Finite Automata

Part Three

Friday Four Square!

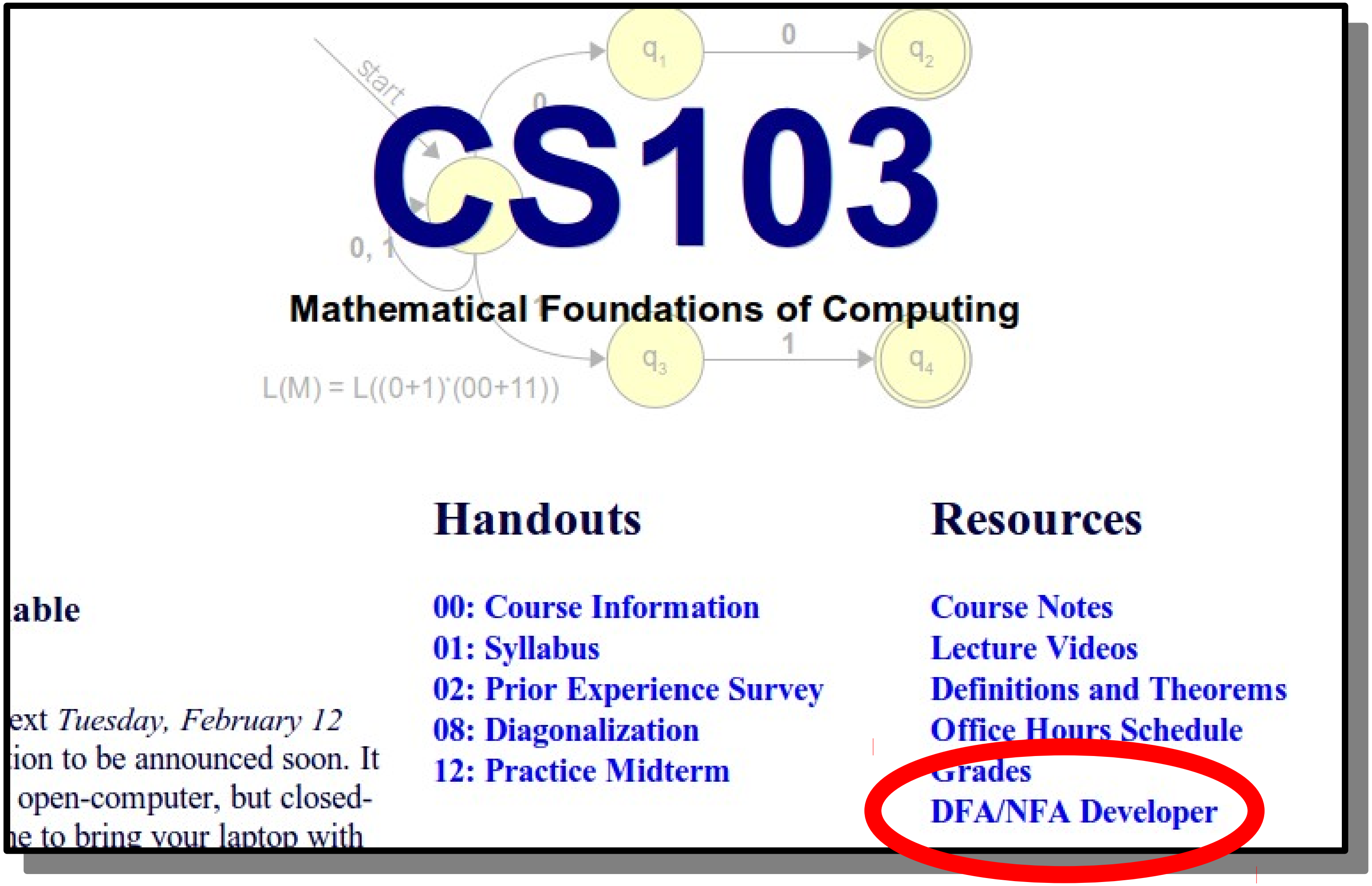
Today at 4:15PM, Outside Gates.

Announcements

- Problem Set 4 due at 2:15PM today.
  - We'll be around after lecture.
- Problem Set 5 out, due next Friday, February 15.
  - Play around with finite automata and regular languages.
  - **No checkpoint problems**.

Midterm

- Midterm is next **Tuesday, February 12** in **Hewlett 200** / **Hewlett 201**.
  - Can show up to either room.
- Covers material up through and including DFAs.
- Review session this **Sunday, February 10** in **Gates 104** from **5PM – 7PM**.
  - Show up with questions, leave with answers!


CS103
Mathematical Foundations of Computing
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q2
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q4
L(M) = L((0+1)*(00+11))
Handouts
00: Course Information
01: Syllabus
02: Prior Experience Survey
08: Diagonalization
12: Practice Midterm
Resources
Course Notes
Lecture Videos
Definitions and Theorems
Office Hours Schedule
Grades
DFA/NFA Developer
able
ext Tuesday, February 12
ion to be announced soon. It
open-computer, but closed-
e to bring your laptop with

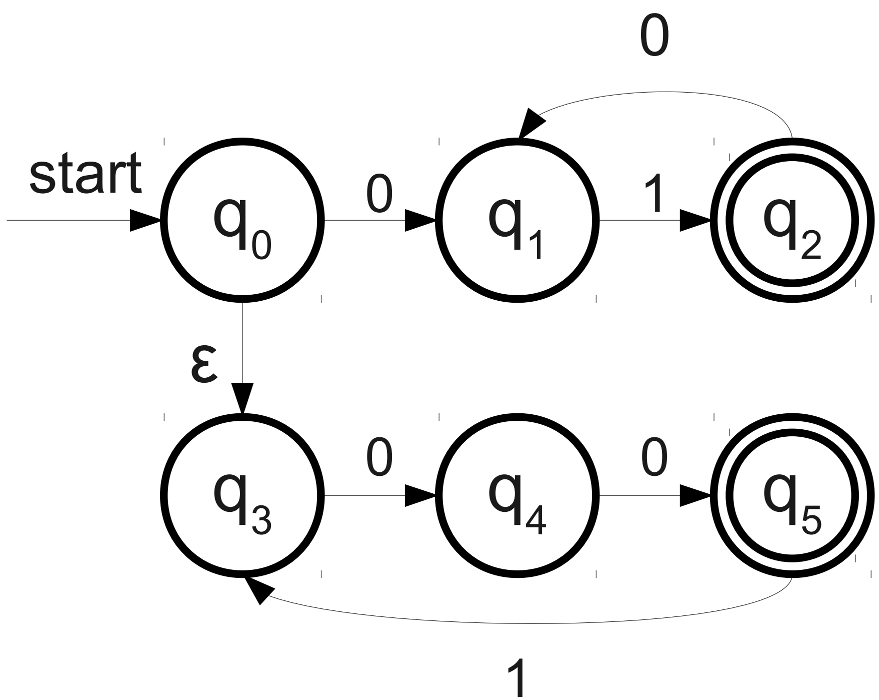
NFAs

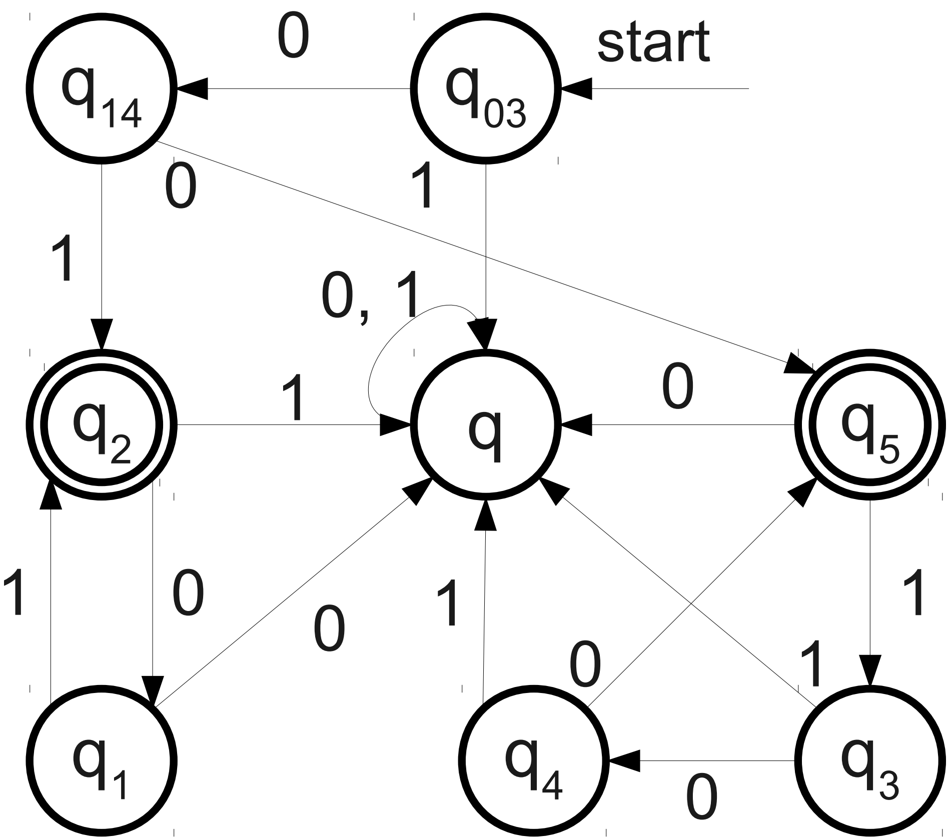
- An **NFA** is a
    - **N**ondeterministic
    - **F**inite
    - **A**utomaton
- Conceptually similar to a DFA, but equipped with the vast power of **nondeterminism**.
- There can be many or no transitions defined on certain inputs.
- An NFA accepts a string if *any* series of choices causes the string to enter an accepting state.

Three Intuitions for Nondeterminism

Tree Computation
Massive Parallelism
Perfect Guessing

Simulating an NFA with a DFA





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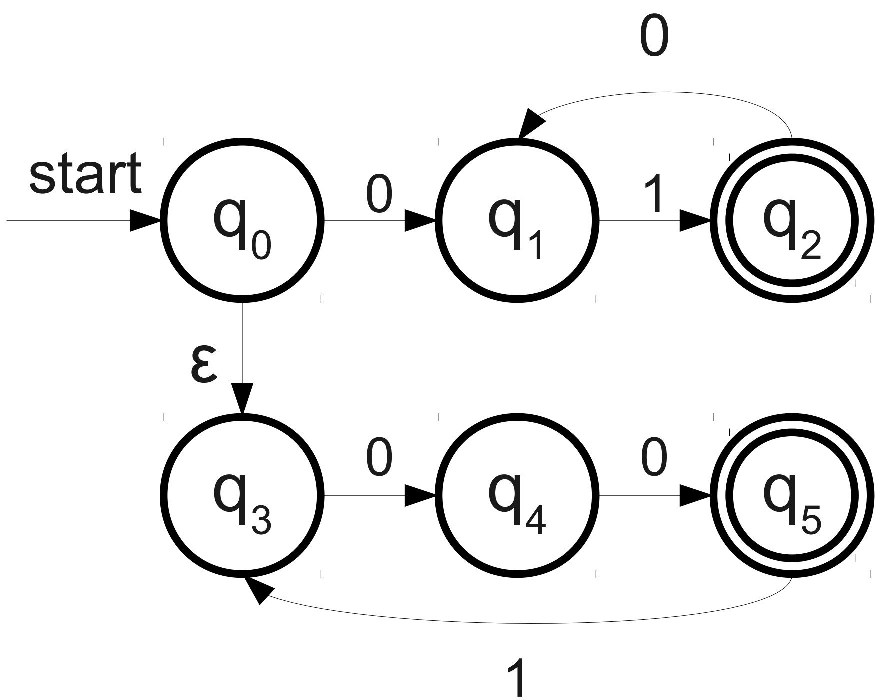

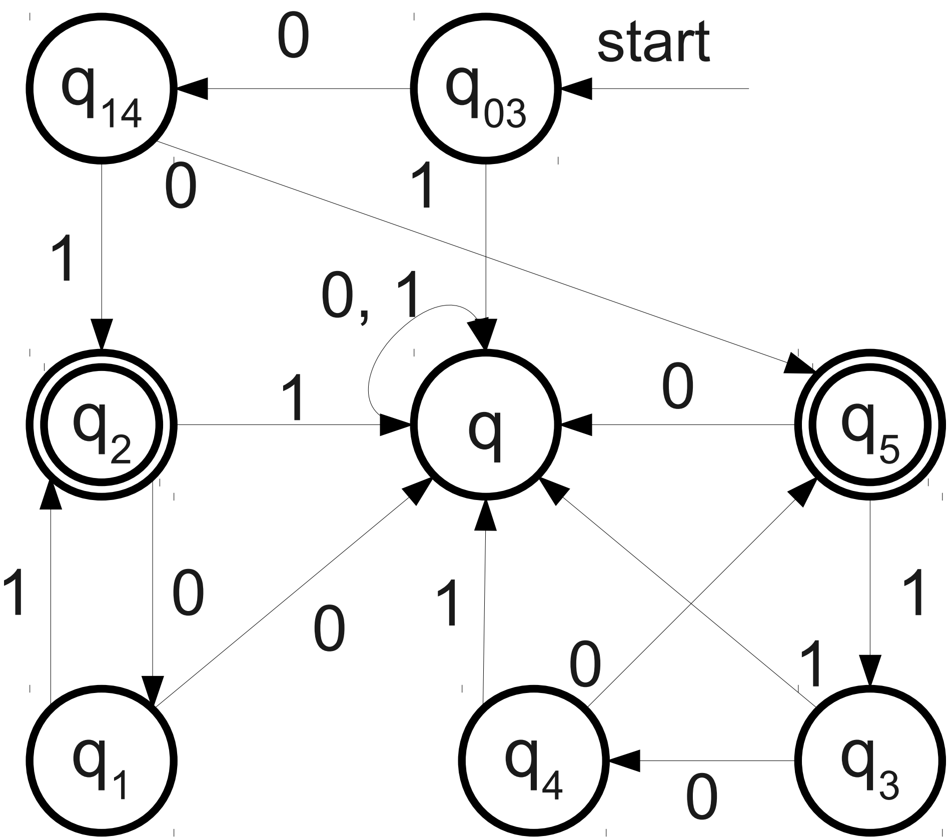

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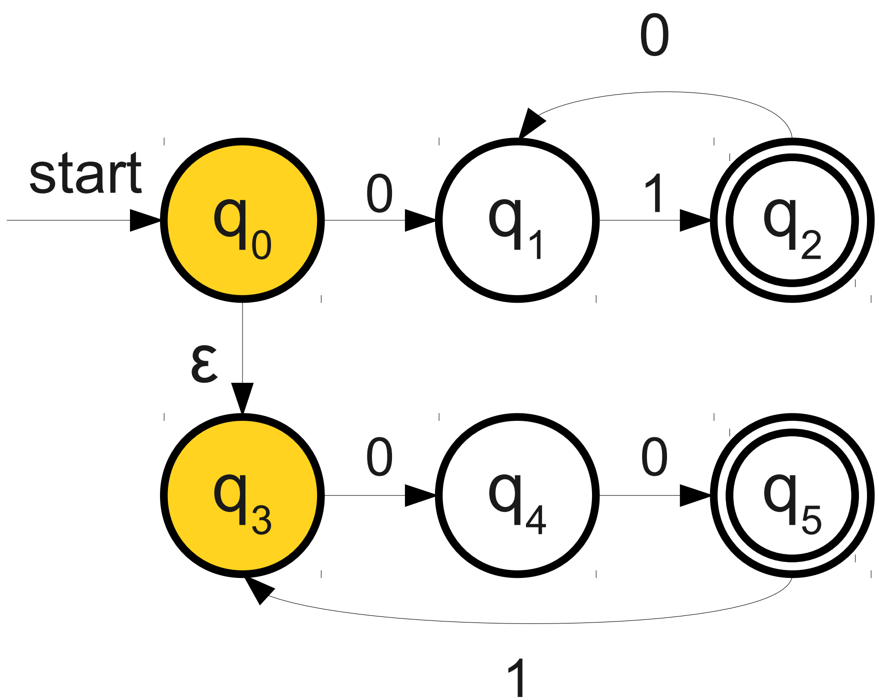

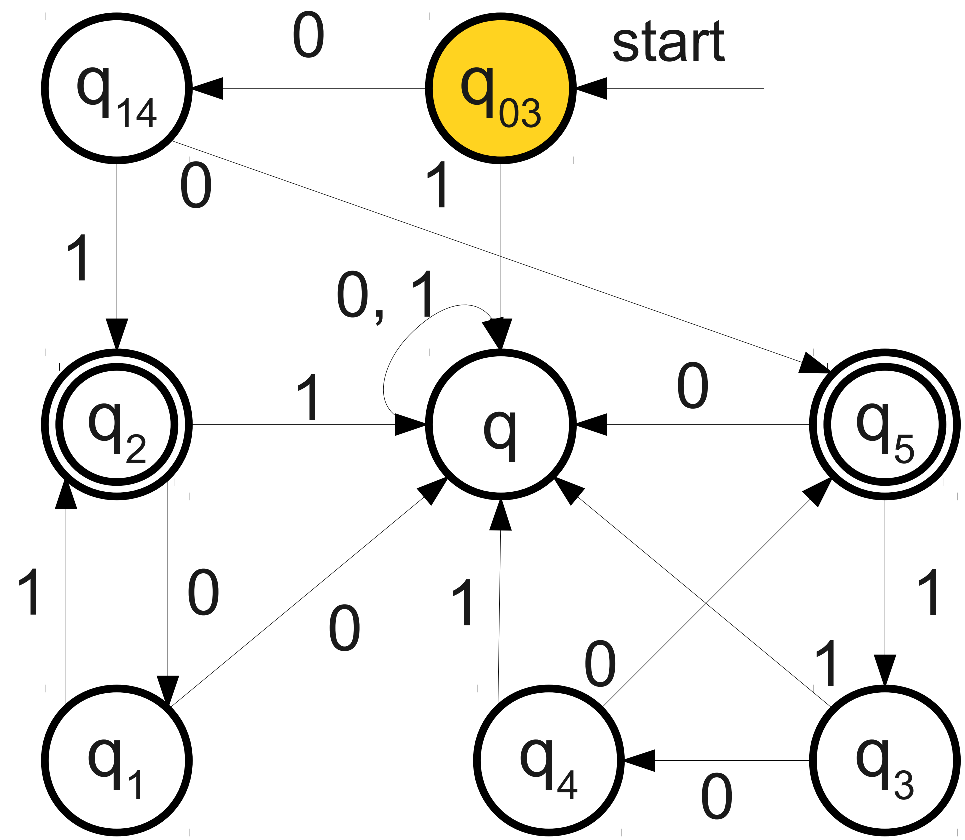

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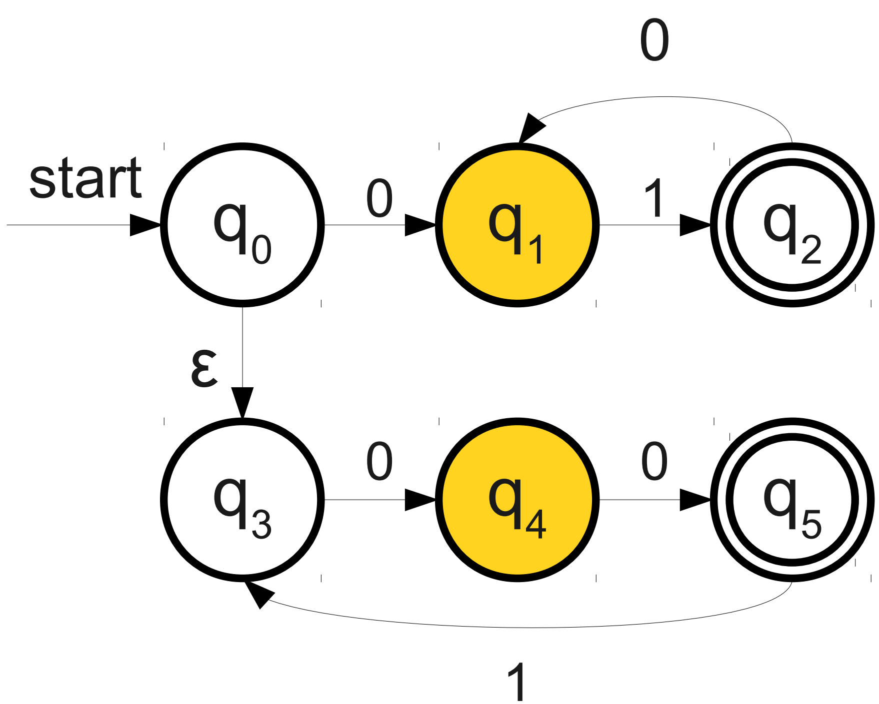

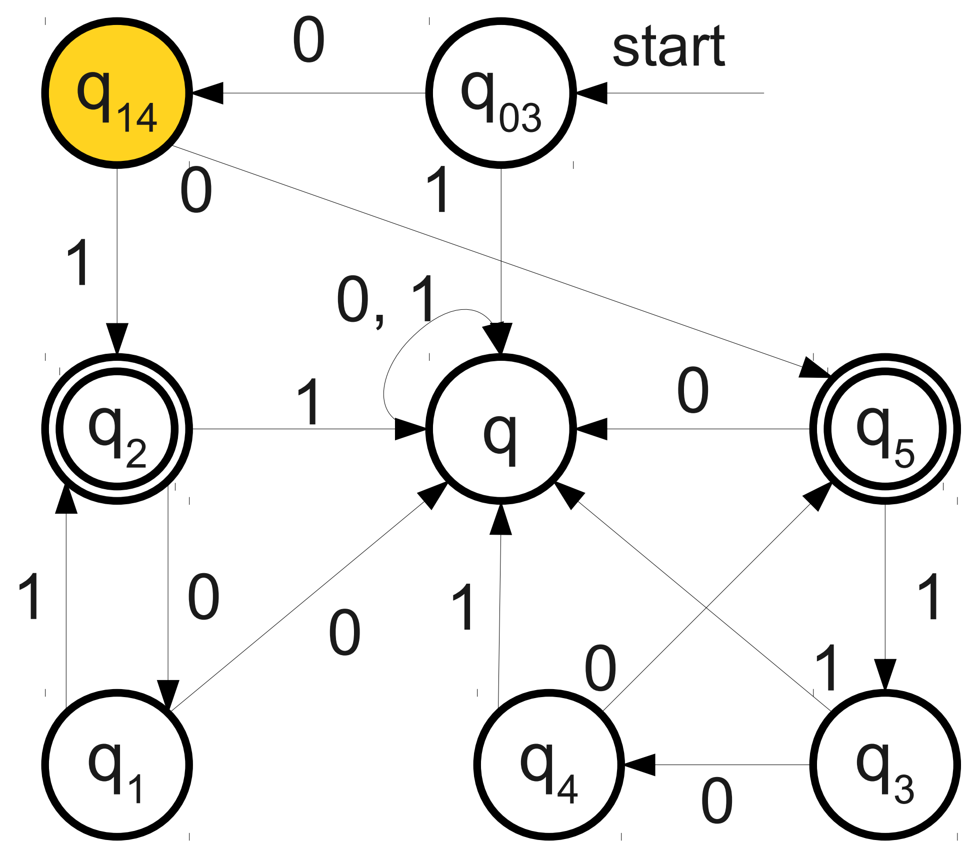

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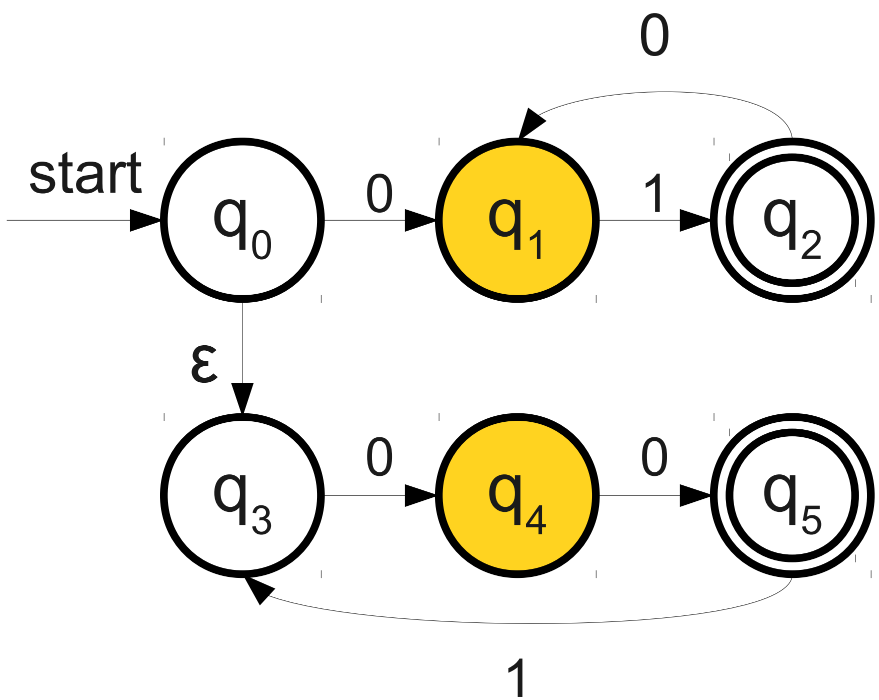

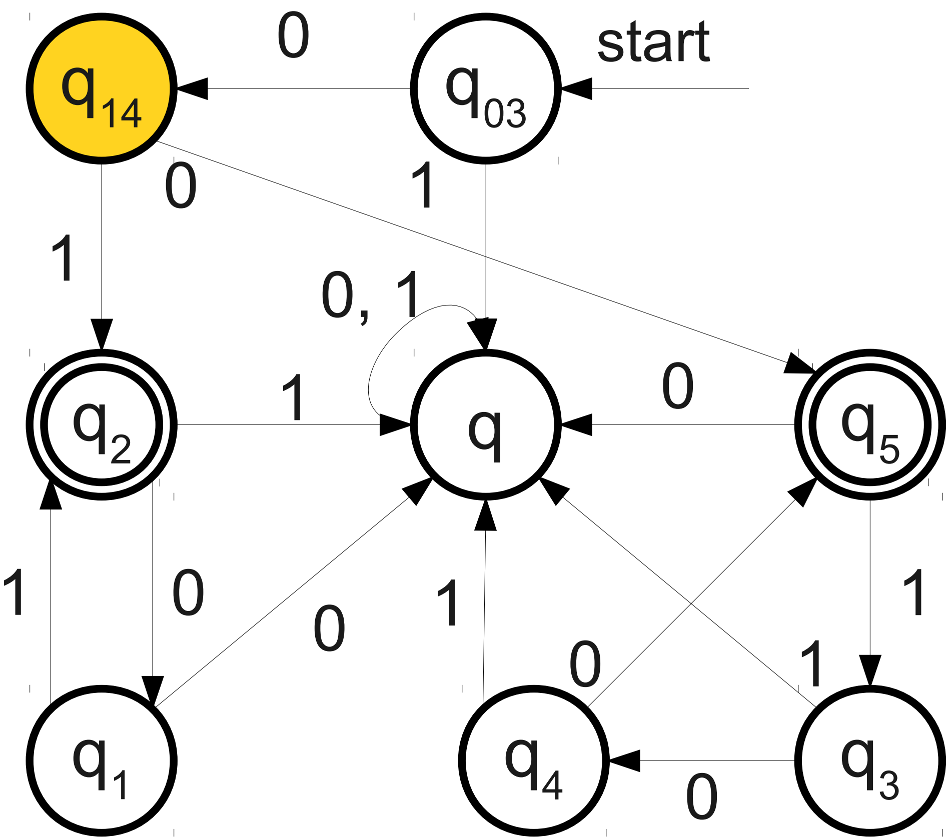

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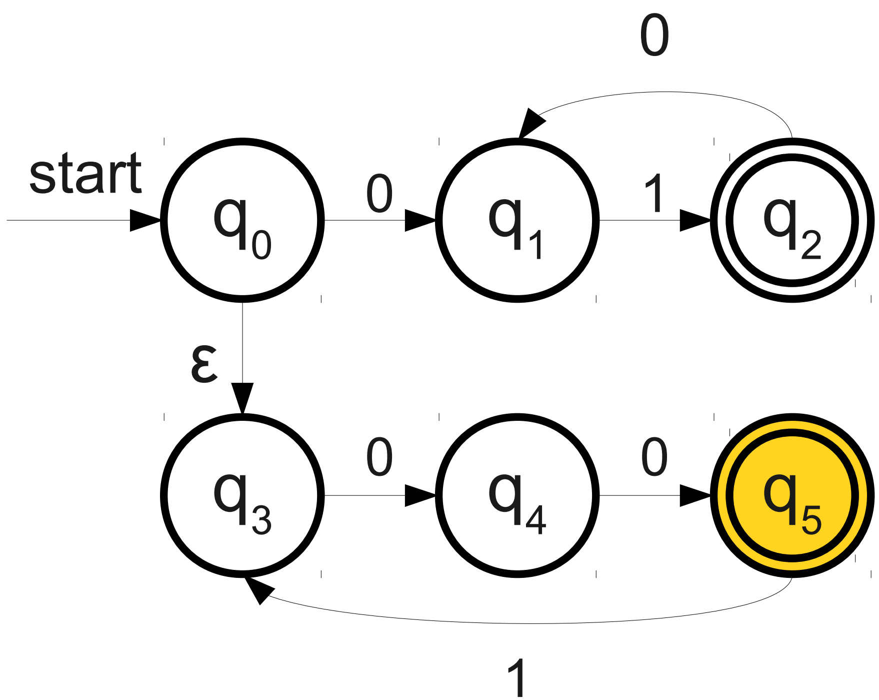

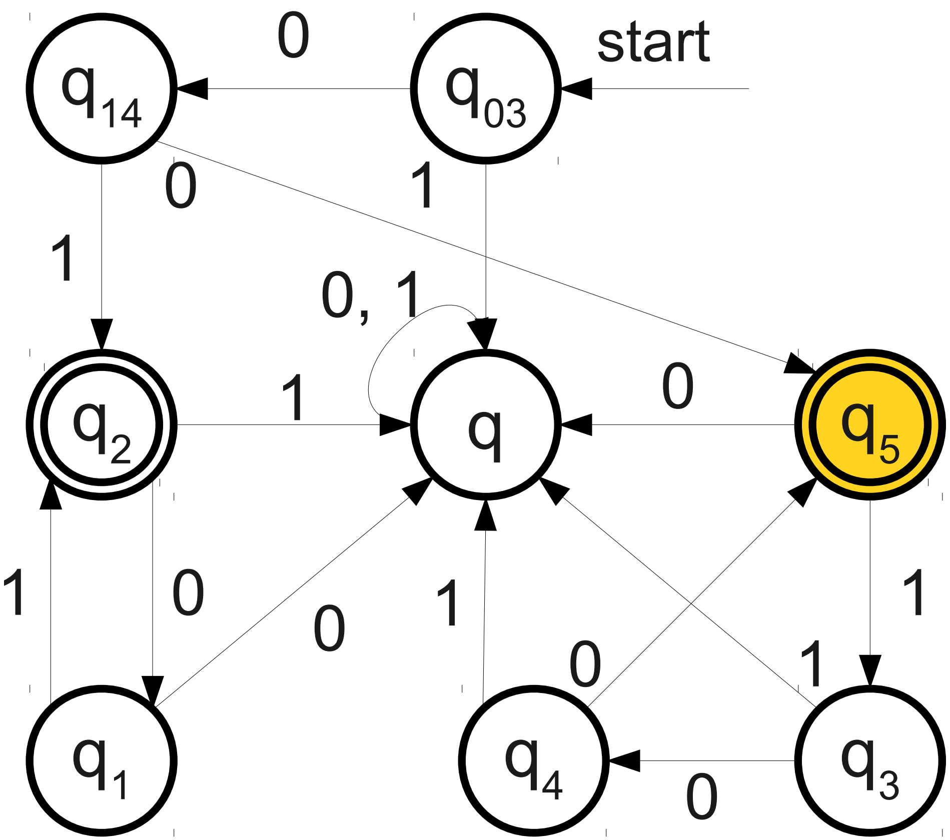

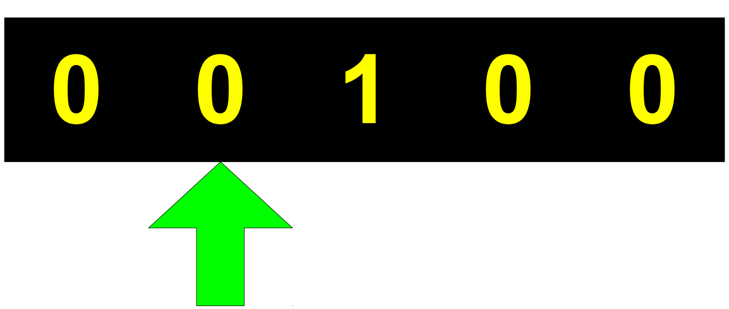

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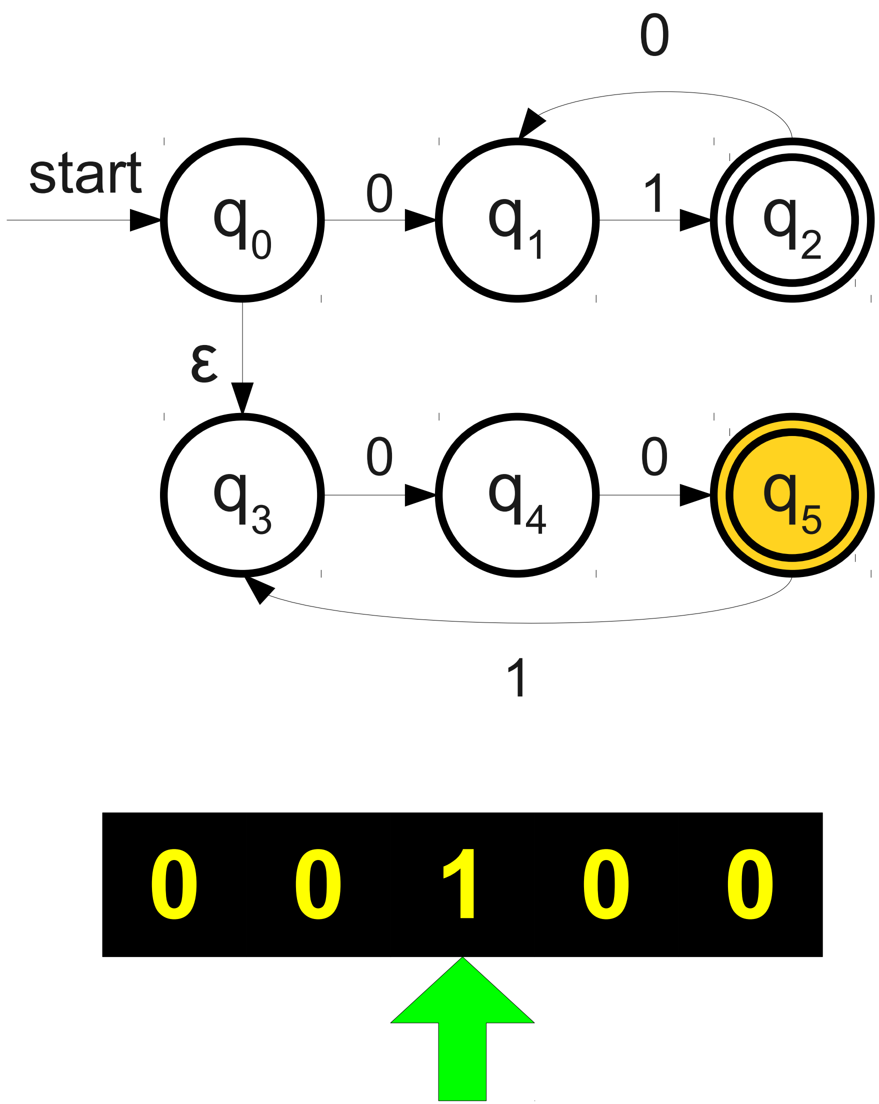

