

Finite Automata

Part Two

Announcements

- Practice midterm solutions available.
- Second practice midterm available soon.
- Problem Set 3 and Problem Set 4
Checkpoints graded; will be returned at
end of lecture.

A Friendly Reminder

\forall goes with \rightarrow

\exists goes with \wedge

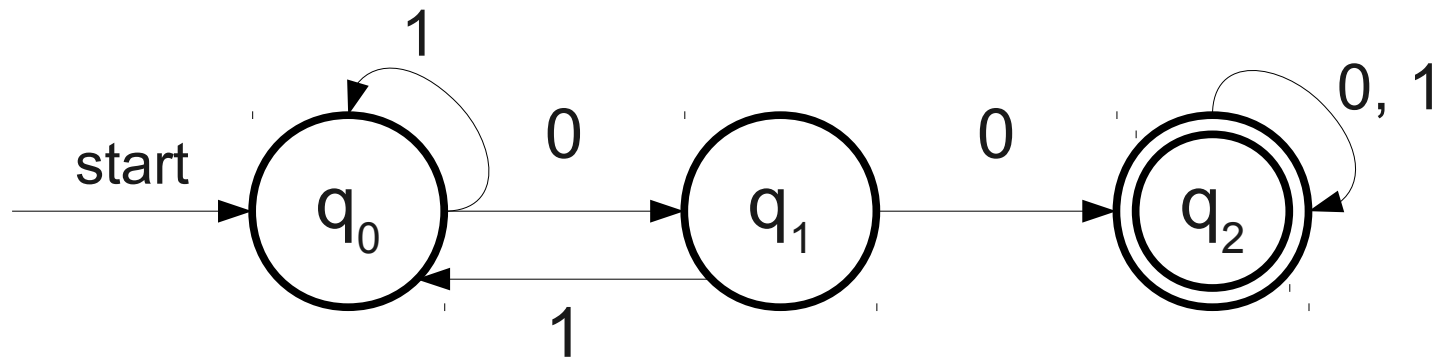
Finite Automata

DFA's, Informally

- A DFA is defined relative to some alphabet Σ .
- For each state in the DFA, there must be **exactly one** transition defined for each symbol in the alphabet.
 - This is the “deterministic” part of DFA.
- There is a **unique** start state.
- There may be multiple accepting states.

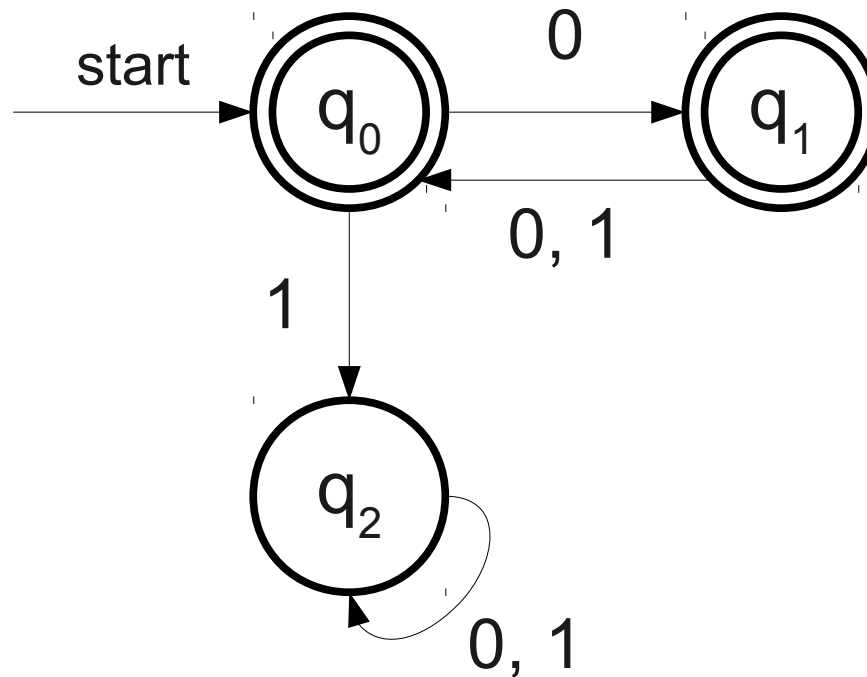
Recognizing Languages with DFAs

$L = \{ w \in \{0, 1\}^* \mid w \text{ contains } 00 \text{ as a substring} \}$



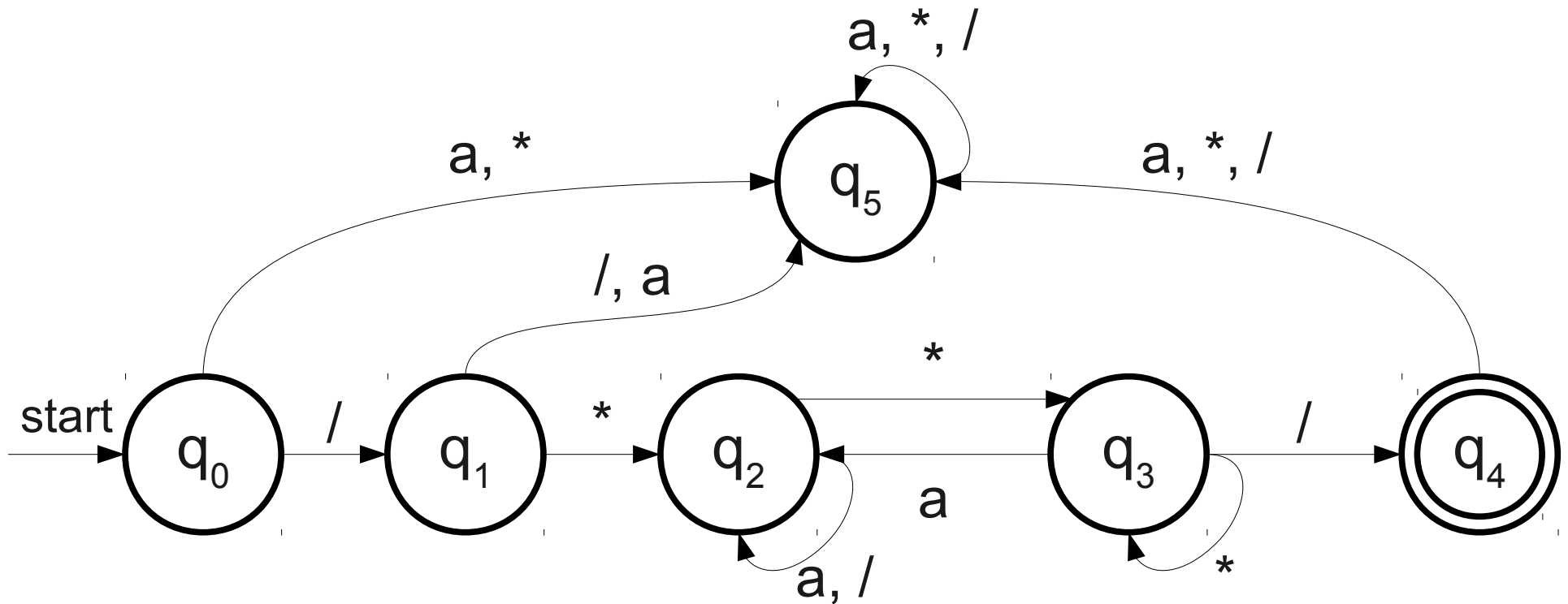
Recognizing Languages with DFAs

$L = \{ w \in \{0, 1\}^* \mid \text{all even-numbered characters of } w \text{ are } 0 \}$



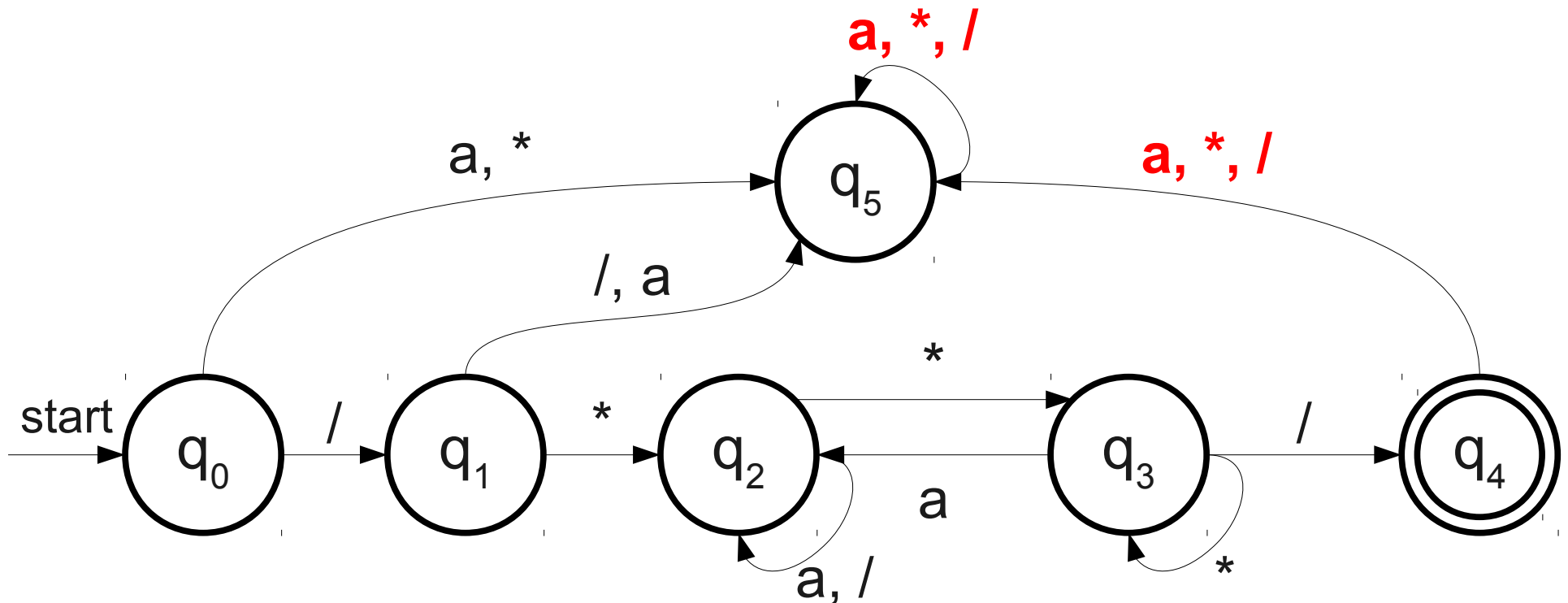
More Elaborate DFAs

$L = \{ w \mid w \text{ is a C-style comment} \}$



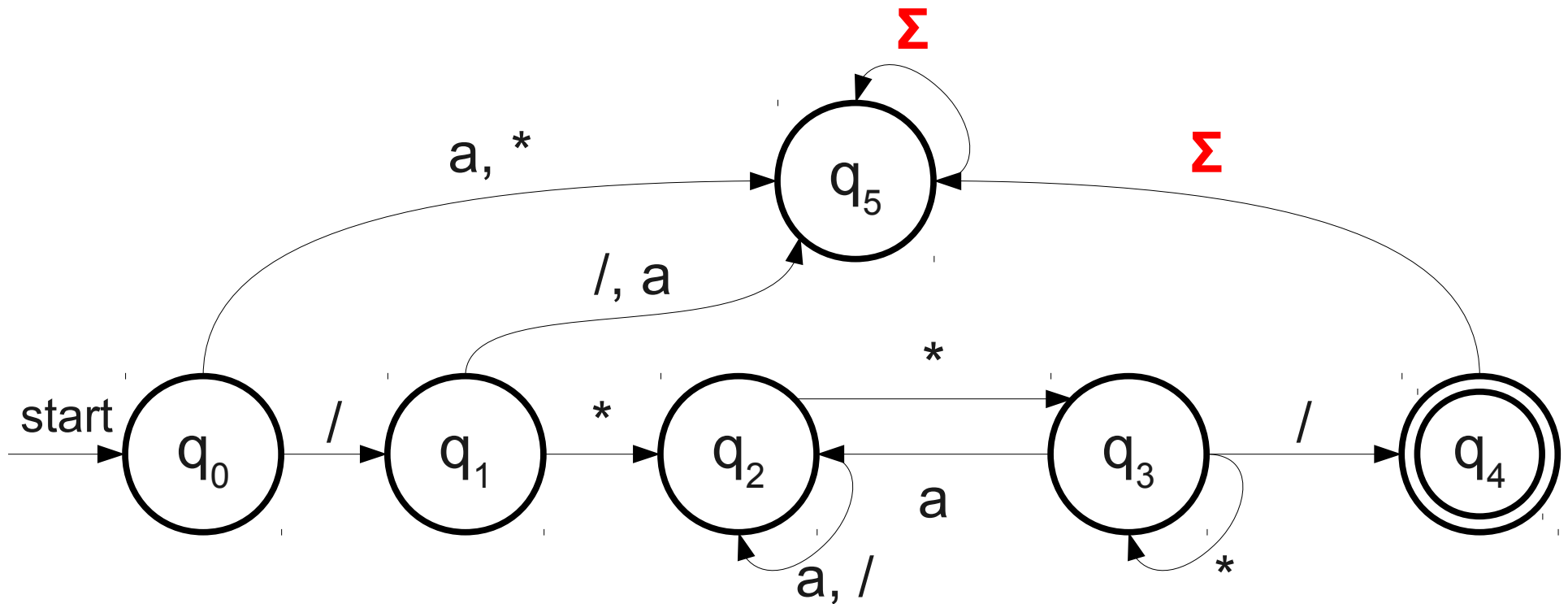
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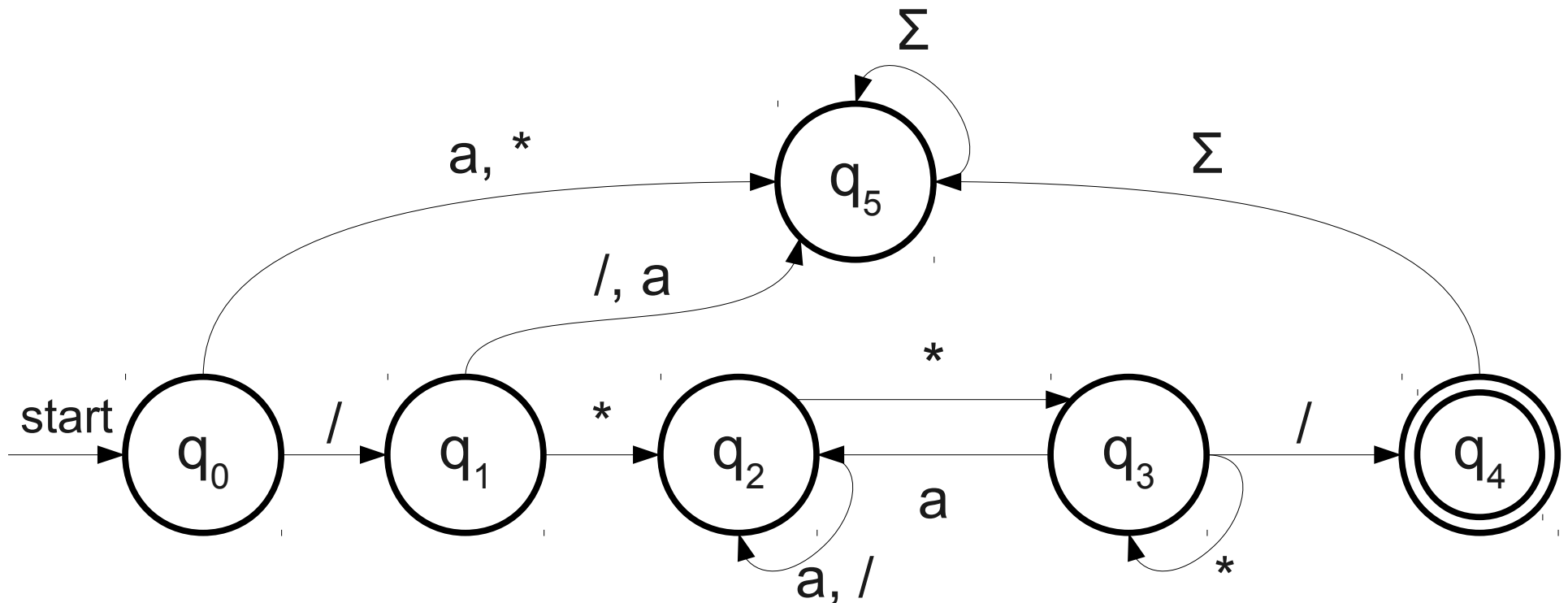
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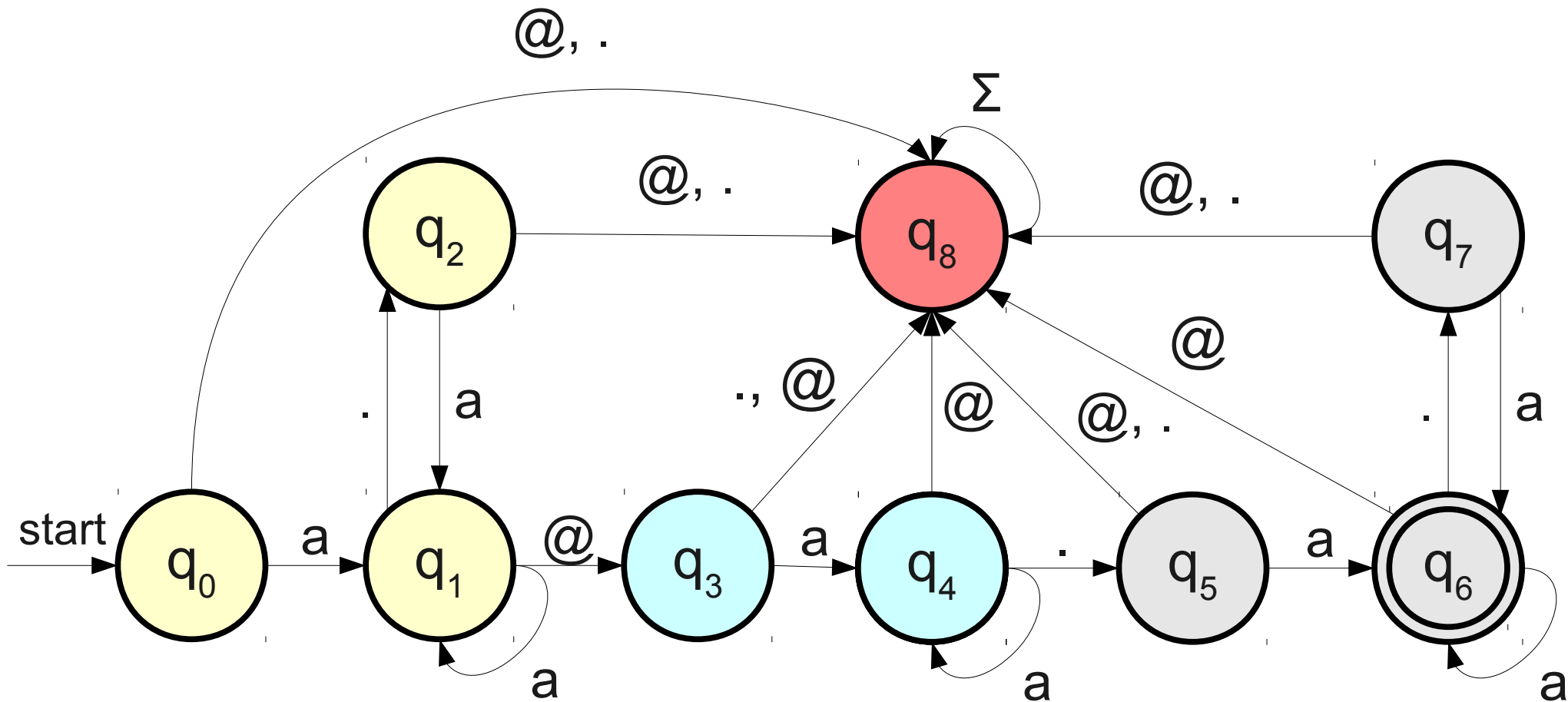


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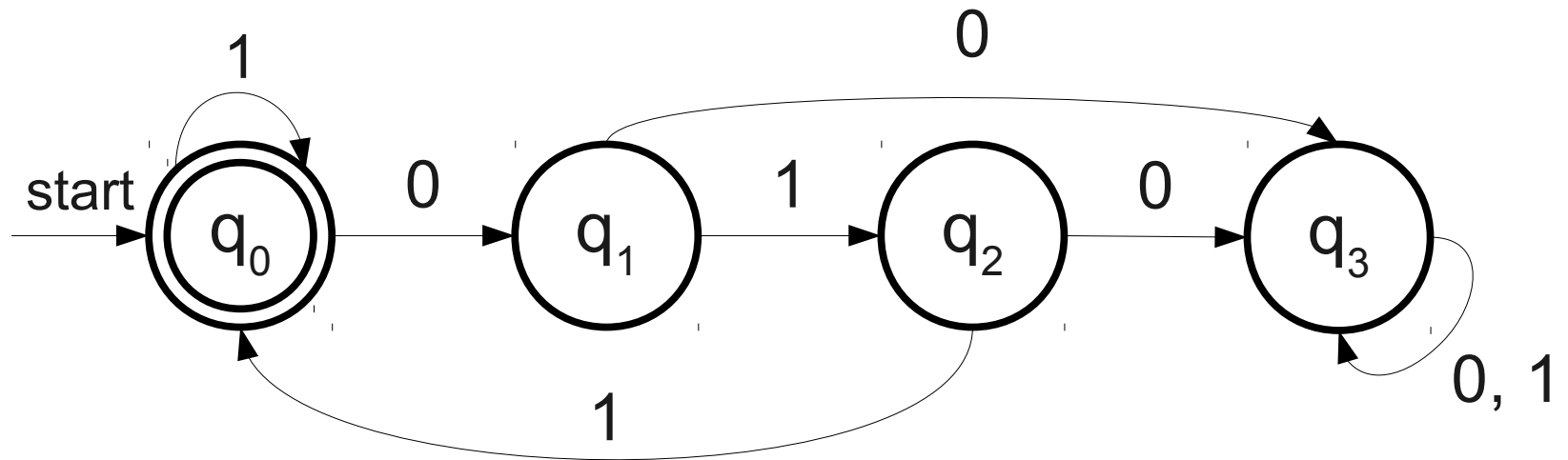
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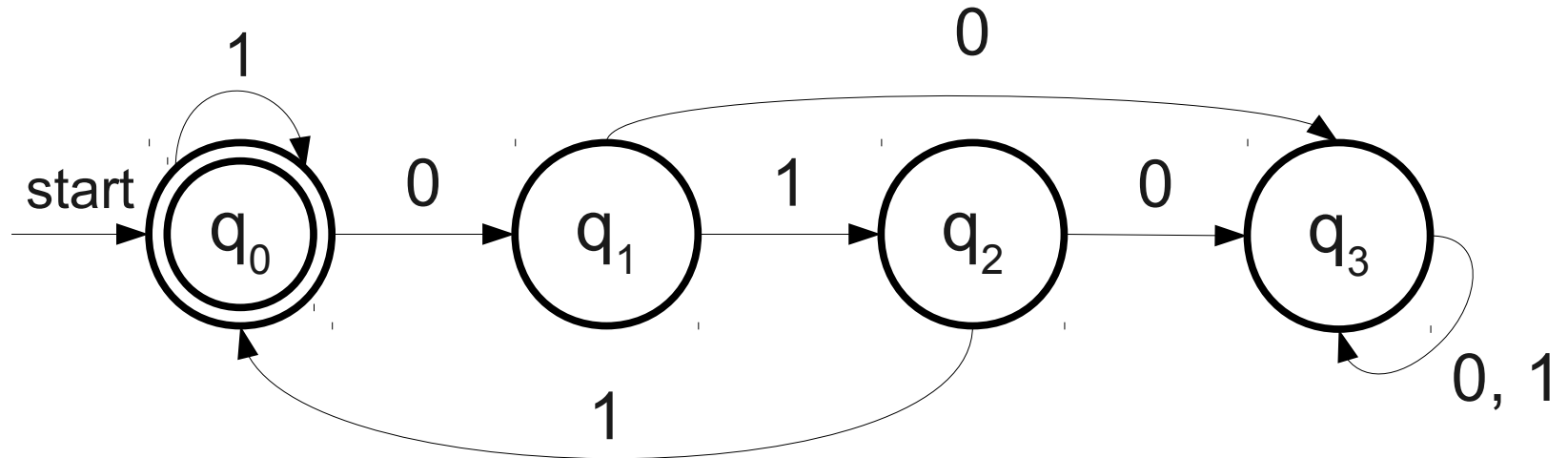
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Tabular DFAs

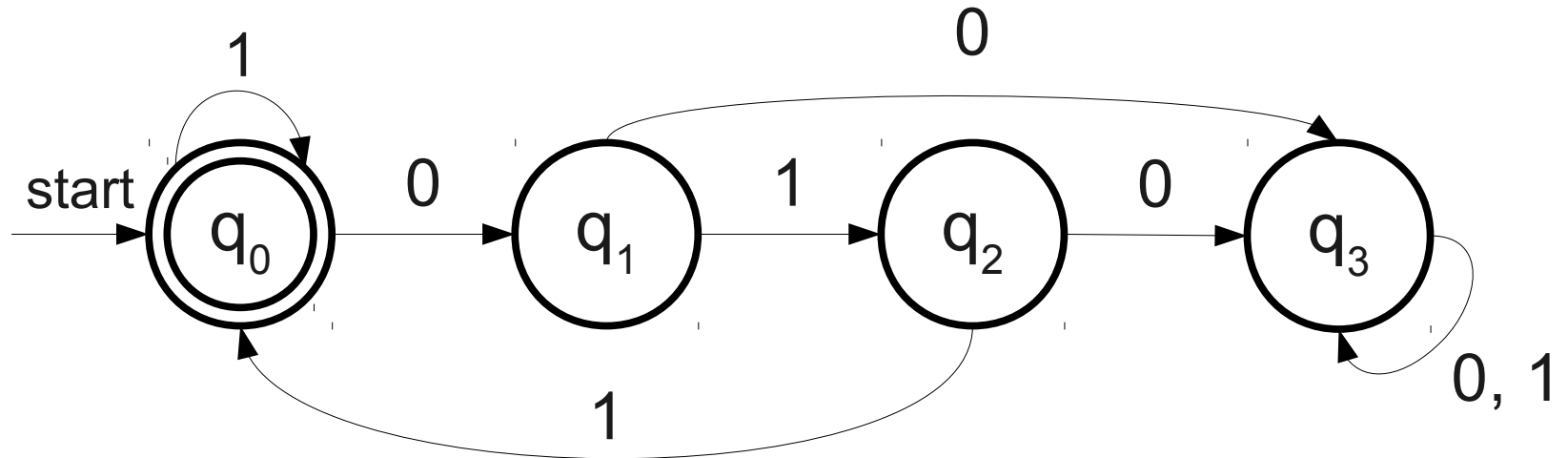


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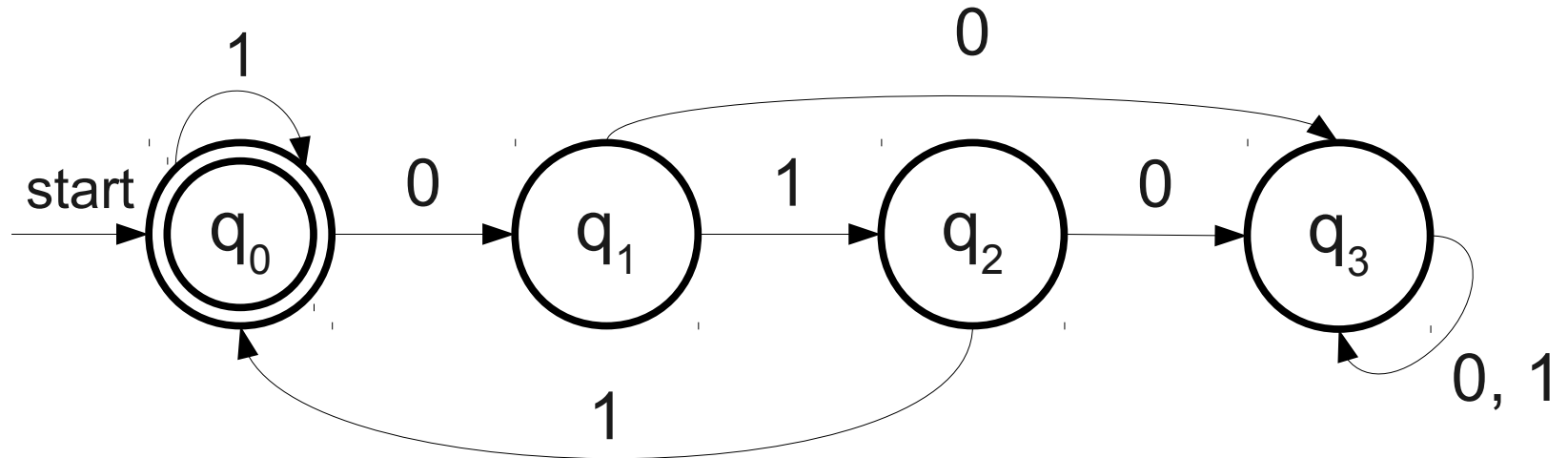
	0	1
q_0		
q_1		
q_2		
q_3		

Tabular DFAs



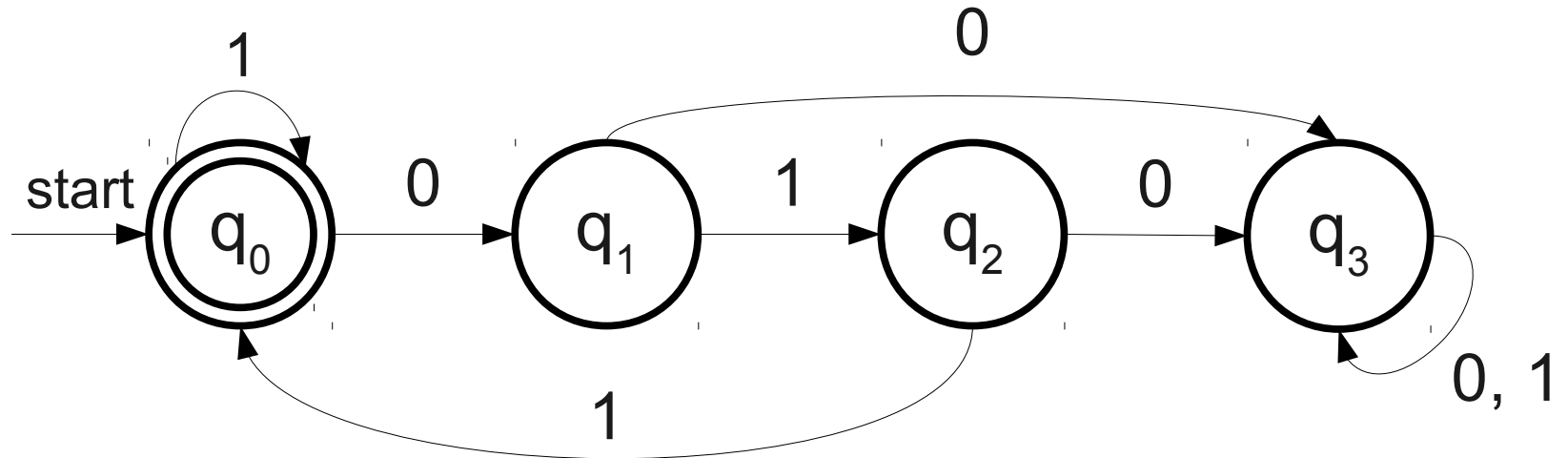
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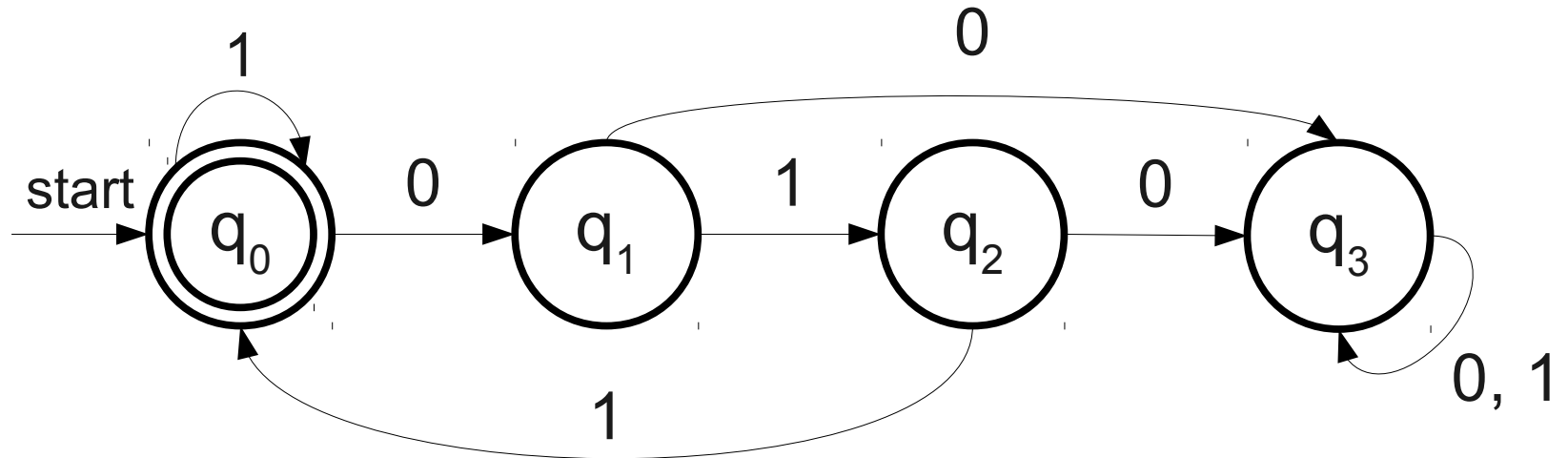
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q_0	q_1	q_0
q_1		
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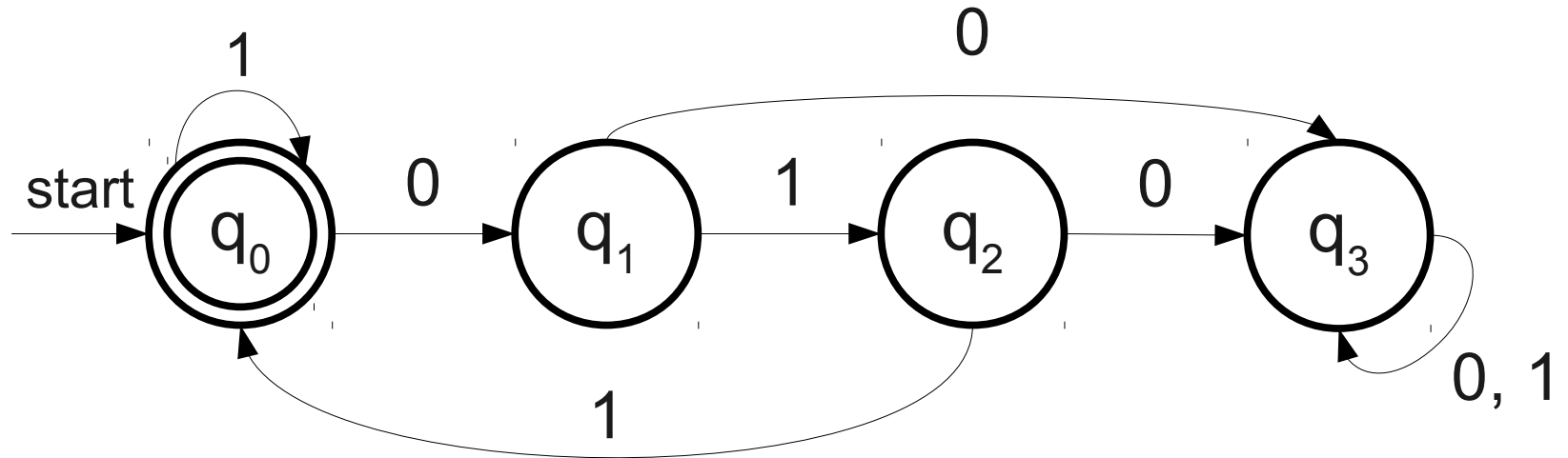
	0	1
q_0	q_1	q_0
q_1	q_3	
q_2		
q_3		

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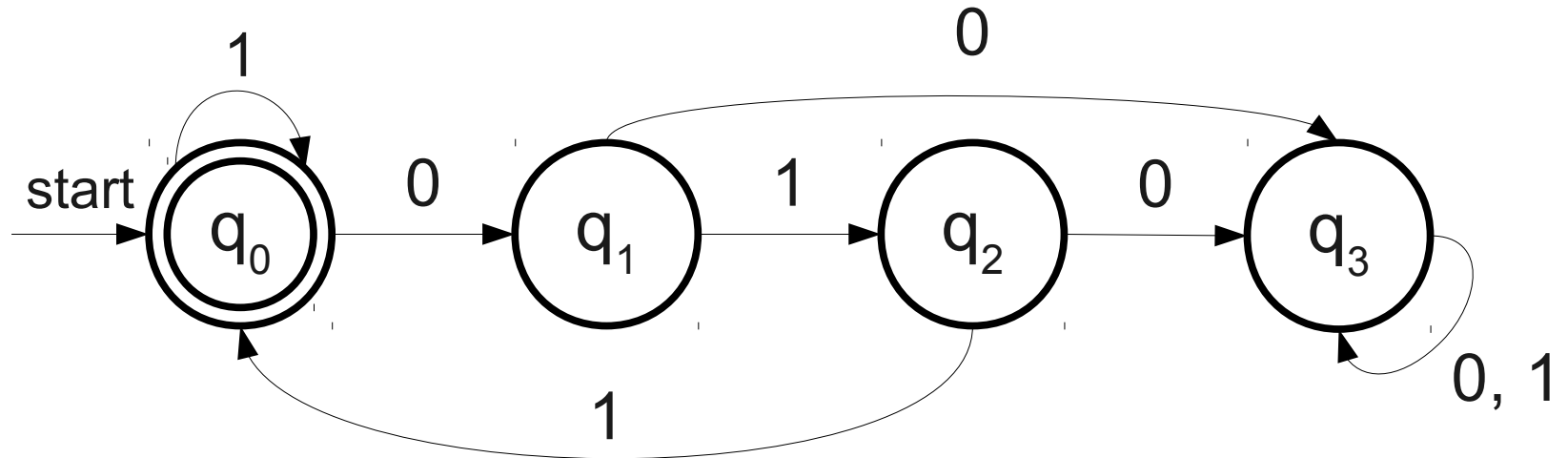
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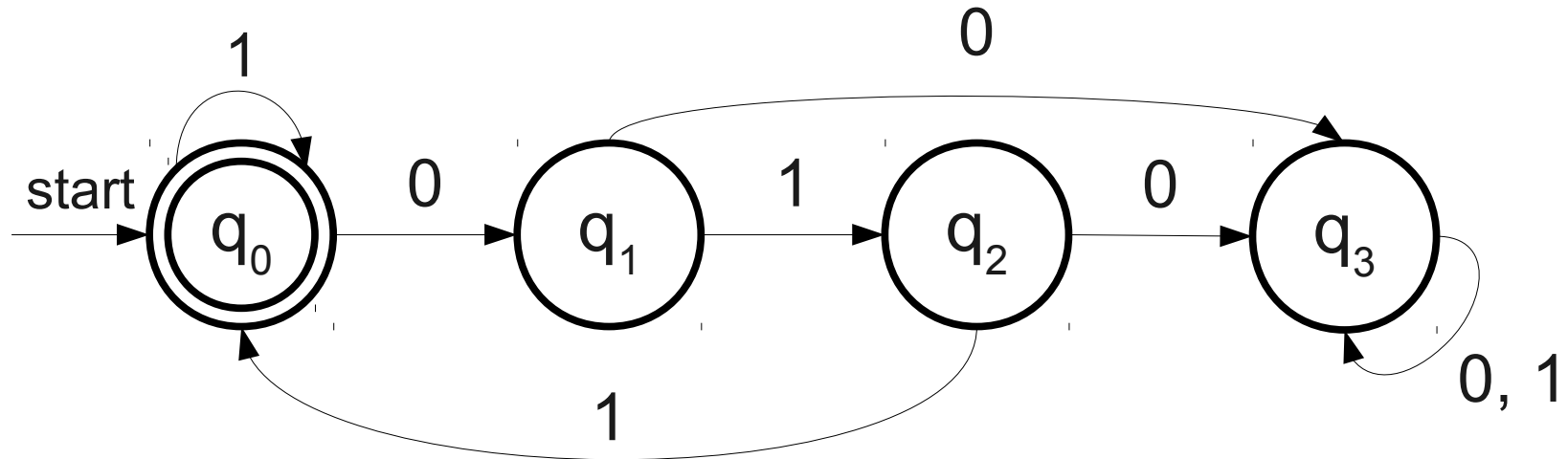
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q_0	q_1	q_0
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q_2	q_3	
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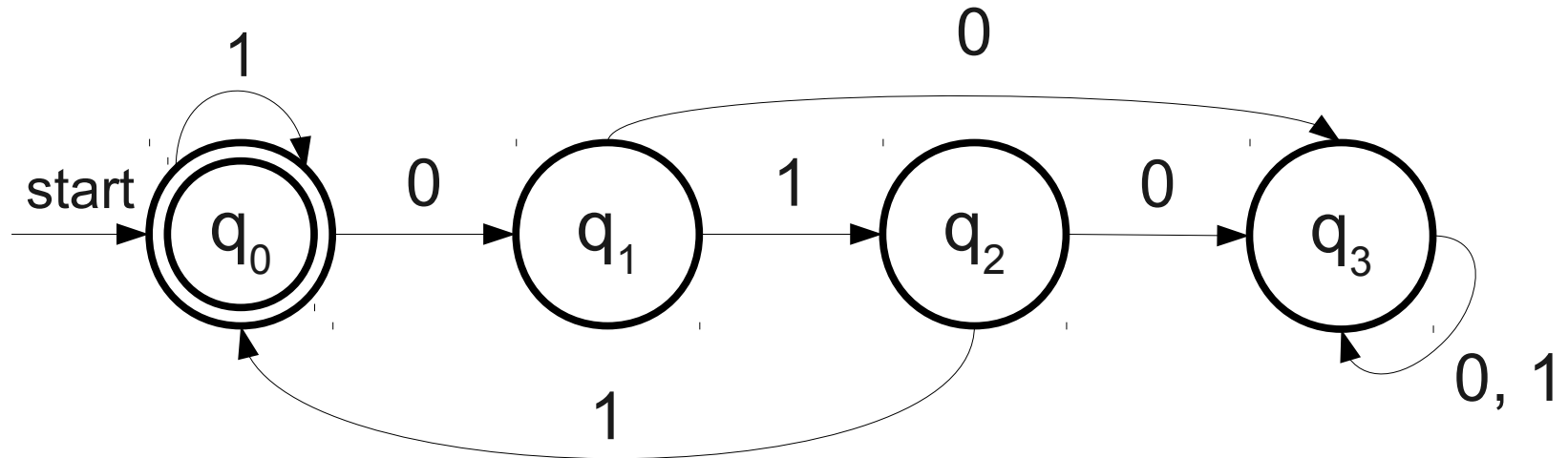
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q_0	q_1	q_0
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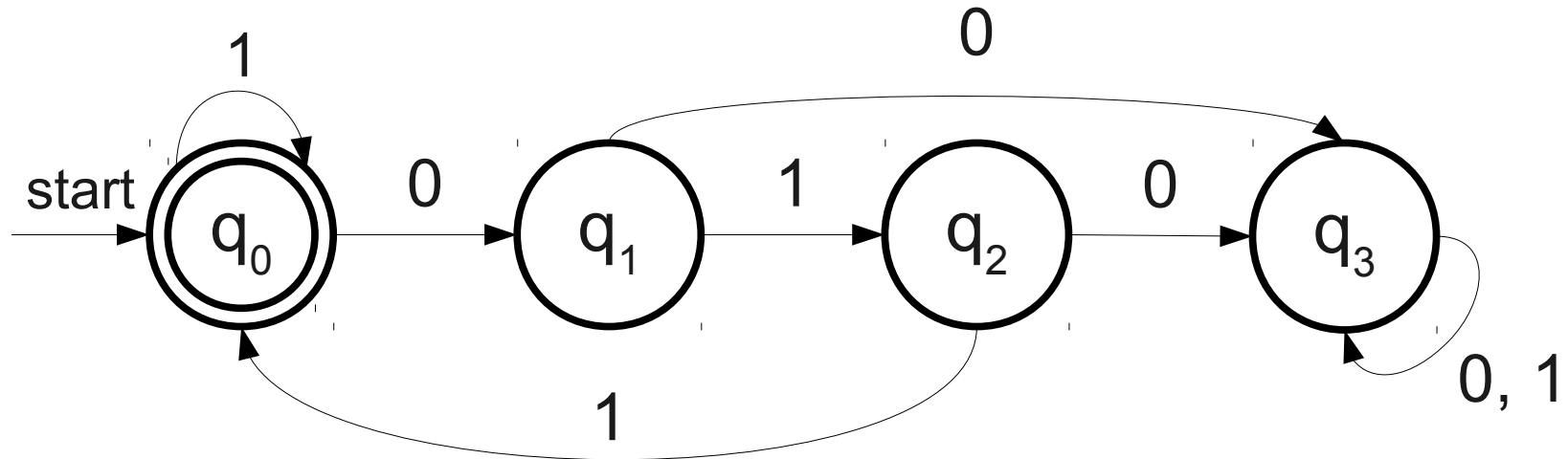
	0	1
q_0	q_1	q_0
q_1	q_3	q_2
q_2	q_3	q_0
q_3	q_3	

Tabular DFAs



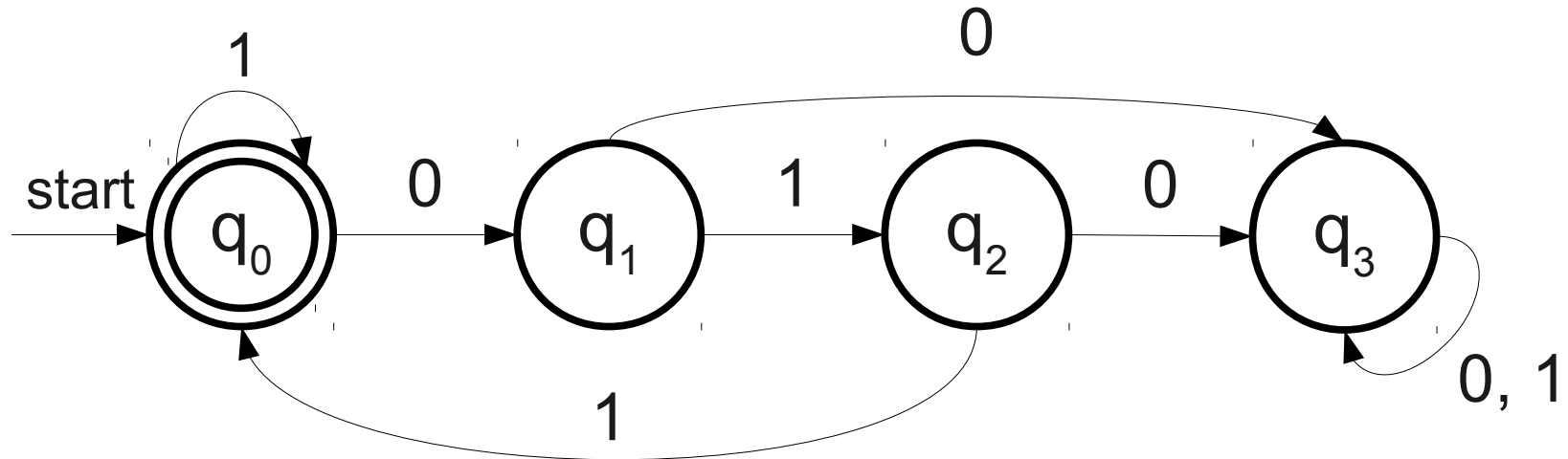
	0	1
q_0	q_1	q_0
q_1	q_3	q_2
q_2	q_3	q_0
q_3	q_3	q_3

Tabular DFAs



	0	1
* q_0	q_1	q_0
q_1	q_3	q_2
q_2	q_3	q_0
q_3	q_3	q_3

Tabular DFAs



	0	1
*q ₀	q ₁	q ₀
q ₁	q ₃	q ₂
q ₂	q ₃	q ₀
q ₃	q ₃	q ₃

The star indicates that this is an accepting state.

Code? In a Theory Course?

```
int kTransitionTable[kNumStates][kNumSymbols] = {
    {0, 0, 1, 3, 7, 1, ...},
    ...
};
bool kAcceptTable[kNumStates] = {
    false,
    true,
    true,
    ...
};
bool SimulateDFA(string input) {
    int state = 0;
    for (char ch: input)
        state = kTransitionTable[state][ch];
    return kAcceptTable[state];
}
```

A language L is called a **regular language** iff there exists a DFA D such that $\mathcal{L}(D) = L$.

The Complement of a Language

- Given a language $L \subseteq \Sigma^*$, the **complement** of that language (denoted \bar{L}) is the language of all strings in Σ^* not in L .
- Formally:

$$\bar{L} = \{ w \mid w \in \Sigma^* \wedge w \notin L \}$$

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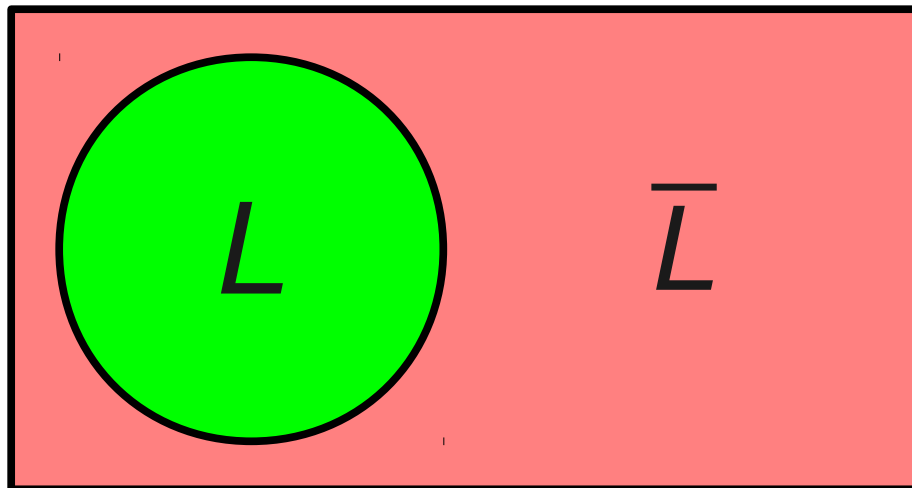
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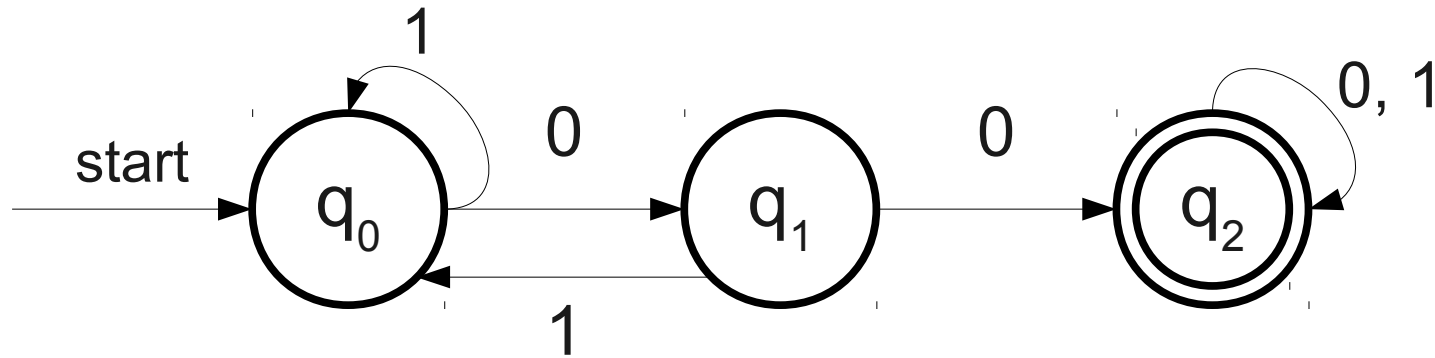


Complementing Regular Languages

- Recall: A **regular language** is a language accepted by some DFA.
- **Question:** If L is a regular language, is \bar{L} a regular language?
- If the answer is “yes,” then there must be some way to construct a DFA for \bar{L} .
- If the answer is “no,” then some language L can be accepted by a DFA, but \bar{L} cannot be accepted by any DFA.

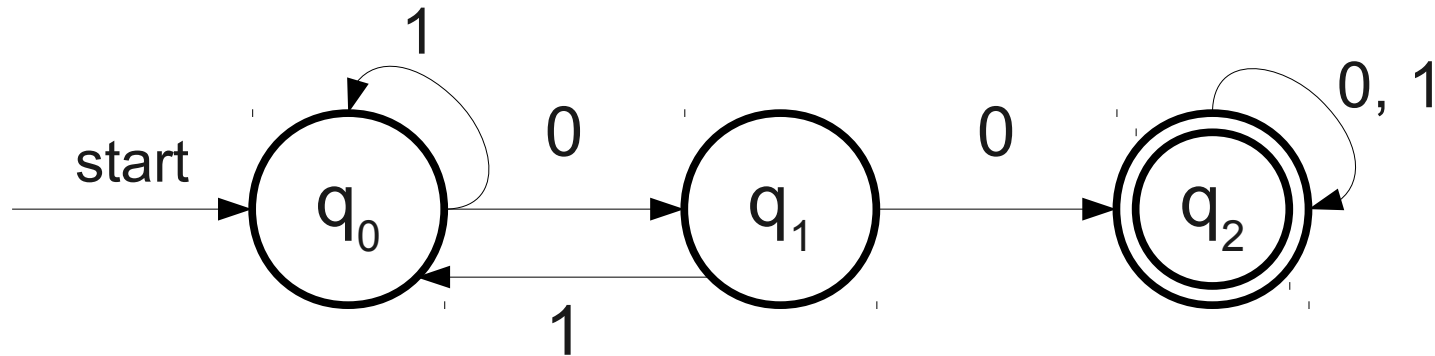
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$L = \{ w \in \{0, 1\}^* \mid w \text{ contains } 00 \text{ as a substring} \}$



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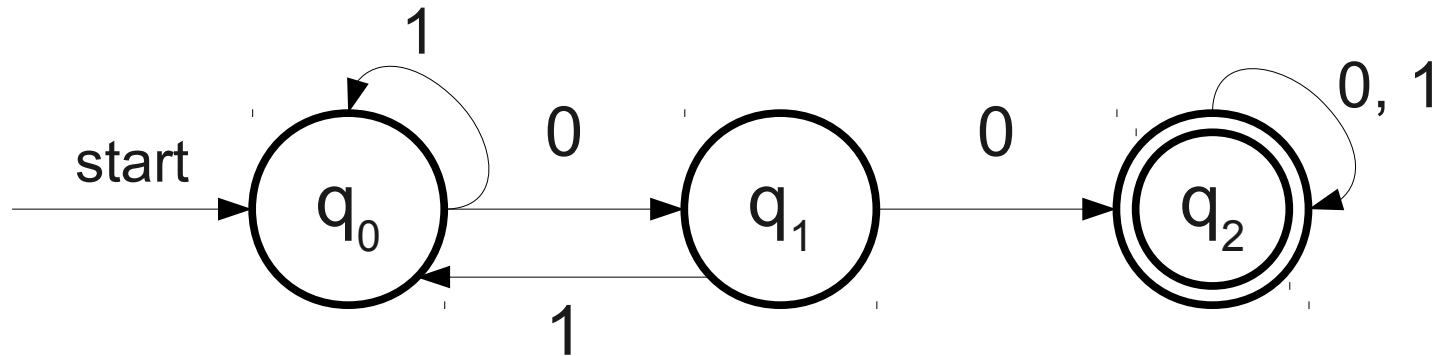
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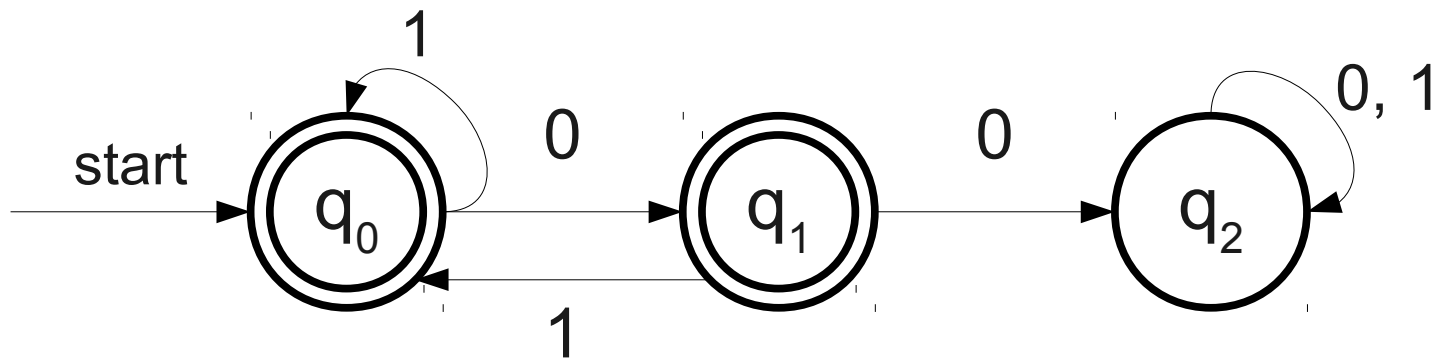
$$\bar{L} = \{ w \in \{0, 1\}^* \mid w \text{ **does not contain** } 00 \text{ as a substring} \}$$

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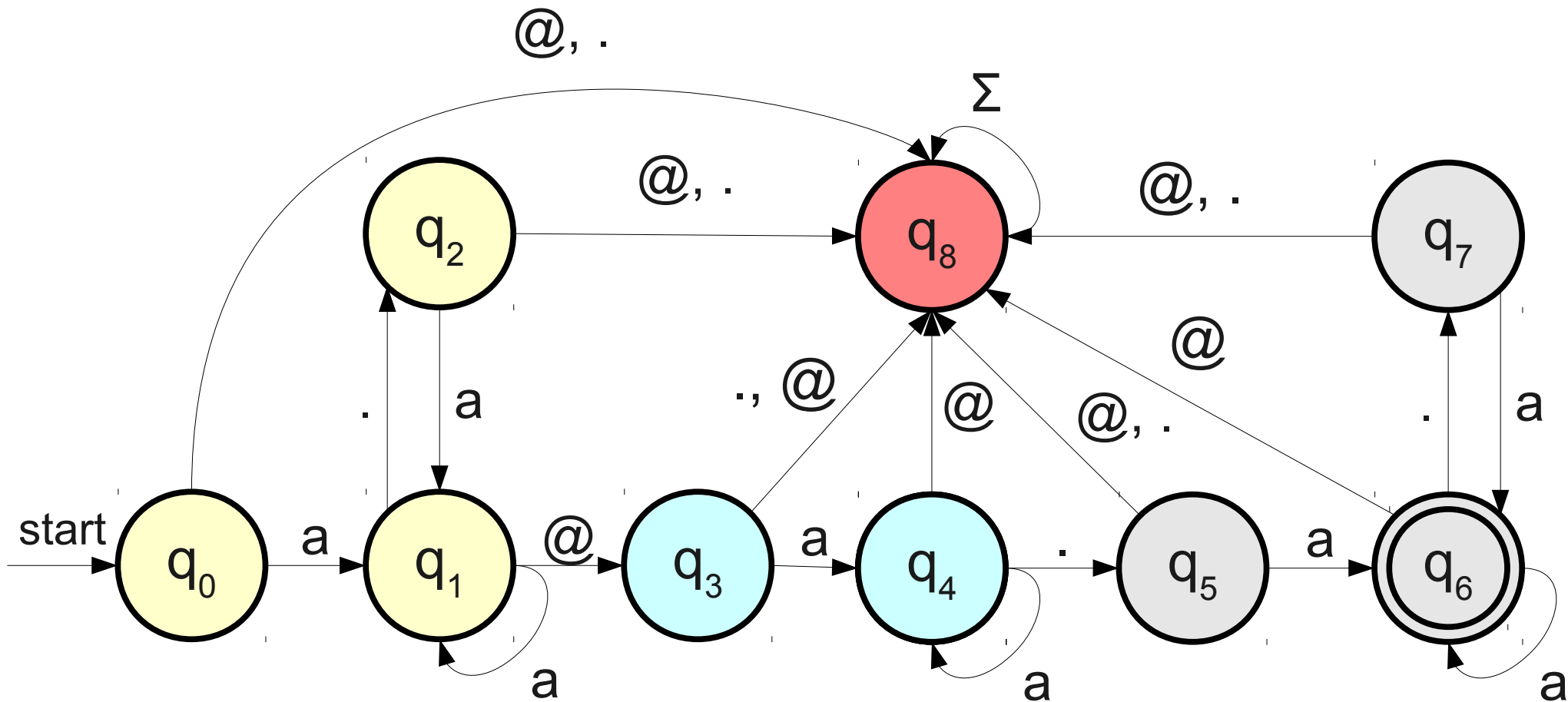


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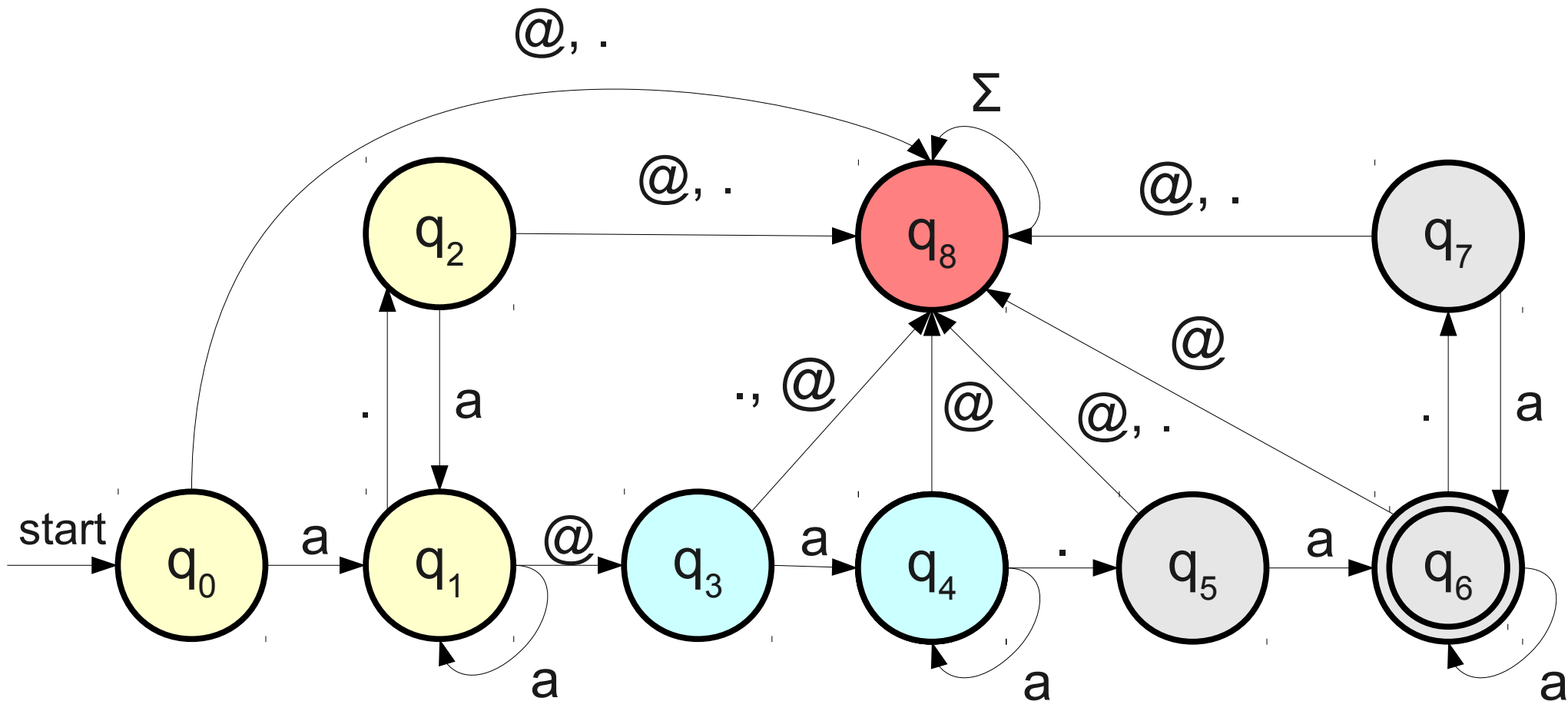
Complementing Regular Languages

$L = \{ w \mid w \text{ is a legal email address} \}$



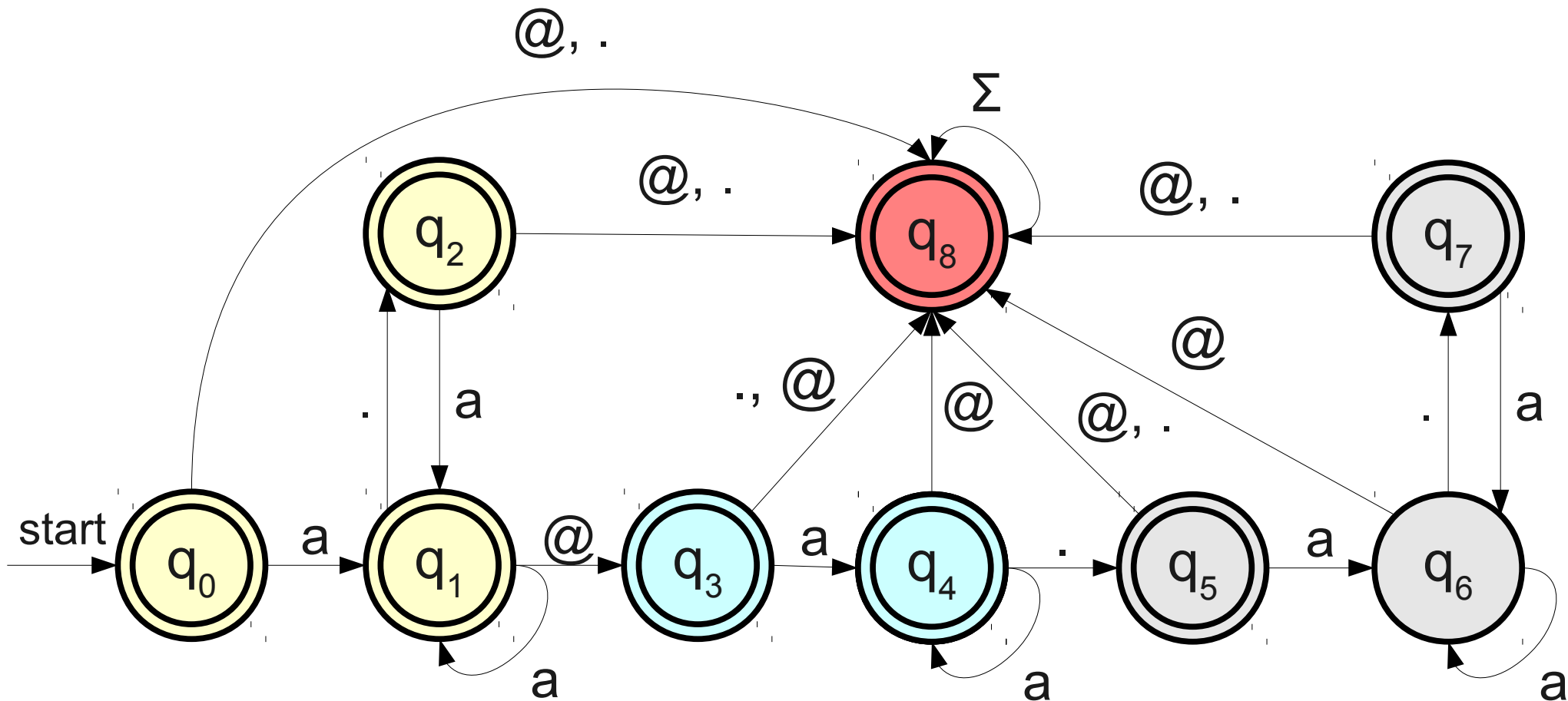
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Constructions on Automata

- Much of our discussion of automata will consider constructions that transform one automaton into another.
- Exchanging accepting and rejecting states is a simple construction sometimes called the **complement construction**.

Closure Properties

- If L is a regular language, \bar{L} is a regular language.
- If we begin with a regular language and complement it, we end up with a regular language.
- This is an example of a **closure property of regular languages**.
 - The regular languages are **closed under complementation**.
 - We'll see more such properties later on.

NFAS

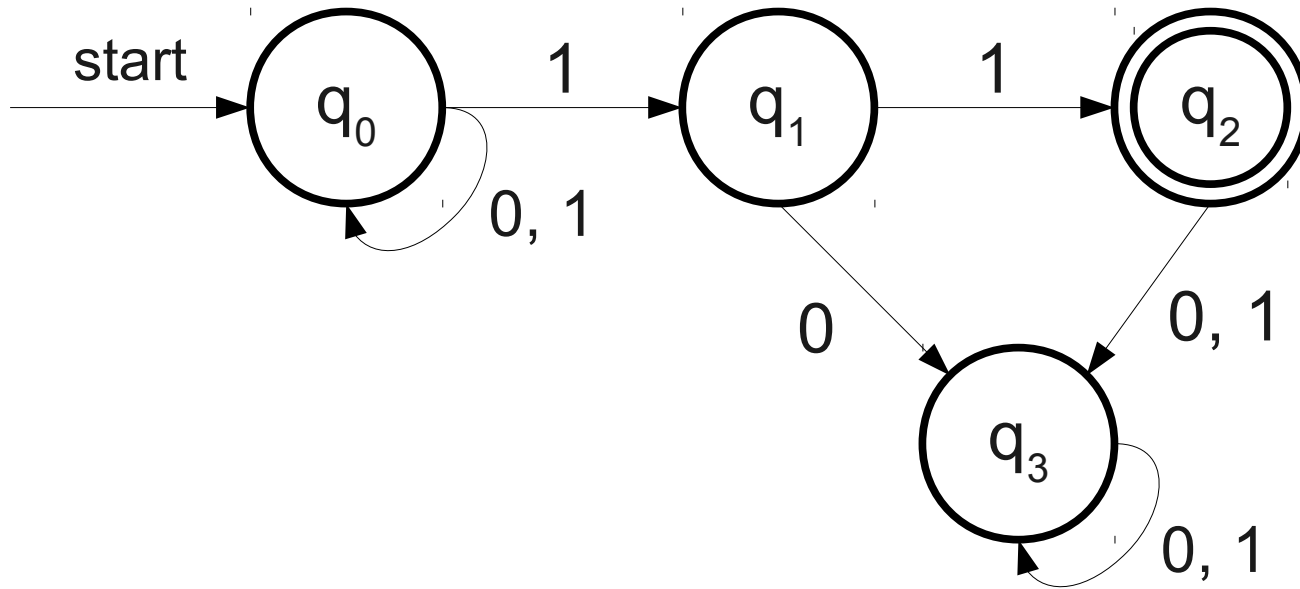
NFAs

- An **NFA** is a
 - **N**ondeterministic
 - **F**inite
 - **A**utomaton
- Conceptually similar to a DFA, but equipped with the vast power of **nondeterminism**.

