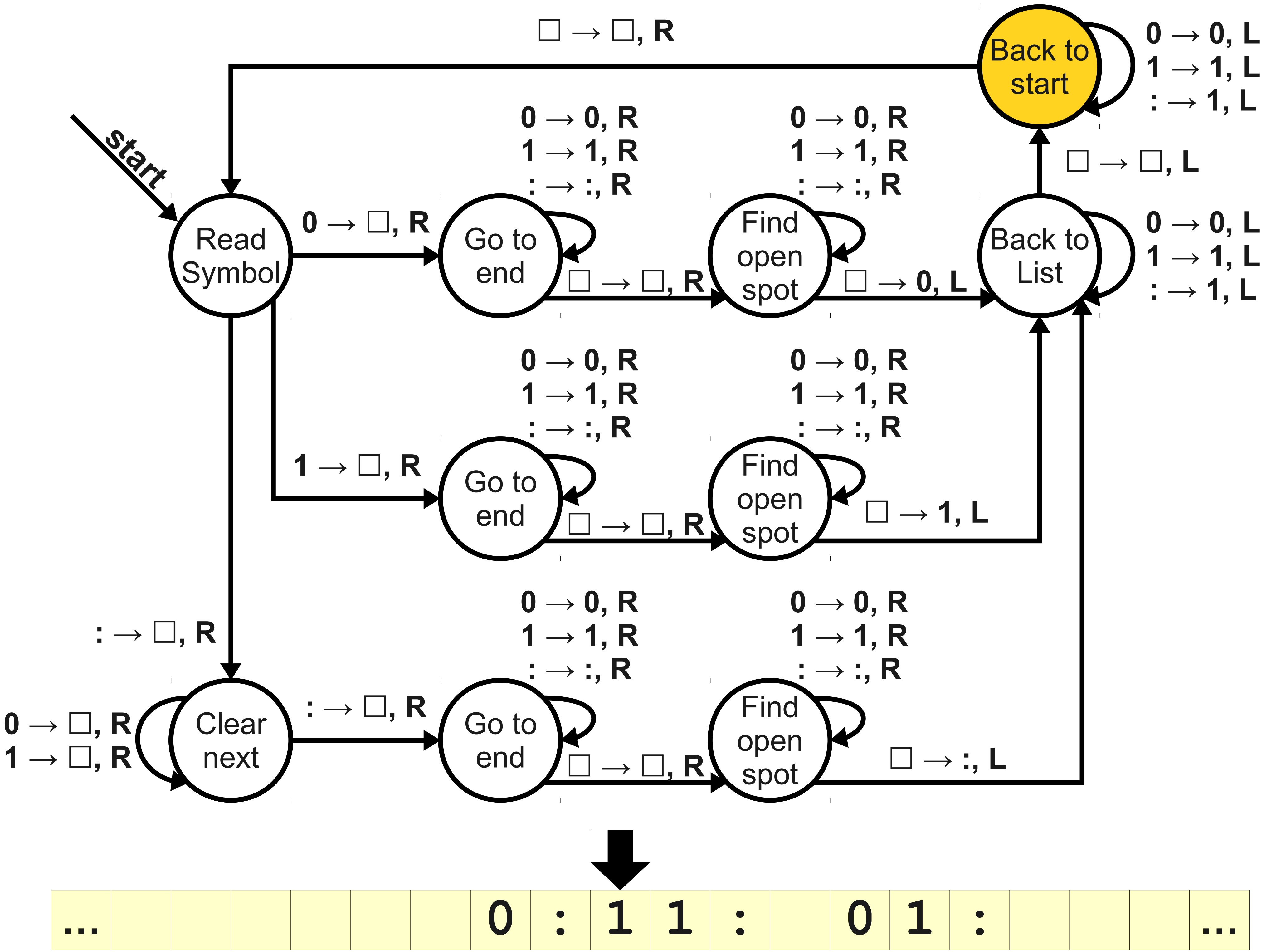
Find open spot
□ → :, L
... 0 : 1 1 : 0 1 : ...



start
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Go to end
Find open spot
Back to List
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Clear next
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1 → 1, L
: → 1, L
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1 → 1, R
: → :, R
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0 → □, R
□ → □, R
□ → 0, L
1 → □, R
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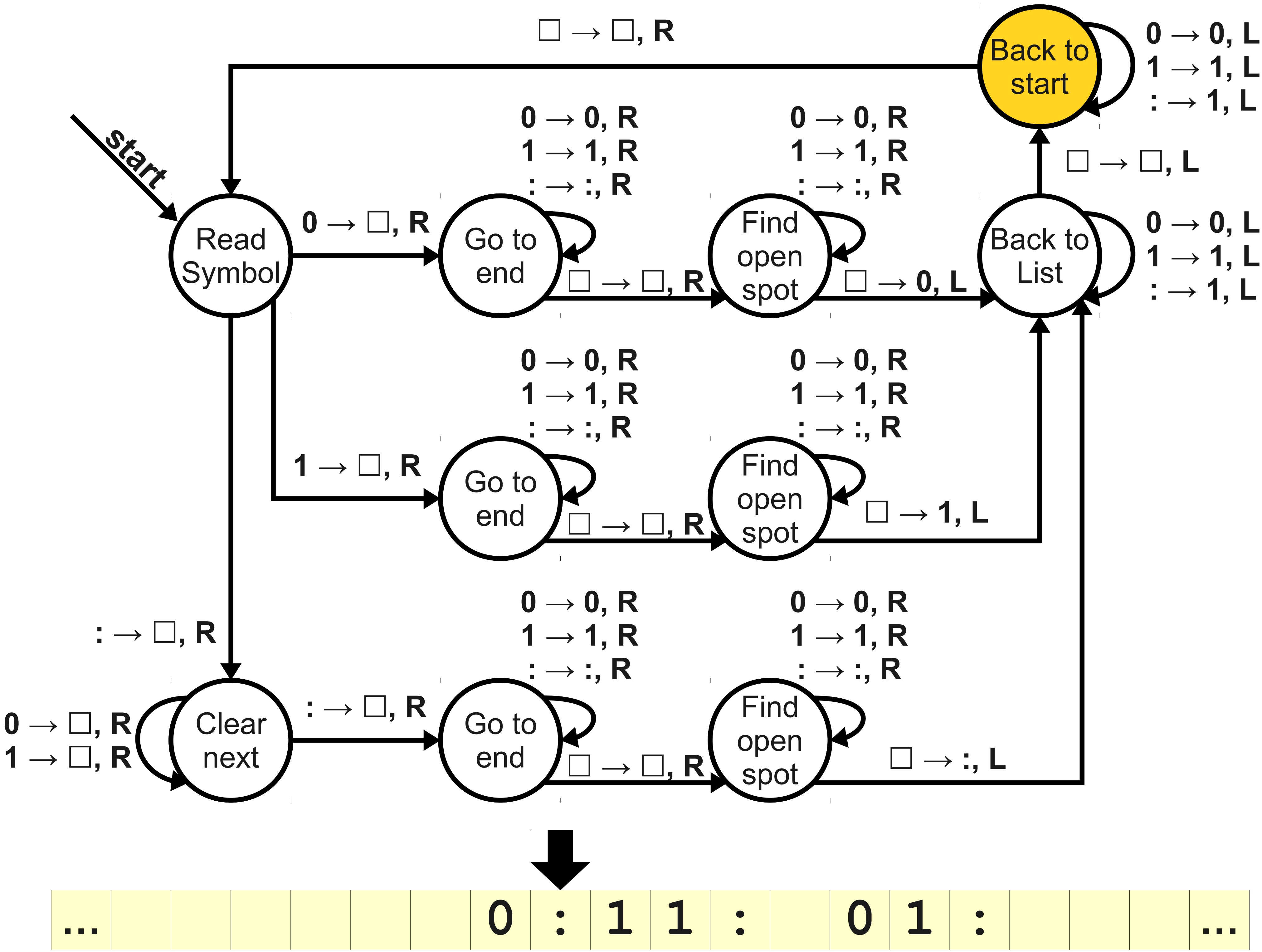
1 → □, R
□ → :, L
...
0 : 1 1 : 0 1 :
...



□ → □, R
Back to start
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: → 1, L
start
Read Symbol
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Go to end
0 → 0, R
1 → 1, R
: → :, R
□ → □, R
Find open spot
0 → 0, R
1 → 1, R
: → :, R
□ → 0, L
Back to List
□ → □, L
0 → 0, L
1 → 1, L
: → 1, L
1 → □, R
Go to end
0 → 0, R
1 → 1, R
: → :, R
□ → □, R
Find open spot
0 → 0, R
1 → 1, R
: → :, R
□ → 1, L
: → □, R
Clear next
0 → □, R
1 → □, R
: → □, R
Go to end
0 → 0, R
1 → 1, R
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Find open spot
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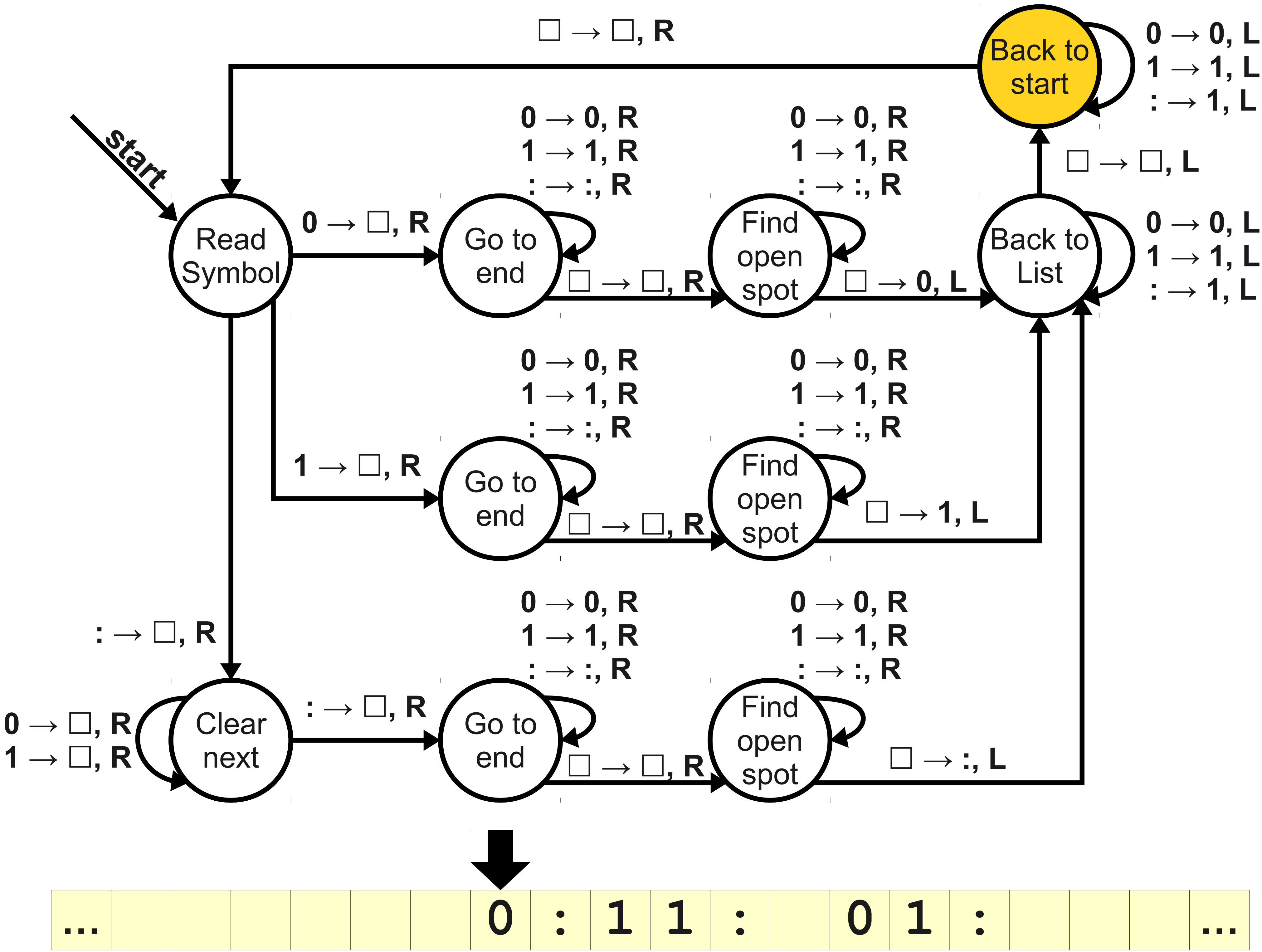
1 → 1, R
: → :, R
□ → :, L
... 0 : 1 1 : 0 1 : ...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Back to start
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: → 1, L
start
Read Symbol
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Go to end
0 → 0, R
1 → 1, R
: → :, R
□ → □, R
Find open spot
□ → 0, L
□ → □, L
Back to List
1 → □, R
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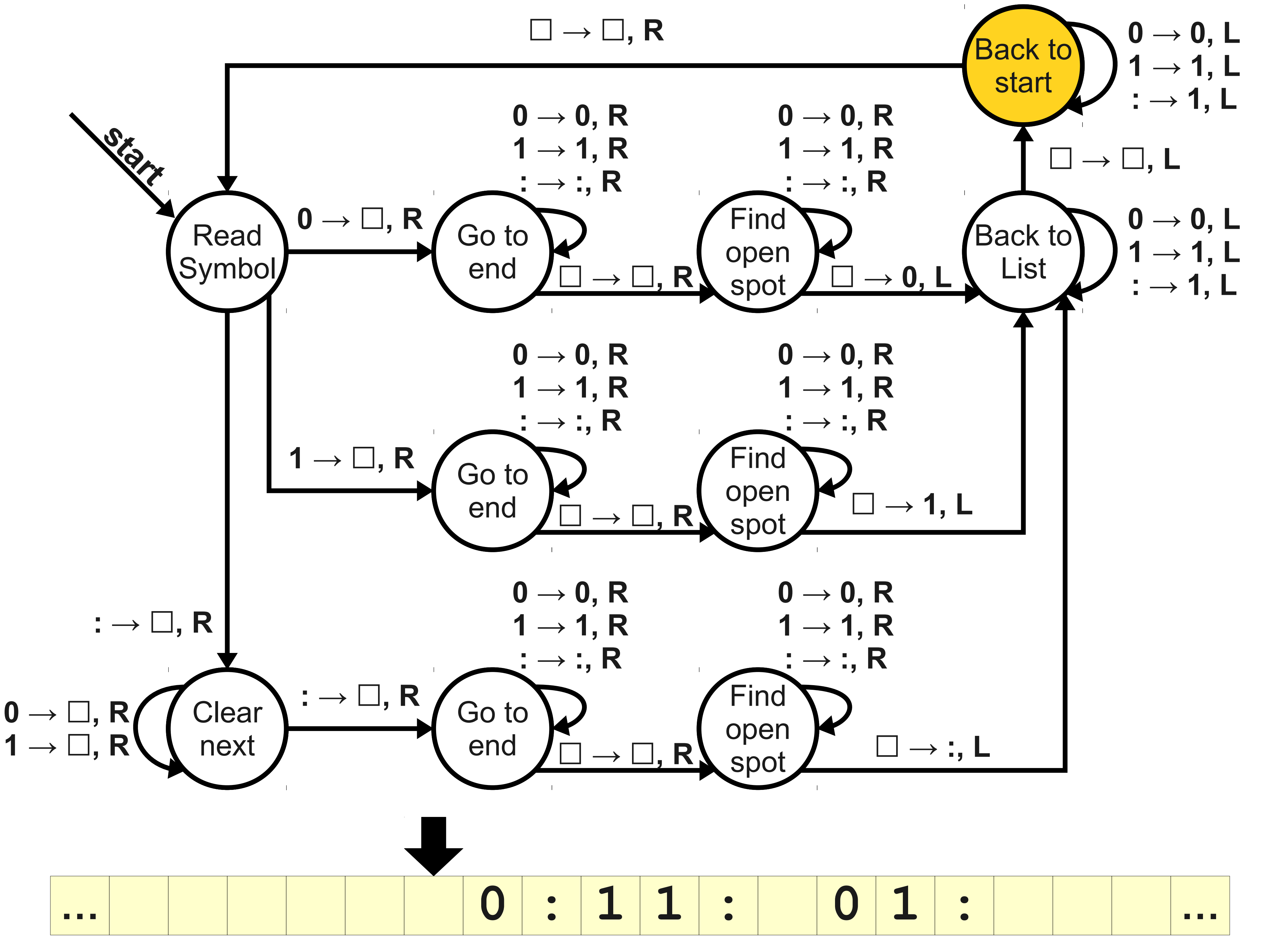
: → □, R
Clear next
0 → □, R
1 → □, R
□ → :, L
... 0 : 1 1 : 0 1 : ...


start
Read Symbol
Go to end
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Clear next
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: → :, R
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□ → □, R
□ → 0, L
□ → □, L
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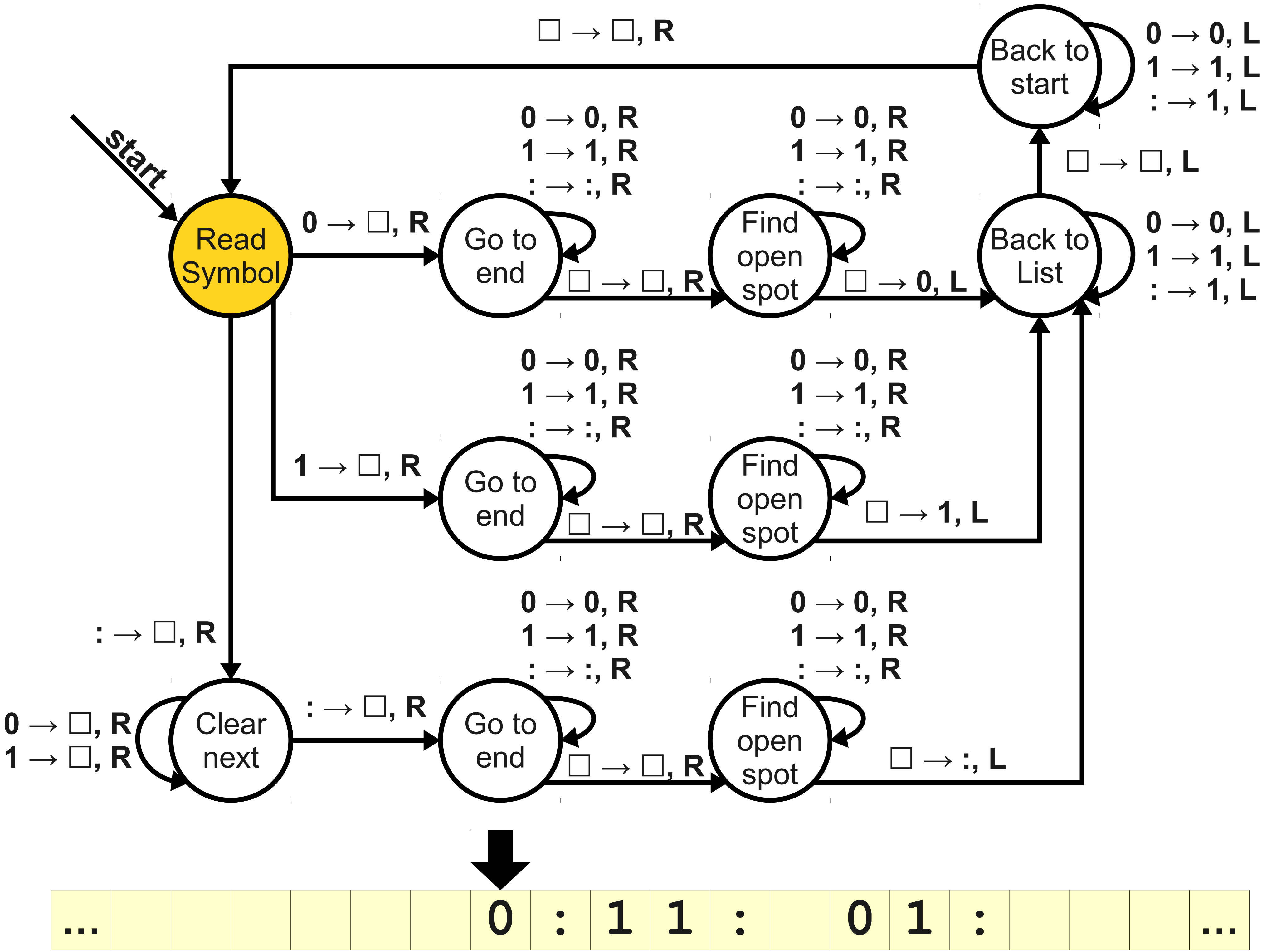
□ → :, L
... 0 : 1 1 : 0 1 : ...



