Turing Machines

Part II

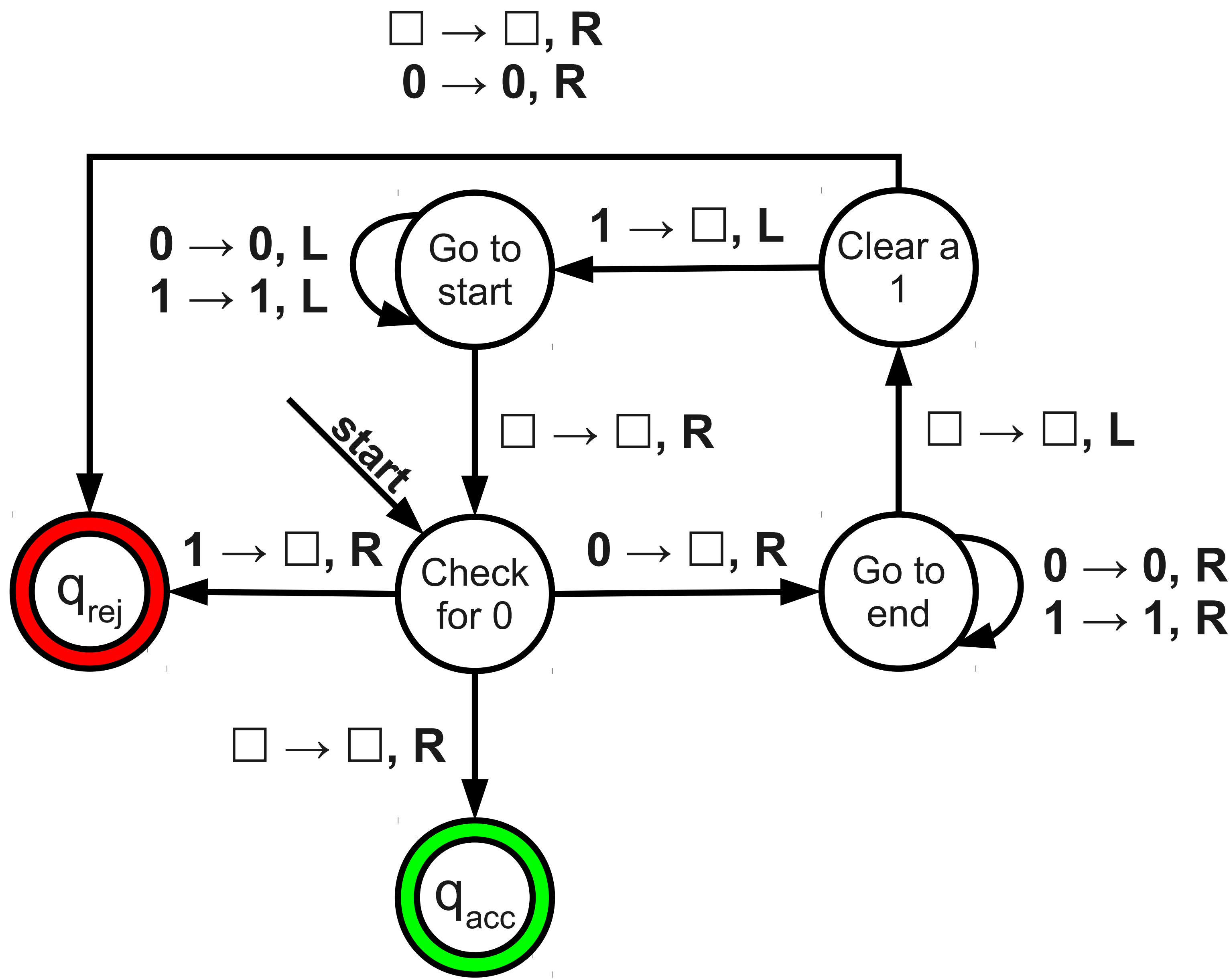
Friday Four Square!
Today at 4:15PM, Outside Gates

Announcements

- Problem Set 6 due next Monday, February 25, at 12:50PM.
    - Email the staff list with questions! (**cs103-win1213-staff@lists.stanford.edu**)

The Turing Machine

- A Turing machine consists of three parts:
  - A **finite-state control** that issues commands,
  - an **infinite tape** for input and scratch space, and
  - a **tape head** that can read and write a single tape cell.
- At each step, the Turing machine
  - Writes a symbol to the tape cell under the tape head,
  - changes state, and
  - moves the tape head to the left or to the right.



□ → □, R
0 → 0, R
0 → 0, L
1 → 1, L
Go to start
1 → □, L
Clear a 1
start
□ → □, R
□ → □, L
q_{rej}
1 → □, R
Check for 0
0 → □, R
Go to end
0 → 0, R
1 → 1, R
□ → □, R
q_{acc}

Key Idea: Subroutines

- A **subroutine** of a Turing machine is a small set of states in the TM such that performs a small computation.
- Usually, a single entry state and a single exit state.
- Many very complicated tasks can be performed by TMs by breaking those tasks into smaller subroutines.

Turing Machine Memory

- Turing machines often contain many seemingly replicated states in order to store a finite amount of extra information.
- A Turing machine can remember one of k different constants by copying its states k times, once for each possible value, and wiring those states appropriately.

The Power of Turing Machines

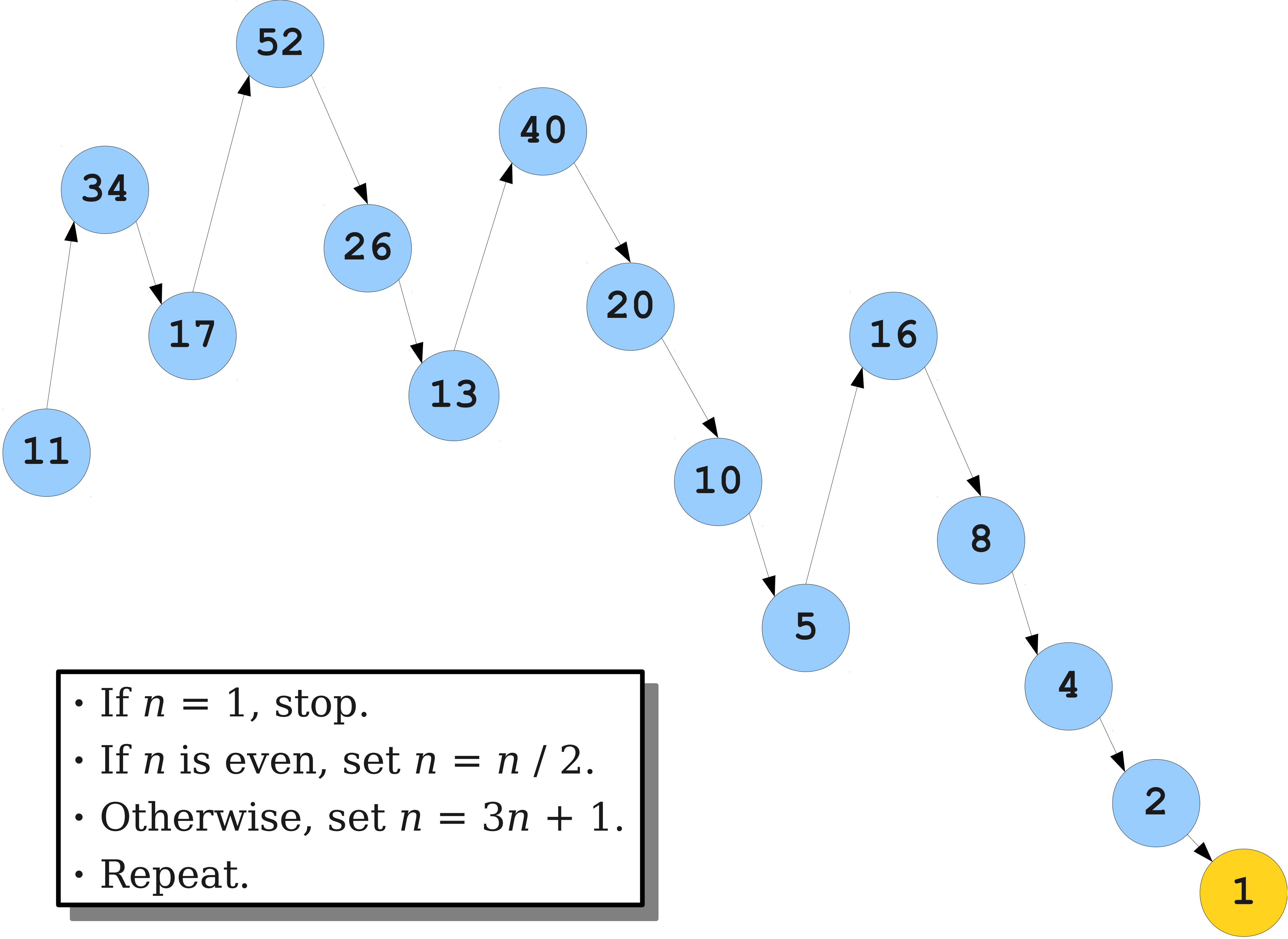
- Turing machines can
  - Perform standard arithmetic operations (addition, subtraction, multiplication, division, exponentiation, etc.)
  - Manipulate lists of elements (searching, sorting, reversing, etc.)
- What else can Turing machines do?

Outline for Today

- **Exhaustive Search**
  - A fundamentally different approach to designing Turing machines.
- **Nondeterministic Turing Machines**
  - What does a Turing machine with Magic Superpowers look like?
- **The Church-Turing Thesis**
  - Just how powerful *are* Turing machines?

The Hailstone Sequence

- Consider the following procedure, starting with some $n \in \mathbb{N}$, where $n > 0$:
  - If $n = 1$, you are done.
  - If n is even, set $n = n / 2$.
  - Otherwise, set $n = 3n + 1$.
  - Repeat.
- Question: Given a number n, does this process terminate?



- If $n = 1$, stop.
- If n is even, set $n = n / 2$.
- Otherwise, set $n = 3n + 1$.
- Repeat.

The Hailstone Sequence

- Let $\Sigma = \{\mathtt{1}\}$ and consider the language

$$L = \{\ \mathtt{1}^n \mid n > 0 \text{ and the hailstone sequence terminates for } n\ \}.$$

- Could we build a TM for L?

The Hailstone Turing Machine

- Intuitively, we can build a TM for the hailstone language as follows: the machine M does the following:
  - If the input is ε, reject.
  - While the input is not `1`:
    - If the input has even length, halve the length of the string.
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  - Accept.

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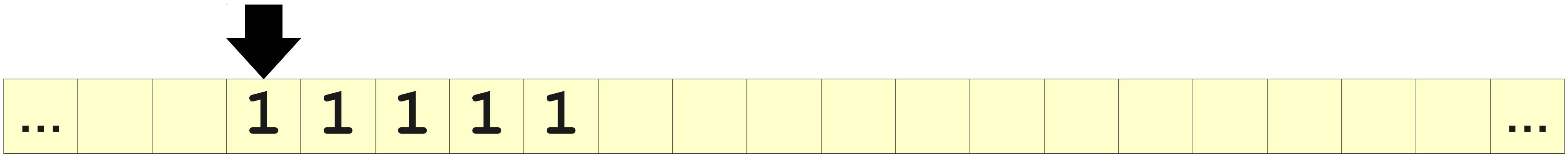

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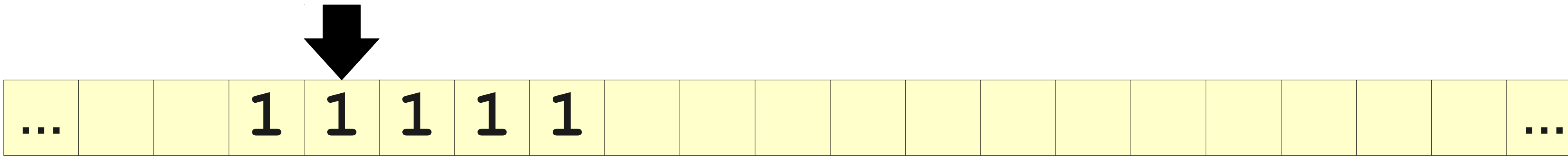

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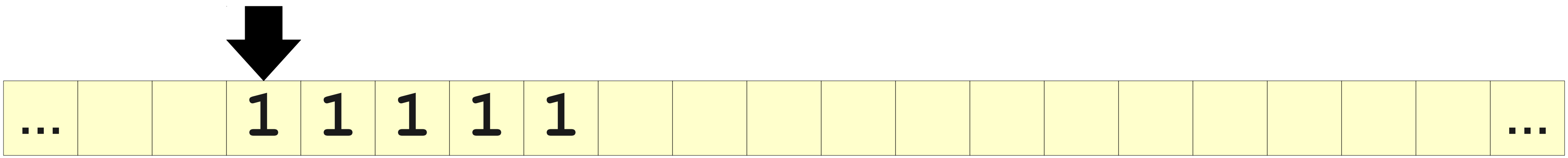

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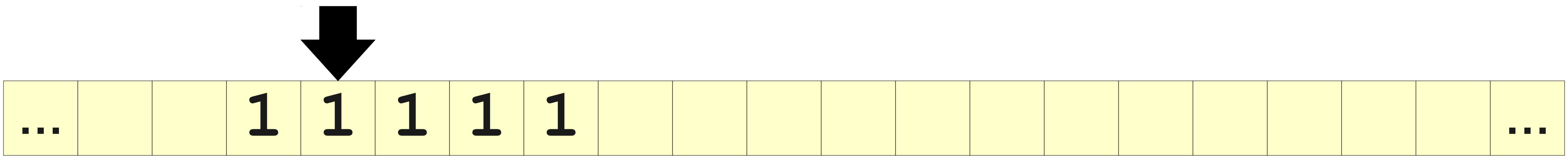

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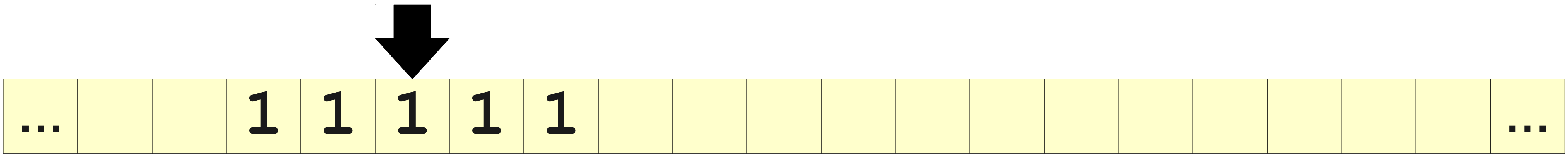

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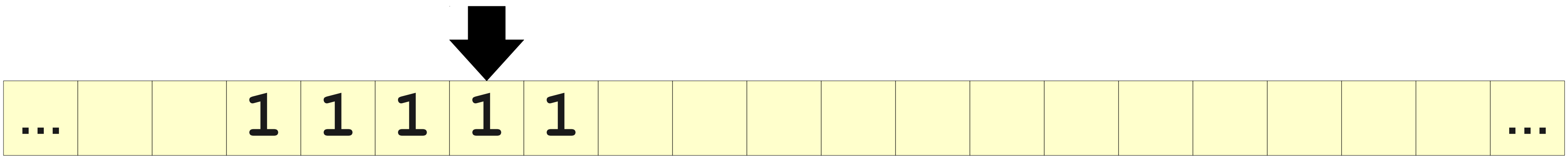

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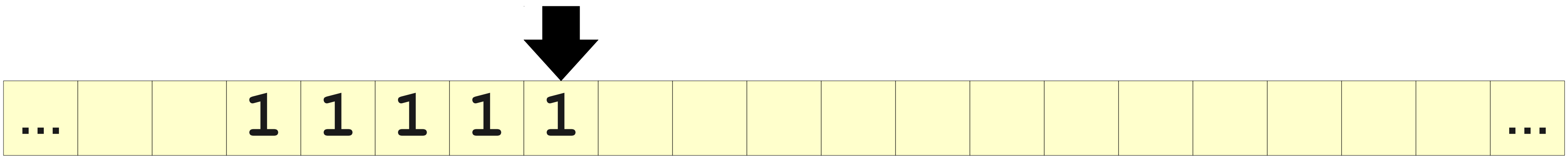

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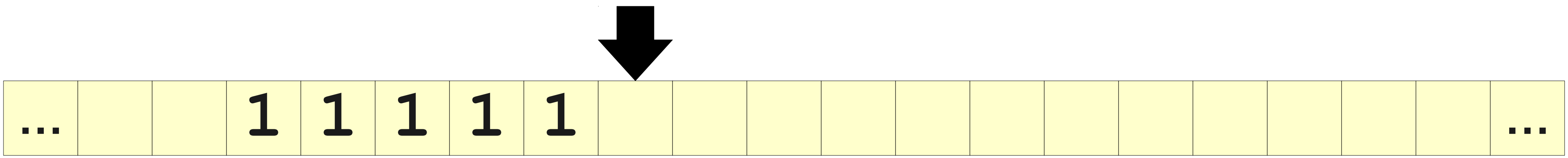

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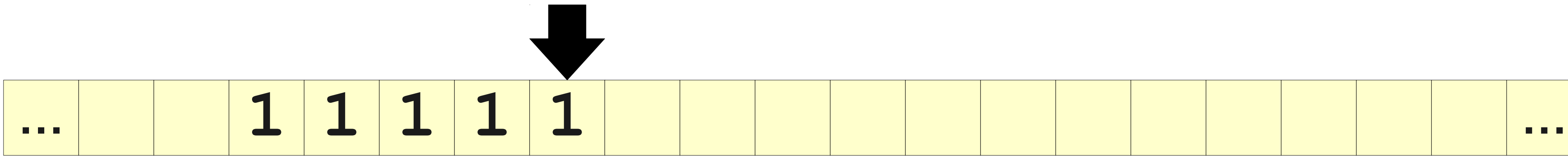

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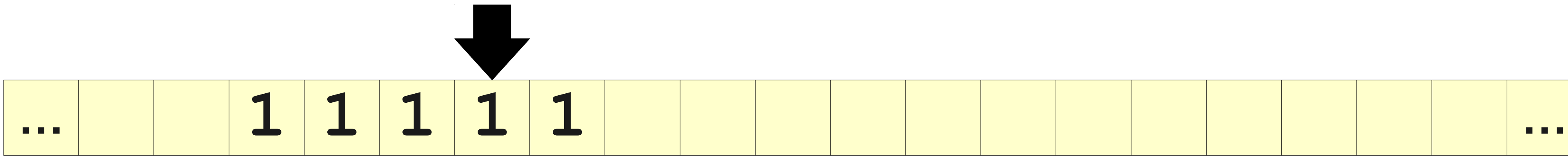

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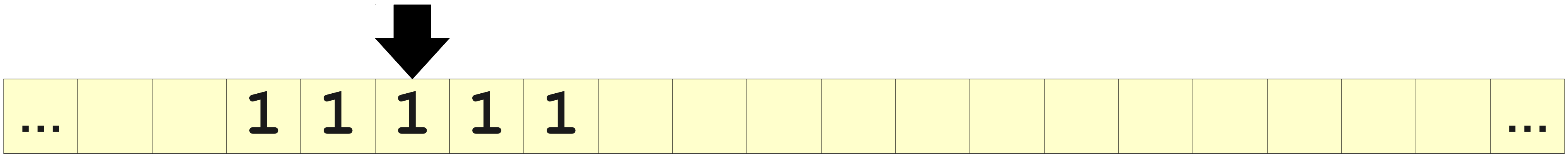

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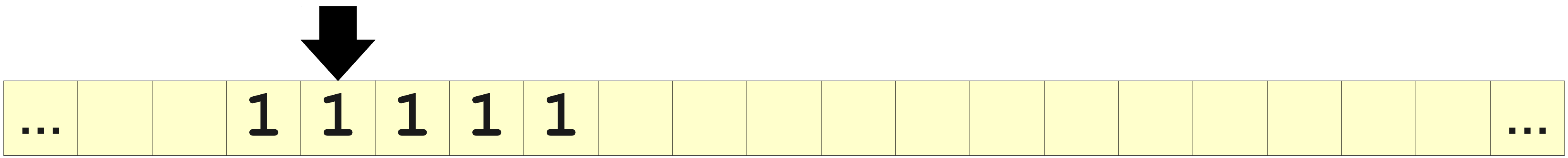

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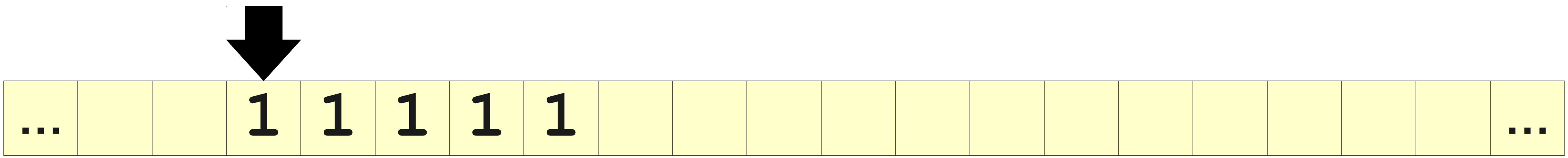

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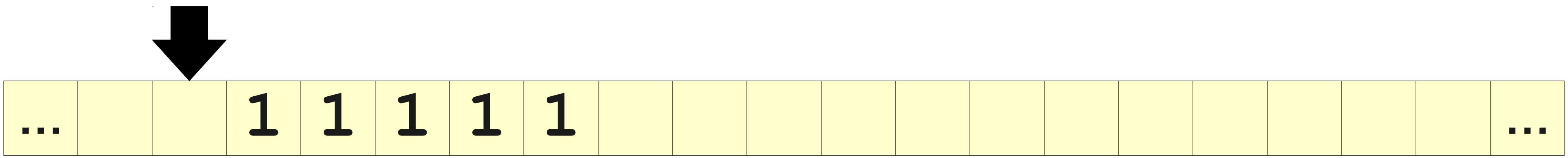

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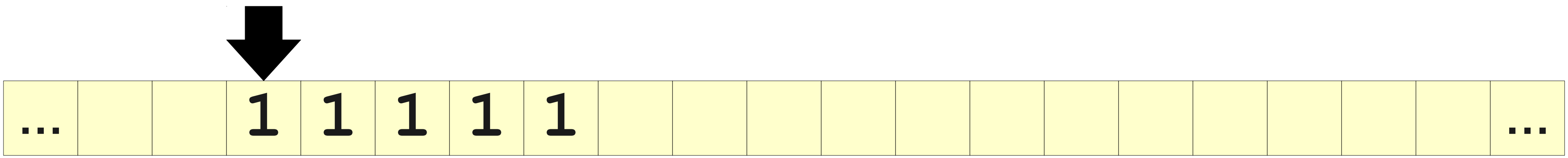

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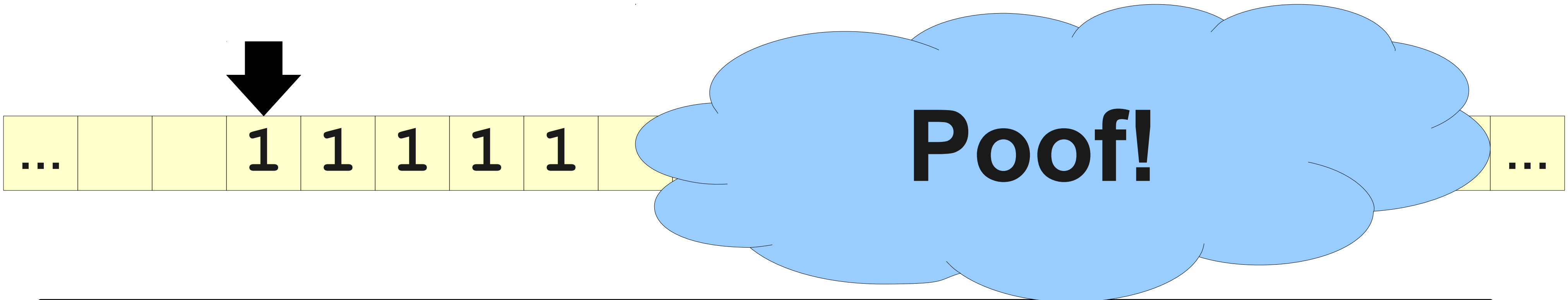

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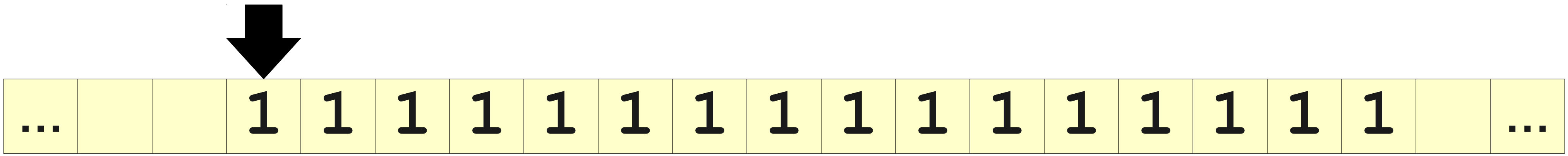

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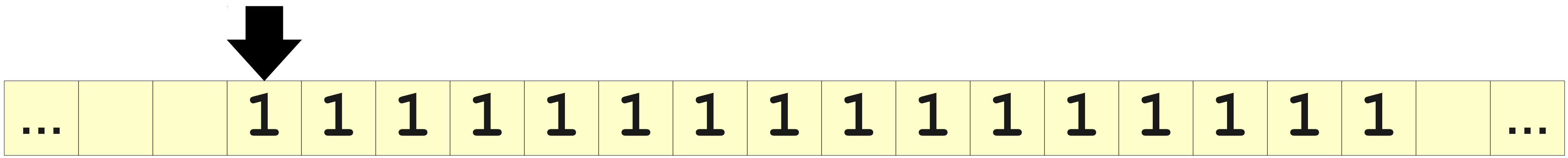

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Interesting problem: Build a TM that, starting with n 1s on its tape, ends with $3n + 1$ 1s on its tape.

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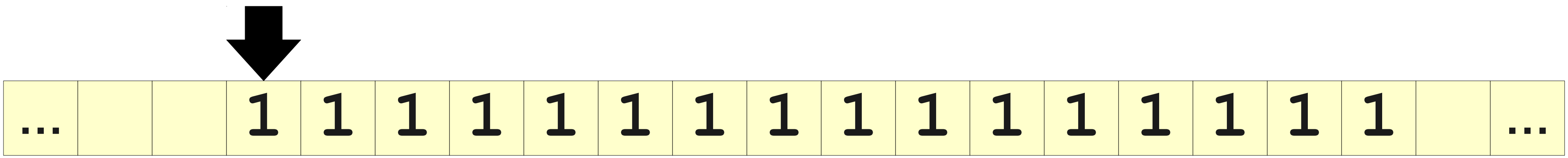

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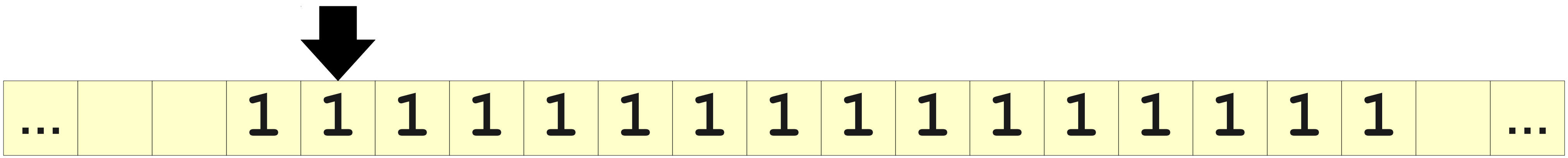

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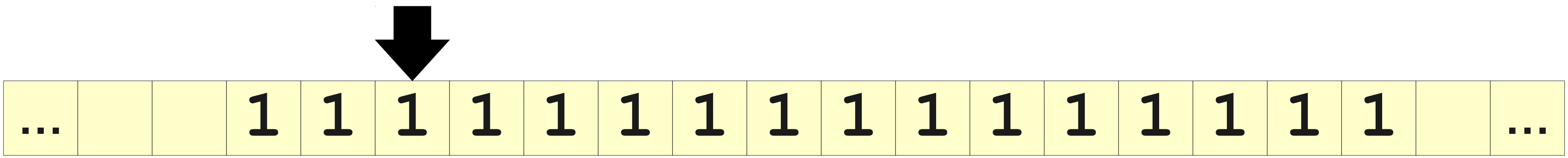

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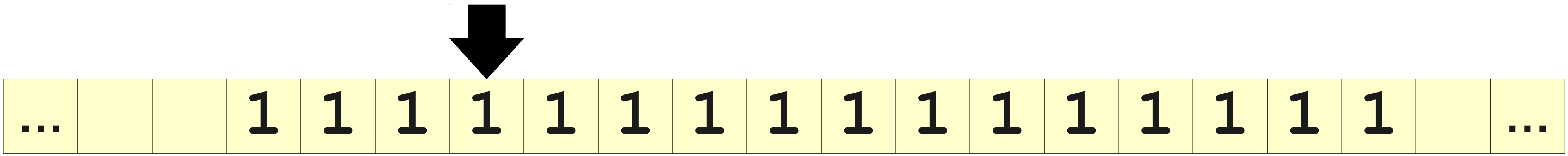

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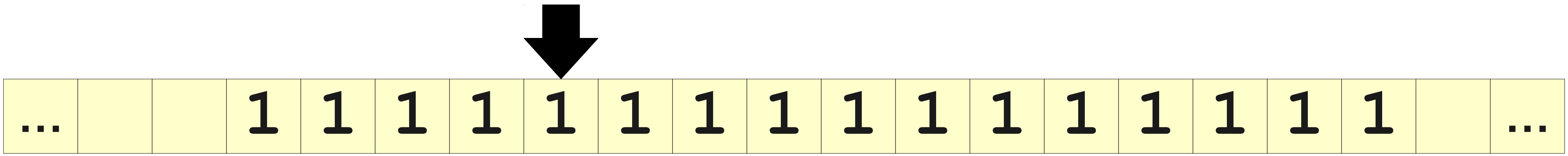

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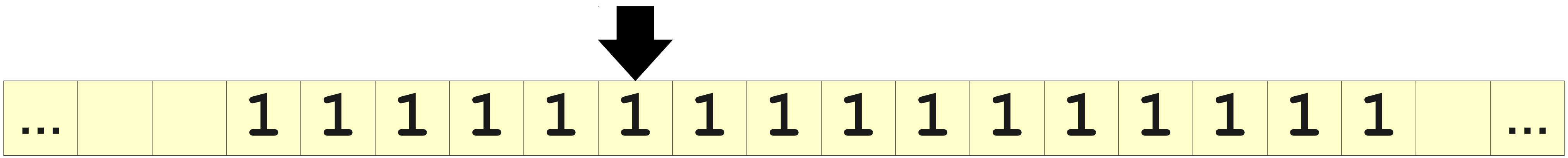

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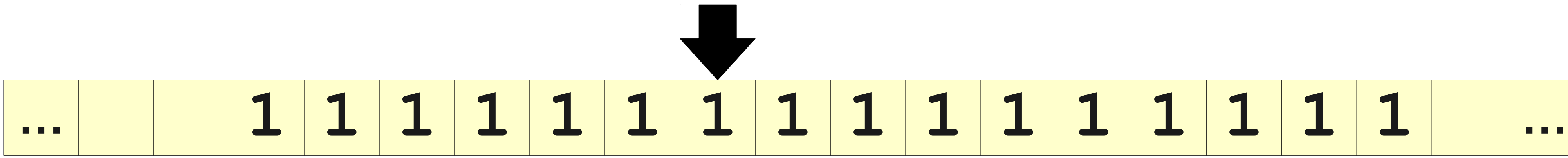

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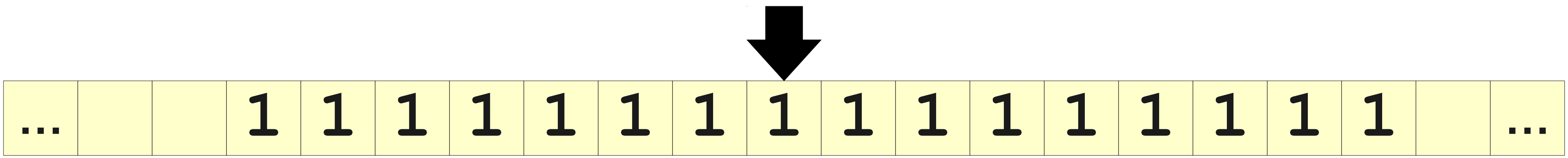

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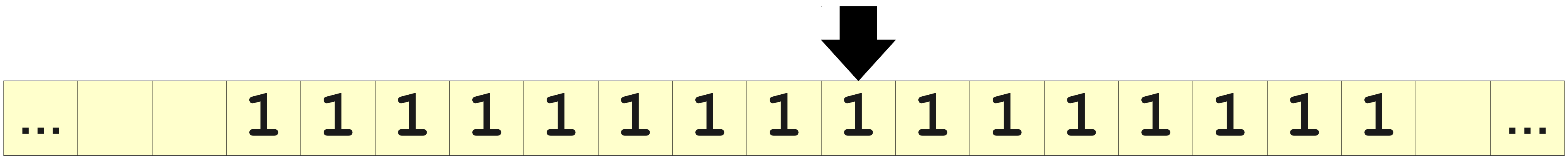

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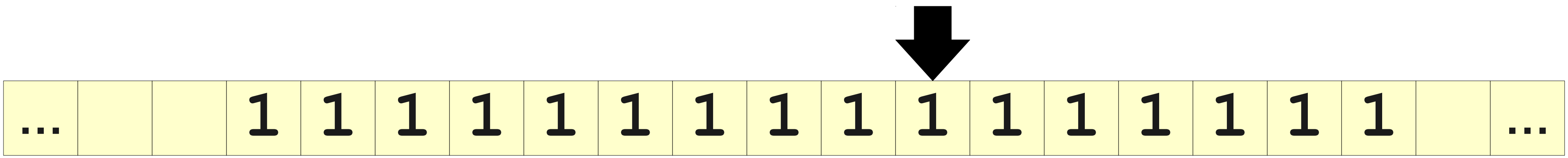

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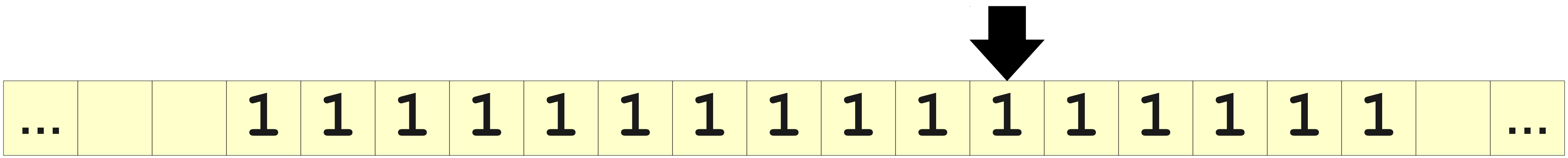

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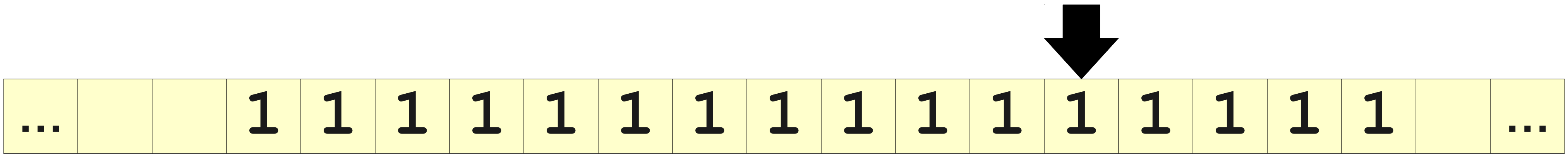

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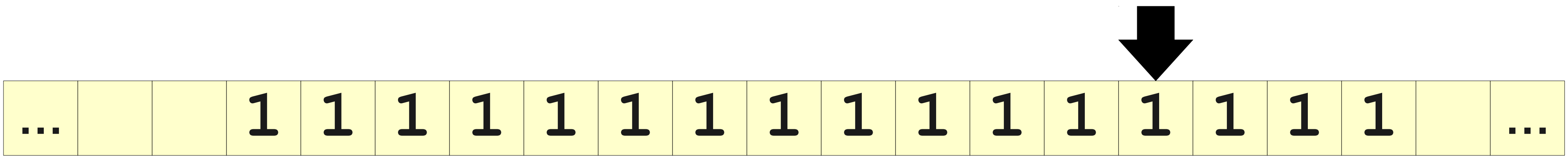

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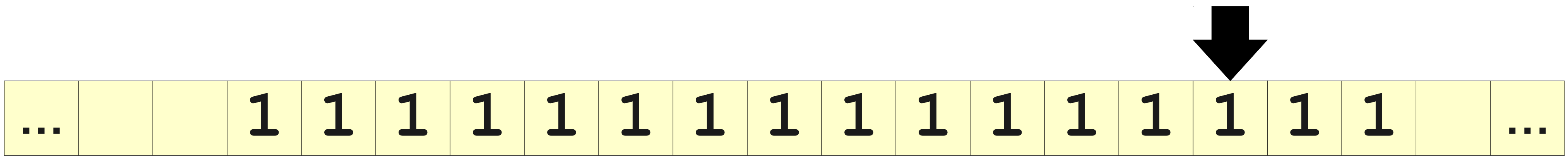

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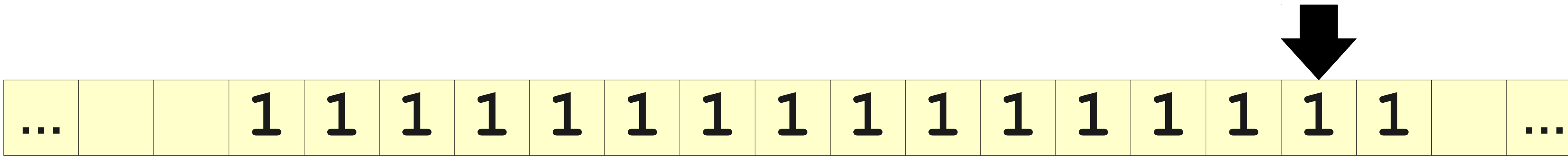

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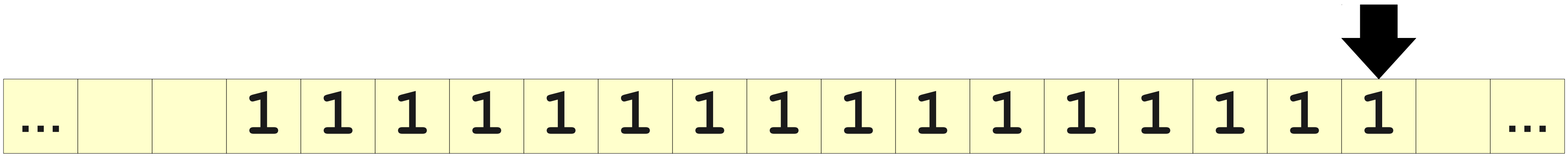

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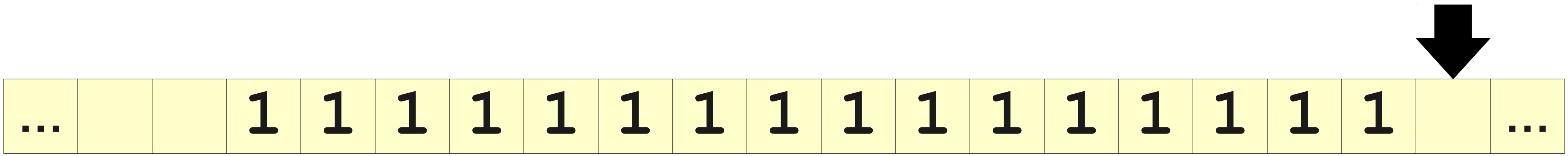

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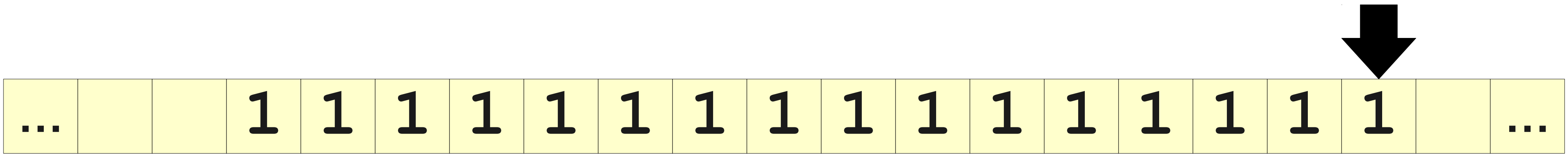

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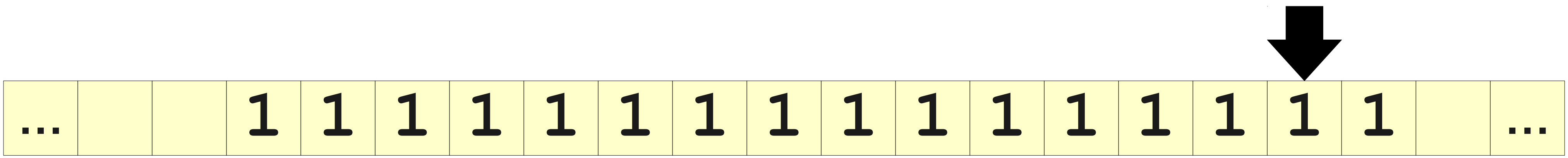

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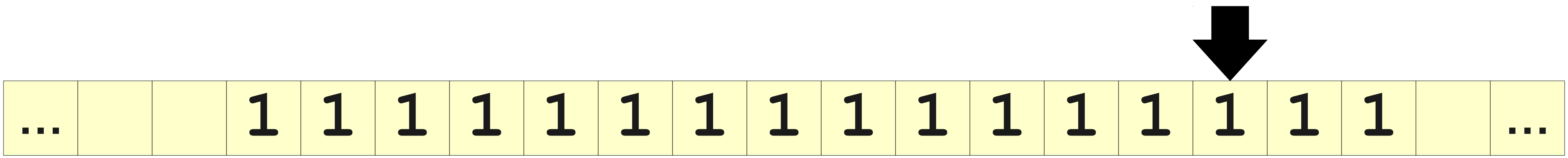

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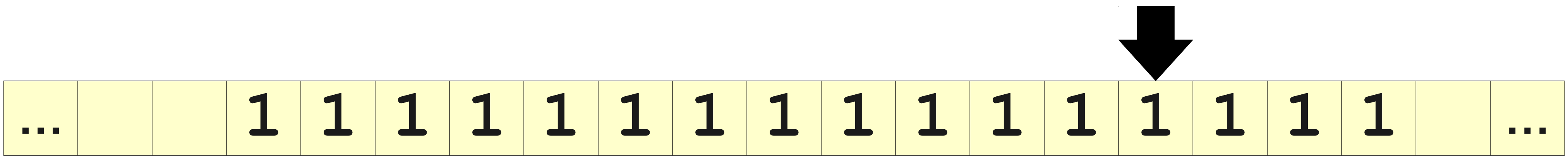

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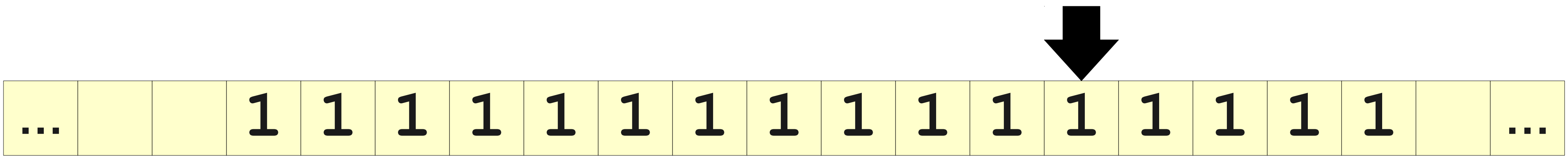

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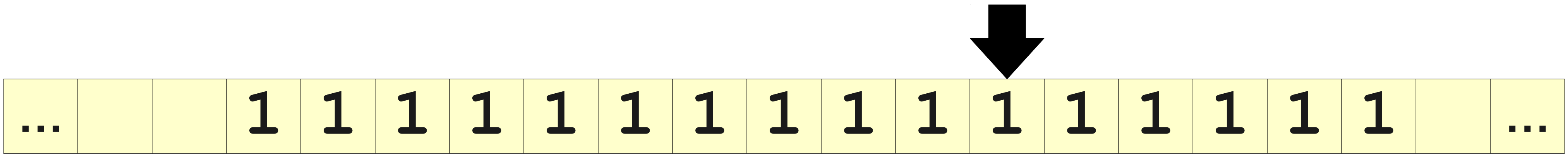

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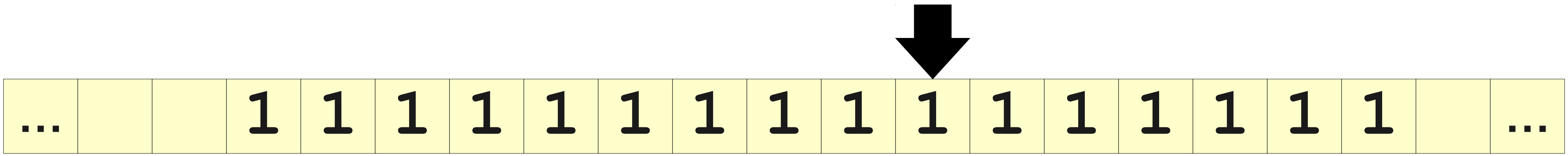

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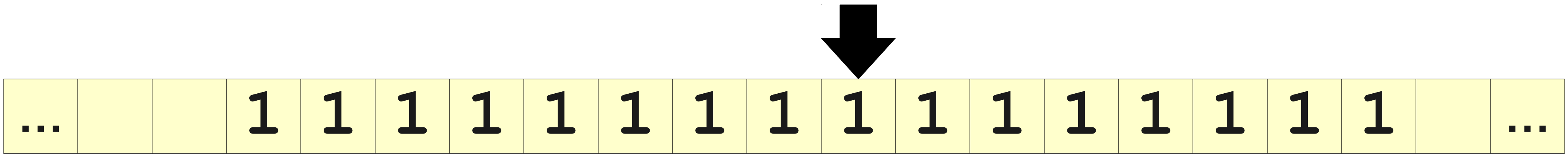

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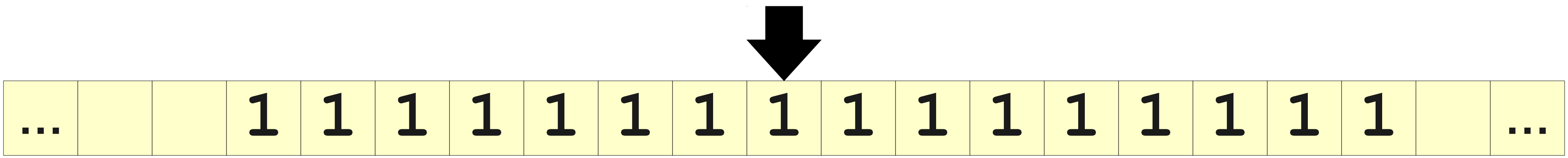

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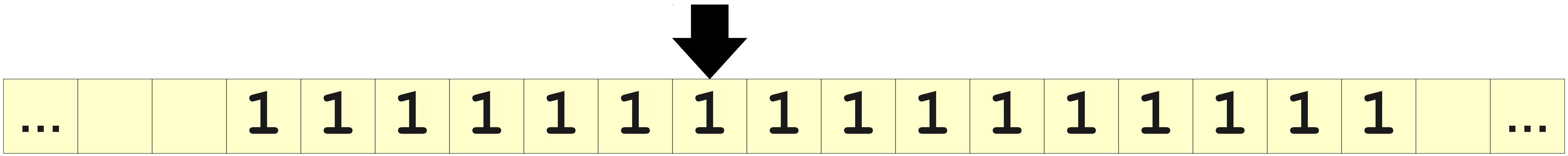

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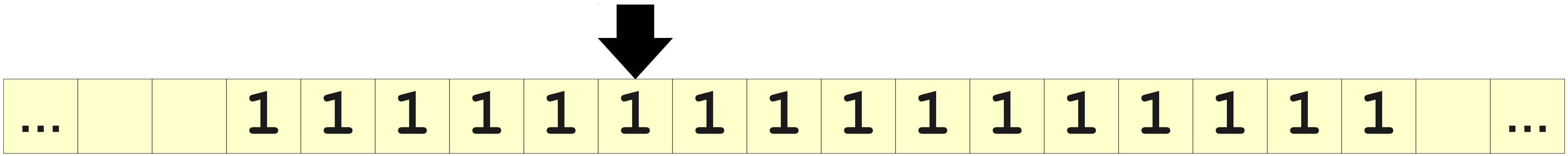

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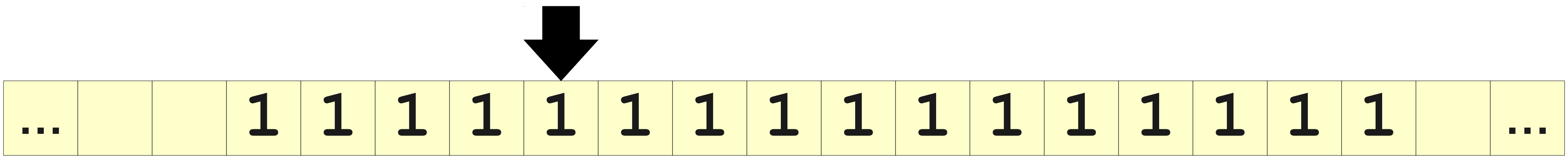

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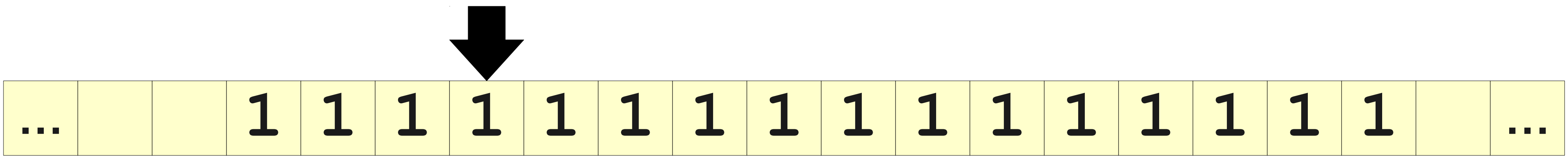

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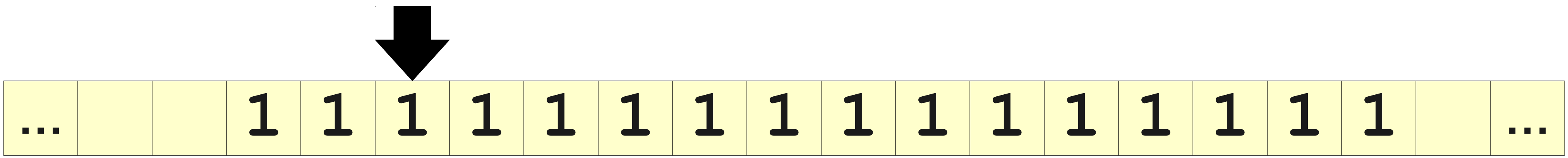

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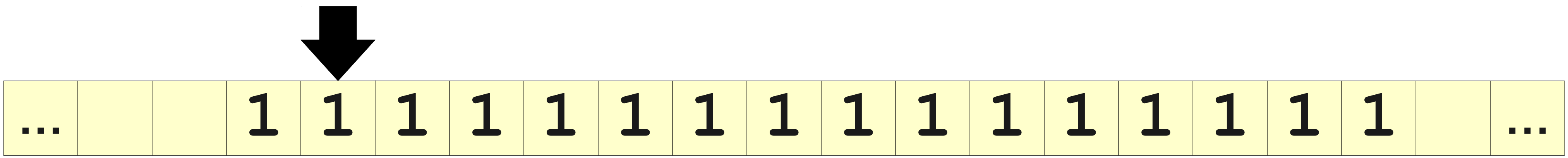

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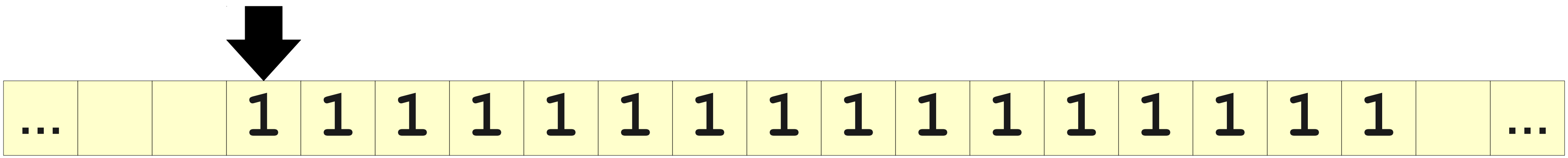

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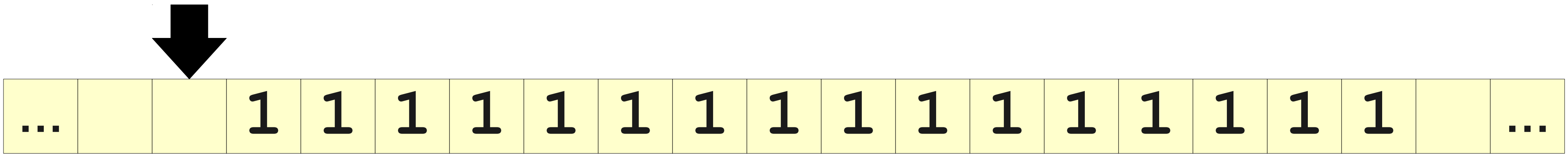

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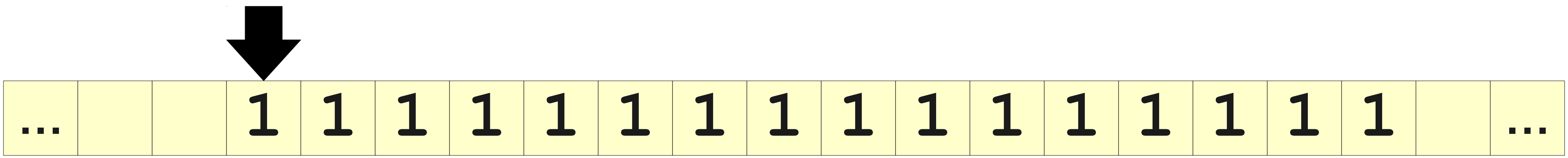

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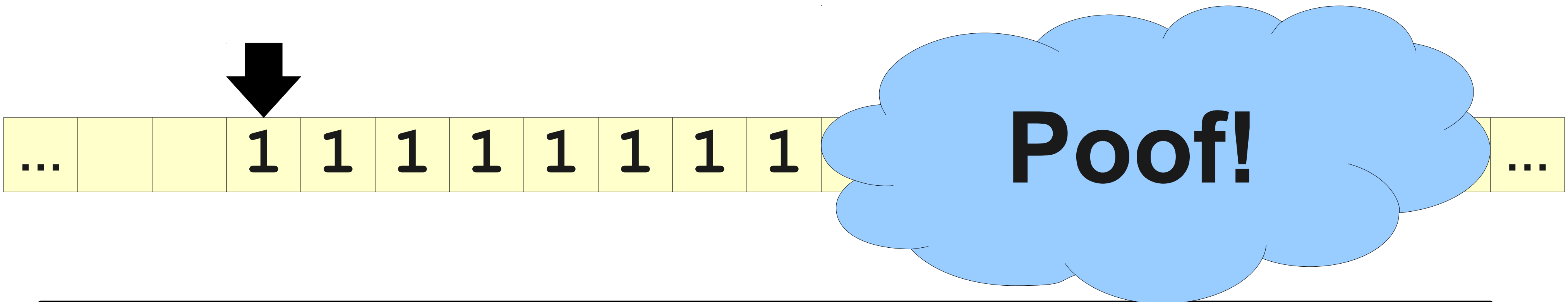

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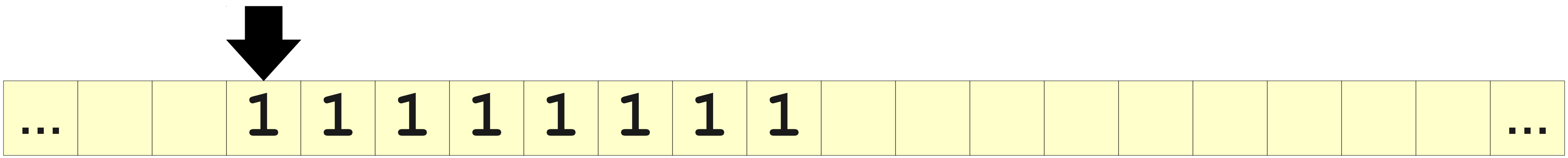

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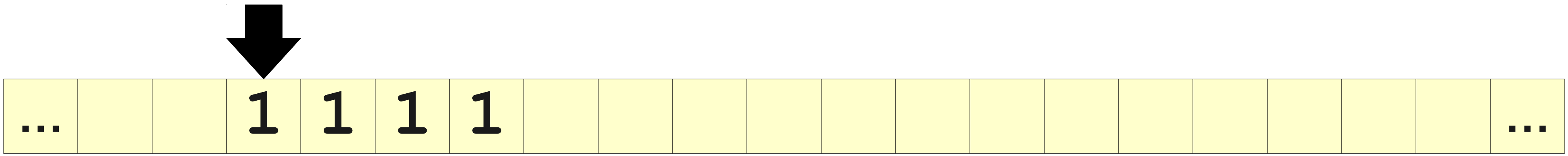

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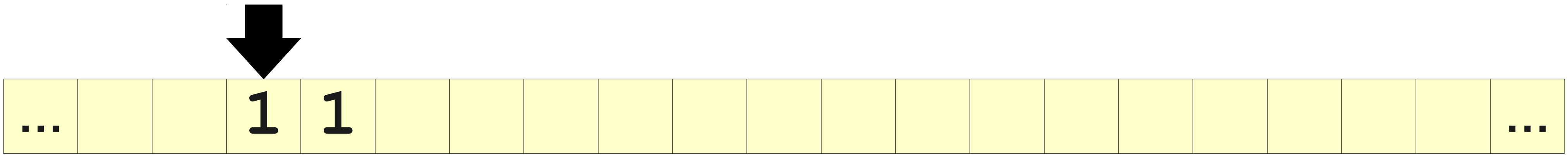

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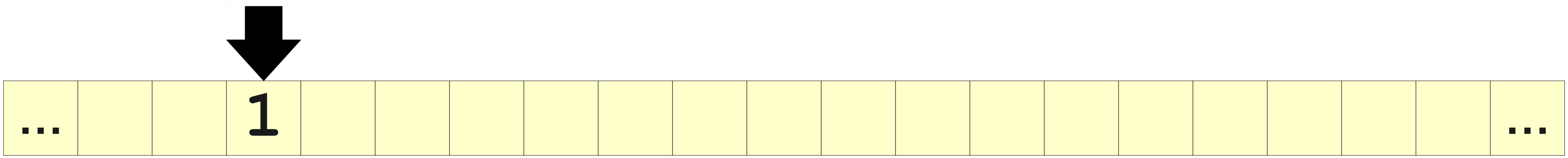

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Accept.

Does this Turing machine always accept?

The Collatz Conjecture

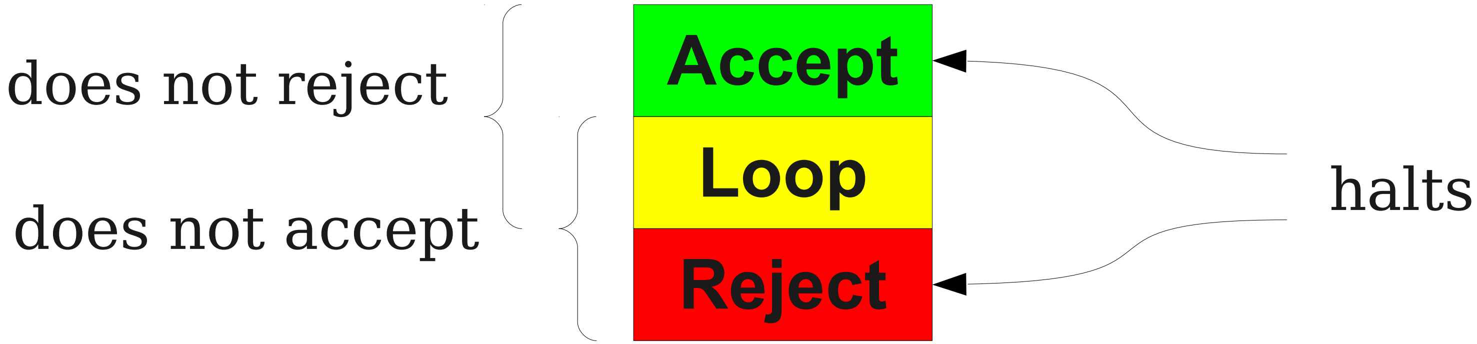
- It is *unknown* whether this process will terminate for all natural numbers.
- In other words, ***no one knows whether the TM described in the previous slides will always stop running!***
- The conjecture (claim) that this always terminates is called the **Collatz Conjecture**.

An Important Observation

- Unlike the other automata we've seen so far, Turing machines choose for themselves whether to accept or reject.
- It is therefore possible for a TM to run forever without accepting or rejecting.

Some Important Terminology

- Let M be a Turing machine.
- M **accepts** a string w if it enters the accept state when run on w.
- M **rejects** a string w if it enters the reject state when run on w.
- M **loops infinitely** (or just **loops**) on a string w if when run on w it enters neither the accept or reject state.
- M **does not accept *w*** if it either rejects w or loops infinitely on w.
- M **does not reject *w*** w if it either accepts w or loops on w.
- M **halts on *w*** if it accepts w or rejects w.



The Language of a TM

- The language of a Turing machine M, denoted $\mathscr{L}(M)$, is the set of all strings that M accepts:

$$\boldsymbol{\mathscr{L}(M) = \{ w \in \Sigma^* \mid M \textbf{ accepts } w \}}$$

- For any $w \in \mathscr{L}(M)$, M accepts w.
- For any $w \notin \mathscr{L}(M)$, M does not accept w.
  - It might loop forever, or it might explicitly reject.
- A language is called **recognizable** iff it is the language of some TM.
- Notation: **RE** is the set of all recognizable languages.

$$L \in \mathbf{RE} \quad \text{iff} \quad L \text{ is recognizable}$$

A) Double the contents of the string after **`M`**.

B) Replace **`III`** with **`U`**.

C) Remove **`UU`**

D) Append **`U`** if the string ends in **`I`**.

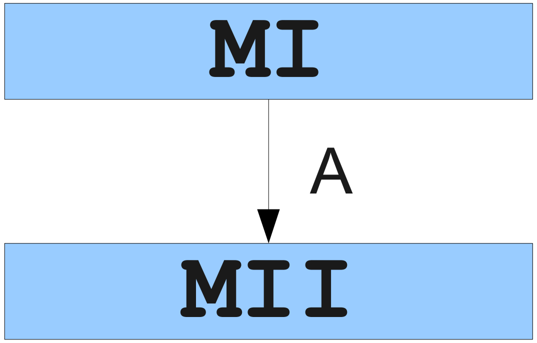
MI

A) Double the contents of the string after **M**.

B) Replace **III** with **U**.

C) Remove **UU**

D) Append **U** if the string ends in **I**.



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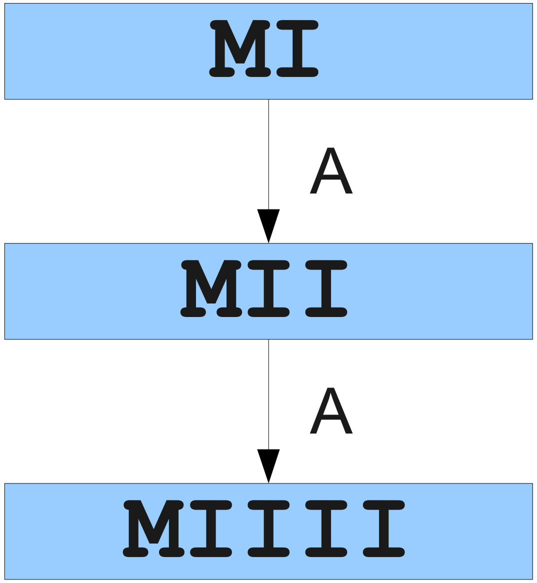
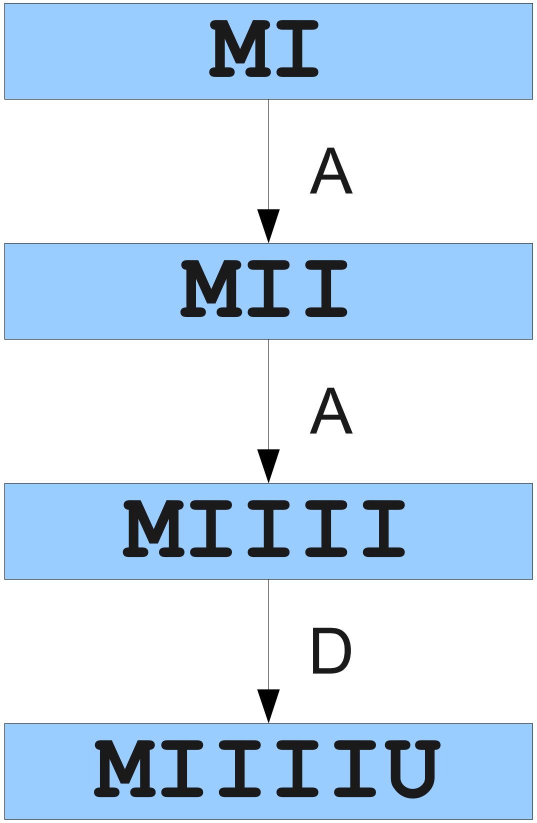


A) Double the contents of the string after **M**.

B) Replace **III** with **U**.

C) Remove **UU**

D) Append **U** if the string ends in **I**.



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C) Remove **UU**

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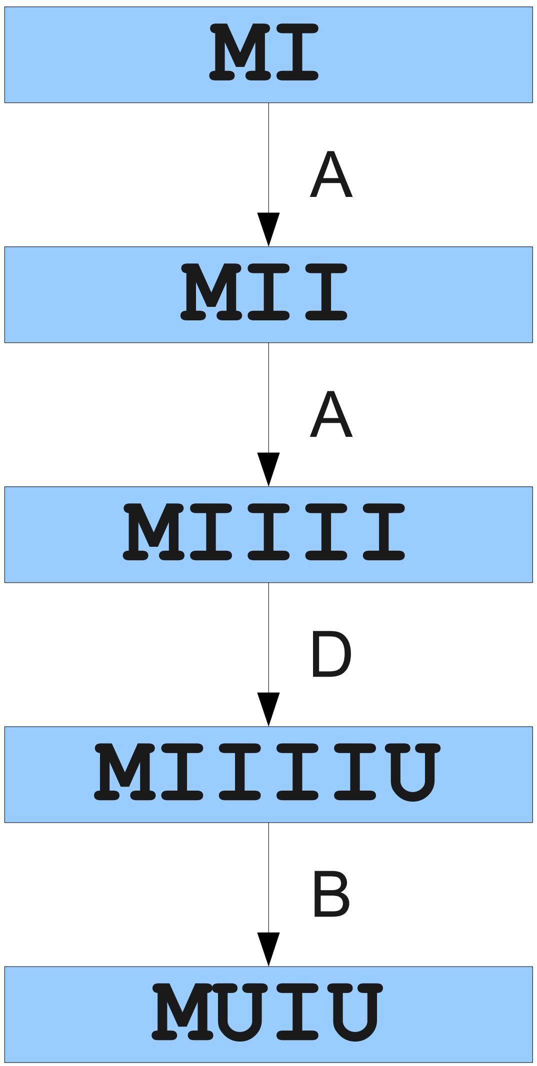

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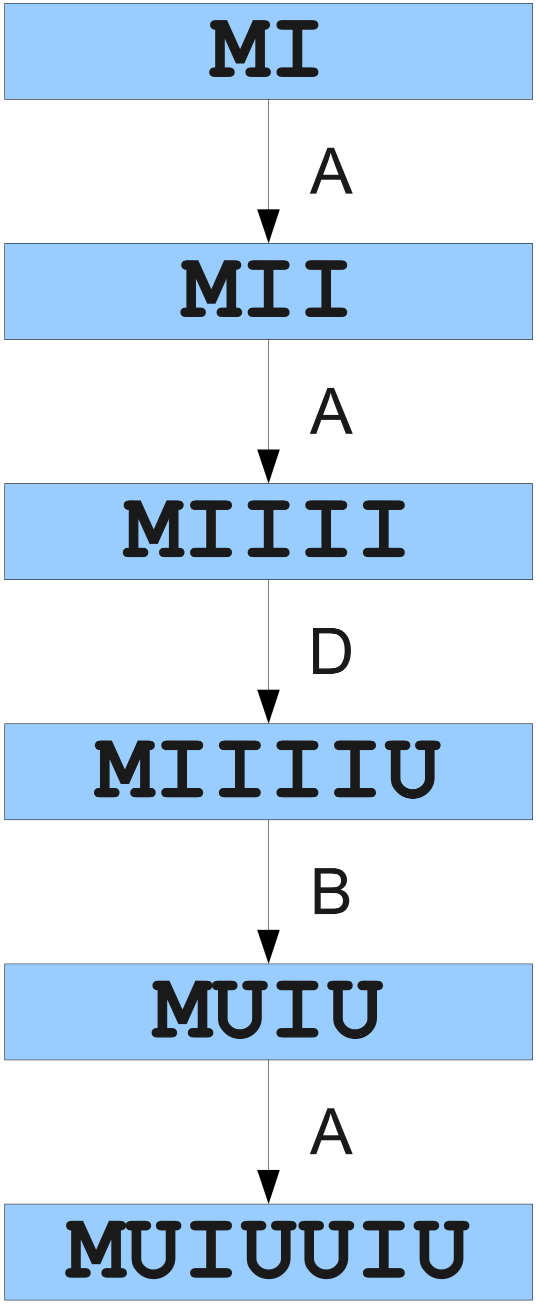

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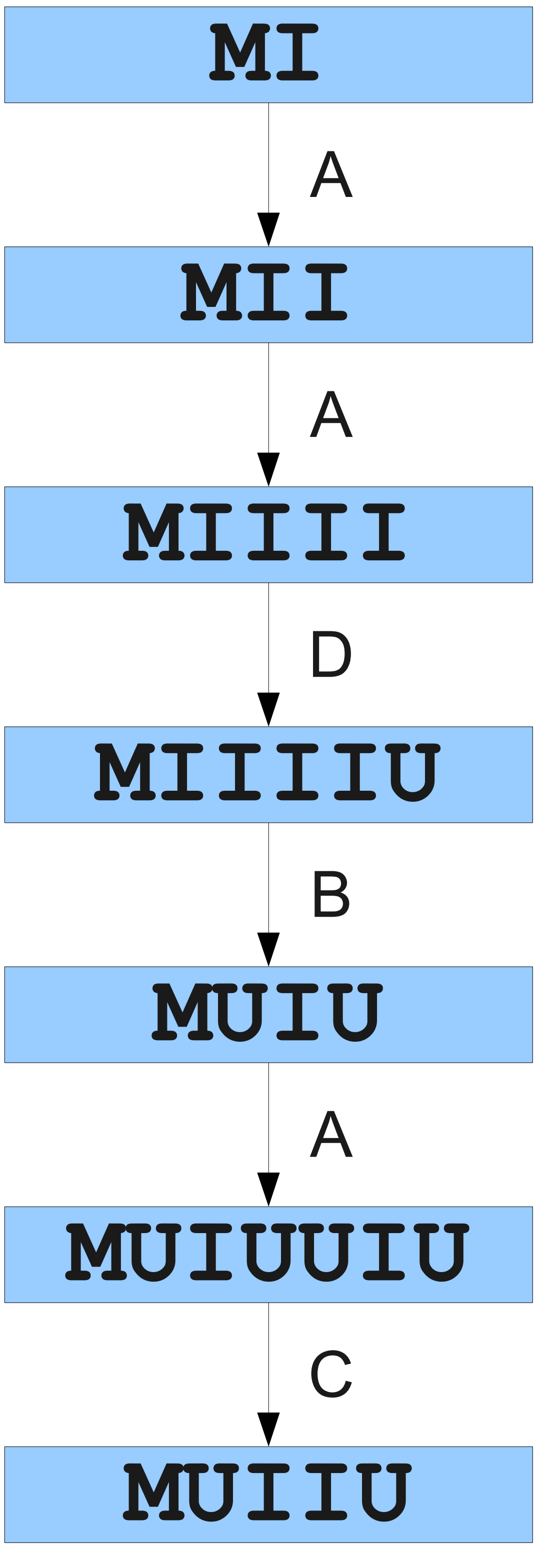


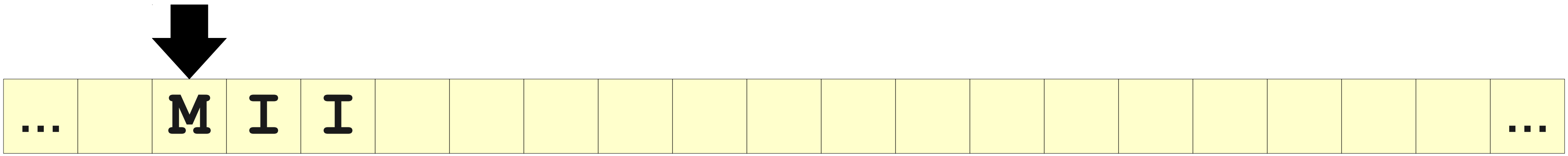
A Recognizable Language

- Let $\Sigma = \{$ `M`, `I`, `U` $\}$ and consider the language $L = \{ w \in \Sigma^* \mid$ Using the four provided rules, it is possible to convert w into `MU` $\}$
- Some strings are in this language (for example, `MU` $\in L$, `MIII` $\in L$, `MUUU` $\in L$).
- Some strings are not in this language (for example, `I` $\notin L$ `MI` $\notin L$ `MIIU` $\notin L$).
- Could we build a Turing machine for L?

TM Design Trick: Worklists

- It is possible to design TMs that search over an infinite space using a worklist.
- Conceptually, the TM
  - Finds all possible options one step away from the original input,
  - Appends each of them to the end of the worklist,
  - Clears the current option, then
  - Grabs the next element from the worklist to process.
- **This Turing machine is not guaranteed to halt.**

A Sketch of the Turing Machine



To recognize L, our TM M does the following:

- While M has not found the string **`MU`**:
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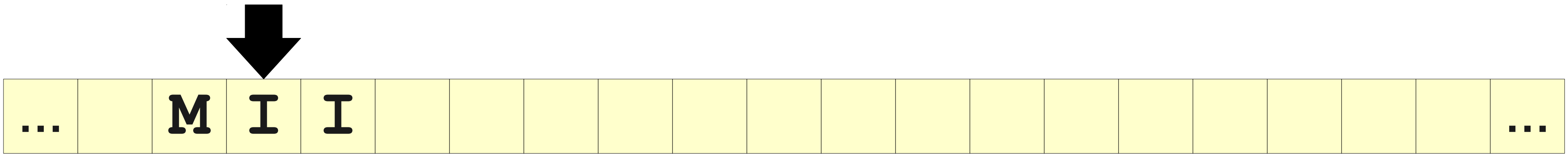

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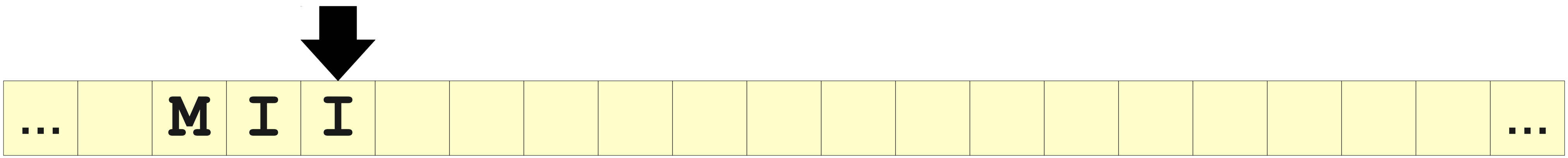

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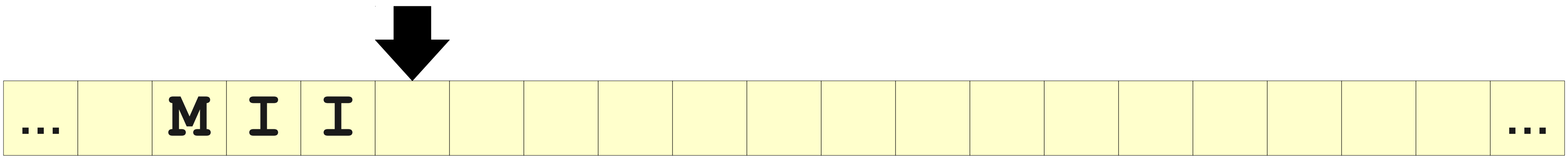

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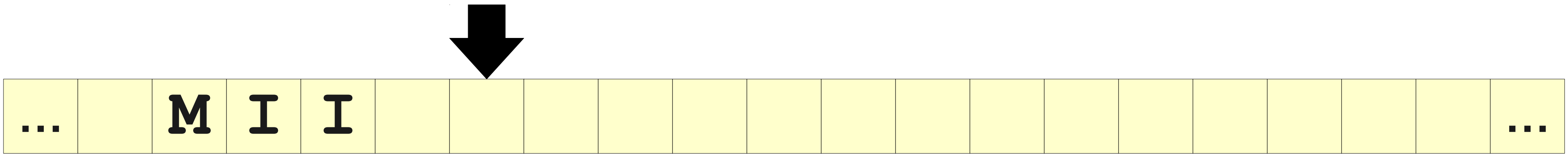

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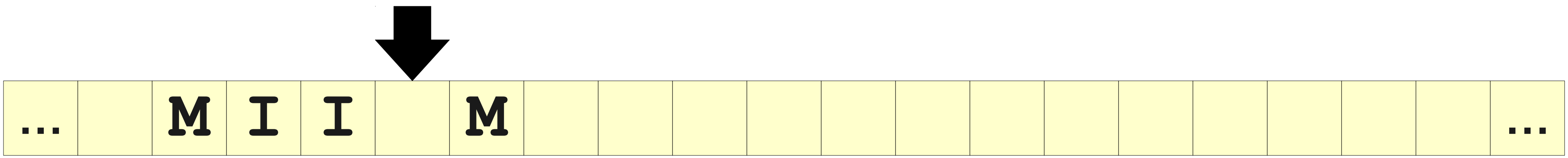

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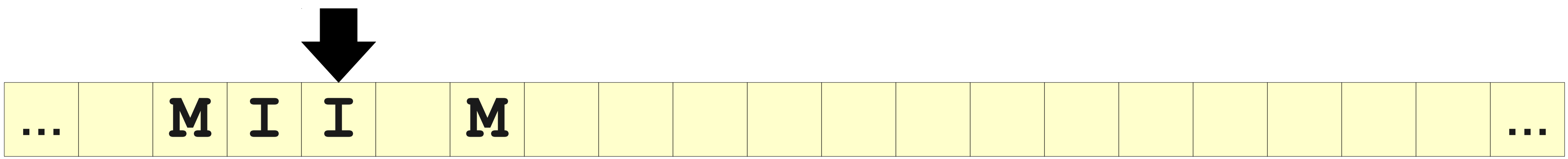

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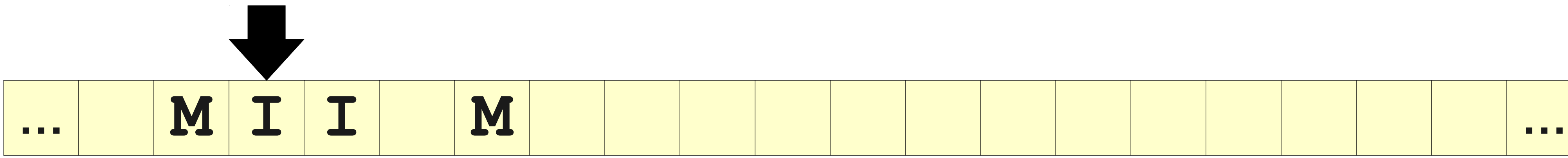

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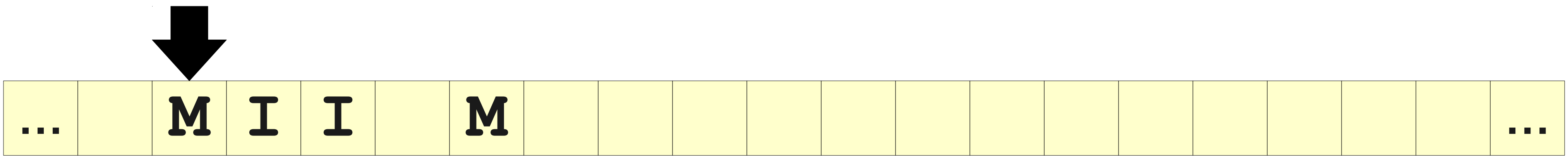

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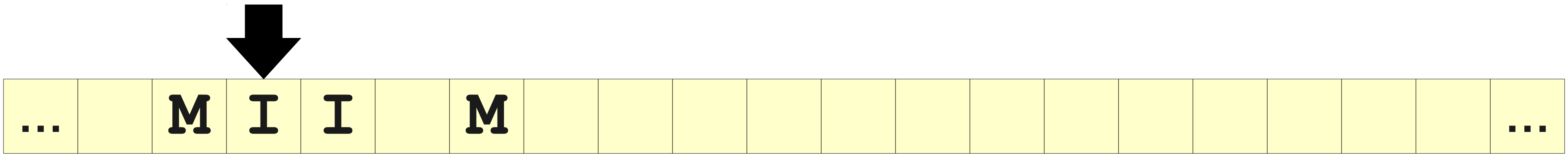

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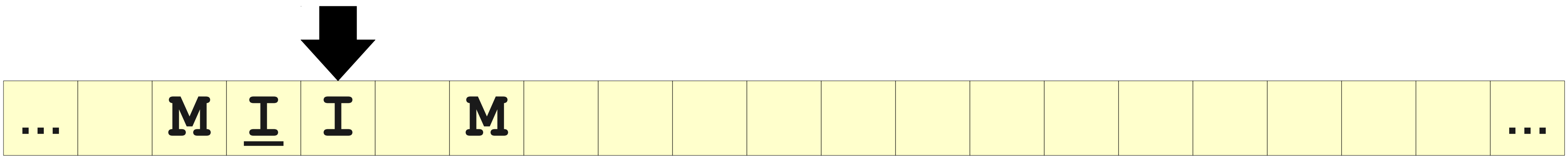

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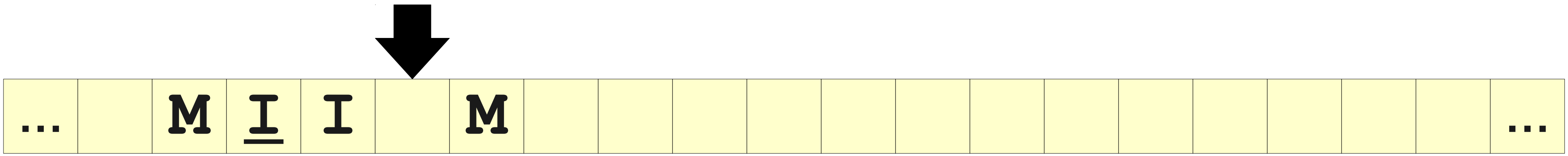

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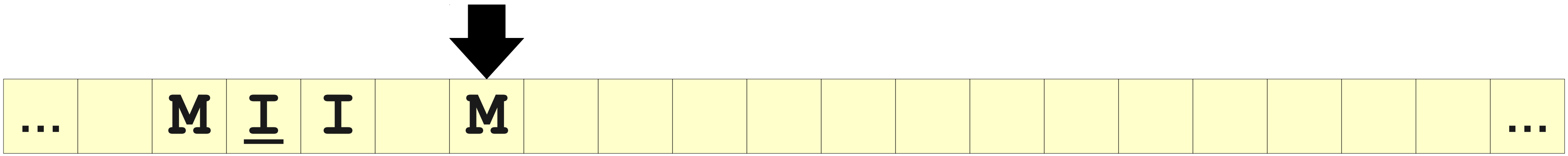

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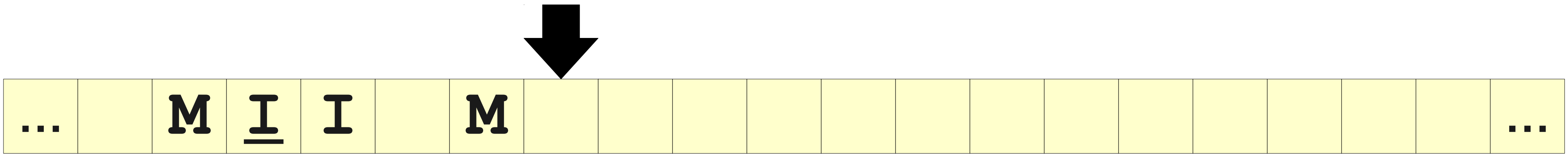

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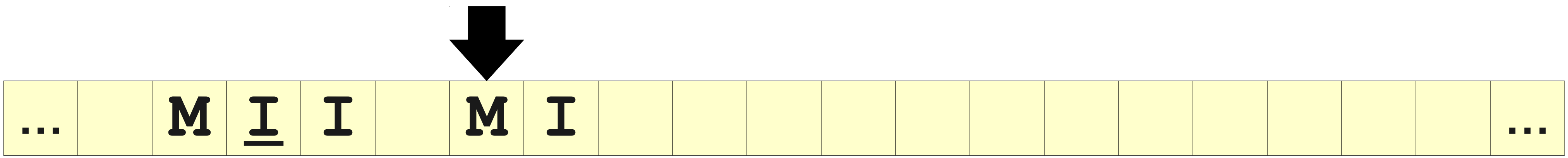

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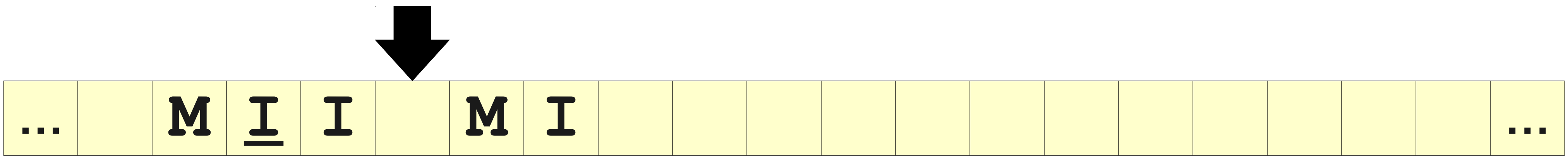

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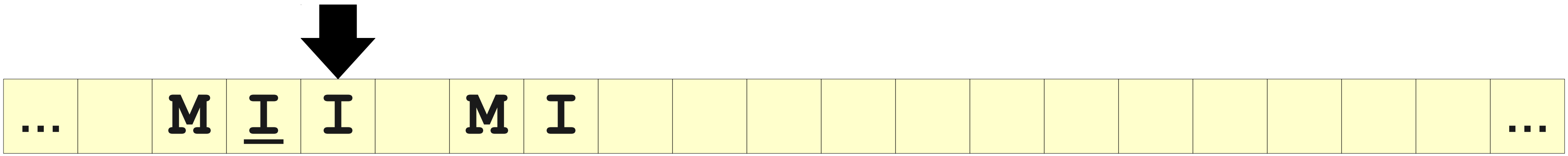

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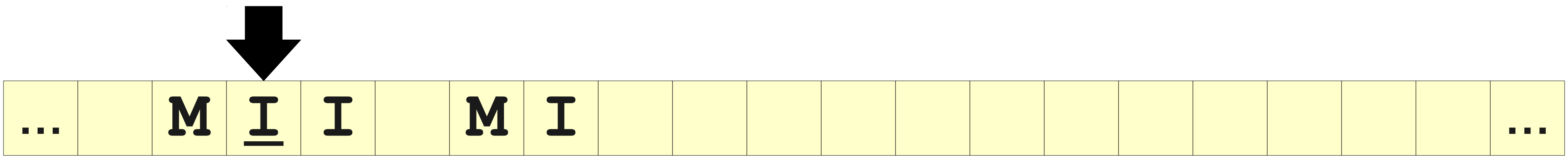

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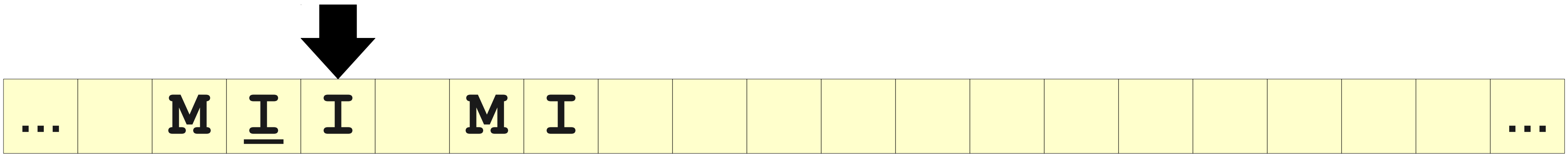

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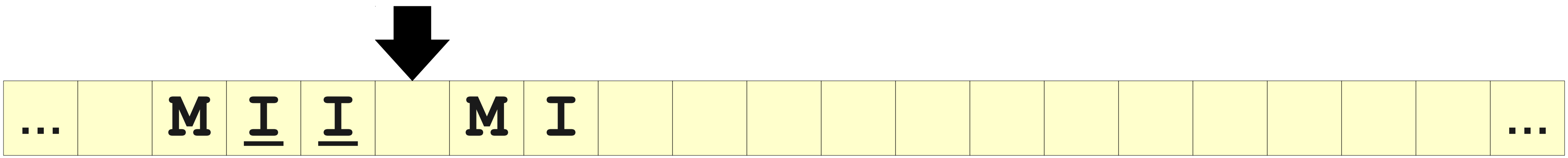

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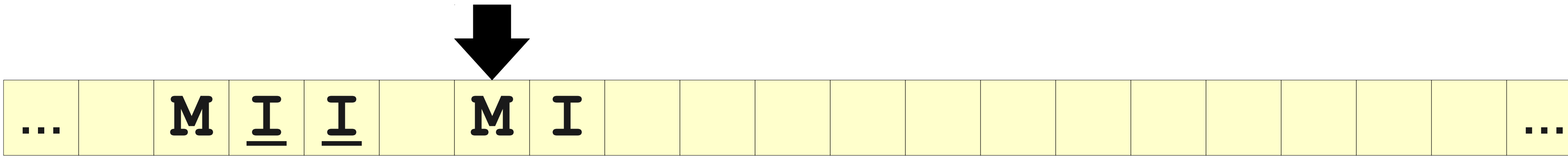

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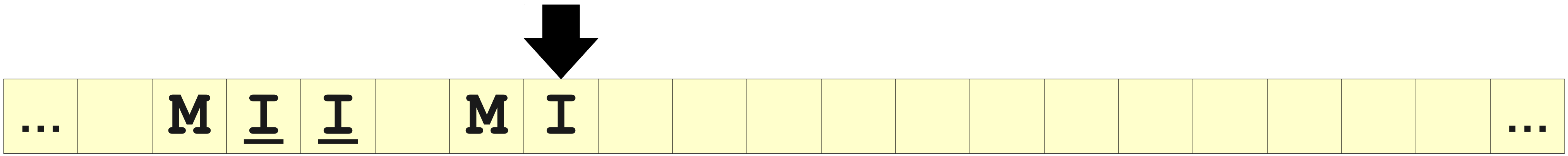

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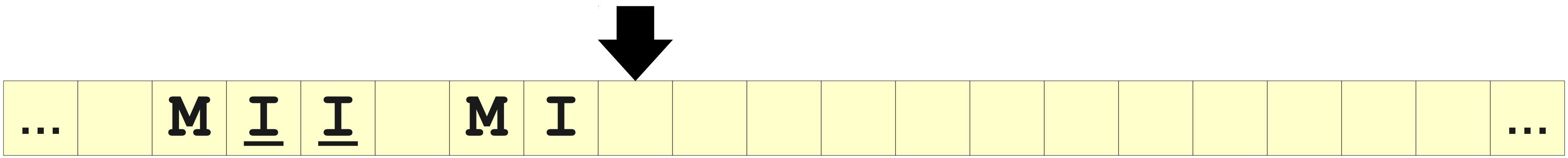

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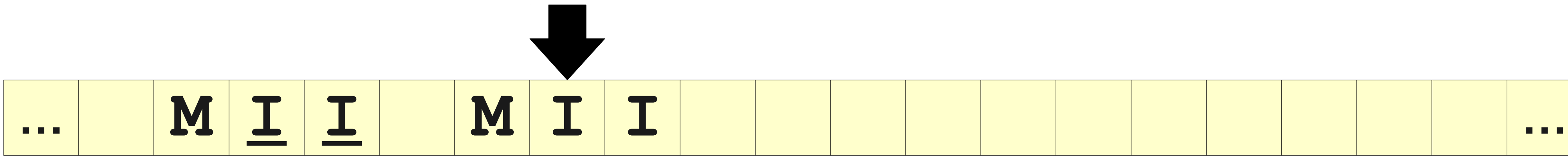

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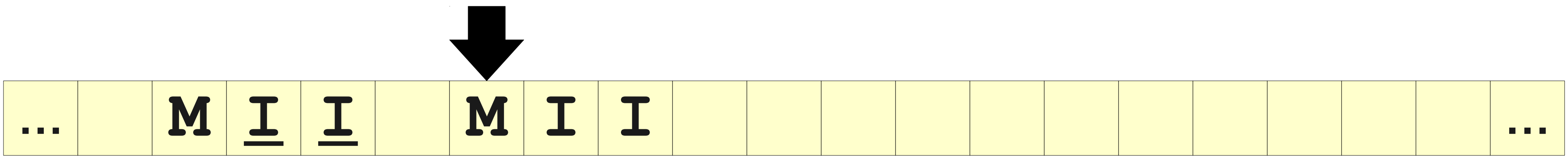

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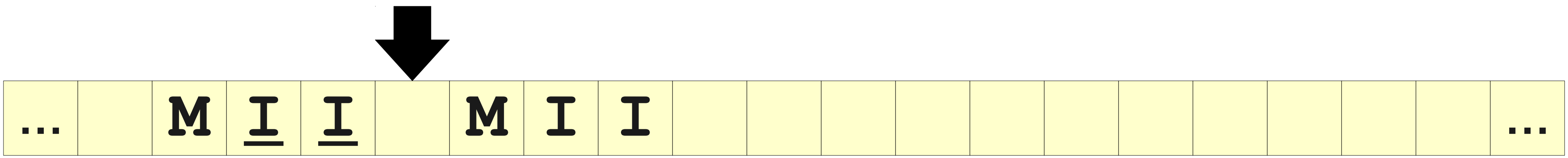