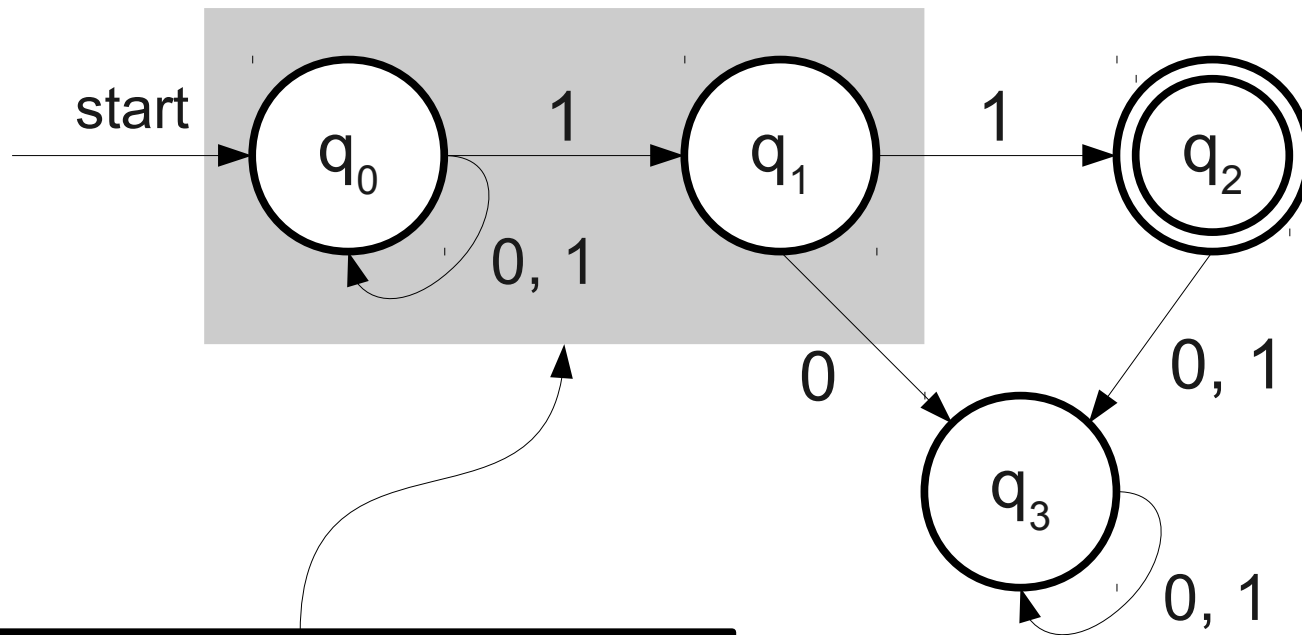
(Non)determinism

- A model of computation is **deterministic** if at every point in the computation, there is exactly one choice that can make.
- The machine accepts if that series of choices leads to an accepting state.
- A model of computation is **nondeterministic** if the computing machine may have multiple decisions that it can make at one point.
- The machine accepts if **any** series of choices leads to an accepting state.

A Simple NFA

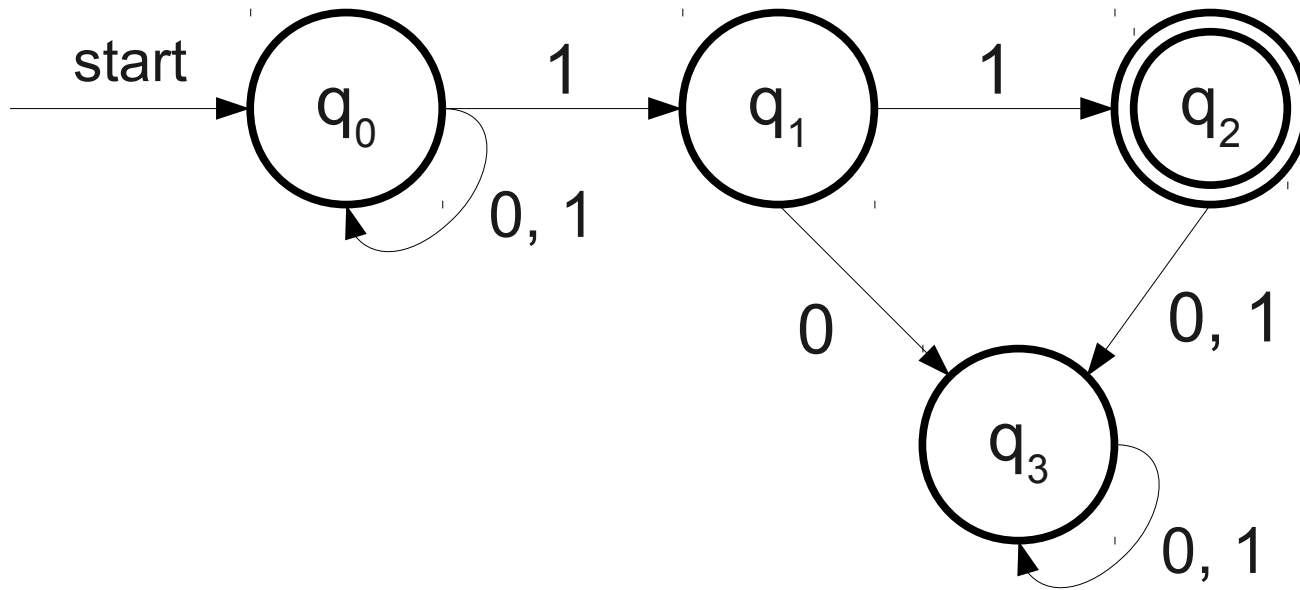


A Simple NFA



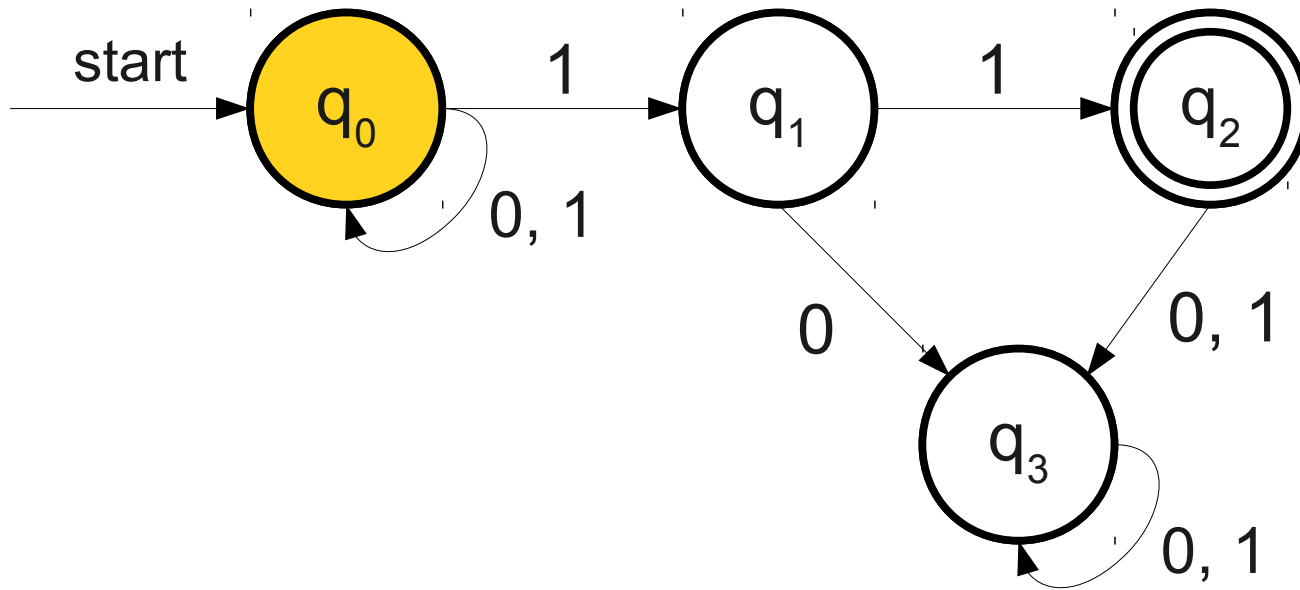
q_0 has two transitions
defined on 1!

A Simple NFA



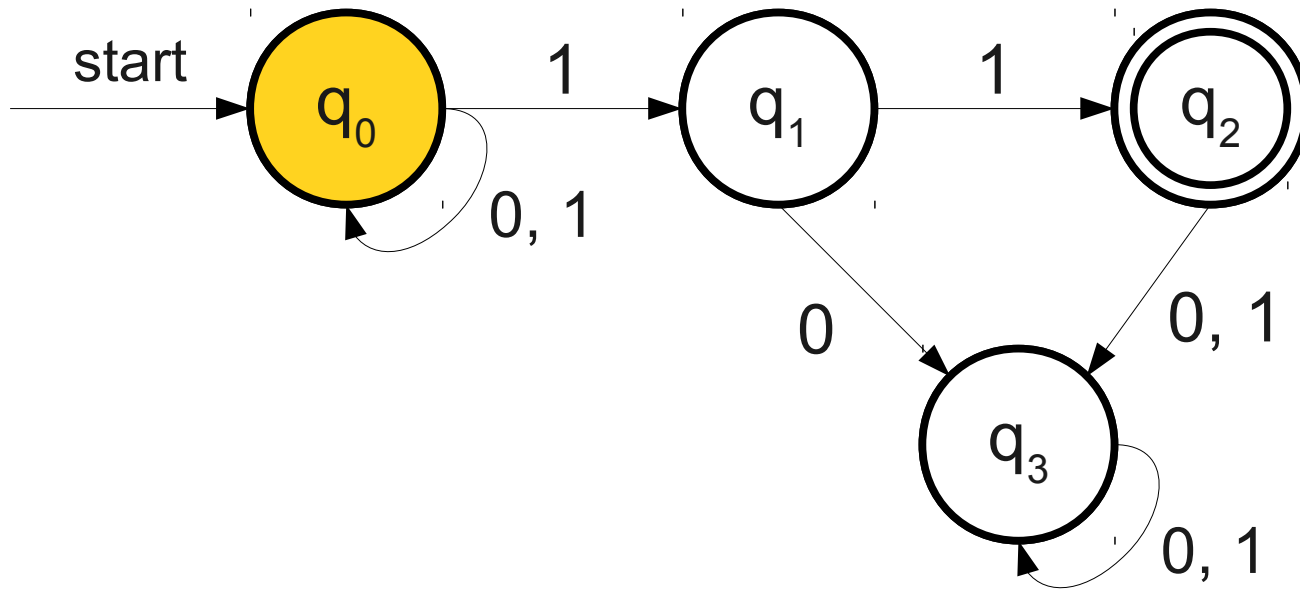
0 1 0 1 1

A Simple NFA



0 1 0 1 1

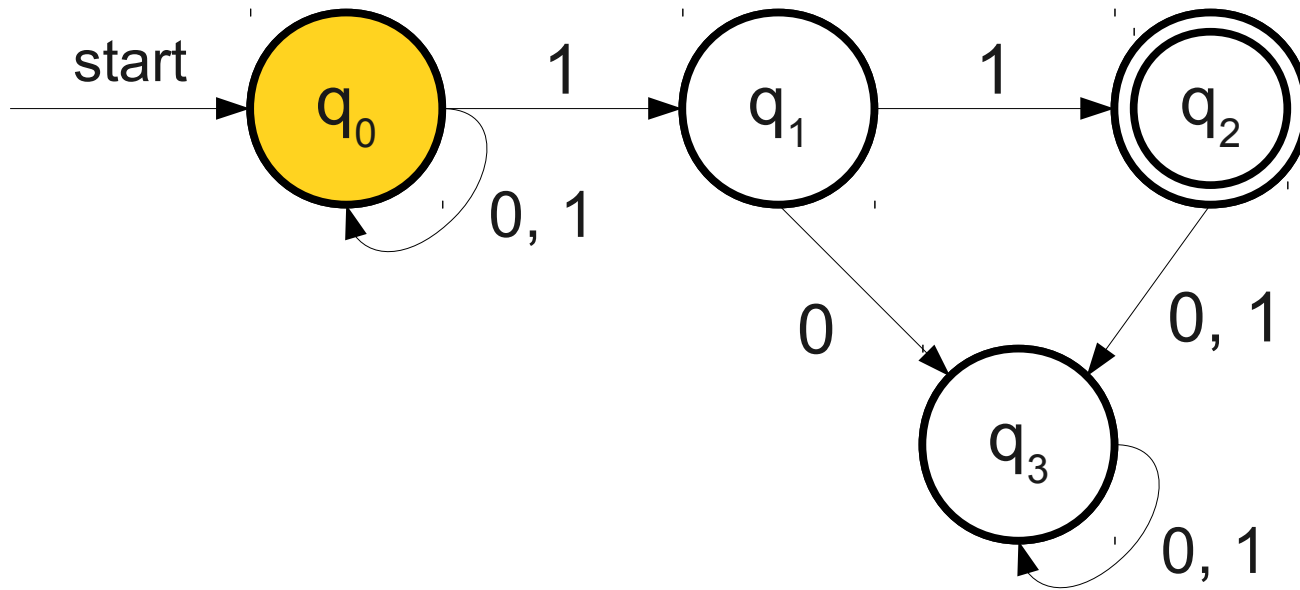
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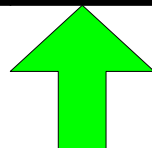
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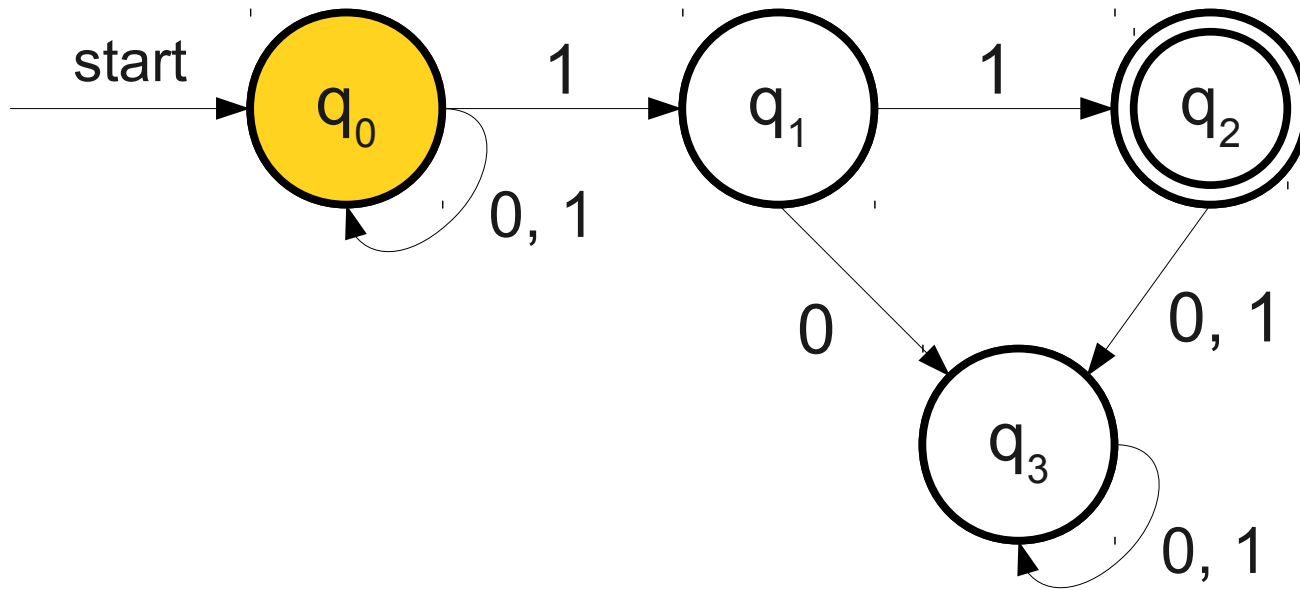
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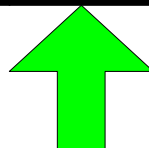
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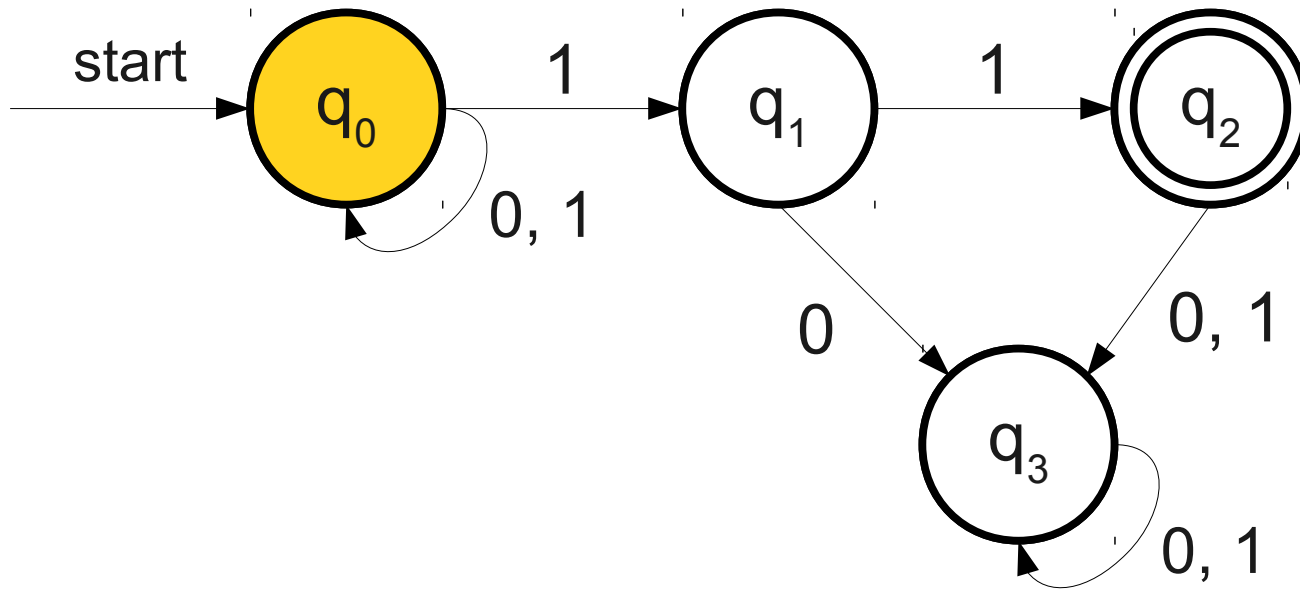
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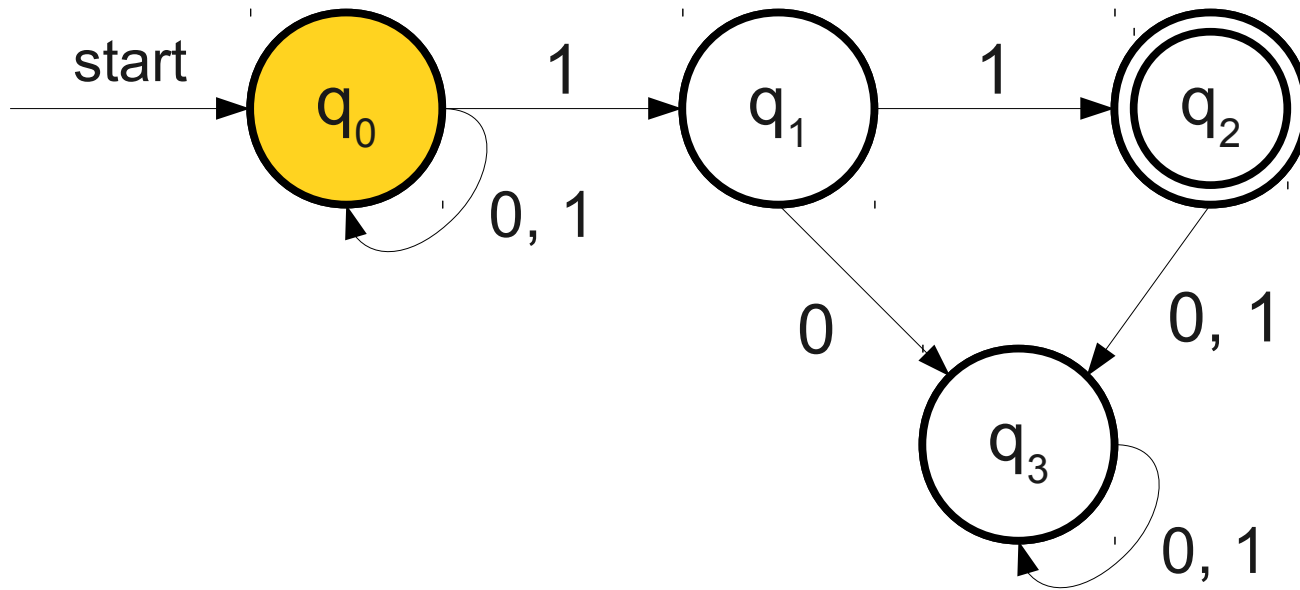
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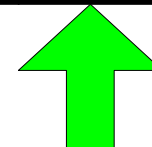
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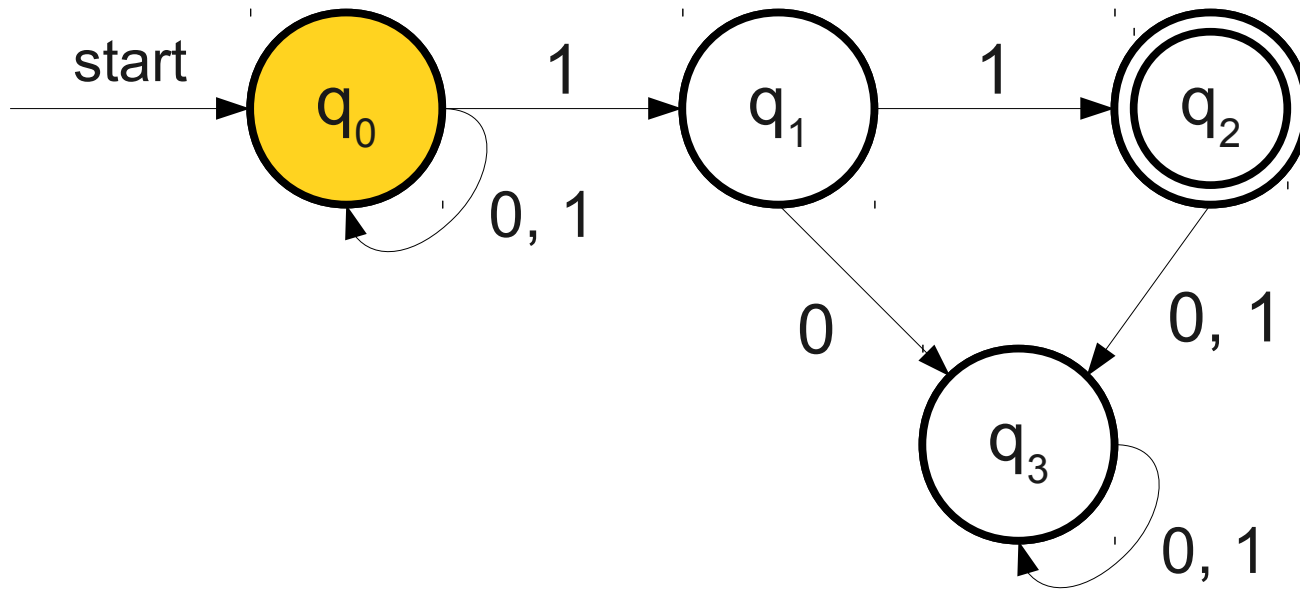
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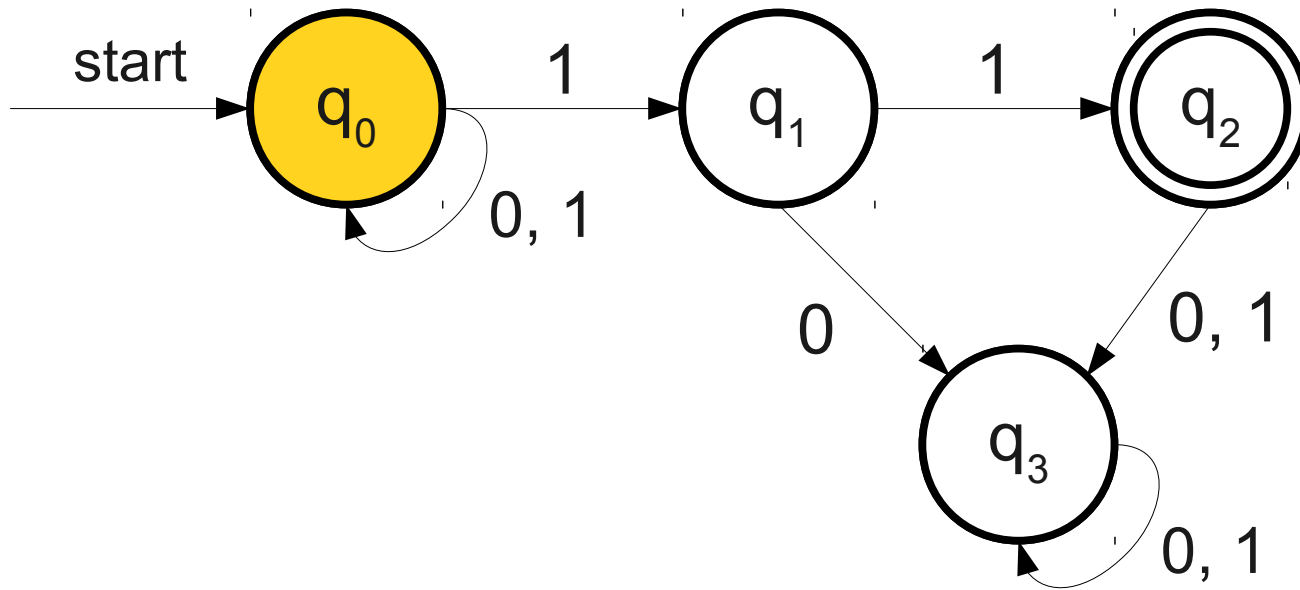


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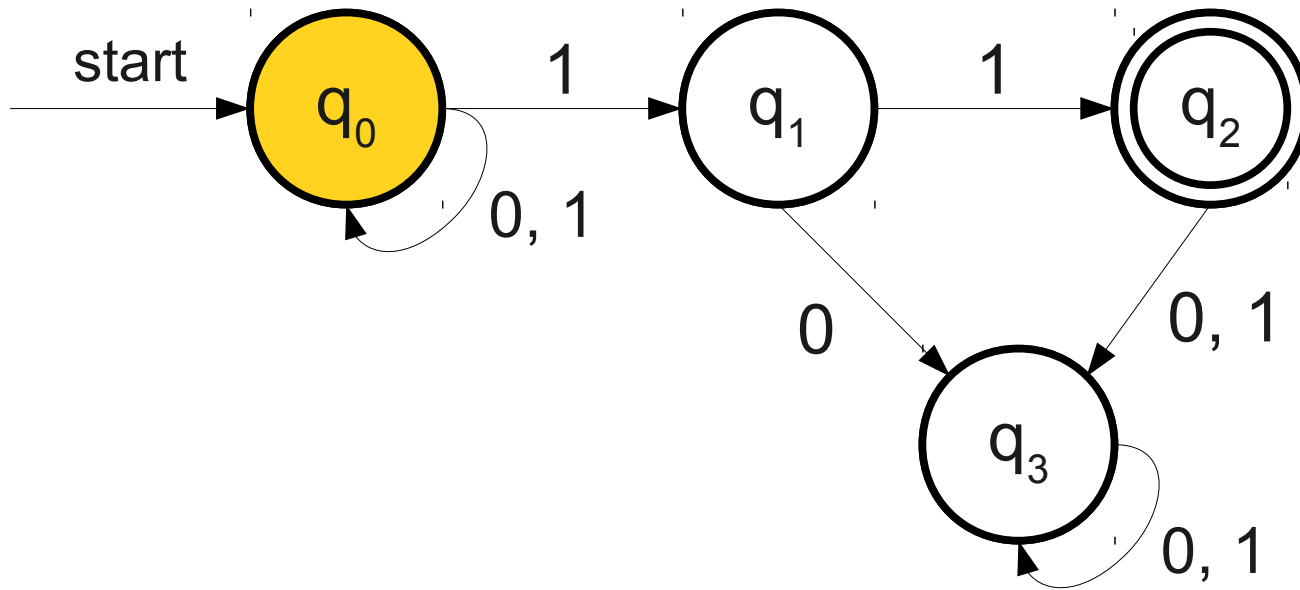
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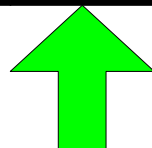
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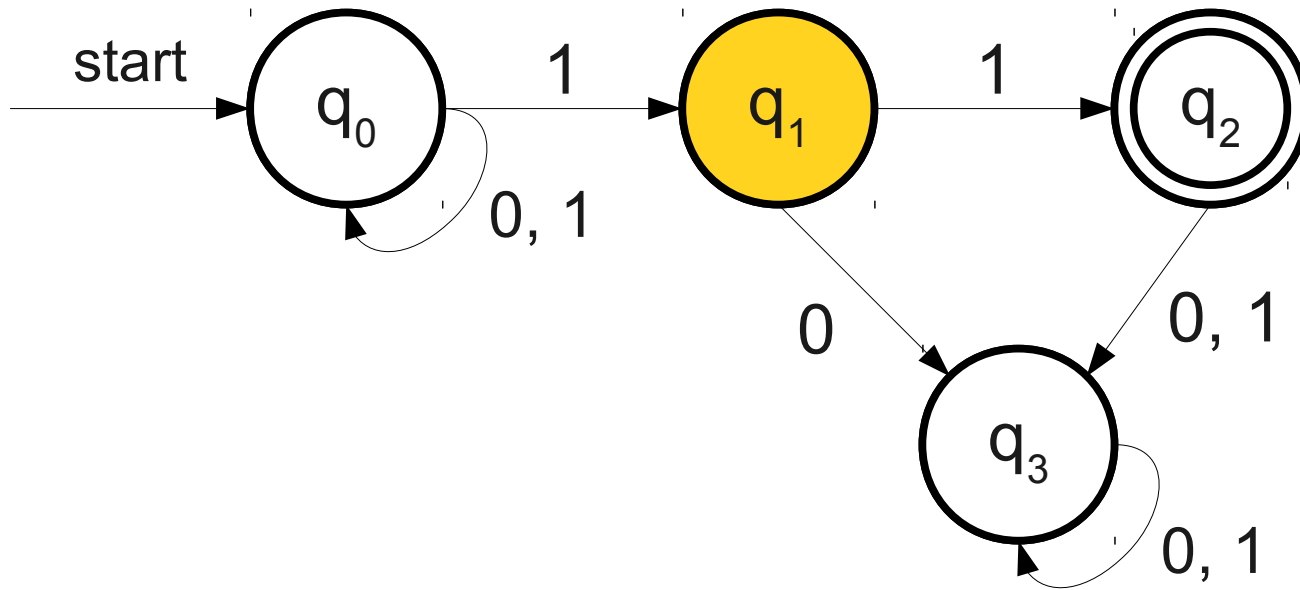
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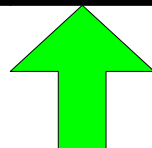
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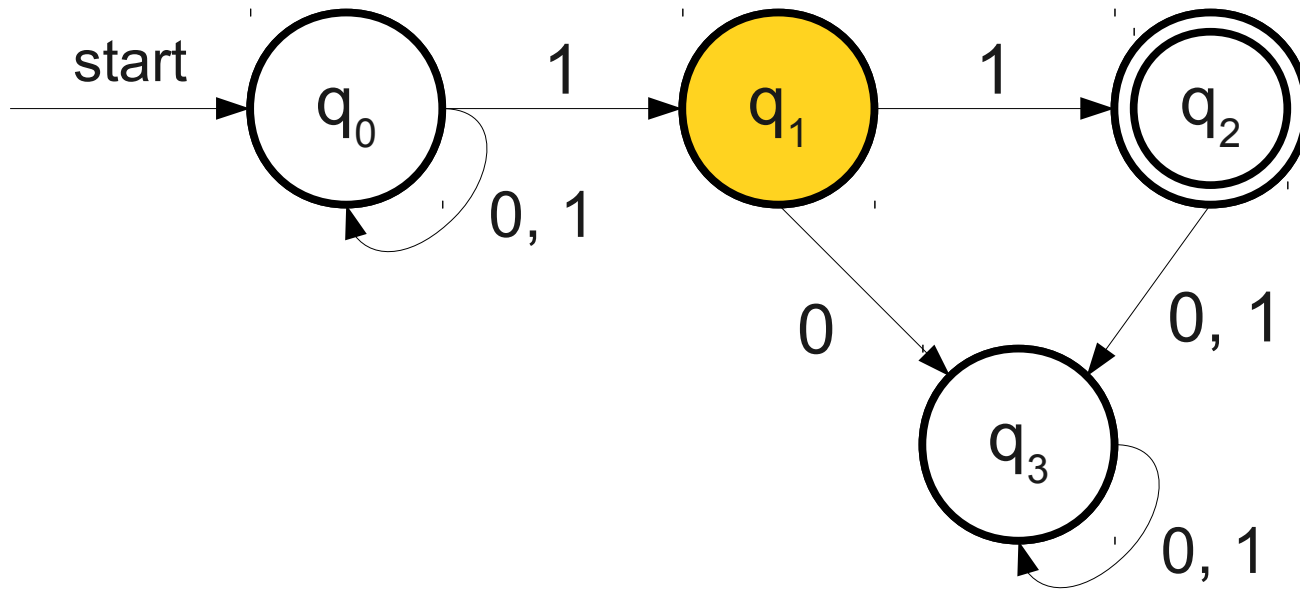
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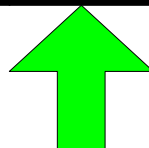
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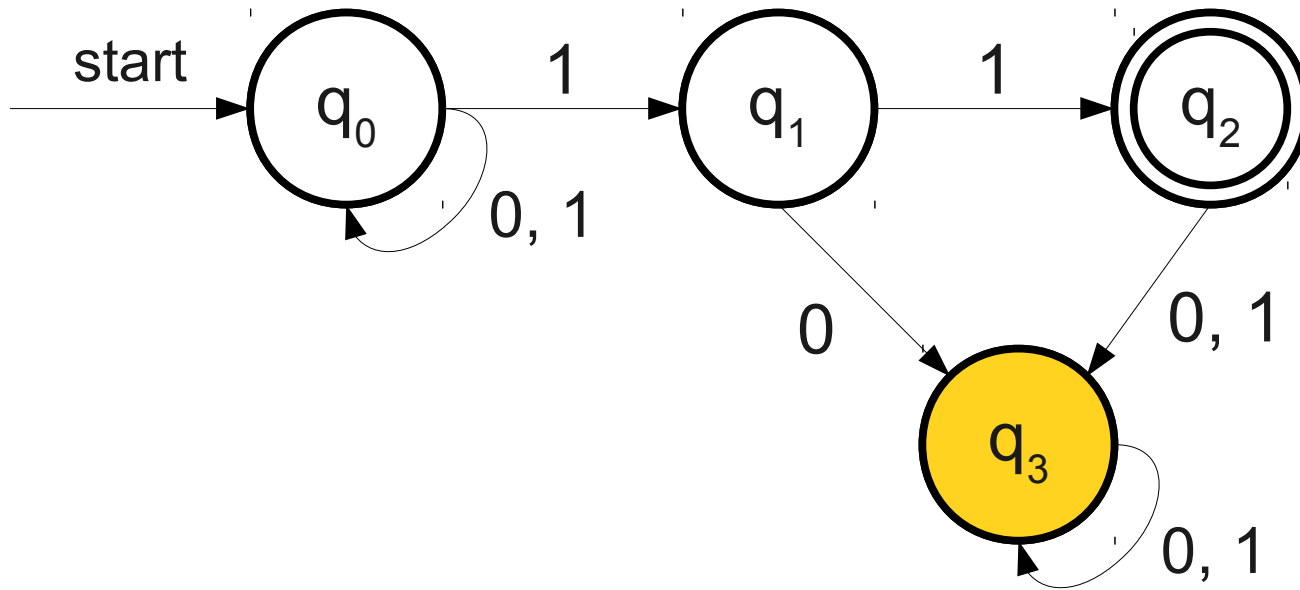
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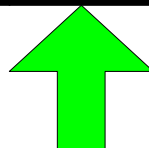
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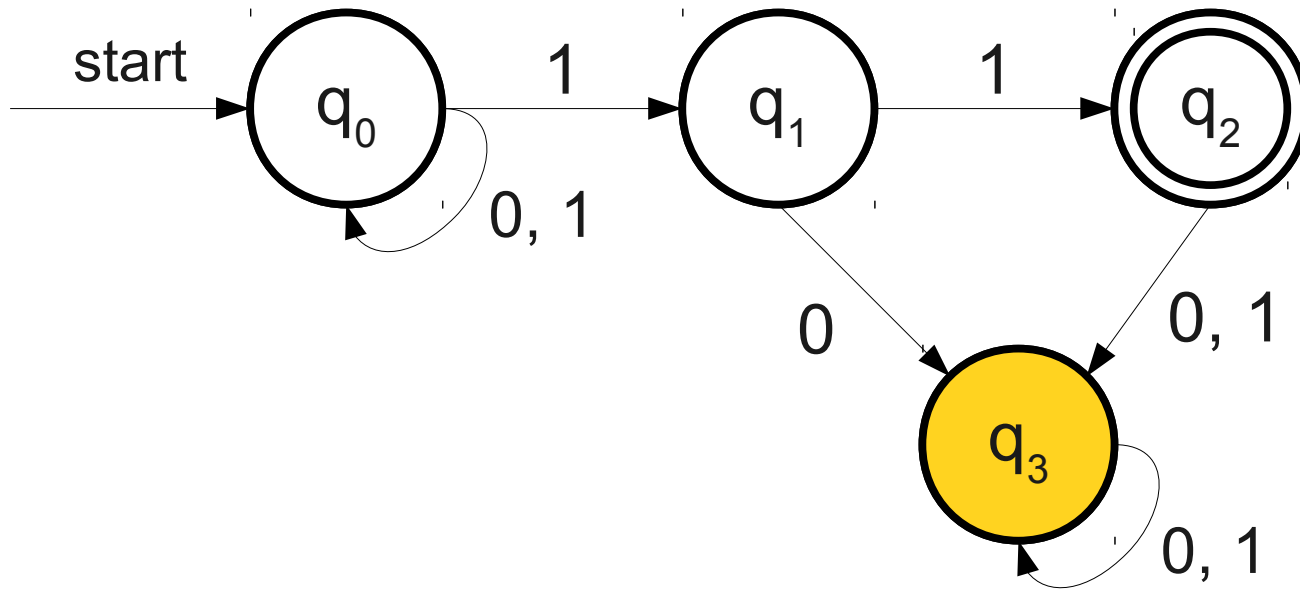
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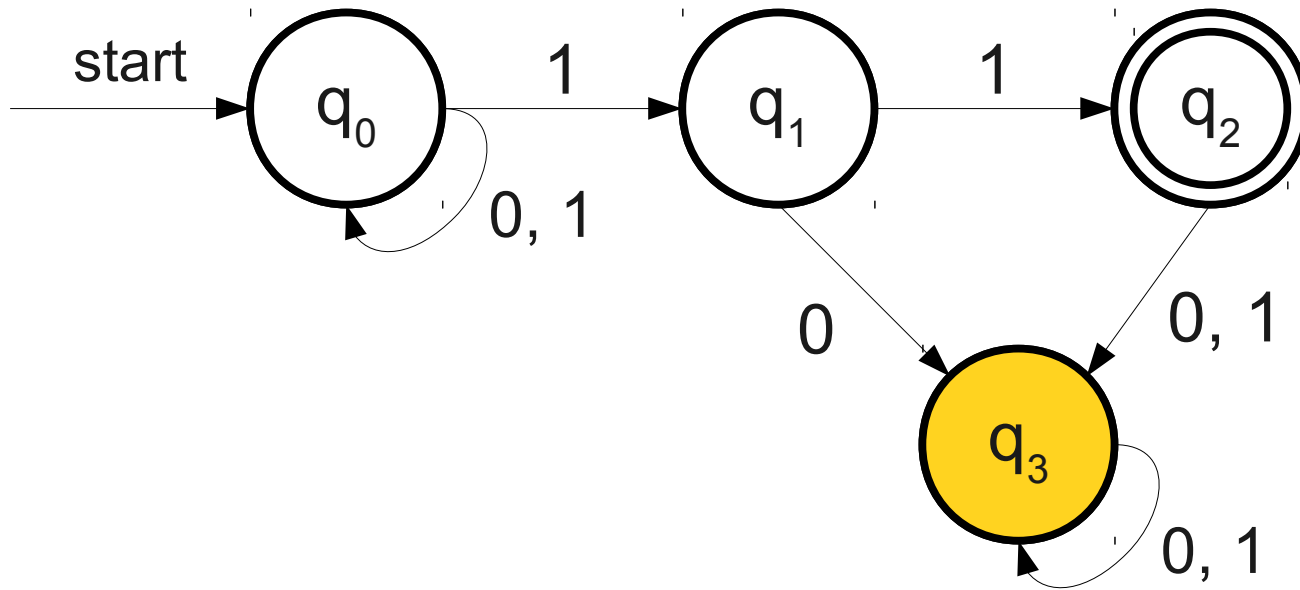
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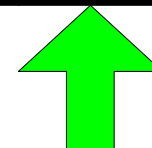
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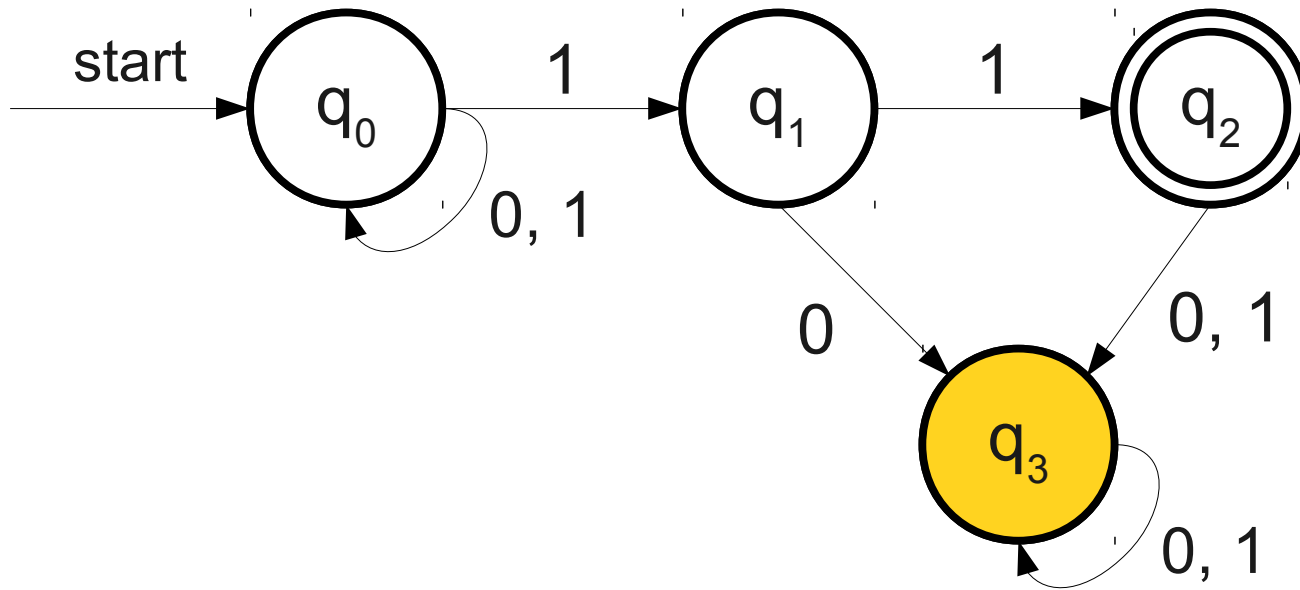
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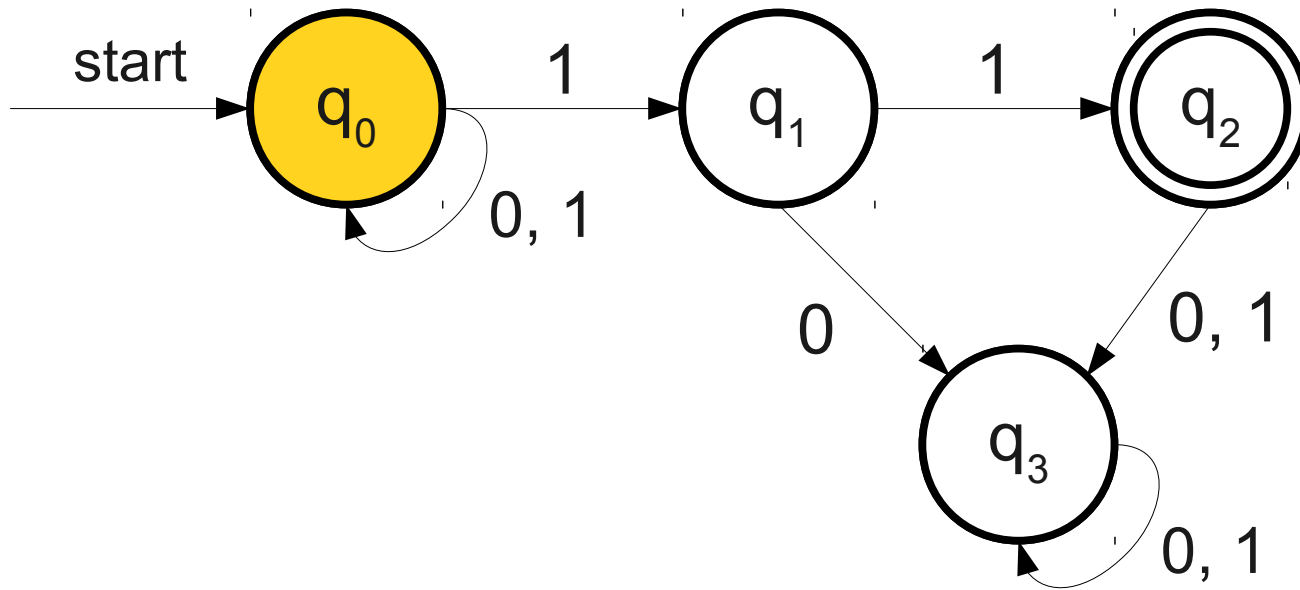


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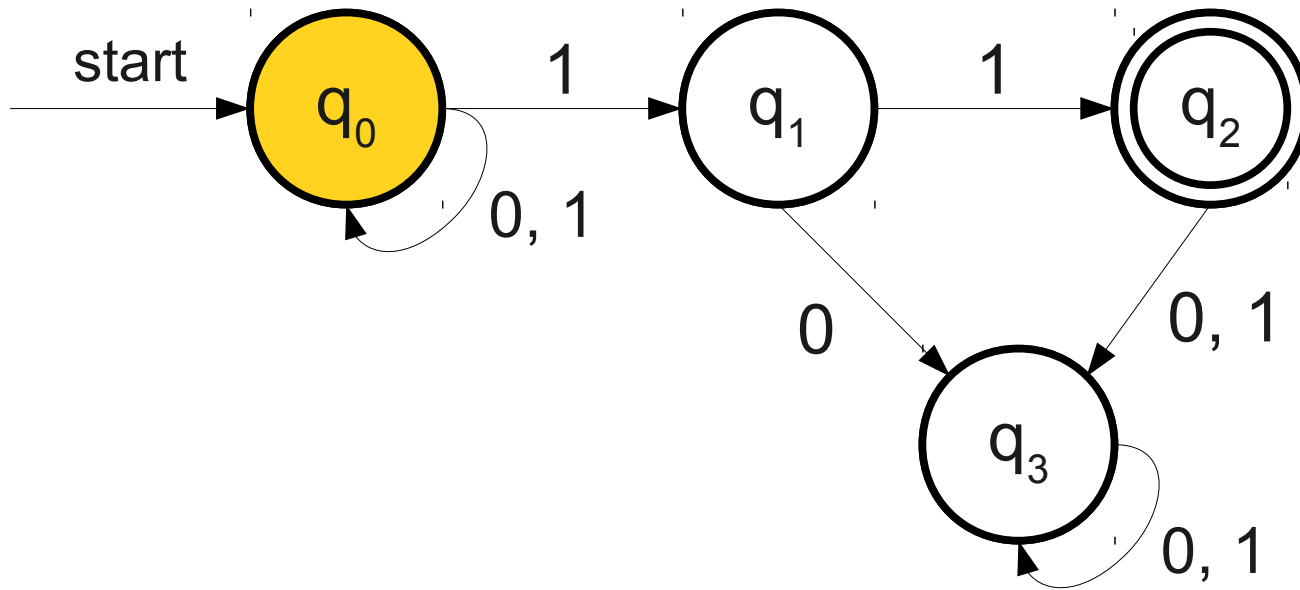
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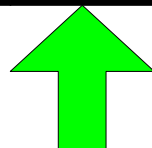
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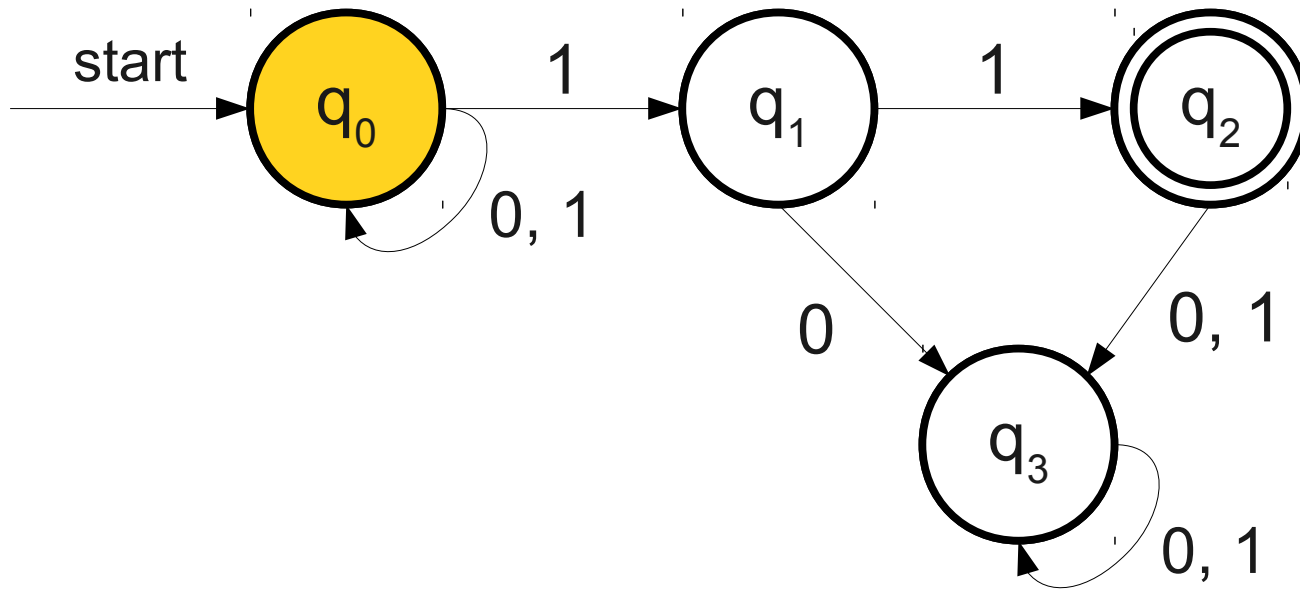
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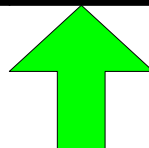
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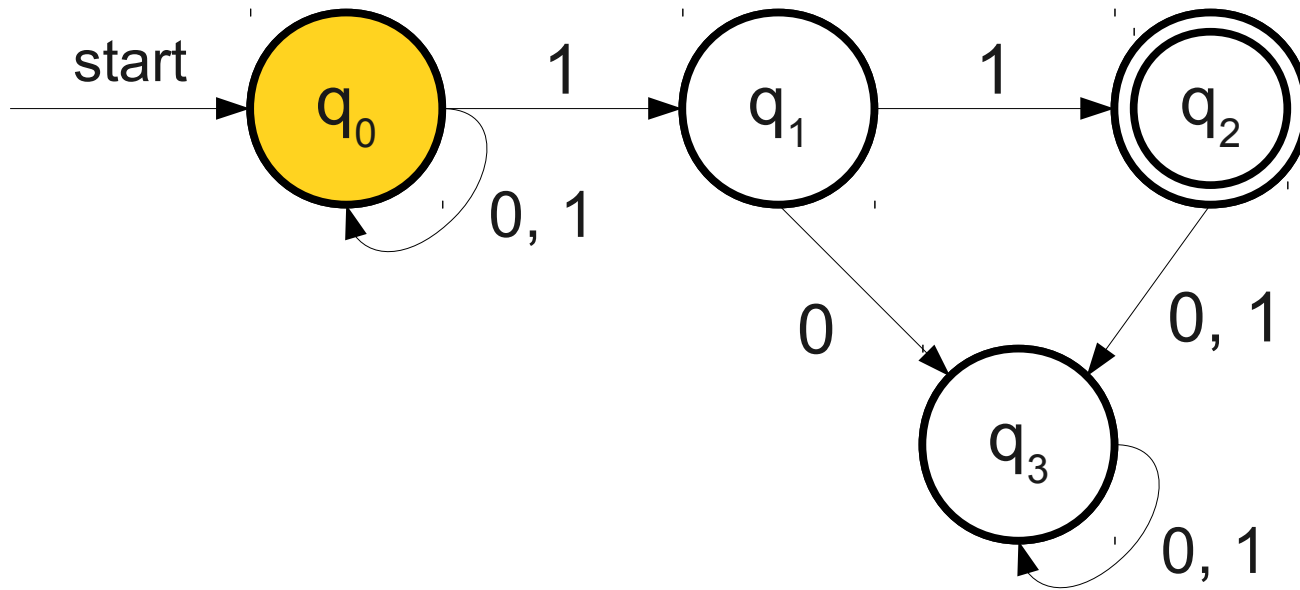
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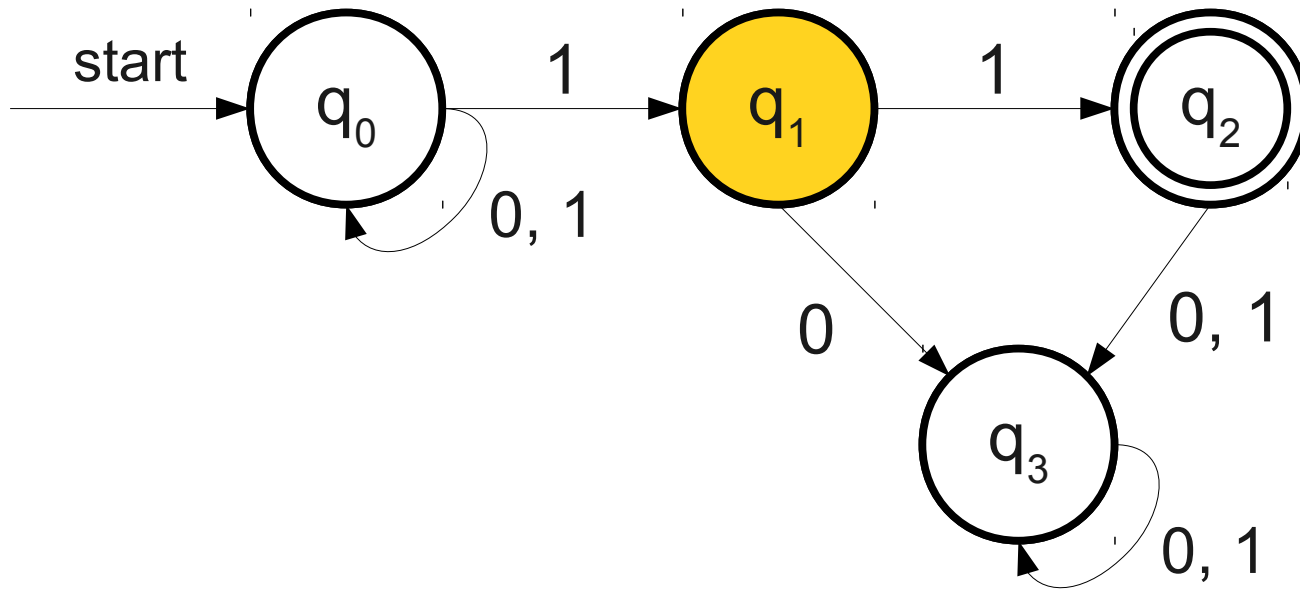
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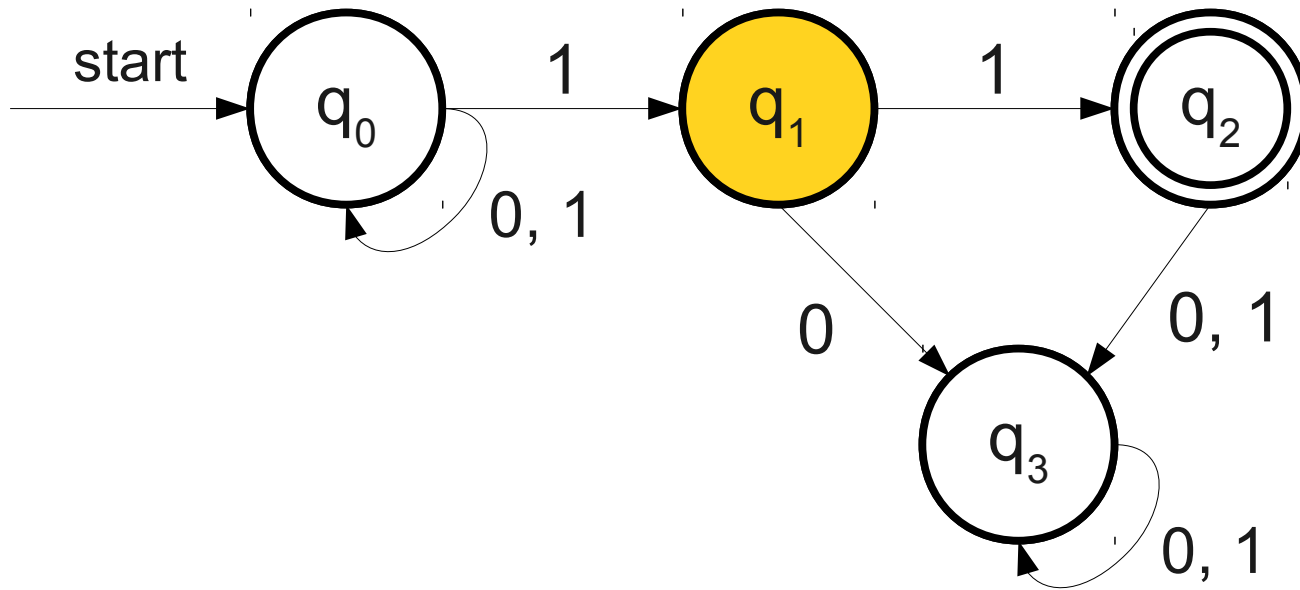
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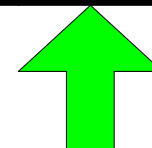
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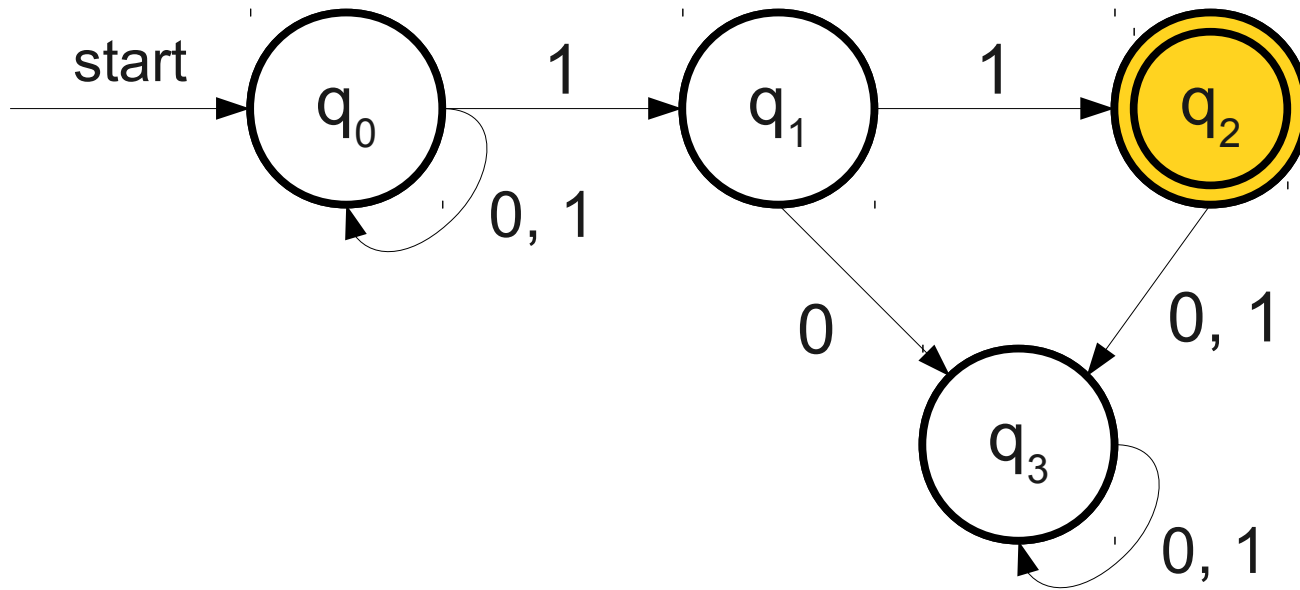
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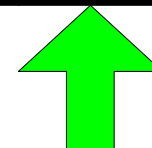
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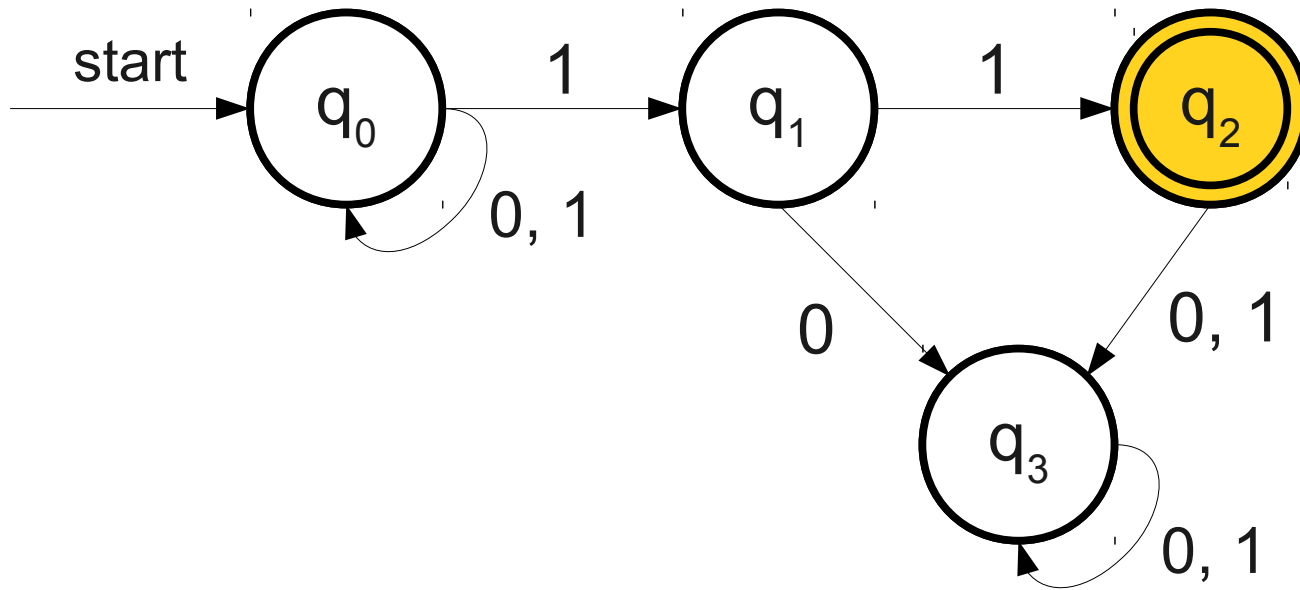
A Simple NFA



0 1 0 1 1

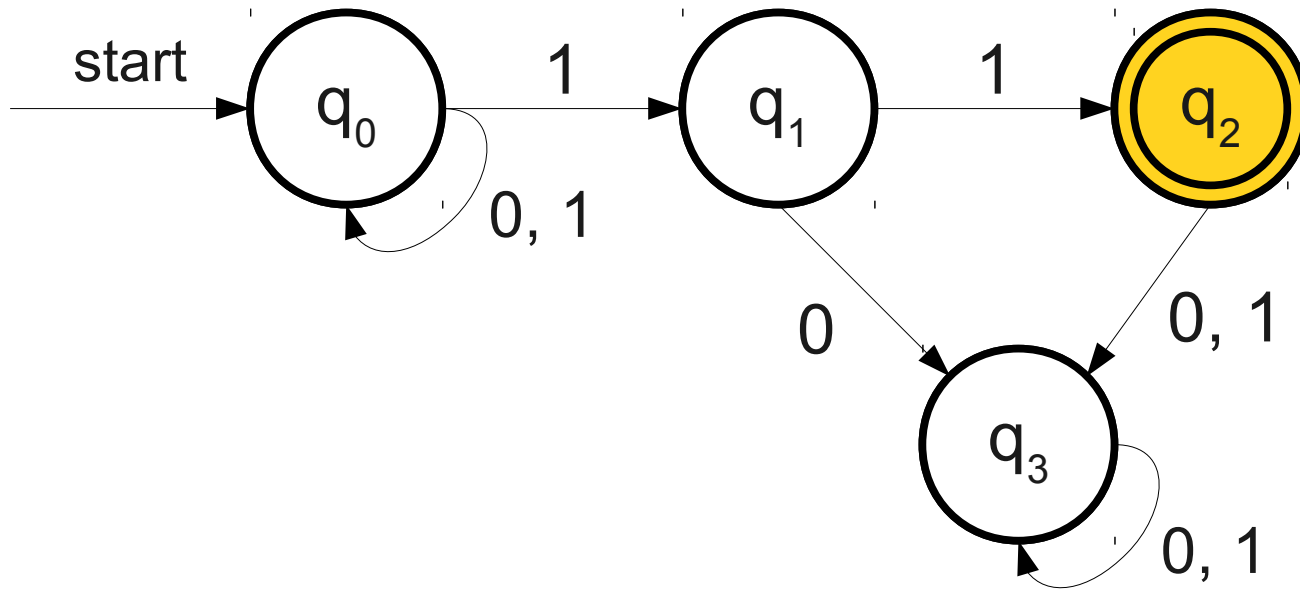


A Simple NFA



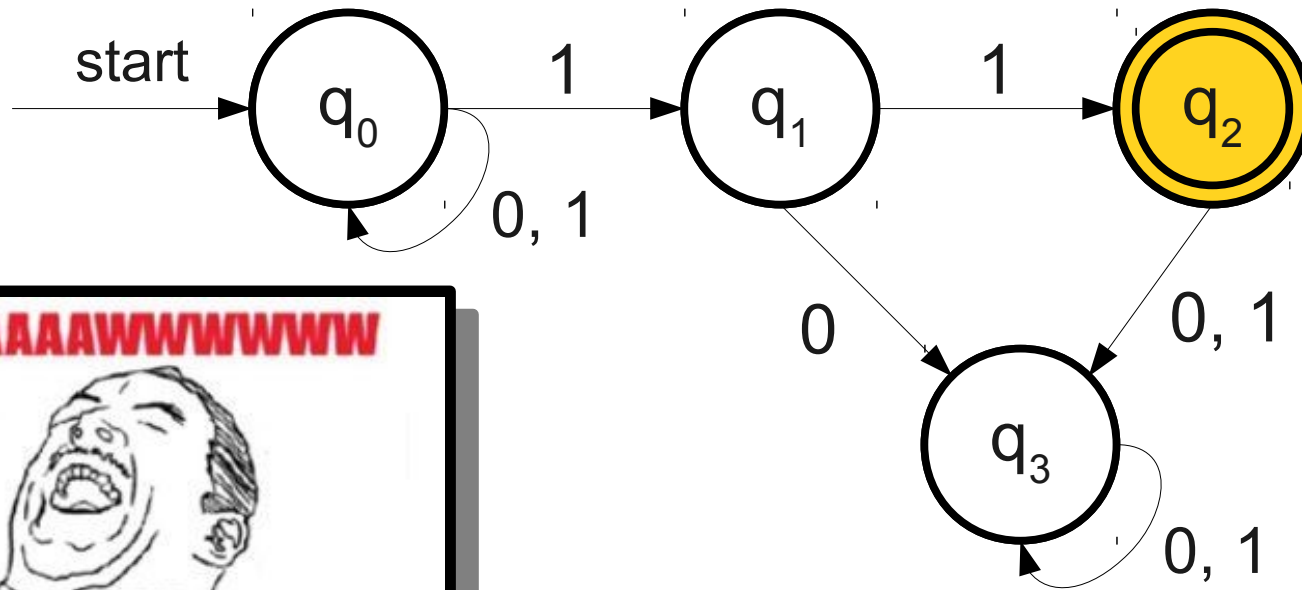
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A Simple NFA



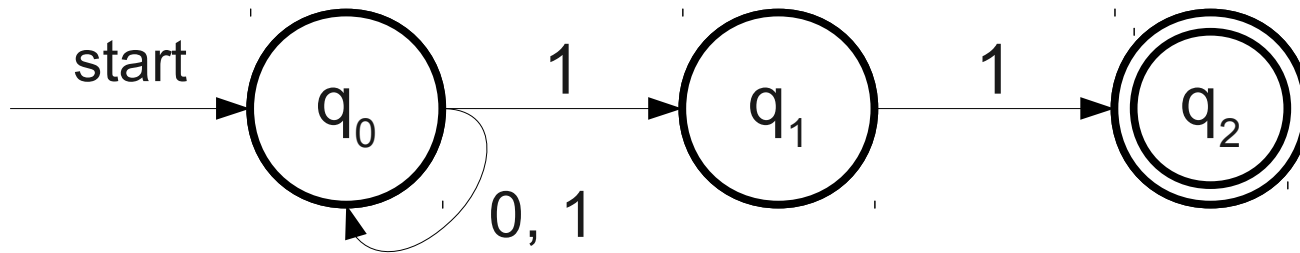
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A Simple NFA

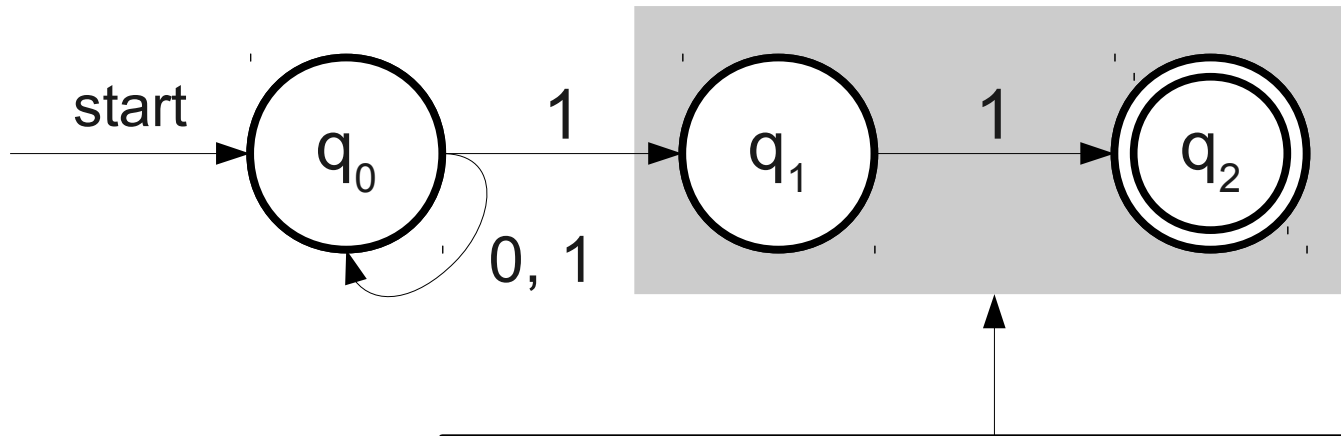


0 1 0 1 1

A More Complex NFA

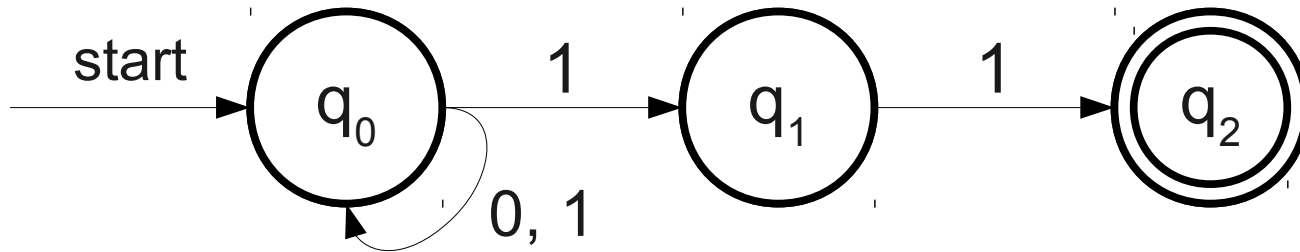


A More Complex NFA



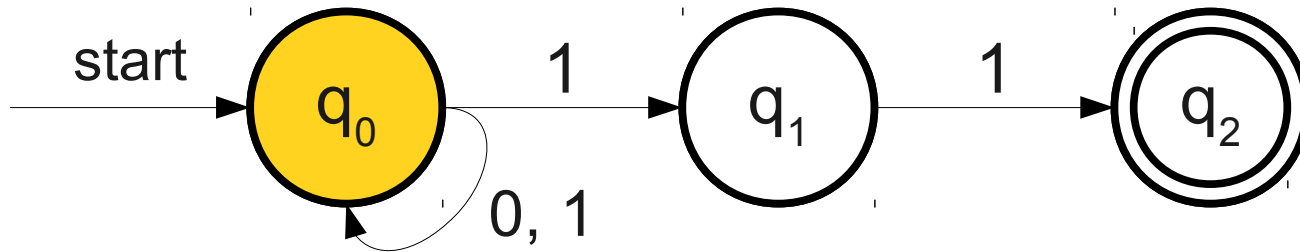
If a NFA needs to make a transition when no transition exists, the automaton **dies** and that particular path rejects.