□ → □, R
Back to start
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: → 1, L
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Read Symbol
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Go to end
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: → :, R
□ → □, R
Find open spot
0 → 0, R
1 → 1, R
: → :, R
□ → 0, L
□ → □, L
Back to List
0 → 0, L
1 → 1, L
: → 1, L
1 → □, R
Go to end
0 → 0, R
1 → 1, R
: → :, R
□ → □, R
Find open spot
0 → 0, R
1 → 1, R
: → :, R
□ → 1, L
: → □, R
Clear next
0 → □, R
1 → □, R
: → □, R
Go to end
0 → 0, R
1 → 1, R
: → :, R
□ → □, R
Find open spot
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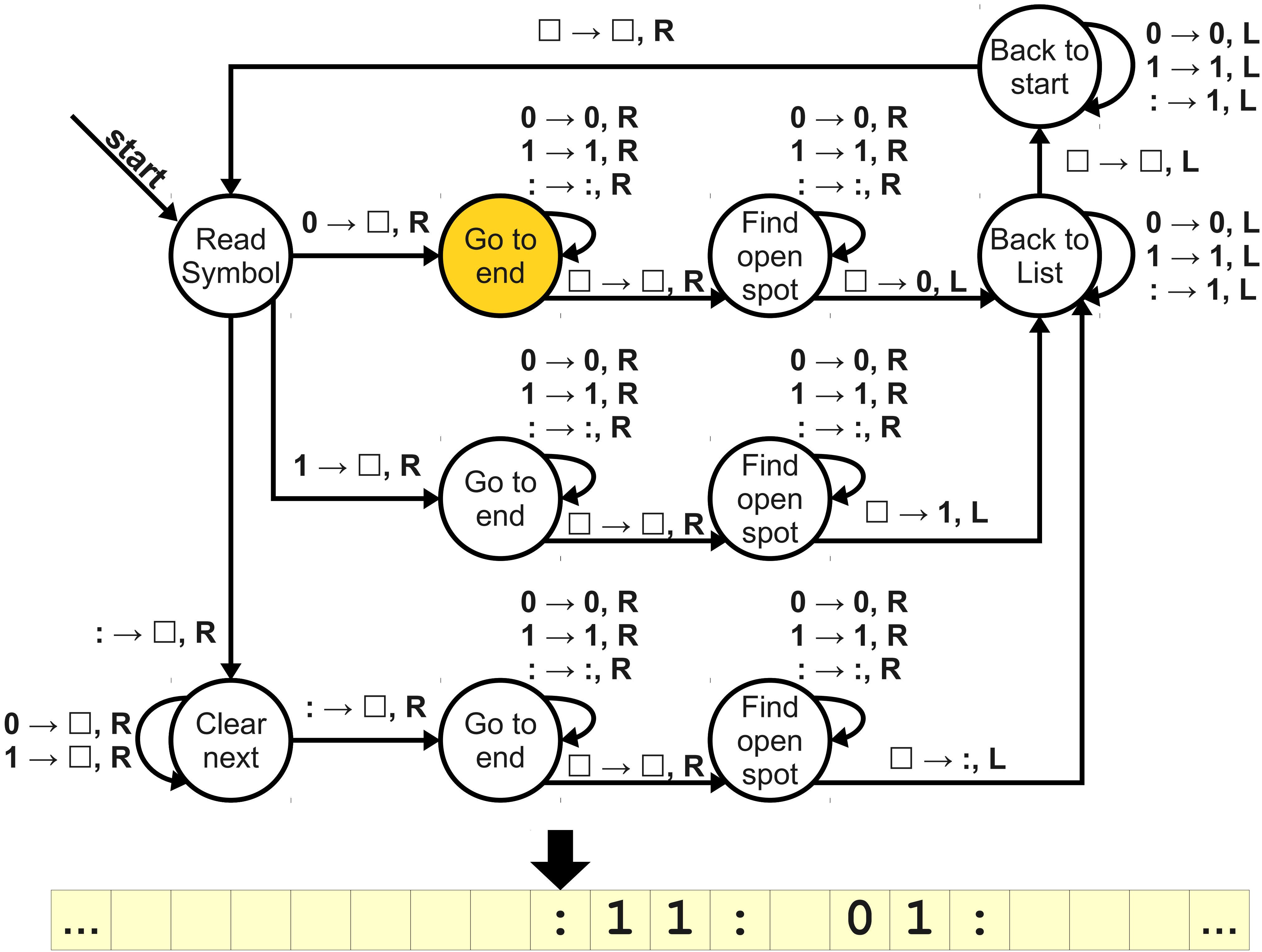

start
Read Symbol
Go to end
Find open spot
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Back to start
Clear next
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1 → 1, R
: → :, R
□ → □, R
□ → 0, L
□ → 1, L
□ → :, L
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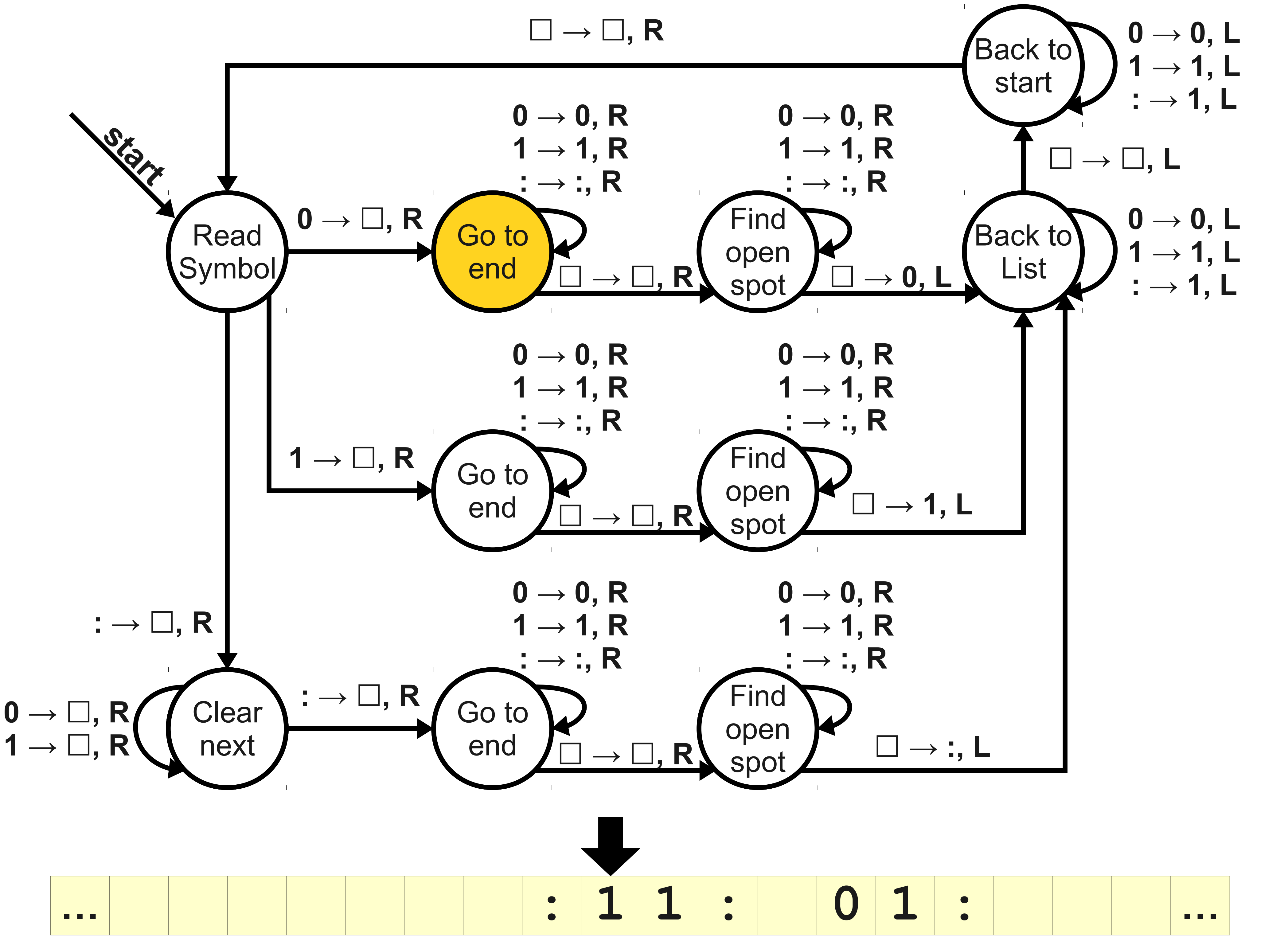
1 → □, R
... : 1 1 : 0 1 : ...


start
Read Symbol
Go to end
Find open spot
Back to List
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Clear next
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: → :, R
□ → □, R
□ → 0, L
□ → 1, L
□ → :, L
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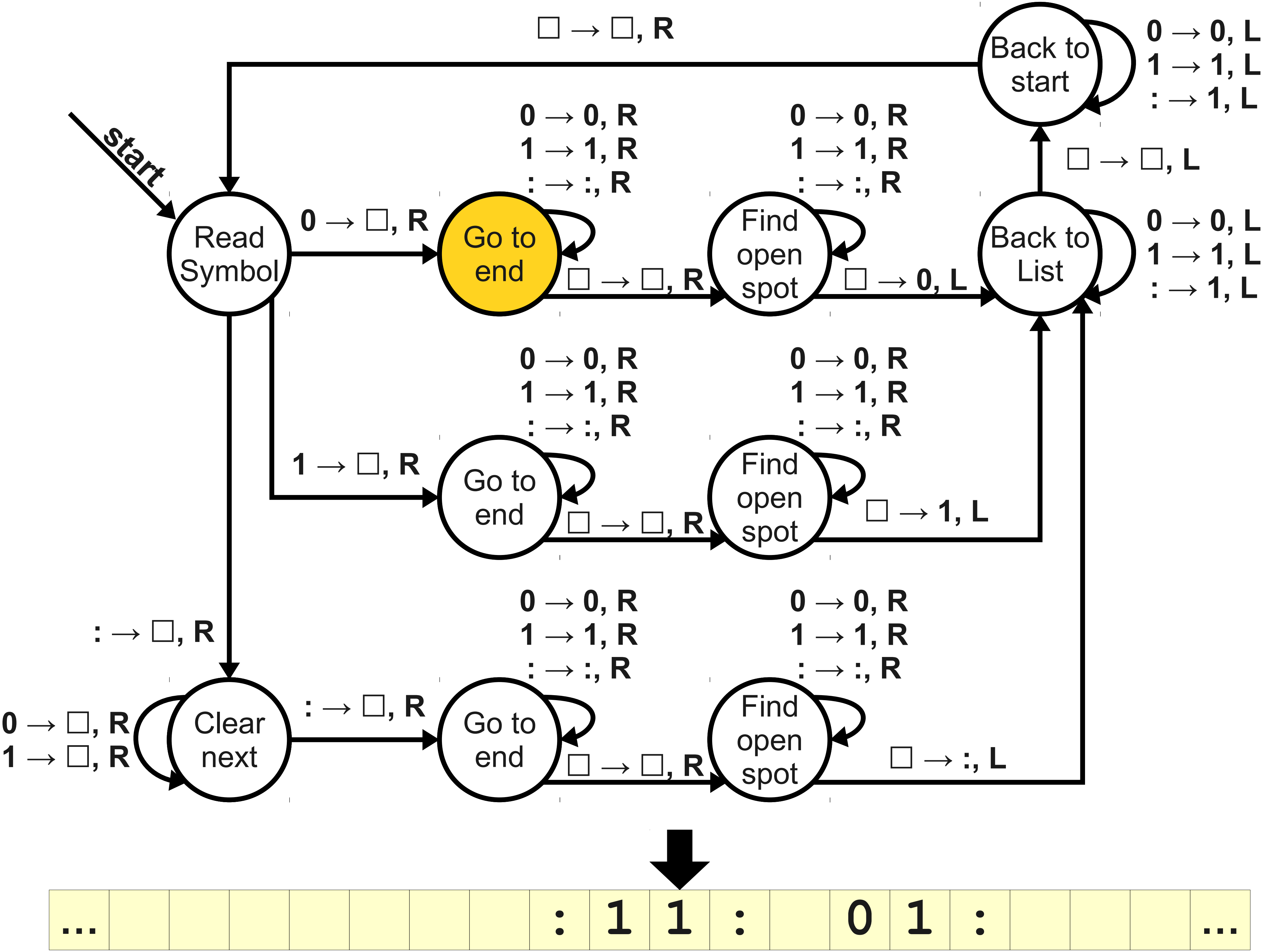
: → 1, L
□ → □, L
0 → □, R
1 → □, R
... : 1 1 : 0 1 : ...



start
Read Symbol
Go to end
Find open spot
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Back to start
Clear next
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: → 1, L
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1 → 1, R
: → :, R
□ → □, L
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: → □, R
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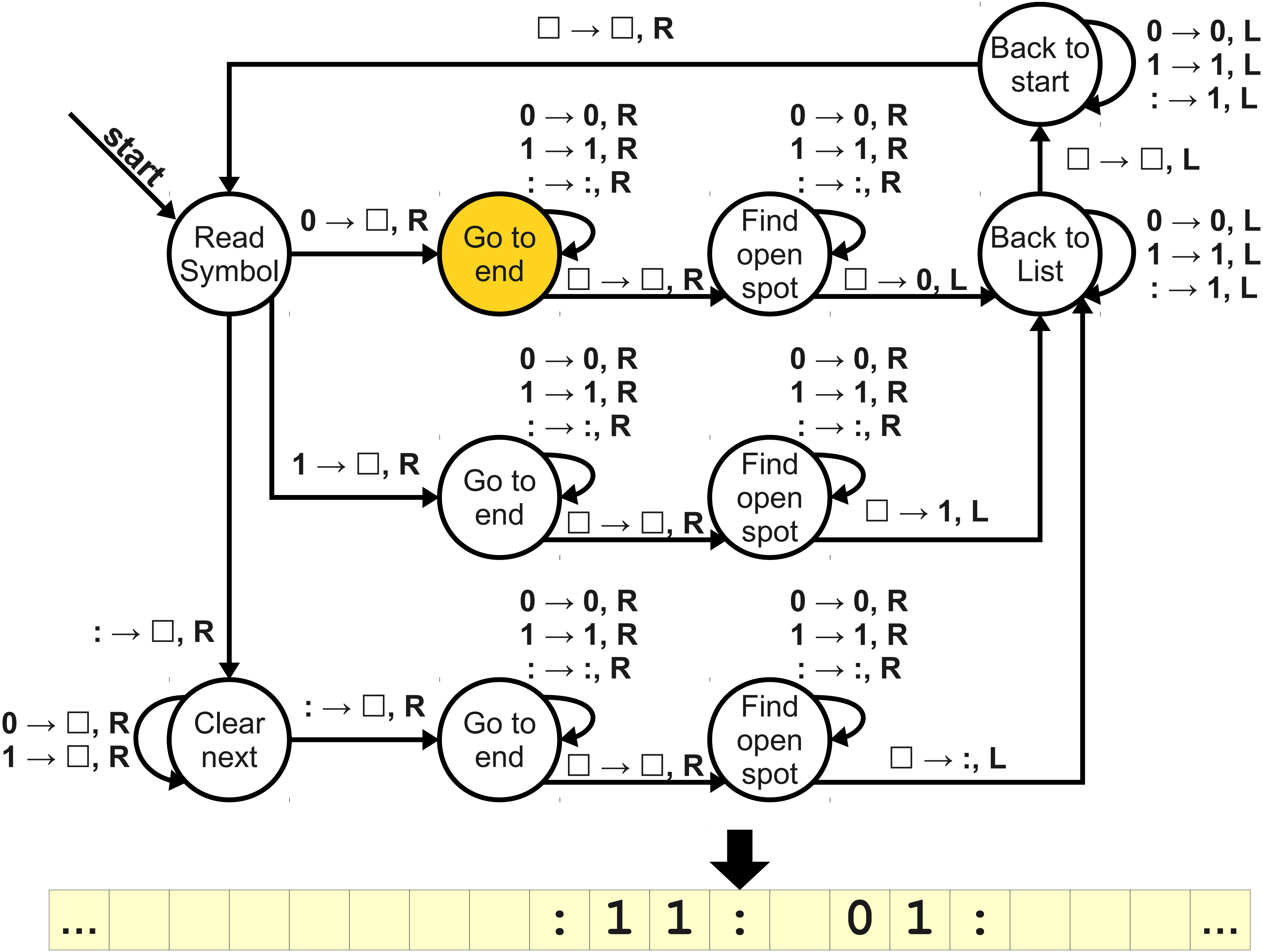
□ → 1, L
□ → :, L
... : 1 1 : 0 1 : ...



start
Read Symbol
Go to end
Find open spot
Back to List
Back to start
Clear next
0 → □, R
1 → □, R
: → □, R
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: → :, R
□ → □, R
□ → 0, L
□ → 1, L
□ → :, L
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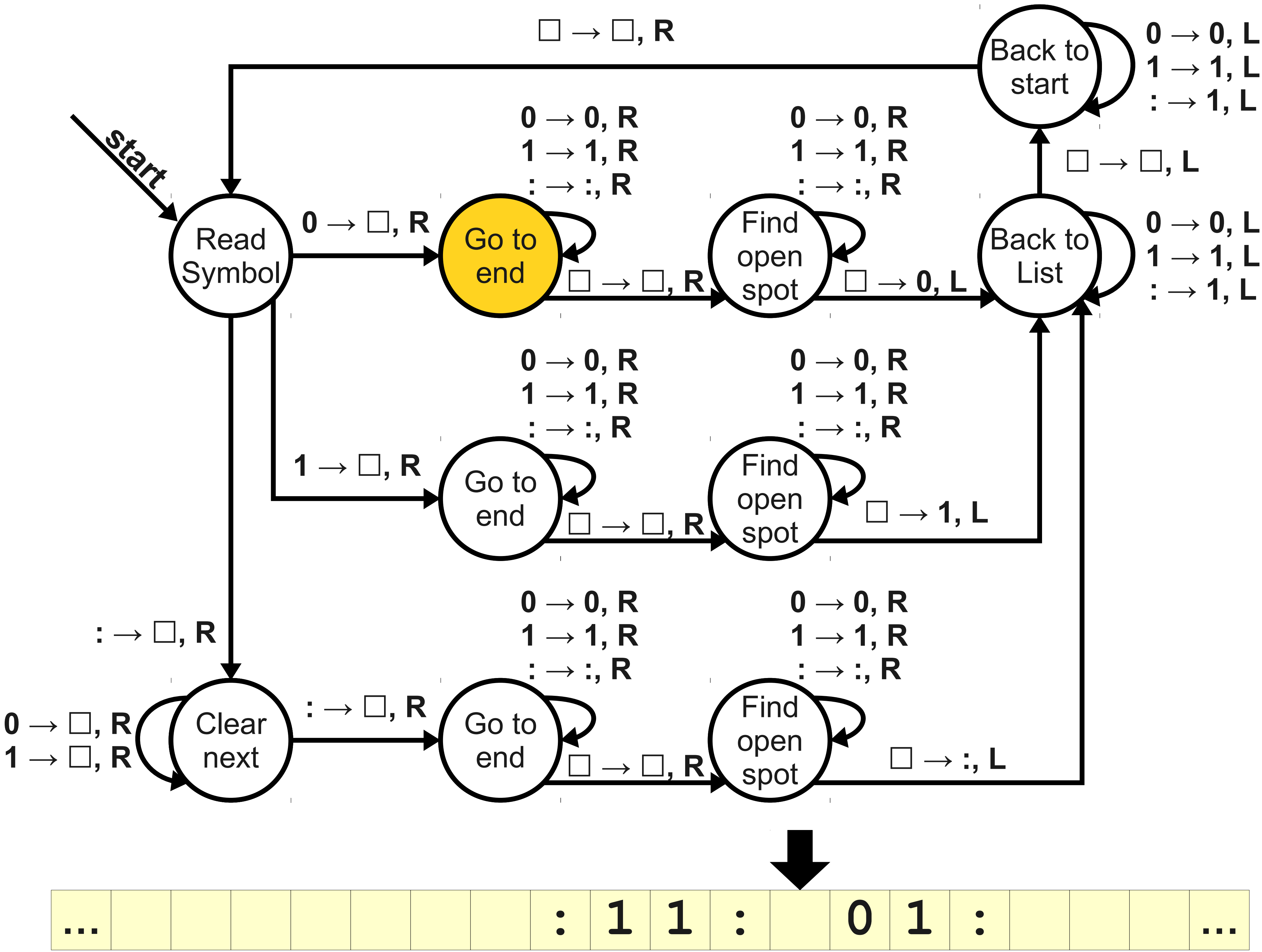
1 → □, R
… : 1 1 : 0 1 : …



□ → □, R
Back to start
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start
Read Symbol
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Go to end
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1 → 1, R
: → :, R
□ → □, R
Find open spot
0 → 0, R
1 → 1, R
: → :, R
□ → 0, L
Back to List
□ → □, L
0 → 0, L
1 → 1, L
: → 1, L
1 → □, R
Go to end
0 → 0, R
1 → 1, R
: → :, R
□ → □, R
Find open spot
0 → 0, R
1 → 1, R
: → :, R
□ → 1, L
: → □, R
Clear next
0 → □, R
1 → □, R
: → □, R
Go to end
0 → 0, R
1 → 1, R
: → :, R
□ → □, R
Find open spot
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: → :, R
□ → :, L
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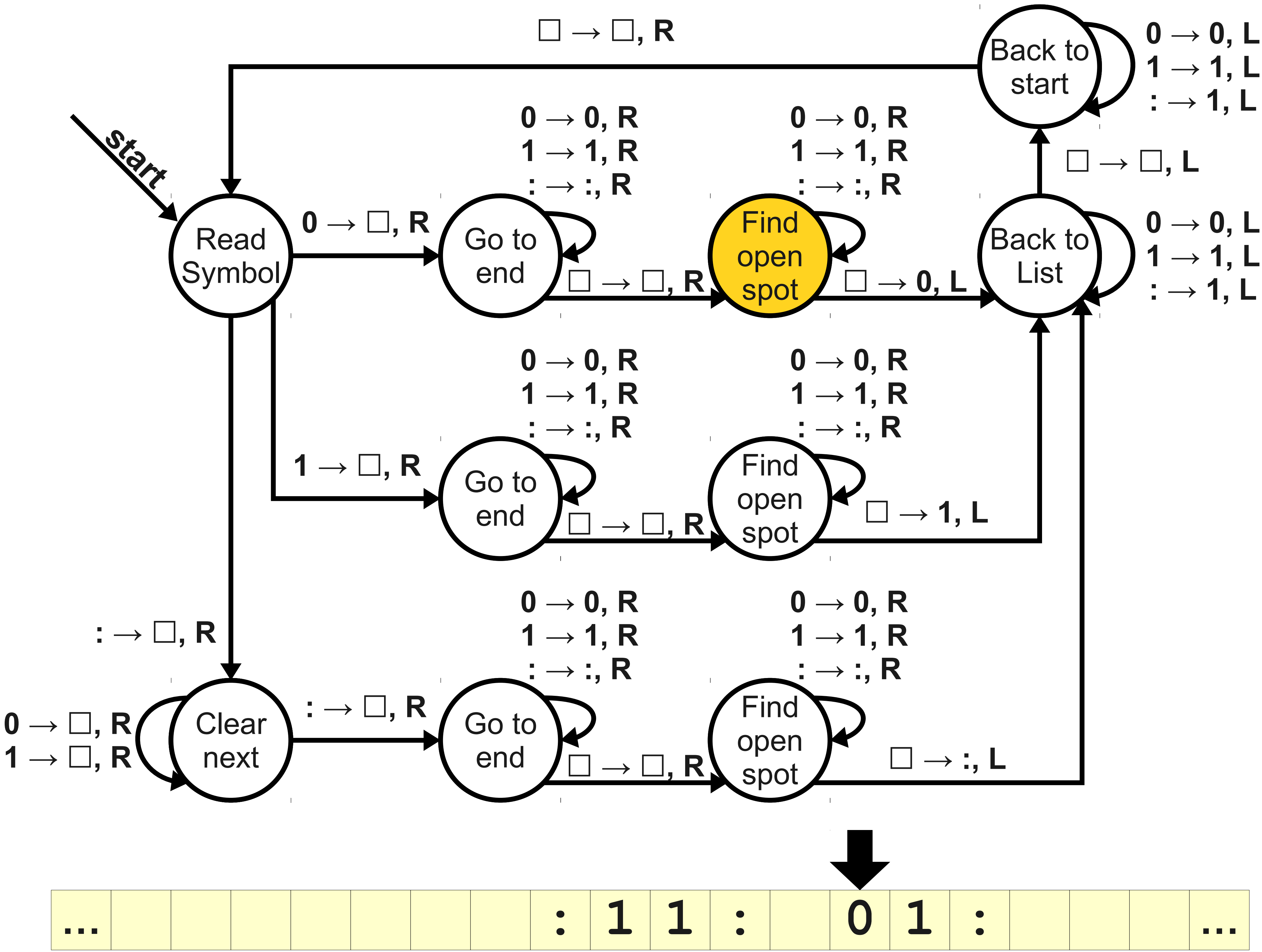
start
Read Symbol
Go to end
Find open spot
Back to List
Back to start
Clear next
0 → □, R
1 → □, R
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□ → 1, L
□ → :, L
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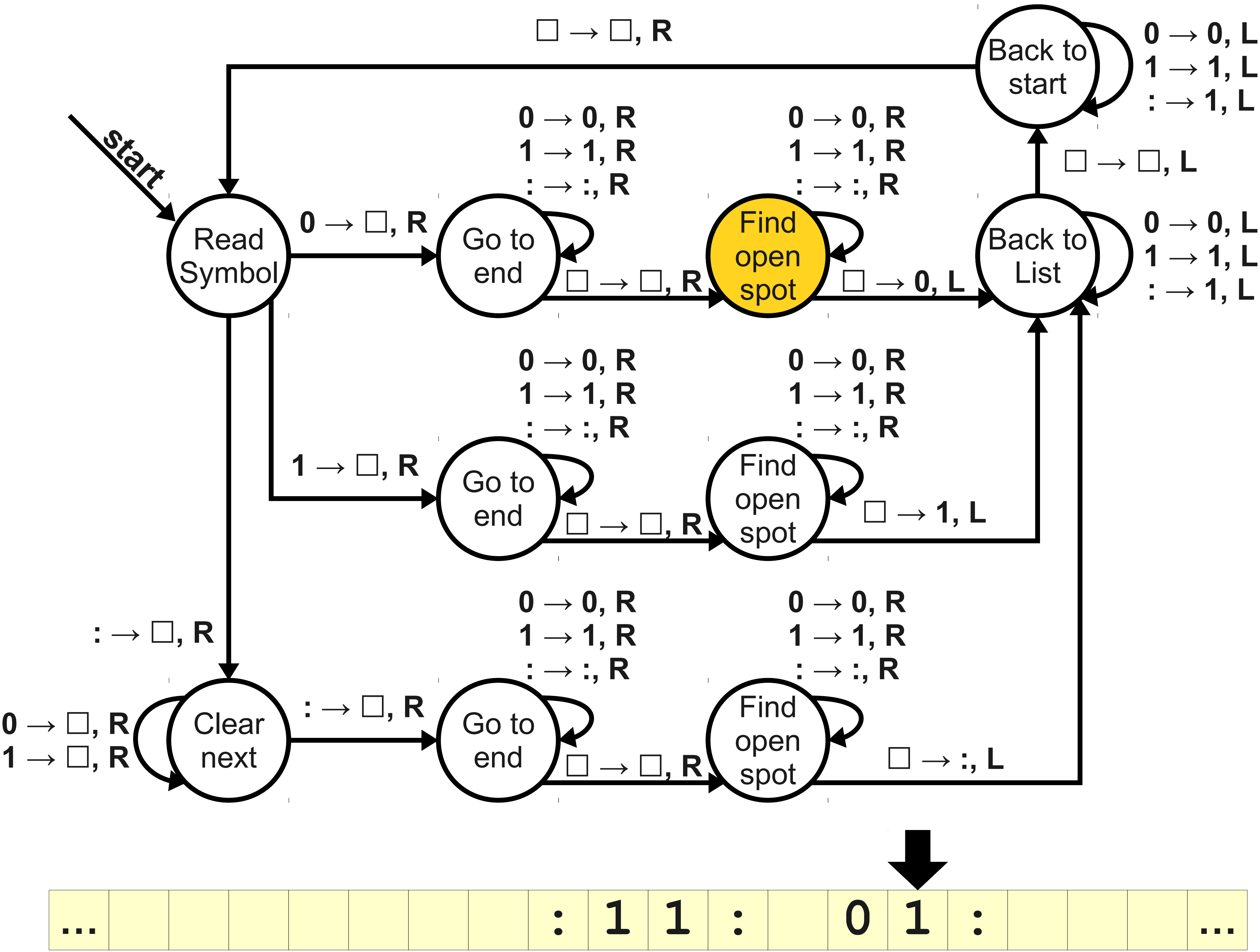
1 → 1, L
: → 1, L
… : 1 1 : 0 1 : …