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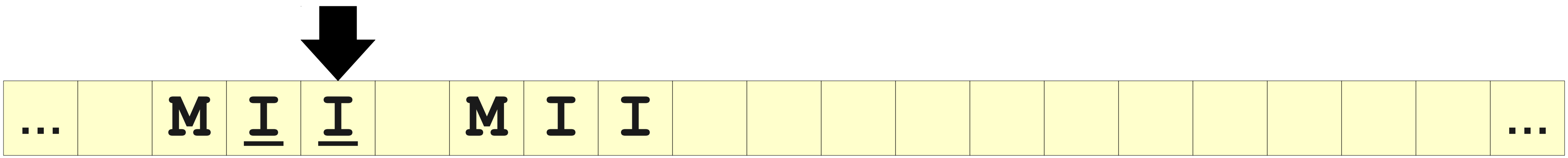

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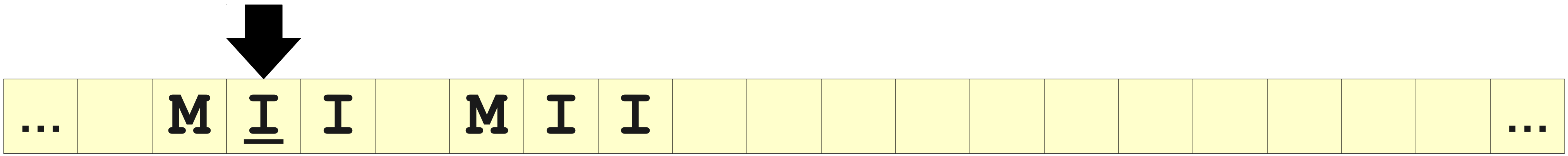

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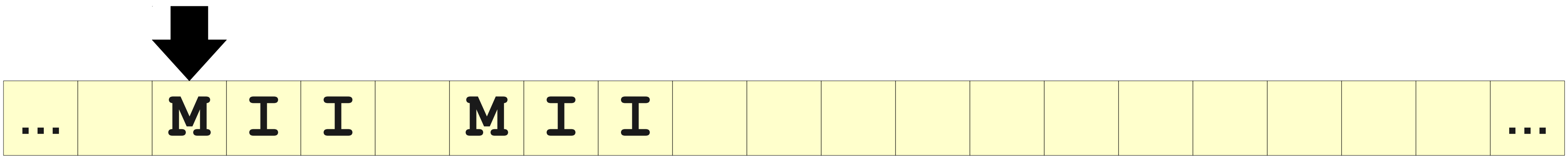

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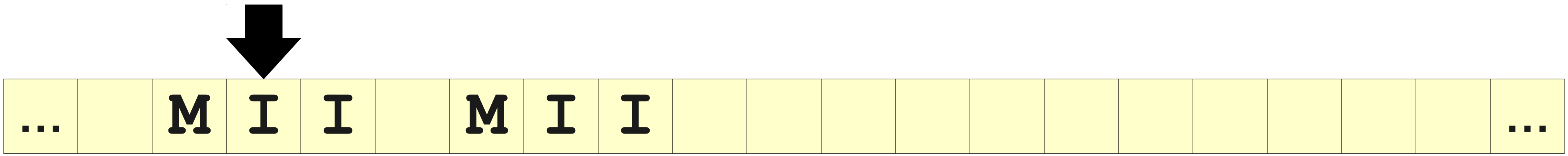

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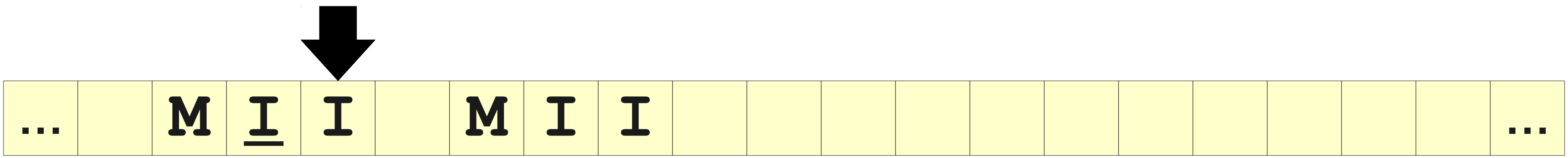

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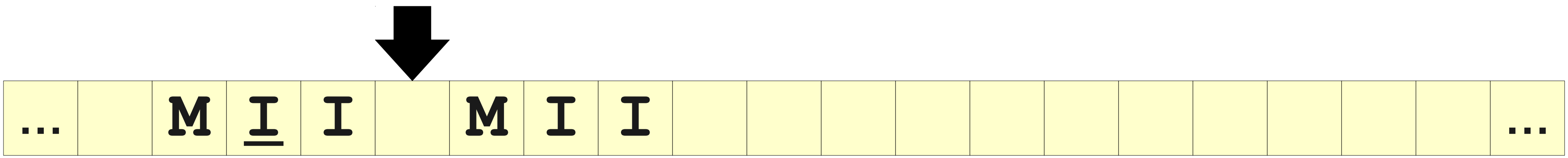

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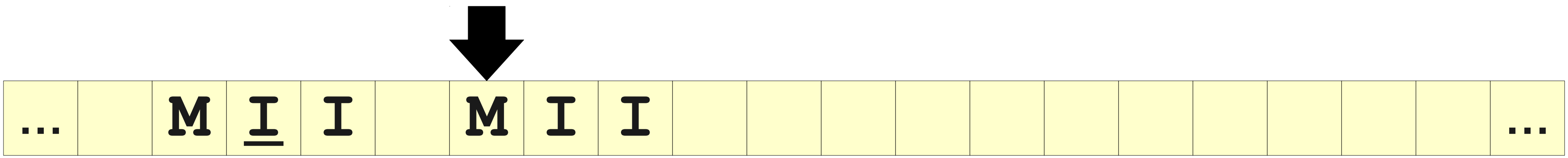

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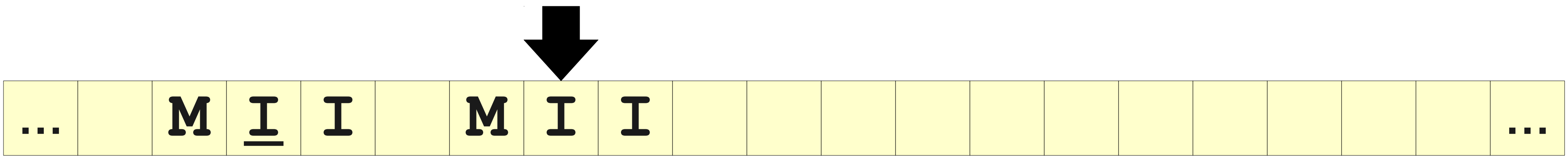

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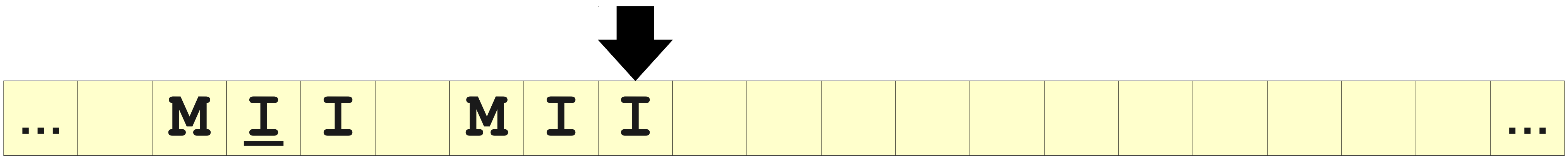

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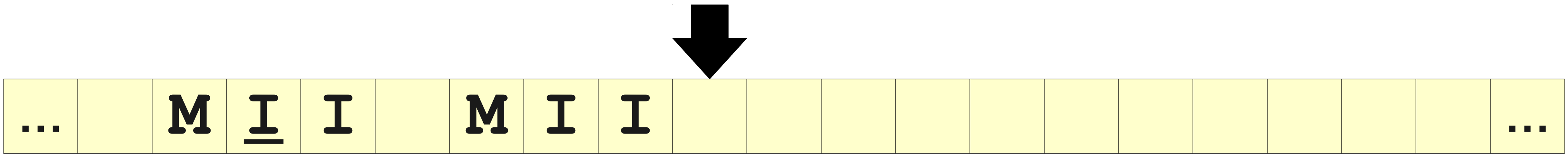

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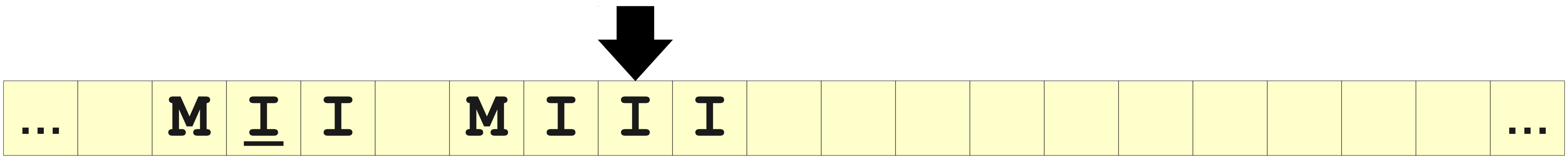

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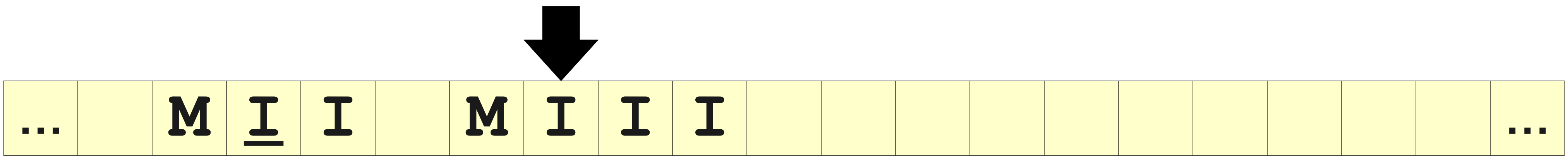

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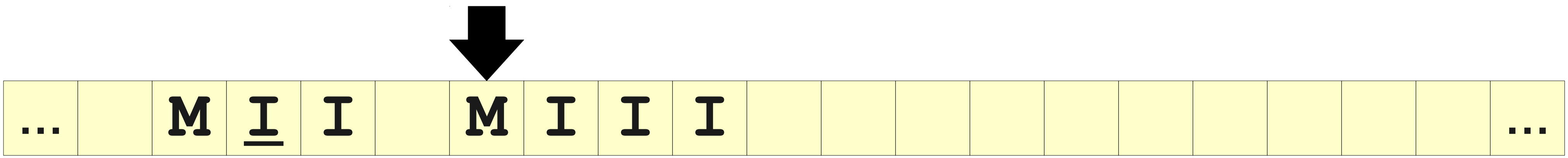

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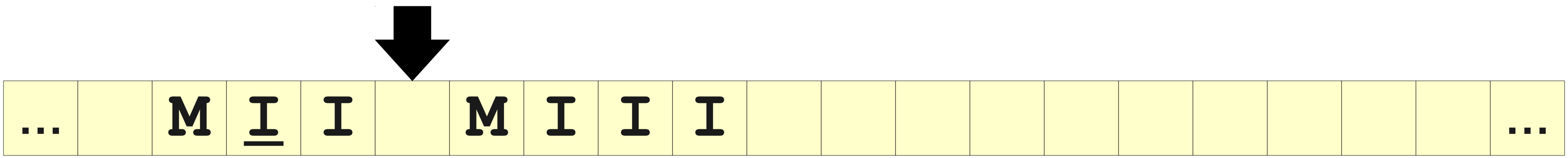

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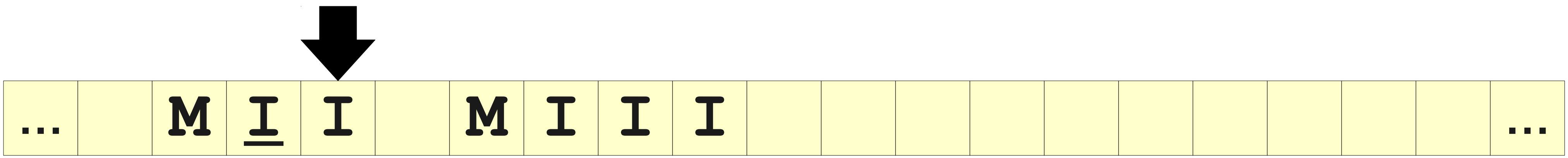

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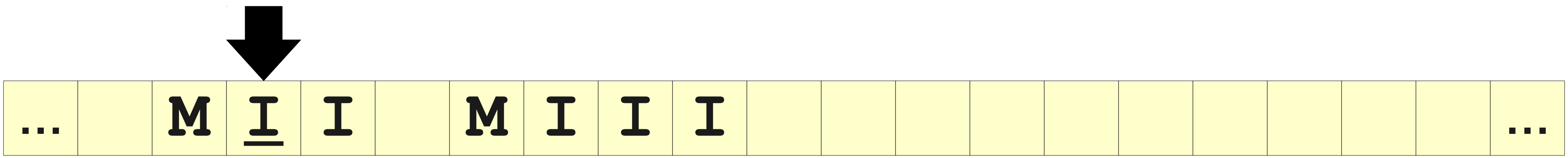

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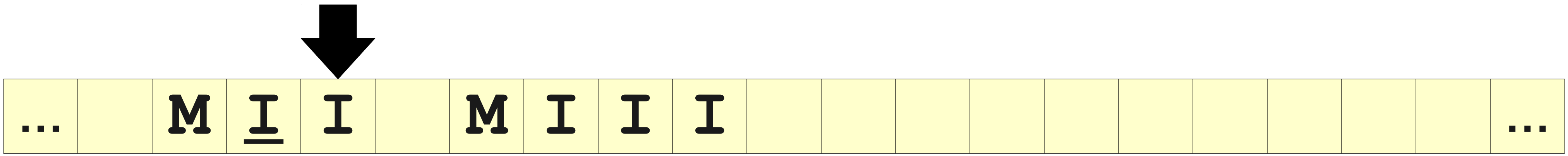

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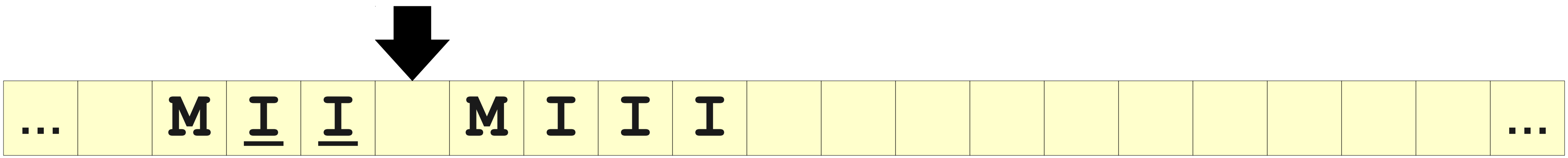

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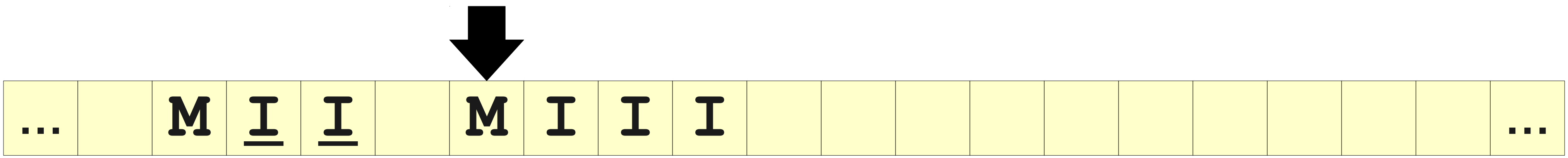

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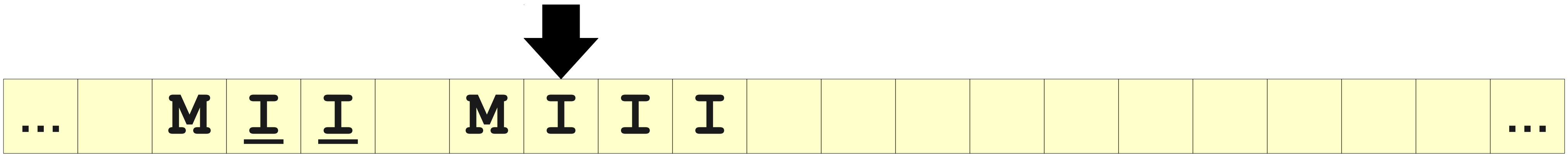

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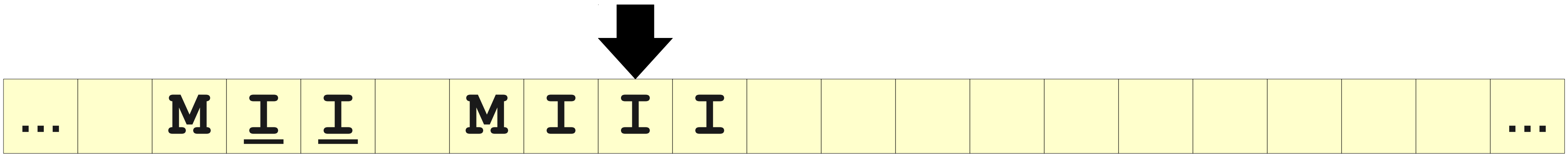

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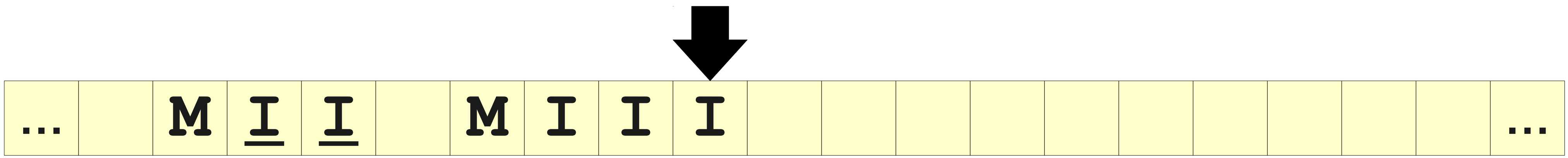

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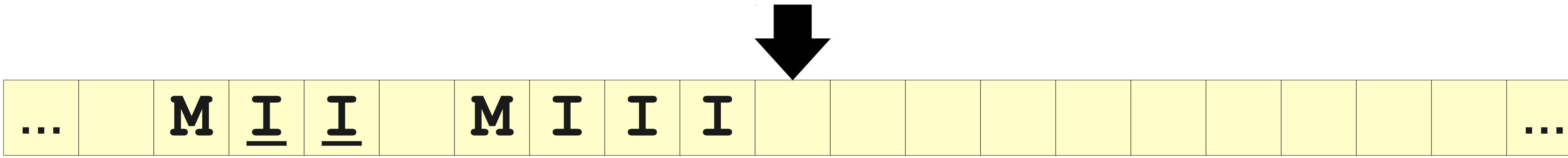

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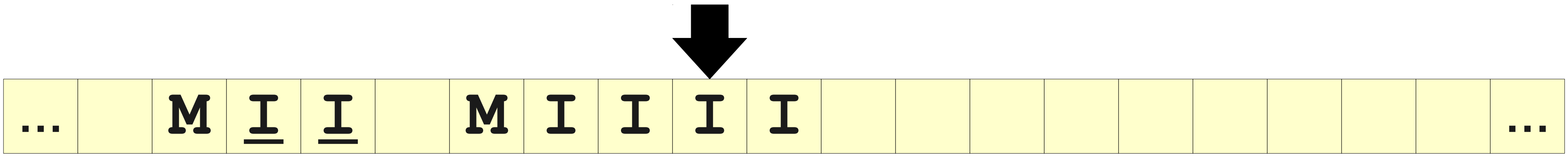

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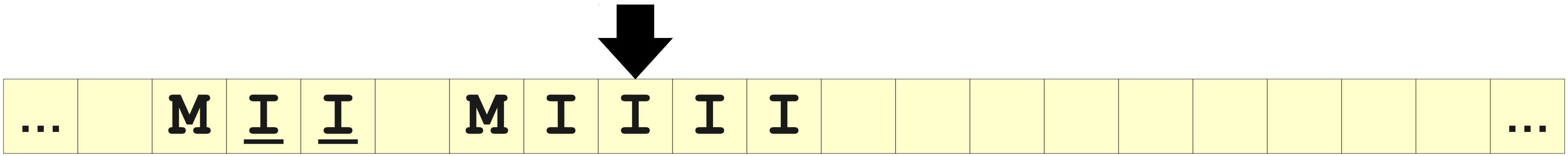

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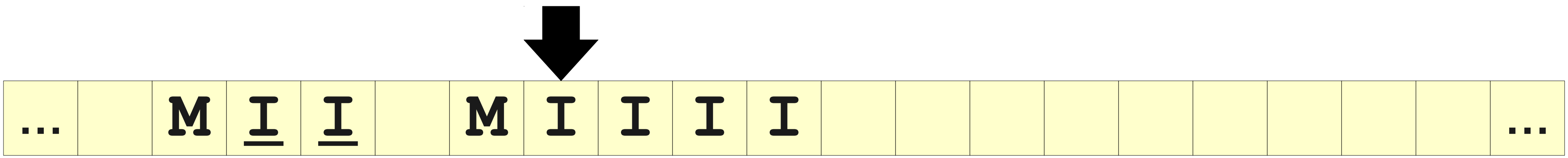

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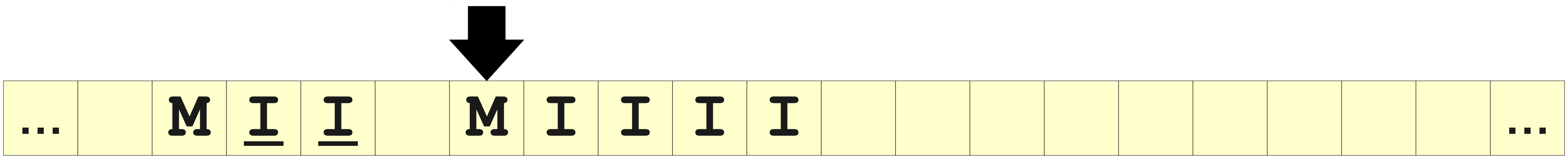

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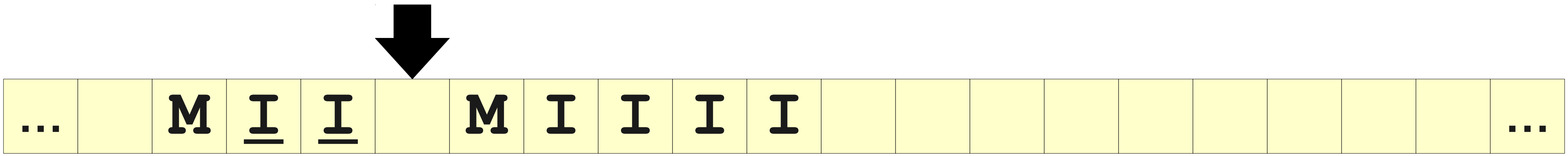

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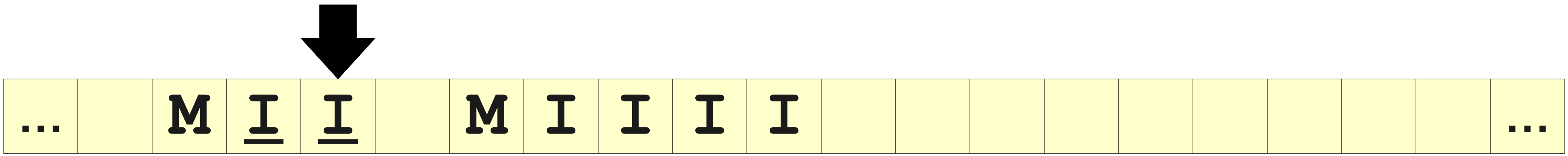

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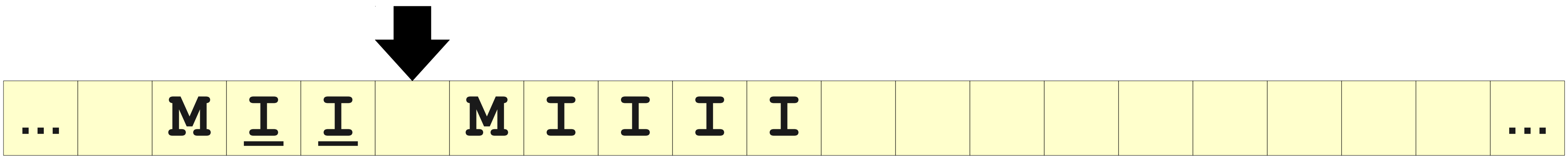

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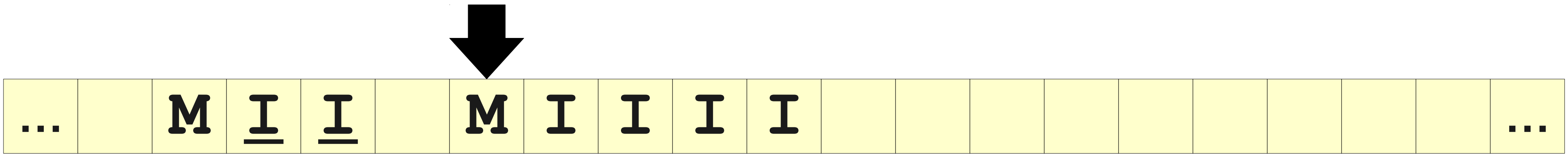

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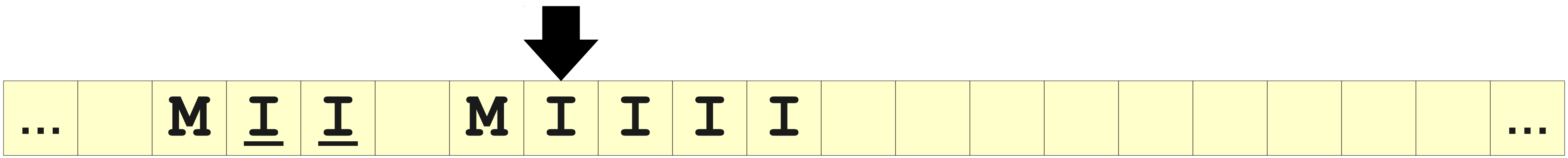

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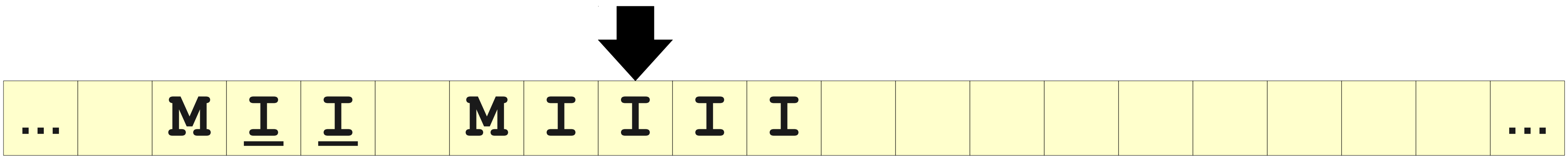

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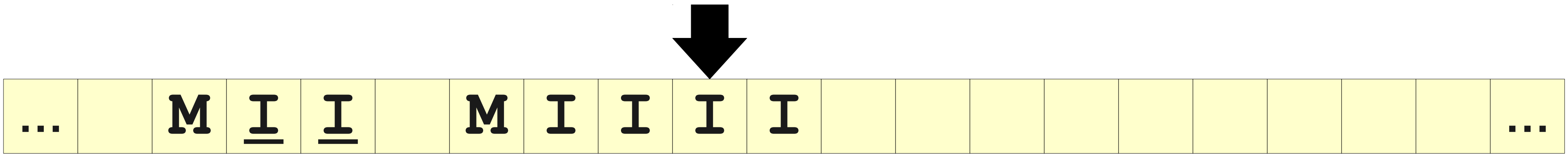

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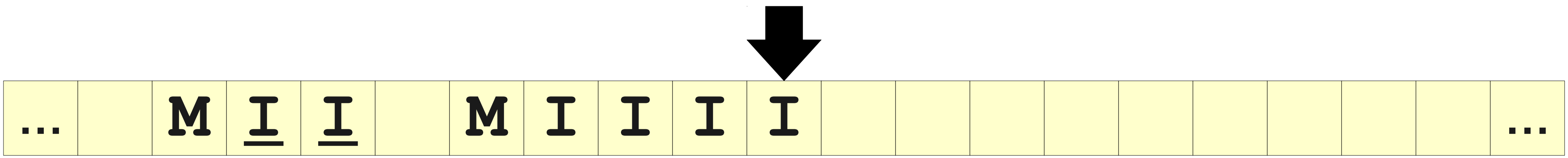

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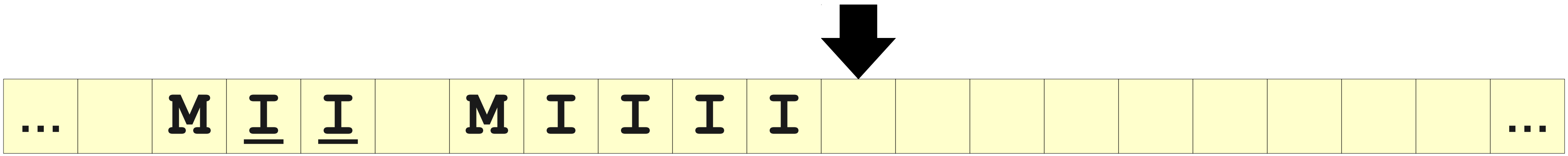

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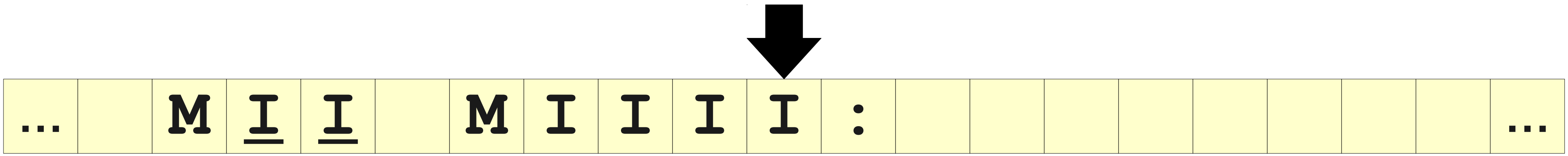

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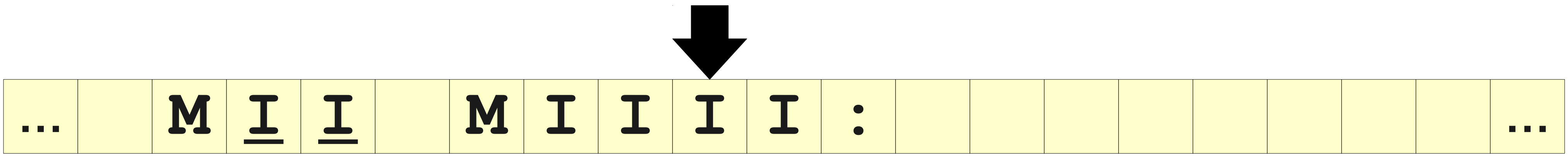

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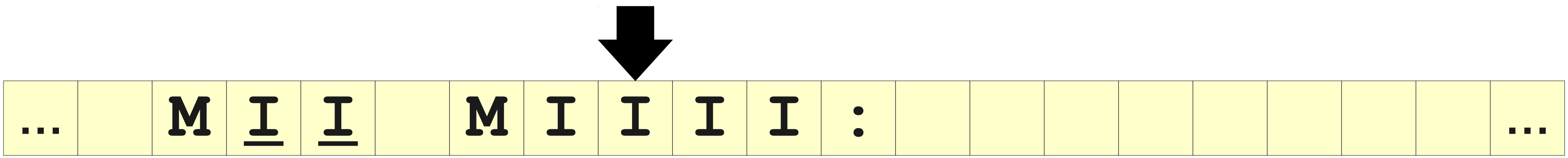

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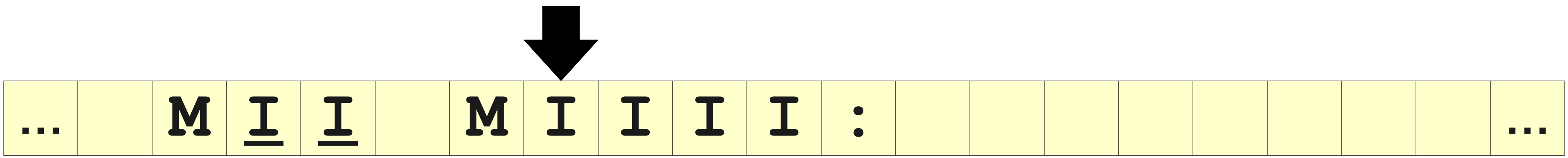

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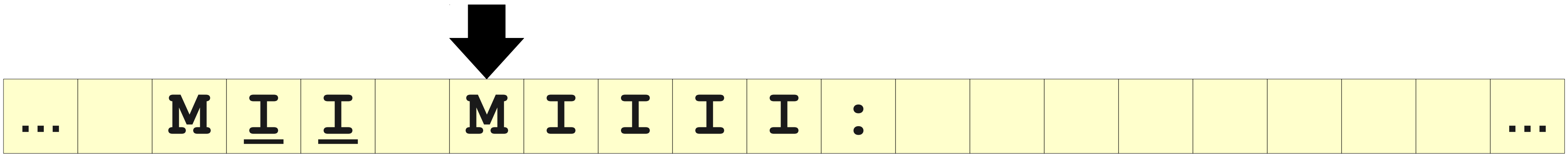

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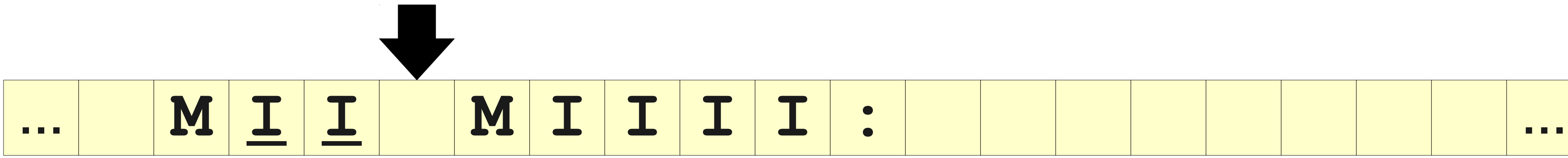

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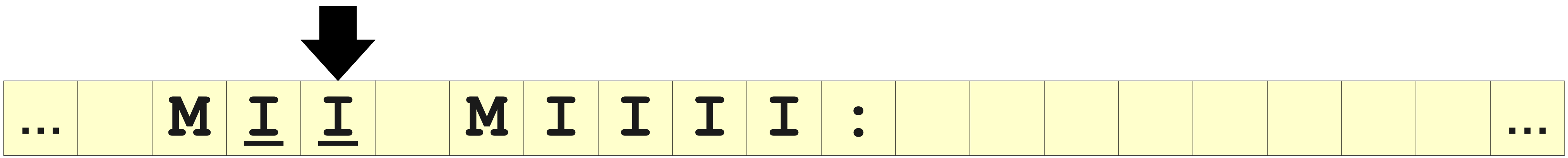

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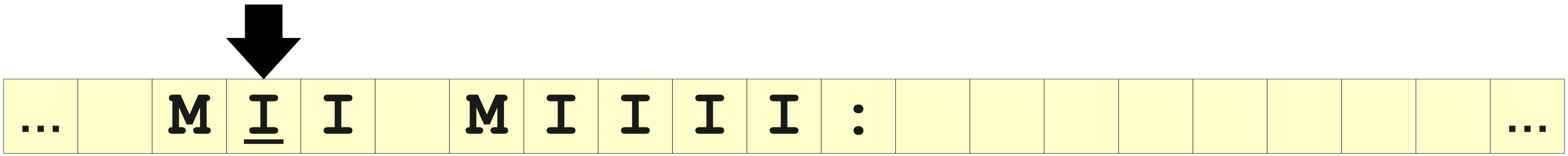

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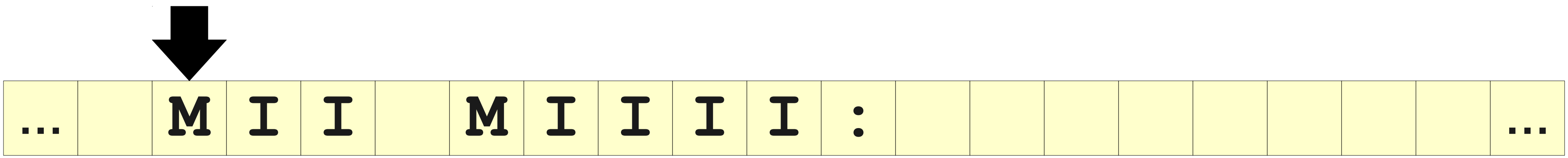

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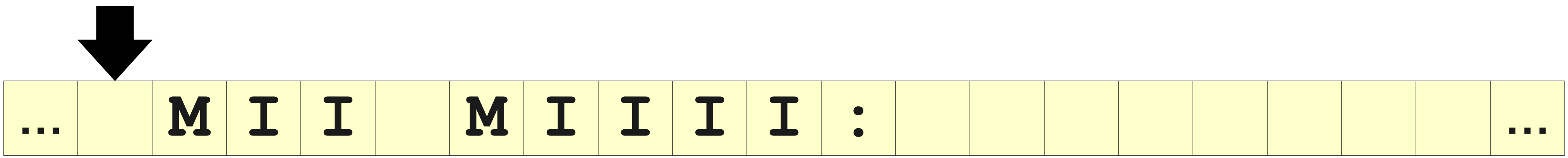

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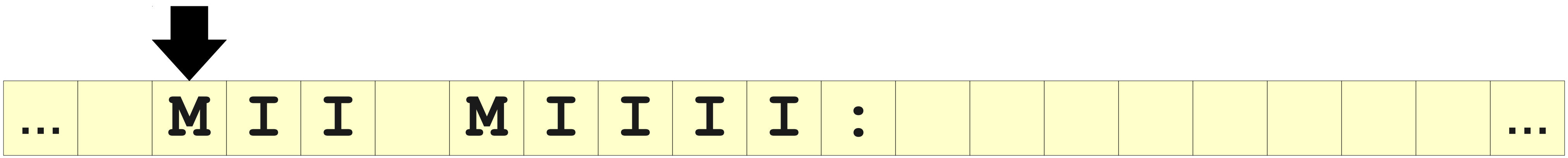

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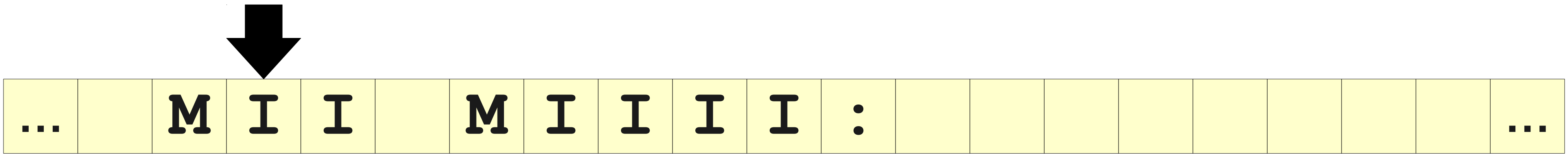

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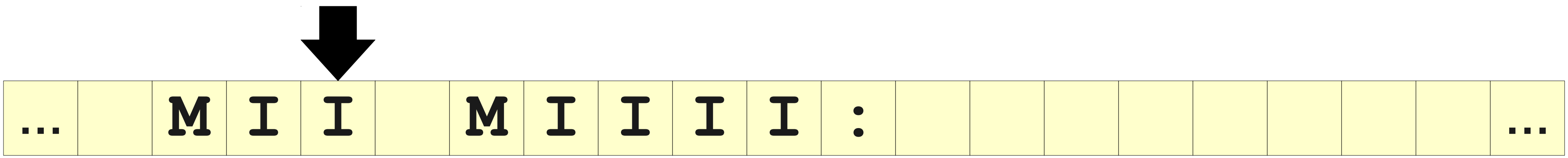

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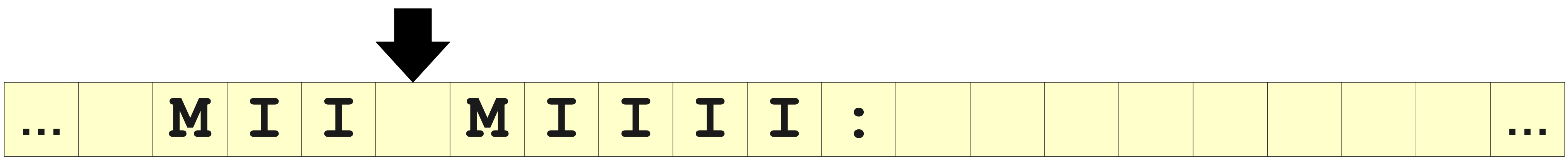

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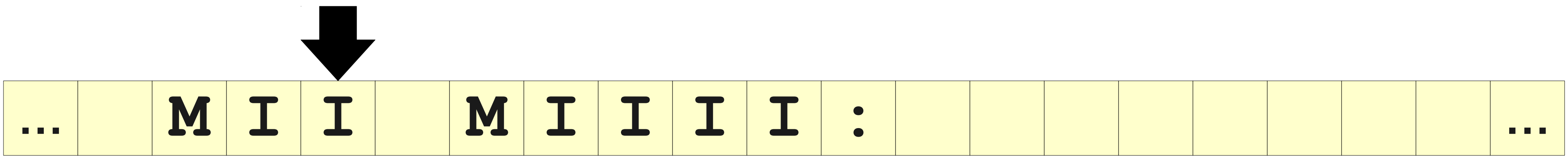

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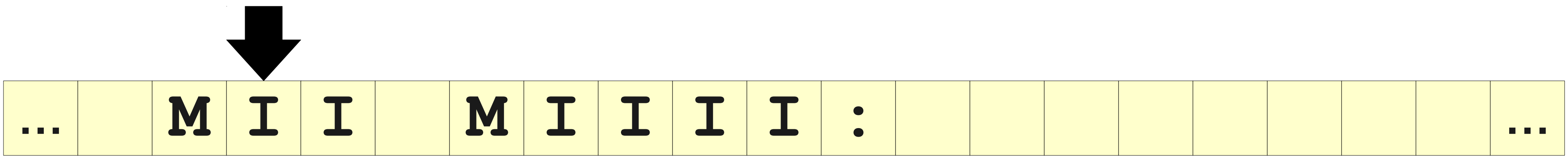

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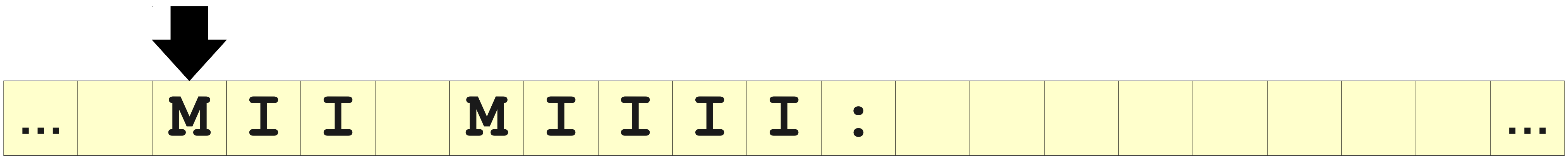

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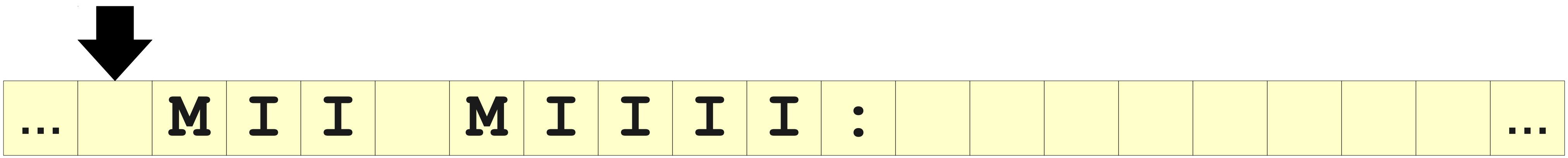

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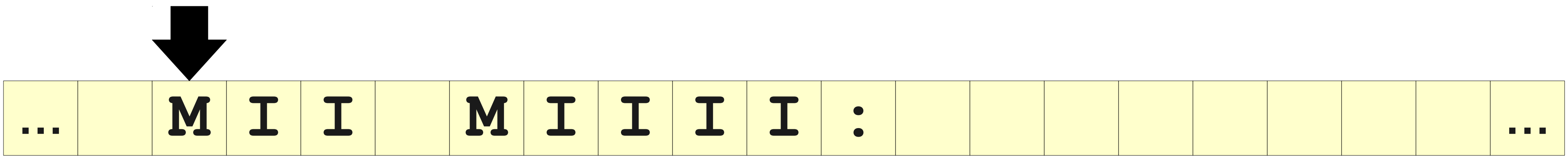

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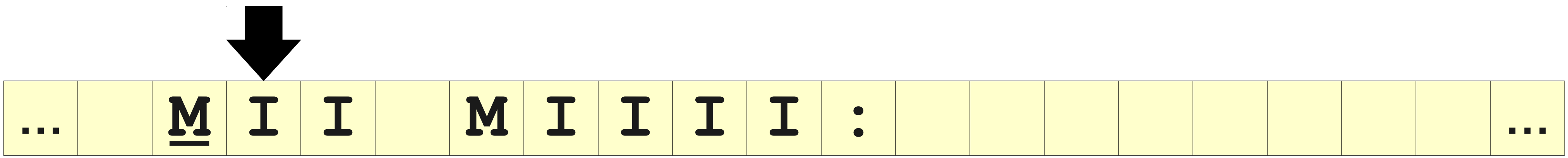

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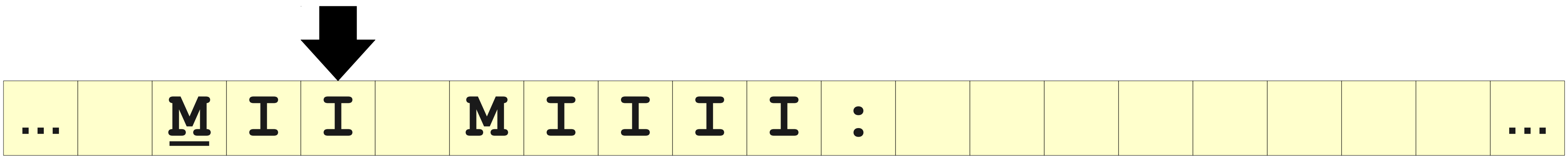

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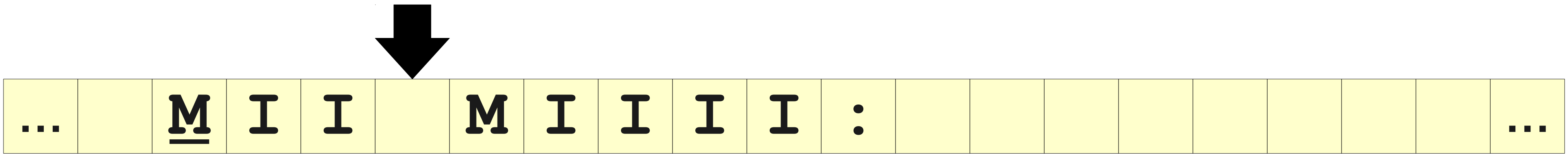

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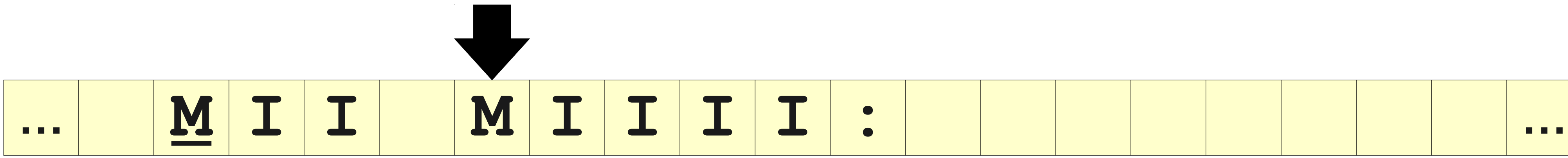

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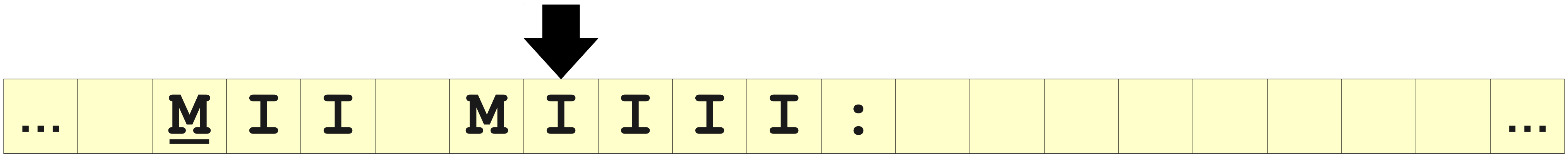

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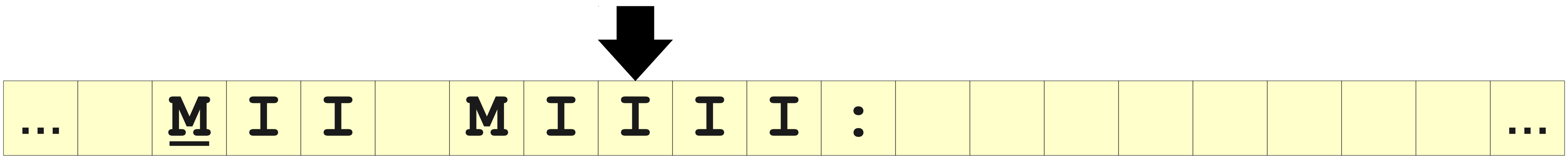

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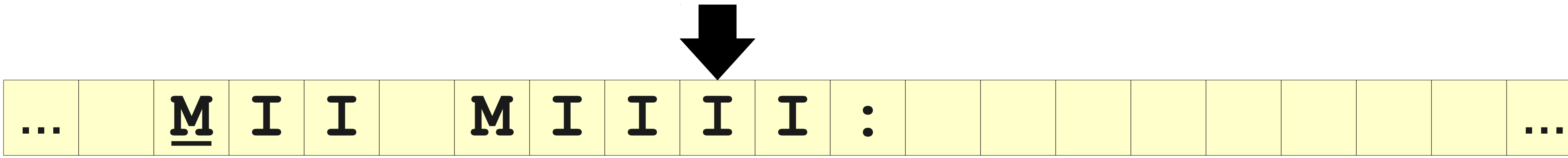

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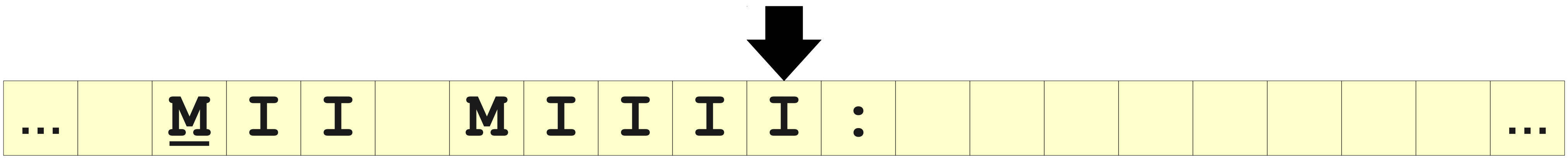

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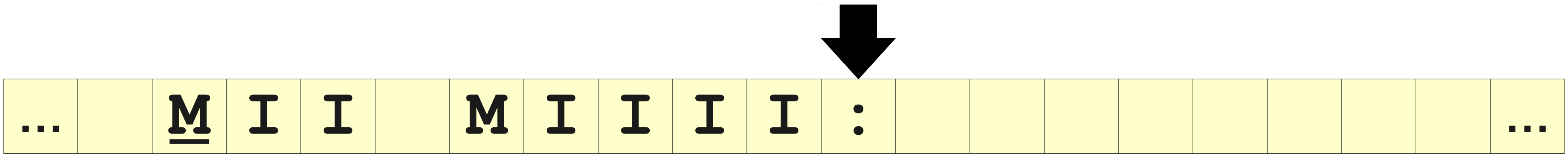

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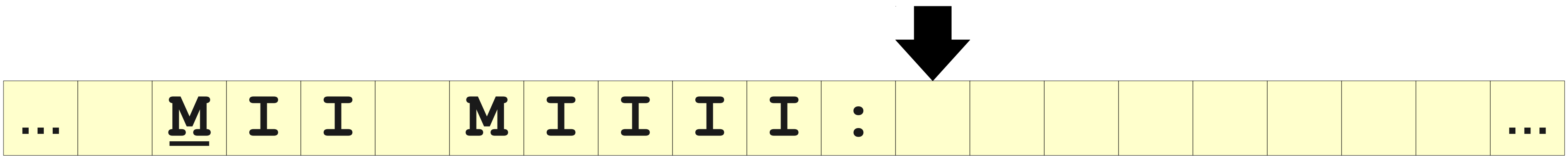

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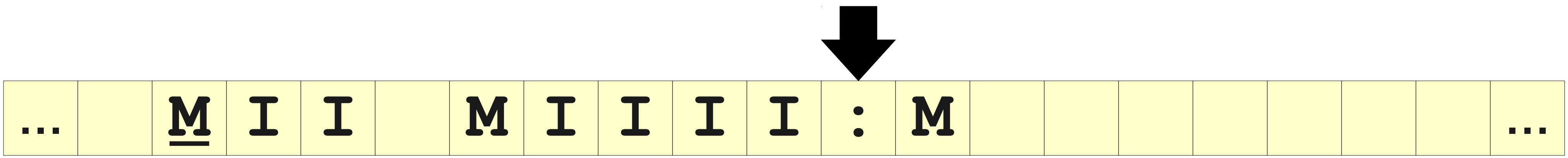

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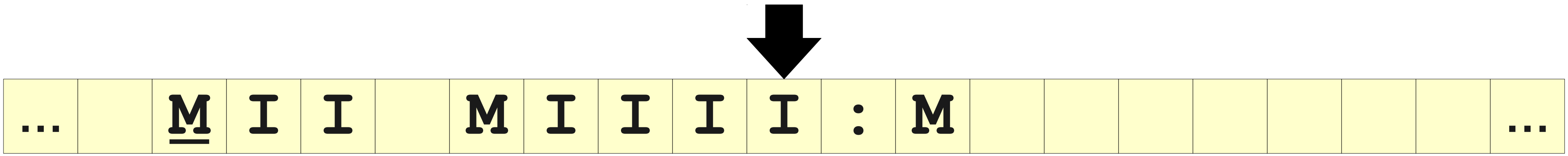

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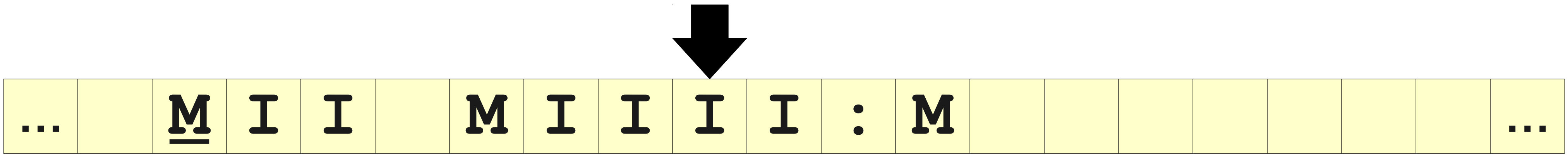

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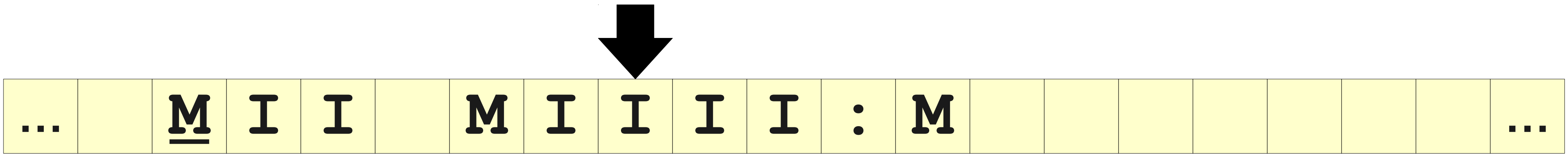

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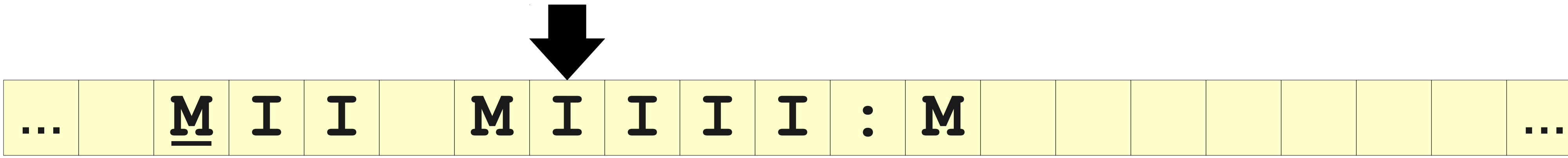

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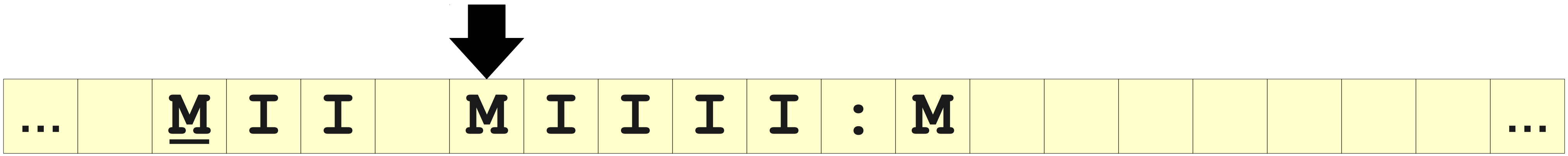

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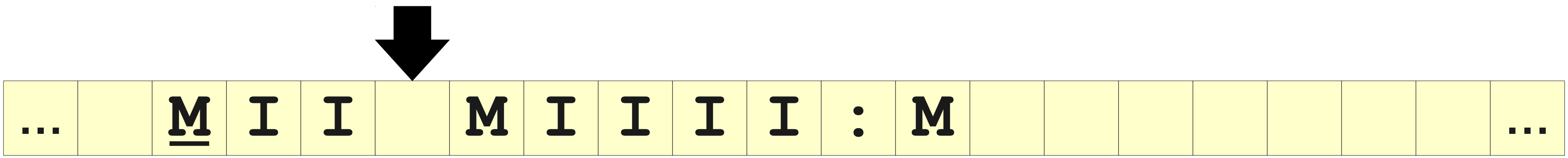

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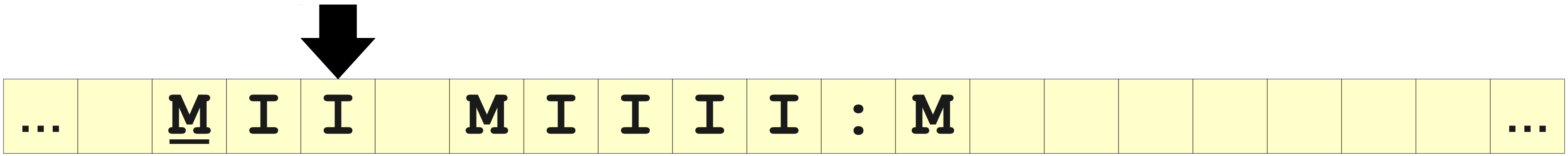

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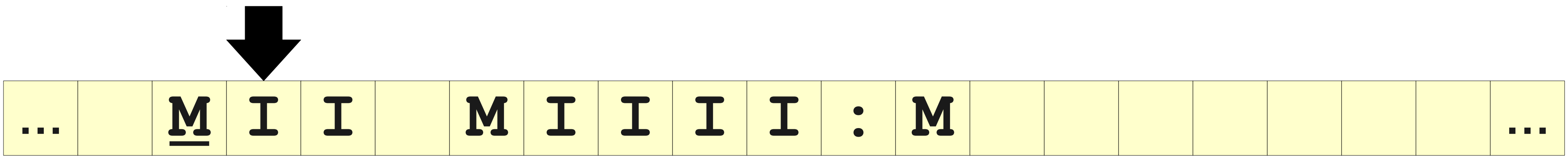

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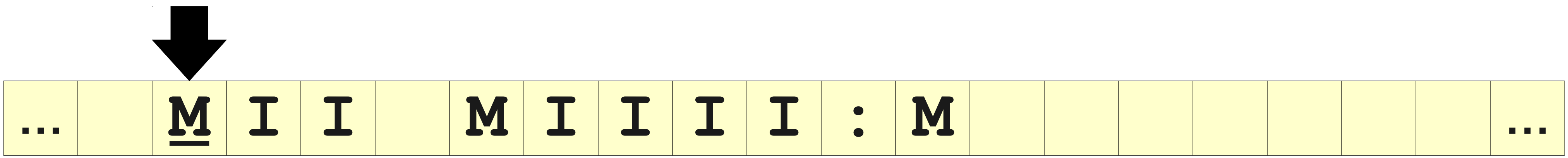

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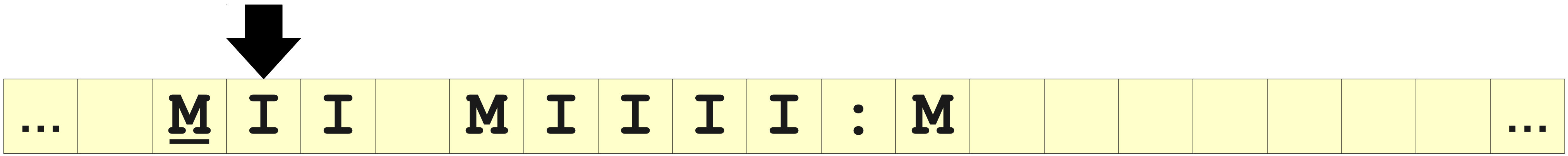

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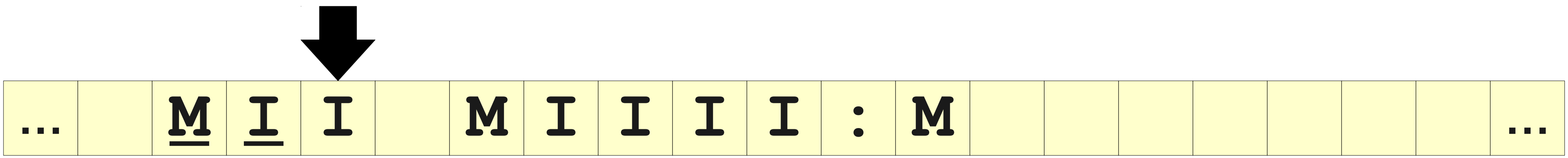

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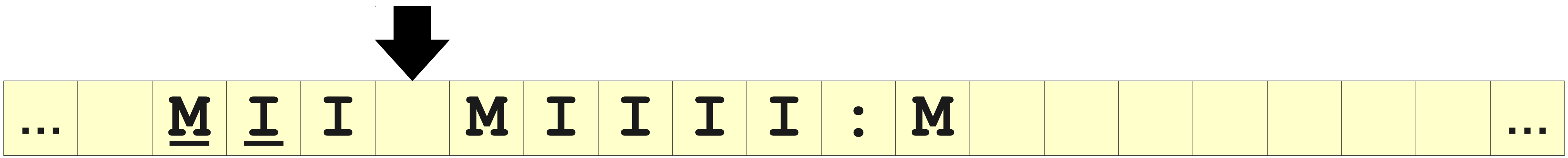

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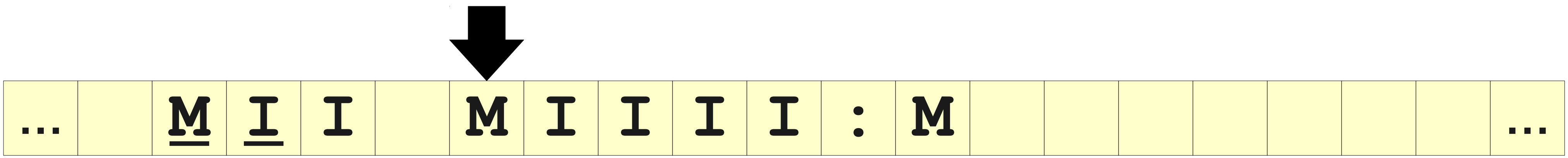

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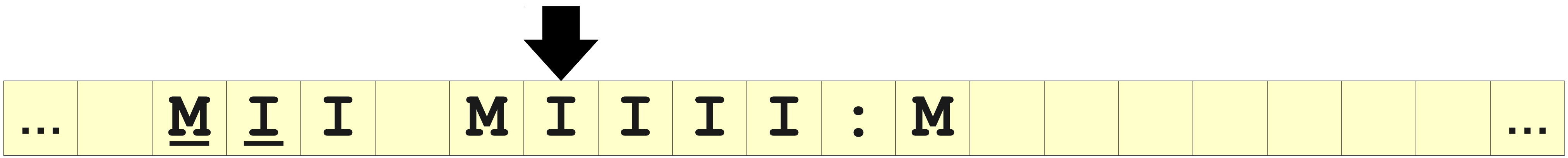

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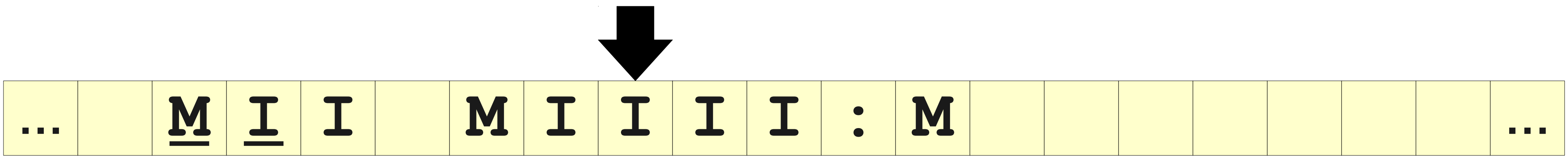

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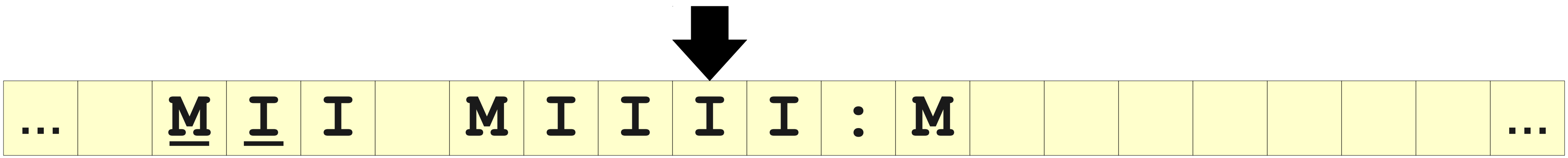

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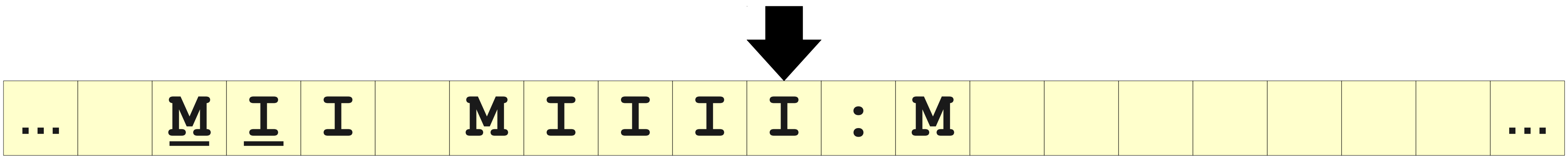

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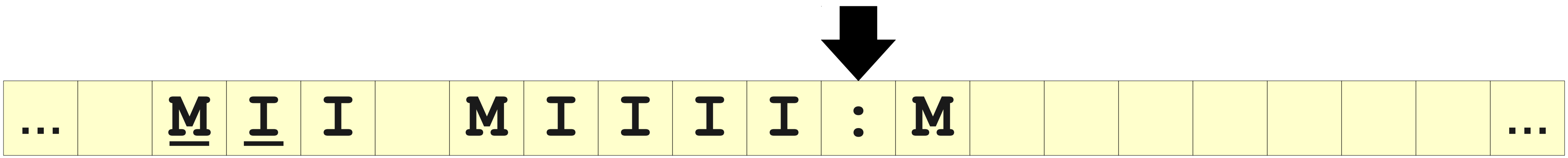

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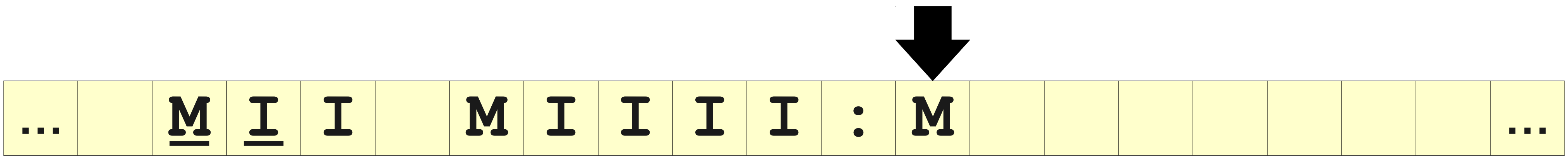

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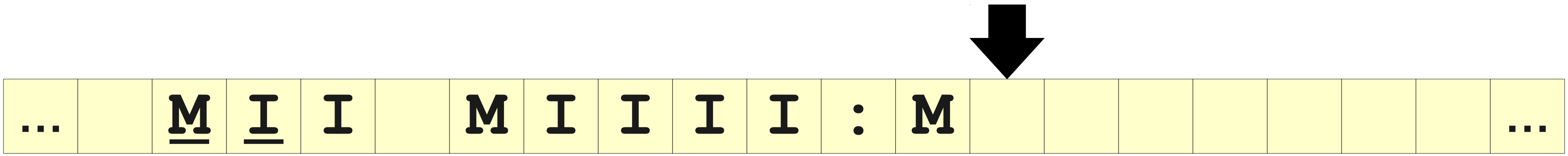

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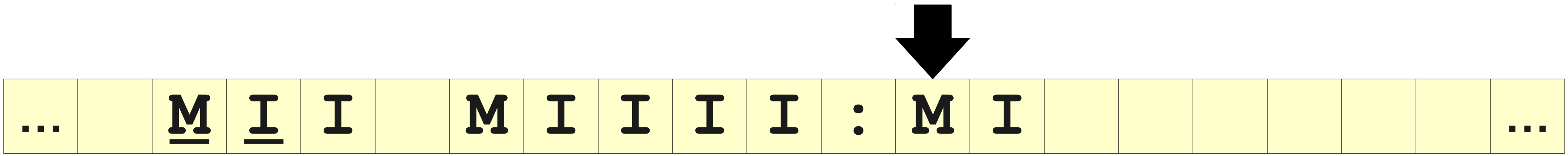

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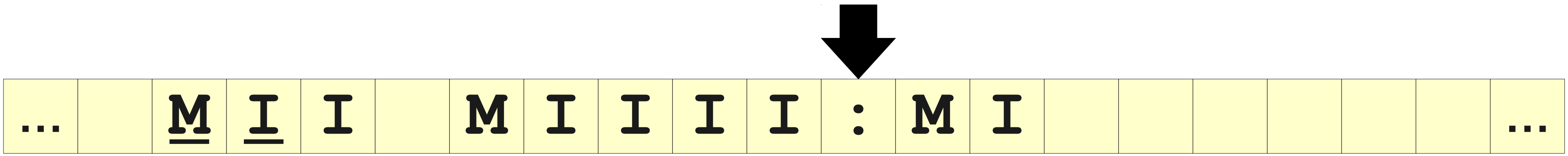

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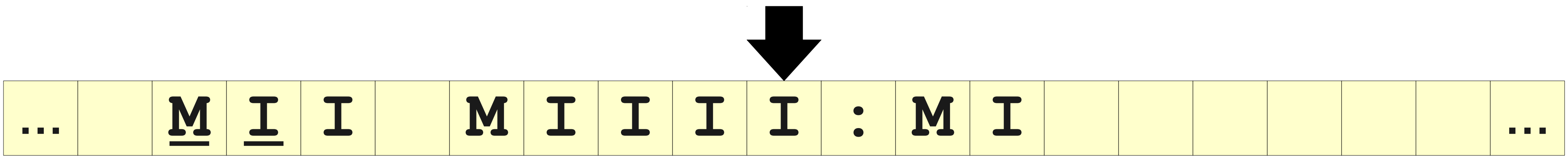

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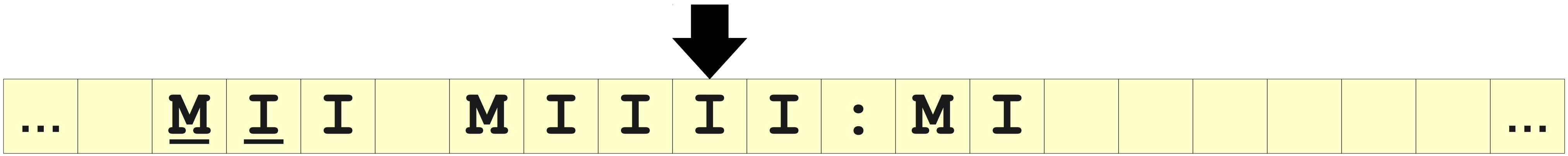

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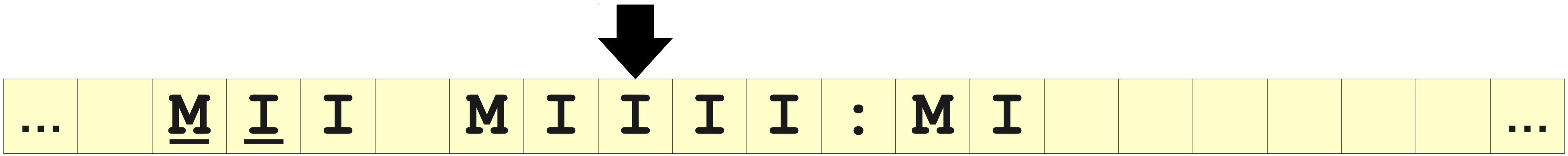

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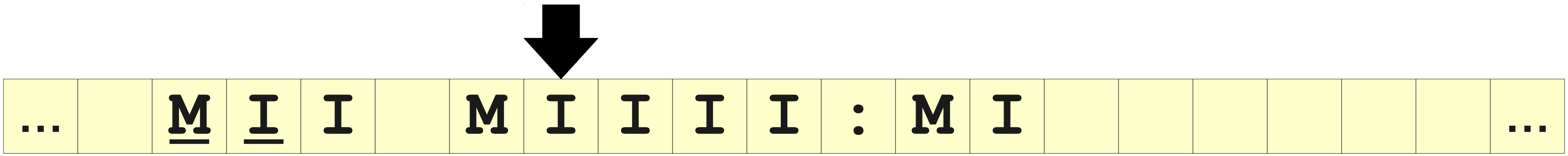

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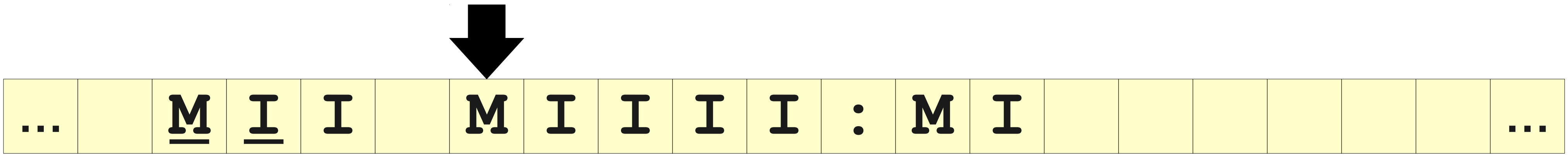

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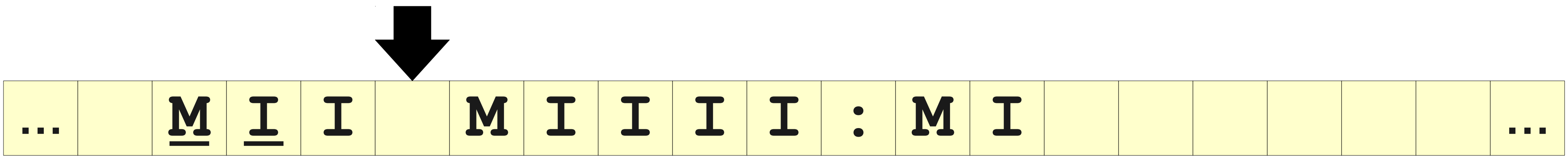

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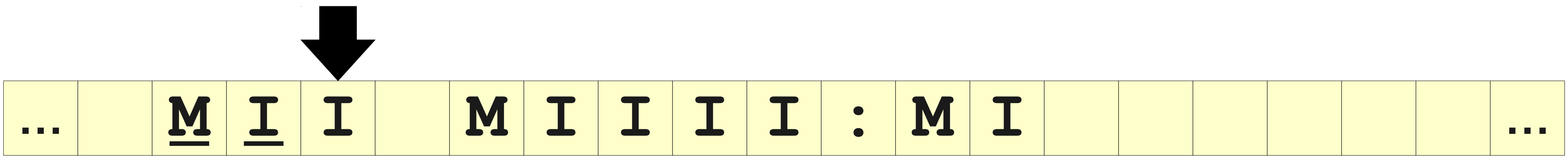

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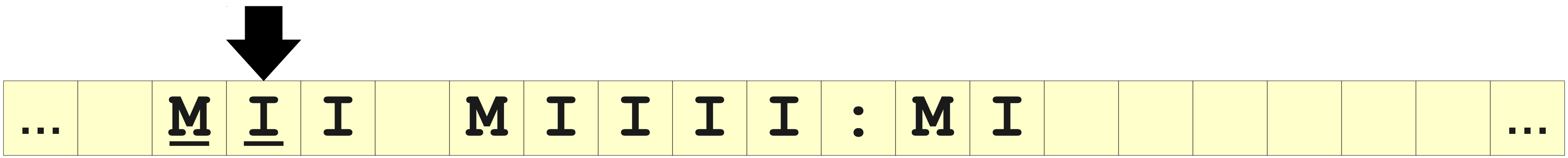

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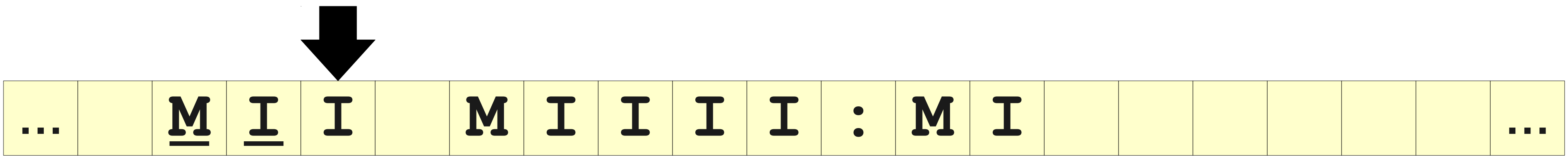

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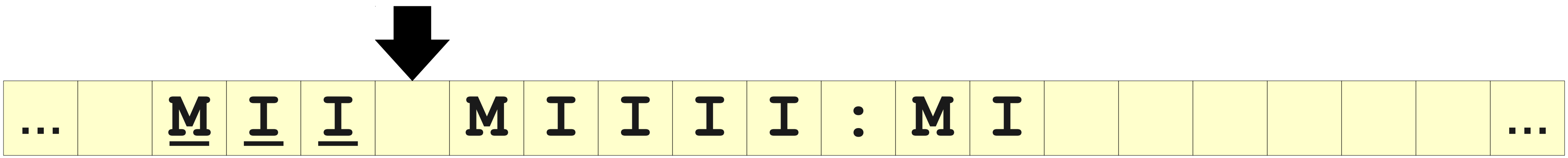

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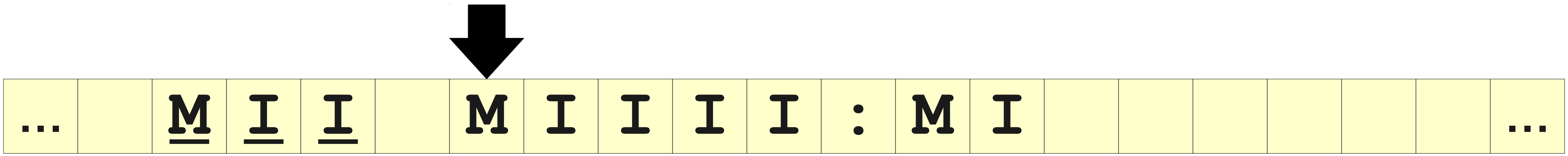

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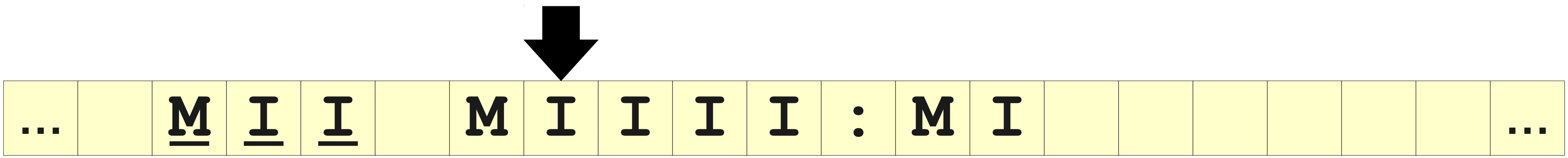

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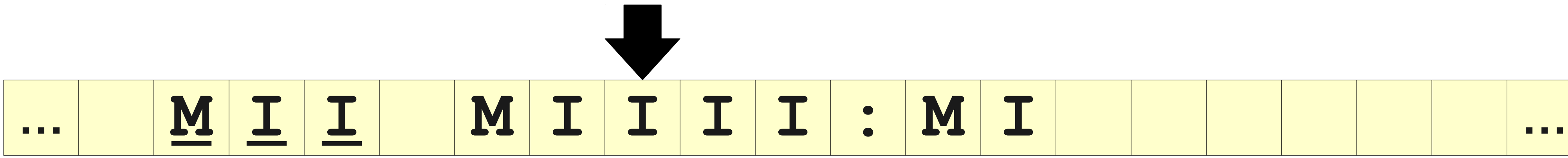

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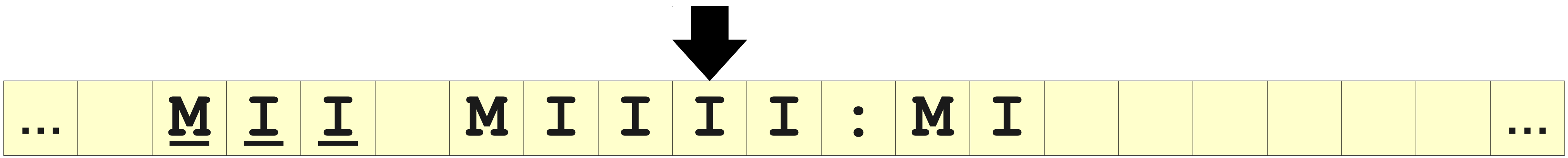

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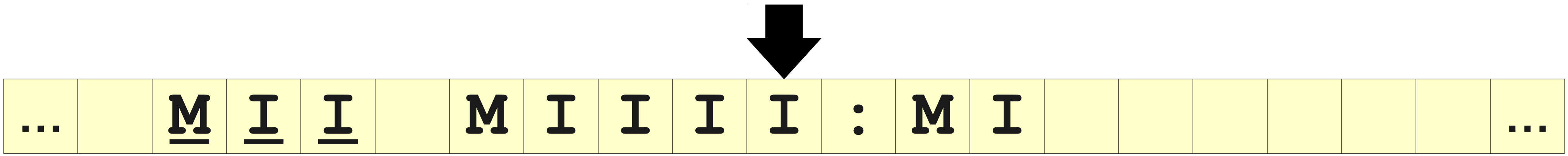

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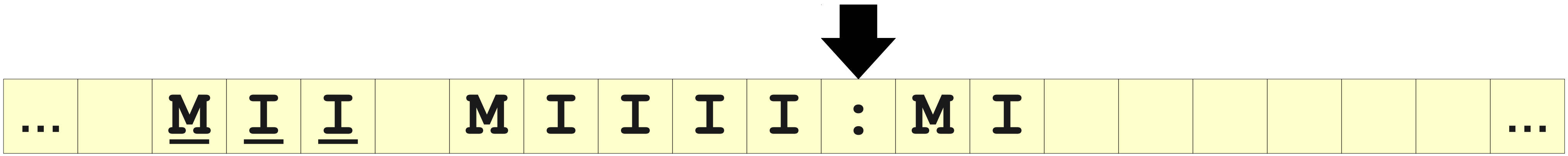

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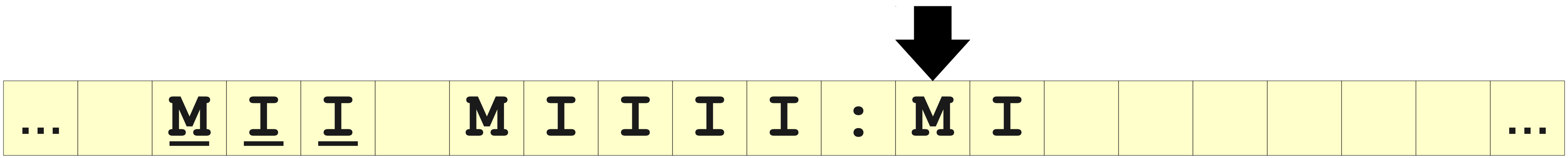

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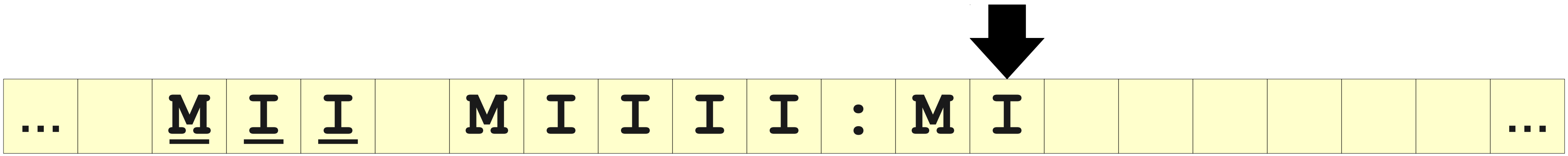

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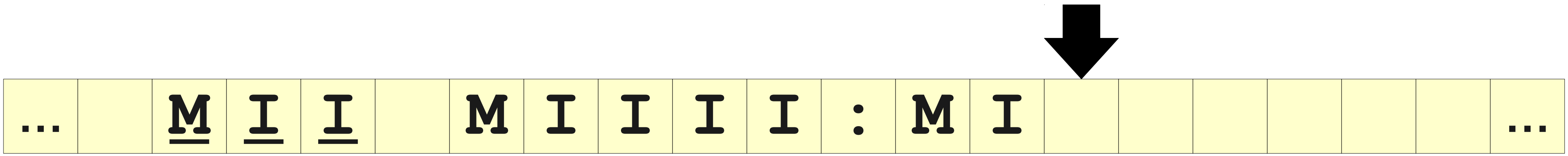

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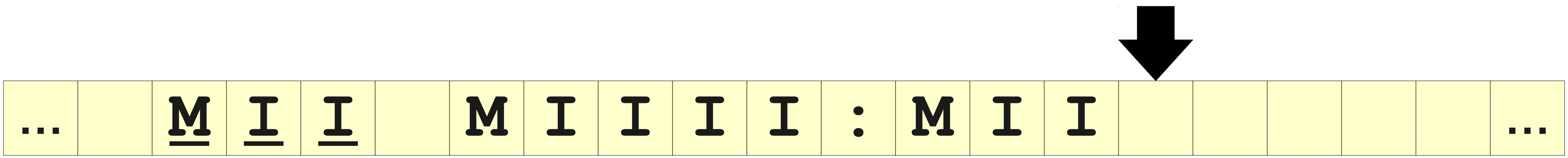

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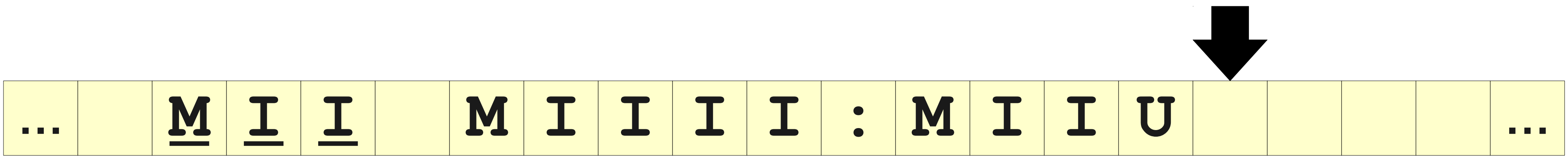

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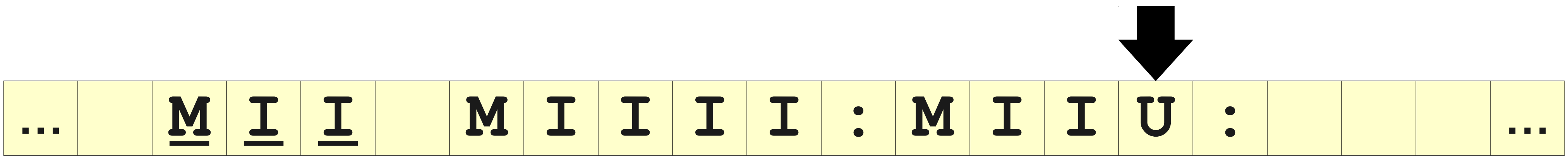

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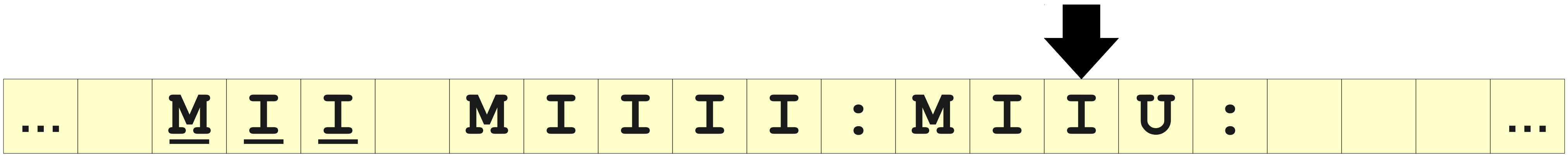

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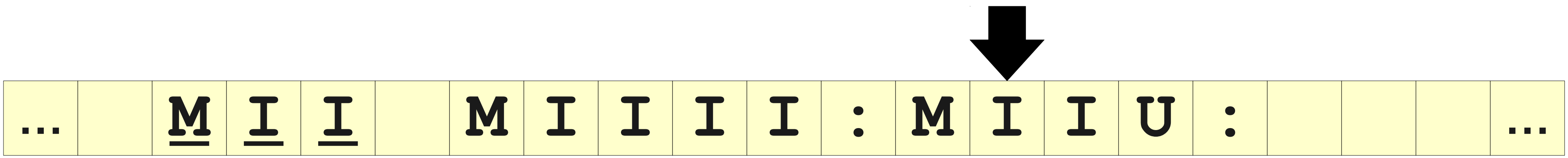

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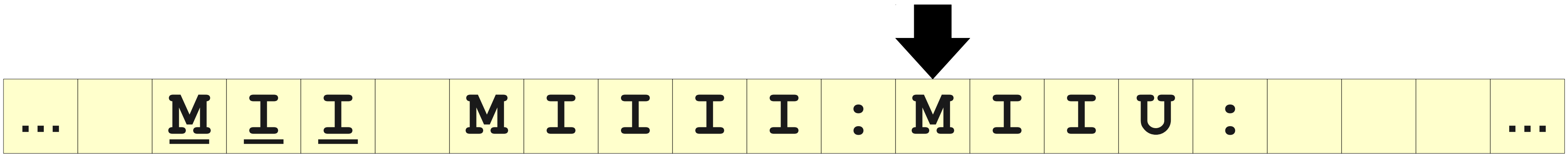

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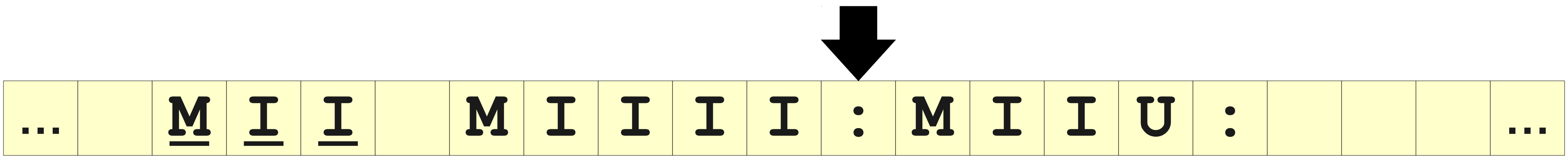

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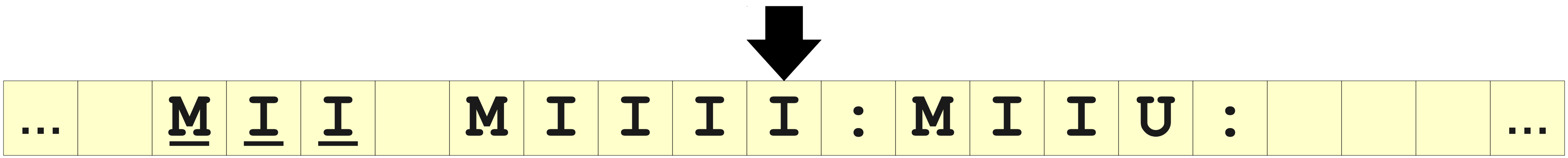

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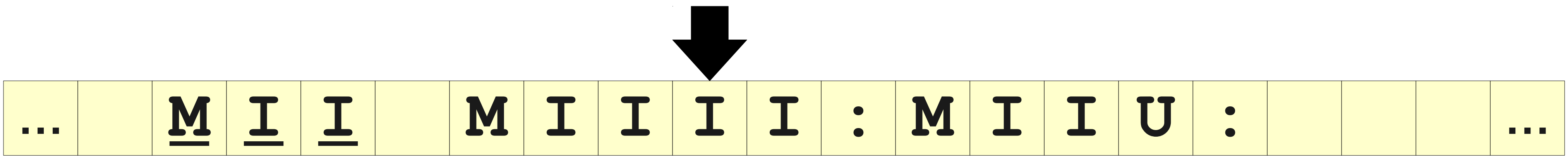