A More Complex NFA



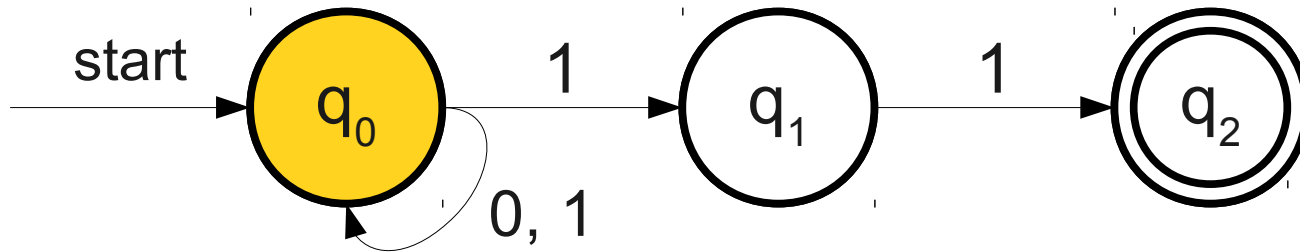
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A More Complex NFA

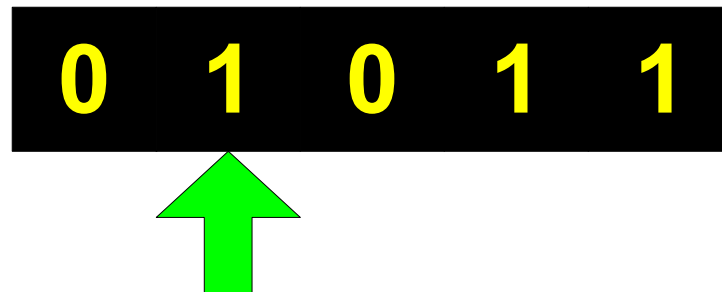
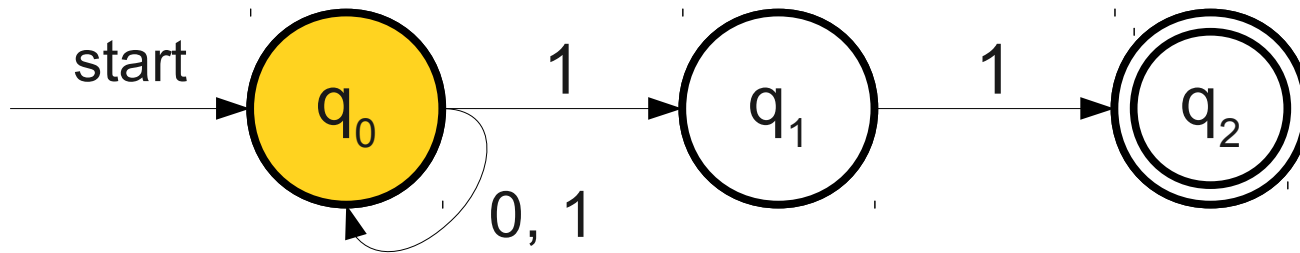


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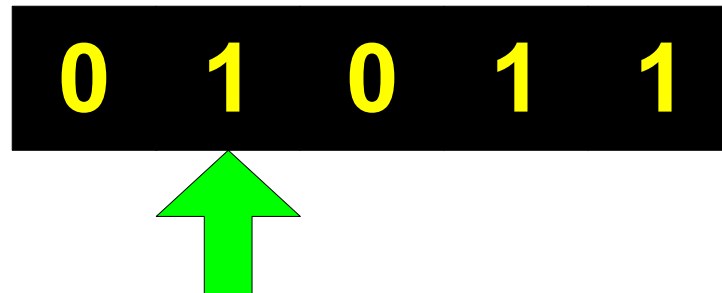
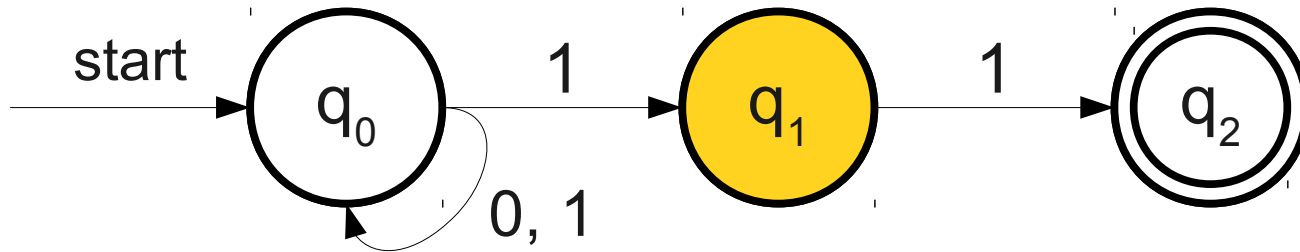
A More Complex NFA



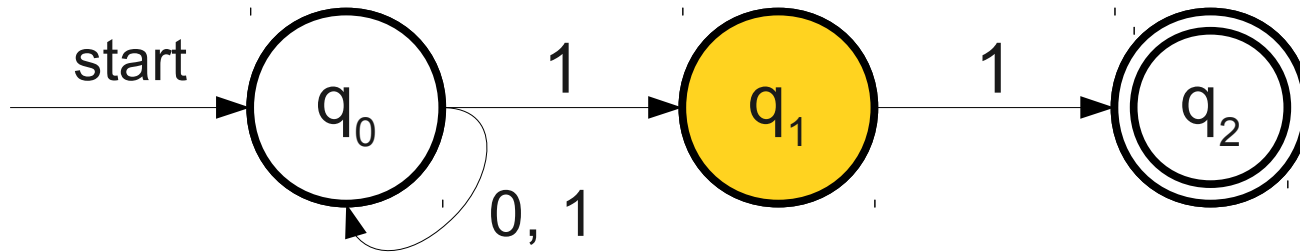
A More Complex NFA



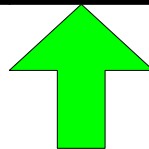
A More Complex NFA



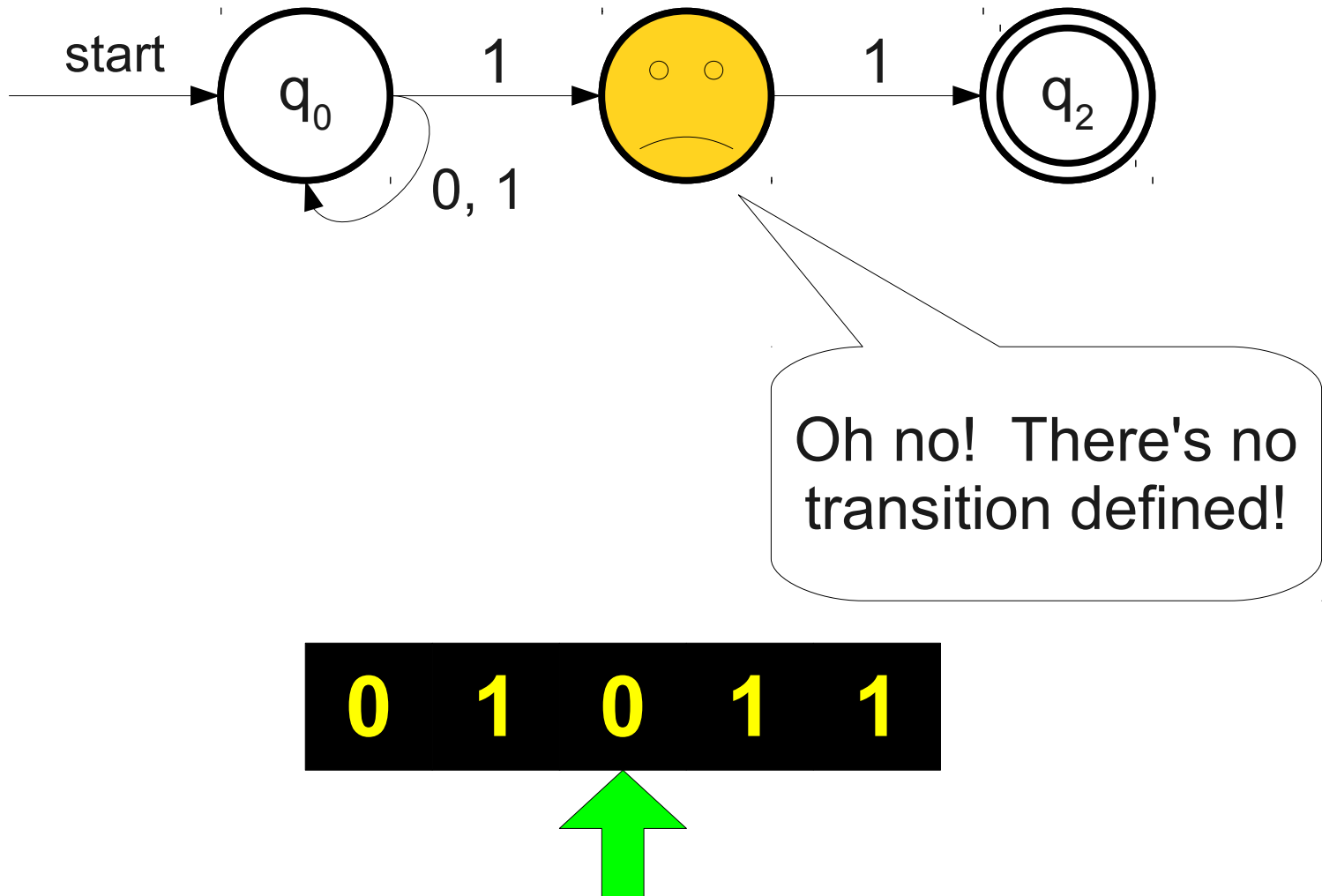
A More Complex NFA



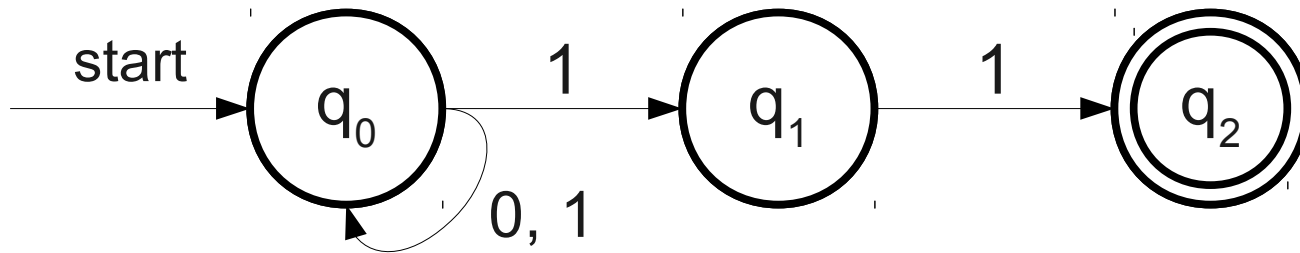
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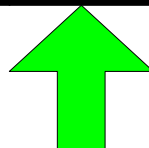
A More Complex NFA



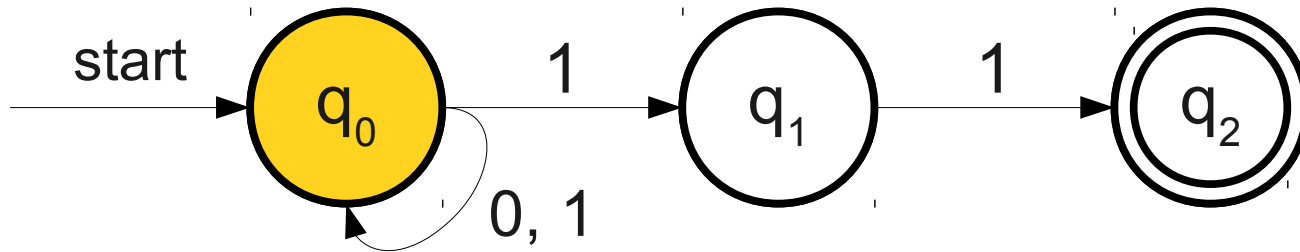
A More Complex NFA



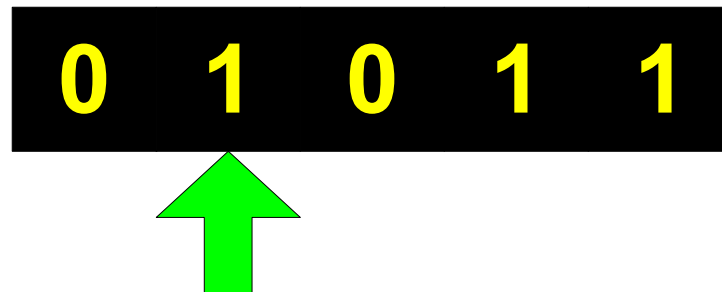
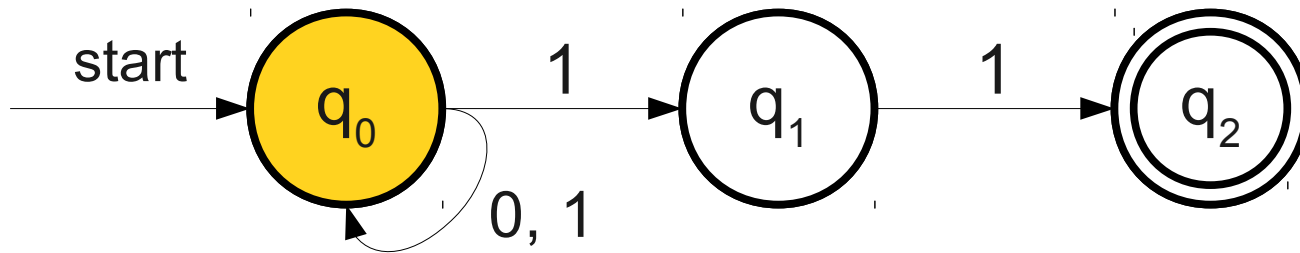
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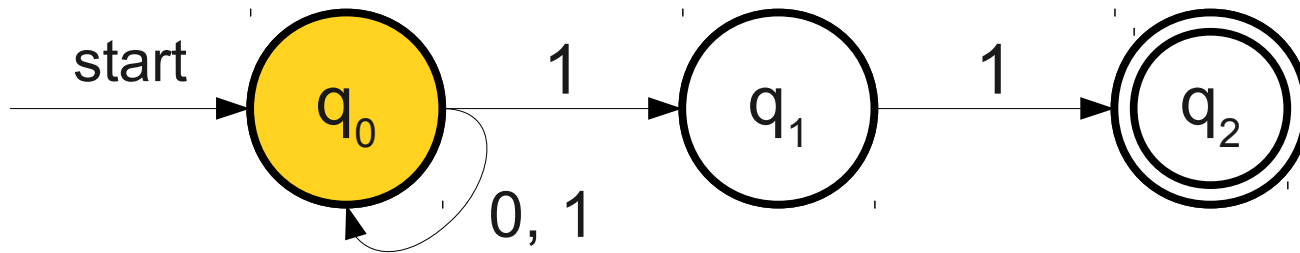
A More Complex NFA



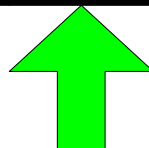
A More Complex NFA



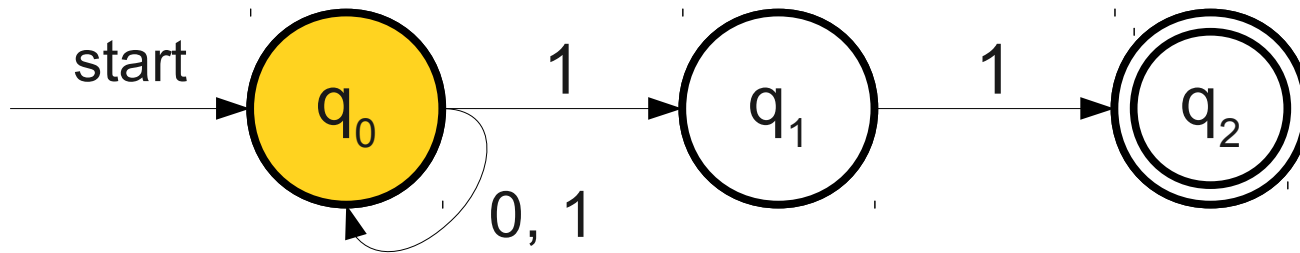
A More Complex NFA



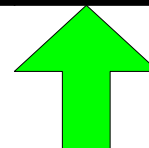
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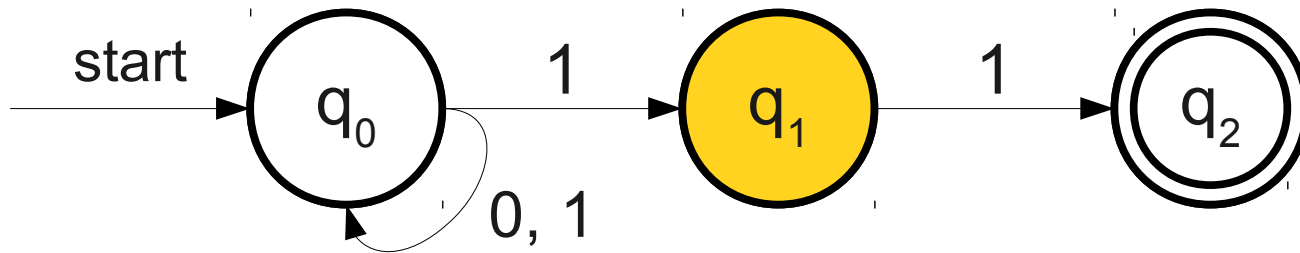
A More Complex NFA



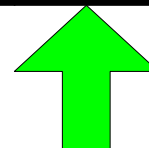
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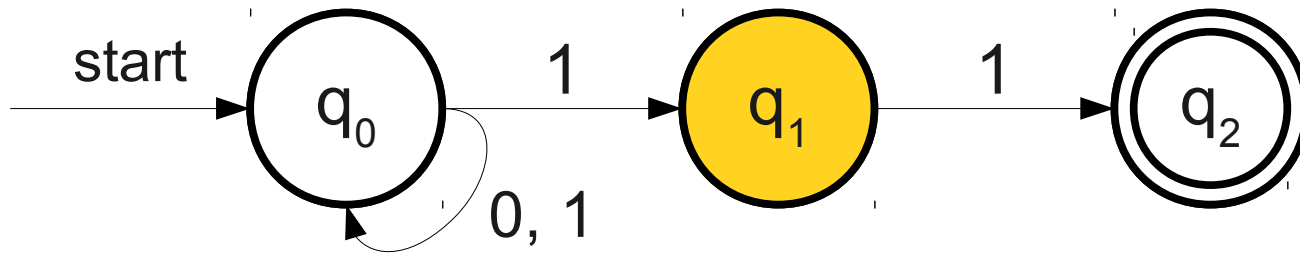
A More Complex NFA



0 1 0 1 1



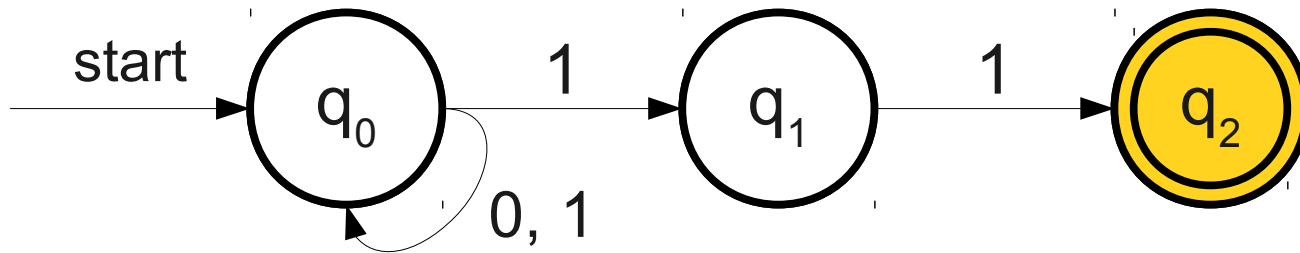
A More Complex NFA



0 1 0 1 1



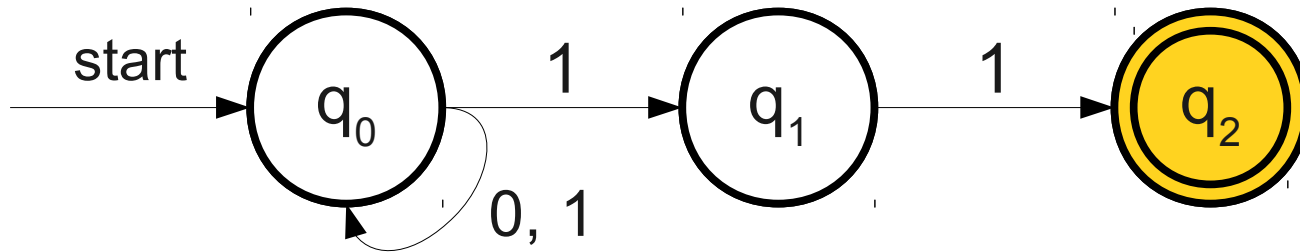
A More Complex NFA



0 1 0 1 1

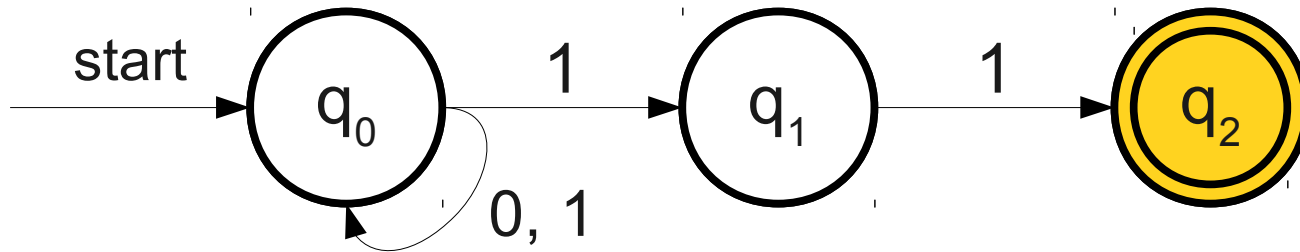


A More Complex NFA



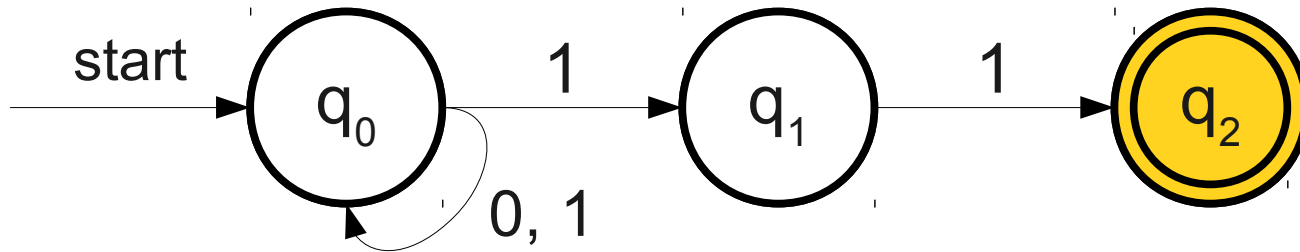
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A More Complex NFA



0 1 0 1 1

A More Complex NFA

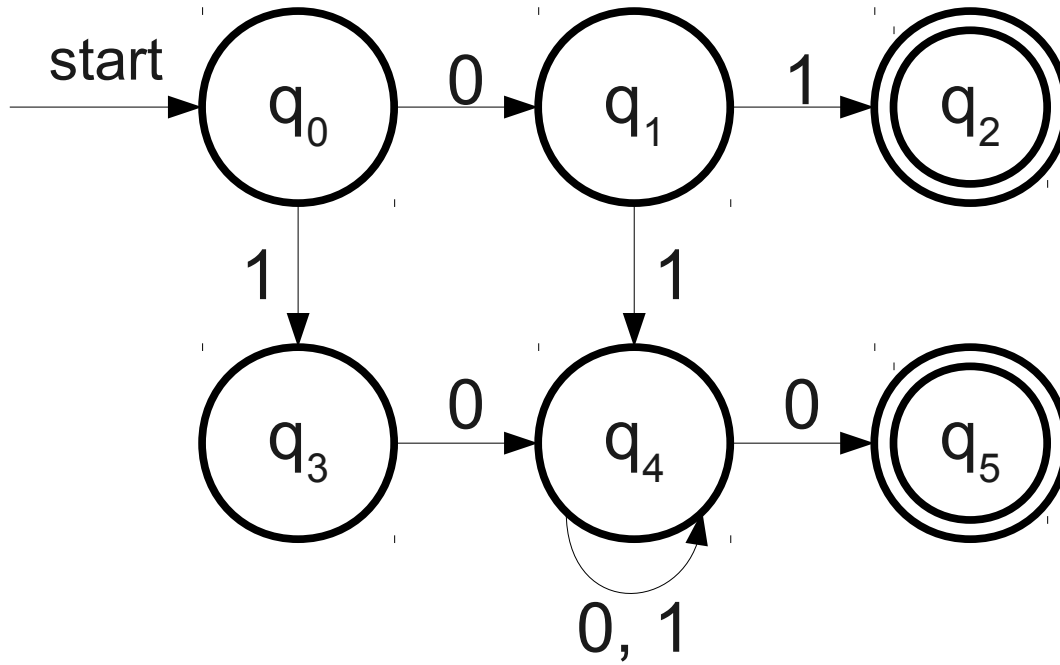


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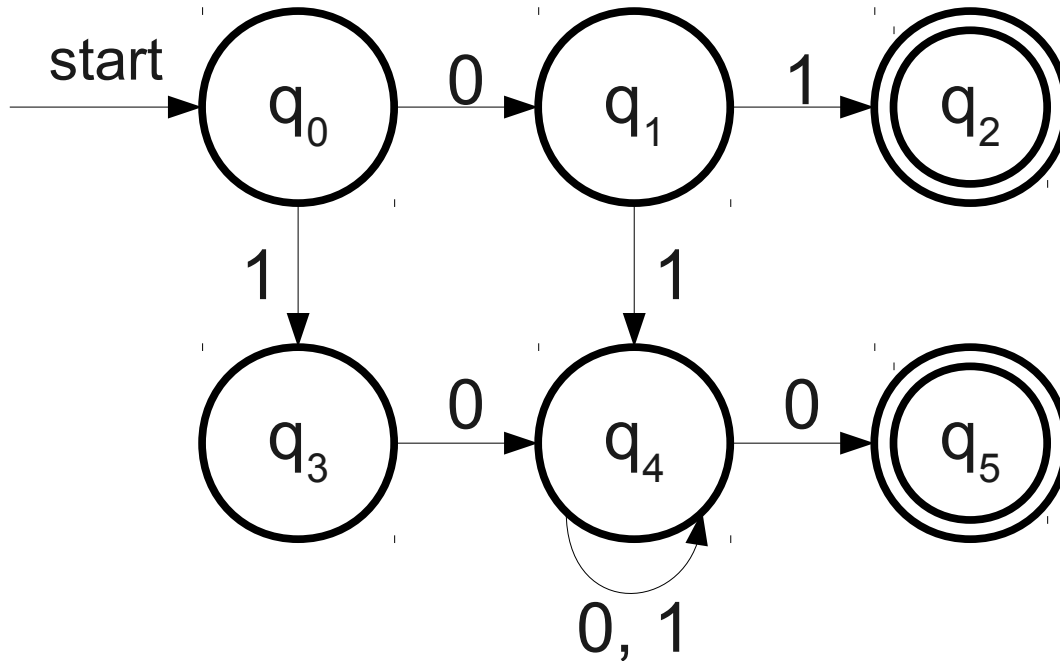
Intuiting Nondeterminism

- Nondeterministic machines are a serious departure from physical computers.
- How can we build up an intuition for them?
- Three approaches:
 - **Tree computation**
 - **Perfect guessing**
 - **Massive parallelism**

Tree Computation

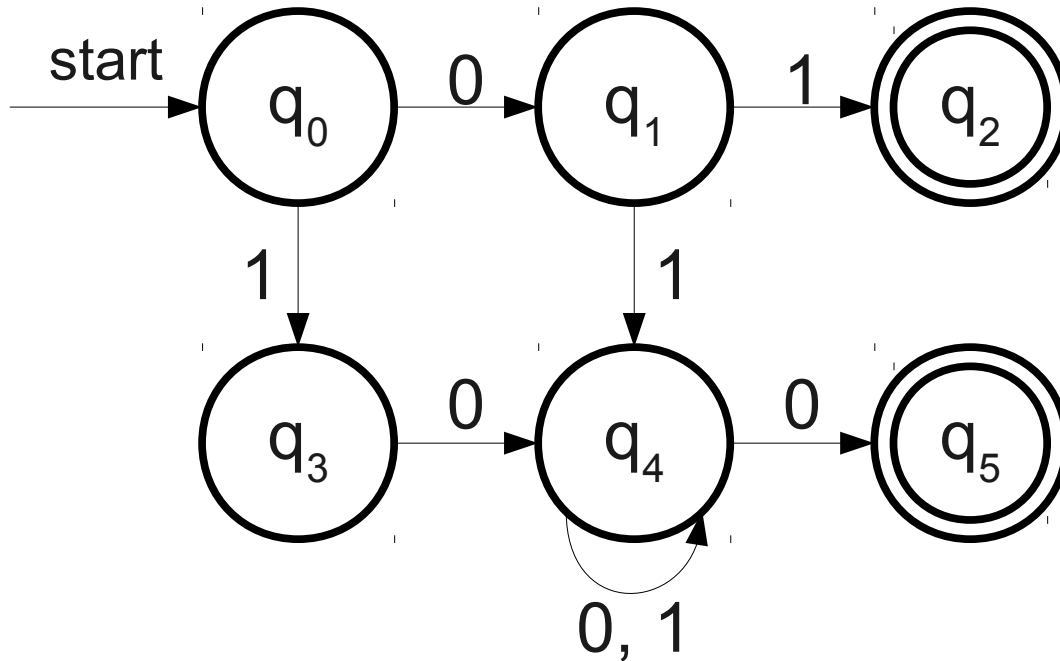
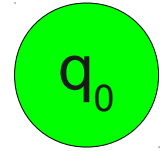


Tree Computation



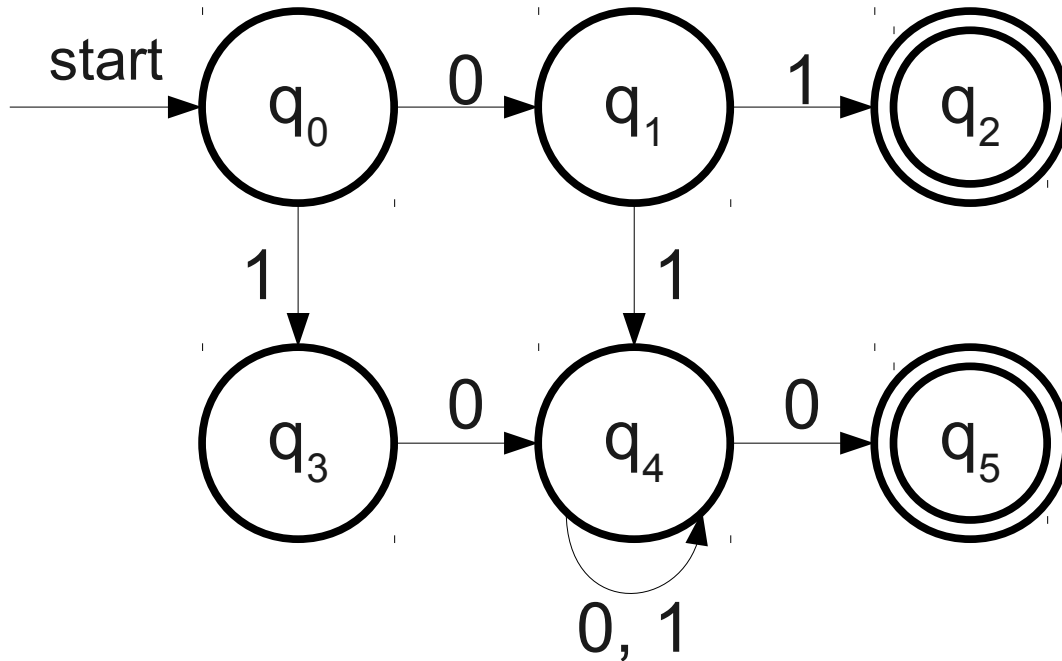
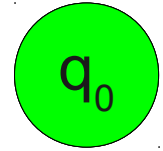
0 1 0 1 0

Tree Computation



0 1 0 1 0

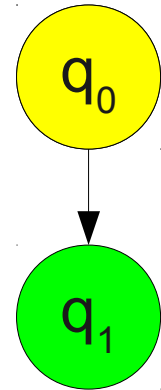
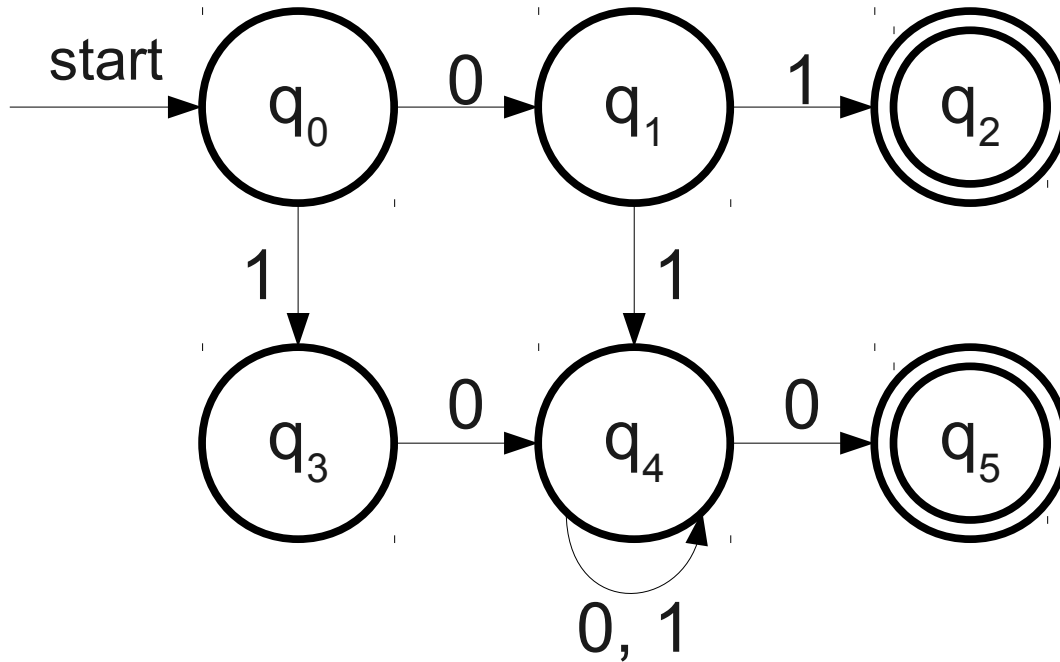
Tree Computation



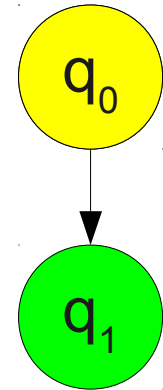
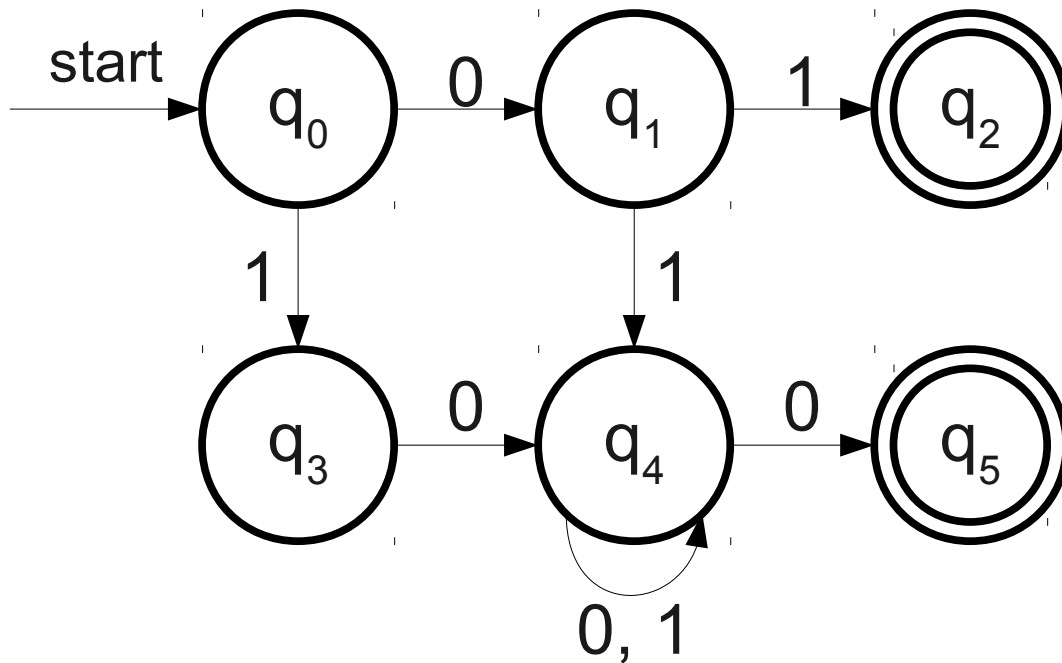
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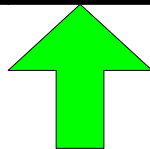
Tree Computation



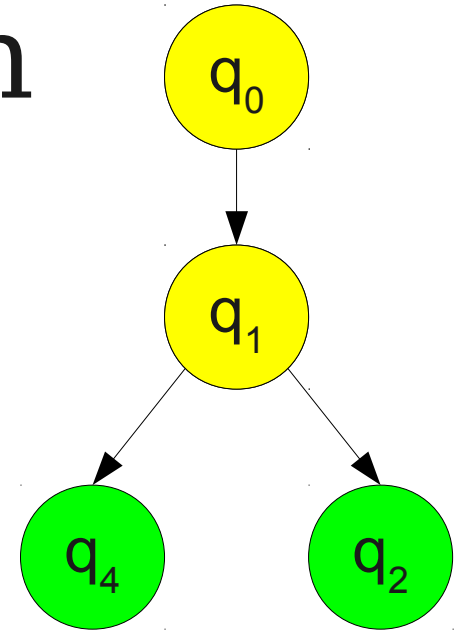
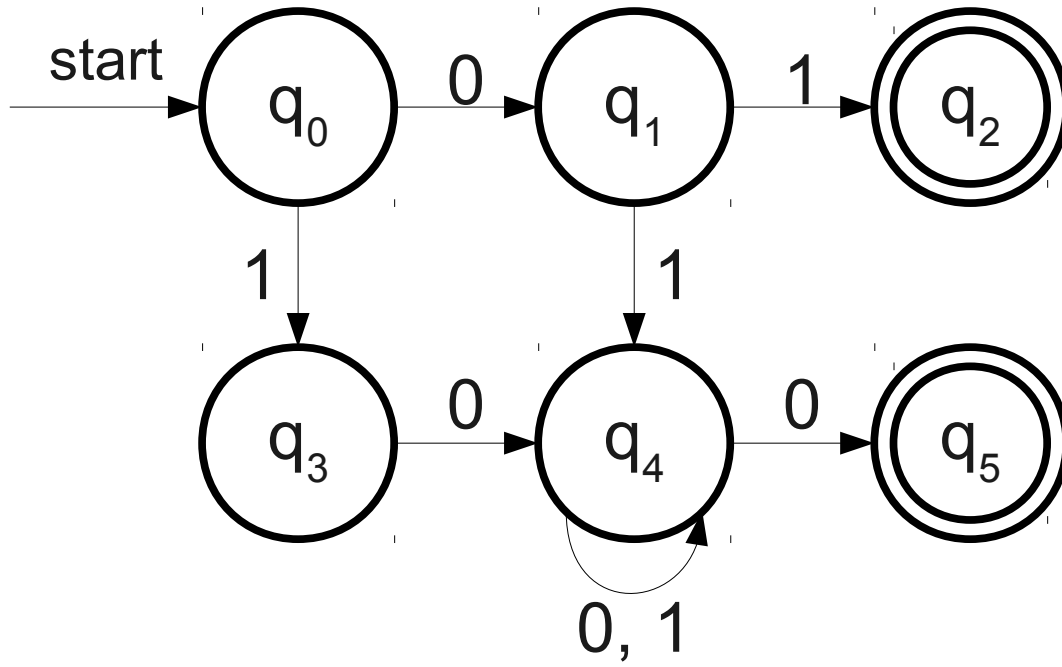
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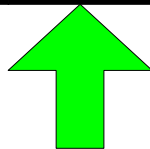
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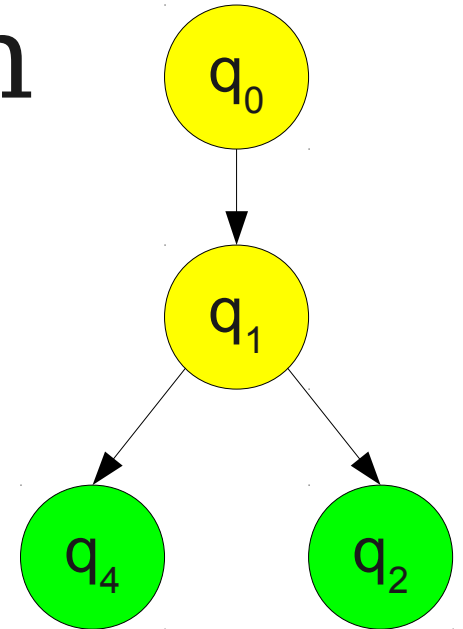
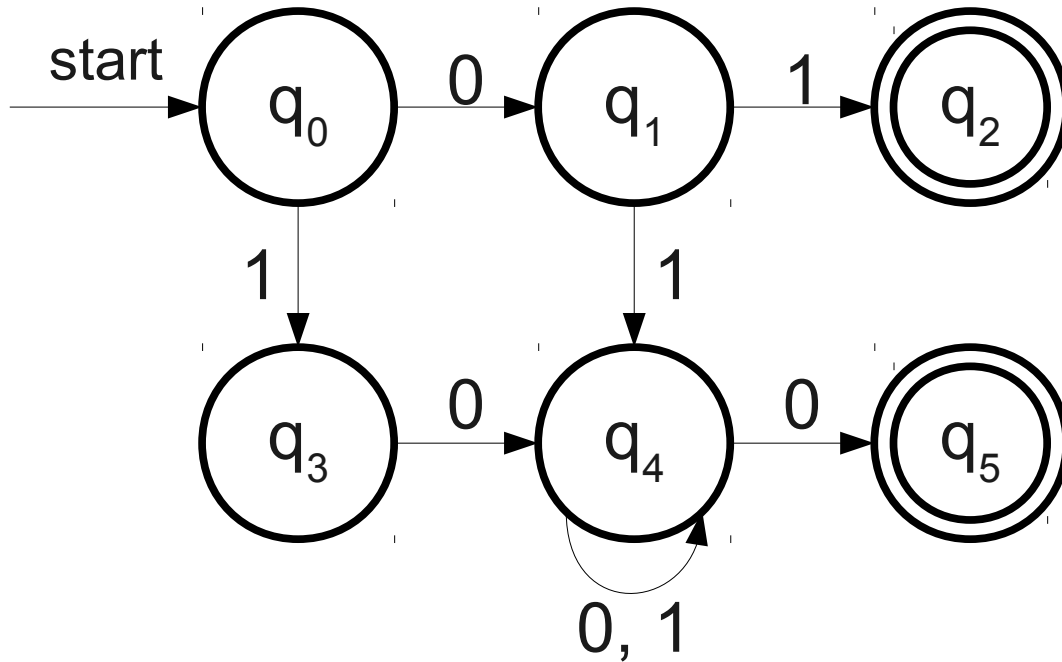
Tree Computation



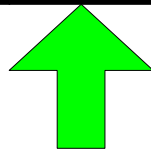
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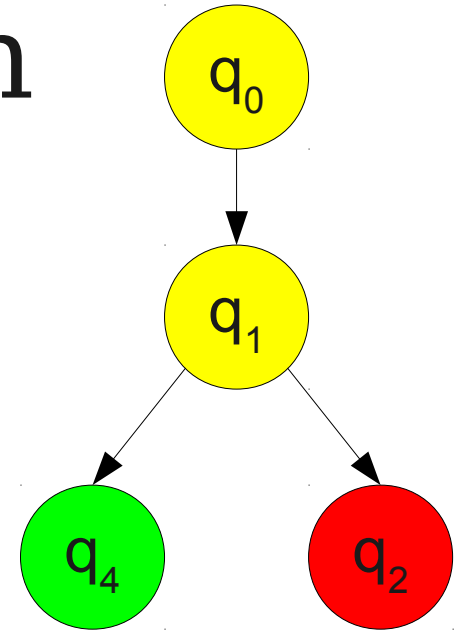
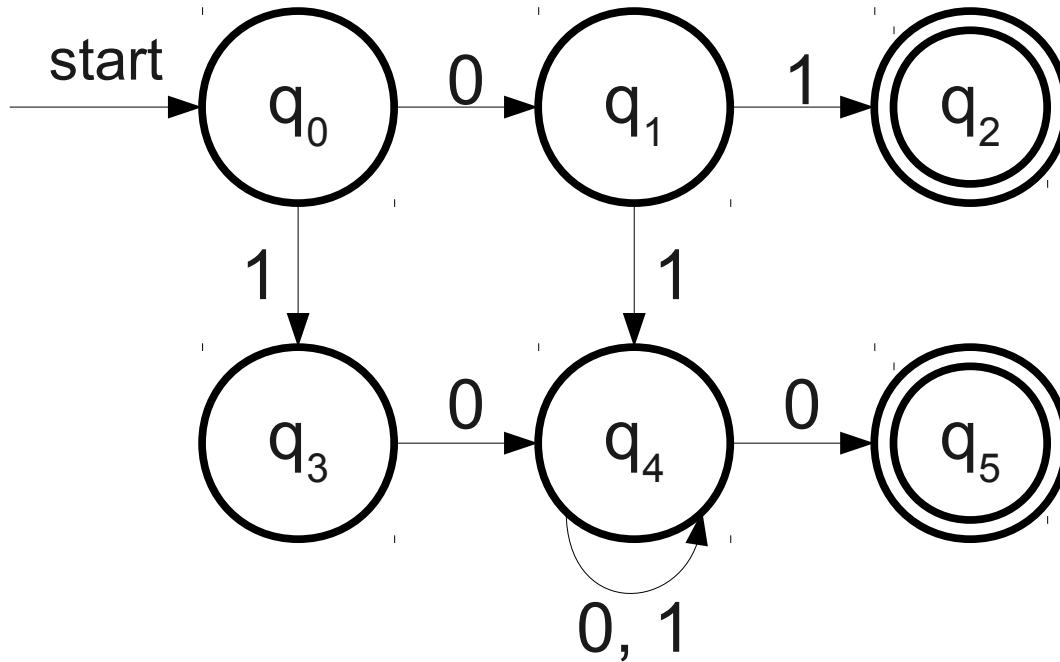
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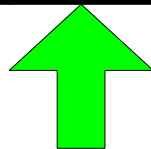
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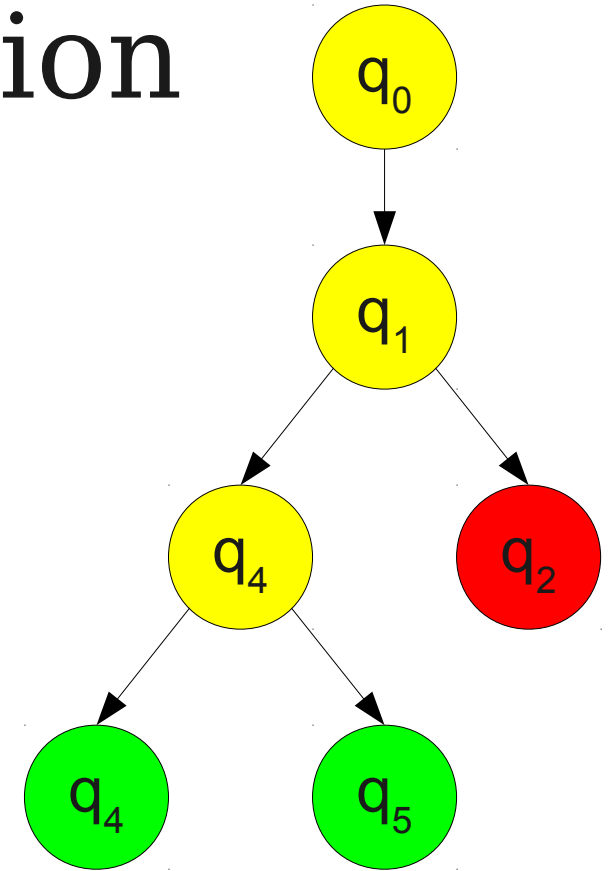
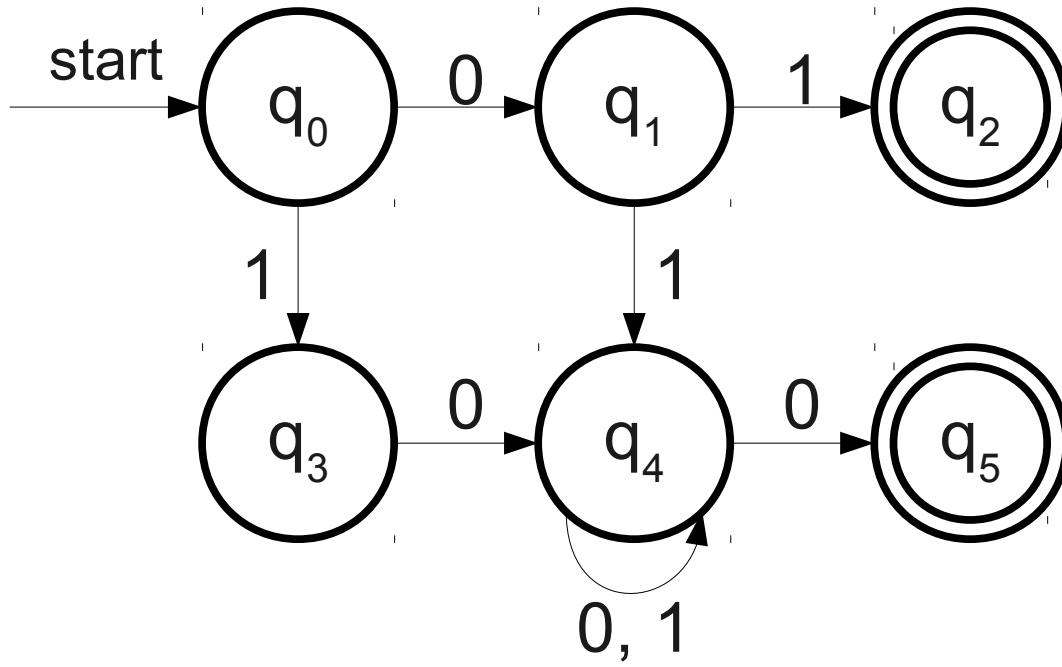
Tree Computation



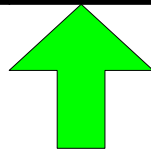
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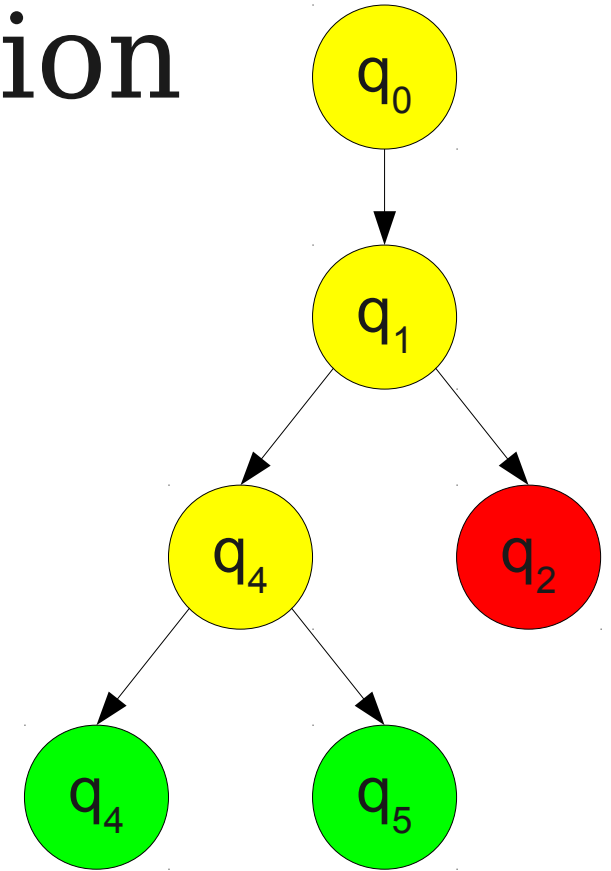
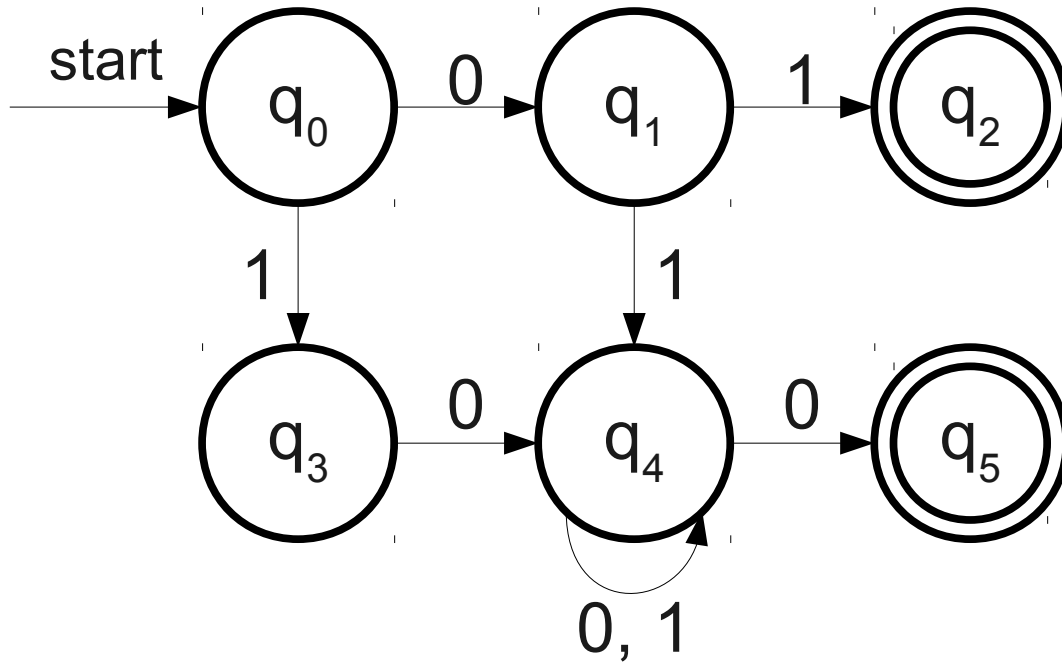
Tree Computation