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Back to start
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start
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: → :, R
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1 → 1, R
: → :, R
□ → □, L
Read Symbol
0 → □, R
Go to end
□ → □, R
Find open spot
□ → 0, L
Back to List
0 → 0, L
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: → 1, L
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1 → 1, R
: → :, R
0 → 0, R
1 → 1, R
: → :, R
1 → □, R
Go to end
□ → □, R
Find open spot
□ → 1, L
0 → 0, R
1 → 1, R
: → :, R
0 → 0, R
1 → 1, R
: → :, R
: → □, R
0 → □, R
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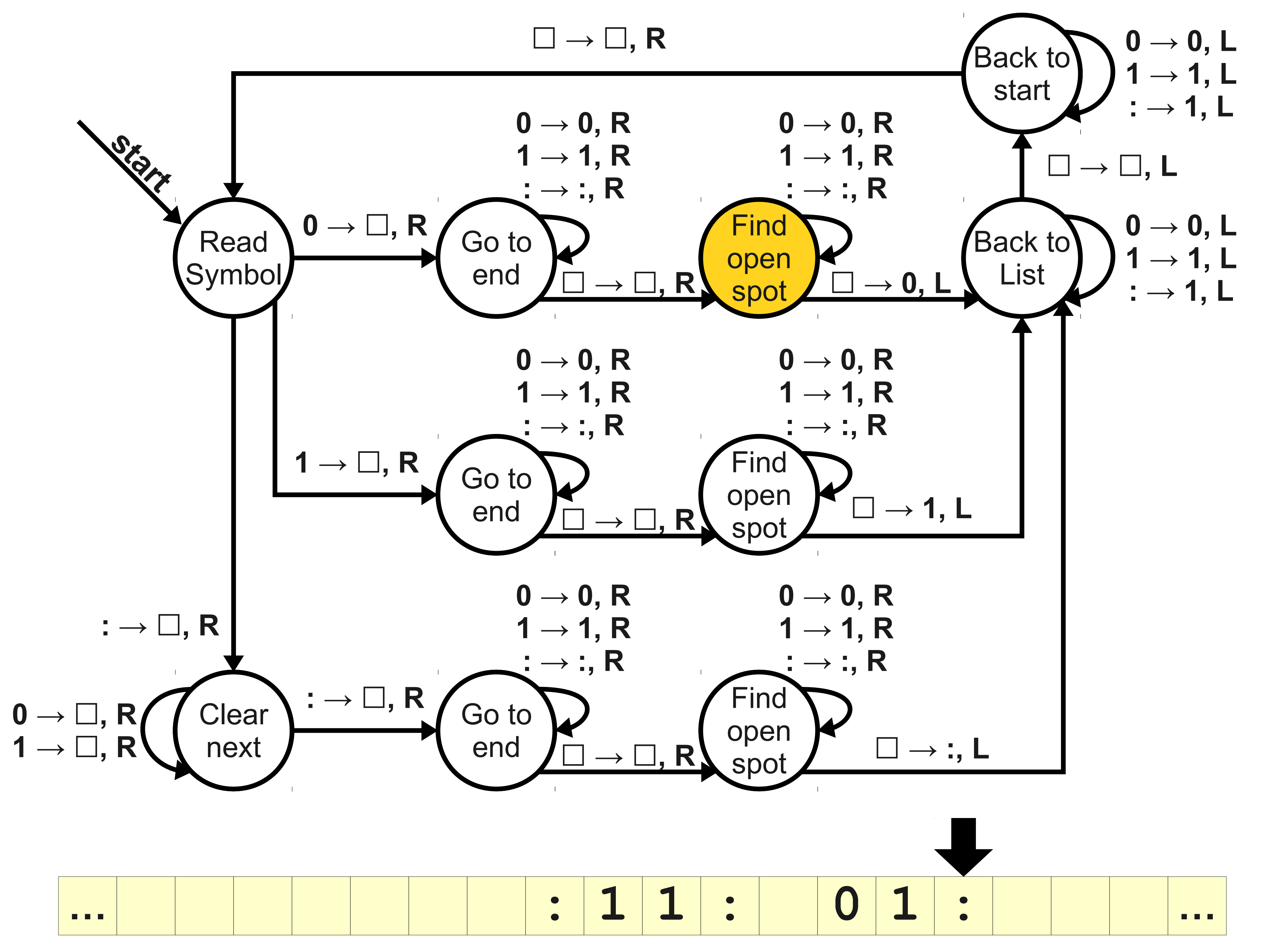
Clear next
: → □, R
Go to end
□ → □, R
Find open spot
□ → :, L
… : 1 1 : 0 1 : …



start
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Go to end
Find open spot
Back to List
Back to start
Clear next
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1 → □, R
: → □, R
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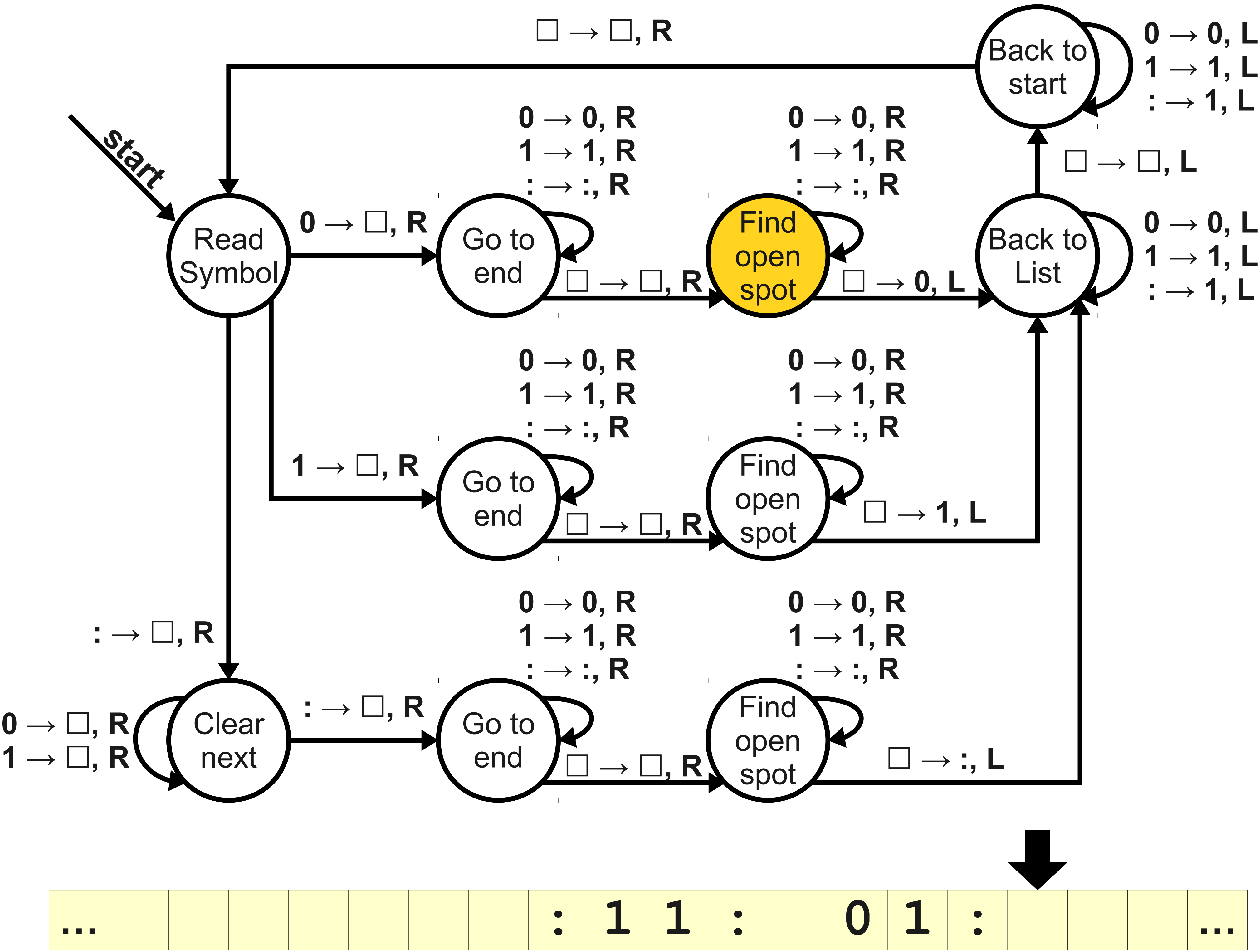
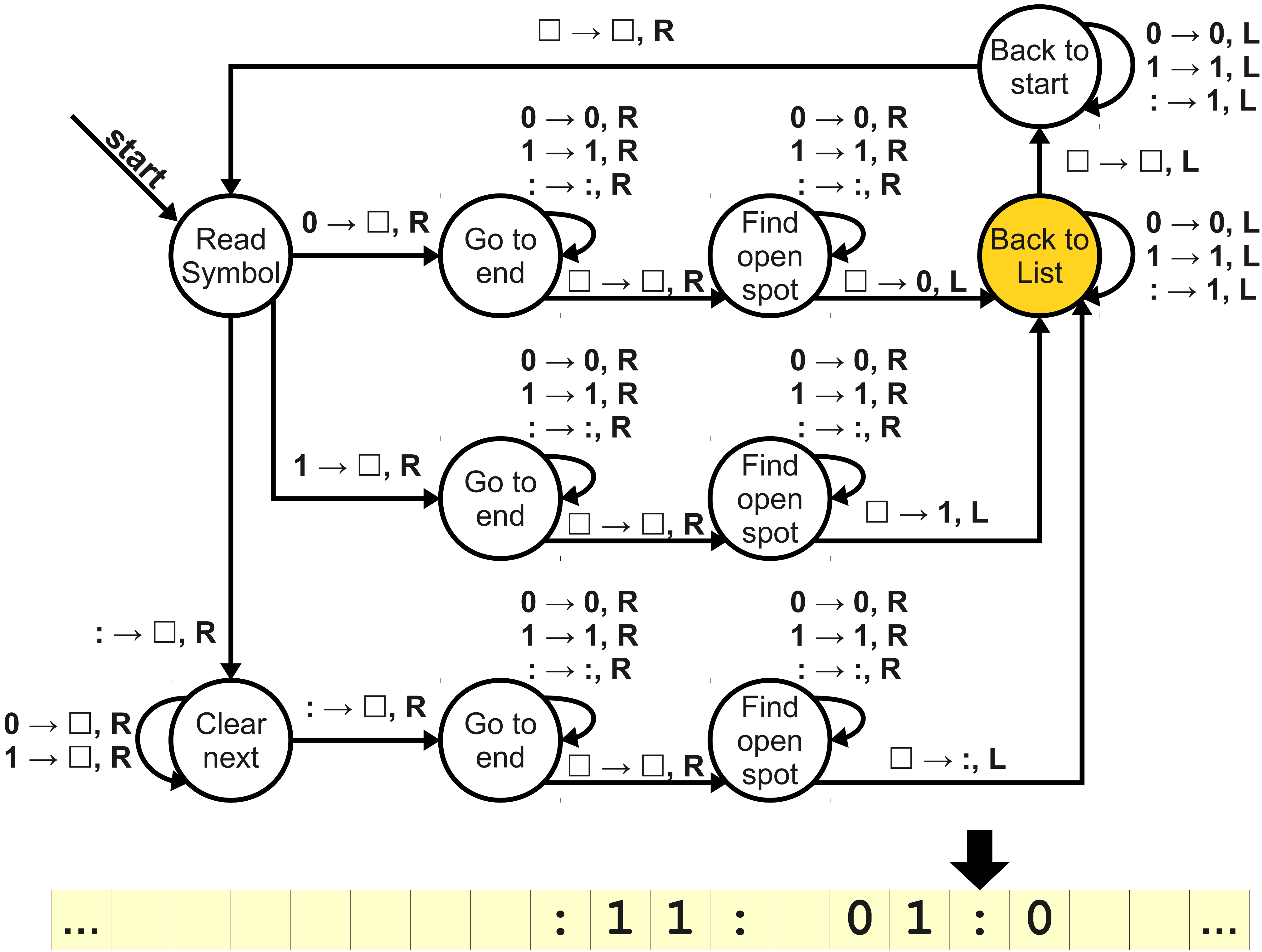
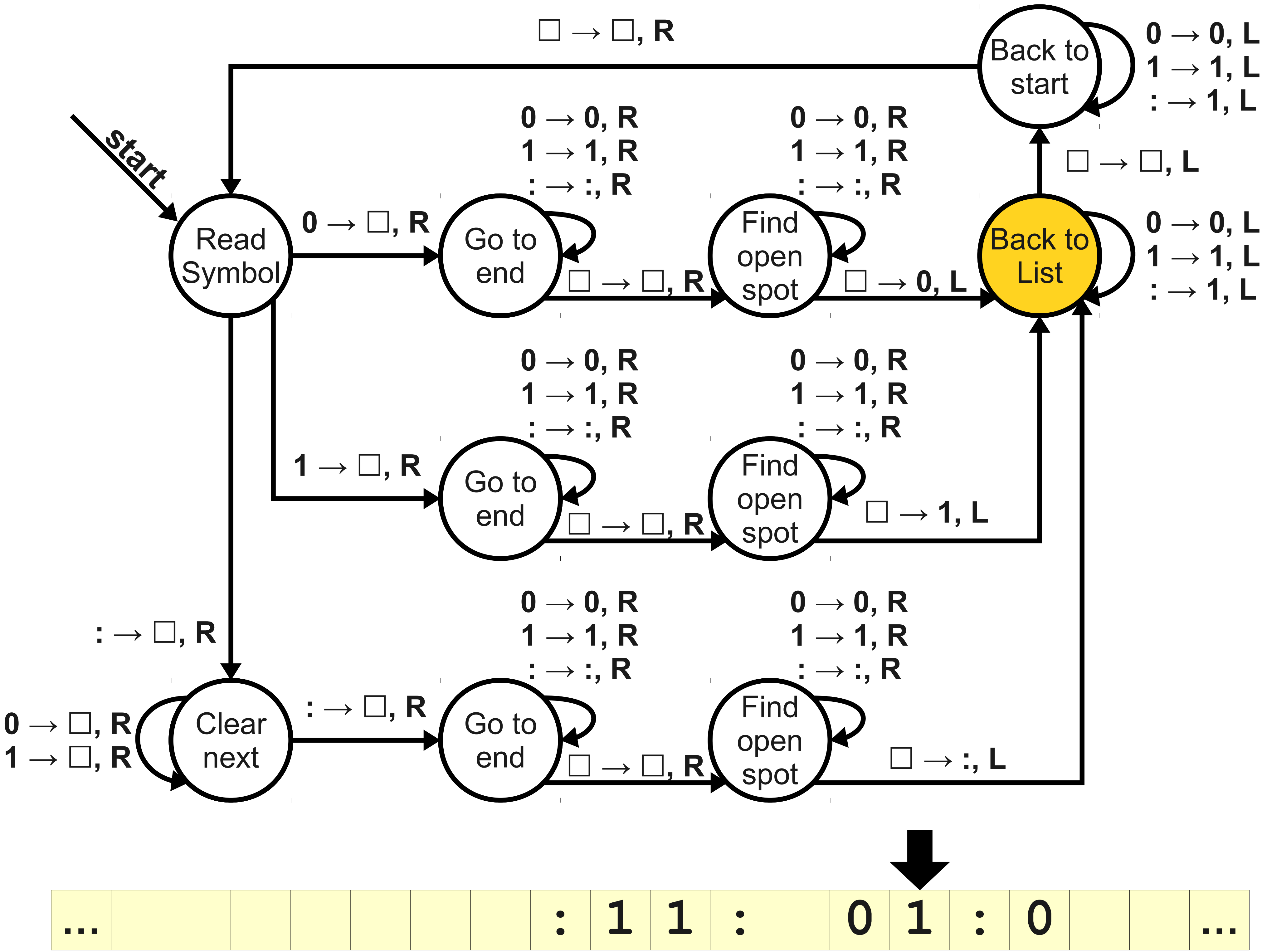
1 → 1, R
: → :, R
□ → □, R
□ → 0, L
□ → 1, L
□ → :, L
0 → 0, L
1 → 1, L
: → 1, L
□ → □, L
... : 1 1 : 0 1 : ...

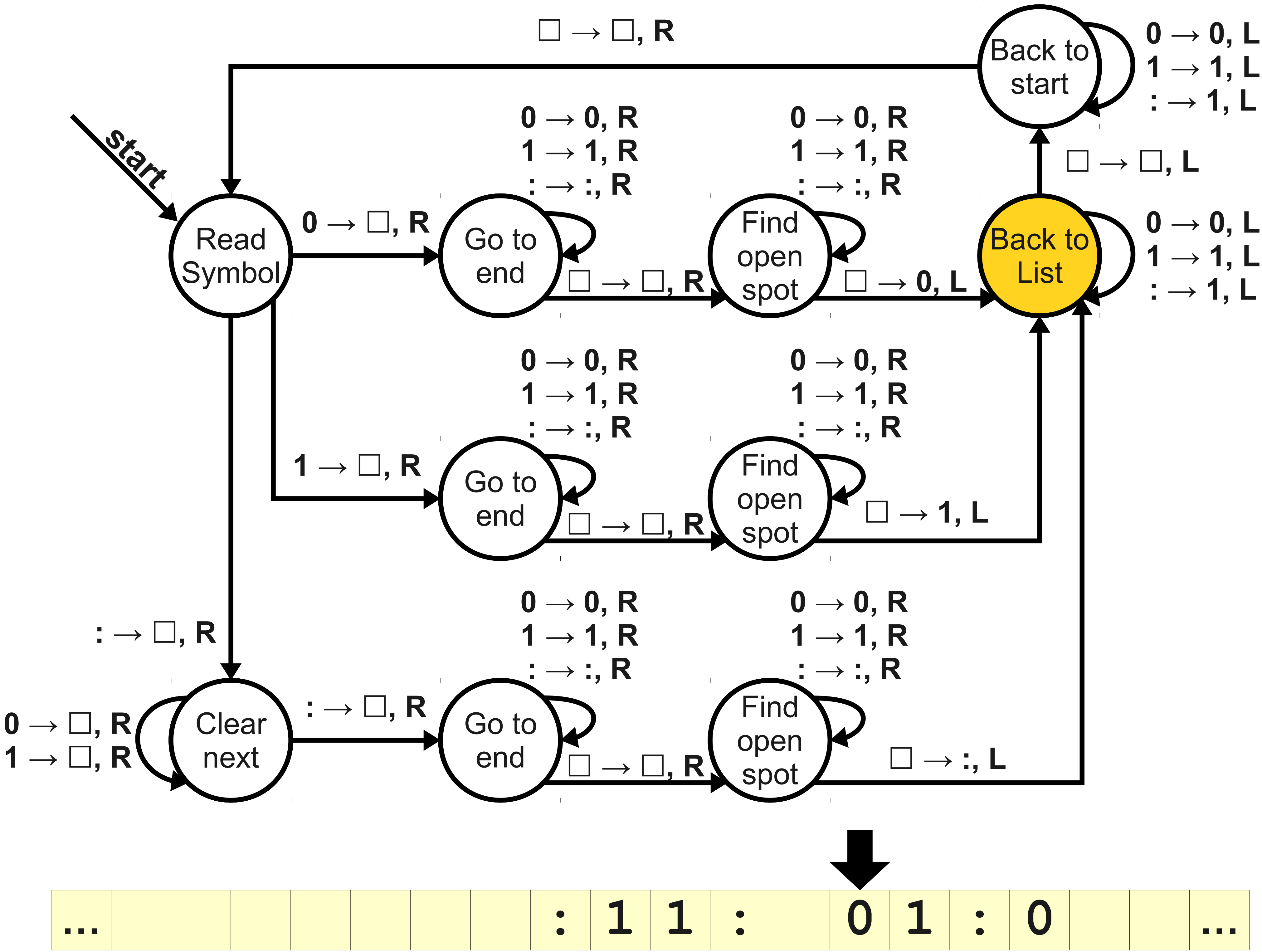


start
Read Symbol
Go to end
Find open spot
Back to List
Back to start
Clear next
□ → □, R
0 → □, R
1 → □, R
: → □, R
0 → 0, R
1 → 1, R
: → :, R
□ → 0, L
□ → 1, L
□ → :, L
□ → □, L
0 → 0, L
1 → 1, L
: → 1, L
0 → □, R
1 → □, R
... : 1 1 : 0 1 : ...