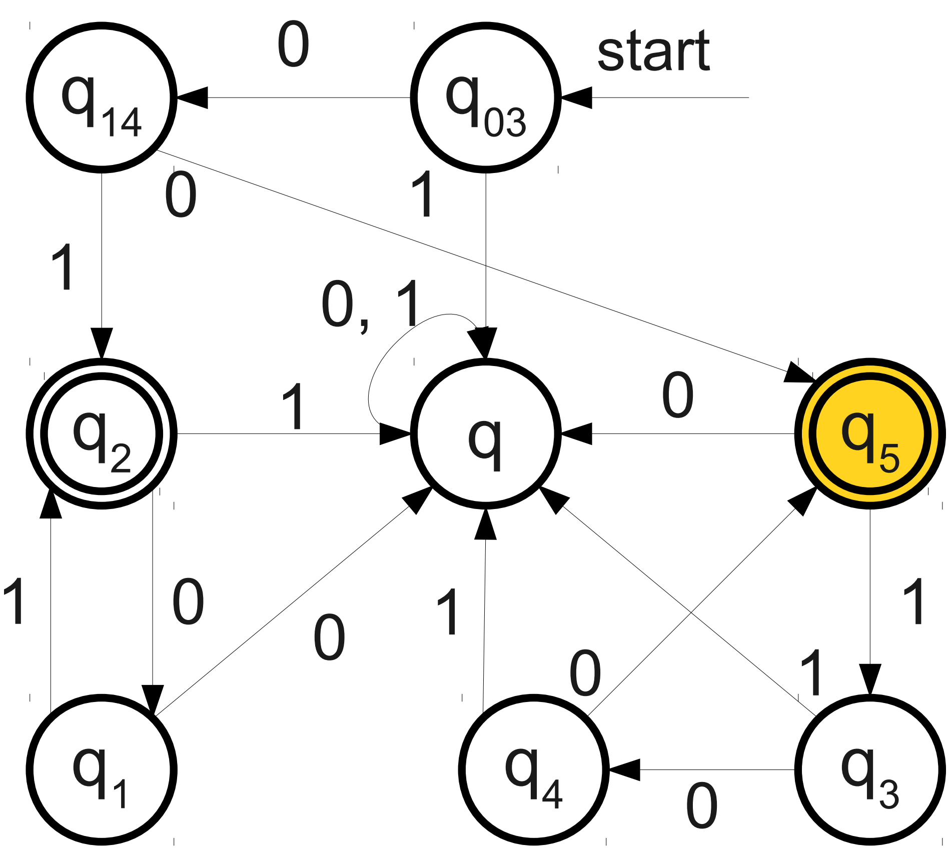

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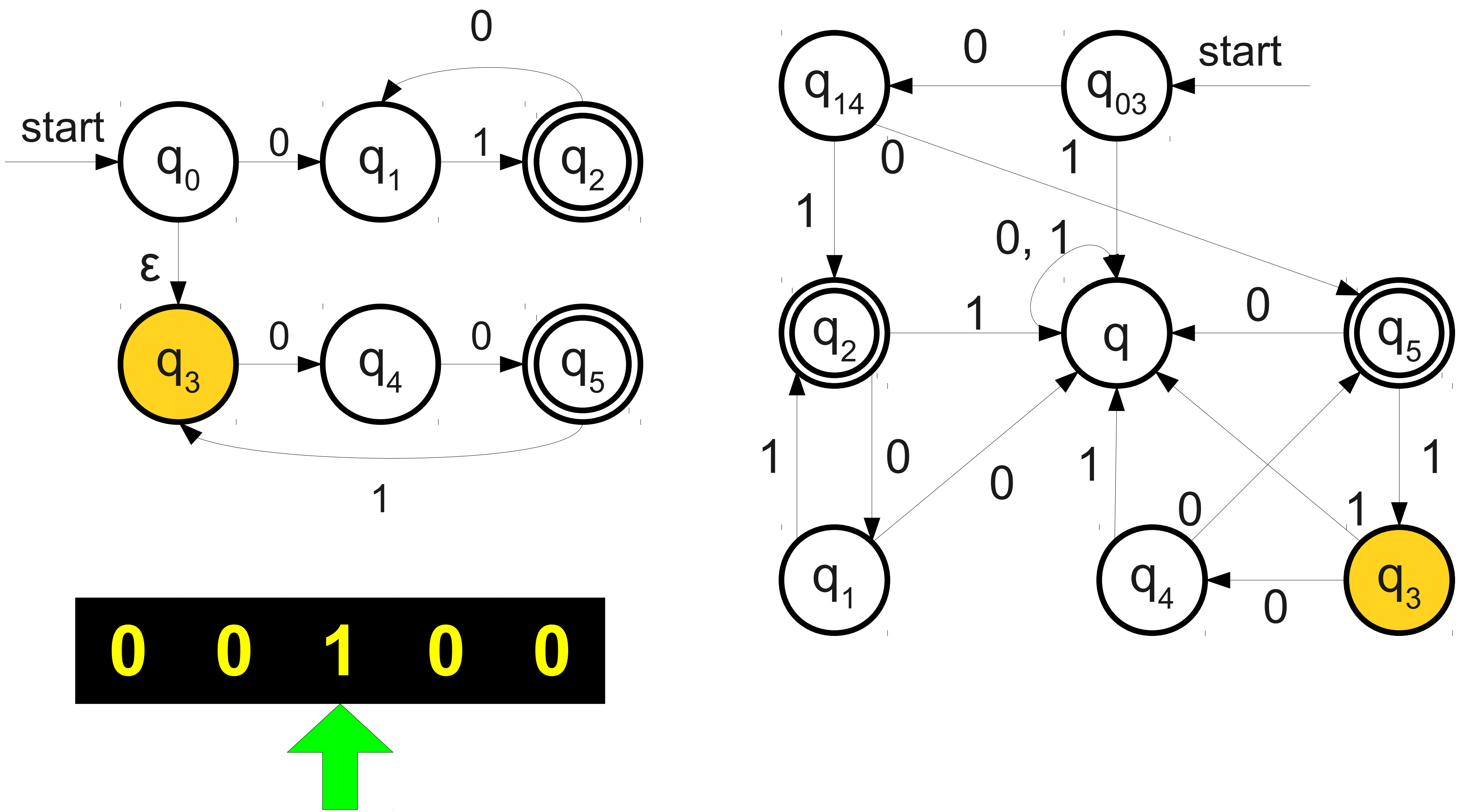

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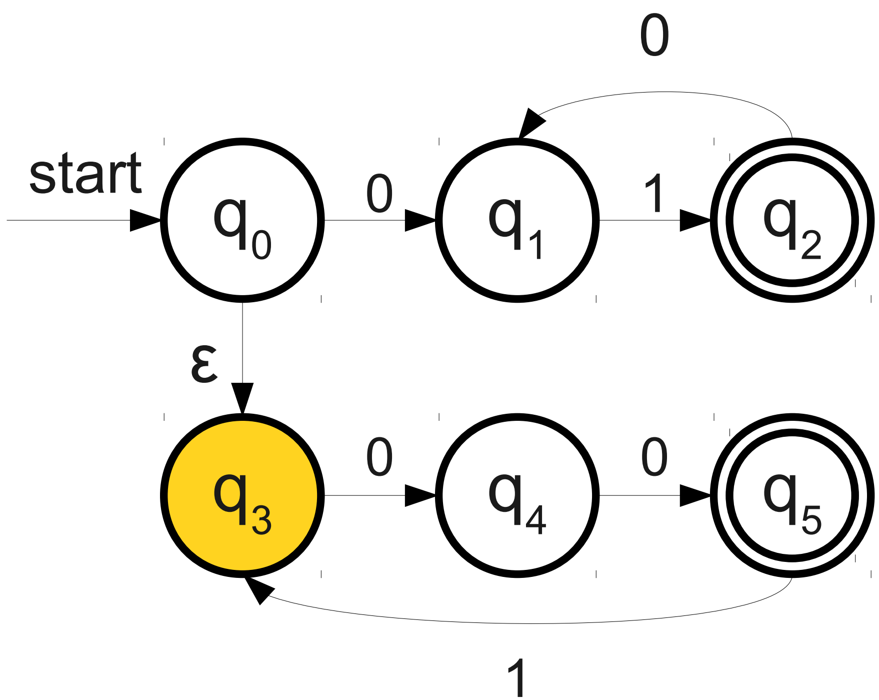

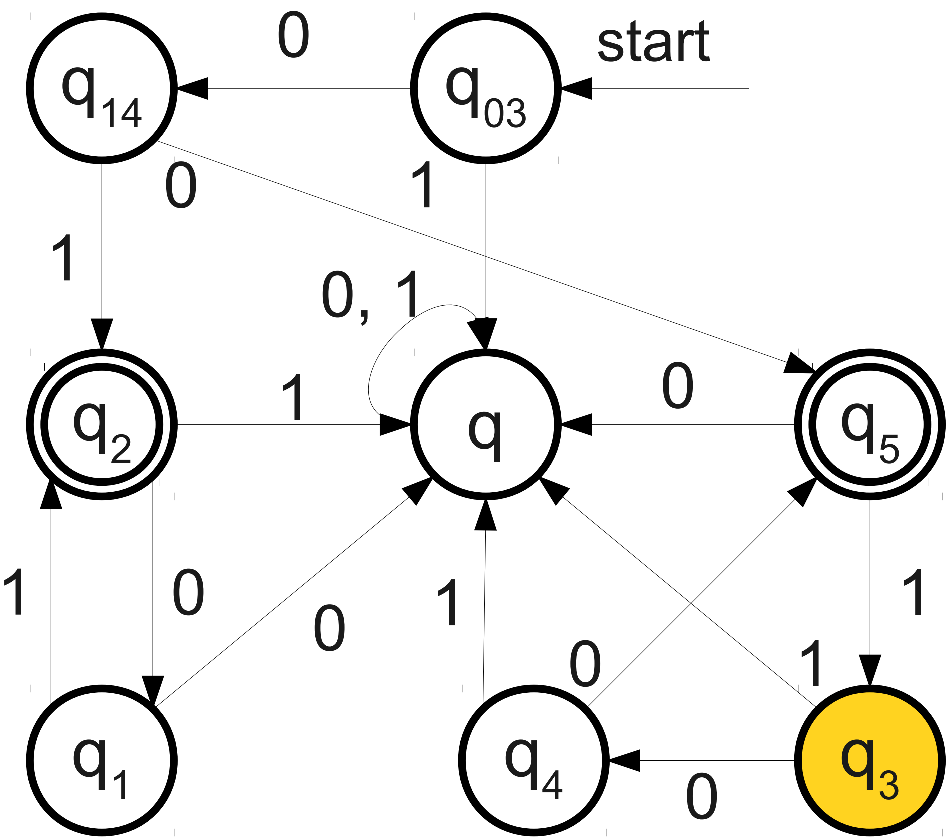

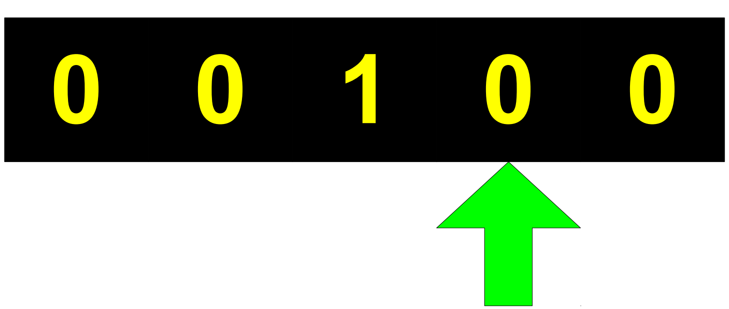

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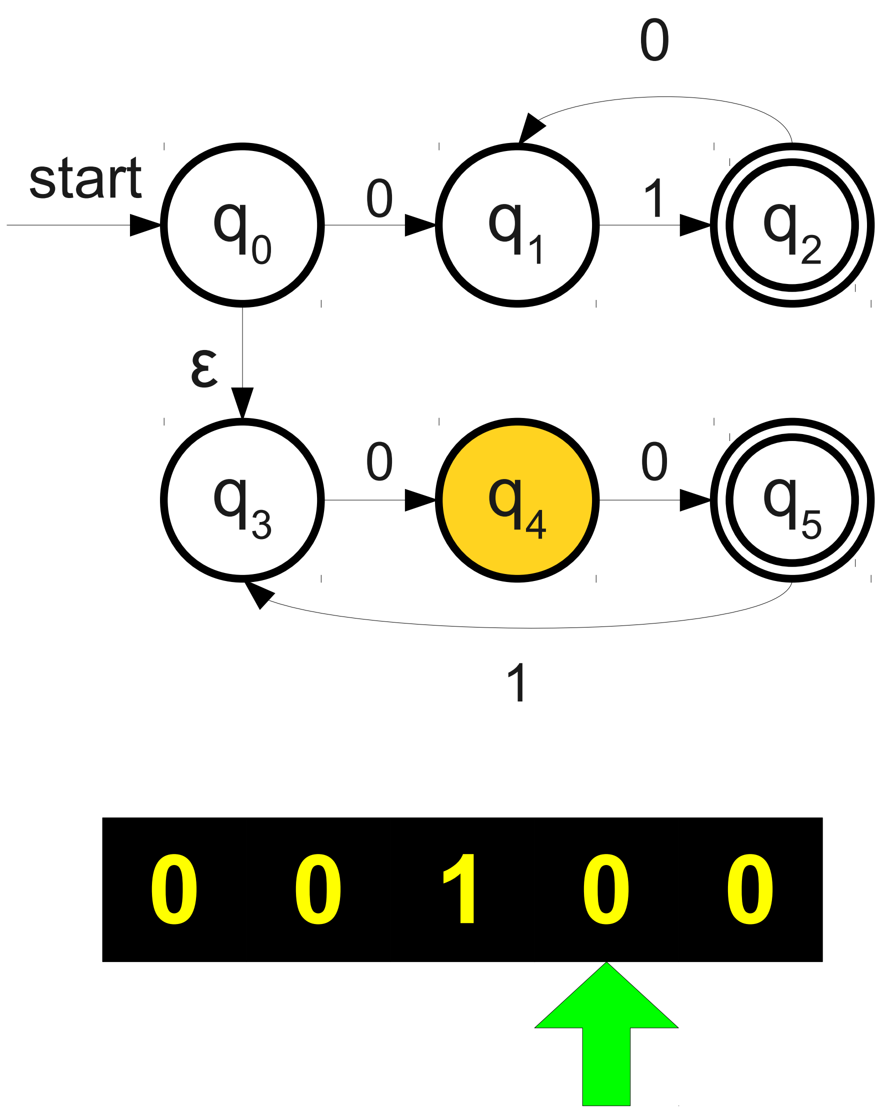

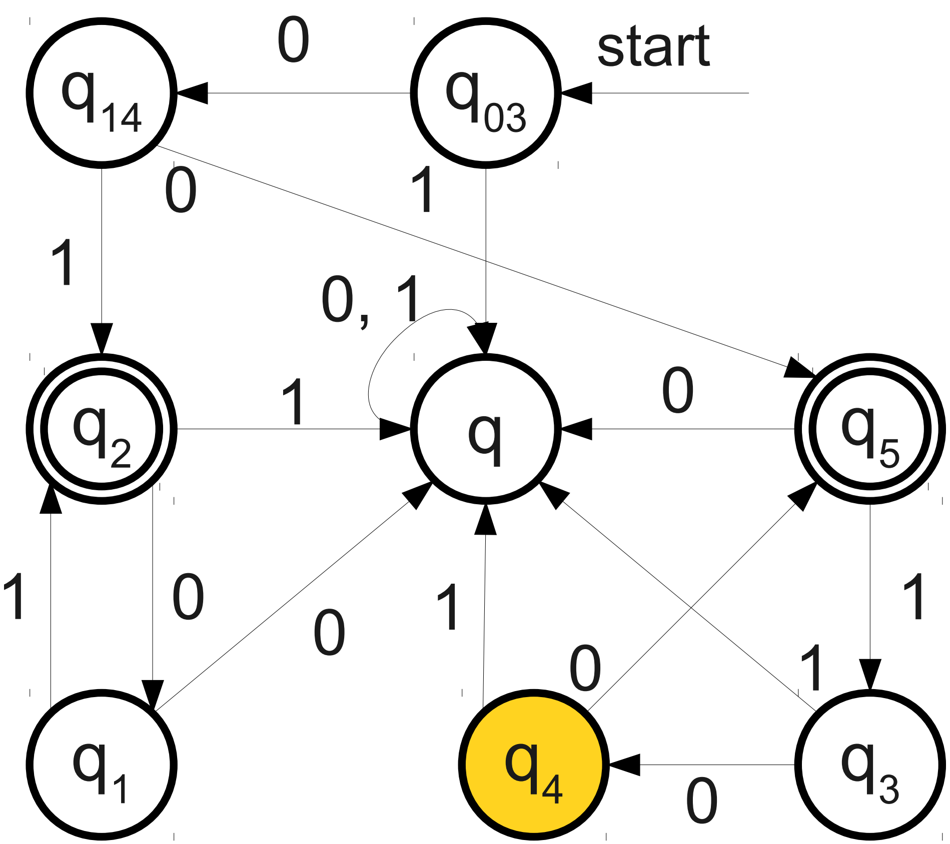

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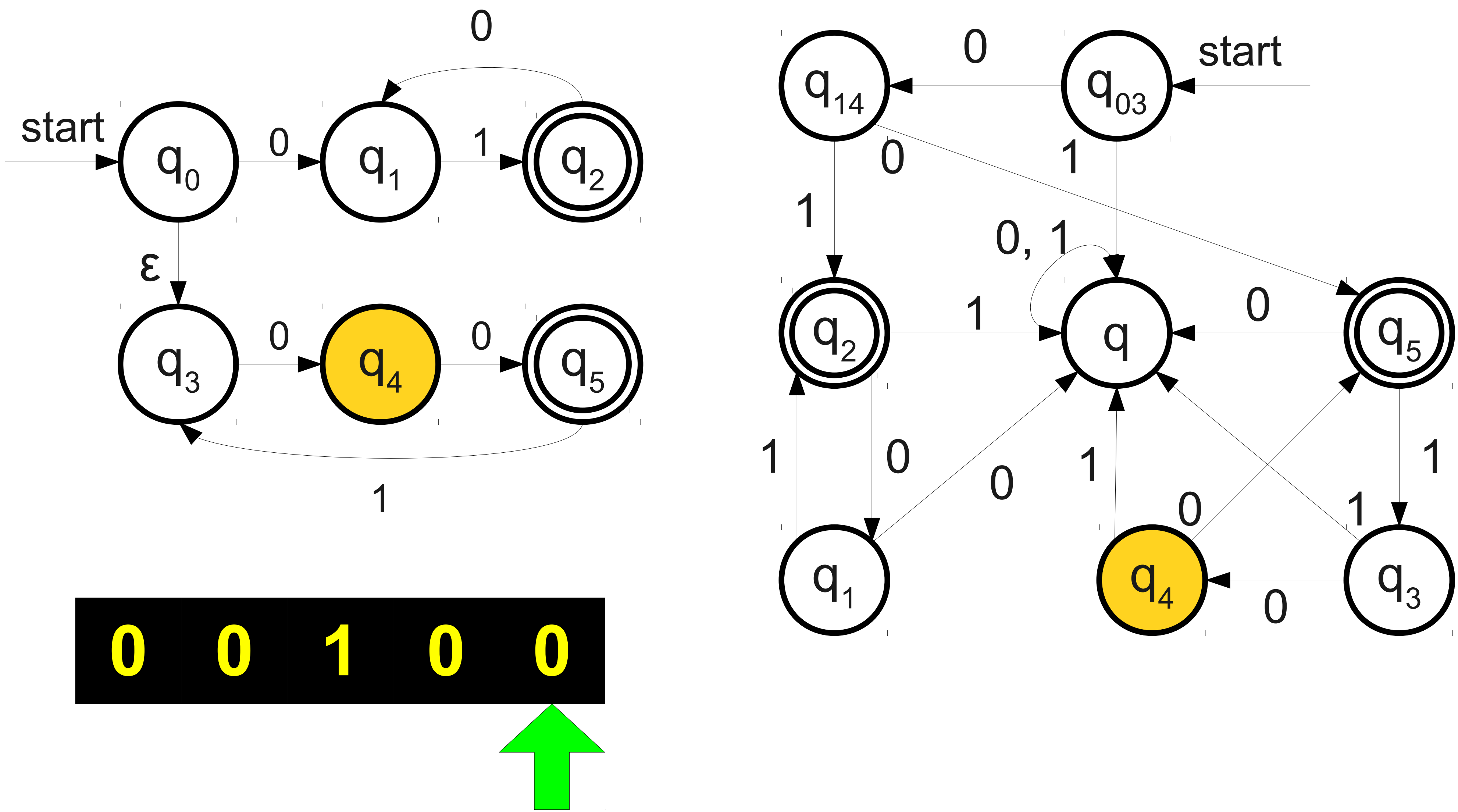

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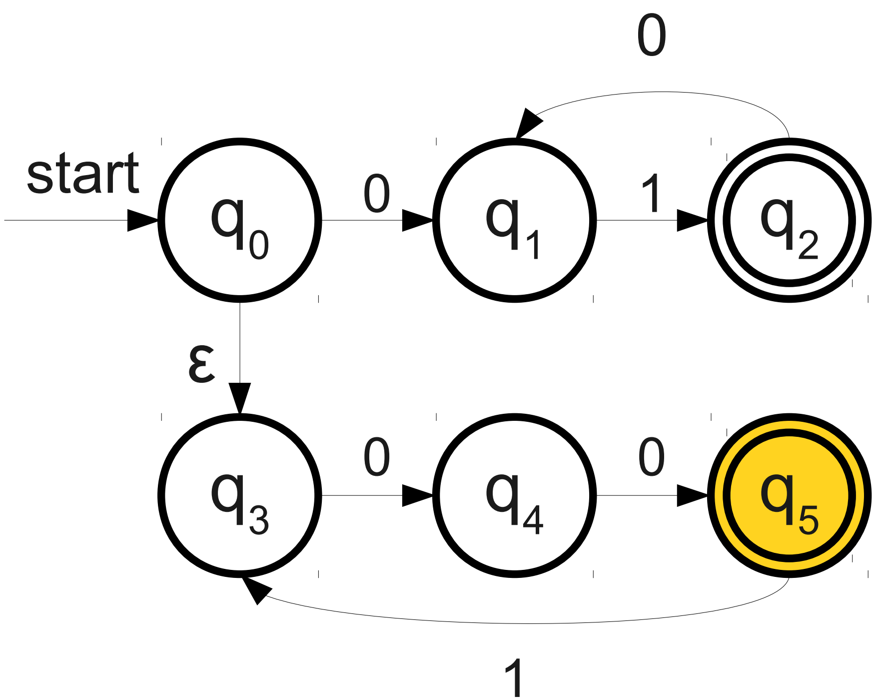

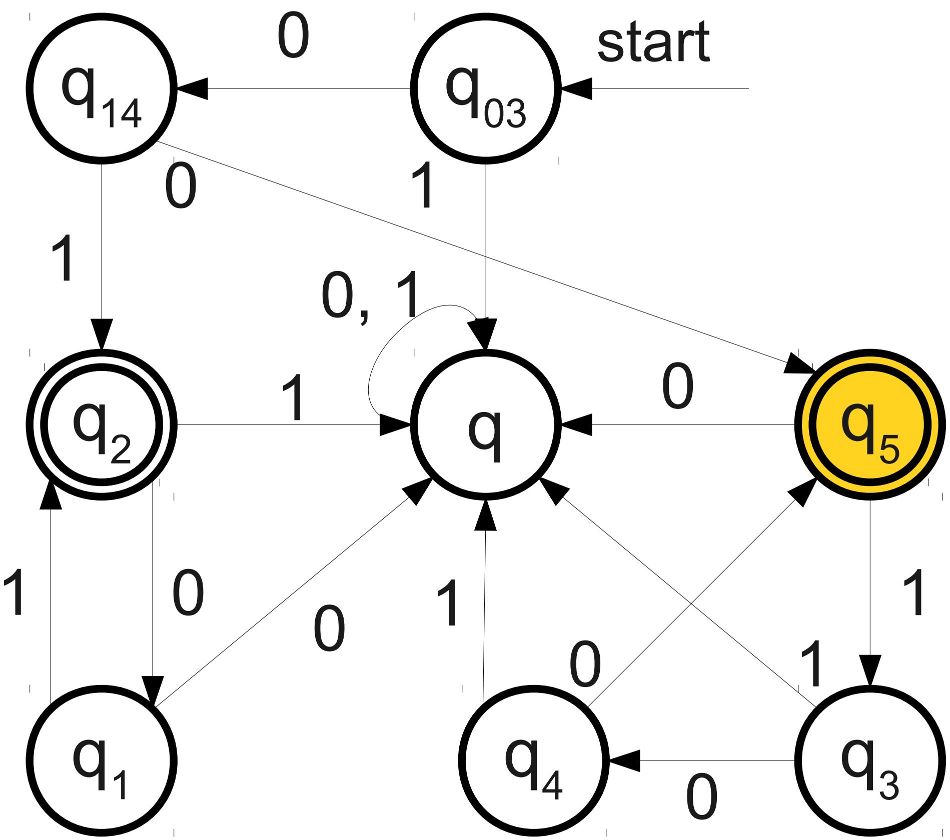

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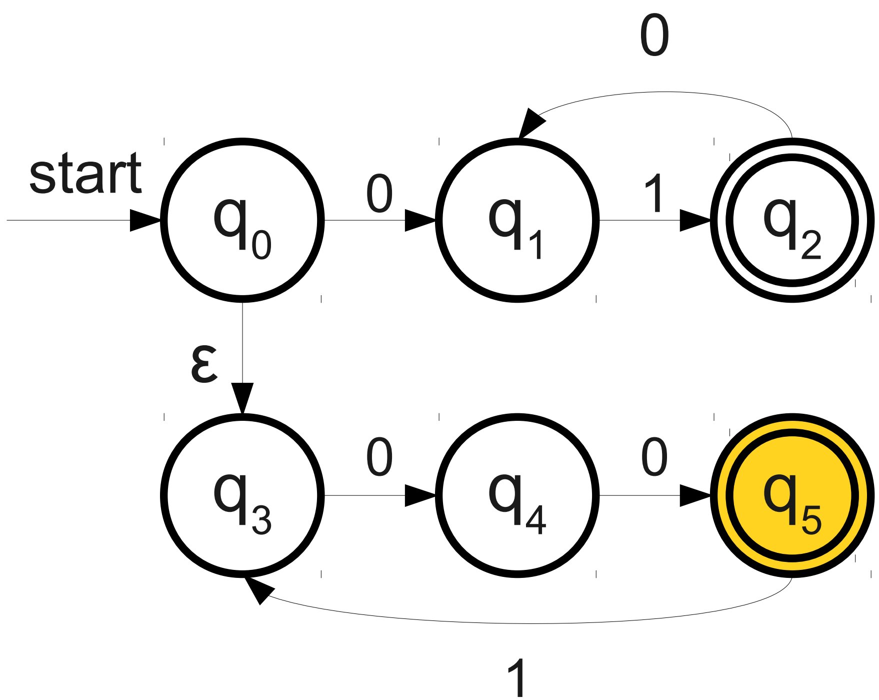

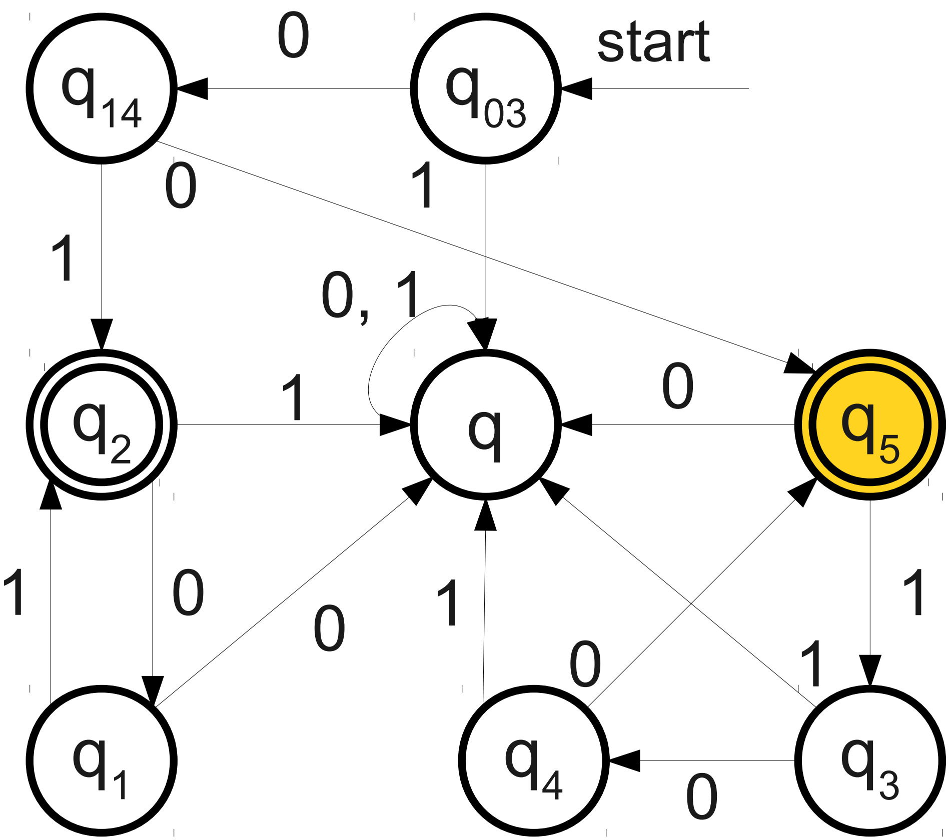

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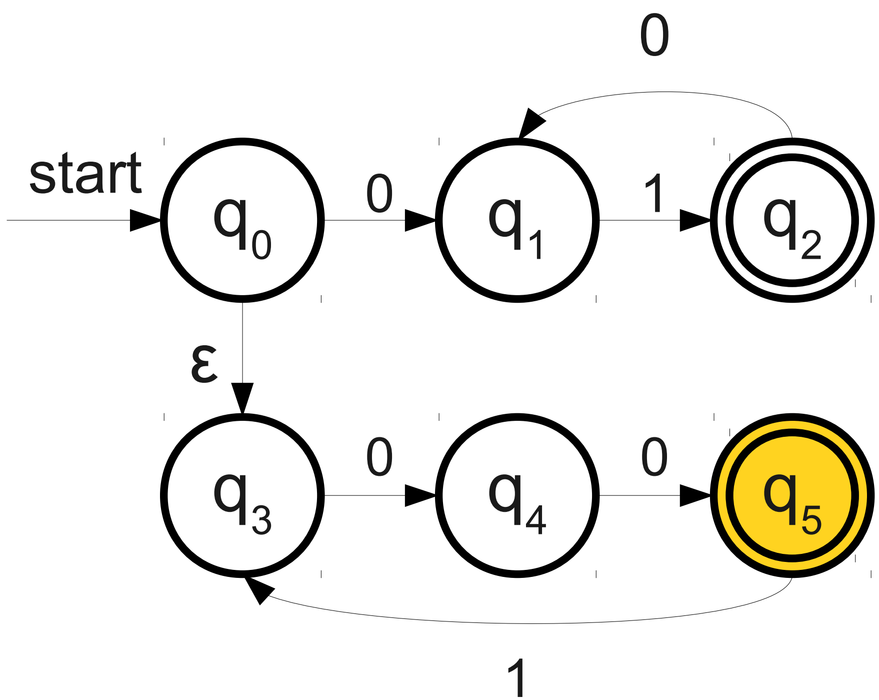

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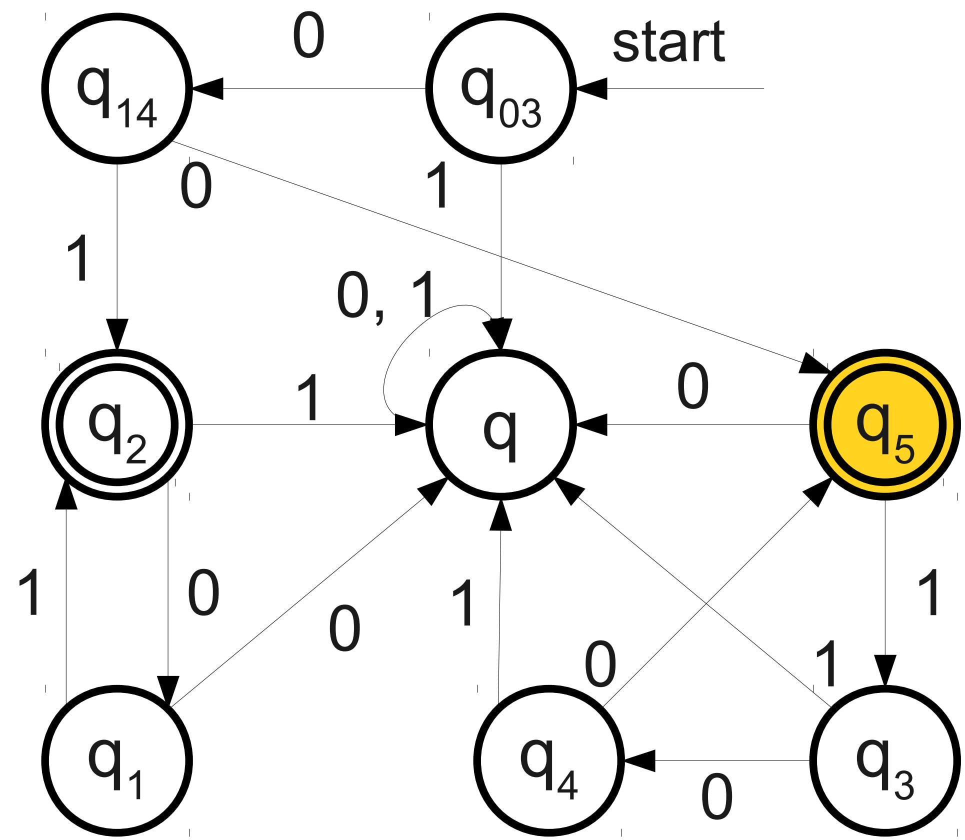

The Subset Construction

- This construction for transforming an NFA into a DFA is called the **subset construction** (or sometimes the **powerset construction**).
- Intuitively:
  - States of the new DFA correspond to *sets of states* of the NFA.
  - The initial state is the start state, plus all states reachable from the start state via ε-transitions.
  - Transition on state *S* on character **`a`** is found by following all possible transitions on **`a`** for each state in *S*, then taking the set of states reachable from there by ε-transitions.
  - Accepting states are any set of states where *some* state in the set is an accepting state.
- **Read Sipser for a formal account.**

The Subset Construction

- In converting an NFA to a DFA, the DFA's states correspond to sets of NFA states.
- Fact: $|\wp(S)| = 2^{|S|}$ for any finite set S.
- In the worst-case, the construction can result in a DFA that is *exponentially larger* than the original NFA.
- Interesting challenge: Find a language for which this worst-case behavior occurs (there are infinitely many of them!)