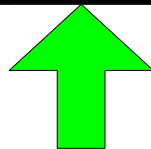
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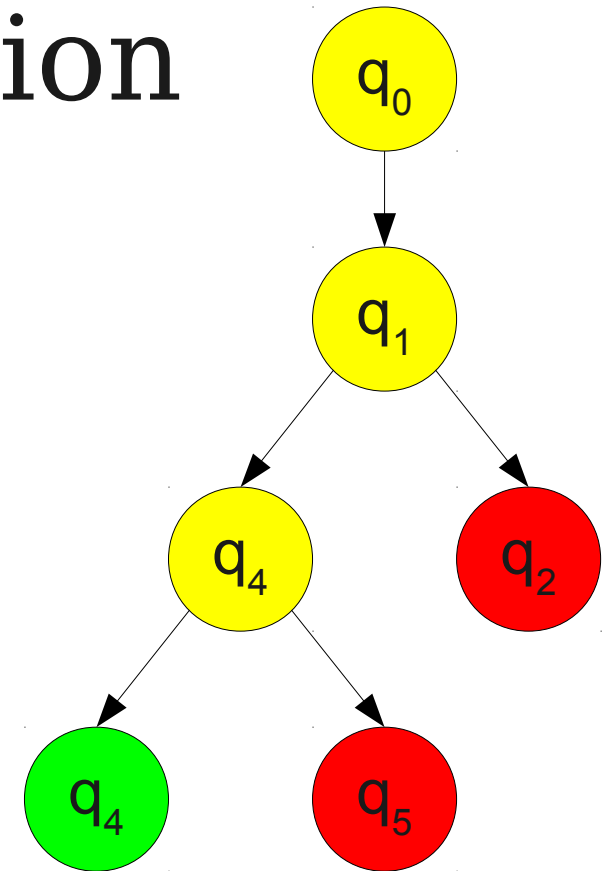
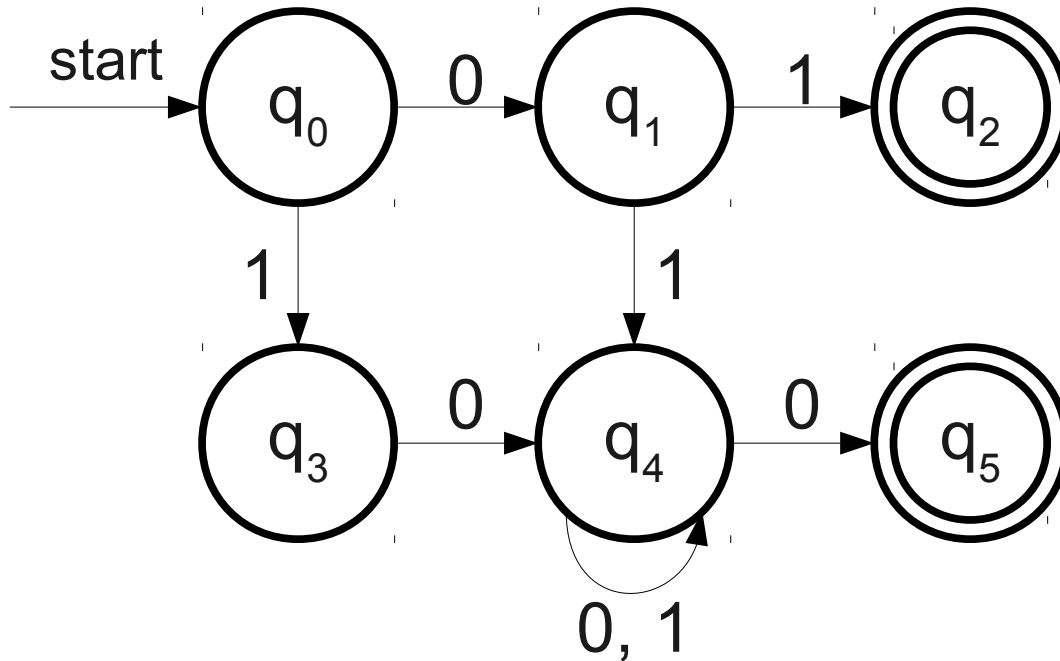
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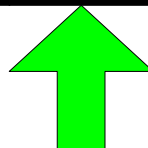
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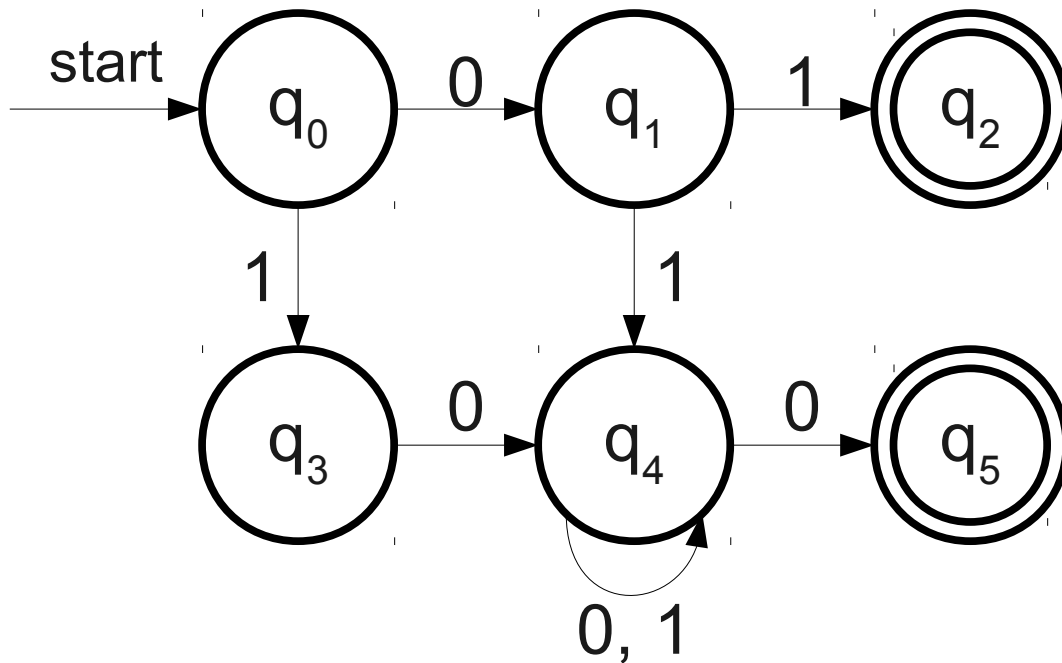
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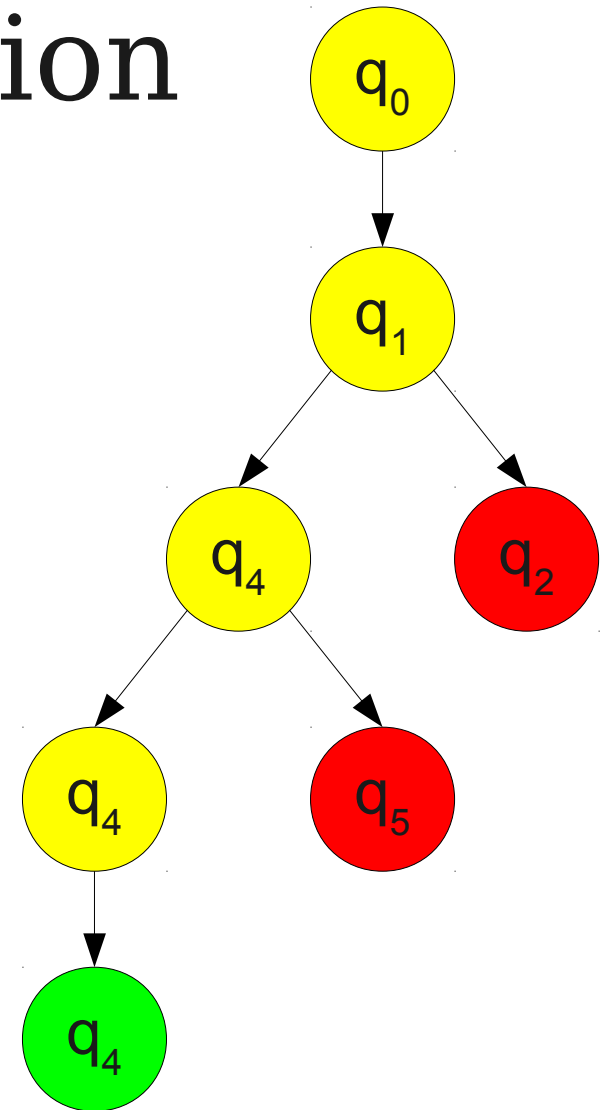
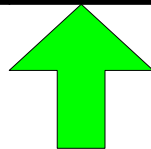
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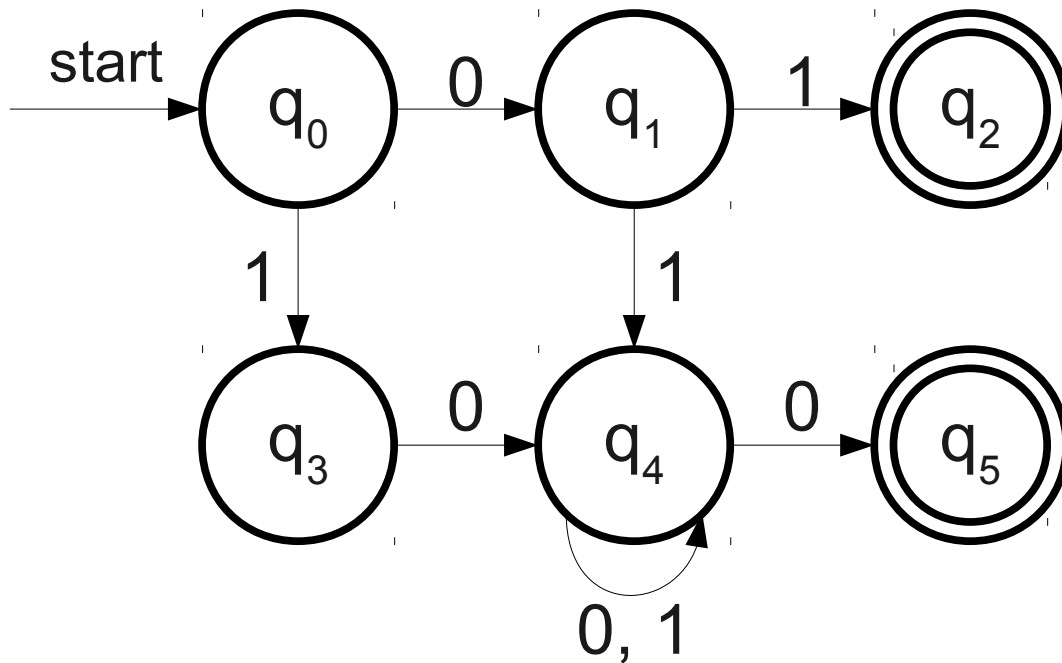
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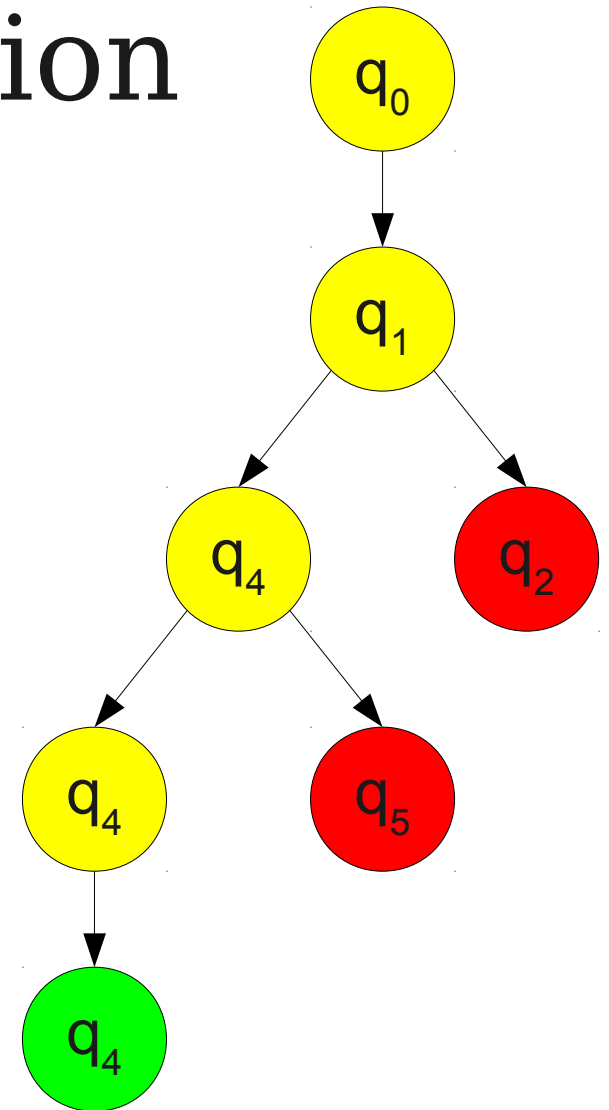
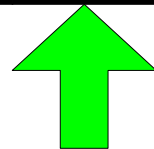
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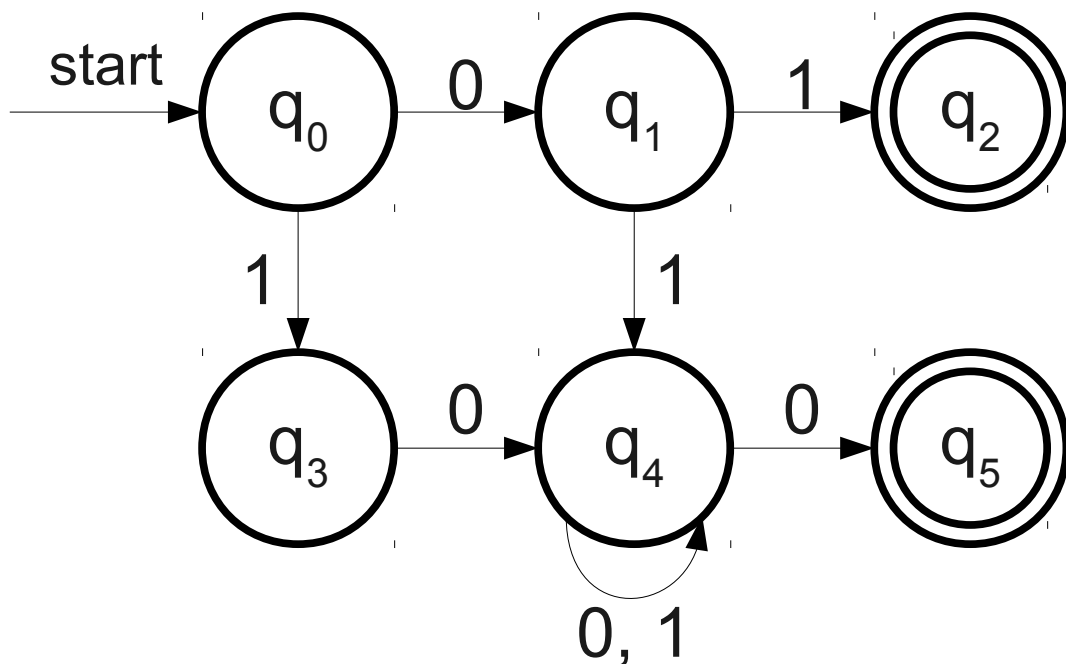
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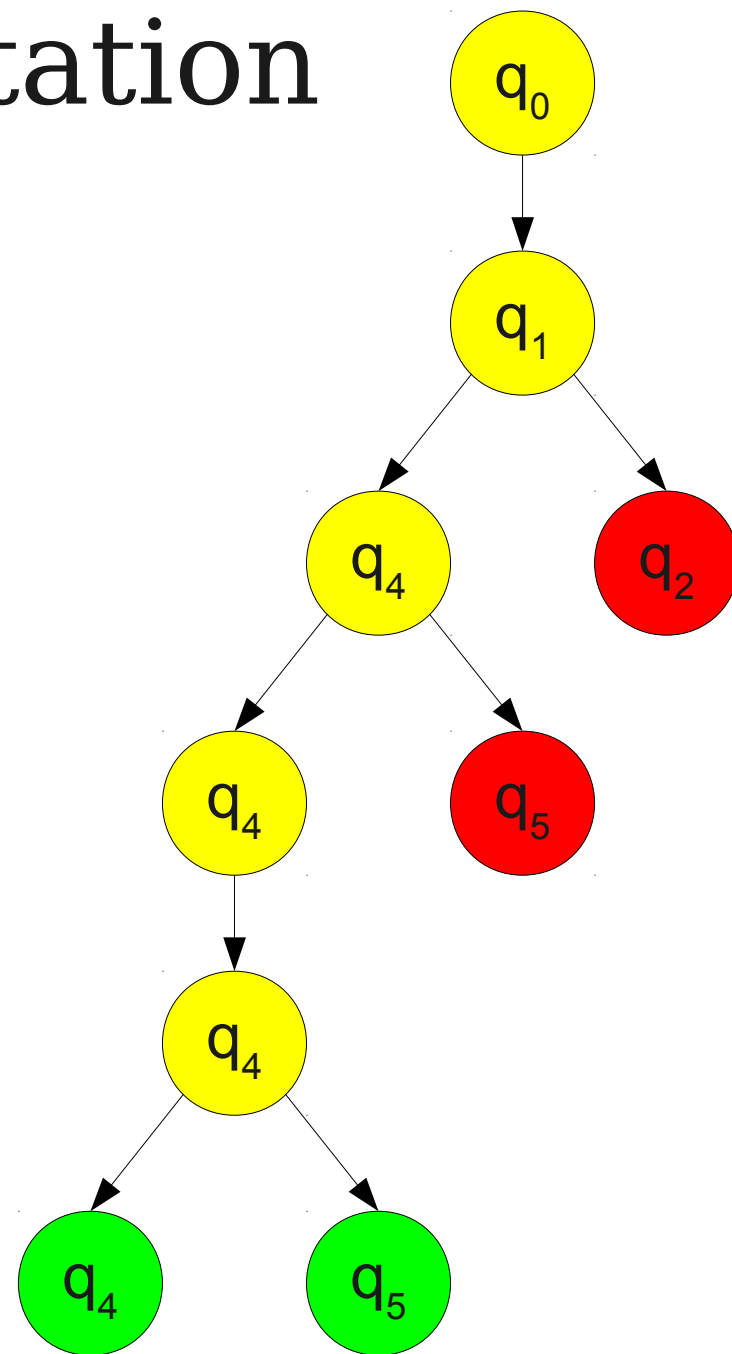
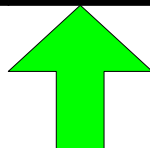
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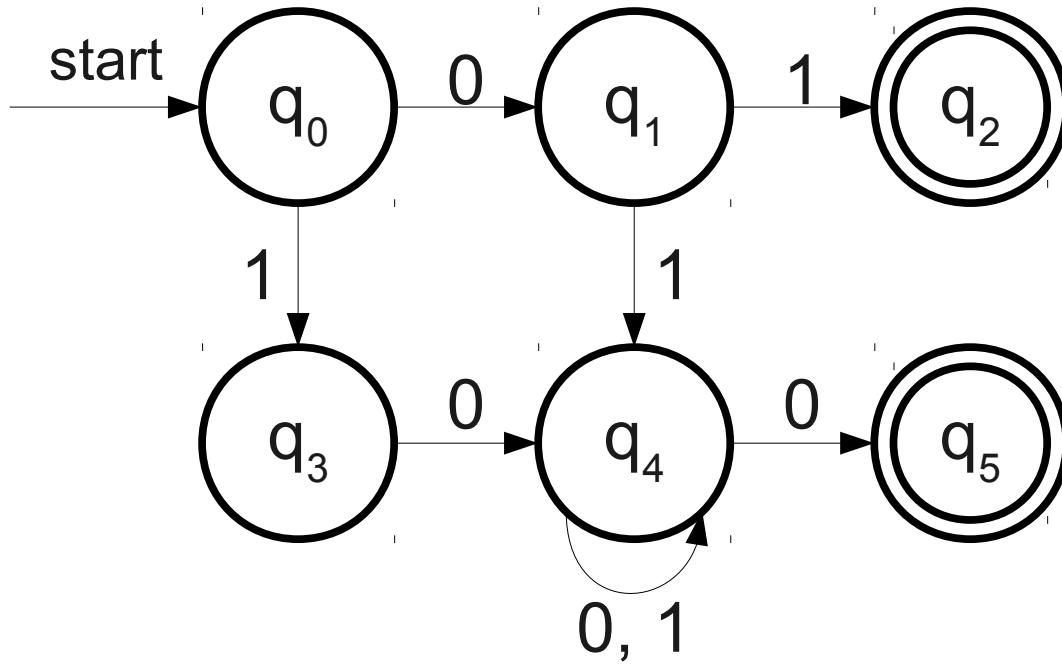
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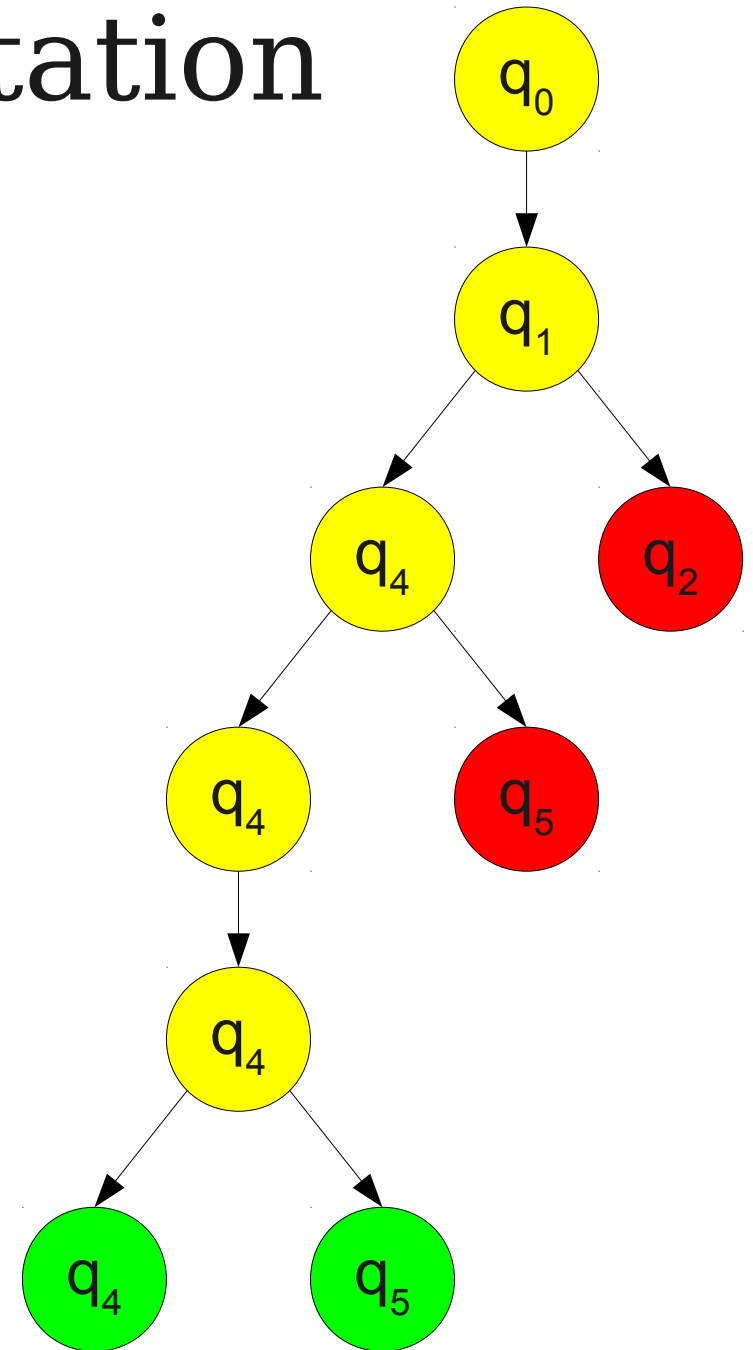
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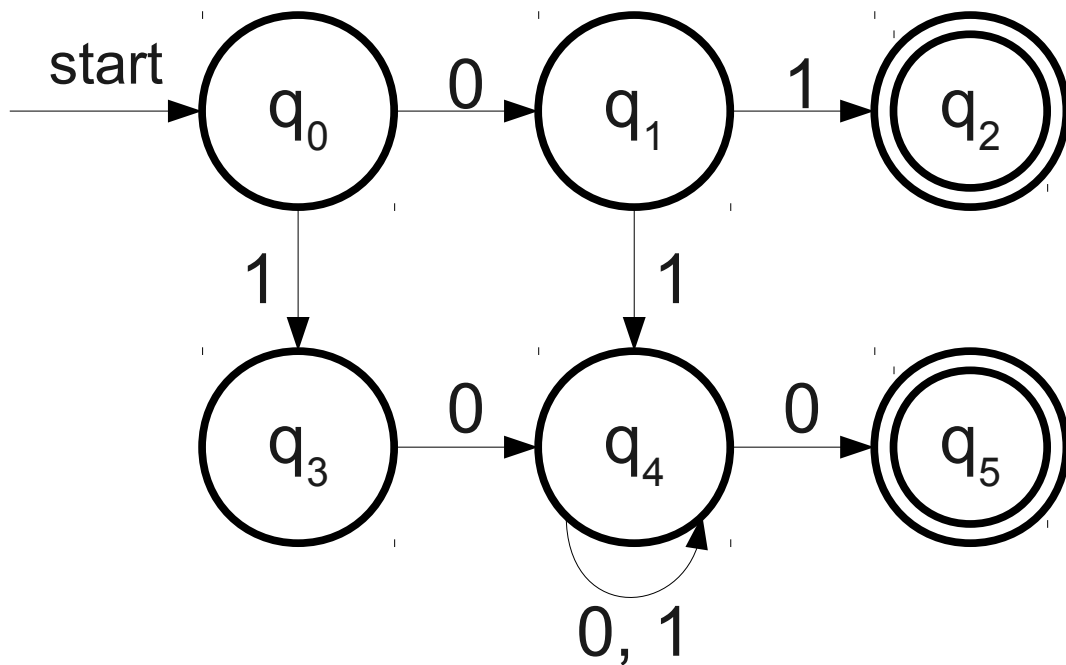
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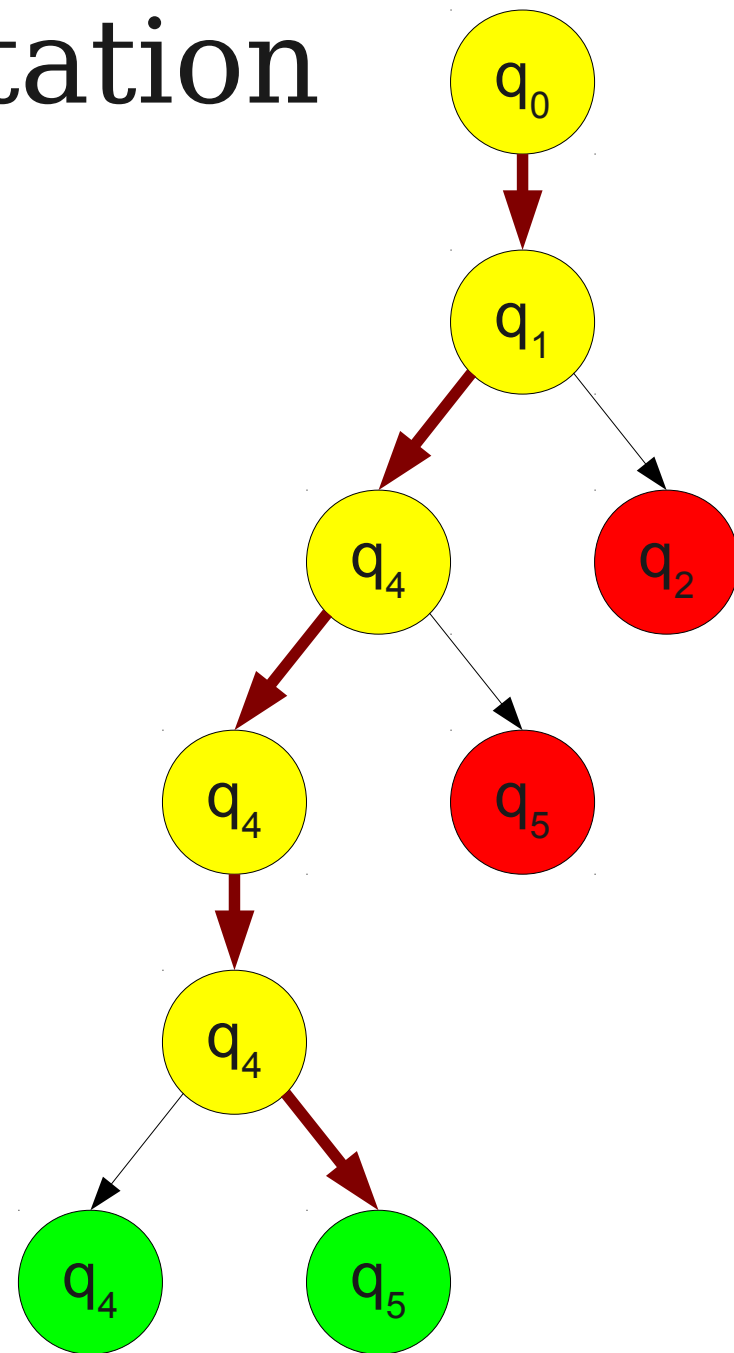
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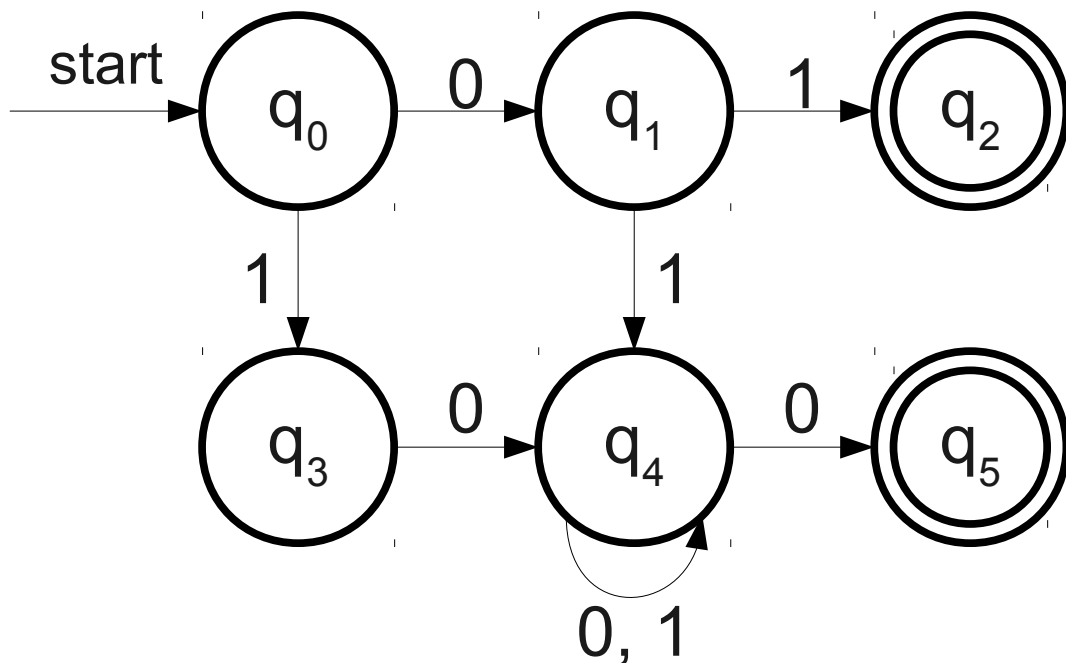
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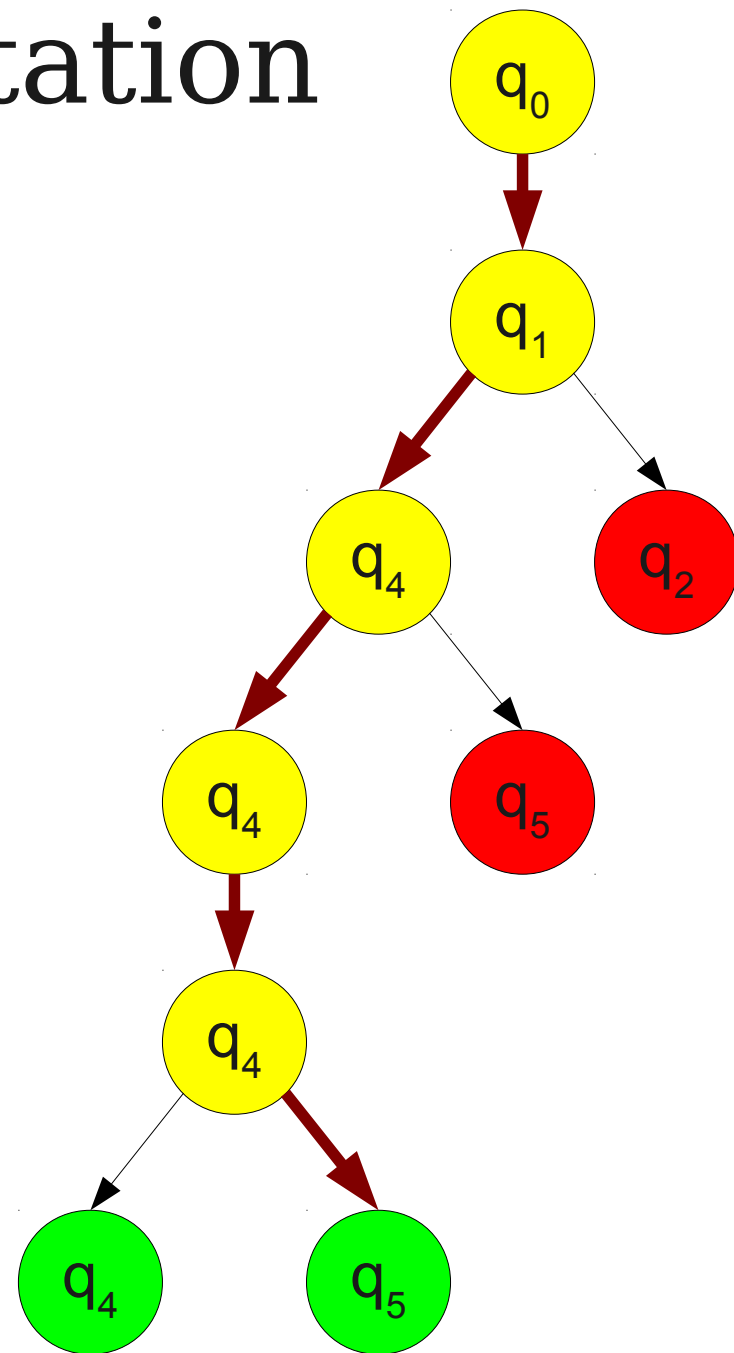
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Tree Computation



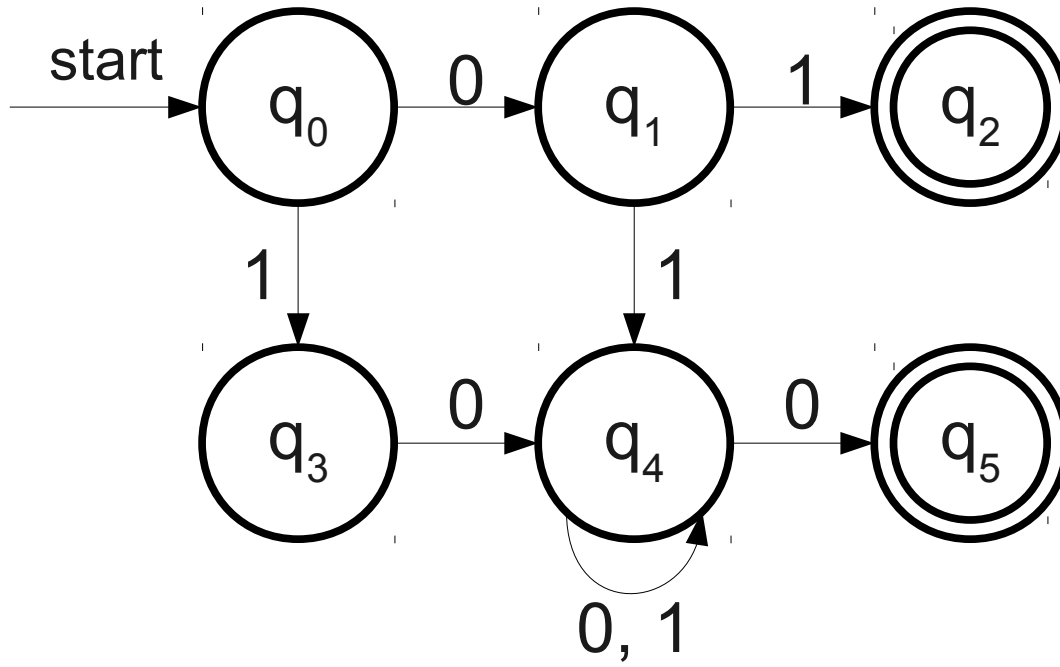
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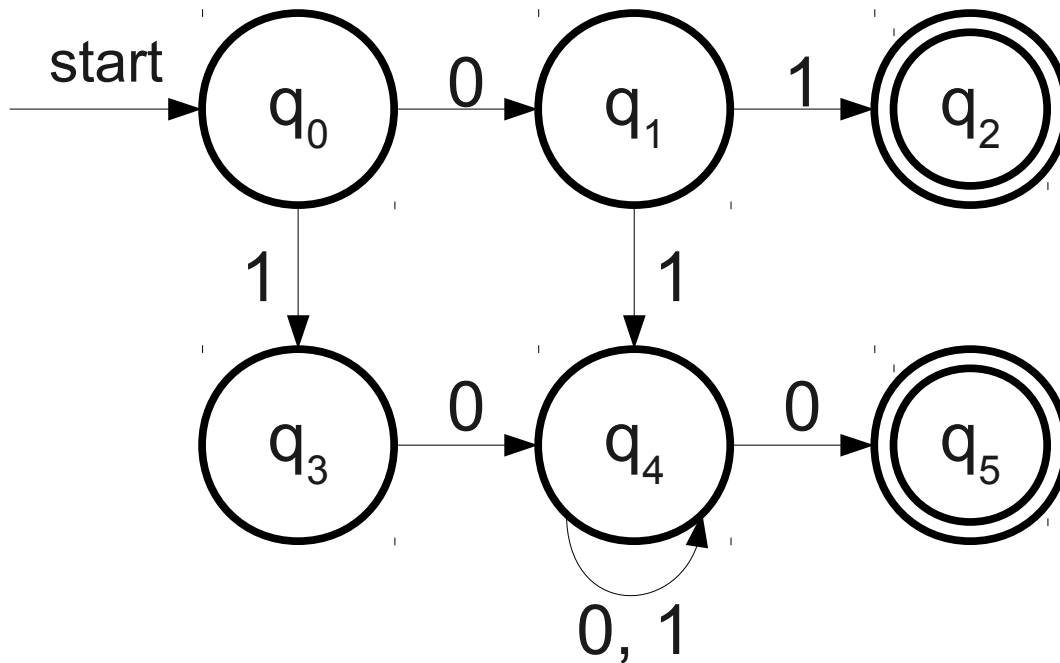
Nondeterminism as a Tree

- At each decision point, the automaton clones itself for each possible decision.
- The series of choices forms a directed, rooted tree.
- At the end, if any active accepting states remain, we accept.

Perfect Guessing

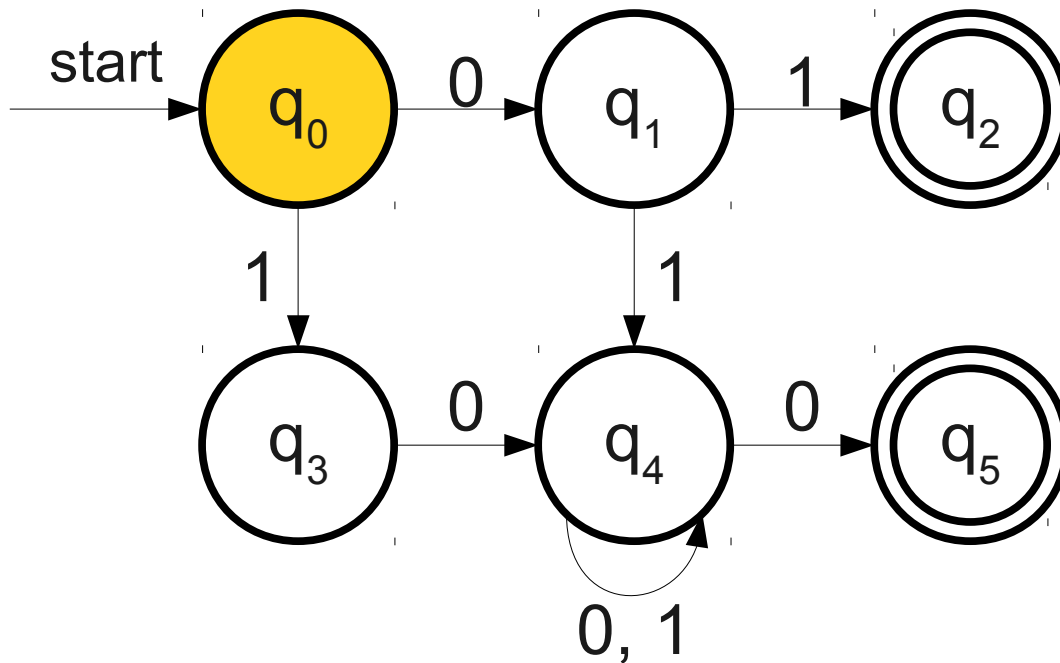


Perfect Guessing



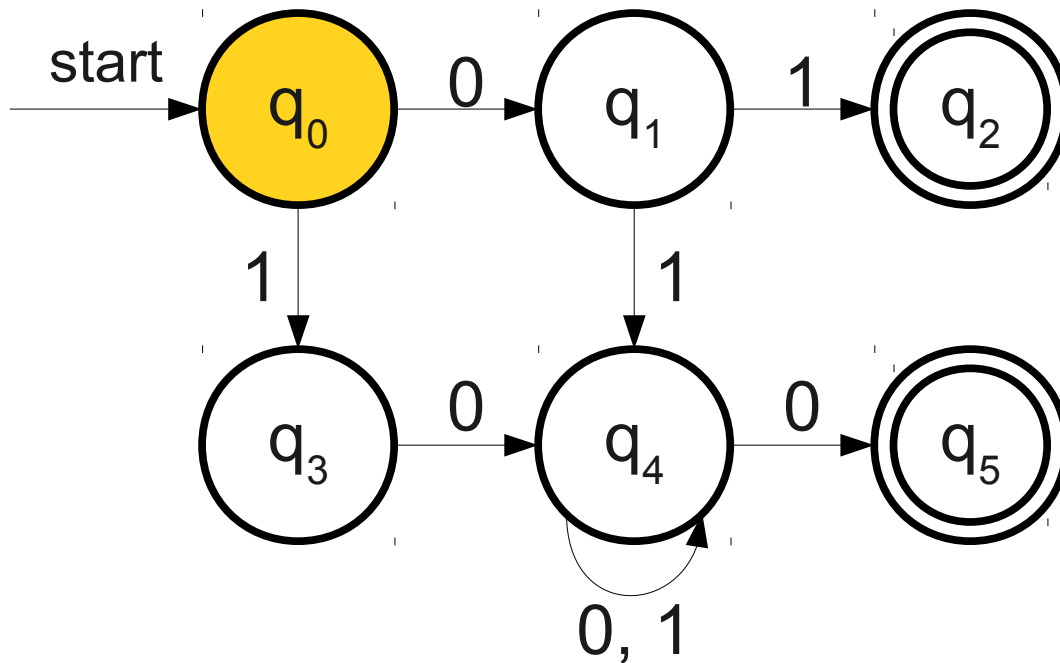
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Perfect Guessing



0 1 0 1 0

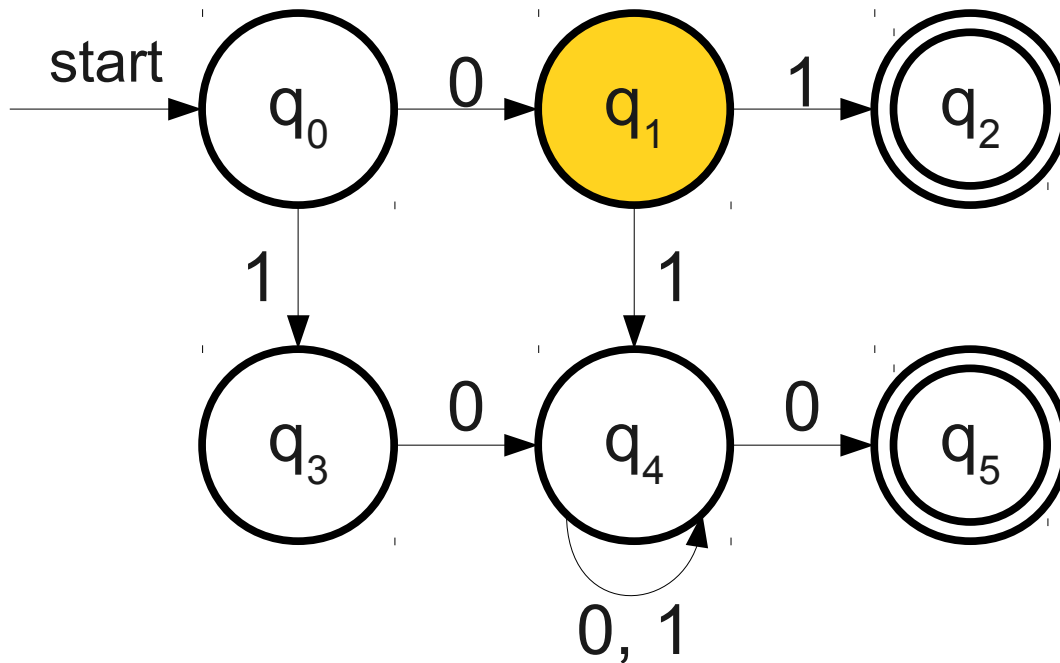
Perfect Guessing



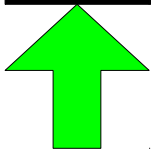
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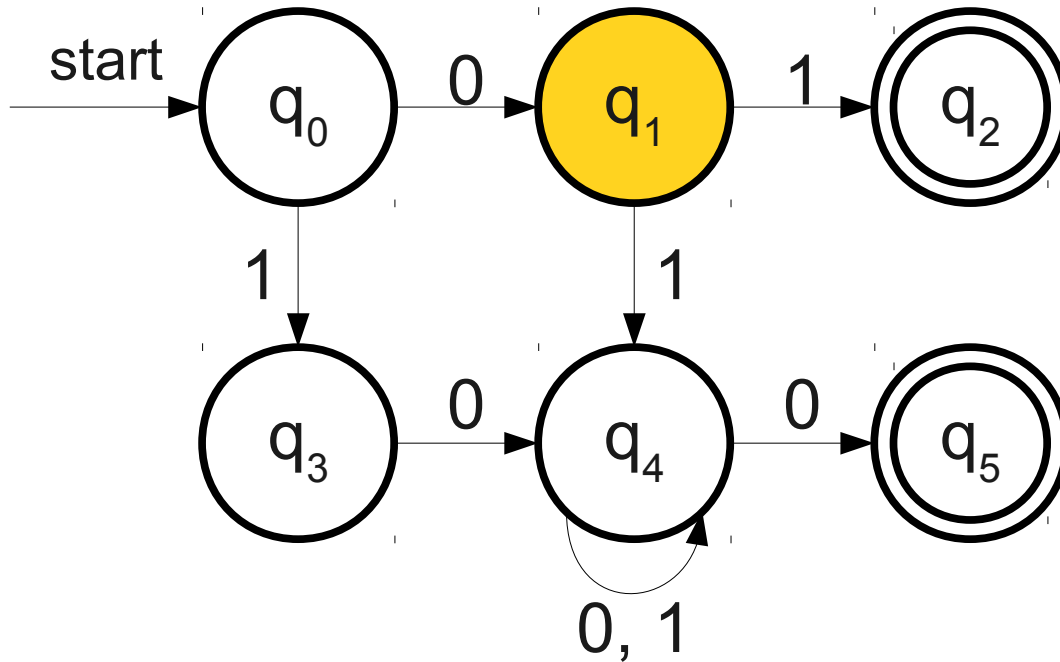
Perfect Guessing



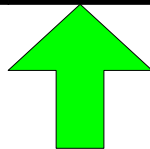
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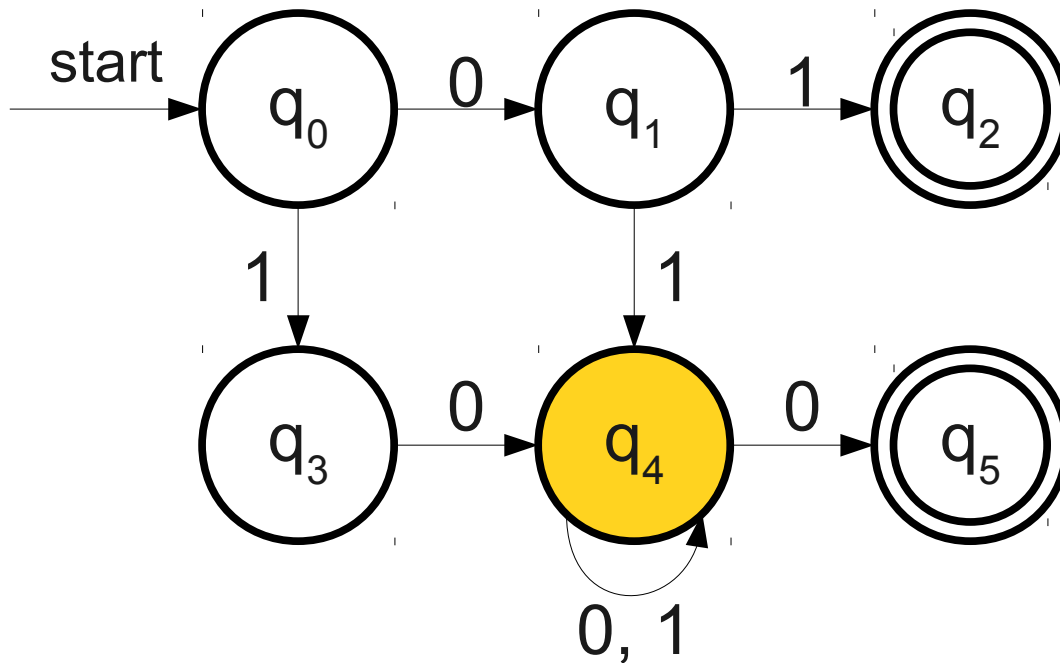
Perfect Guessing



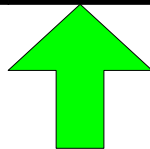
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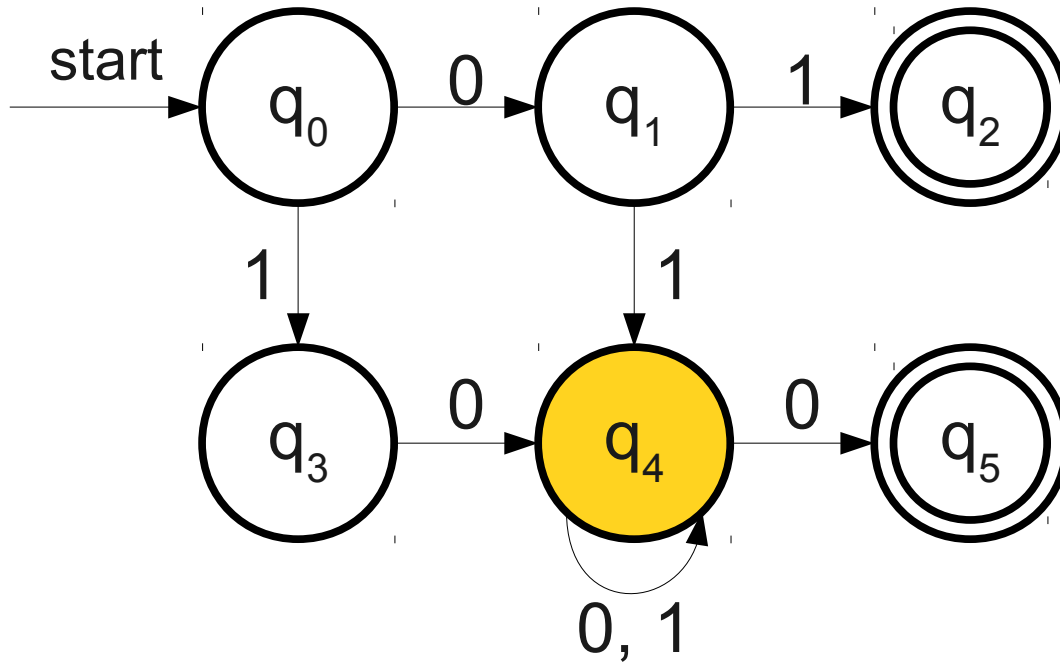
Perfect Guessing



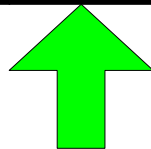
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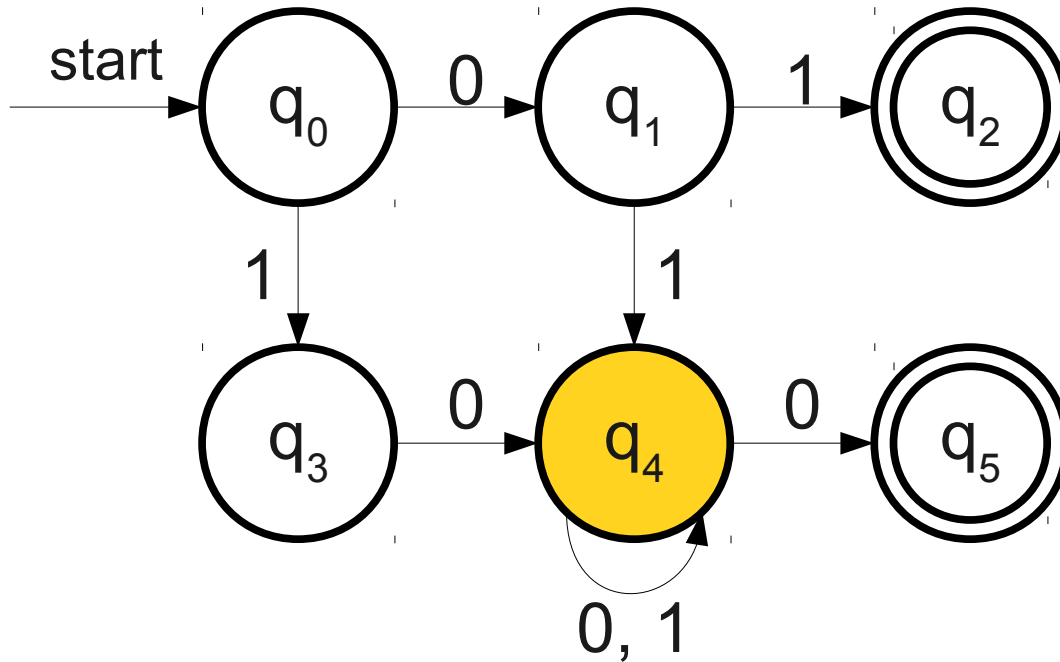
Perfect Guessing



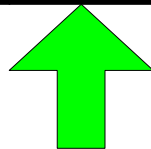
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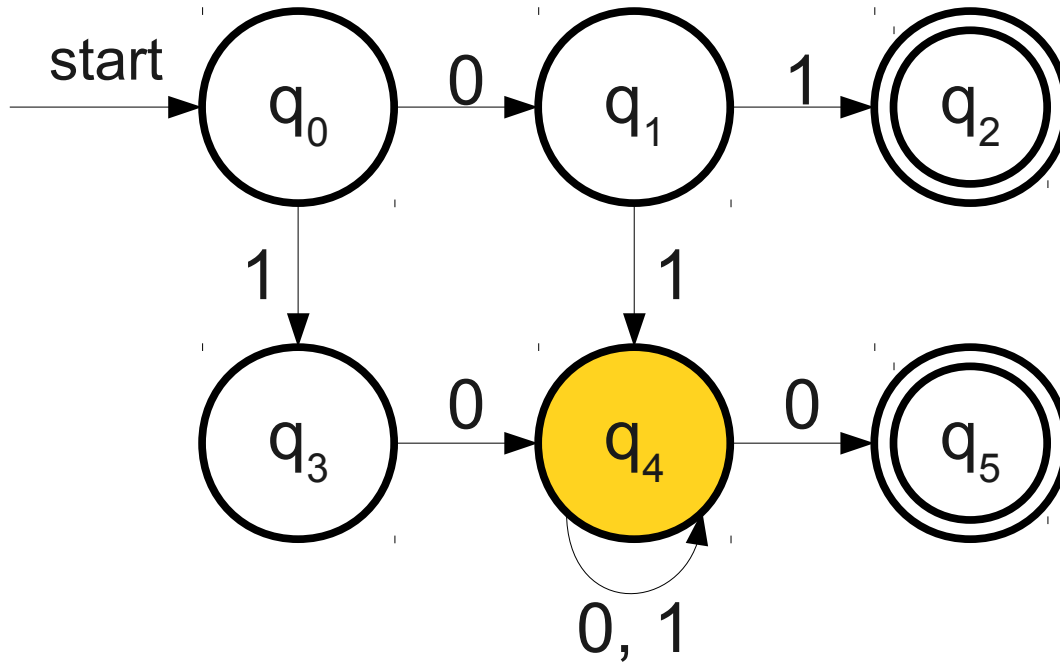
Perfect Guessing



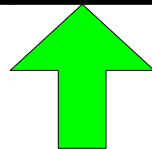
0 1 0 1 0



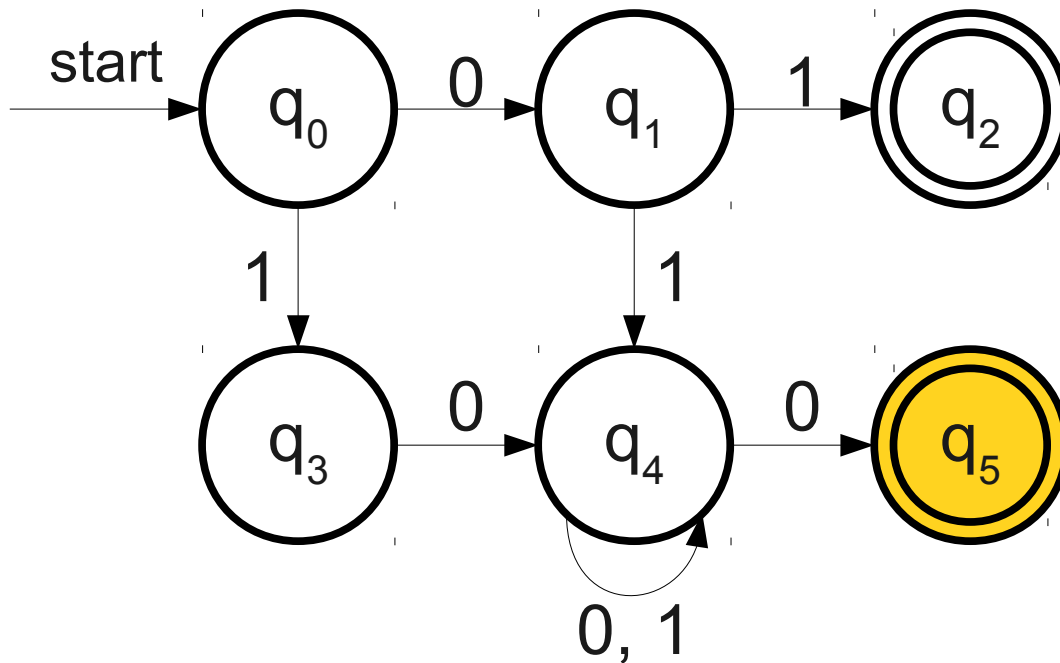
Perfect Guessing



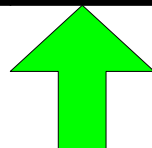
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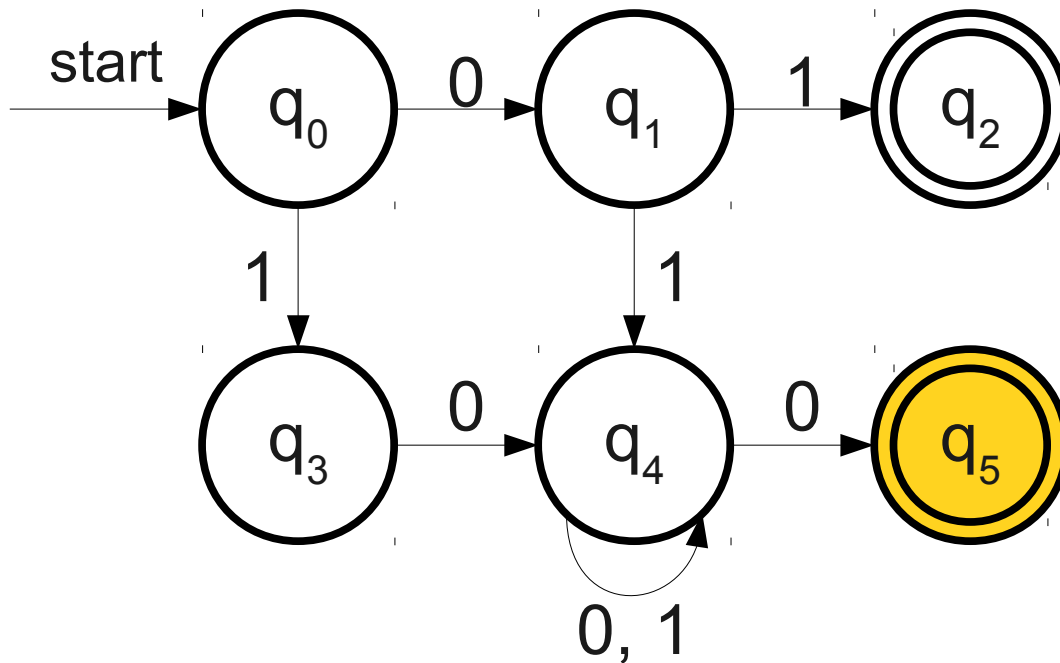
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0 1 0 1 0

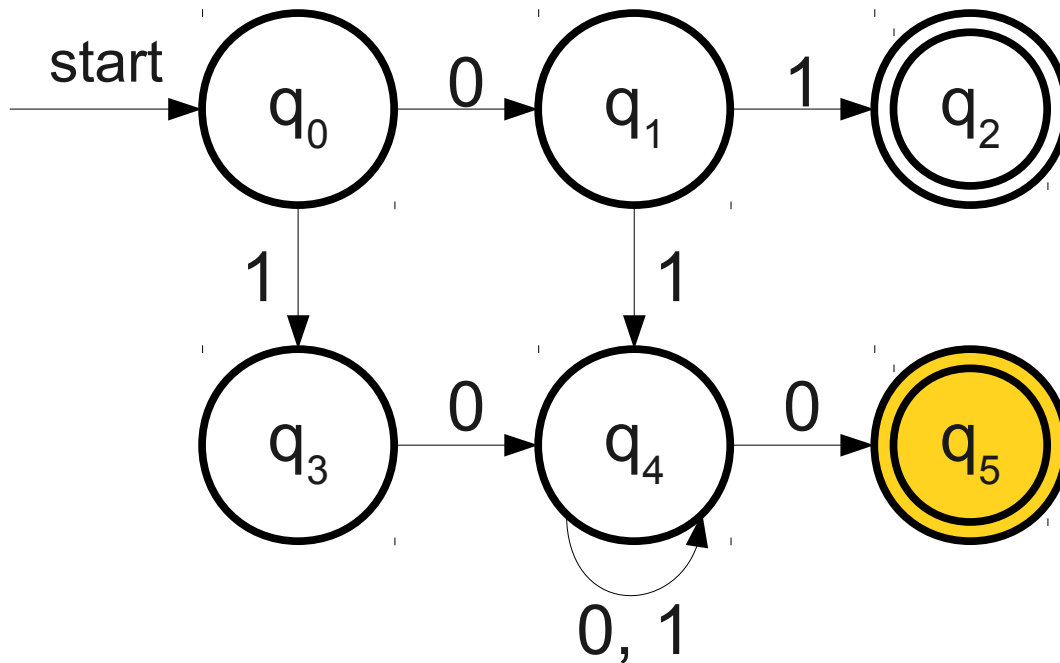


Perfect Guessing



0 1 0 1 0

Perfect Guessing

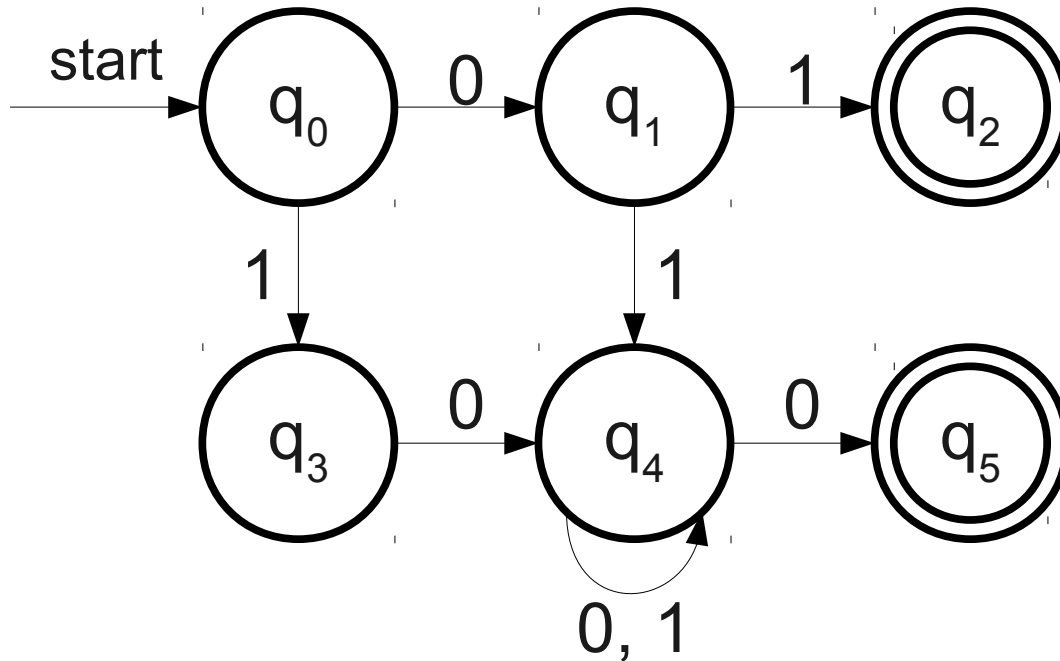


0 1 0 1 0

Perfect Guessing

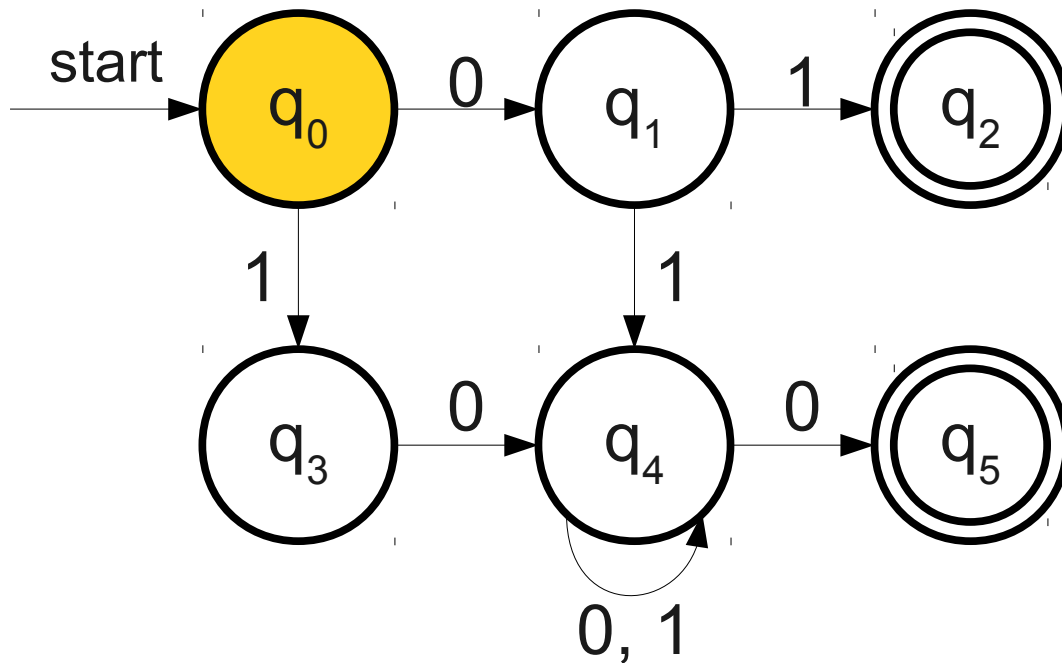
- We can view nondeterministic machines as having **Magic Superpowers** that enable them to guess the correct choice of moves to make.
- Idea: Machine can always guess the right choice if one exists.
- No physical analog for something of this sort.
 - (Those of you thinking quantum computing – this is not the same thing. We actually don't fully know the relation between quantum and nondeterministic computation.)

Massive Parallelism



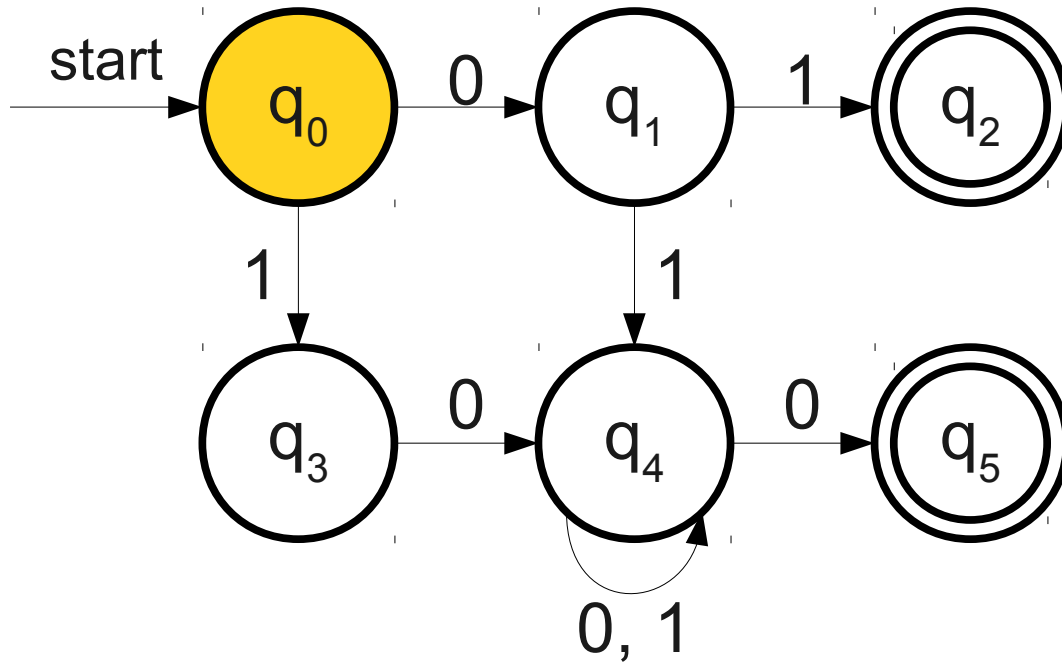
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Massive Parallelism



0 1 0 1 0

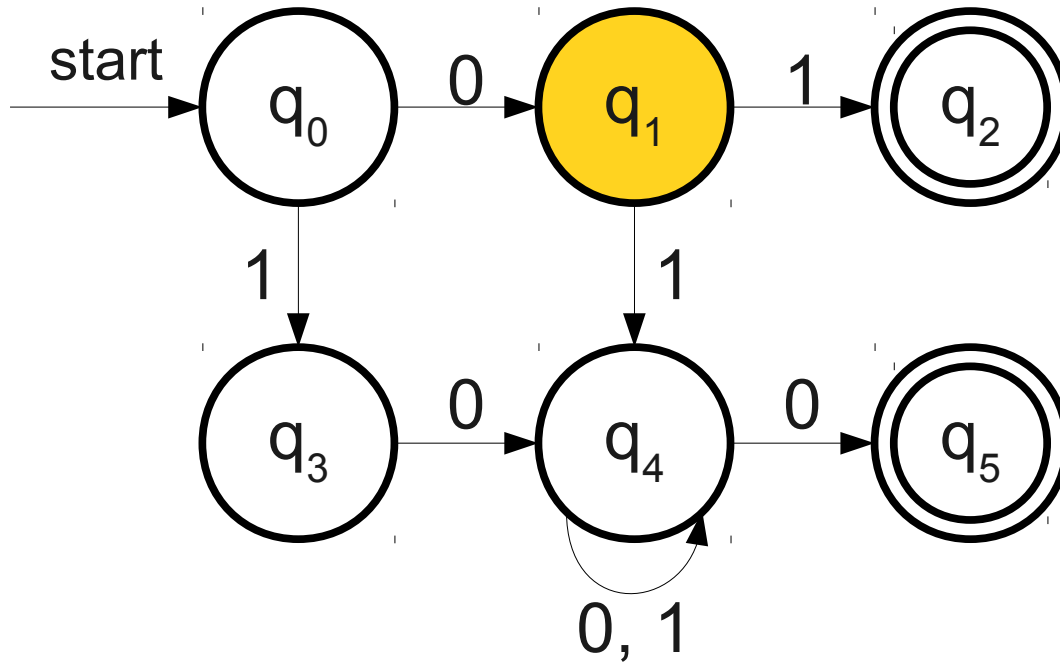
Massive Parallelism



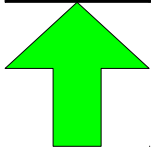
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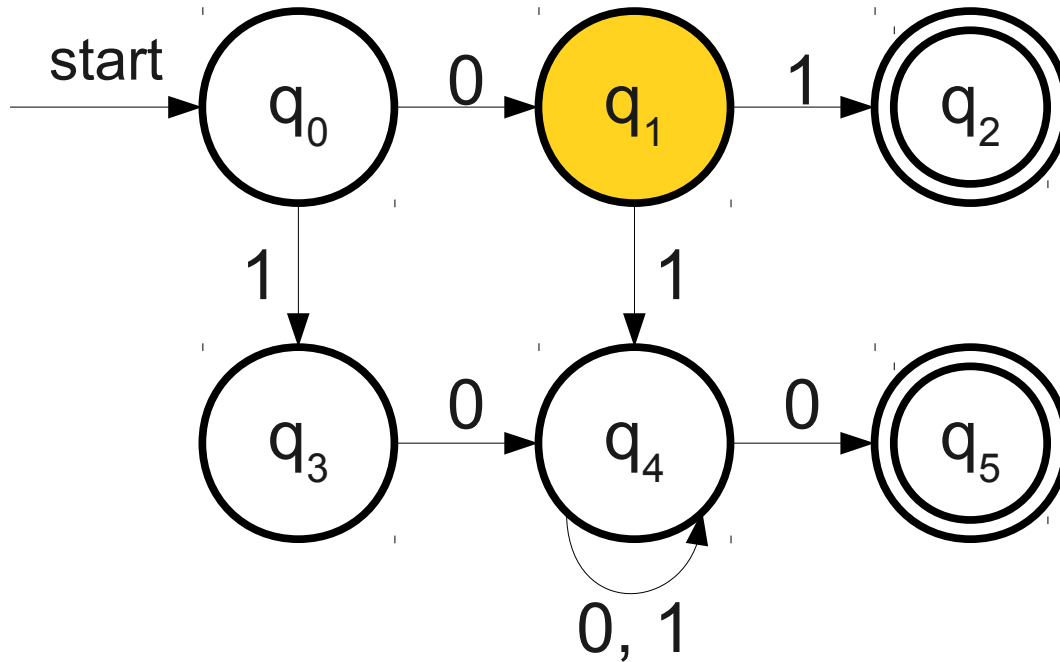
Massive Parallelism



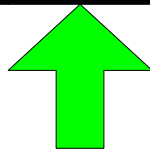
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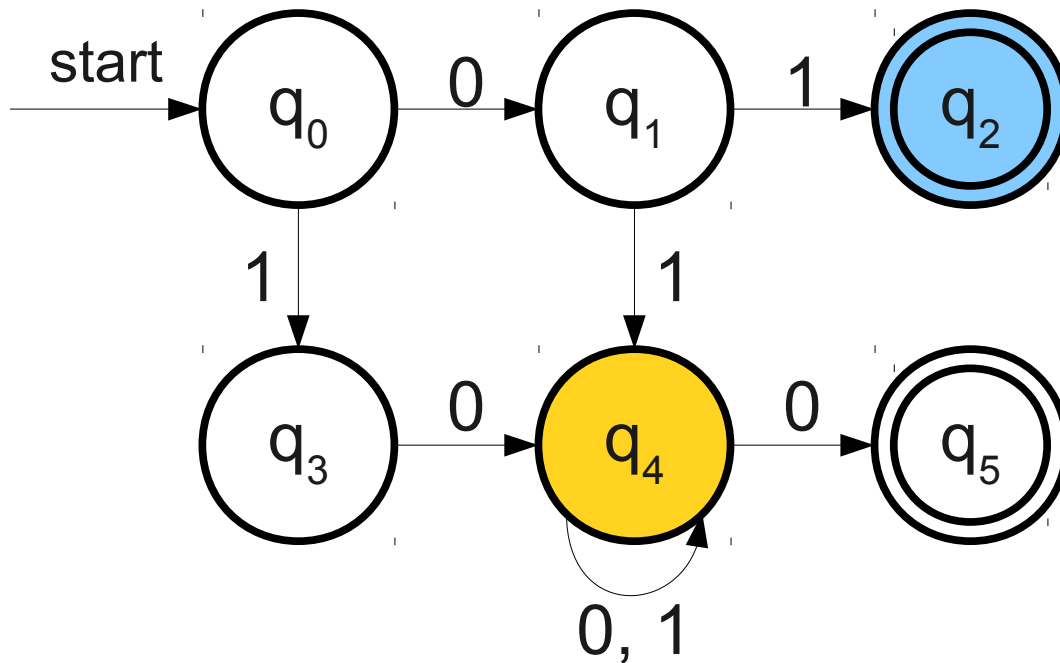
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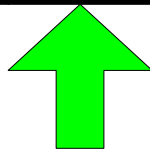
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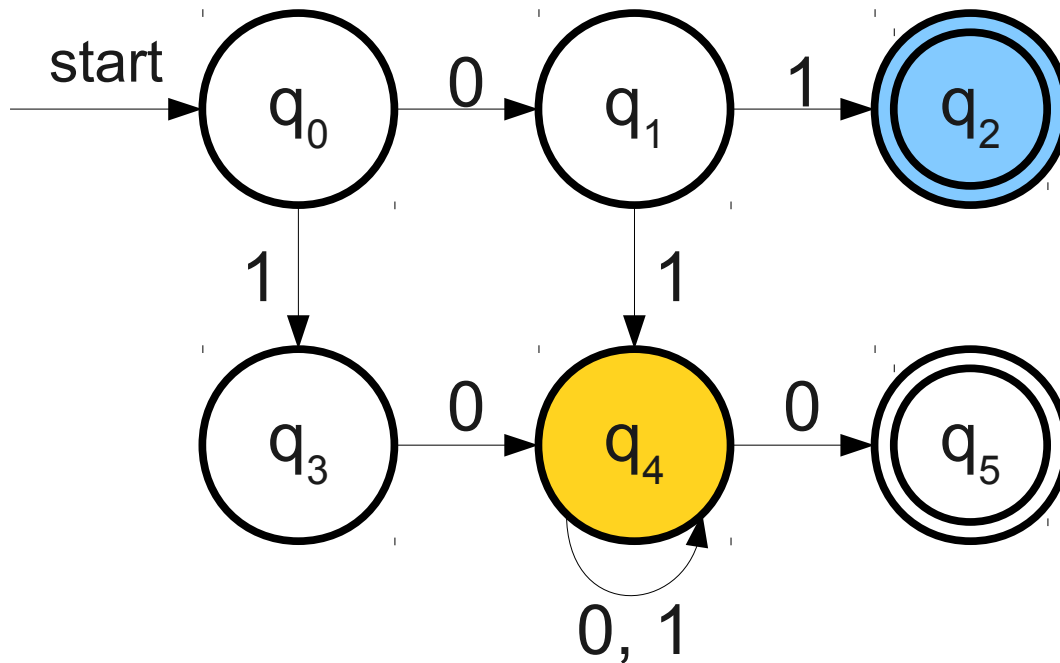
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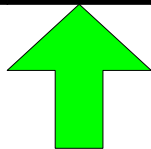
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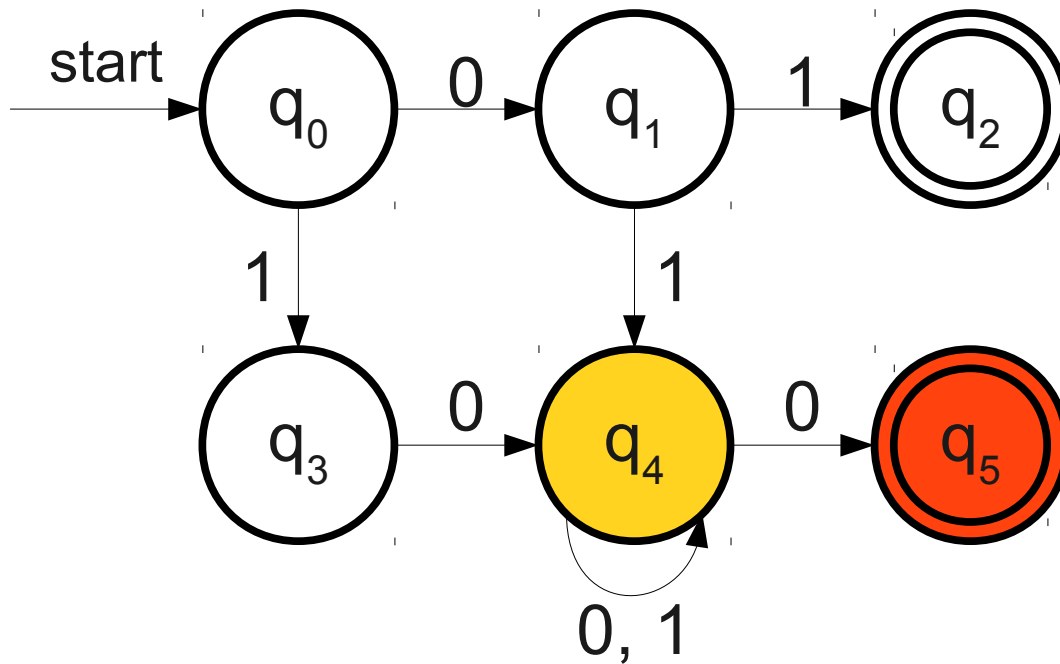
Massive Parallelism