□ → □, R
Back to start
0 → 0, L
1 → 1, L
: → 1, L
start
Read Symbol
0 → □, R
Go to end
0 → 0, R
1 → 1, R
: → :, R
□ → □, R
Find open spot
0 → 0, R
1 → 1, R
: → :, R
□ → 0, L
Back to List
□ → □, L
0 → 0, L
1 → 1, L
: → 1, L
1 → □, R
Go to end
0 → 0, R
1 → 1, R
: → :, R
□ → □, R
Find open spot
0 → 0, R
1 → 1, R
: → :, R
□ → 1, L
: → □, R
Clear next
0 → □, R
1 → □, R
: → □, R
Go to end
0 → 0, R
1 → 1, R
: → :, R
□ → □, R
Find open spot
0 → 0, R
1 → 1, R
: → :, R
□ → :, L
... : 1 1 : 0 1 : 0 ...



□ → □, R
Back to start
0 → 0, L
1 → 1, L
: → 1, L
start
Read Symbol
0 → □, R
Go to end
0 → 0, R
1 → 1, R
: → :, R
□ → □, R
Find open spot
0 → 0, R
1 → 1, R
: → :, R
□ → 0, L
□ → □, L
Back to List
0 → 0, L
1 → 1, L
: → 1, L
1 → □, R
Go to end
0 → 0, R
1 → 1, R
: → :, R
□ → □, R
Find open spot
0 → 0, R
1 → 1, R
: → :, R
□ → 1, L
: → □, R
Clear next
0 → □, R
1 → □, R
: → □, R
Go to end
0 → 0, R
1 → 1, R
: → :, R
□ → □, R
Find open spot
0 → 0, R
1 → 1, R
: → :, R
□ → :, L
... : 1 1 : 0 1 : 0 ...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Back to start
0 → 0, L
1 → 1, L
: → 1, L
start
0 → 0, R
1 → 1, R
: → :, R
0 → 0, R
1 → 1, R
: → :, R
□ → □, L
Read Symbol
0 → □, R
Go to end
□ → □, R
Find open spot
□ → 0, L
Back to List
0 → 0, L
1 → 1, L
: → 1, L
0 → 0, R
1 → 1, R
: → :, R
0 → 0, R
1 → 1, R
: → :, R
1 → □, R
Go to end
□ → □, R
Find open spot
□ → 1, L
: → □, R
0 → 0, R
1 → 1, R
: → :, R
0 → 0, R
1 → 1, R
: → :, R
0 → □, R
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Go to end
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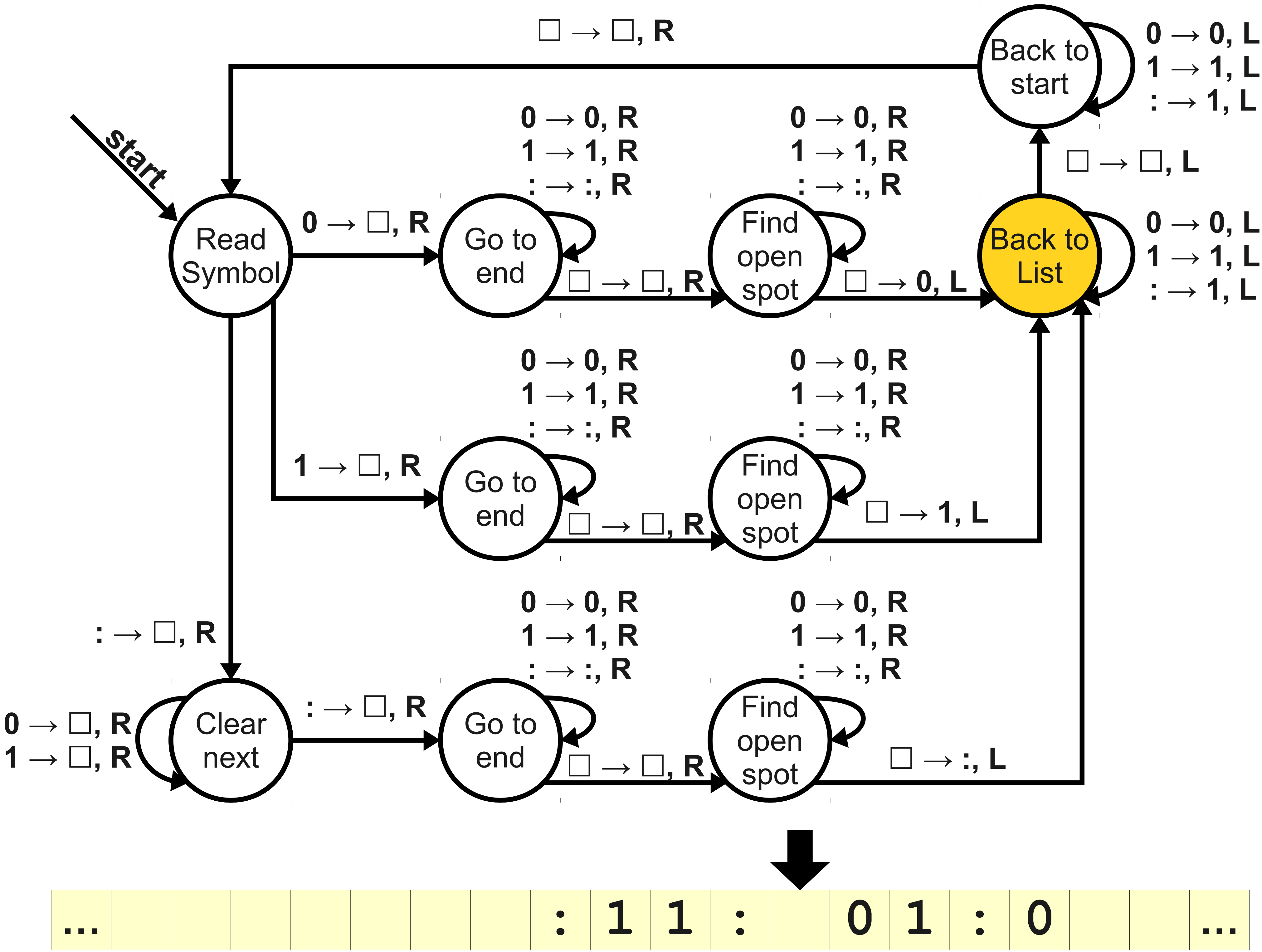
Find open spot
□ → :, L
... : 1 1 : 0 1 : 0 ...


start
Read Symbol
Go to end
Find open spot
Back to List
Back to start
Clear next
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1 → □, R
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0 → 0, R
1 → 1, R
: → :, R
□ → □, R
□ → 0, L
□ → 1, L
□ → :, L
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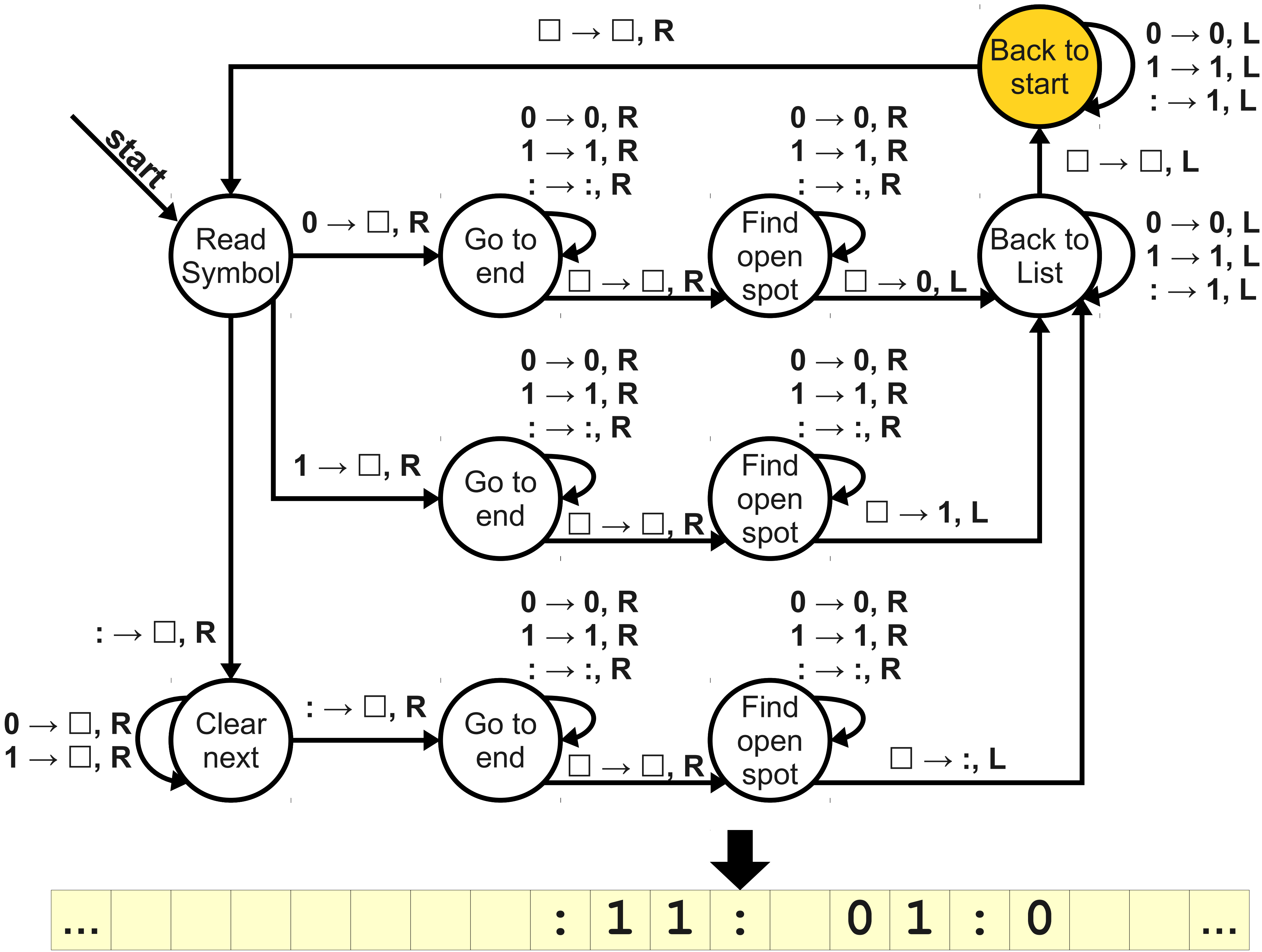
□ → □, L
0 → □, R
1 → □, R
… : 1 1 : 0 1 : 0 …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Back to start
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1 → 1, L
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start
Read Symbol
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Go to end
0 → 0, R
1 → 1, R
: → :, R
□ → □, R
Find open spot
□ → 0, L
Back to List
□ → □, L
1 → □, R
□ → 1, L
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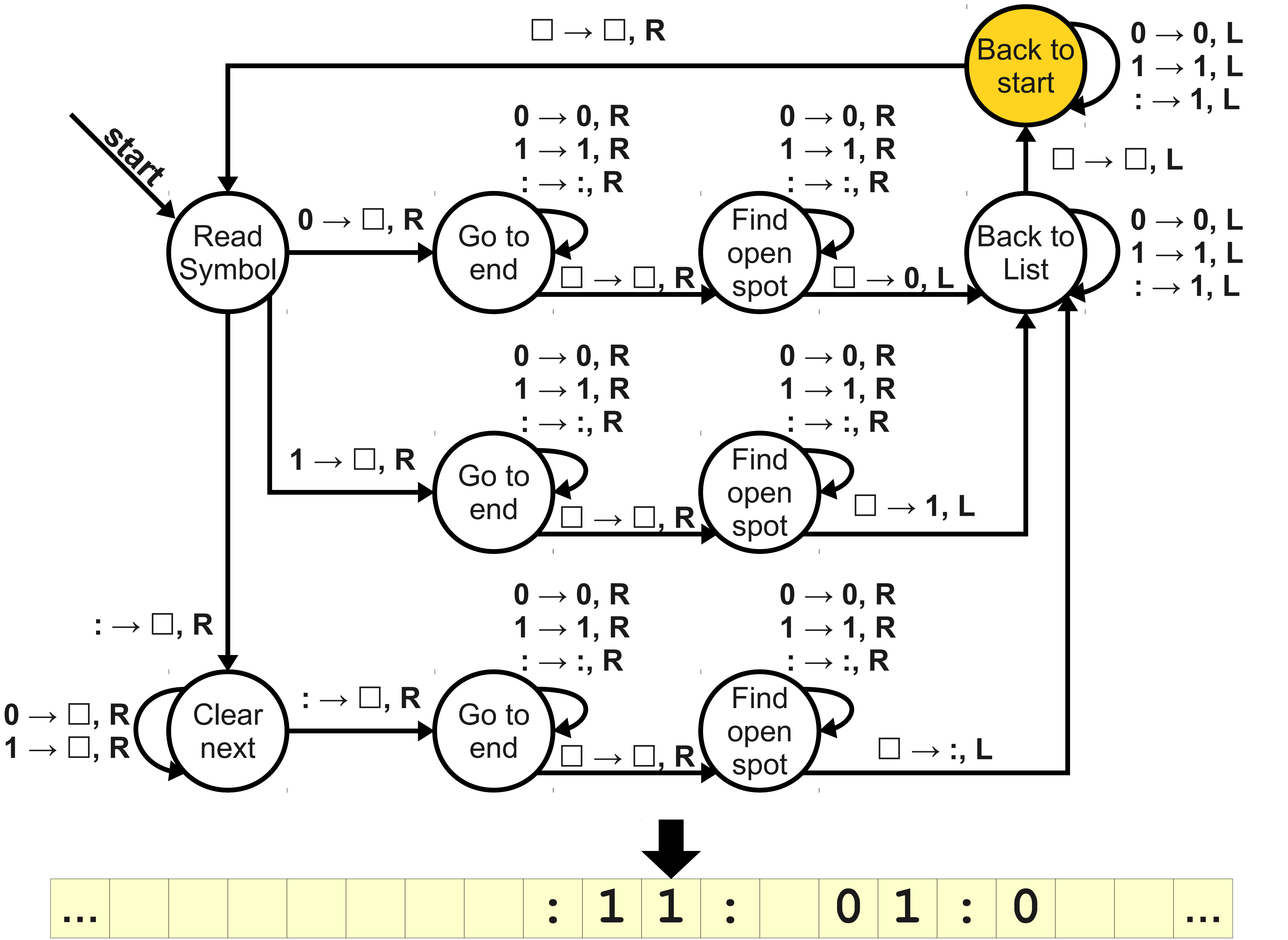
Clear next
0 → □, R
1 → □, R
: → □, R
□ → :, L
... : 1 1 : 0 1 : 0 ...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start
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1 → 1, R
: → :, R
□ → □, L
Read Symbol
0 → □, R
Go to end
□ → □, R
Find open spot
□ → 0, L
Back to List
0 → 0, L
1 → 1, L
: → 1, L
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1 → 1, R
: → :, R
0 → 0, R
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Go to end
□ → □, R
Find open spot
□ → 1, L
: → □, R
0 → 0, R
1 → 1, R
: → :, R
0 → 0, R
1 → 1, R
: → :, R
0 → □, R
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Clear next
: → □, R
Go to end
□ → □, R
Find open spot
□ → :, L
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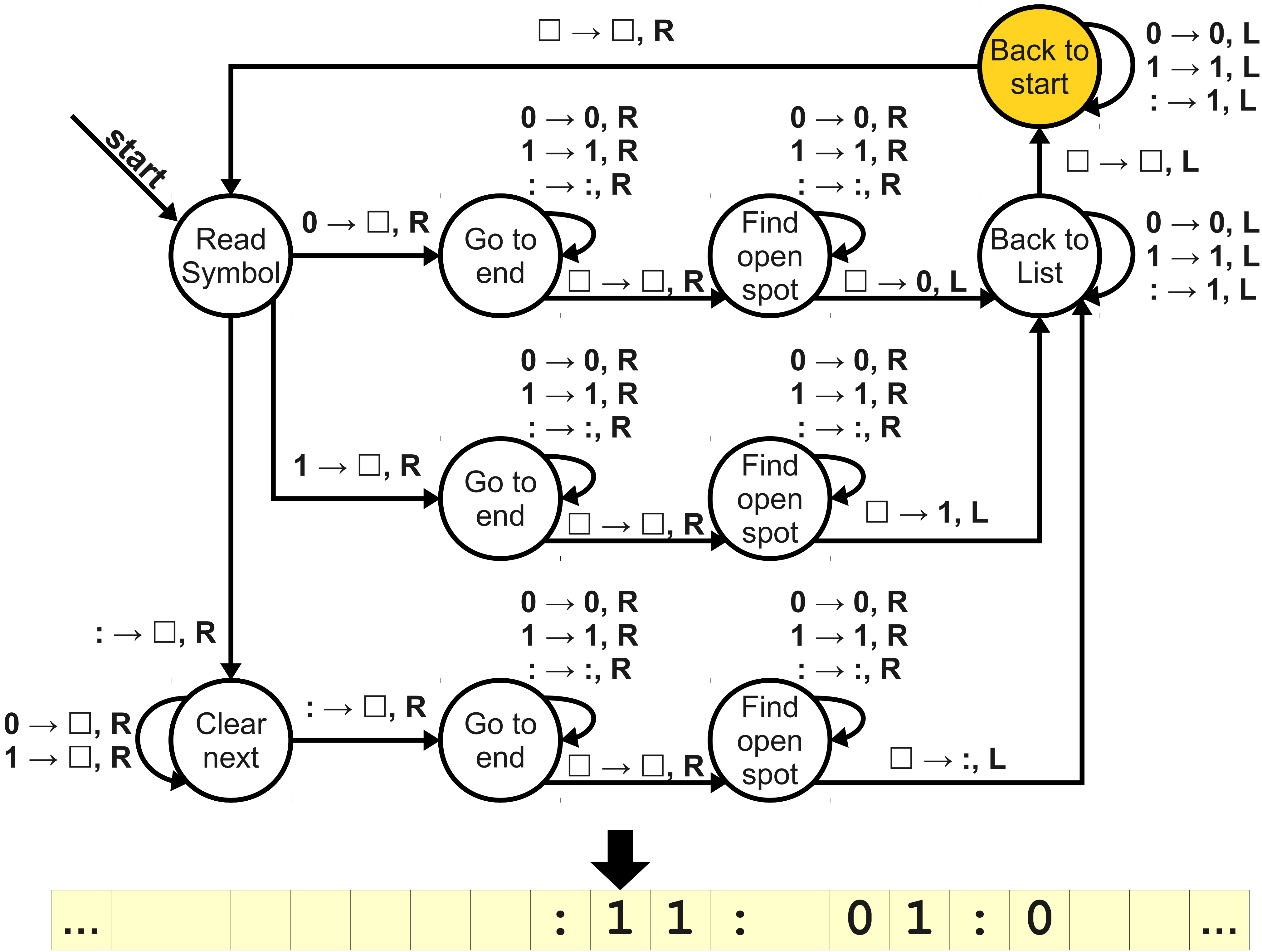
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Back to start
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start
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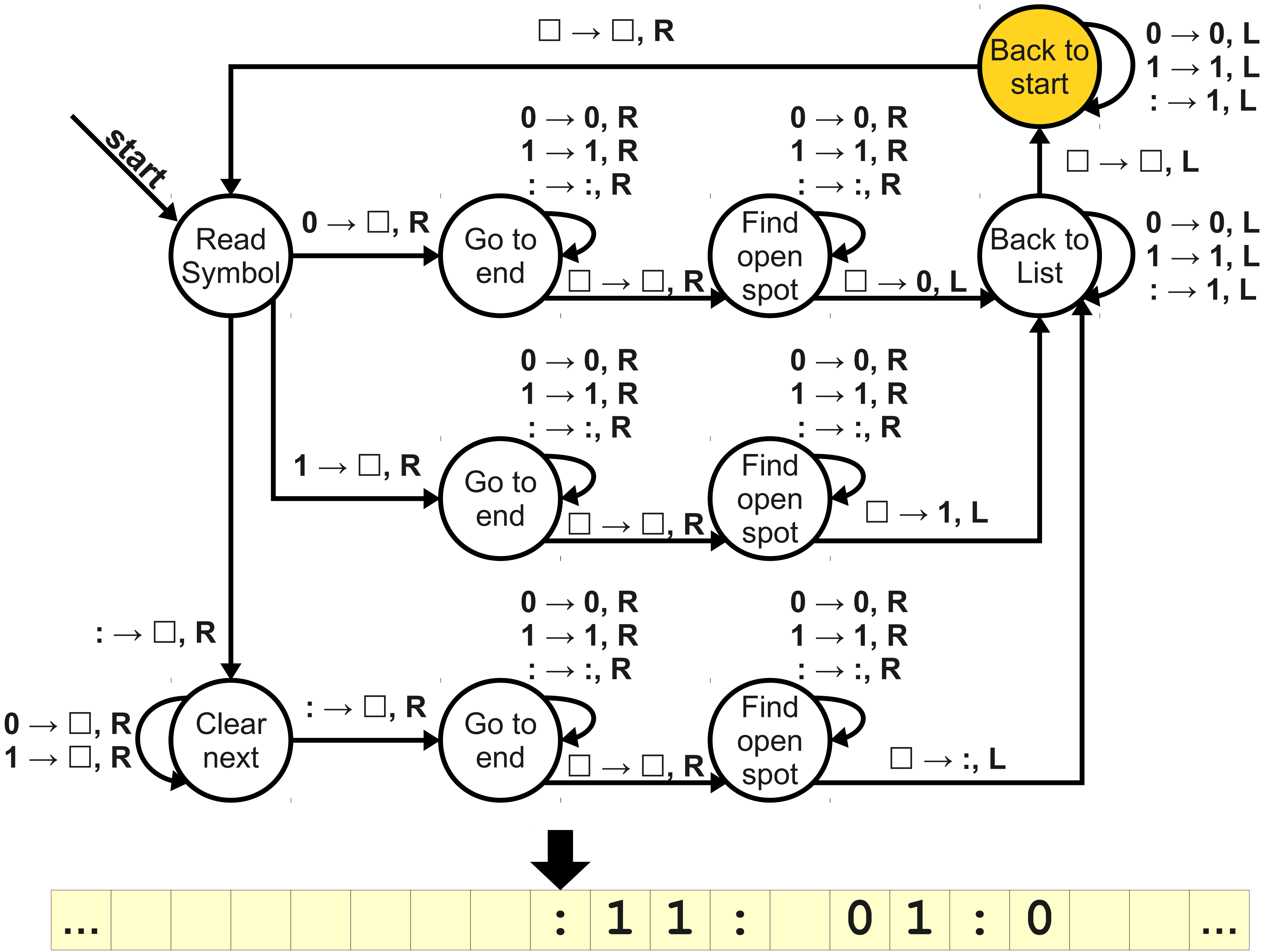
1 → 1, R
: → :, R
□ → □, L
Read Symbol
0 → □, R
Go to end
□ → □, R
Find open spot
□ → 0, L
Back to List
0 → 0, L
1 → 1, L
: → 1, L
0 → 0, R
1 → 1, R
: → :, R
0 → 0, R
1 → 1, R
: → :, R
1 → □, R
Go to end
□ → □, R
Find open spot
□ → 1, L
: → □, R
0 → 0, R
1 → 1, R
: → :, R
0 → 0, R
1 → 1, R
: → :, R
0 → □, R
1 → □, R
Clear next
: → □, R
Go to end
□ → □, R
Find open spot
□ → :, L
... : 1 1 : 0 1 : 0 ...