A language L is called a **regular language** iff there exists a DFA D such that $\mathscr{L}(D) = L$.

An Important Result

Theorem: A language L is regular iff there is some NFA N such that $\mathscr{L}(N) = L$.

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Why This Matters

- We now have two perspectives on regular languages:
  - Regular languages are languages accepted by DFAs.
  - Regular languages are languages accepted by NFAs.
- We can now reason about the regular languages in two different ways.

The Union of Two Languages

- If L_1 and L_2 are languages over the alphabet Σ, the language $L_1 \cup L_2$ is the language of all strings in at least one of the two languages.
- If L_1 and L_2 are regular languages, is $L_1 \cup L_2$?

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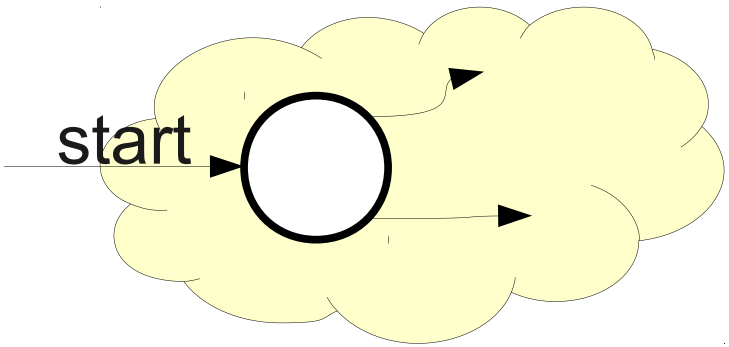
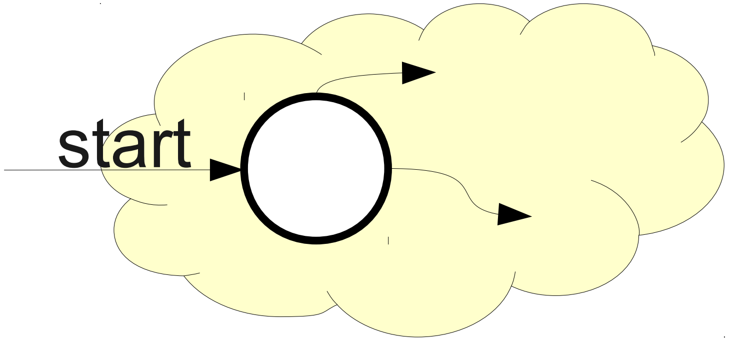


Machine for L_1



Machine for L_2

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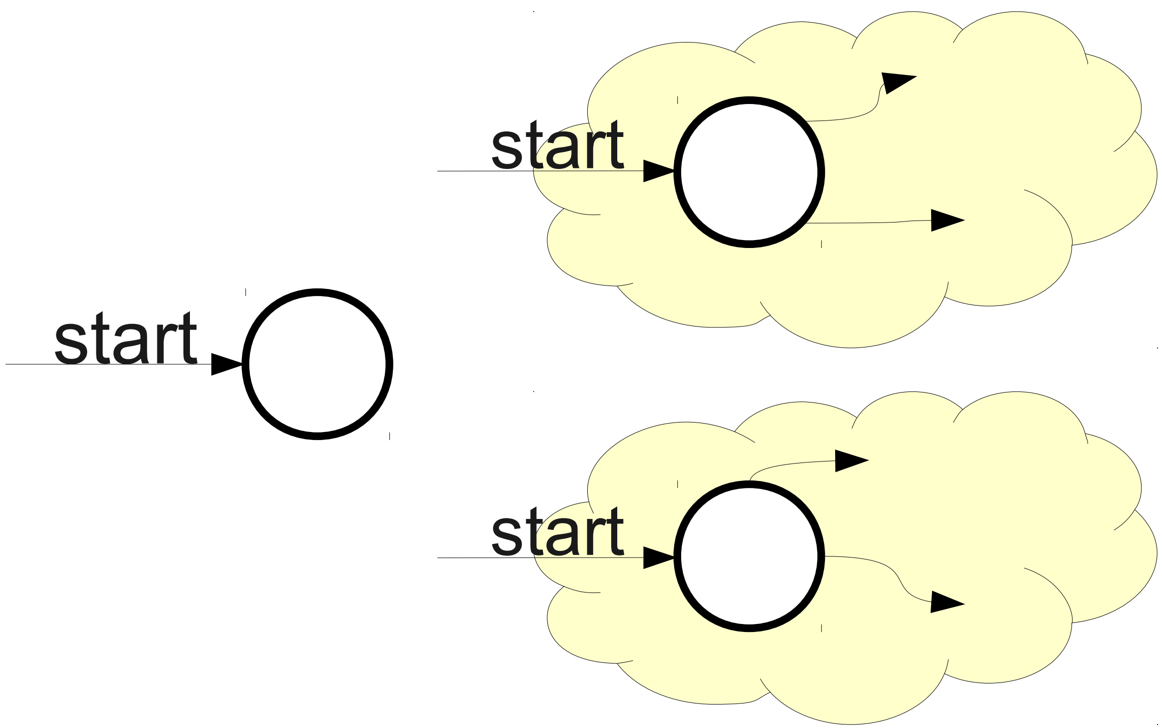

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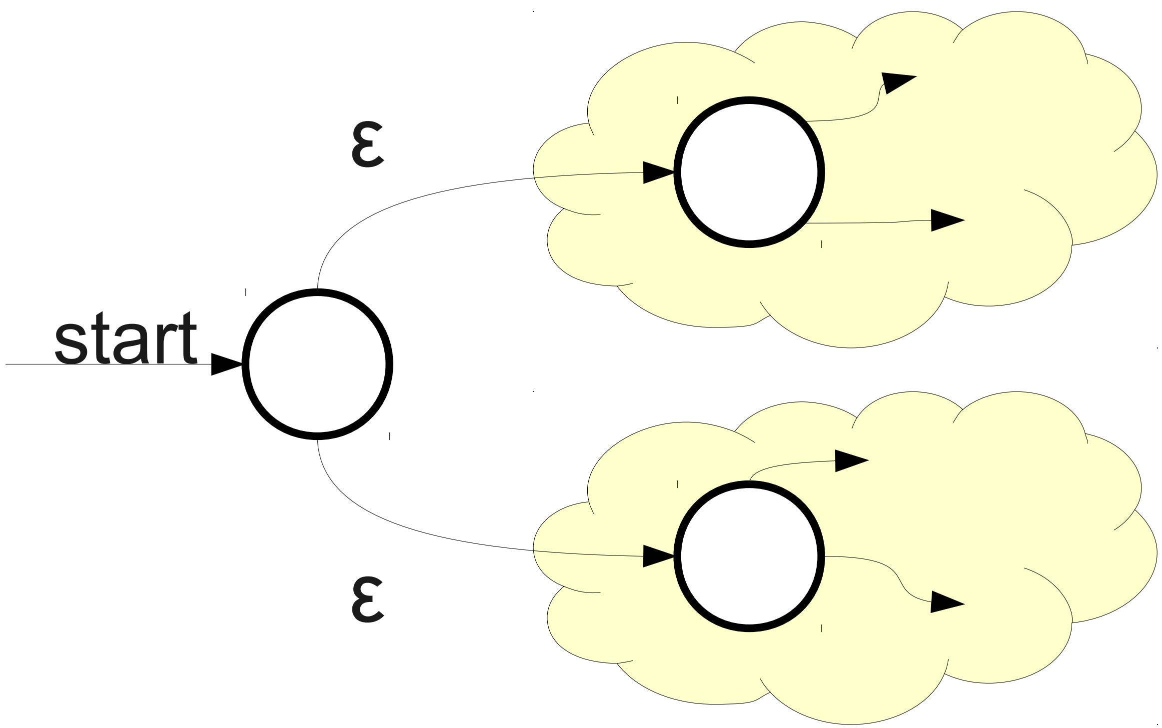

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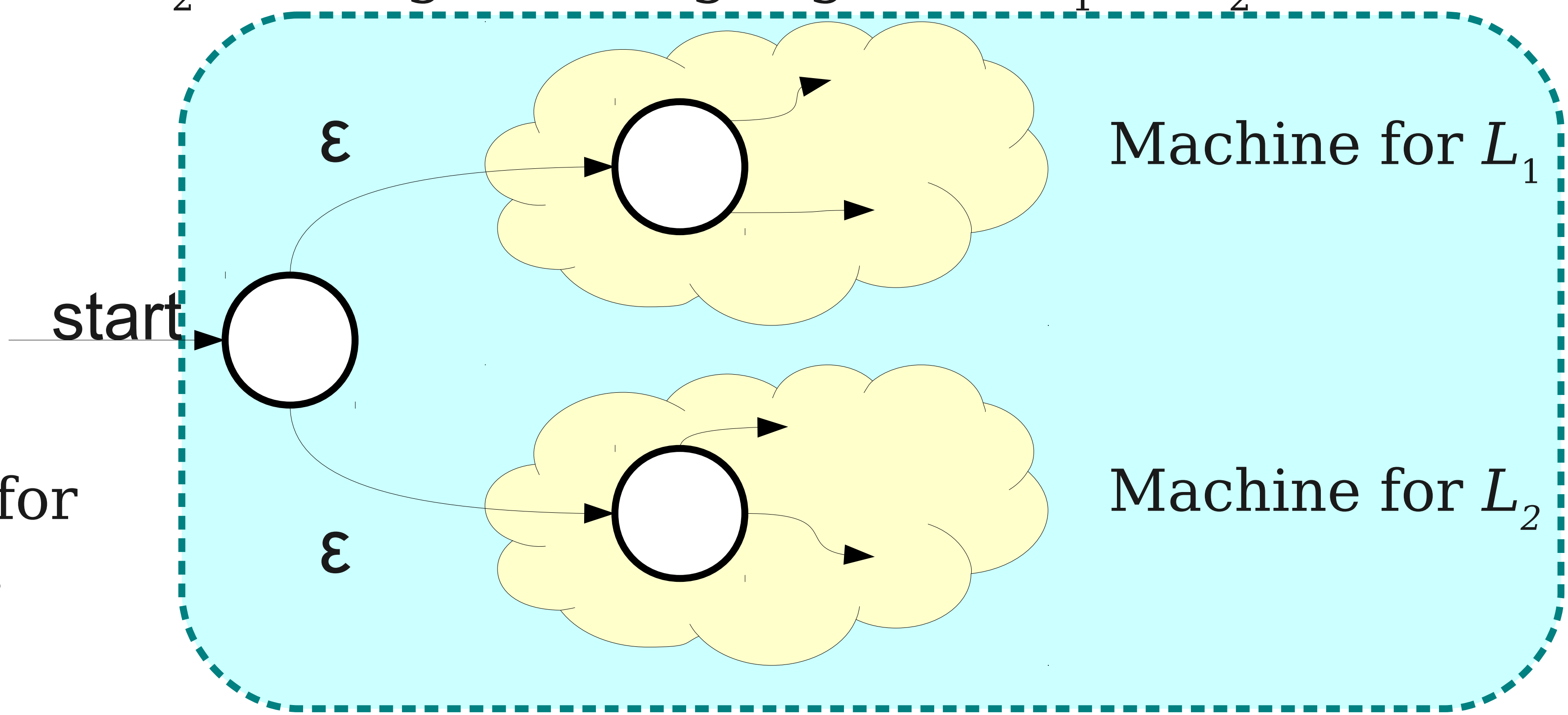

Machine for $L_1 \cup L_2$

The Intersection of Two Languages

- If L_1 and L_2 are languages over Σ, then $L_1 \cap L_2$ is the language of strings in both L_1 and L_2.
- Question: If L_1 and L_2 are regular, is $L_1 \cap L_2$ regular as well?

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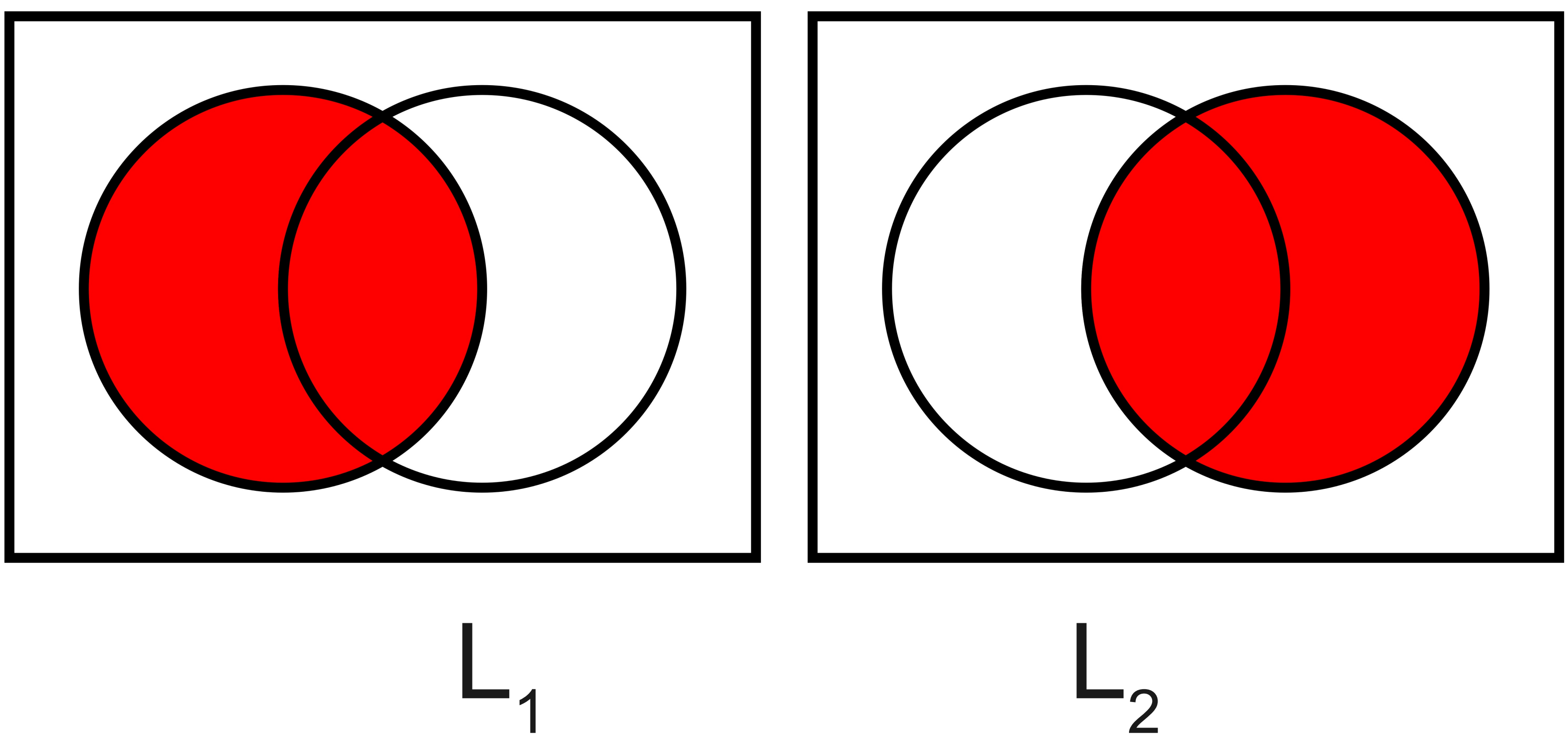

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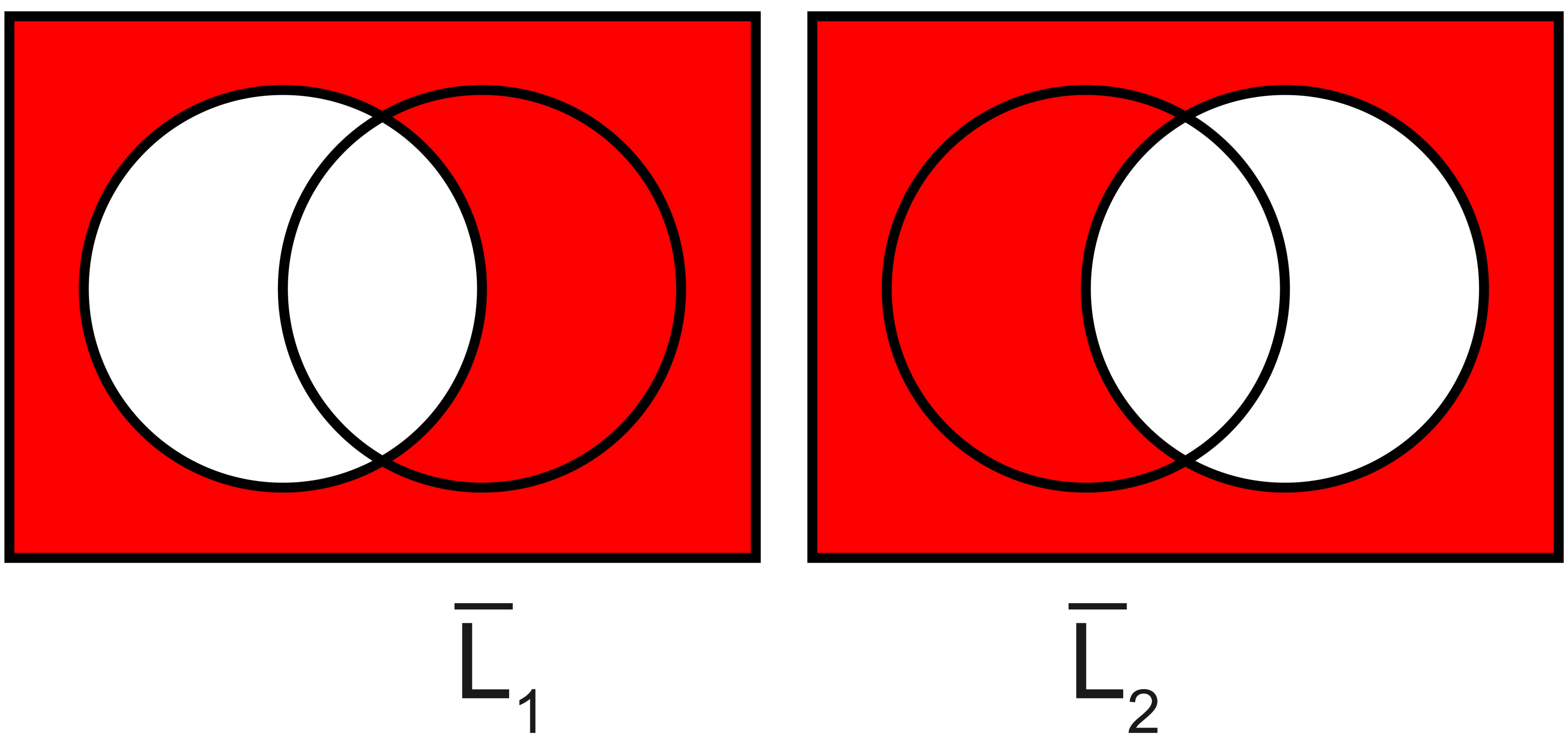

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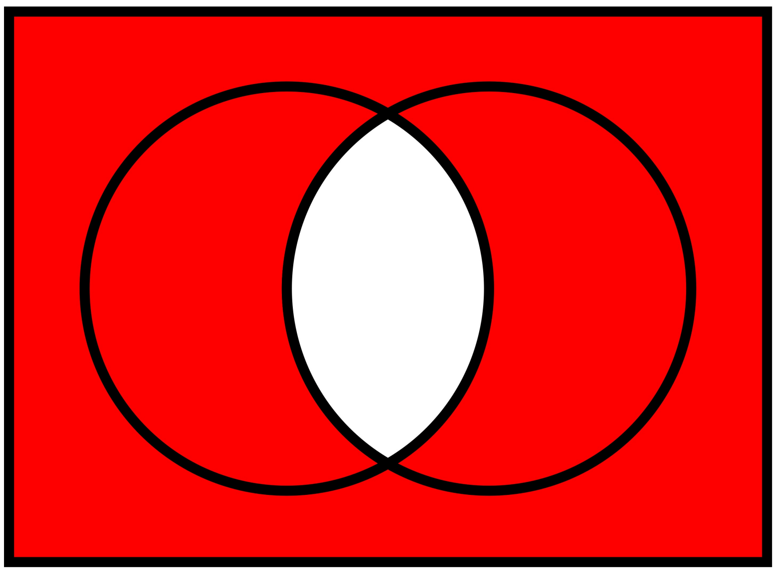

$\overline{L}_1 \cup \overline{L}_2$

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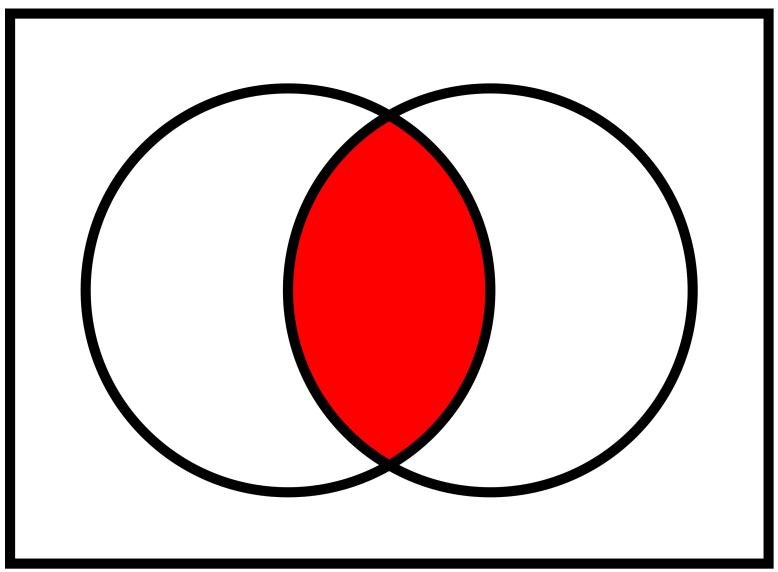

$$\overline{\overline{L_1} \cup \overline{L_2}}$$

Hey, it's De Morgan's laws!

Concatenation

- The **concatenation** of two languages L_1 and L_2 over the alphabet Σ is the language

$$\boldsymbol{L_1 L_2 = \{\, wx \in \Sigma^* \mid w \in L_1 \land x \in L_2 \,\}}$$

- The set of strings that can be split into two pieces: a piece from L_1 and a piece from L_2.
- Conceptually similar to the Cartesian product of two sets, only with strings.

Concatenation Example

- Let Σ = { **a**, **b**, …, **z**, **A**, **B**, …, **Z** } and consider these languages over Σ:
  - ***Noun*** = { **Puppy**, **Rainbow**, **Whale**, … }
  - ***Verb*** = { **Hugs**, **Juggles**, **Loves**, … }
  - ***The*** = { **The** }
- The language ***TheNounVerbTheNoun*** is

  { **ThePuppyHugsTheWhale**,
  TheWhaleLovesTheRainbow,
  TheRainbowJugglesTheRainbow, … }

Concatenating Regular Languages

- If L_1 and L_2 are regular languages, is L_1L_2?
- Intuition – can we split a string w into two strings xy such that $x \in L_1$ and $y \in L_2$?

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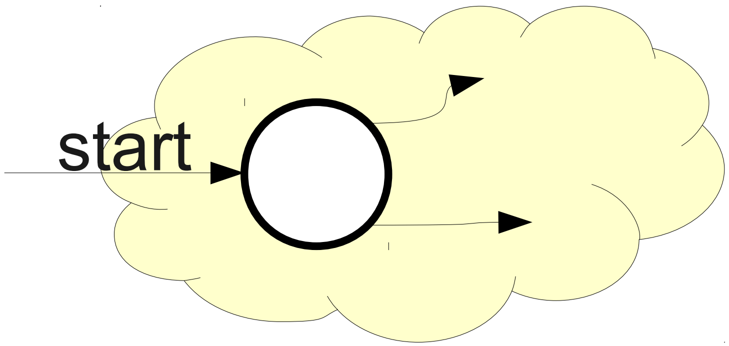
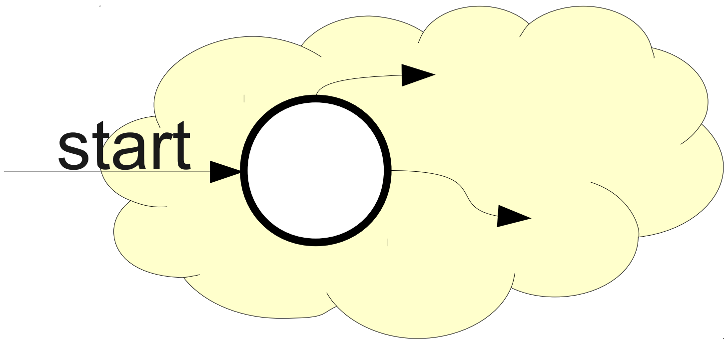


Machine for L_1



Machine for L_2

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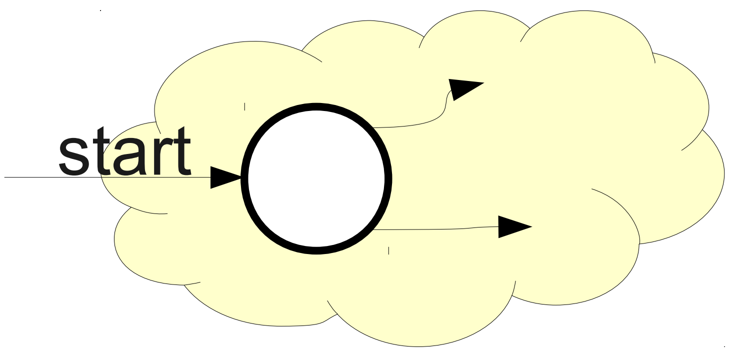

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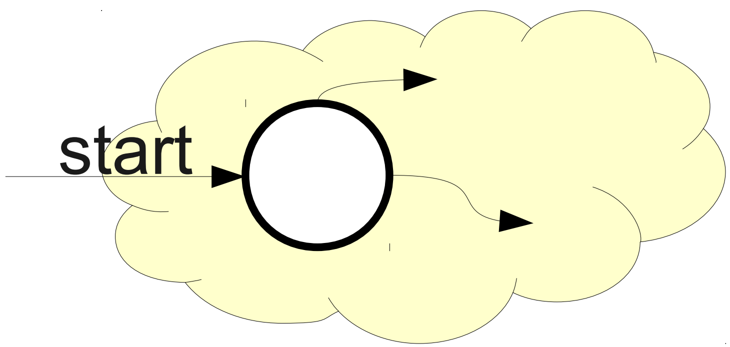

Machine for L_2

b	**o**	**o**	**k**	**k**	**e**	**e**	**p**	**e**	**r**

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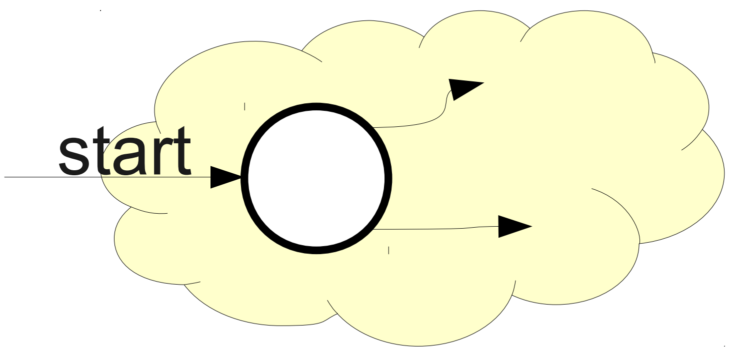

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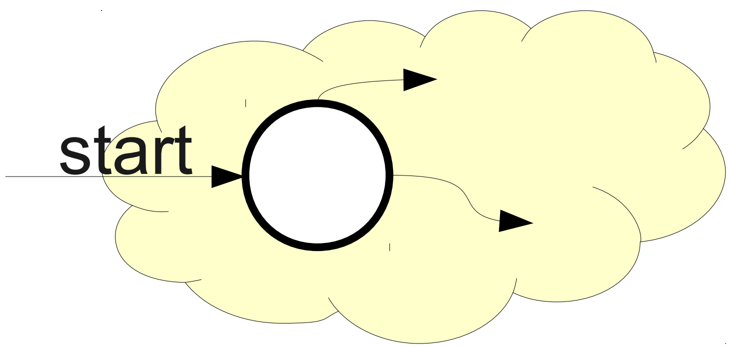

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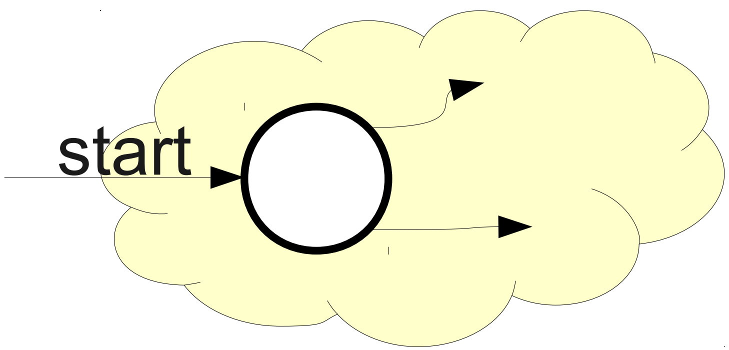

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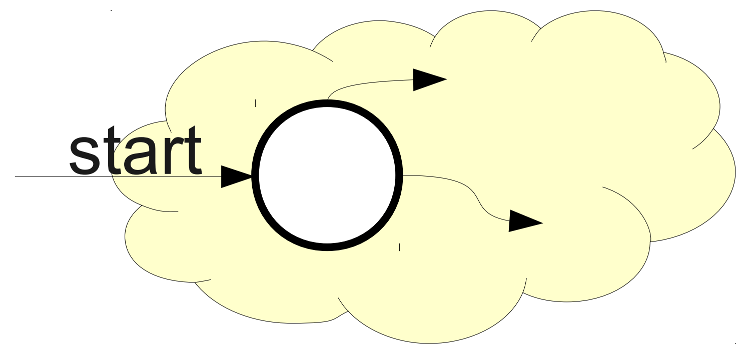

Machine for L_2

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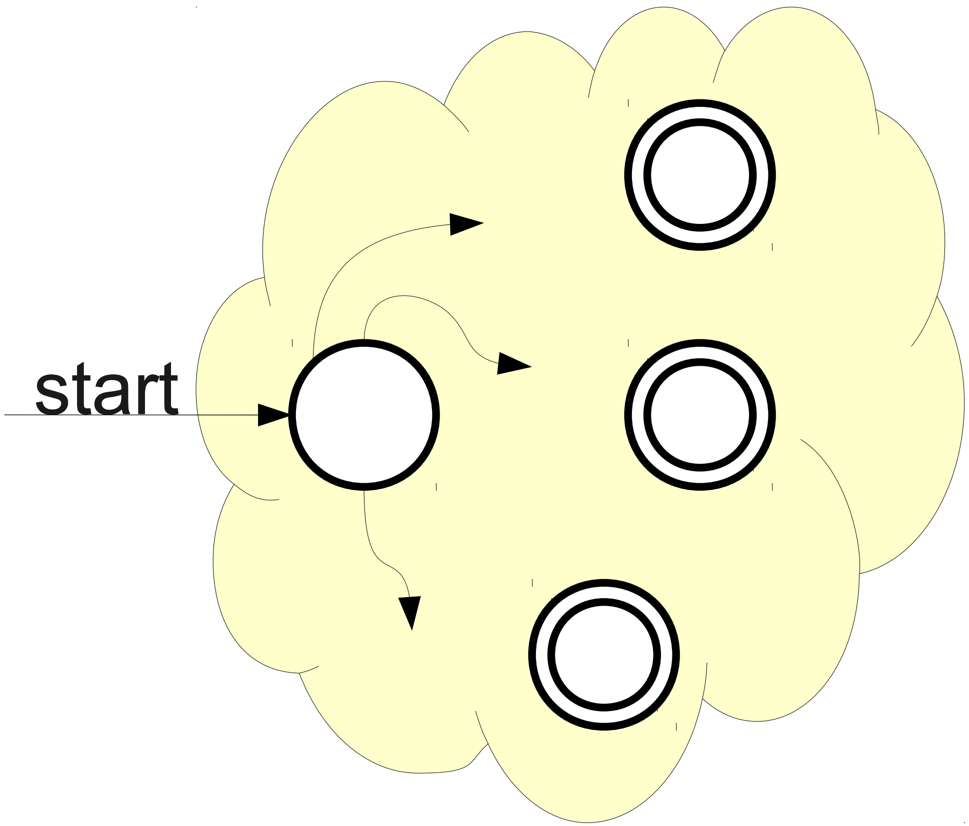
- Intuition – can we split a string w into two strings xy such that $x \in L_1$ and $y \in L_2$?
- **Idea**: Run the automaton for L_1 on w, and whenever L_1 reaches an accepting state, optionally hand the rest off w to L_2.
  - If L_2 accepts the remainder, then L_1 accepted the first part and the string is in L_1L_2.
  - If L_2 rejects the remainder, then the split was incorrect.