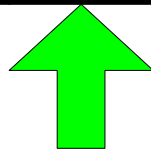
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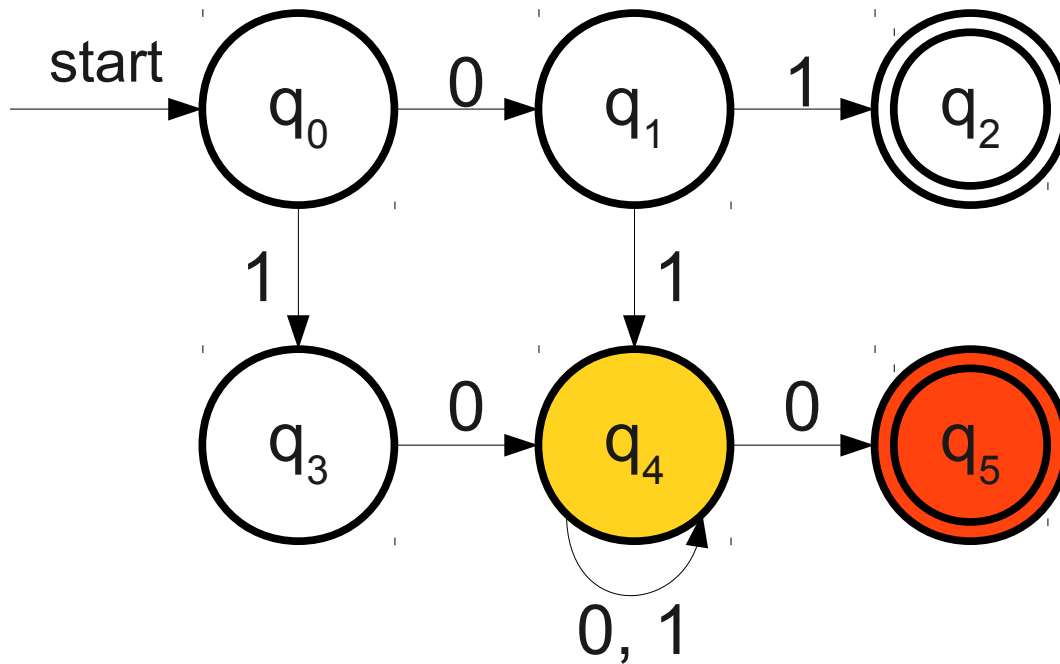
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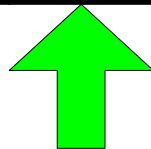
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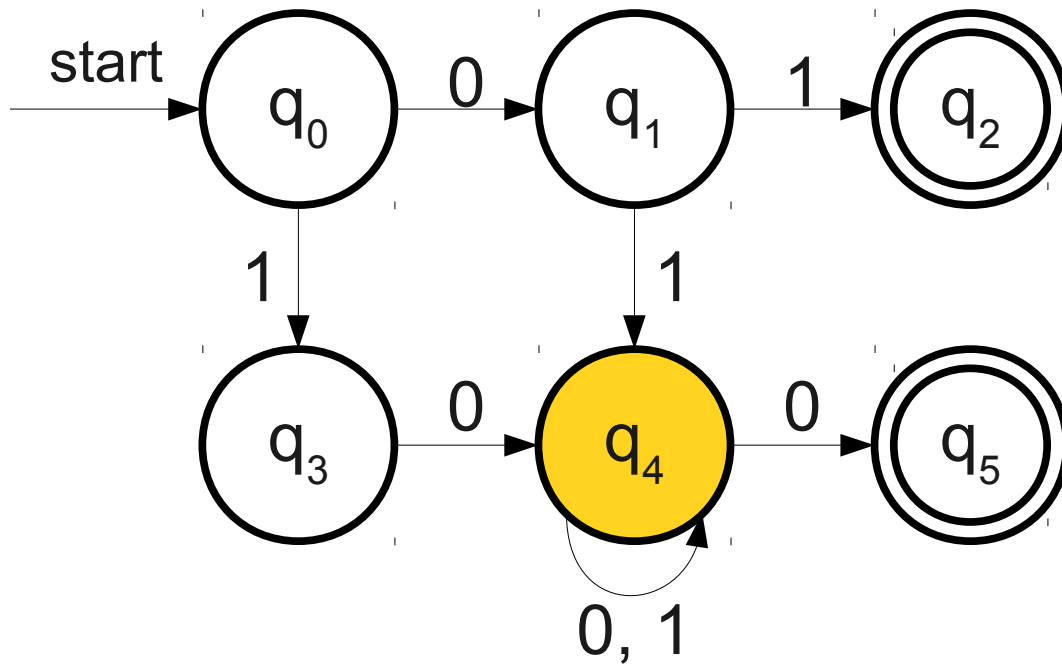
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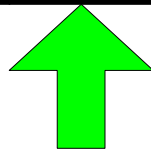
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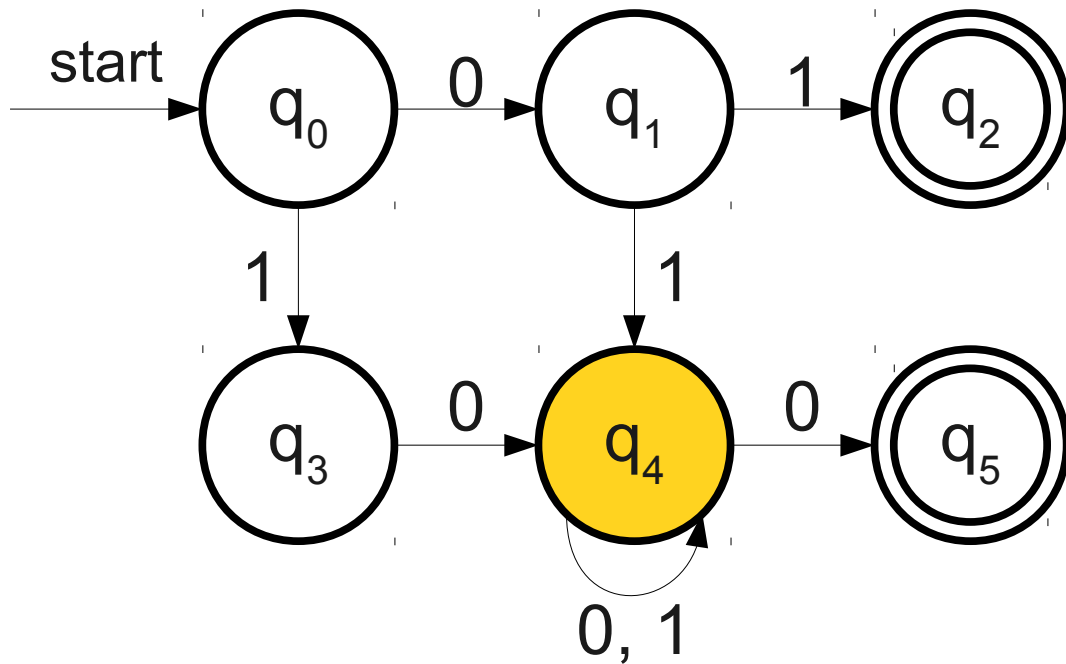
Massive Parallelism



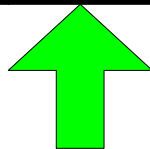
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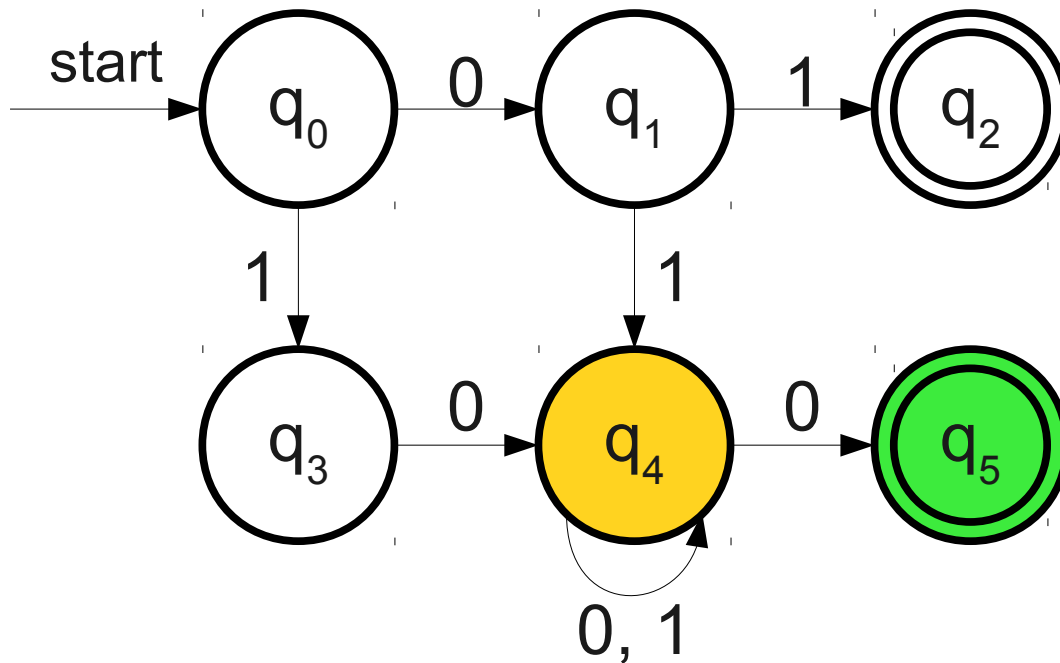
Massive Parallelism



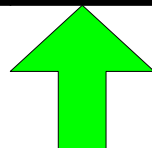
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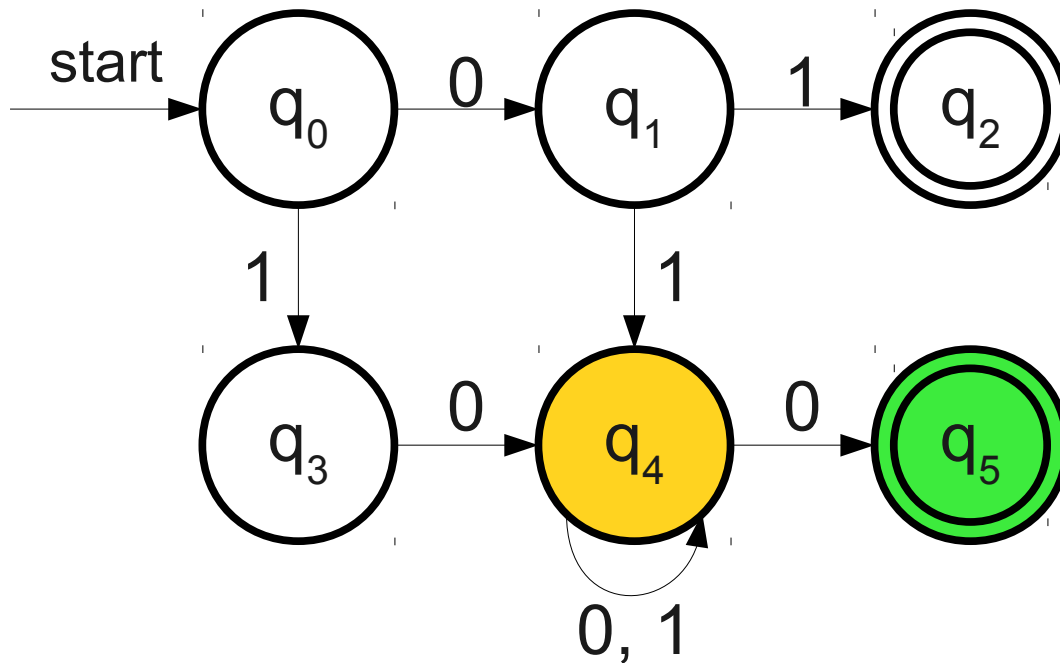
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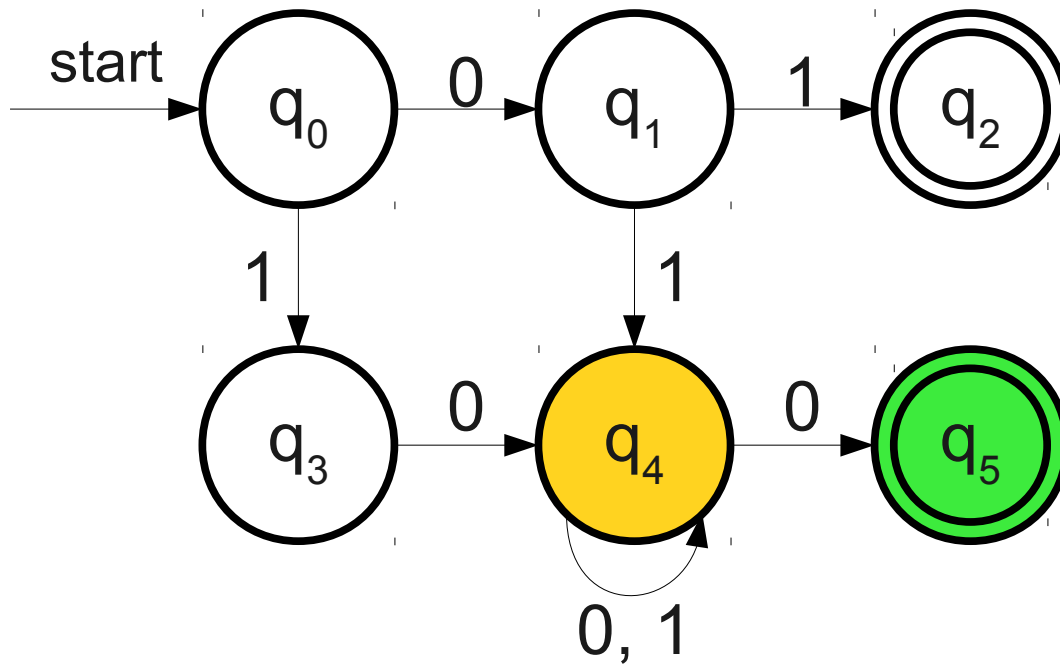


Massive Parallelism



0 1 0 1 0

Massive Parallelism



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Massive Parallelism

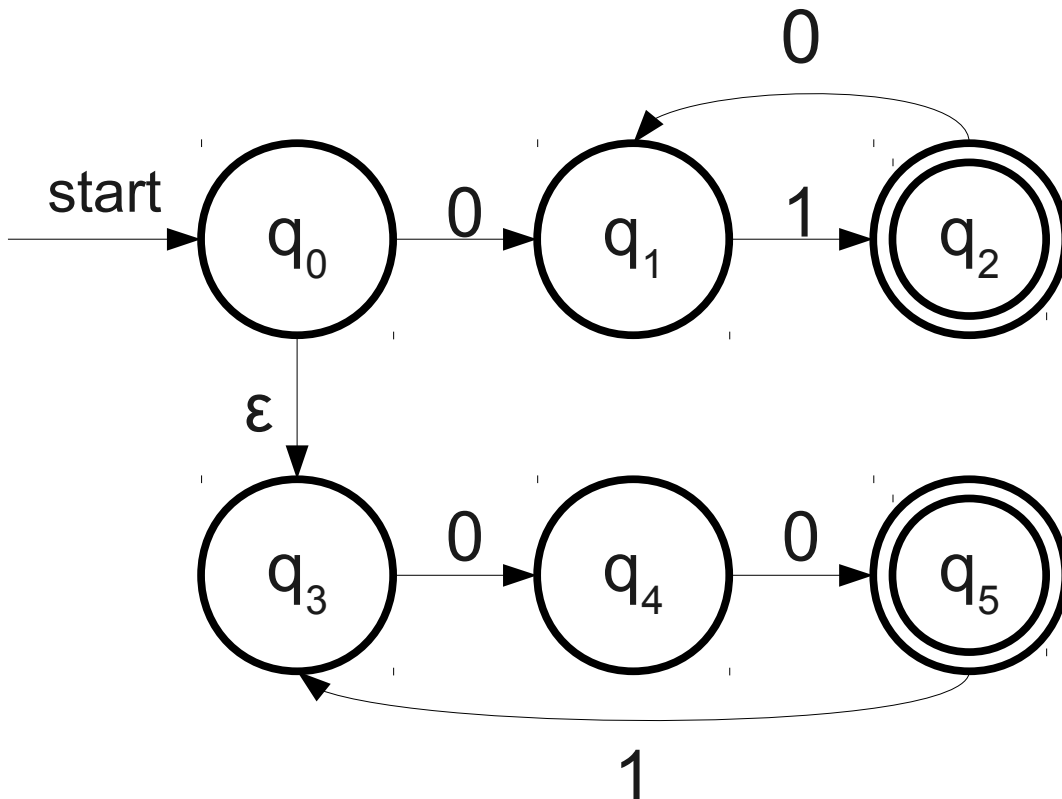
- An NFA can be thought of as a DFA that can be in many states at once.
- Each symbol read causes a transition on every active state into each potential state that could be visited.
- Nondeterministic machines can be thought of as machines that can try any number of options in parallel.
 - No fixed limit on processors; makes multicore machines look downright wimpy!

So What?

- We will turn to these three intuitions for nondeterminism more later in the quarter.
- Nondeterministic machines may not be feasible, but they give a great basis for interesting questions:
 - Can any problem that can be solved by a nondeterministic machine be solved by a deterministic machine?
 - Can any problem that can be solved by a nondeterministic machine be solved **efficiently** by a deterministic machine?
- The answers vary from automaton to automaton.

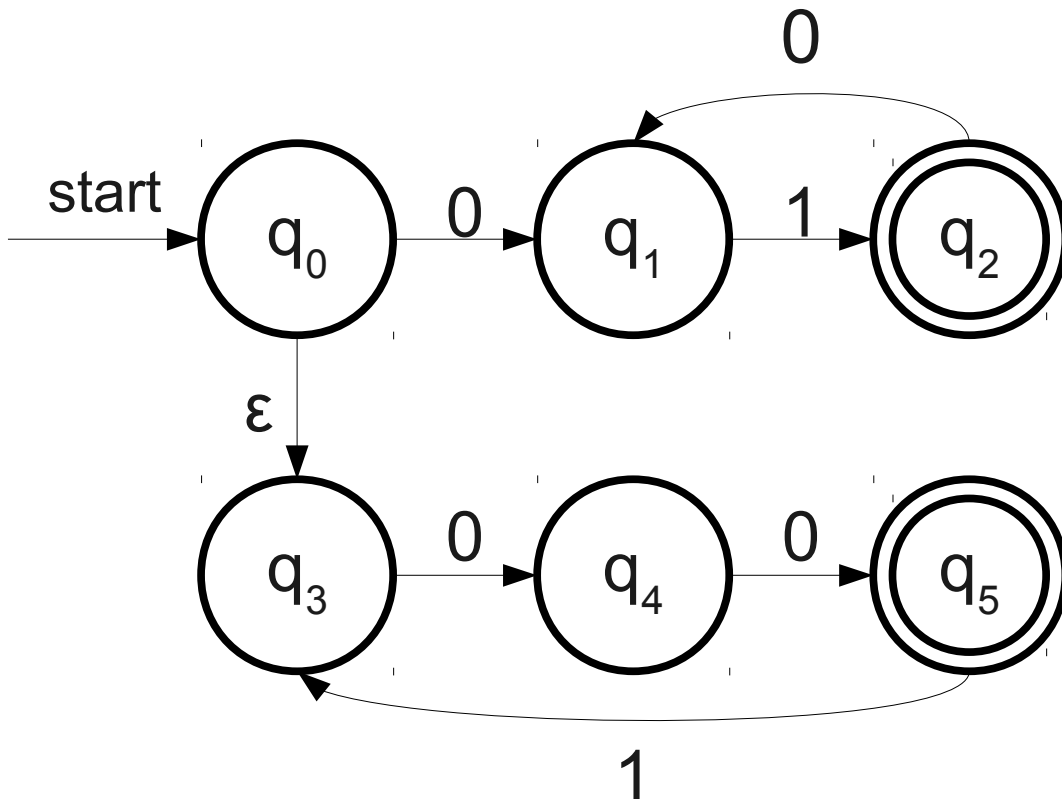
ϵ -Transitions

- NFAs have a special type of transition called the **ϵ -transition**.
- An NFA may follow any number of ϵ -transitions at any time without consuming any input.



ϵ -Transitions

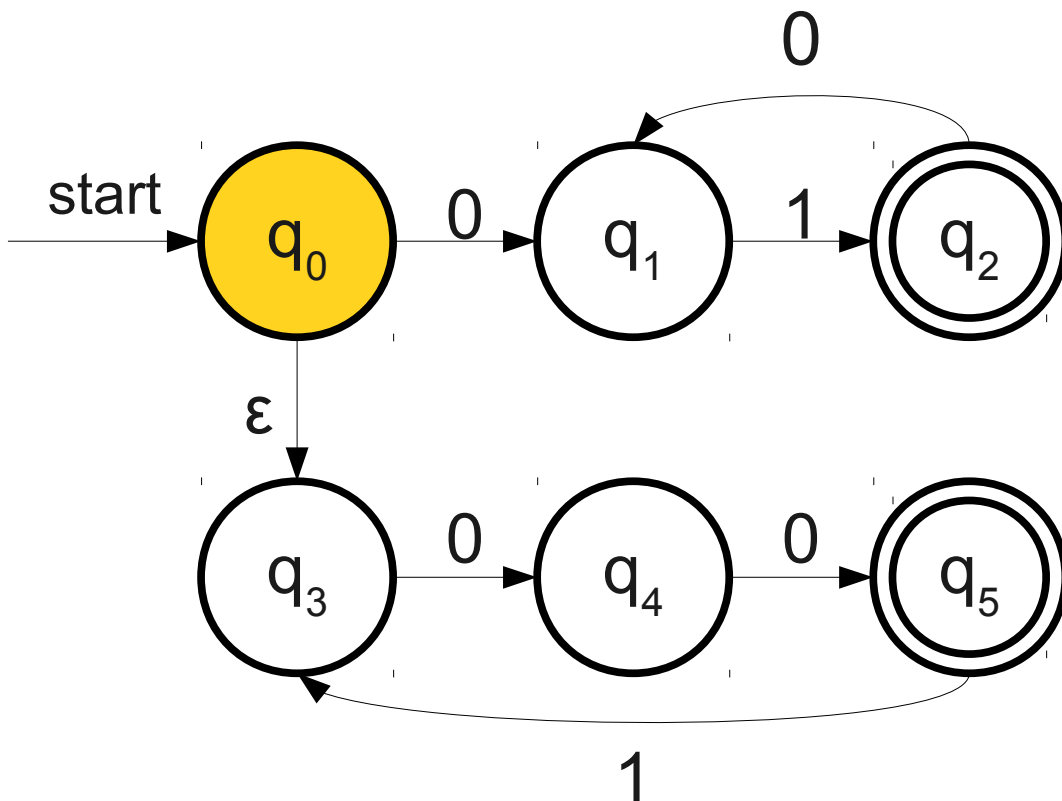
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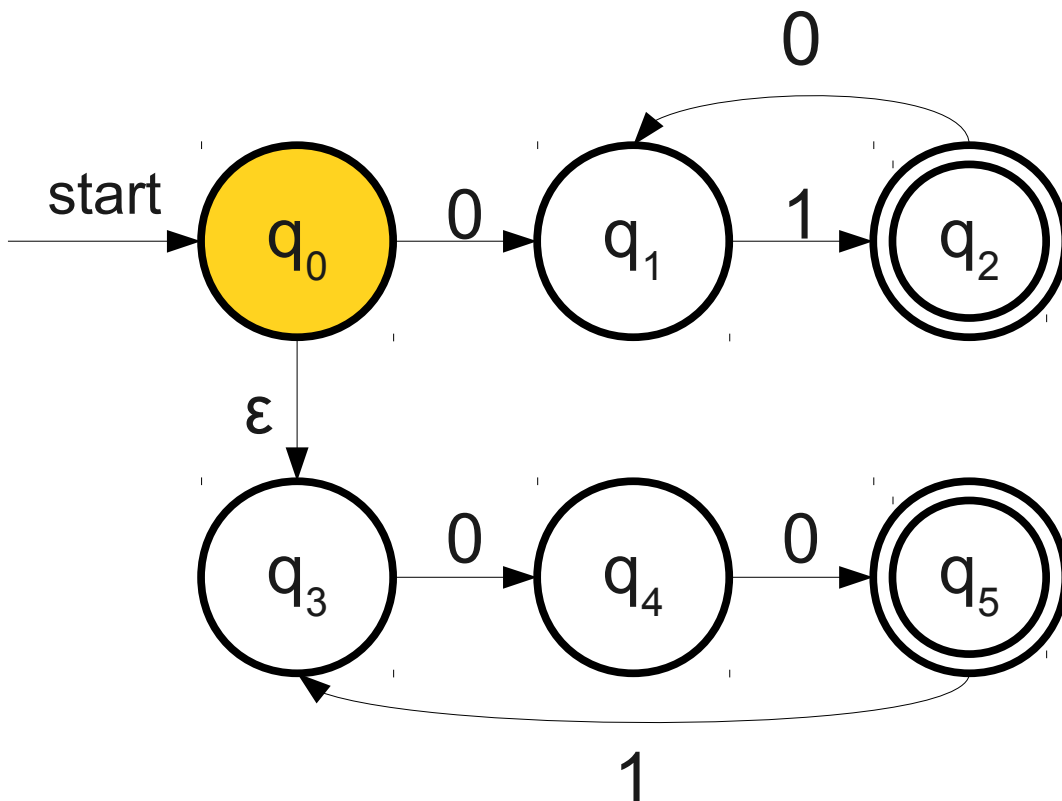
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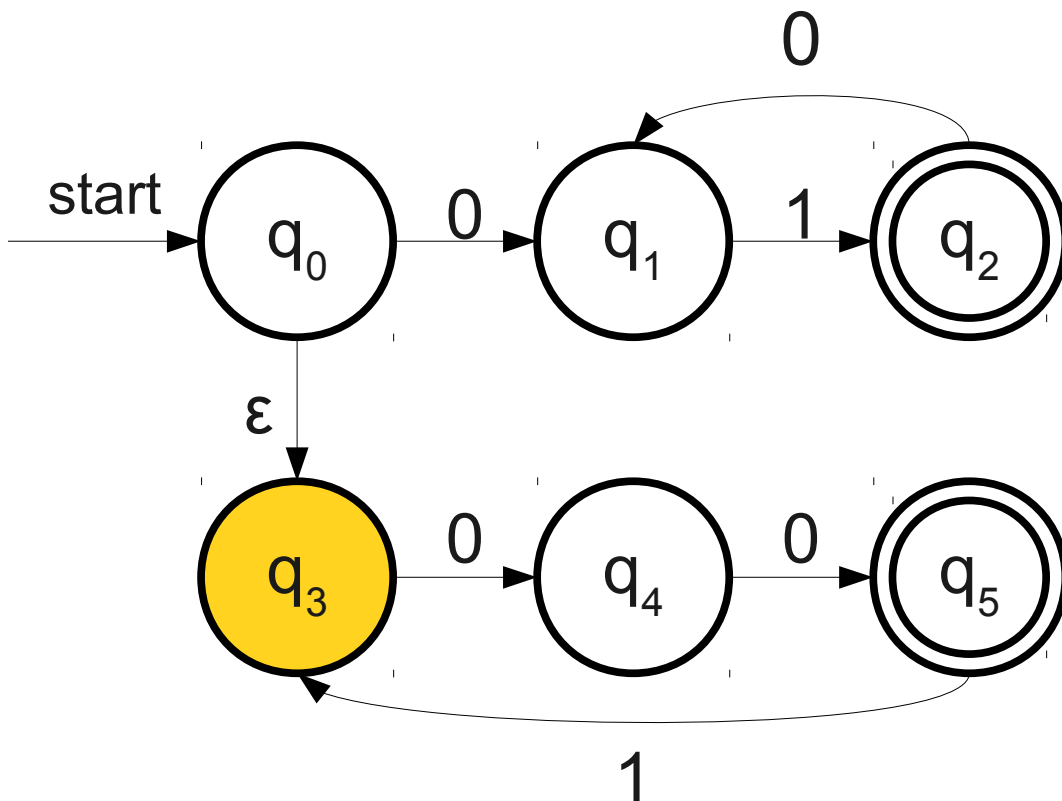
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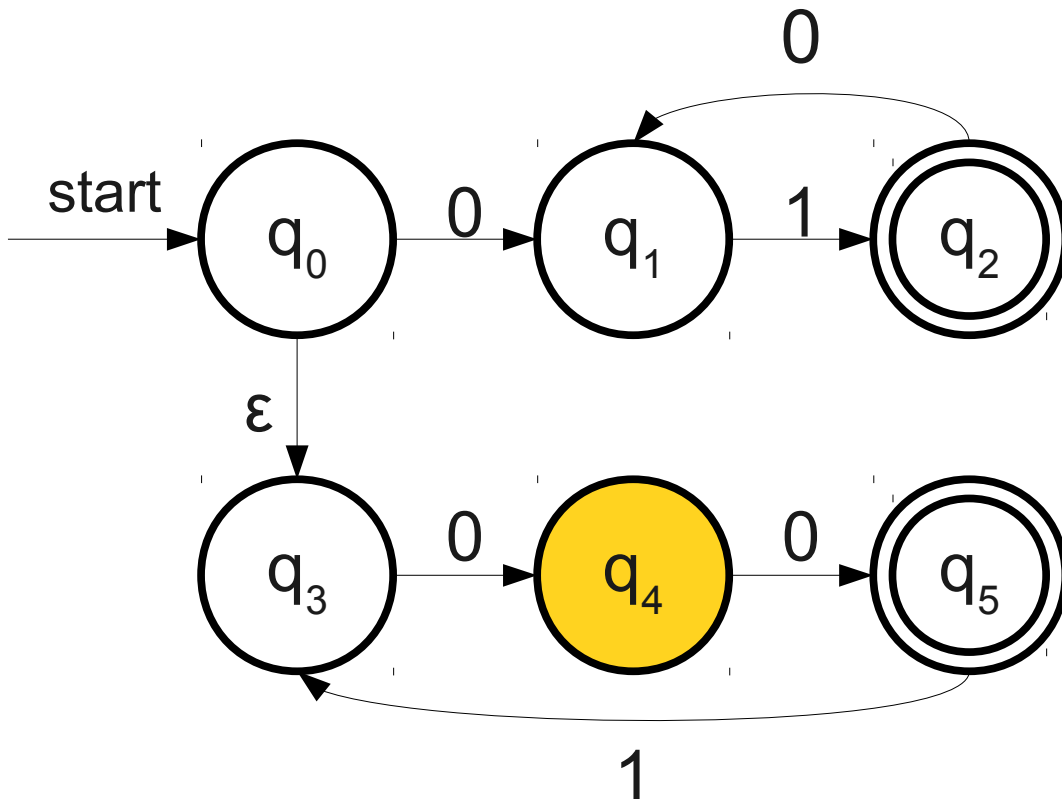
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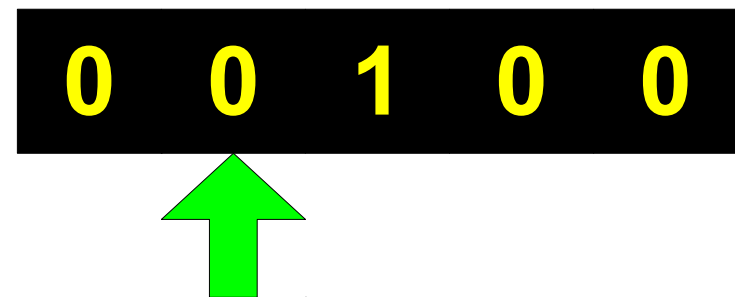
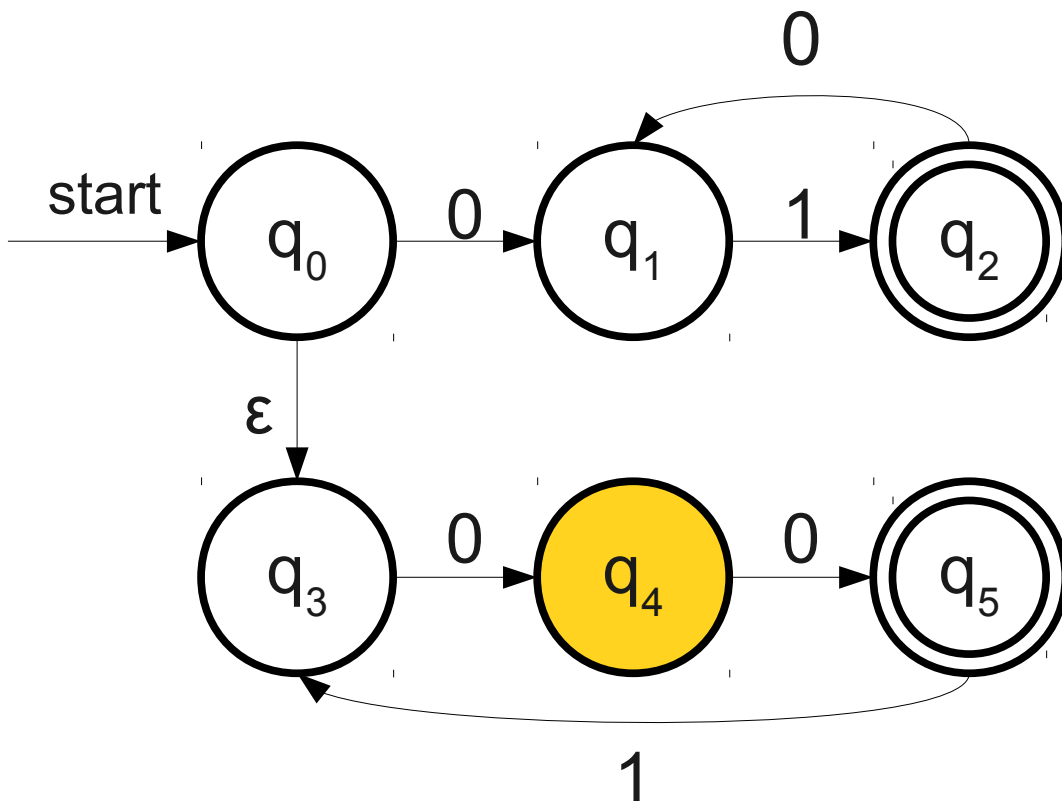
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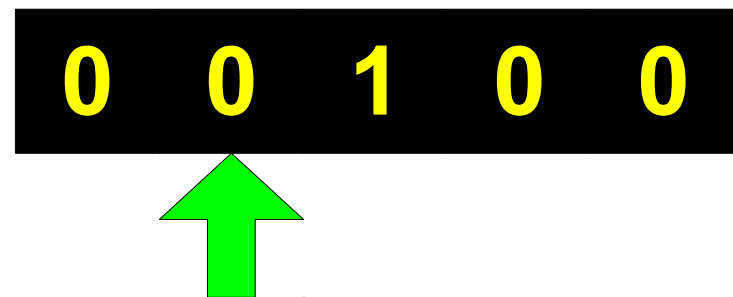
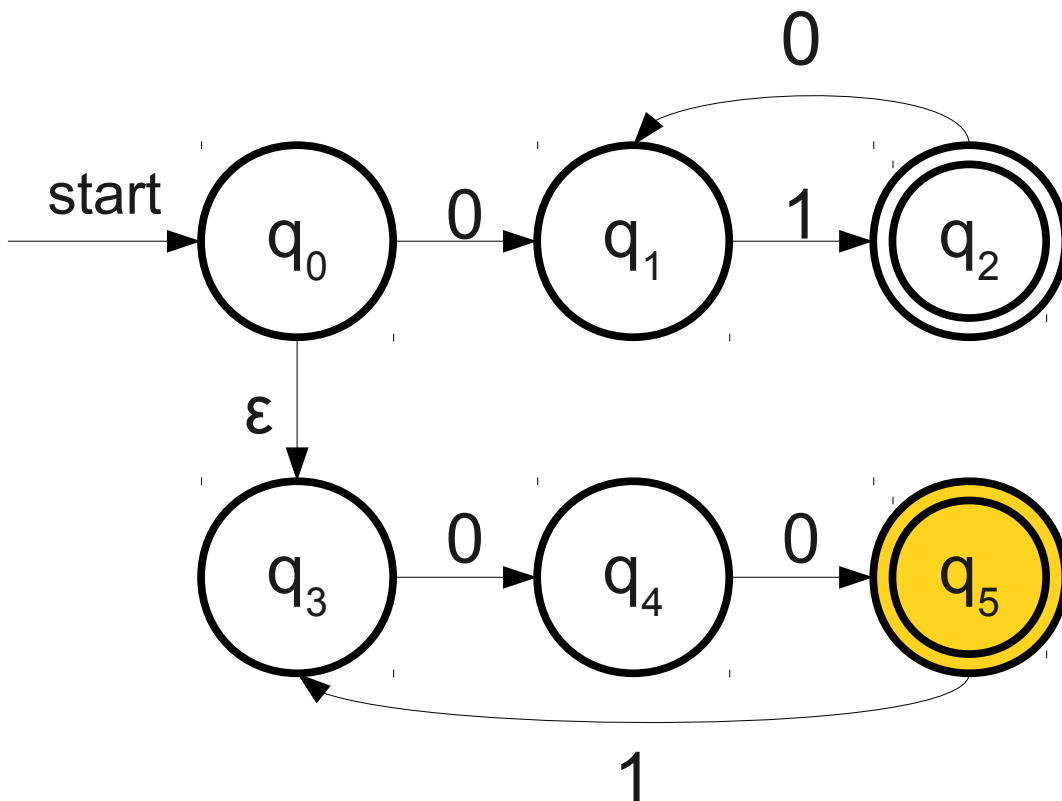
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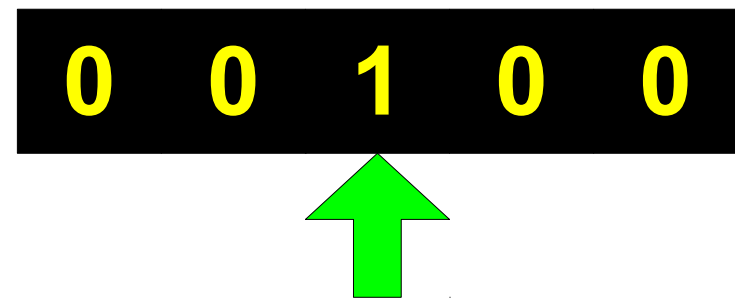
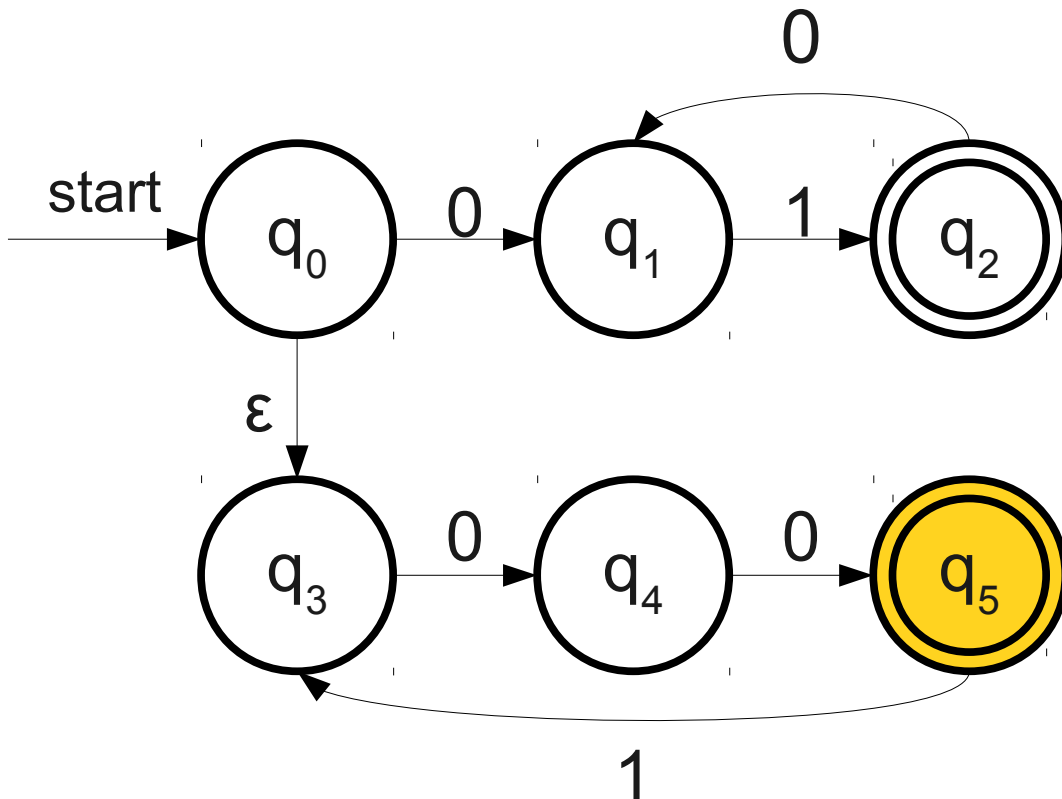
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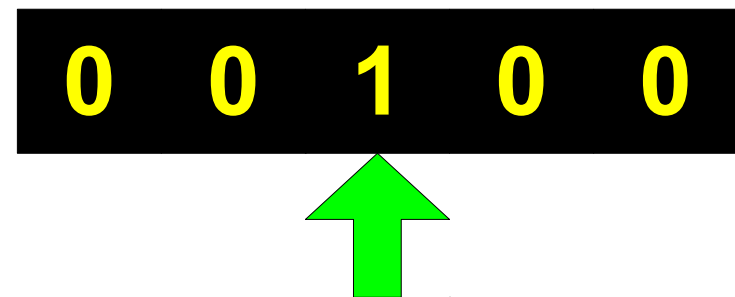
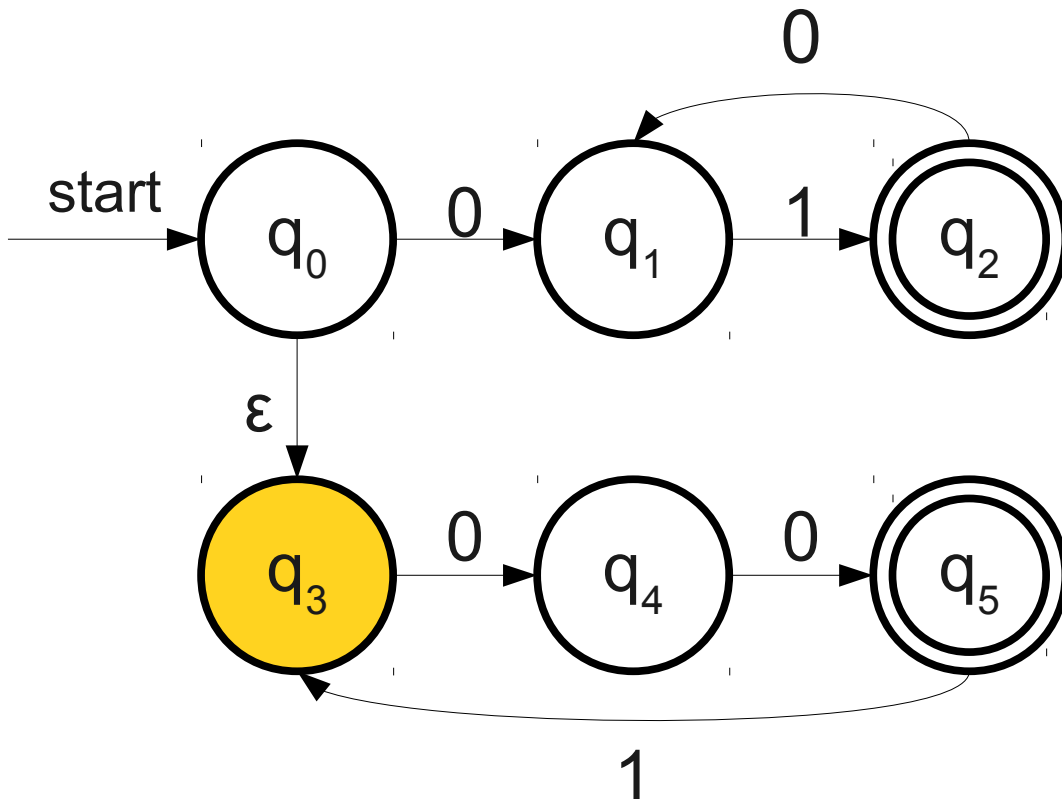
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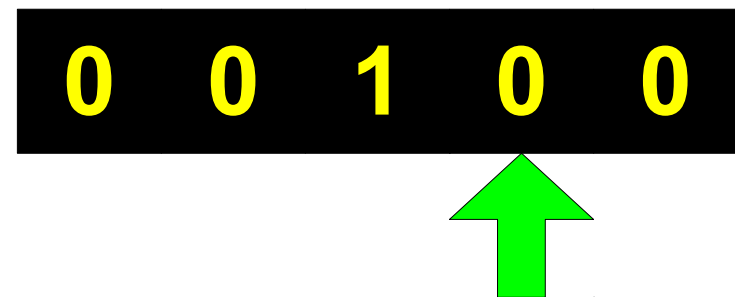
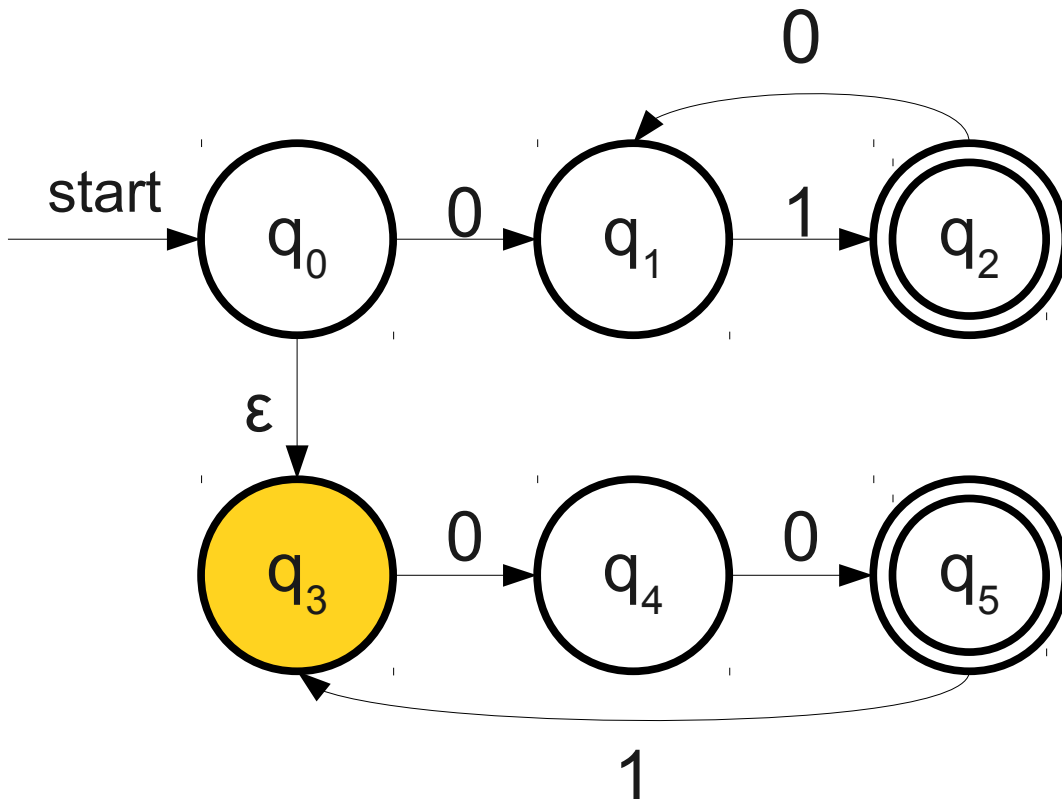
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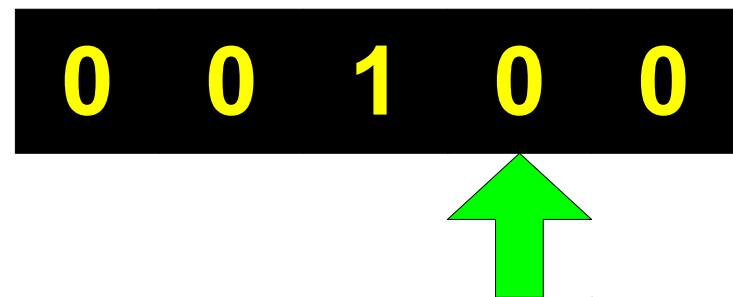
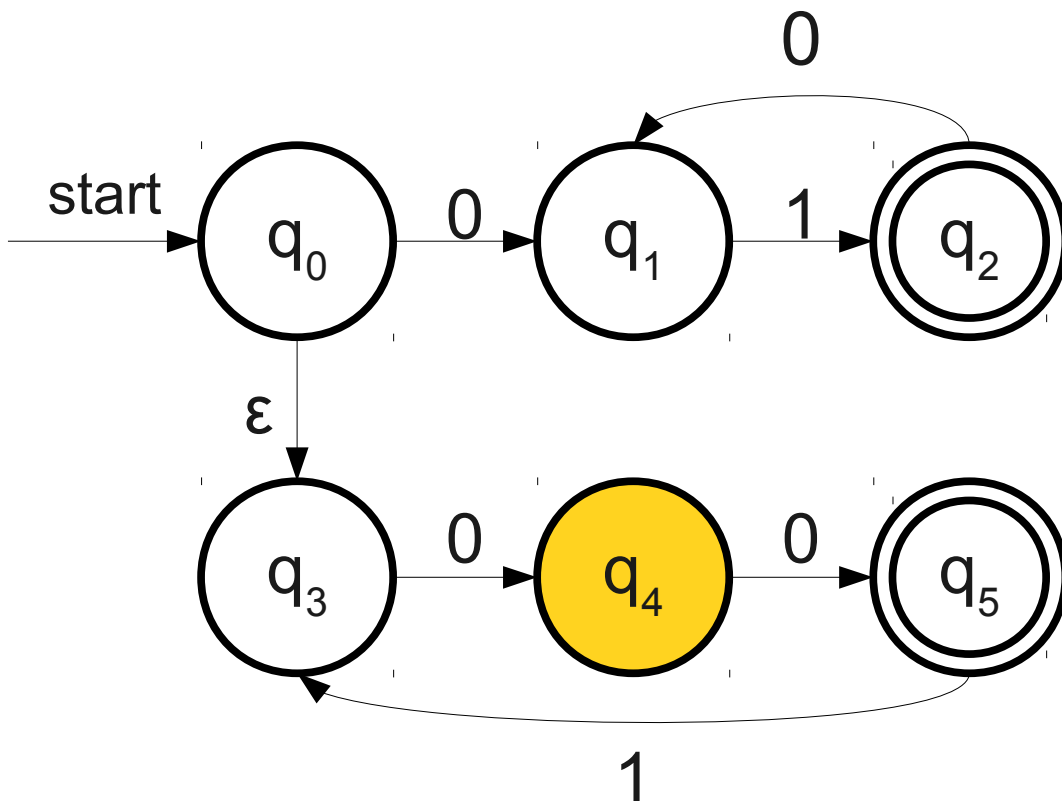
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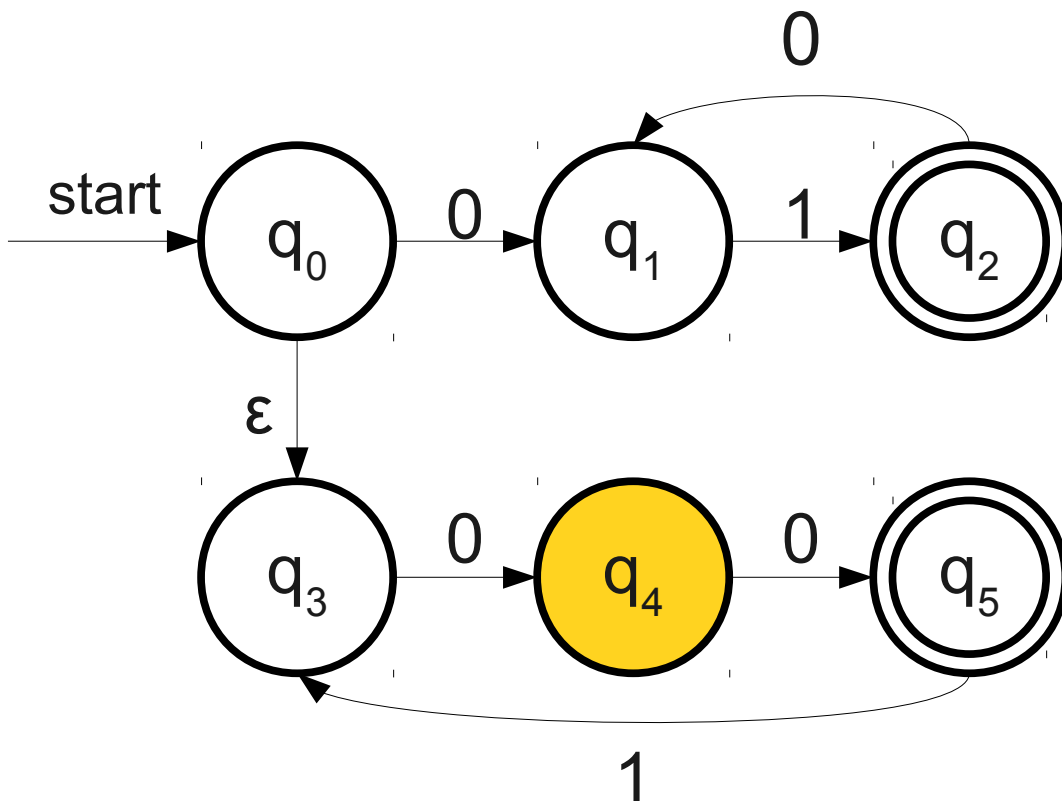
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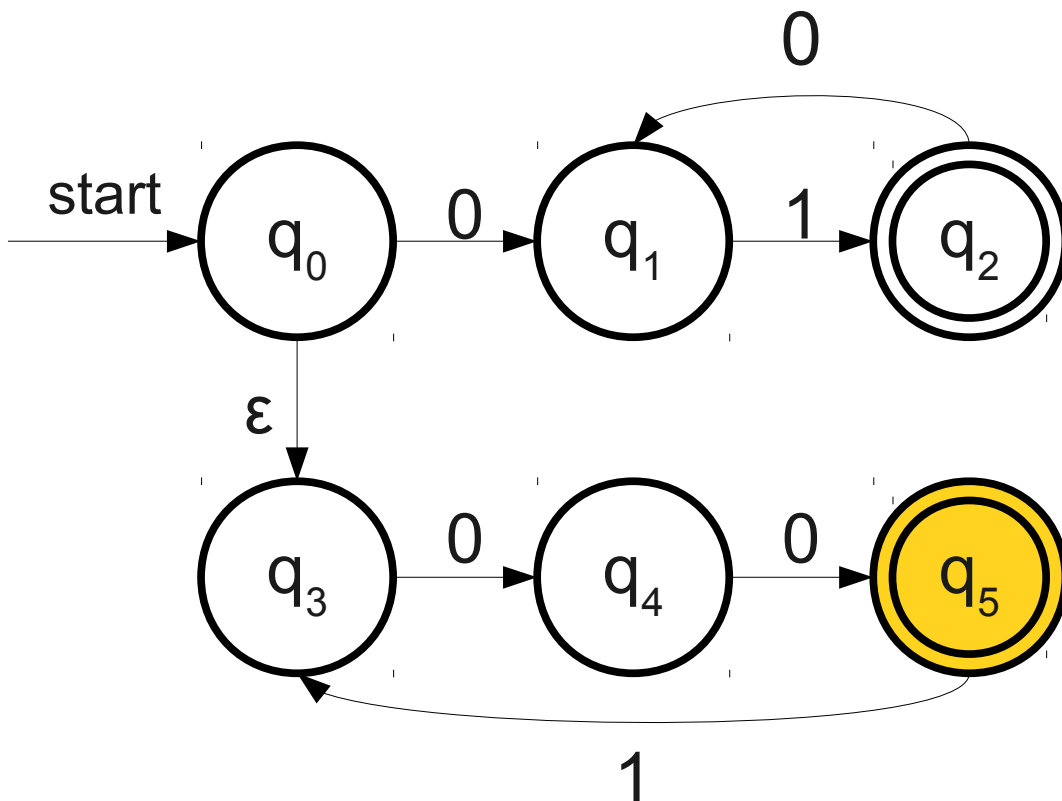
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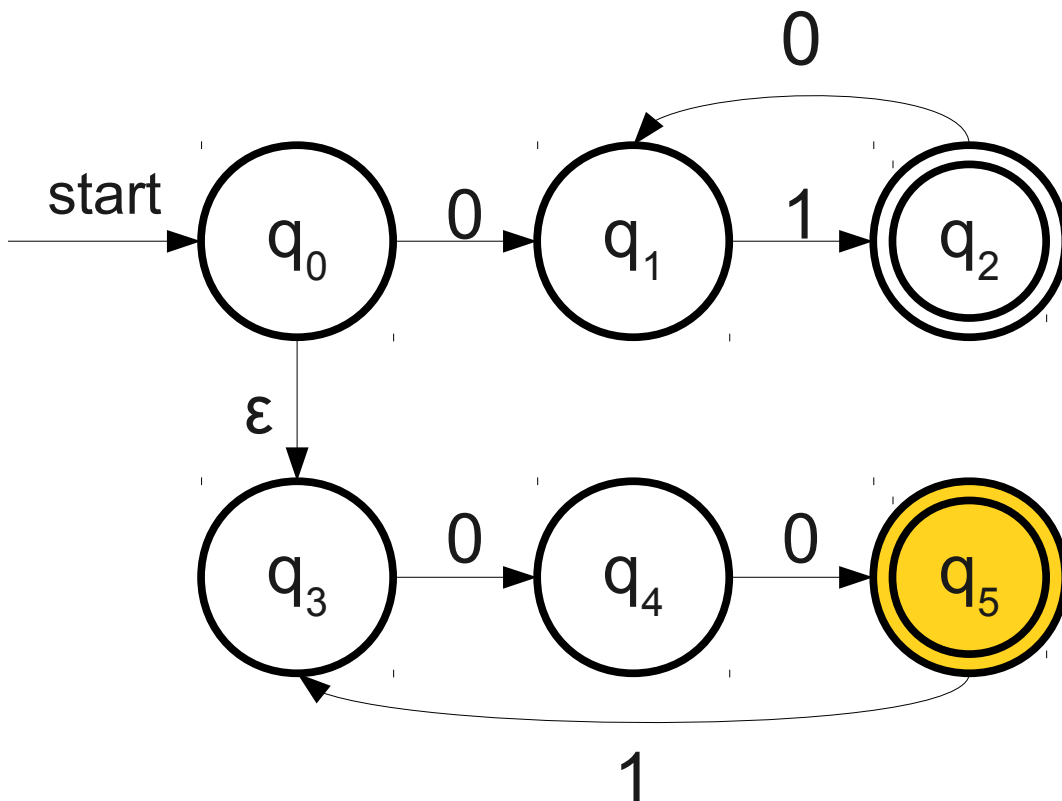
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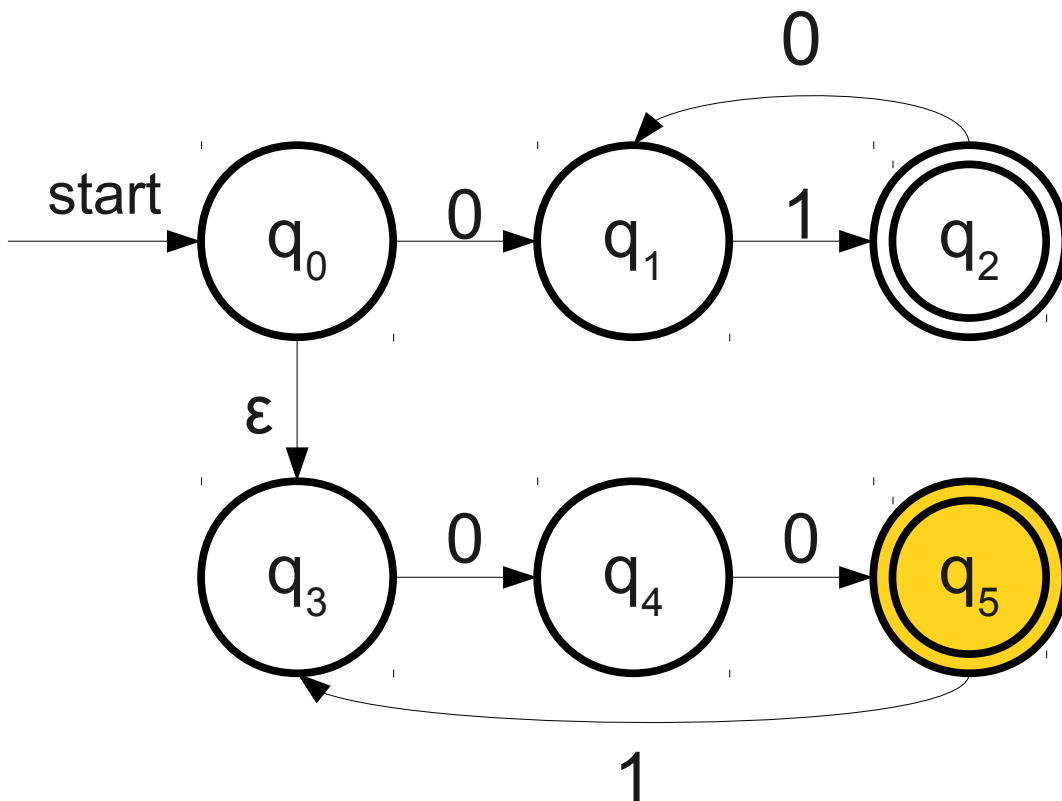
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0 0 1 0 0

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0 0 1 0 0

Designing NFAs

Designing NFAs

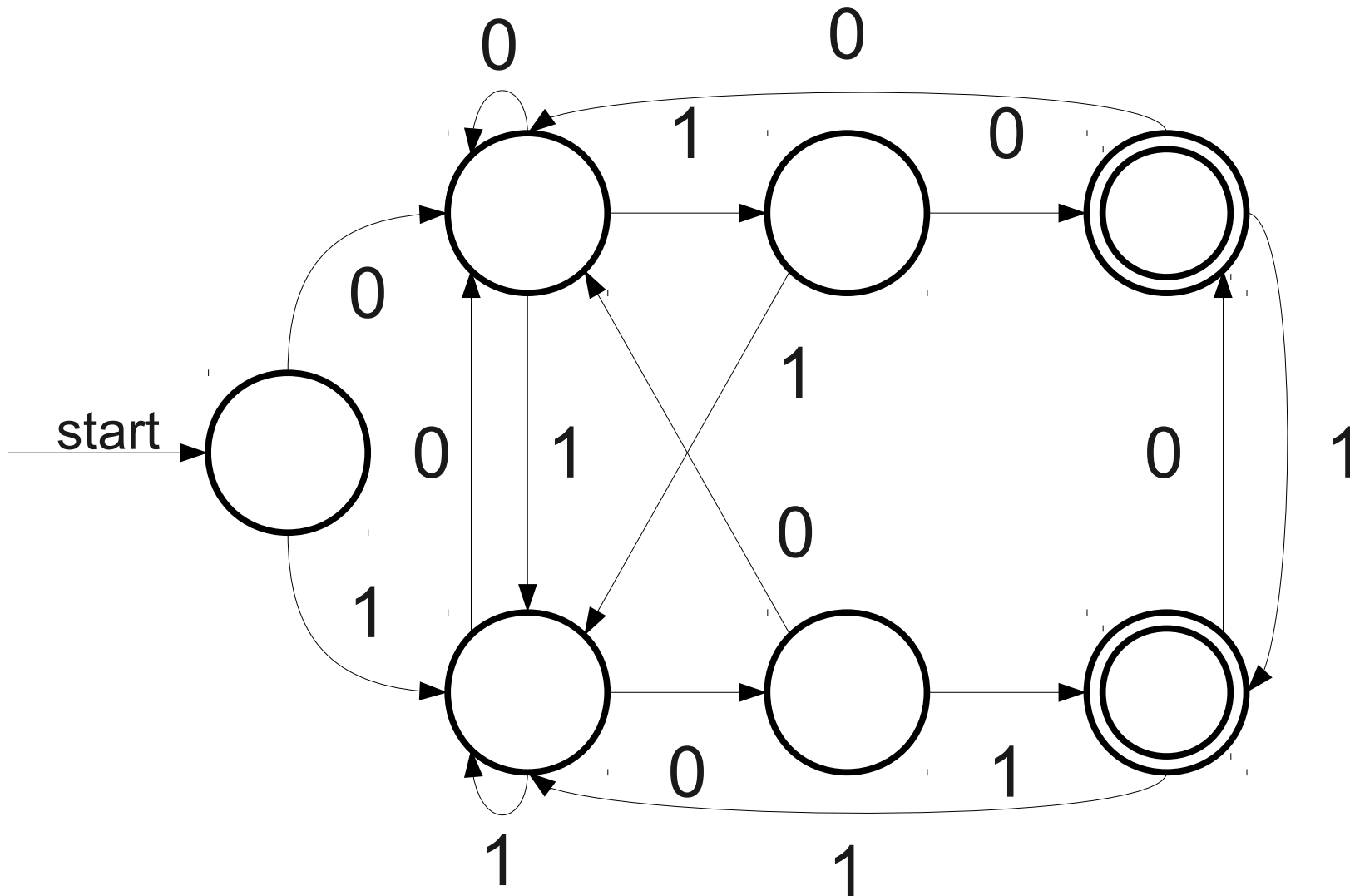
- When designing NFAs, *embrace the nondeterminism!*
- Good model: **Guess-and-check:**
 - Have the machine *nondeterministically guess* what the right choice is.
 - Have the machine *deterministically check* that the choice was correct.