□ → □, R
Back to start
0 → 0, L
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start
Read Symbol
0 → □, R
Go to end
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1 → 1, R
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□ → □, R
Find open spot
0 → 0, R
1 → 1, R
: → :, R
□ → 0, L
Back to List
□ → □, L
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1 → 1, L
: → 1, L
1 → □, R
Go to end
0 → 0, R
1 → 1, R
: → :, R
□ → □, R
Find open spot
0 → 0, R
1 → 1, R
: → :, R
□ → 1, L
: → □, R
Clear next
0 → □, R
1 → □, R
: → □, R
Go to end
0 → 0, R
1 → 1, R
: → :, R
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Find open spot
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1 → 1, R
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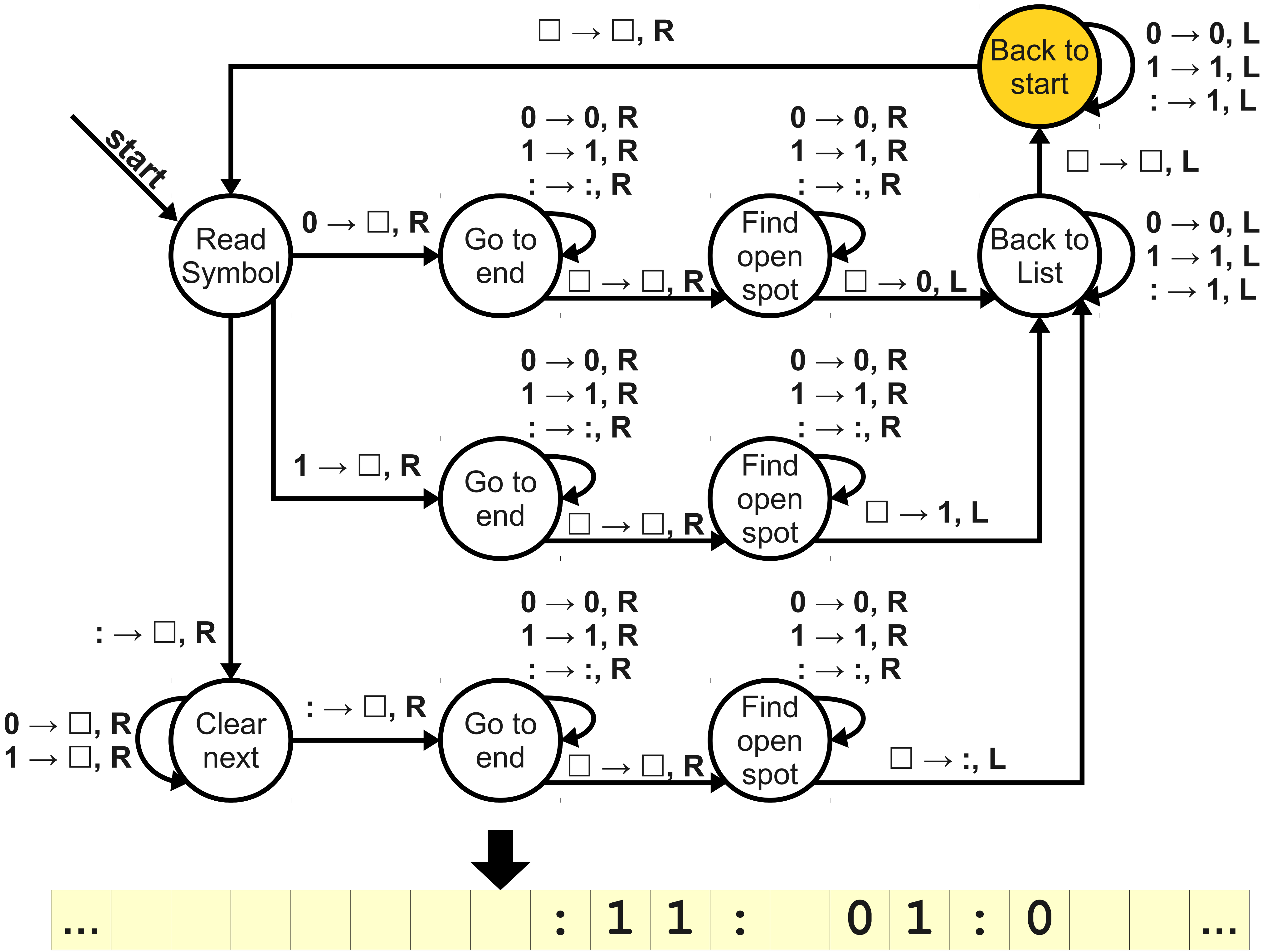
… : 1 1 : 0 1 : 0 …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start
Read Symbol
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Go to end
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1 → 1, R
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□ → □, R
Find open spot
0 → 0, R
1 → 1, R
: → :, R
□ → 0, L
Back to List
0 → 0, L
1 → 1, L
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□ → □, L
1 → □, R
Go to end
0 → 0, R
1 → 1, R
: → :, R
□ → □, R
Find open spot
0 → 0, R
1 → 1, R
: → :, R
□ → 1, L
: → □, R
Clear next
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Go to end
0 → 0, R
1 → 1, R
: → :, R
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Find open spot
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1 → 1, R
: → :, R
□ → :, L
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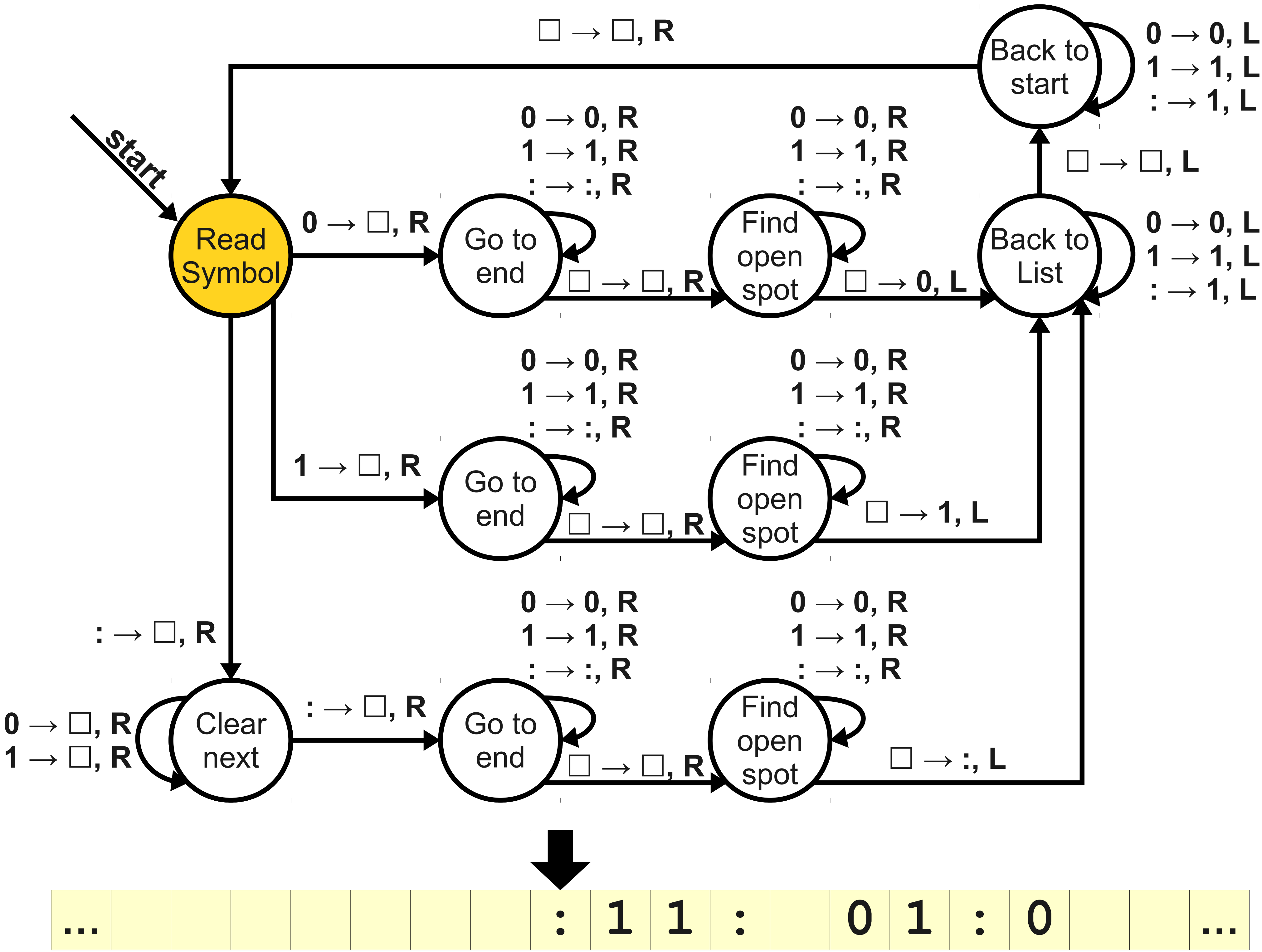

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Go to end
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Find open spot
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: → :, R
□ → 0, L
□ → □, L
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Go to end
0 → 0, R
1 → 1, R
: → :, R
□ → □, R
Find open spot
0 → 0, R
1 → 1, R
: → :, R
□ → 1, L
: → □, R
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Go to end
0 → 0, R
1 → 1, R
: → :, R
□ → □, R
Find open spot
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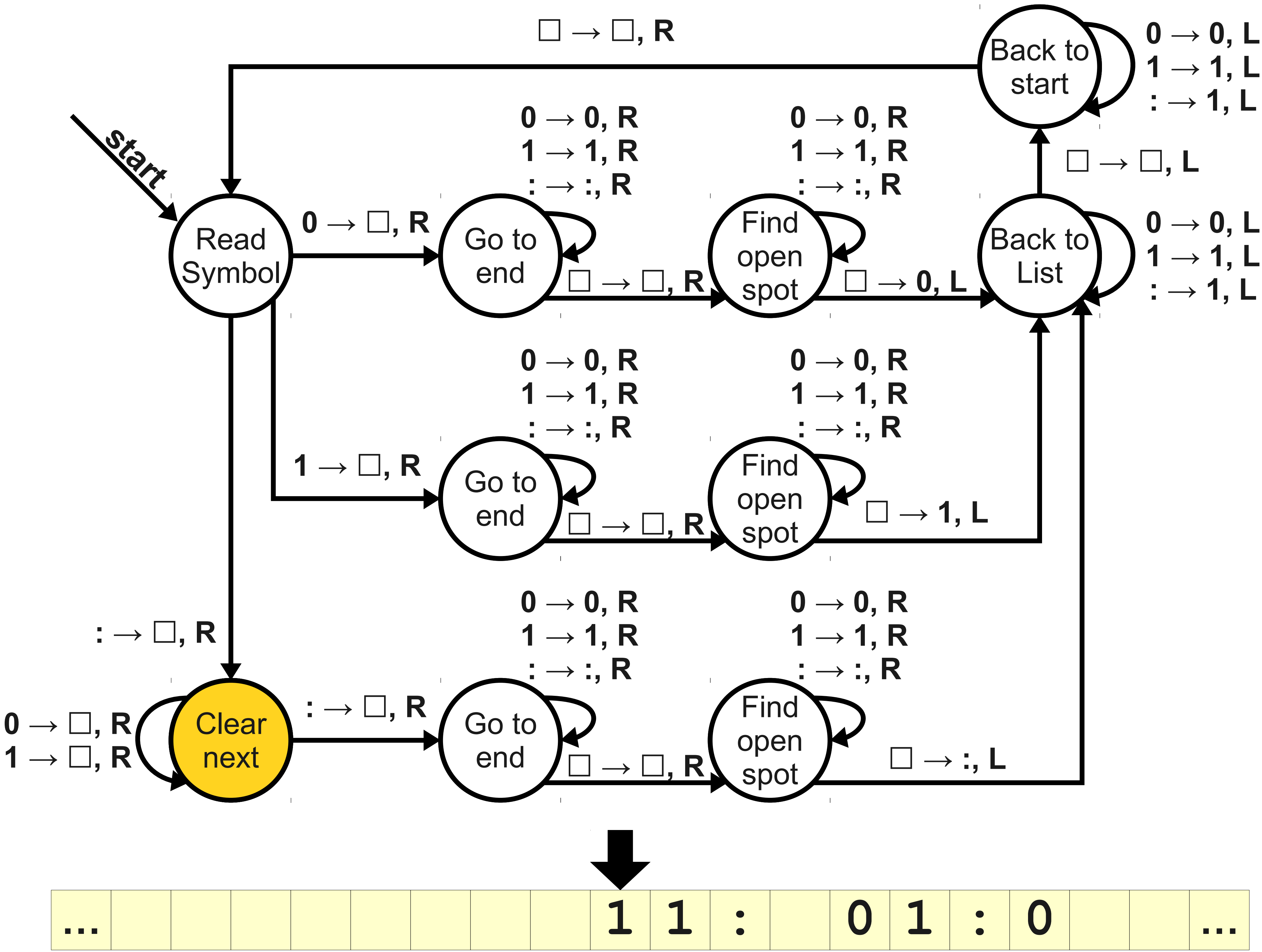

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: → :, R
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Read Symbol
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Go to end
□ → □, R
Find open spot
□ → 0, L
Back to List
1 → □, R
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: → □, R
Clear next
0 → □, R
1 → □, R
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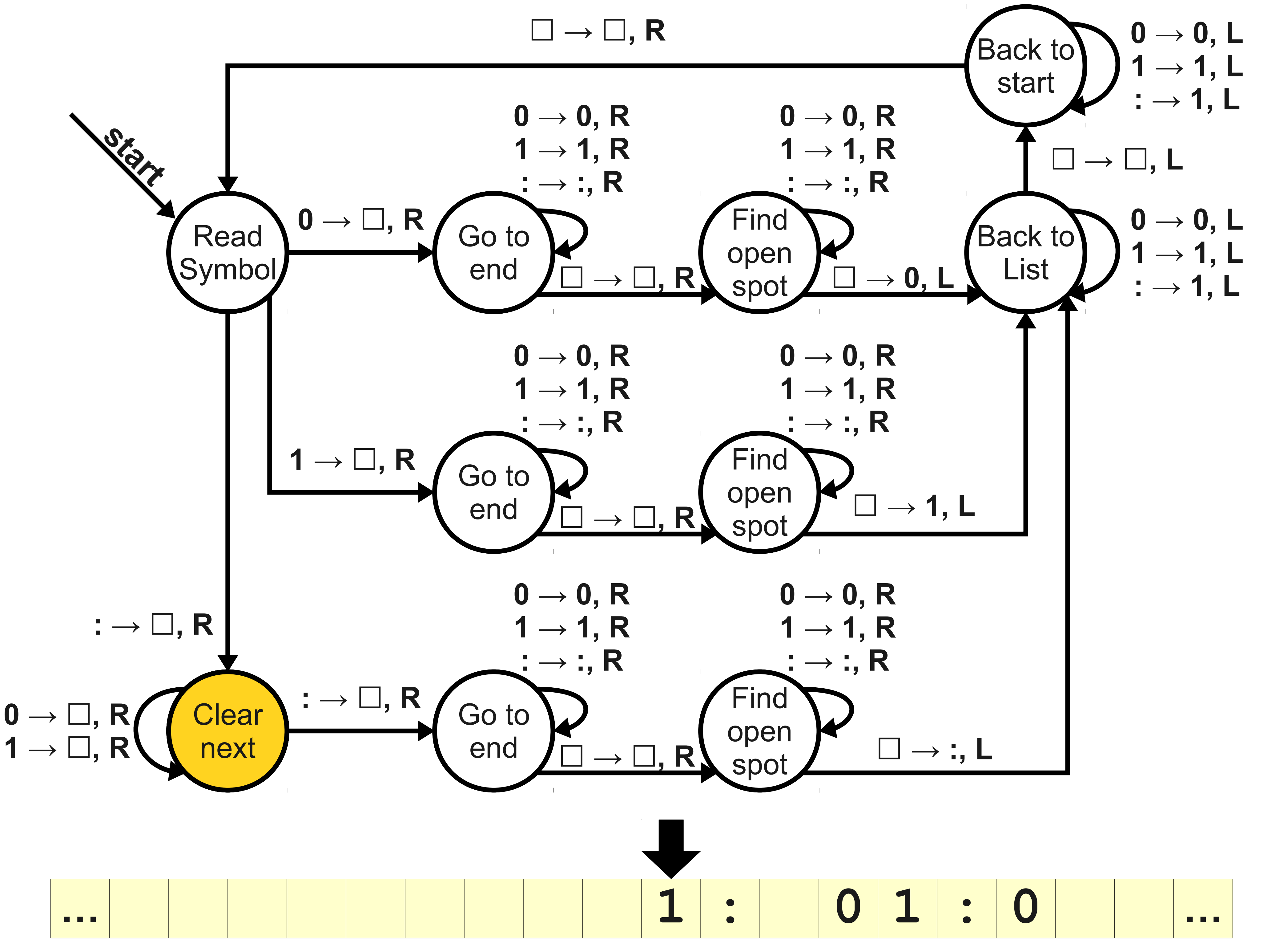

□ → □, R
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Read Symbol
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Go to end
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□ → □, R
Find open spot
0 → 0, R
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: → :, R
□ → 0, L
Back to List
0 → 0, L
1 → 1, L
: → 1, L
□ → □, L
1 → □, R
Go to end
0 → 0, R
1 → 1, R
: → :, R
□ → □, R
Find open spot
0 → 0, R
1 → 1, R
: → :, R
□ → 1, L
: → □, R
Clear next
0 → □, R
1 → □, R
: → □, R
Go to end
0 → 0, R
1 → 1, R
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Find open spot
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1 → 1, R
: → :, R
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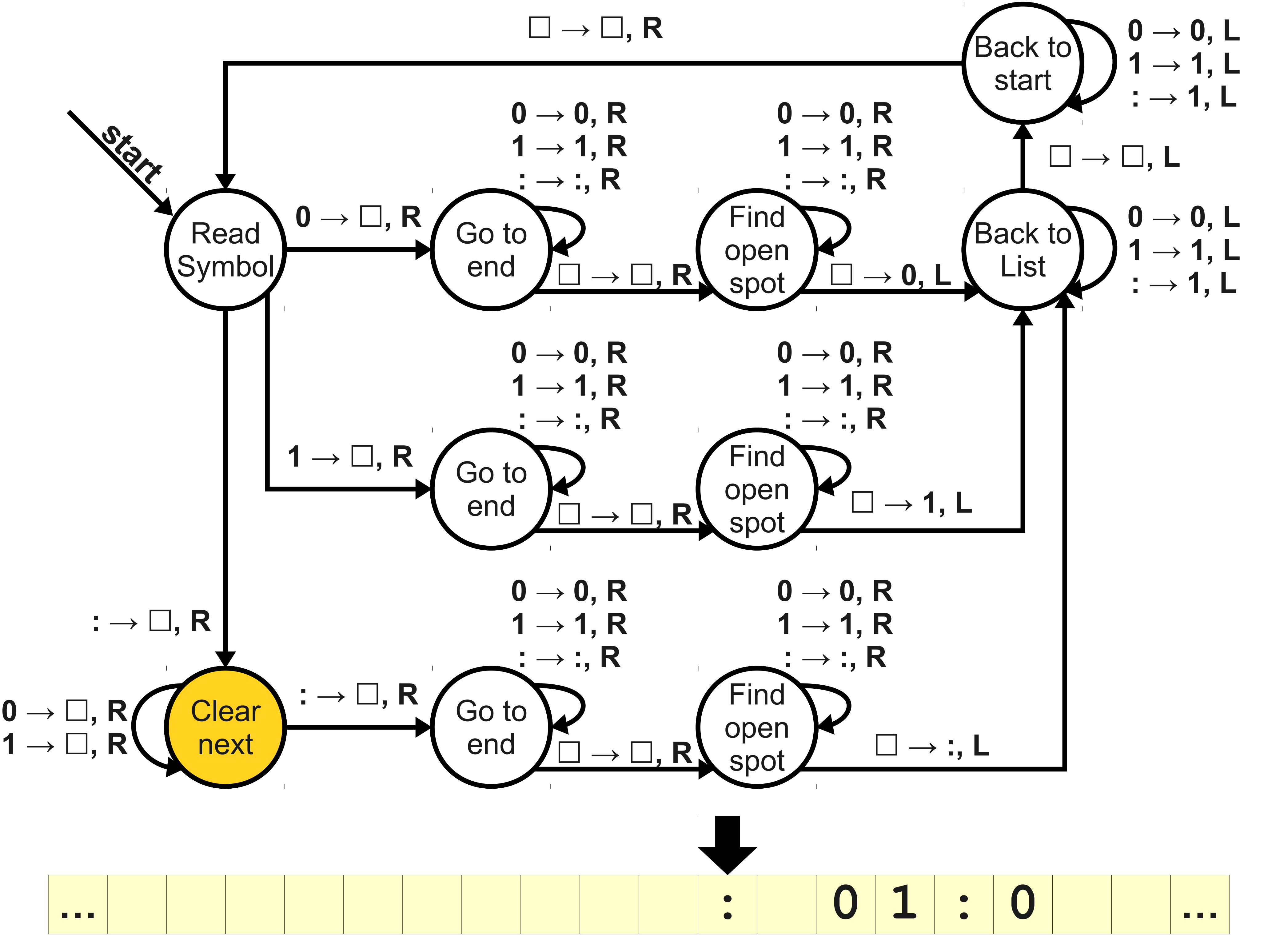
start
Read Symbol
Go to end
Find open spot
Back to List
Back to start
Clear next
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: → :, R
□ → □, R
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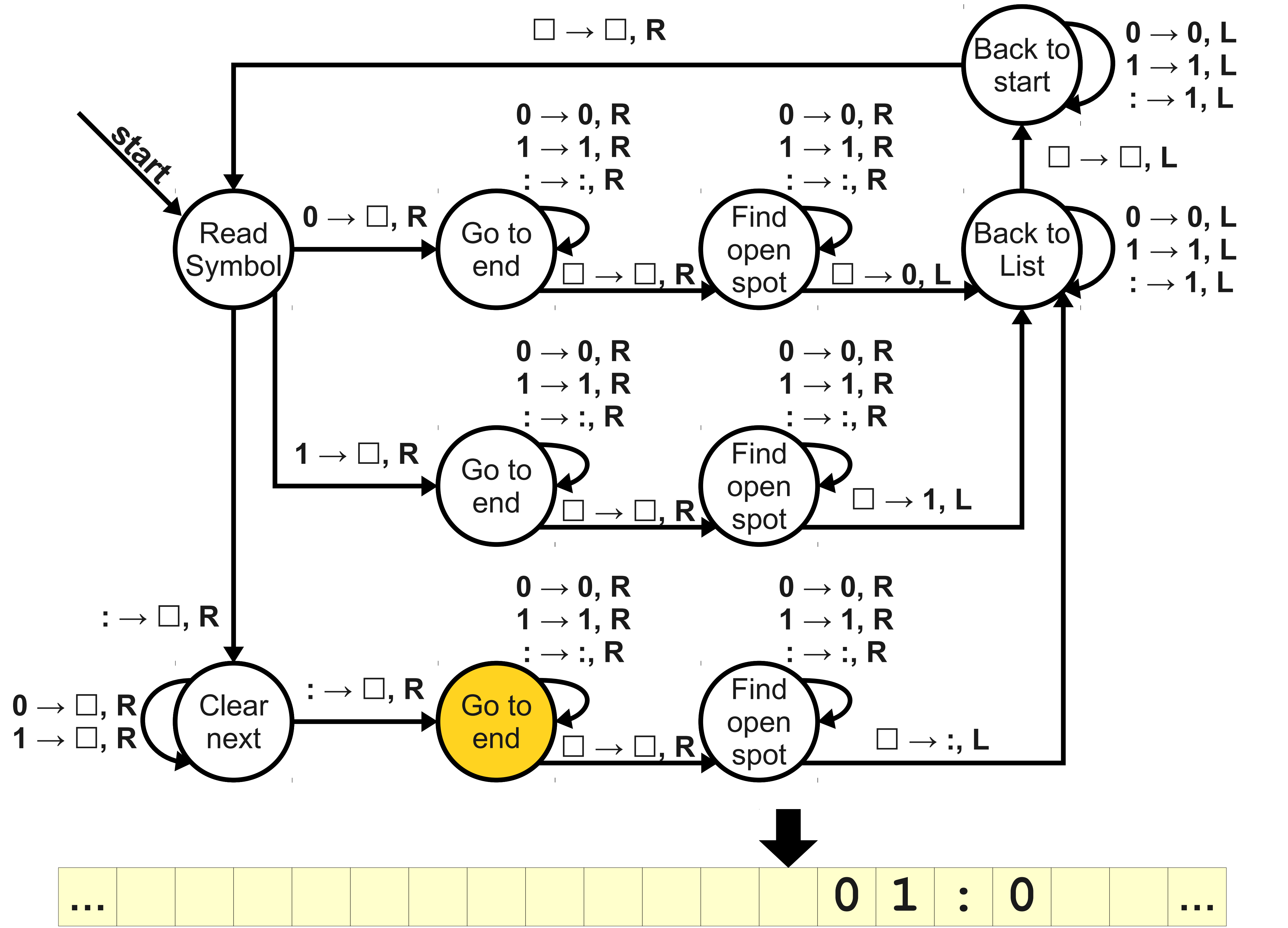