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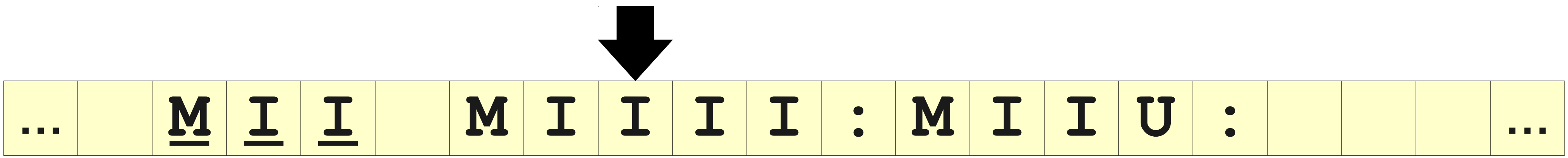

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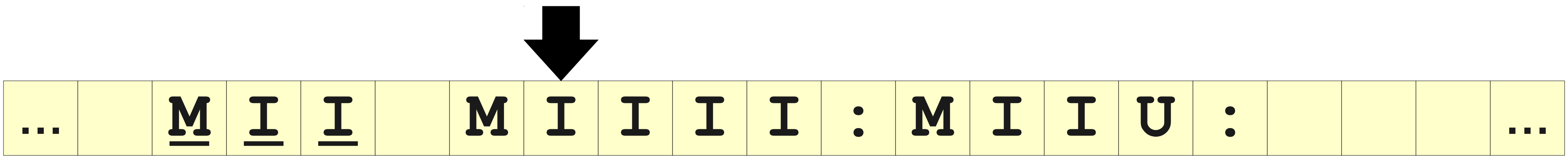

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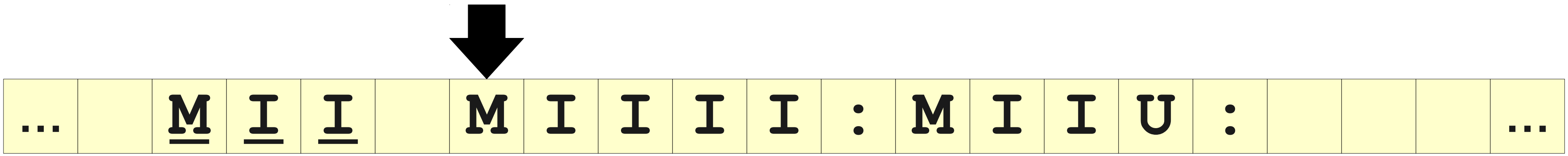

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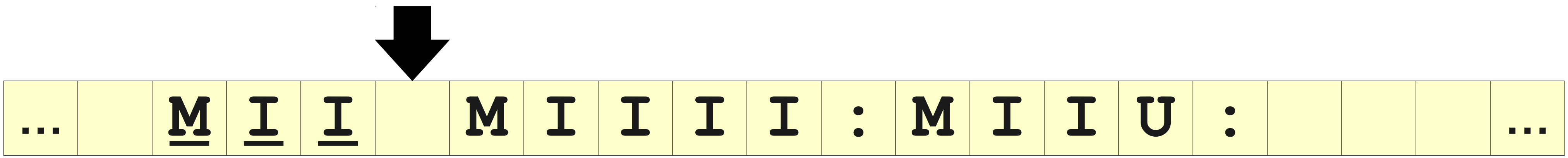

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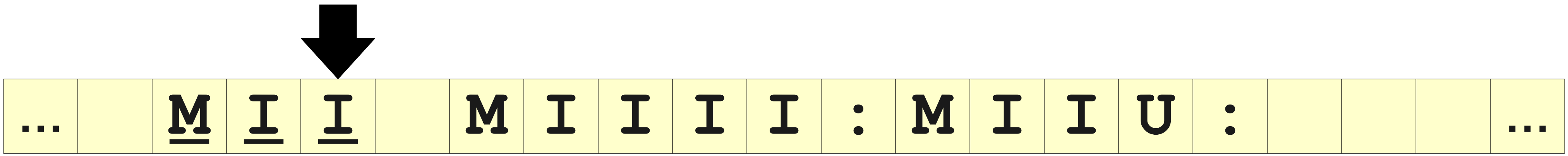

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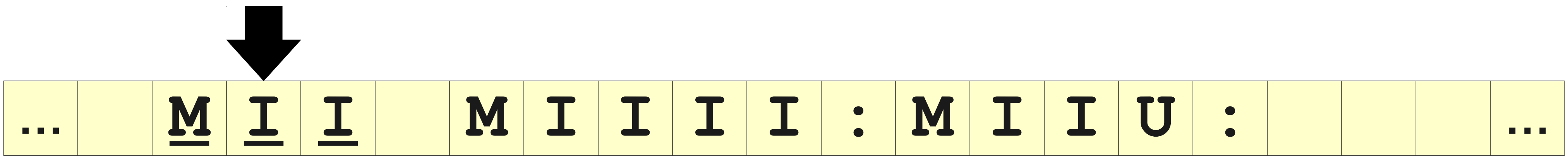

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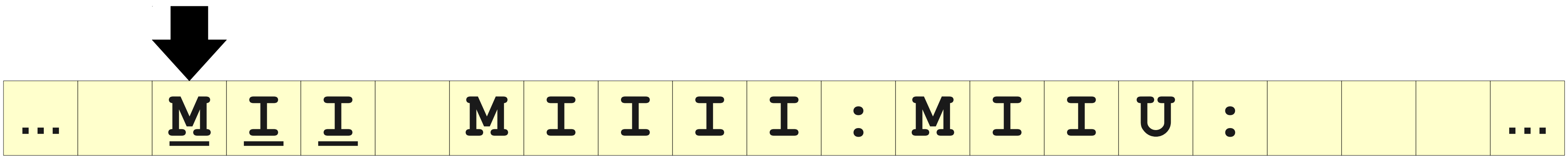

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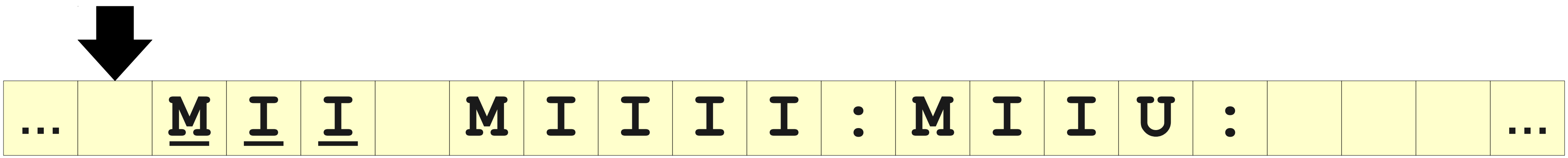

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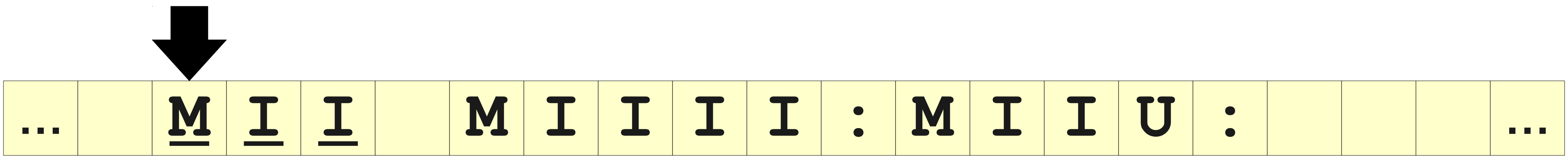

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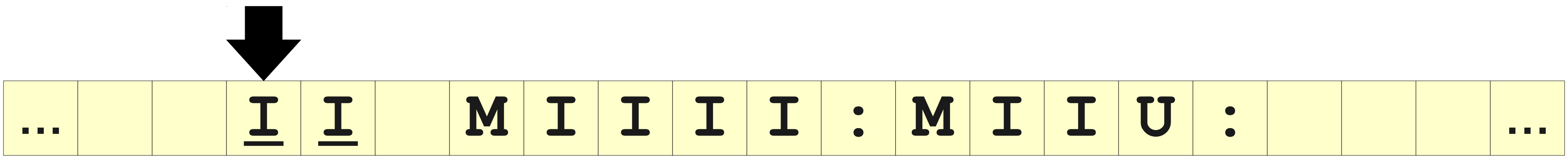

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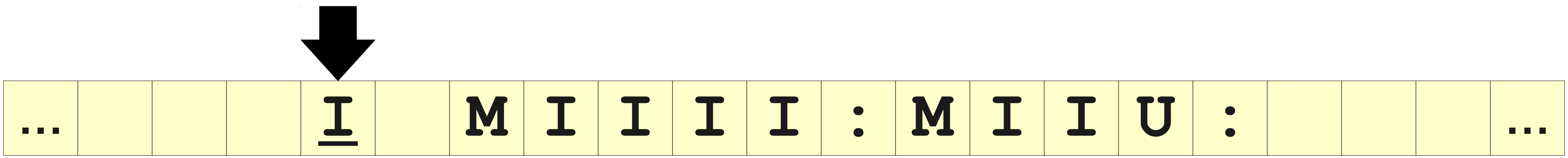

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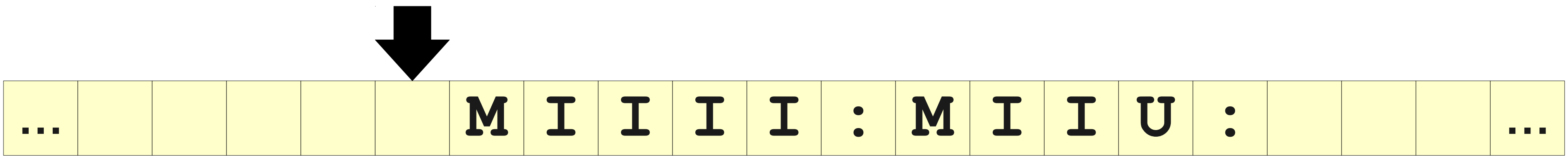

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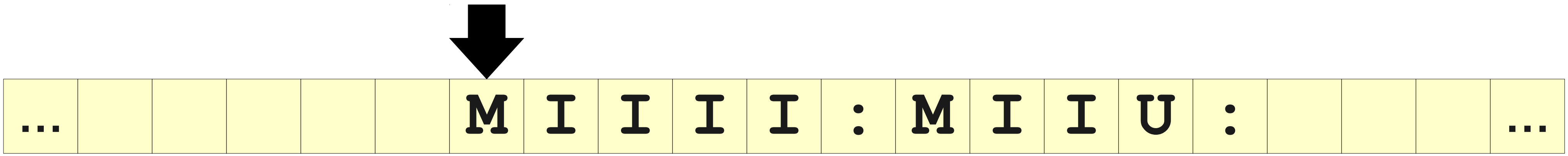

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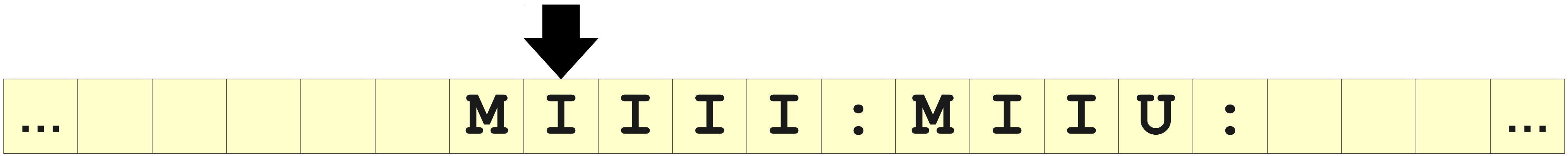

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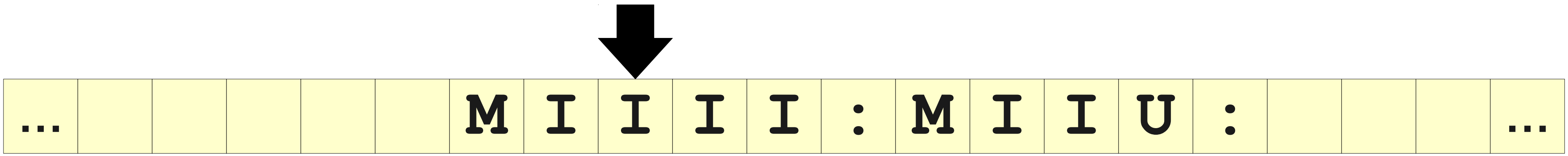

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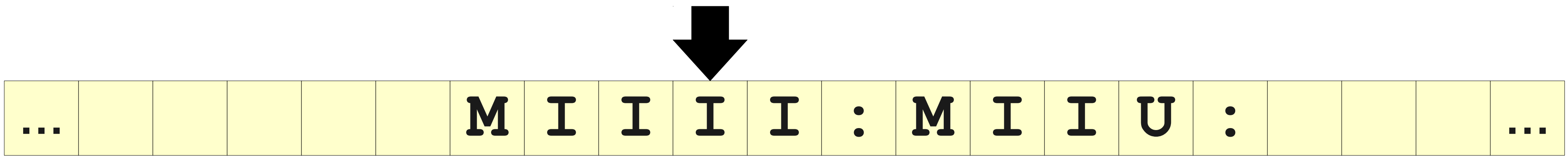

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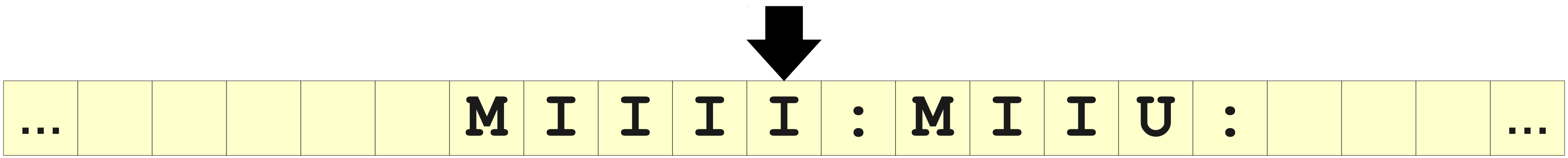

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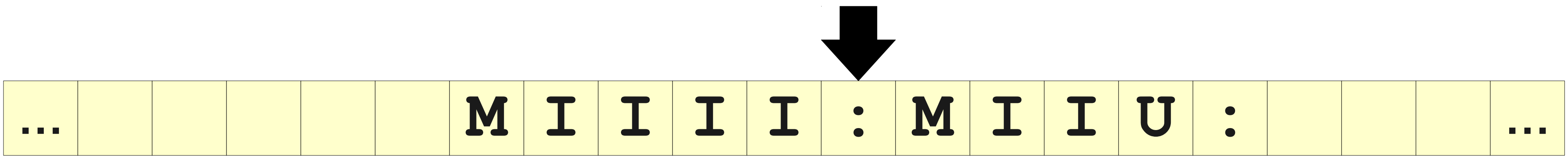

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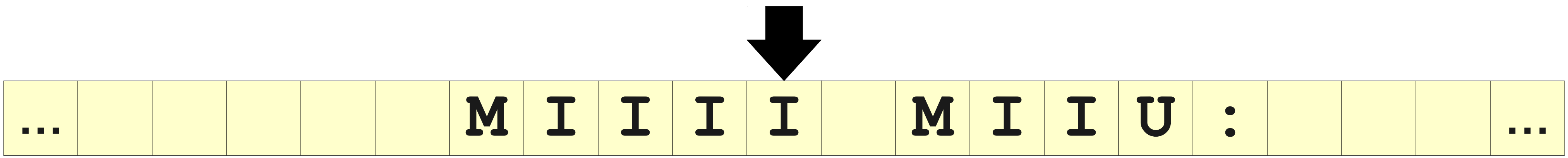

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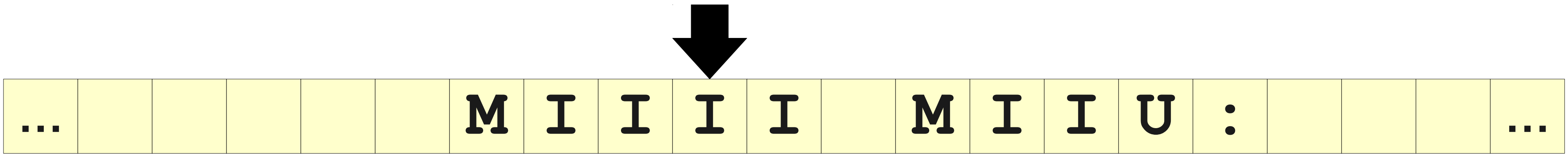

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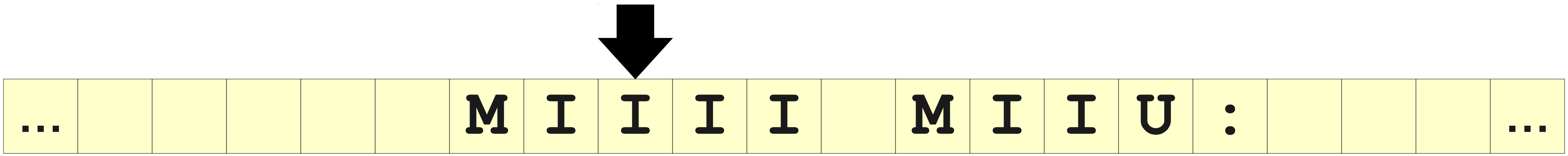

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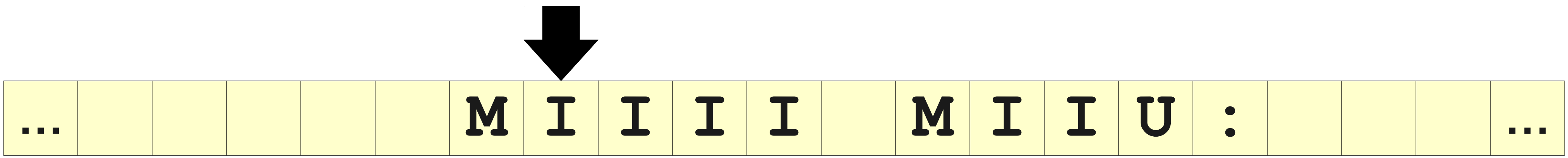

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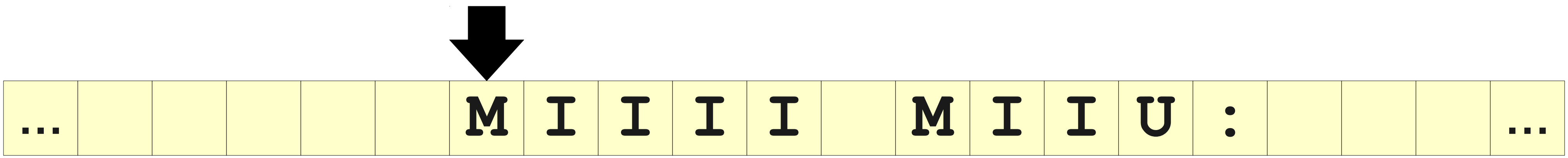

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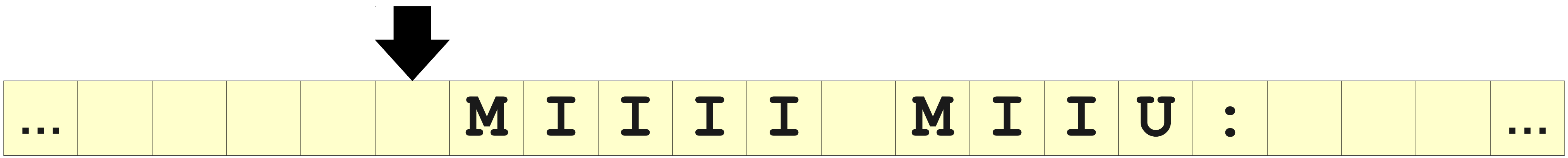

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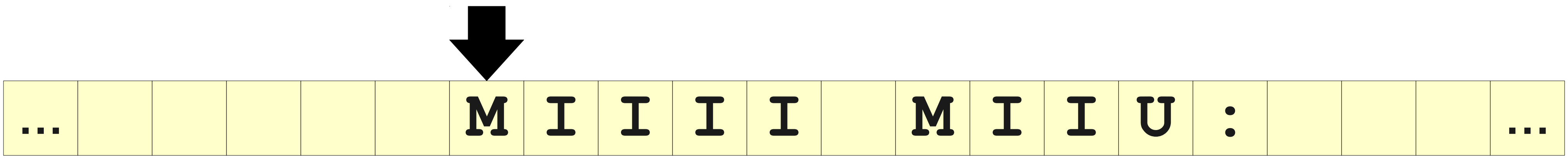

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The Power of TMs

- The worklist approach makes that all of the following languages are recognizable:
  - Any context-free language: simulate all possible production rules and see if the target string can be derived.
  - Solving a maze – use the worklist to explore all paths of length 0, 1, 2, … until a solution is found.
  - Determining whether a polynomial has an integer zeros: try 0, -1, +1, -2, +2, -3, +3, … until a result is found.

Searching and Guessing

Nondeterminism Revisited

- Recall: One intuition for nondeterminism is **perfect guessing**.
  - The machine has many options, and somehow magically knows which guess to make.
- With regular languages, we could *simulate* perfect guessing by building an enormous DFA to try out each option in parallel.
  - Only finitely many possible options.
- With context-free languages, some NPDAs do not have corresponding DPDAs.
  - Cannot simulate all possible stacks with just one stack.

Nondeterministic TMs

- A **nondeterministic Turing machine** (or **NTM**) is a variant on a Turing machine where there can be any number of transitions for a given state/tape symbol combination.
  - Notation: “Turing machine” or “TM” refers to a deterministic Turing machine unless specified otherwise. The term **DTM** specifically represents a deterministic TM.
- The NTM accepts iff there is *some possible series of choices* it can make such that it accepts.

Questions for Now

- How can we build an intuition for nondeterministic Turing machines?
- What sorts of problems can we solve with NTMs?
- What is the relative power of NTMs and DTMs?

Designing NTMs

- When designing NTMs, it is often useful to use the approach of guess and check:
  - **Nondeterministically** guess some object that can "prove" that $w \in L$.
  - **Deterministically** verify that you have guessed the right object.
- If $w \in L$, there will be some guess that causes the machine to accept.
- If $w \notin L$, then no guess will ever cause the machine to accept.

Composite Numbers

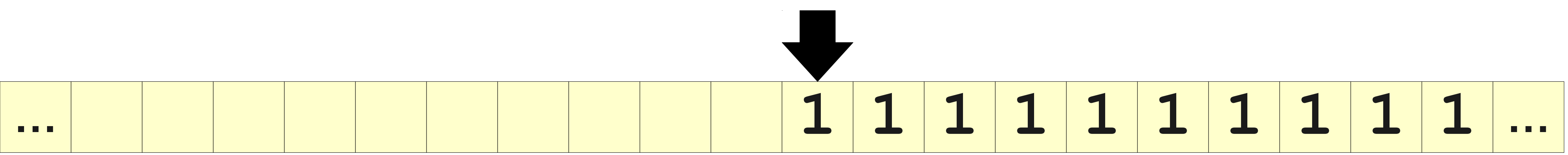
- A natural number $n \geq 2$ is called **composite** iff it has a factor other than 1 and n.
- Equivalently: there are two natural numbers $r \geq 2$ and $s \geq 2$ such that $rs = n$.
- Let $\Sigma = \{\mathtt{1}\}$ and consider the language

$$L = \{\ \mathtt{1}^n \mid n \text{ is composite }\}$$

- How might we design an NTM for L?

A Sketch of the NTM

- We saw how to build a TM that checks for correct multiplication.
- Have our NTM
  - **Nondeterministically** guess two factors, then
  - **Deterministically** run the multiplication TM.



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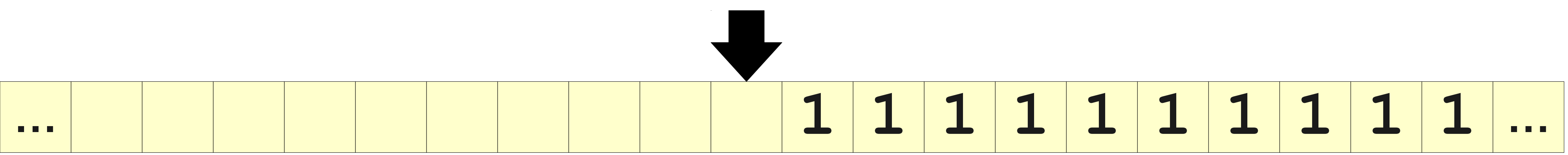

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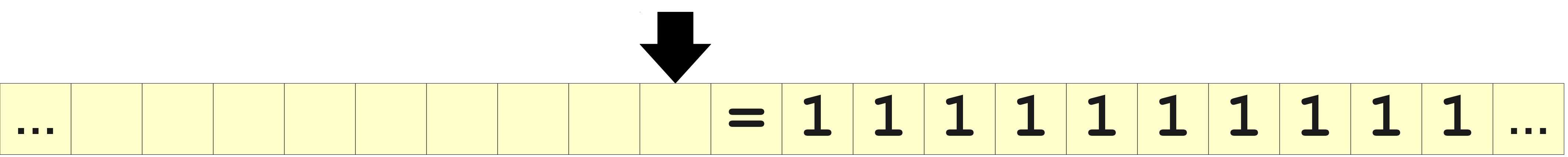

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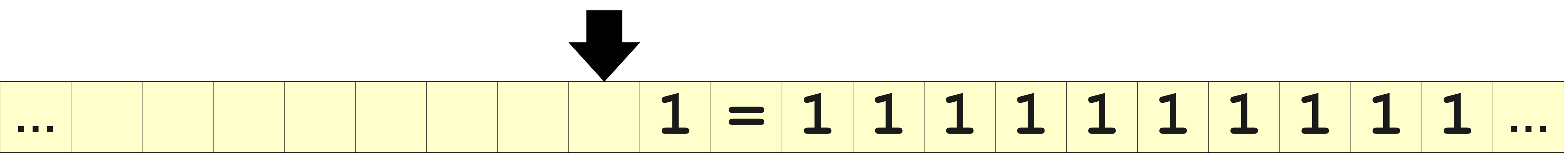

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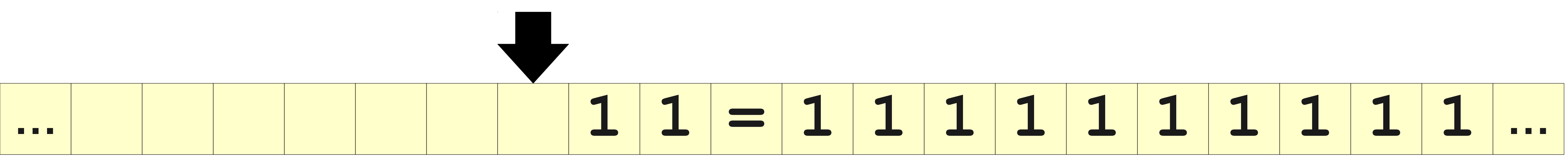

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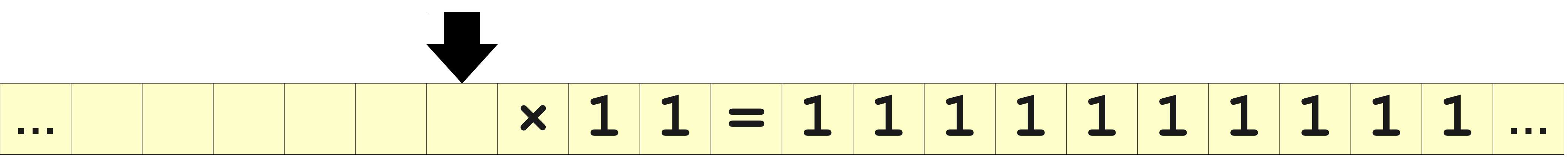

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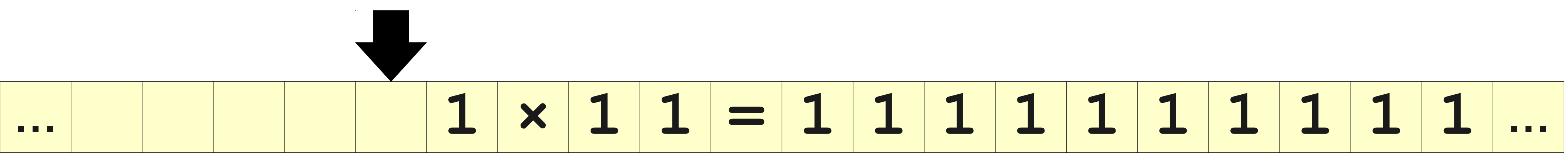

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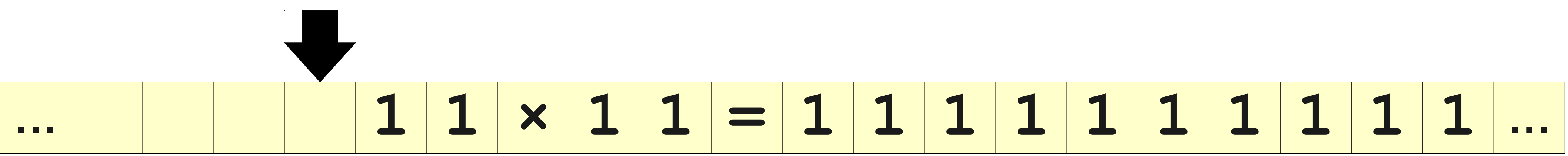

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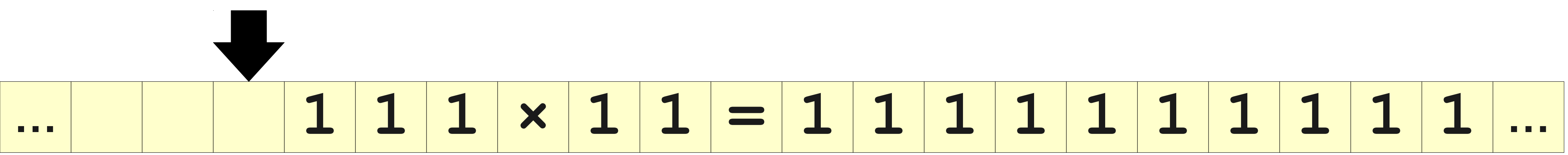

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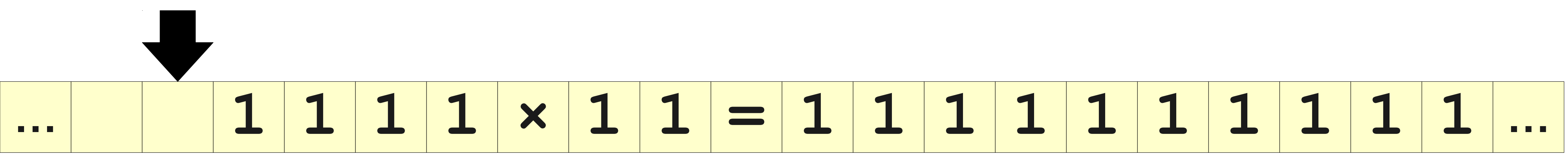

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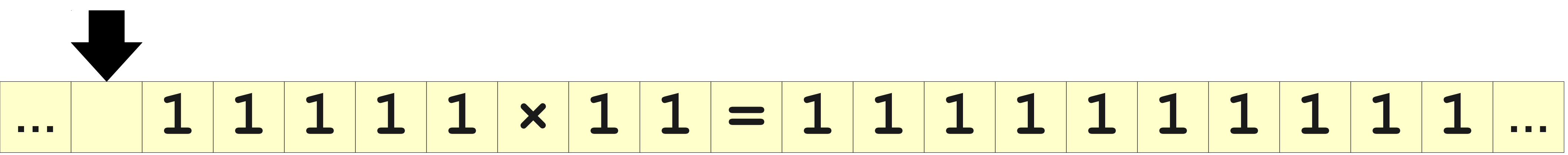

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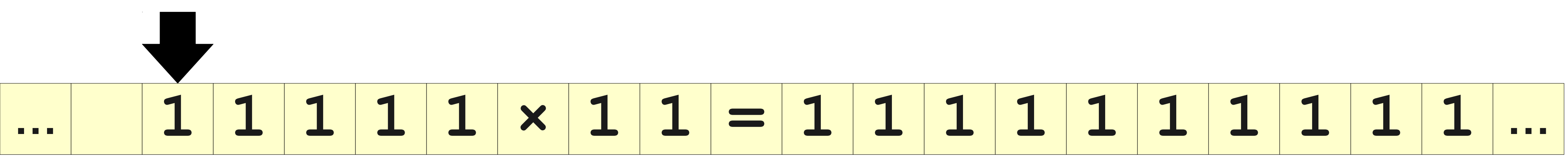

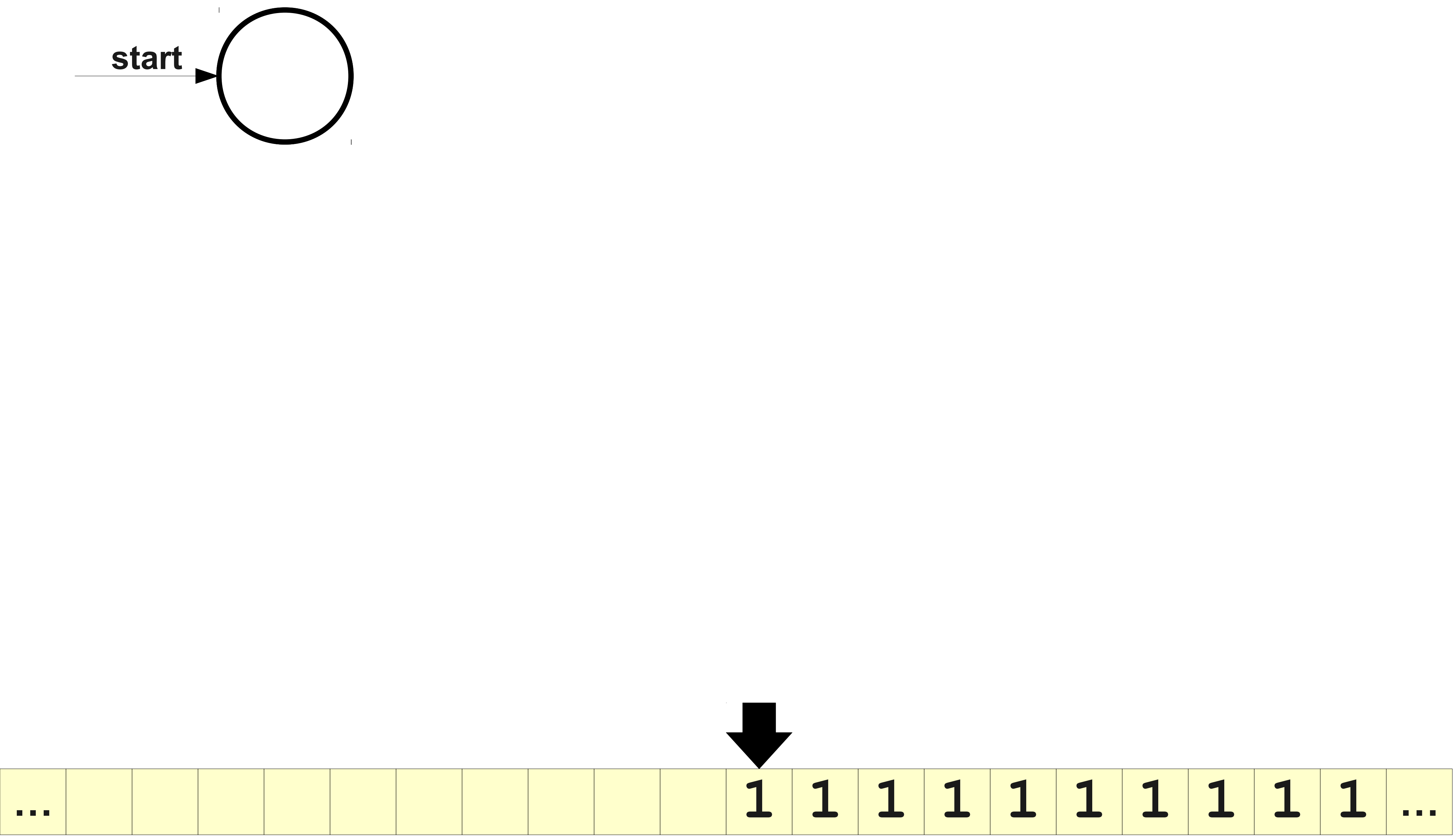
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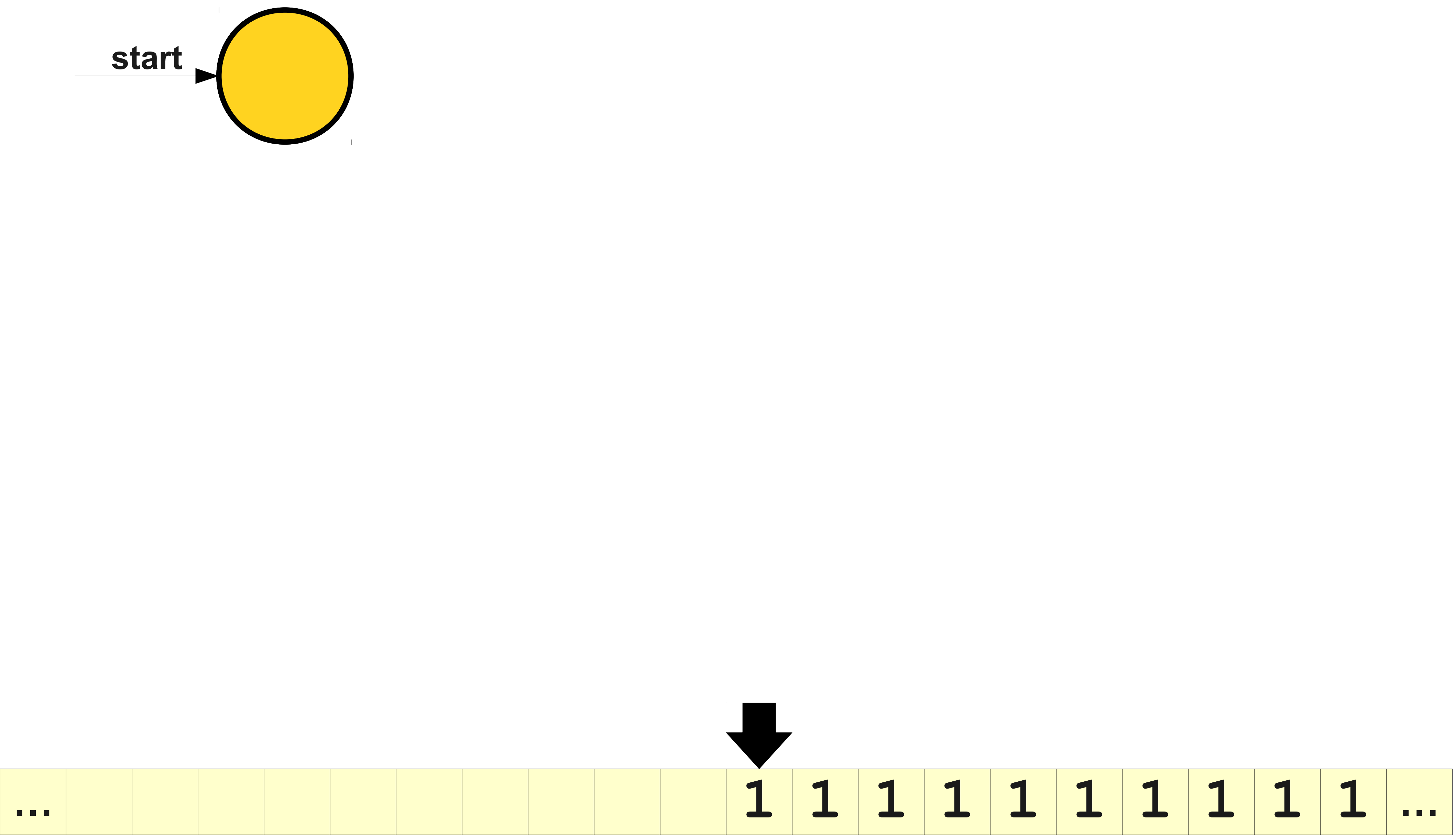
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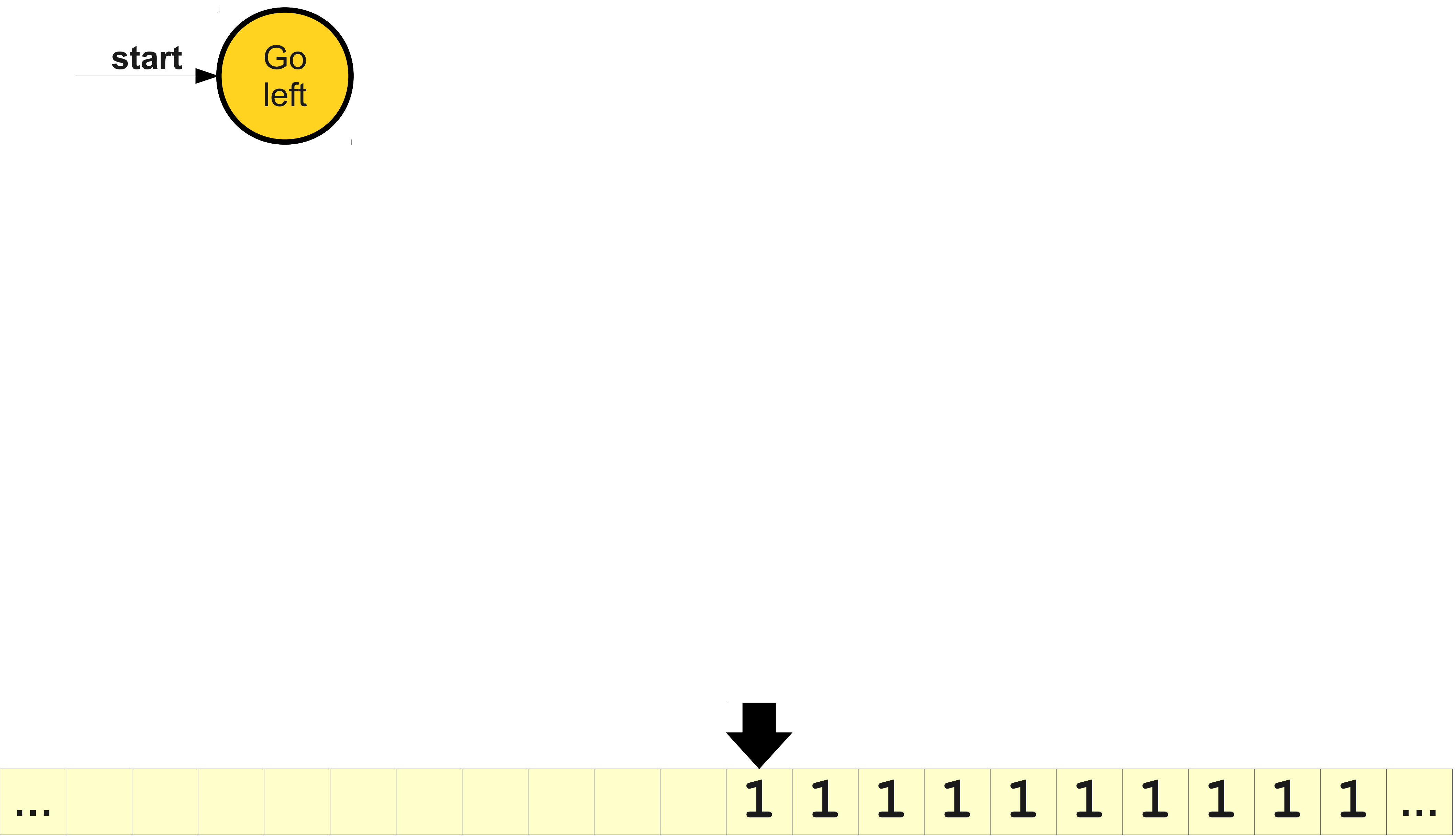
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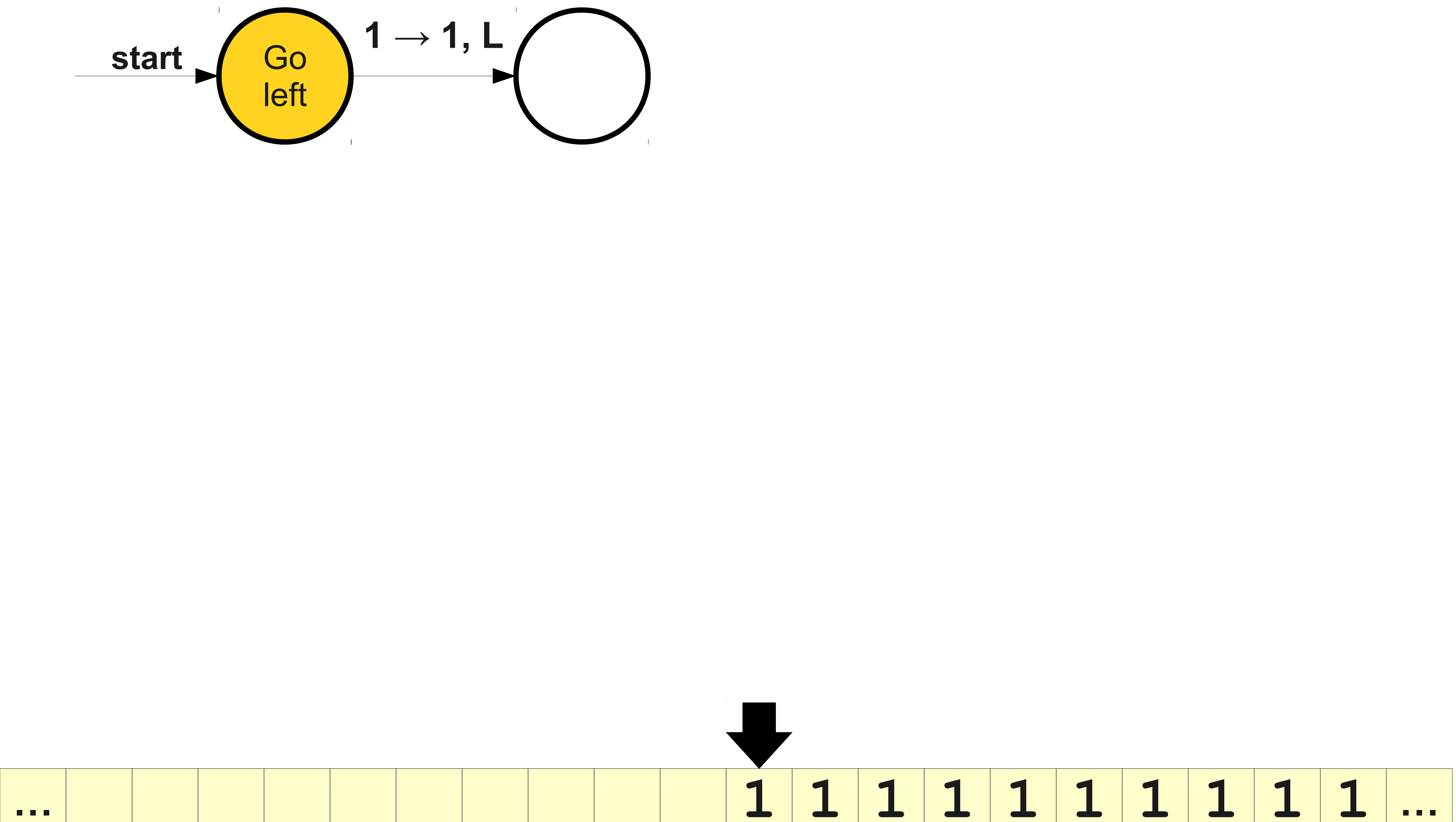
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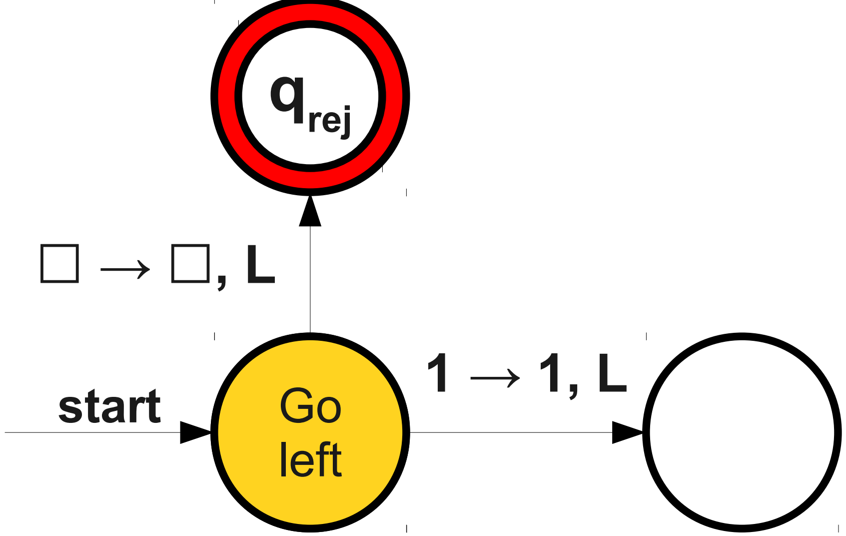

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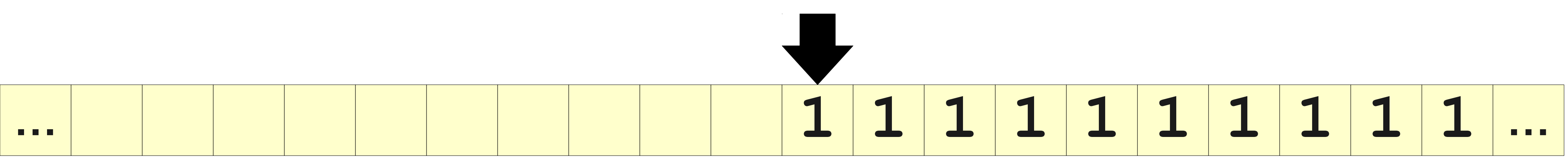

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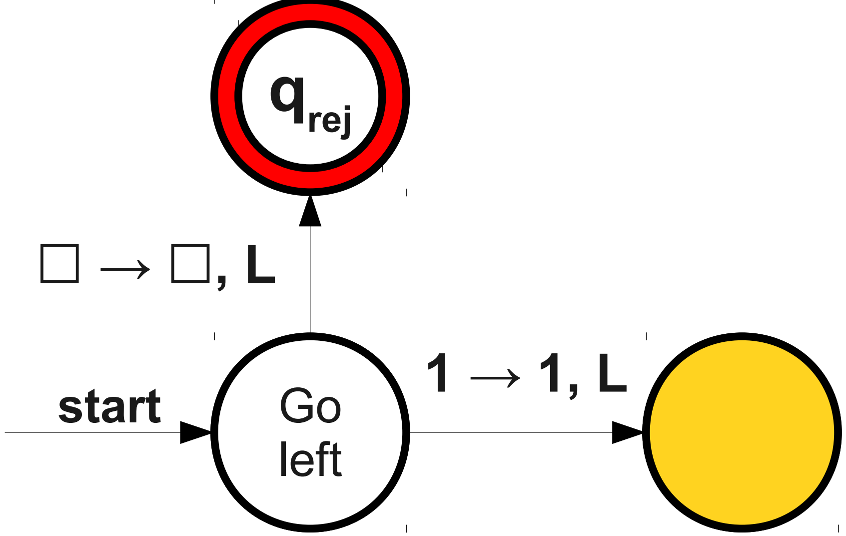

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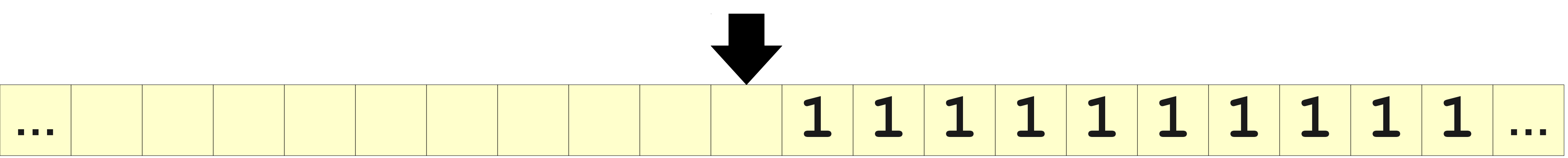

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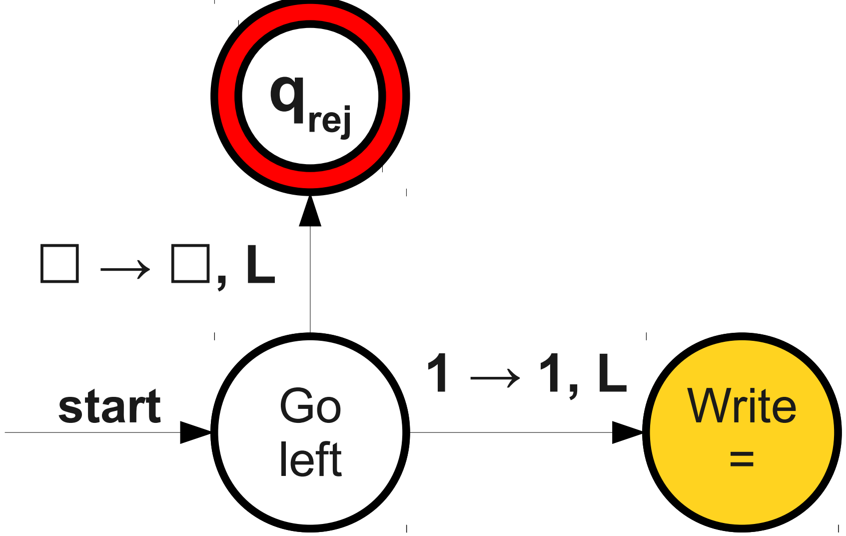
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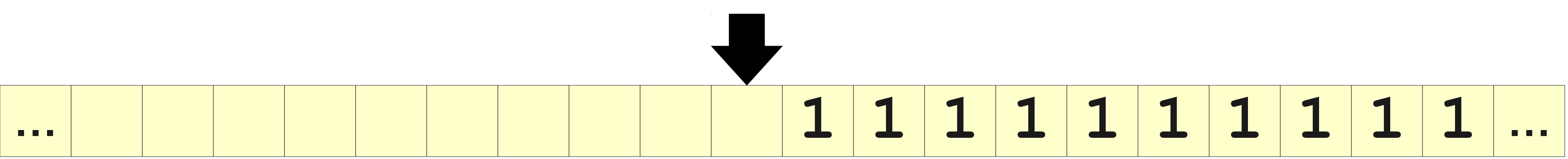
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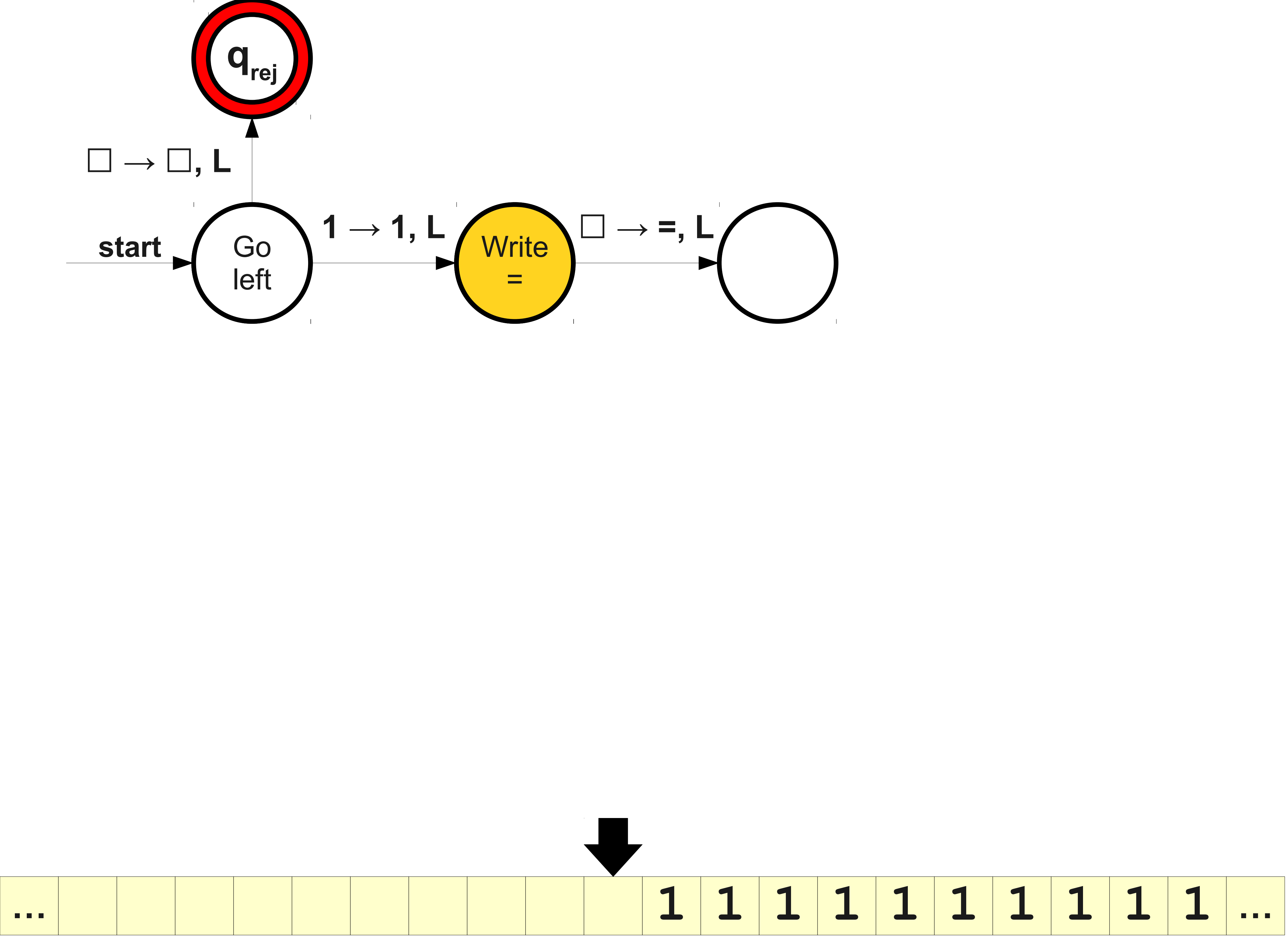

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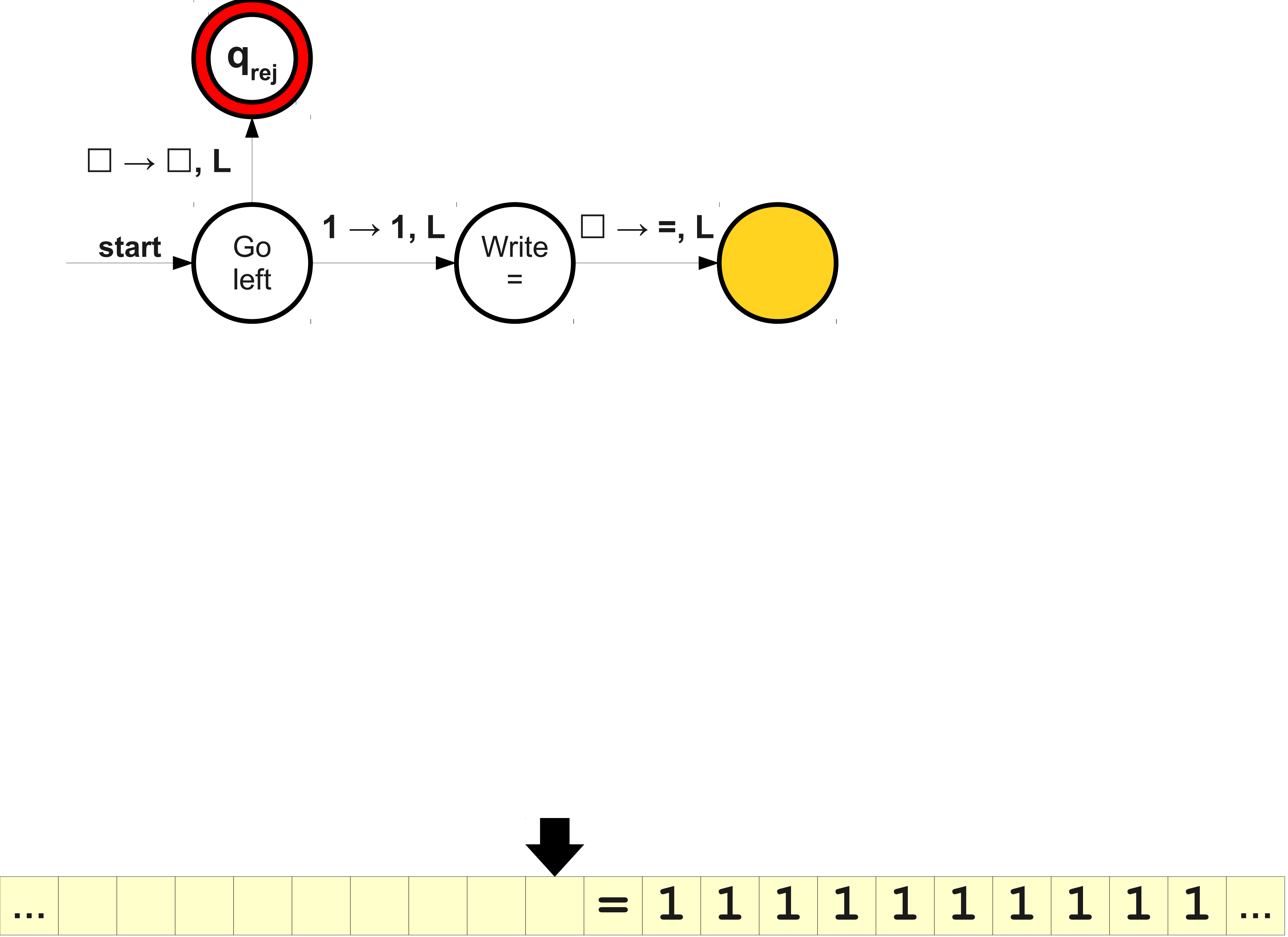

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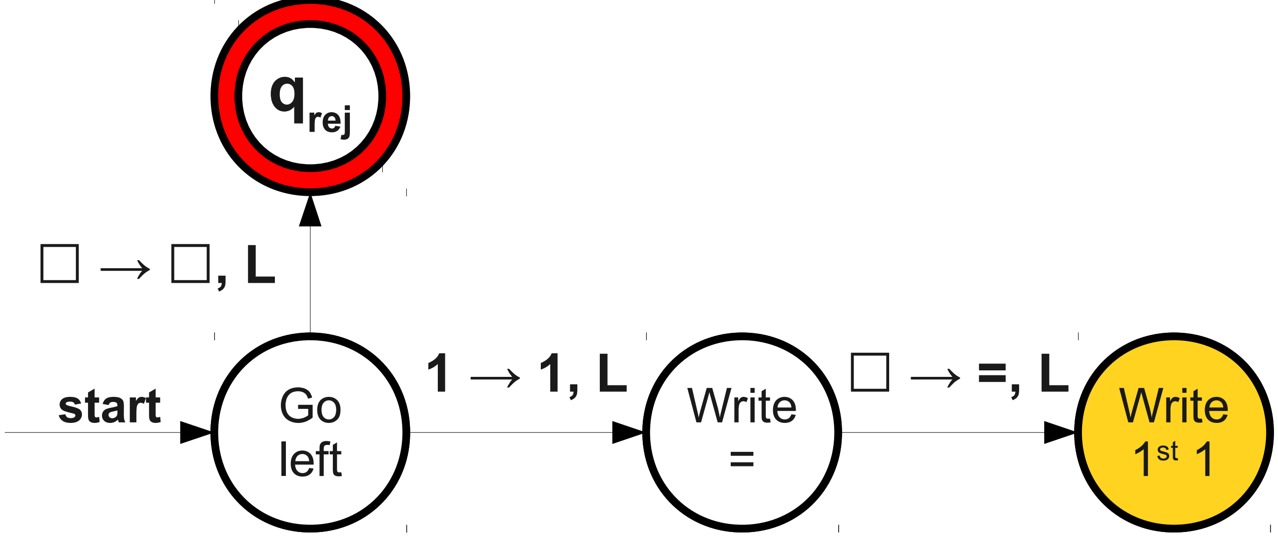

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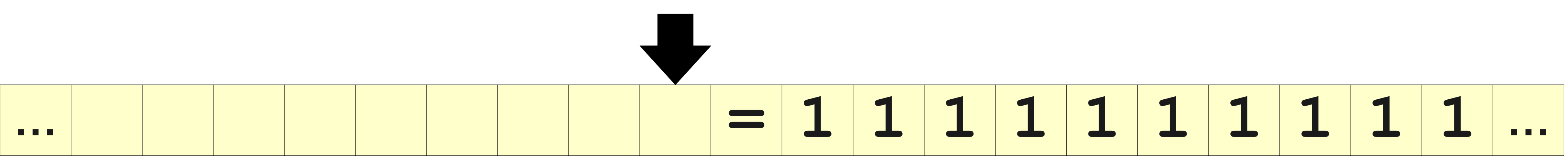

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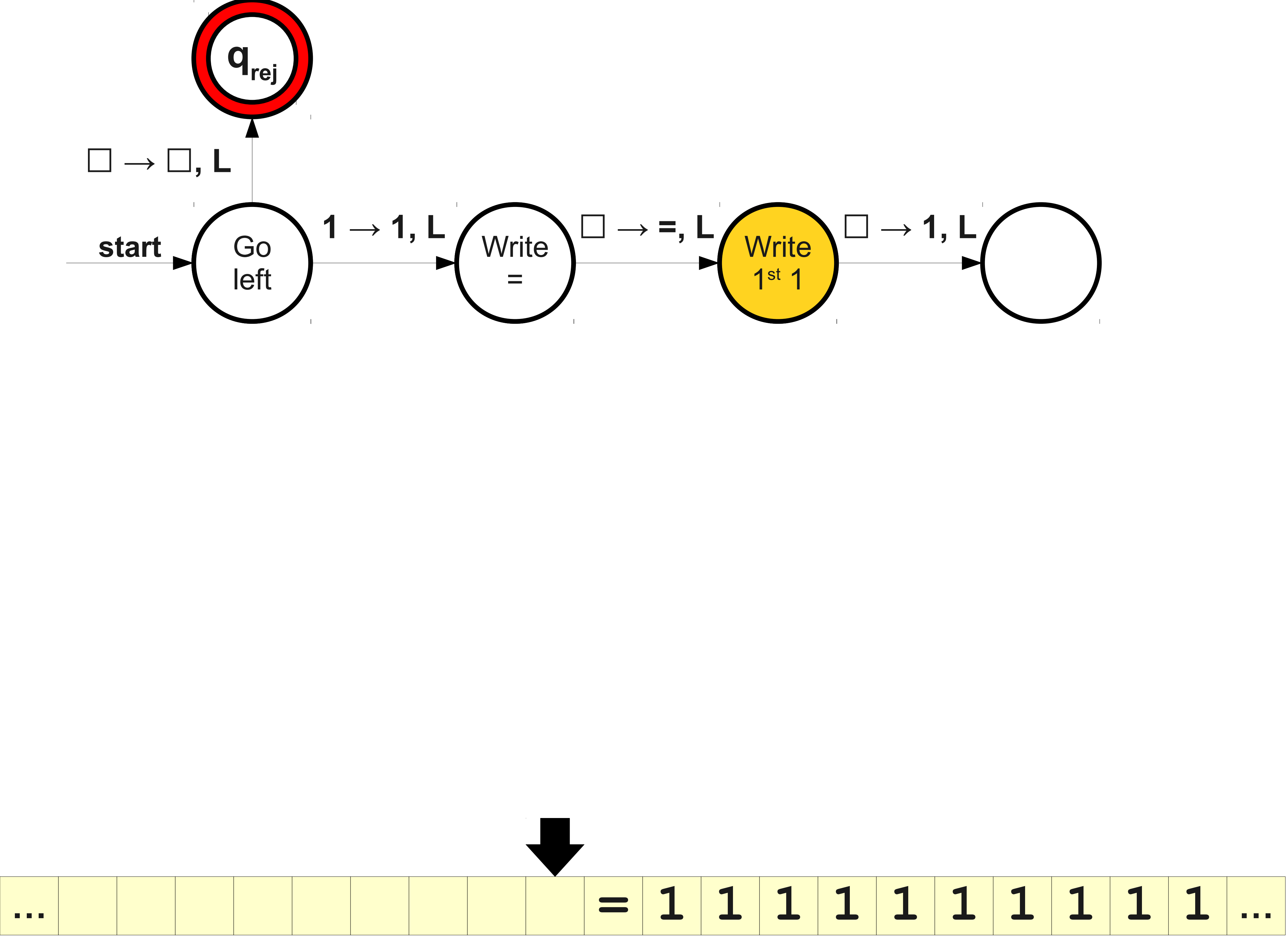

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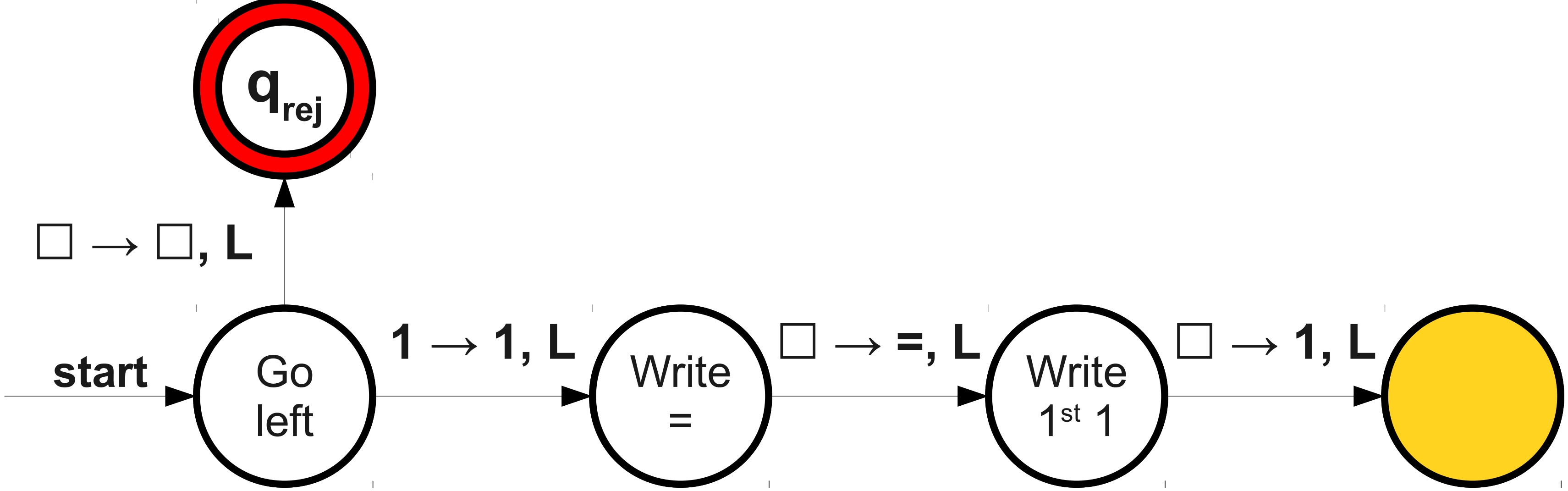

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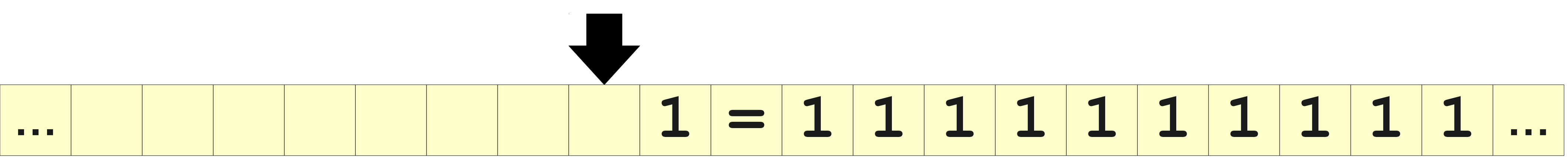

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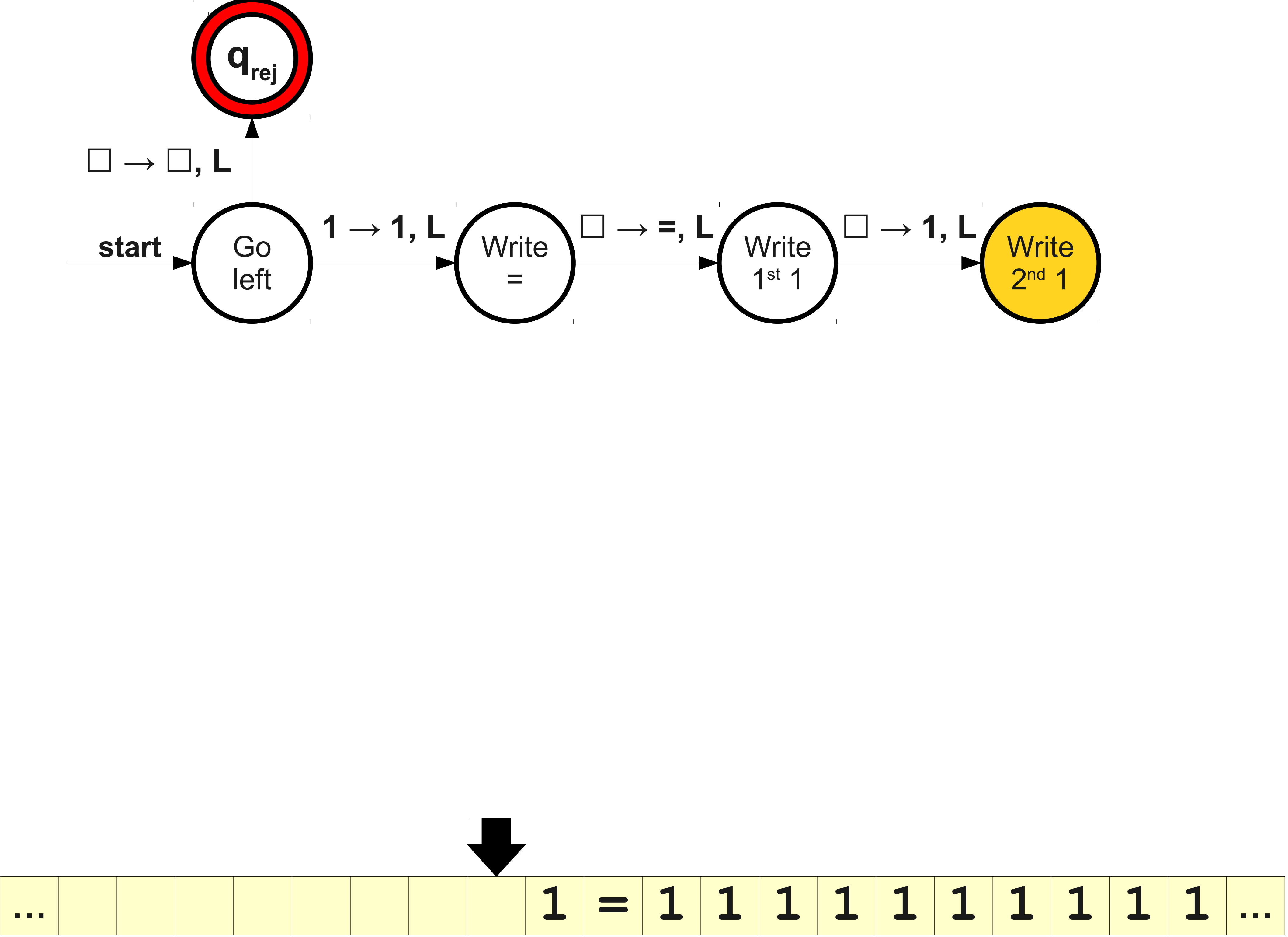

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Write 1st 1
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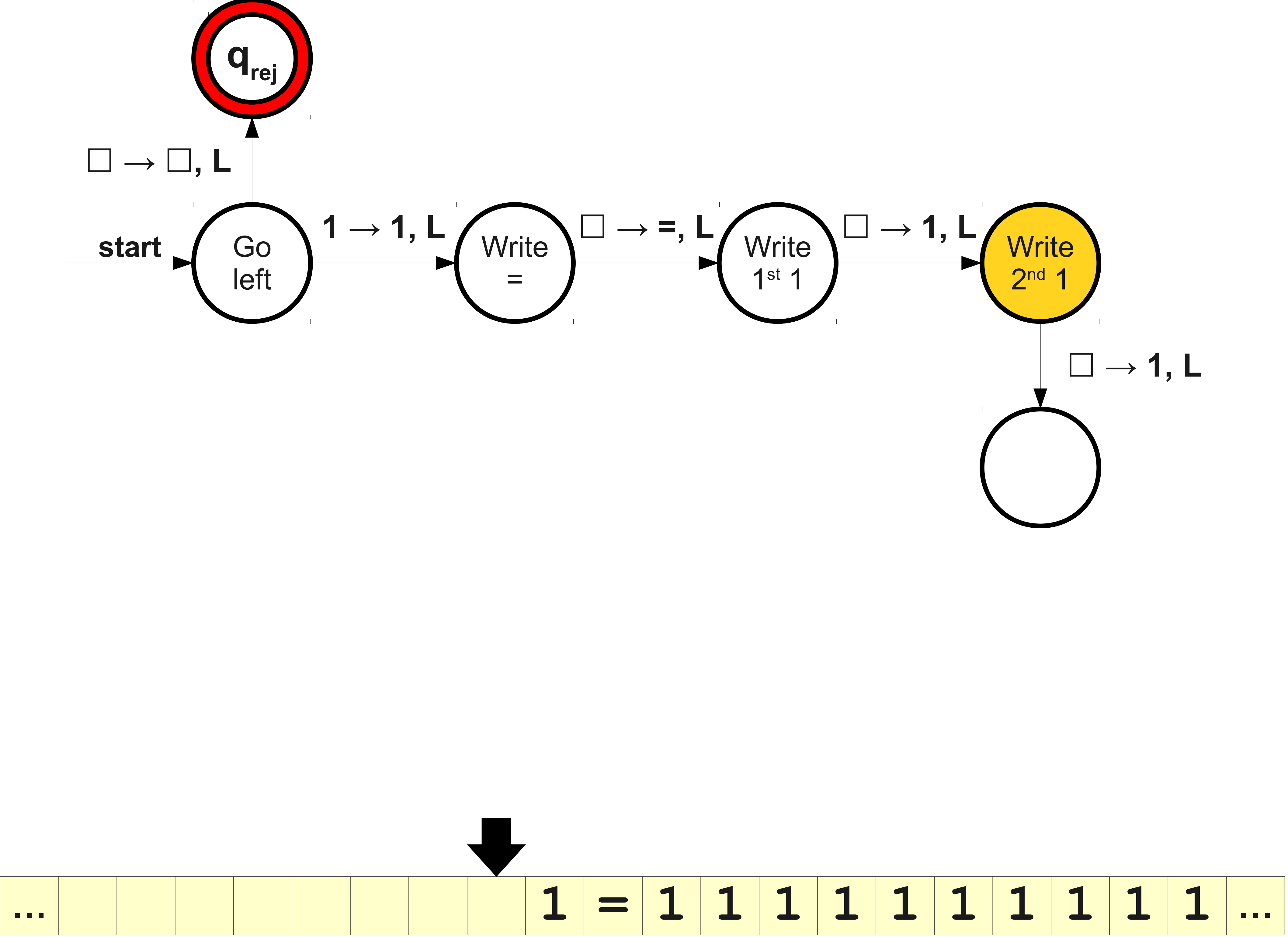
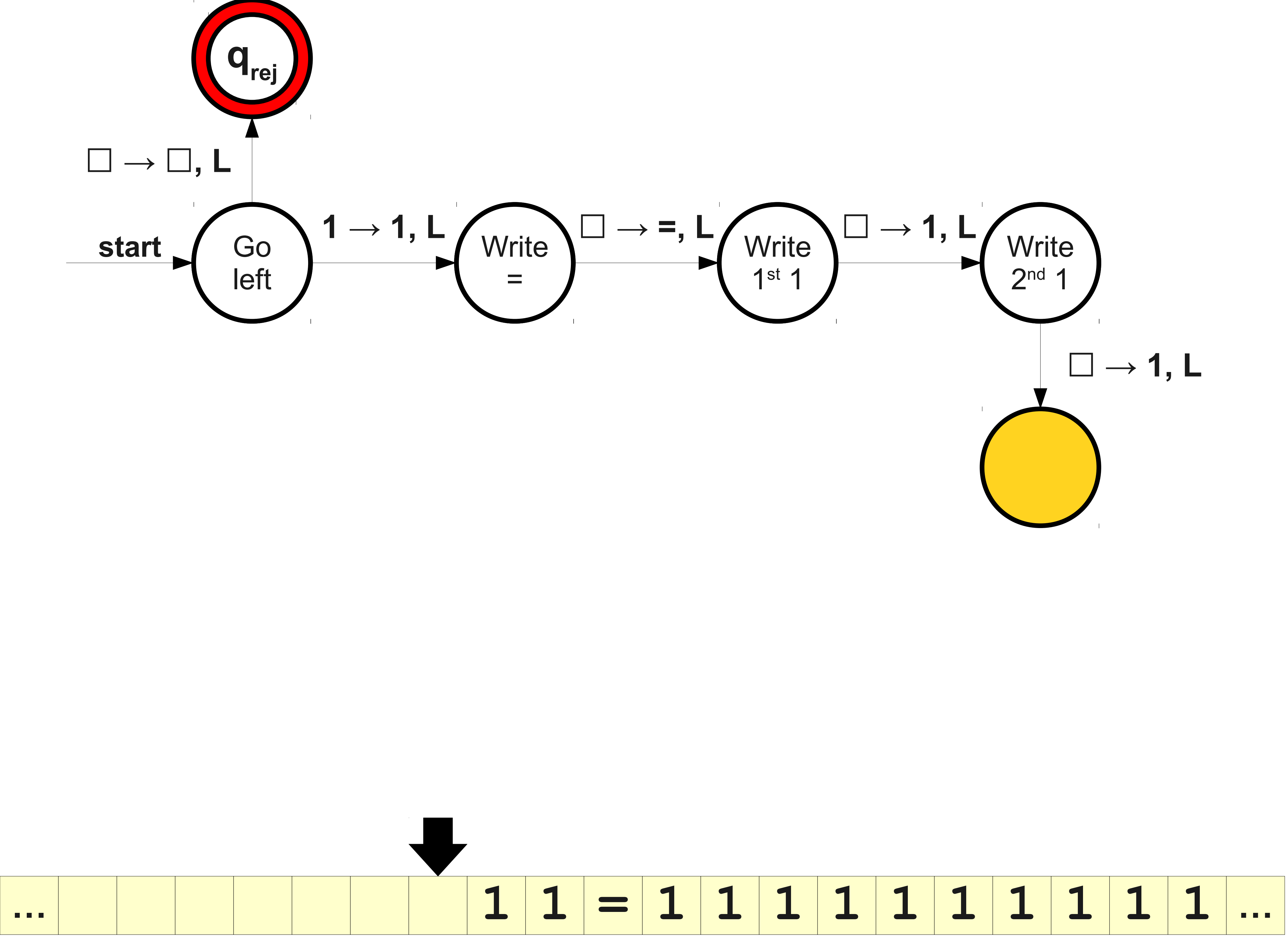
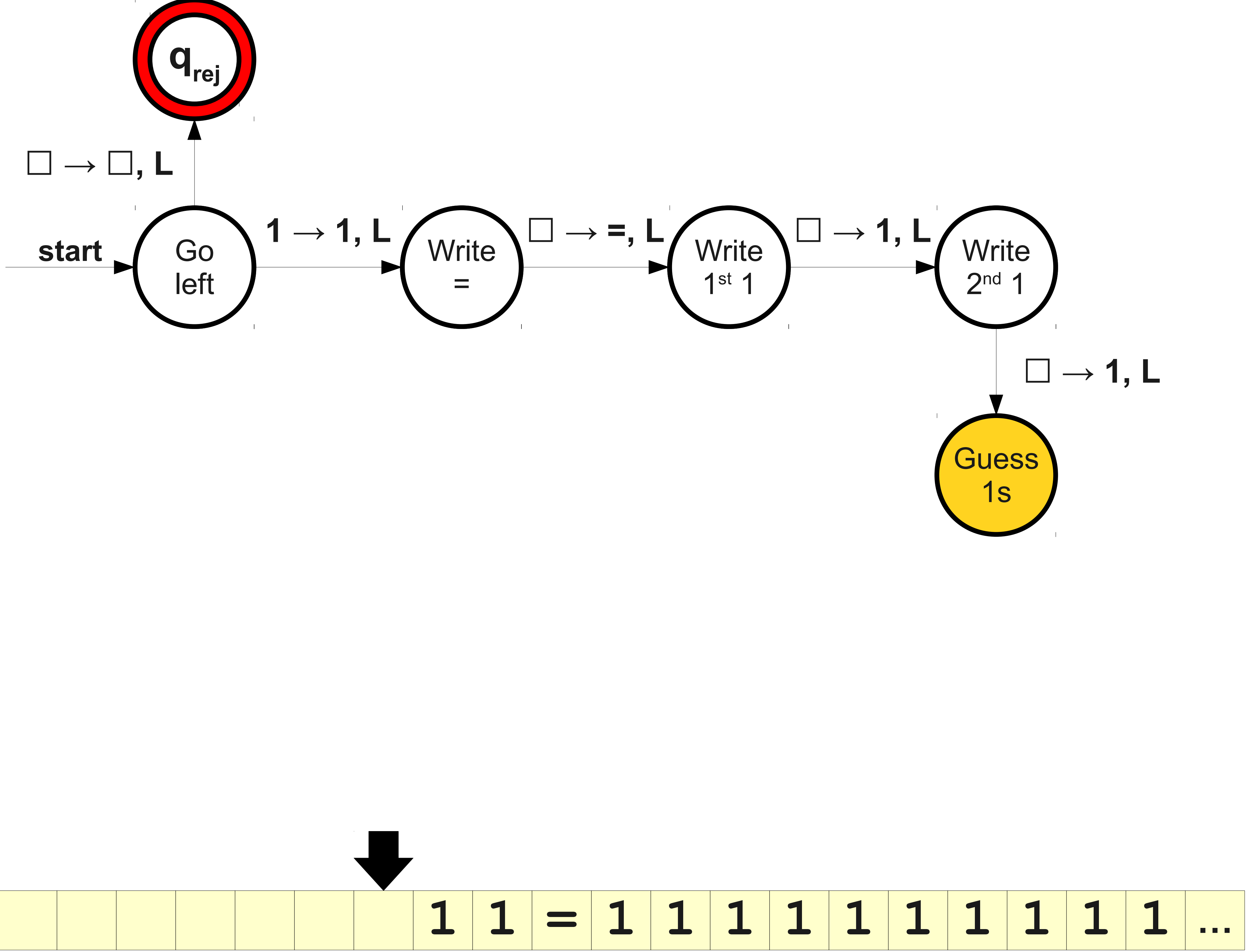
Write 2nd 1
... 1 = 1 1 1 1 1 1 1 1 1 1 ...

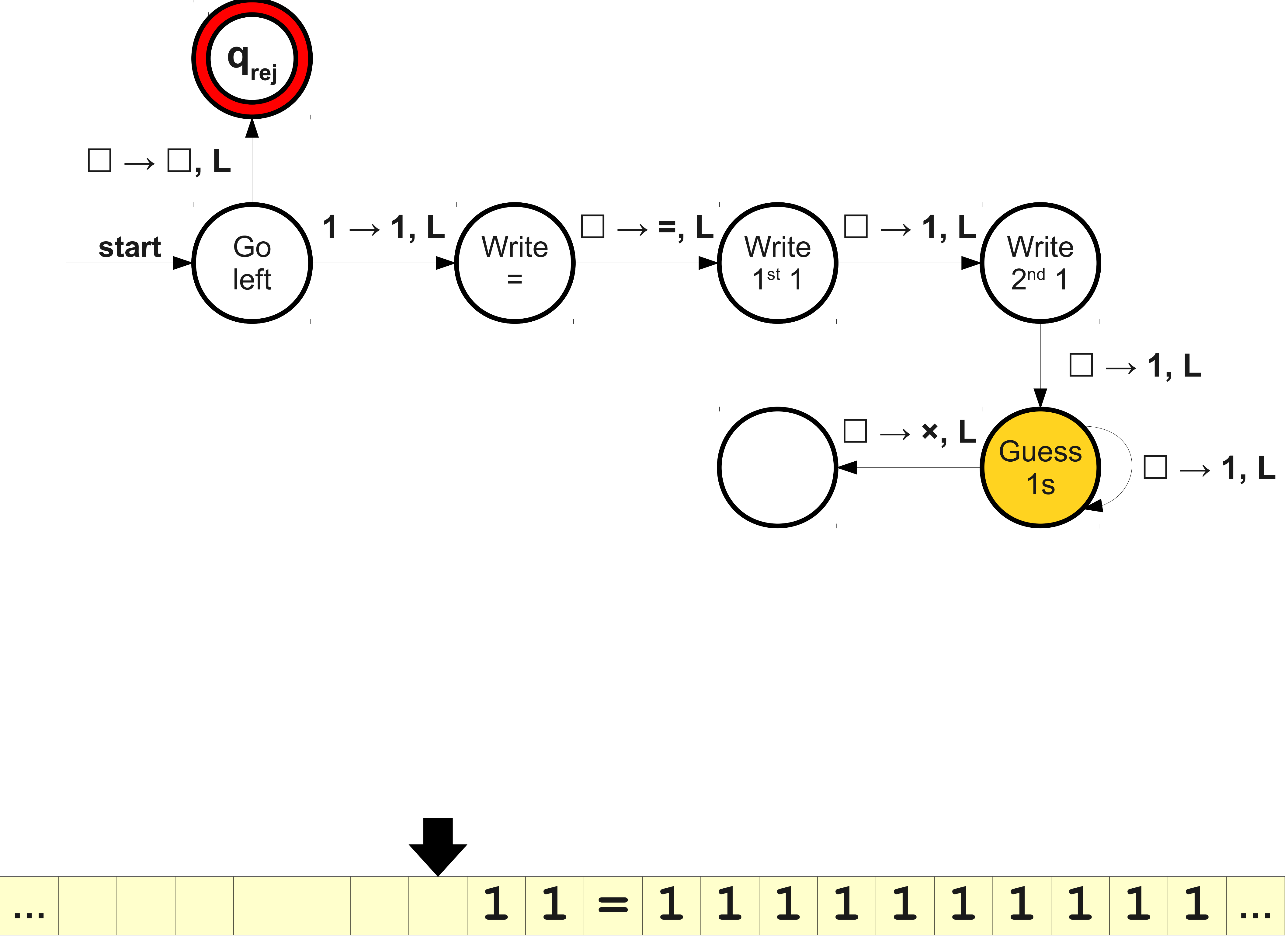

q_rej
□ → □, L
start
Go left
1 → 1, L
Write =
□ → =, L
Write 1st 1
□ → 1, L
Write 2nd 1
□ → 1, L
...
1 = 1 1 1 1 1 1 1 1 1 1
...