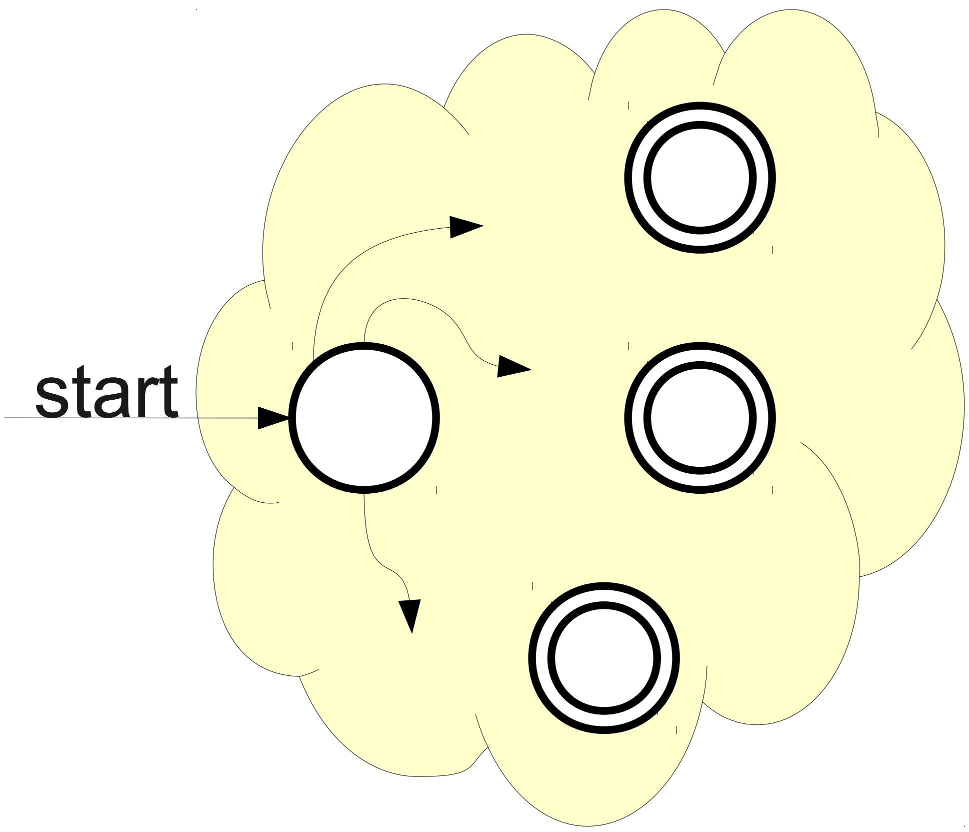
Concatenating Regular Languages

Concatenating Regular Languages

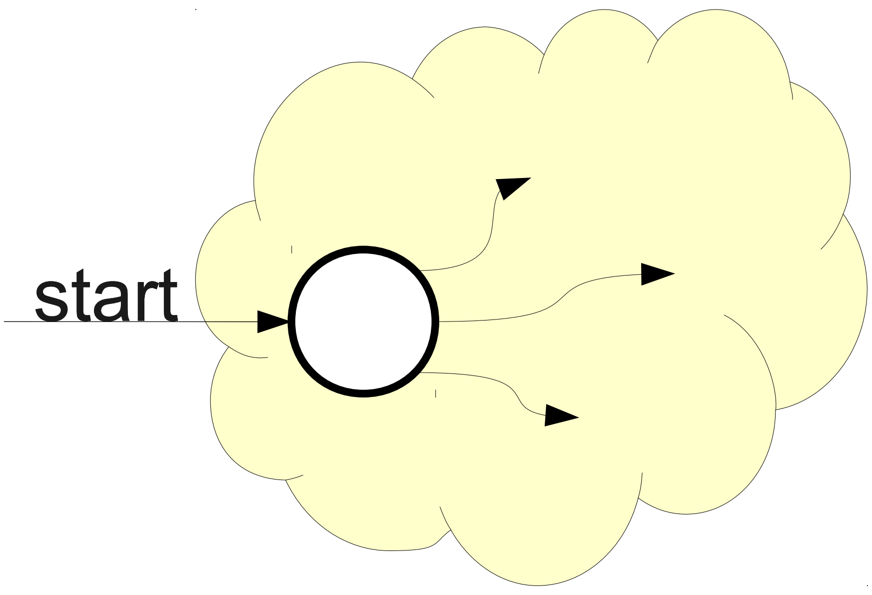


Machine for
L_1

Concatenating Regular Languages

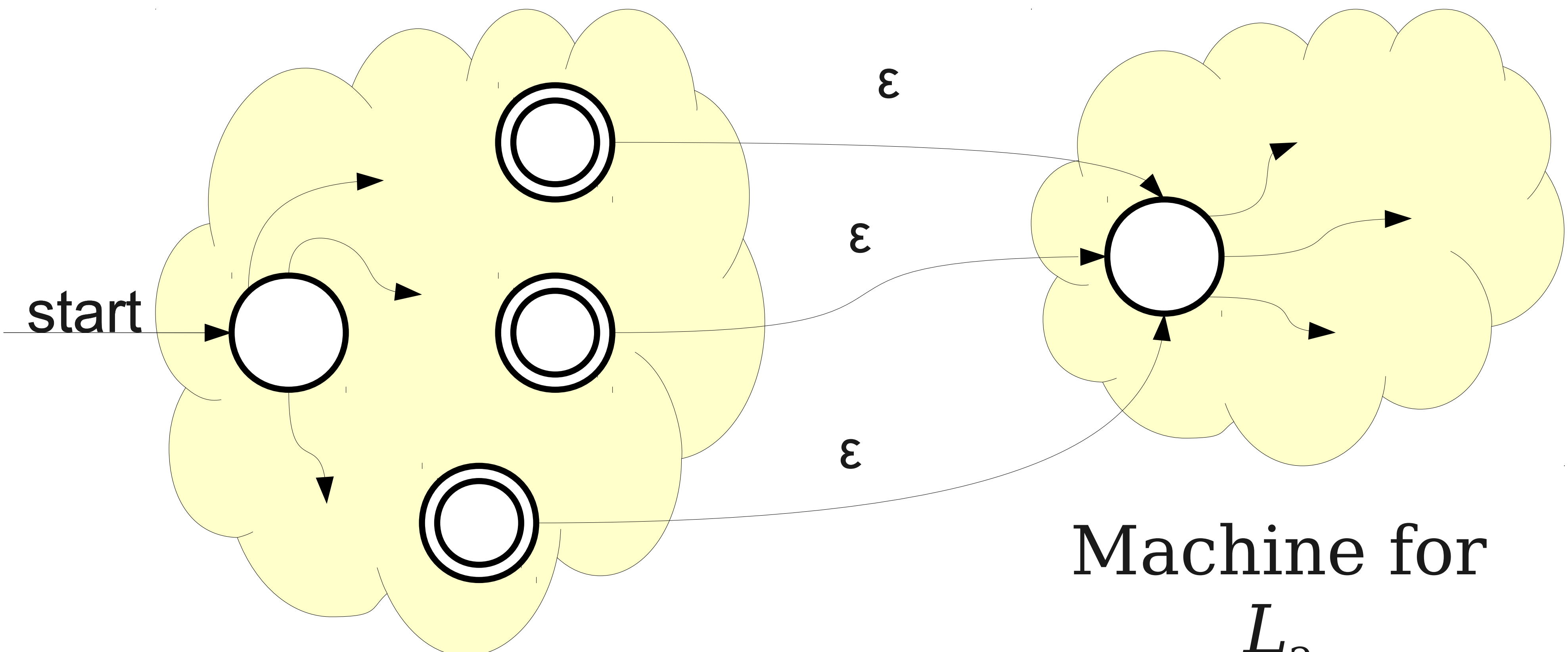


Machine for L_1



Machine for L_2

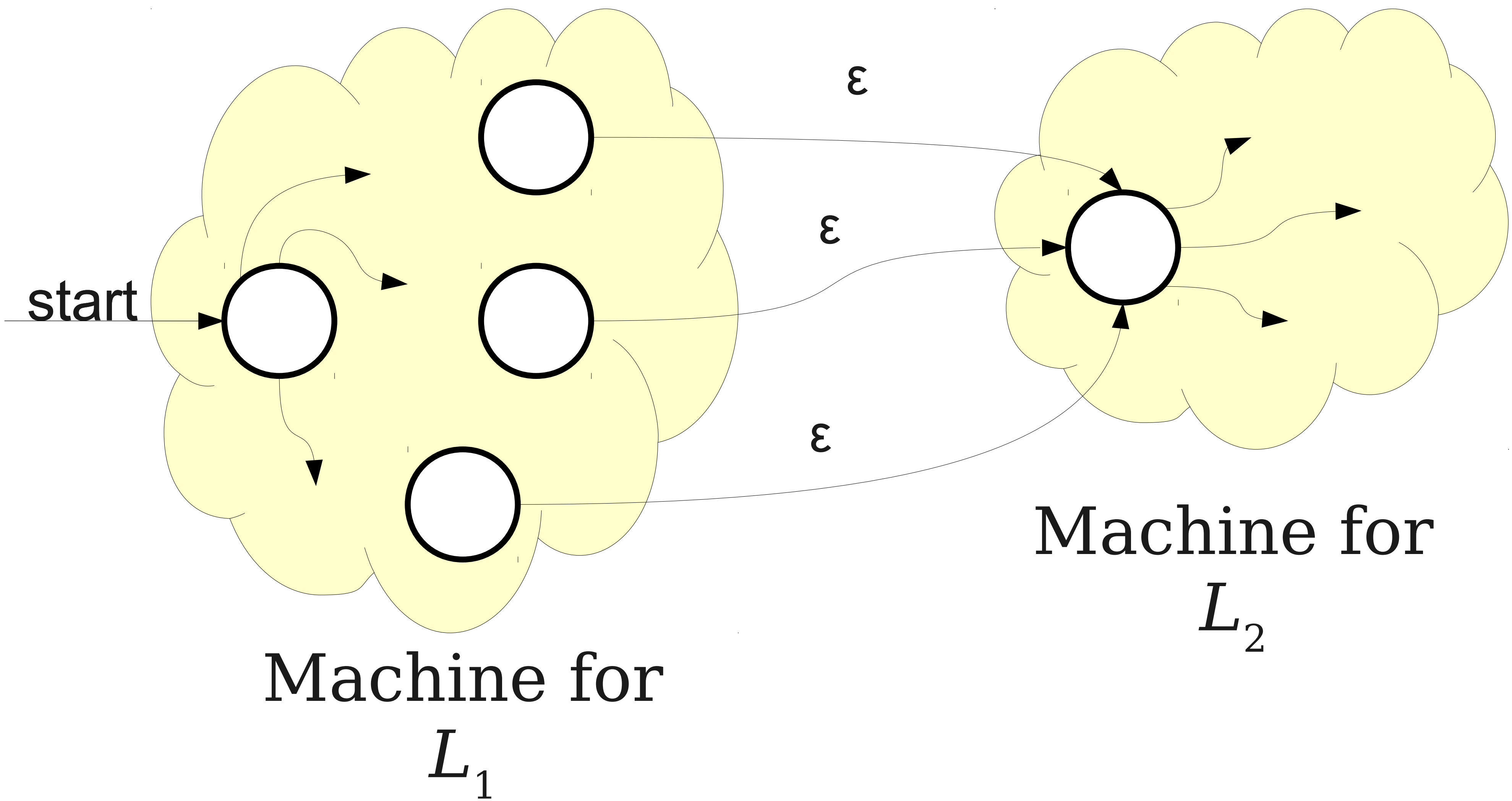
Concatenating Regular Languages



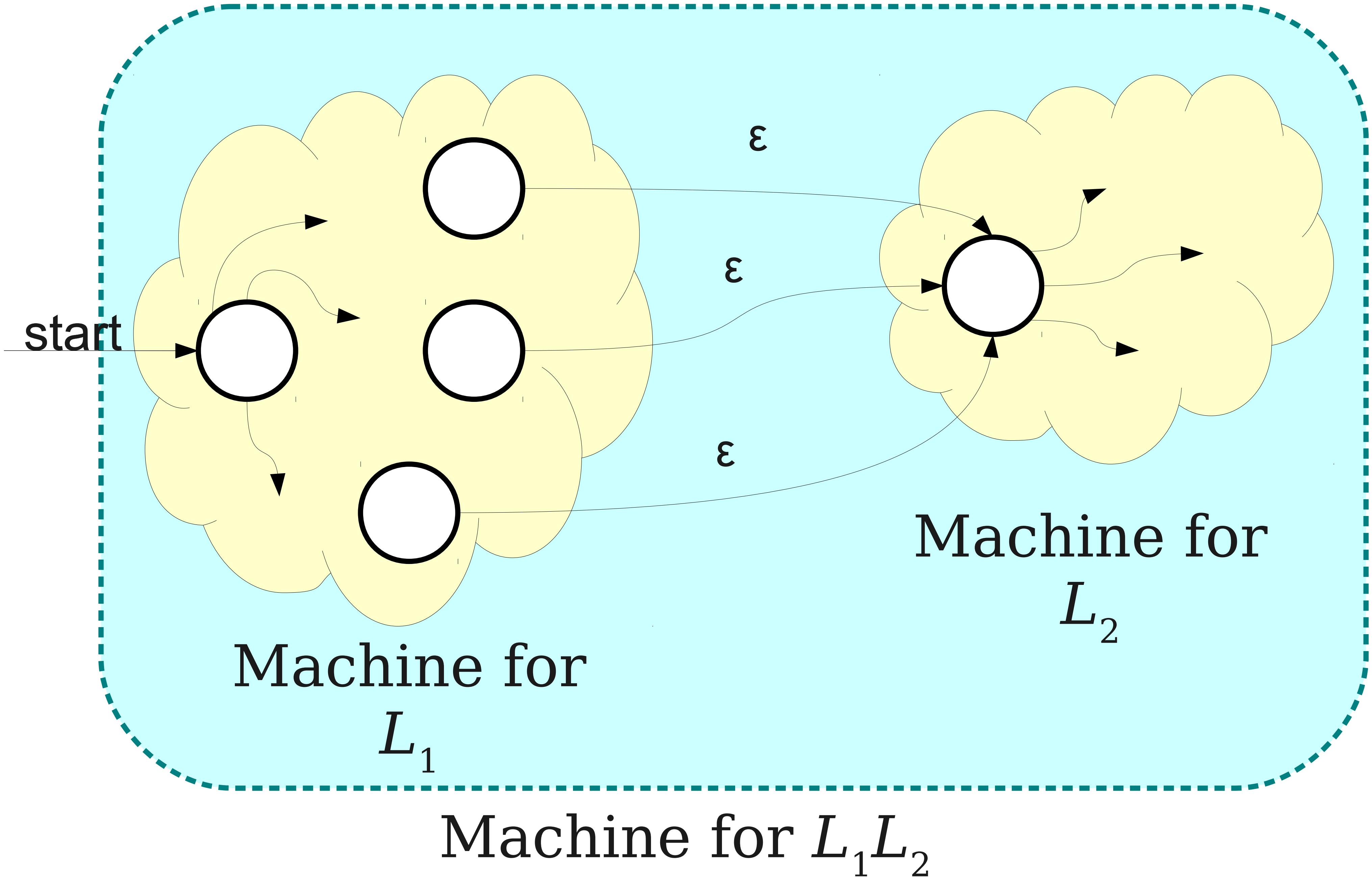
Machine for L_1

Machine for L_2

Concatenating Regular Languages



Concatenating Regular Languages



Lots and Lots of Concatenation

- Consider the language L = { `aa`, `b` }
- LL is the set of strings formed by concatenating pairs of strings in L.

{ `aaaa`, `aab`, `baa`, `bb` }

- LLL is the set of strings formed by concatenating triples of strings in L.

{ `aaaaaa`, `aaaab`, `aabaa`, `aabb`, `baaaa`, `baab`, `bbaa`, `bbb`}

- $LLLL$ is the set of strings formed by concatenating quadruples of strings in L.

{ `aaaaaaaa`, `aaaaaab`, `aaaabaa`, `aaaabb`, `aabaaaa`, `aabaab`, `aabbaa`, `aabbb`, `baaaaaa`, `baaaab`, `baabaa`, `baabb`, `bbaaaa`, `bbaab`, `bbbaa`, `bbbb`}

Language Exponentiation

- We can define what it means to "exponentiate" a language as follows:
- $L^0 = \{ \varepsilon \}$
  - The set containing just the empty string.
  - Idea: Any string formed by concatenating zero strings together is the empty string.
- $L^{n+1} = LL^n$
  - Idea: Concatenating $(n + 1)$ strings together works by concatenating n strings, then concatenating one more.

The Kleene Closure

- An important operation on languages is the **Kleene Closure**, which is defined as

$$L^* = \bigcup_{i=0}^{\infty} L^i$$

- Mathematically:

$$\boldsymbol{w \in L^*} \quad \textbf{iff} \quad \boldsymbol{\exists n \in \mathbb{N}.\ w \in L^n}$$

- Intuitively, all possible ways of concatenating any number of copies of strings in L together.

The Kleene Closure

If L = { **a**, **bb** }, then L^* = {

ε,

a, **bb**,

aa, **abb**, **bba**, **bbbb**,

aaa, **aabb**, **abba**, **abbbb**, **bbaa**, **bbabb**, **bbbba**, **bbbbbb**,

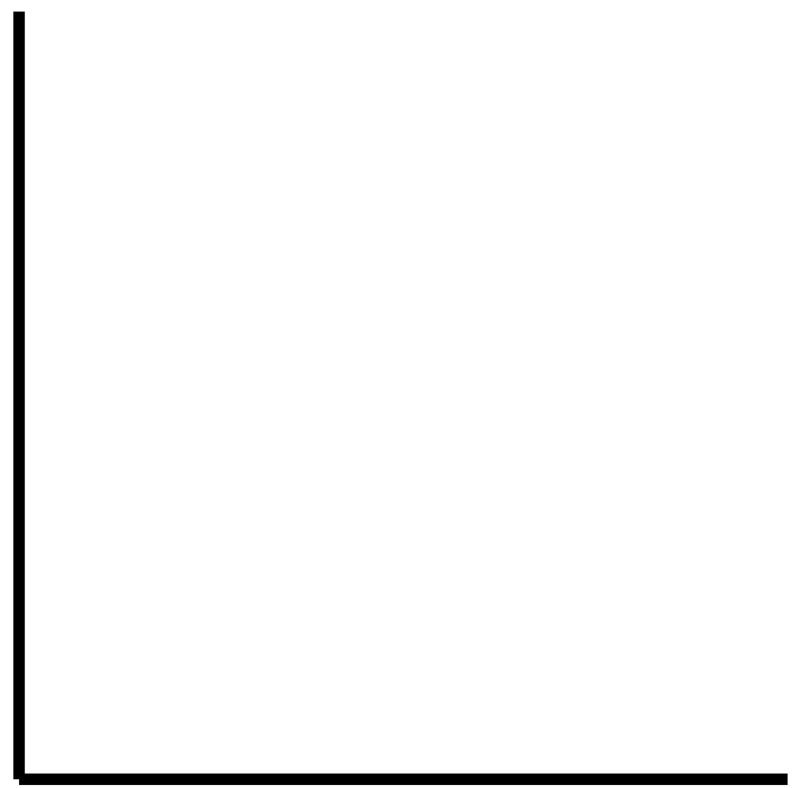
…

}

Reasoning about Infinity

- If L is regular, is L^* necessarily regular?
- **A Bad Line of Reasoning**:
  - $L^0 = \{ \varepsilon \}$ is regular.
  - $L^1 = L$ is regular.
  - $L^2 = LL$ is regular
  - $L^3 = L(LL)$ is regular
  - …
  - Regular languages are closed under union.
  - So the union of all these languages is regular.

Reasoning about Infinity



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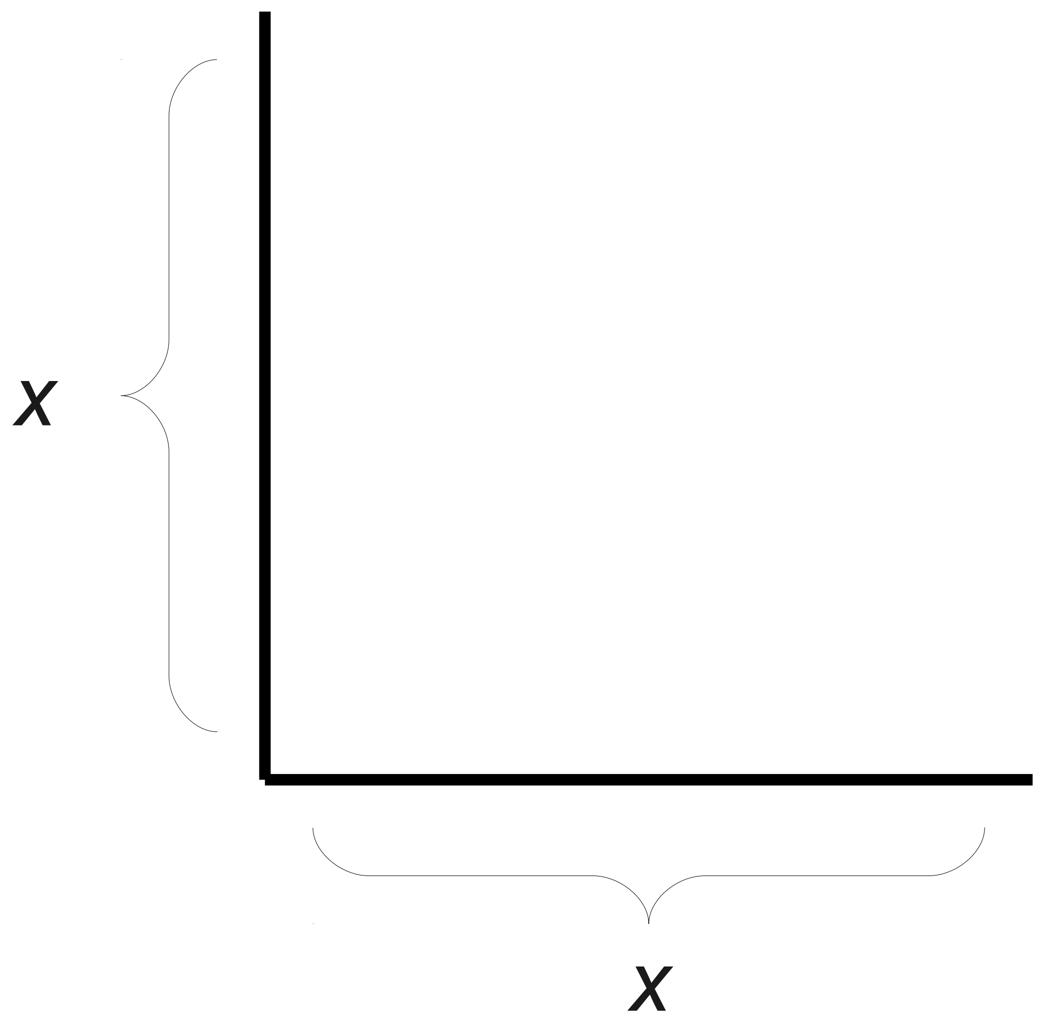

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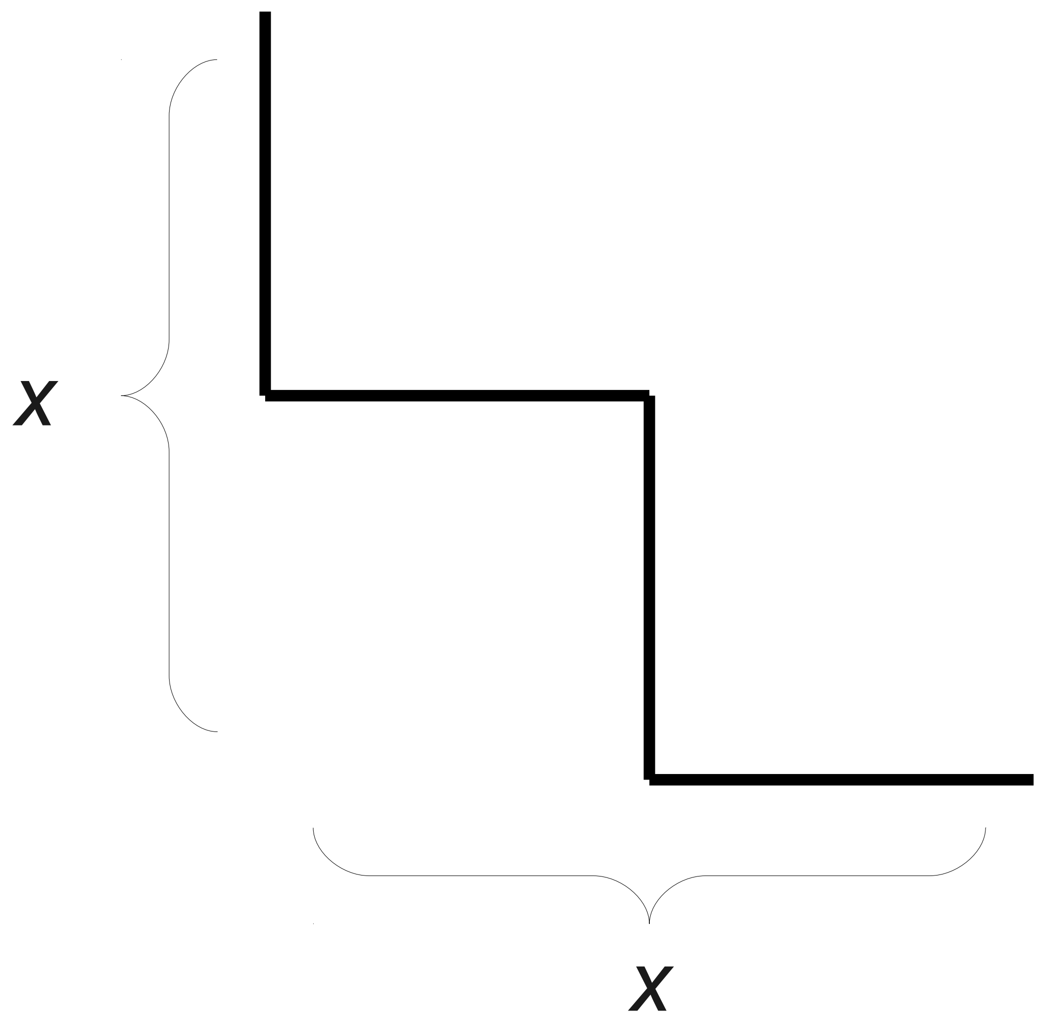

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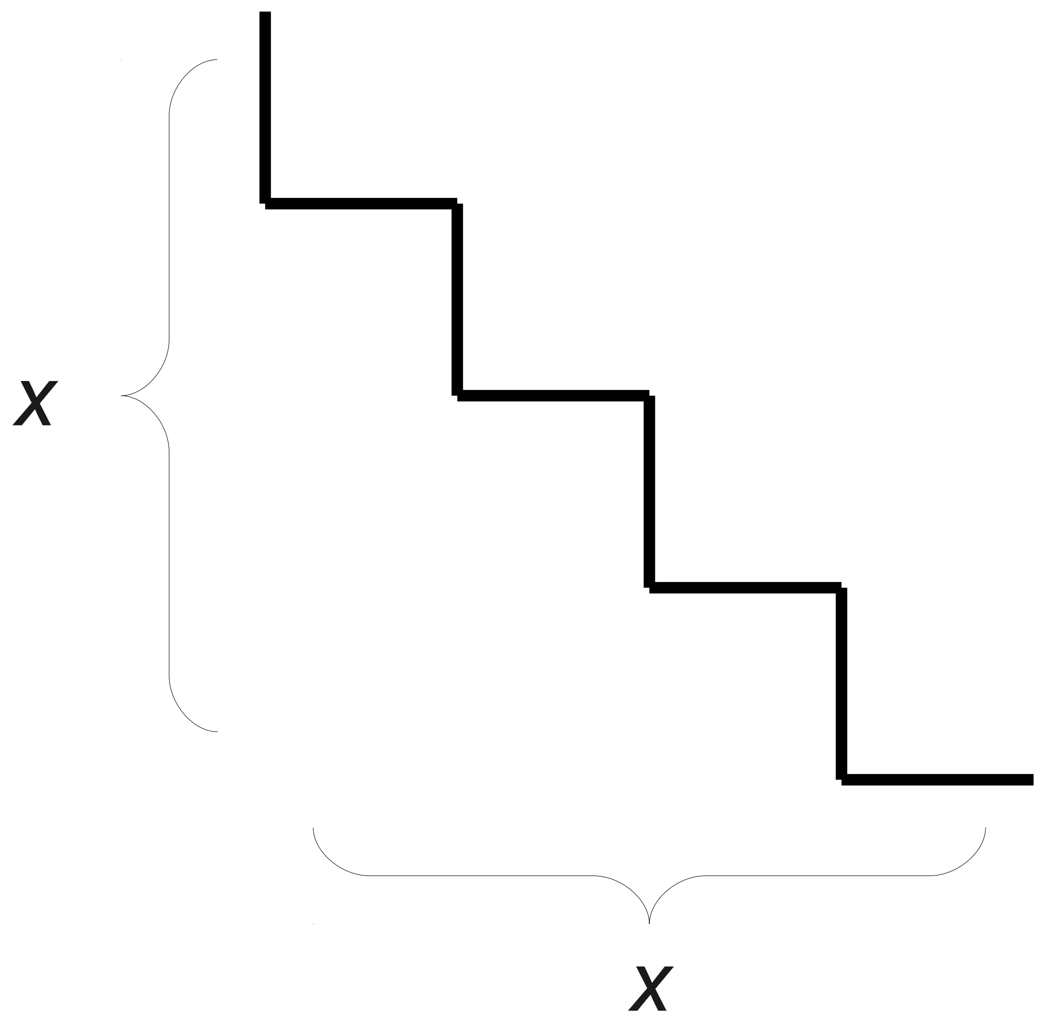

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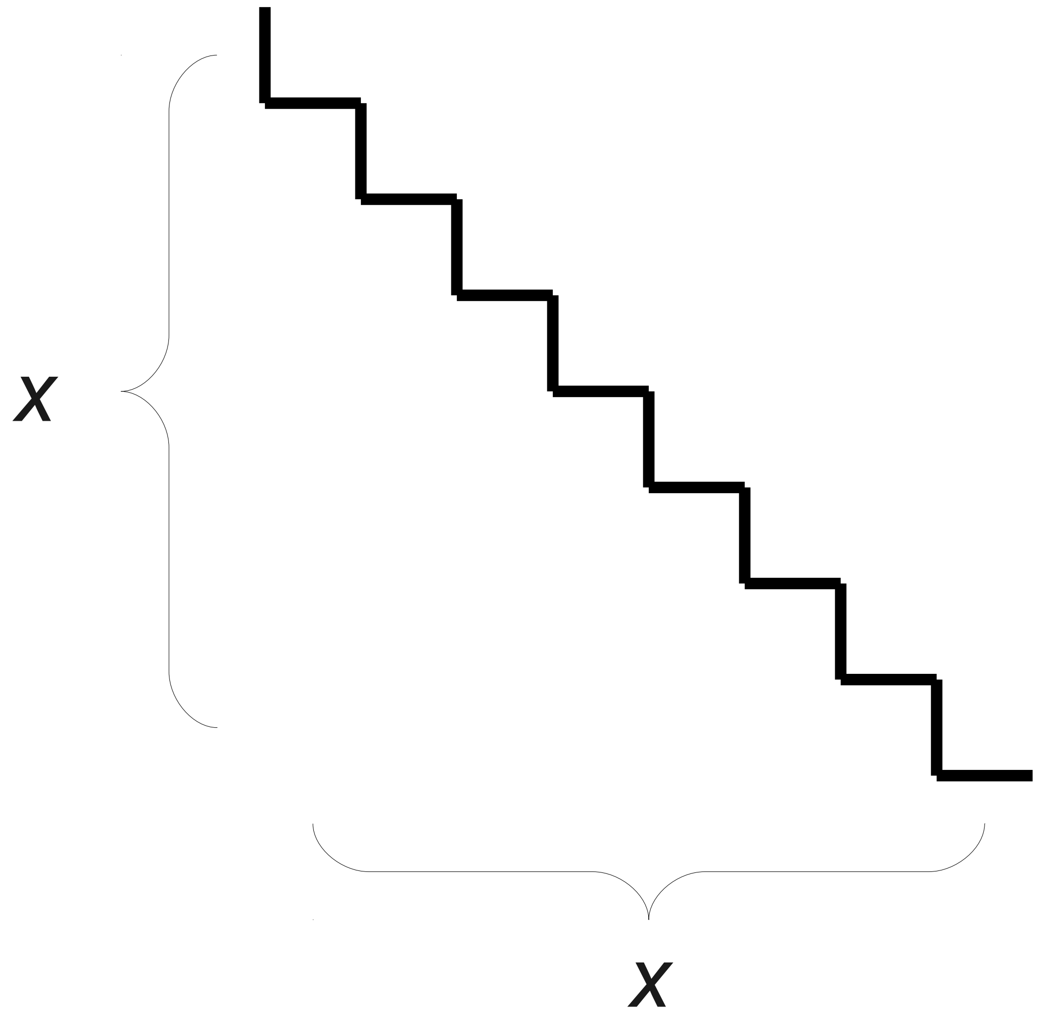

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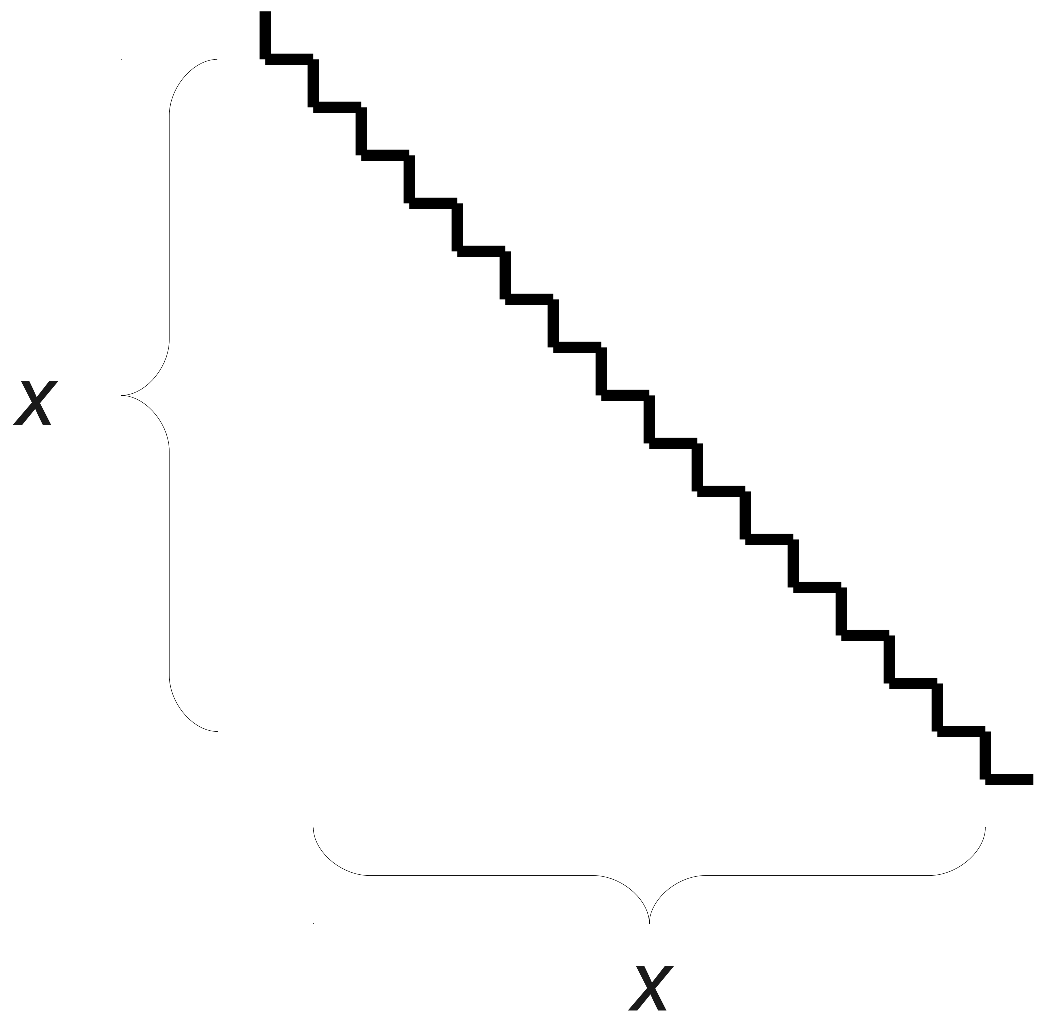