Guess-and-Check

$$L = \{ w \in \{0, 1\}^* \mid w \text{ ends in } 010 \text{ or } 101 \}$$

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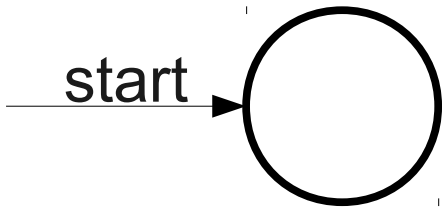


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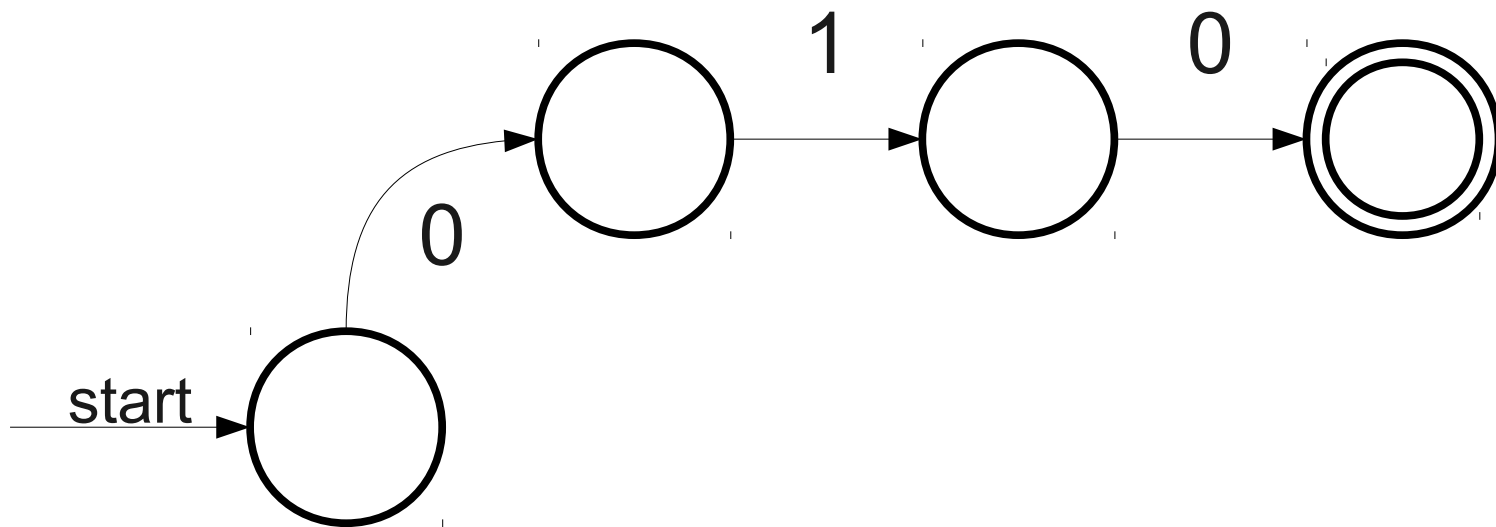
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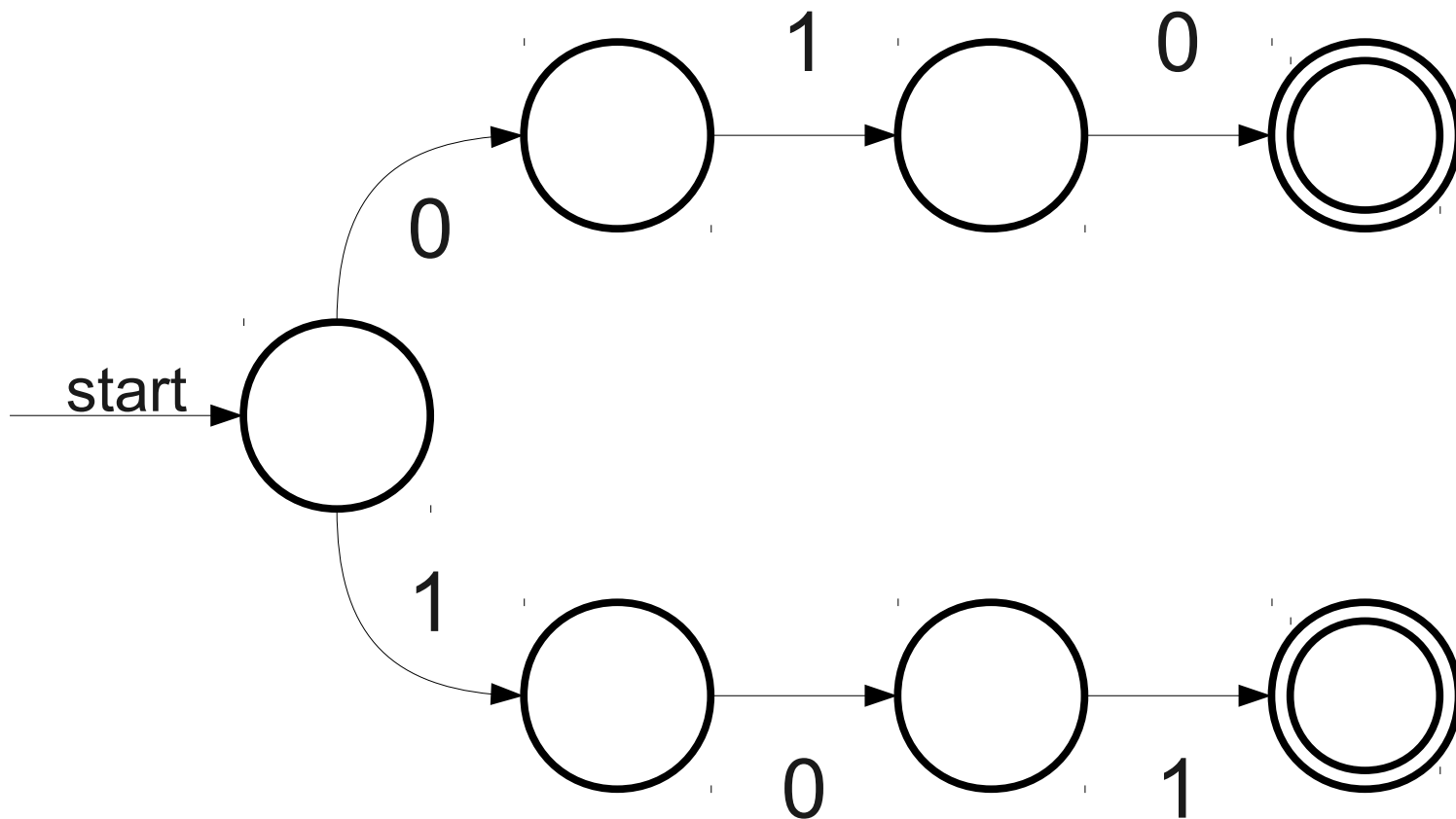
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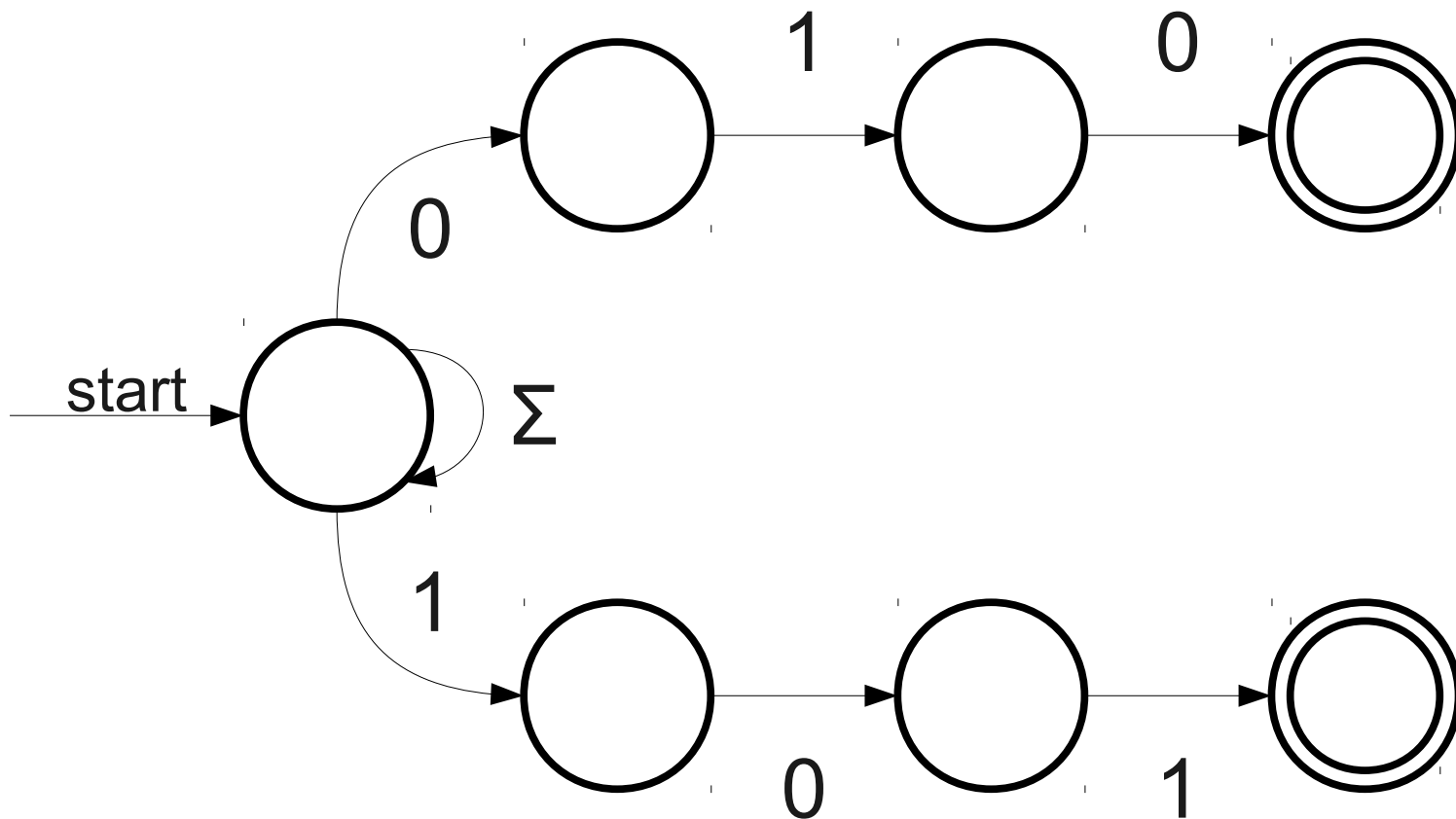
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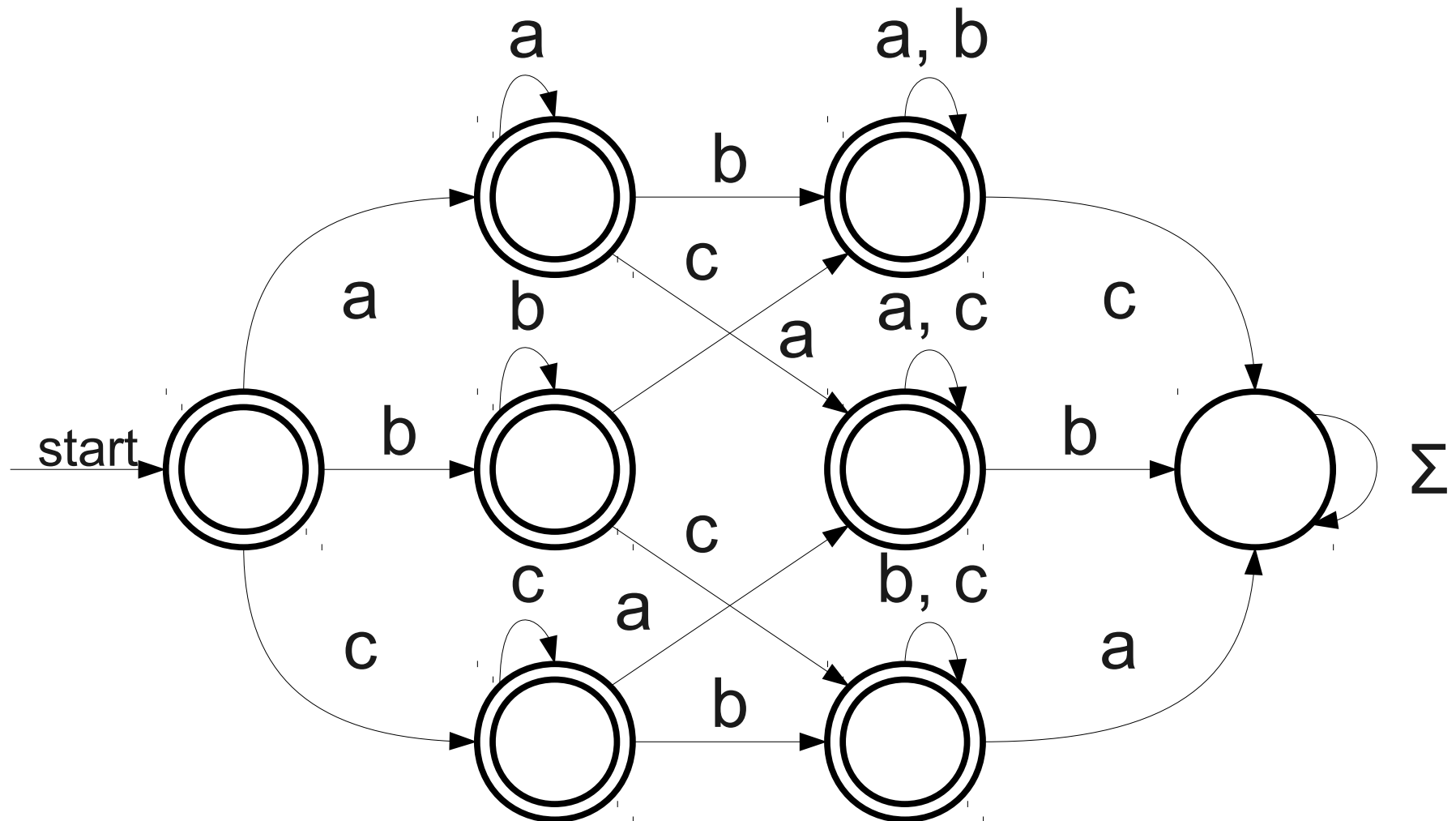


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$L = \{ w \in \{a, b, c\}^* \mid \text{at least one of } a, b, \text{ or } c \text{ is not in } w \}$

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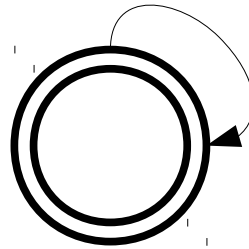


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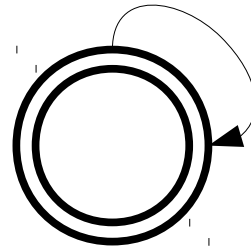
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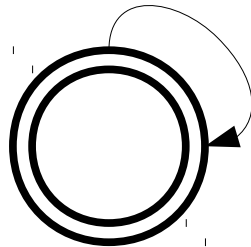
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a, b



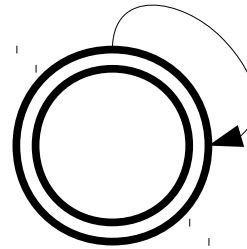
a, c



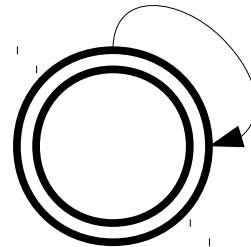
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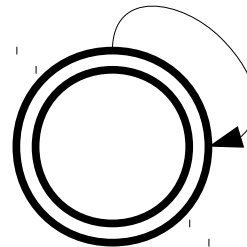
a, b



a, c



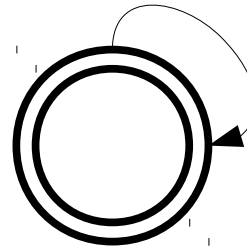
b, c



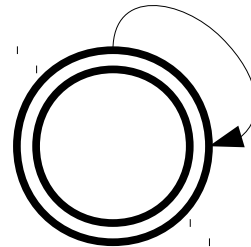
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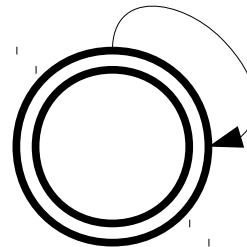
a, b



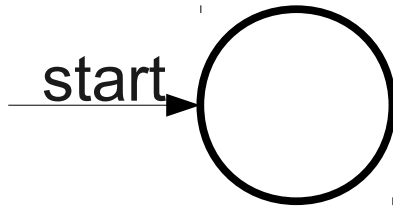
a, c



b, c

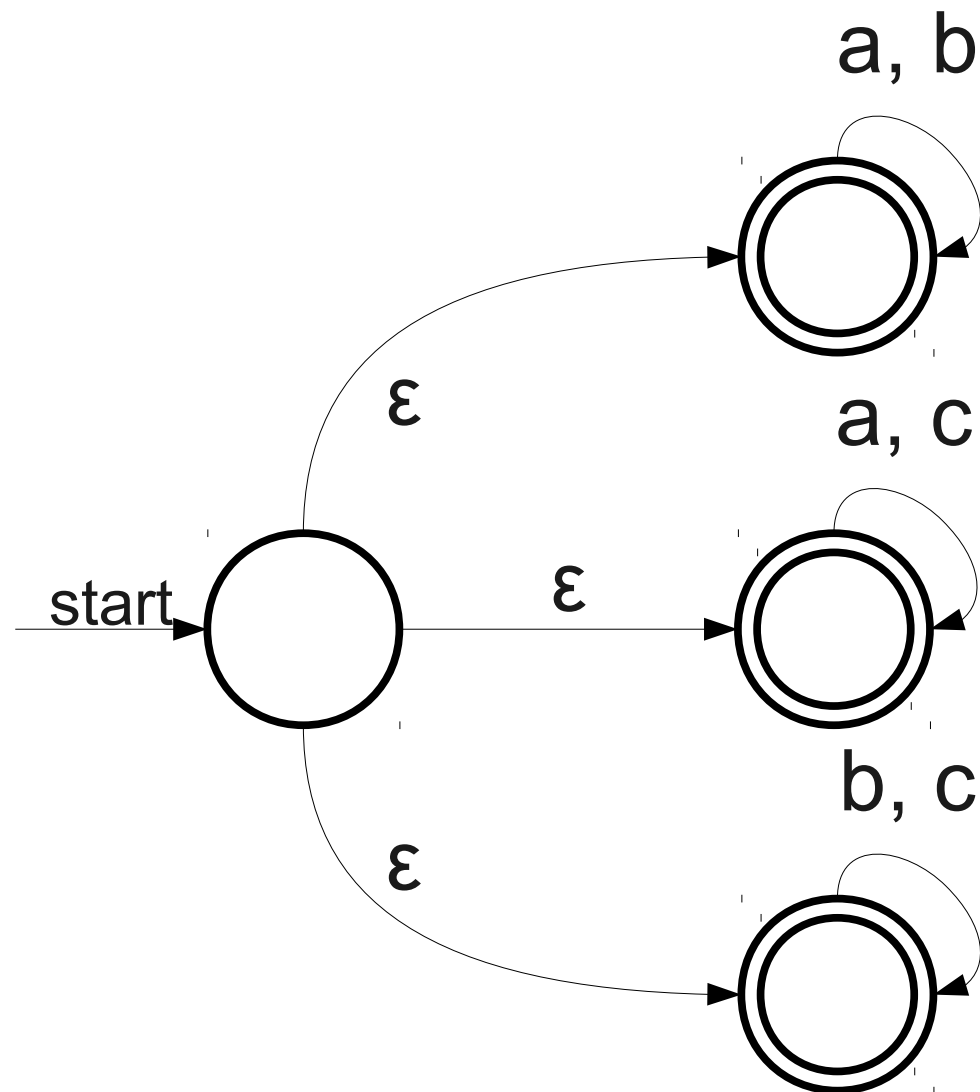


start



Guess-and-Check

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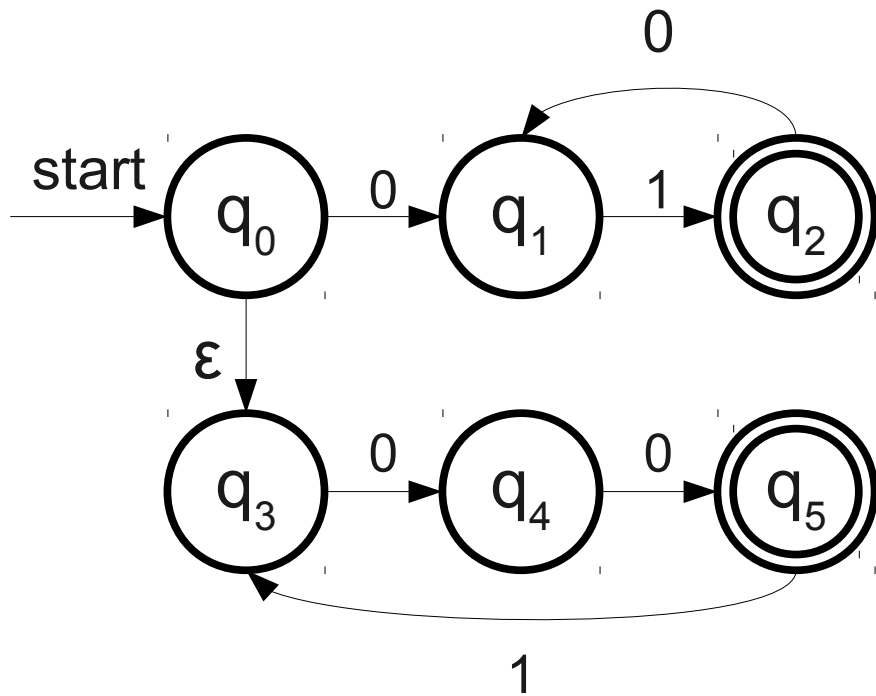
NFAs and DFAs

- Any language that can be accepted by a DFA can be accepted by an NFA.
- Why?
 - Just use the same set of transitions as before.
- **Question:** Can any language accepted by an NFA also be accepted by a DFA?
- Surprisingly, the answer is **yes!**

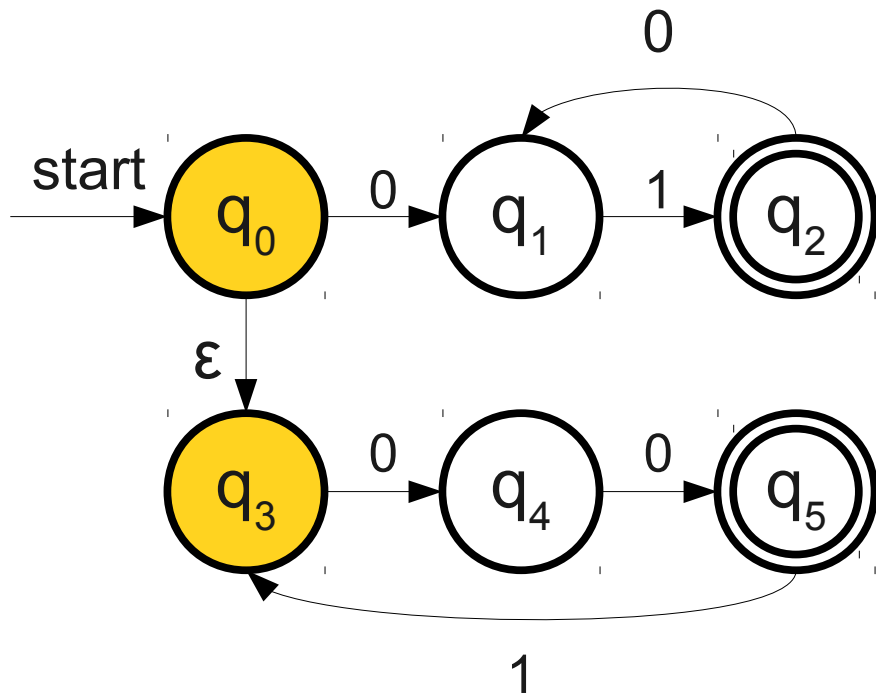
Simulation

- **Simulation** is a key technique in computability theory.
- If we can build an automaton A' whose behavior *simulates* that of another automaton A , then we can make a connection between A and A' .
- To show that any language accepted by an NFA can be accepted by a DFA, we will show how to make a DFA that *simulates* the execution of an NFA.

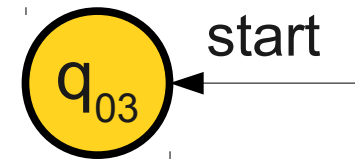
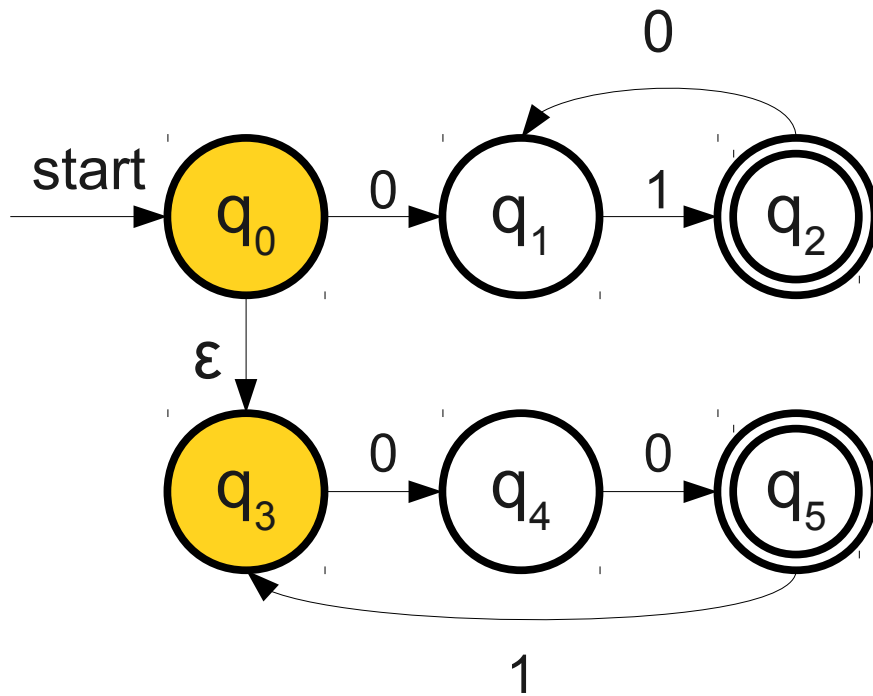
Simulating an NFA with a DFA



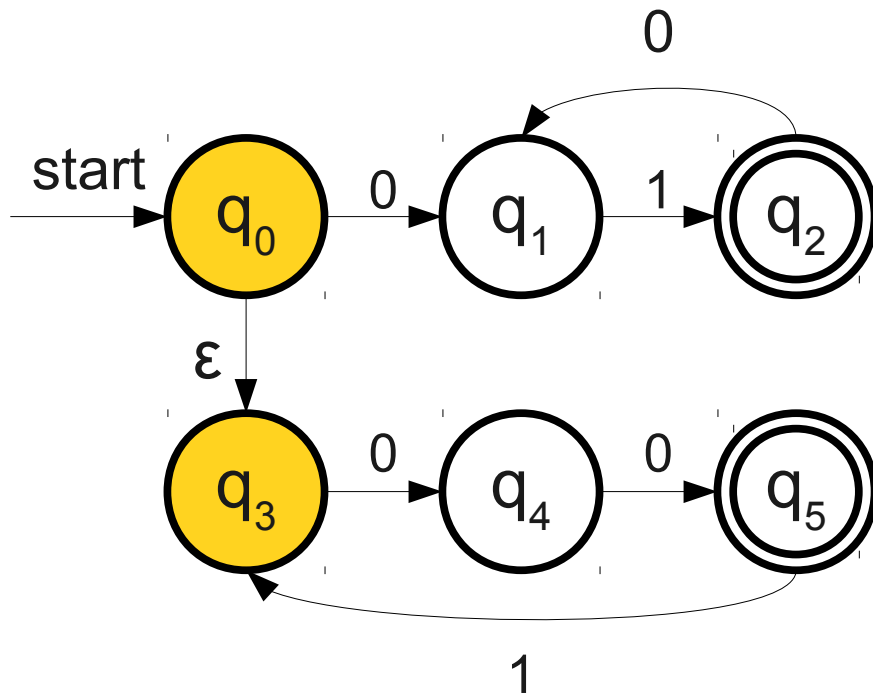
Simulating an NFA with a DFA



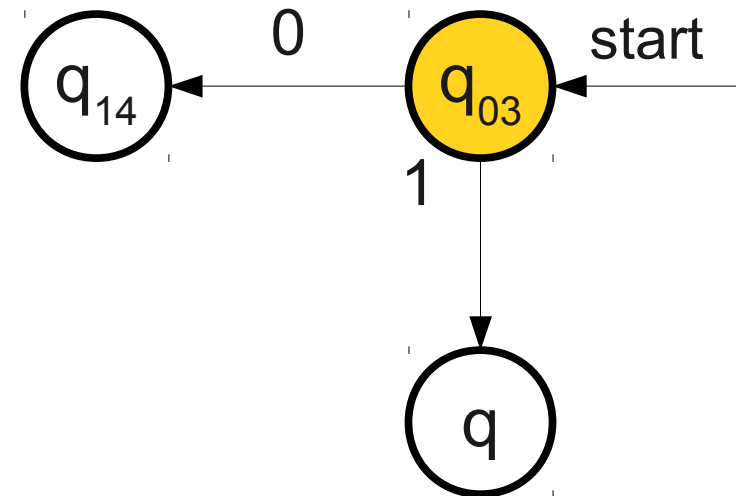
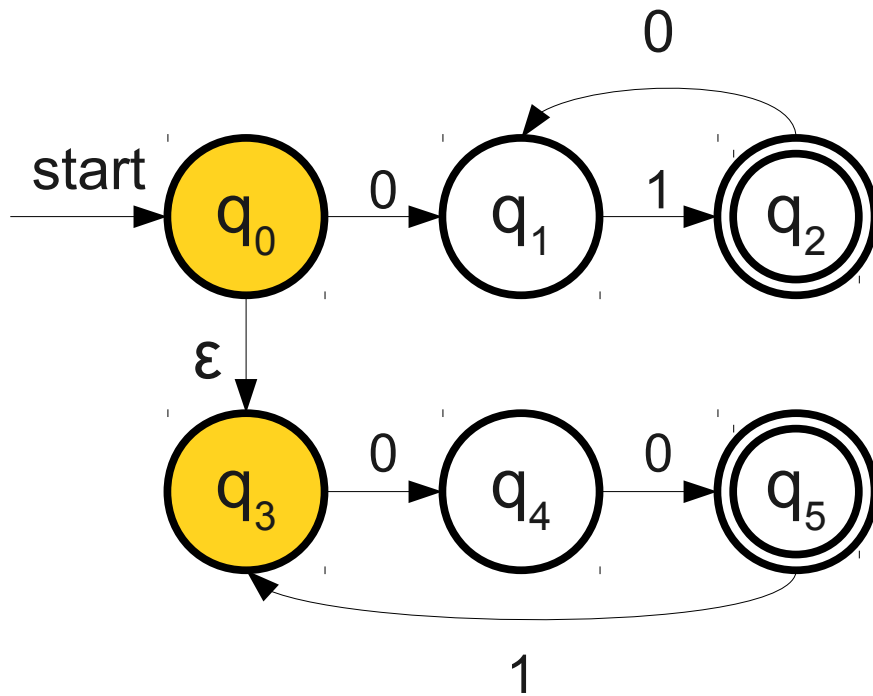
Simulating an NFA with a DFA



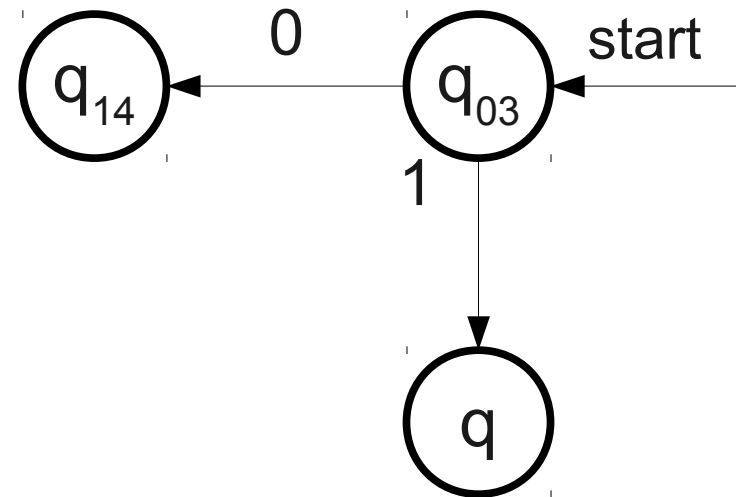
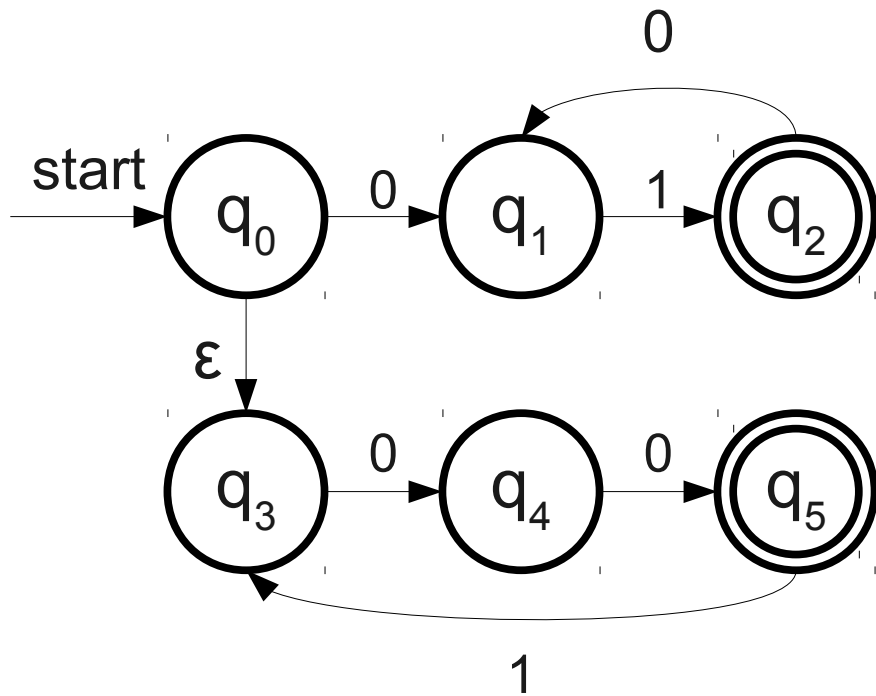
Simulating an NFA with a DFA