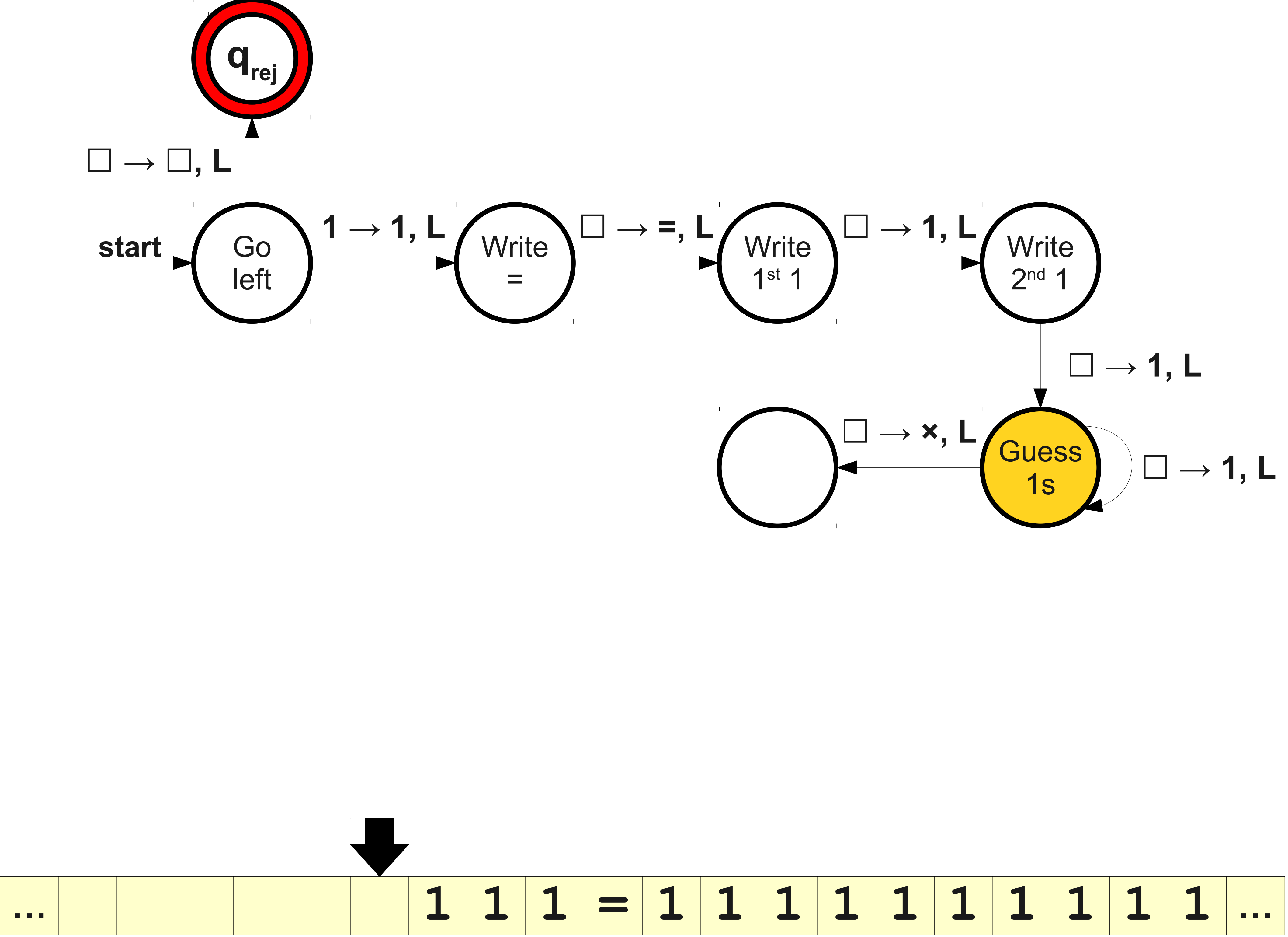
q_rej
☐ → ☐, L
start
Go left
1 → 1, L
Write =
☐ → =, L
Write 1st 1
☐ → 1, L
Write 2nd 1
☐ → 1, L
… 1 1 = 1 1 1 1 1 1 1 1 1 1 …



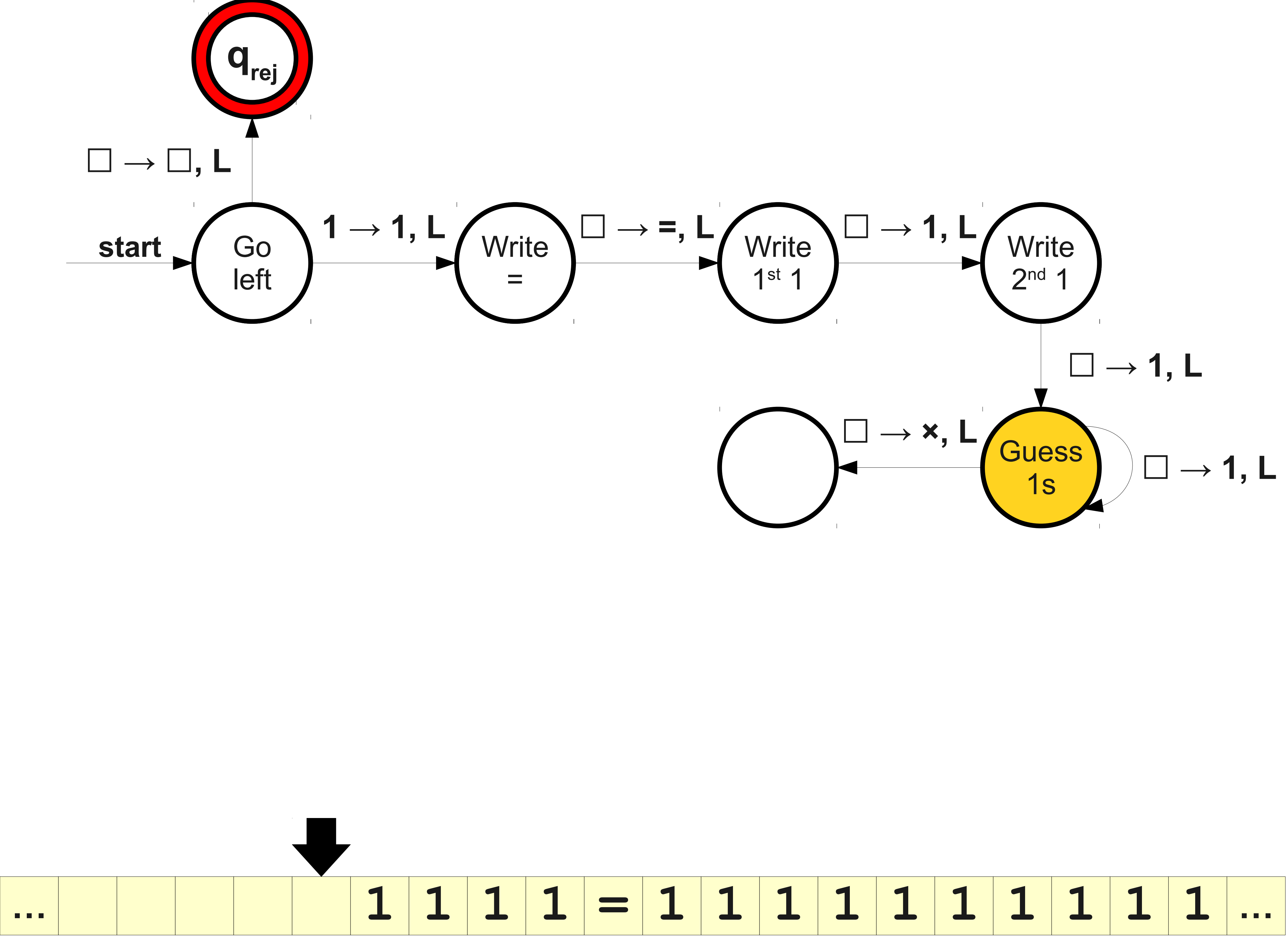
q rej
□ → □, L
start
Go left
1 → 1, L
Write =
□ → =, L
Write 1st 1
□ → 1, L
Write 2nd 1
□ → 1, L
Guess 1s
... 1 1 = 1 1 1 1 1 1 1 1 1 1 ...



q_{rej}
□ → □, L
start
Go left
1 → 1, L
Write =
□ → =, L
Write 1st 1
□ → 1, L
Write 2nd 1
□ → 1, L
□ → ×, L
Guess 1s
□ → 1, L
... 1 1 = 1 1 1 1 1 1 1 1 1 1 ...



q_rej
□ → □, L
start
Go left
1 → 1, L
Write =
□ → =, L
Write 1st 1
□ → 1, L
Write 2nd 1
□ → 1, L
Guess 1s
□ → 1, L
□ → ×, L
...
1 1 1 = 1 1 1 1 1 1 1 1 1 1
...



q_rej
□ → □, L
start
Go left
1 → 1, L
Write =
□ → =, L
Write 1st 1
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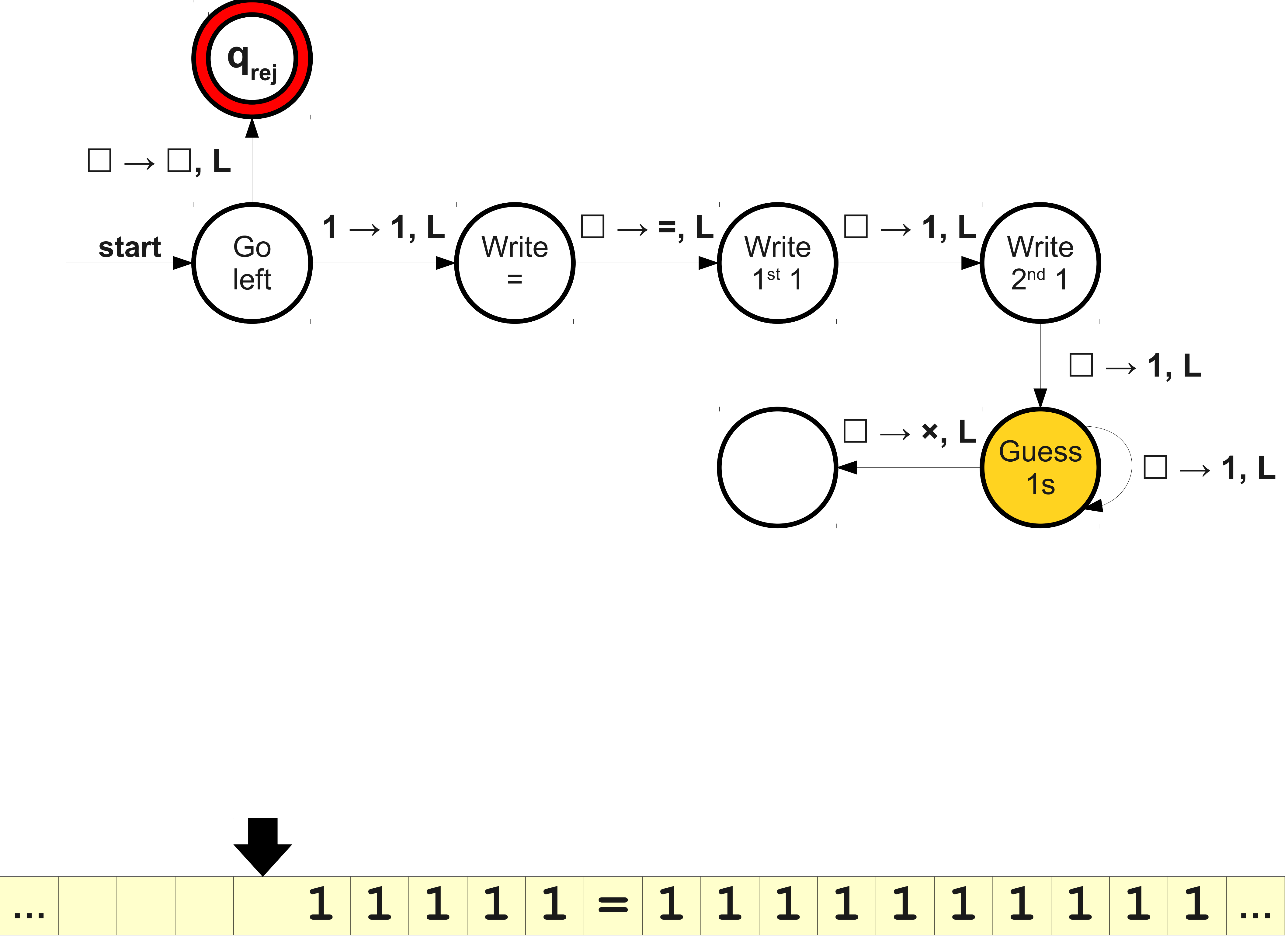
Write 2nd 1
□ → 1, L
□ → ×, L
Guess 1s
□ → 1, L
… 1 1 1 1 = 1 1 1 1 1 1 1 1 1 1 …



q_{rej}
□ → □, L
start
Go left
1 → 1, L
Write =
□ → =, L
Write 1st 1
□ → 1, L
Write 2nd 1
□ → 1, L
Guess 1s
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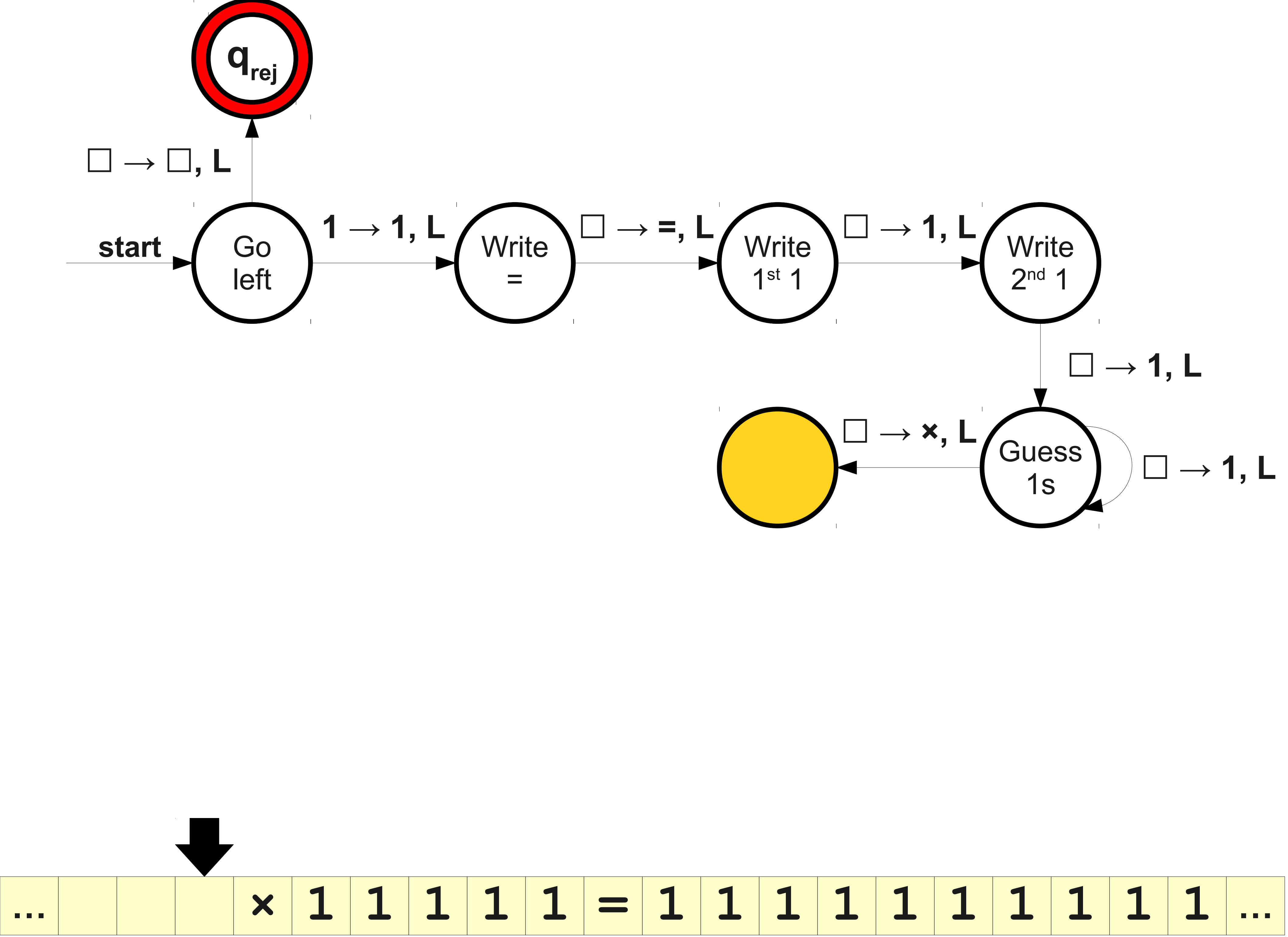
… 1 1 1 1 1 = 1 1 1 1 1 1 1 1 1 1 …


q_rej
□ → □, L
start
Go left
1 → 1, L
Write =
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Guess 1s
□ → 1, L
□ → ×, L
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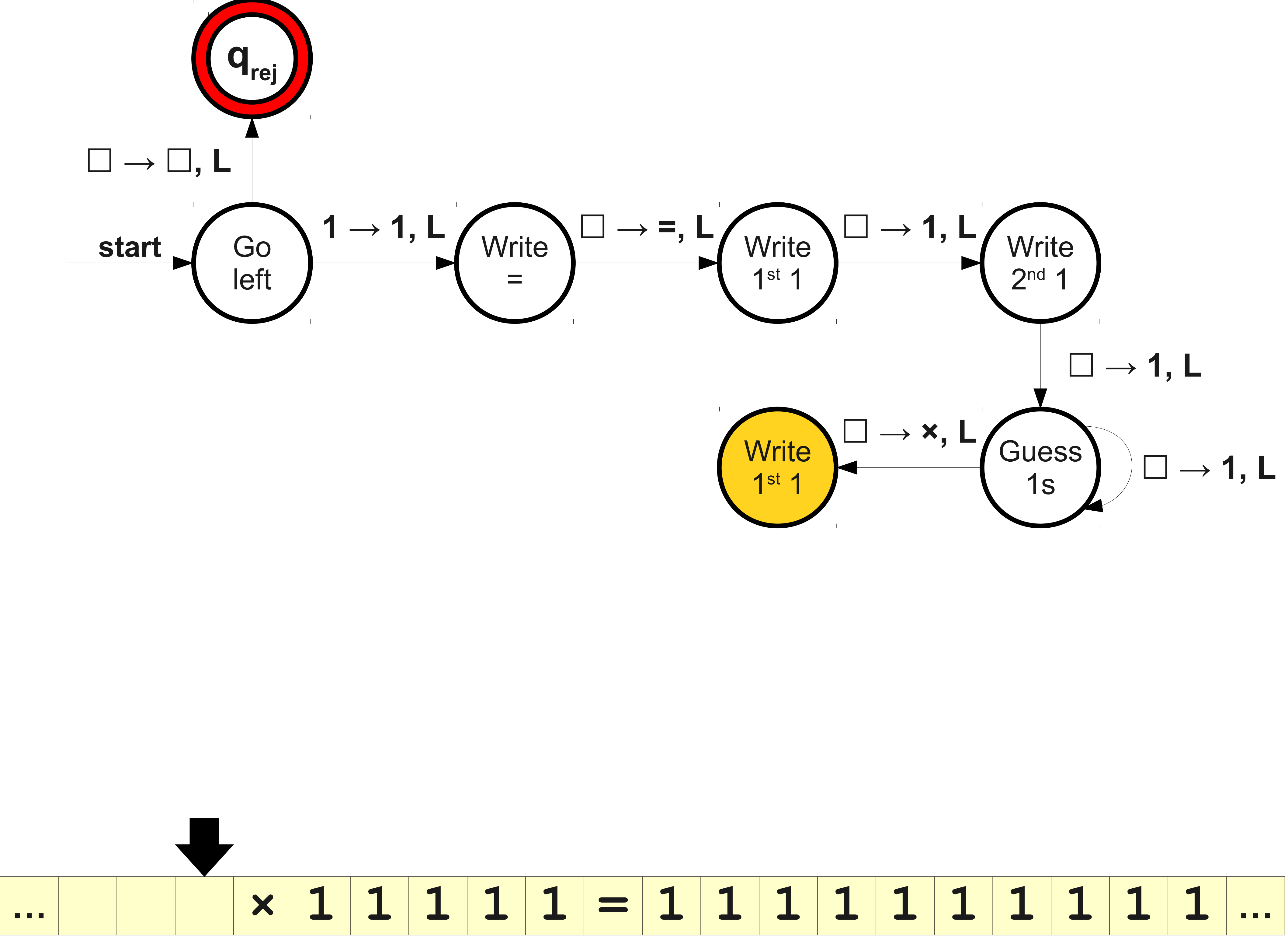
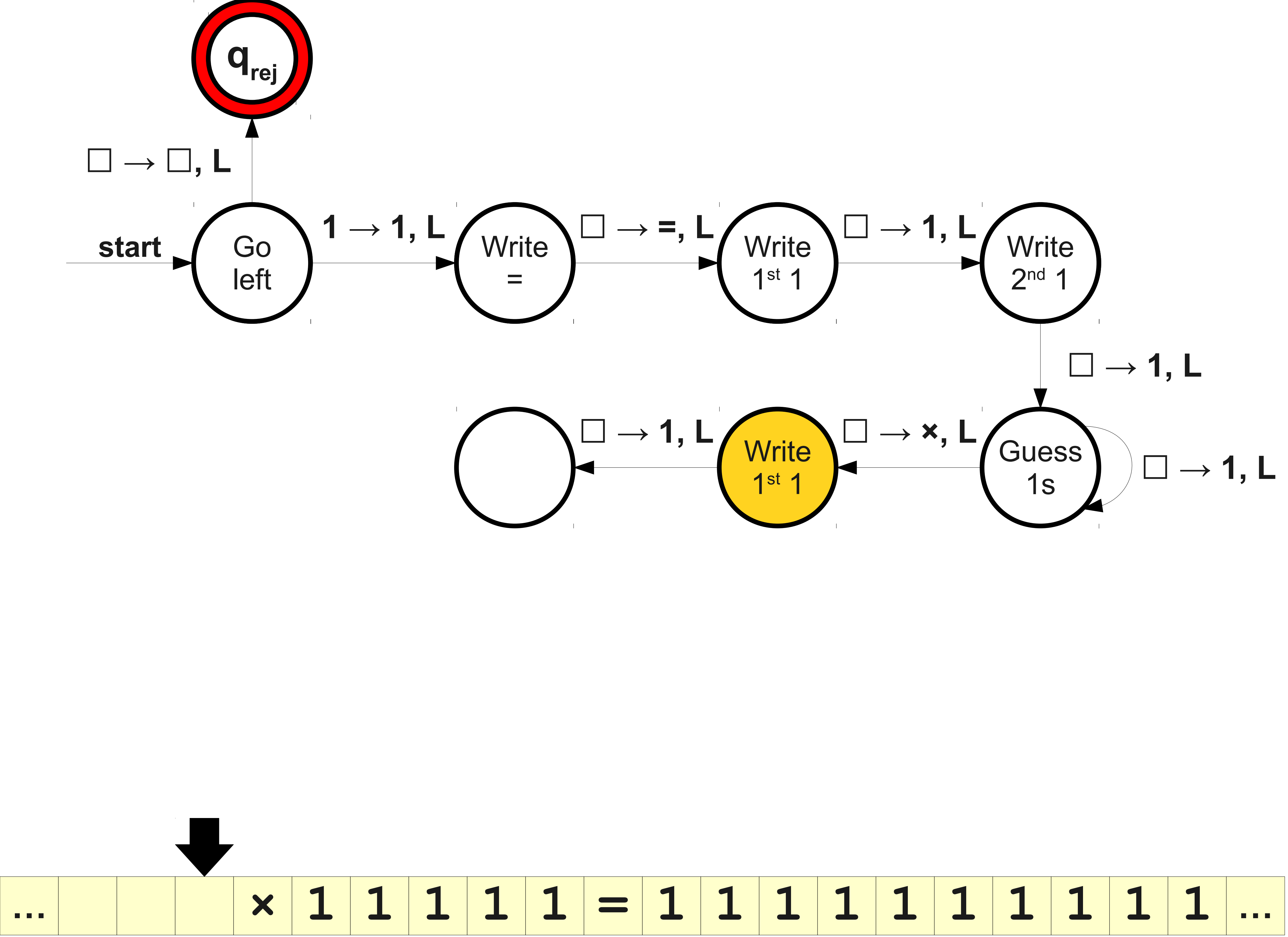


q_rej
□ → □, L
start
Go left
1 → 1, L
Write =
□ → =, L
Write 1st 1
□ → 1, L
Write 2nd 1
□ → 1, L
Guess 1s
□ → 1, L
□ → ×, L
Write 1st 1
… × 1 1 1 1 1 = 1 1 1 1 1 1 1 1 1 1 …


q_rej
□ → □, L
start
Go left
1 → 1, L
Write =
□ → =, L
Write 1st 1
□ → 1, L
Write 2nd 1
□ → 1, L
Guess 1s
□ → 1, L
□ → ×, L
Write 1st 1
□ → 1, L
...
×
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=
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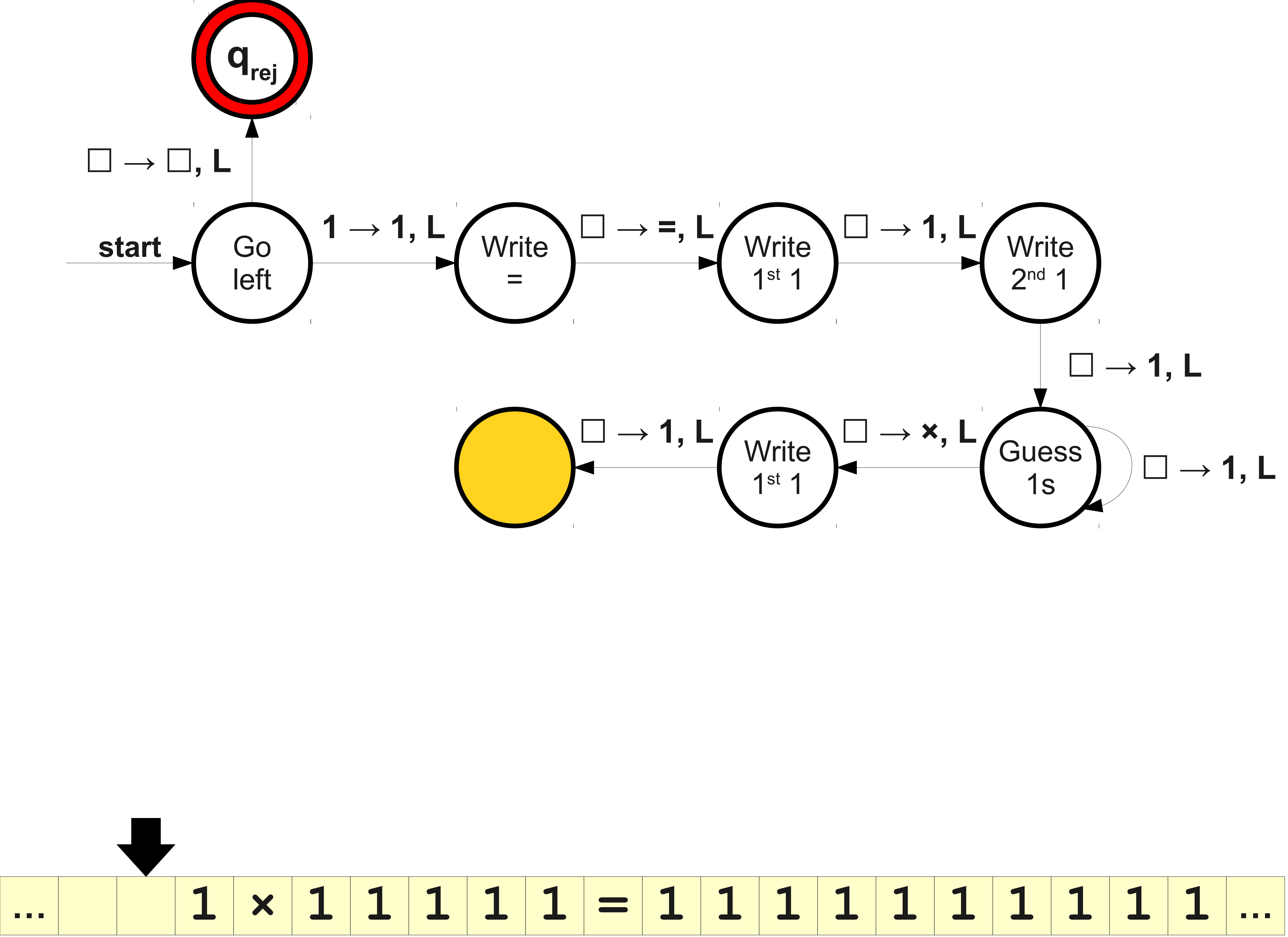
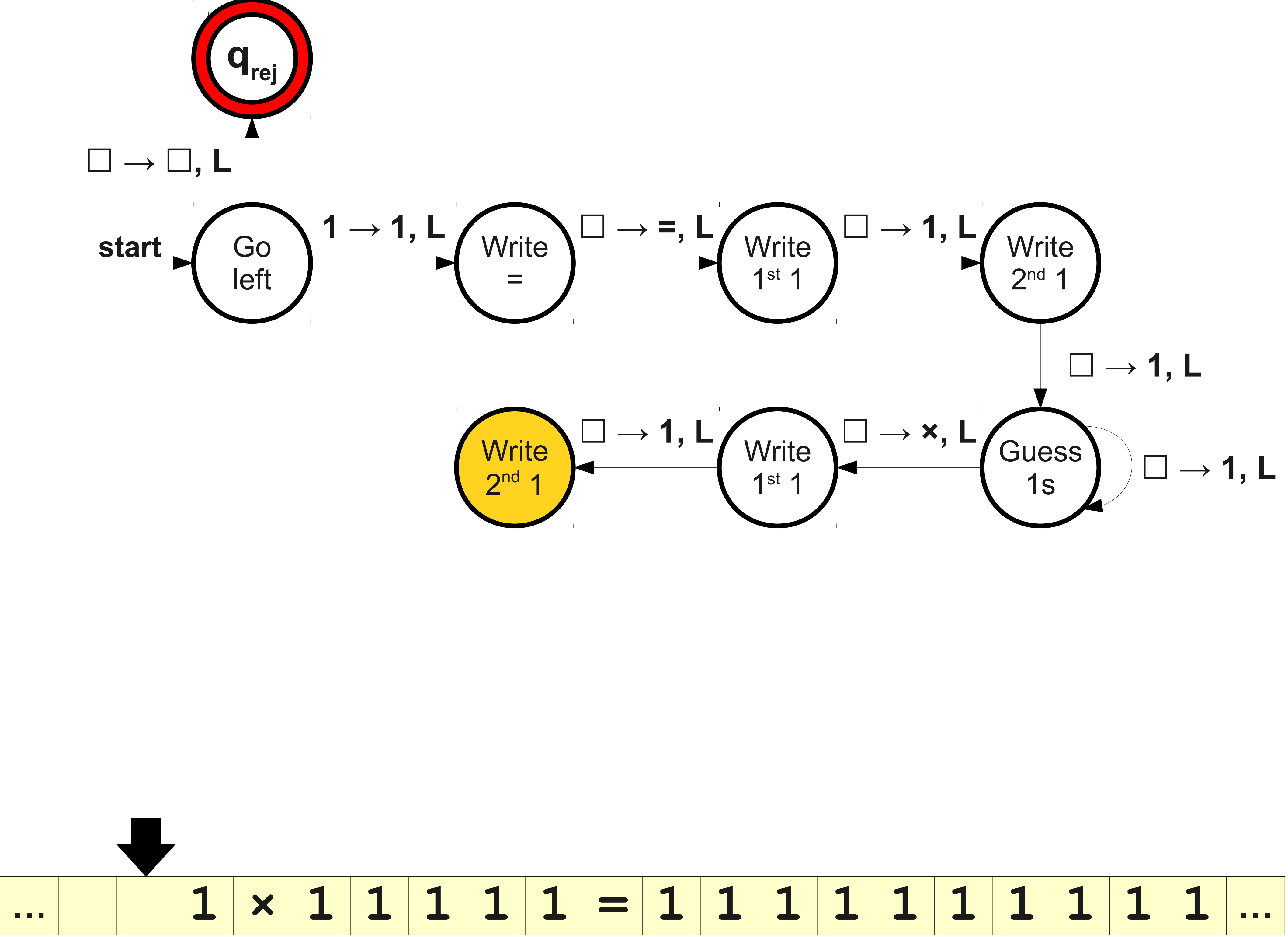
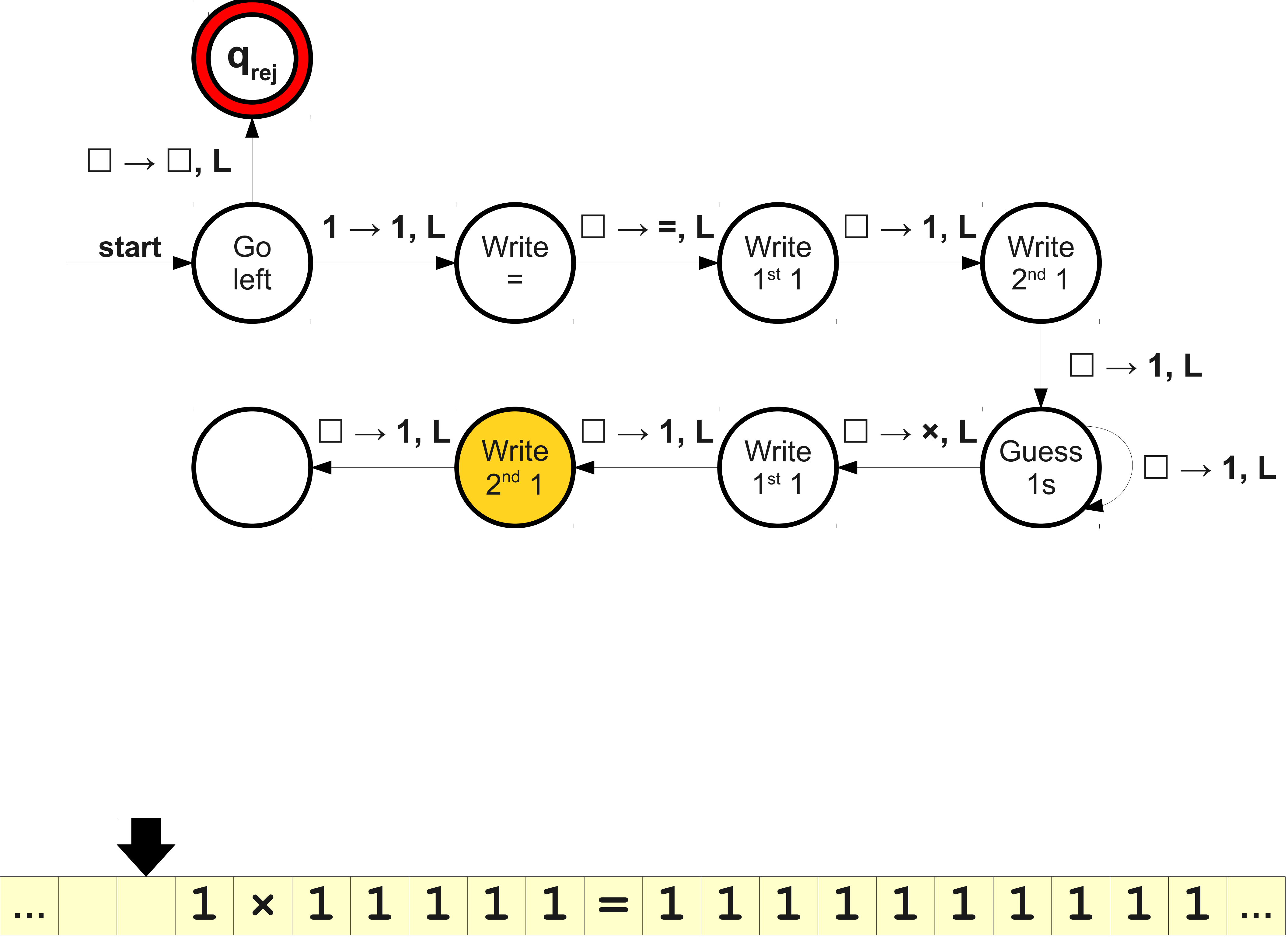
1
1
1
1
1
1
1
1
...

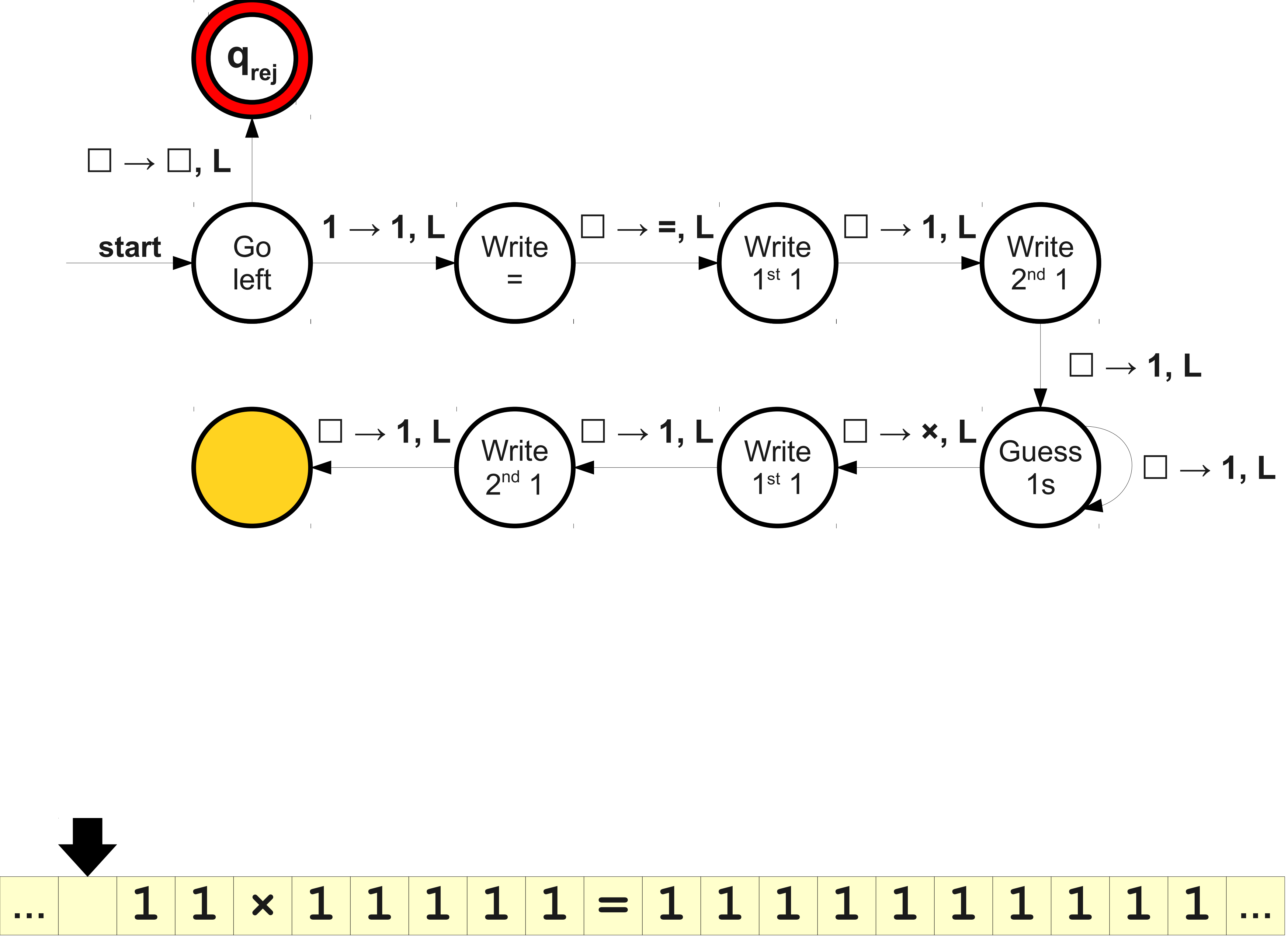

q rej
□ → □, L
start
Go left
1 → 1, L
Write =
□ → =, L
Write 1st 1
□ → 1, L
Write 2nd 1
□ → 1, L
Guess 1s
□ → 1, L
□ → ×, L
Write 1st 1
□ → 1, L
... 1 × 1 1 1 1 1 = 1 1 1 1 1 1 1 1 1 1 ...



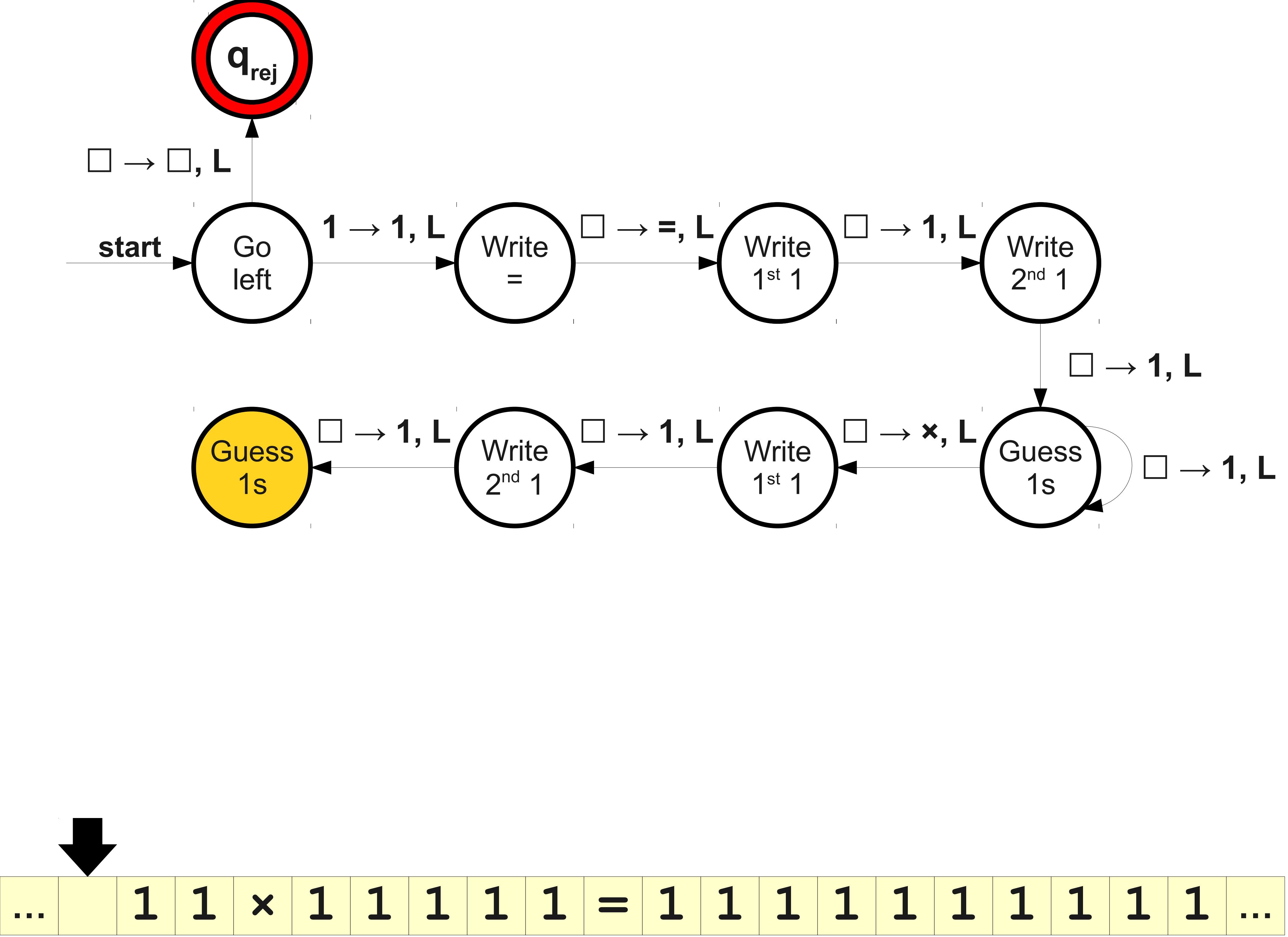
q_rej
□ → □, L
start
Go left
1 → 1, L
Write =
□ → =, L
Write 1st 1
□ → 1, L
Write 2nd 1
□ → 1, L
Guess 1s
□ → 1, L
□ → ×, L
Write 1st 1
□ → 1, L
Write 2nd 1
… 1 × 1 1 1 1 1 = 1 1 1 1 1 1 1 1 1 1 …

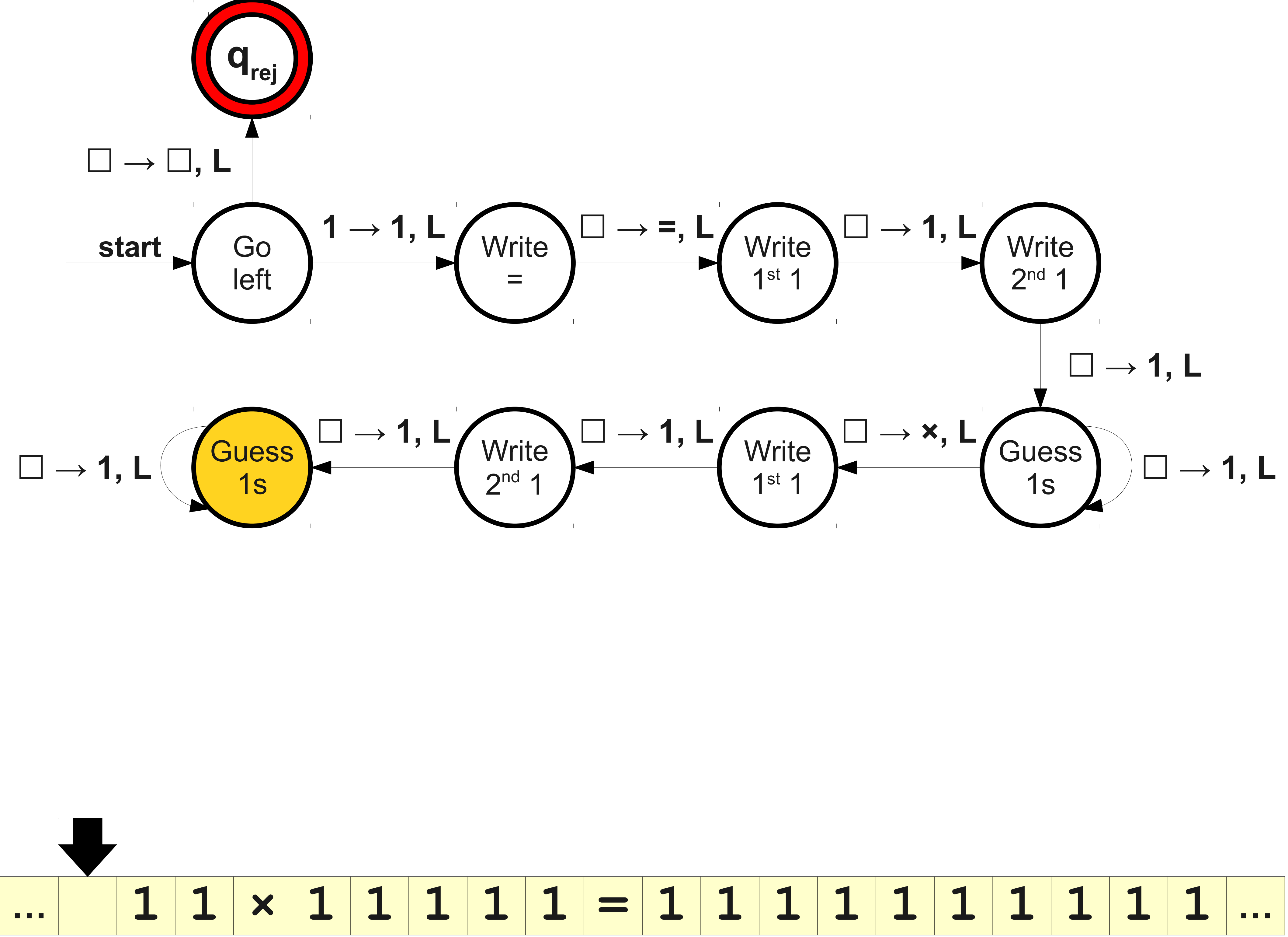


q_rej
□ → □, L
start
Go left
1 → 1, L
Write =
□ → =, L
Write 1st 1
□ → 1, L
Write 2nd 1
□ → 1, L
Guess 1s
□ → 1, L
□ → ×, L
Write 1st 1
□ → 1, L
Write 2nd 1
□ → 1, L
… 1 × 1 1 1 1 1 = 1 1 1 1 1 1 1 1 1 1 …

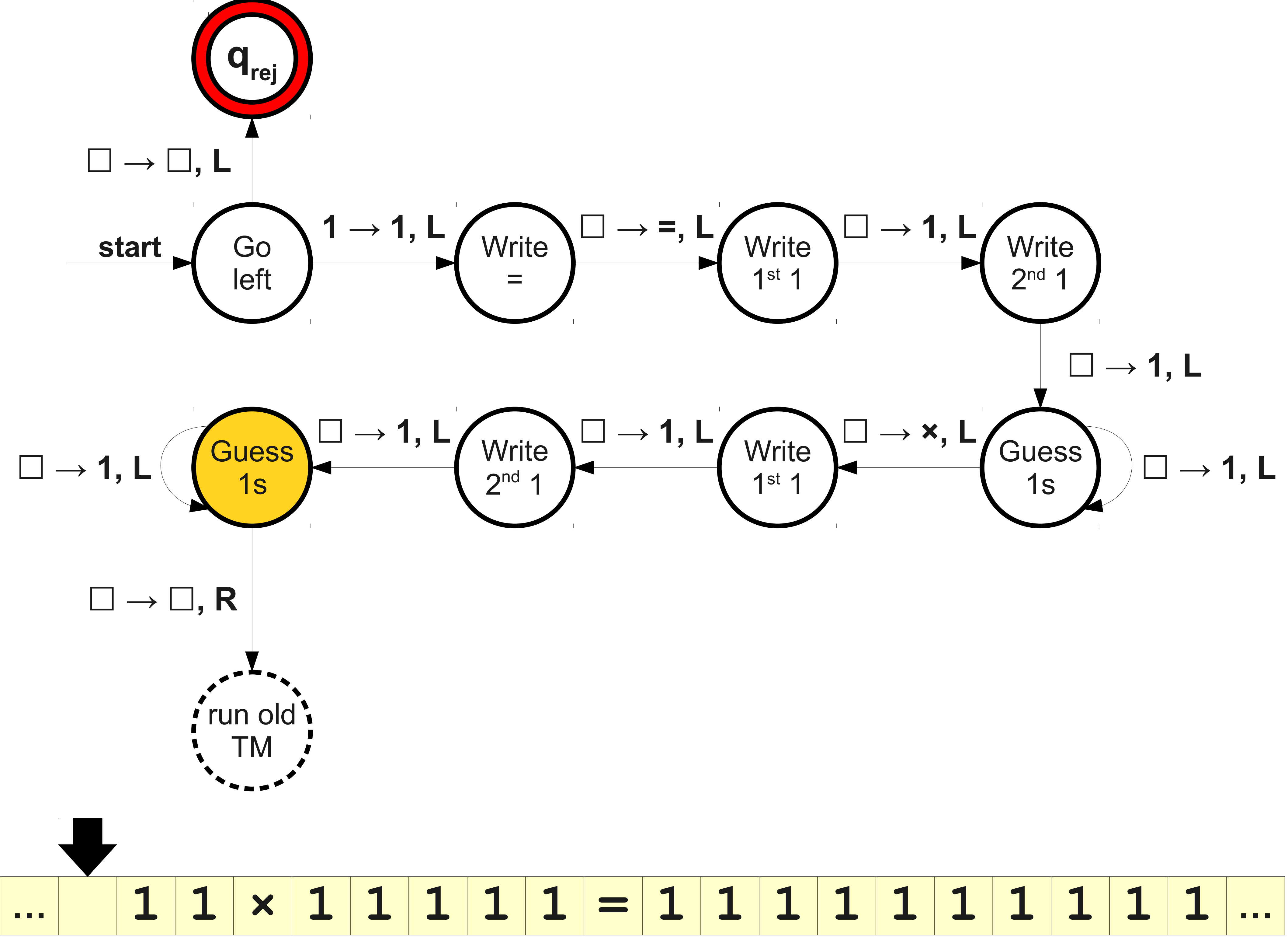


q rej
□ → □, L
start
Go left
1 → 1, L
Write =
□ → =, L
Write 1st 1
□ → 1, L
Write 2nd 1
□ → 1, L
Guess 1s
□ → 1, L
□ → ×, L
Write 1st 1
□ → 1, L
Write 2nd 1
□ → 1, L
... 1 1 × 1 1 1 1 1 = 1 1 1 1 1 1 1 1 1 1 ...

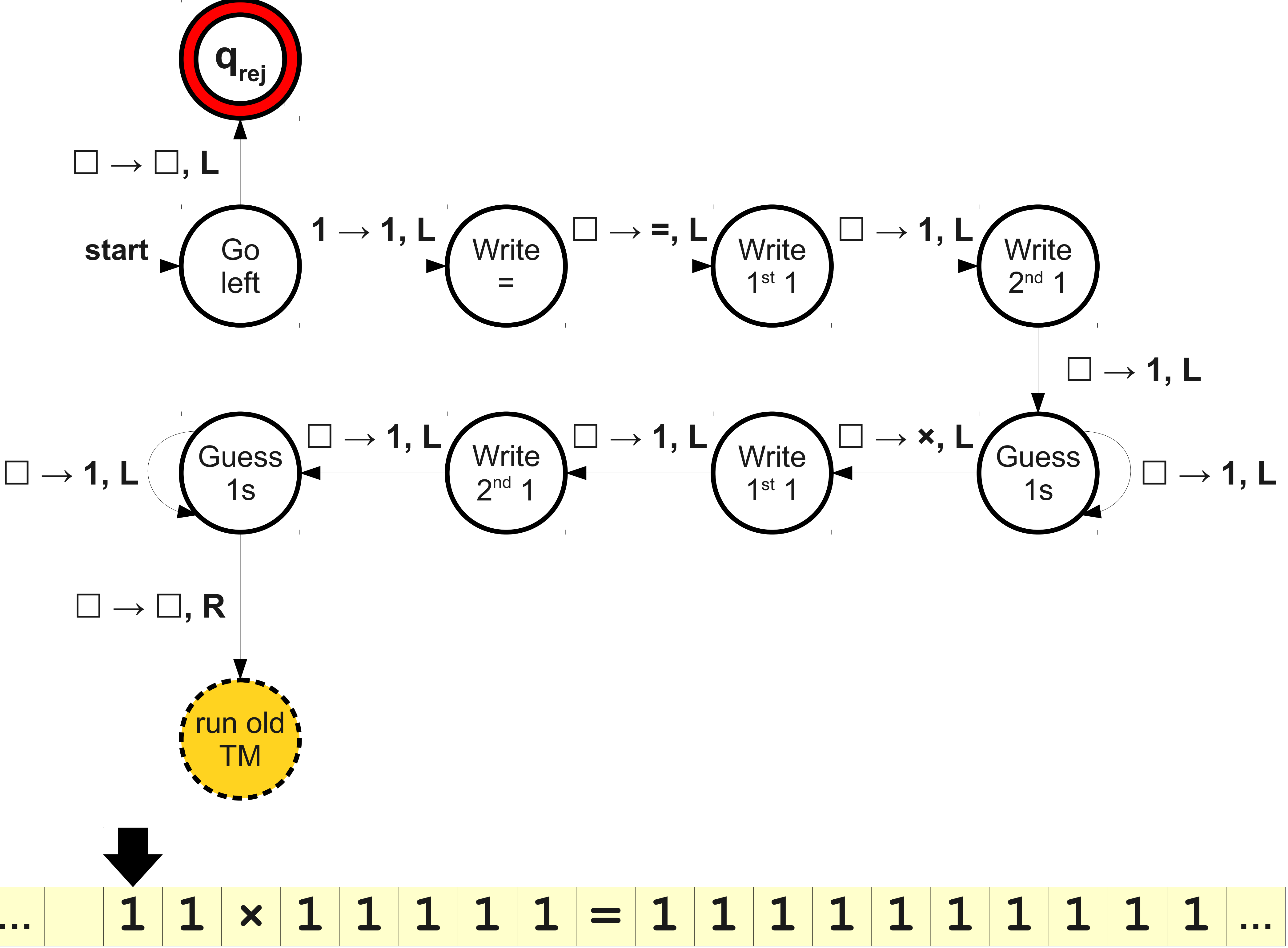

q_rej
□ → □, L
start
Go left
1 → 1, L
Write =
□ → =, L
Write 1st 1
□ → 1, L
Write 2nd 1
□ → 1, L
Guess 1s
□ → 1, L
□ → ×, L
Write 1st 1
□ → 1, L
Write 2nd 1
□ → 1, L
Guess 1s
... 1 1 × 1 1 1 1 1 = 1 1 1 1 1 1 1 1 1 1 ...



q_rej
□ → □, L
start
Go left
1 → 1, L
Write =
□ → =, L
Write 1st 1
□ → 1, L
Write 2nd 1
□ → 1, L
Guess 1s
□ → 1, L
□ → ×, L
Write 1st 1
□ → 1, L
Write 2nd 1
□ → 1, L
Guess 1s
□ → 1, L
… 1 1 × 1 1 1 1 1 = 1 1 1 1 1 1 1 1 1 1 …



q_rej
□ → □, L
start
Go left
1 → 1, L
Write =
□ → =, L
Write 1st 1
□ → 1, L
Write 2nd 1
□ → 1, L
Guess 1s
□ → 1, L
□ → ×, L
Write 1st 1
□ → 1, L
Write 2nd 1
□ → 1, L
Guess 1s
□ → 1, L
□ → □, R
run old TM
… 1 1 × 1 1 1 1 1 = 1 1 1 1 1 1 1 1 1 1 …

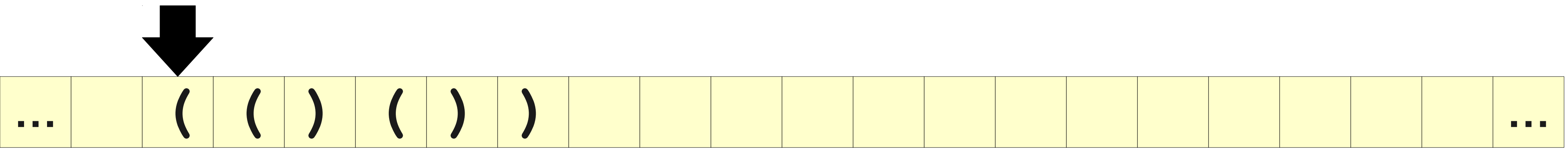


q_{rej}
$\square \rightarrow \square$, L
start
Go left
1 → 1, L
Write =
$\square \rightarrow$ =, L
Write 1^{st} 1
$\square \rightarrow$ 1, L
Write 2^{nd} 1
$\square \rightarrow$ 1, L
Guess 1s
$\square \rightarrow$ 1, L
$\square \rightarrow$ ×, L
Write 1^{st} 1
$\square \rightarrow$ 1, L
Write 2^{nd} 1
$\square \rightarrow$ 1, L
Guess 1s
$\square \rightarrow$ 1, L
$\square \rightarrow \square$, R
run old TM
... 1 1 × 1 1 1 1 1 = 1 1 1 1 1 1 1 1 1 1 ...

Designing NTMs

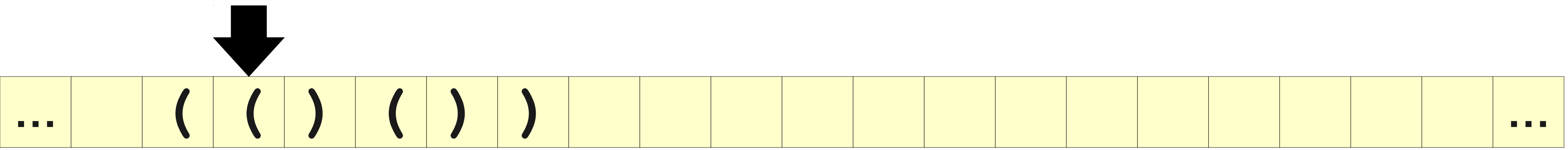
- Suppose that we have a CFG G.
- Can we build a TM M where $\mathscr{L}(M) = \mathscr{L}(G)$?
- **Idea:** Nondeterministically guess which productions ought to be applied.
  - Keep the original string on the input tape.
  - Keep guessing productions until no nonterminals remain.
  - Accept if the resulting string matches.

A Sketch of the Construction



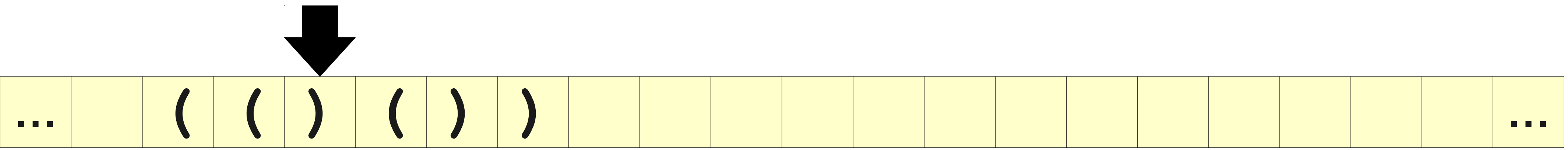
$$S \to SS \mid (S) \mid \varepsilon$$

A Sketch of the Construction



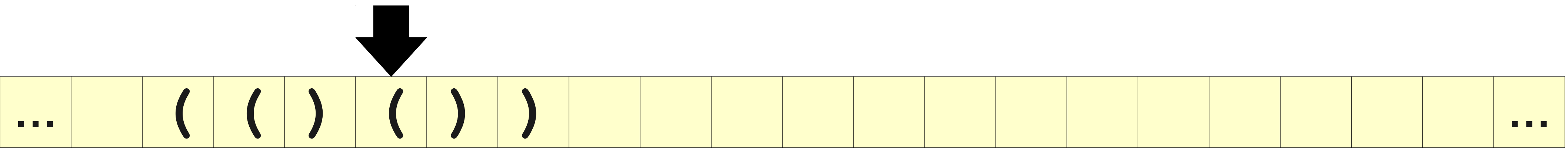
$$\mathbf{S} \to \mathbf{SS} \mid (\mathbf{S}) \mid \varepsilon$$

A Sketch of the Construction



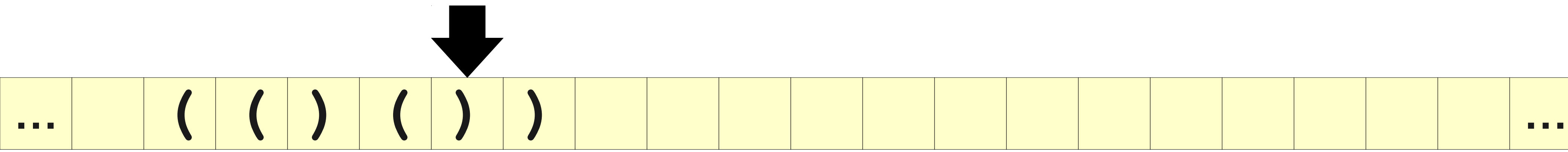
$$S \rightarrow SS \mid (S) \mid \varepsilon$$

A Sketch of the Construction



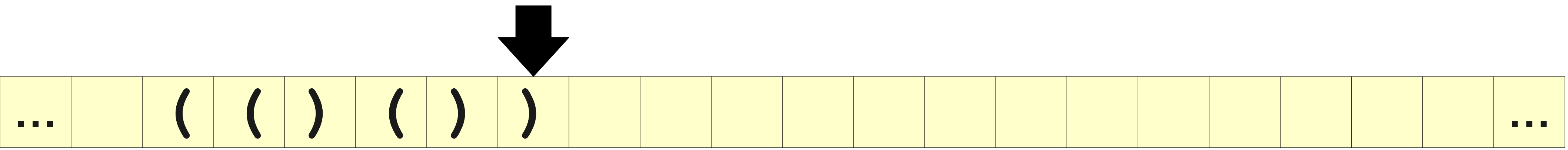
$$S \rightarrow SS \mid (S) \mid \varepsilon$$

A Sketch of the Construction



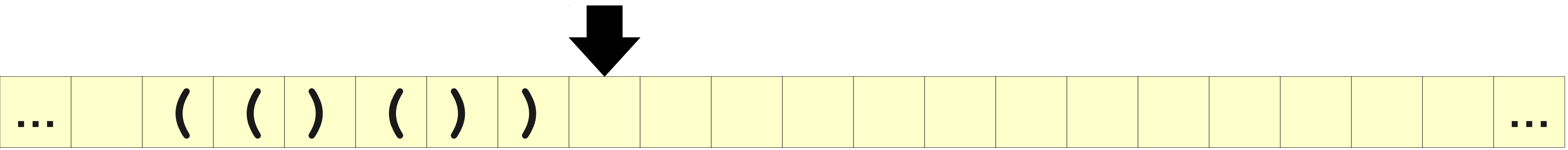
$$S \rightarrow SS \mid (S) \mid \varepsilon$$

A Sketch of the Construction



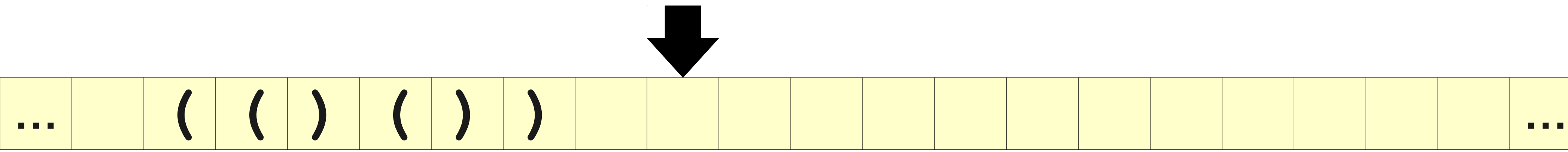
$$S \rightarrow SS \mid (S) \mid \varepsilon$$

A Sketch of the Construction



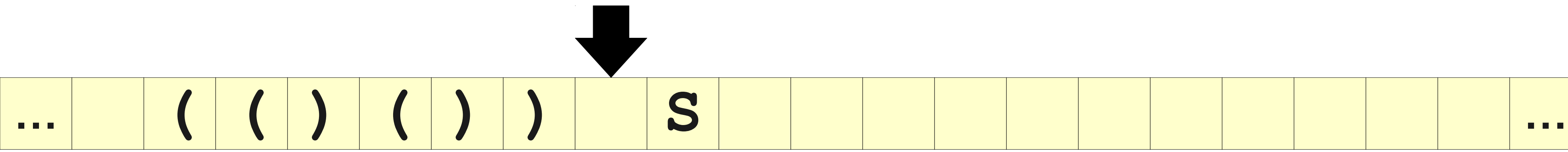
$$S \rightarrow SS \mid (S) \mid \varepsilon$$

A Sketch of the Construction



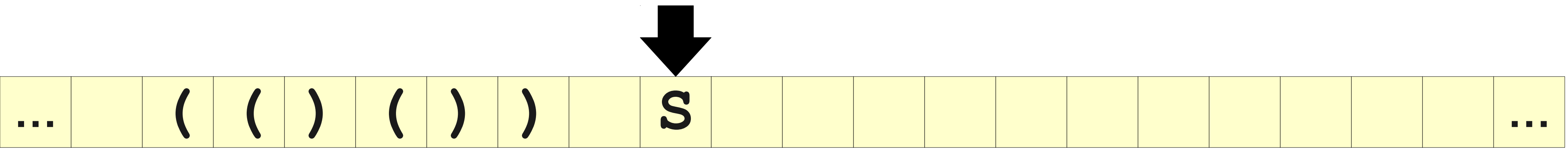
$$S \to SS \mid (S) \mid \varepsilon$$

A Sketch of the Construction



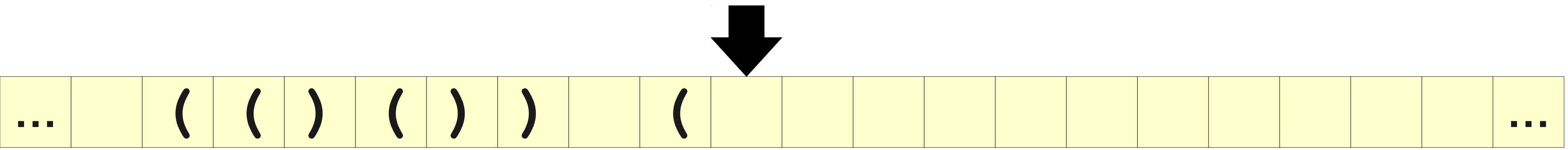
$$S \rightarrow SS \mid (S) \mid \varepsilon$$

A Sketch of the Construction



$$\mathbf{S} \rightarrow \mathbf{SS} \mid \texttt{(}\mathbf{S}\texttt{)} \mid \varepsilon$$

A Sketch of the Construction



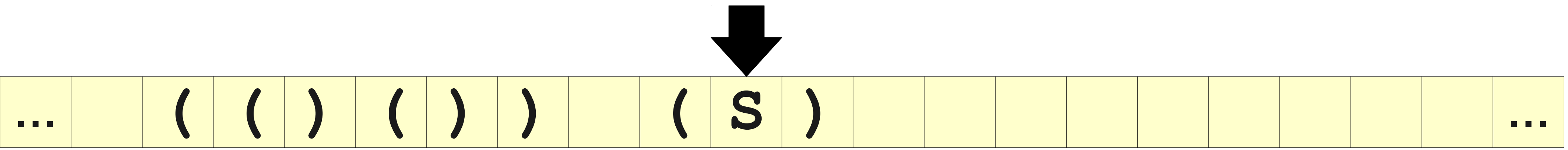
$$\mathbf{S} \rightarrow \mathbf{SS} \mid (\mathbf{S}) \mid \varepsilon$$

A Sketch of the Construction

…		(	(	)	(	)	)		(	S											…

$$\mathbf{S} \rightarrow \mathbf{SS} \mid (\mathbf{S}) \mid \varepsilon$$

A Sketch of the Construction



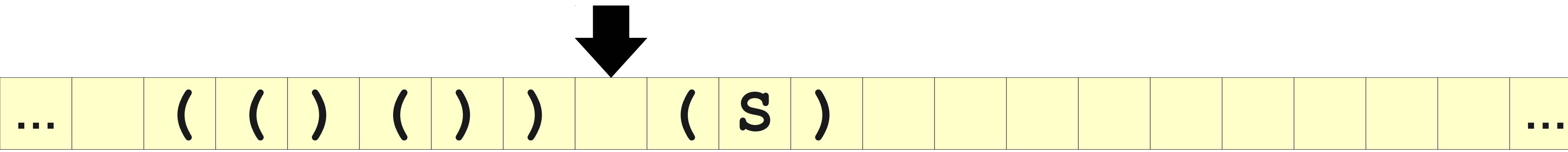
$$\mathbf{S} \rightarrow \mathbf{SS} \mid \texttt{(}\mathbf{S}\texttt{)} \mid \varepsilon$$

A Sketch of the Construction

…		(	(	)	(	)	)		(	S	)										…

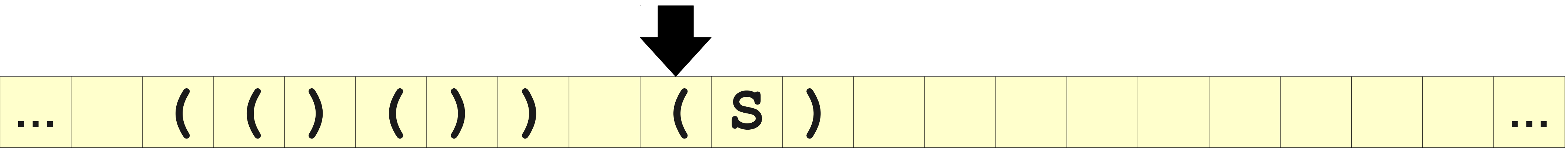
$$\mathbf{S} \rightarrow \mathbf{SS} \mid (\mathbf{S}) \mid \varepsilon$$

A Sketch of the Construction



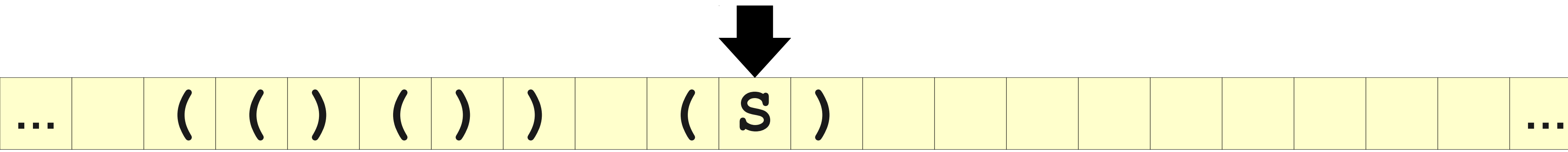
$$S \rightarrow SS \mid (S) \mid \varepsilon$$

A Sketch of the Construction



$$\mathbf{S} \rightarrow \mathbf{SS} \mid (\mathbf{S}) \mid \varepsilon$$

A Sketch of the Construction



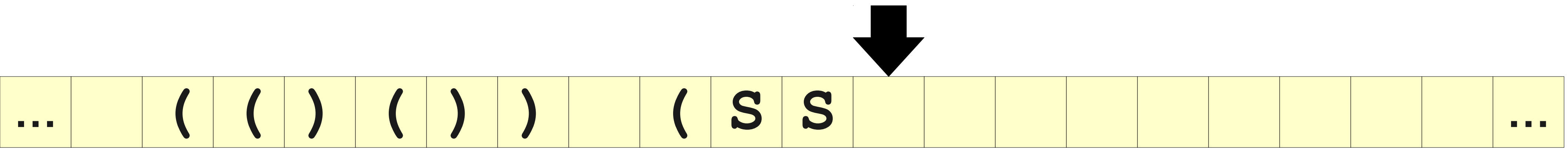
$$\mathbf{S} \rightarrow \mathbf{SS} \mid (\mathbf{S}) \mid \varepsilon$$

A Sketch of the Construction

…		(	(	)	(	)	)		(	S	)										…

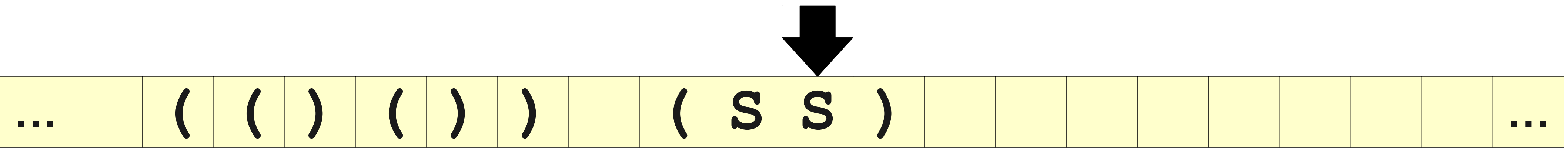
$$\mathbf{S} \rightarrow \mathbf{SS} \mid \texttt{(}\mathbf{S}\texttt{)} \mid \varepsilon$$

A Sketch of the Construction



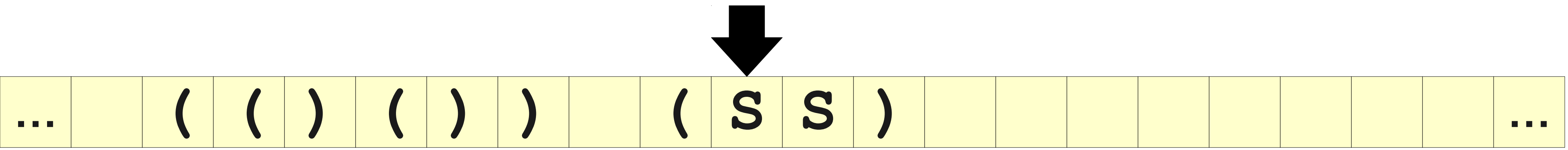
$$S \rightarrow SS \mid (S) \mid \varepsilon$$

A Sketch of the Construction



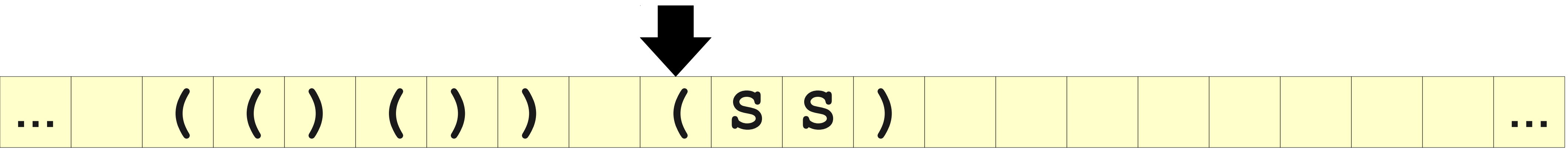
$$S \rightarrow SS \mid (S) \mid \varepsilon$$

A Sketch of the Construction



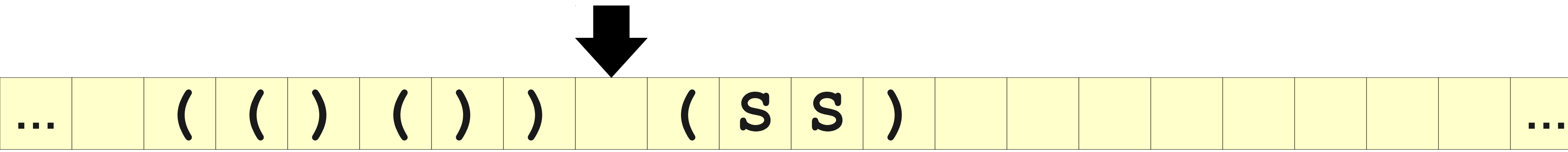
$$\mathbf{S} \rightarrow \mathbf{SS} \mid \mathtt{(}\mathbf{S}\mathtt{)} \mid \varepsilon$$

A Sketch of the Construction



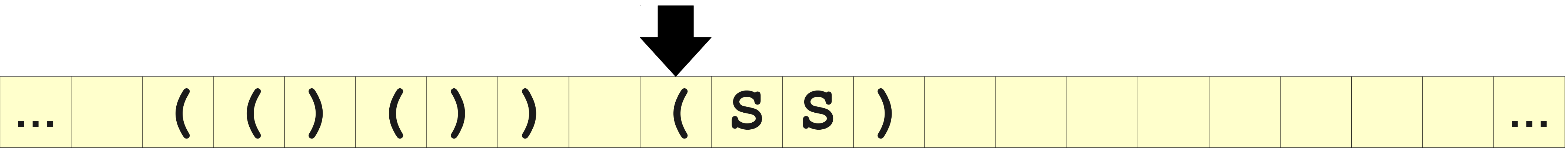
$$\mathbf{S} \rightarrow \mathbf{SS} \mid \mathtt{(}\mathbf{S}\mathtt{)} \mid \varepsilon$$

A Sketch of the Construction



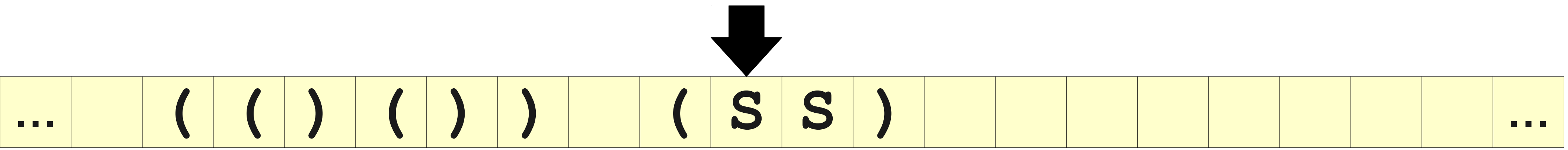
$$S \rightarrow SS \mid (S) \mid \varepsilon$$

A Sketch of the Construction



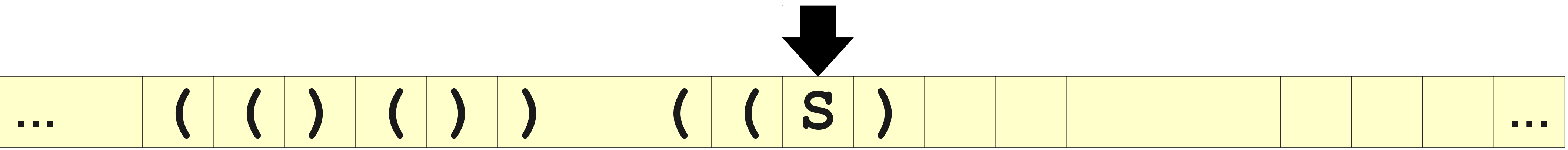
$$\mathbf{S} \rightarrow \mathbf{SS} \mid \mathtt{(}\mathbf{S}\mathtt{)} \mid \varepsilon$$

A Sketch of the Construction



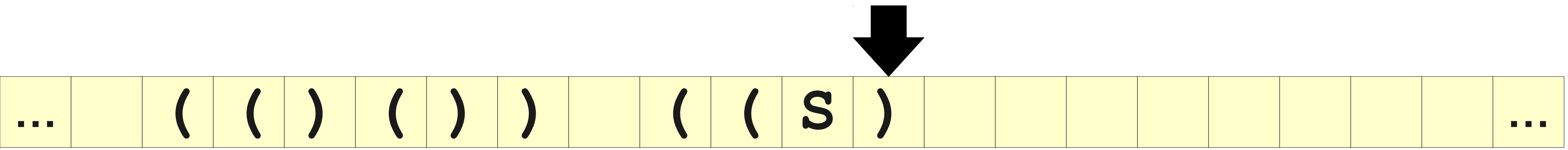
$$\mathbf{S} \rightarrow \mathbf{SS} \mid \texttt{(}\mathbf{S}\texttt{)} \mid \varepsilon$$

A Sketch of the Construction



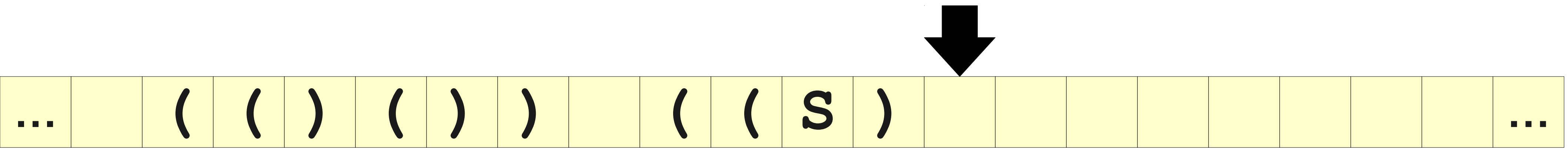
$$\mathbf{S} \rightarrow \mathbf{SS} \mid (\mathbf{S}) \mid \varepsilon$$

A Sketch of the Construction



$$S \rightarrow SS \mid (S) \mid \varepsilon$$

A Sketch of the Construction



$$\mathbf{S} \to \mathbf{SS} \mid \texttt{(}\mathbf{S}\texttt{)} \mid \varepsilon$$

A Sketch of the Construction

…		(	(	)	(	)	)		(	(	S	)	S								…

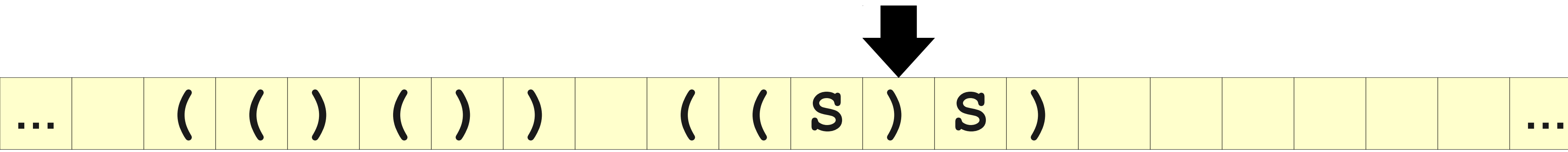
$$\mathbf{S} \rightarrow \mathbf{SS} \mid \mathtt{(}\mathbf{S}\mathtt{)} \mid \varepsilon$$

A Sketch of the Construction

…		(	(	)	(	)	)		(	(	S	)	S	)							…

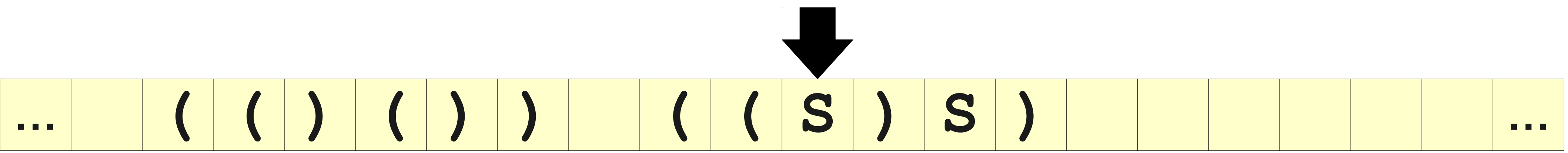
$$\mathbf{S} \rightarrow \mathbf{SS} \mid \mathtt{(}\mathbf{S}\mathtt{)} \mid \varepsilon$$

A Sketch of the Construction



$$S \to SS \mid (S) \mid \varepsilon$$

A Sketch of the Construction



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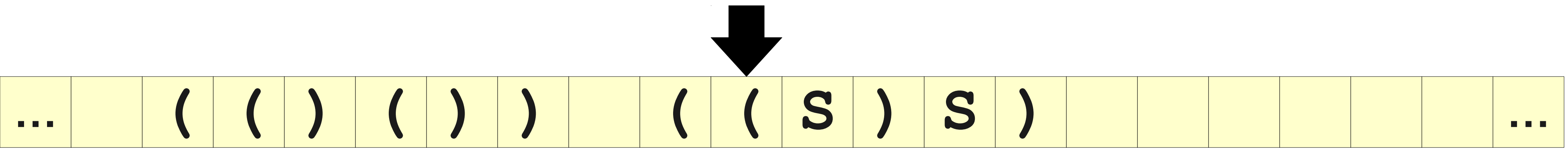
A Sketch of the Construction



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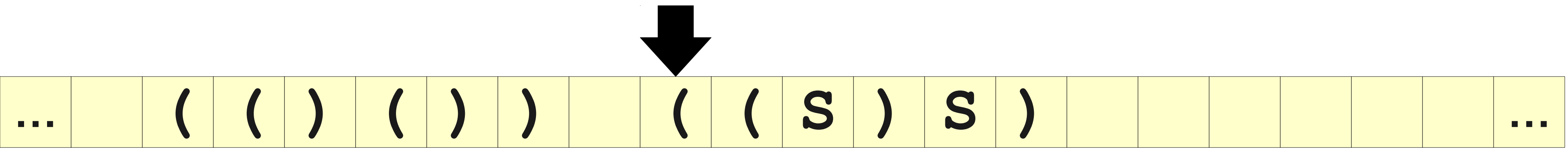

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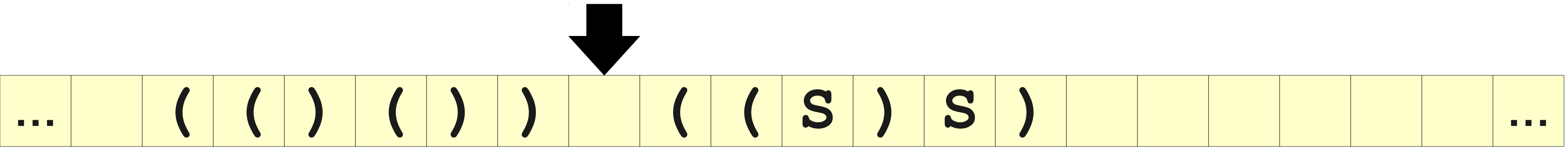

$$S \to SS \mid (S) \mid \varepsilon$$

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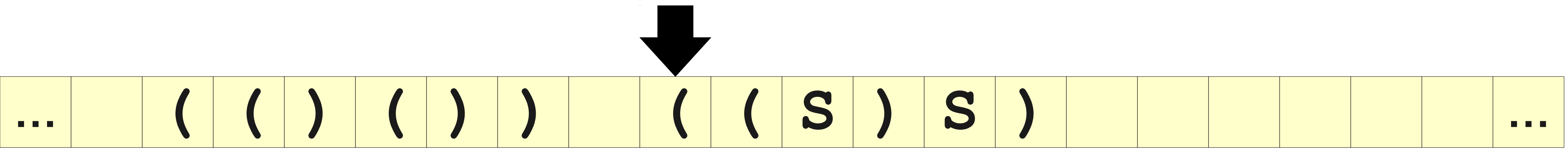

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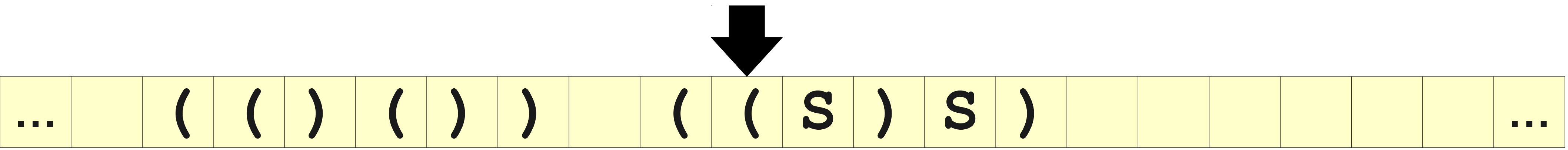
A Sketch of the Construction



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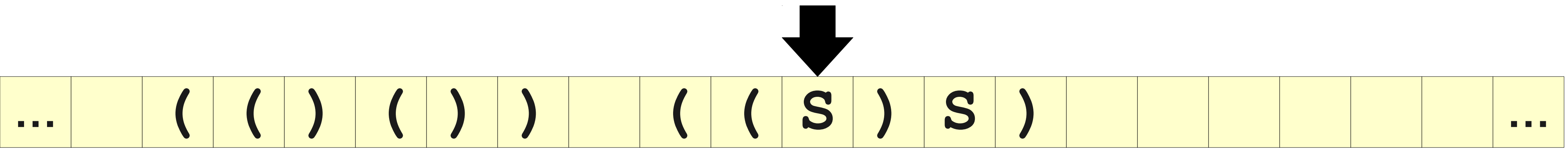

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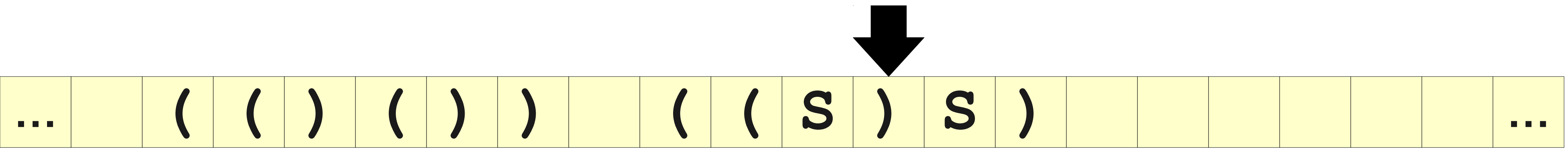

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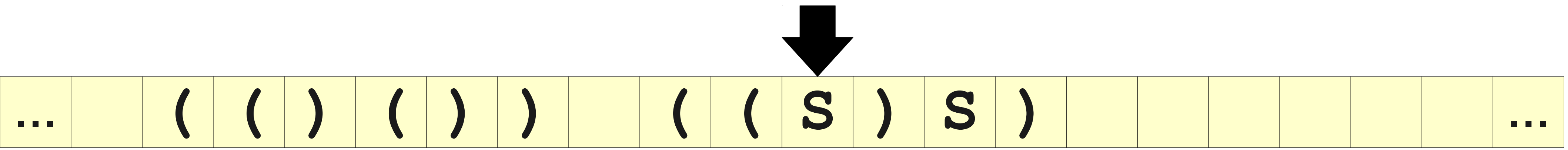

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A Sketch of the Construction

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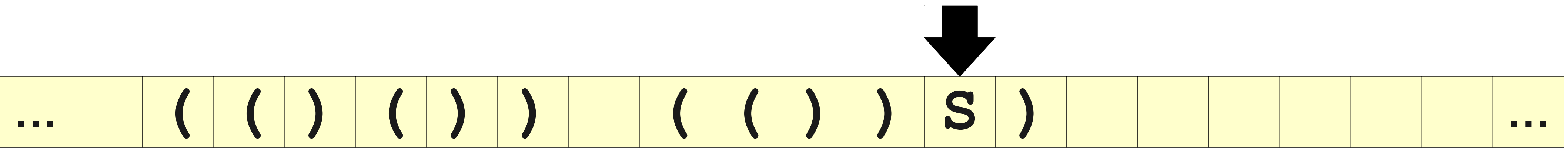

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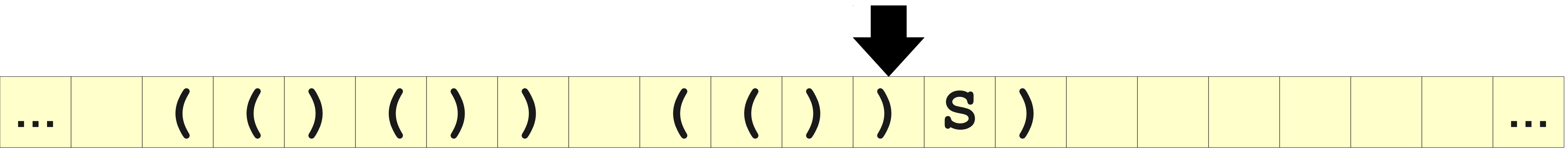

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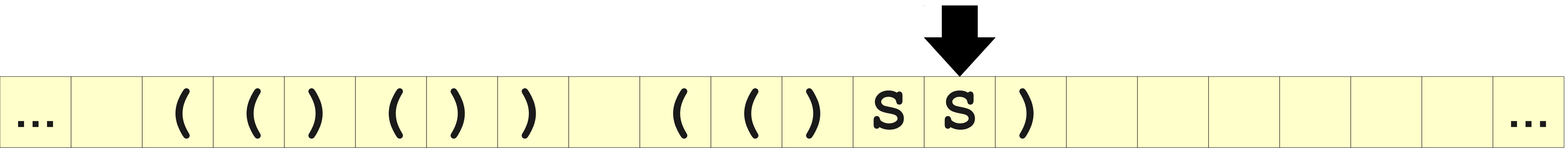

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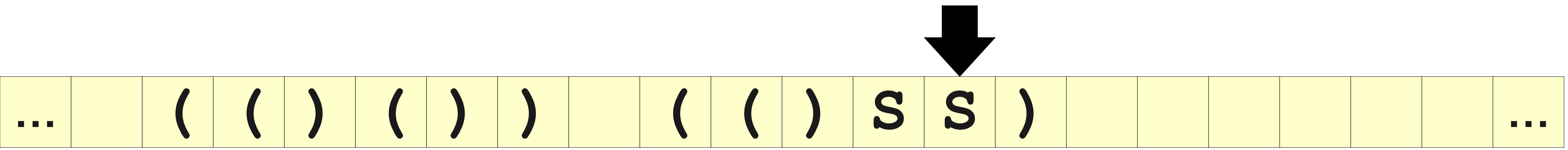

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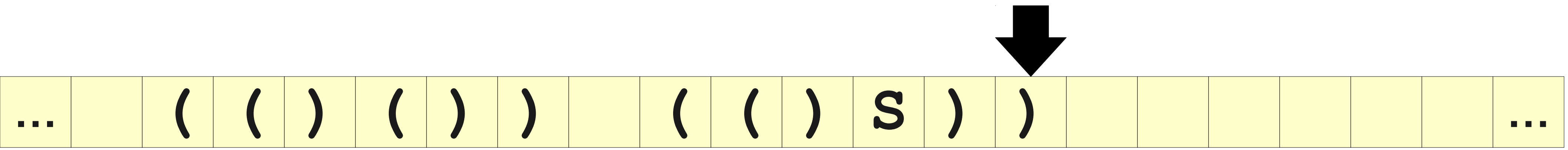

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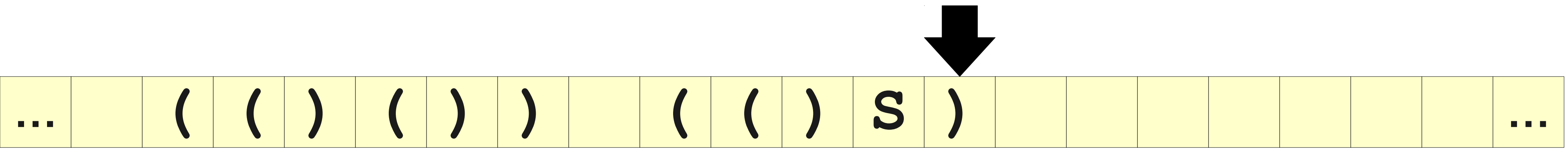

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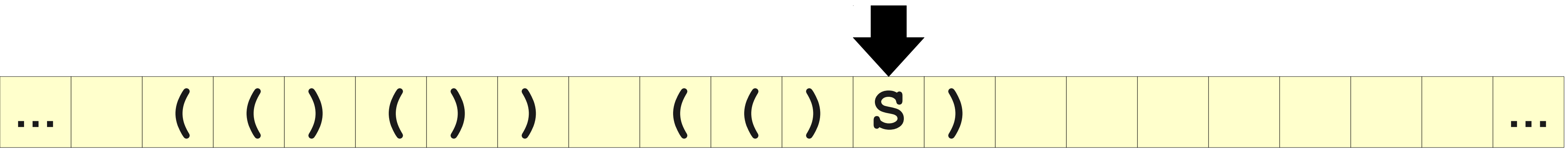