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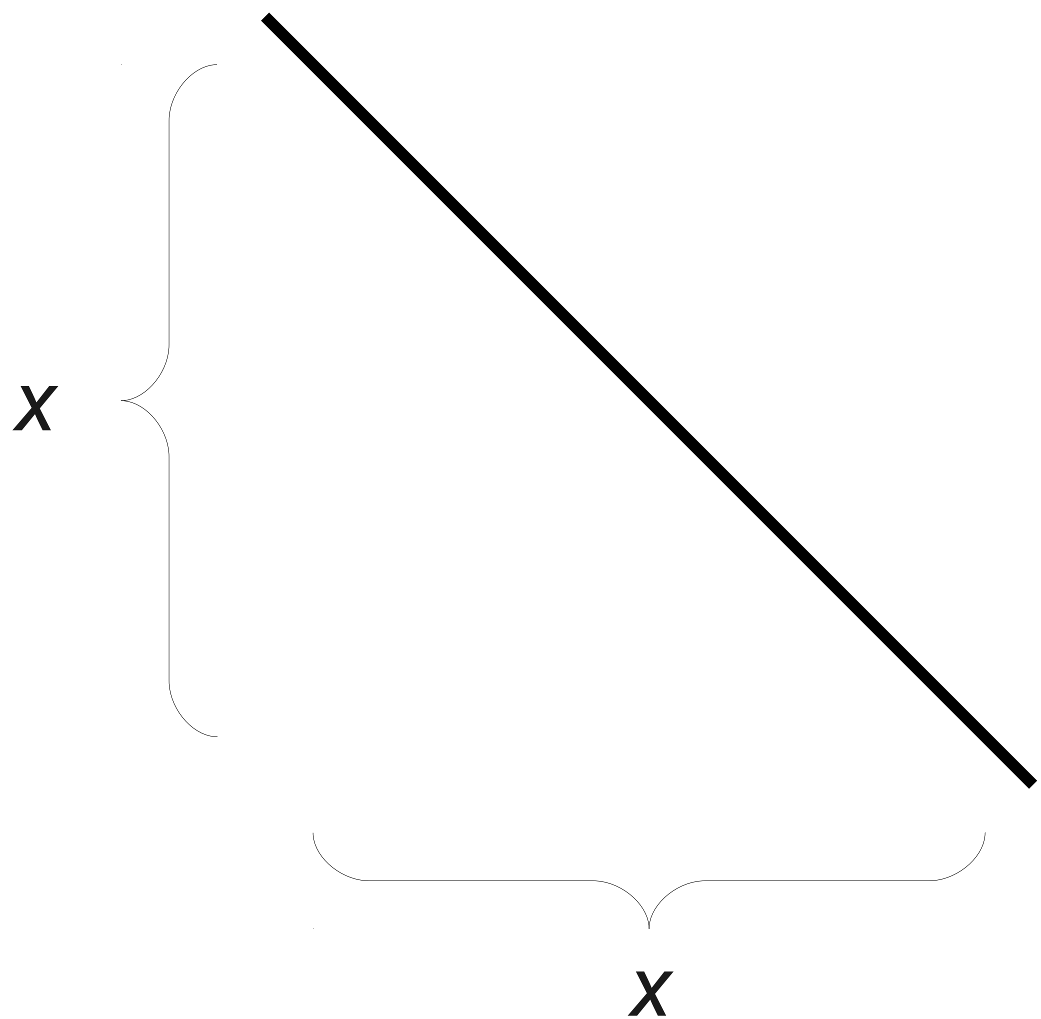
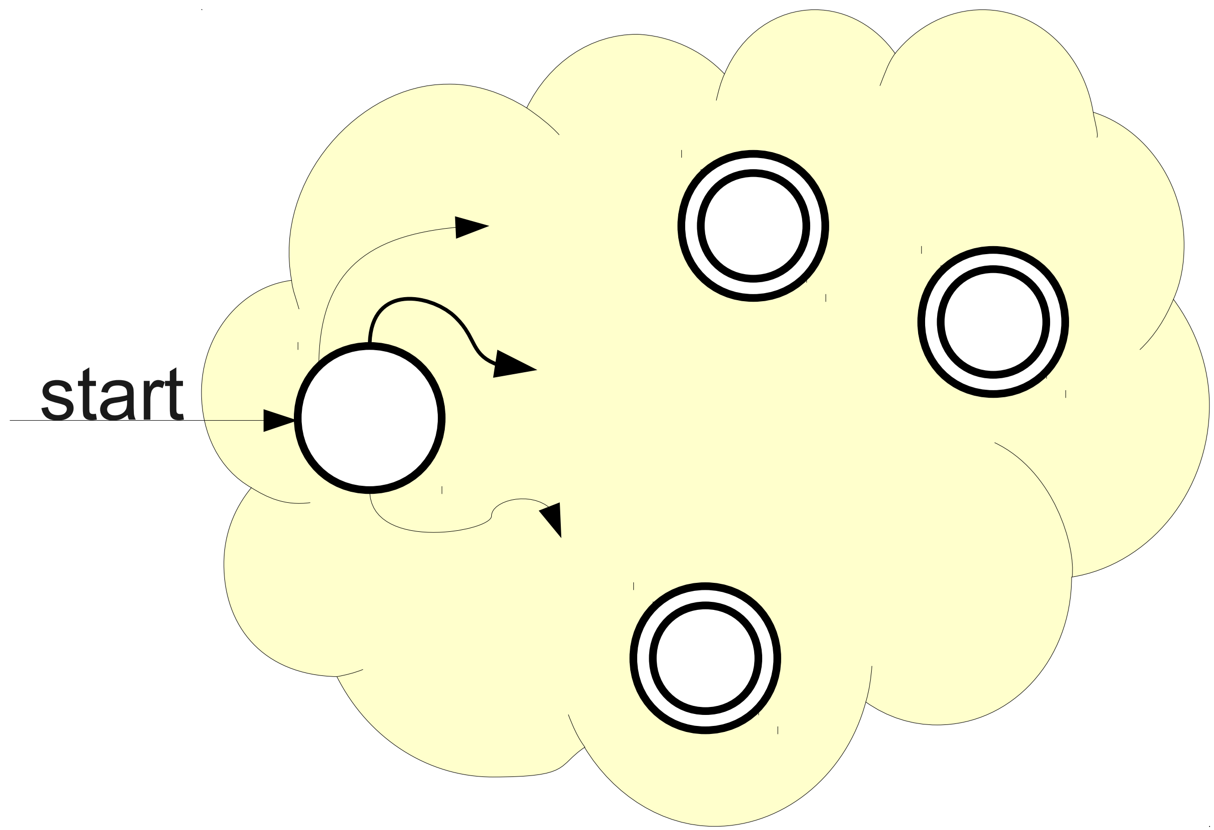


Reasoning About the Infinite

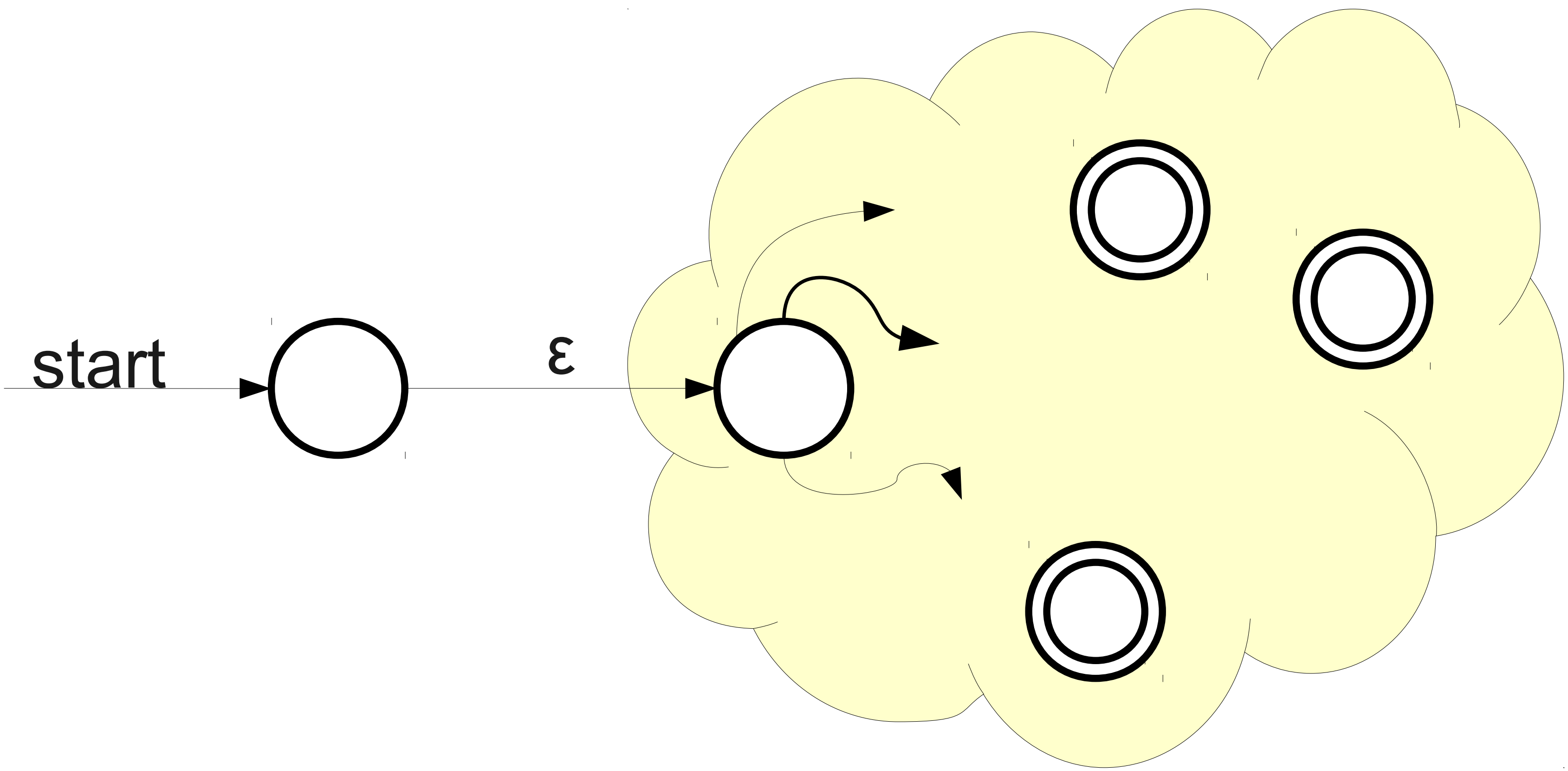
- If a series of finite objects all have some property, their infinite union **does not** necessarily have that property!
  - No matter how many times we zigzag that line, it's never straight.
  - Concluding that it must be equal "in the limit" is not mathematically precise.
  - (This is why calculus is interesting).
- **A better intuition:** Can we convert an NFA for the language L to an NFA for the language L^*?

The Kleene Star



Machine for L

The Kleene Star



Machine for L

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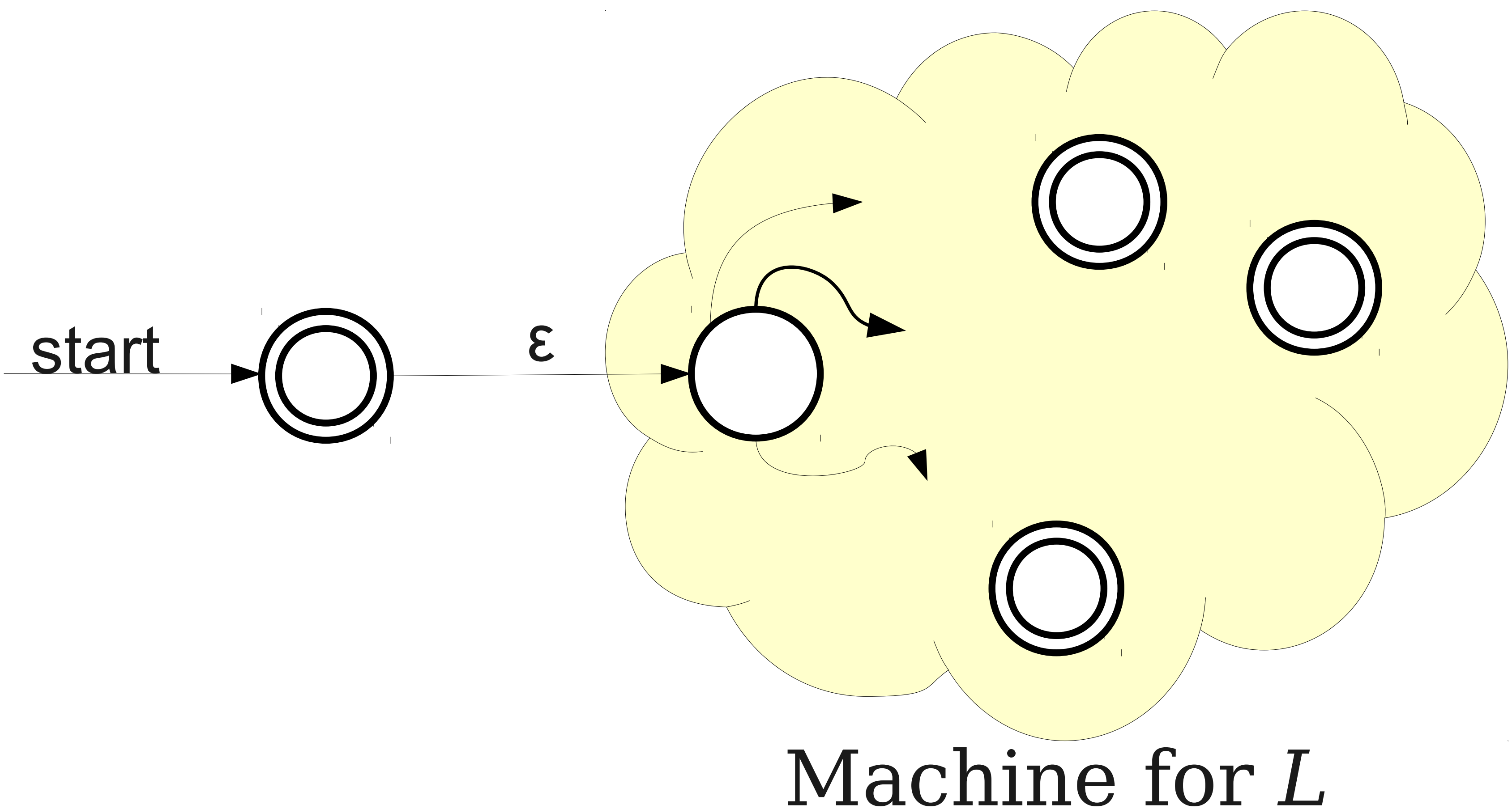

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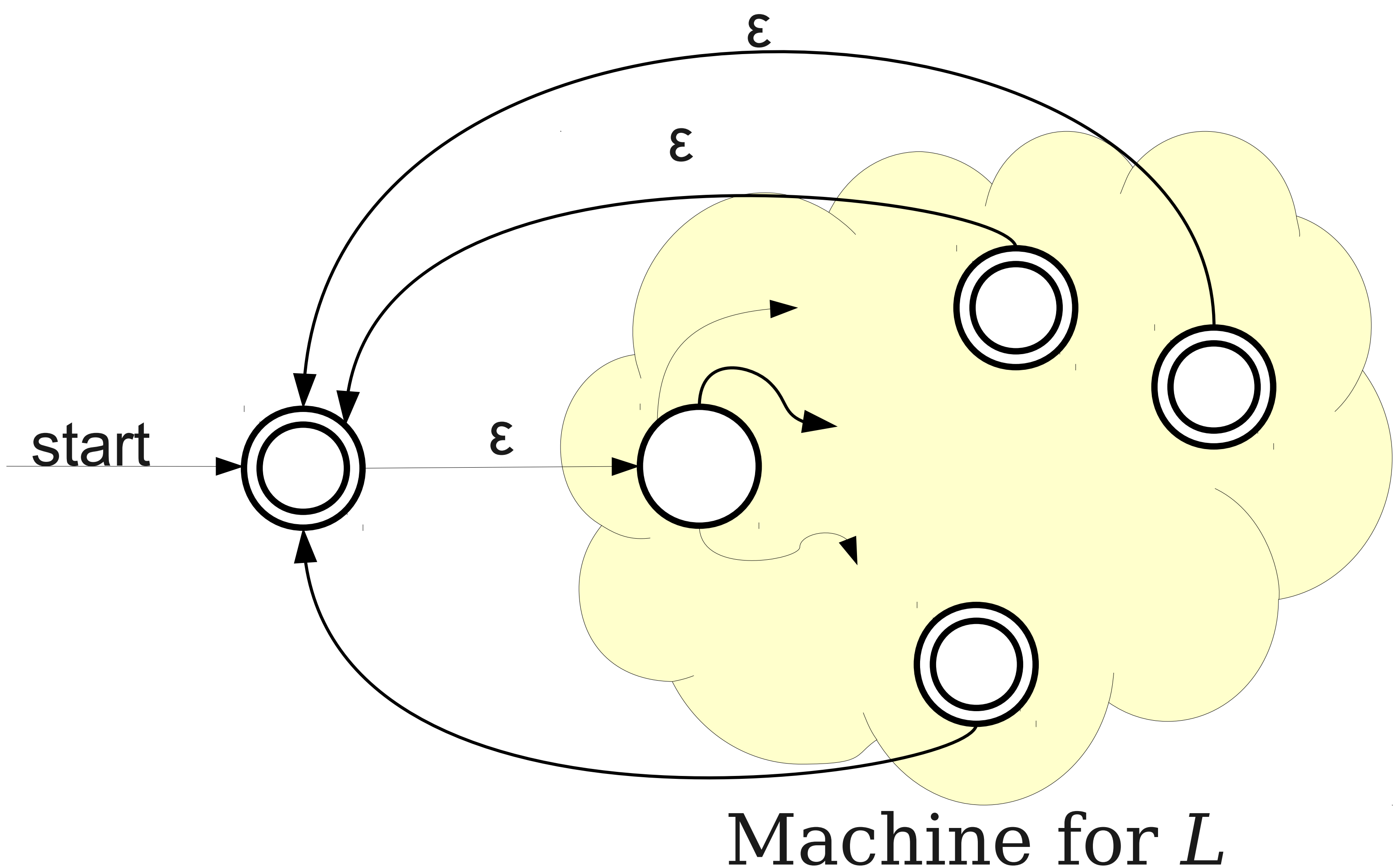

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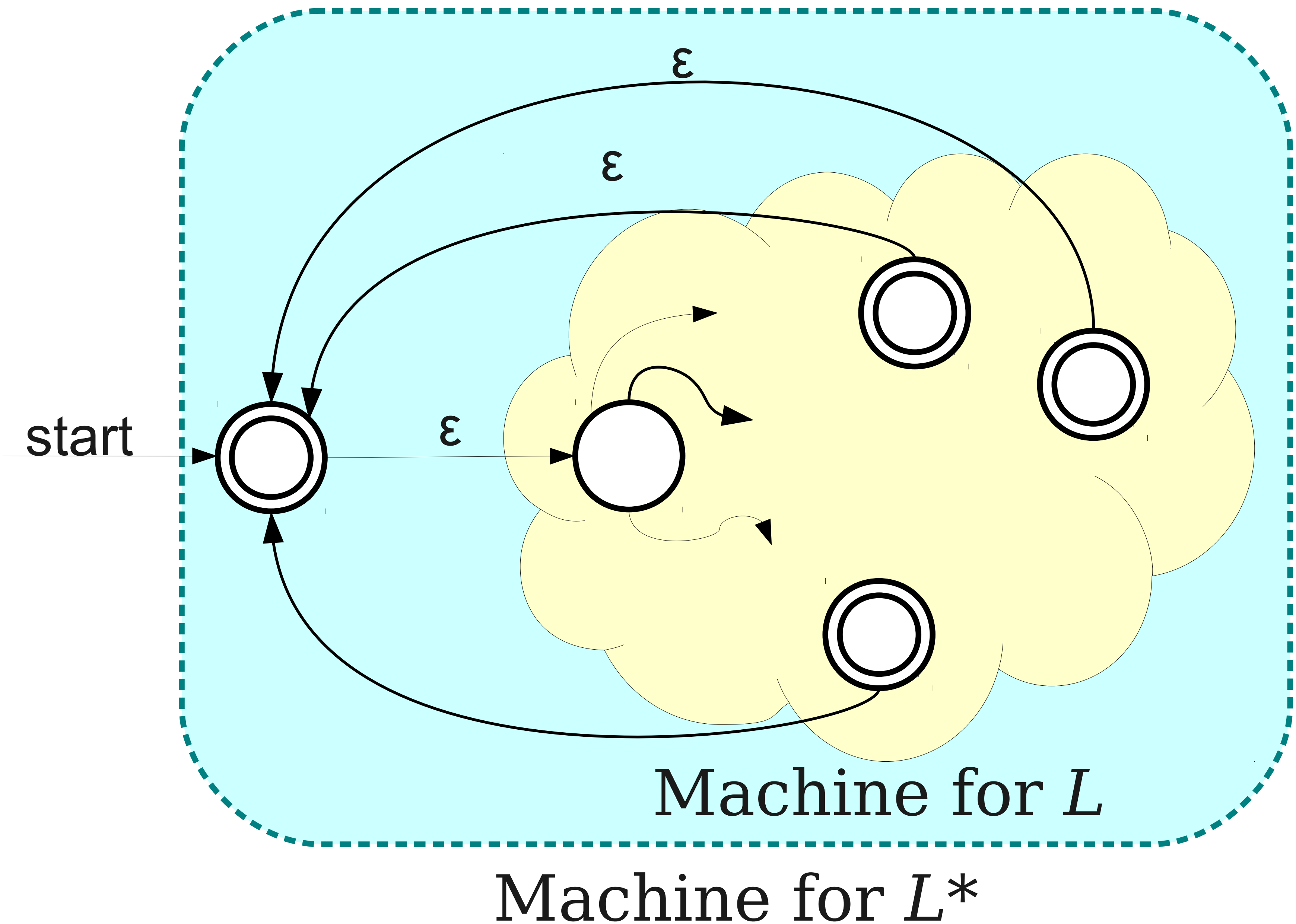
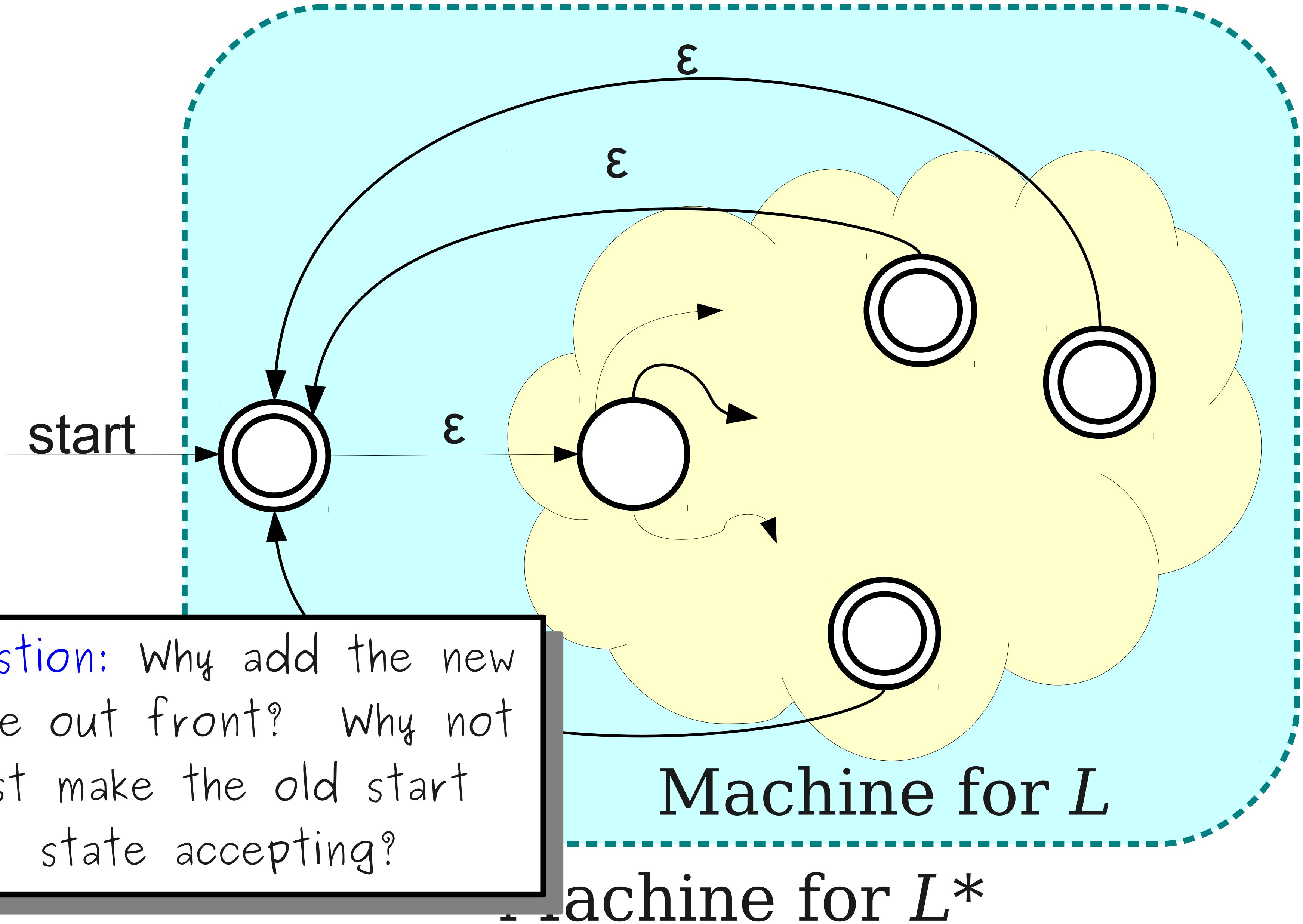


The Kleene Star



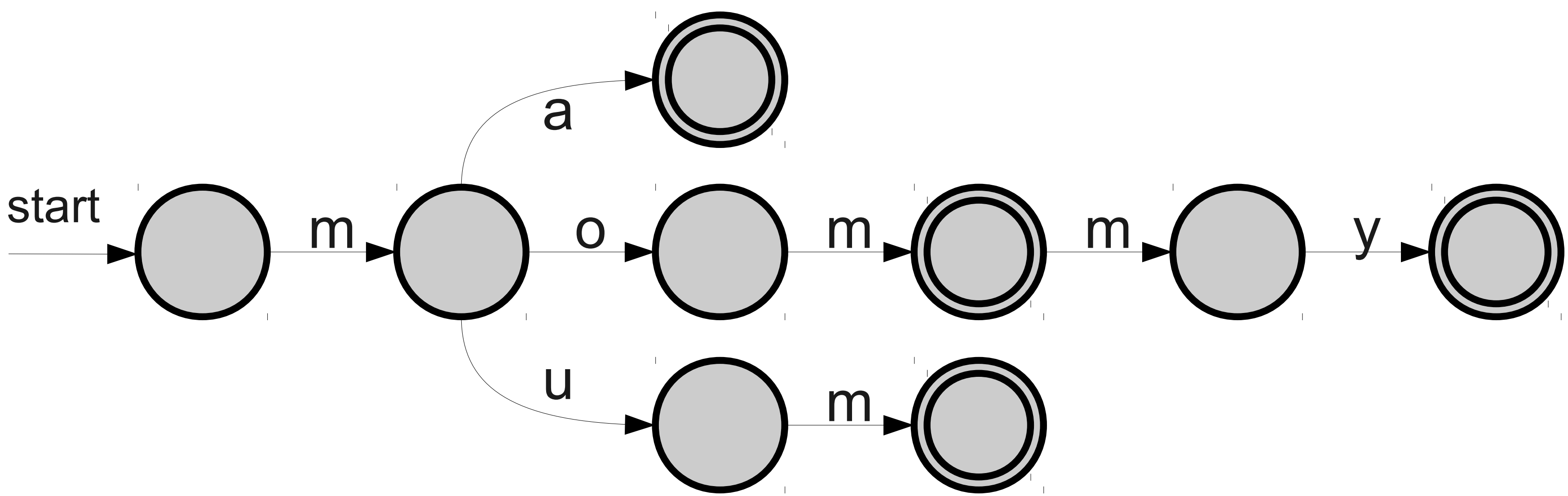
Question: Why add the new state out front? Why not just make the old start state accepting?

Kleene Star in Action

$L = \{$ ma, mom, mommy, mum $\}$

Kleene Star in Action

L = { ma, mom, mommy, mum }



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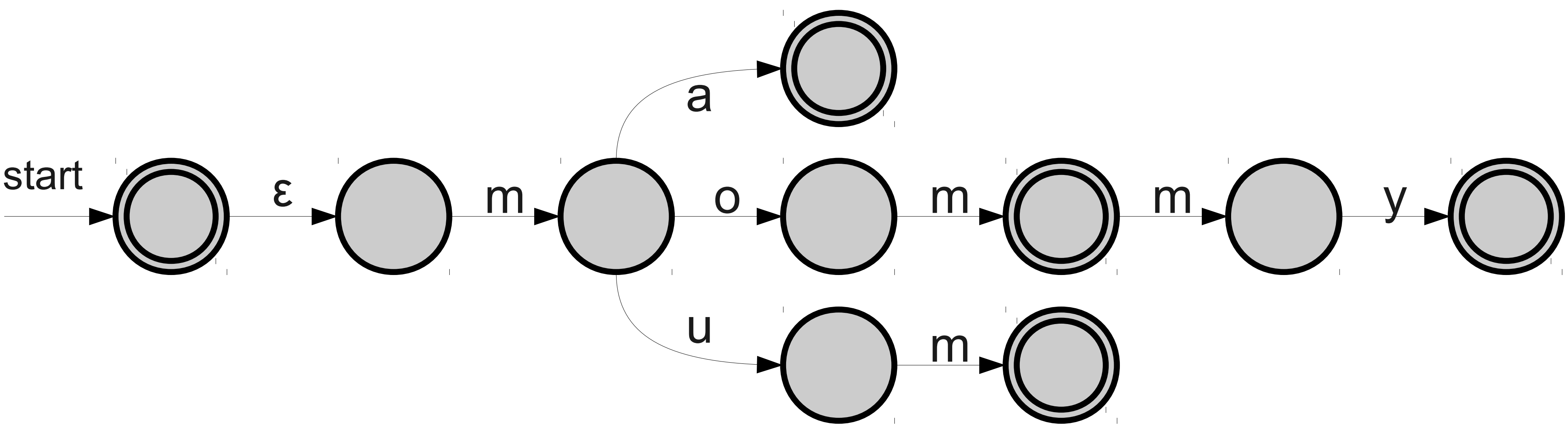

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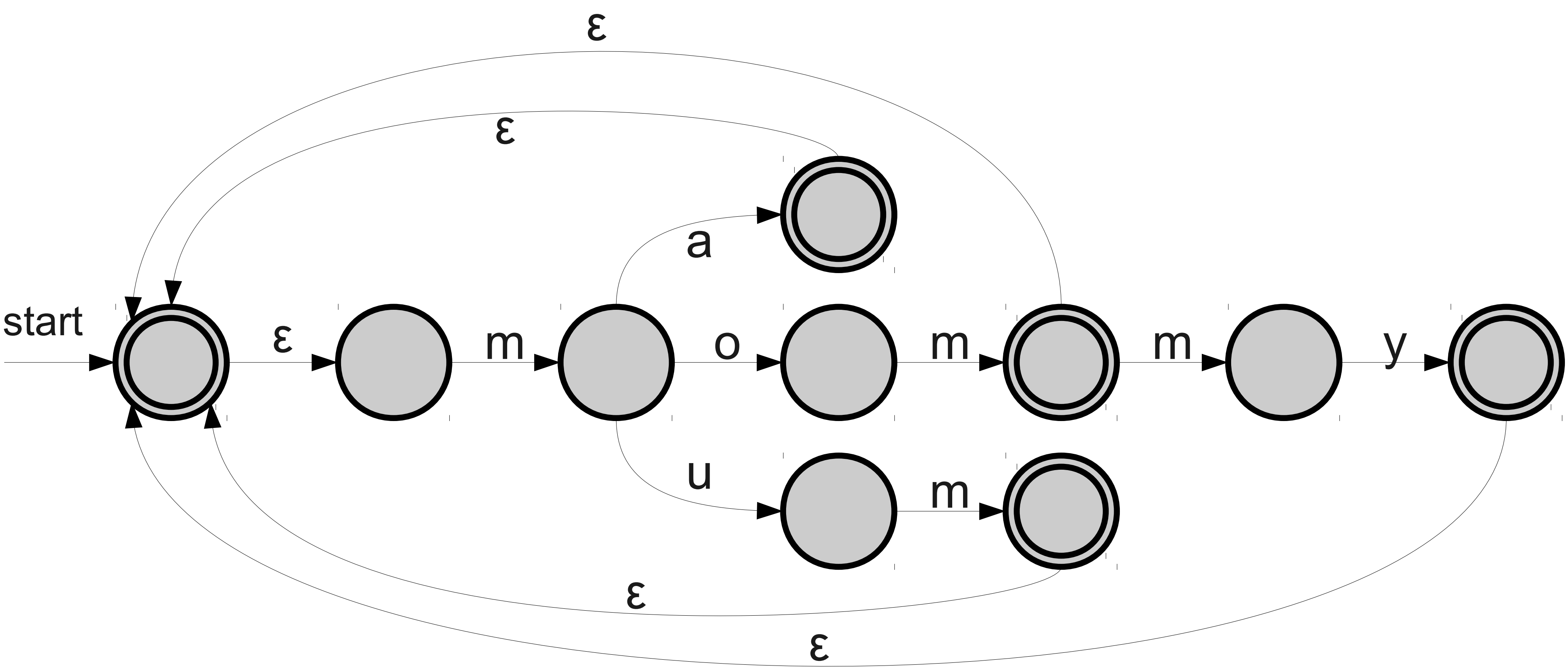
$$L = \{ \text{ma, mom, mommy, mum} \}$$