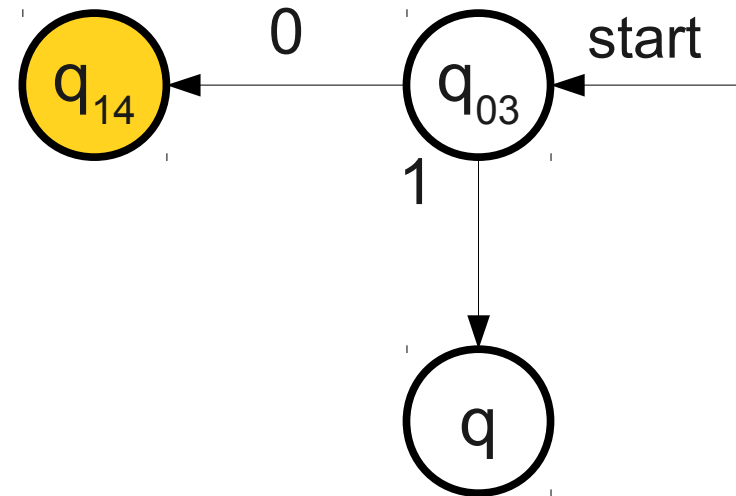
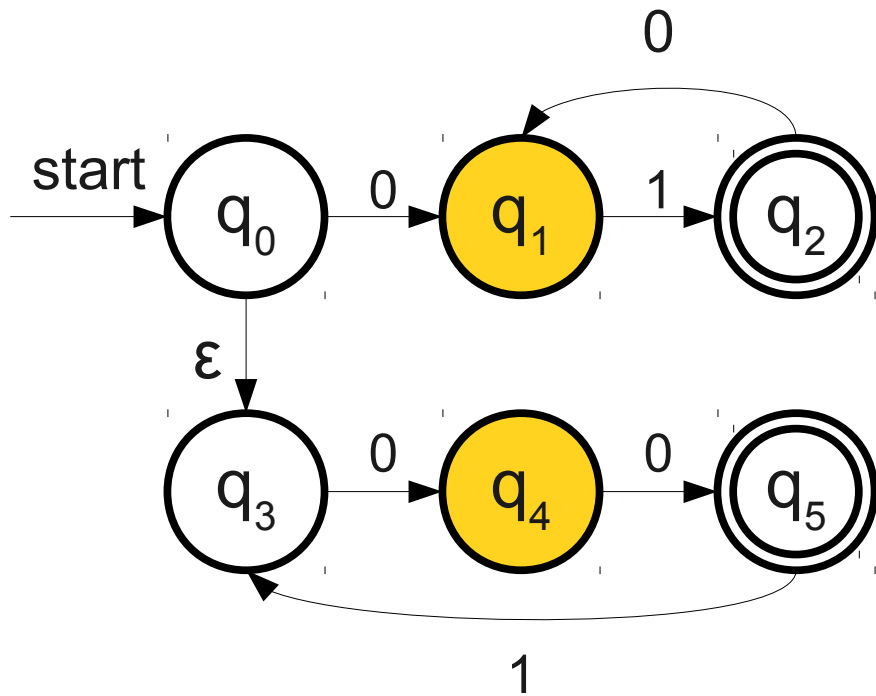
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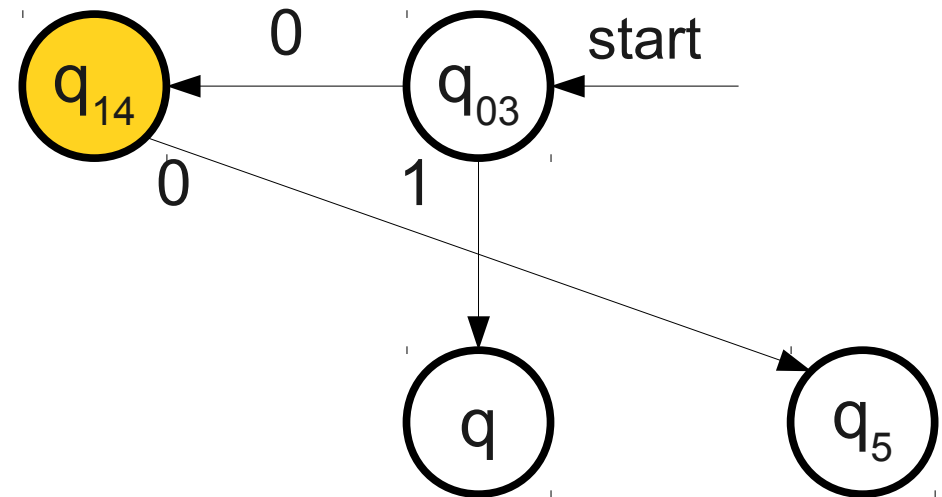
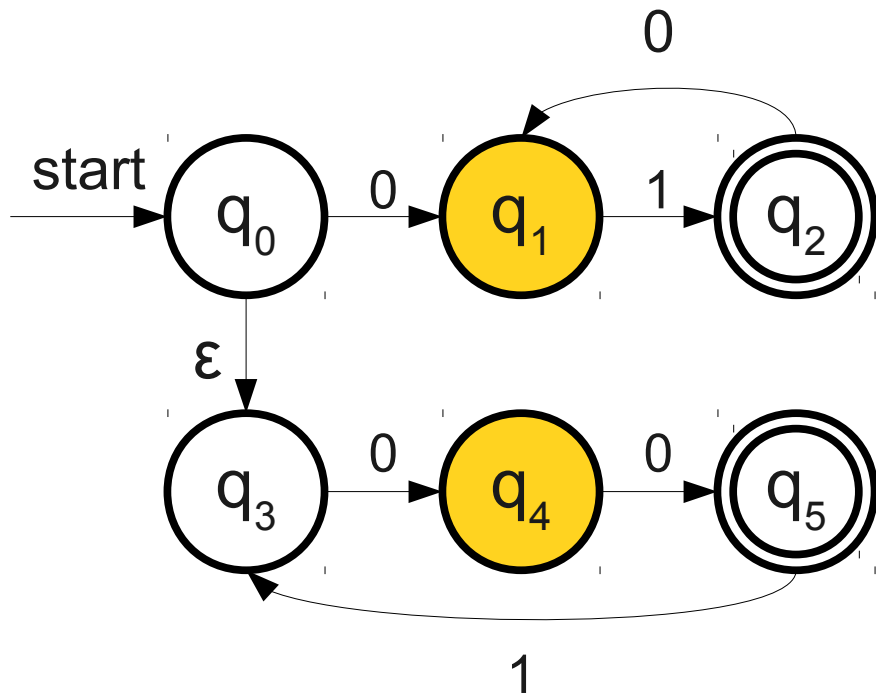
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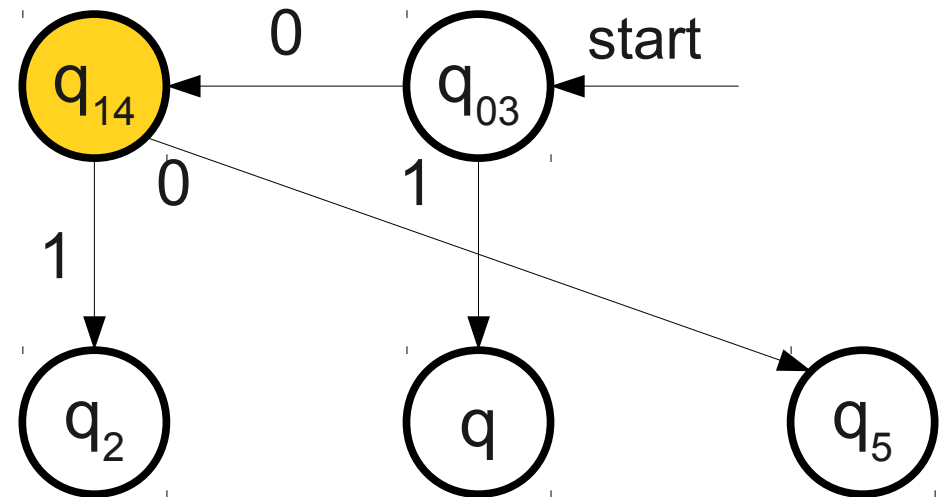
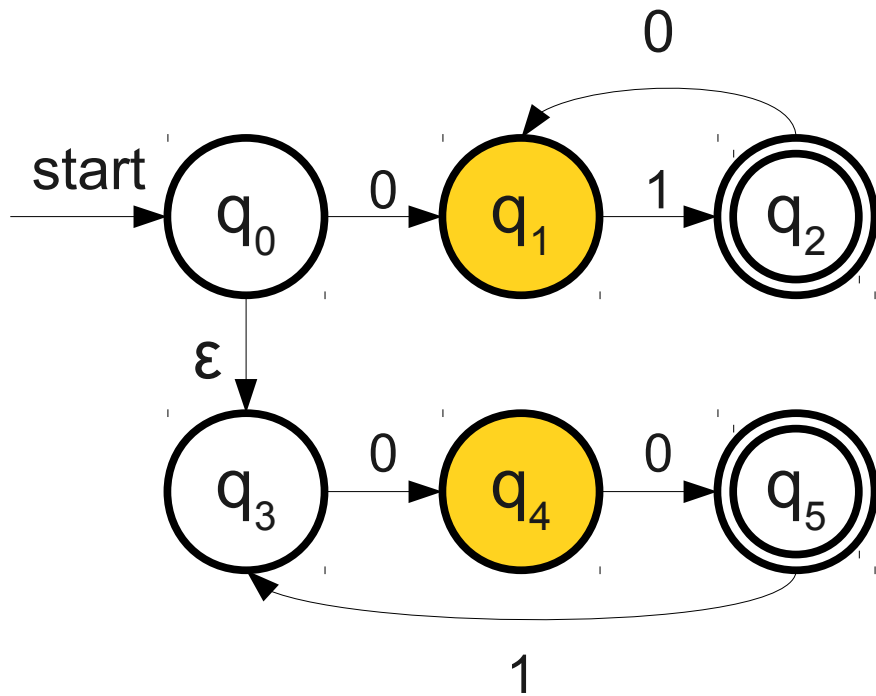
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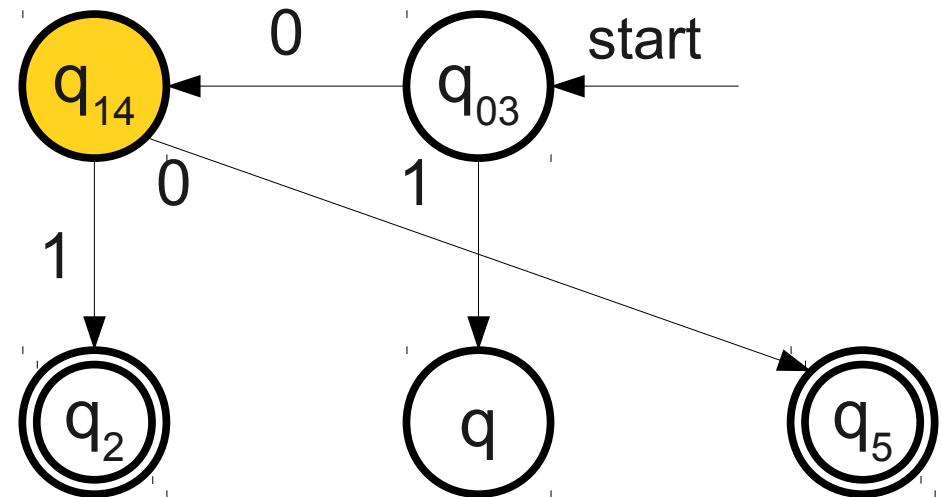
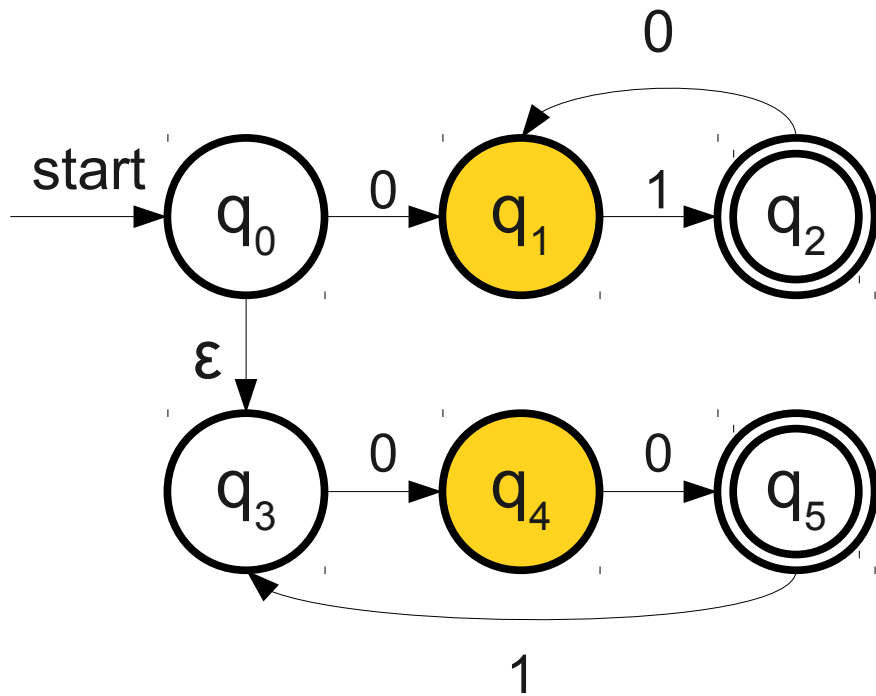
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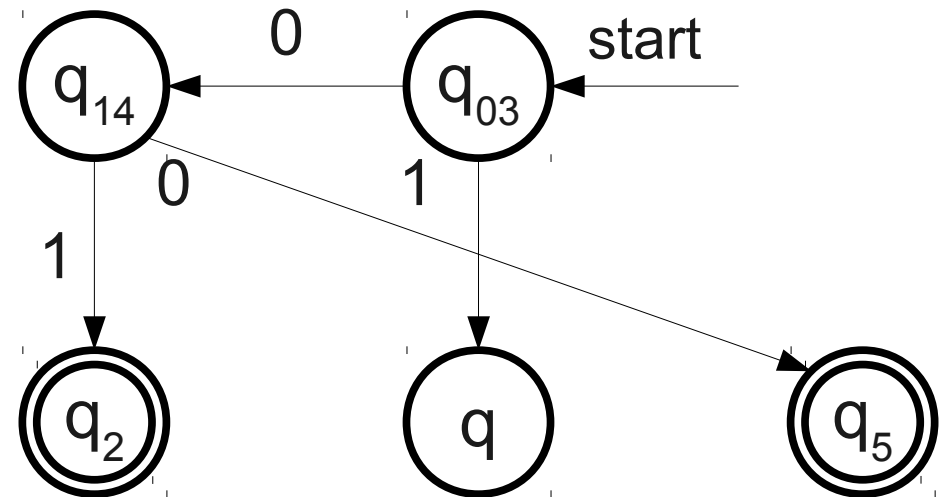
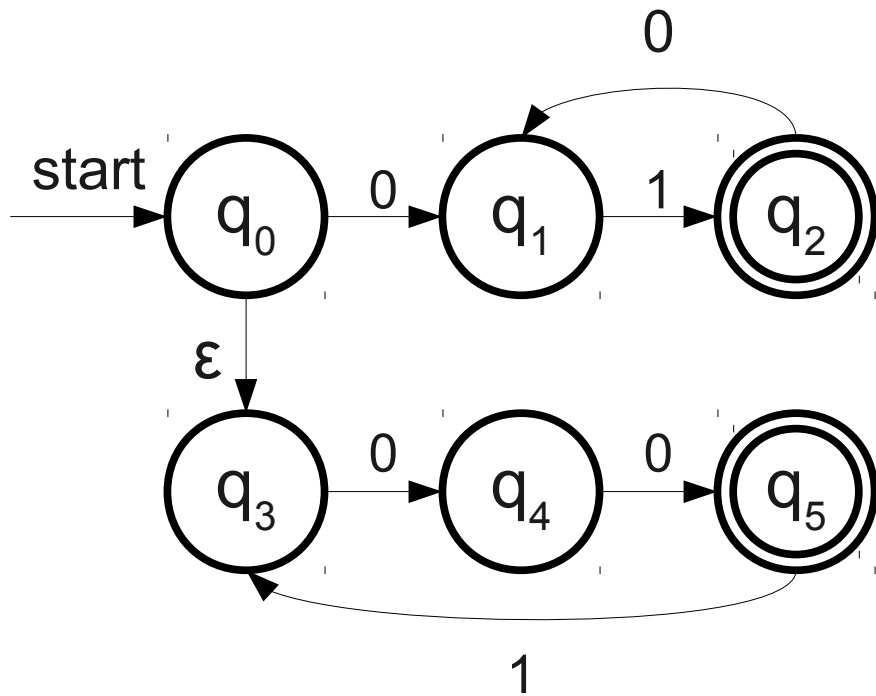
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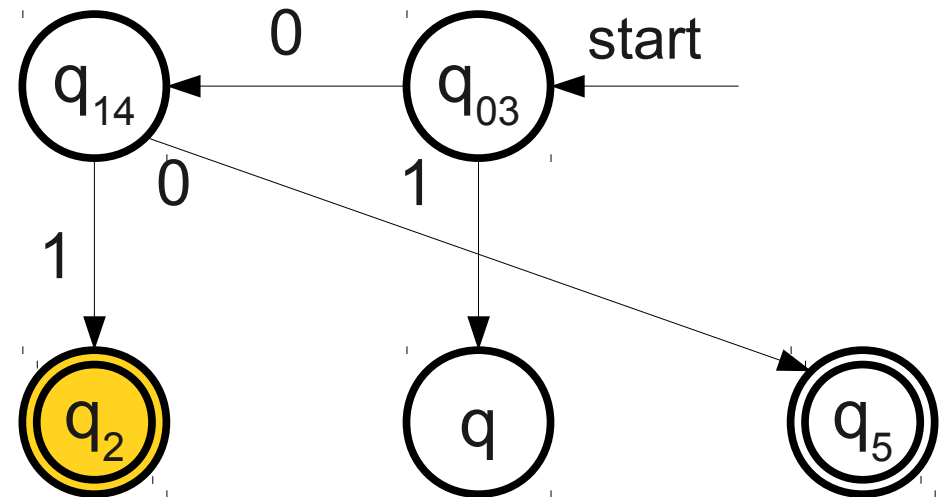
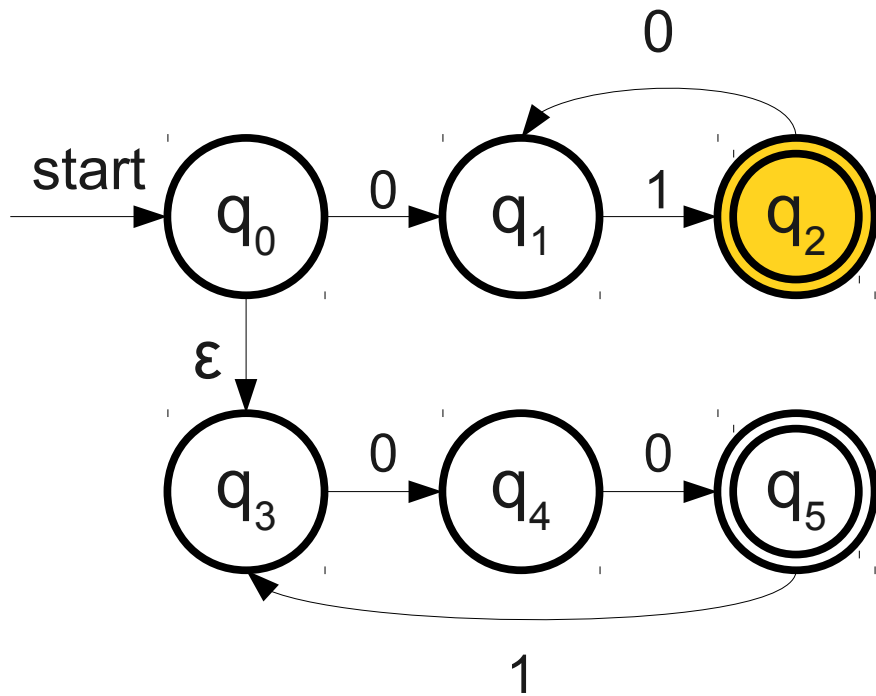
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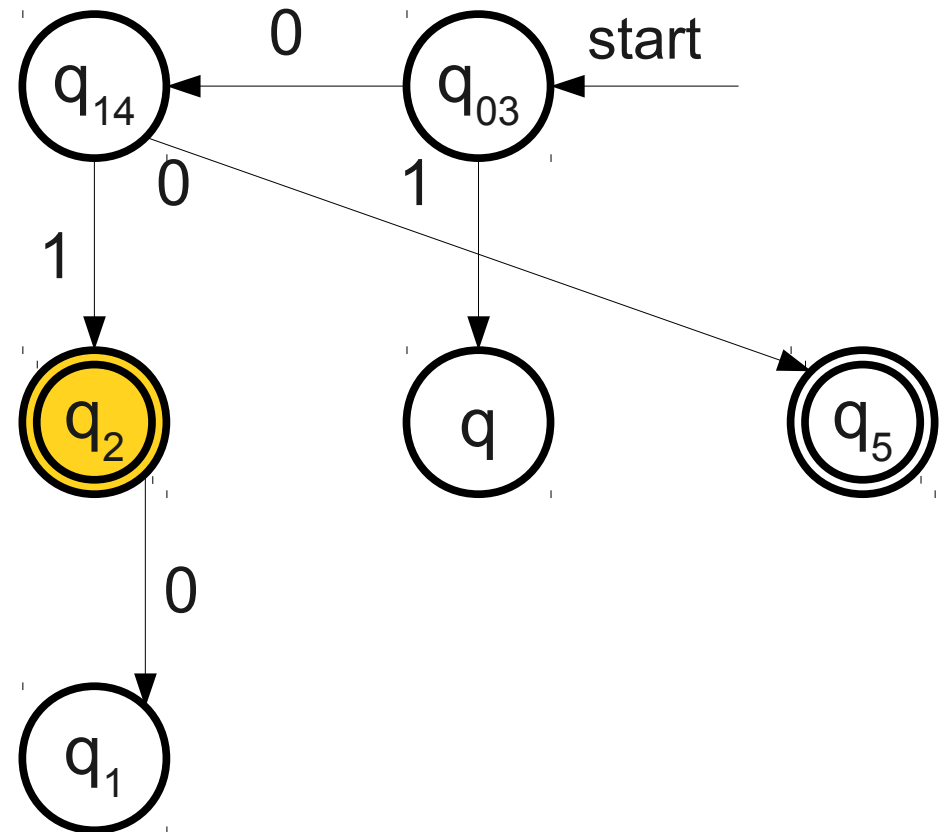
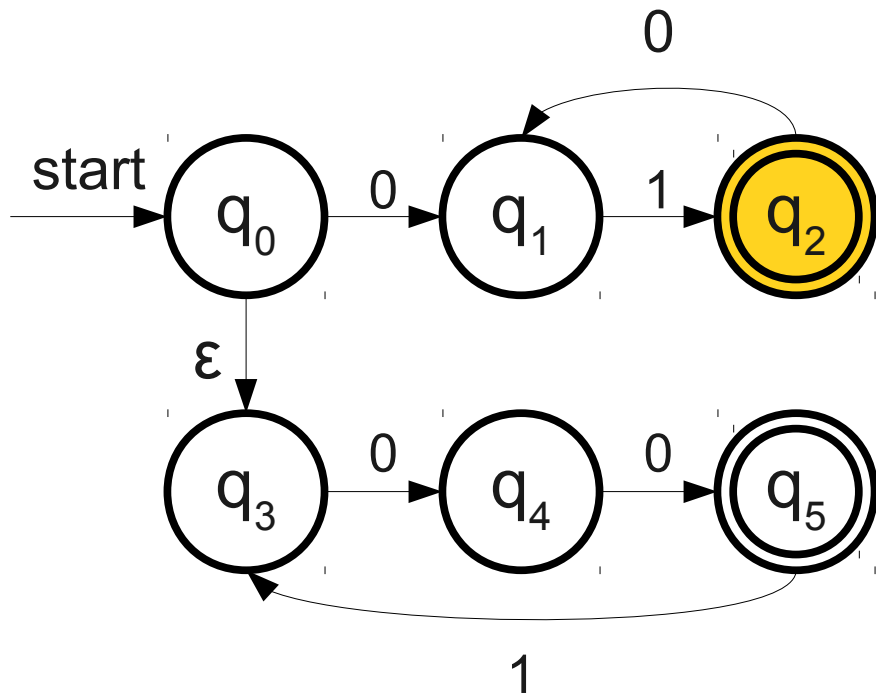
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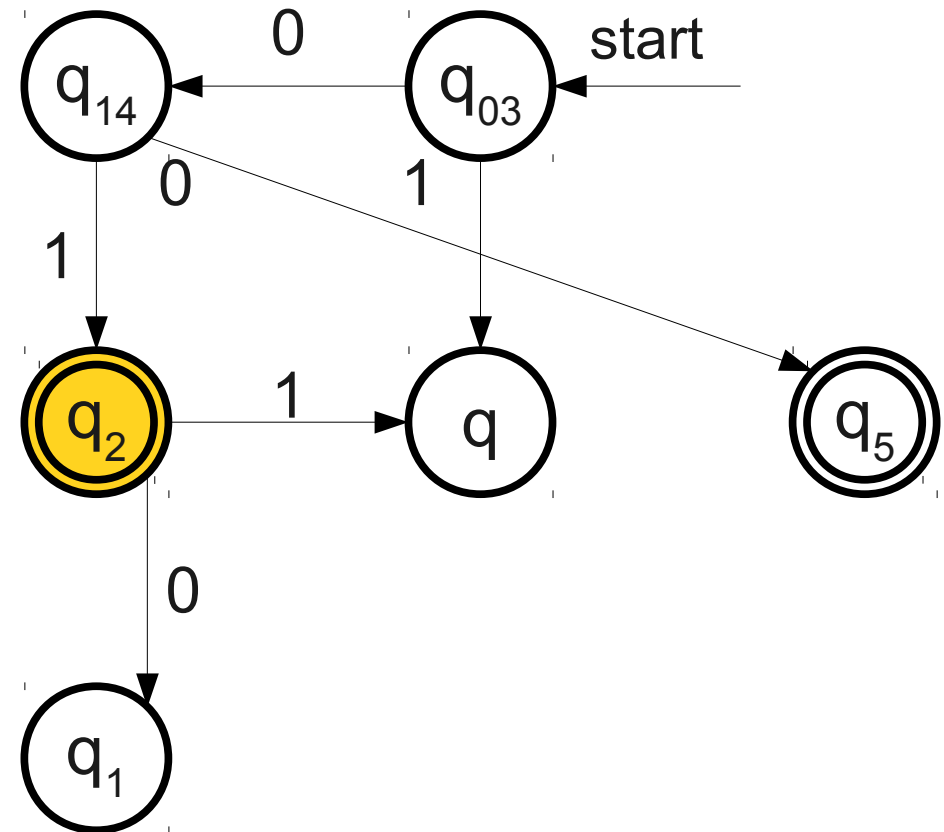
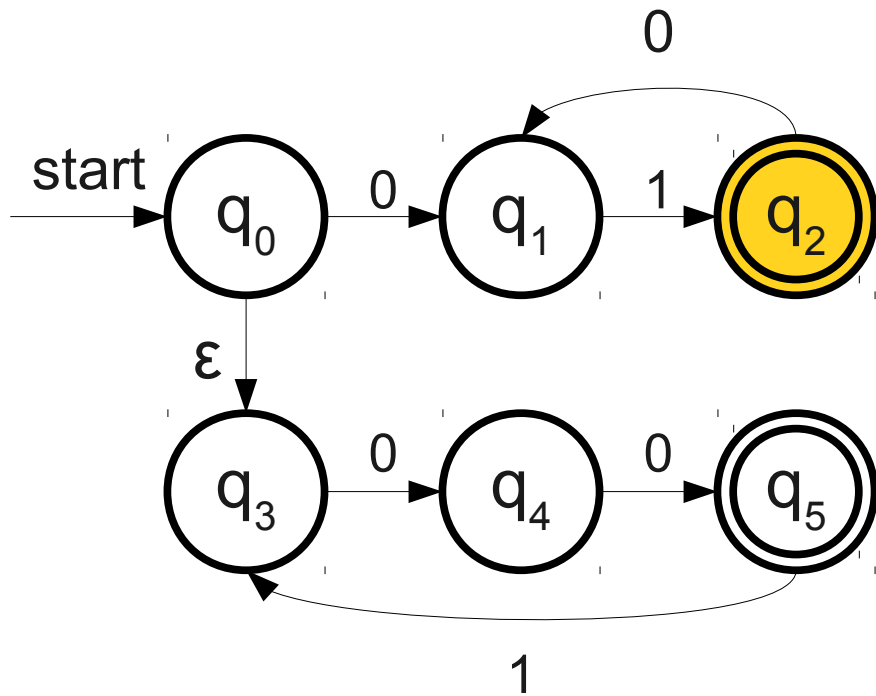
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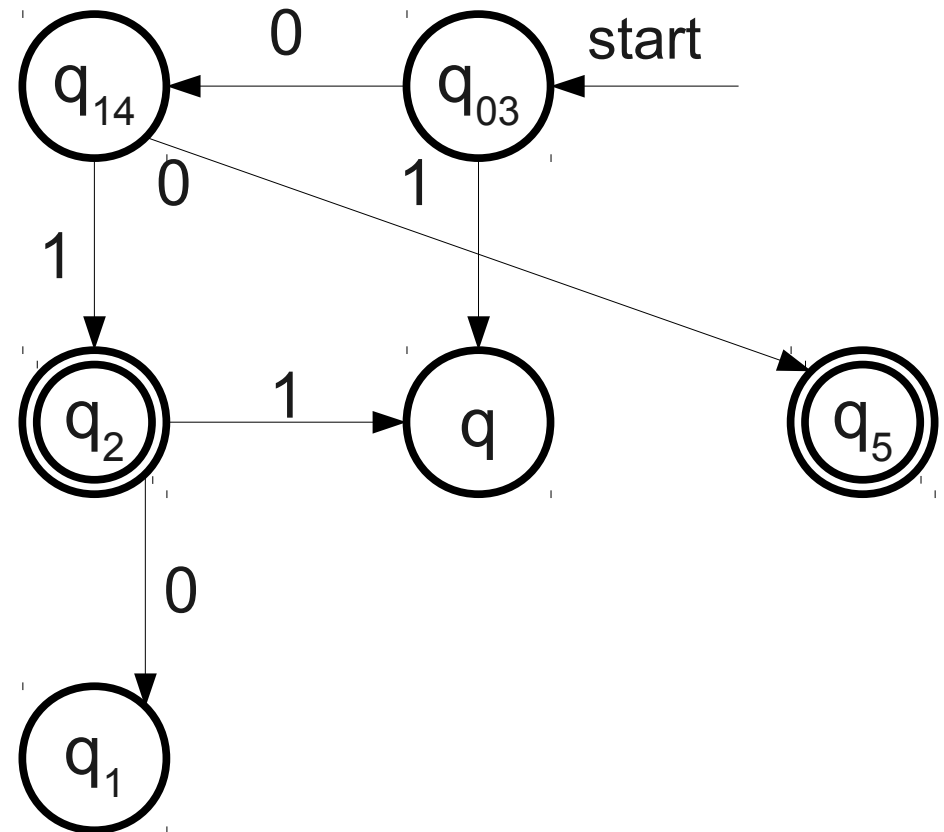
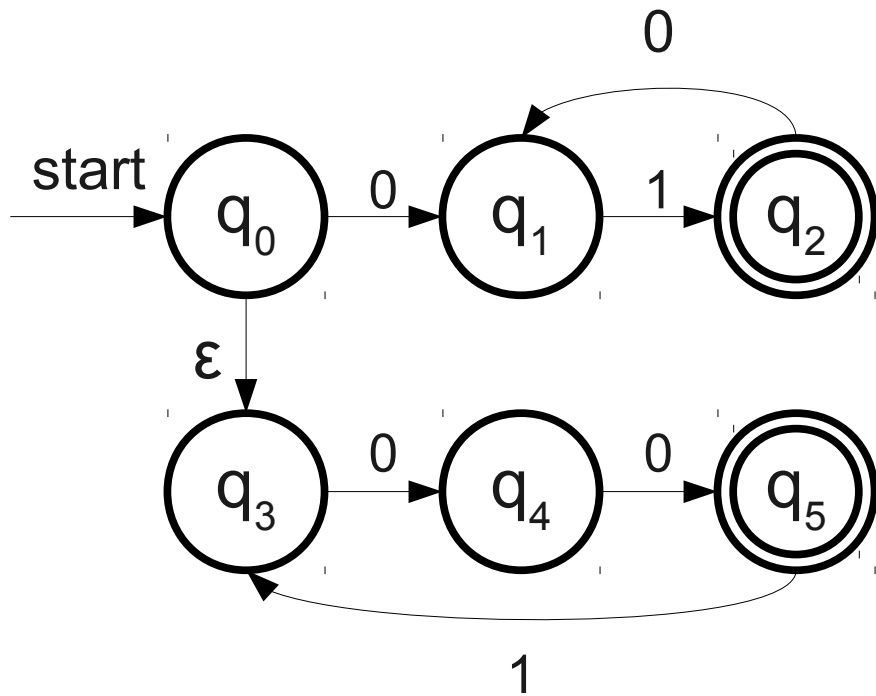
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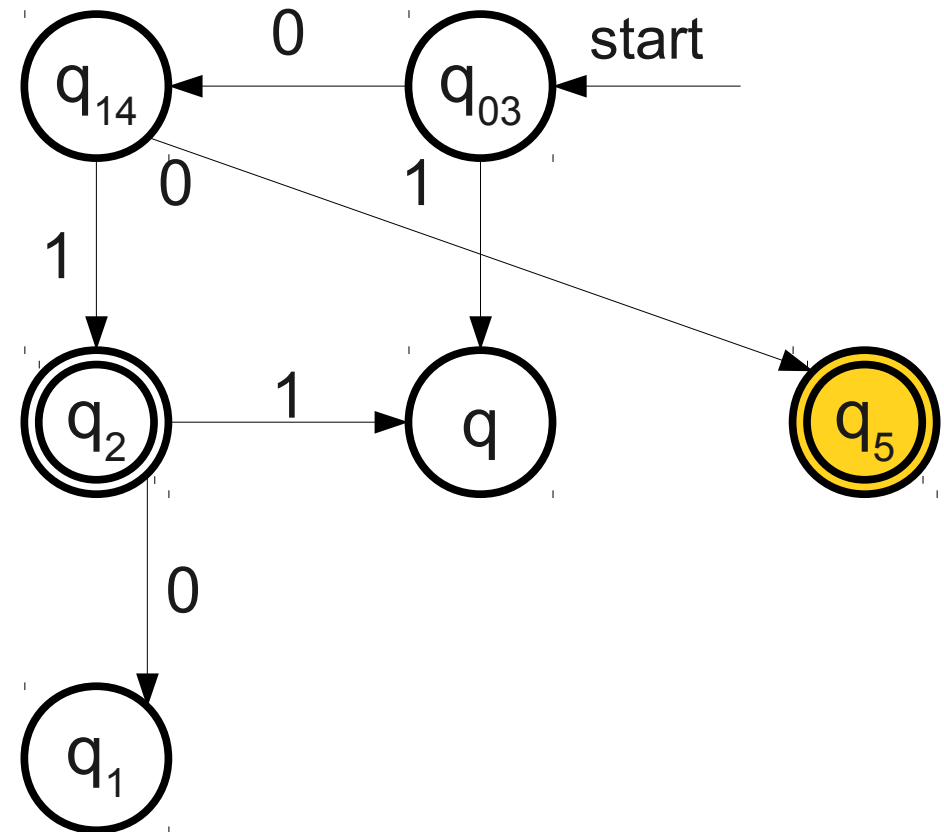
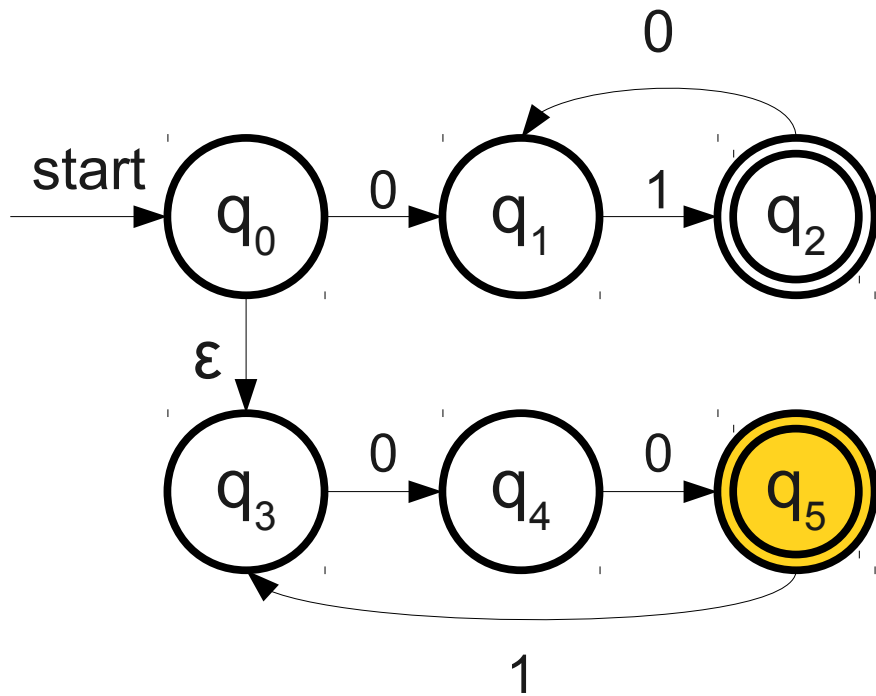
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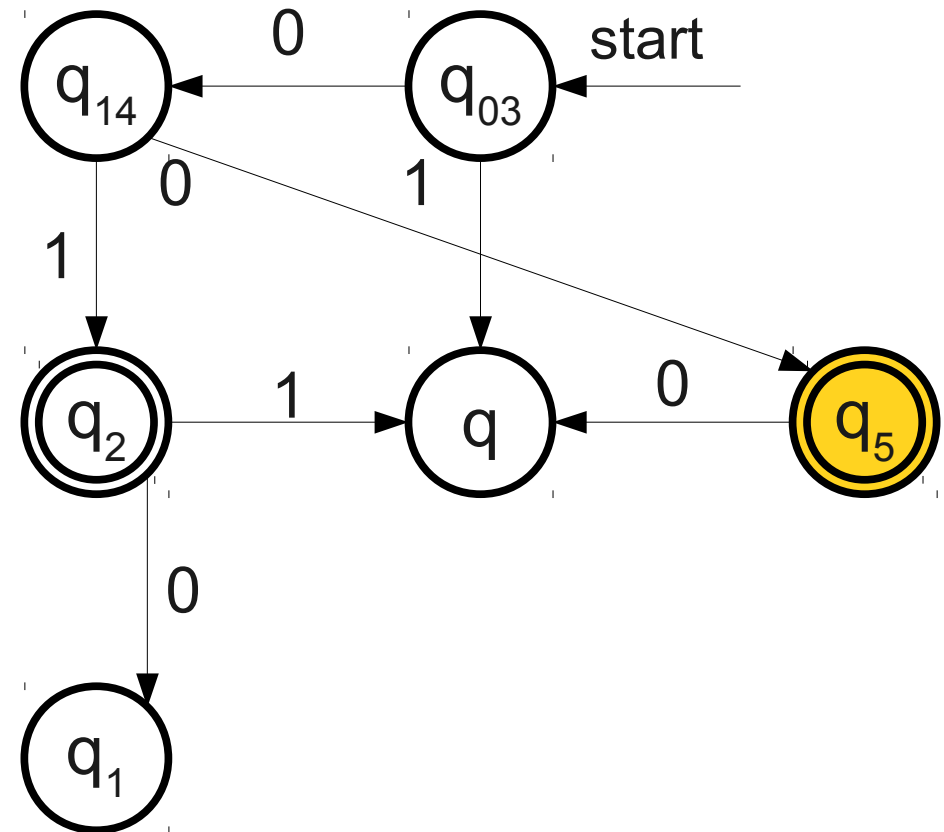
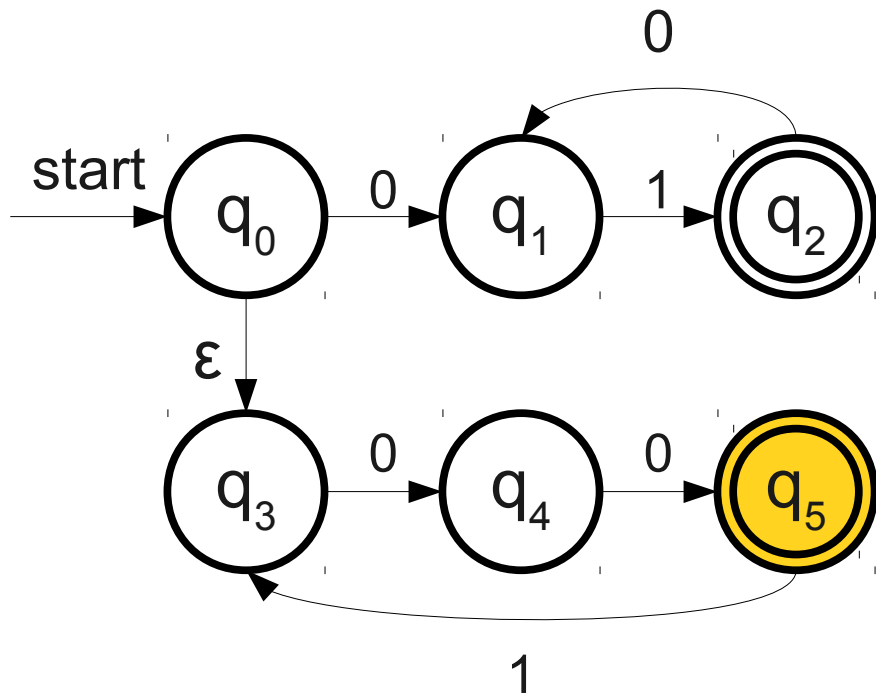
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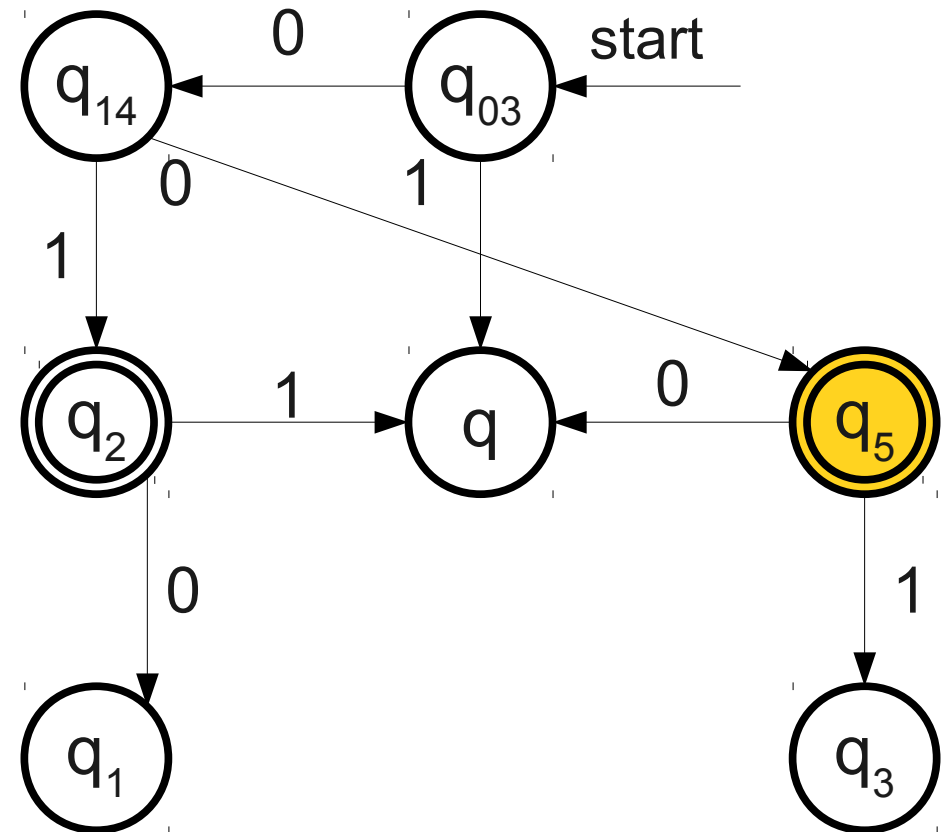
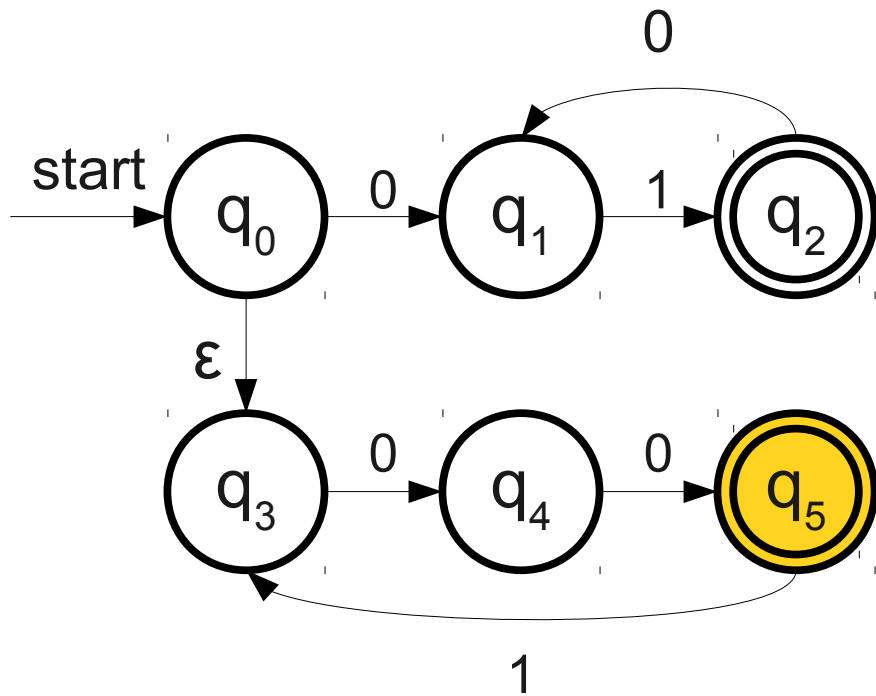
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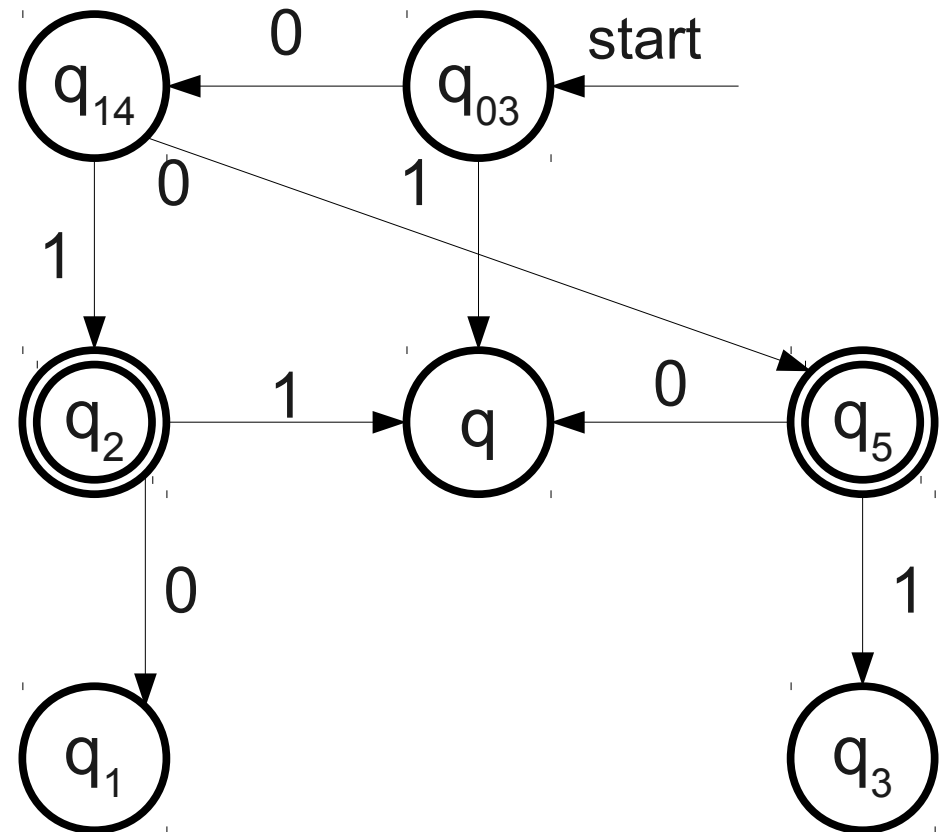
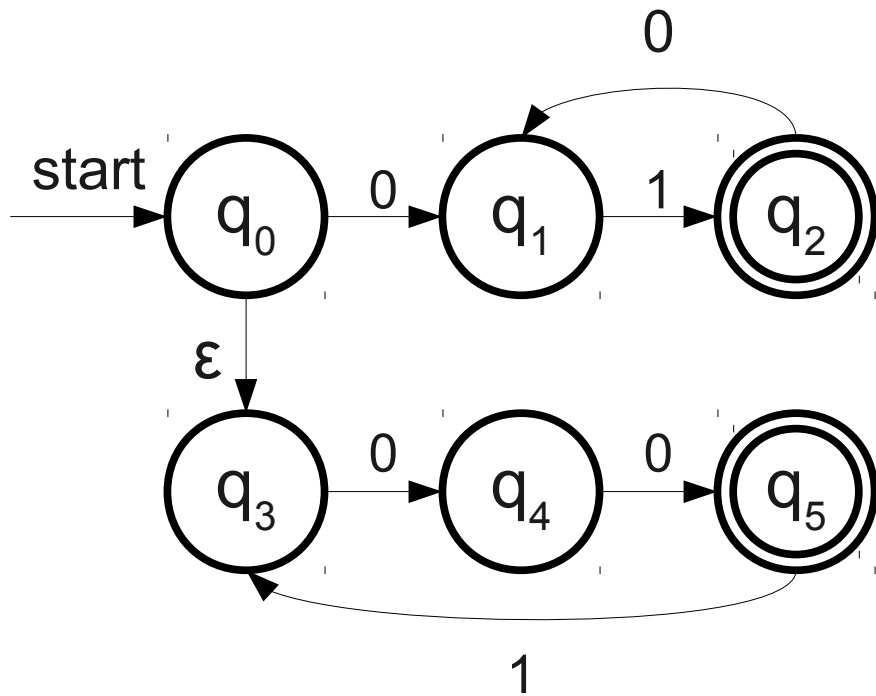
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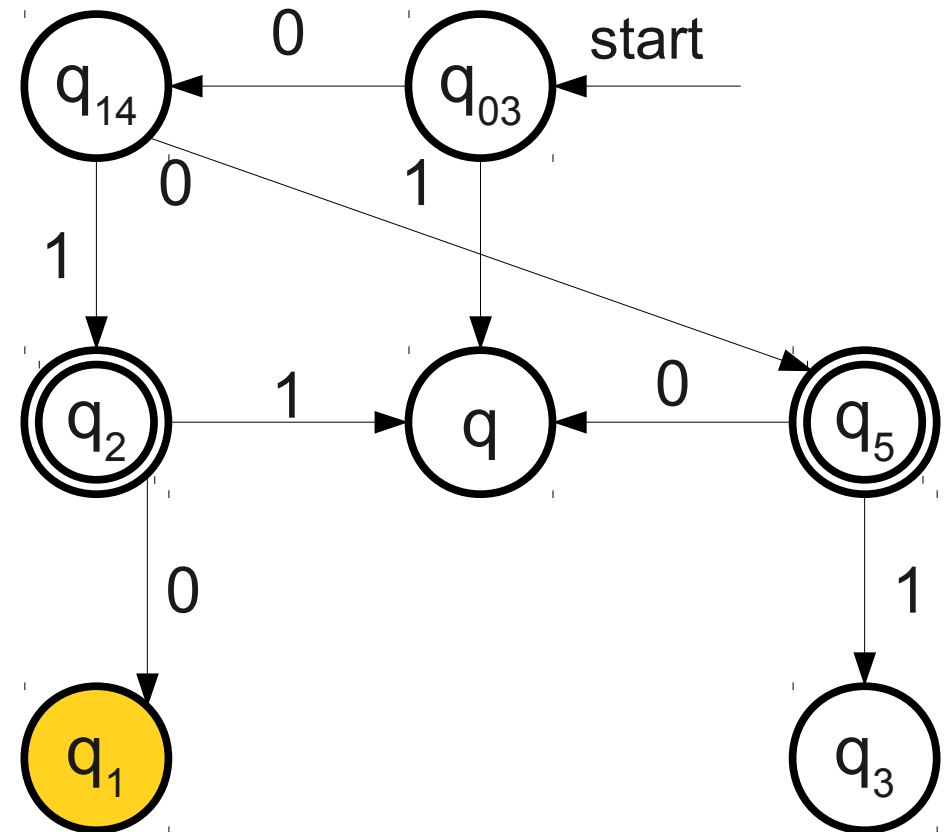
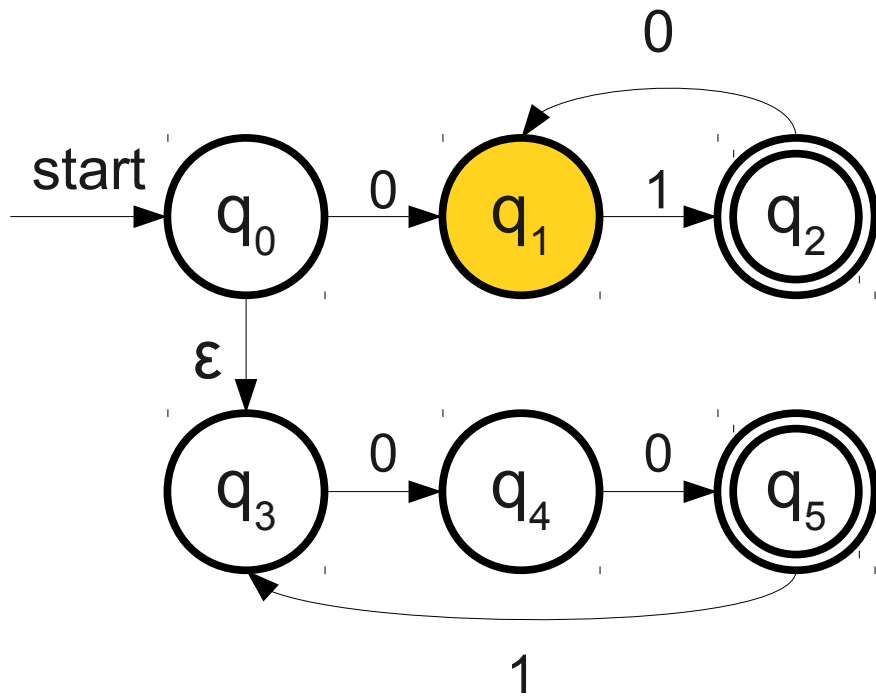
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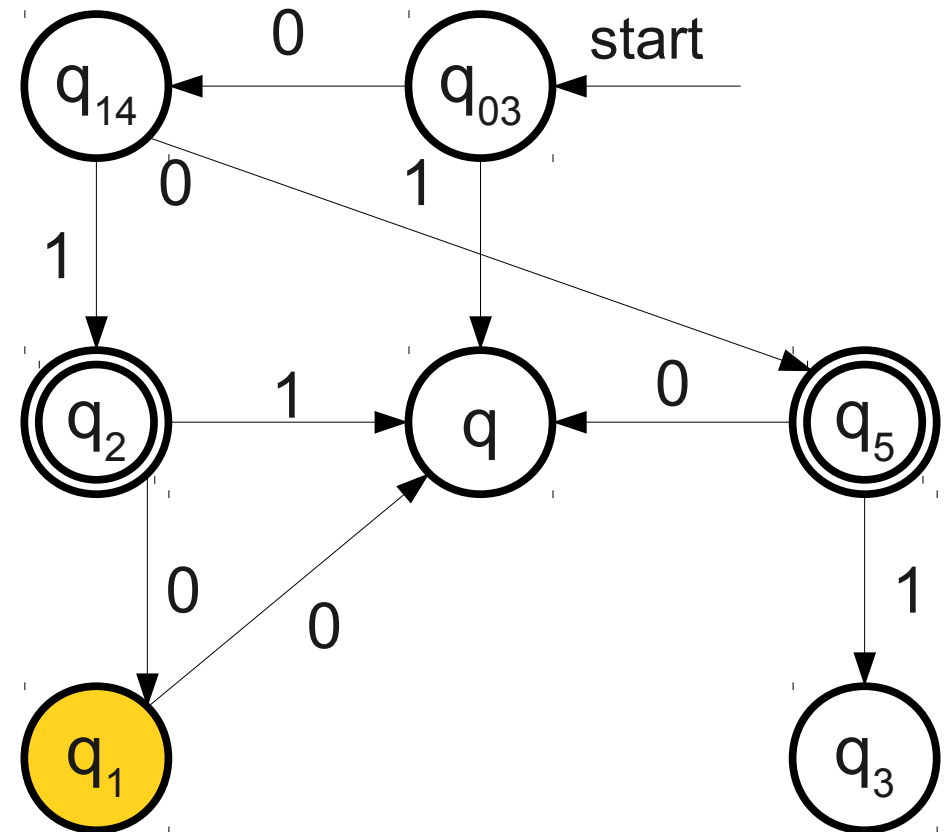
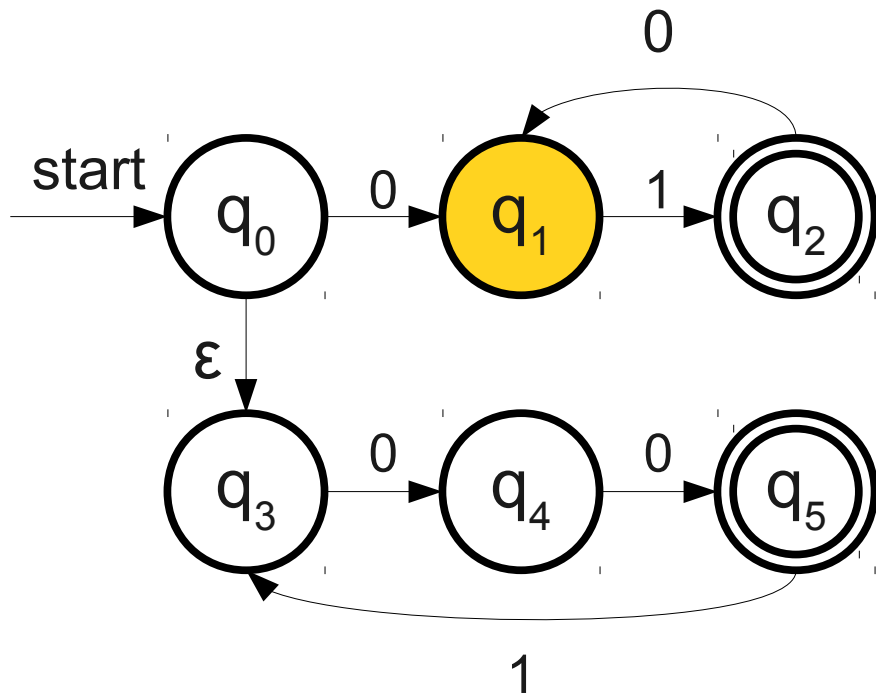
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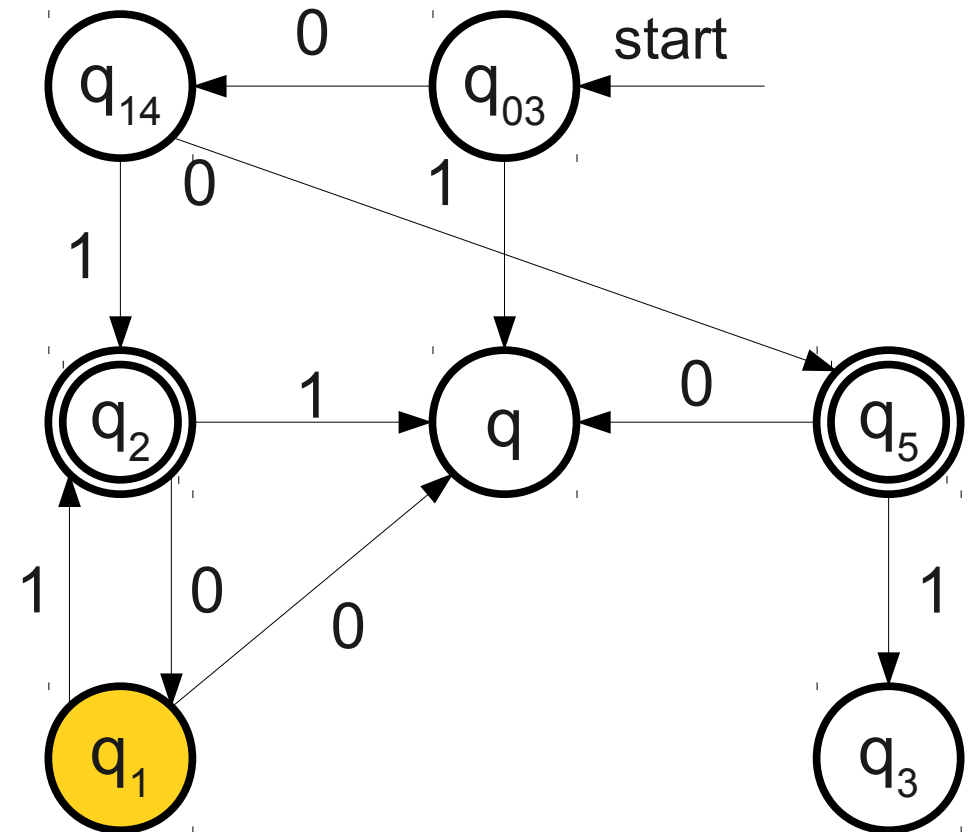
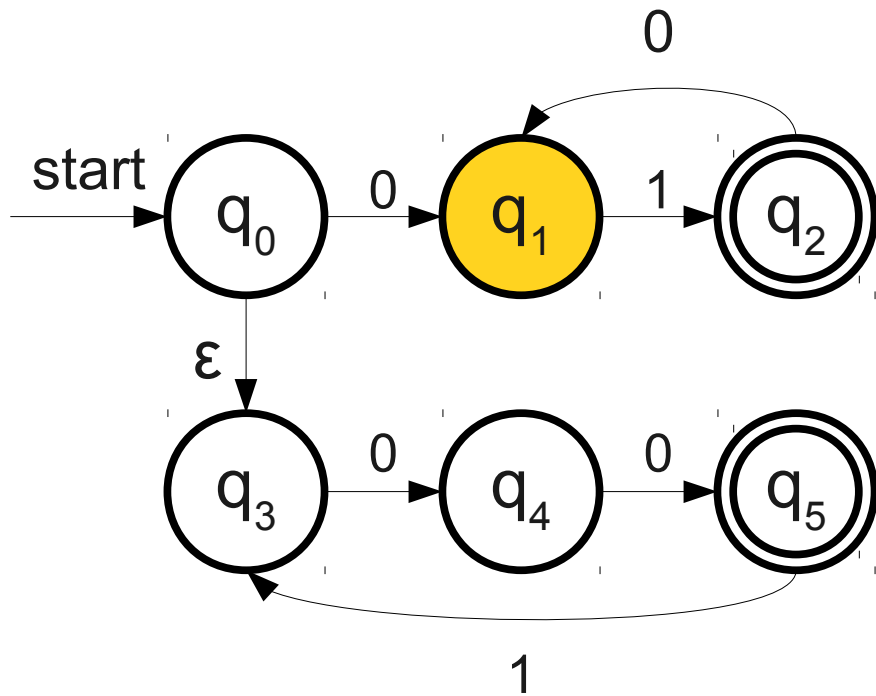
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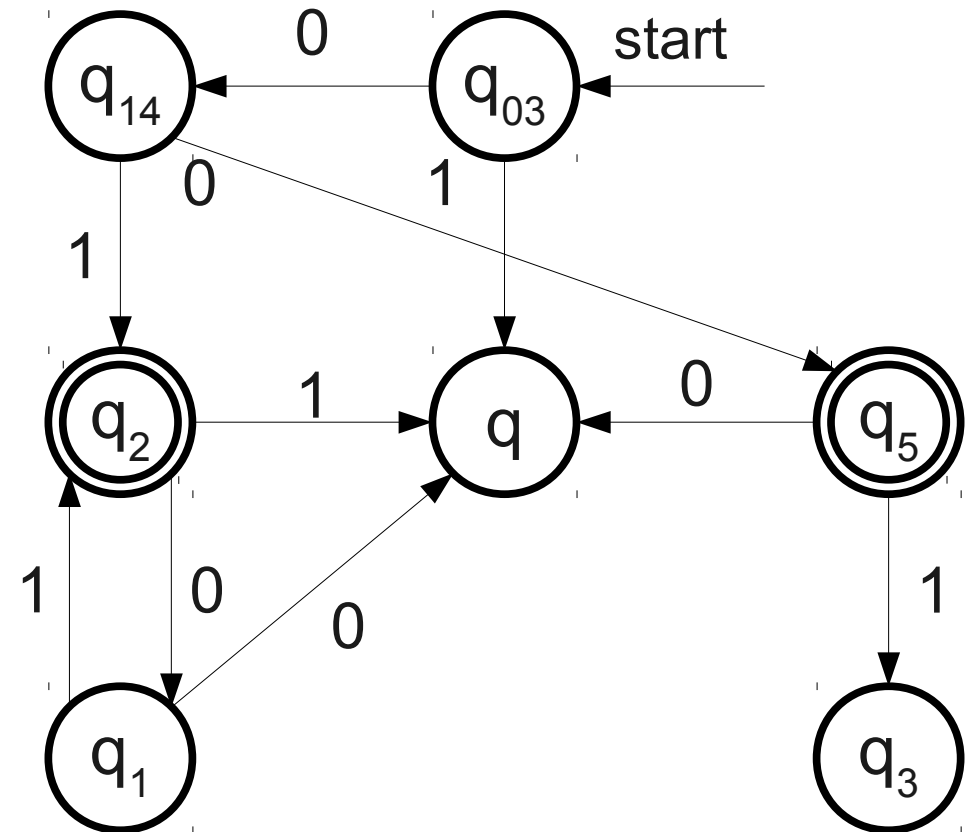
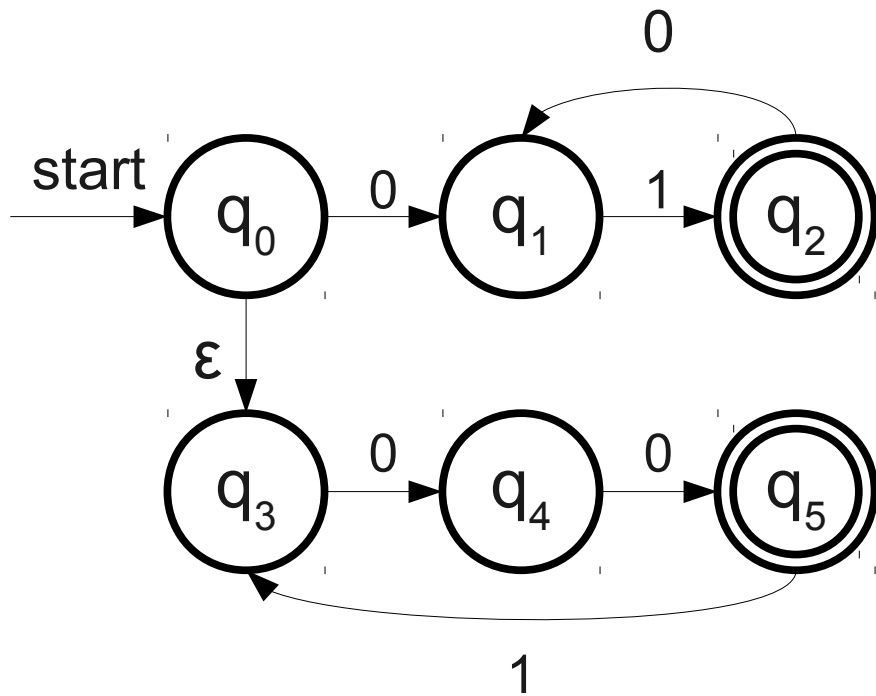
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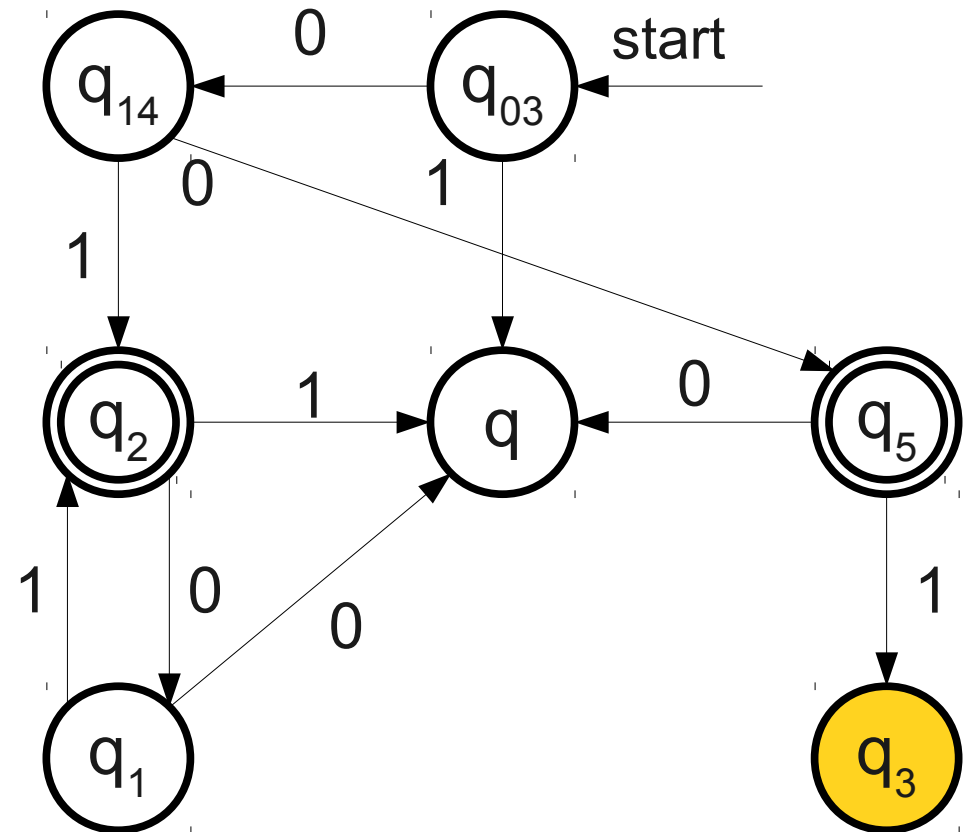
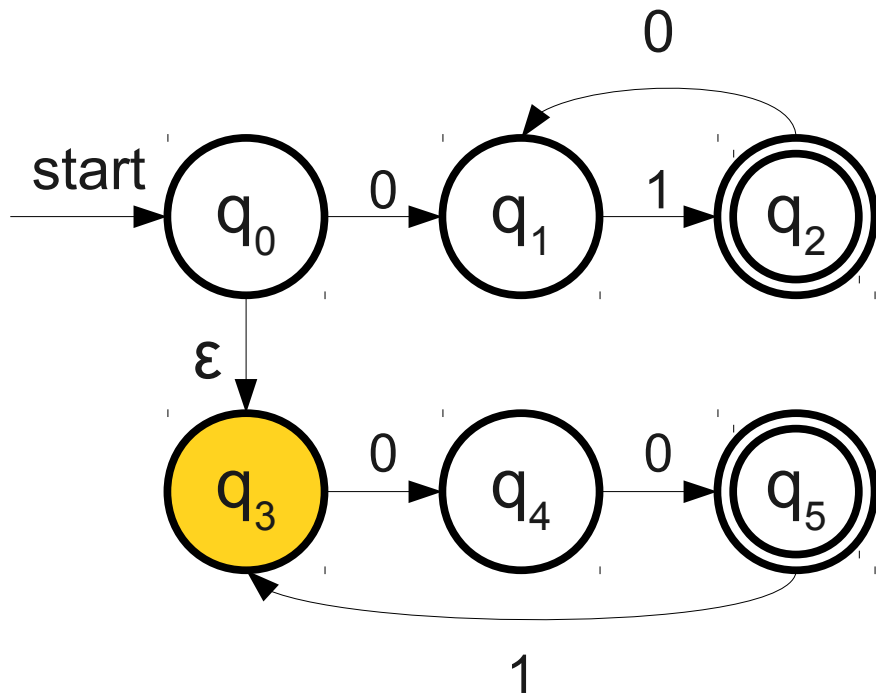
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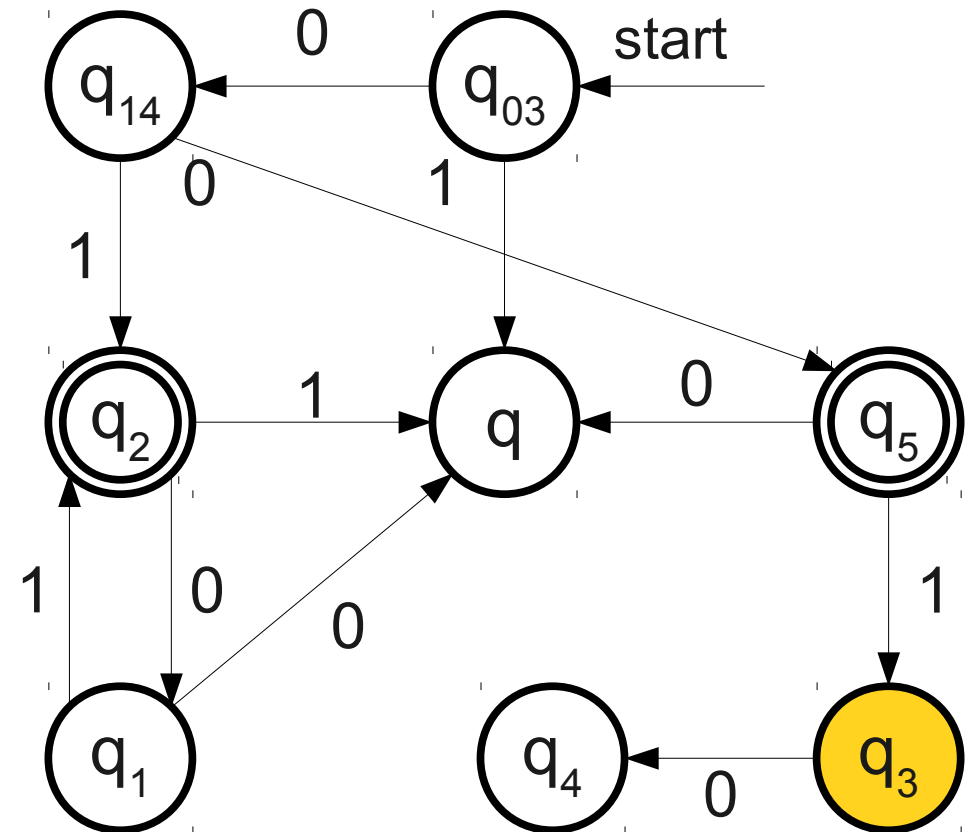
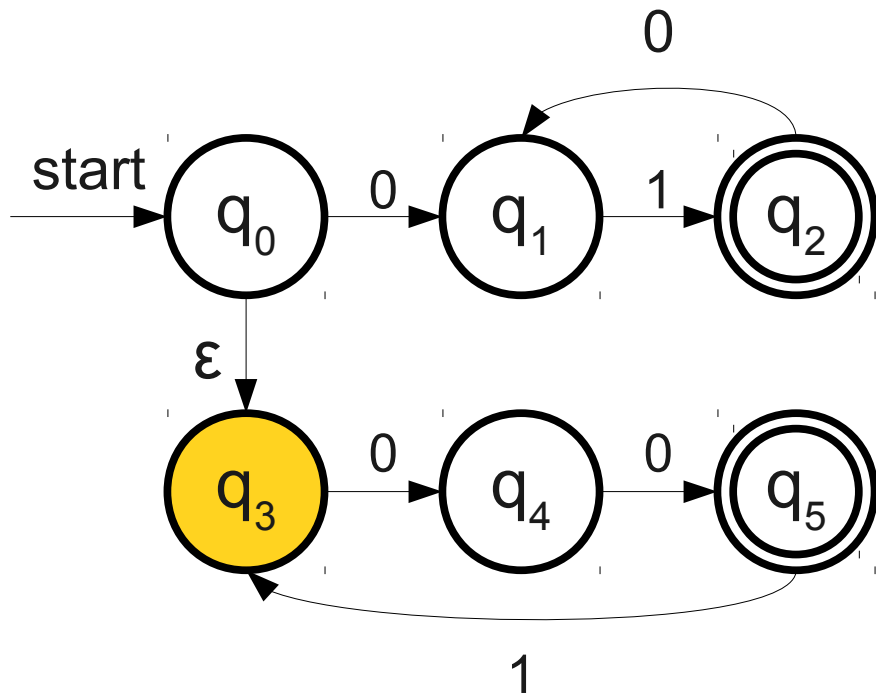
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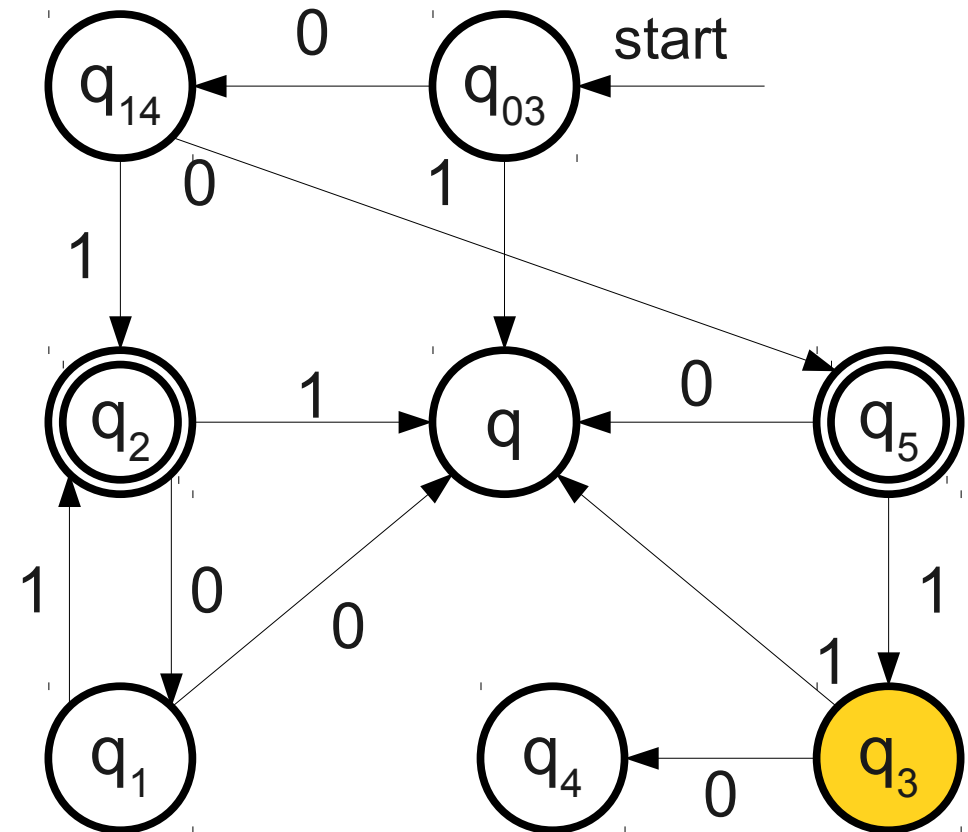
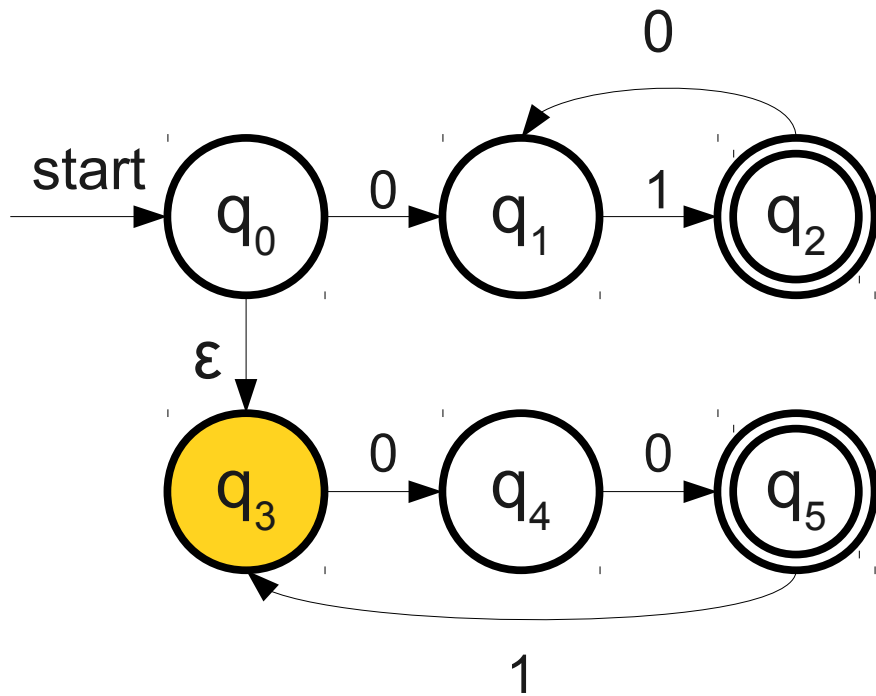
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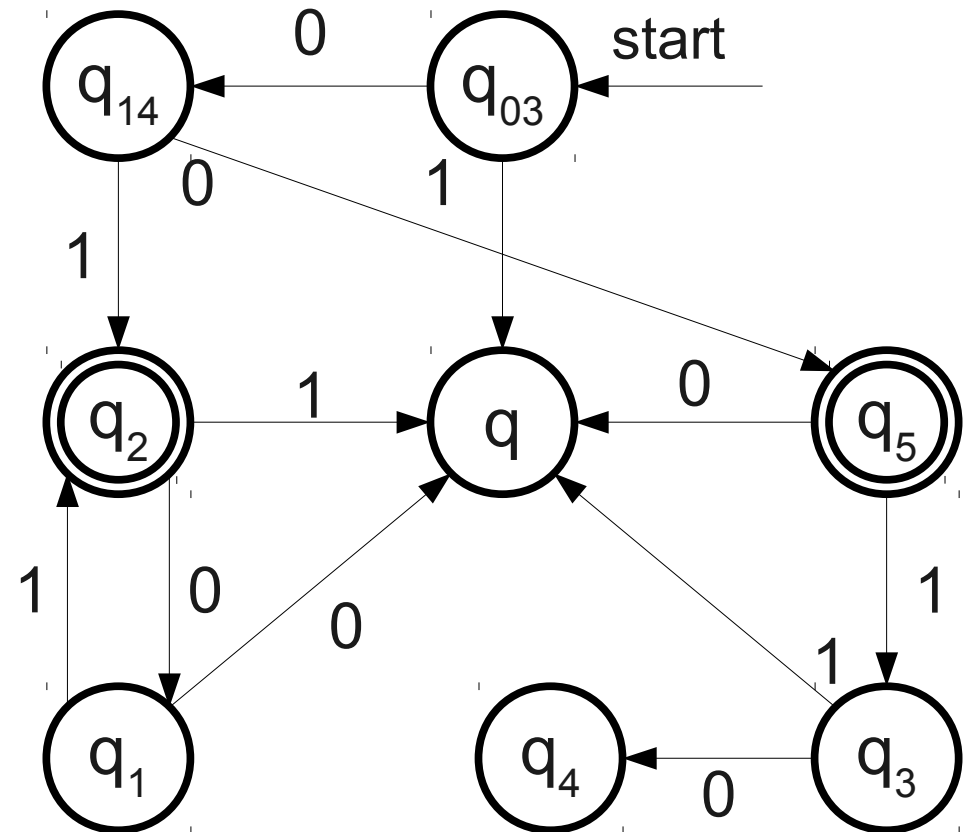
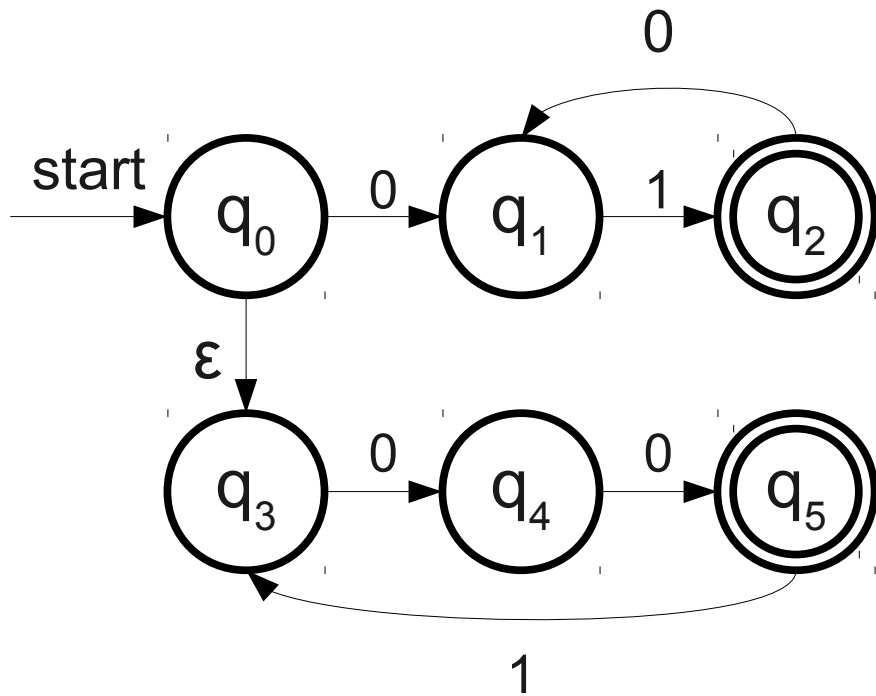
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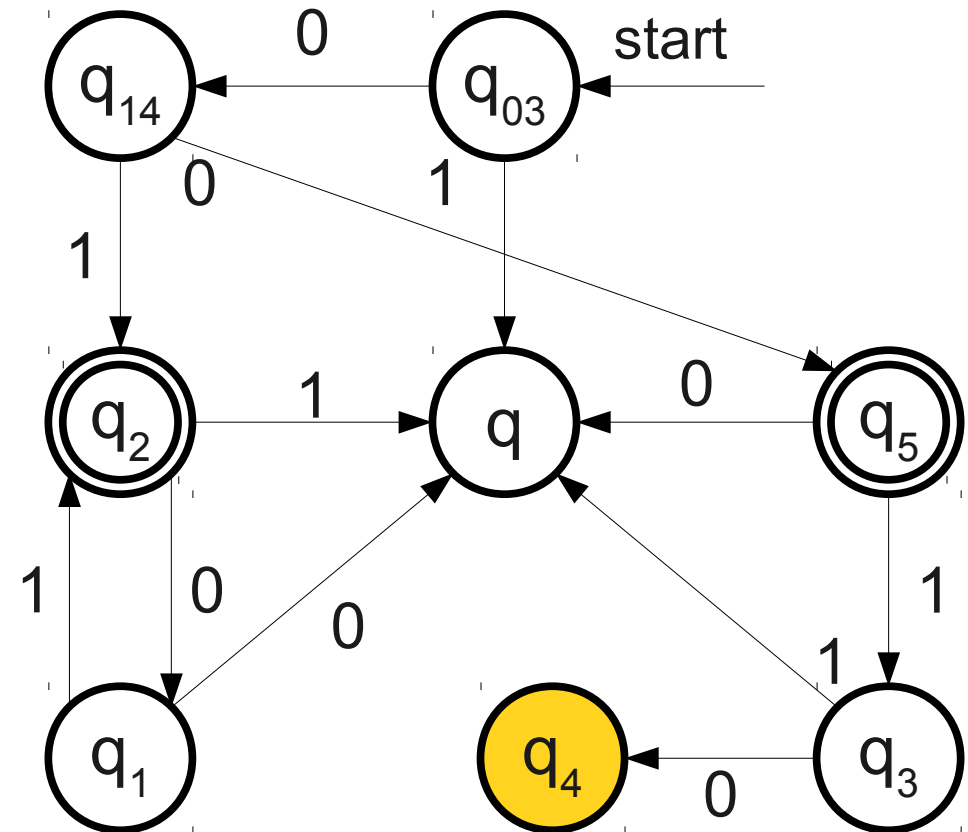
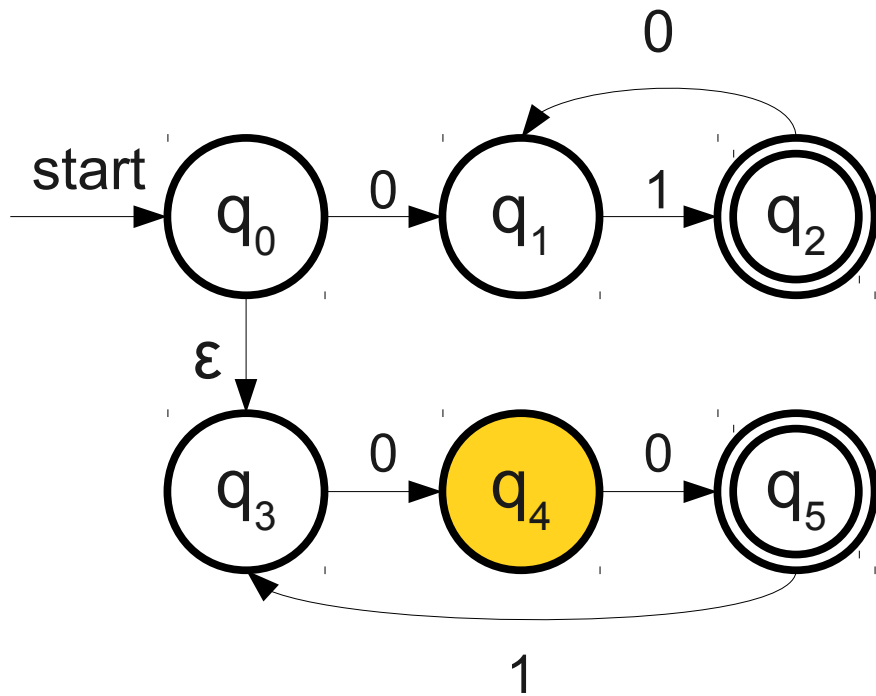
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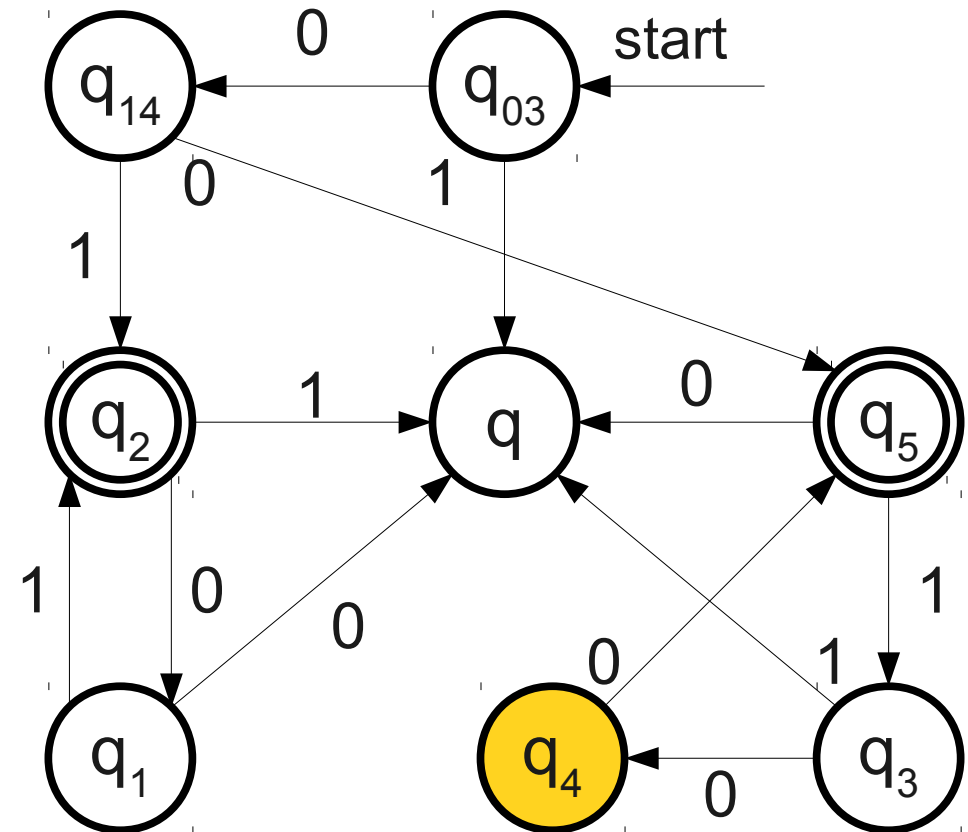
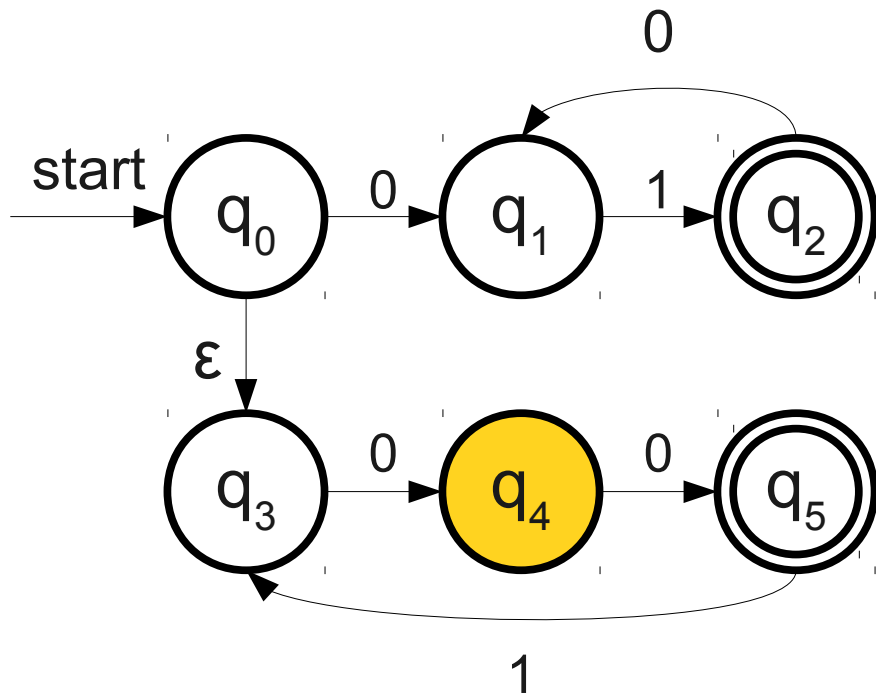
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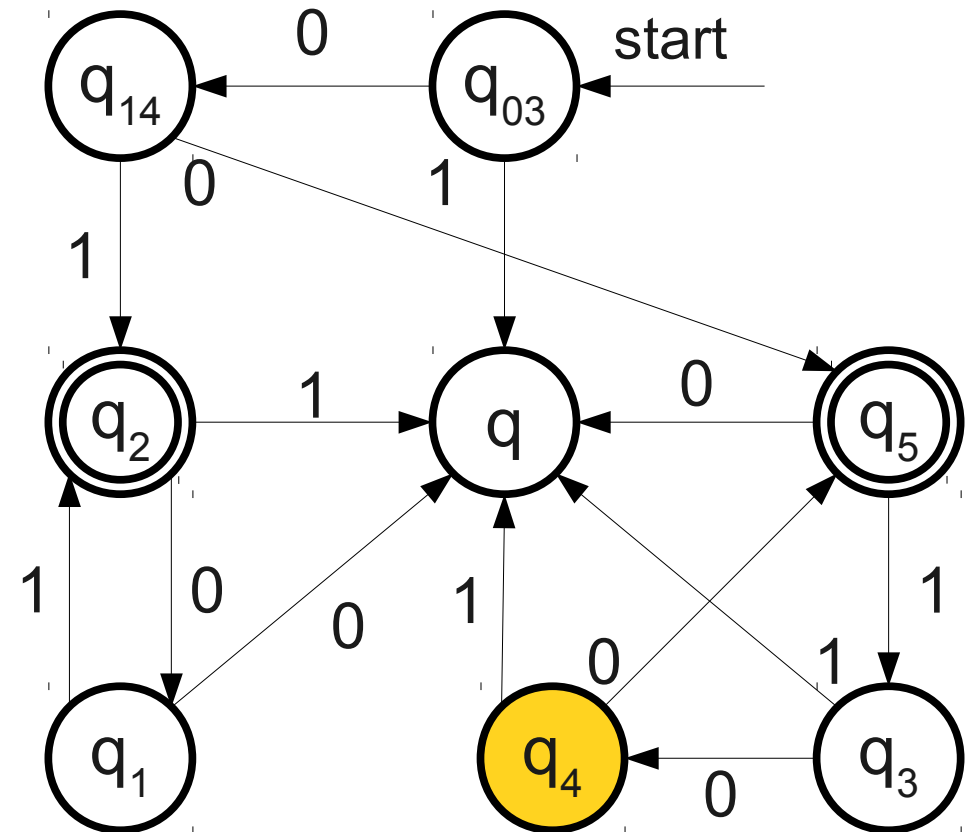
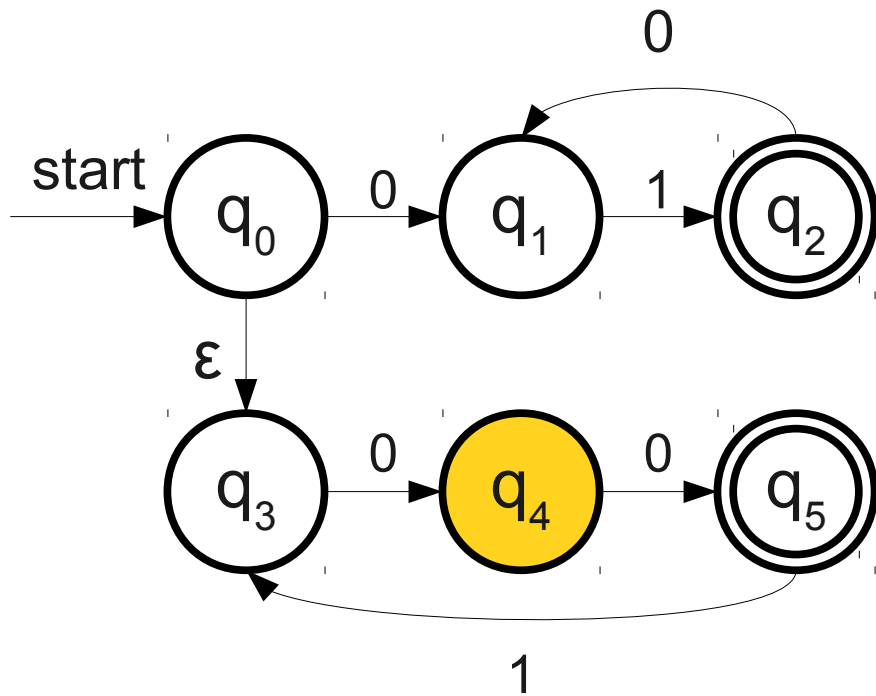
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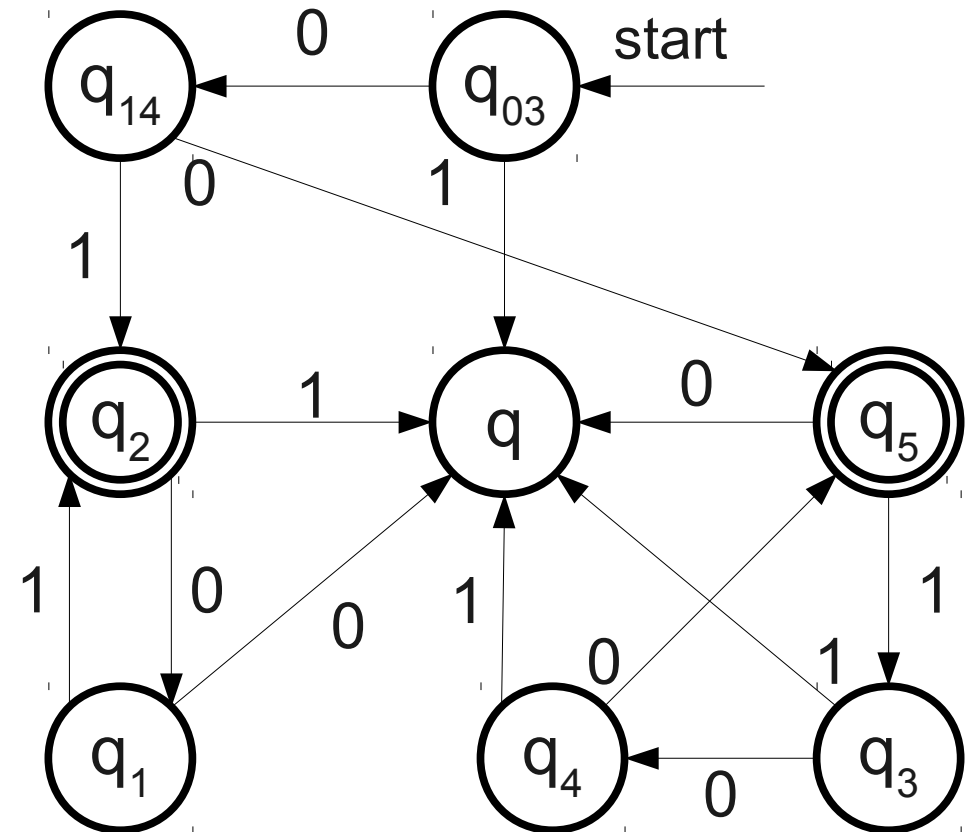
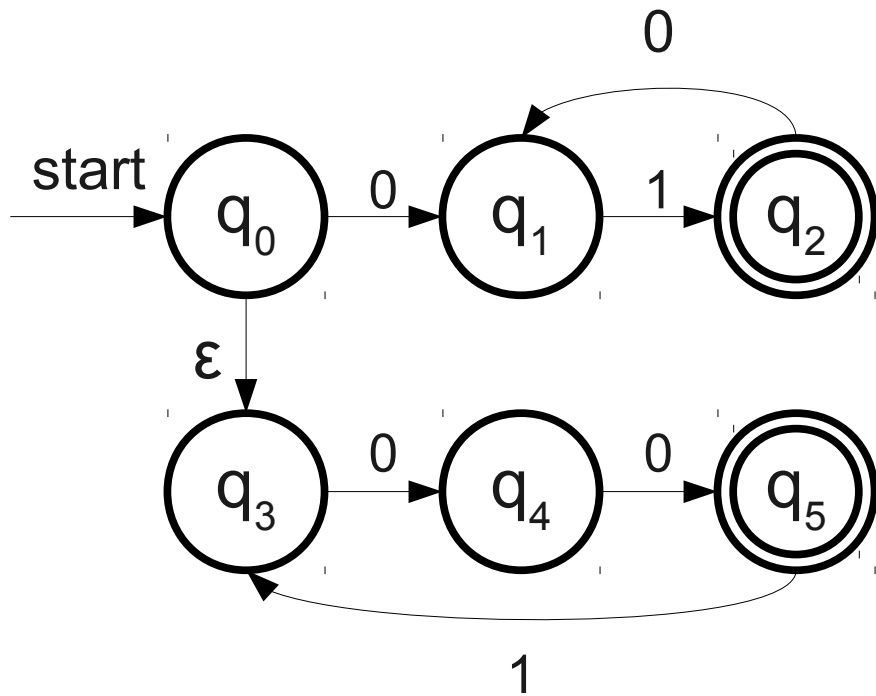
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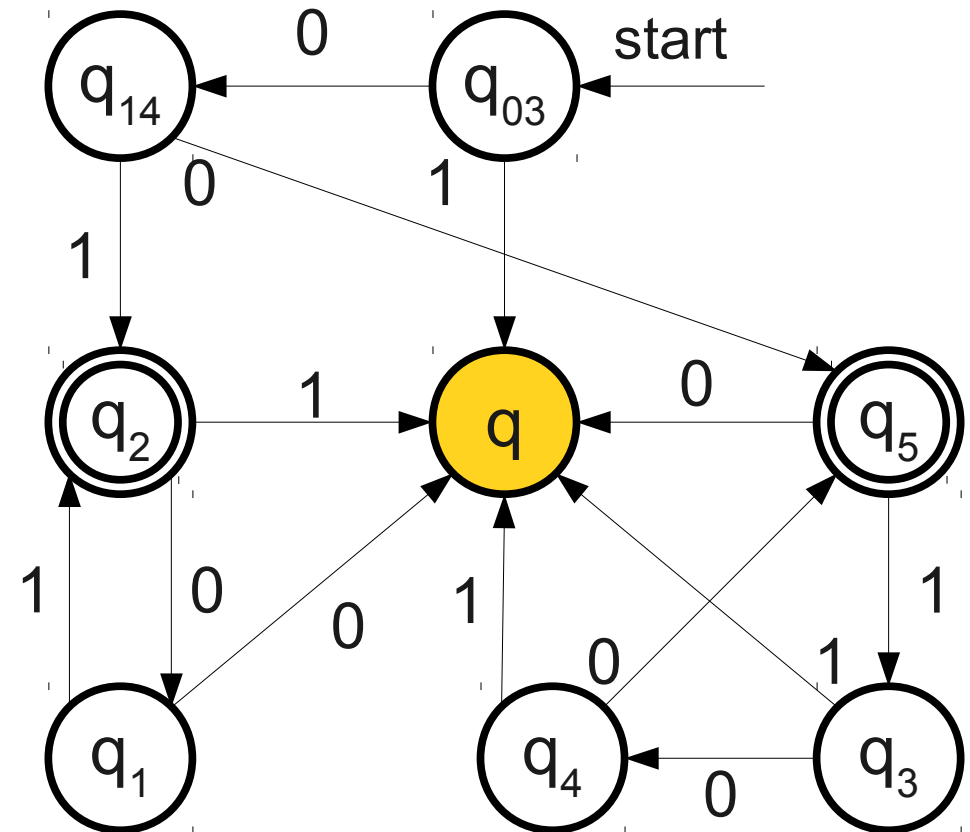
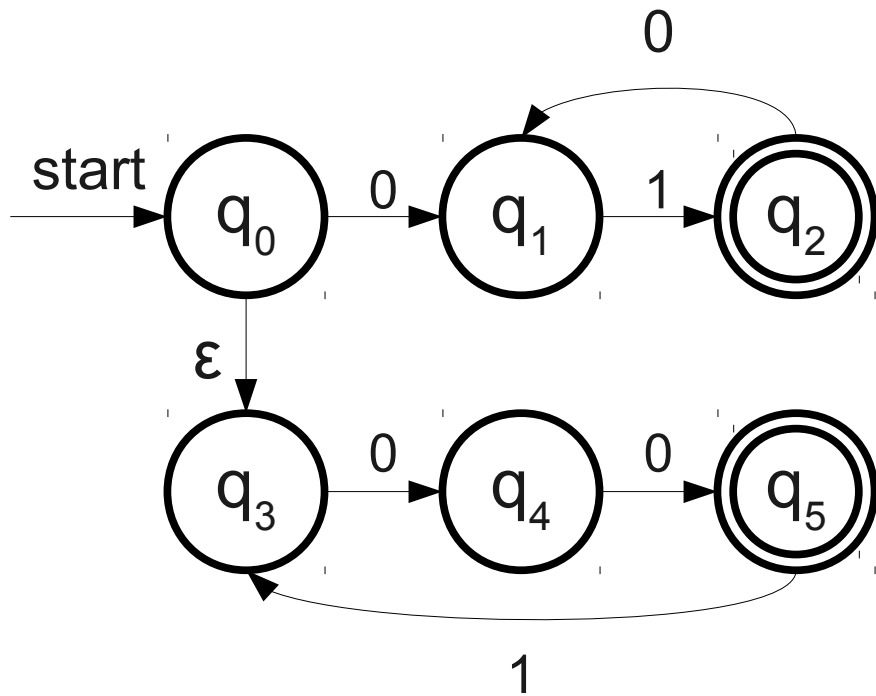
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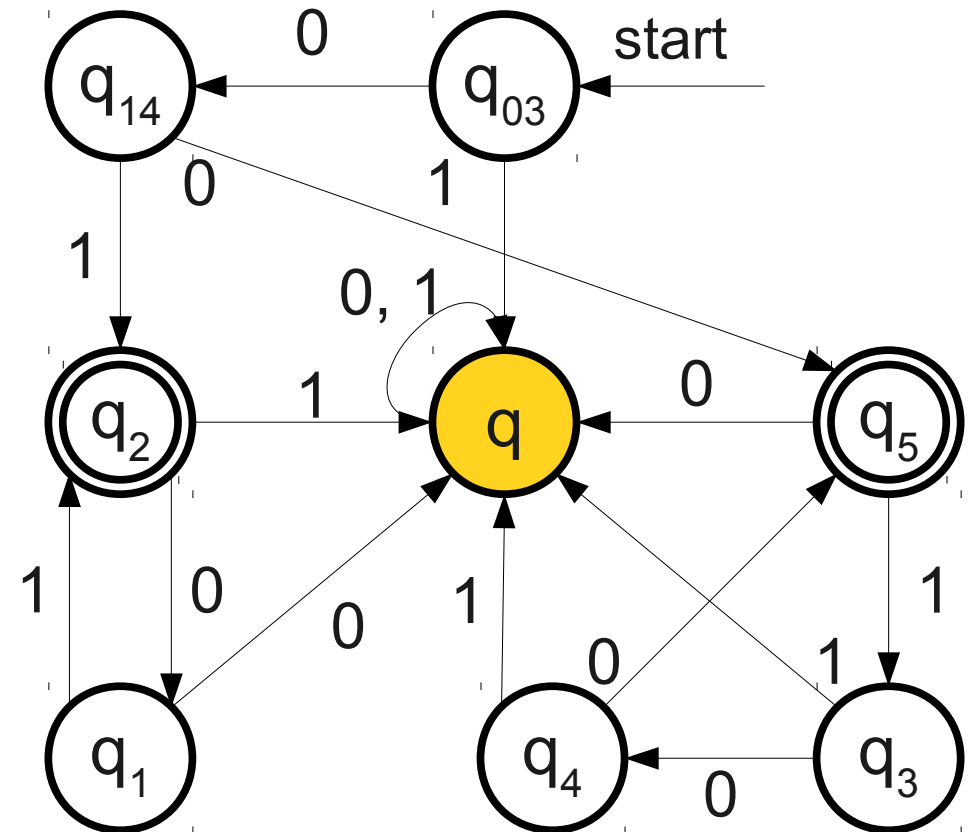
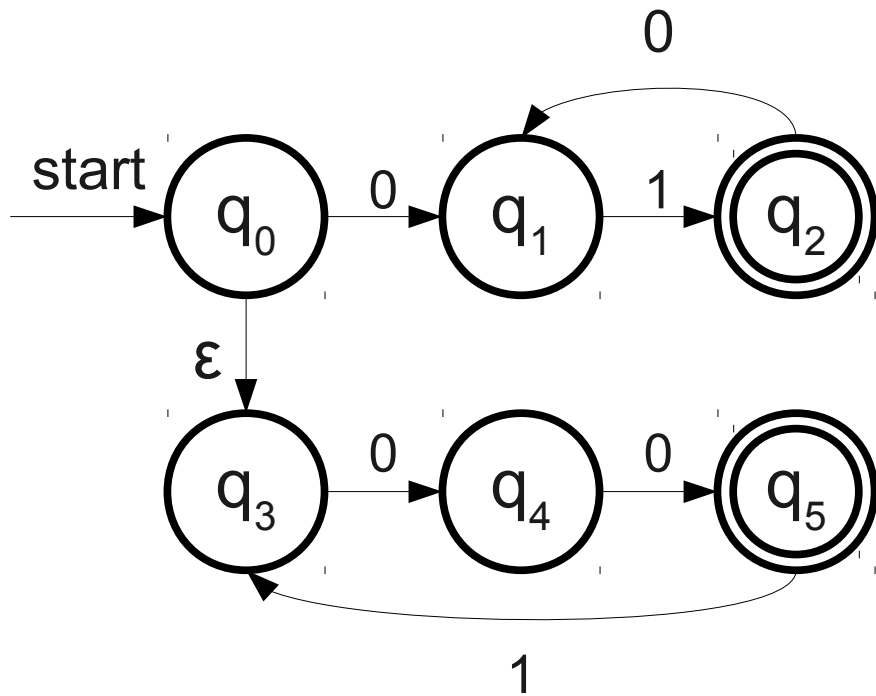
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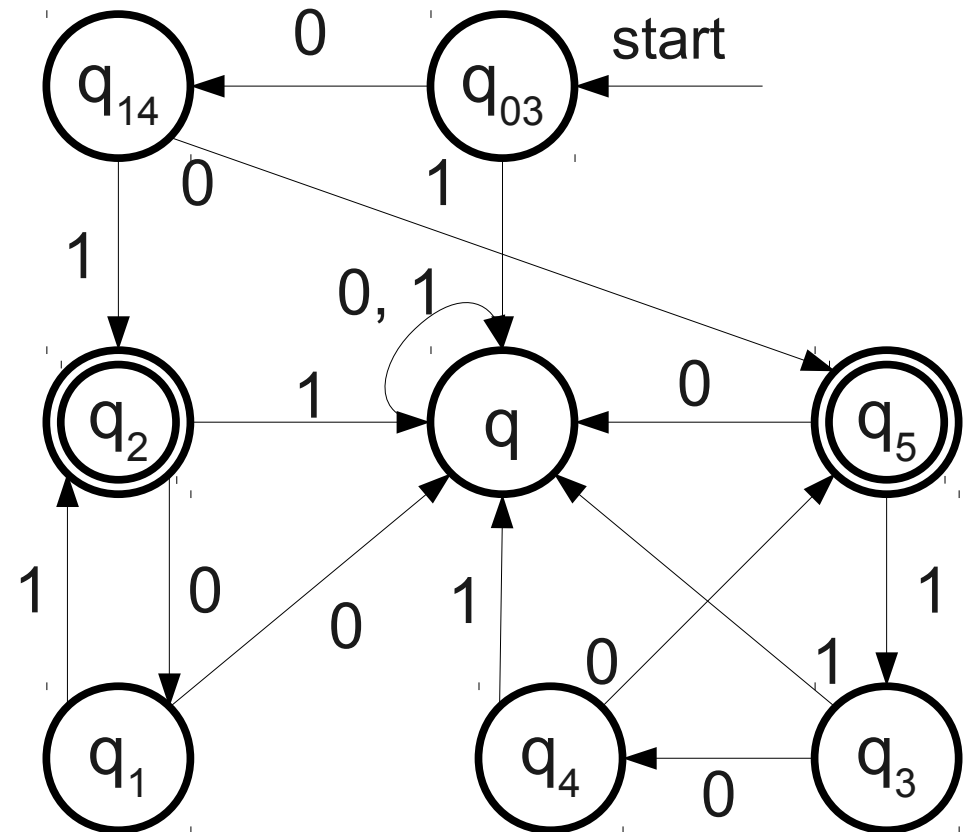
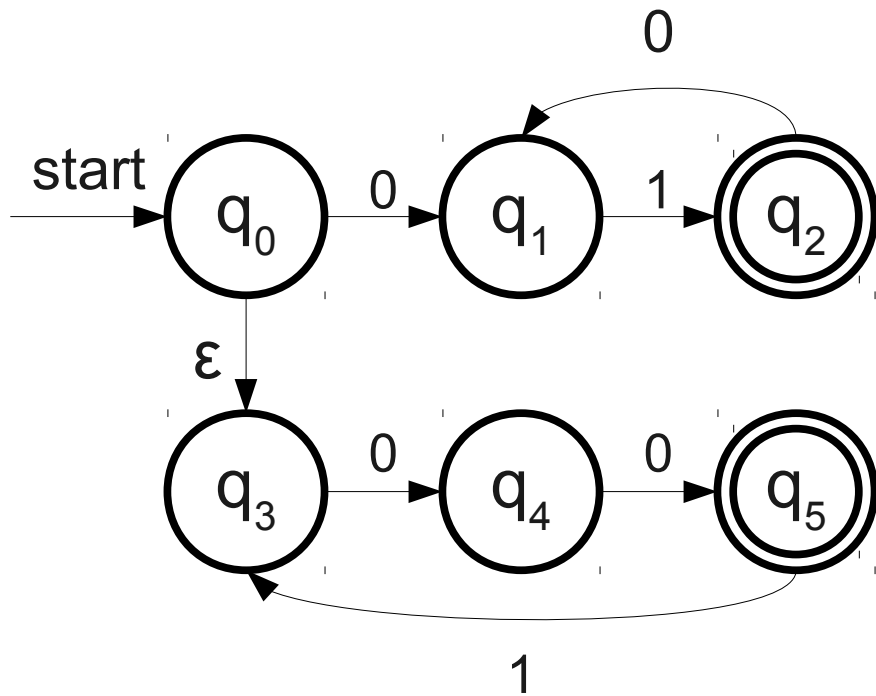
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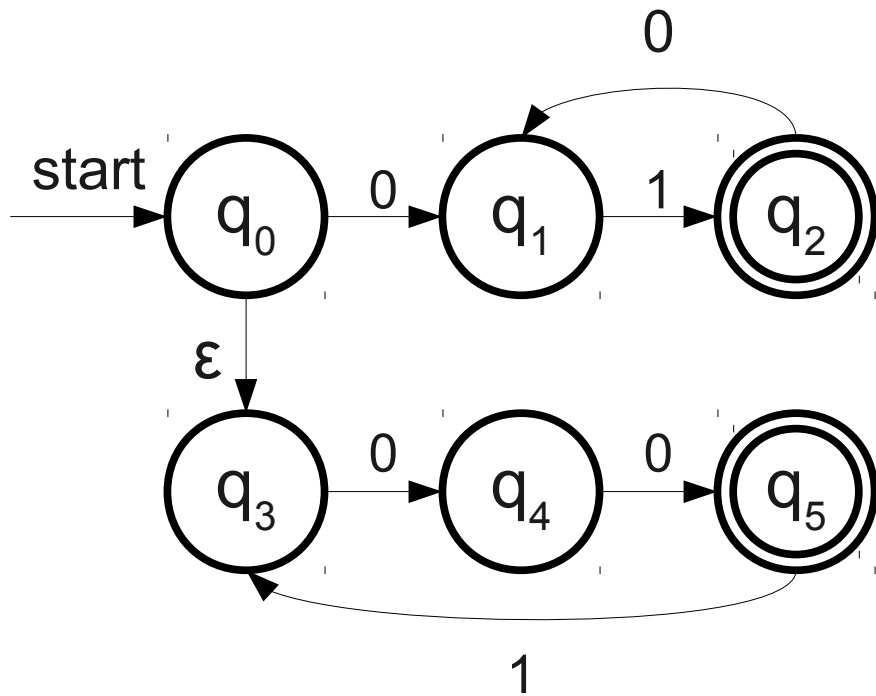
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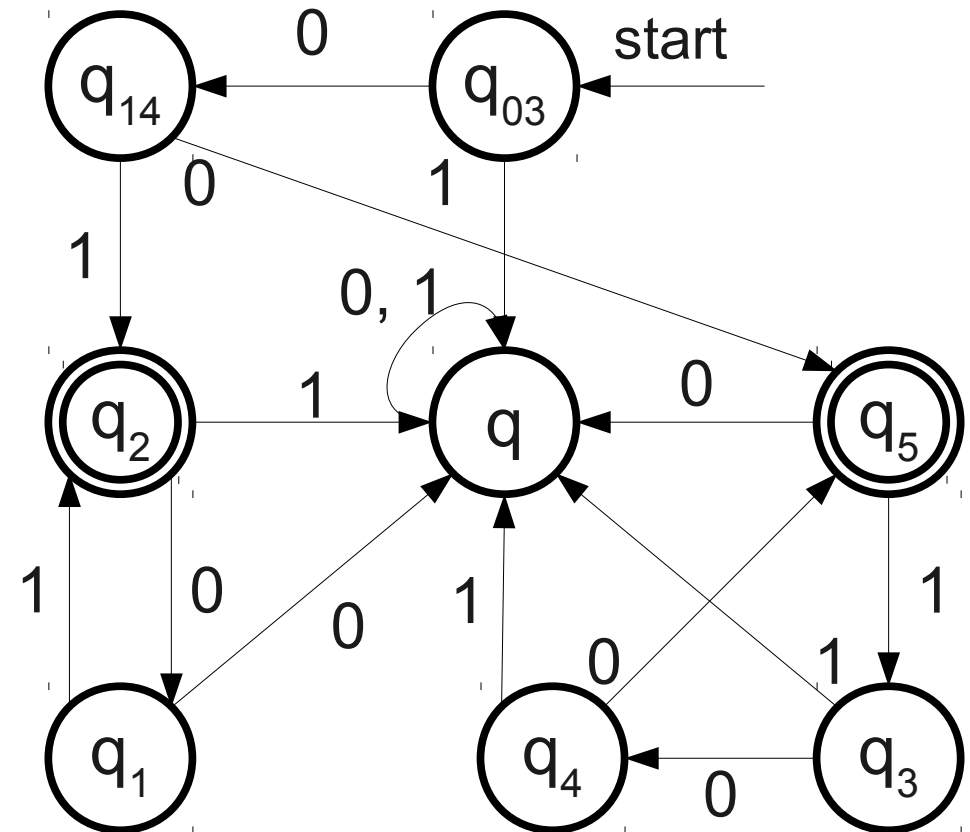
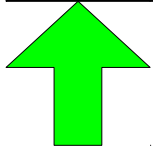
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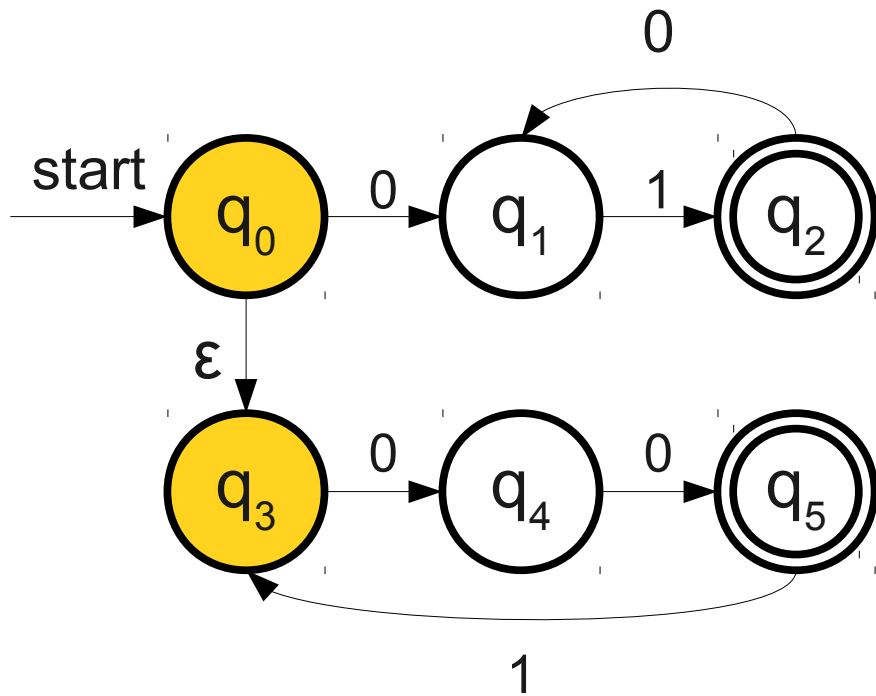
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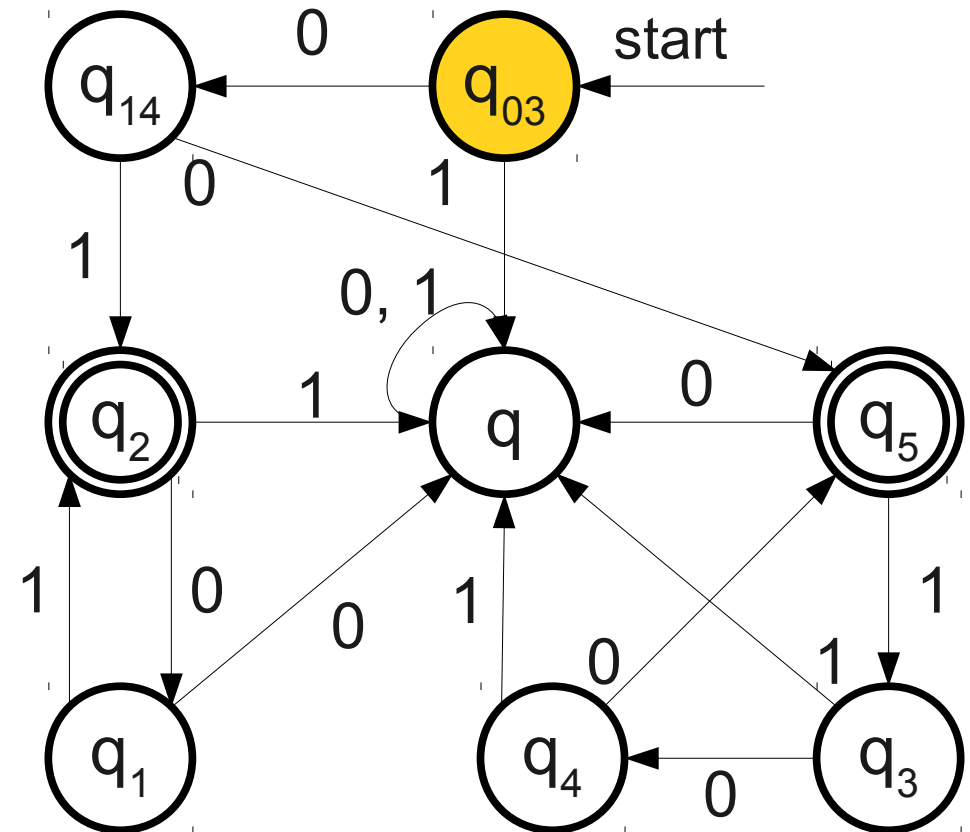
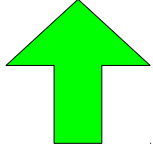
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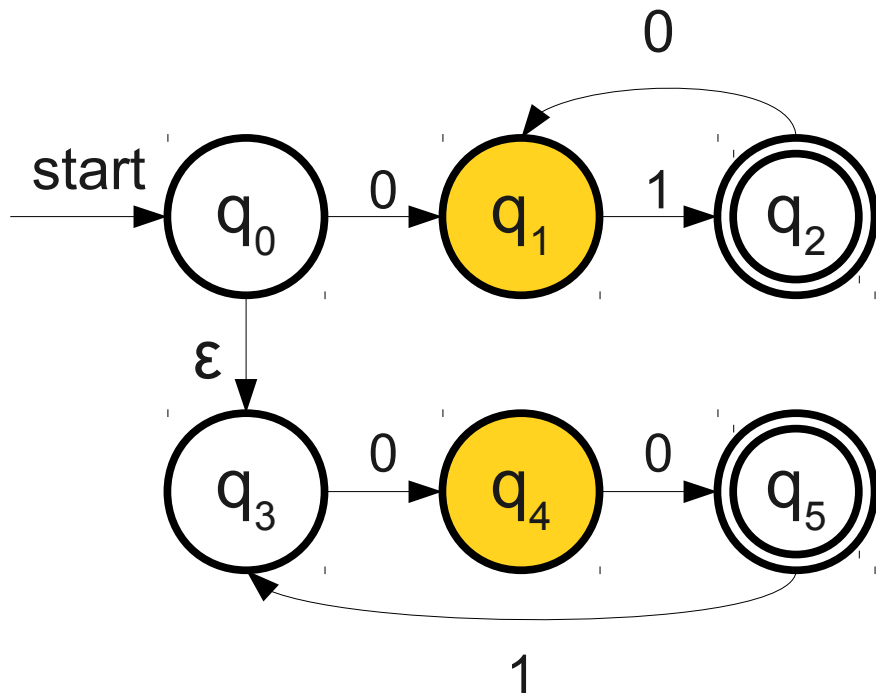
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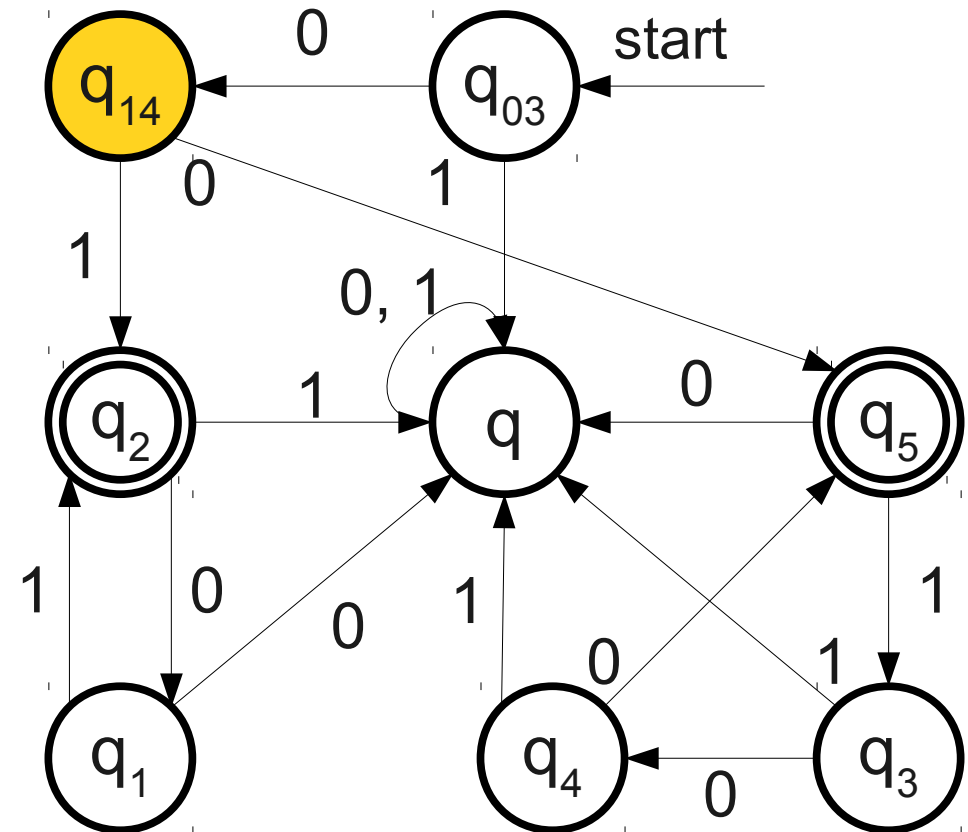
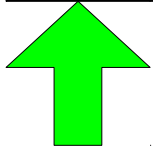
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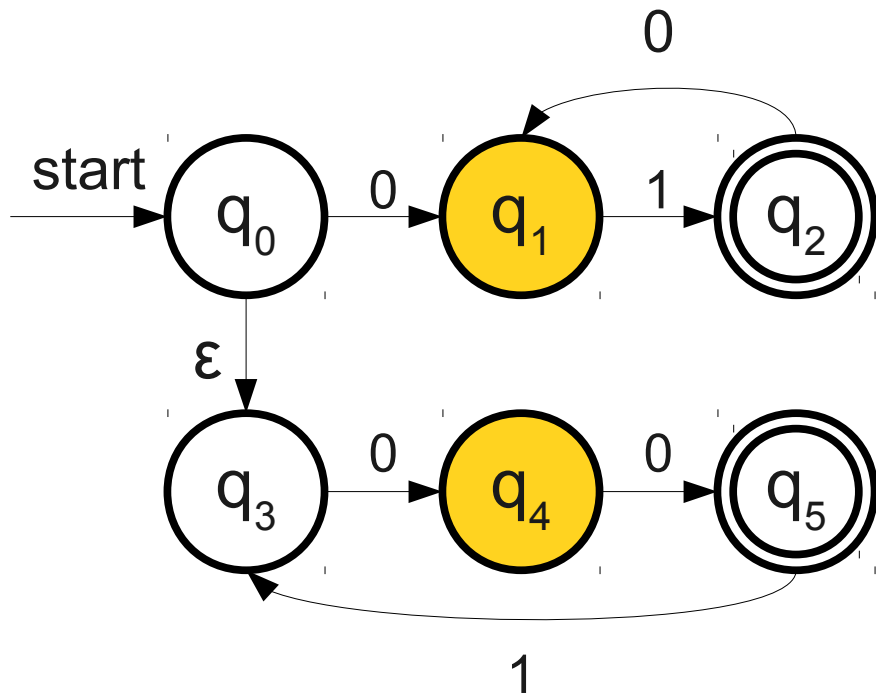
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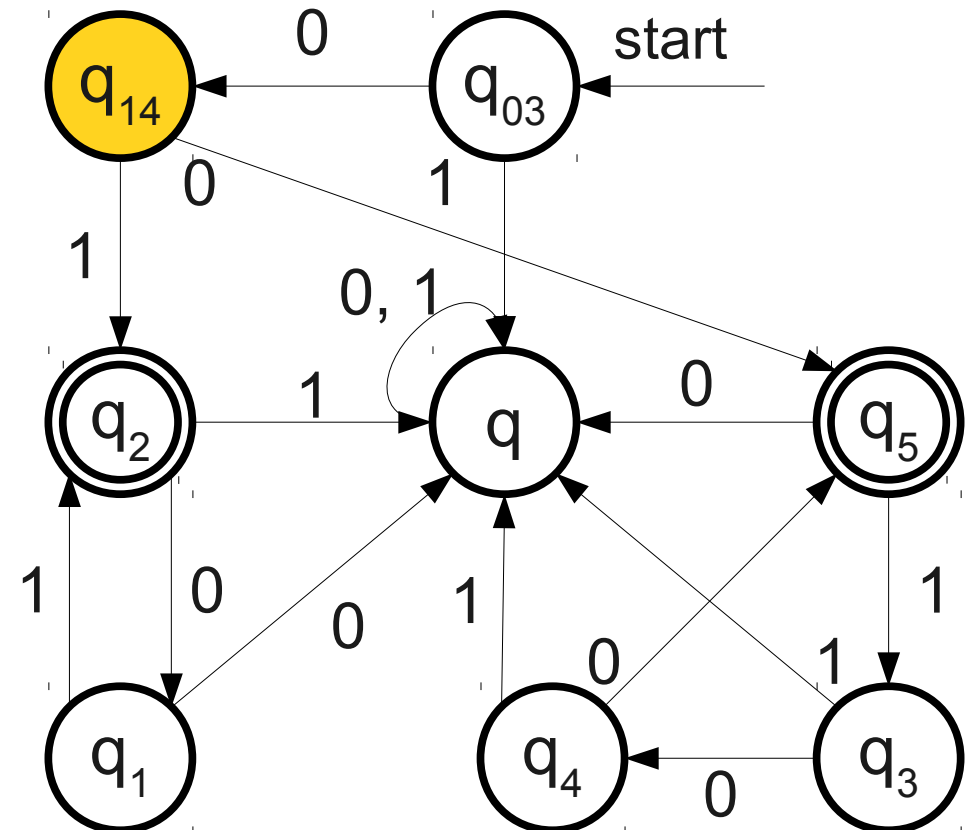
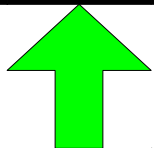
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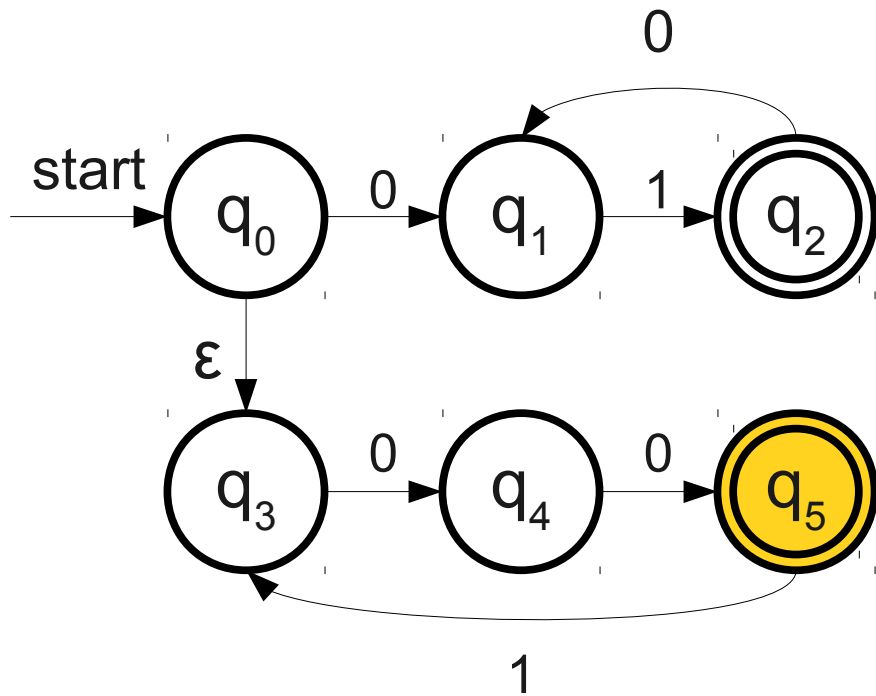
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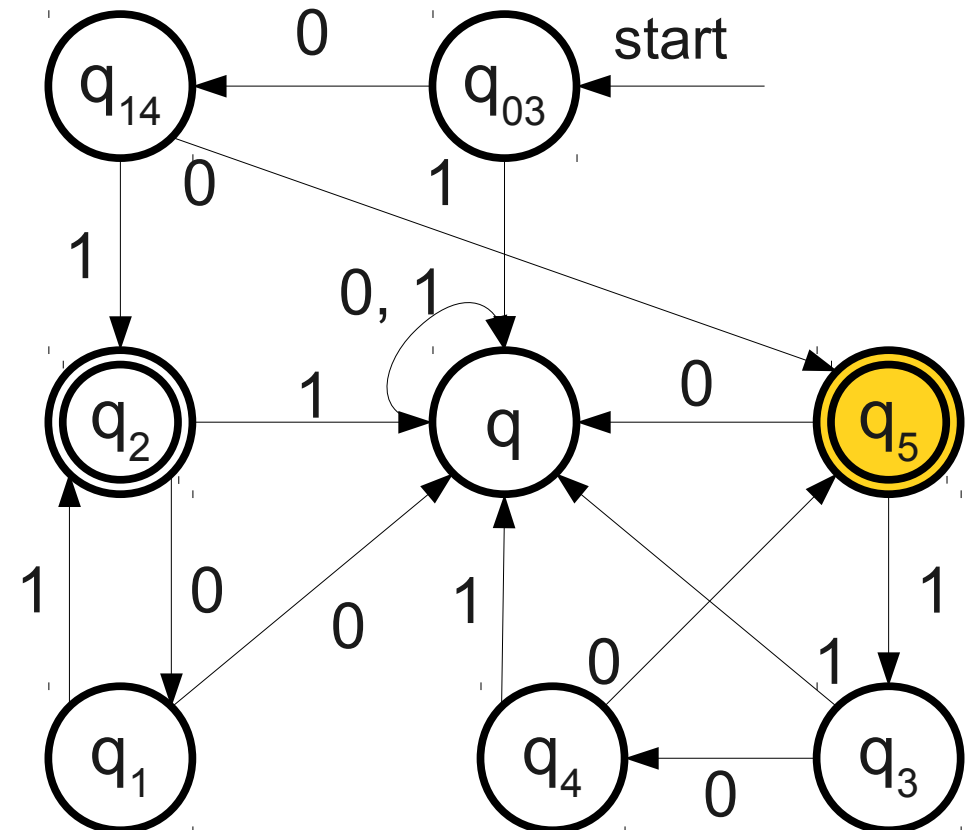
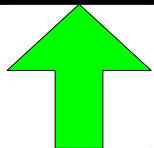
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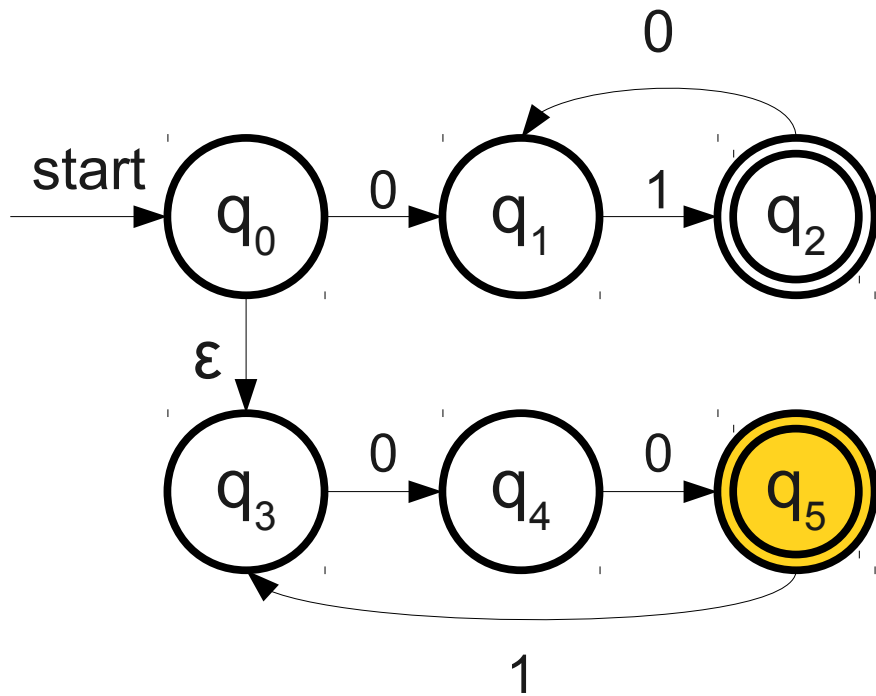
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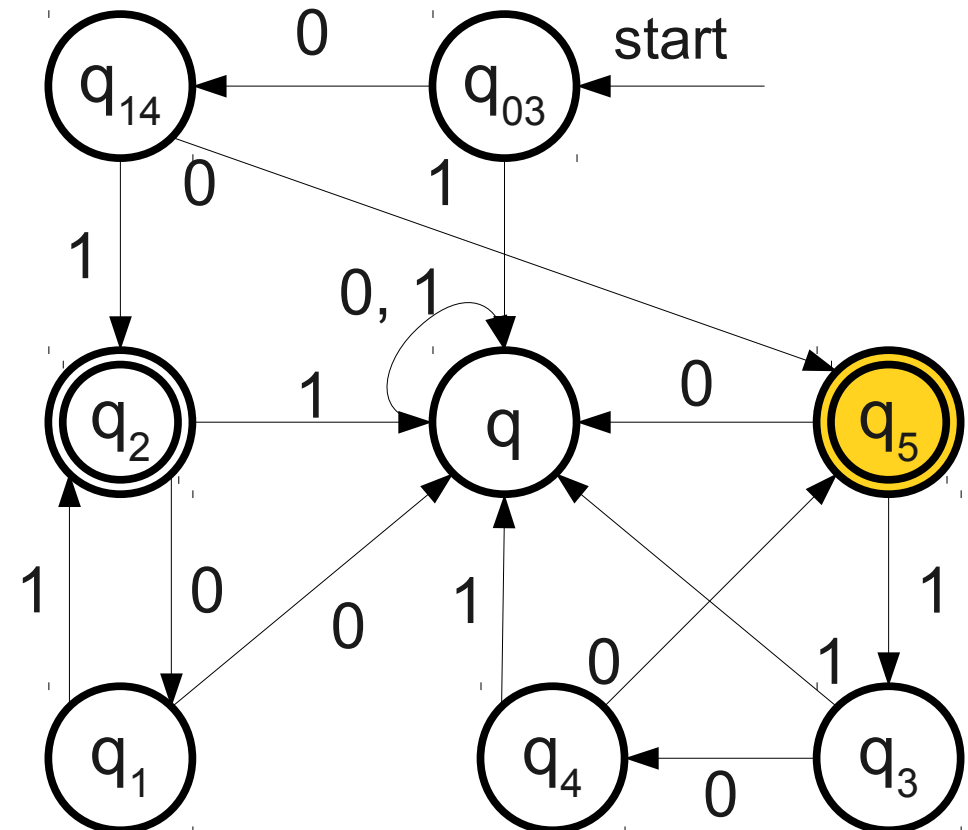
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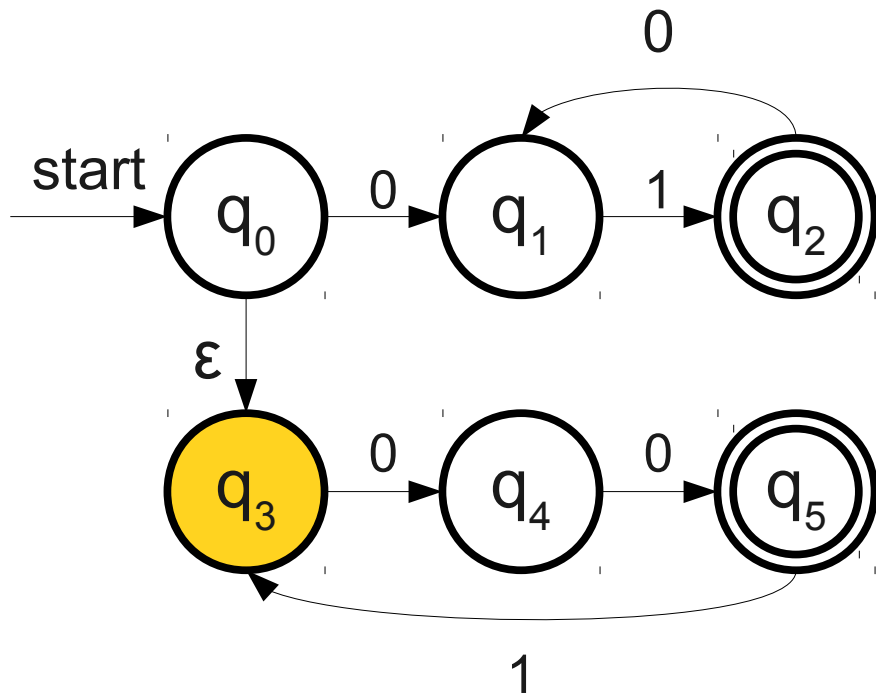
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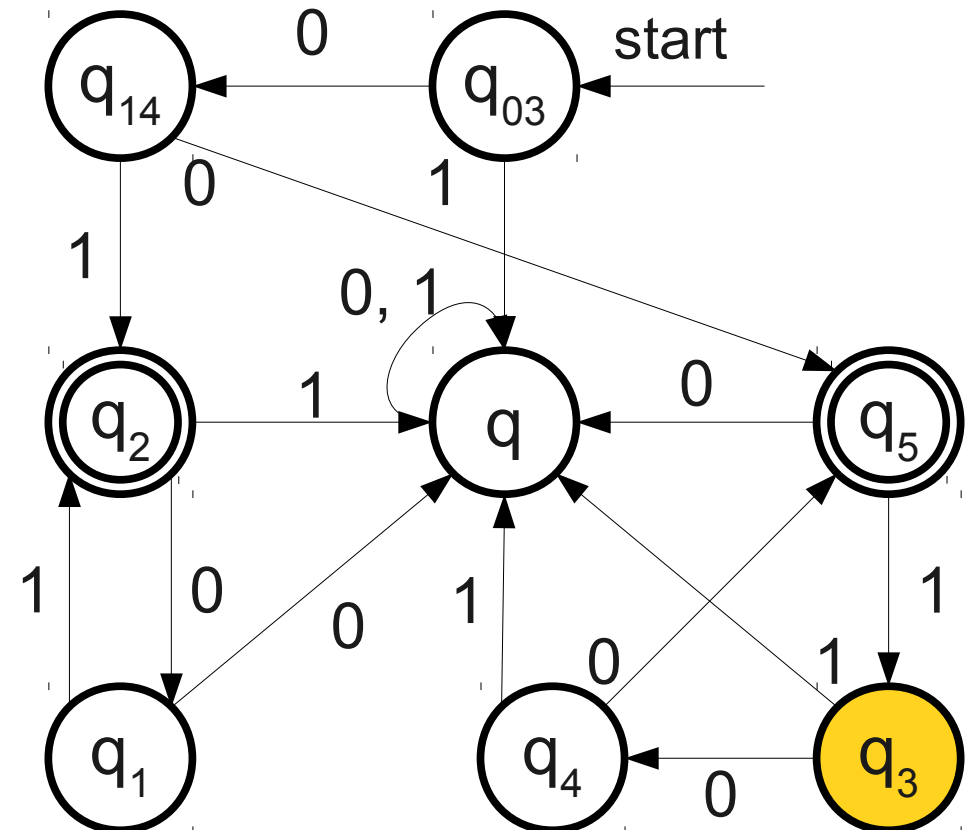
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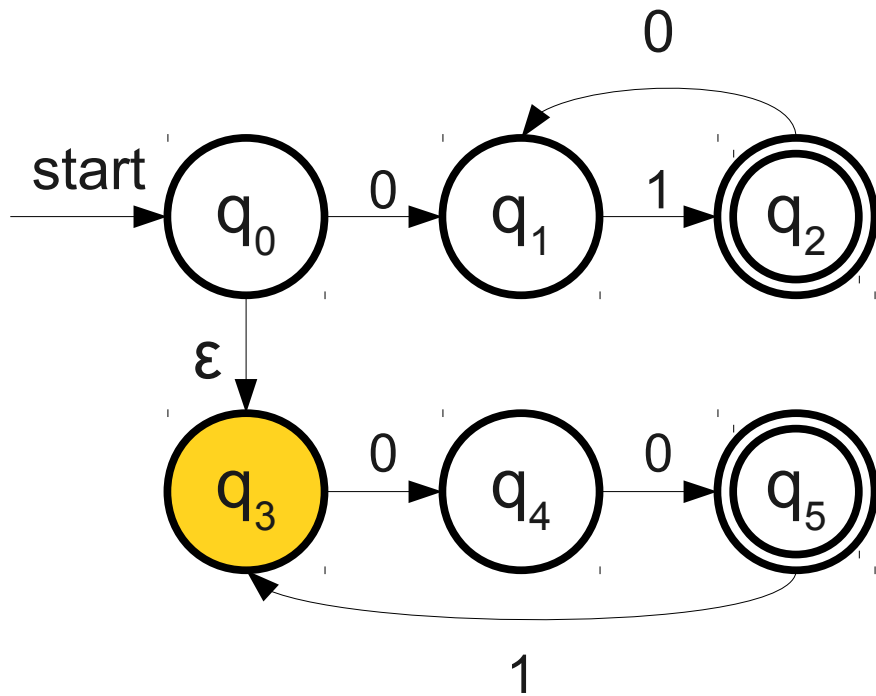
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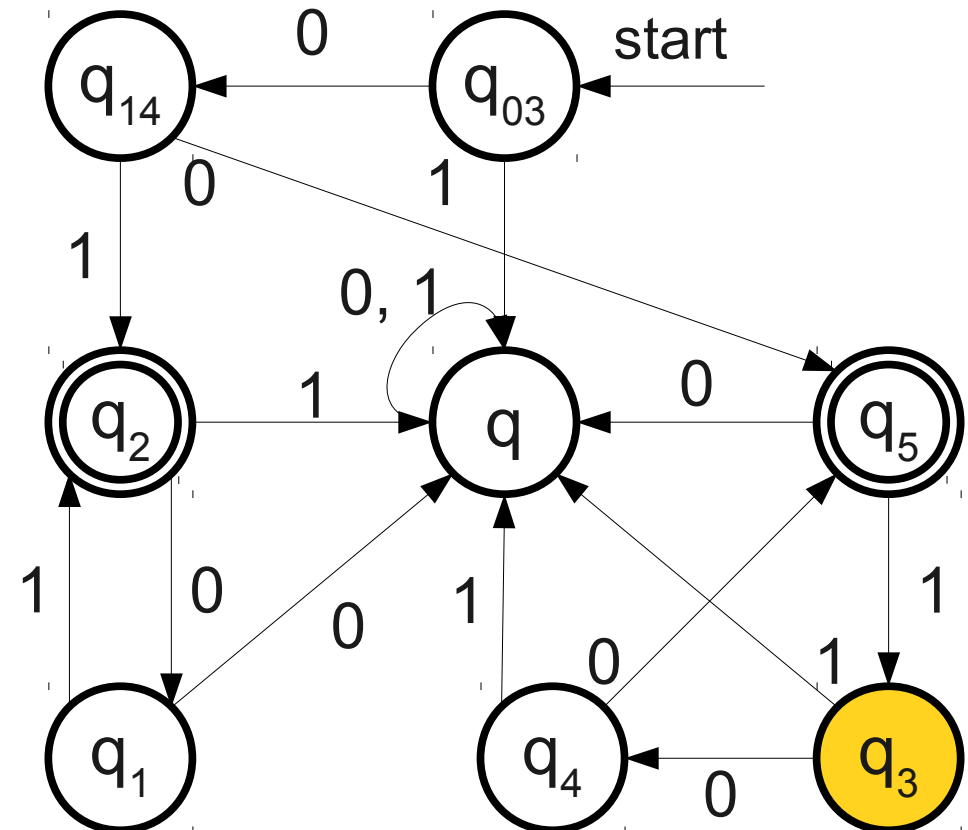
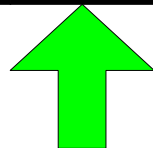
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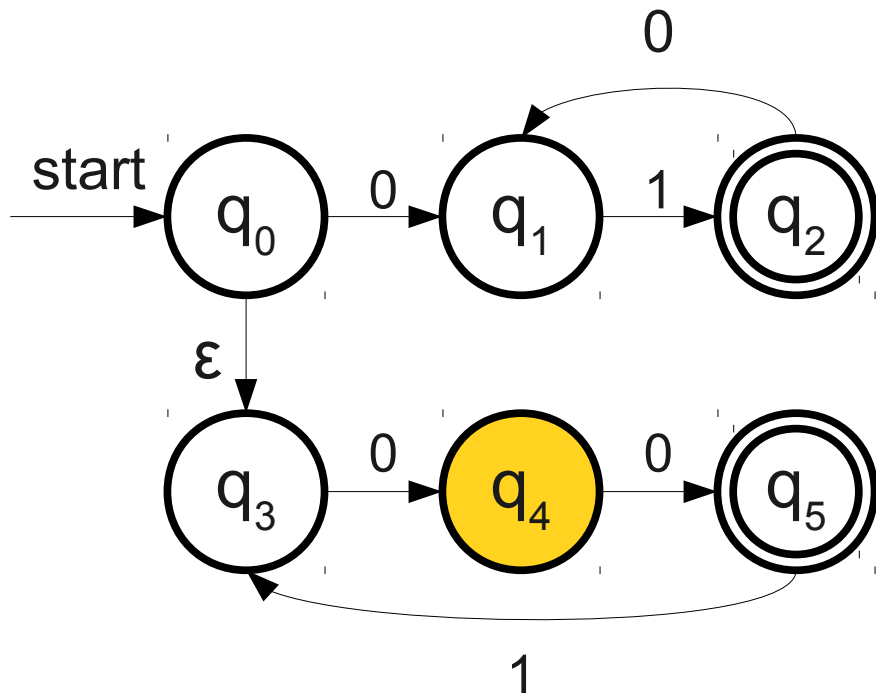
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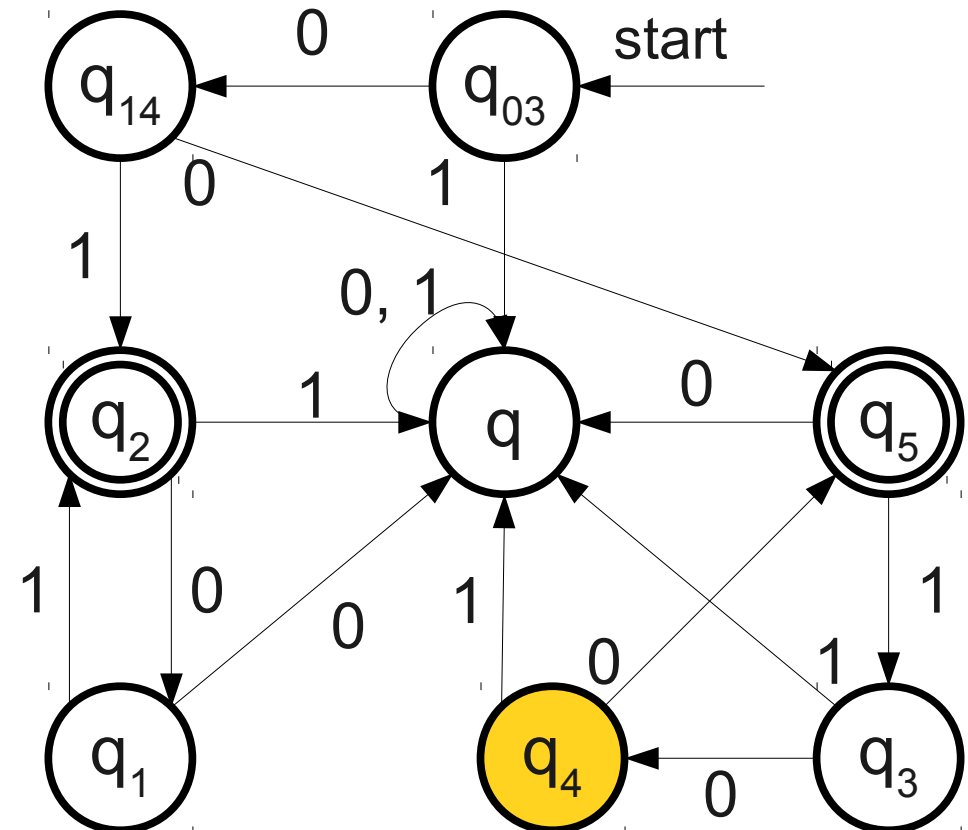
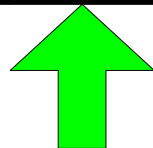
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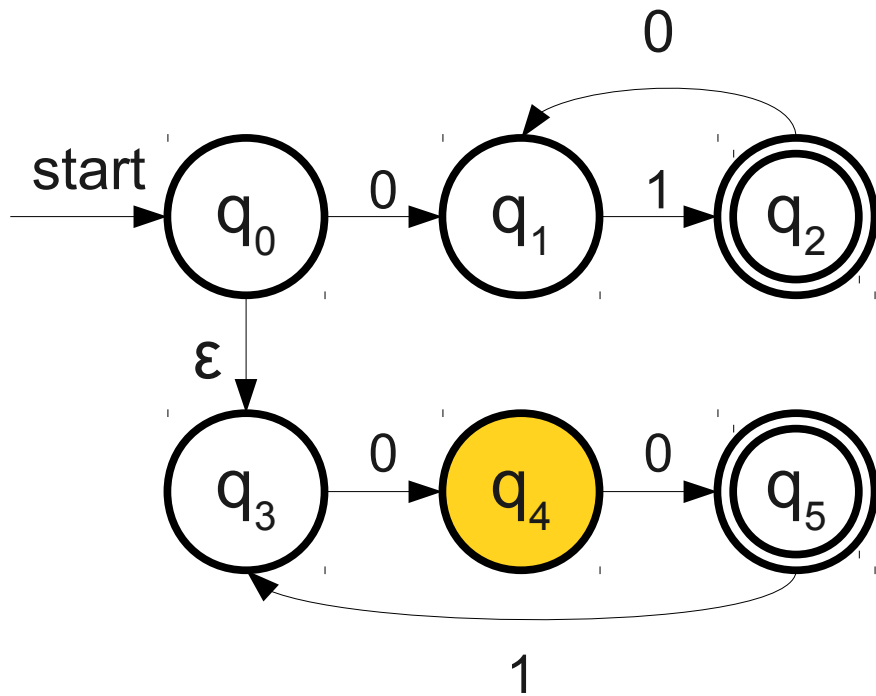
Simulating an NFA with a DFA