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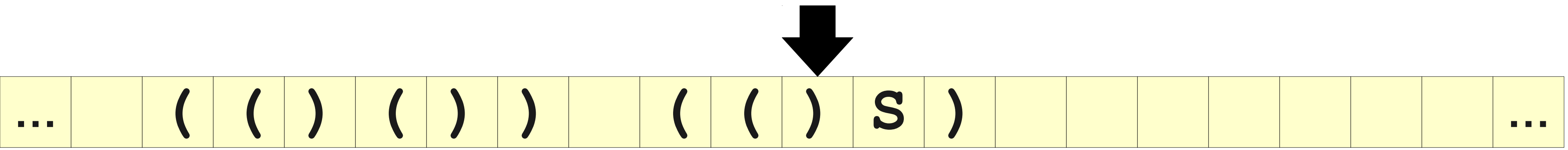

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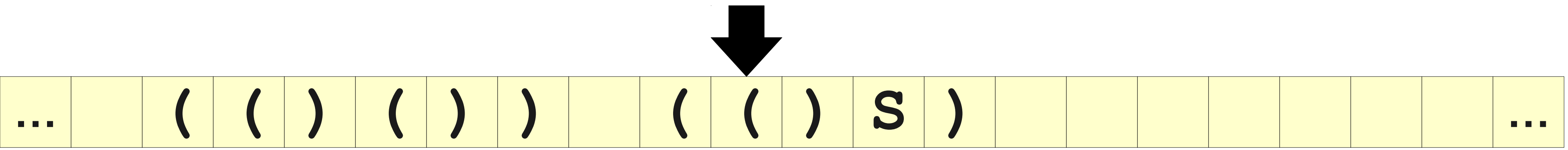

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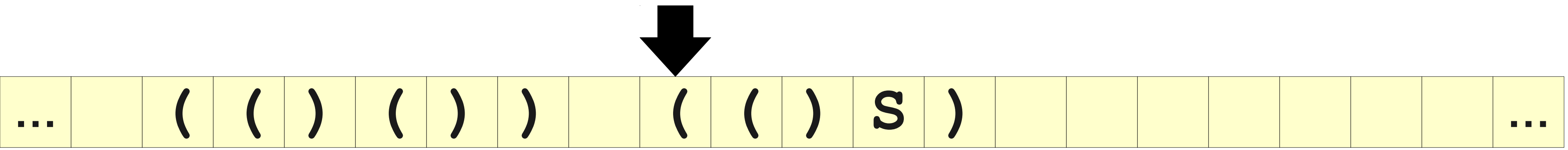

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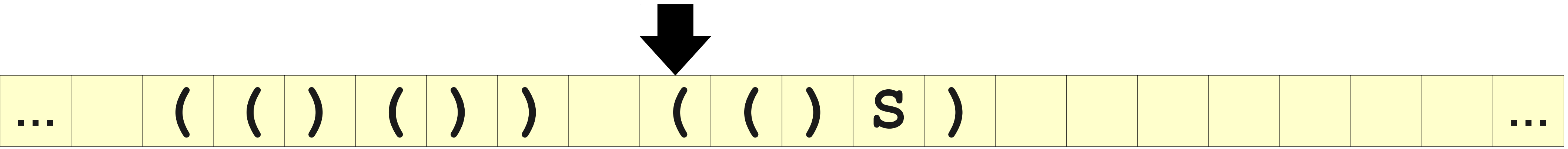

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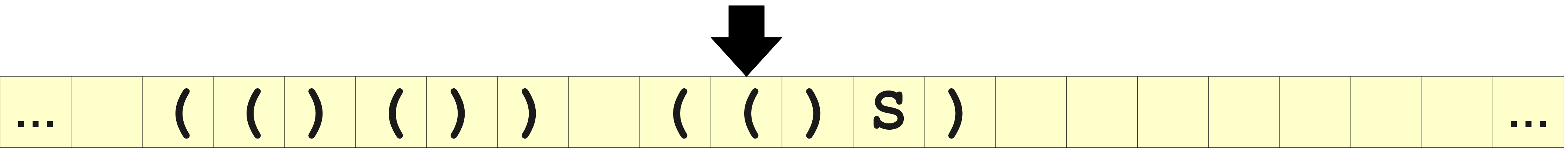

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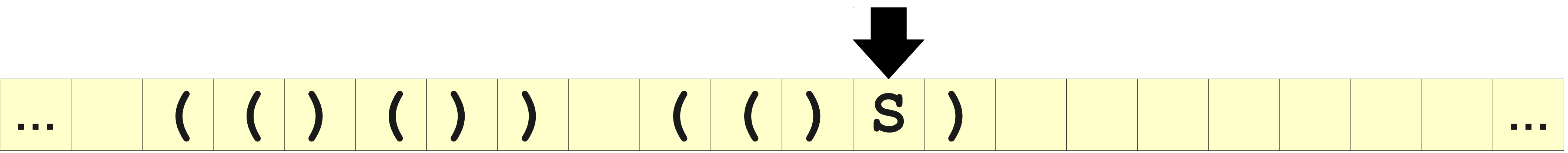

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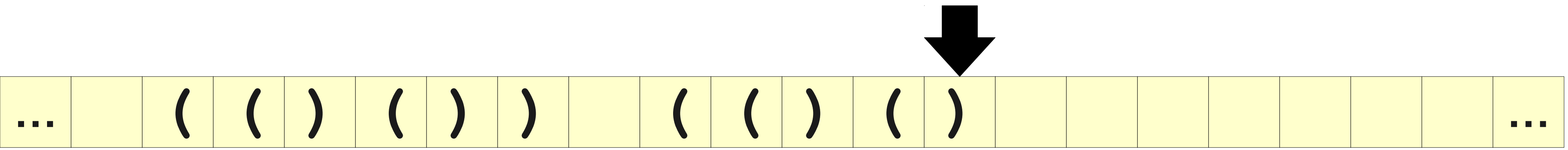

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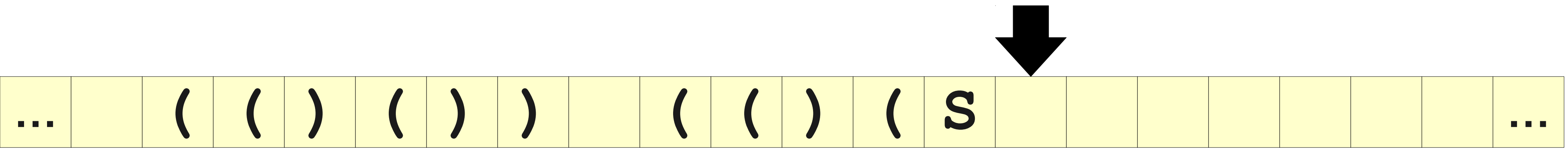

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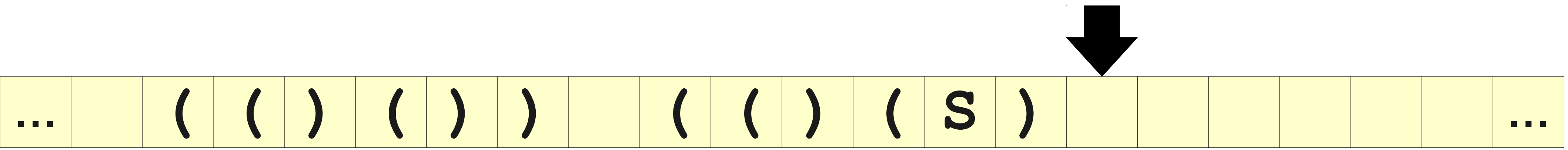

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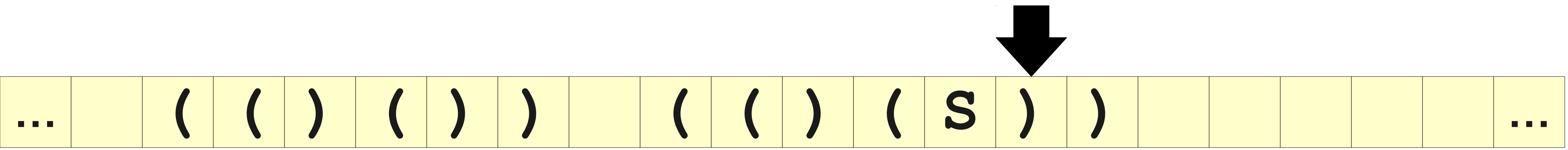

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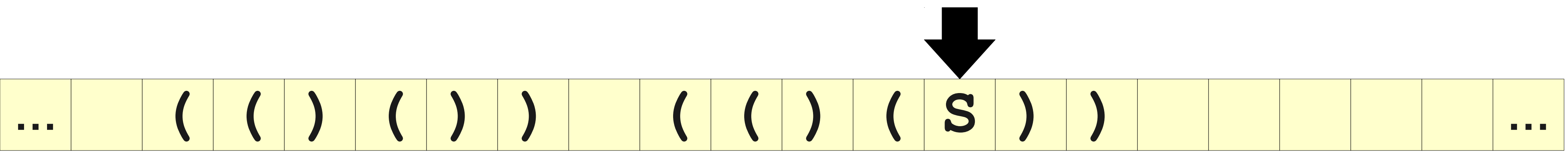

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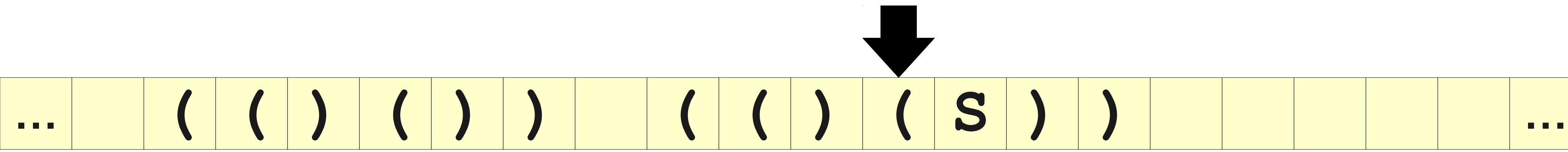

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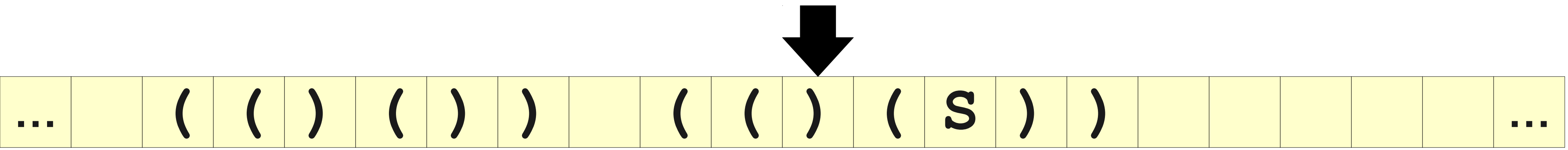

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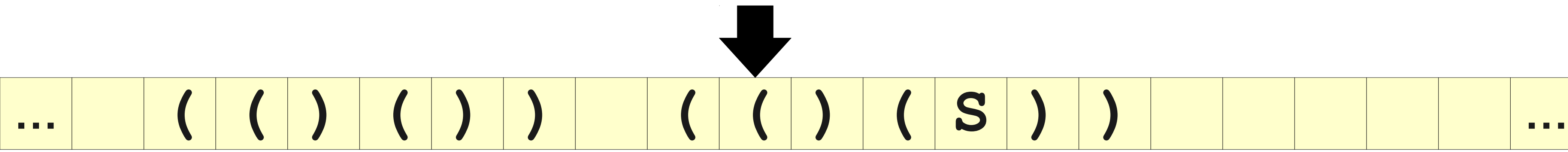

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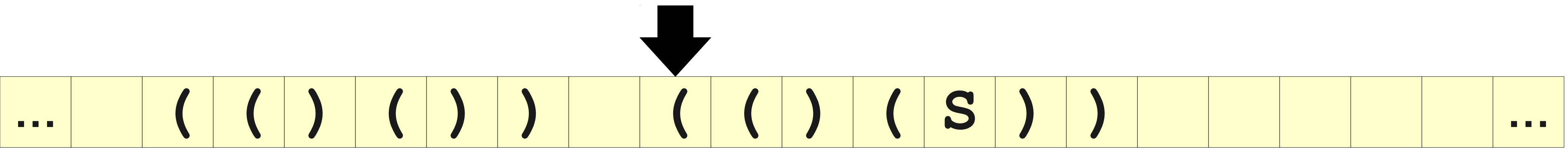

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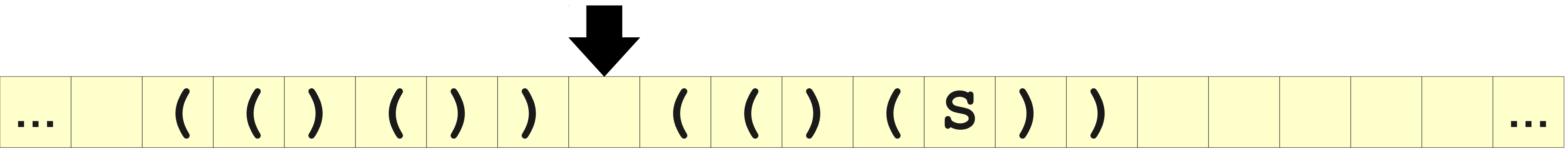

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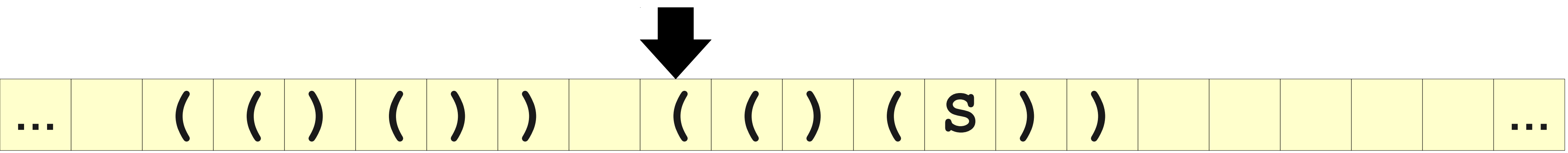

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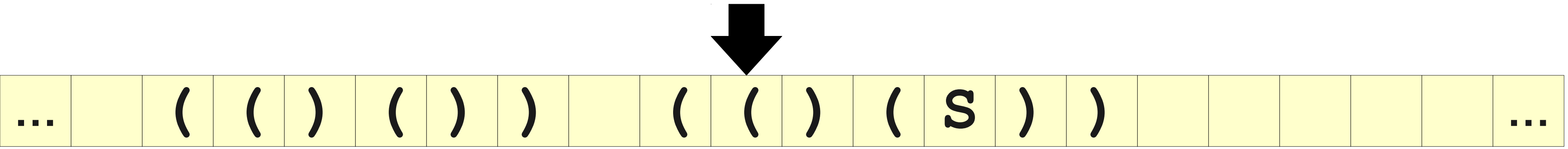

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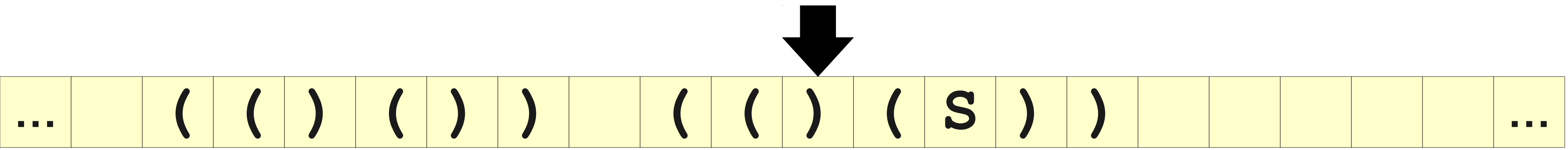

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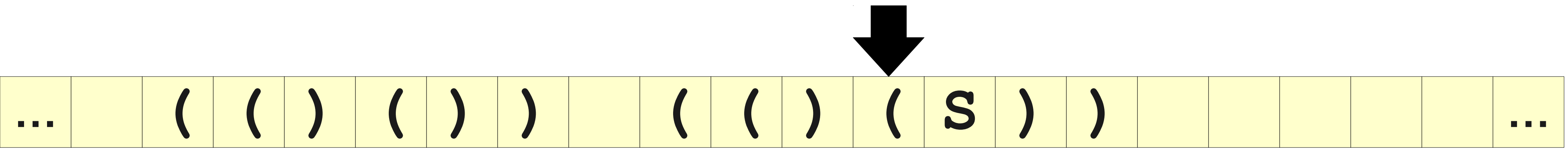

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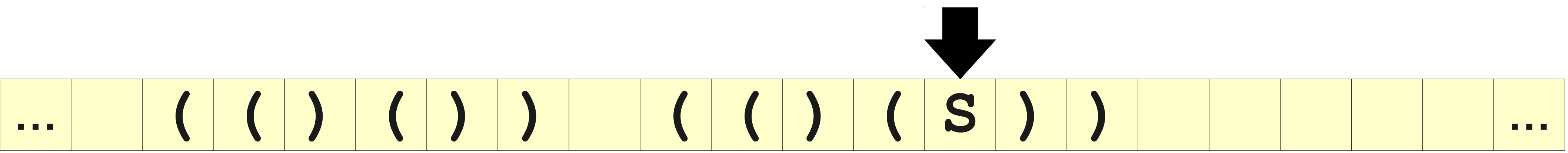

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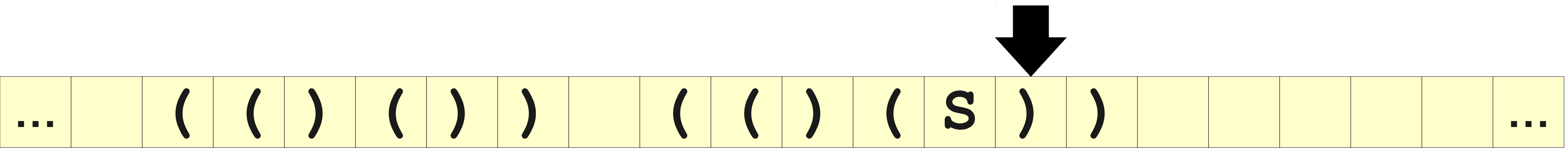

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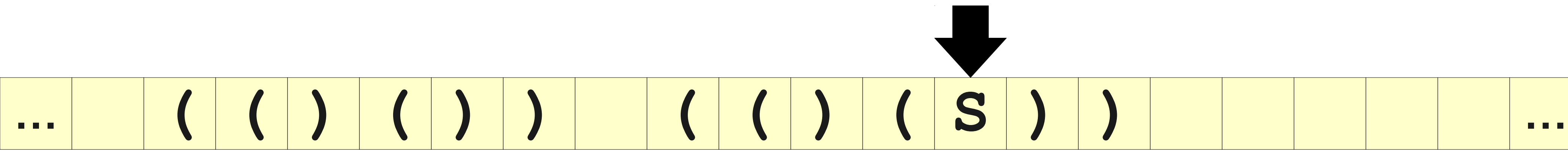

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A Sketch of the Construction

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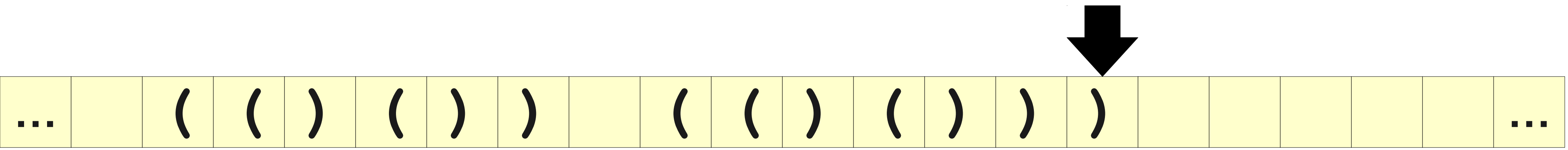

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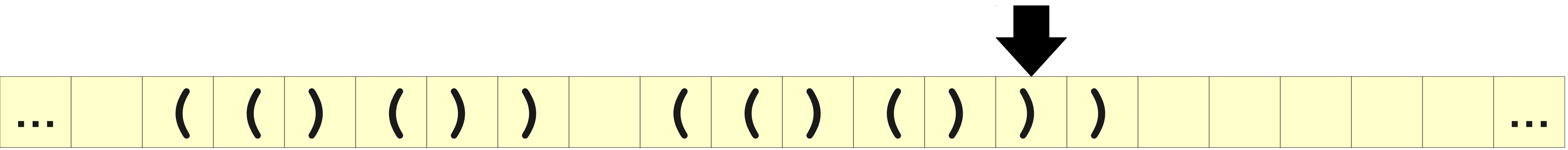

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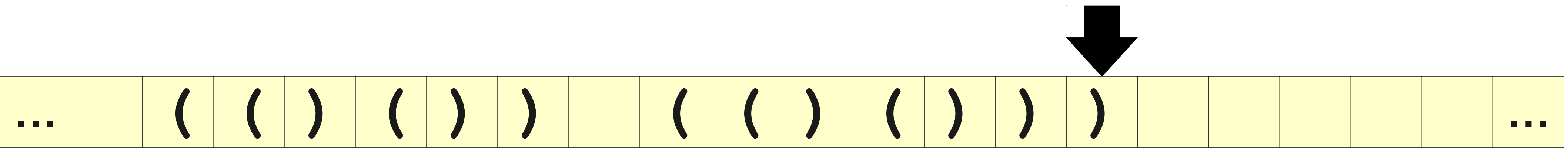

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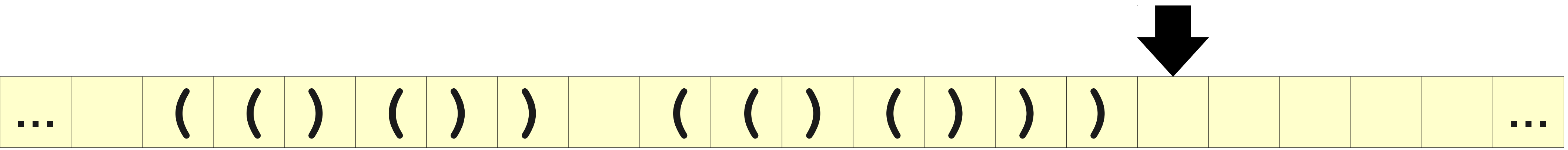

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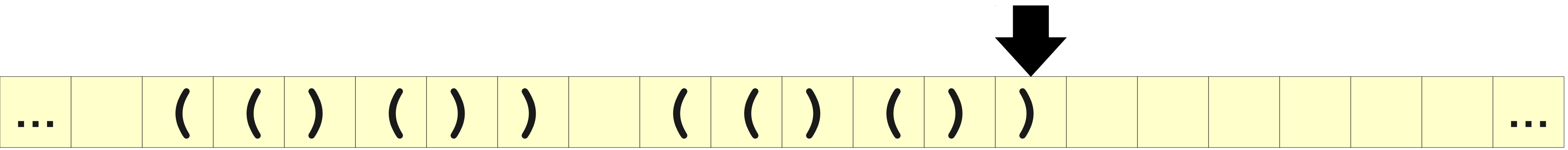

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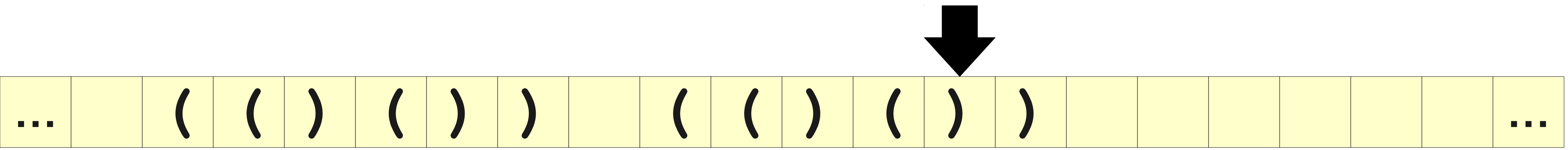

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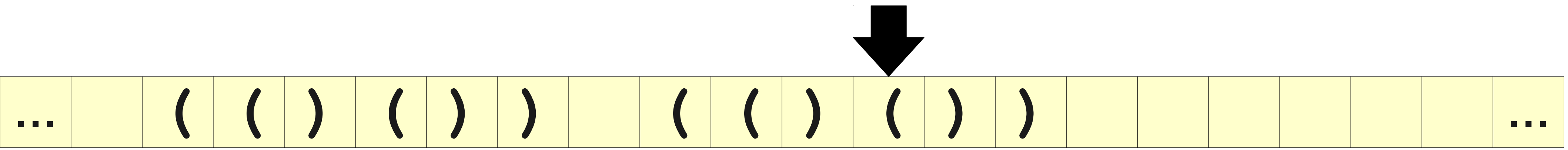

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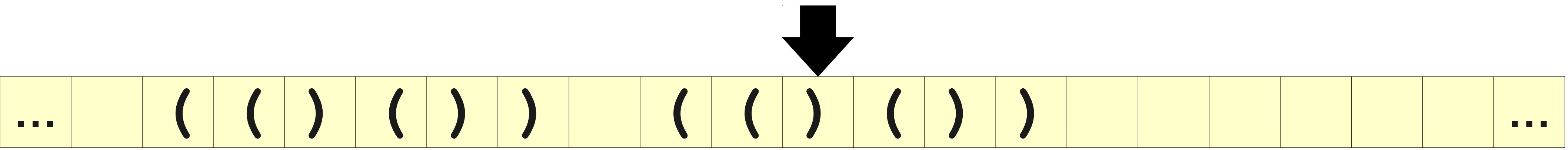

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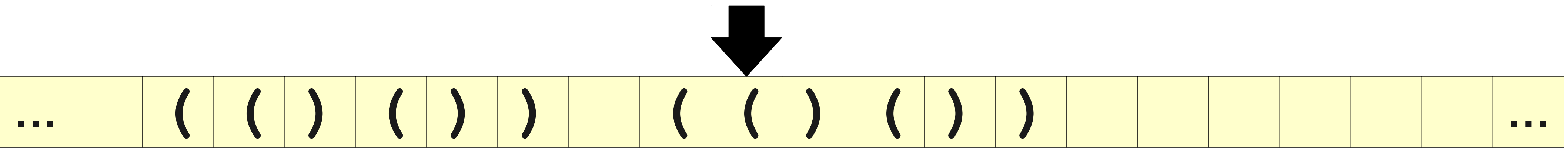

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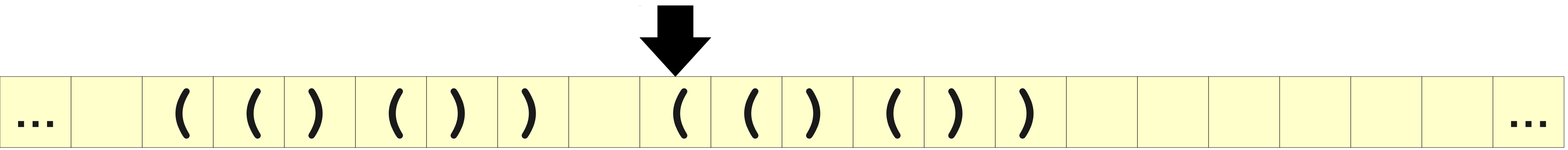

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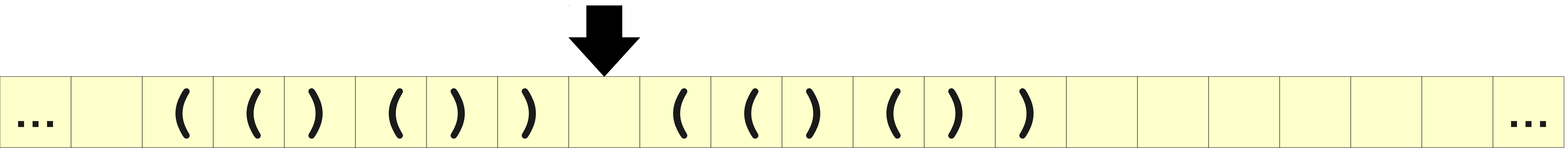

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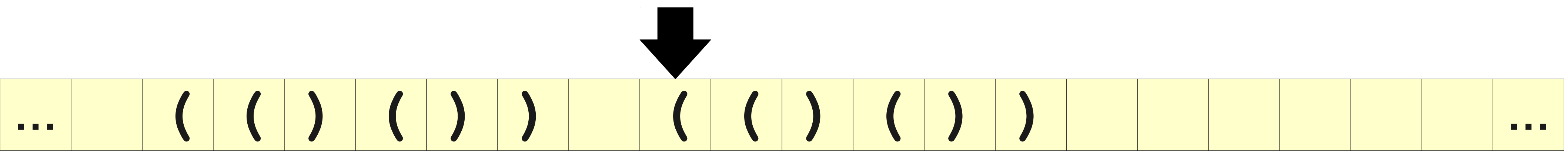

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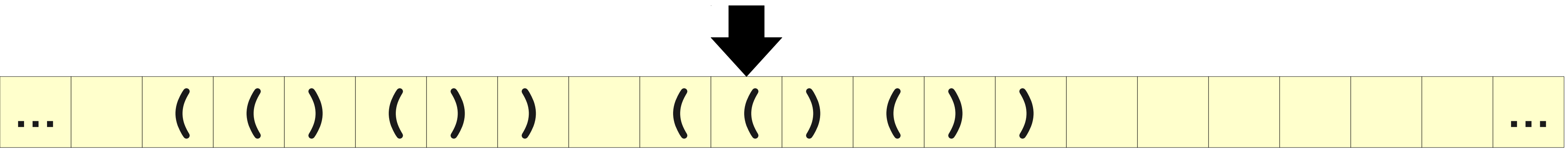

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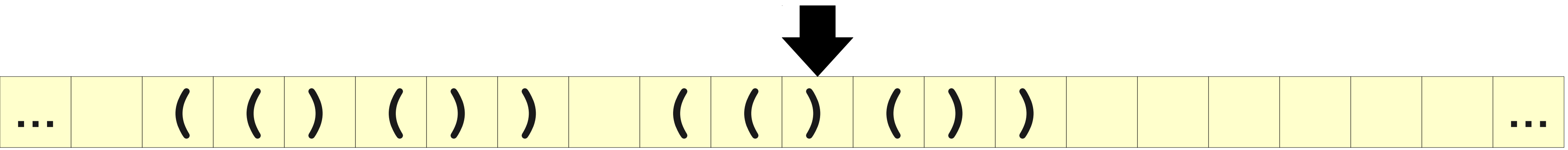

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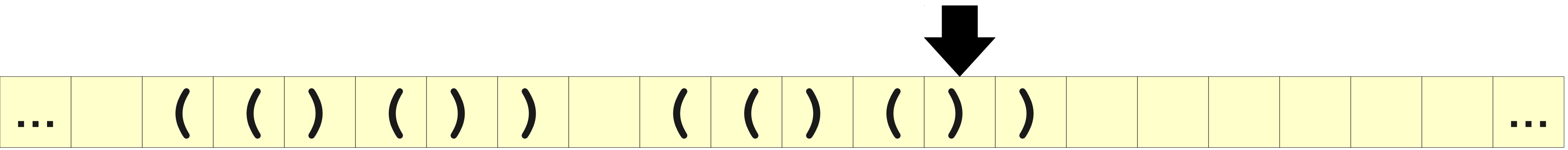

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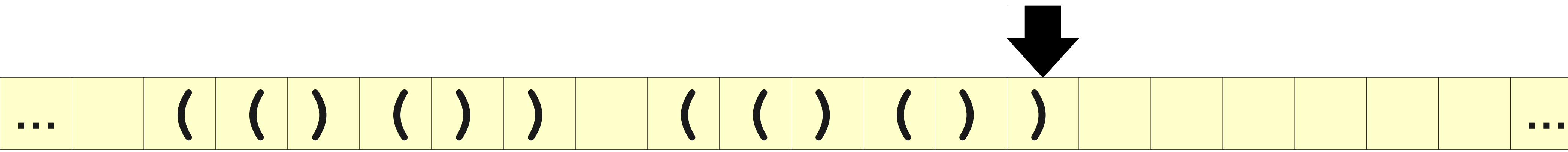

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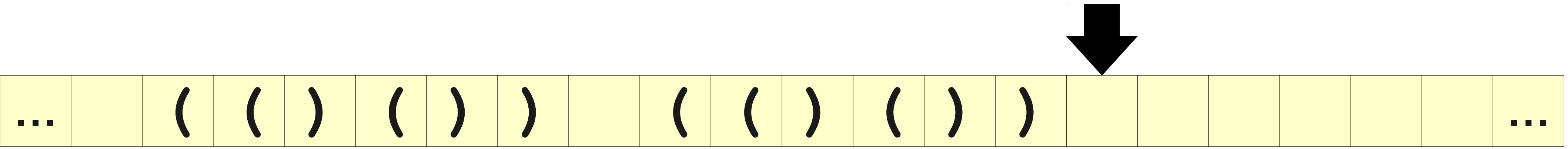


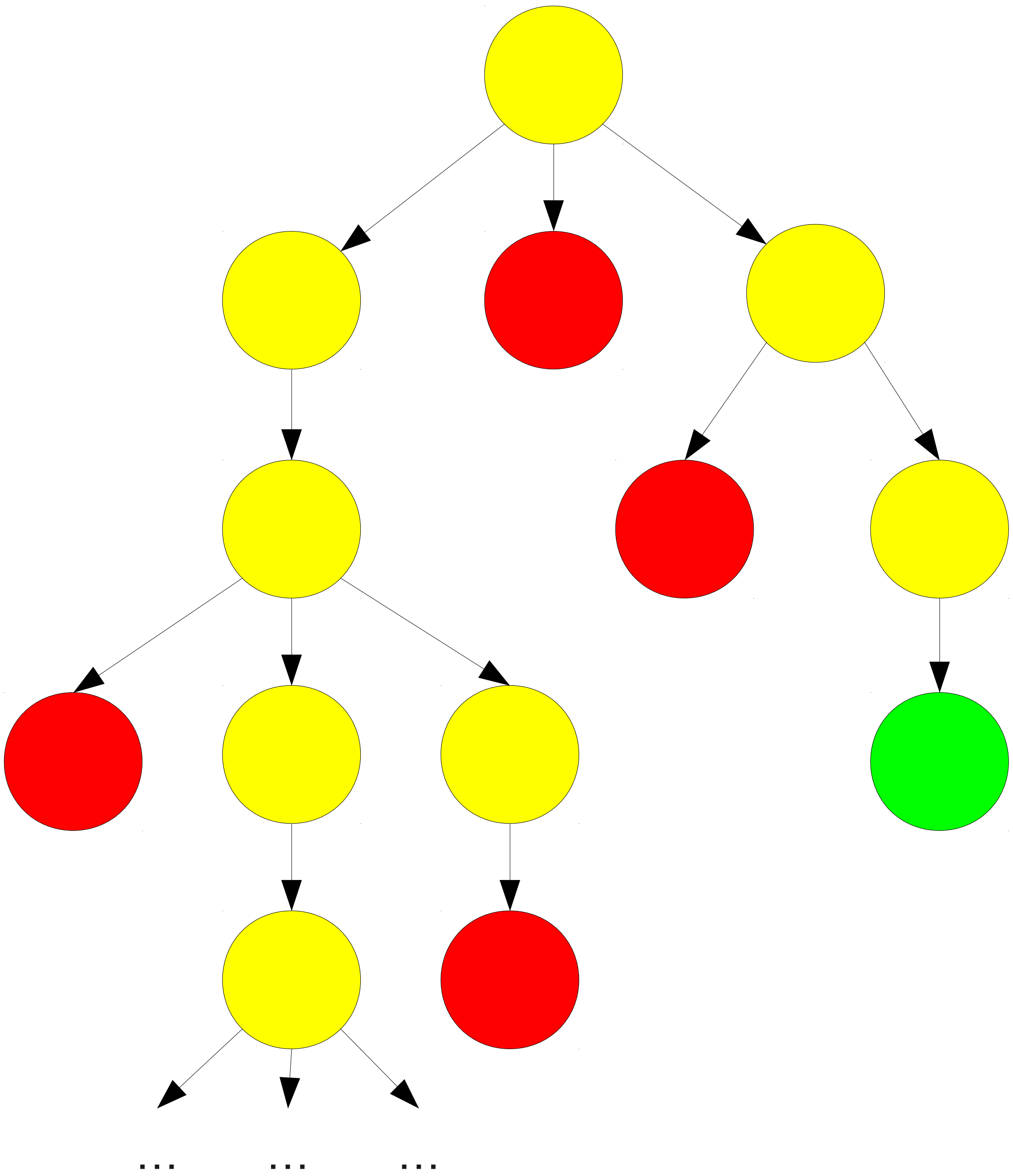
$$S \rightarrow SS \mid (S) \mid \varepsilon$$

The Story So Far

- We now have two different models of solving search problems:
  - Build a worklist and explicitly step through all options.
  - Use a nondeterministic Turing machine.
- Are these two approaches equivalent?
- That is, are NTMs and DTMs equal in power?

Review: Tree Computation

- One interpretation of nondeterminism is as a **tree computation**.
- Each node in the tree has children corresponding to each possible choice for the computation.
- The computation accepts if *any* node in tree enters an accepting state.



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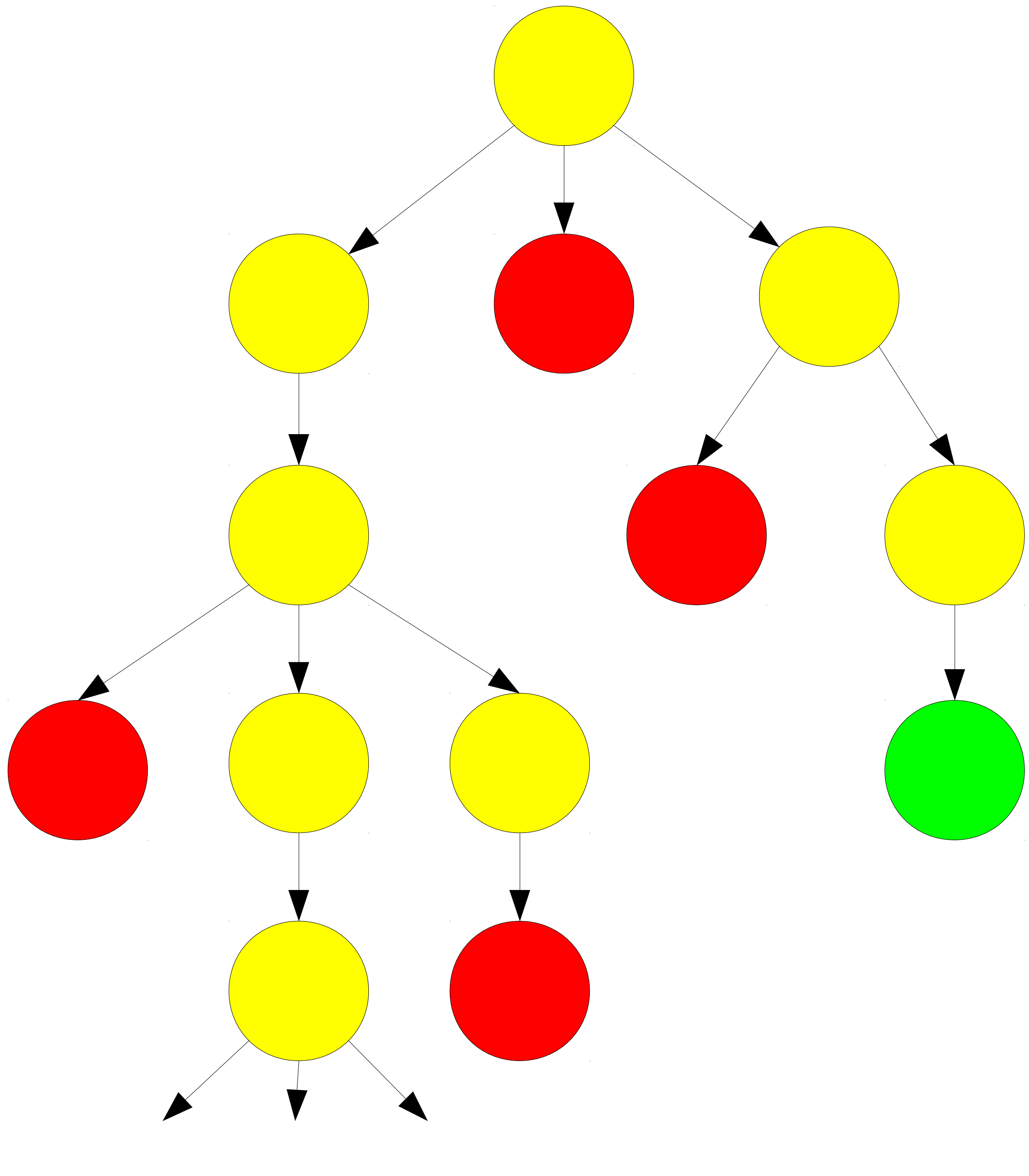

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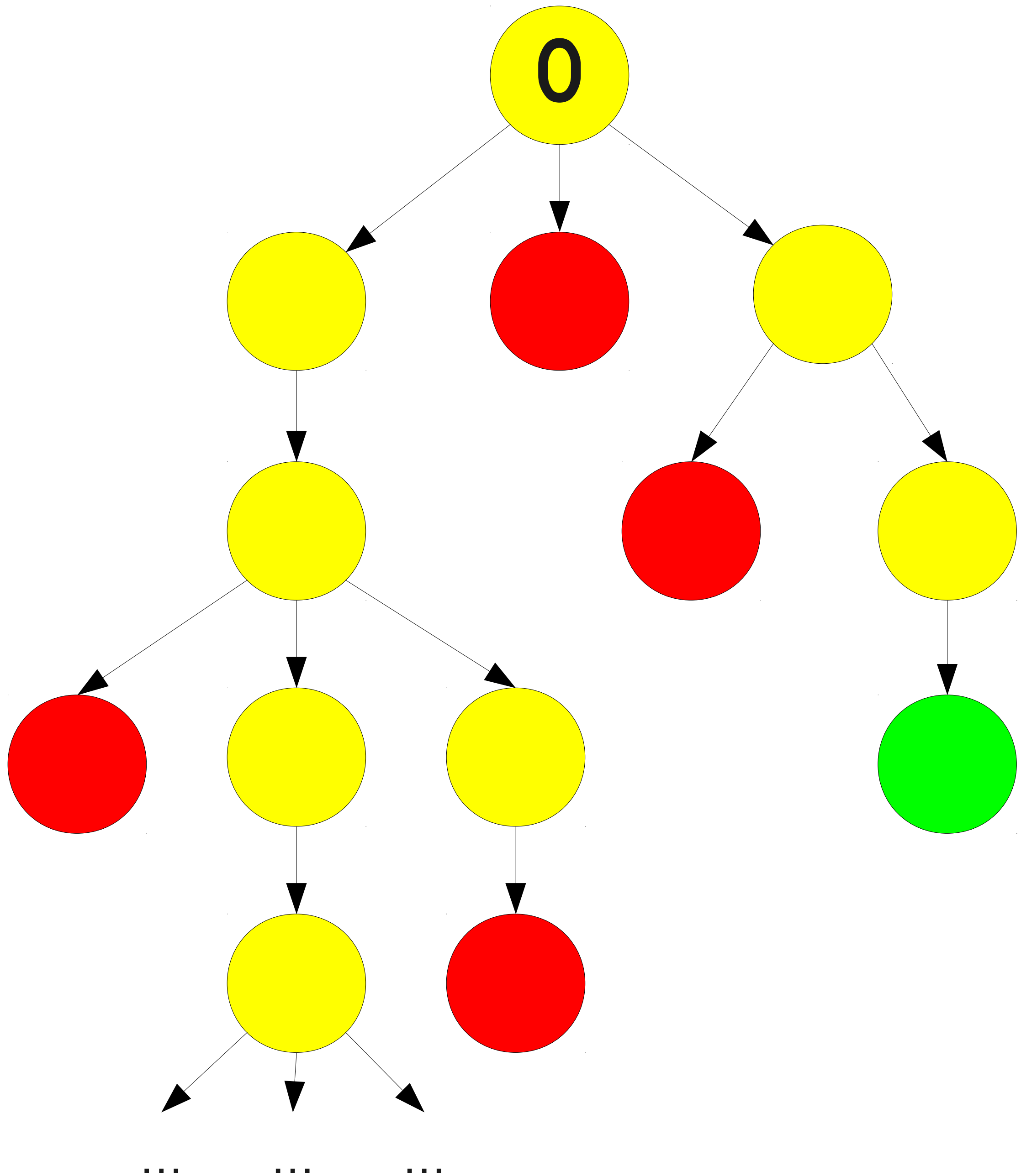

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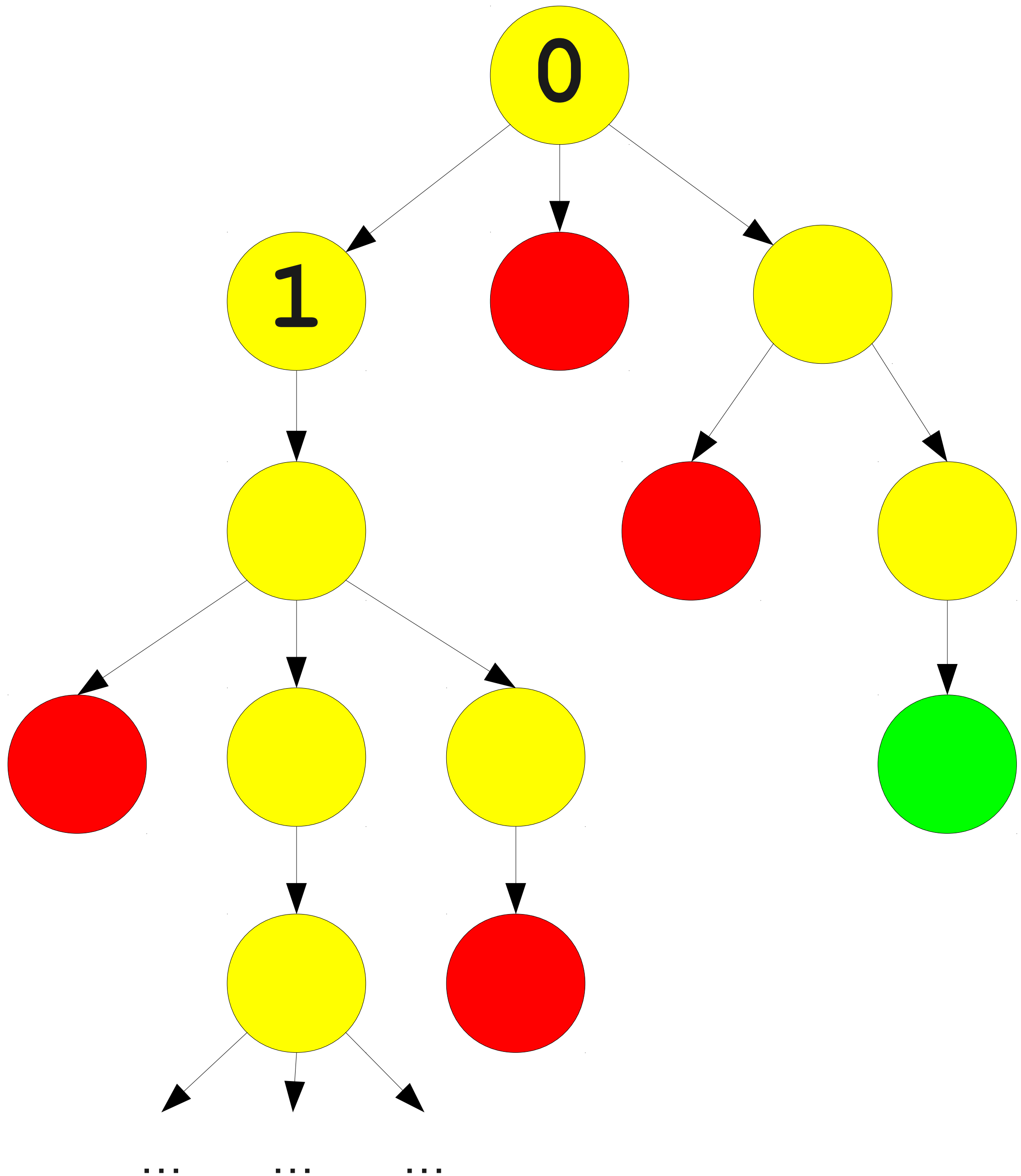

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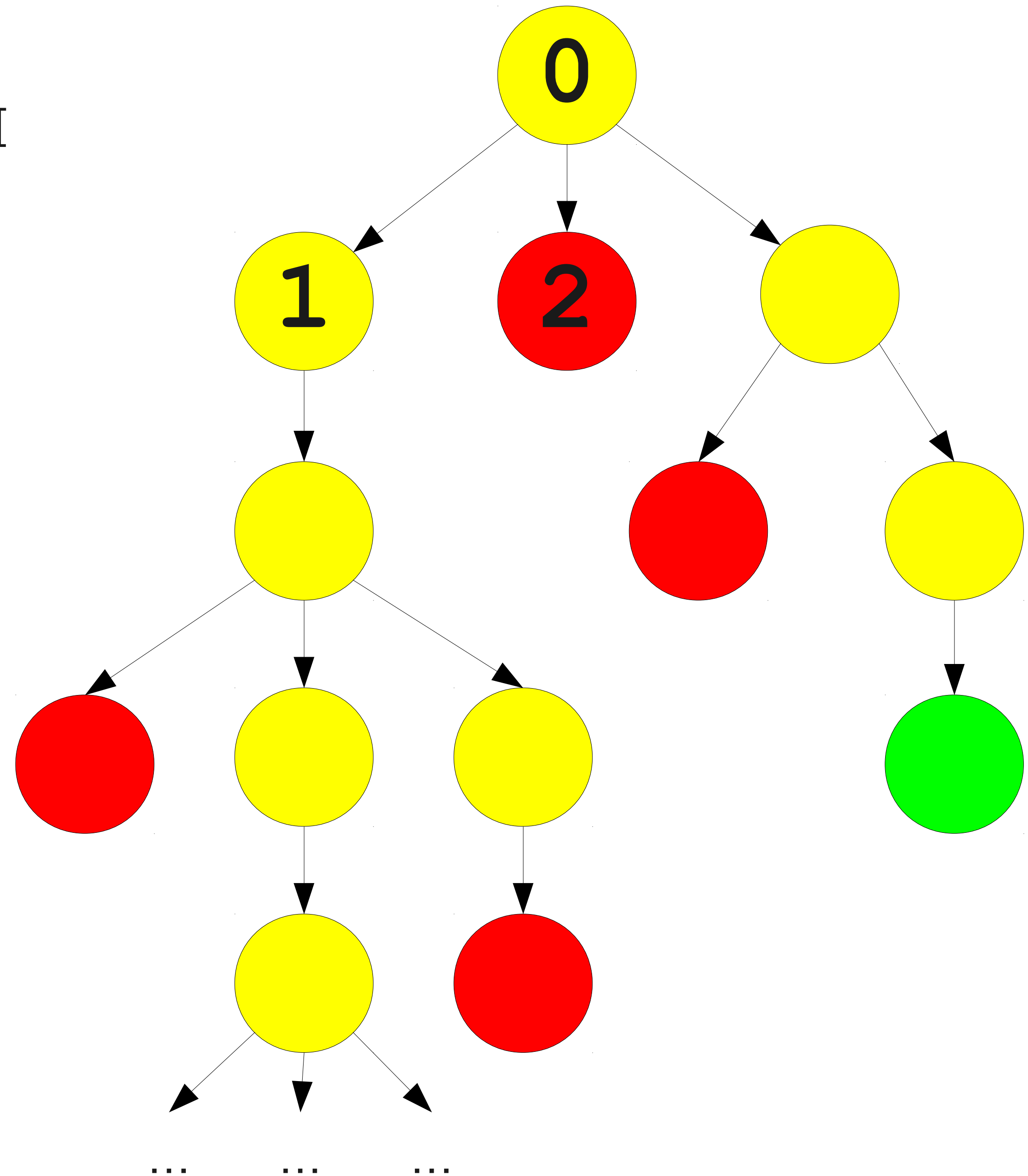

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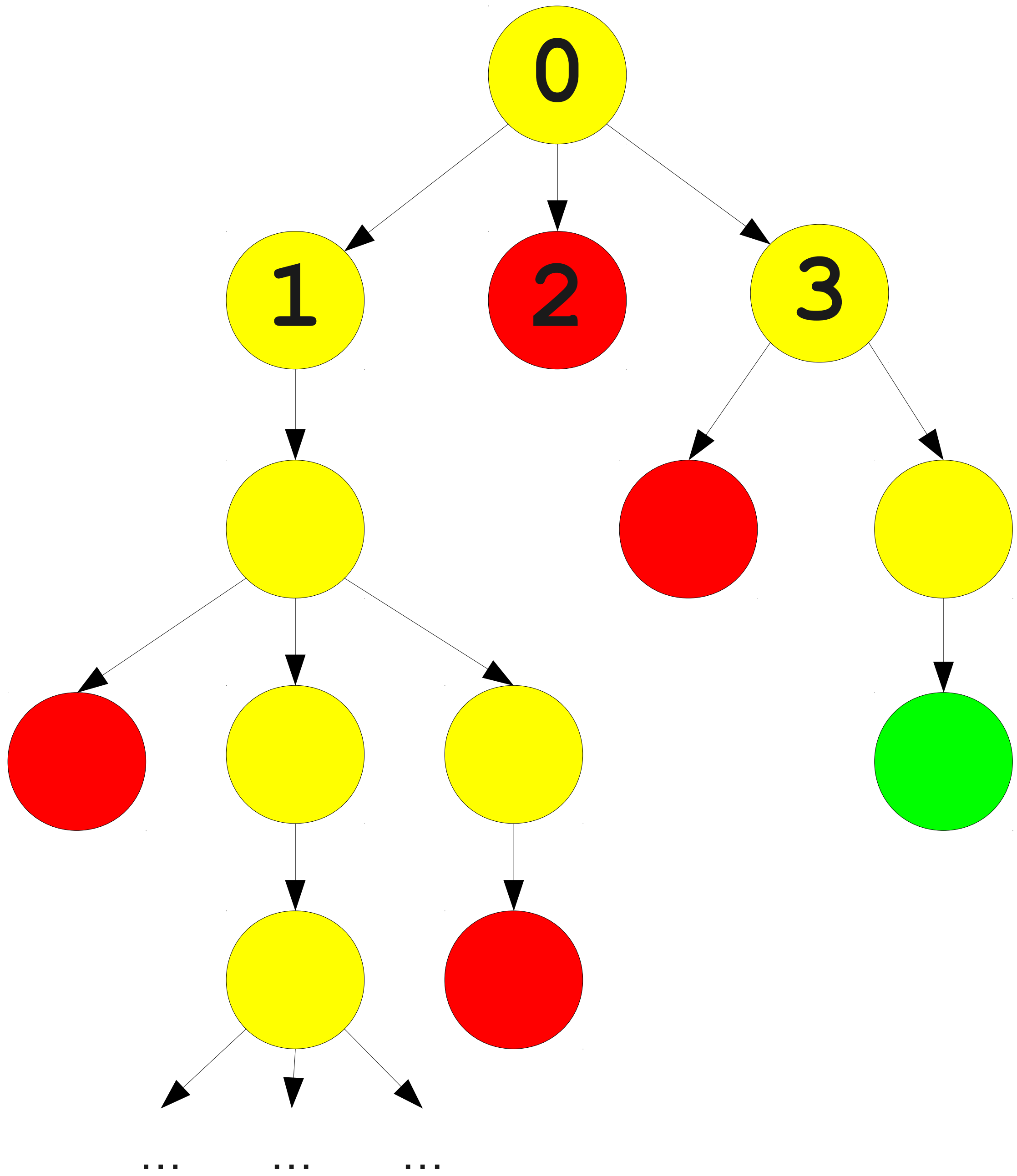

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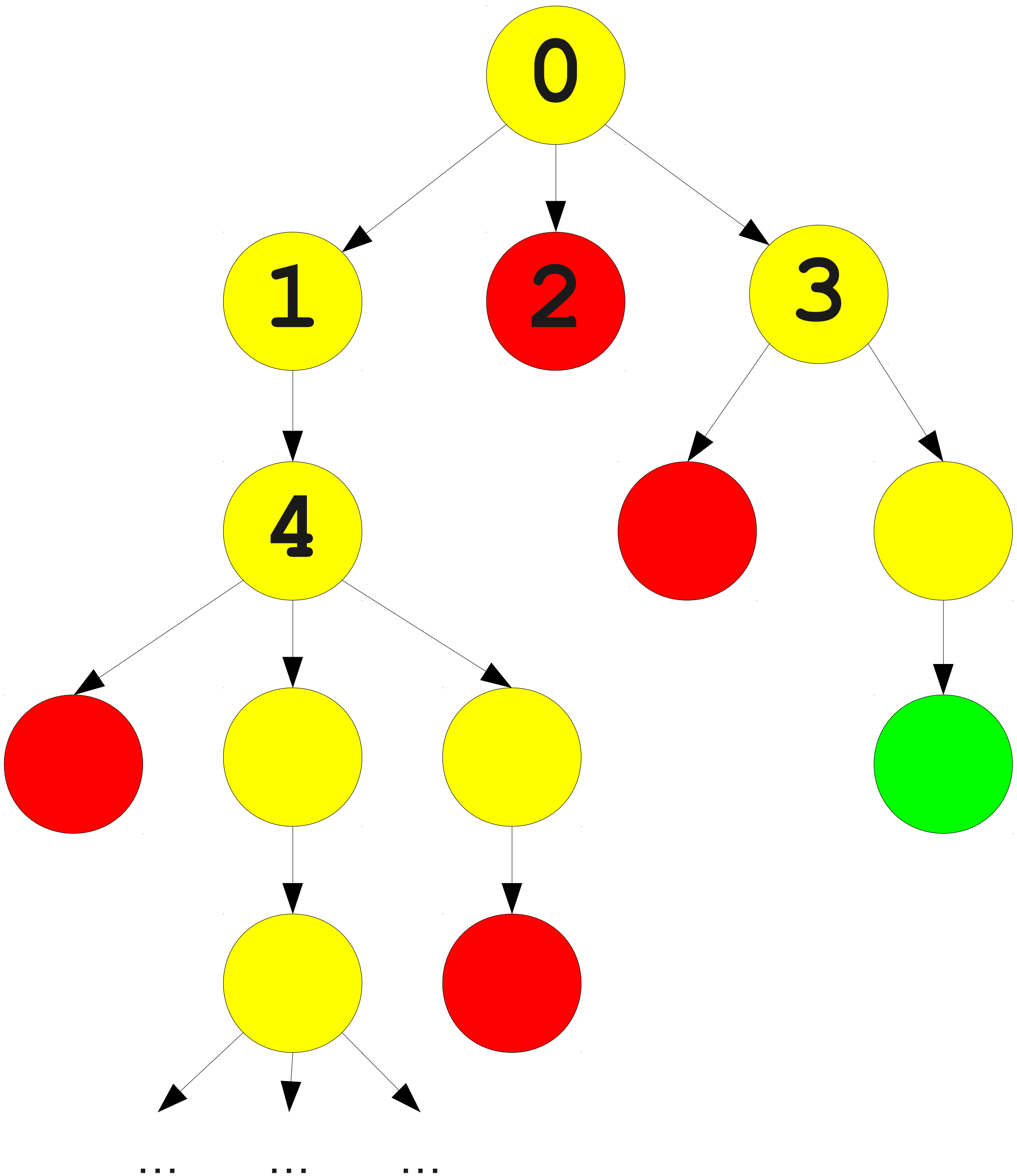

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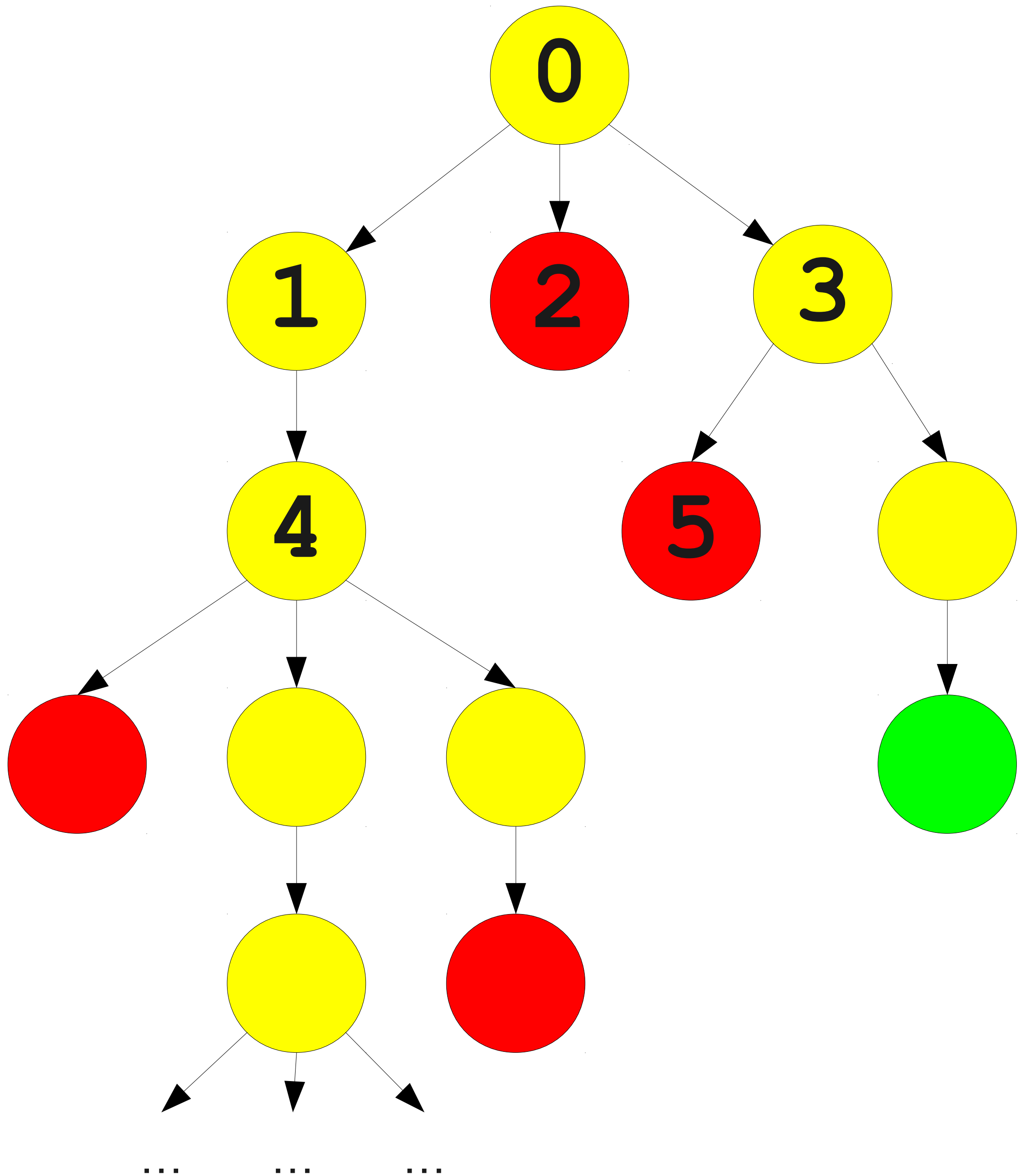

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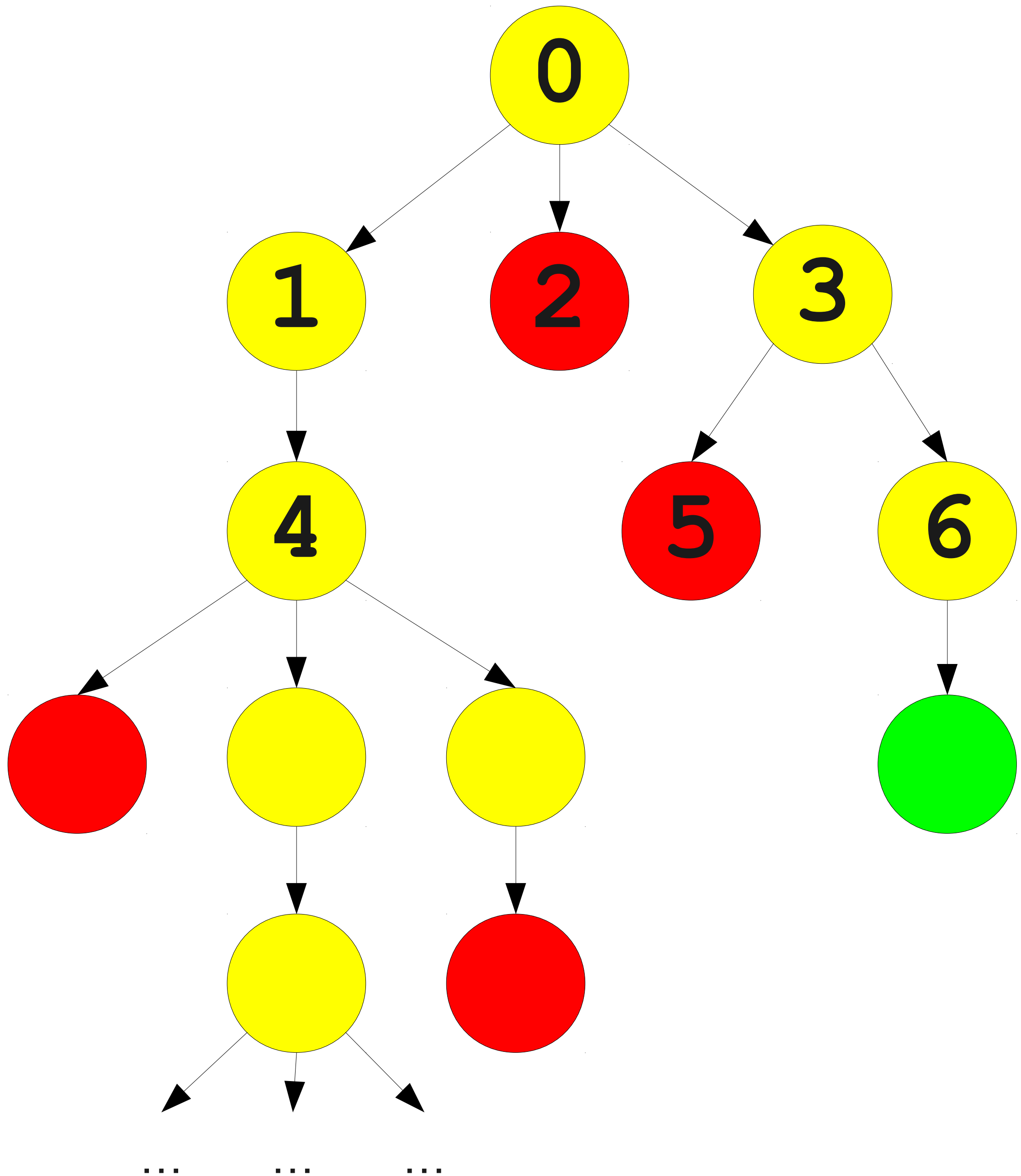

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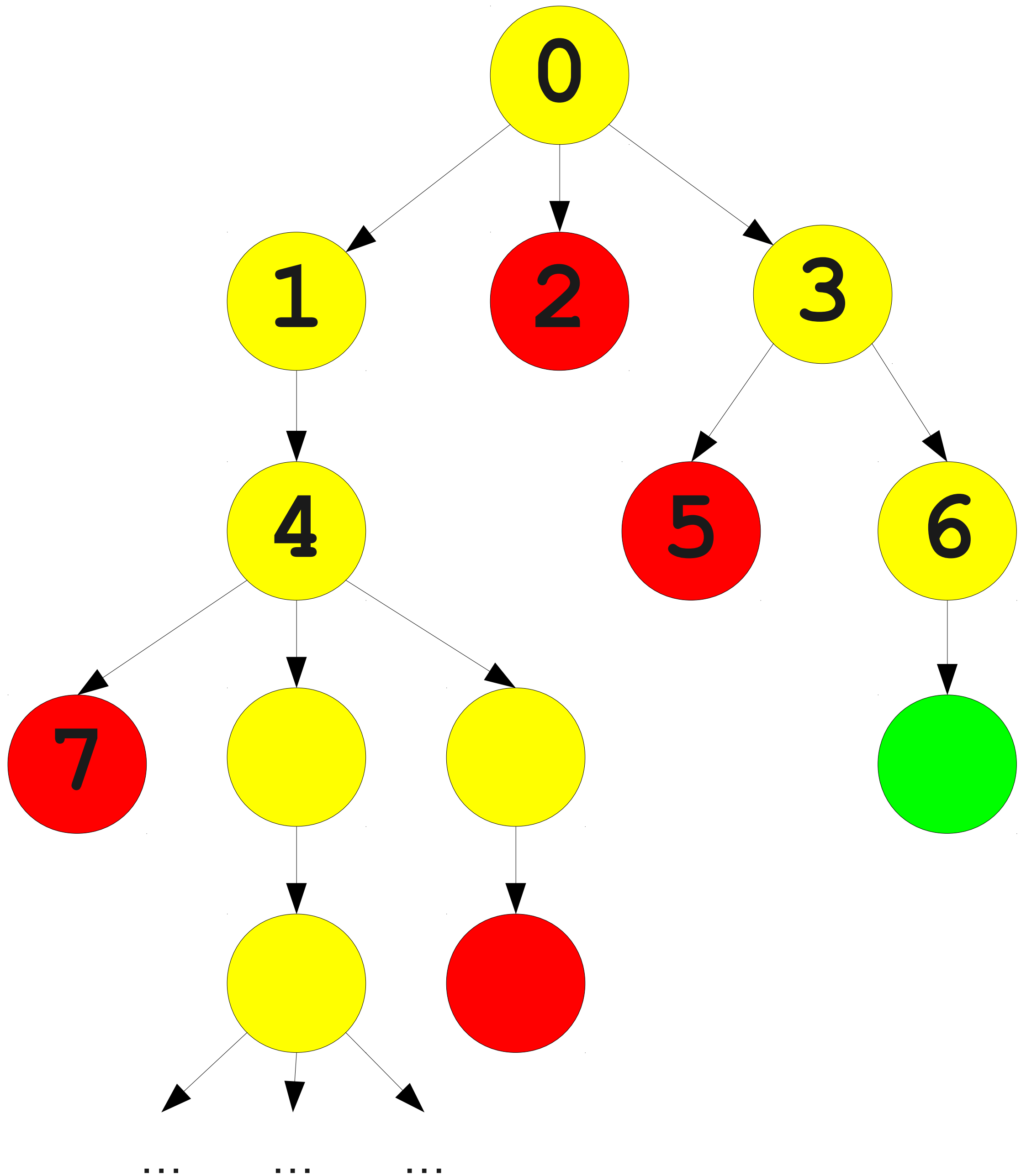

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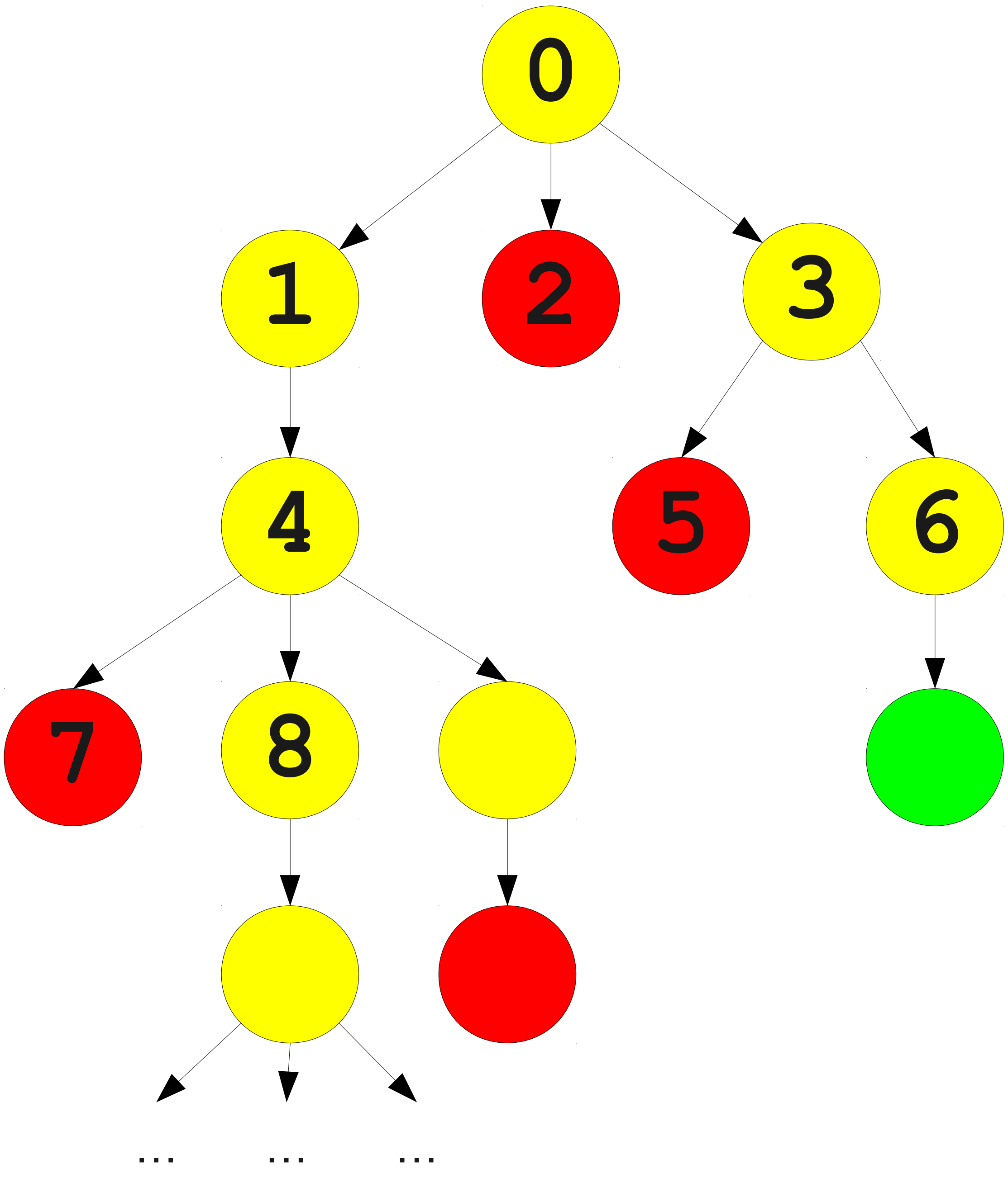

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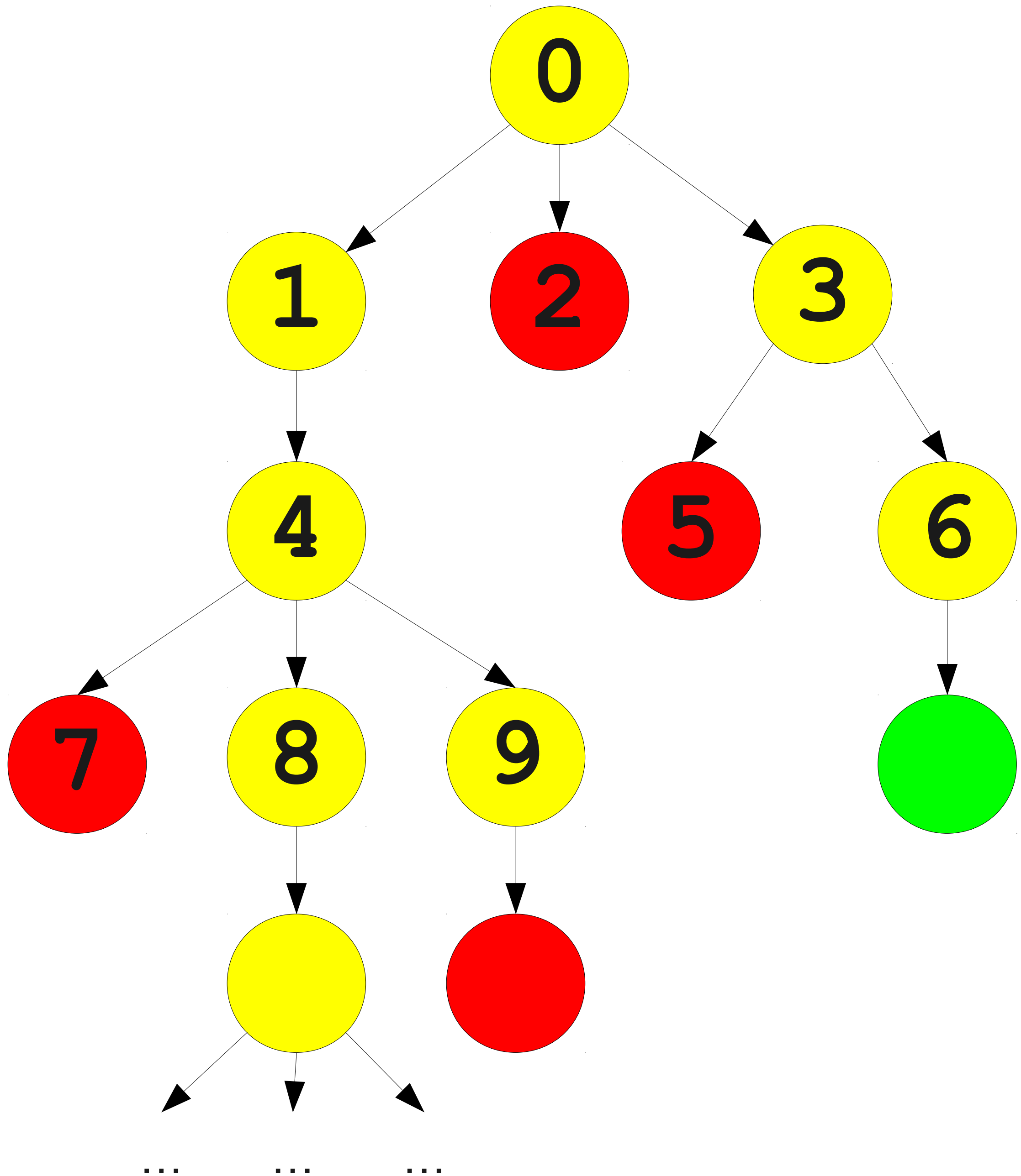

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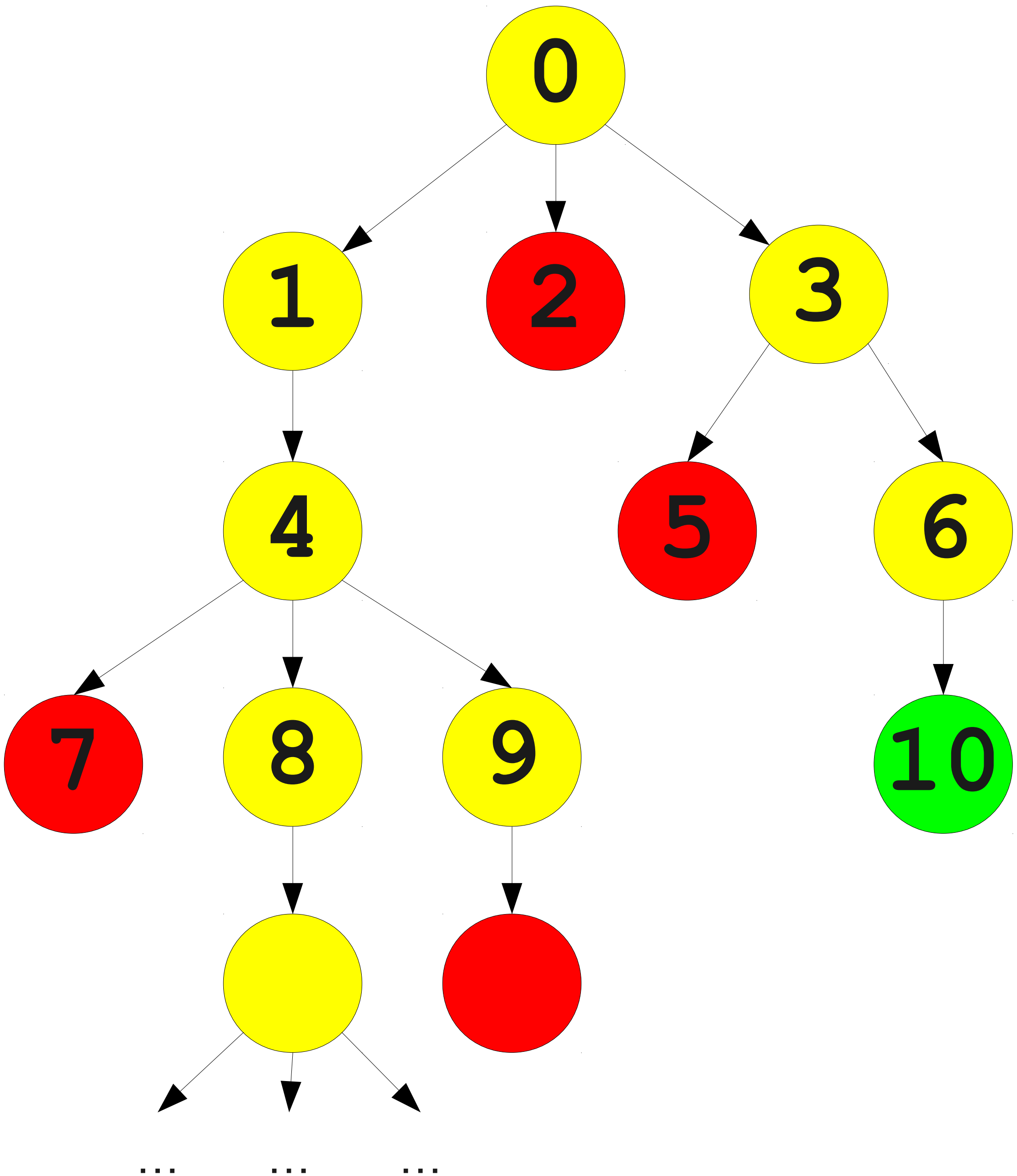

Exploring the Tree

- Each node in this tree consists of one possible configuration that the NTM might be in at any time.
- What does this consist of?
  - The contents of the tape.
  - The position of the tape head.
  - The current state.

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What does this consist of?

- The contents of the tape.
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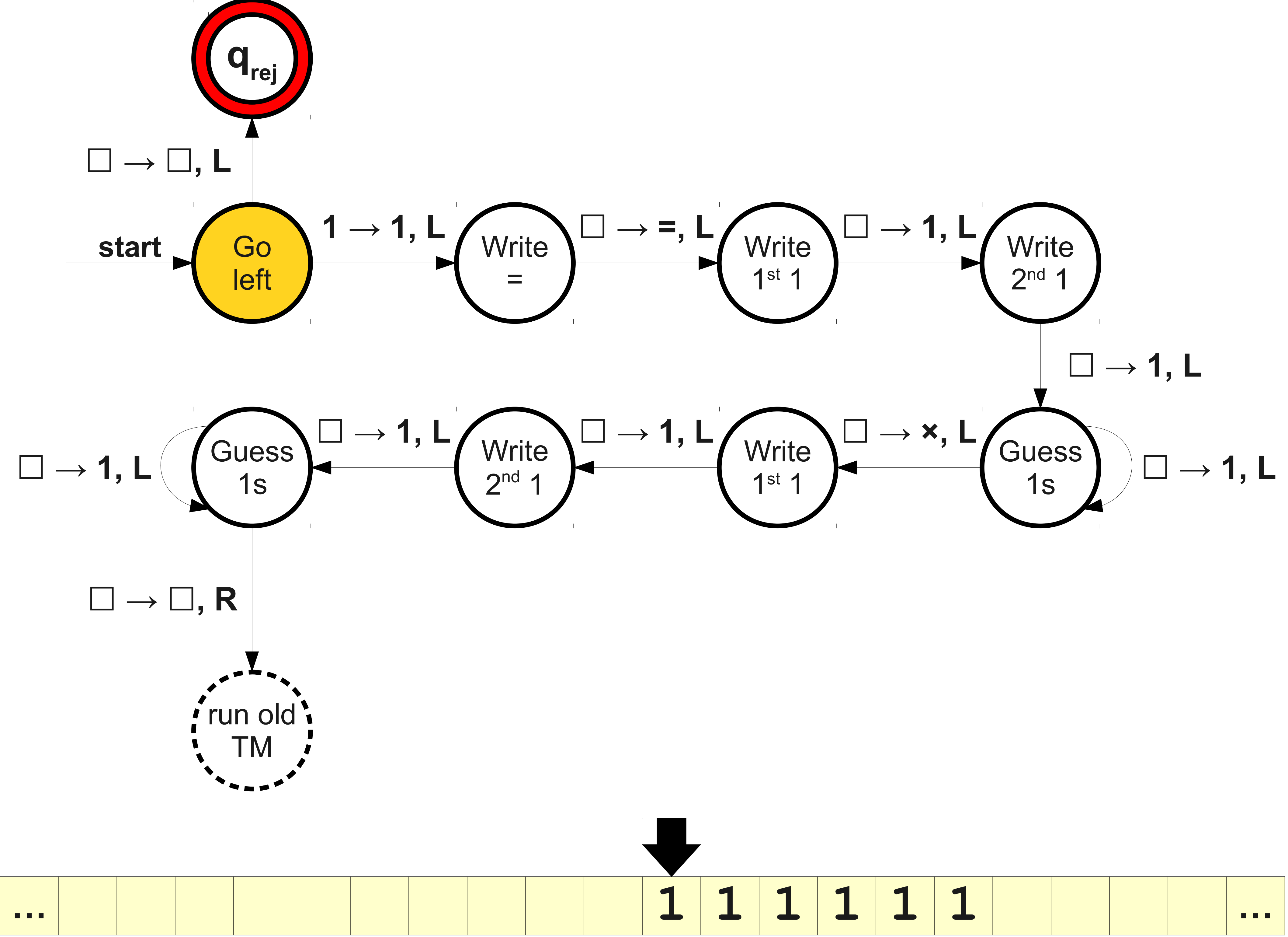
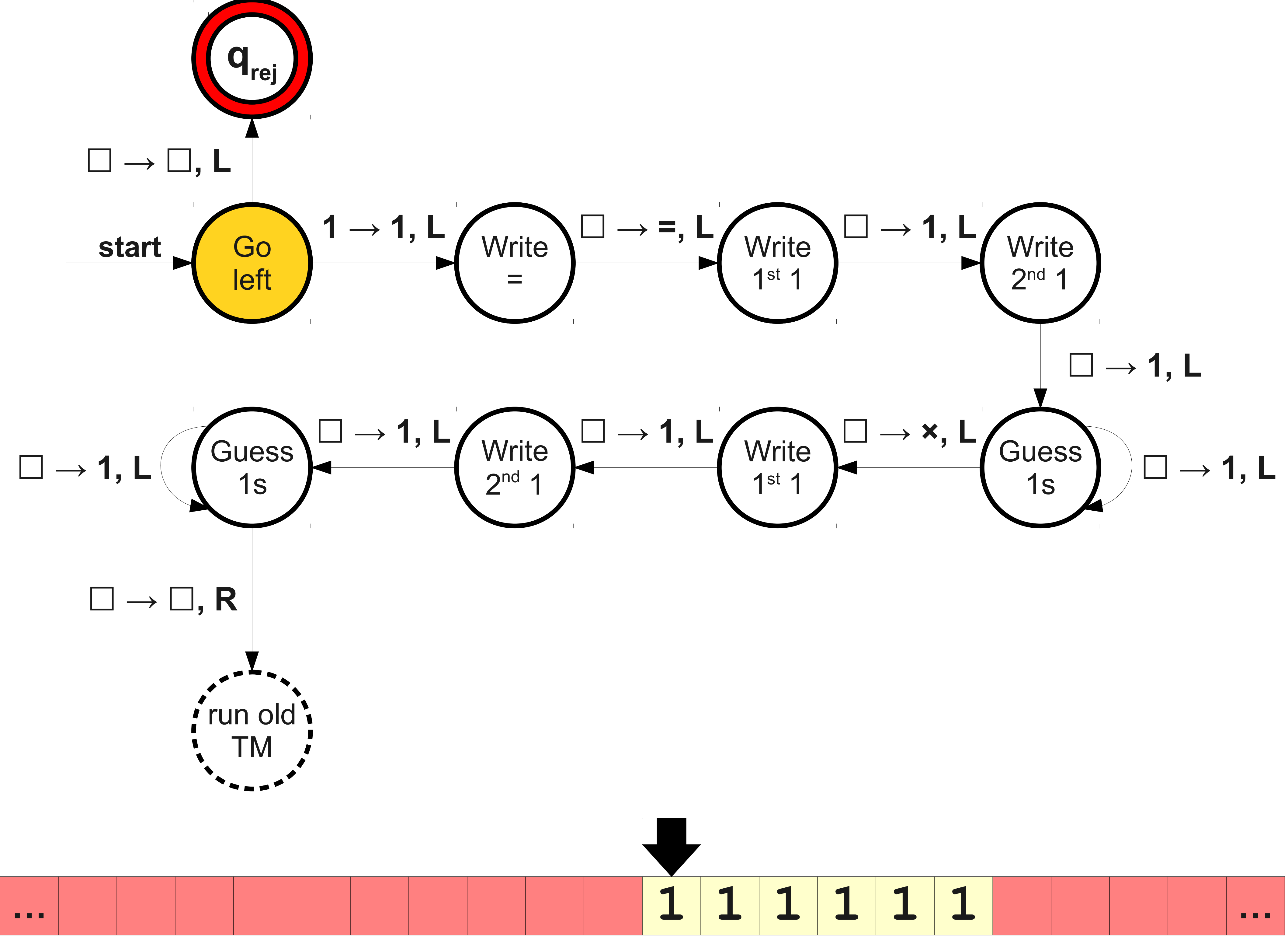
The tape is infinite!



q_rej
□ → □, L
start
Go left
1 → 1, L
Write =
□ → =, L
Write 1st 1
□ → 1, L
Write 2nd 1
□ → 1, L
Guess 1s
□ → 1, L
□ → ×, L
Write 1st 1
□ → 1, L
Write 2nd 1
□ → 1, L
Guess 1s
□ → 1, L
□ → □, R
run old TM
… 1 1 1 1 1 1 …



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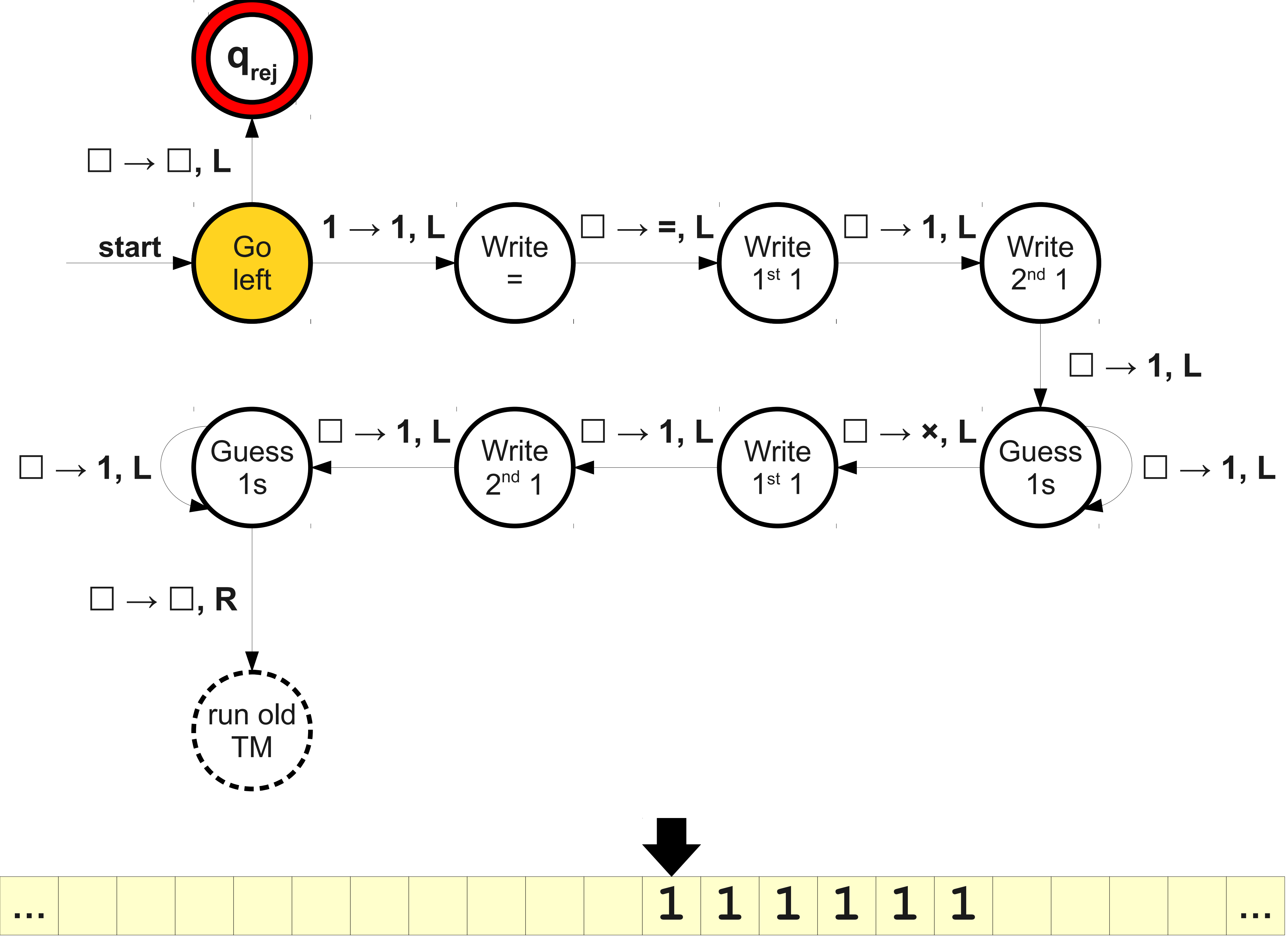
□ → □, R
run old TM
... 1 1 1 1 1 1 ...