start
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Go to end
Find open spot
Back to List
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Clear next
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□ → □, L
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1 → □, R
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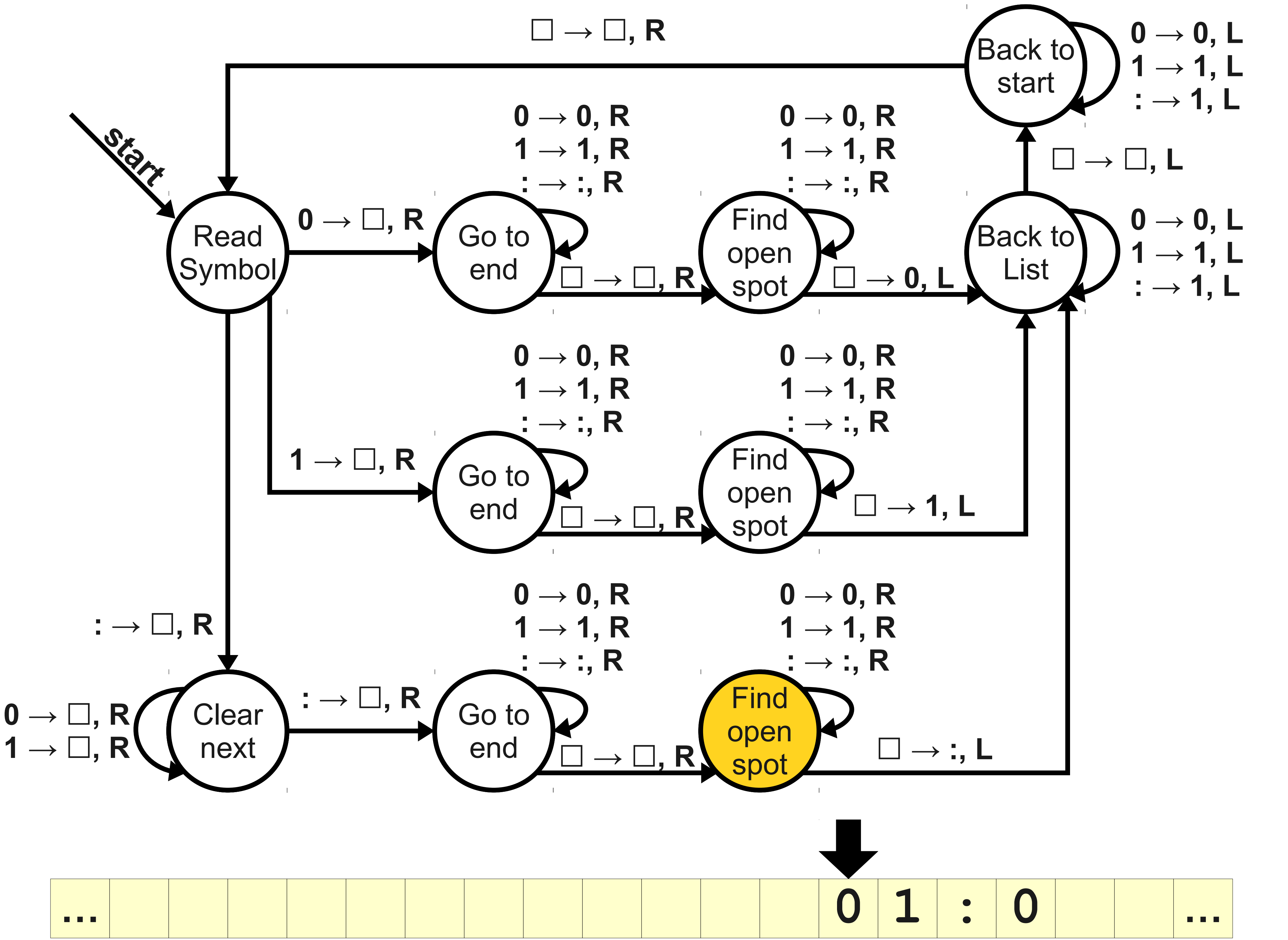

start
Read Symbol
Go to end
Find open spot
Back to List
Back to start
Clear next
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1 → 1, R
: → :, R
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□ → 1, L
□ → :, L
□ → □, L
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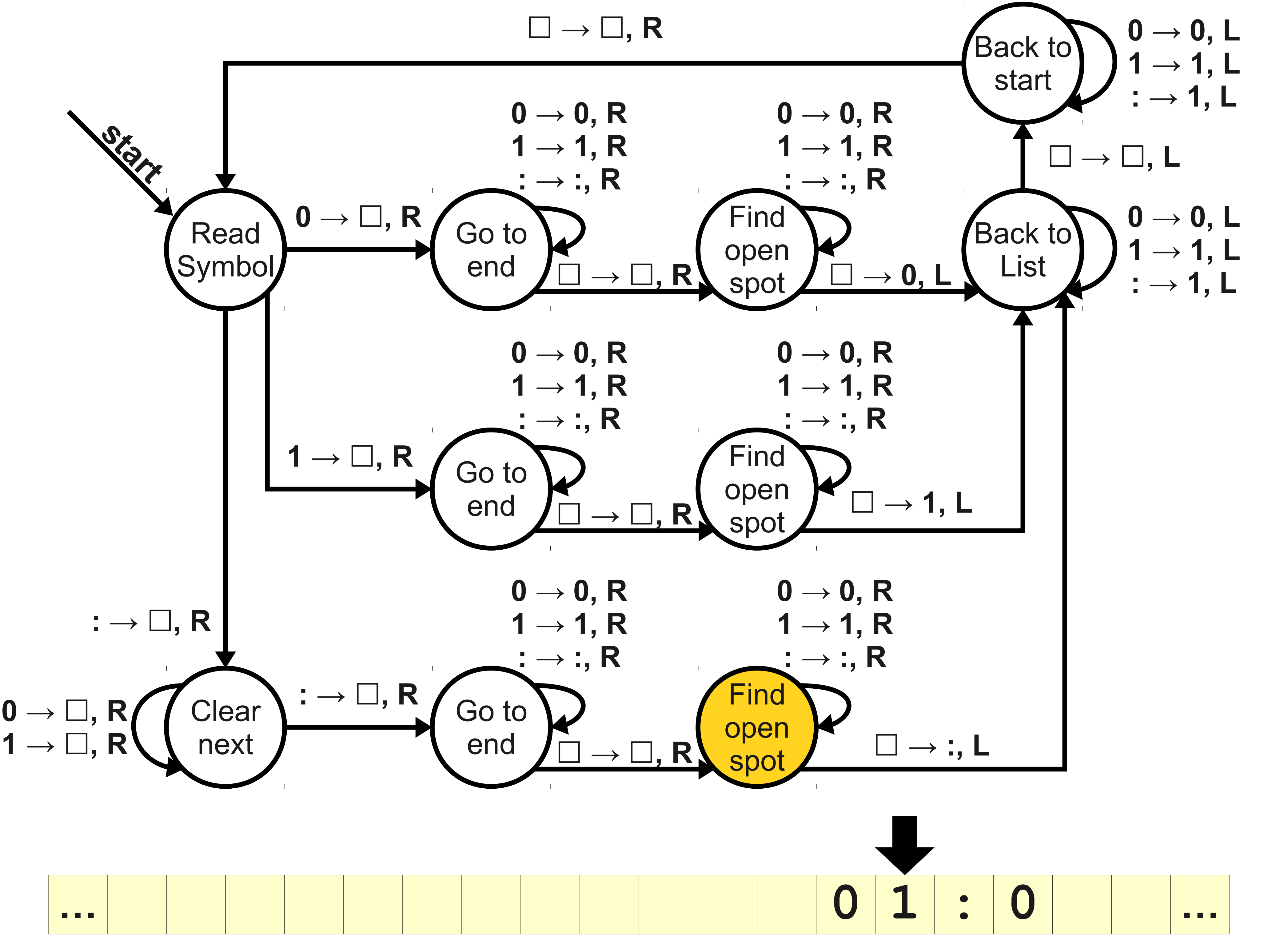

start
Read Symbol
Go to end
Find open spot
Back to List
Back to start
Clear next
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□ → □, R
□ → 0, L
□ → 1, L
□ → :, L
0 → 0, L
1 → 1, L
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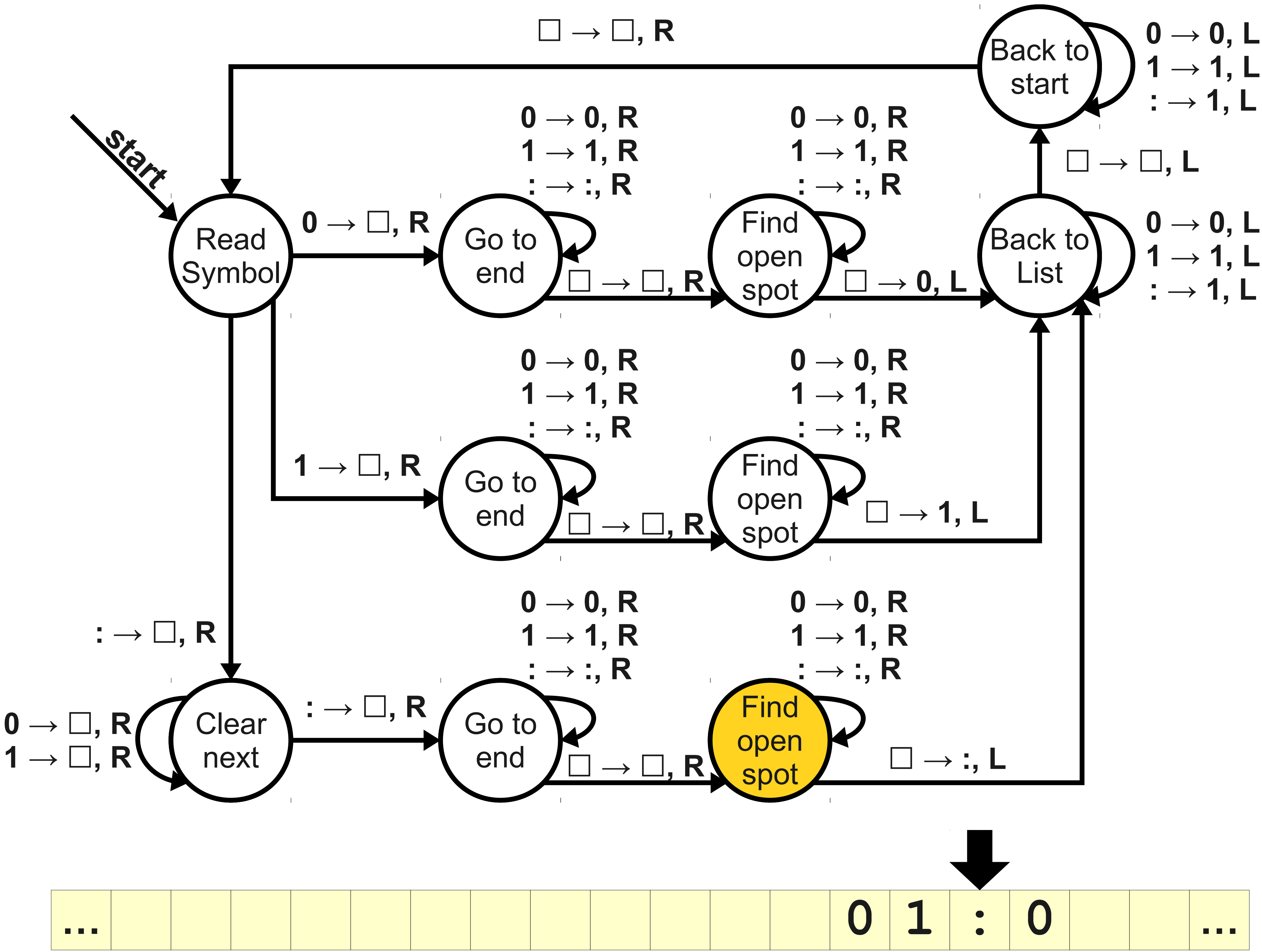

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Back to start
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start
Read Symbol
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Go to end
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1 → 1, R
: → :, R
□ → □, R
Find open spot
0 → 0, R
1 → 1, R
: → :, R
□ → 0, L
Back to List
0 → 0, L
1 → 1, L
: → 1, L
□ → □, L
1 → □, R
Go to end
0 → 0, R
1 → 1, R
: → :, R
□ → □, R
Find open spot
0 → 0, R
1 → 1, R
: → :, R
□ → 1, L
: → □, R
Clear next
0 → □, R
1 → □, R
: → □, R
Go to end
0 → 0, R
1 → 1, R
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Find open spot
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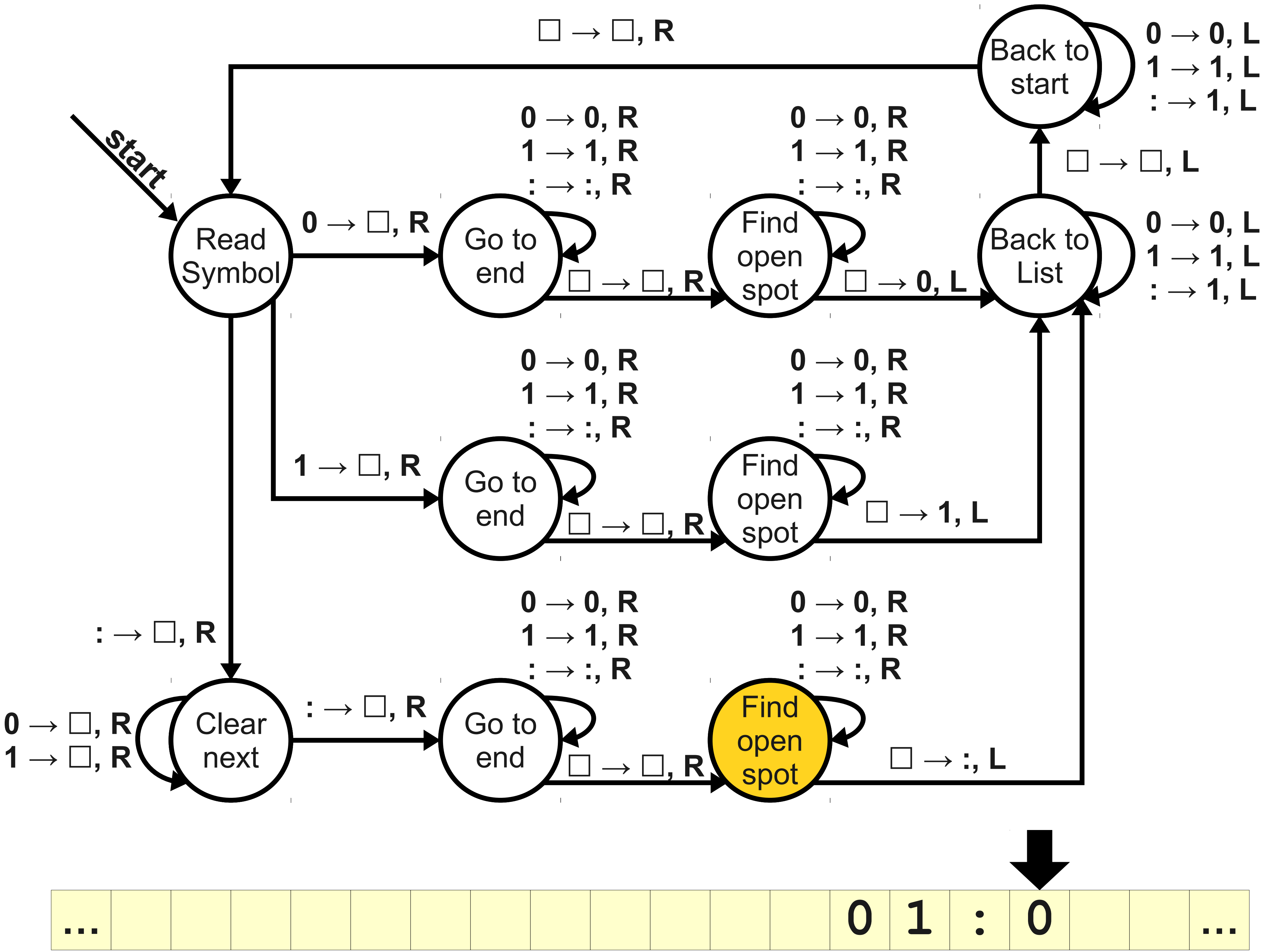
1 → 1, R
: → :, R
□ → :, L
… 0 1 : 0 …