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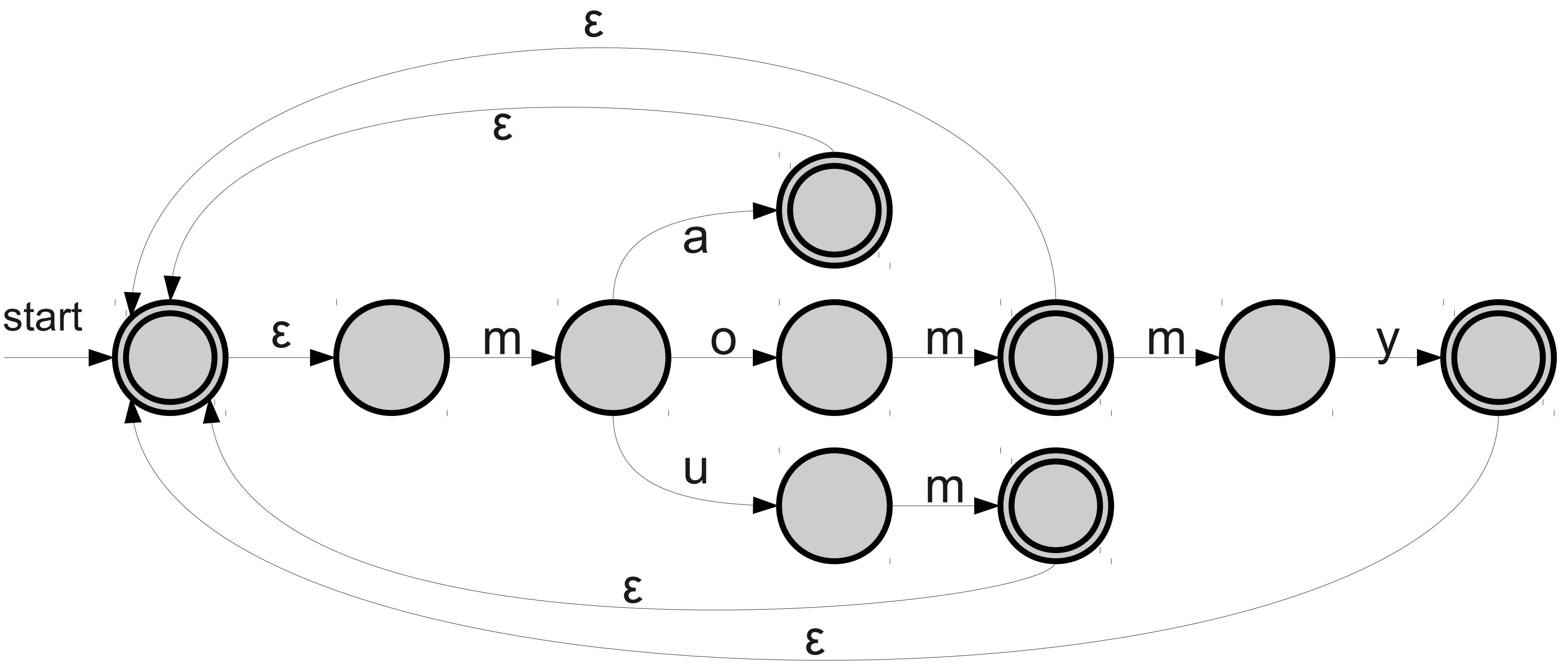

m a m o m m u m

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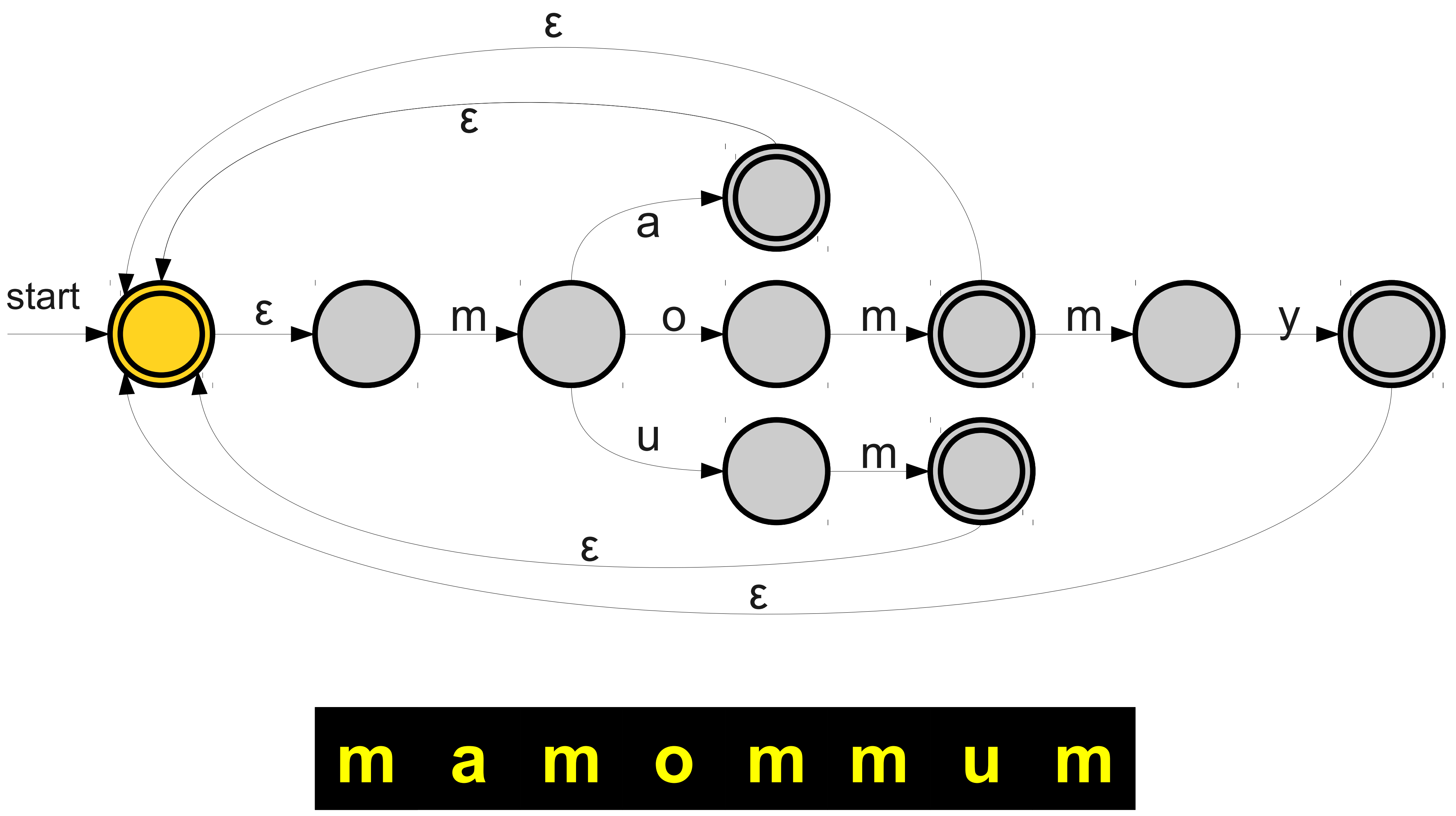

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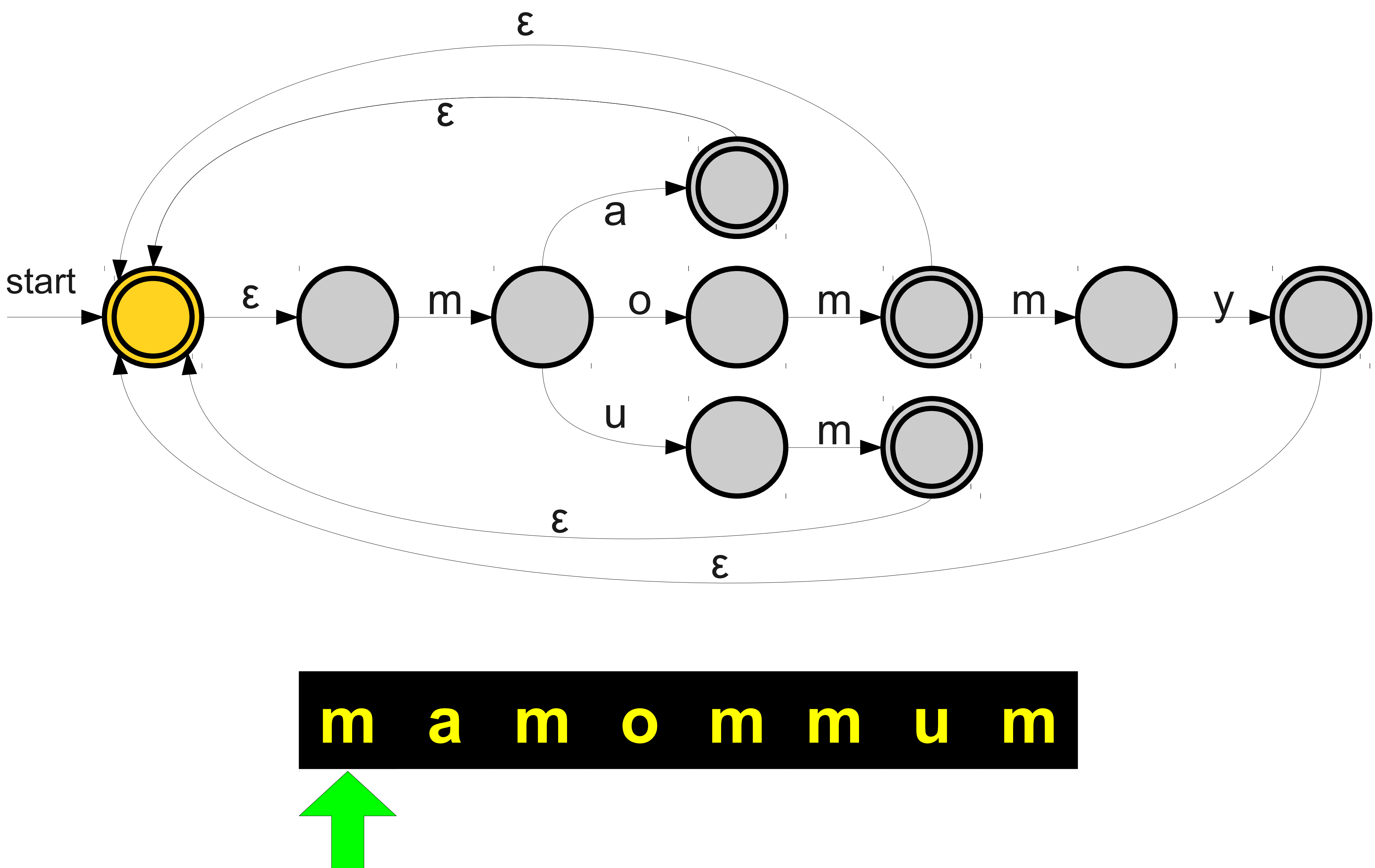

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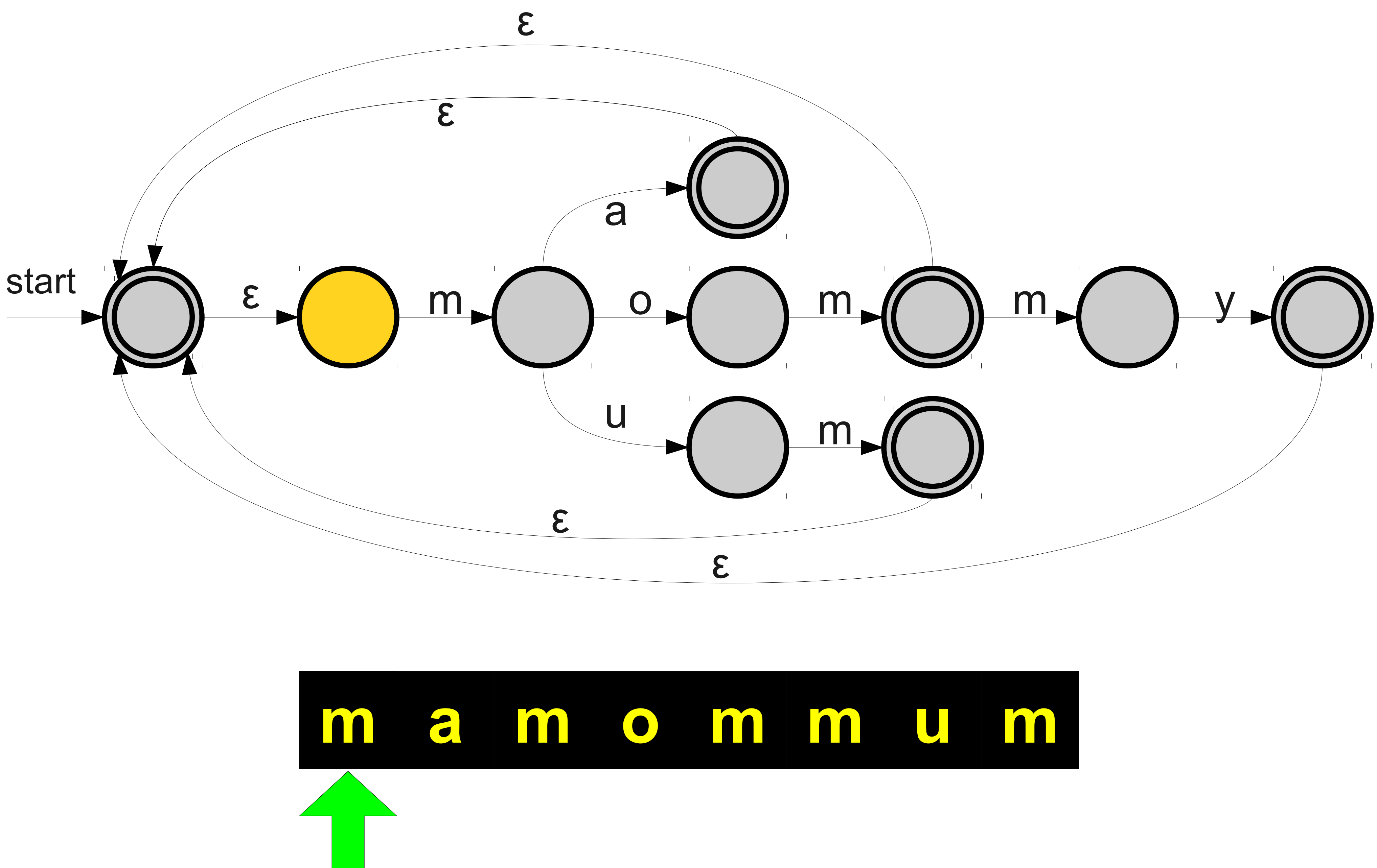

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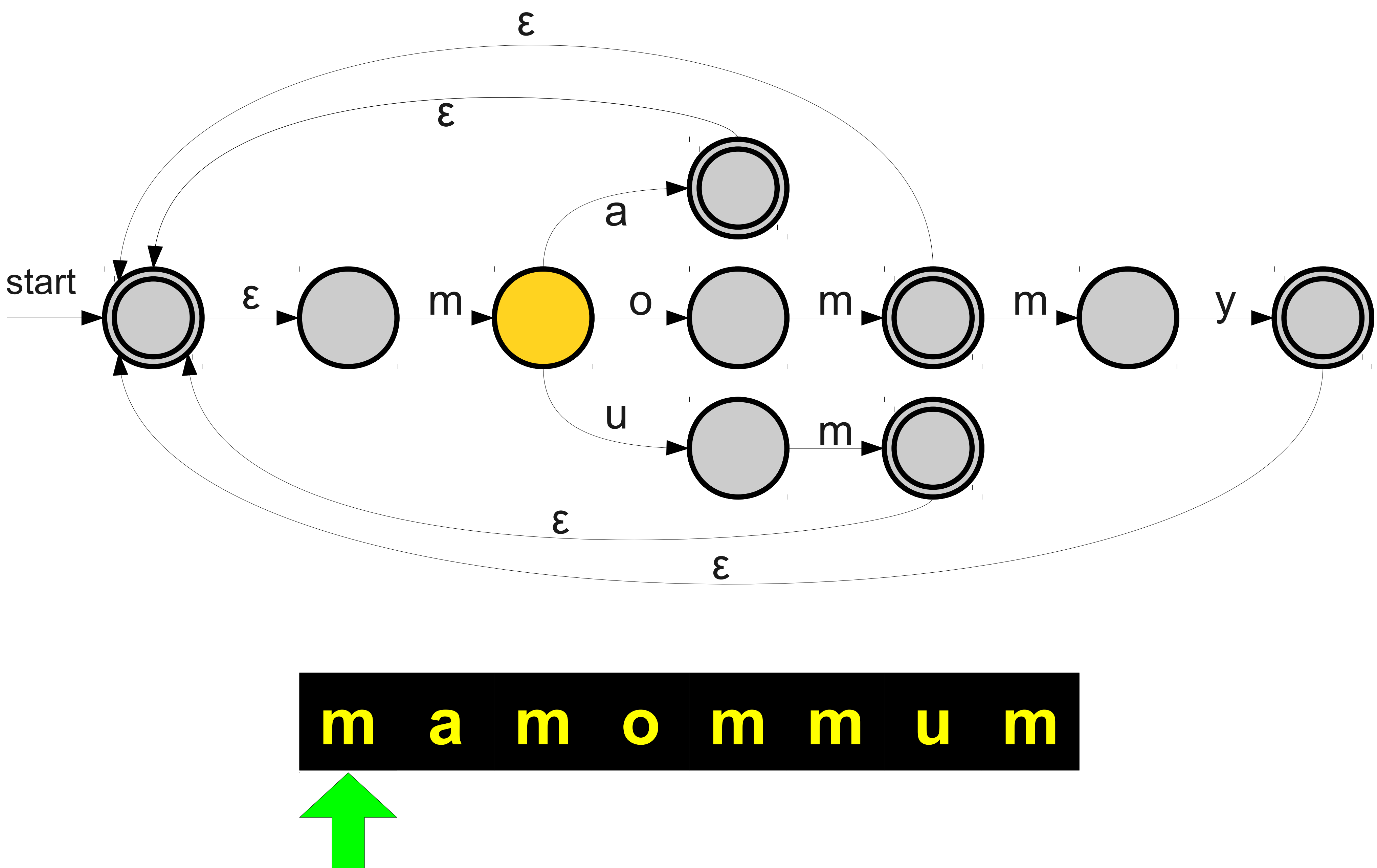

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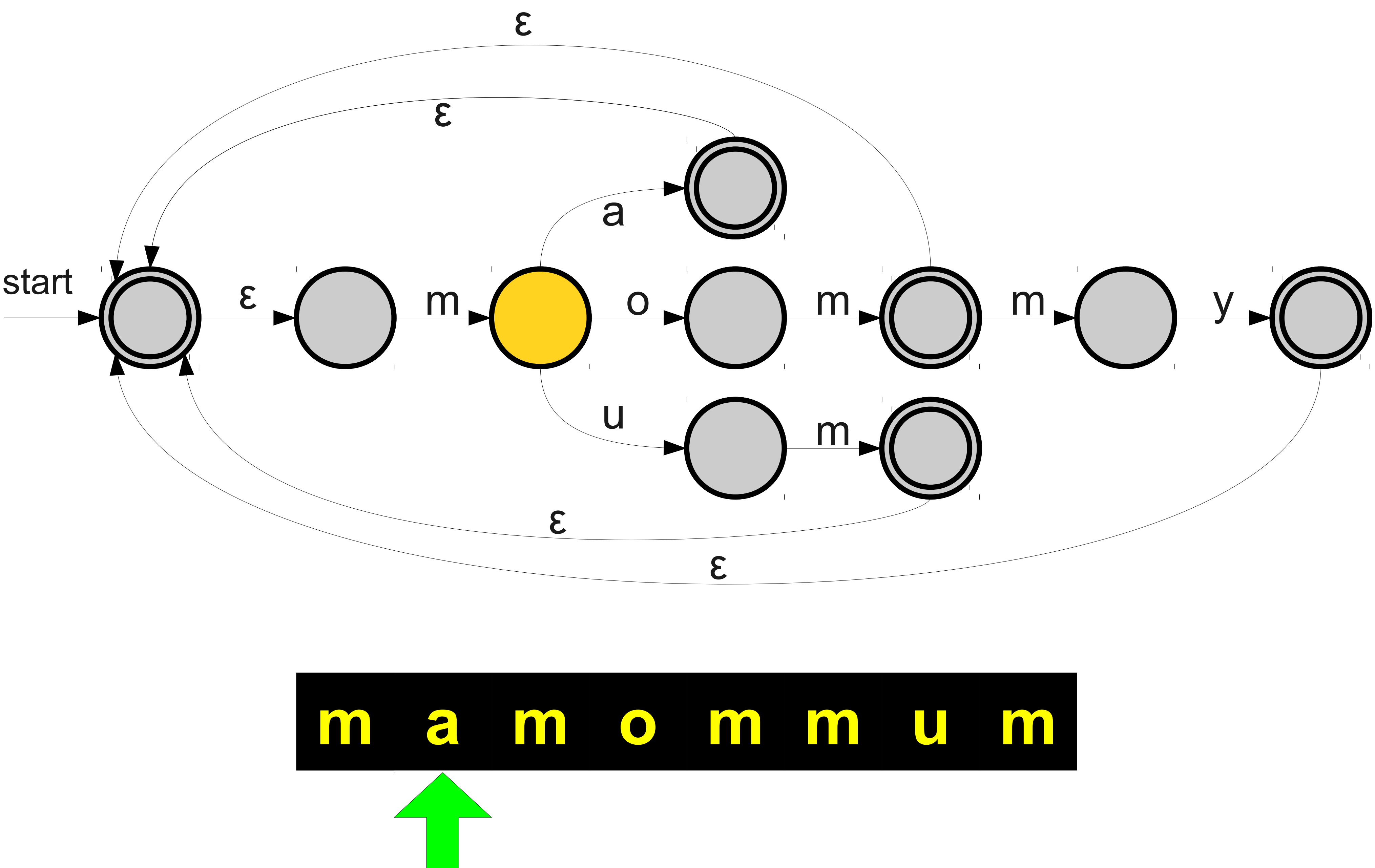

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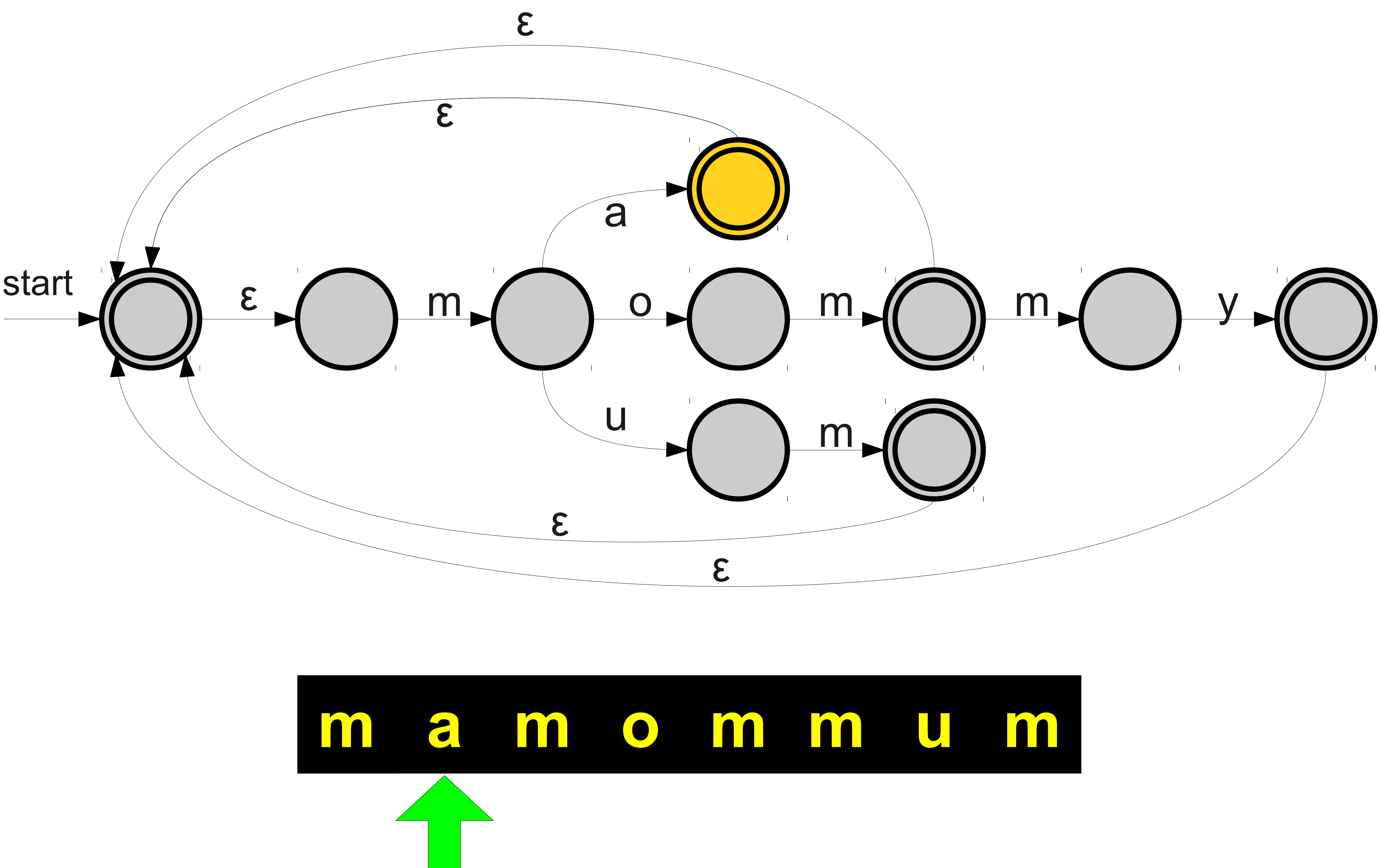

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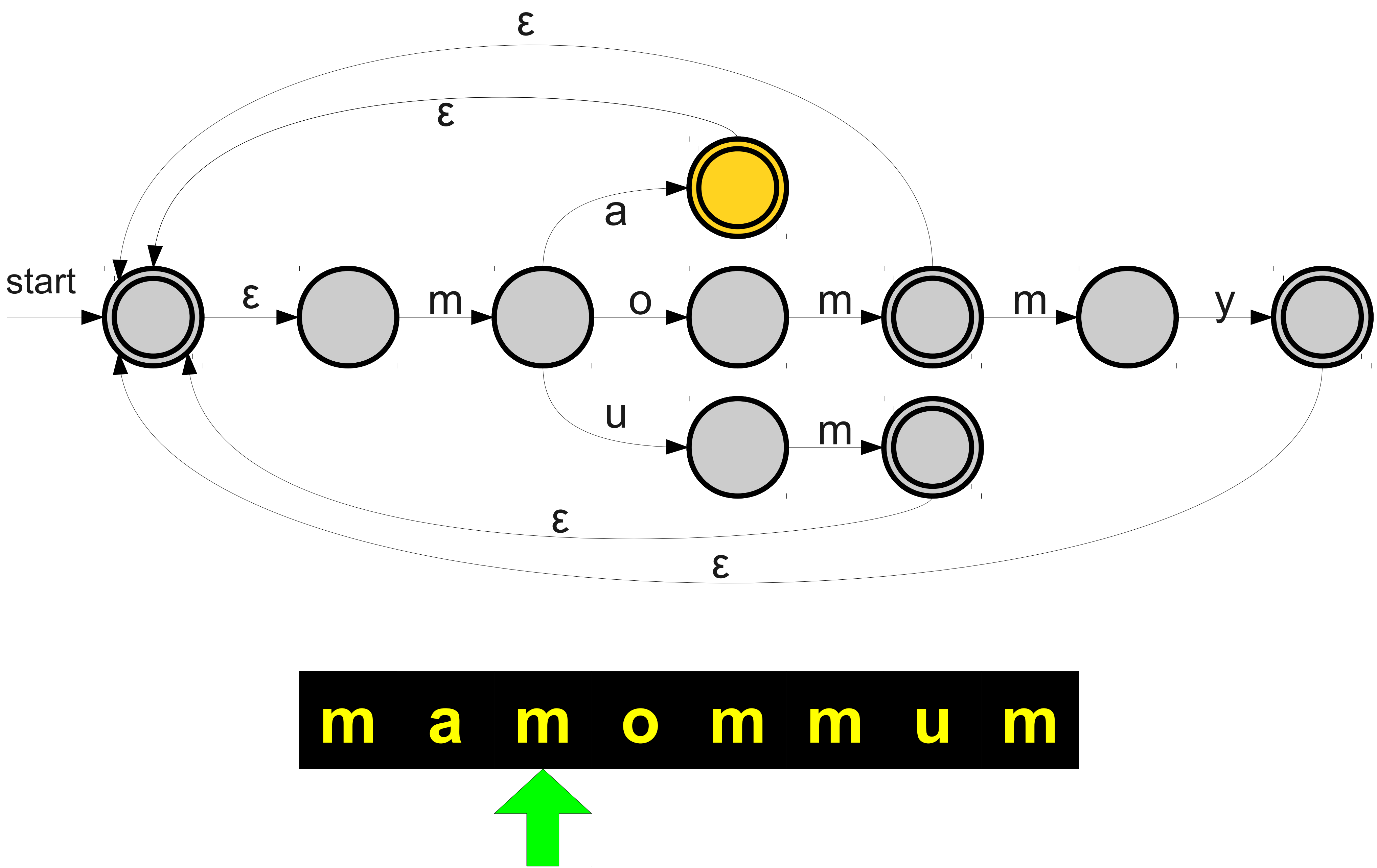

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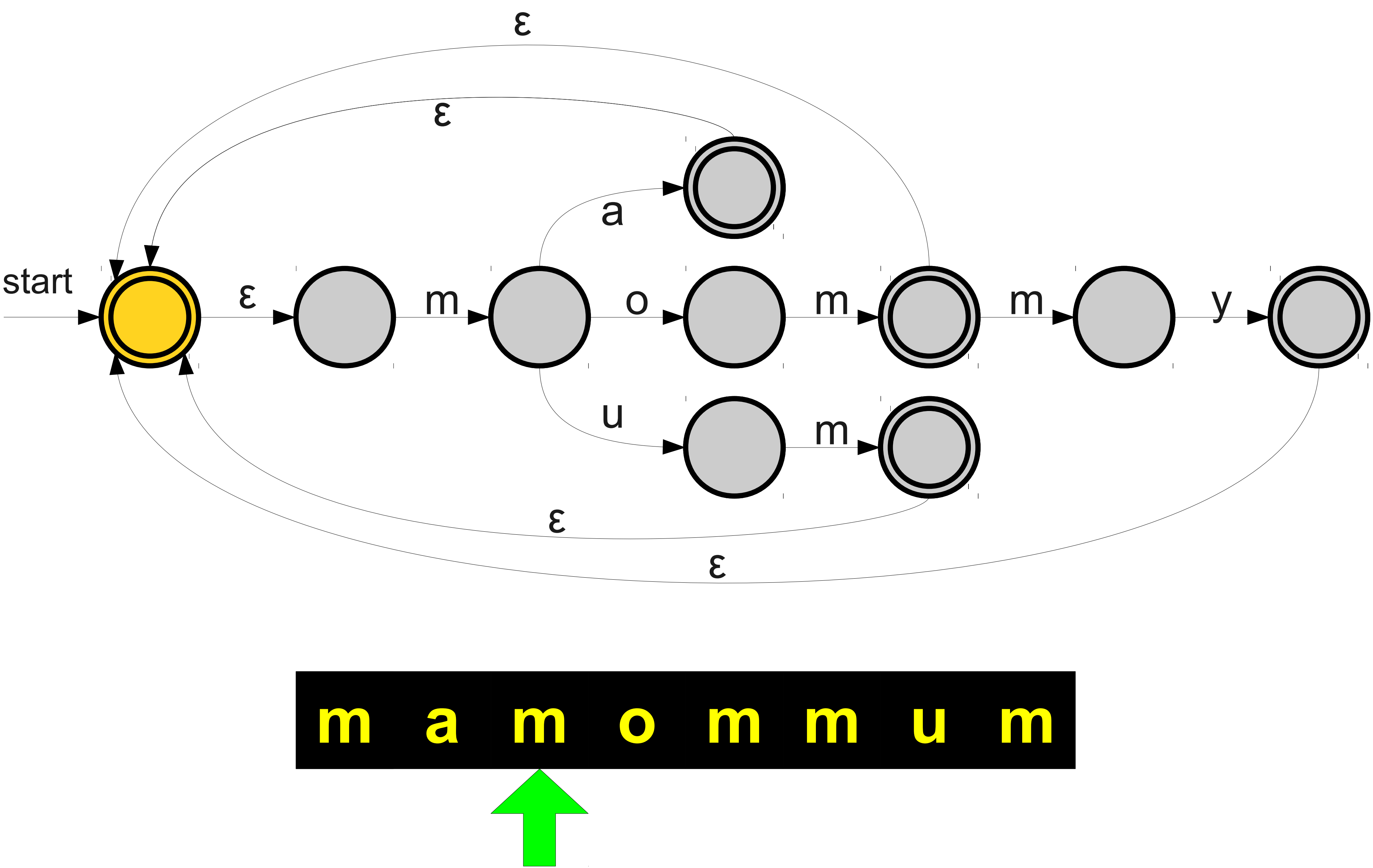

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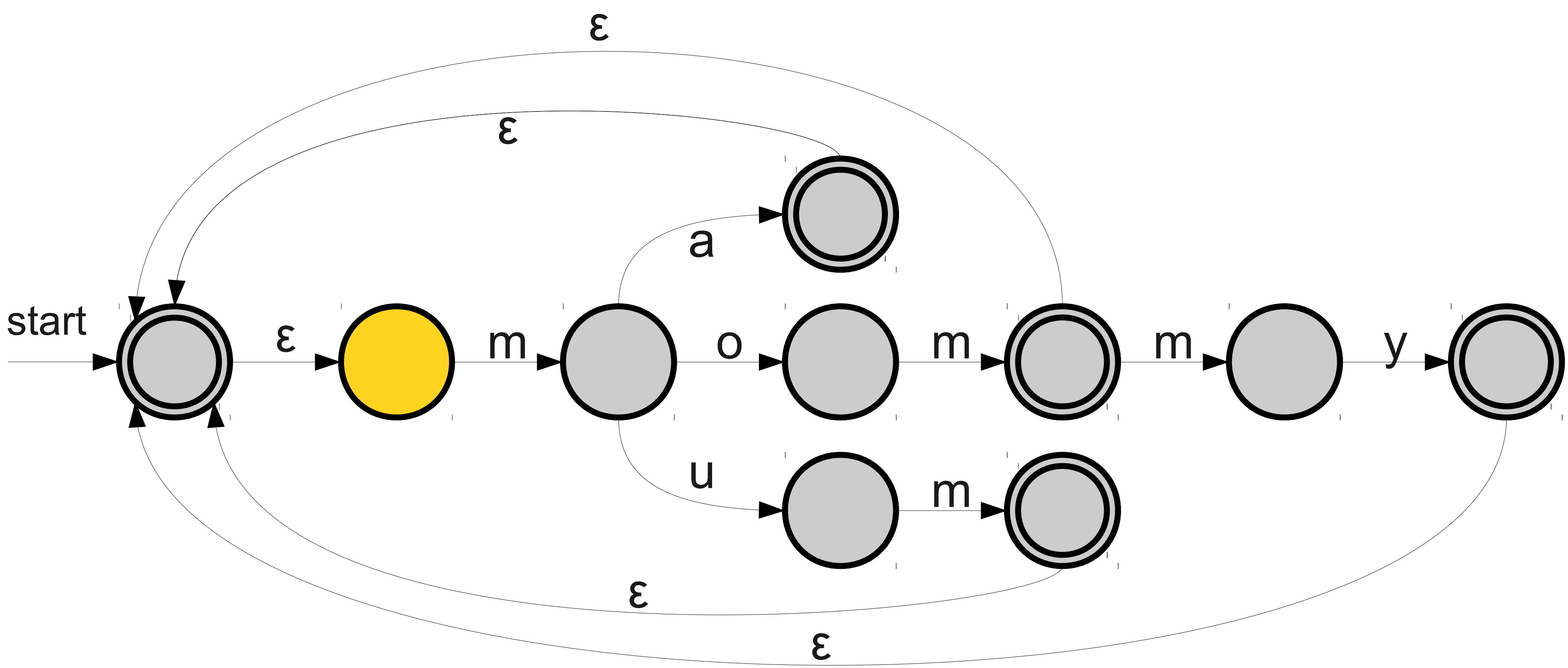