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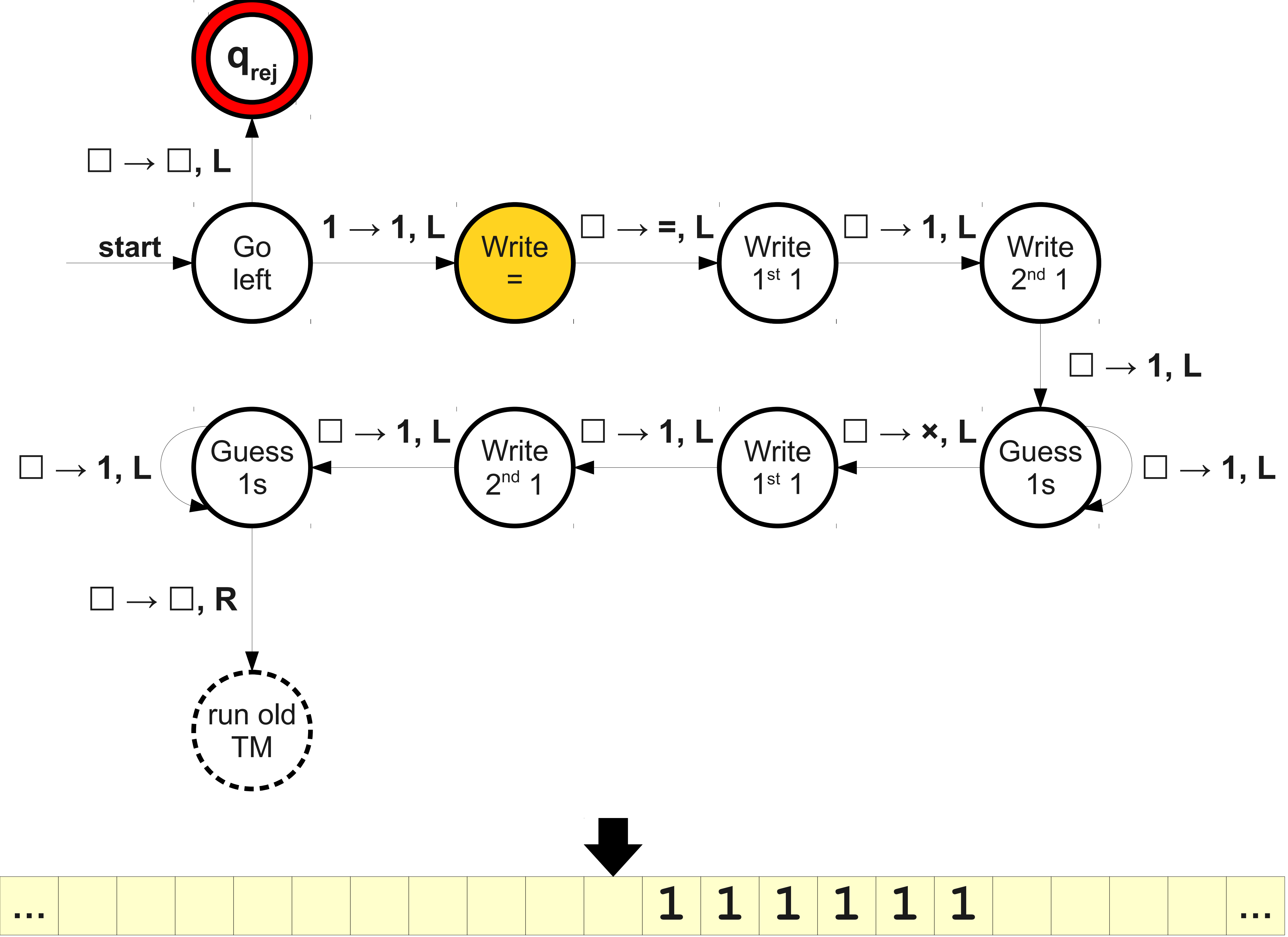

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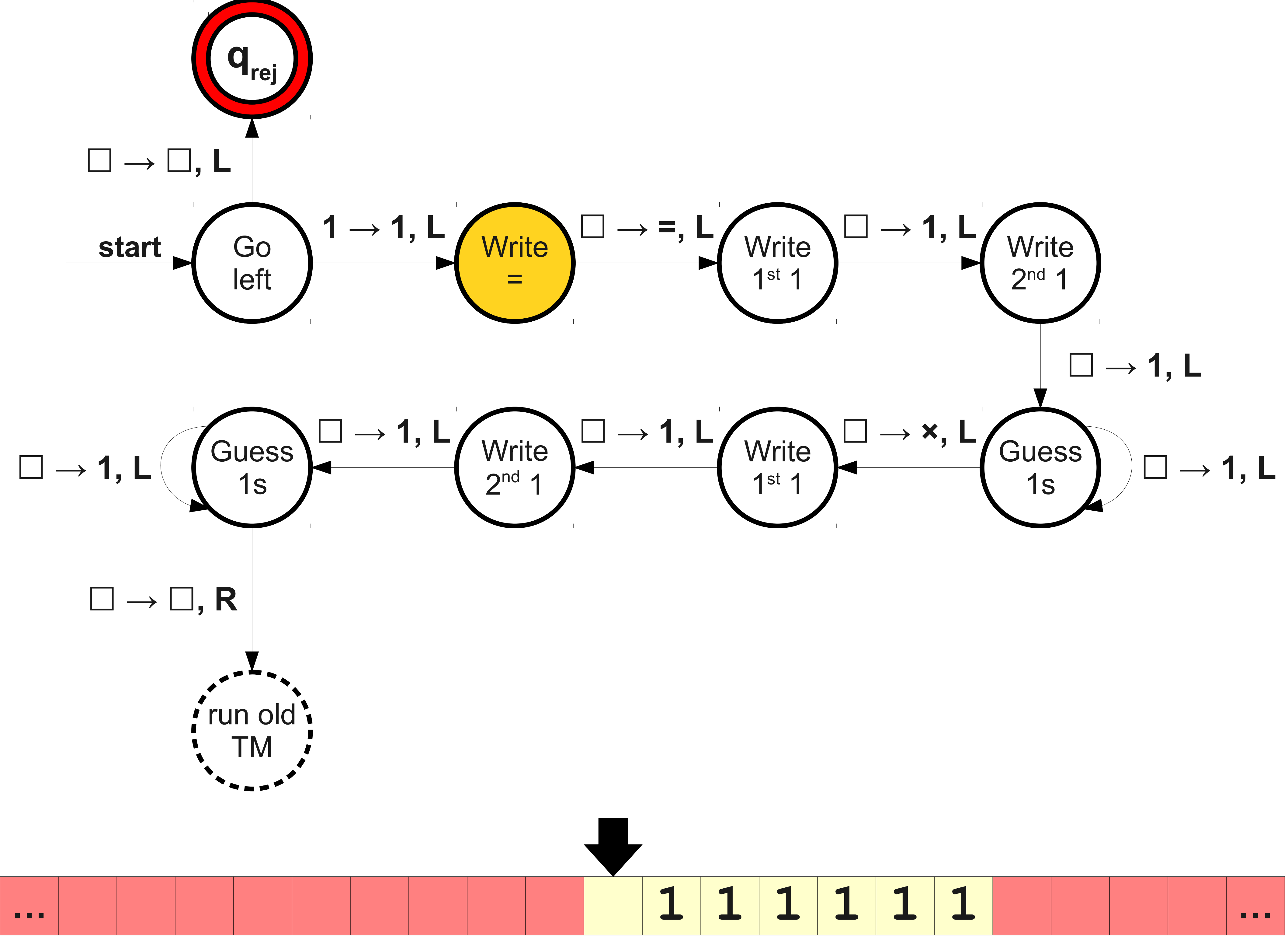
...
1 1 1 1 1 1
...



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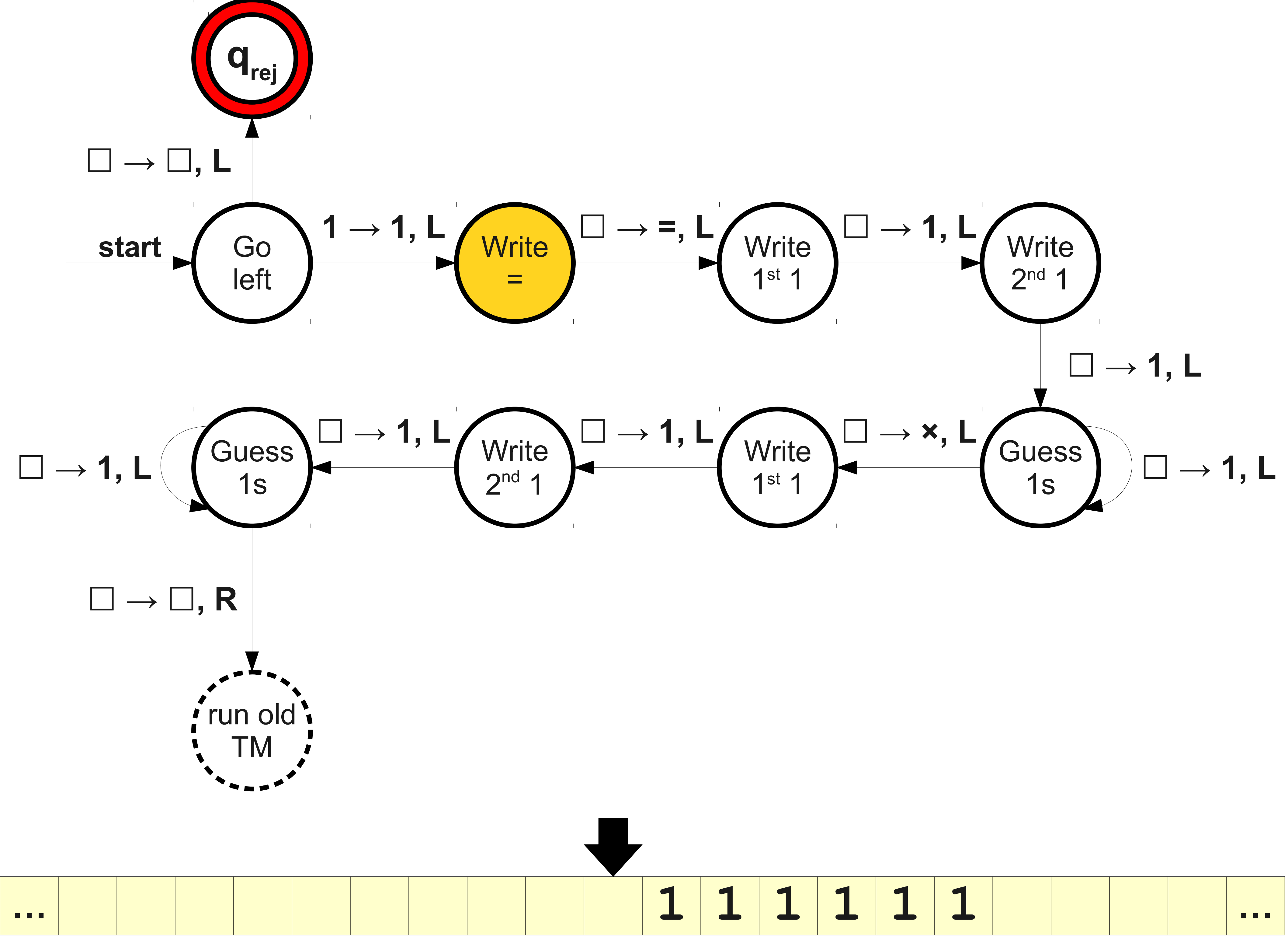
...
1 1 1 1 1 1
...



q_{rej}
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start
Go left
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Write =
□ → =, L
Write 1st 1
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Write 1st 1
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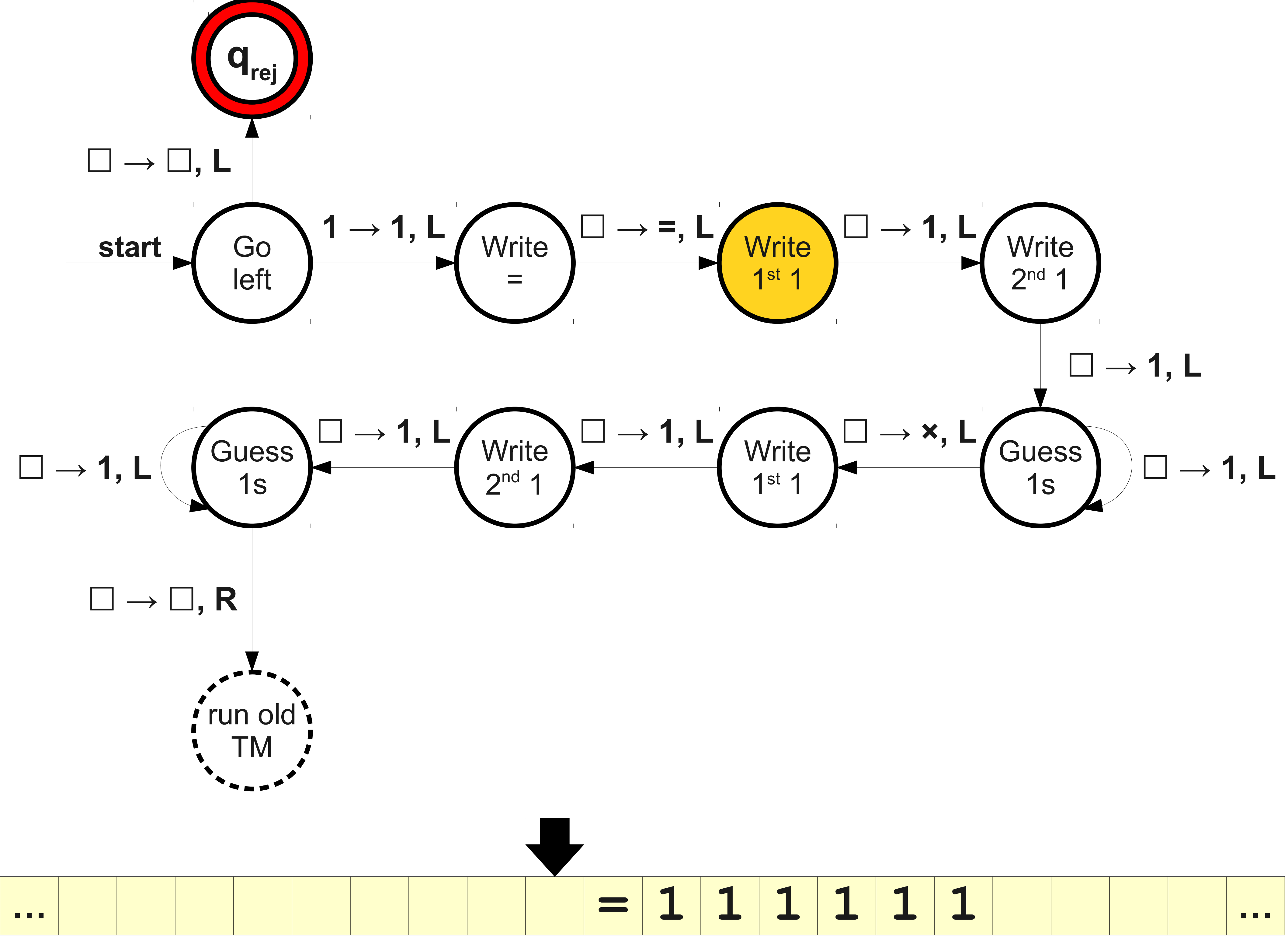
...
1 1 1 1 1 1
...



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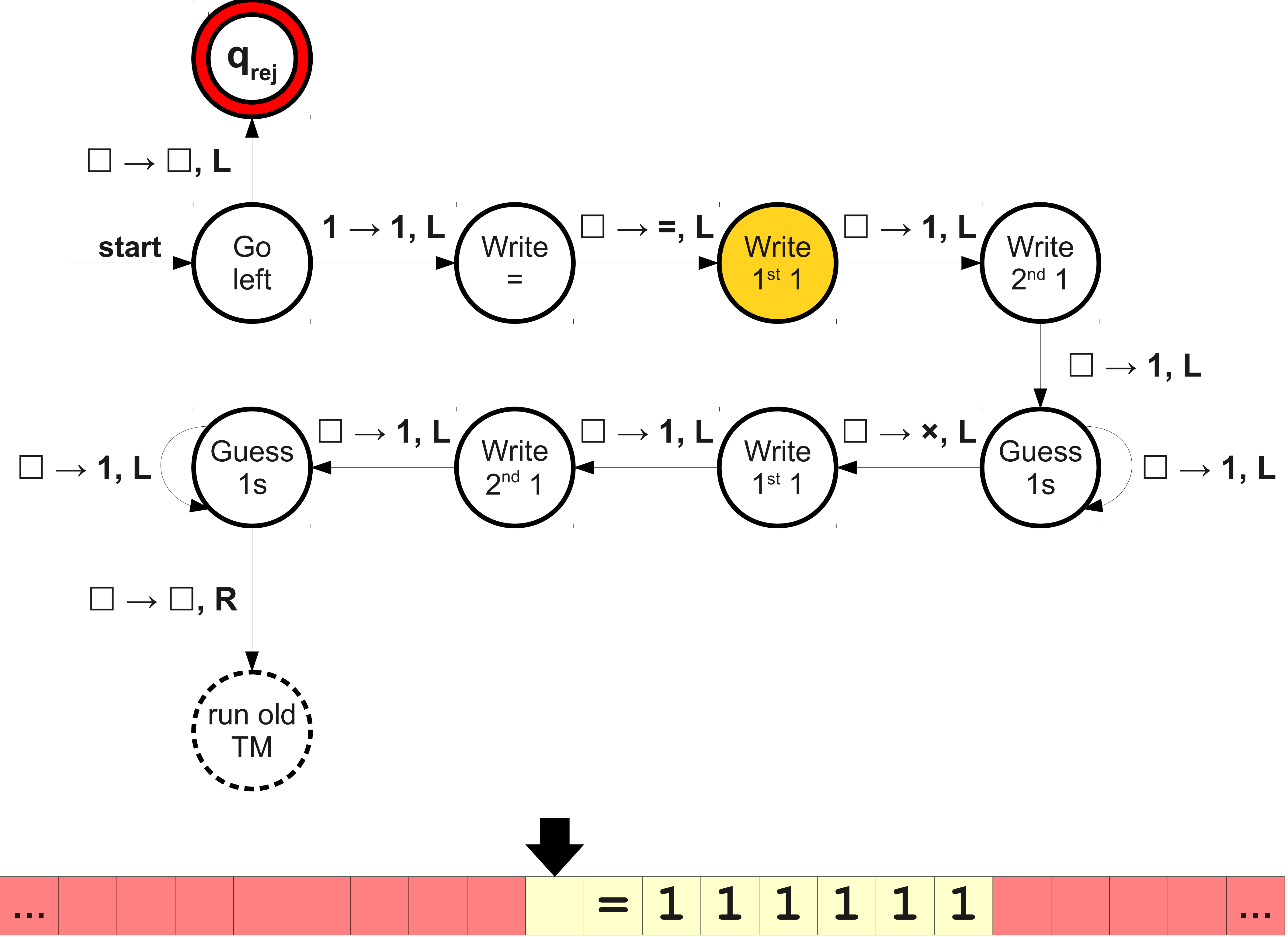
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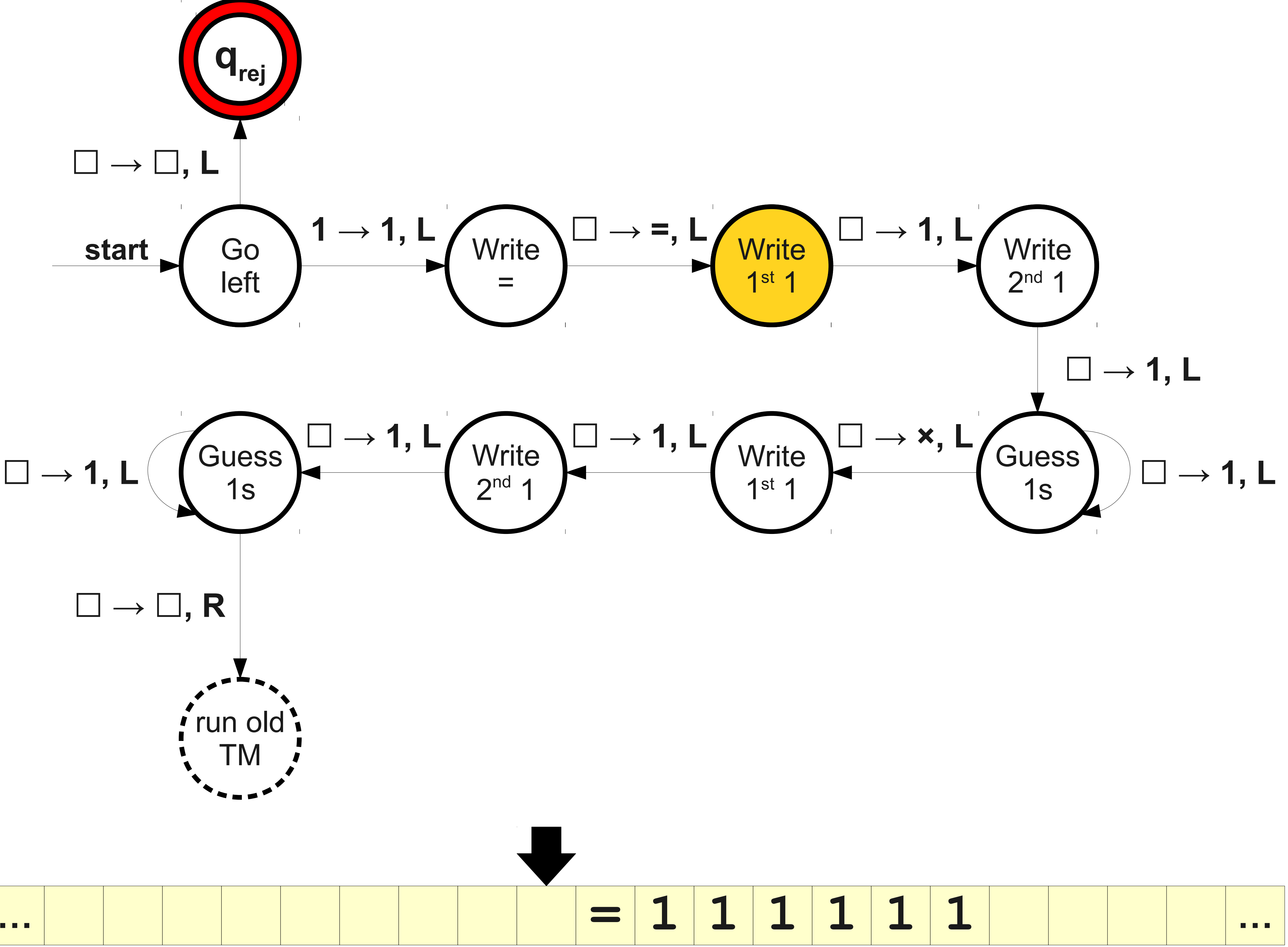
run old TM
…
= 1 1 1 1 1 1
…



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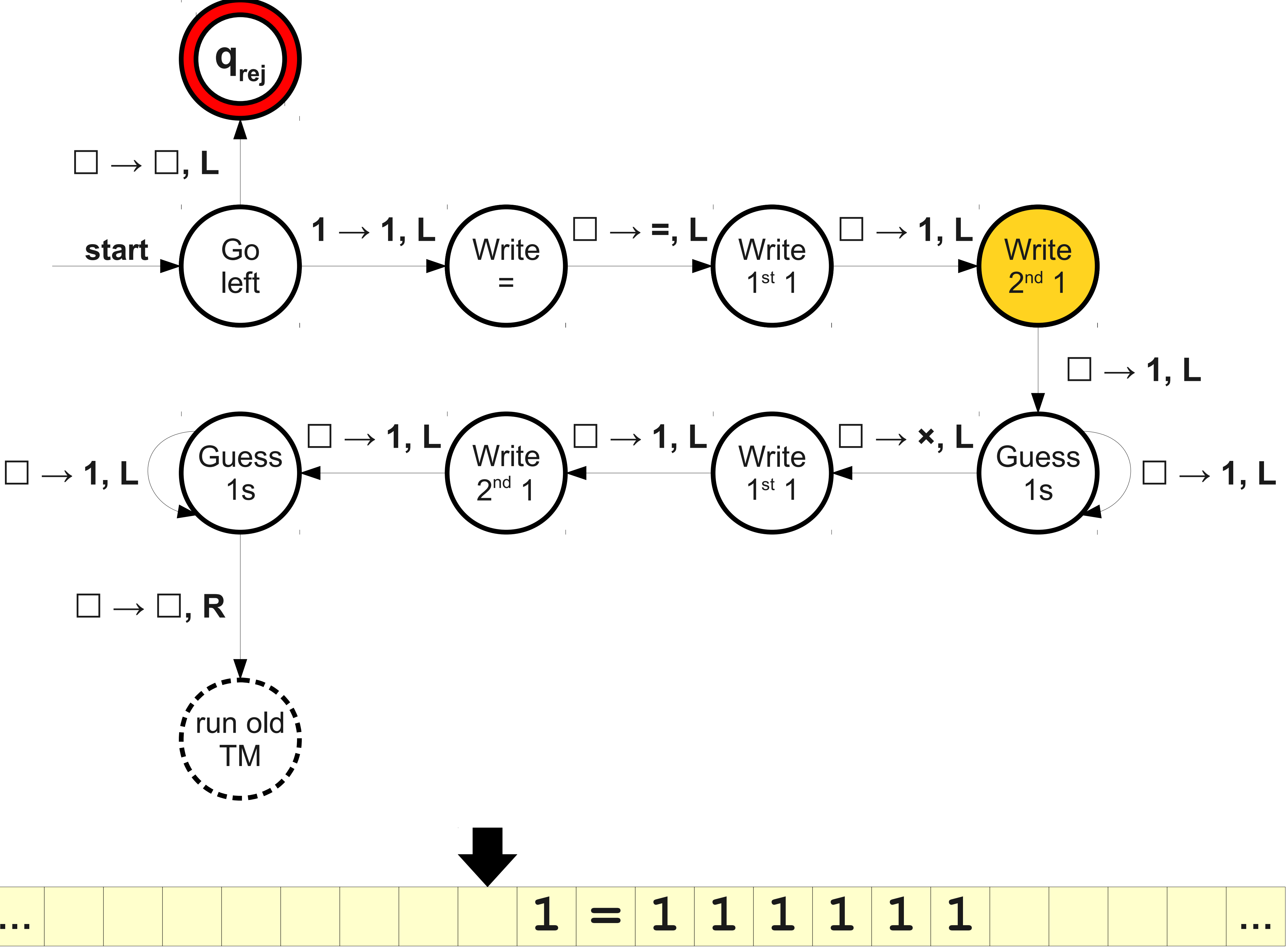

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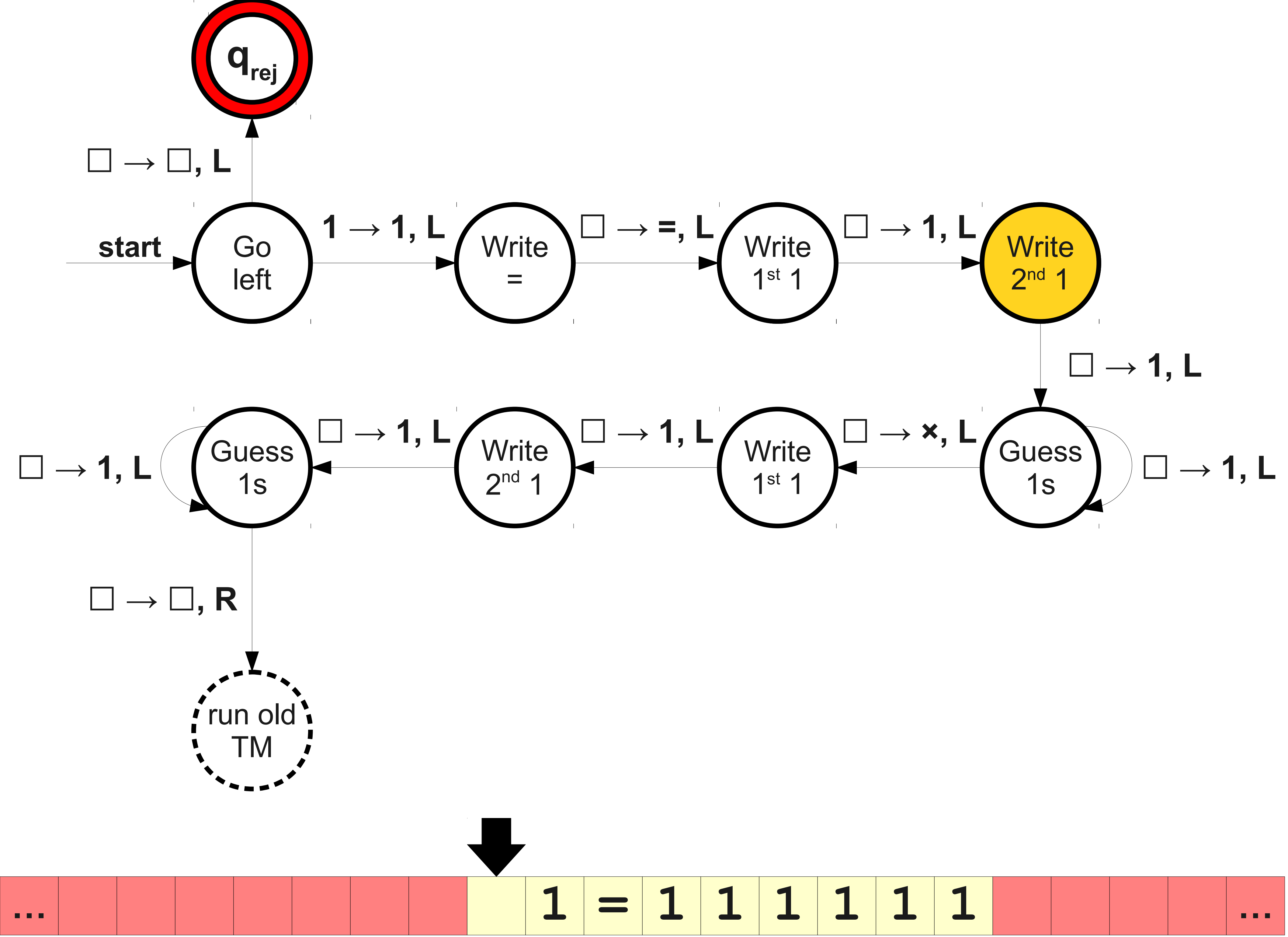
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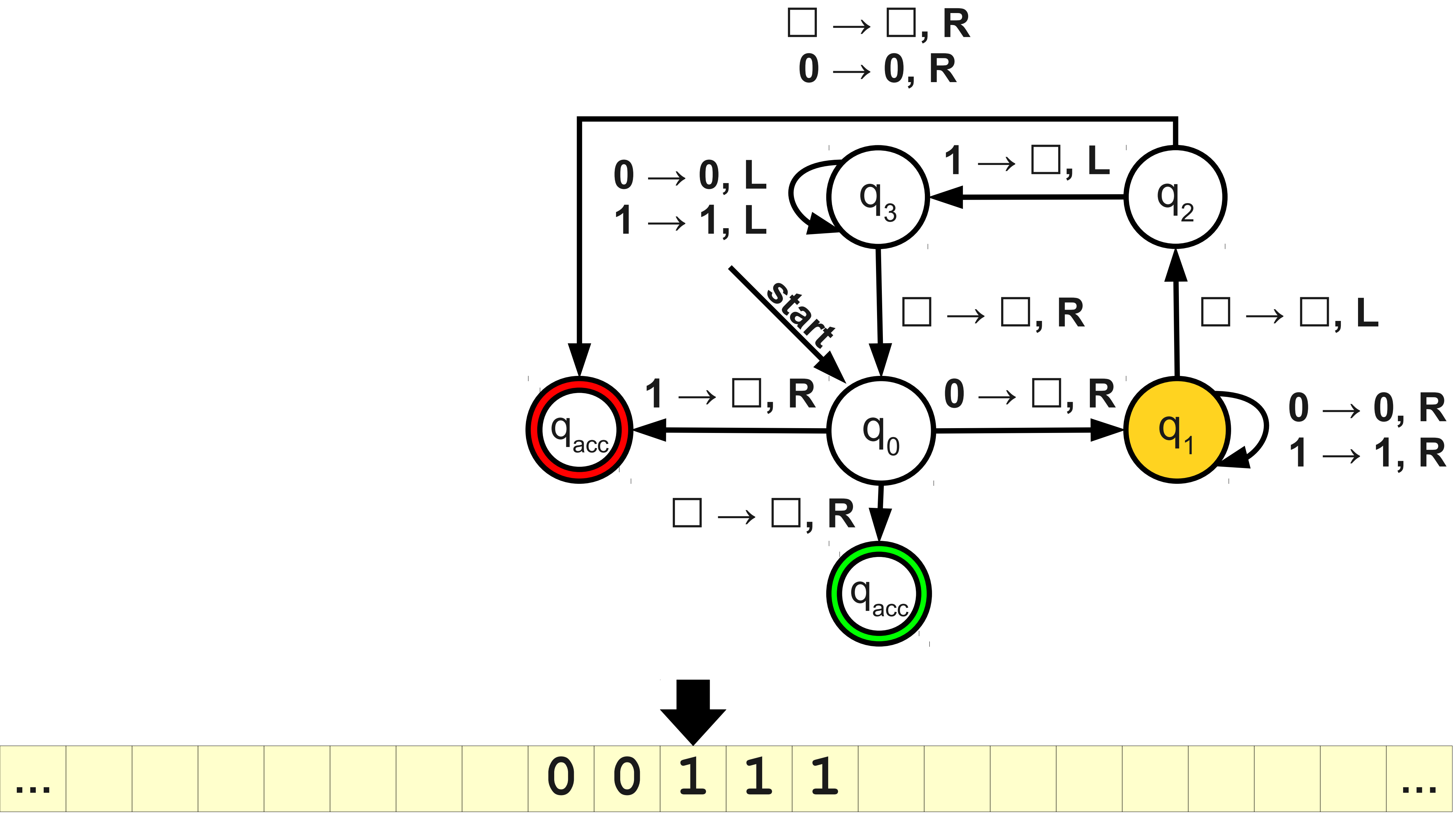
... 1 = 1 1 1 1 1 1 ...

Instantaneous Descriptions

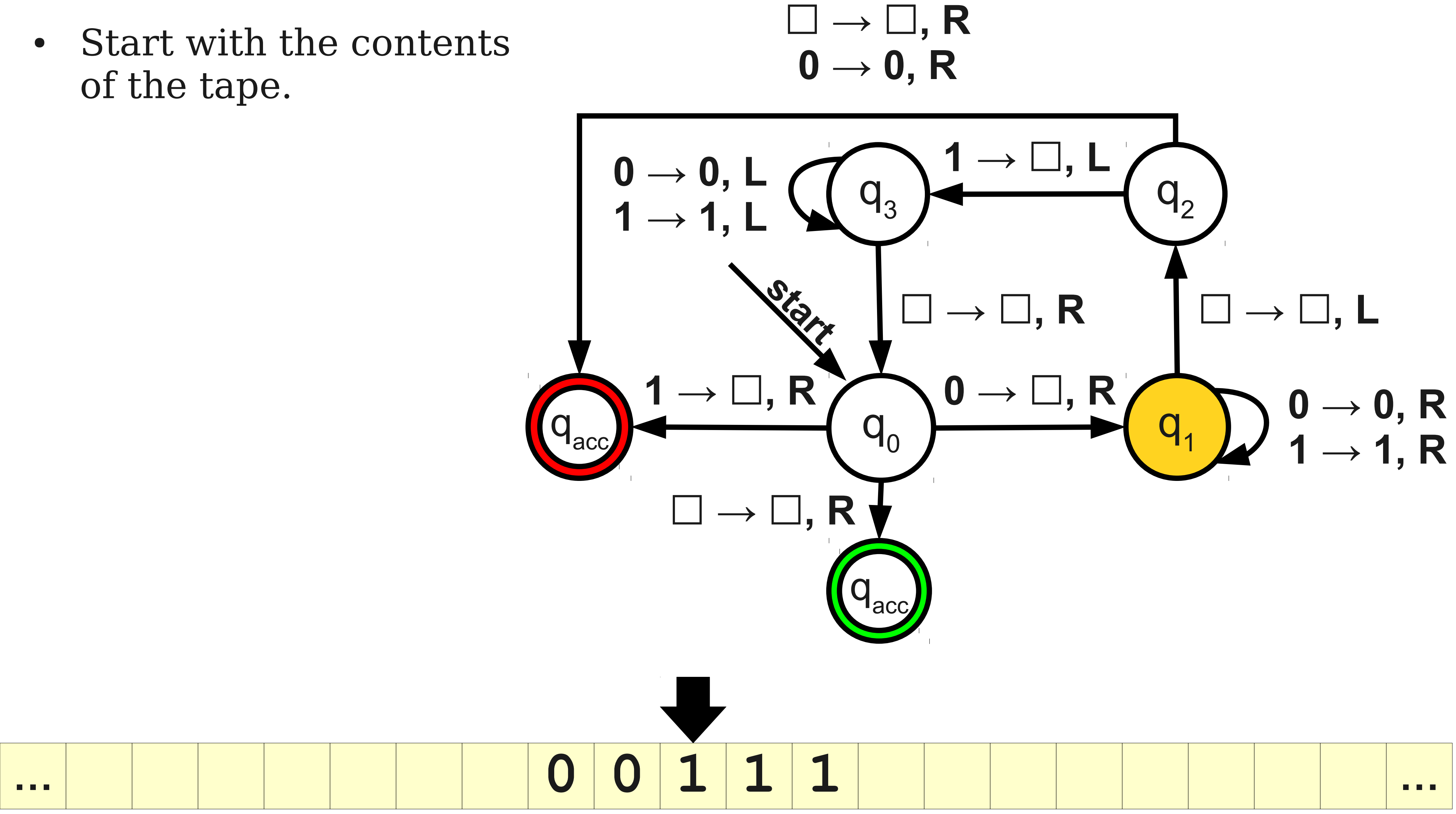
- At any instant in time, only finitely many cells on the TM's tape may be non-blank.
- An **instantaneous description** (ID) of a Turing machine is a string representation of the TM's tape, state, and tape head location.
  - Only store the “interesting” part of the tape.
- There are many ways to encode an ID; we'll see one in a second.

Building an ID



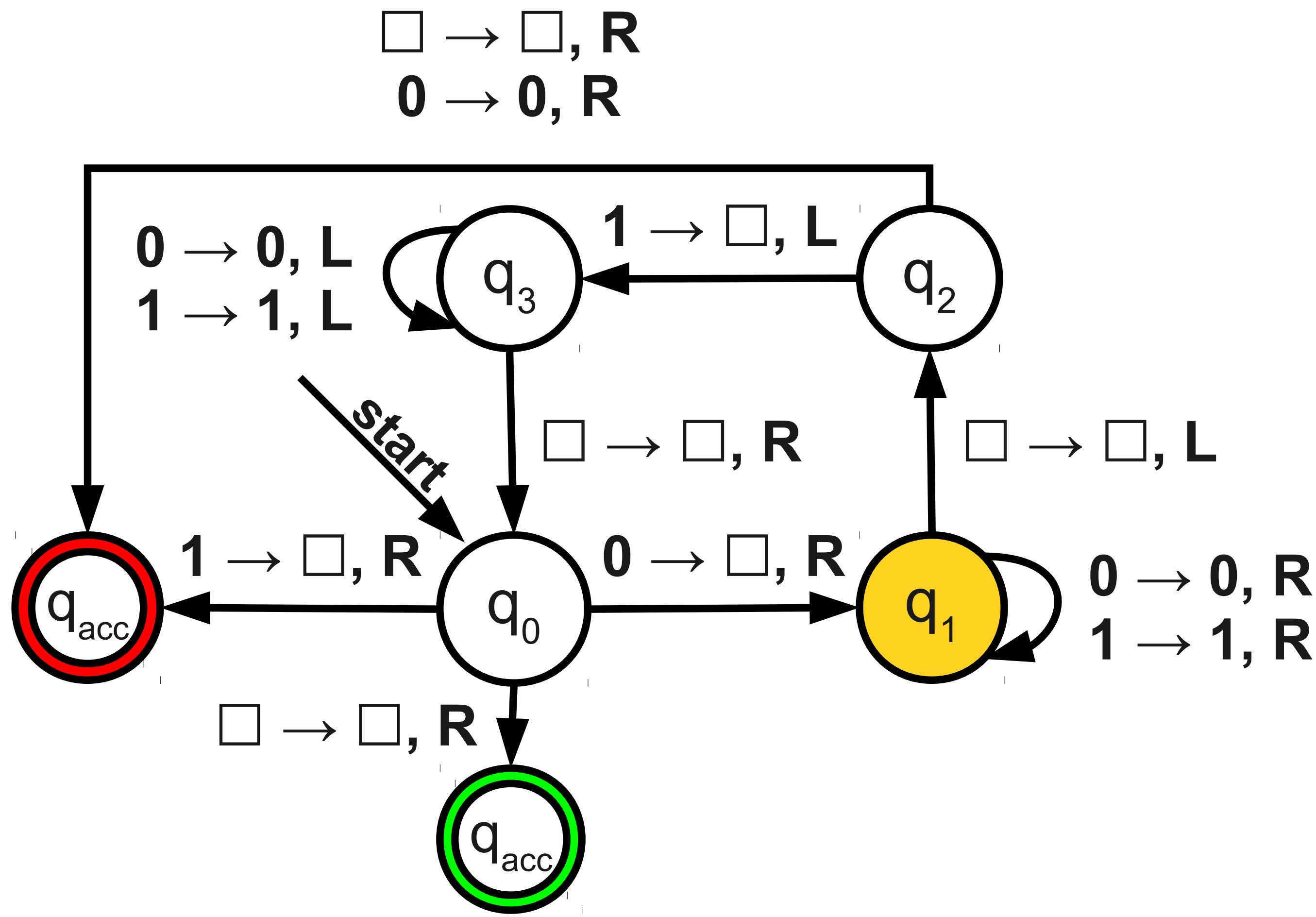
Building an ID

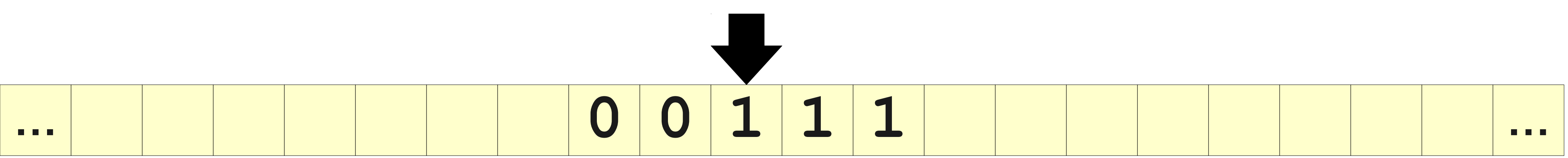
- Start with the contents of the tape.



Building an ID

- Start with the contents of the tape.
- Trim "uninteresting" blank symbols from the ends of the tape (though remember blanks under the tape head).





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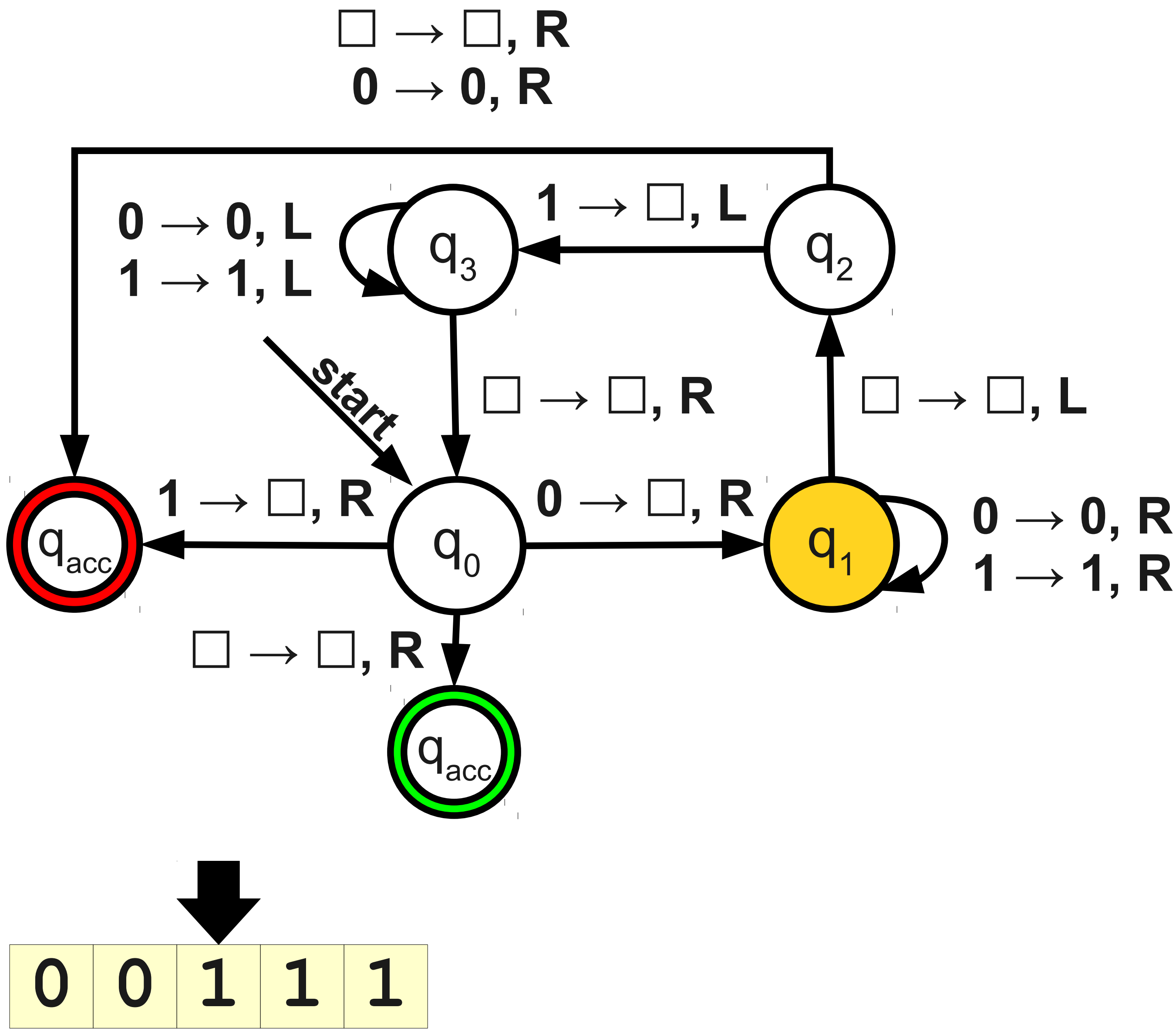

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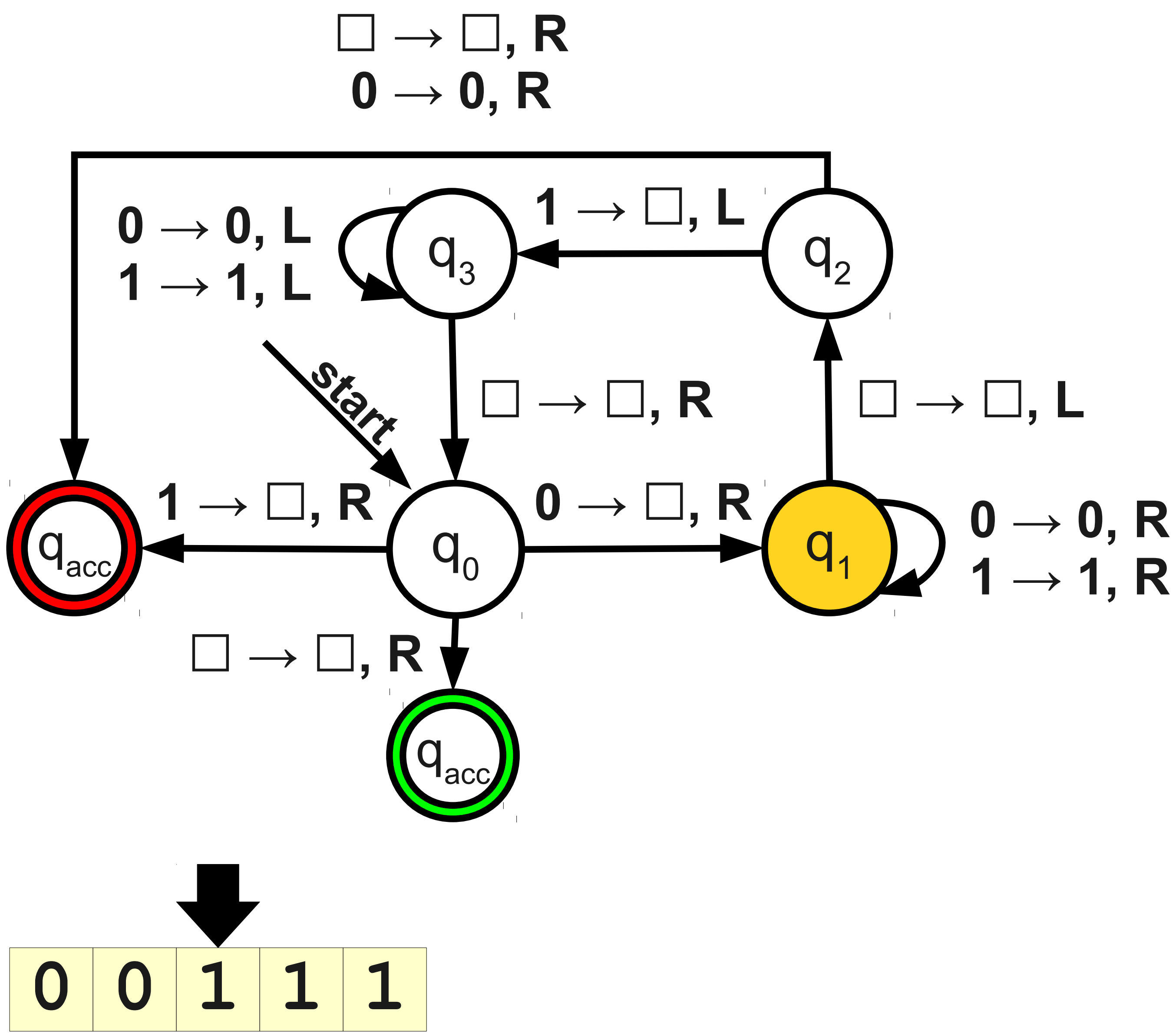

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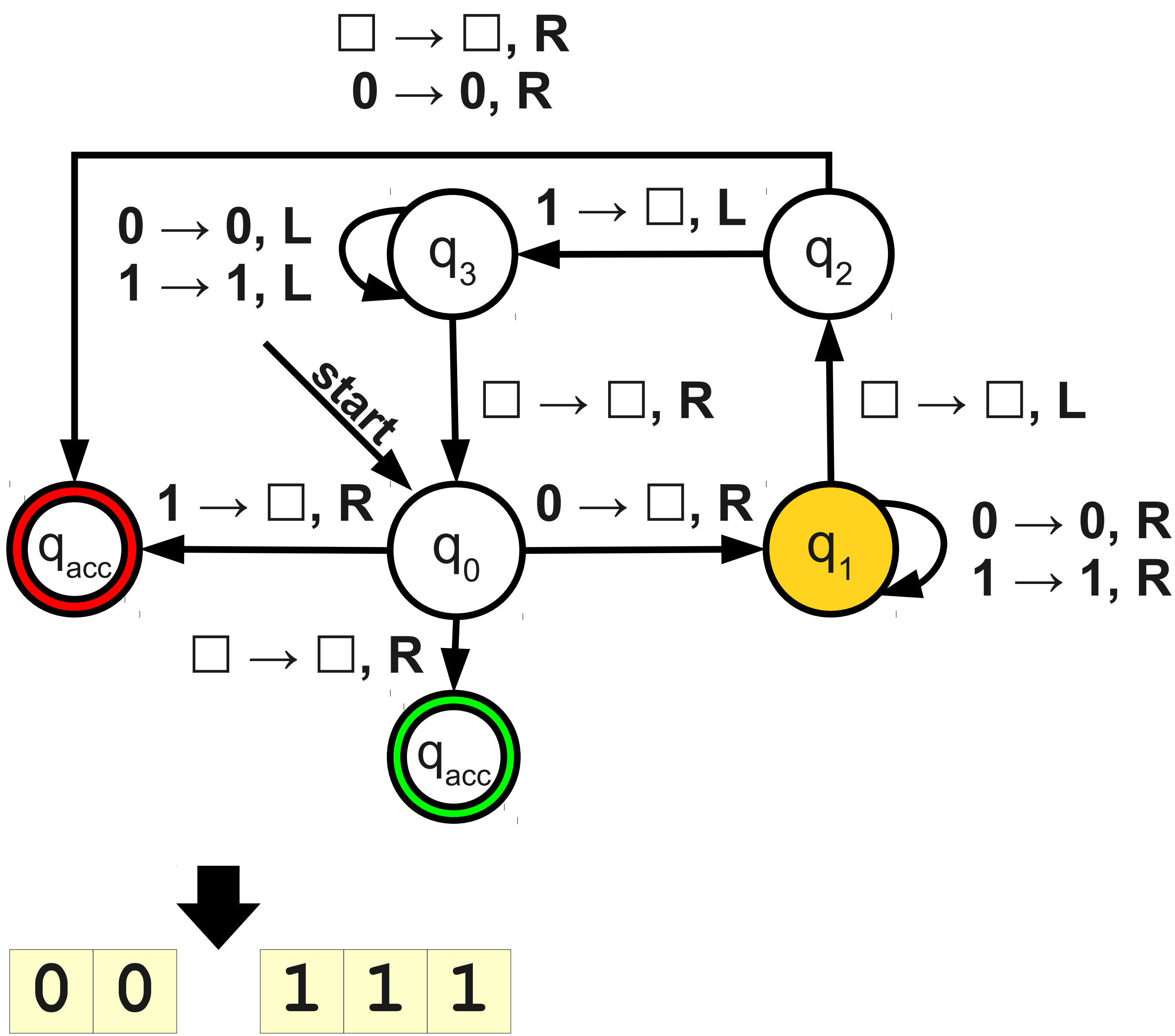

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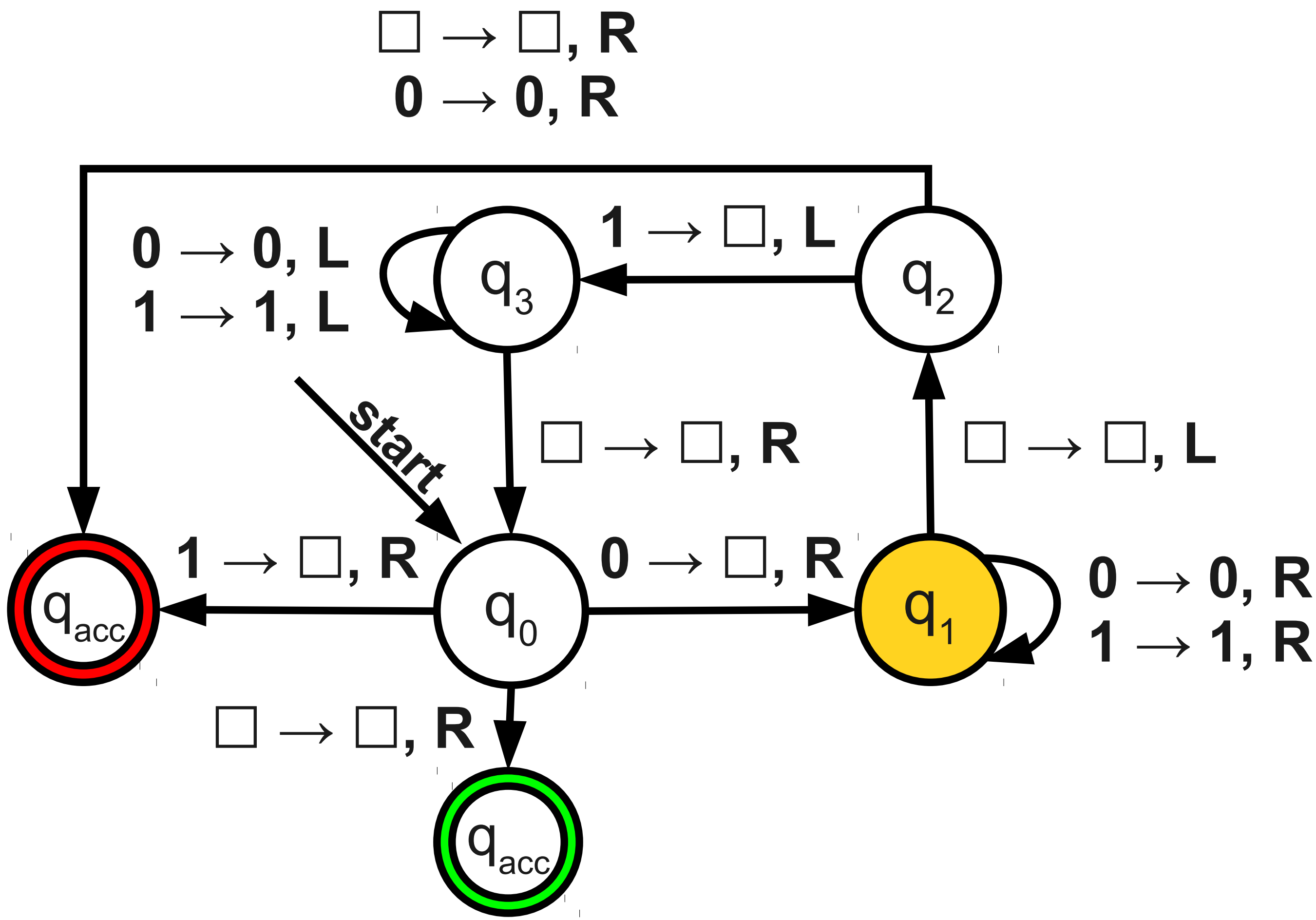

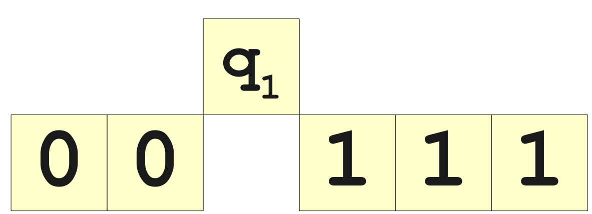

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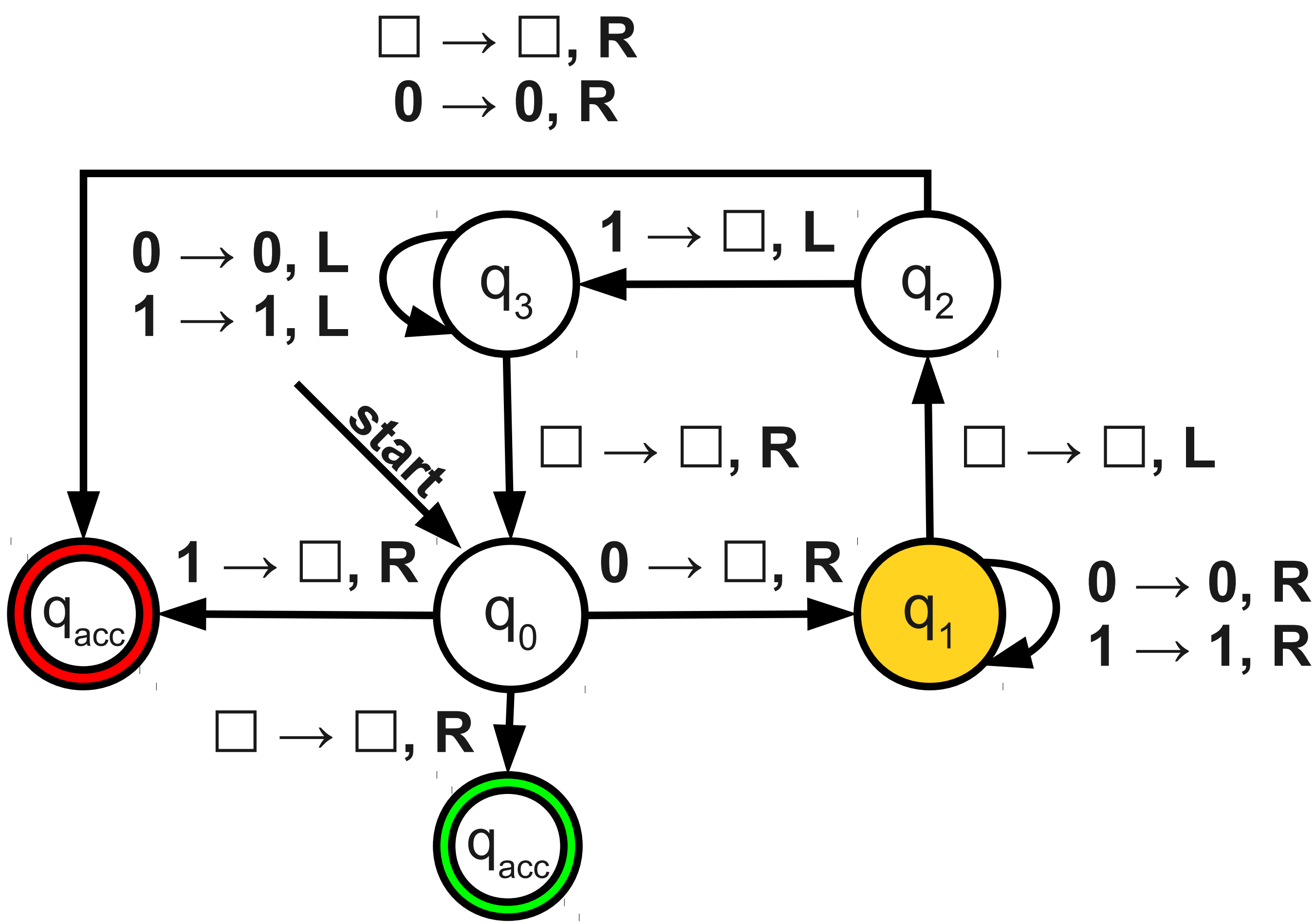
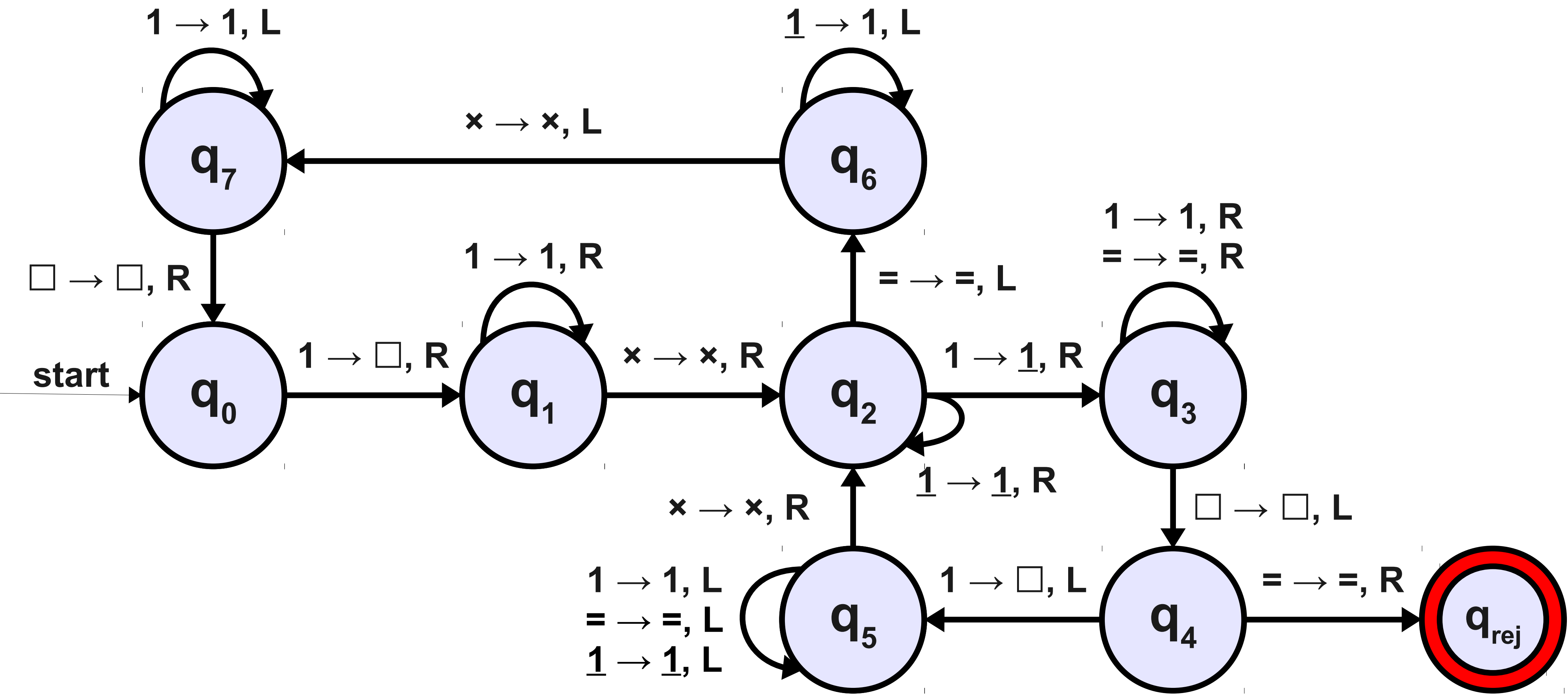


0	0	q_1	1	1	1

Rehydrating IDs



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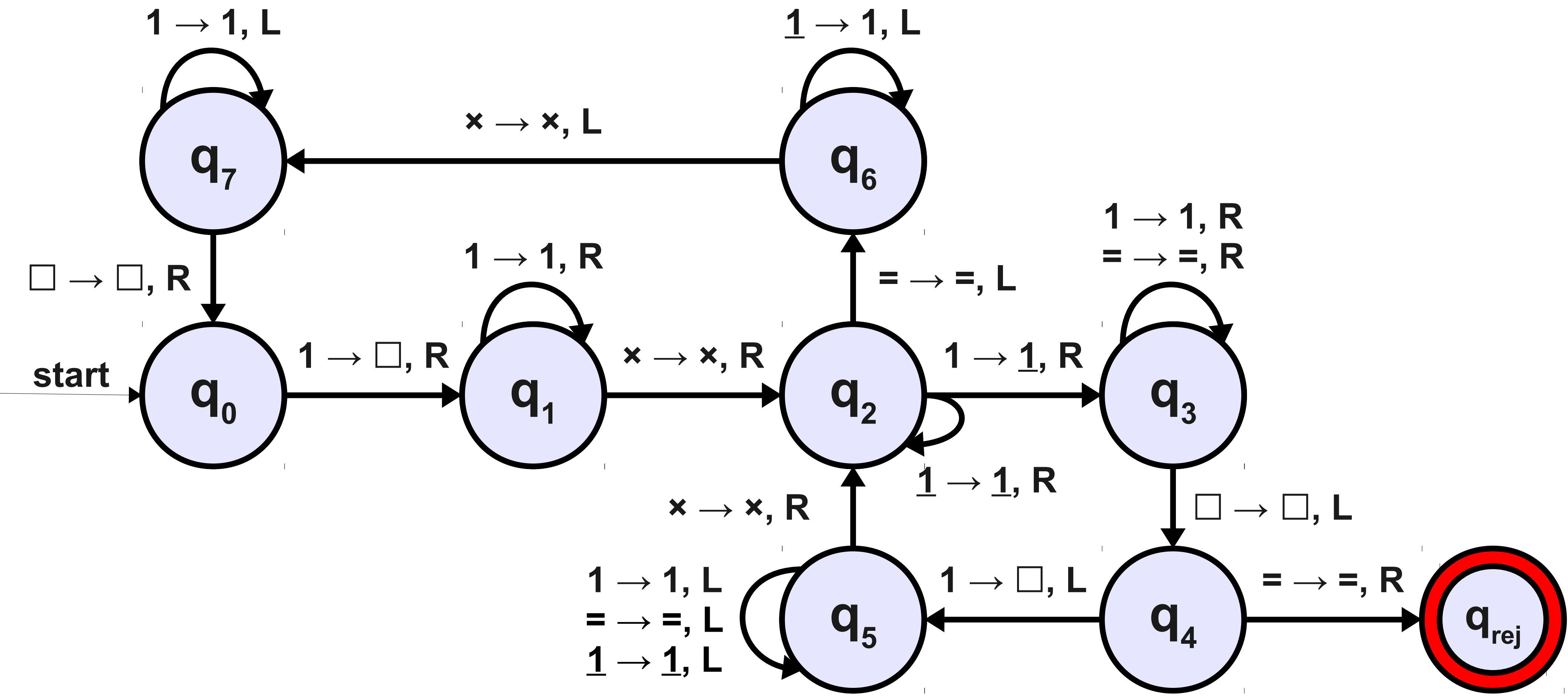
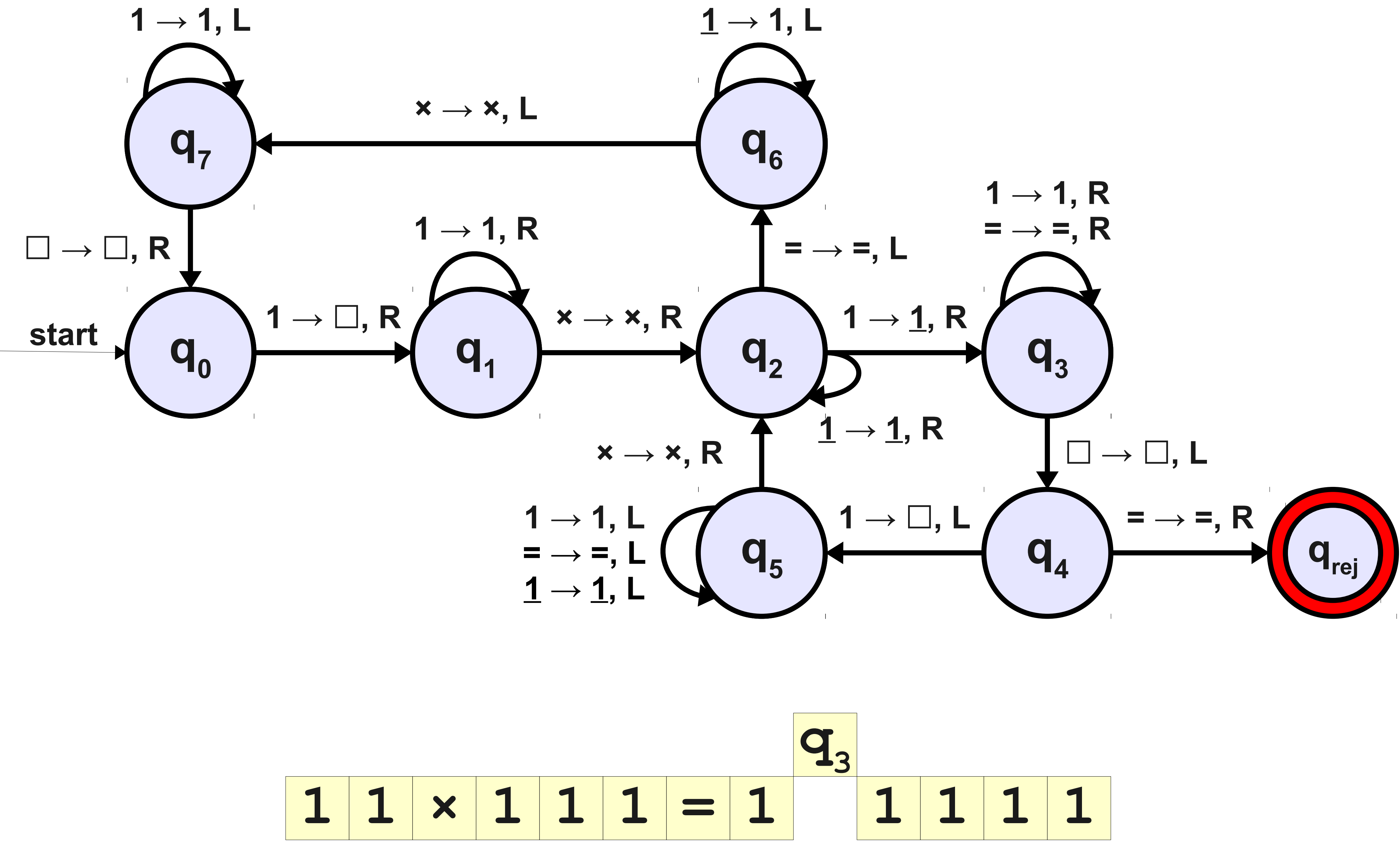


1	1	×	1	1	1	=	1	q_3	1	1	1	1

Rehydrating IDs



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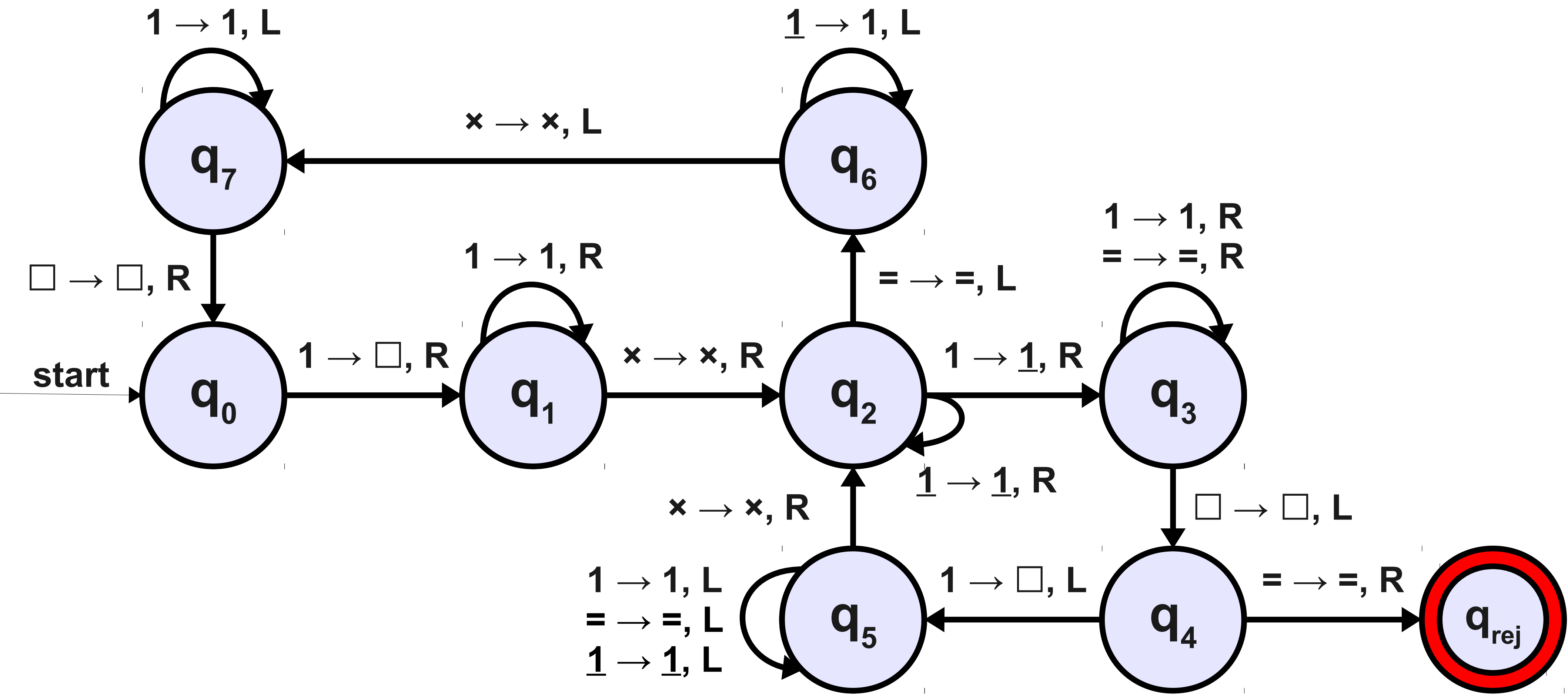

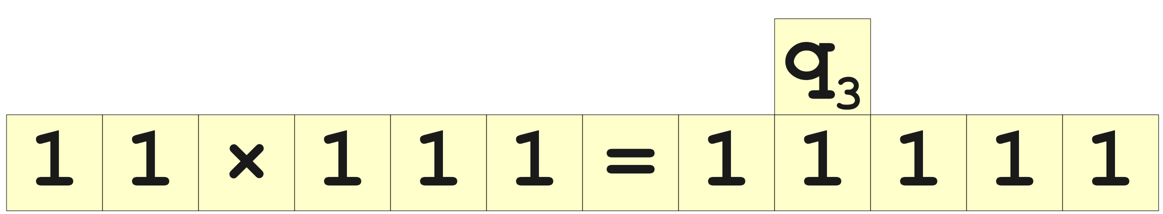

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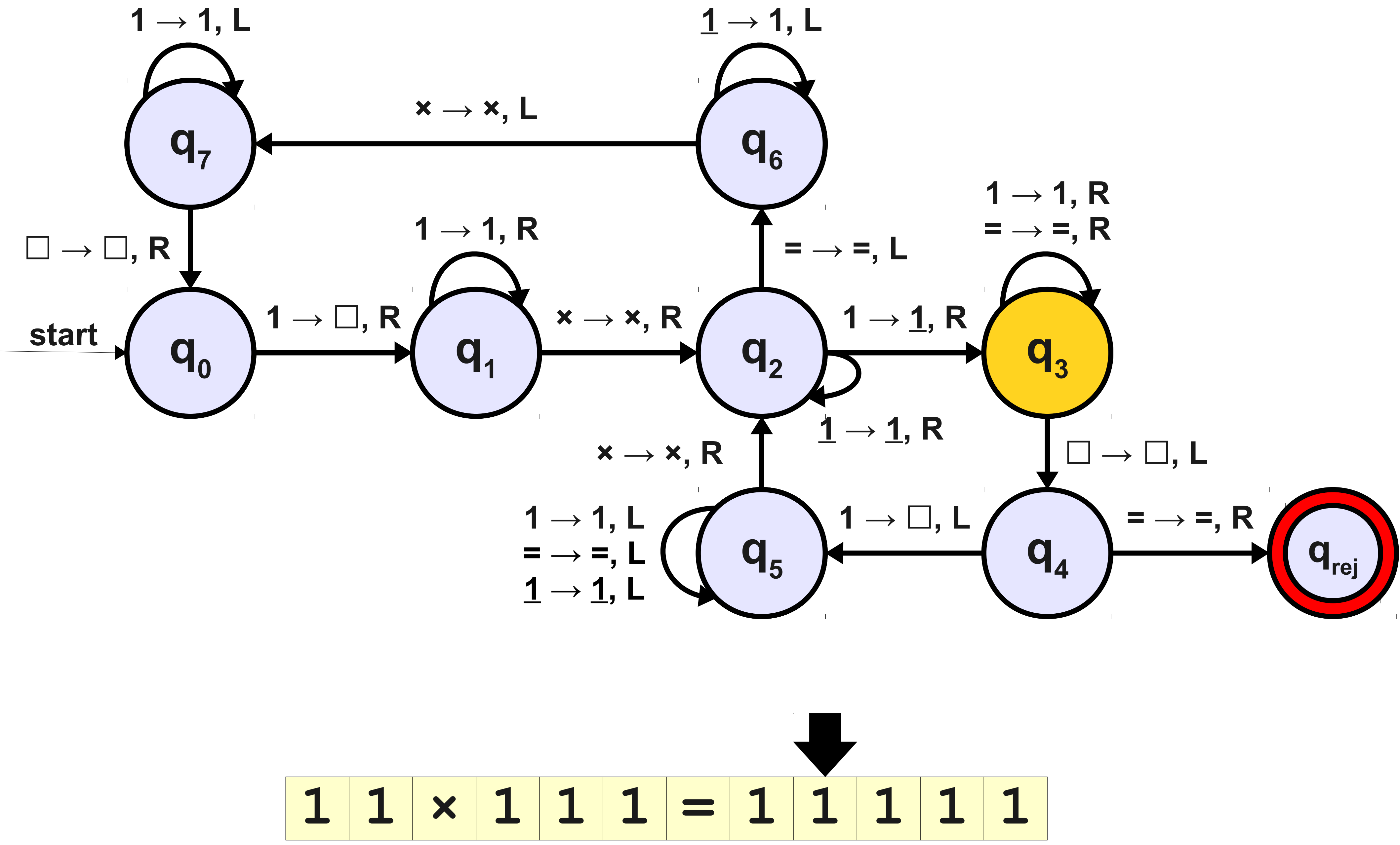

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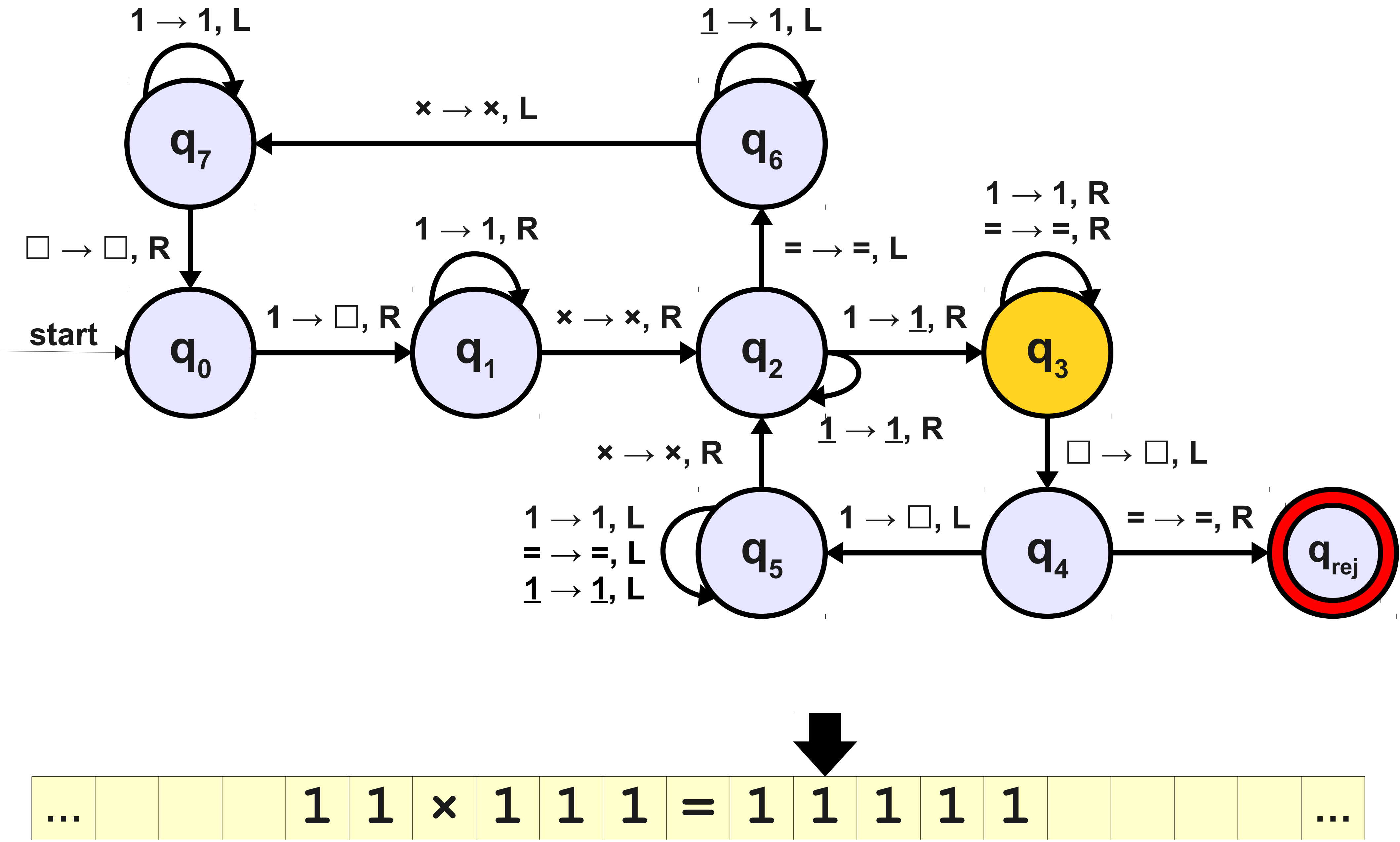

Manipulating IDs

Theorem: For any TM M, it is possible to build a second TM Sim_M that, given a a tape containing an ID of M, simulates one step of M's computation.

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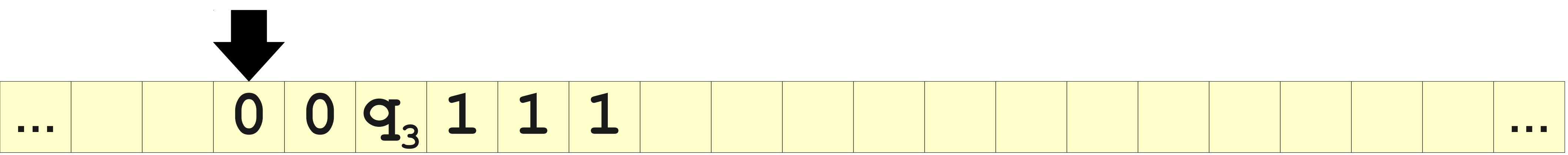

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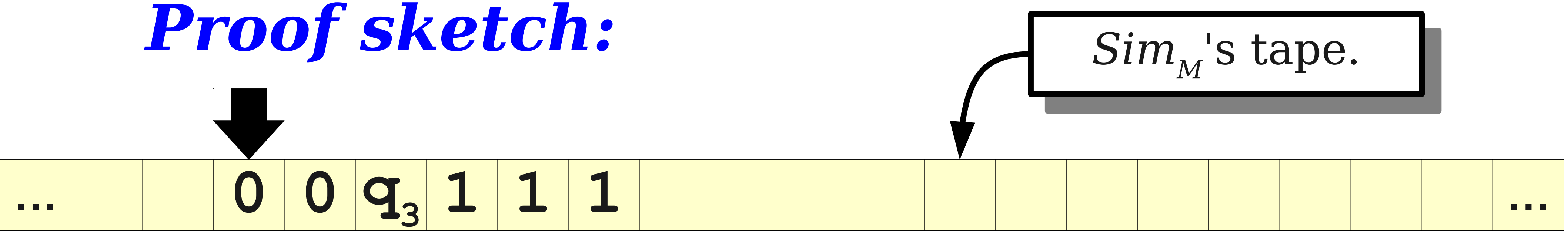

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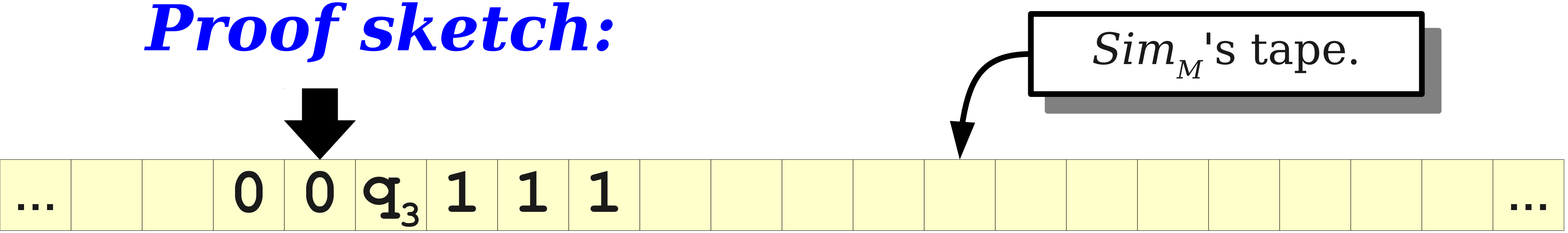

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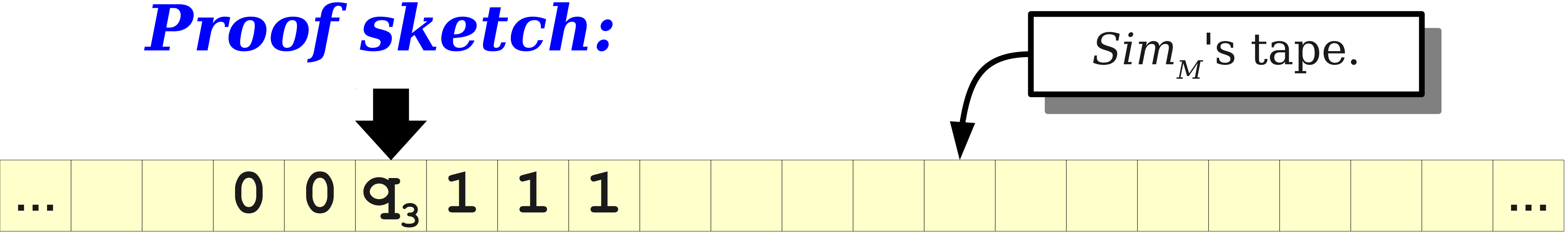
Theorem: For any TM M, it is possible to build a second TM Sim_M that, given a a tape containing an ID of M, simulates one step of M's computation.

Proof sketch:



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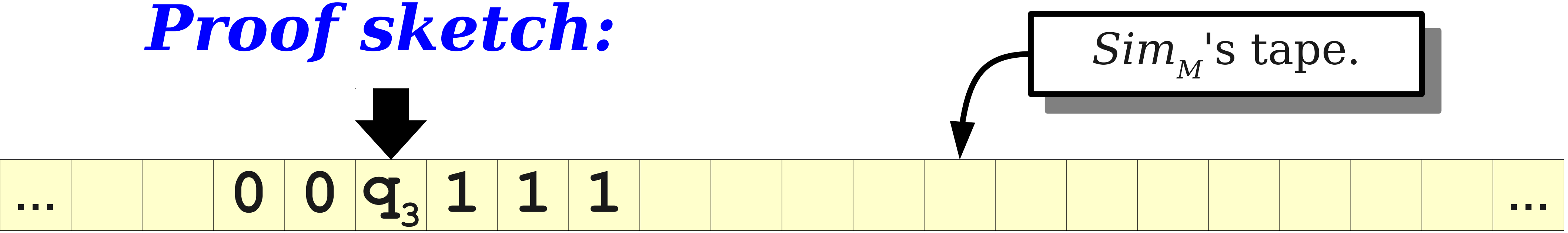

At this point, $\boldsymbol{Sim_M}$ can remember that it's seen state $\boldsymbol{q_3}$. There are only finitely many possible states.

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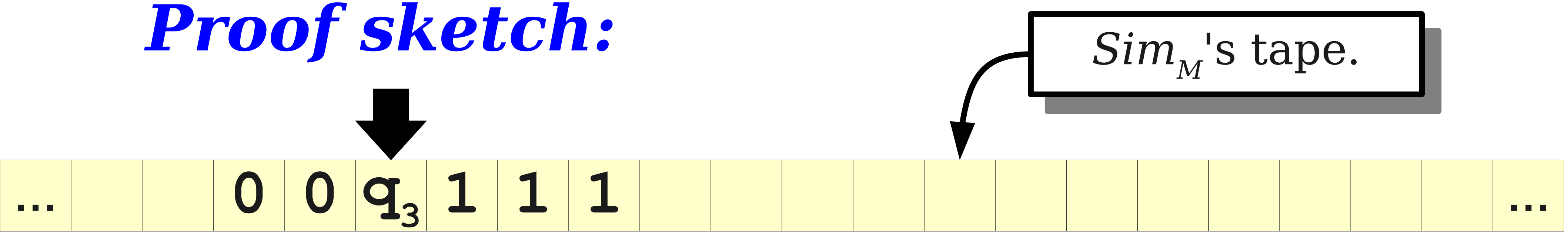

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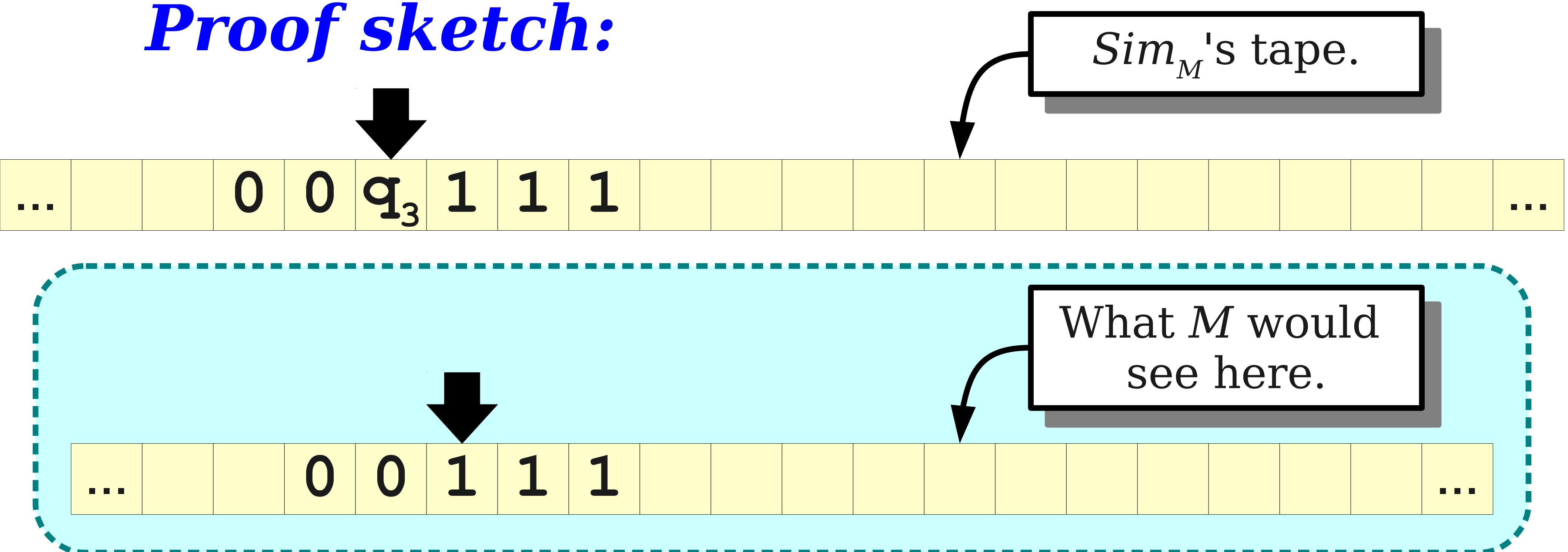

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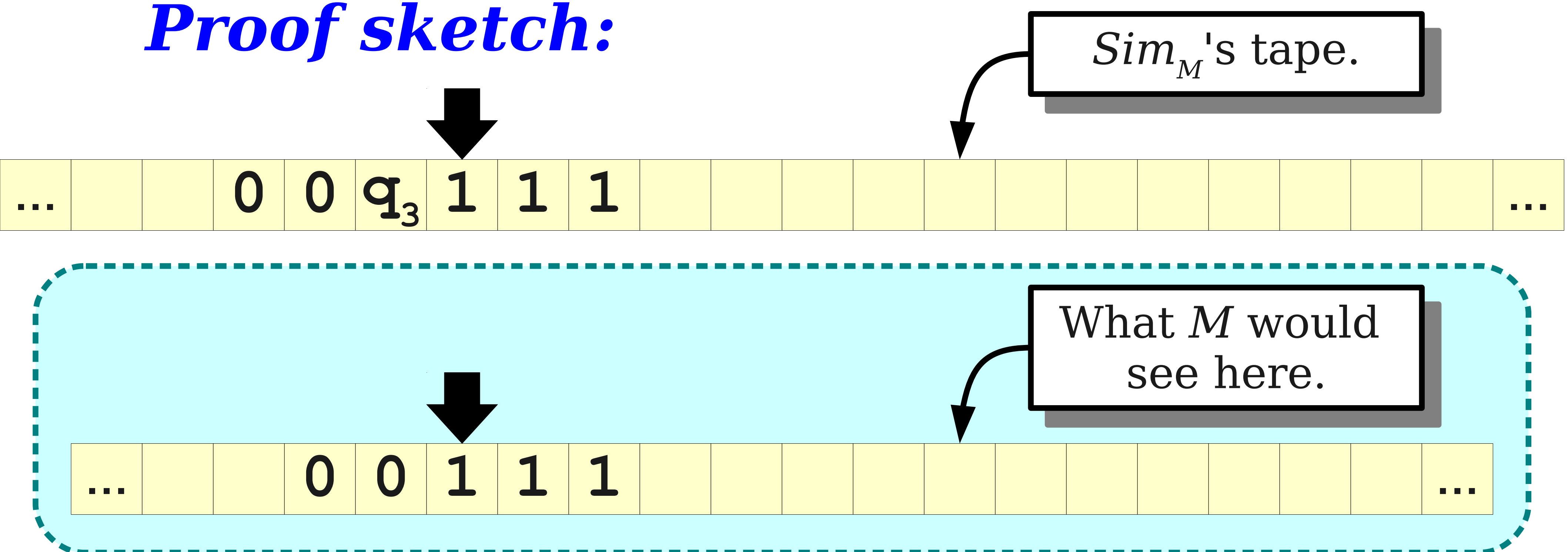

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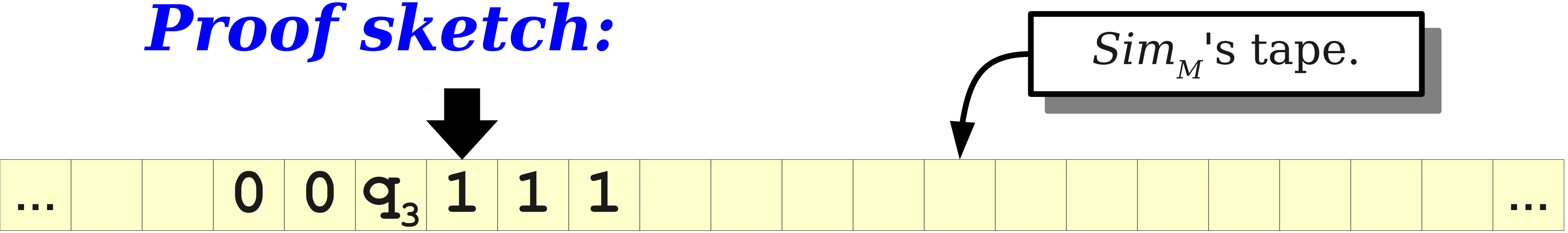

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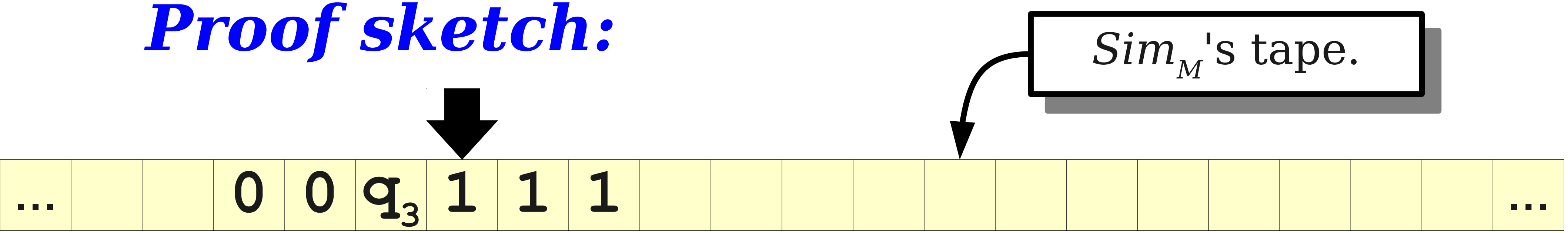

Sim_M now knows that **M** is in state **q_3** and reading **1**.