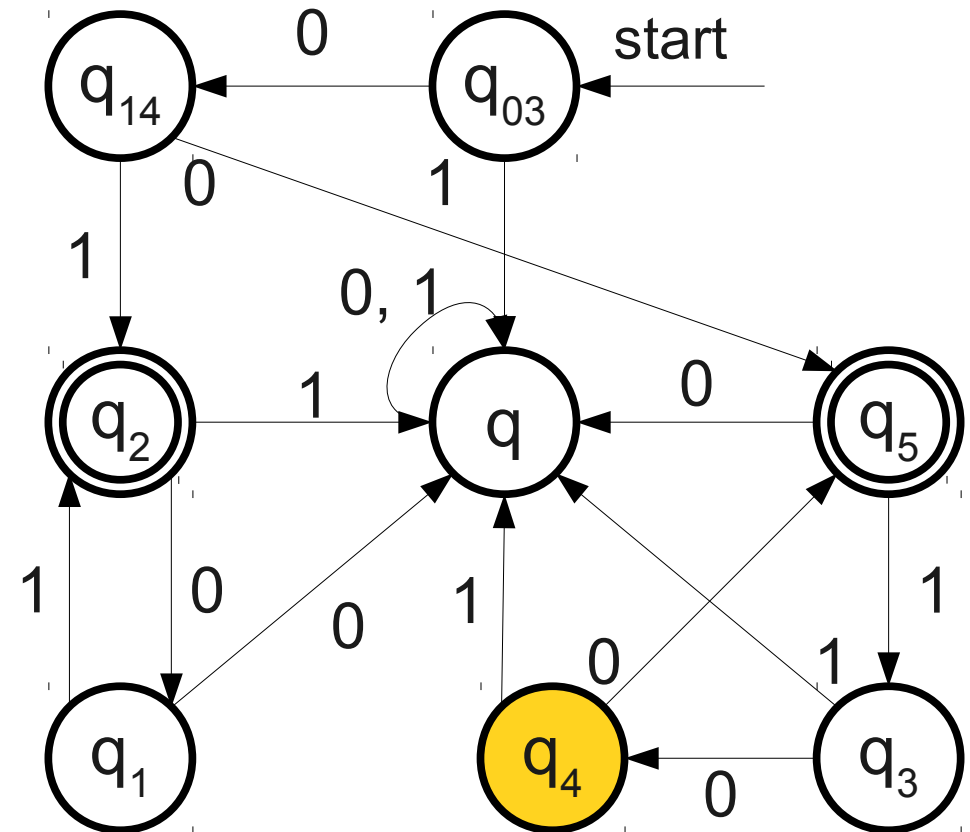
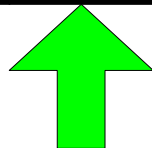
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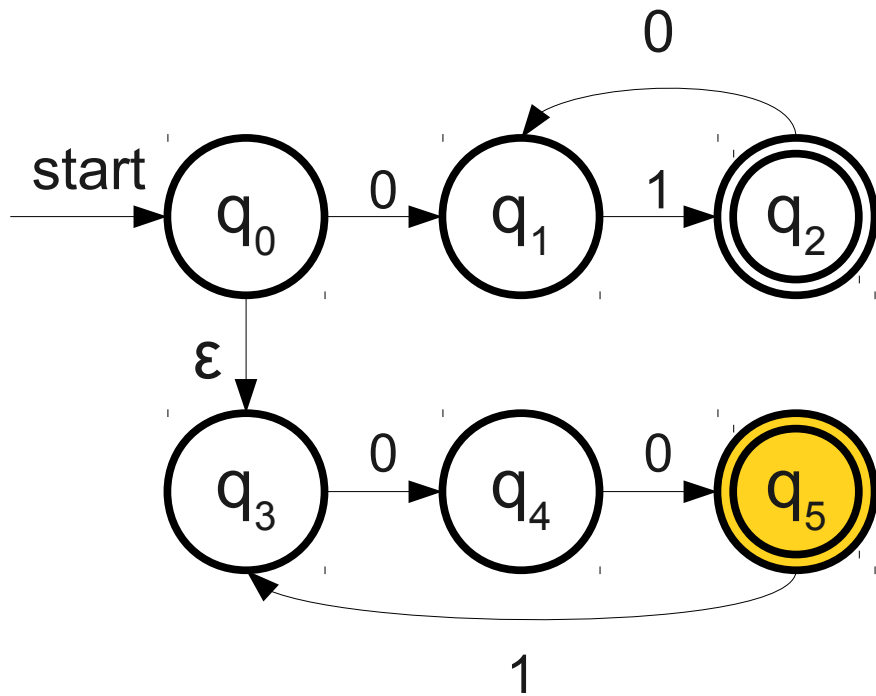
Simulating an NFA with a DFA



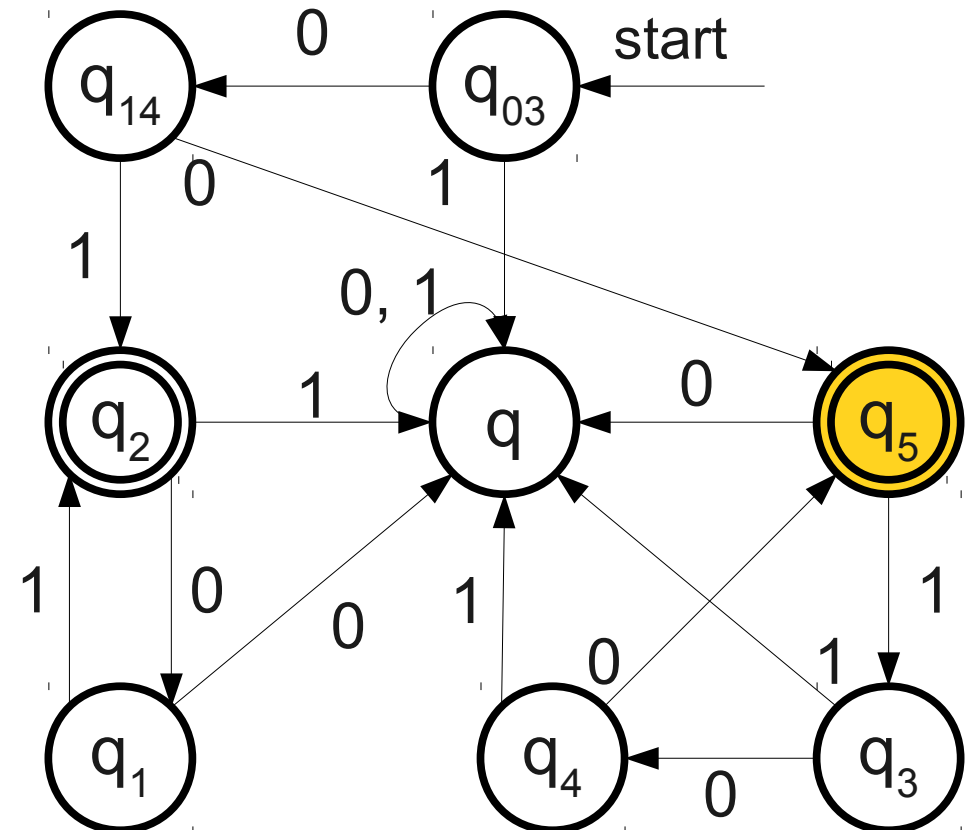
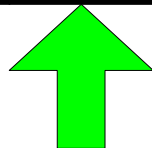
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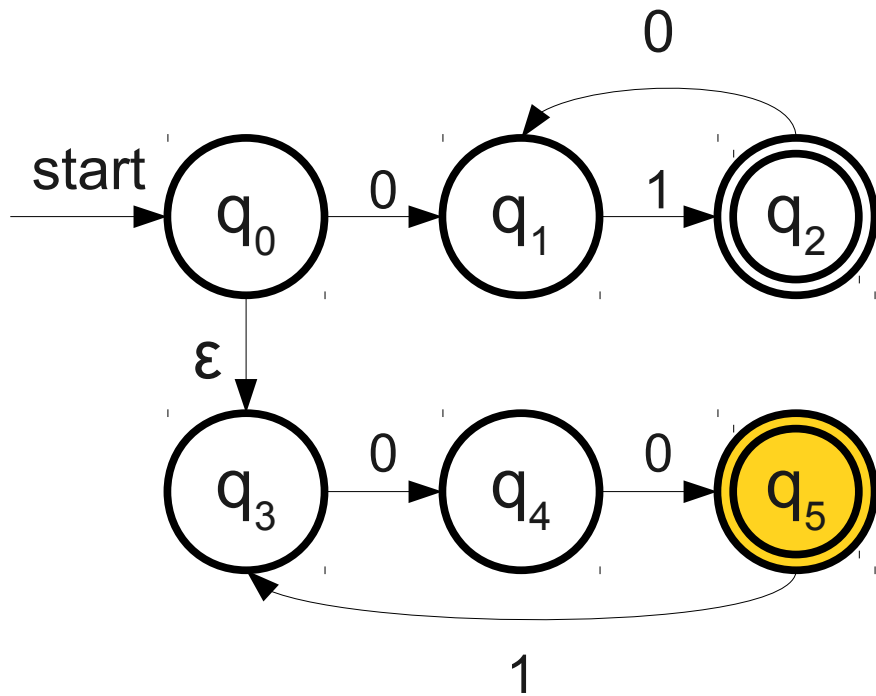
Simulating an NFA with a DFA



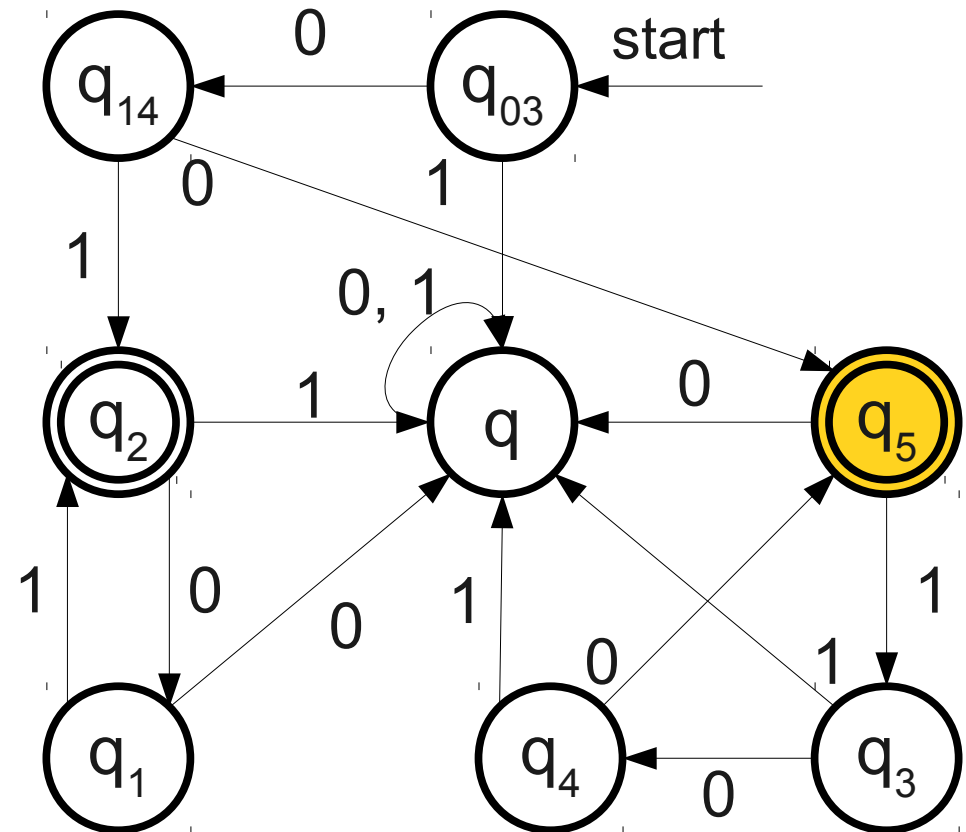
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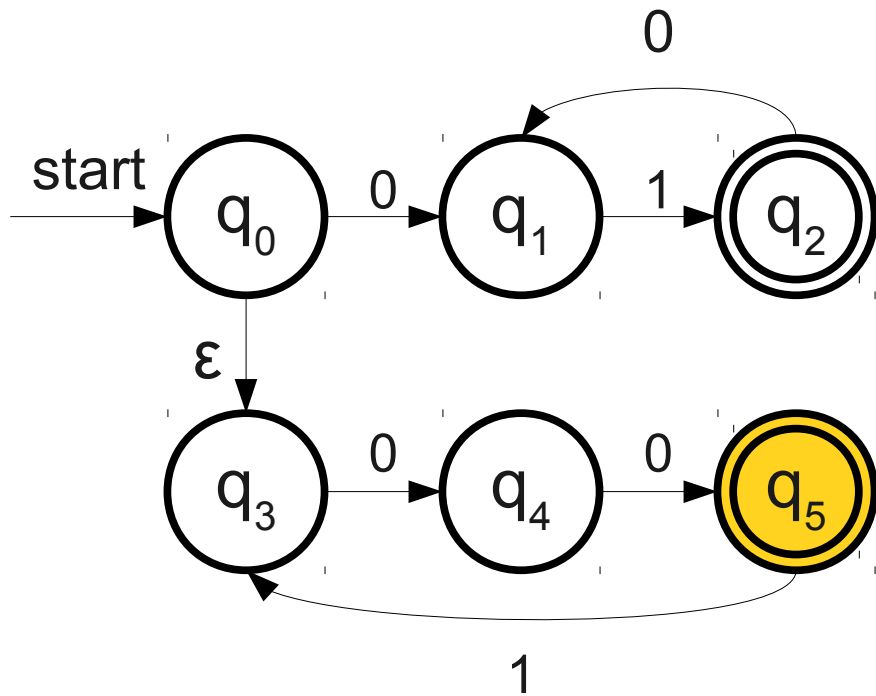
Simulating an NFA with a DFA



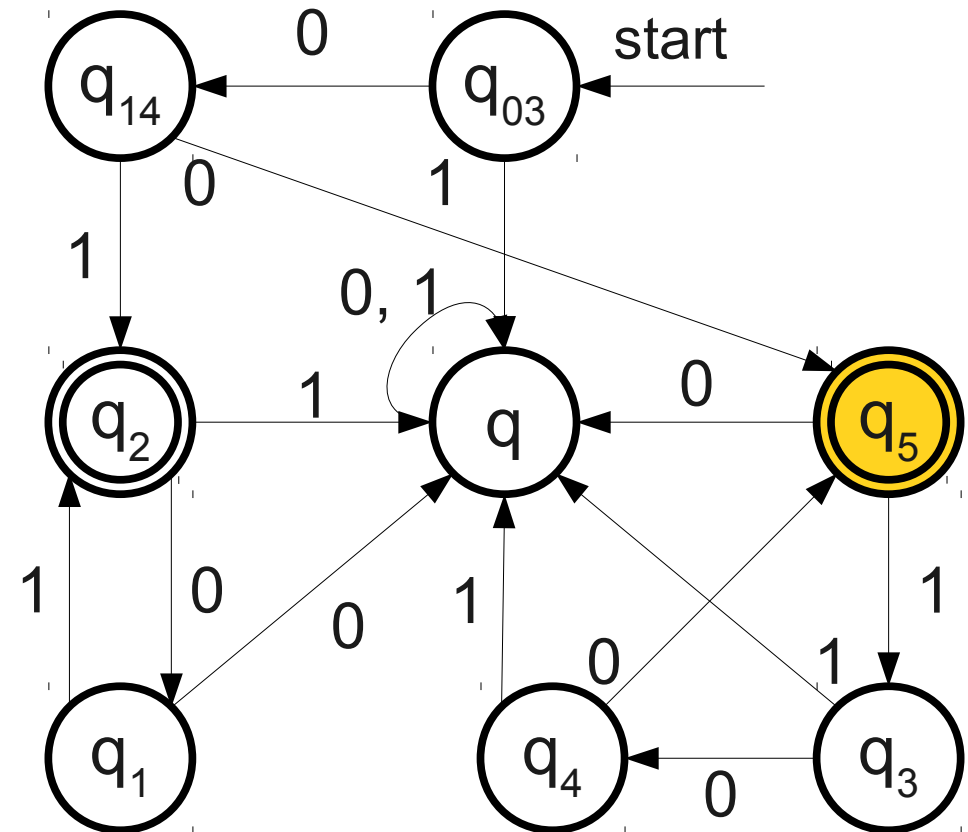
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Simulating an NFA with a DFA



0 0 1 0 0



The Subset Construction

- This construction for transforming an NFA into a DFA is called the **subset construction** (or sometimes the **powerset construction**).
- Intuitively:
 - States of the new DFA correspond to **sets of states** of the NFA.
 - The initial state is the start state, plus all states reachable from the start state via ϵ -transitions.
 - Transition on state S on character a is found by following all possible transitions on a for each state in S , then taking the set of states reachable from there by ϵ -transitions.
 - Accepting states are any set of states where *some* state in the set is an accepting state.
- **Read Sipser for a formal account.**