start
Read Symbol
Go to end
Find open spot
Back to List
Back to start
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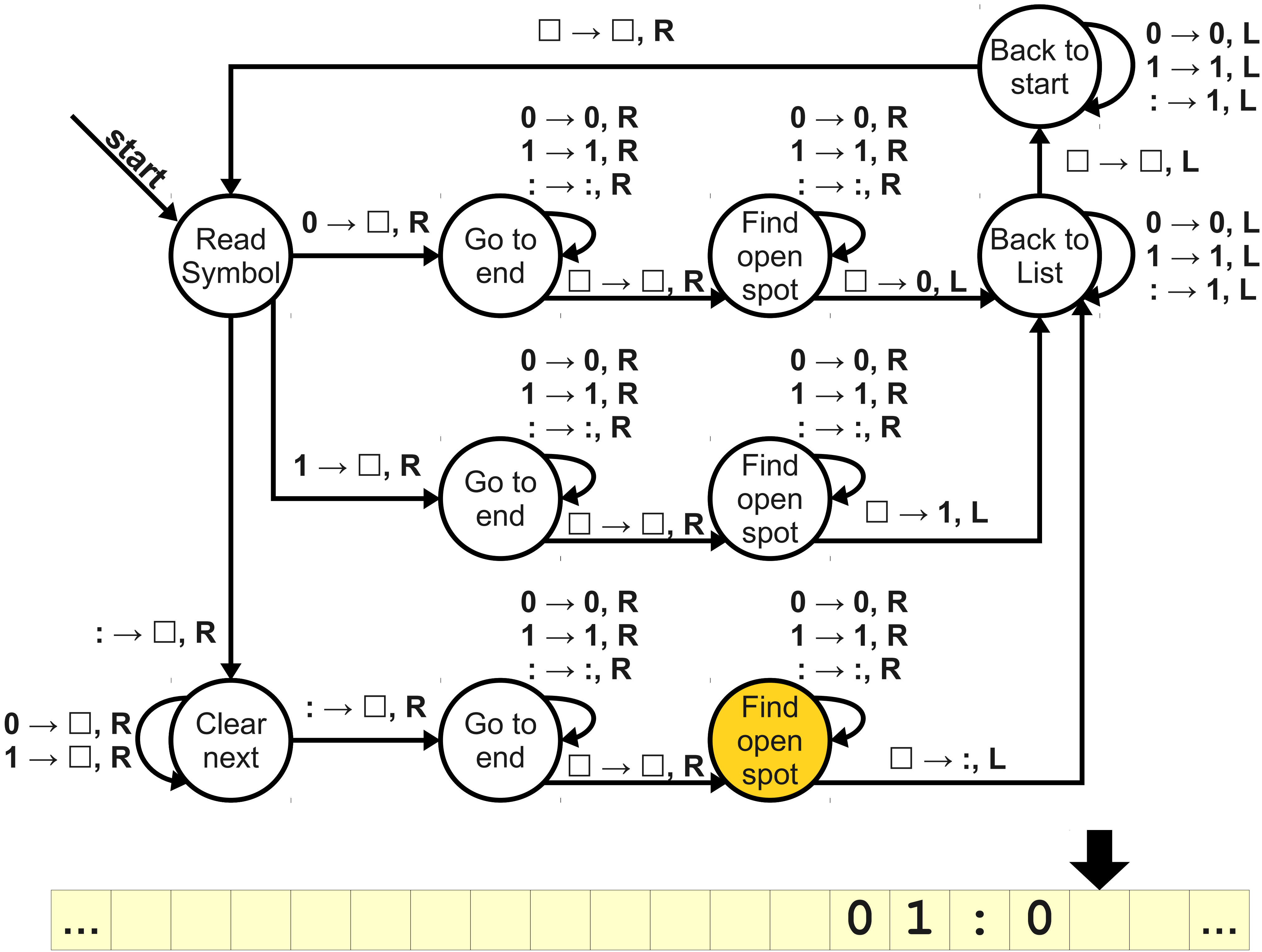
: → 1, L
□ → □, L
... 0 1 : 0 ...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□ → □, L
Read Symbol
0 → □, R
Go to end
□ → □, R
Find open spot
□ → 0, L
Back to List
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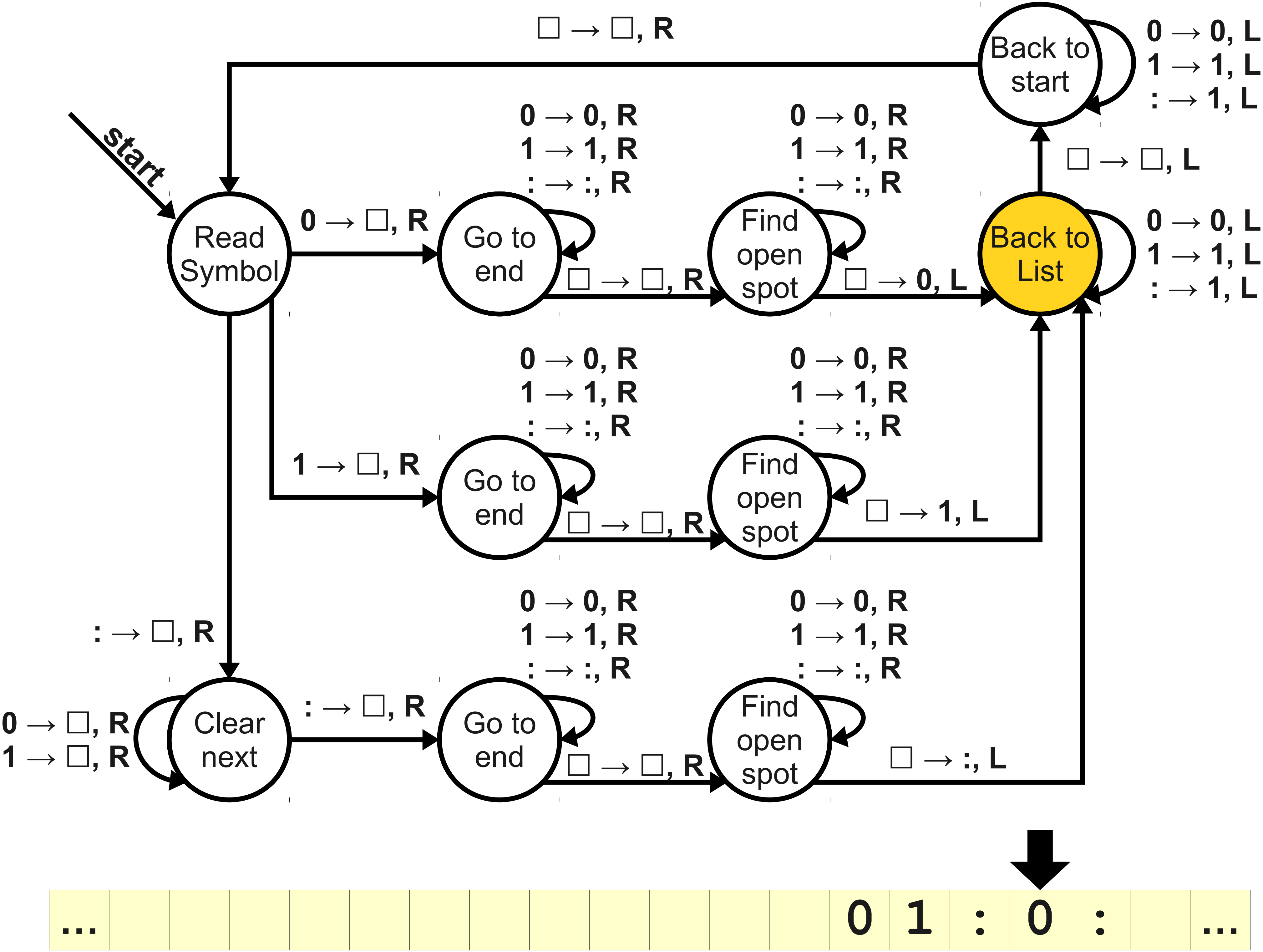
: → □, R
Clear next
0 → □, R
1 → □, R
□ → :, L
... 0 1 : 0 ...


start
Read Symbol
Go to end
Find open spot
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Back to start
Clear next
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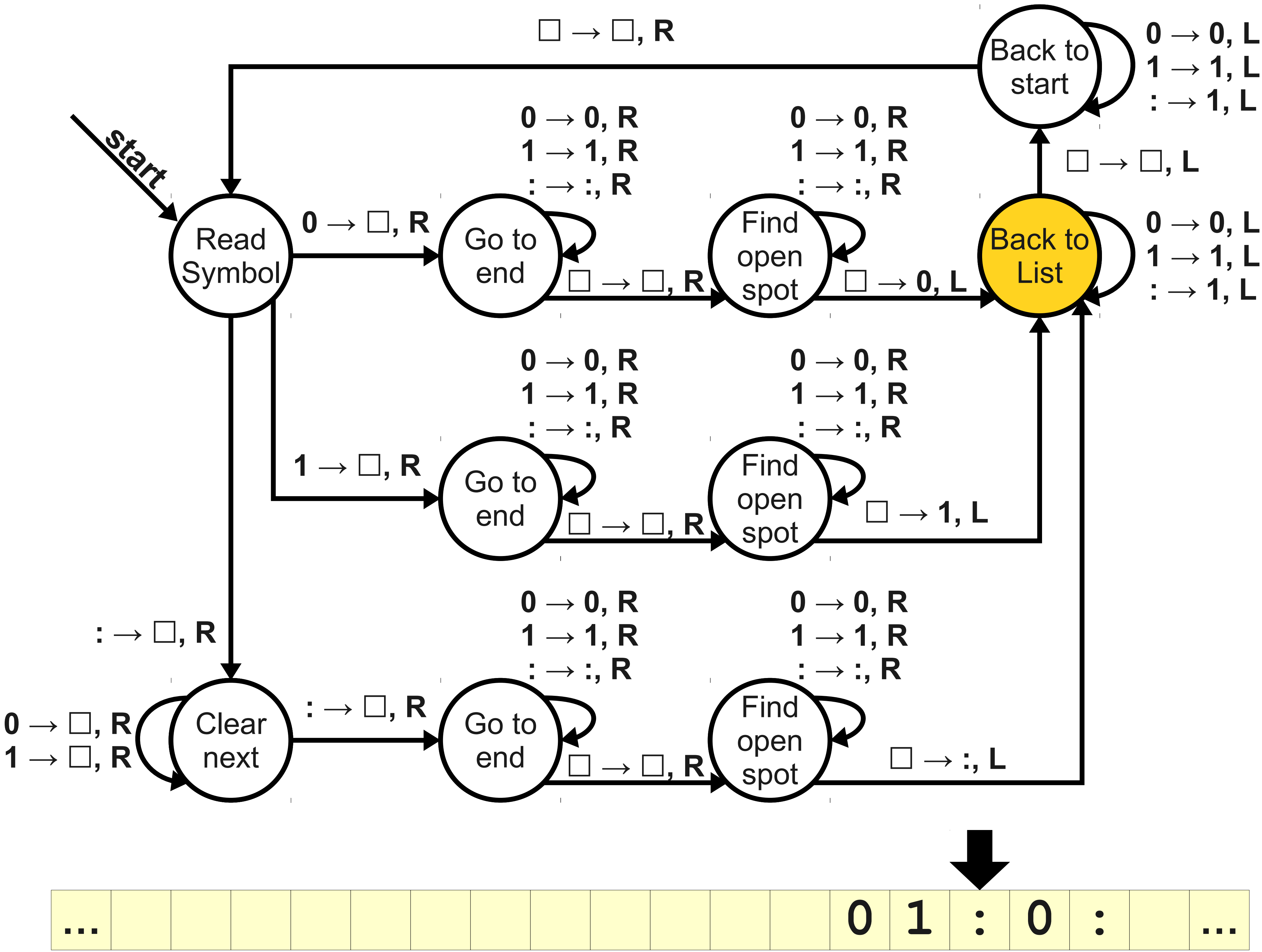
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Back to start
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start
Read Symbol
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Go to end
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□ → □, R
Find open spot
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: → :, R
□ → 0, L
Back to List
□ → □, L
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1 → □, R
Go to end
0 → 0, R
1 → 1, R
: → :, R
□ → □, R
Find open spot
0 → 0, R
1 → 1, R
: → :, R
□ → 1, L
: → □, R
Clear next
0 → □, R
1 → □, R
: → □, R
Go to end
0 → 0, R
1 → 1, R
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Find open spot
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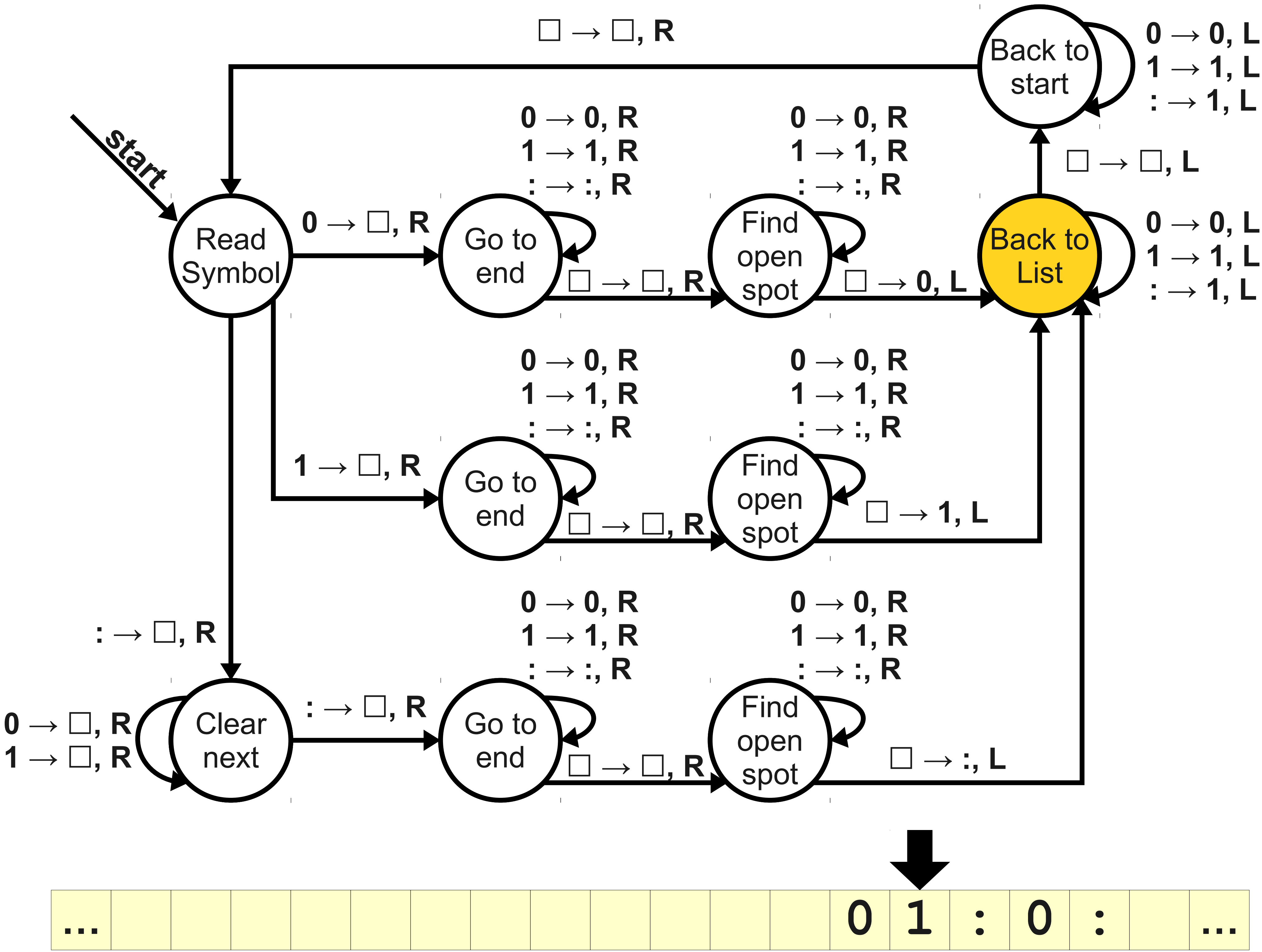
1 → 1, R
: → :, R
□ → :, L
... 0 1 : 0 : ...


□ → □, R
Back to start
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□ → □, R
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Go to end
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Find open spot
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1 → 1, R
: → :, R
0 → □, R
1 → □, R
Clear next
: → □, R
Go to end
□ → □, R
Find open spot
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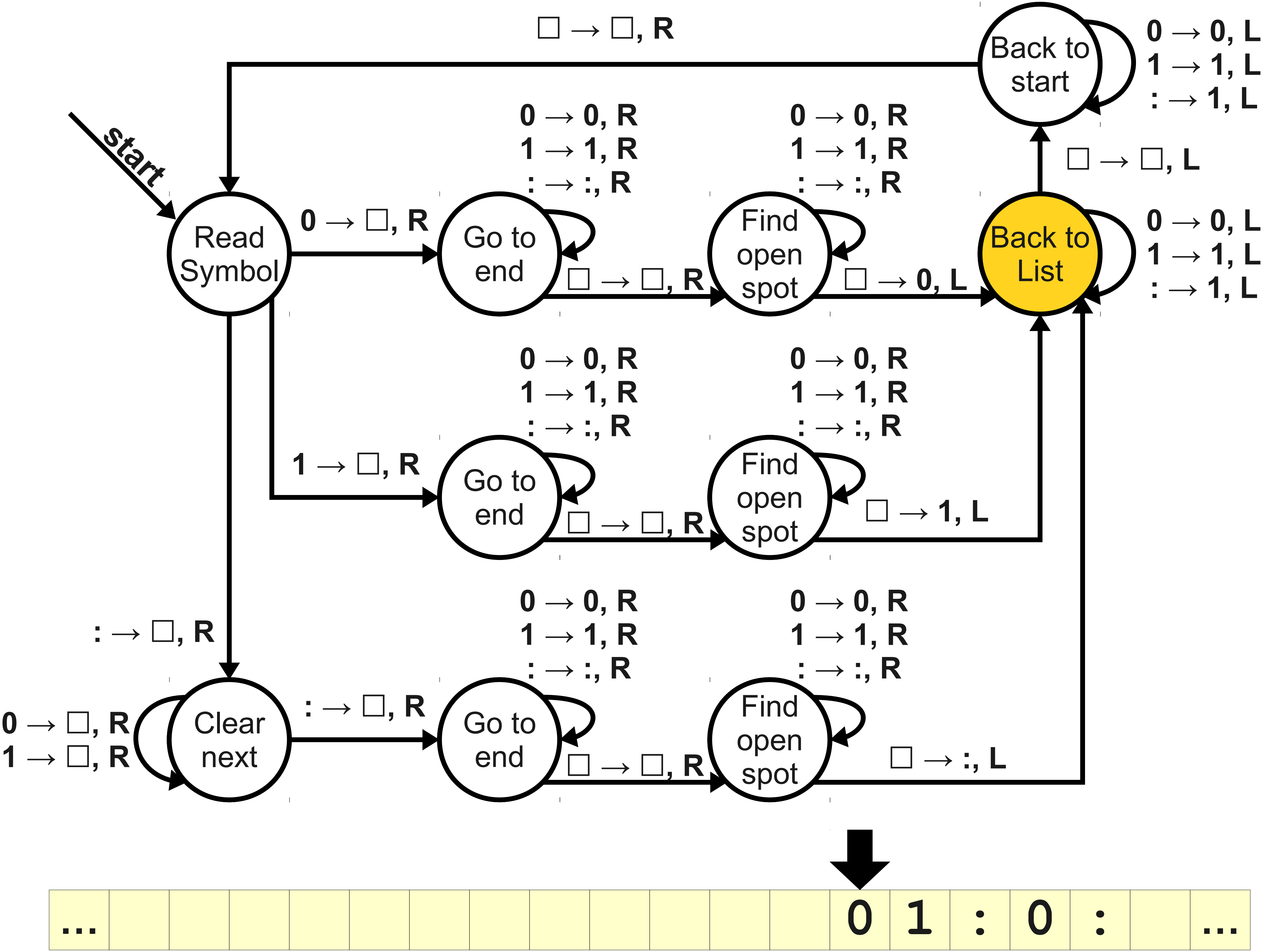

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Back to start
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Read Symbol
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Go to end
□ → □, R
Find open spot
□ → 0, L
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1 → 1, R
: → :, R
0 → 0, R
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1 → □, R
Go to end
□ → □, R
Find open spot
□ → 1, L
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1 → 1, R
: → :, R
0 → 0, R
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Go to end
□ → □, R
Find open spot
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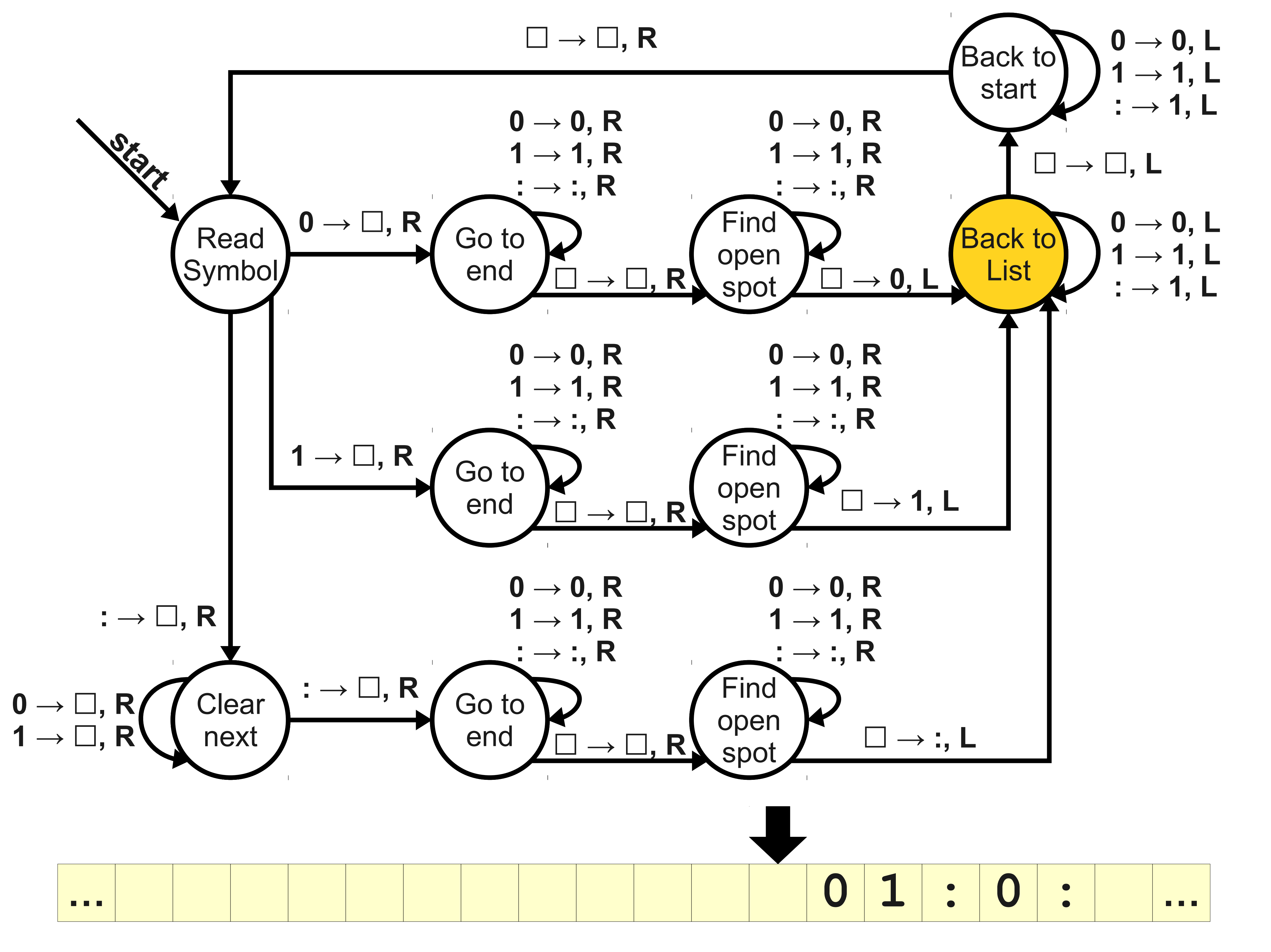

start
Read Symbol
Go to end
Find open spot
Back to List
Back to start
Clear next
□ → □, R
0 → 0, L
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0 → □, R
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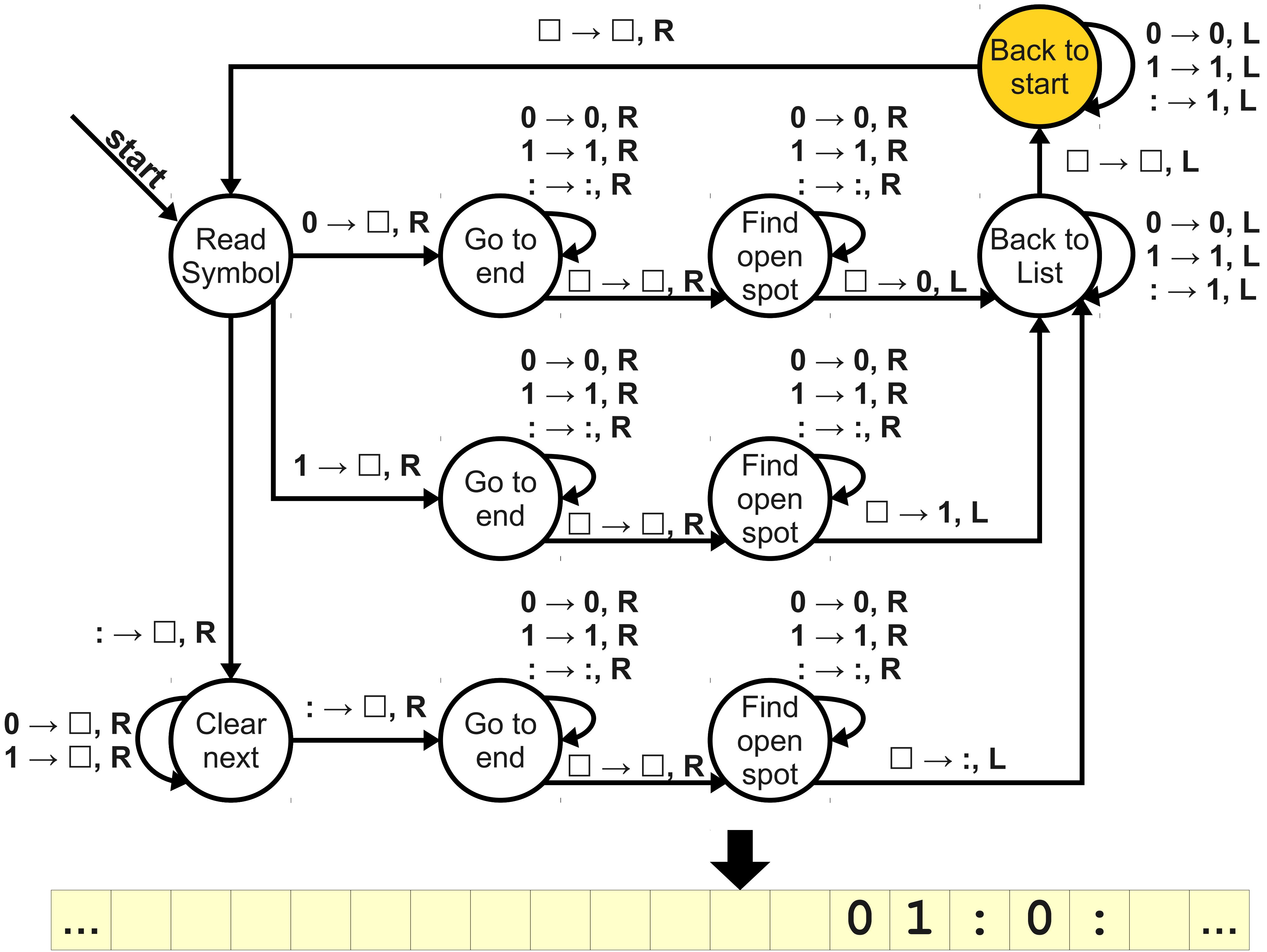