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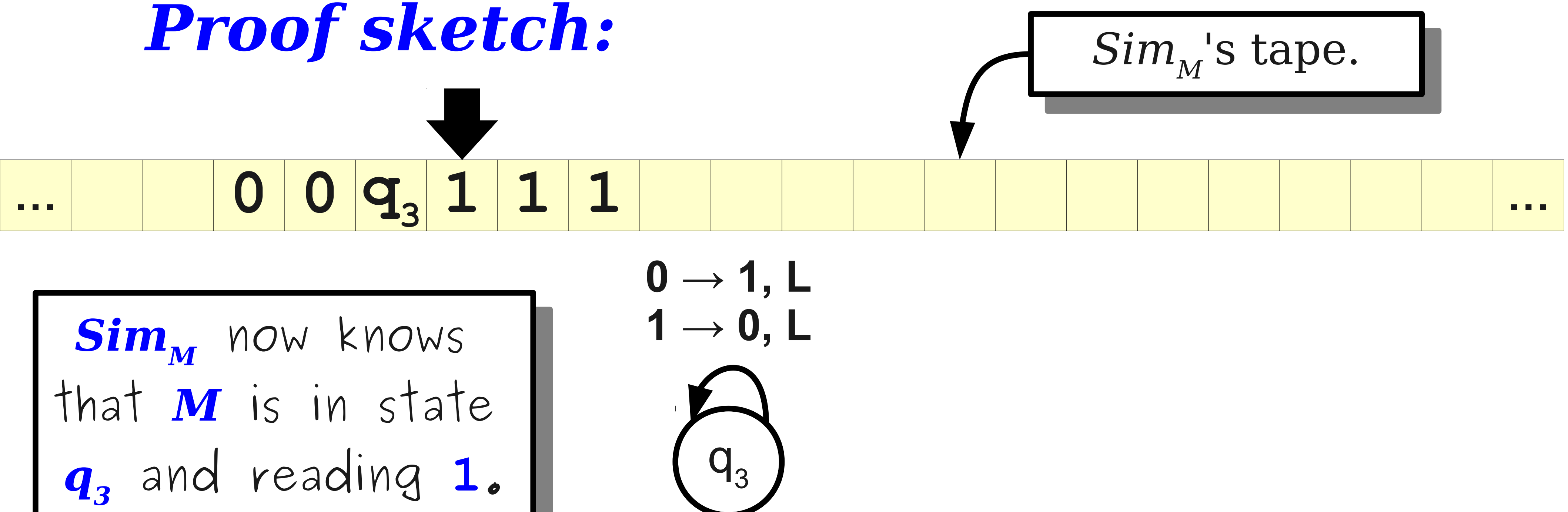

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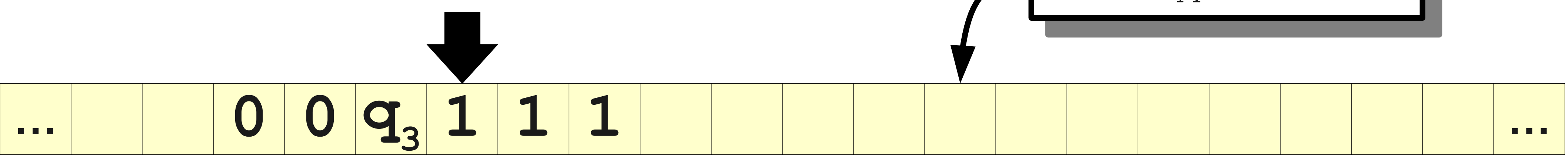
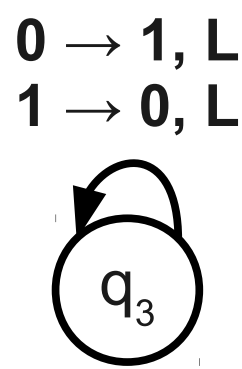


Sim_M now knows that M is in state q_3 and reading **1**.



(Hypothetically, suppose that this is the state q_3 in the machine M)

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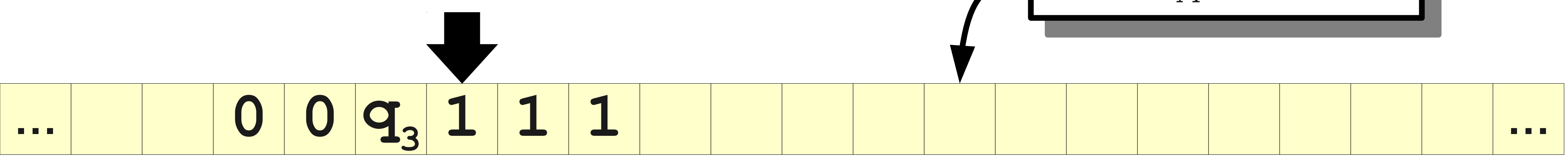

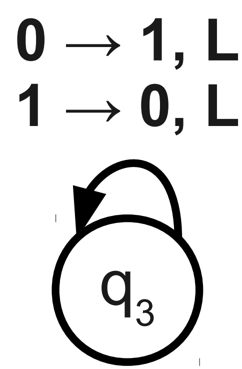

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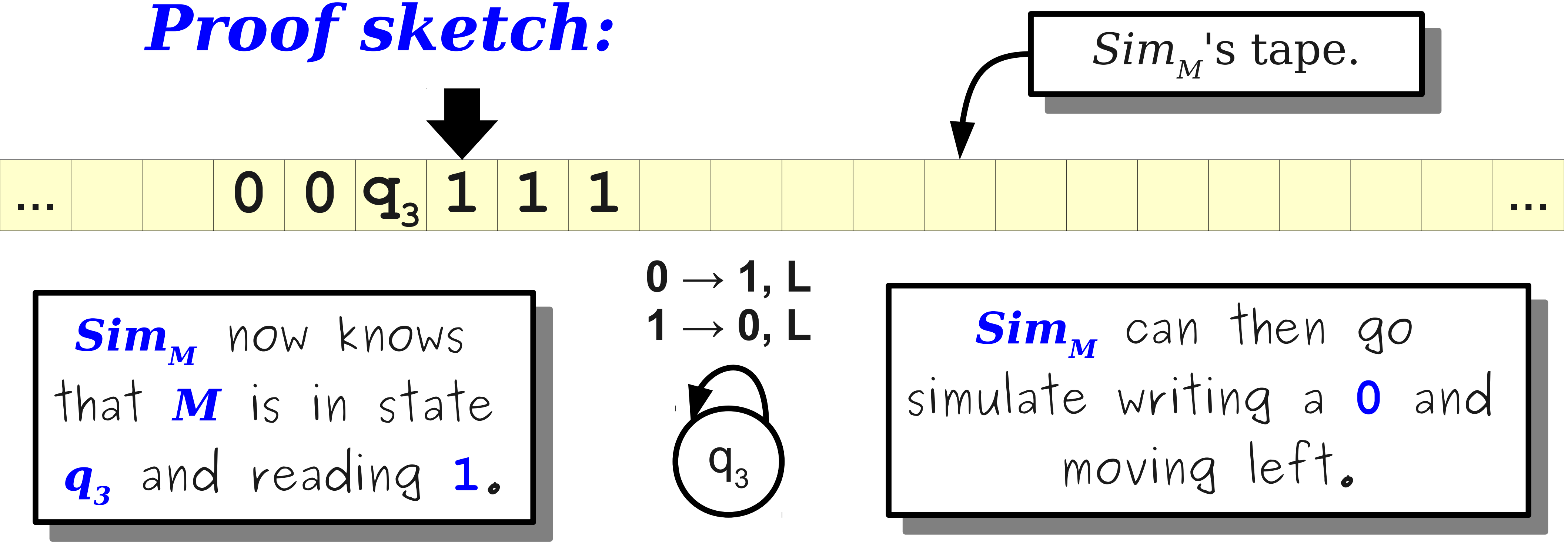

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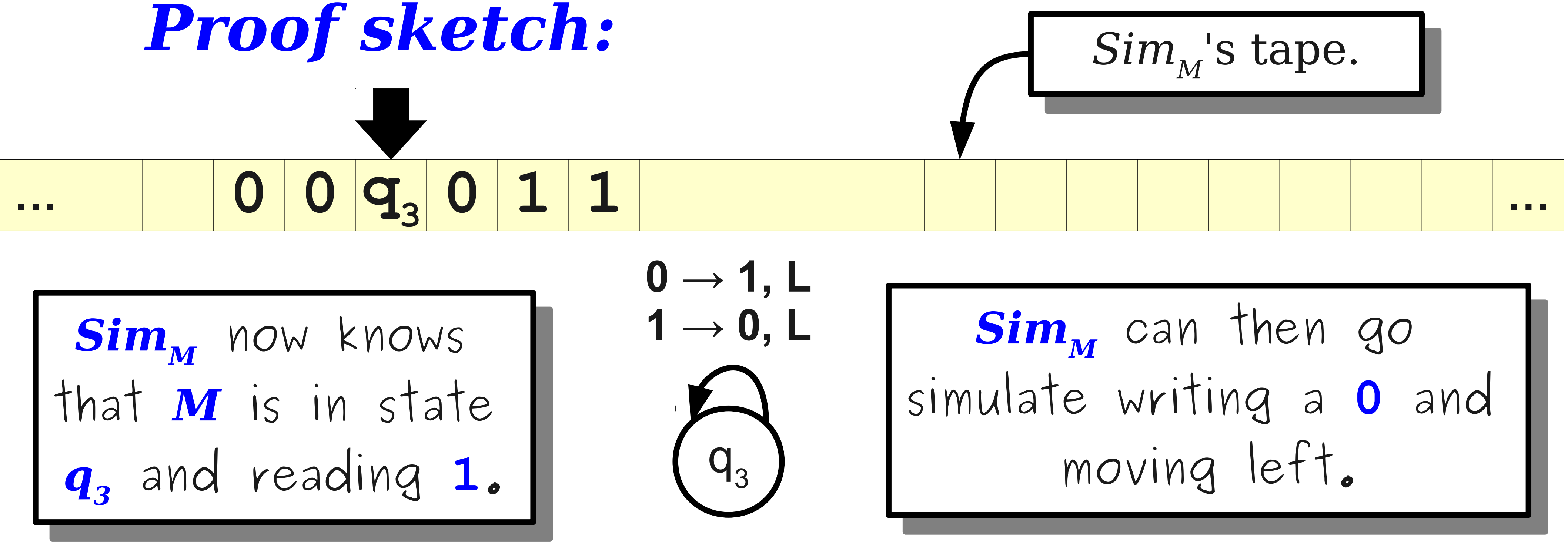

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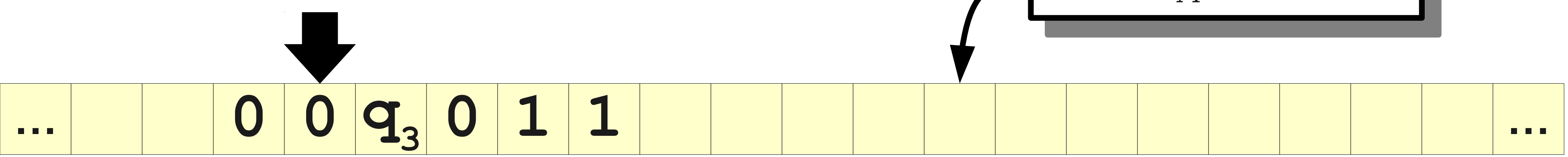

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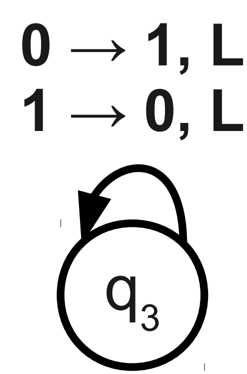

Sim_M can then go simulate writing a **0** and moving left.

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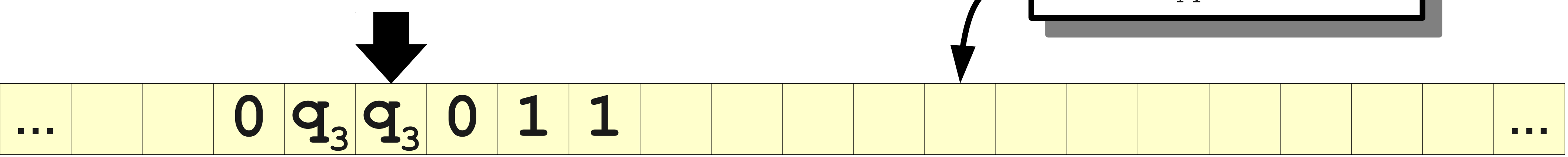

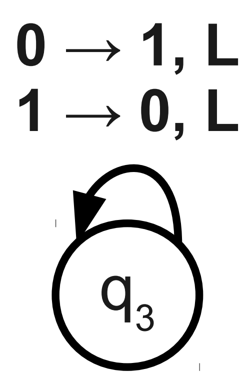

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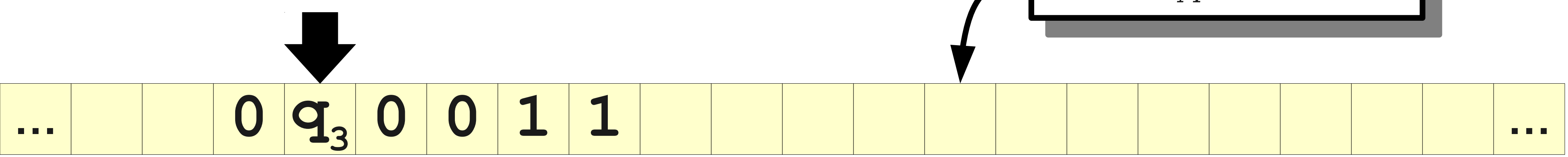

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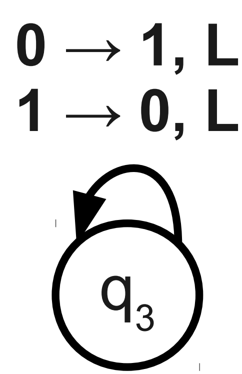

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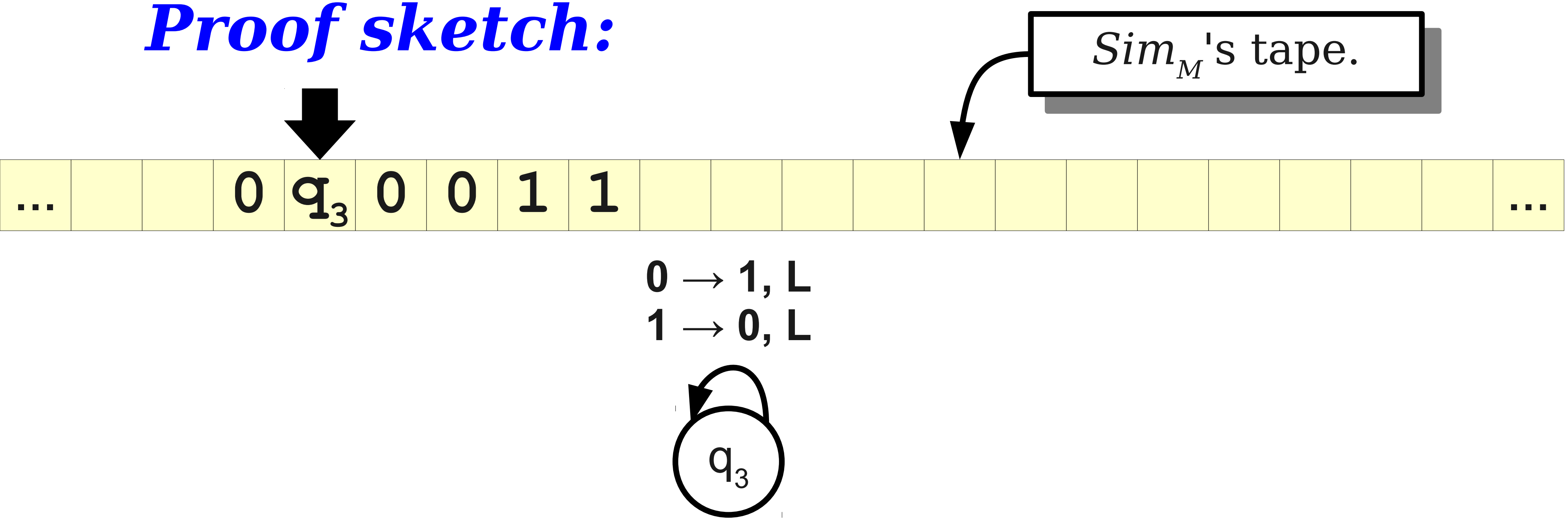

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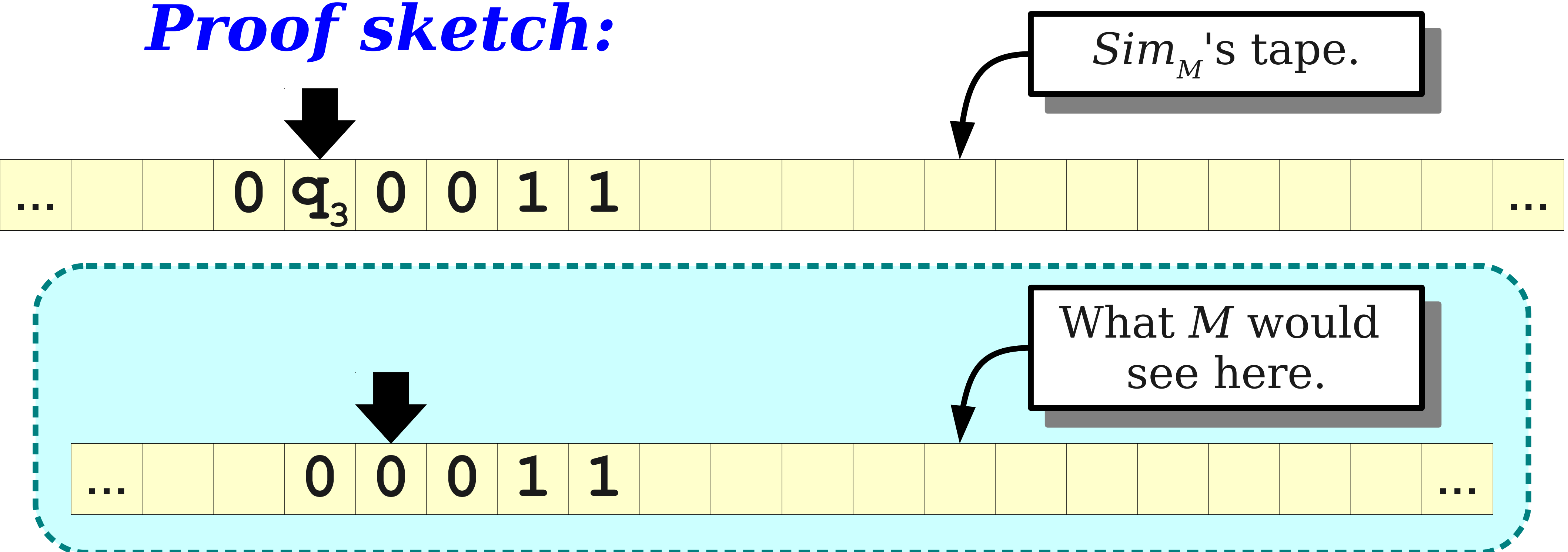

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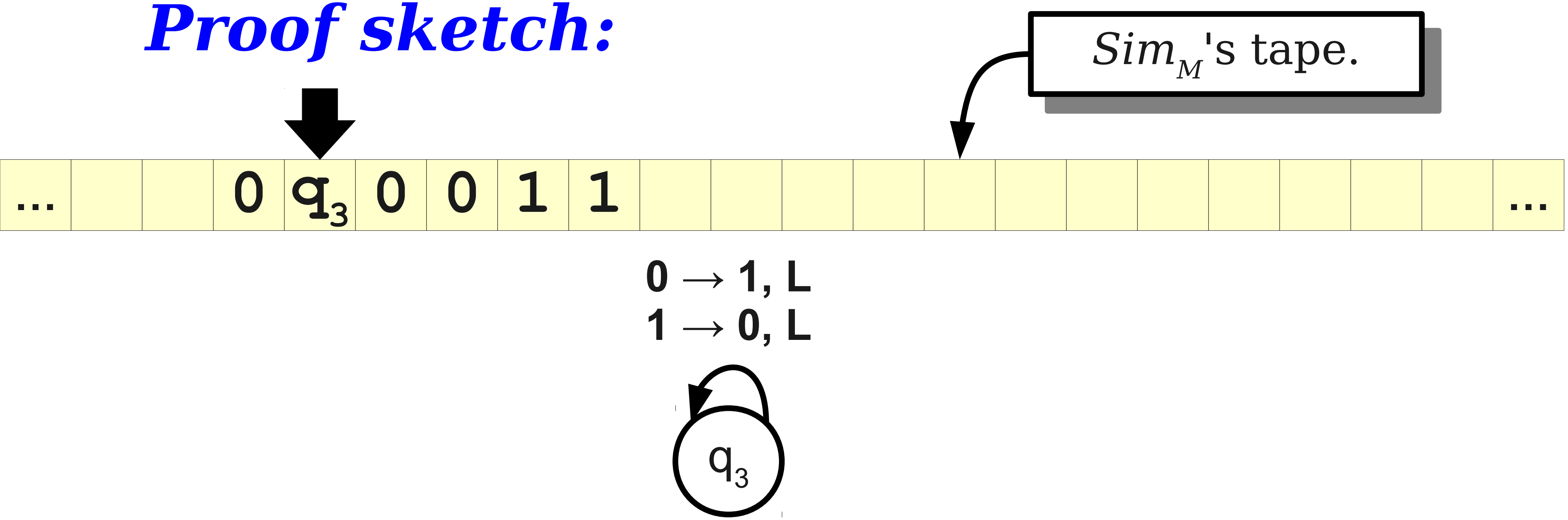

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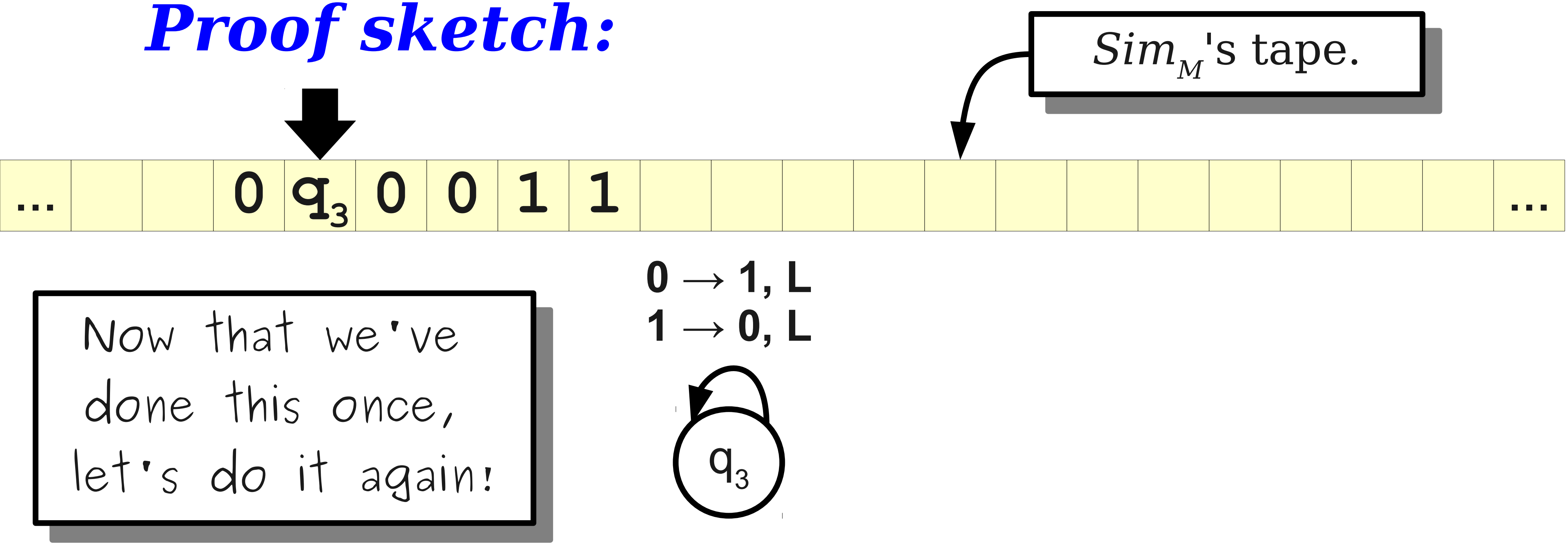

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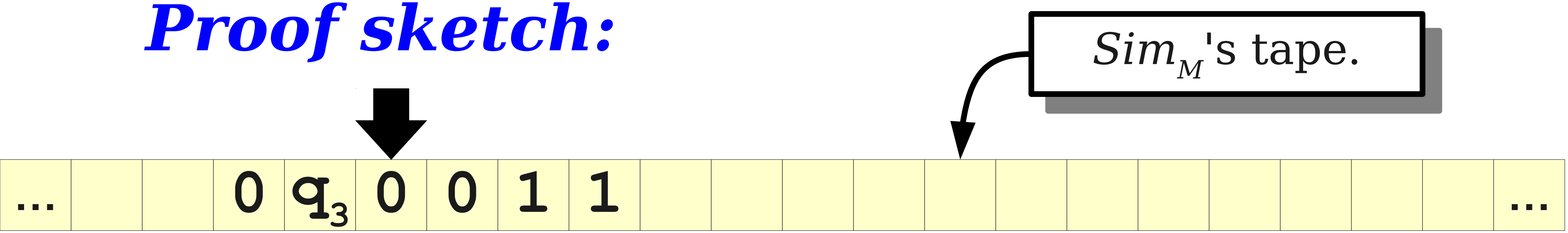

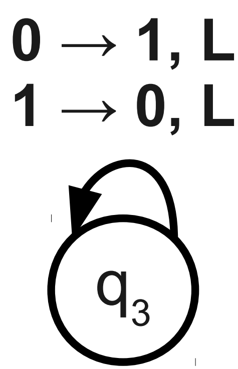

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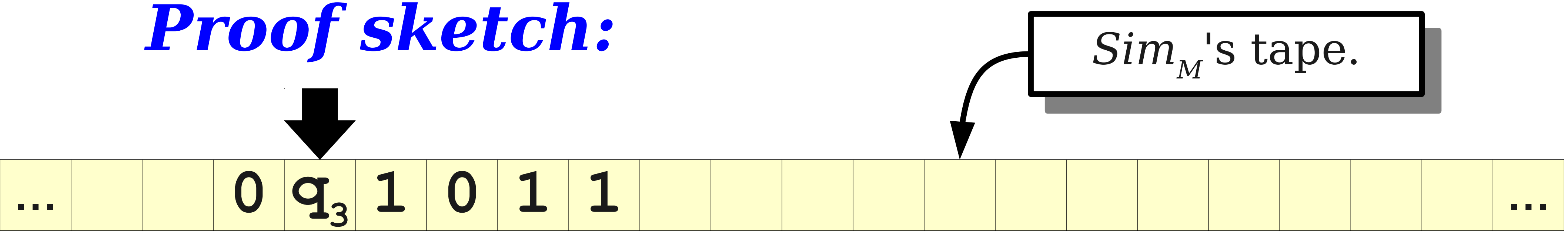

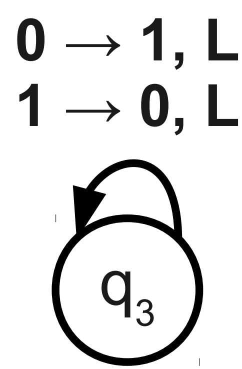

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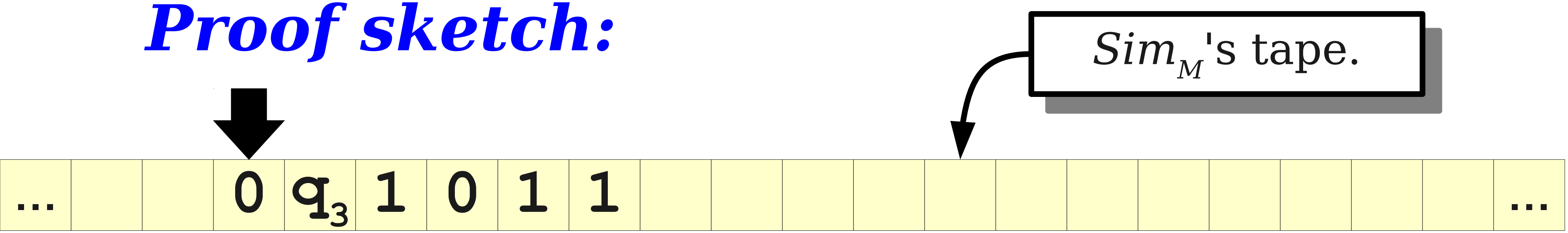

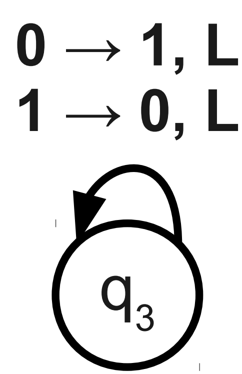

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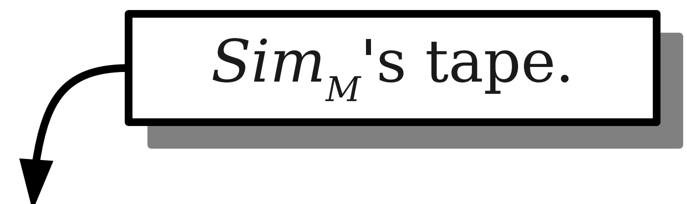

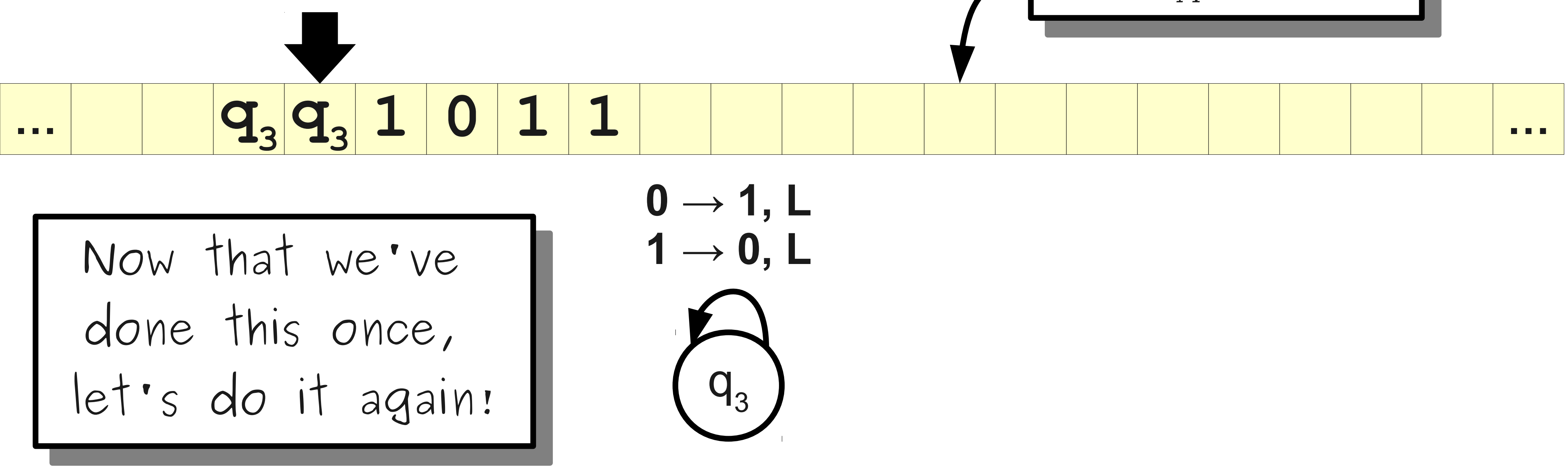

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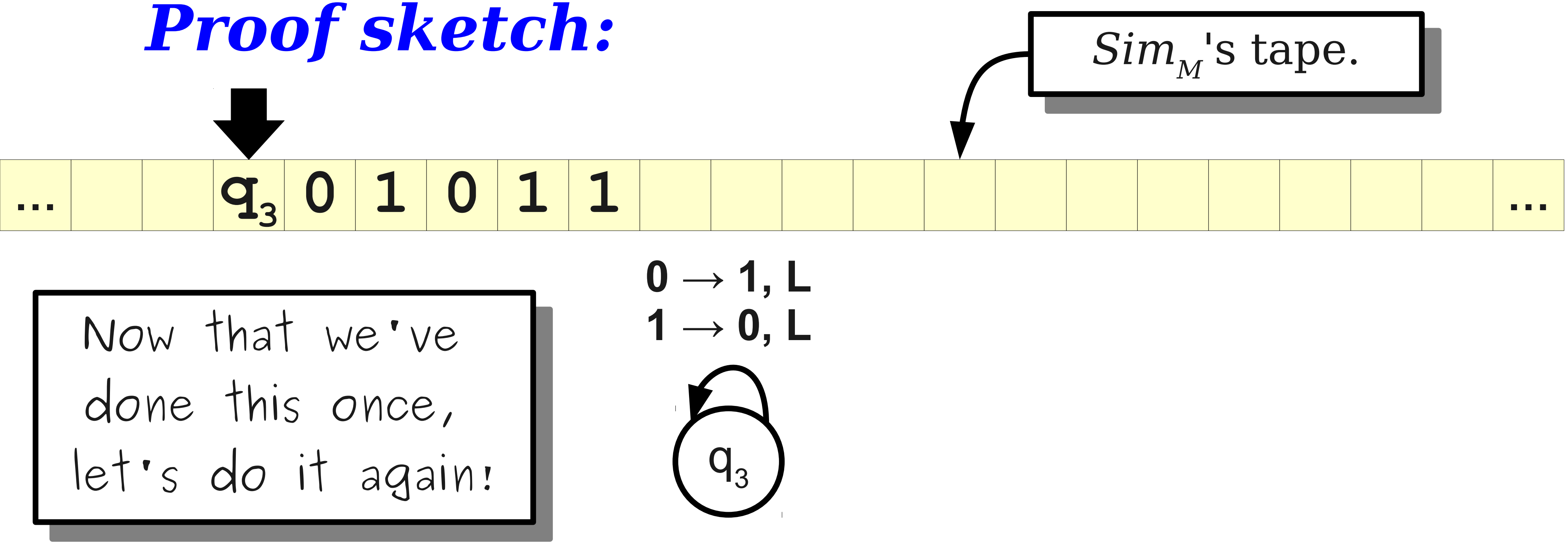

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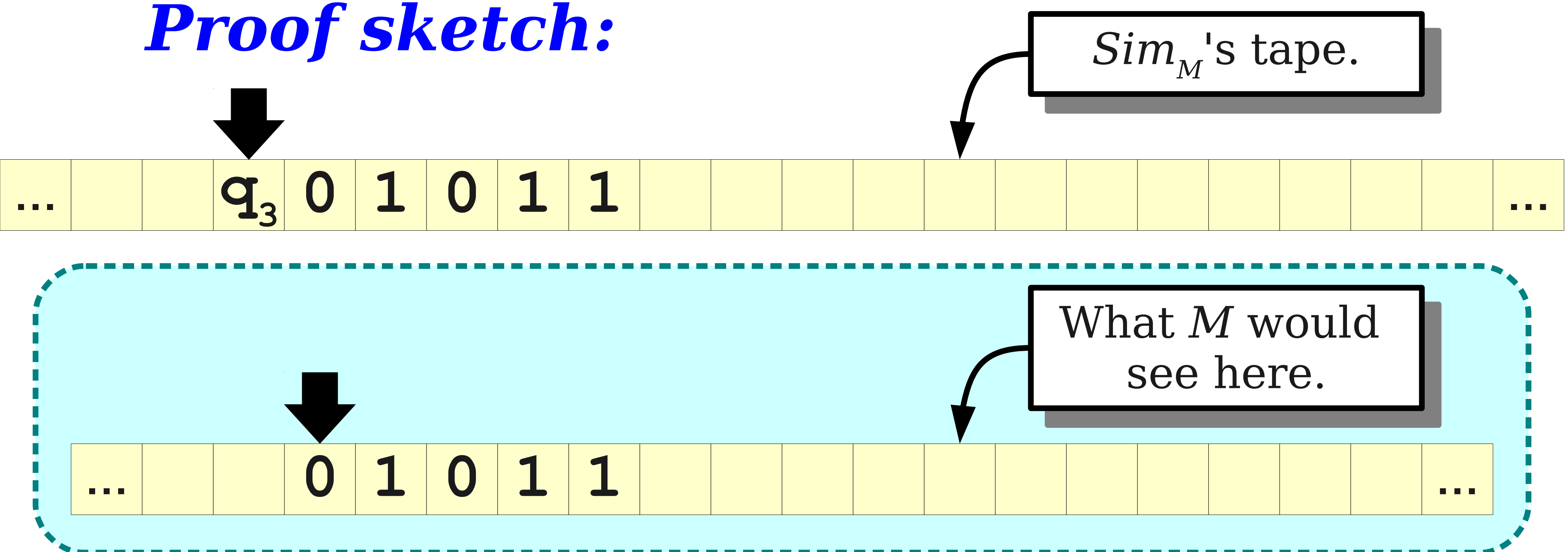

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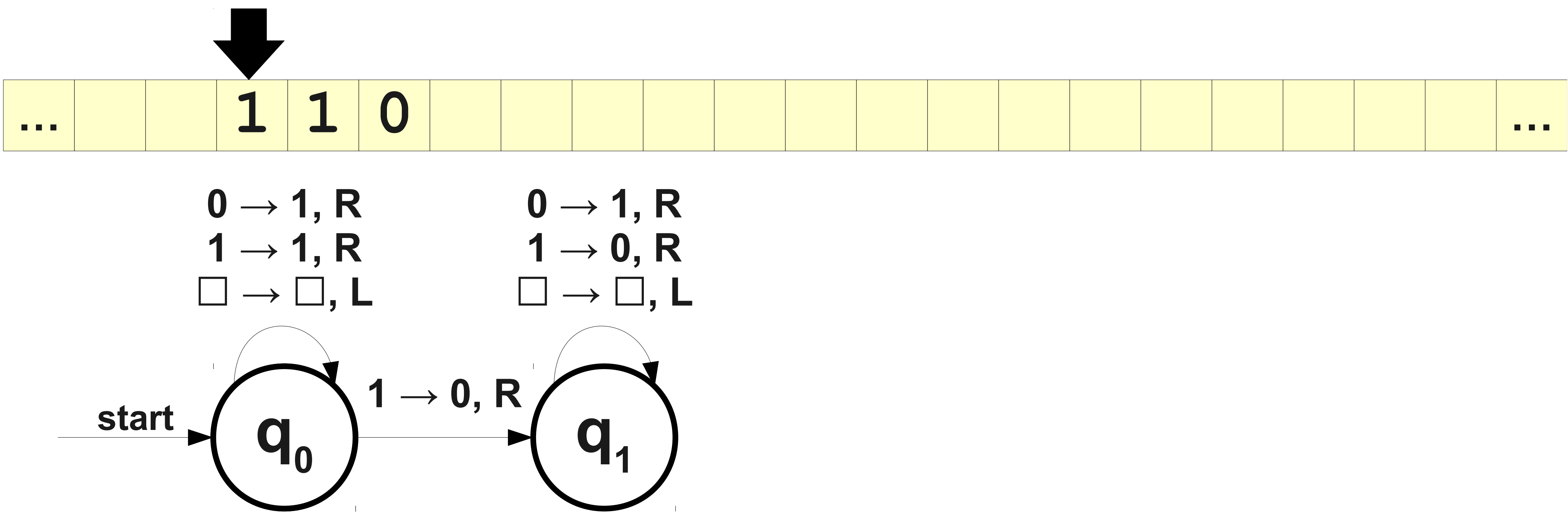
- ***Proof sketch:*** D uses a worklist to exhaustively search over N's computation tree.



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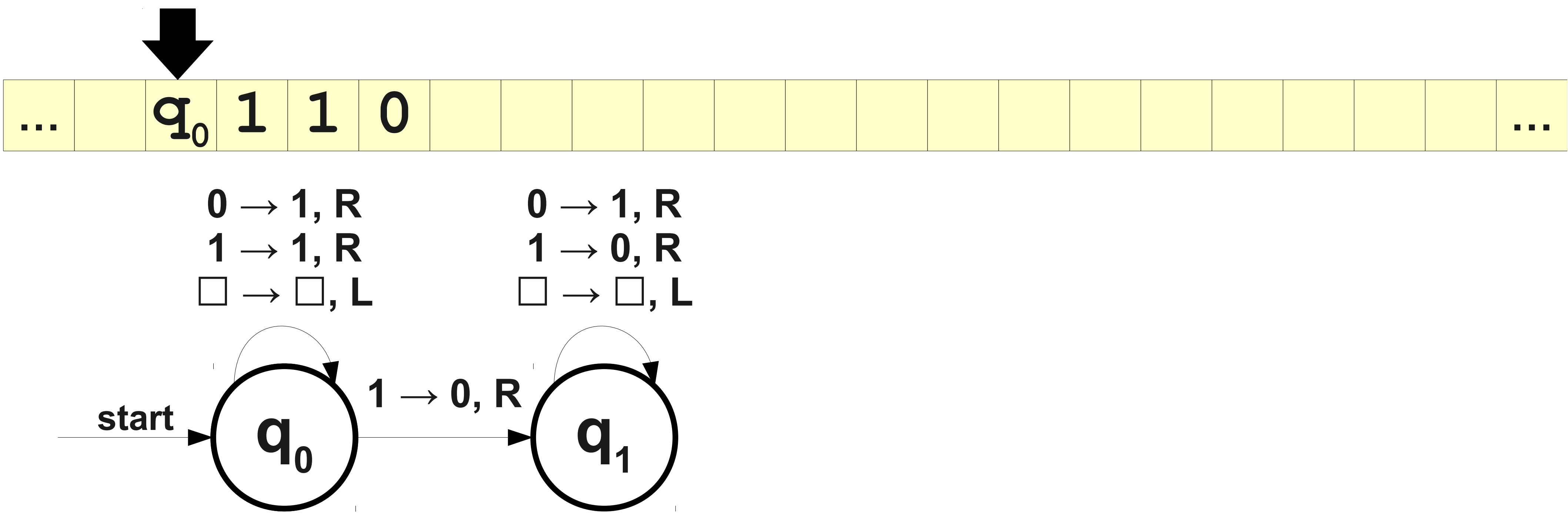

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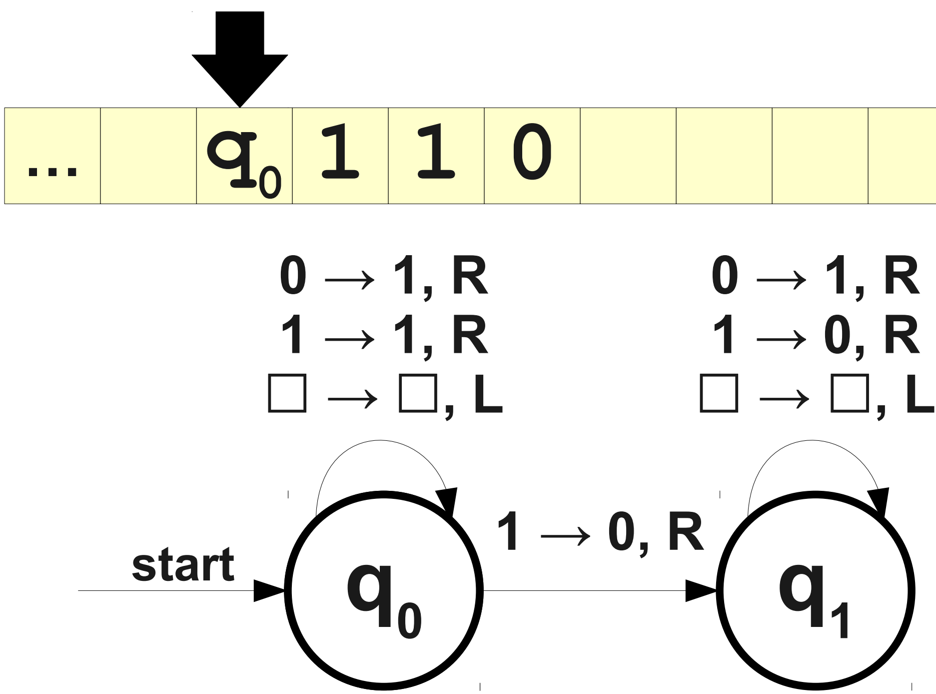

The NTM we're simulating has at most two transitions to choose from. Our DTM therefore makes two copies of the current ID and simulates one step on each of them.

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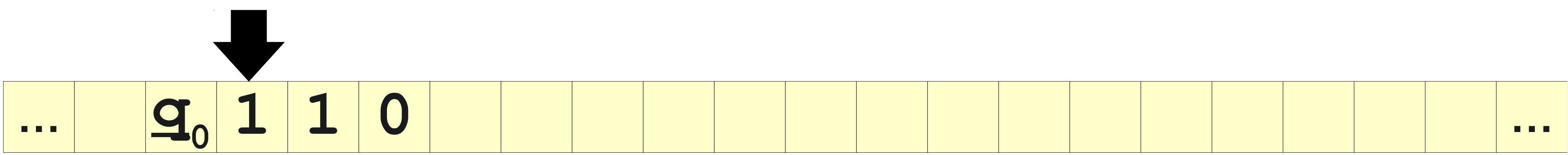

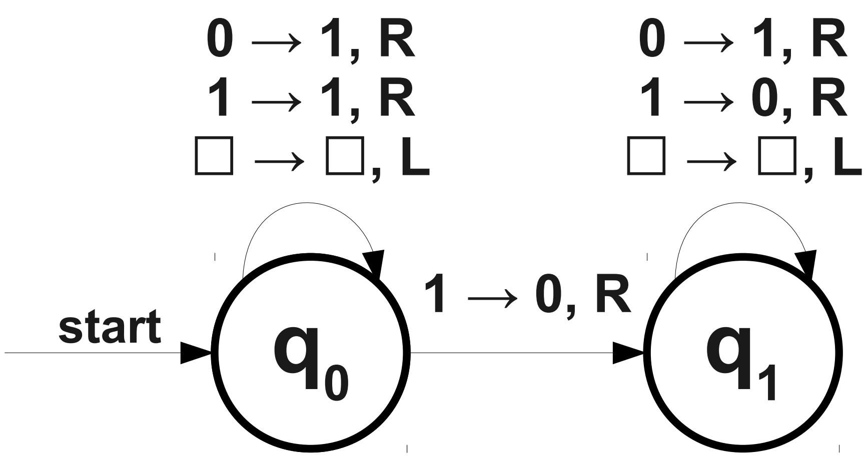

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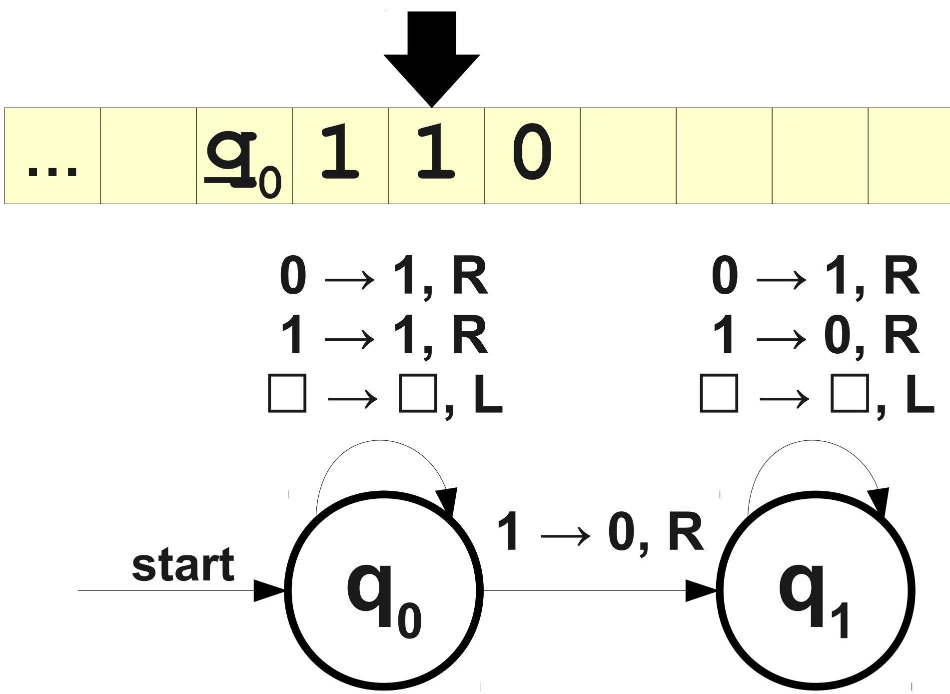

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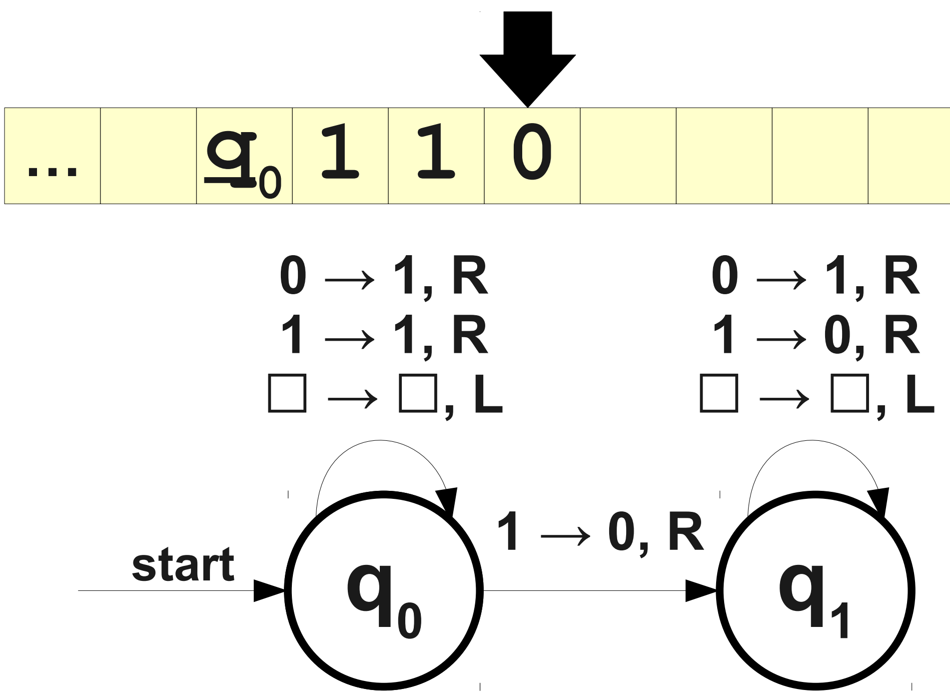

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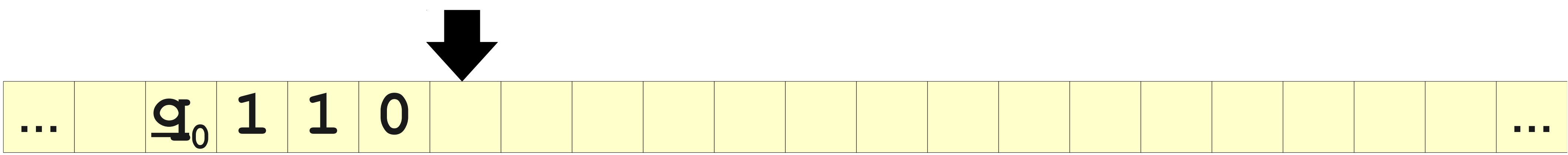

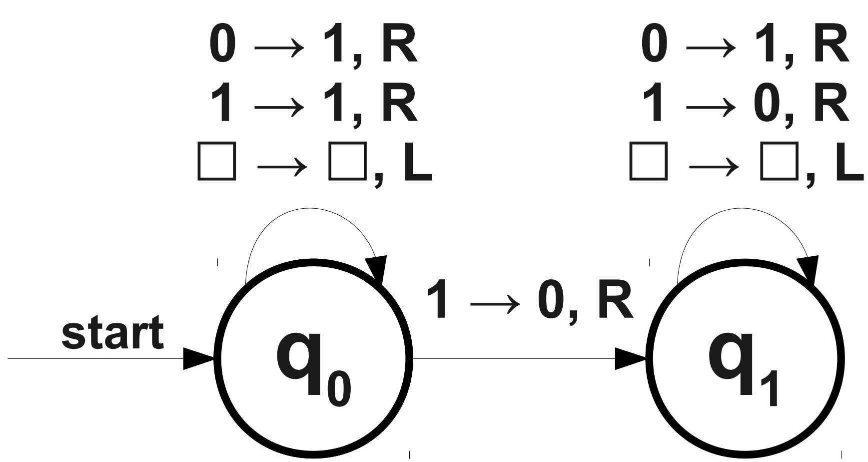

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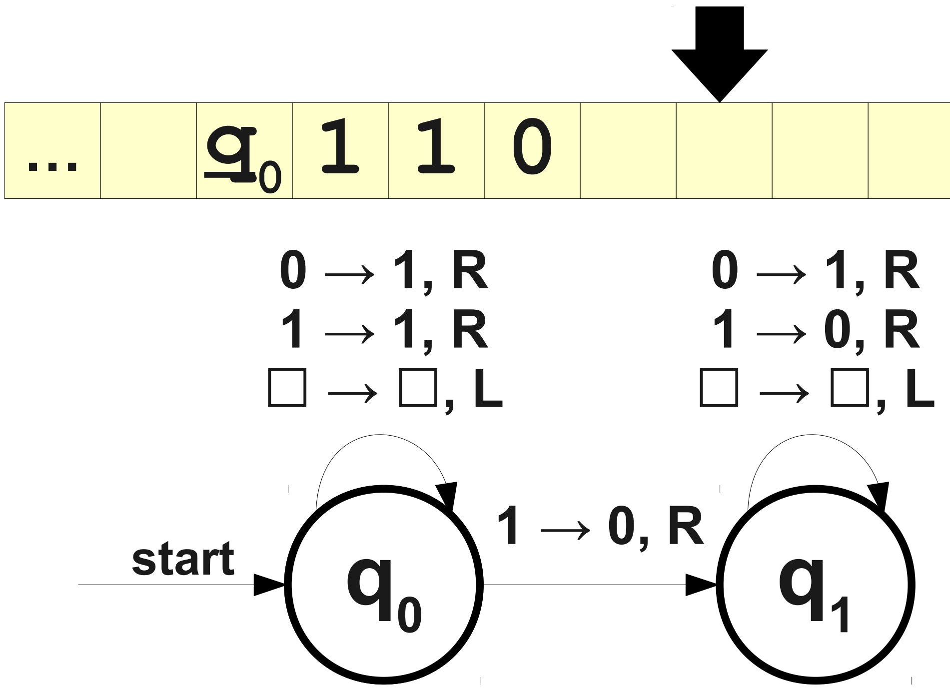

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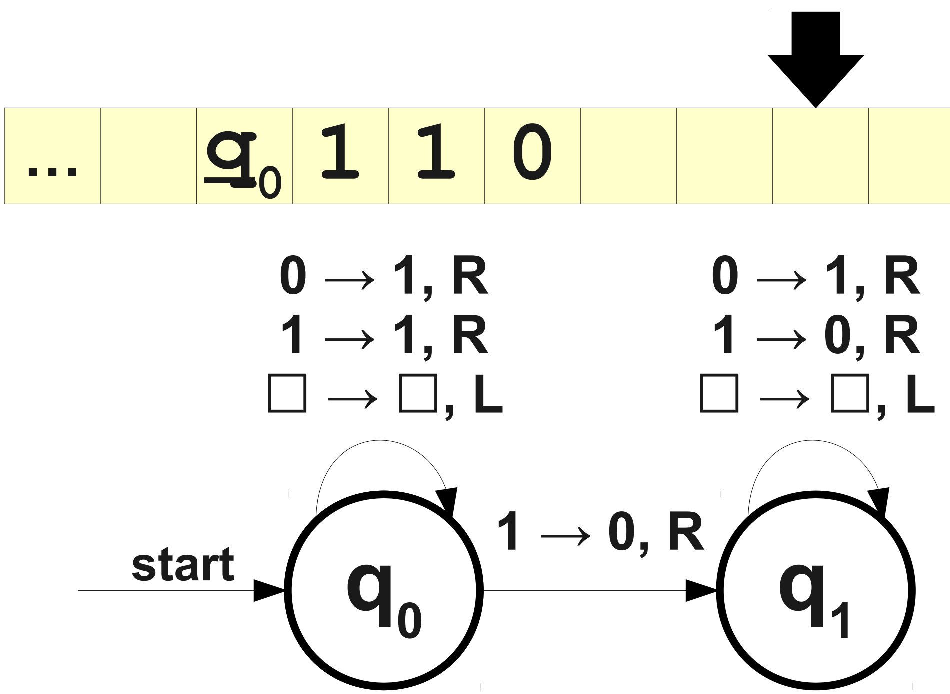

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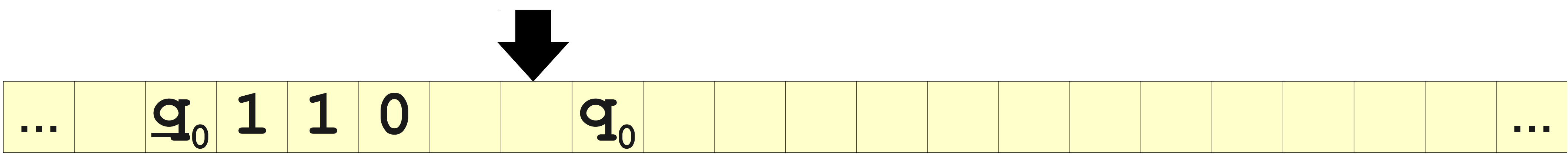

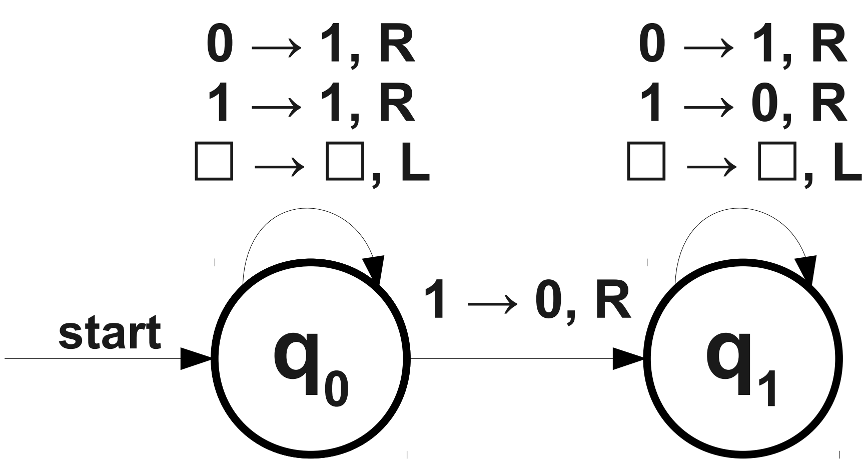

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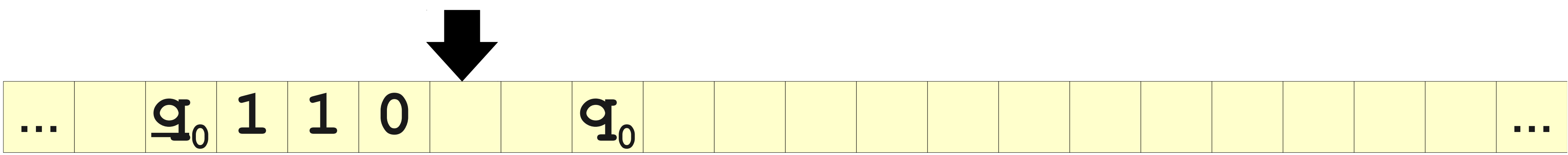

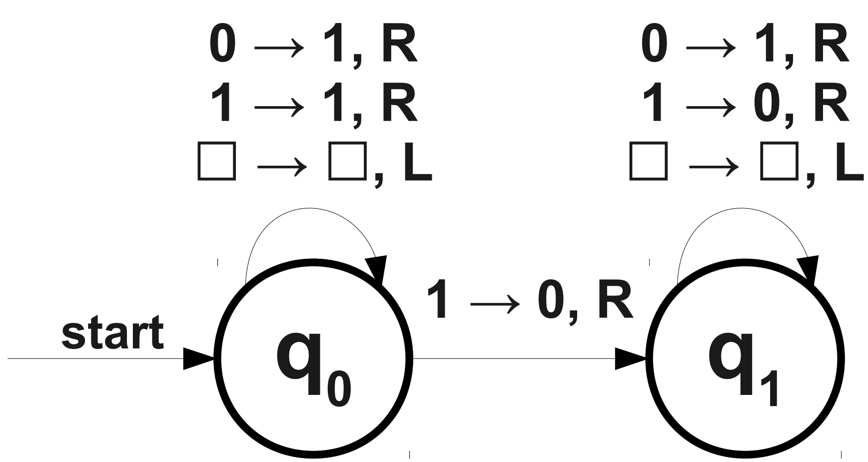

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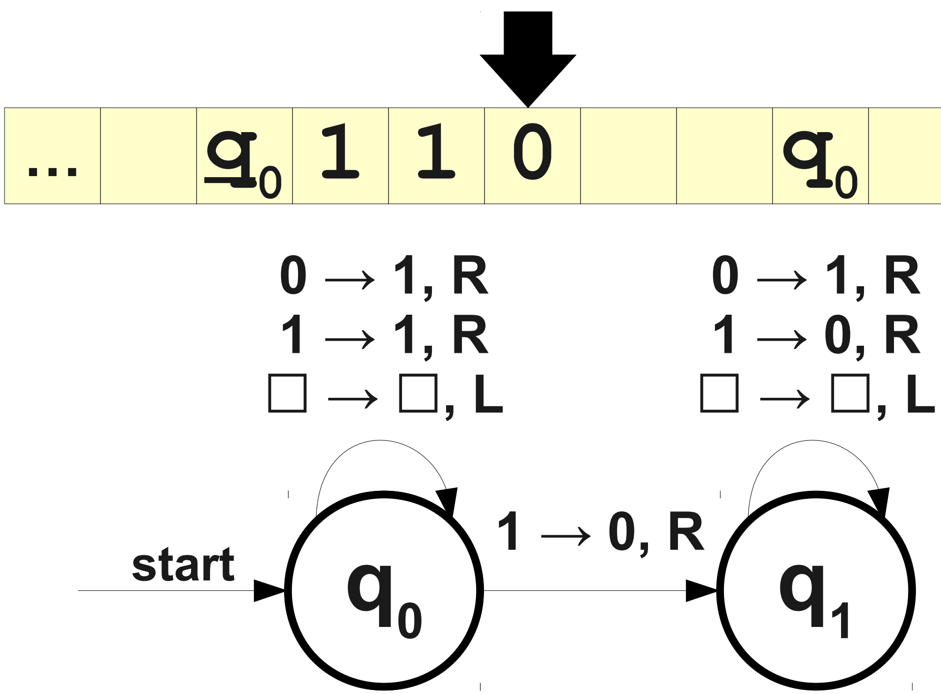

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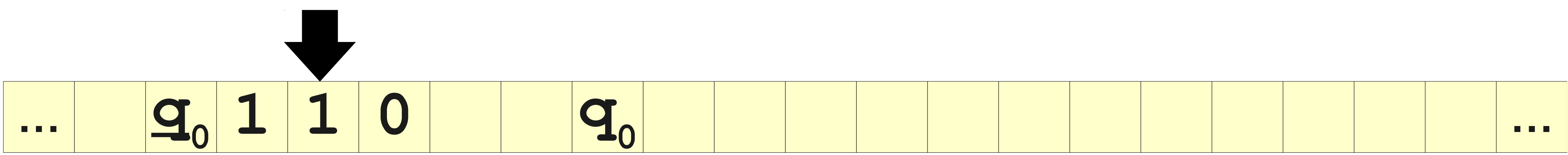

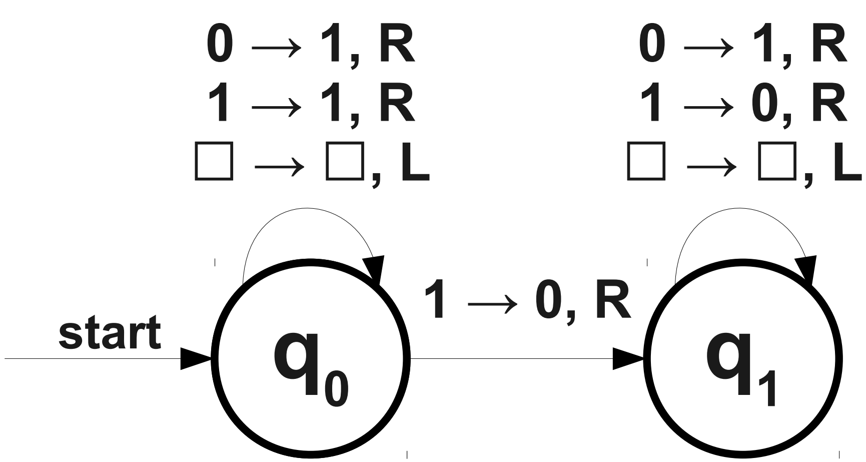

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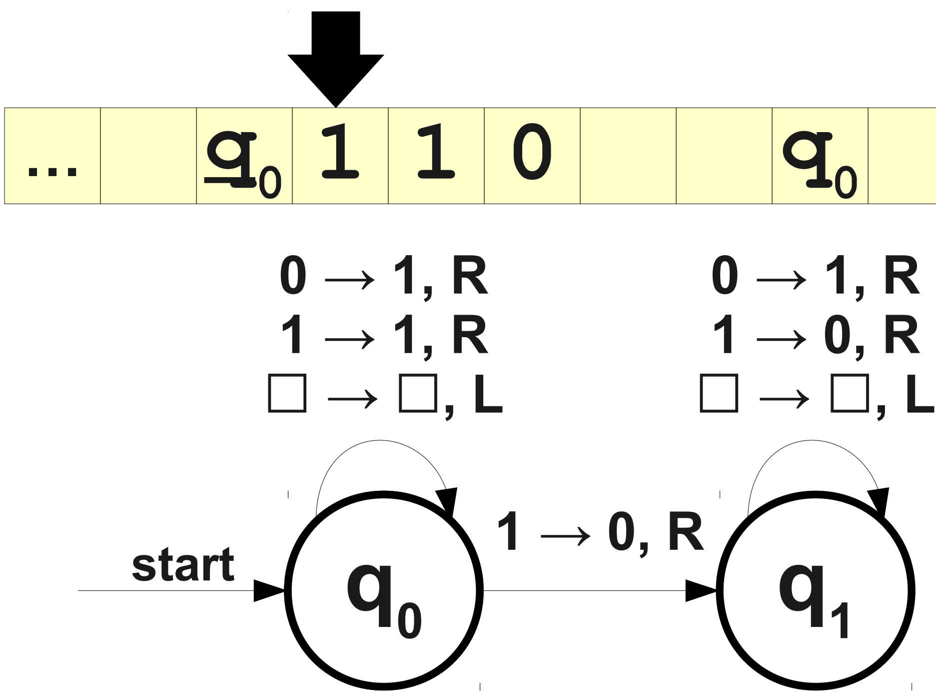

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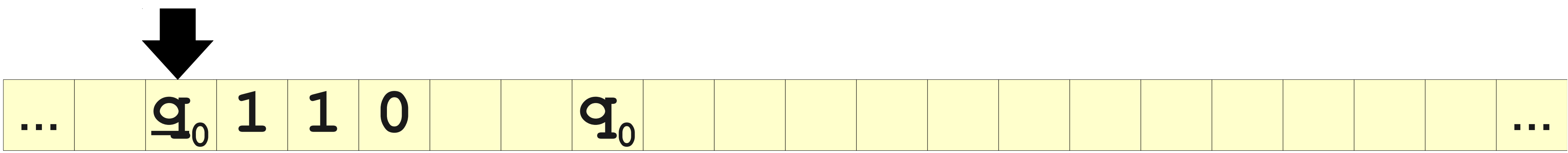

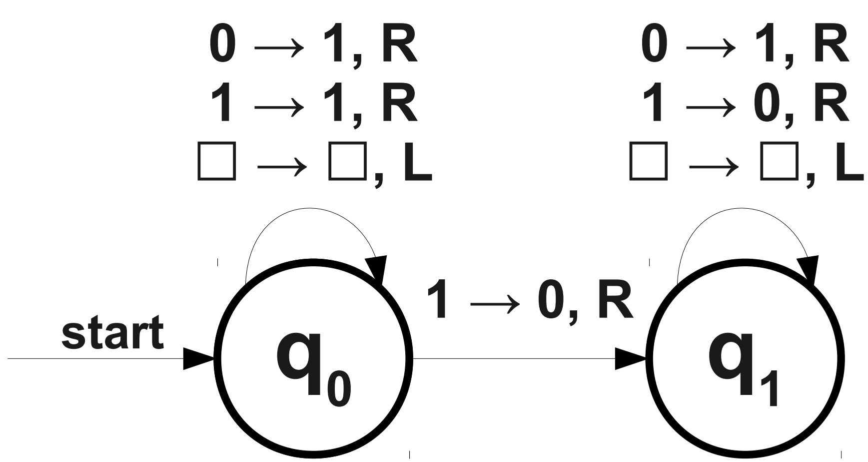

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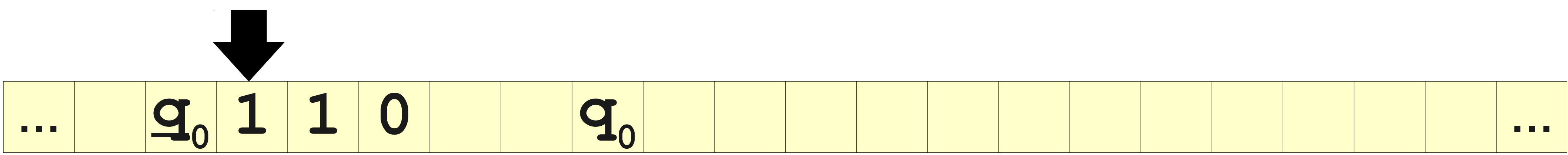

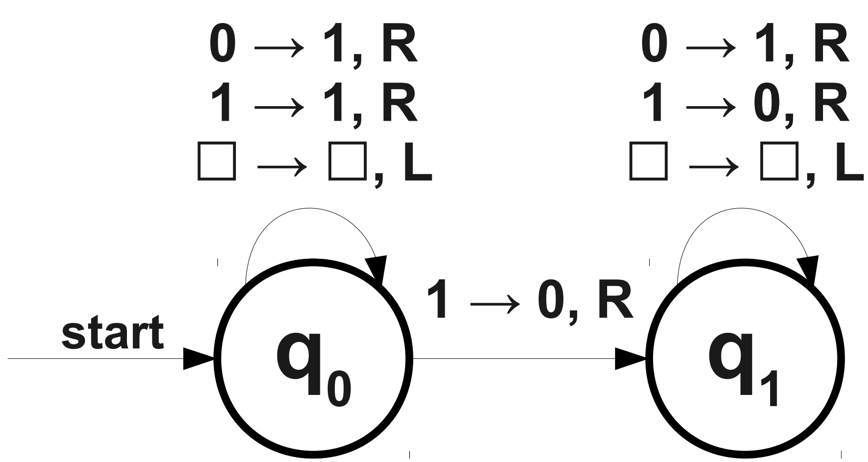

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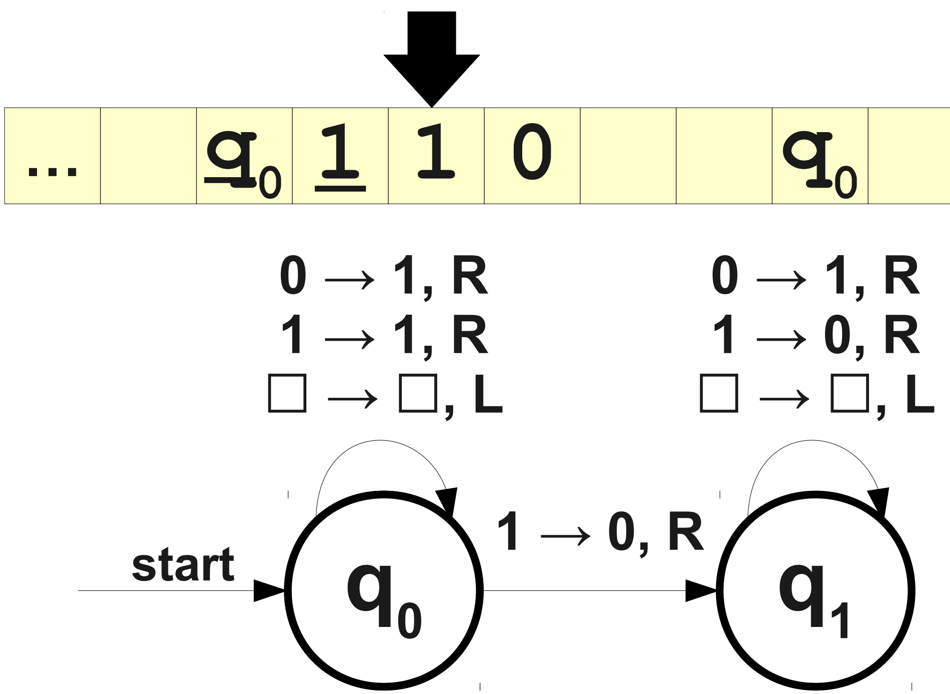

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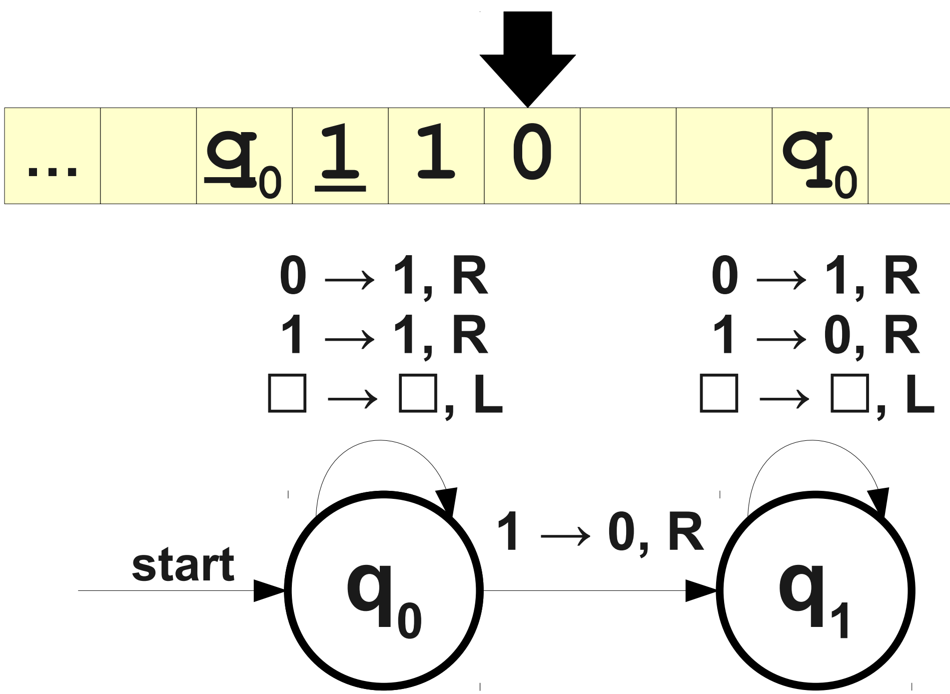

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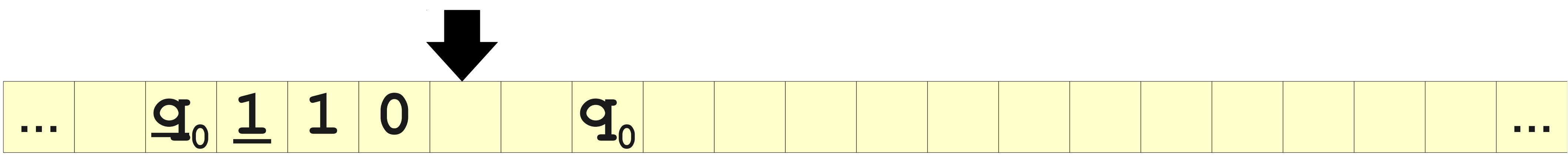

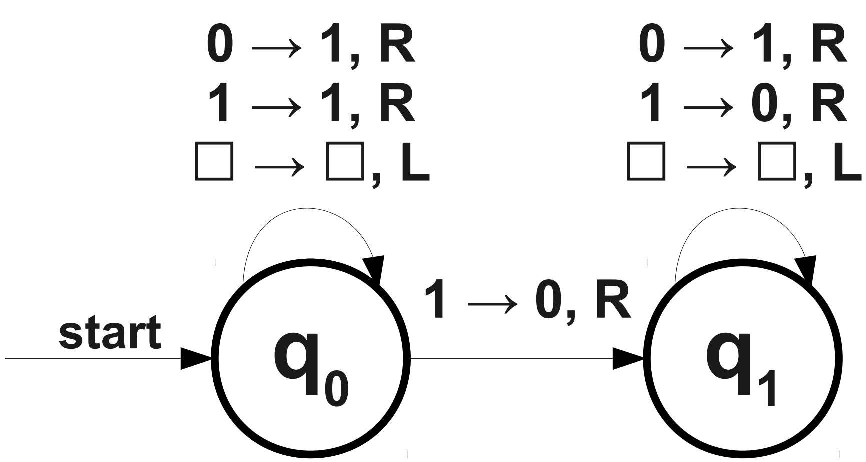

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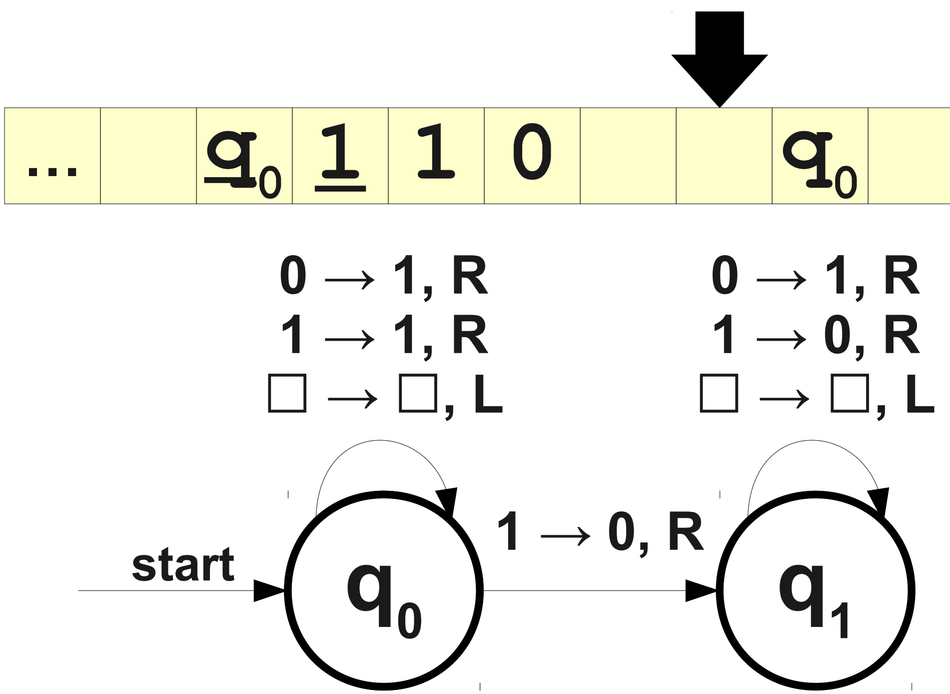

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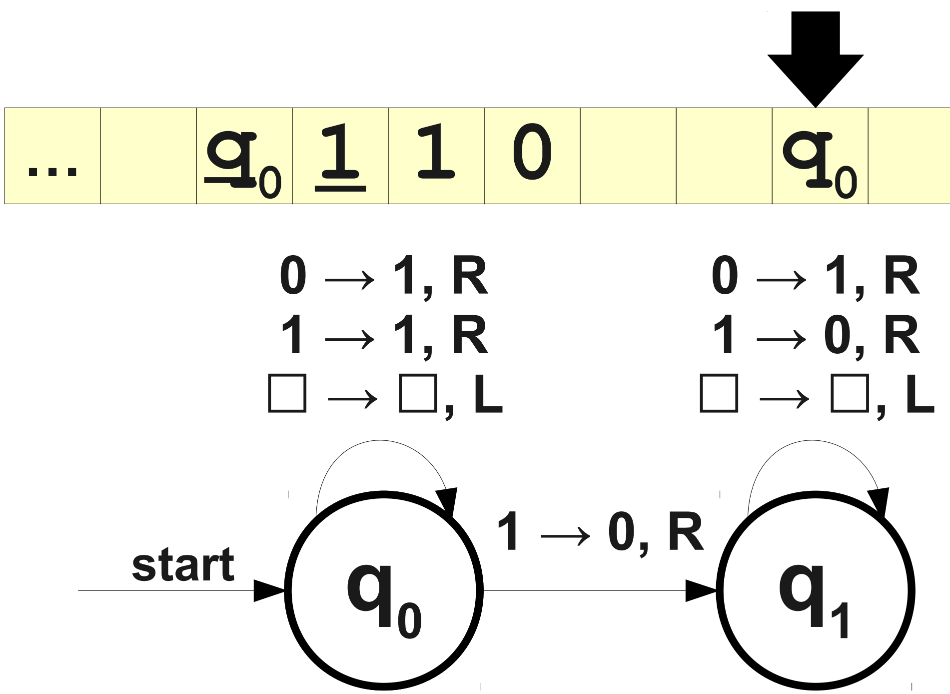

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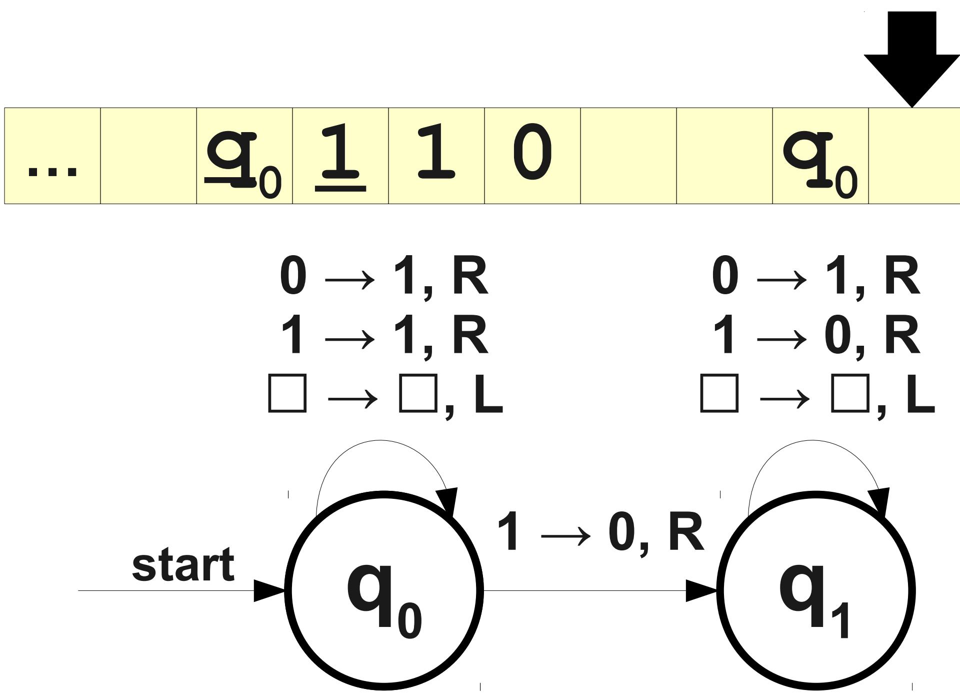

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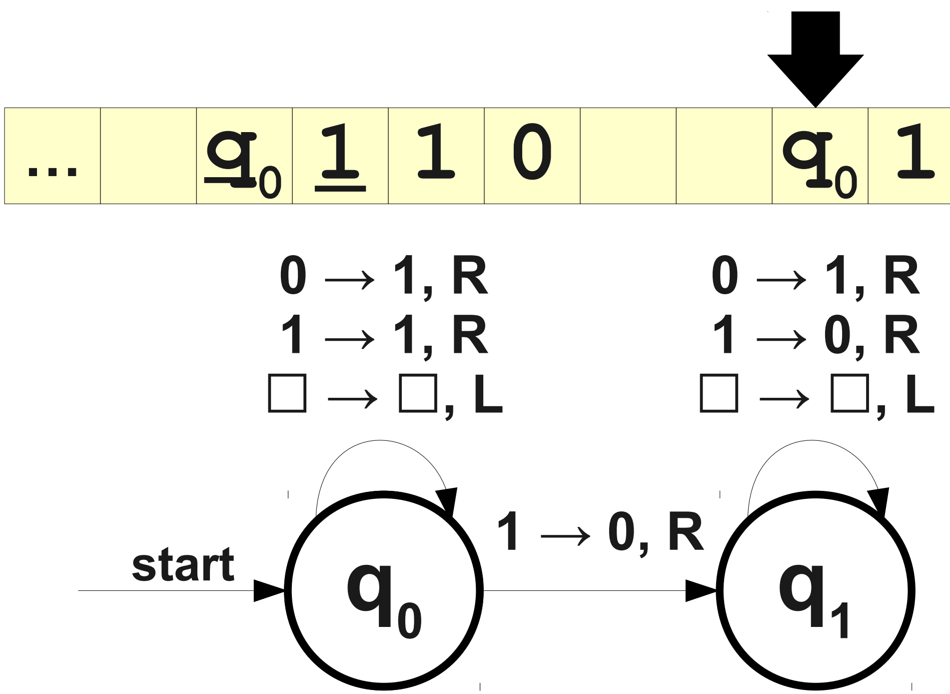

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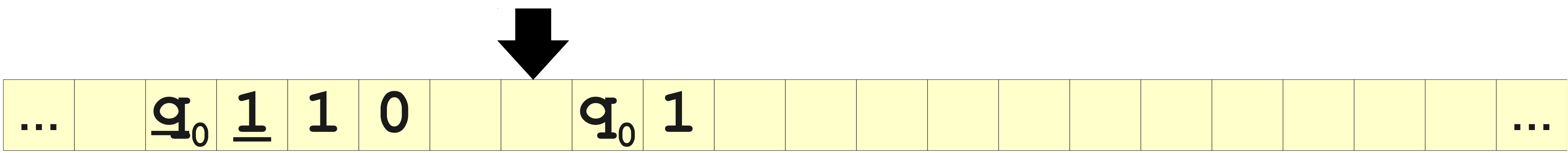

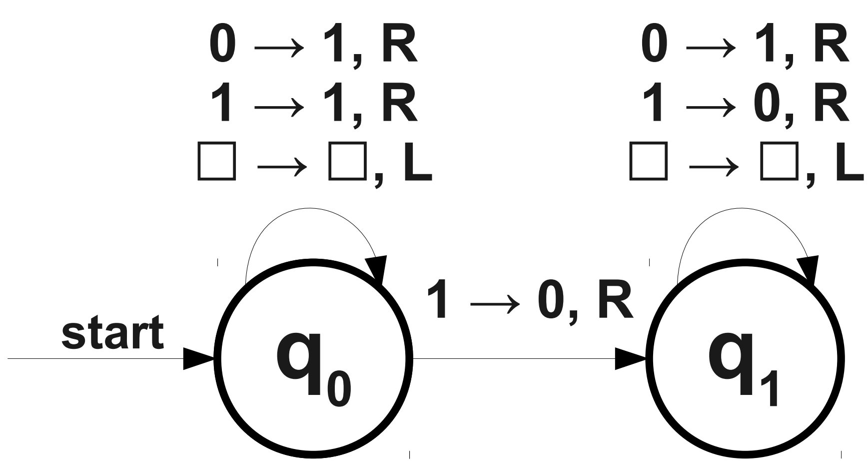

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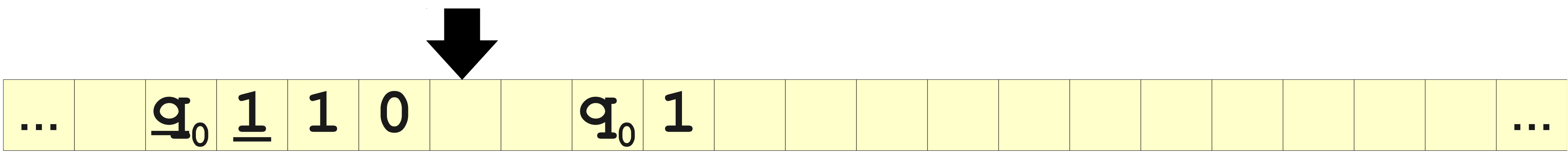