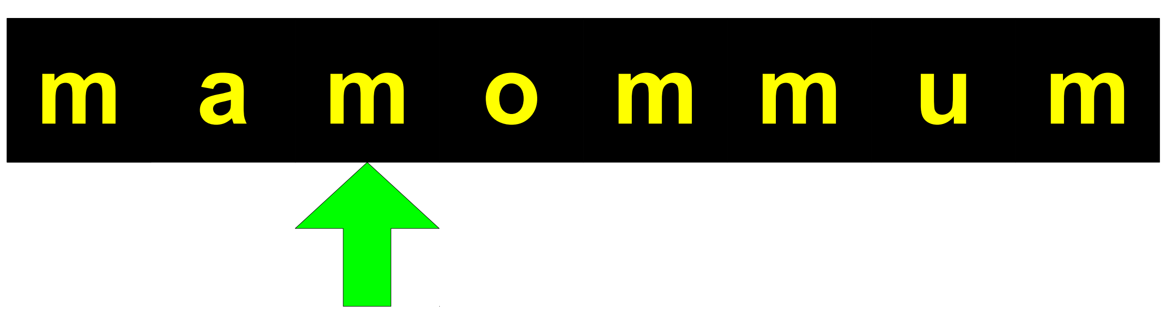

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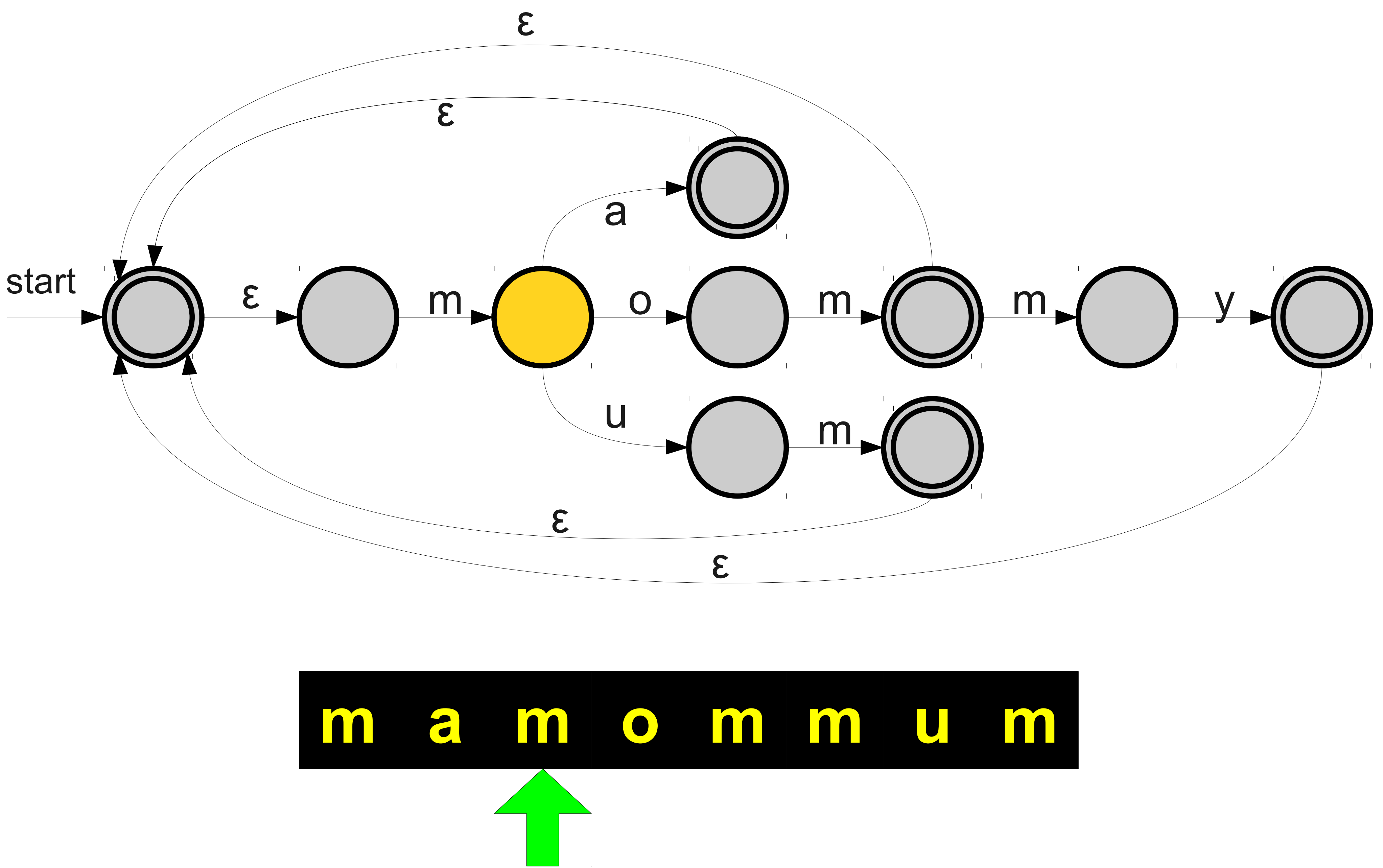

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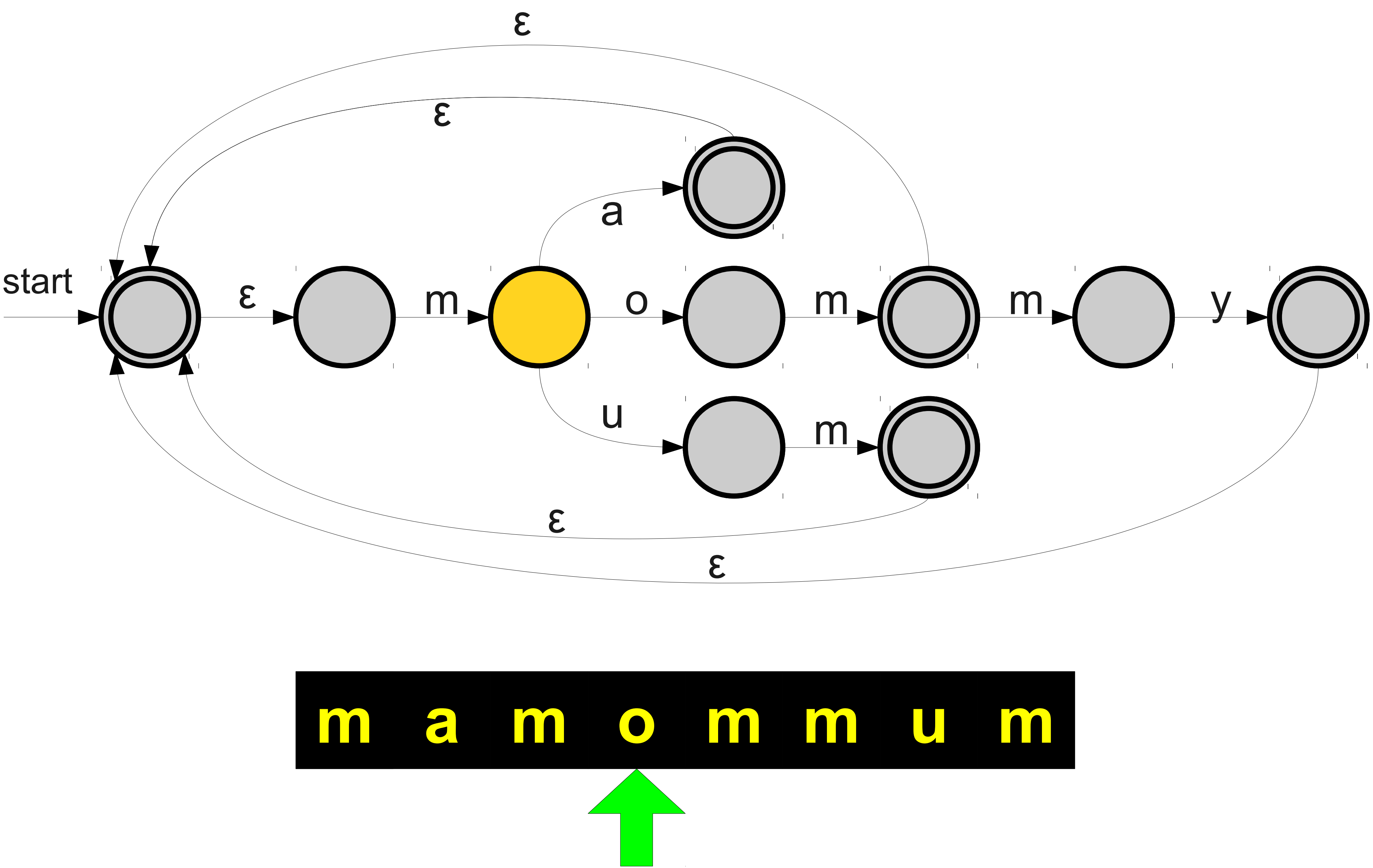

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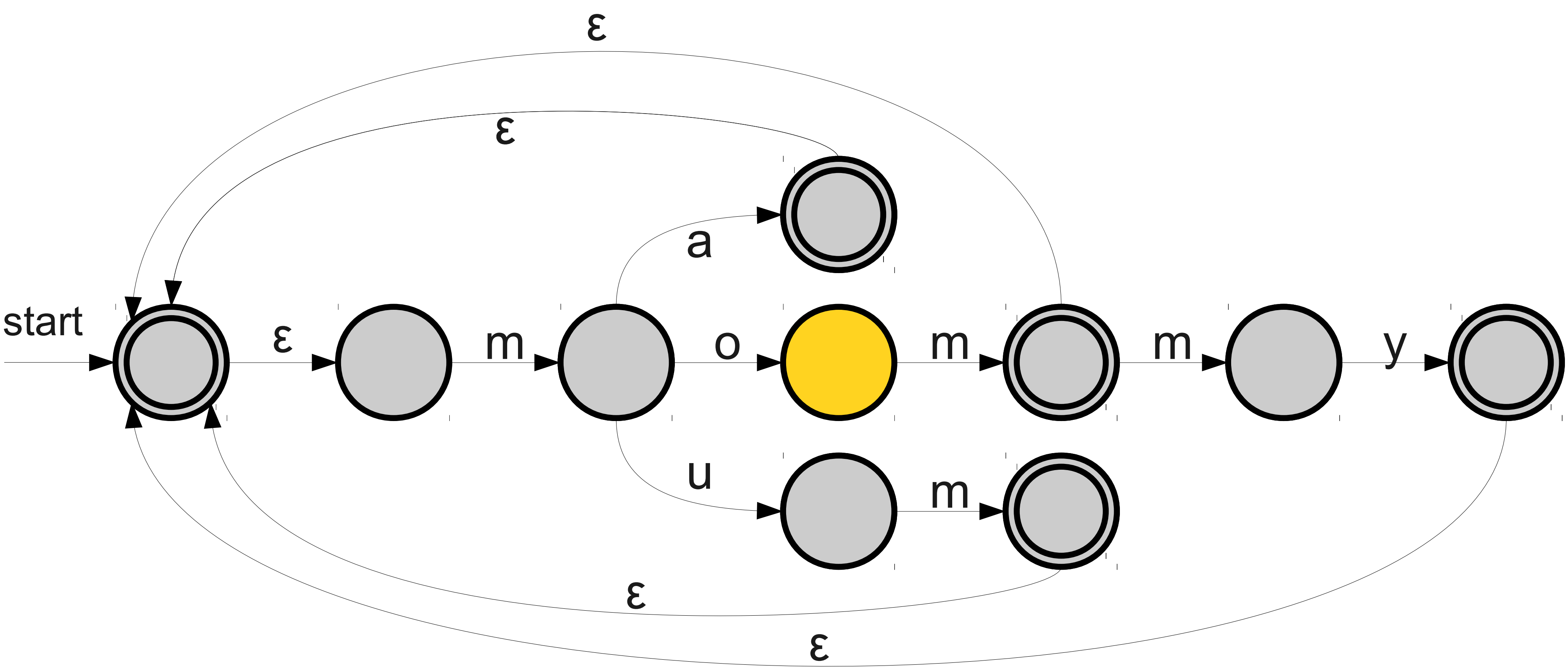

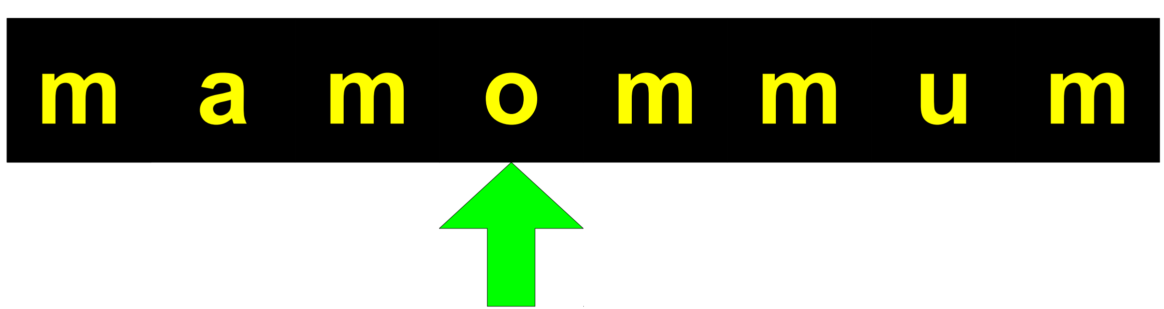

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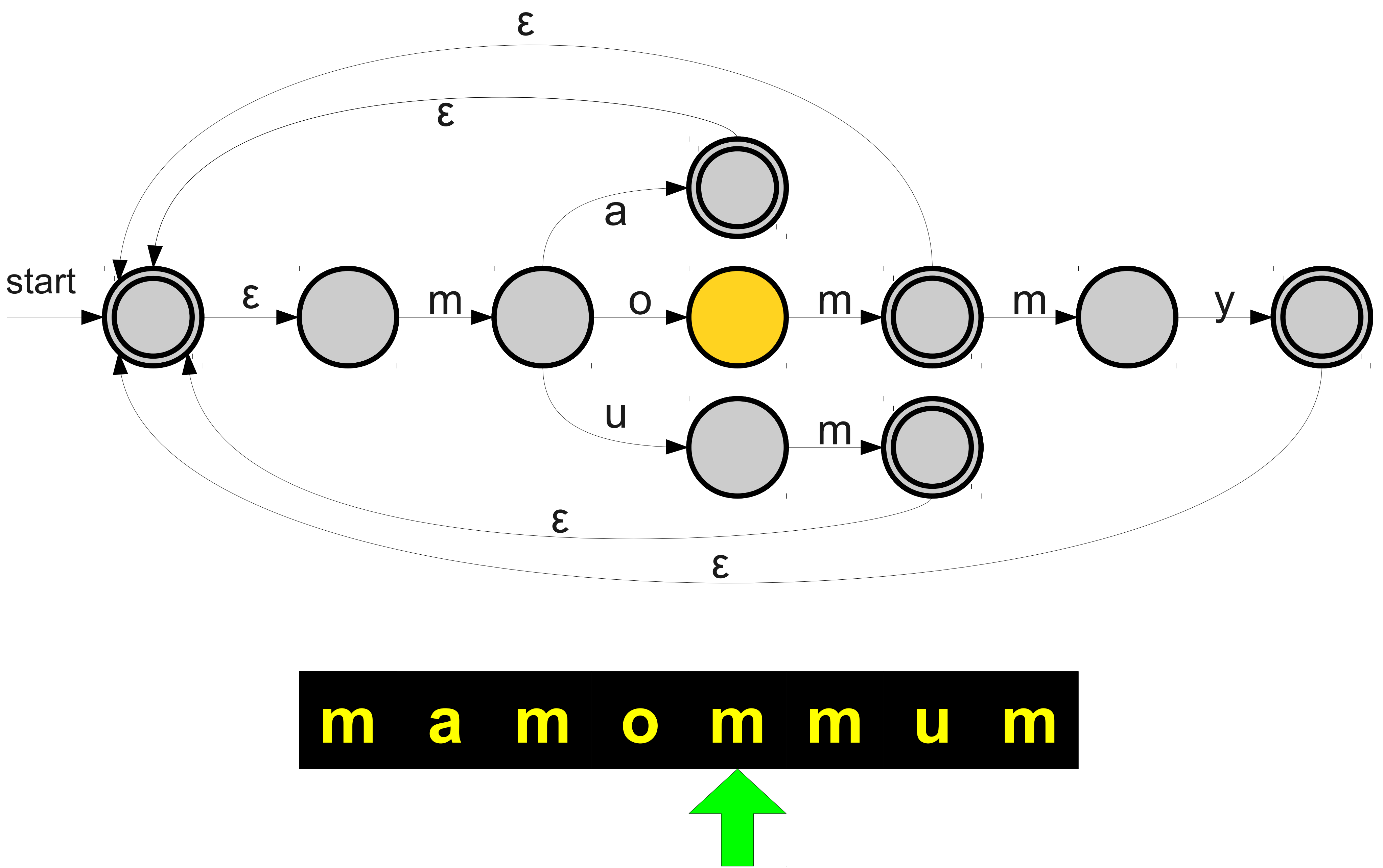

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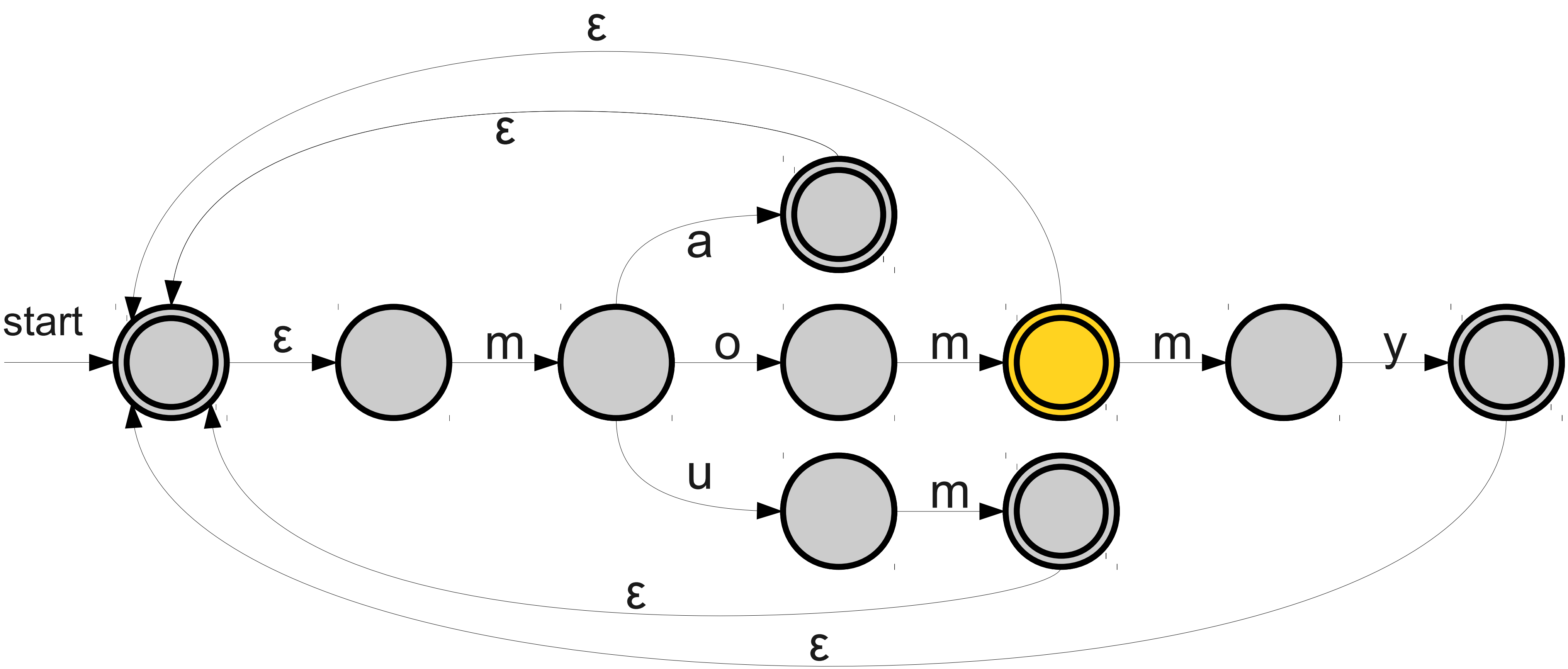

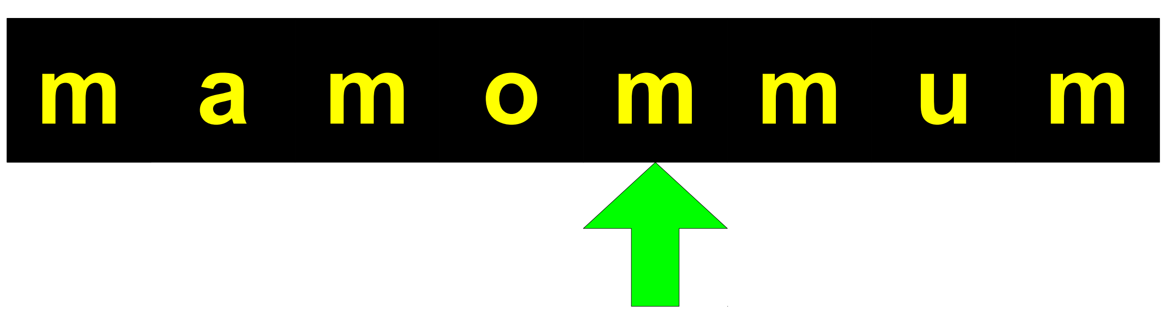

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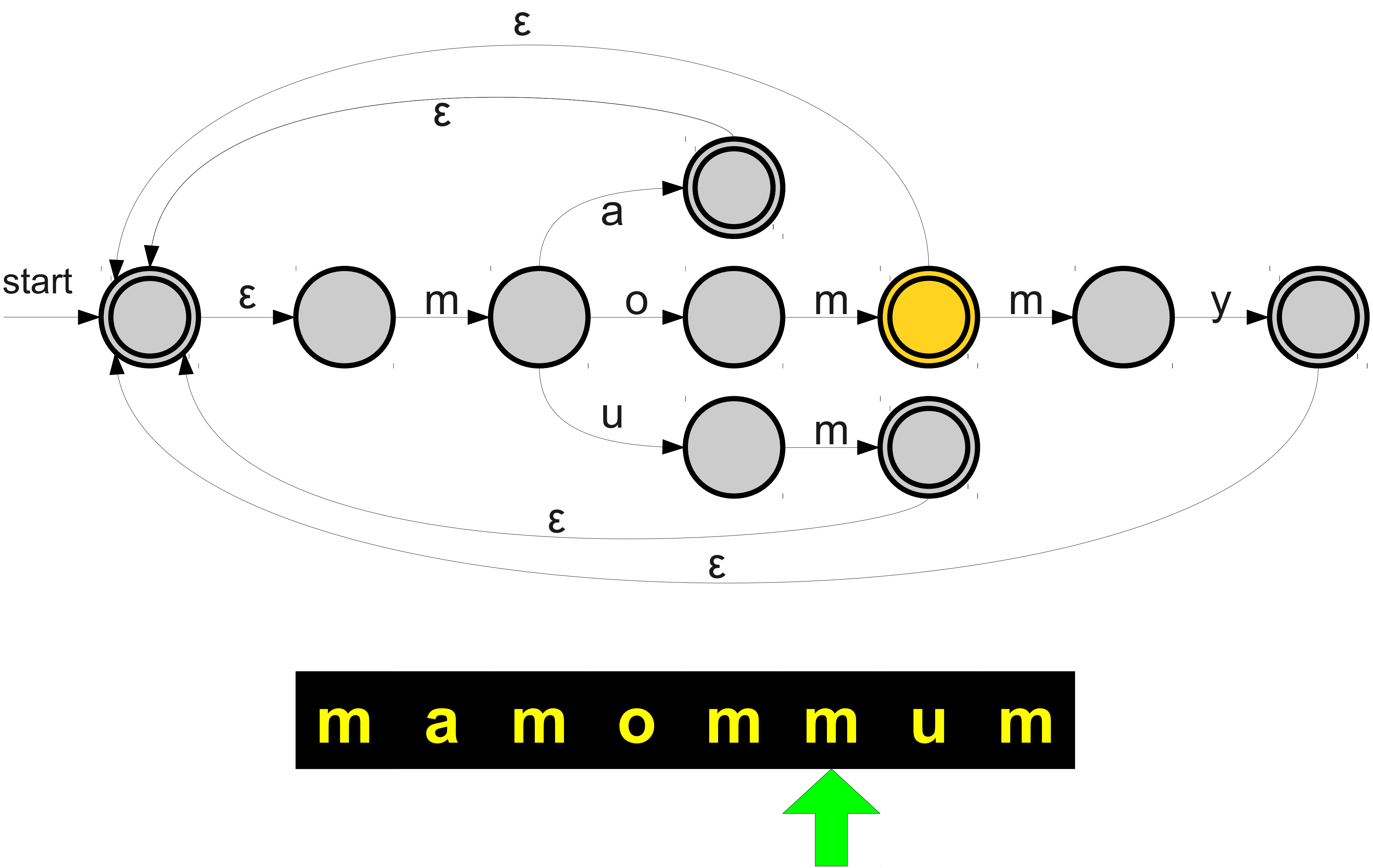

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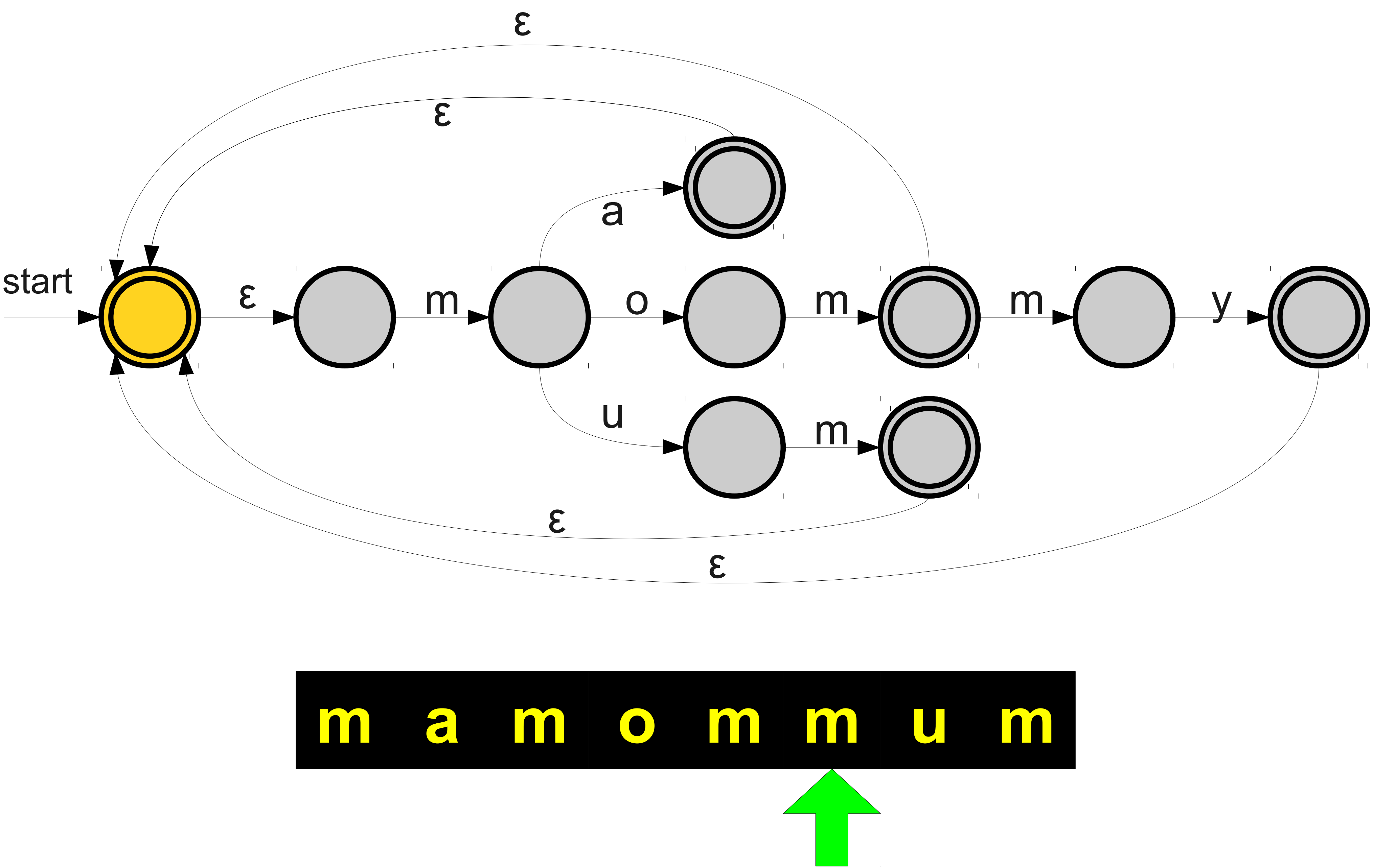

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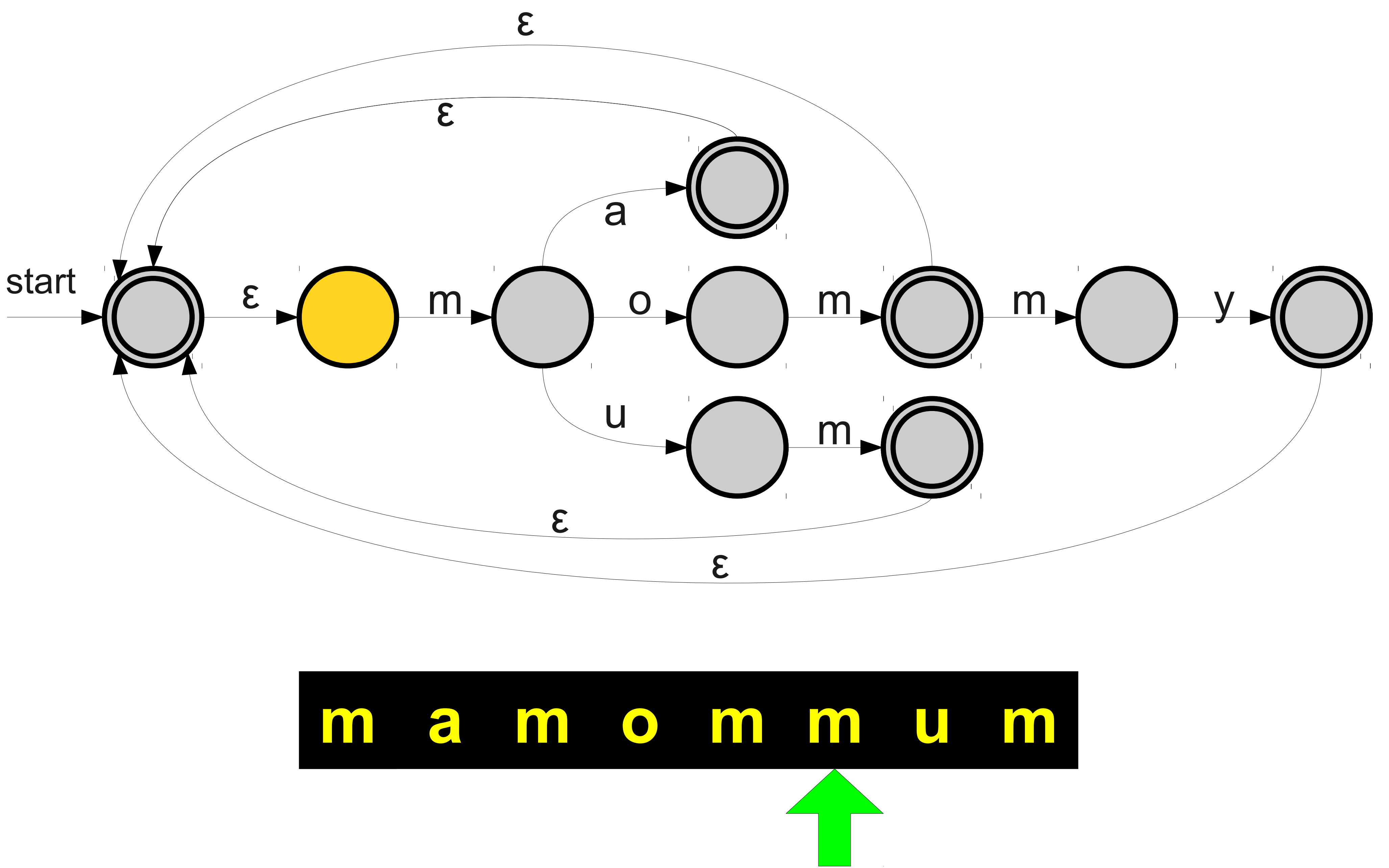

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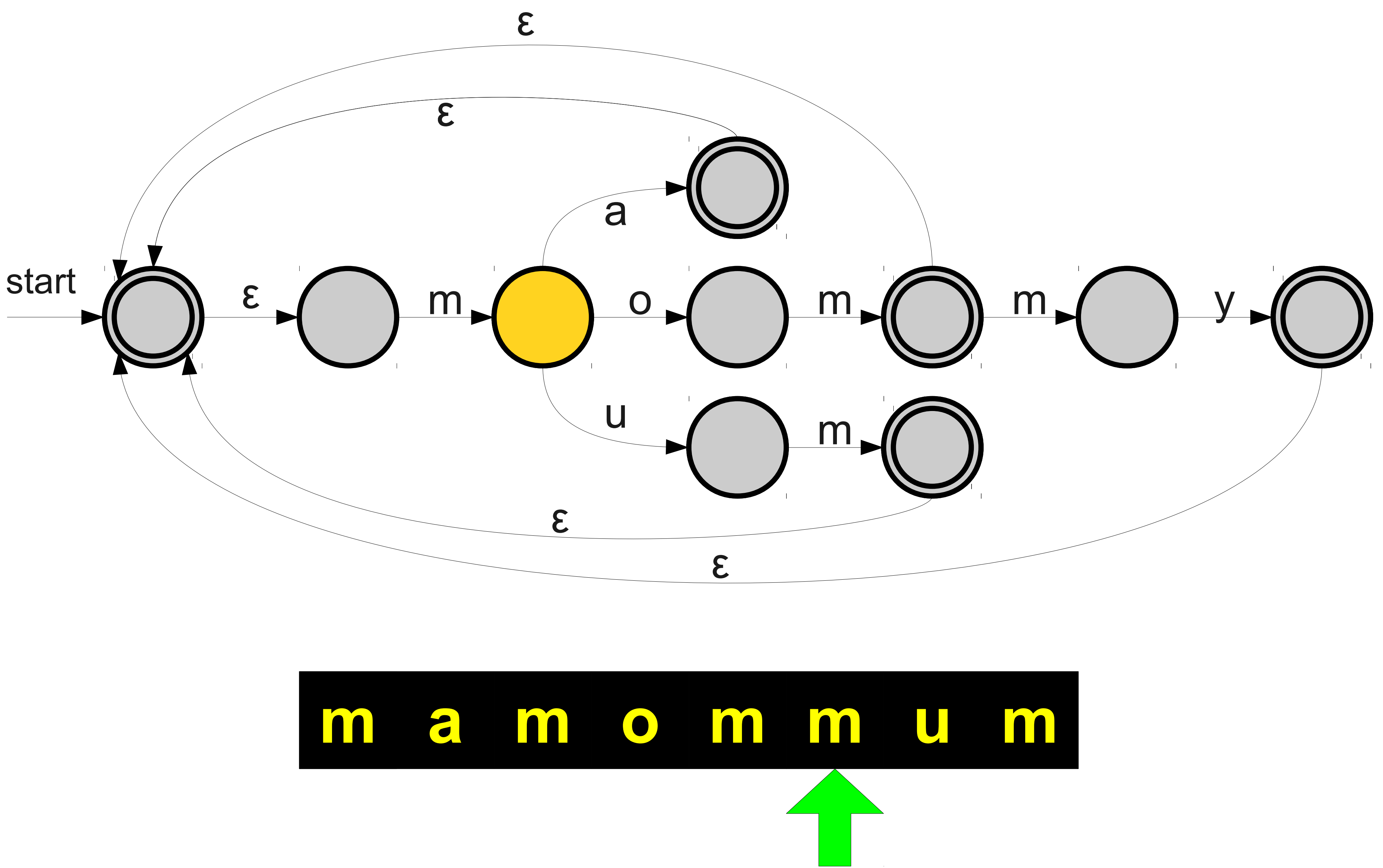