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Find open spot
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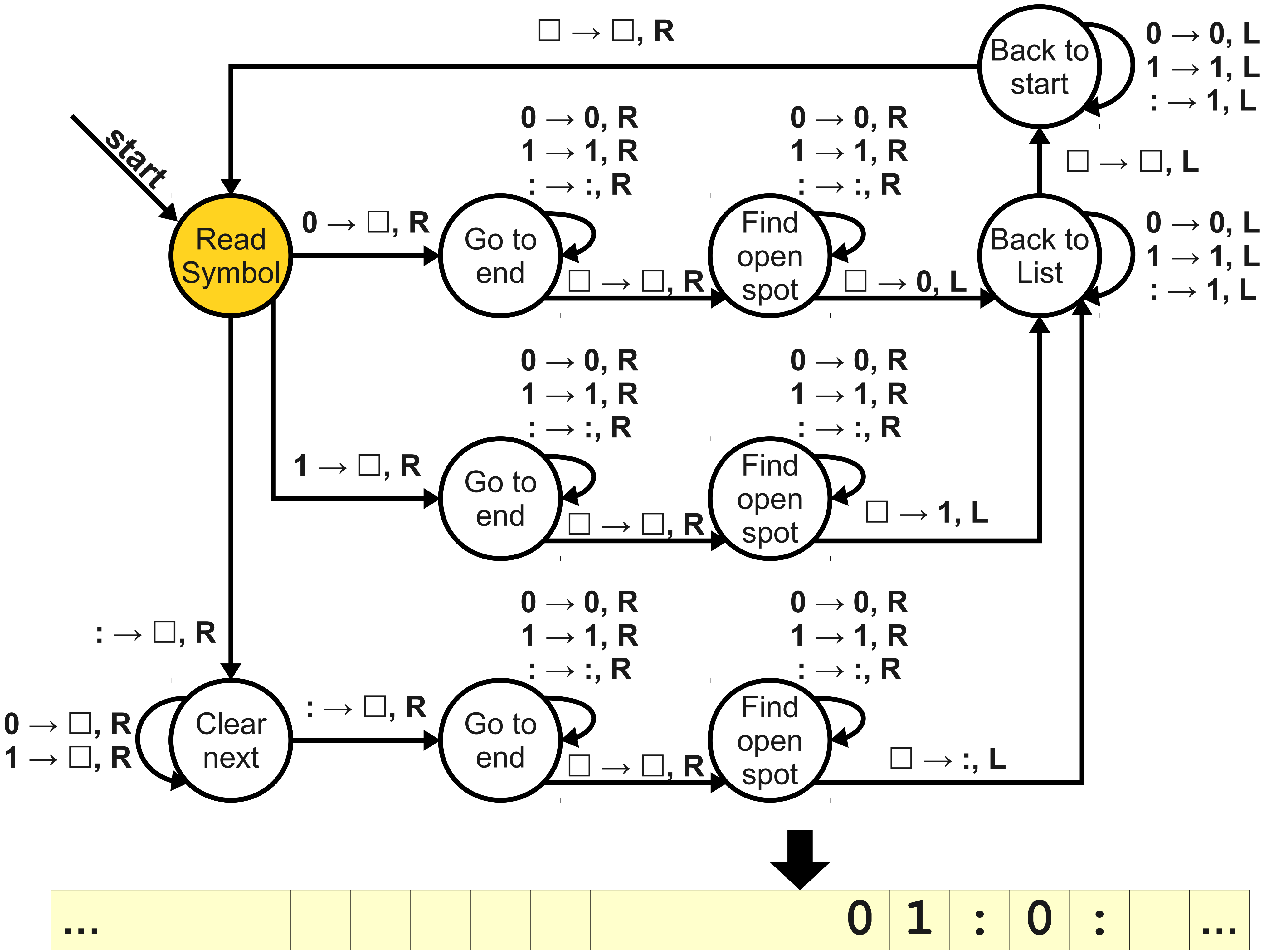
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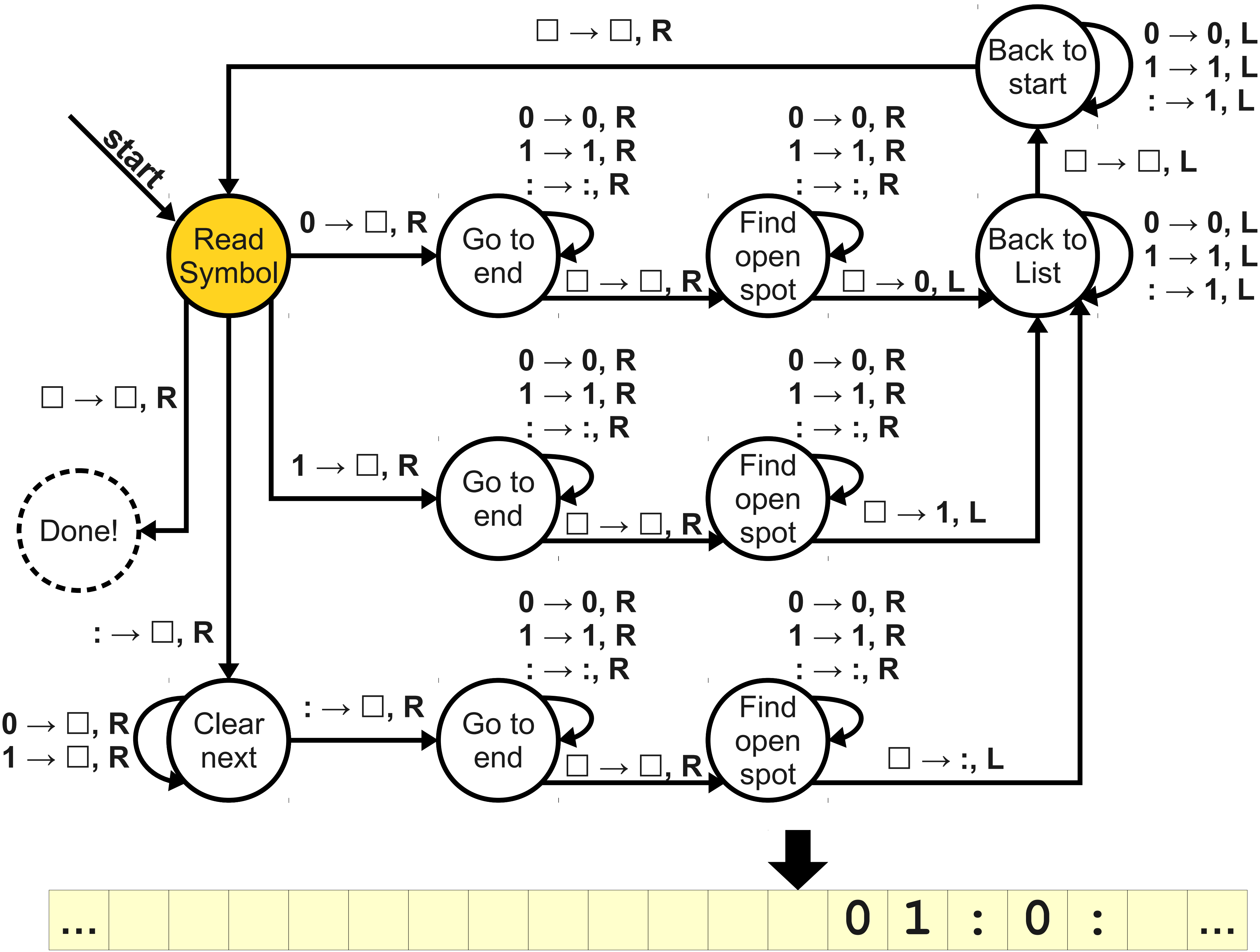
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Read Symbol
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Go to end
□ → □, R
Find open spot
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Back to List
□ → □, R
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Done!
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□ → :, L
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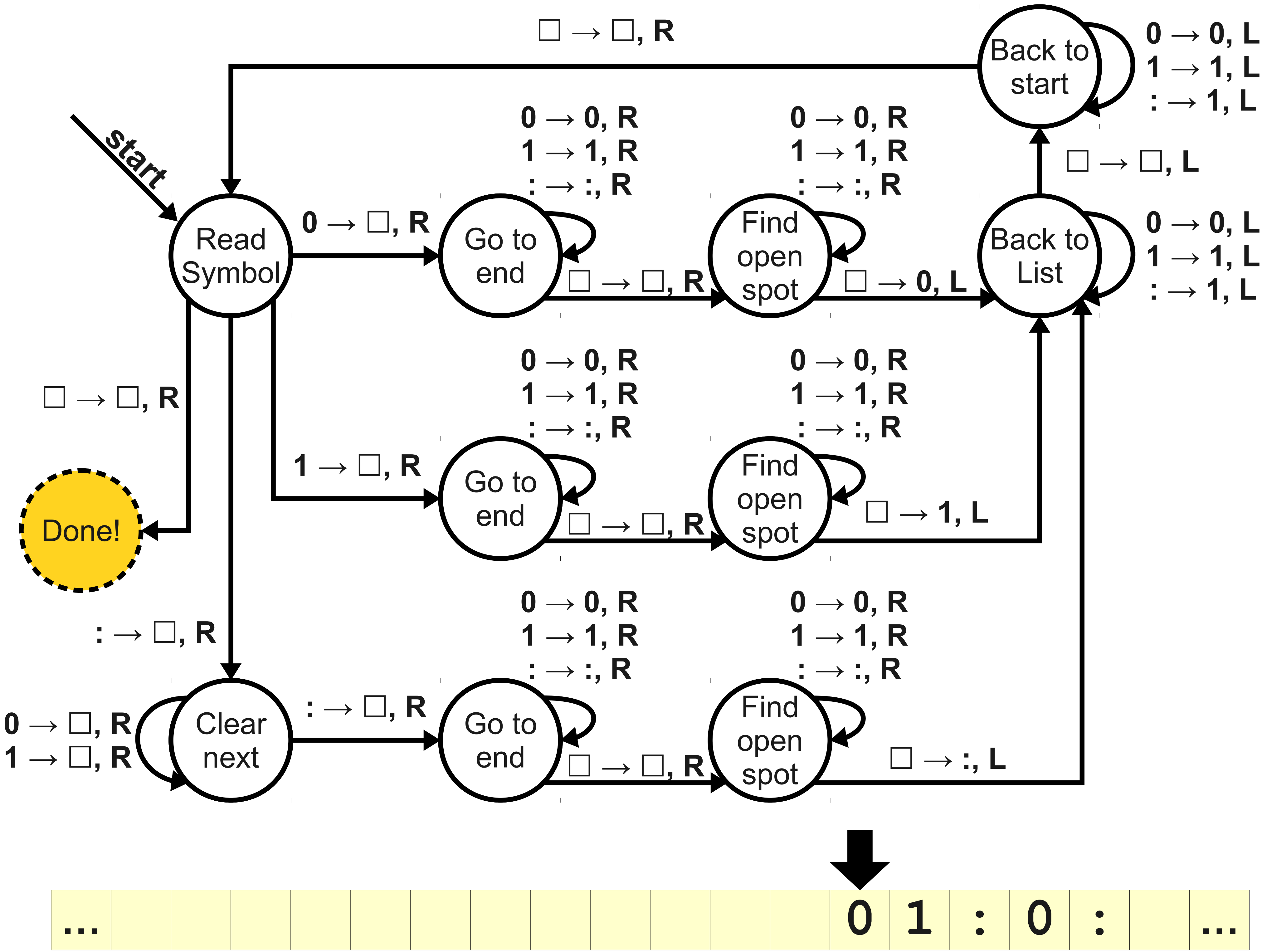
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Go to end
Find open spot
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Back to start
Done!
Clear next
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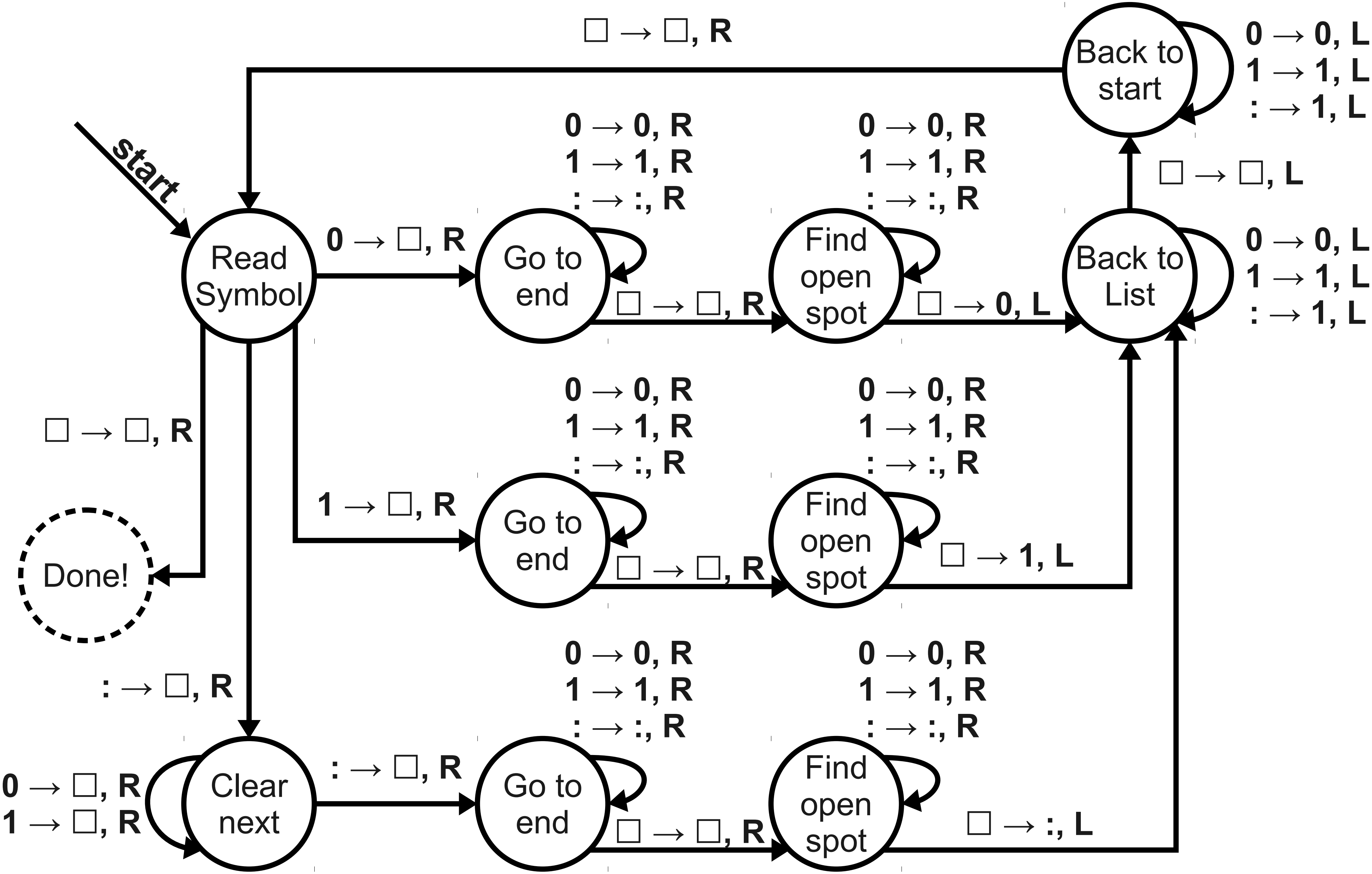

start
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Go to end
Find open spot
Back to List
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Clear next
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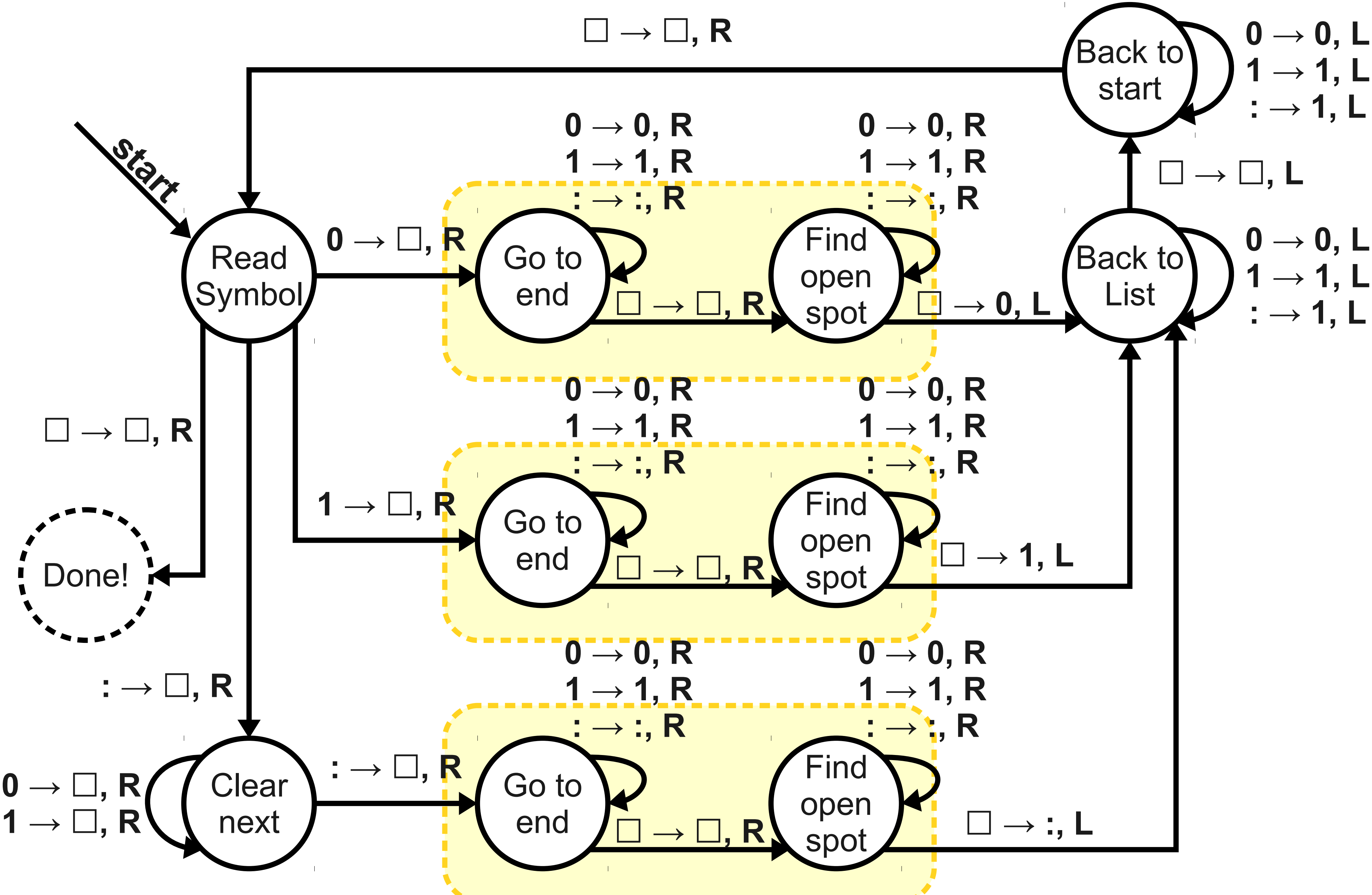

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Read Symbol
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Find open spot
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1 → 1, R
: → :, R
□ → 0, L
Back to List
□ → □, L
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1 → 1, L
: → 1, L
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Done!
1 → □, R
Go to end
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1 → 1, R
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□ → □, R
Find open spot
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1 → 1, R
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□ → 1, L
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Clear next
0 → □, R
1 → □, R
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Go to end
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1 → 1, R
: → :, R
□ → □, R
Find open spot
0 → 0, R
1 → 1, R
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□ → :, L

Turing Machine Memory

- Turing machines often contain many seemingly replicated states in order to store a finite amount of extra information.
- A Turing machine can remember one of k different constants by copying its states k times, once for each possible value, and wiring those states appropriately.
- We will see this used next time.

Turing Machines and Lists

- Turing machines can perform many operations on lists:
  - Concatenate two lists.
  - Reverse a list.
  - Sort a list.
  - Find the maximum element of a list.
  - **And a whole lot more!**

Summary for Today

- Turing machines are powerful computing devices, but can be tricky to program.
- Three useful techniques:
  - Recursion: Try solving problems by recursively simplifying them.
  - Subroutines: Have different parts of the machine do different things.
  - Constant storage: Hold a constant amount of information in the finite-state control.

Next Time

- **The Power of Turing Machines**
  - Recognition vs. Decision.
  - Multitrack Turing machines.
  - Instantaneous Descriptions.
  - Nondeterministic Turing machines.

This is Slide #588.

Thanks for making it this far!