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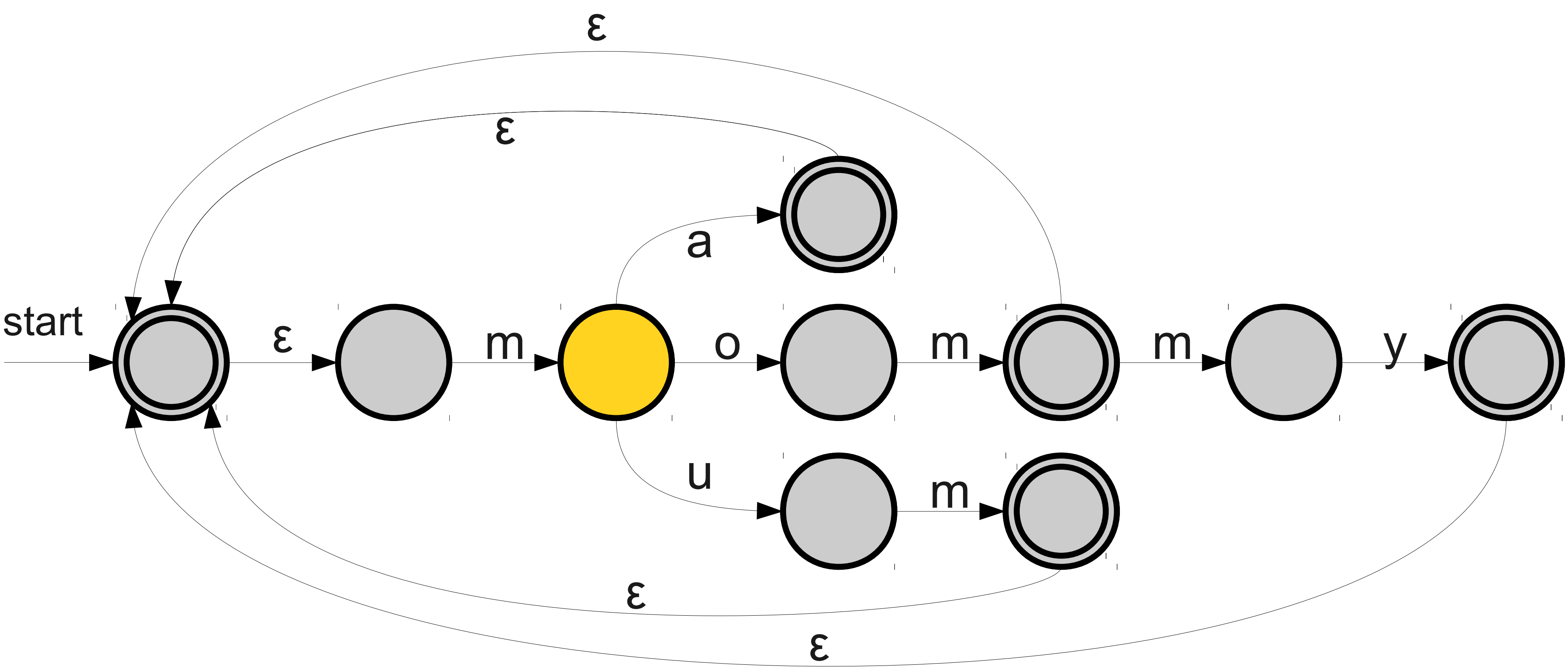

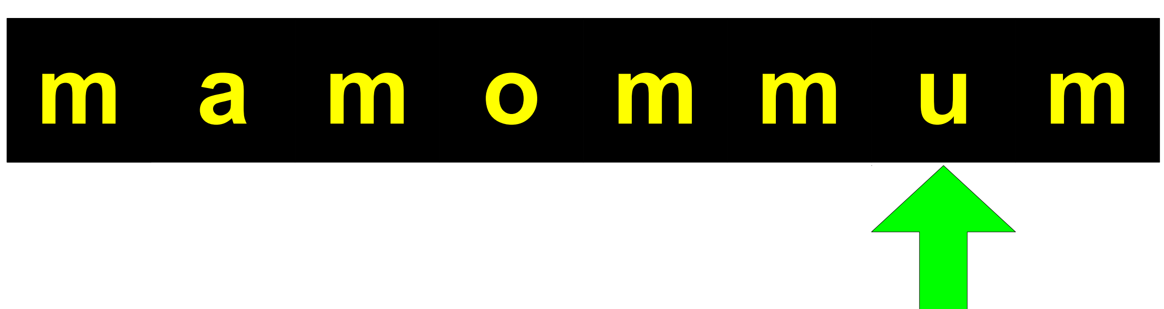

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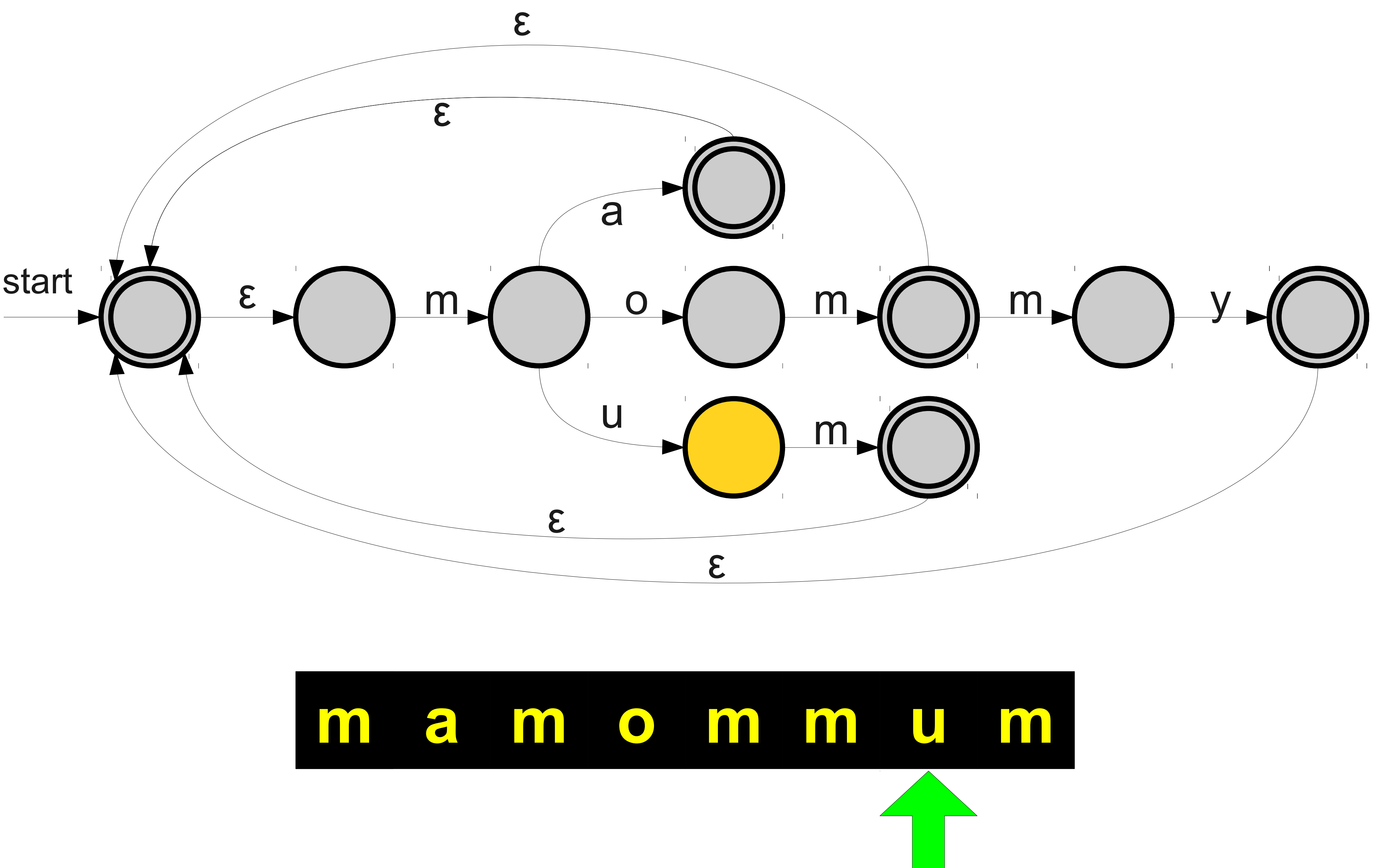

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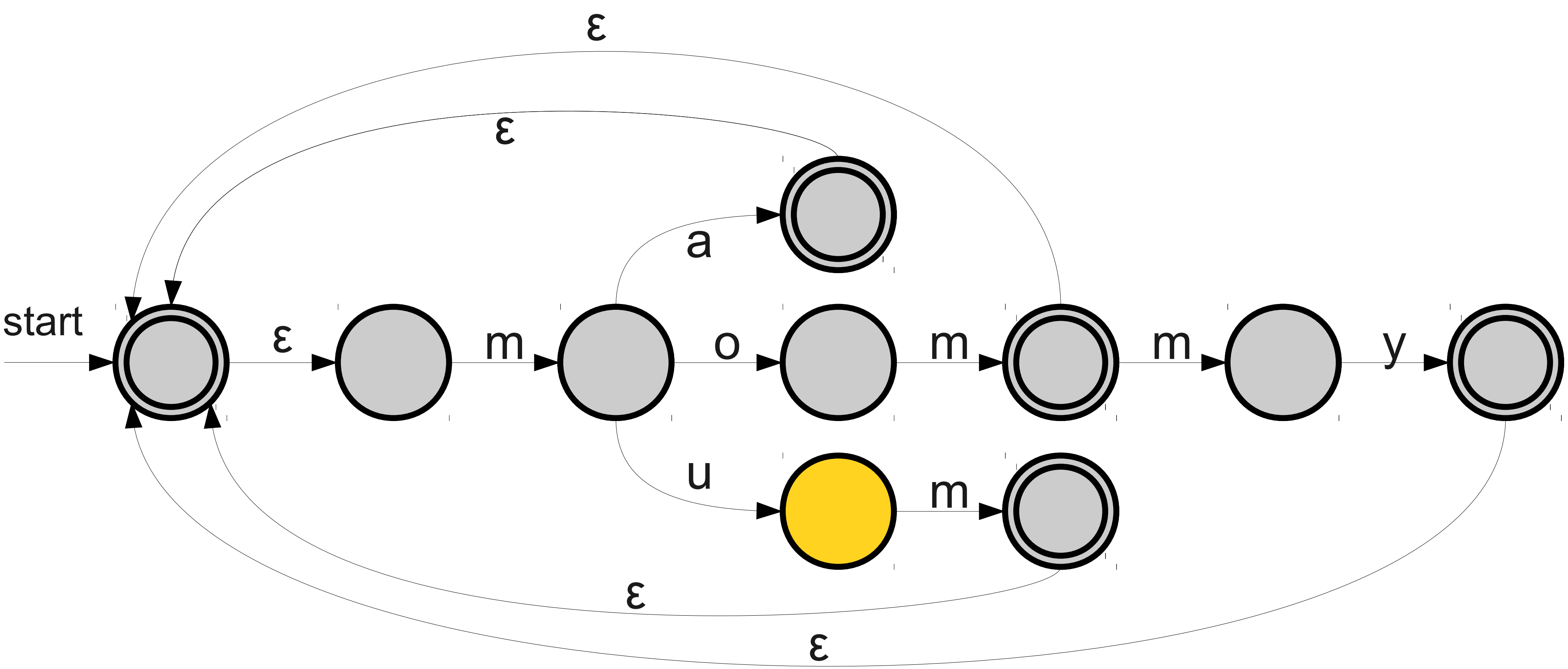

m a m o m m u m

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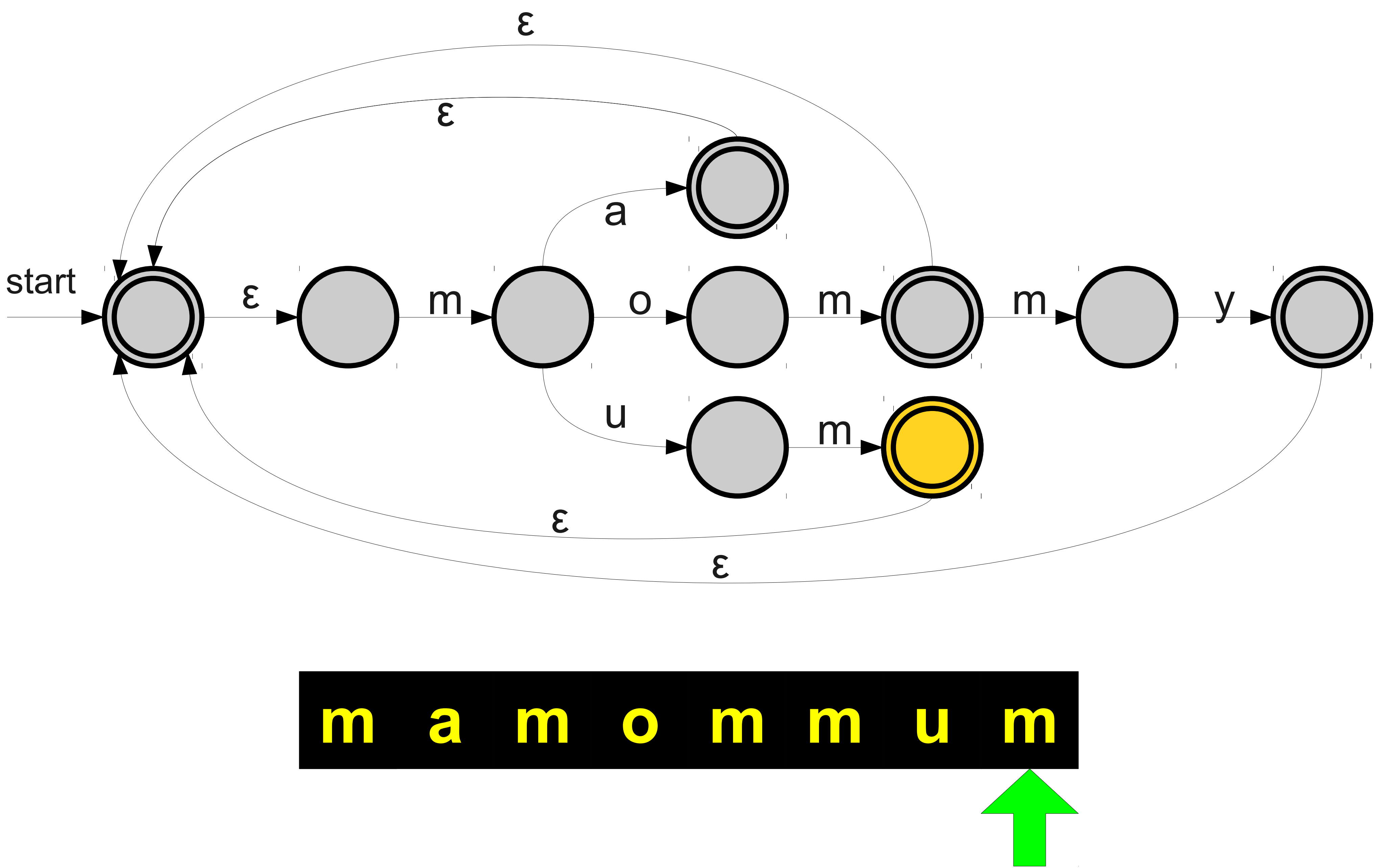

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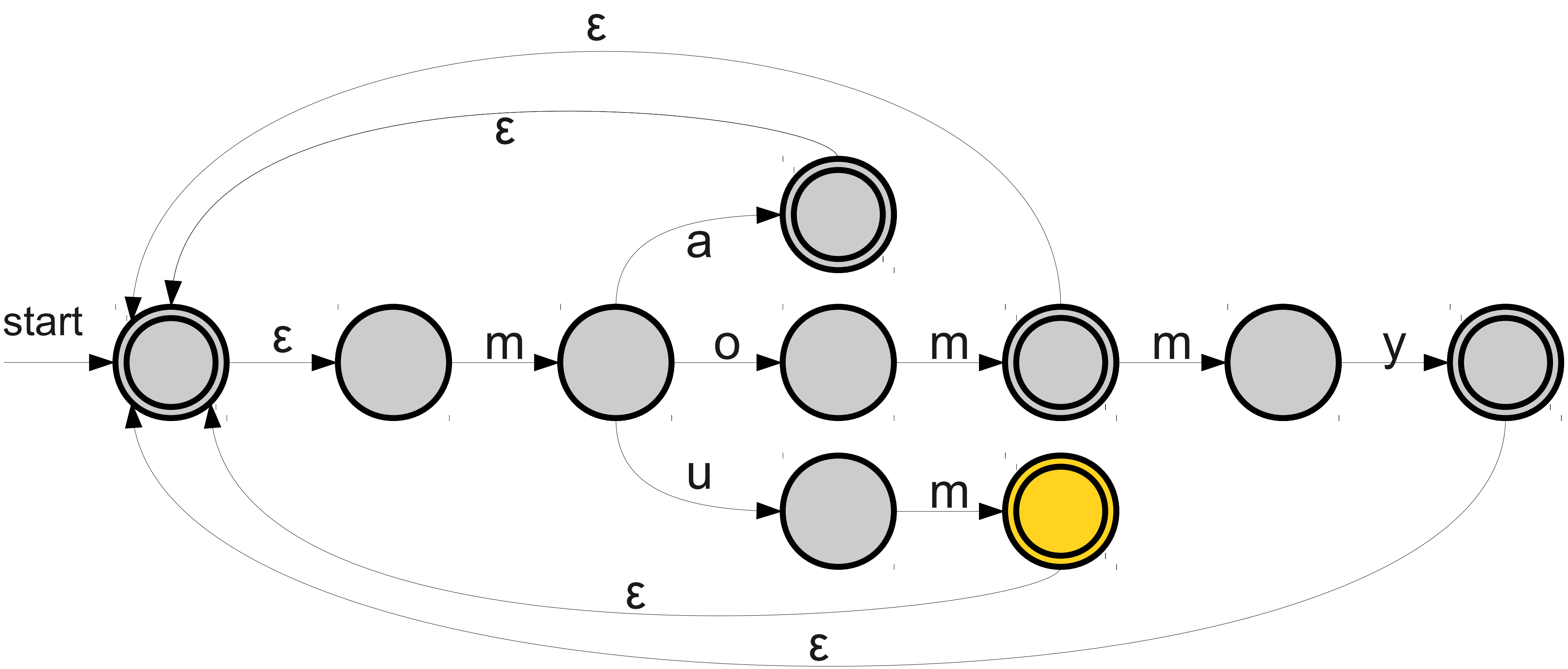

m	a	m	o	m	m	u	m

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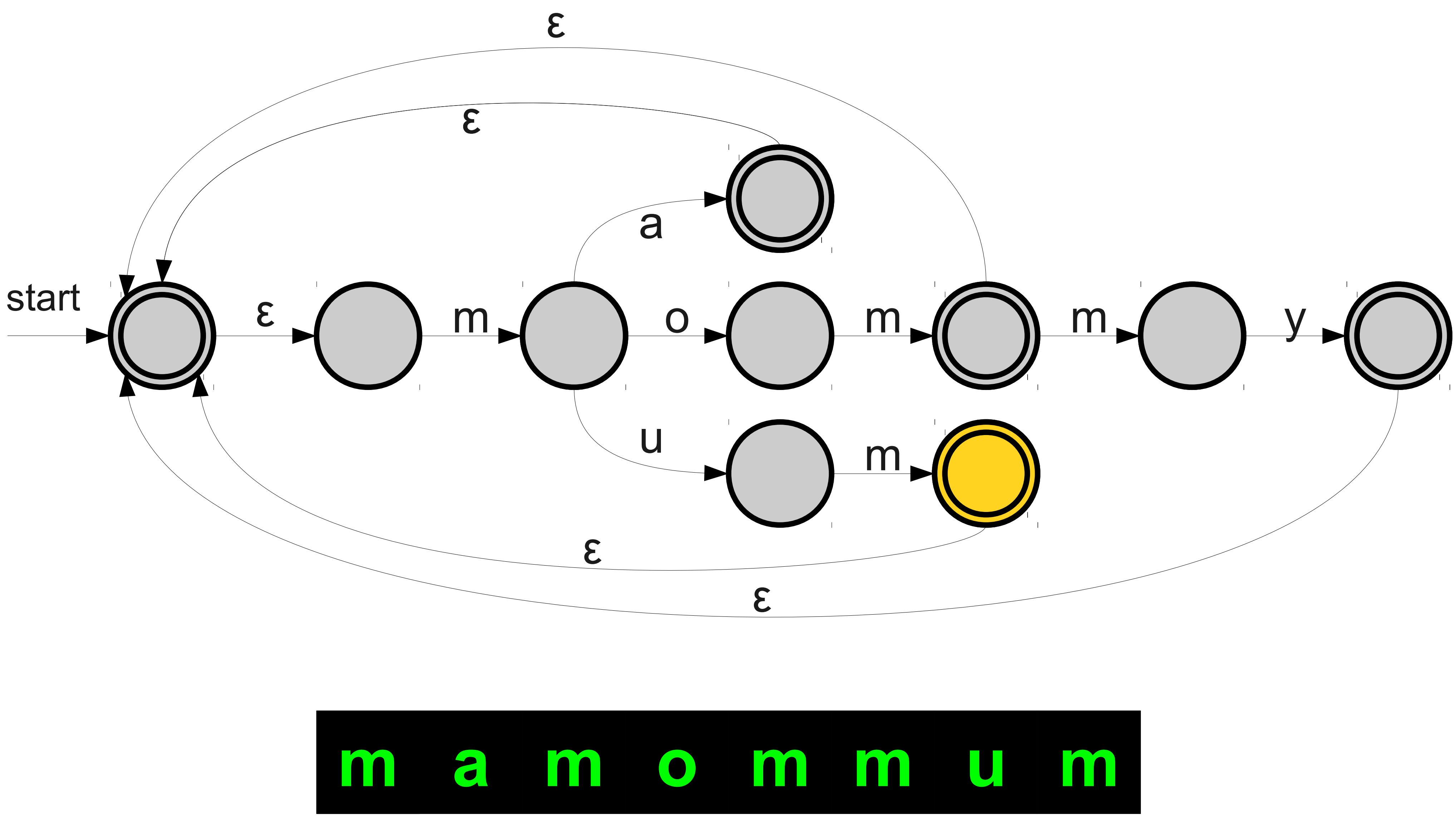

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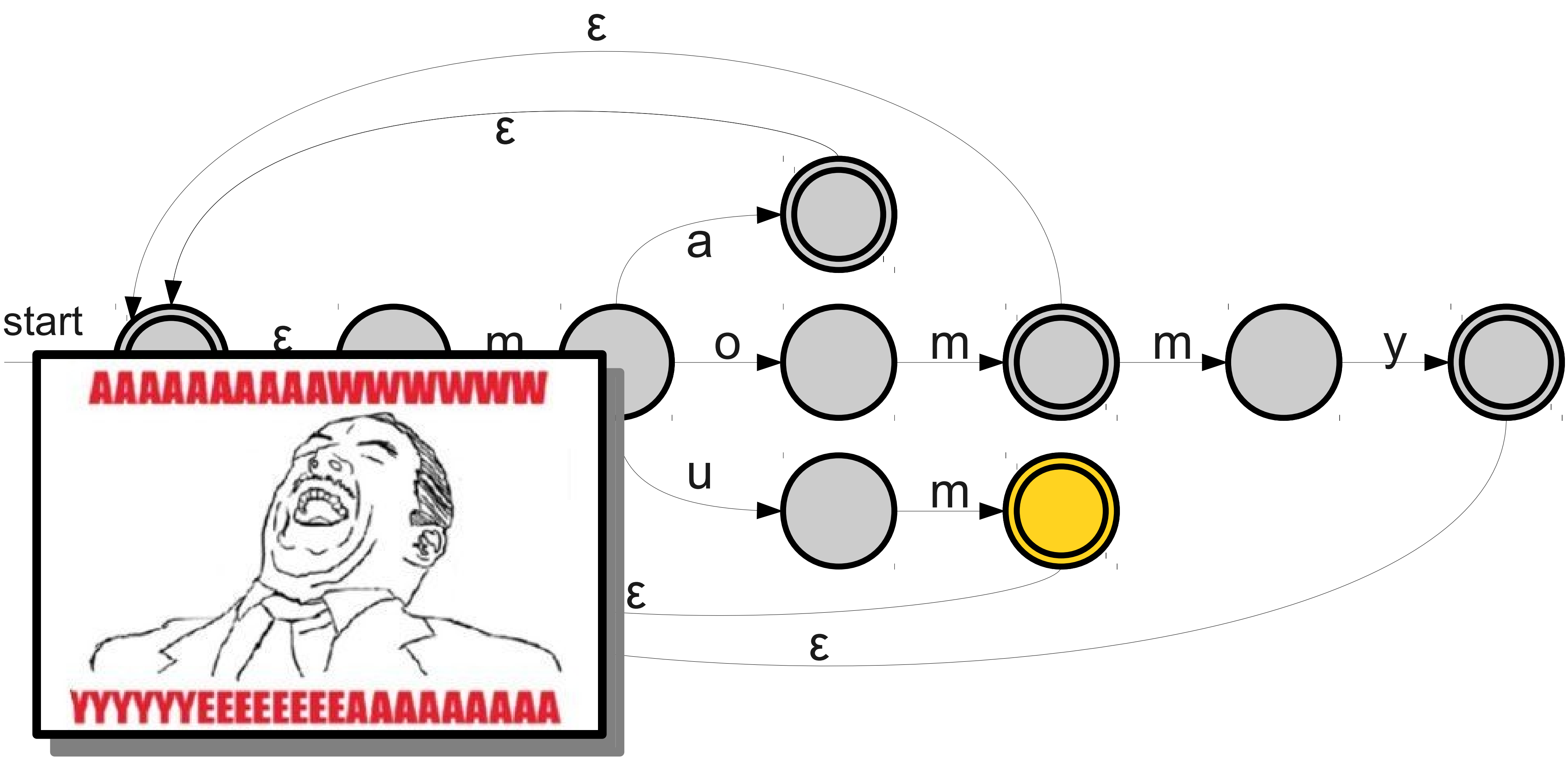

m a m o m m u m

Summary

- NFAs are a powerful type of automaton that allows for **nondeterministic** choices.
- NFAs can also have **ε-transitions** that move from state to state without consuming any input.
- The **subset construction** shows that NFAs are not more powerful than DFAs, because any NFA can be converted into a DFA that accepts the same language.
- The union, intersection, complement, concatenation, and Kleene closure of regular languages are all regular languages.

Another View of Regular Languages

Rethinking Regular Languages

- We currently have several tools for showing a language is regular.
  - Construct a DFA for it.
  - Construct an NFA for it.
  - Apply closure properties to existing languages.
- We have not spoken much of this last idea.

Constructing Regular Languages

- **Idea**: Build up all regular languages as follows:
  - Start with a small set of simple languages we already know to be regular.
  - Using closure properties, combine these simple languages together to form more elaborate languages.
- *A bottom-up approach to the regular languages.*

Regular Expressions

- **Regular expressions** are a family of descriptions that can be used to capture the regular languages.
- Often provide a compact and human-readable description of the language.
- Used as the basis for numerous software systems (Perl, **`flex`**, **`grep`**, etc.)

Atomic Regular Expressions

- The regular expressions begin with three simple building blocks.
- The symbol **Ø** is a regular expression that represents the empty language Ø.
- The symbol **ε** is a regular expression that represents the language { ε }
  - This is not the same as Ø!
- For any **`a`** ∈ Σ, the symbol **`a`** is a regular expression for the language { **`a`** }

Compound Regular Expressions

- We can combine together existing regular expressions in four ways.
- If R_1 and R_2 are regular expressions, $\boldsymbol{R_1 R_2}$ is a regular expression for the **concatenation** of the languages of R_1 and R_2.
- If R_1 and R_2 are regular expressions, $\boldsymbol{R_1 \mid R_2}$ is a regular expression for the **union** of the languages of R_1 and R_2.
- If R is a regular expression, $\boldsymbol{R^*}$ is a regular expression for the **Kleene closure** of the language of R.
- If R is a regular expression, $\boldsymbol{(R)}$ is a regular expression with the same meaning as R.

Operator Precedence

- Regular expression operator precedence is

$$(R)$$

$$R^*$$

$$R_1R_2$$

$$R_1 \mid R_2$$

- So **`ab*c|d`** is parsed as **`((a(b*))c)|d`**

Regular Expression Examples

- The regular expression **`trick|treat`** represents the regular language { **`trick`**, **`treat`** }
- The regular expression **`booo*`** represents the regular language { **`boo`**, **`booo`**, **`boooo`**, … }
- The regular expression **`candy!(candy!)*`** represents the regular language { **`candy!`**, **`candy!candy!`**, **`candy!candy!candy!`**, … }

Regular Expressions, Formally

- The **language of a regular expression** is the language described by that regular expression.
- Formally:
  - $\mathscr{L}(\varepsilon) = \{\varepsilon\}$
  - $\mathscr{L}(\emptyset) = \emptyset$
  - $\mathscr{L}(\texttt{a}) = \{\texttt{a}\}$
  - $\mathscr{L}(R_1R_2) = \mathscr{L}(R_1)\ \mathscr{L}(R_2)$
  - $\mathscr{L}(R_1 \mid R_2) = \mathscr{L}(R_1) \cup \mathscr{L}(R_2)$
  - $\mathscr{L}(R^*) = \mathscr{L}(R)^*$
  - $\mathscr{L}((R)) = \mathscr{L}(R)$

Worthwhile activity: Apply this recursive definition to

`a(b|c)((d))`

and see what you get.

Regular Expressions are Awesome

- Let $\Sigma = \{\texttt{0}, \texttt{1}\}$
- Let $L = \{ w \in \Sigma^* \mid w$ contains `00` as a substring $\}$

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11011100101

0000

11111011110011111

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11011100101
0000
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The length of a string w is denoted $|w|$

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0000
1010
1111
1000

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(0|1)(0|1)(0|1)(0|1)

0000
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1111
1000

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$$(0|1)^4$$

0000
1010
1111
1000

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$(0|1)^4$

0000
1010
1111
1000

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1*(0 | ε)1*

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$$1^*(0 \mid \varepsilon)1^*$$

Regular Expressions are Awesome

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1*(0 | ε)1*

11110111
111111
0111
0

Regular Expressions are Awesome

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1*(0 | ε)1*

11110111
111111
0111
0

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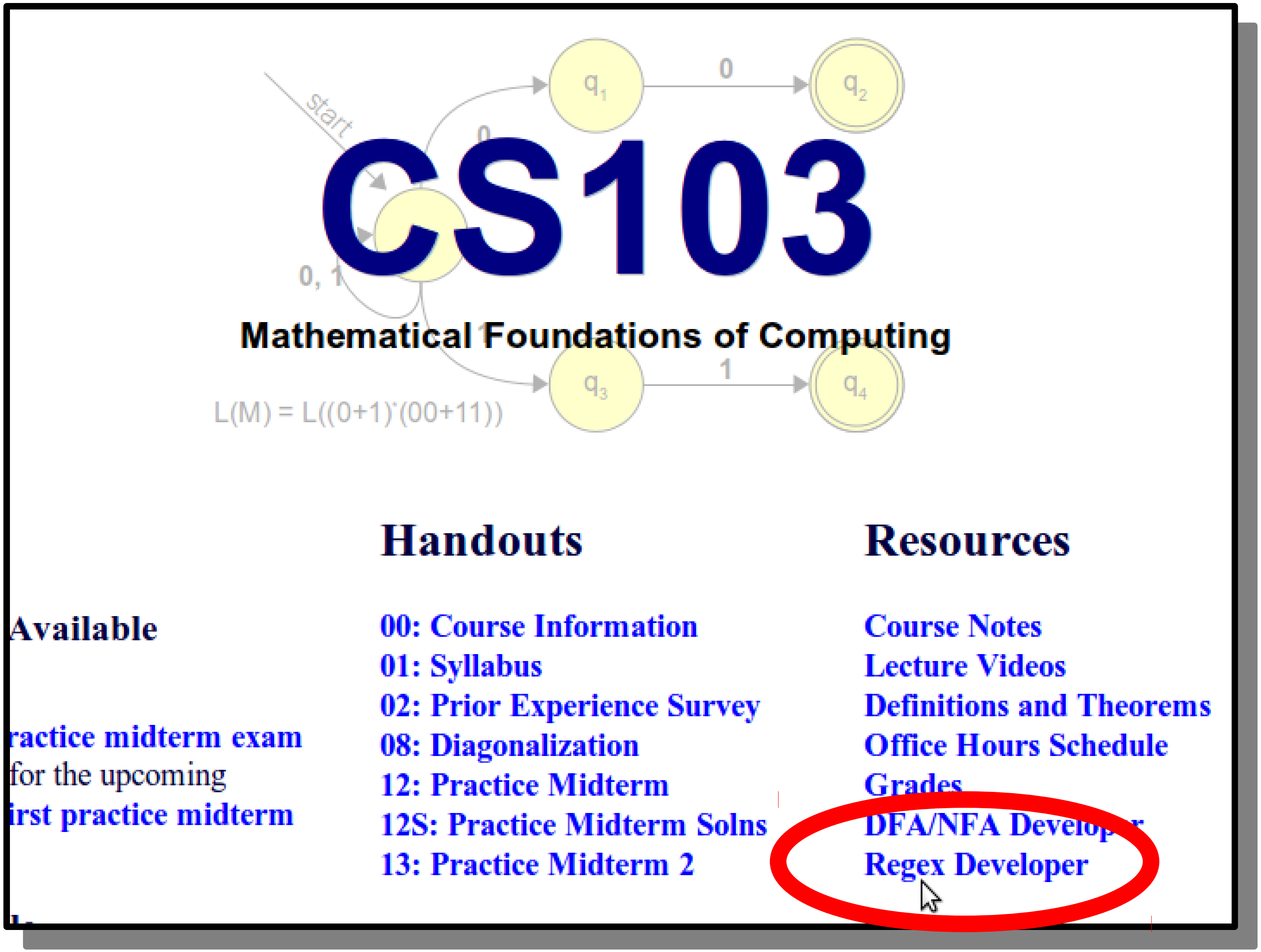
1*0?1*

11110111
111111
0111
0


start
0
0, 1
1
L(M) = L((0+1)*(00+11))
CS103
Mathematical Foundations of Computing
Handouts
Resources
Available
00: Course Information
01: Syllabus
02: Prior Experience Survey
08: Diagonalization
12: Practice Midterm
12S: Practice Midterm Solns
13: Practice Midterm 2
ractice midterm exam
for the upcoming
irst practice midterm
Course Notes
Lecture Videos
Definitions and Theorems
Office Hours Schedule
Grades
Regex Developer