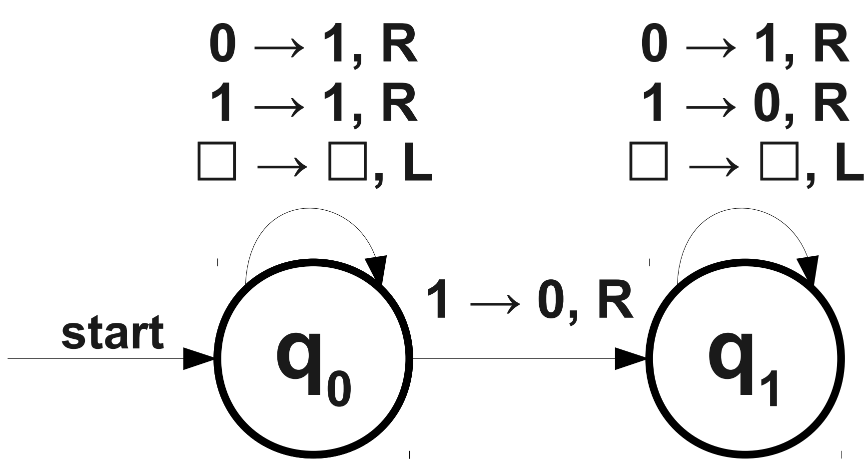

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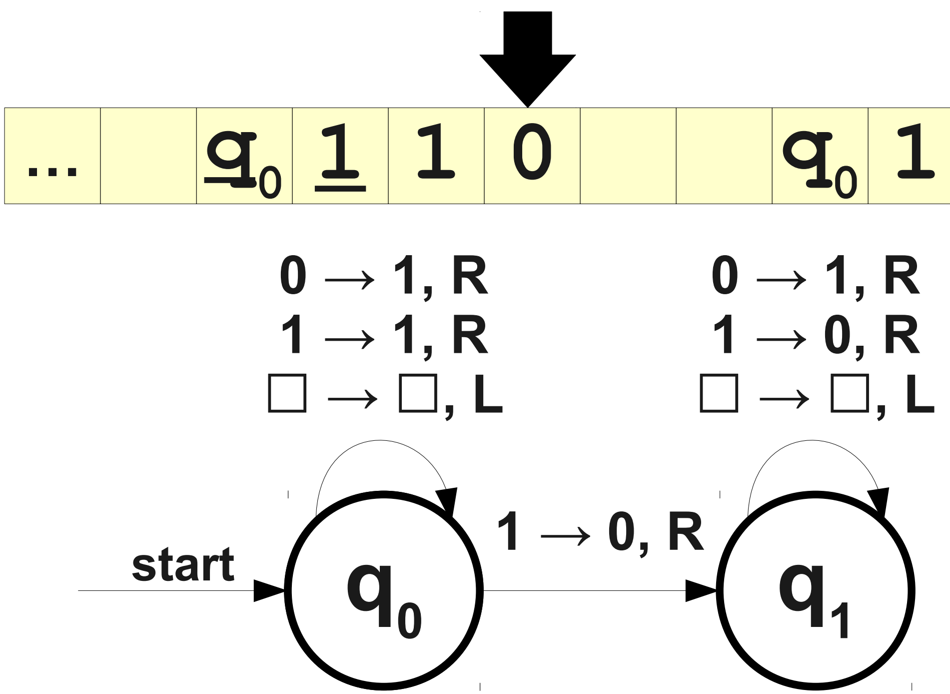

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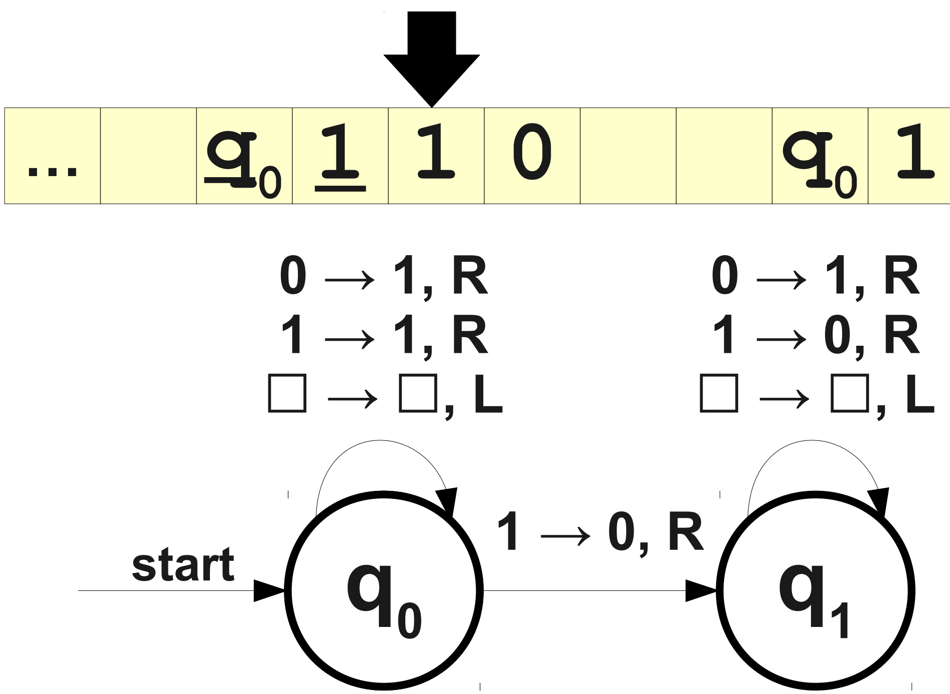

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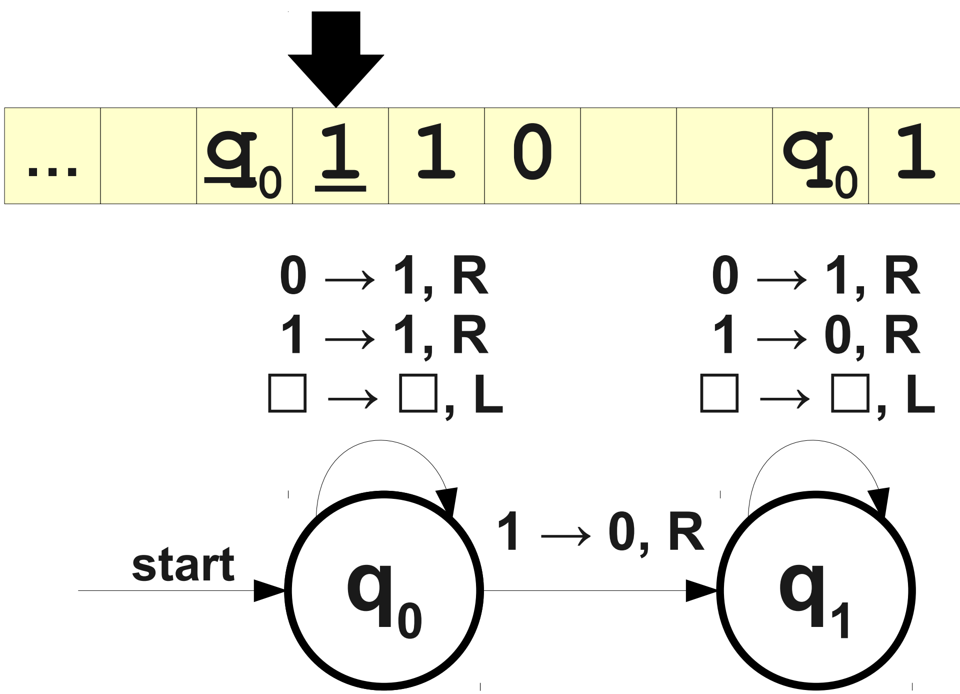

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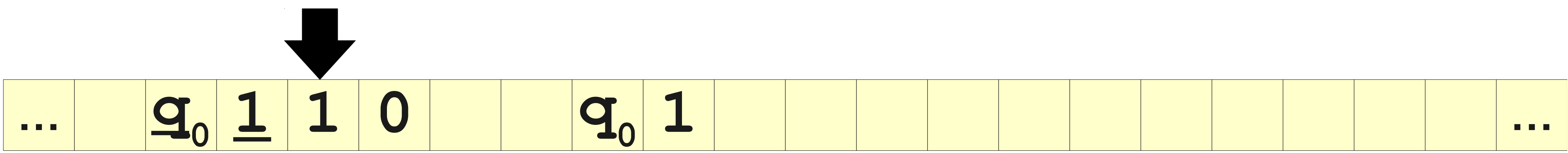

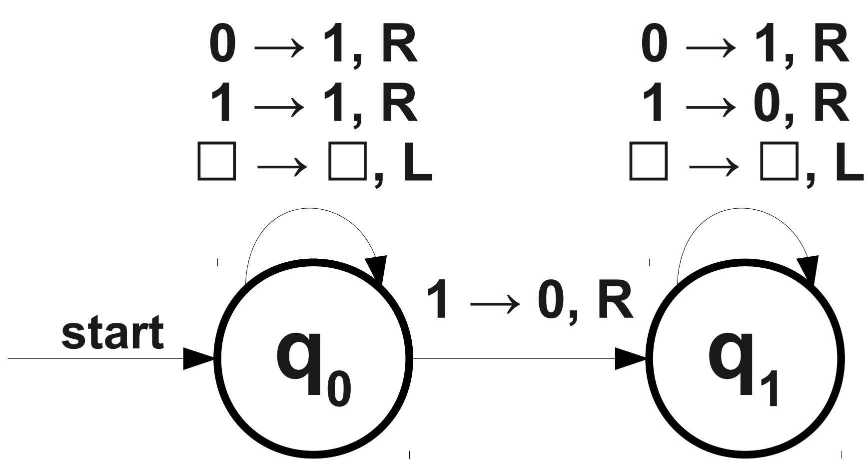

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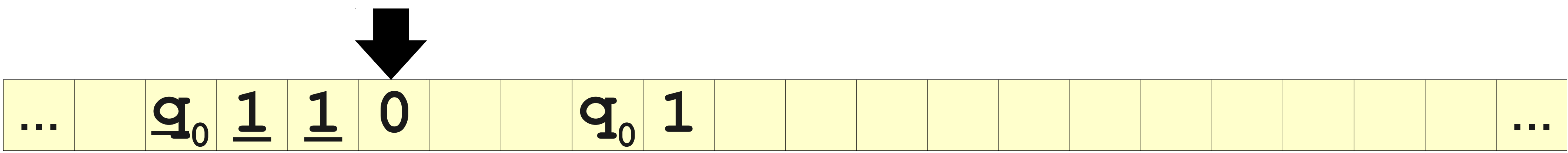

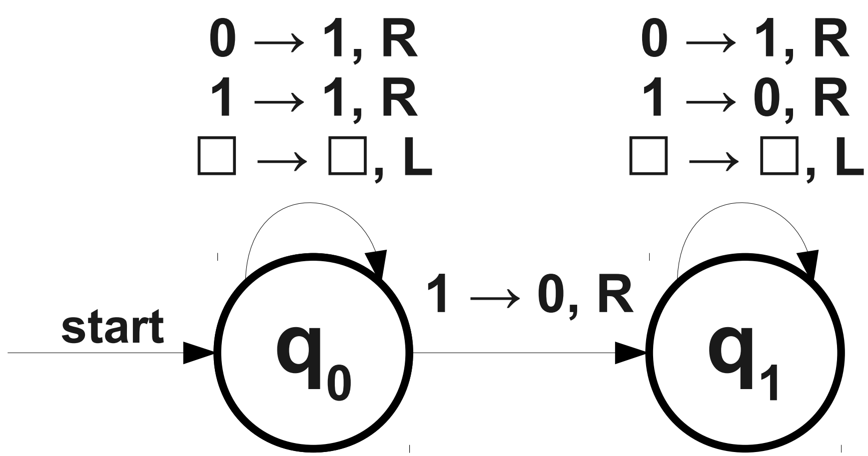

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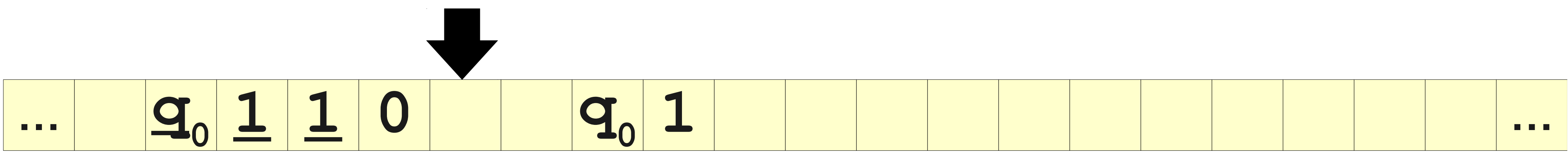

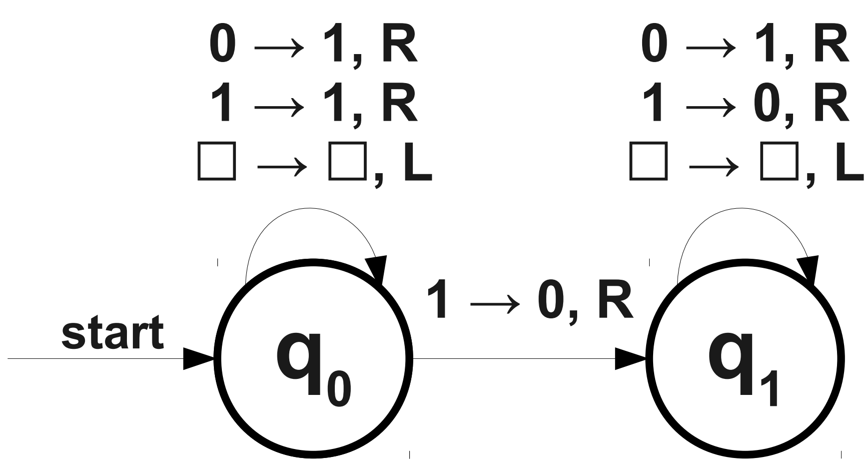

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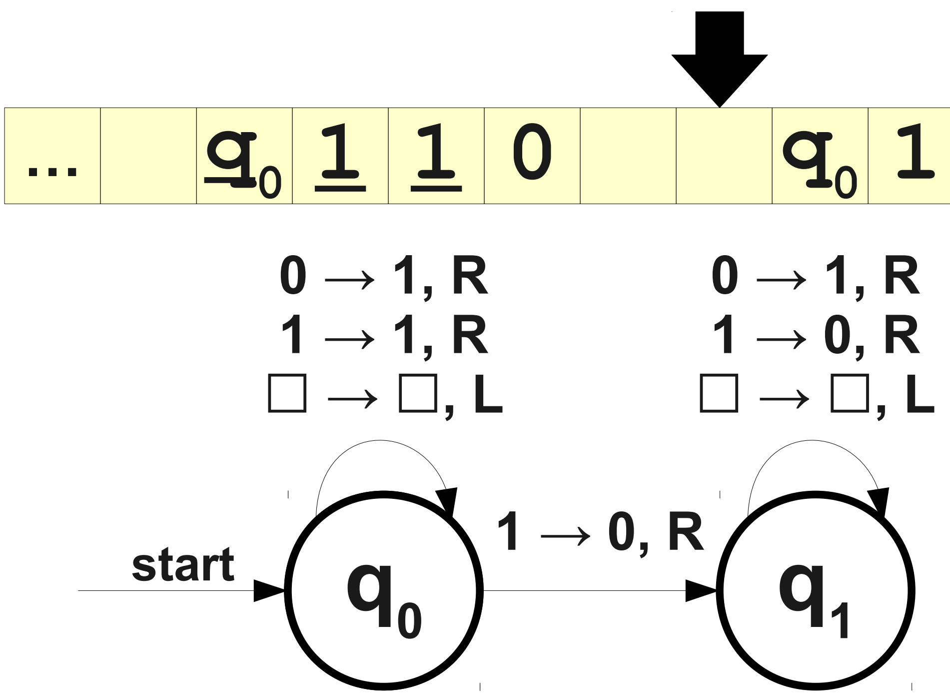

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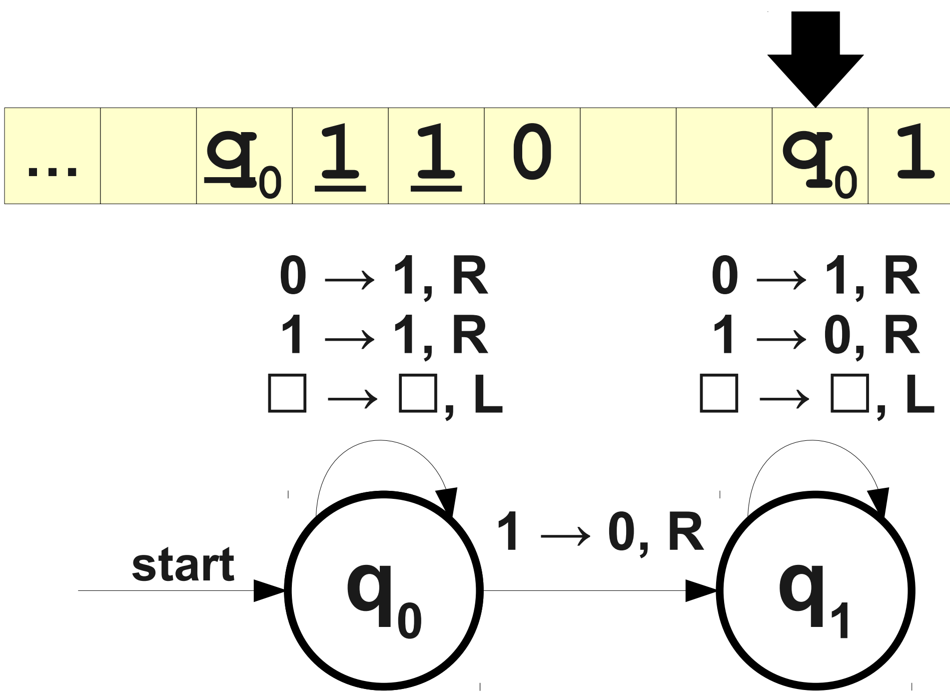

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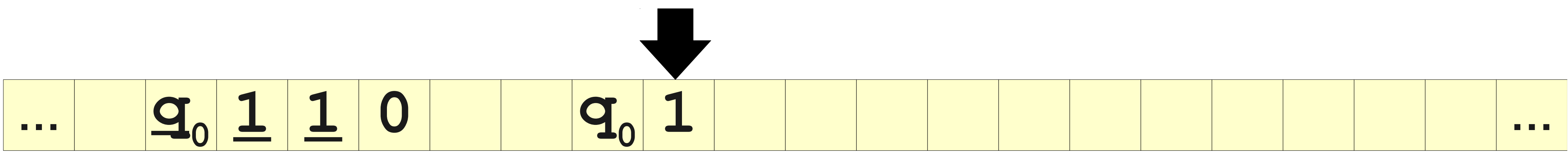

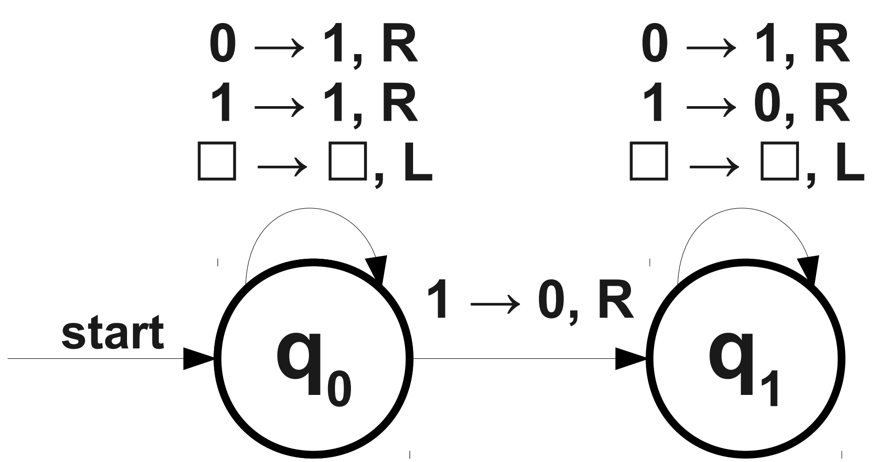

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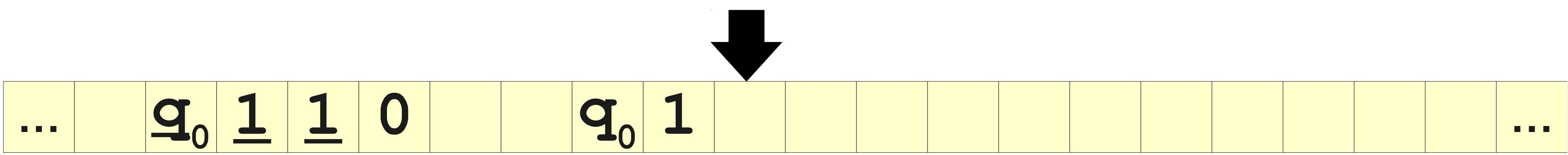

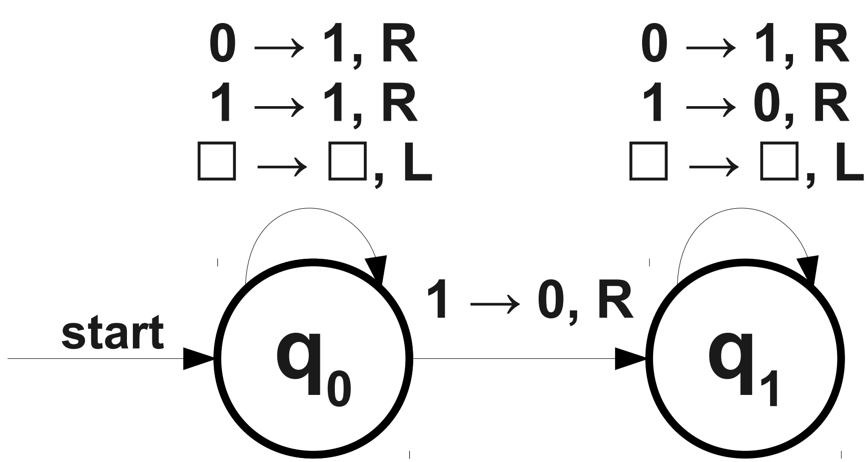

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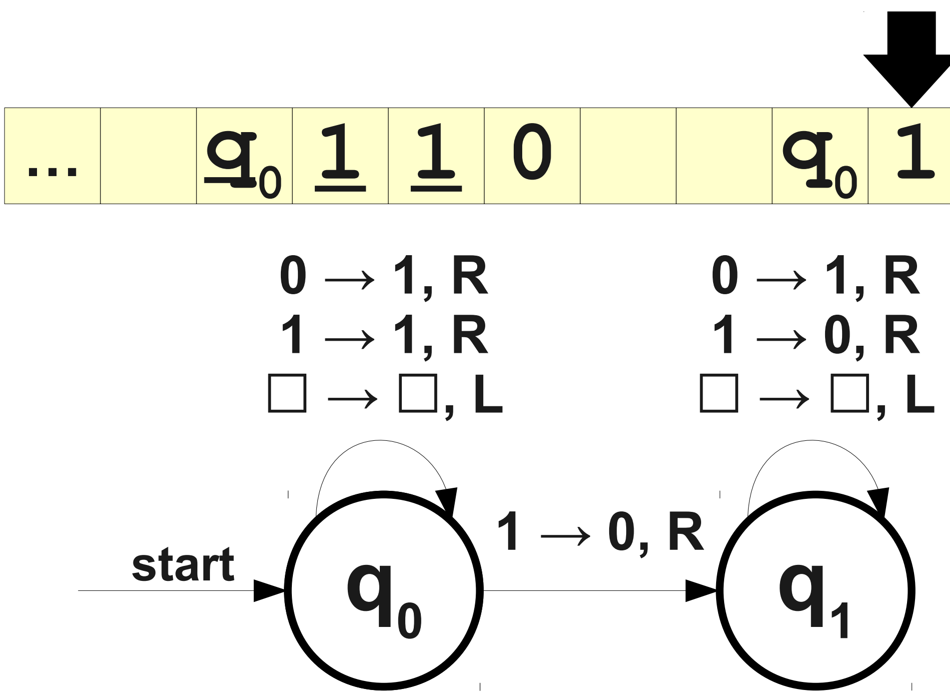

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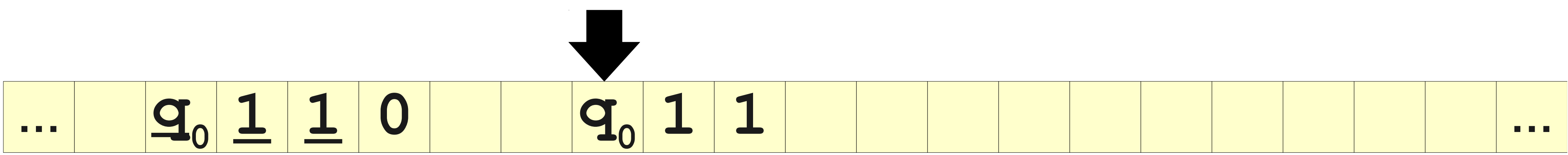

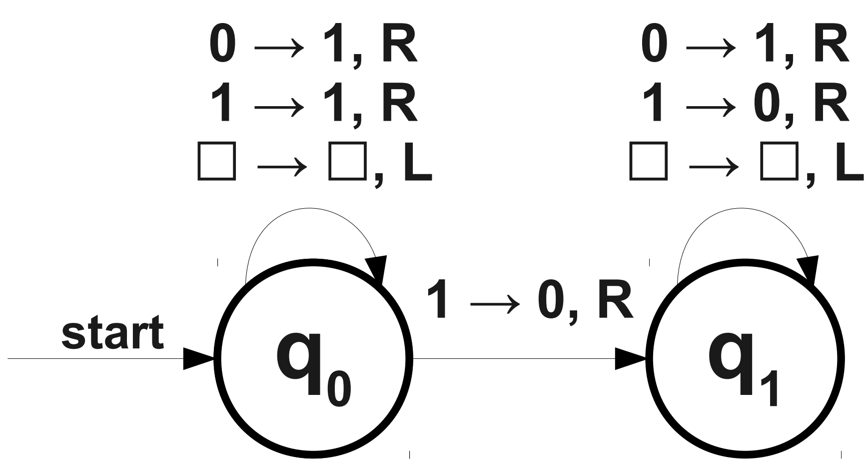

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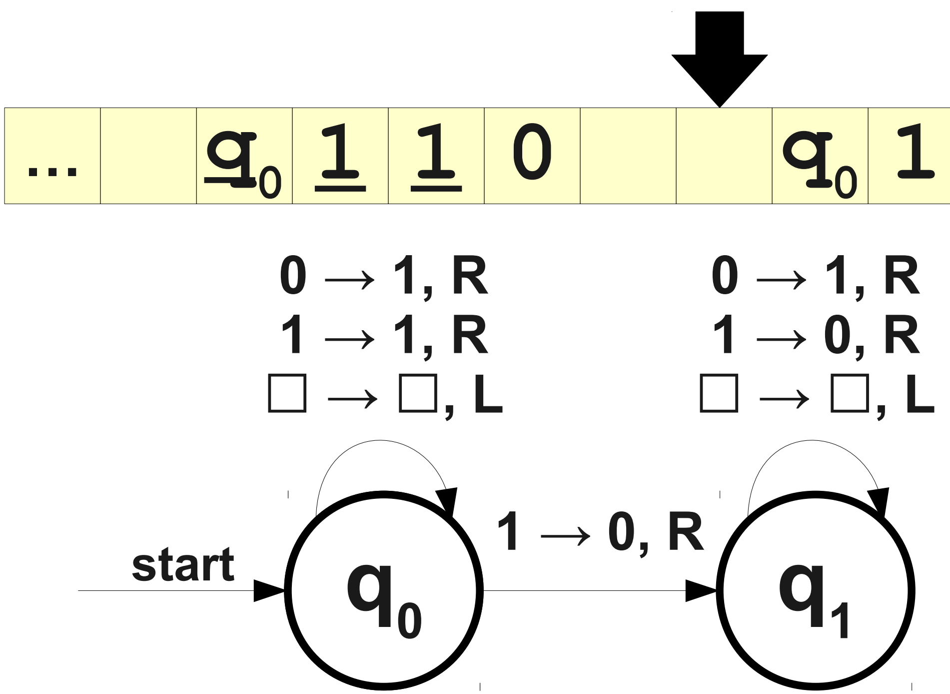

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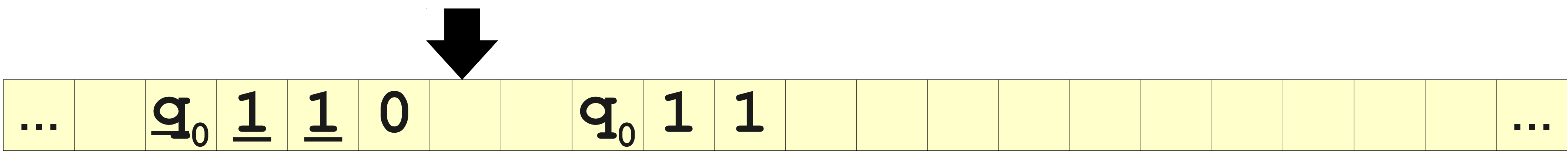

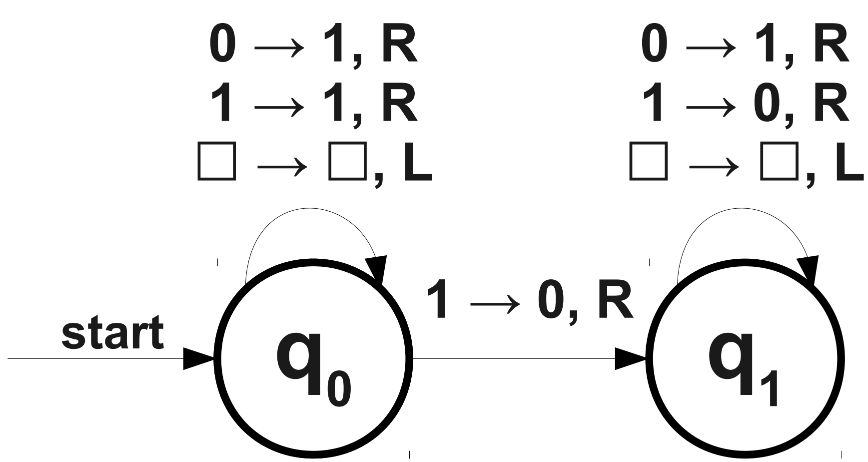

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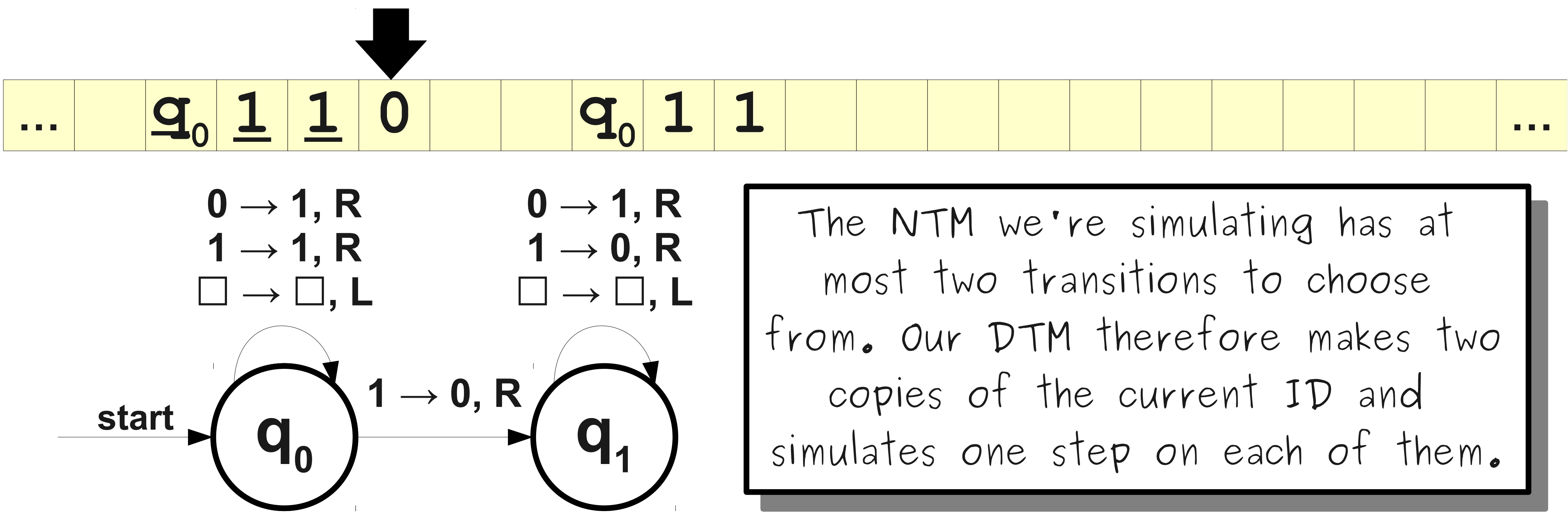

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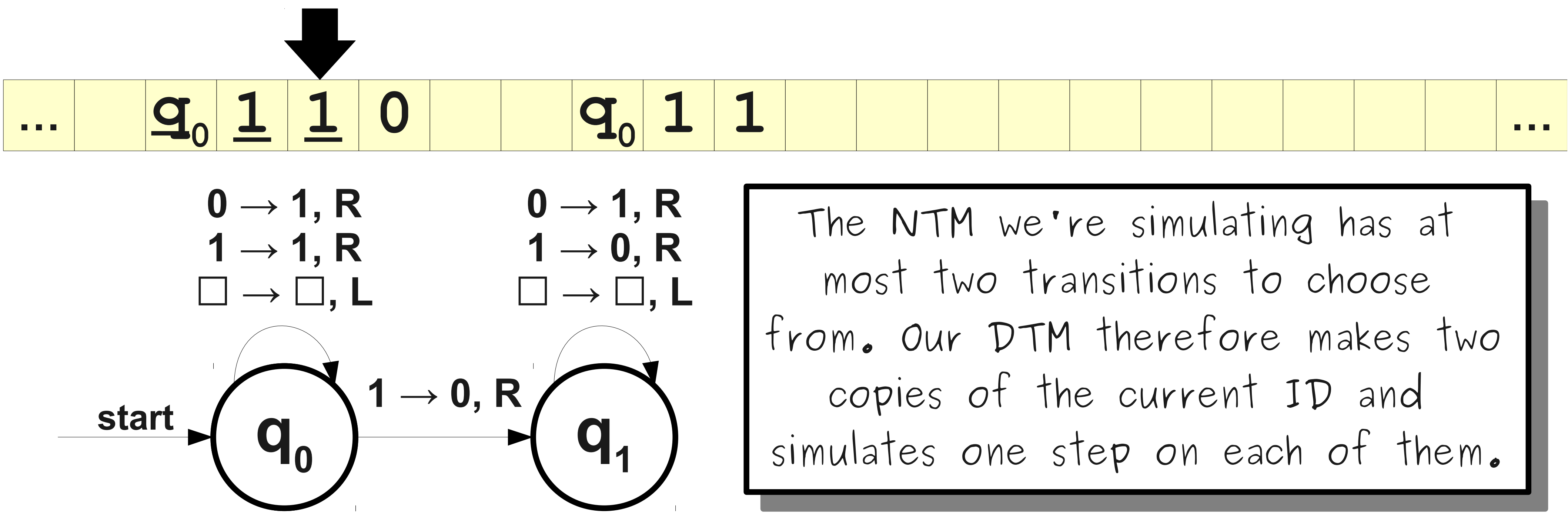

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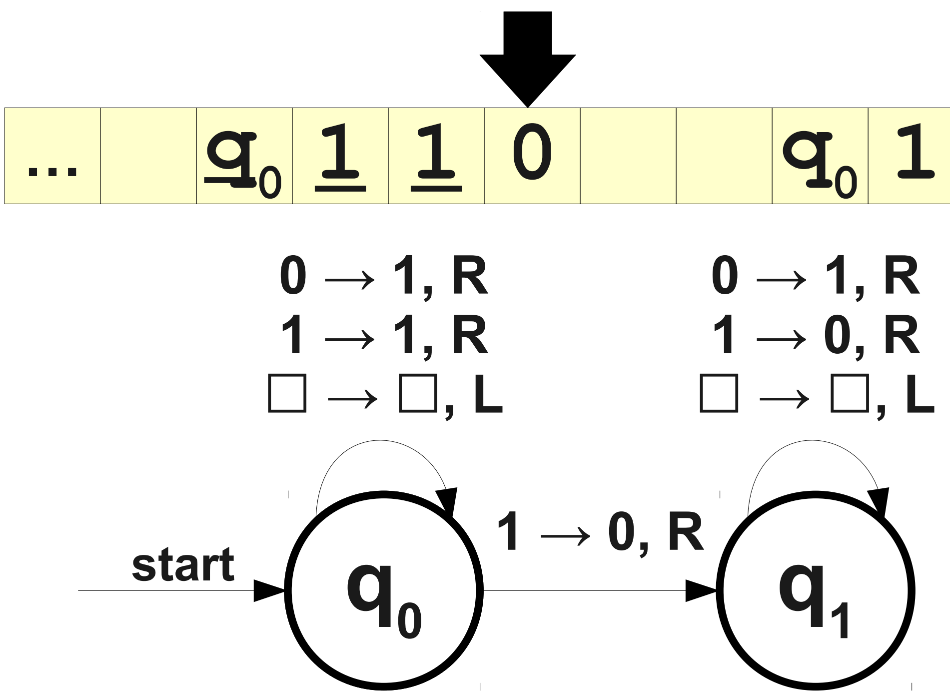

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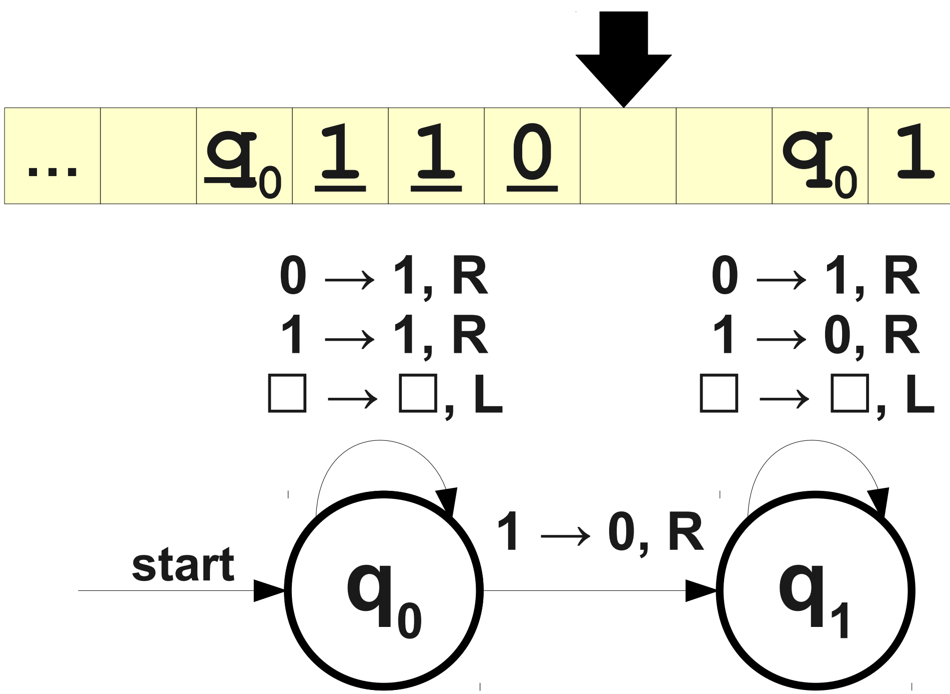

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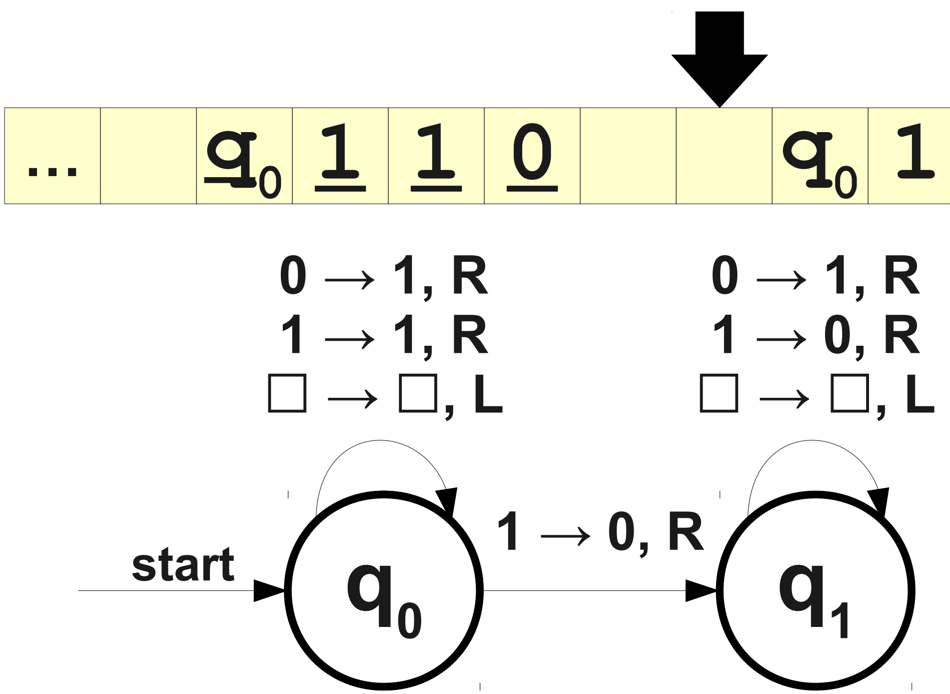

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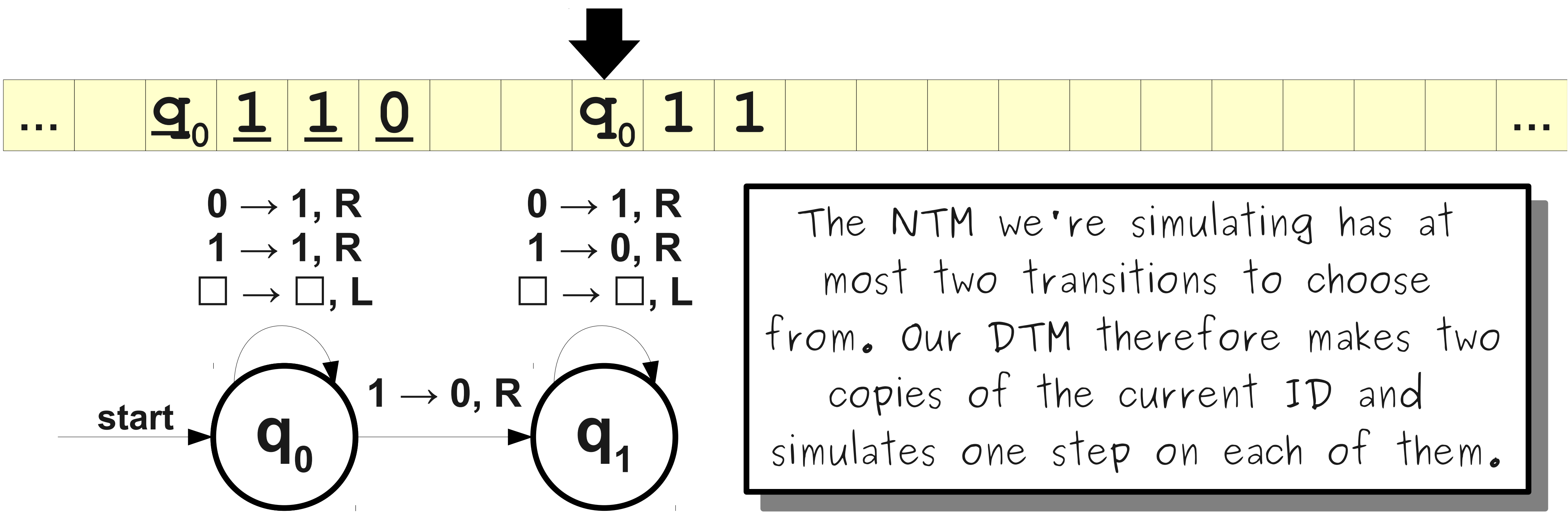

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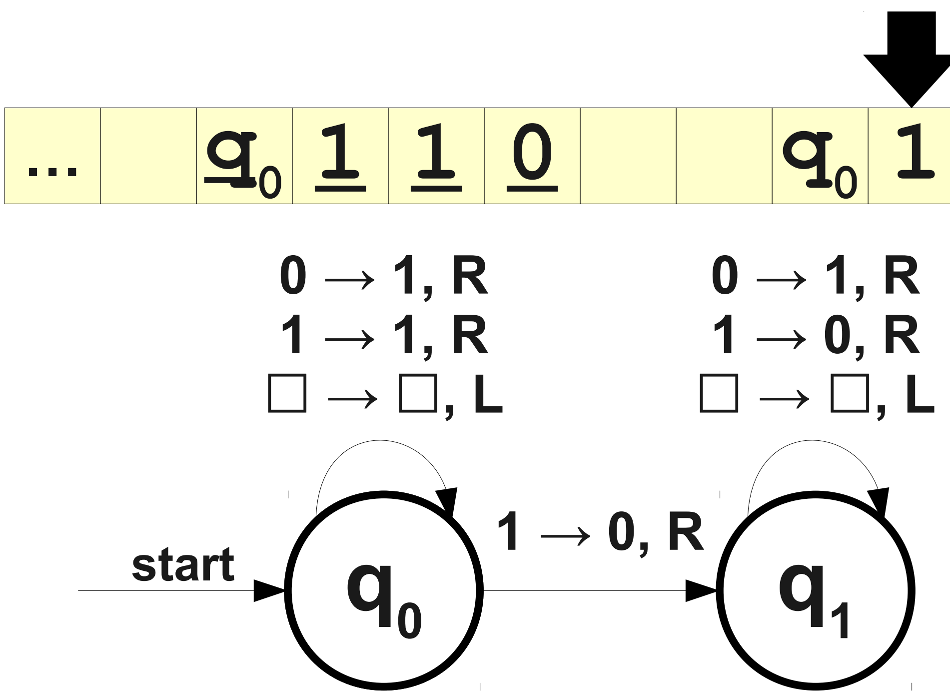

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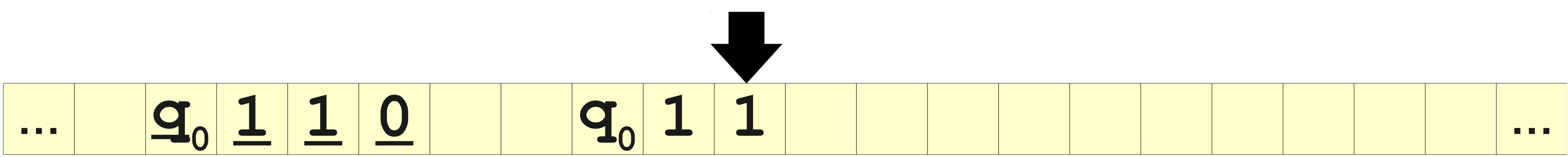

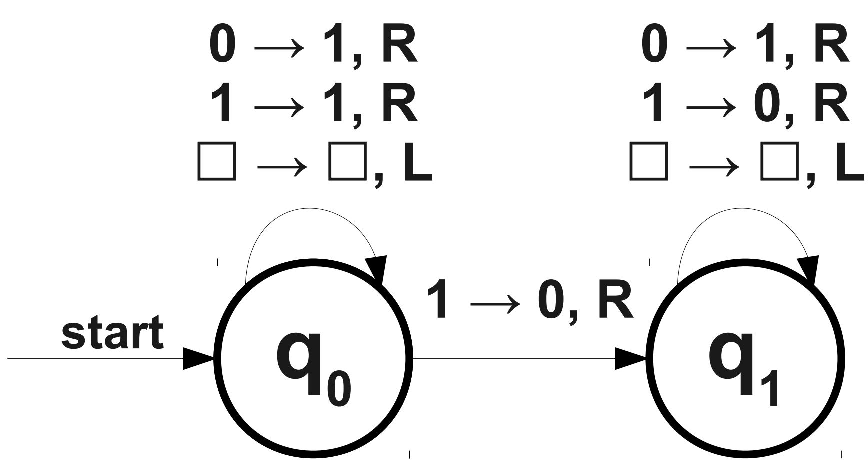

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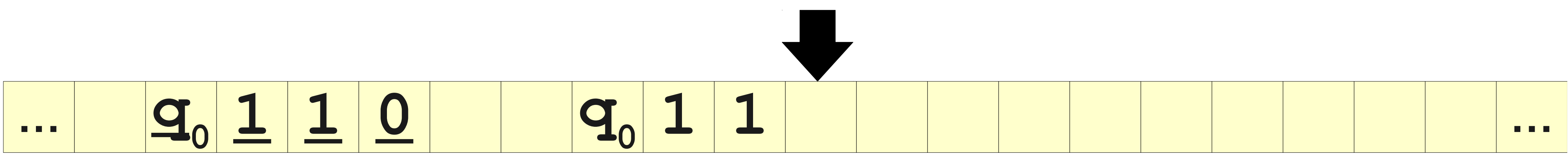

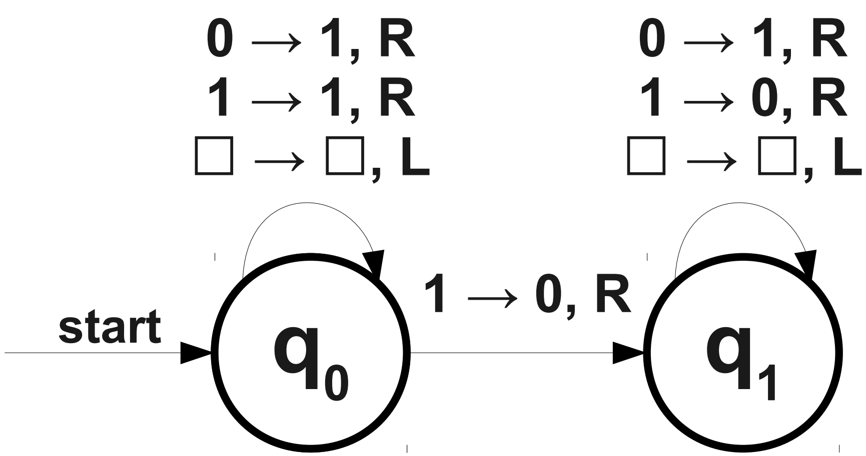

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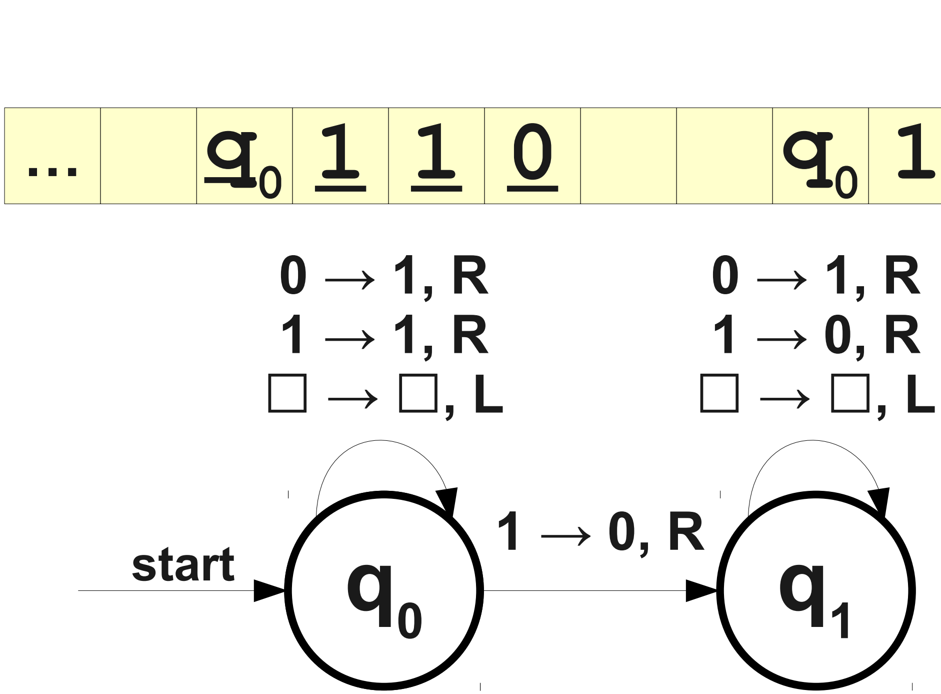

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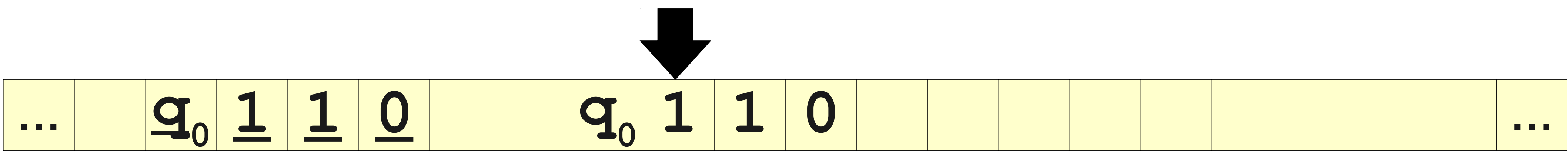

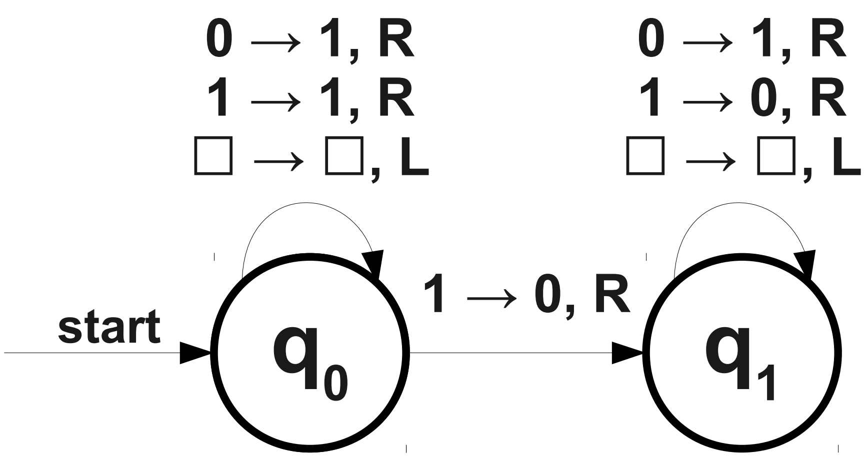

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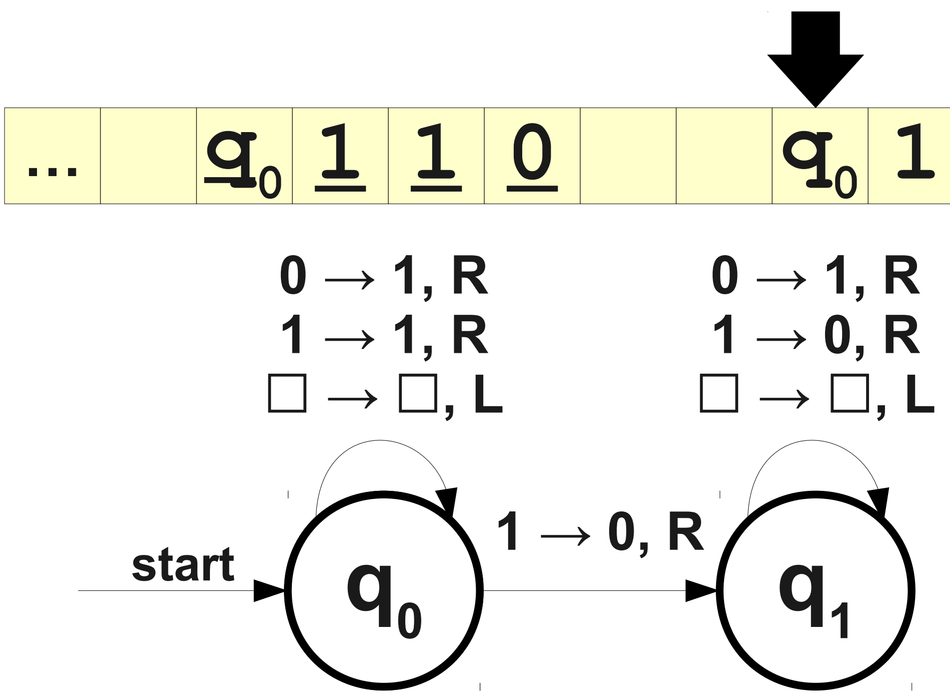

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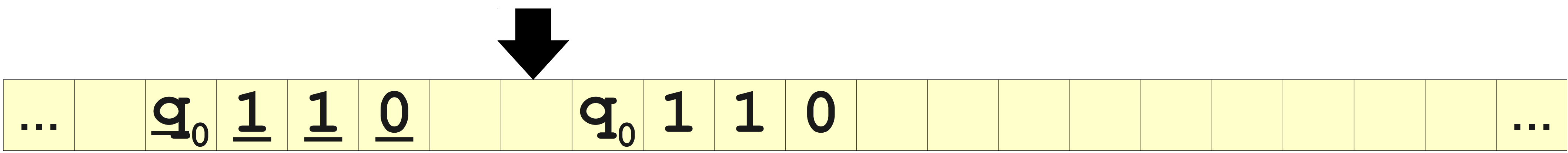

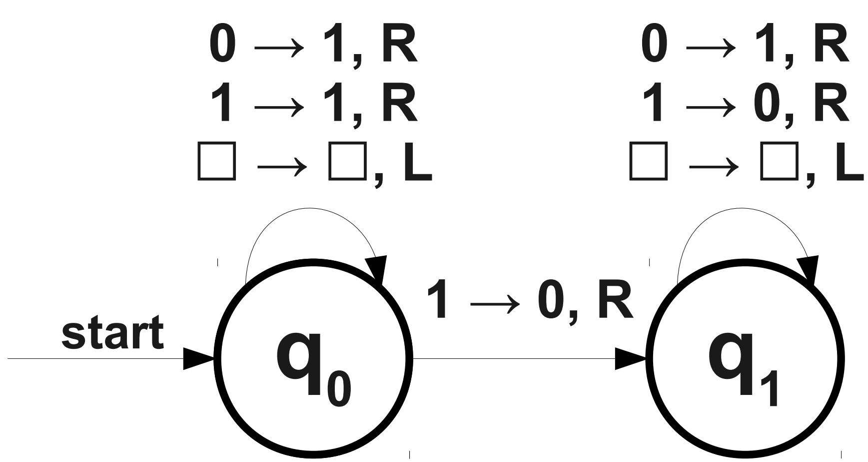

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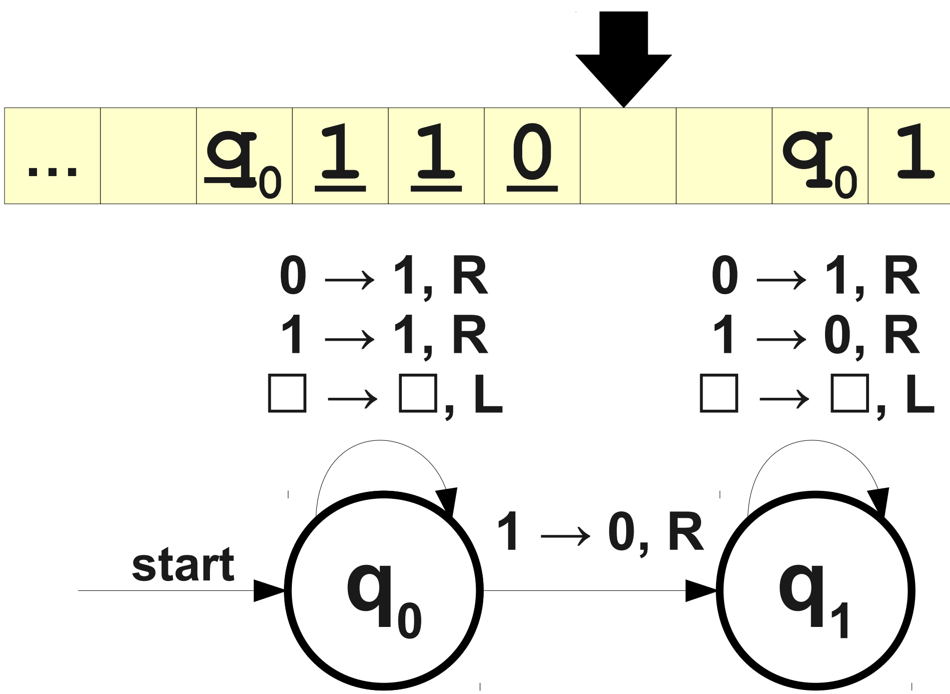

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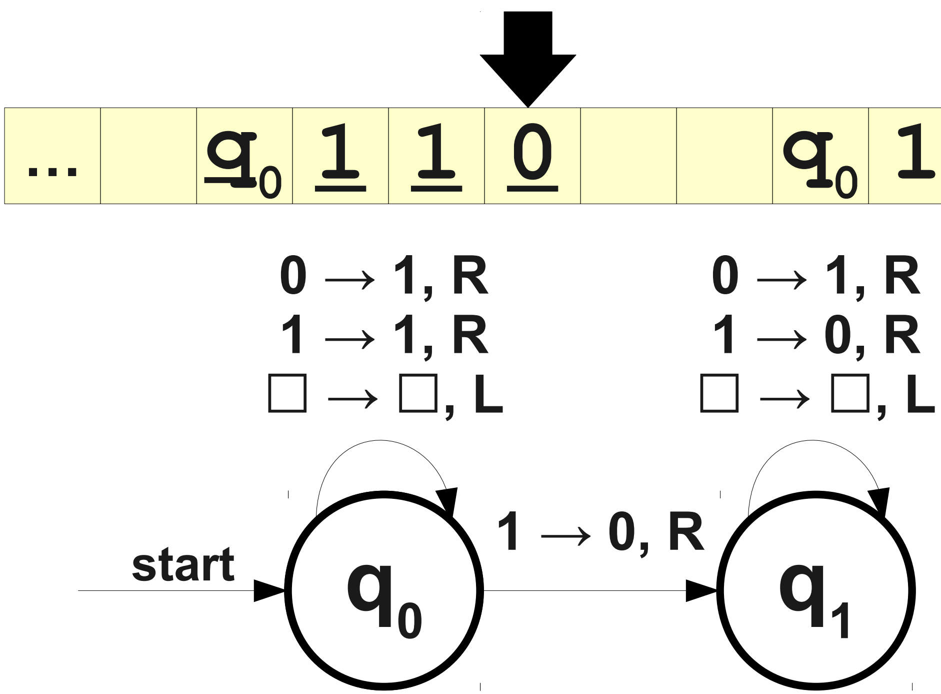

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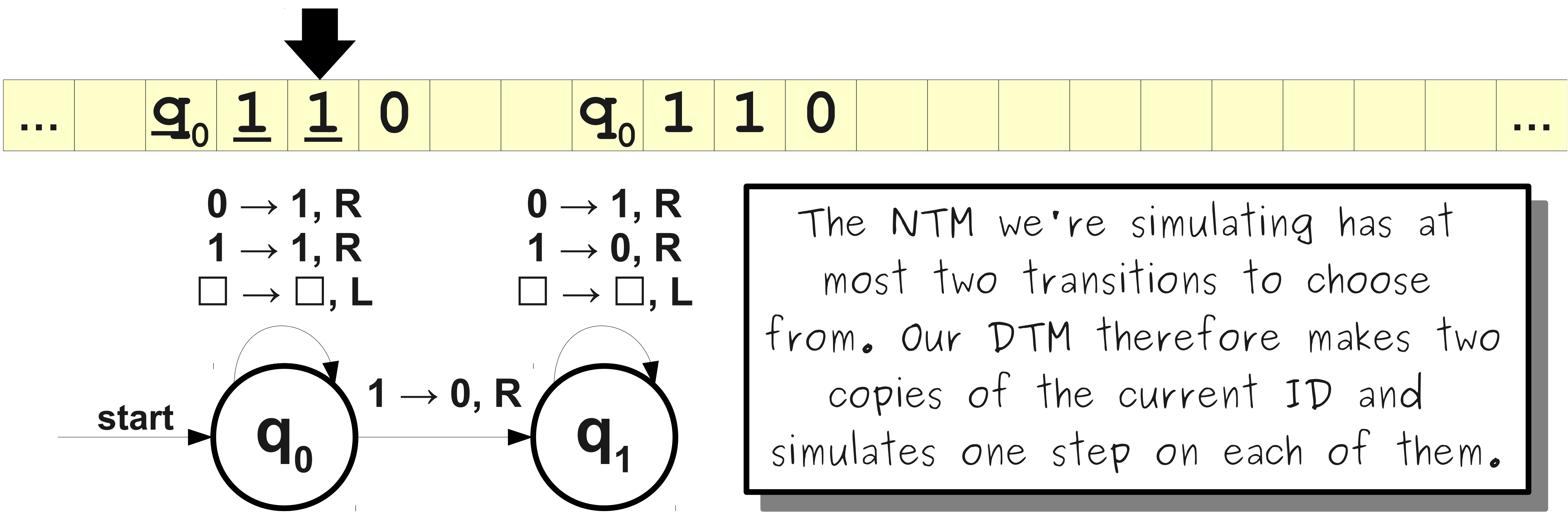

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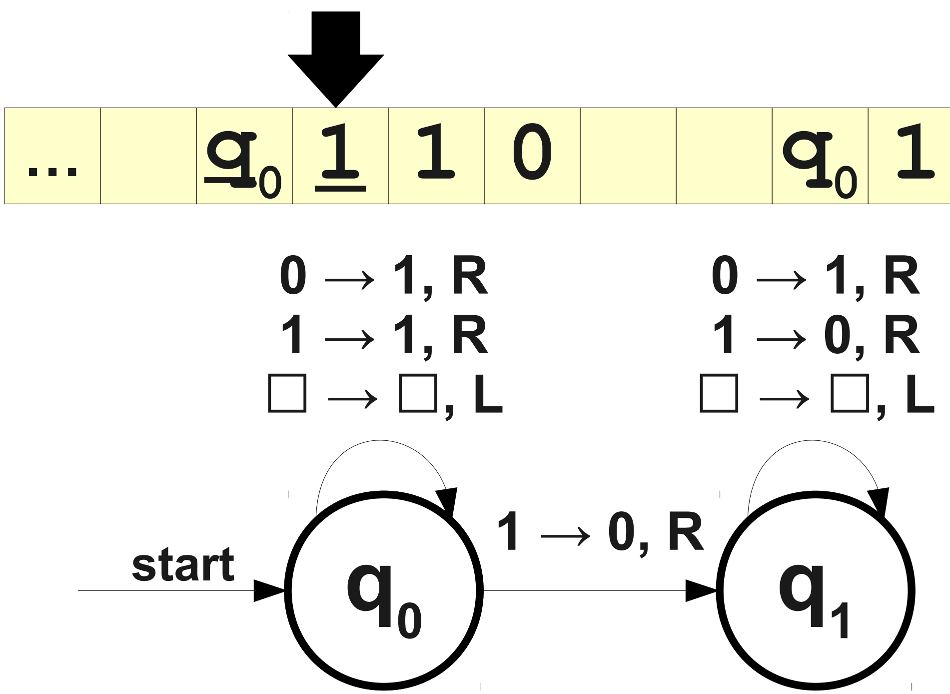

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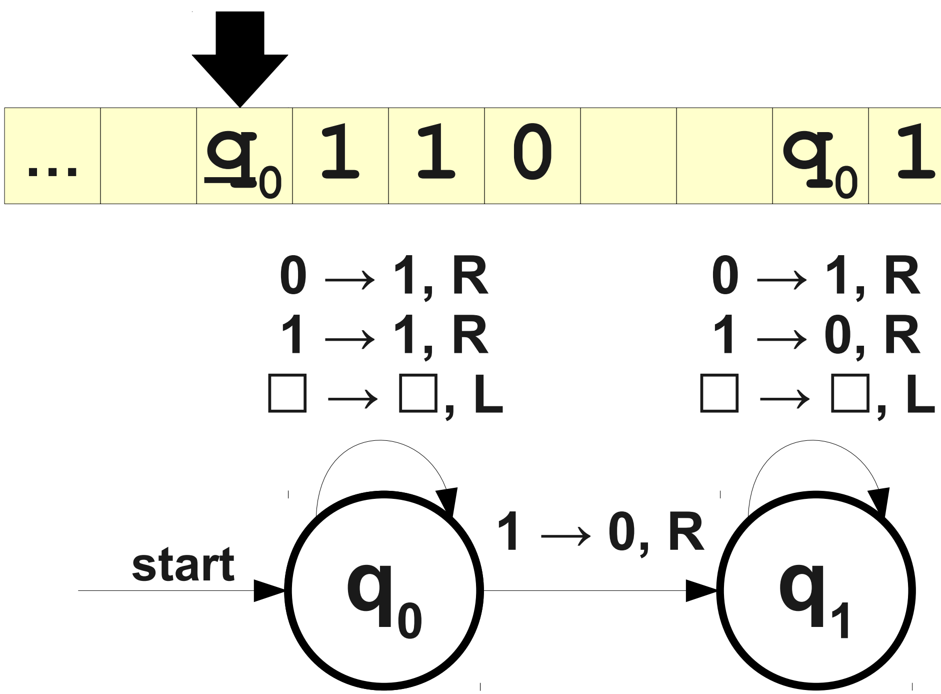

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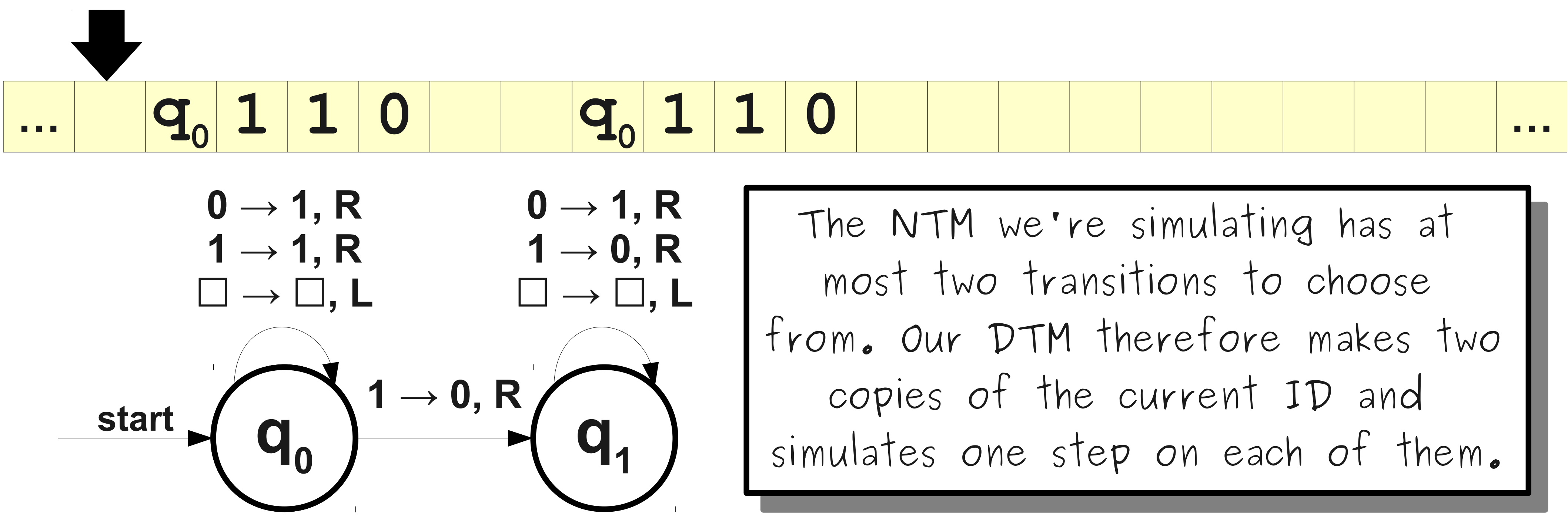

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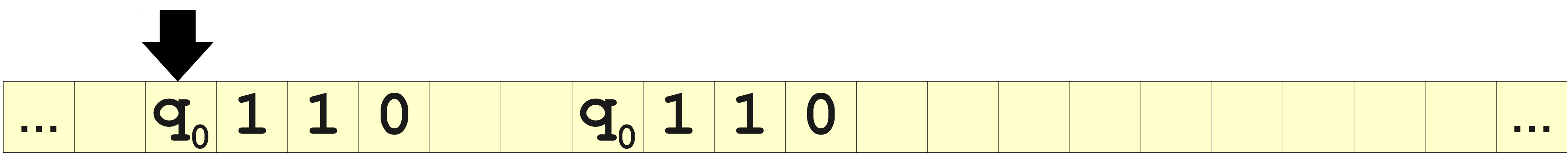

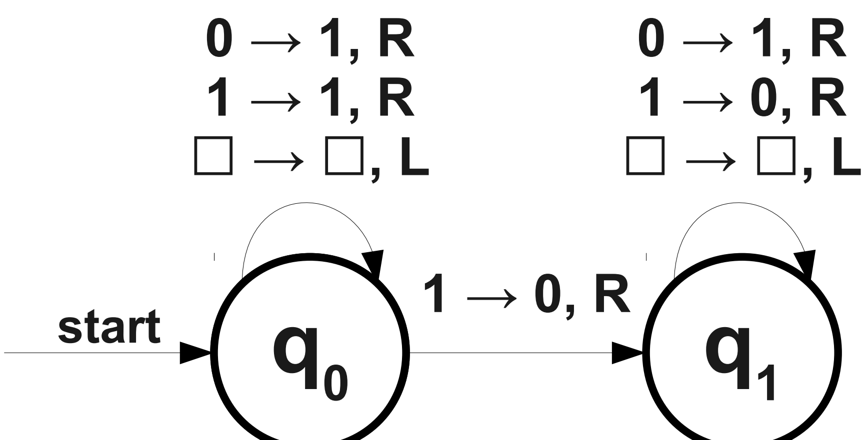

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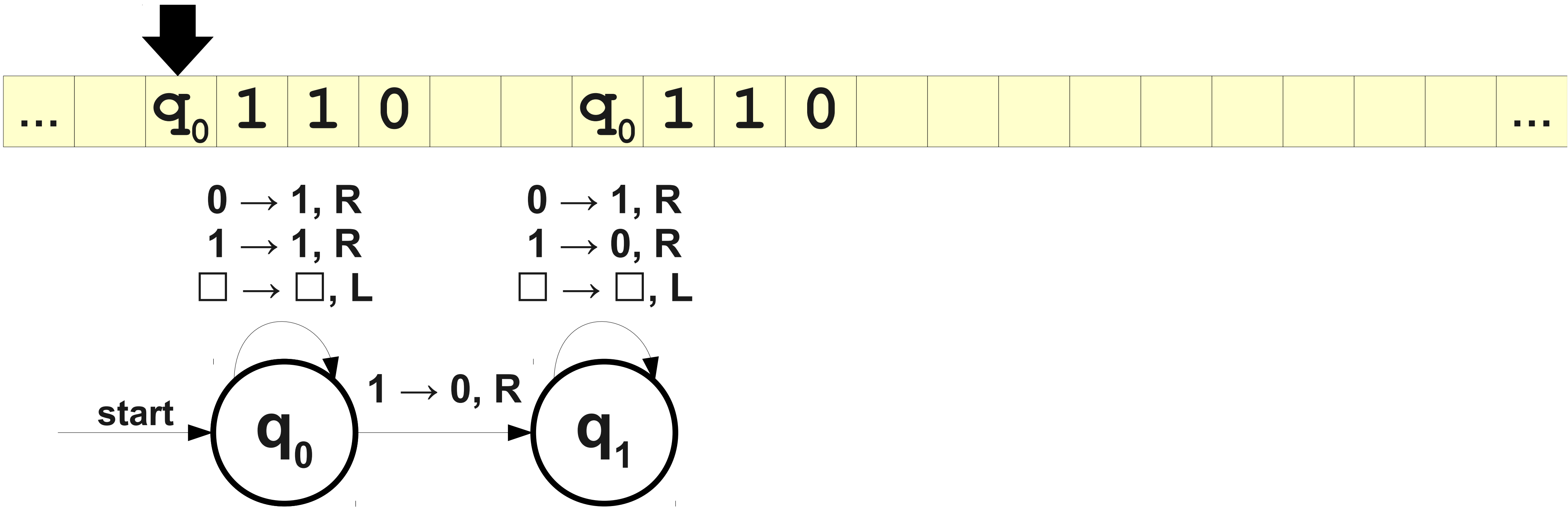

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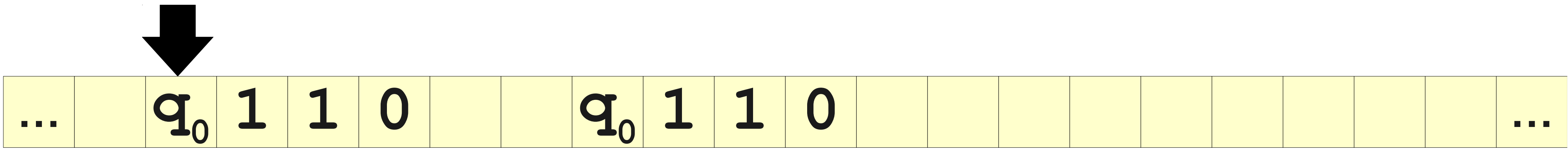

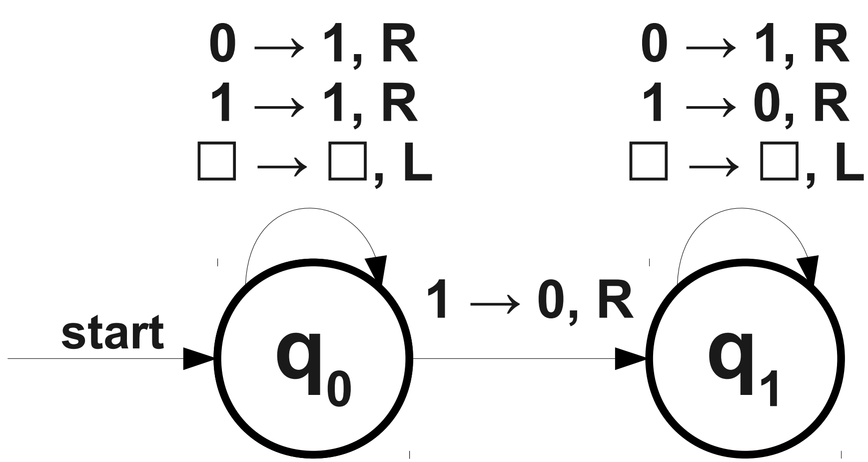

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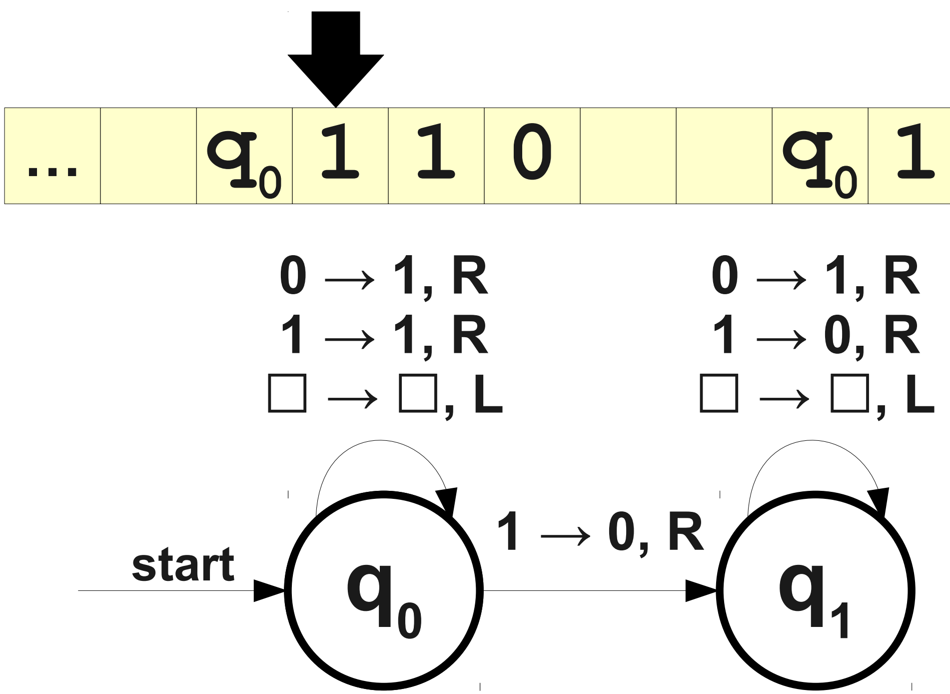

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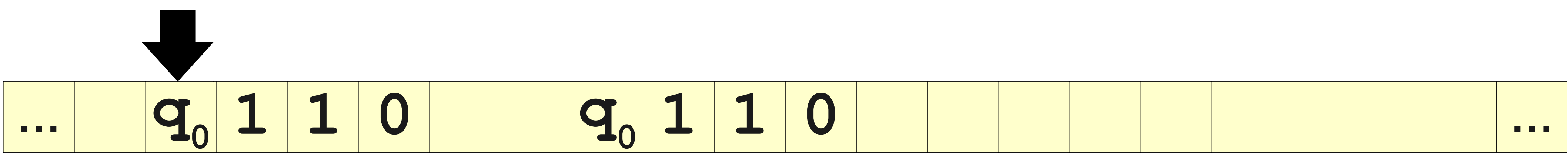

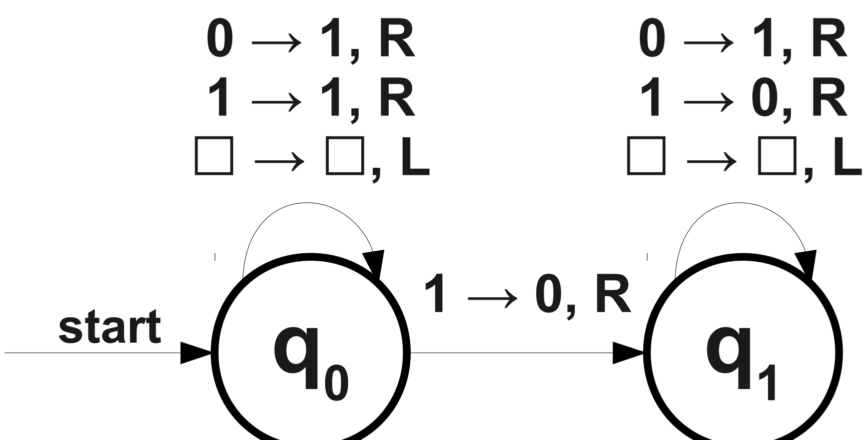

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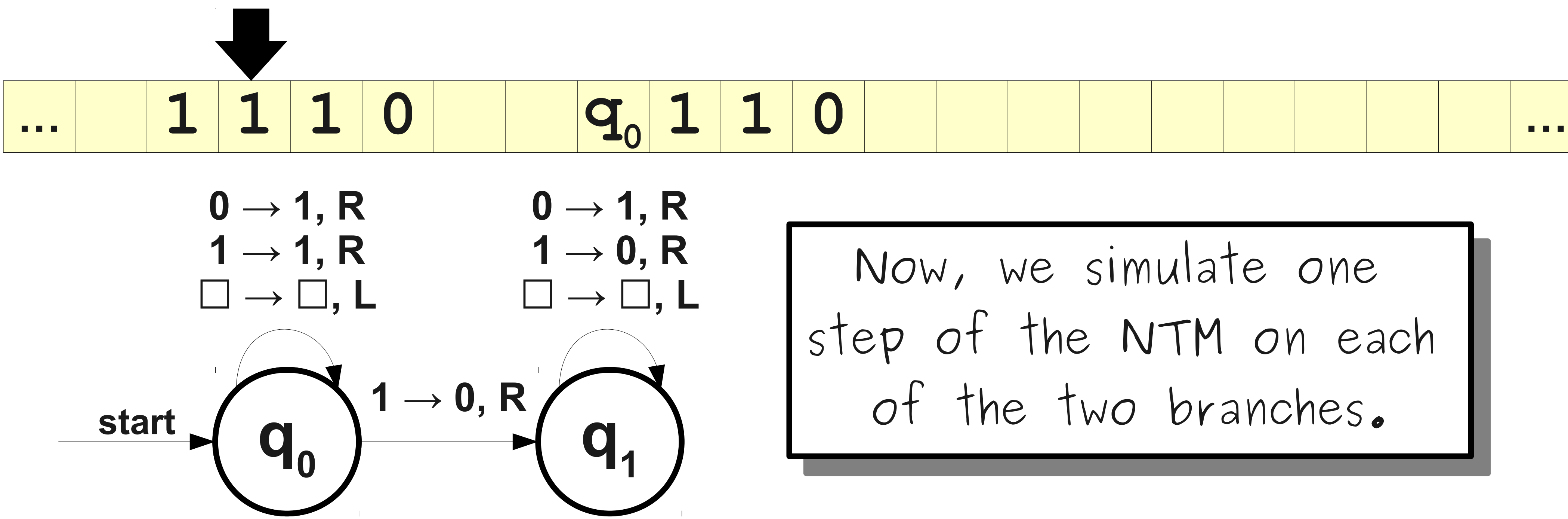

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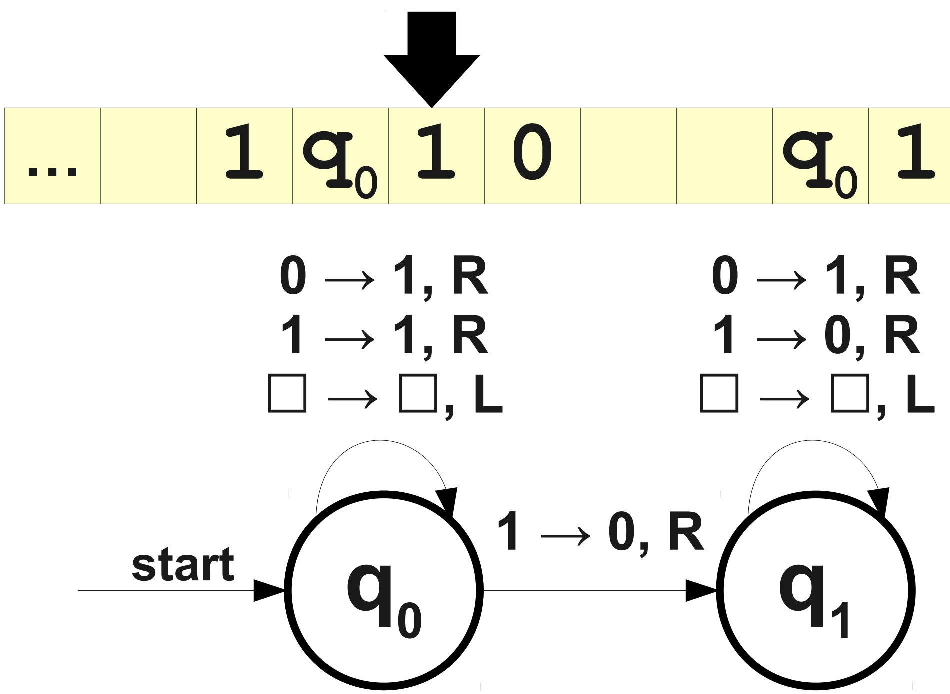

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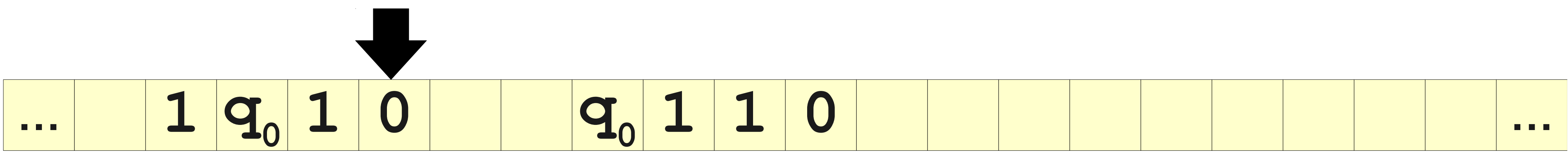

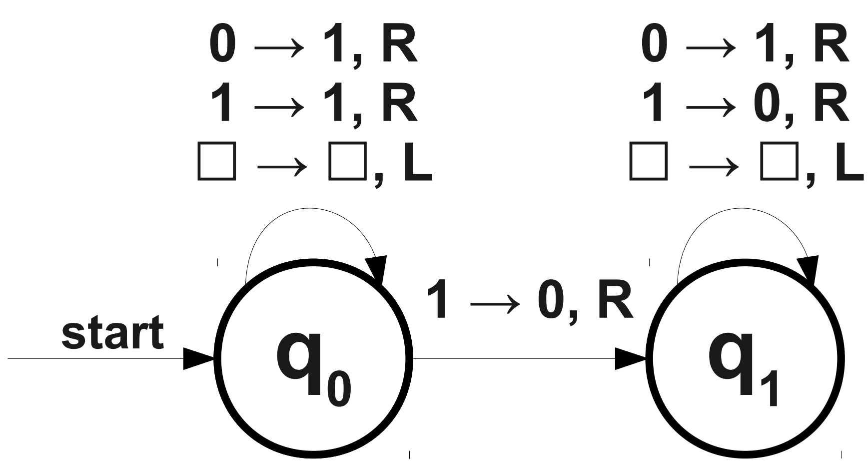

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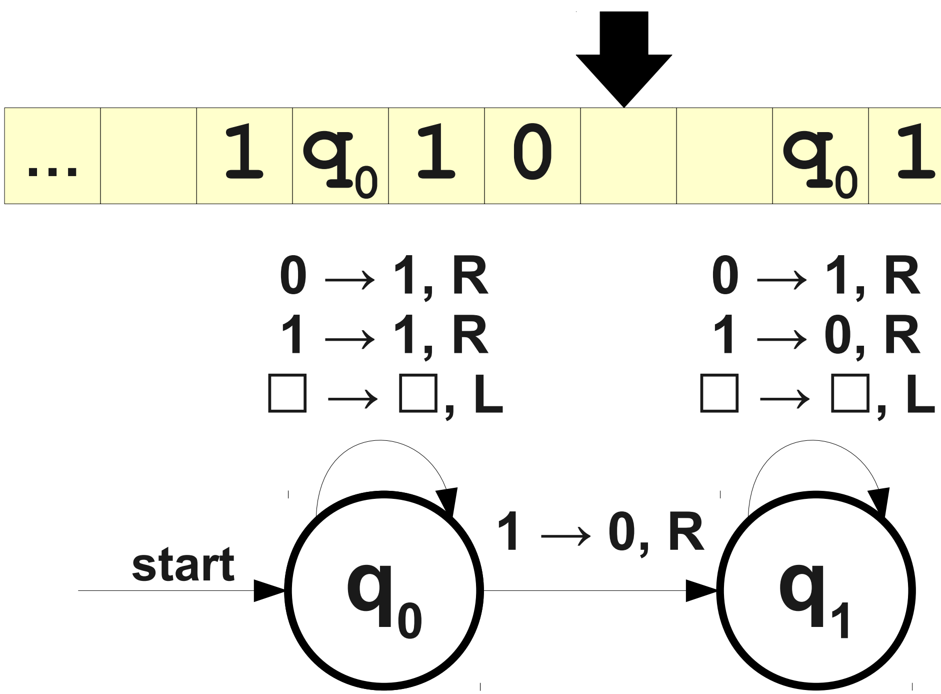

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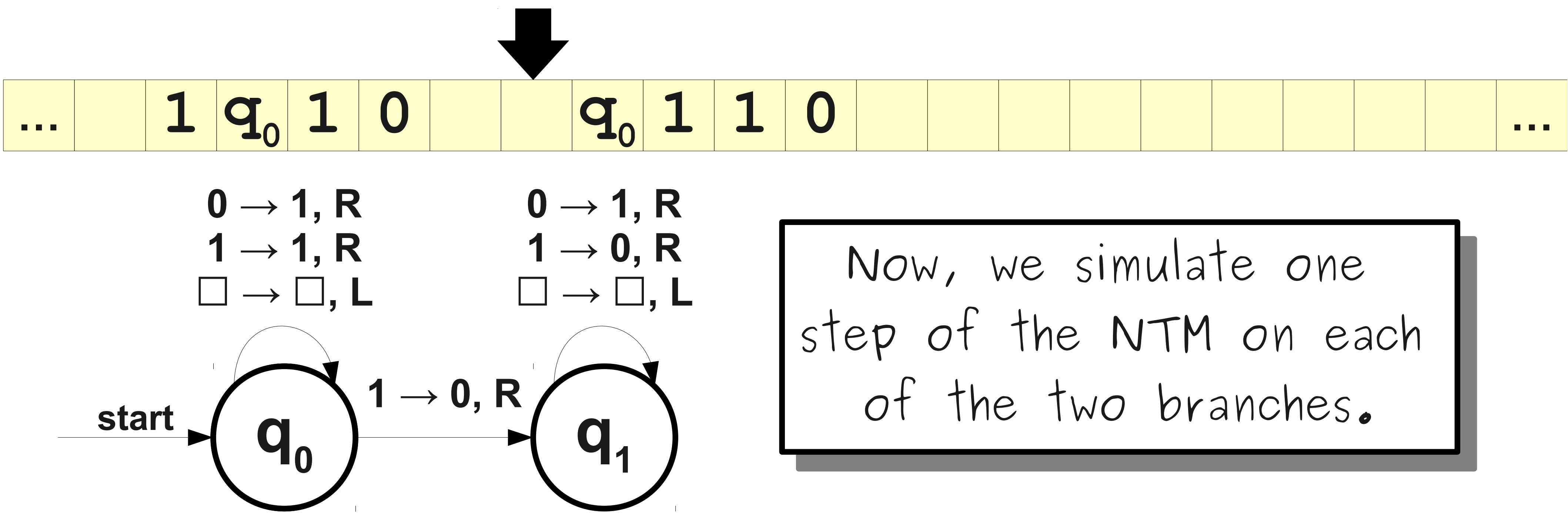

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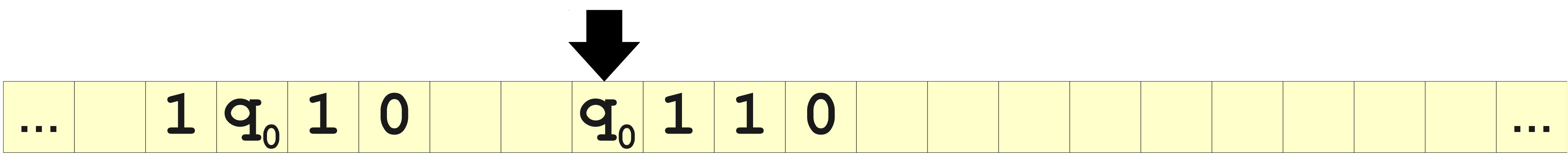

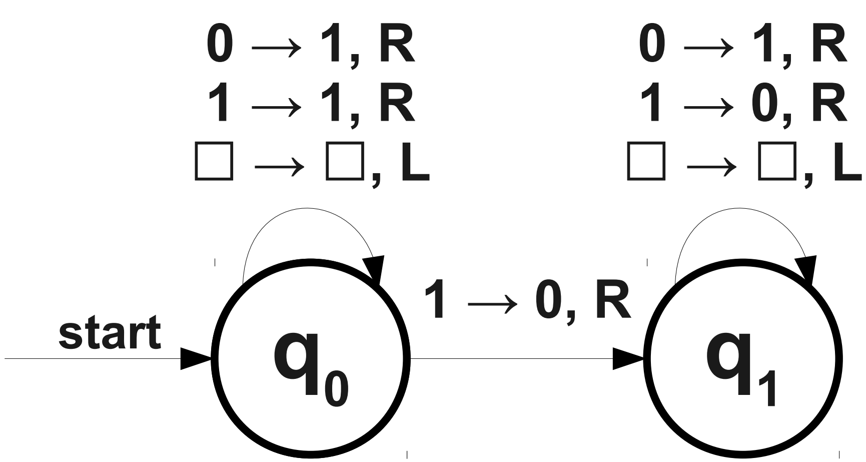

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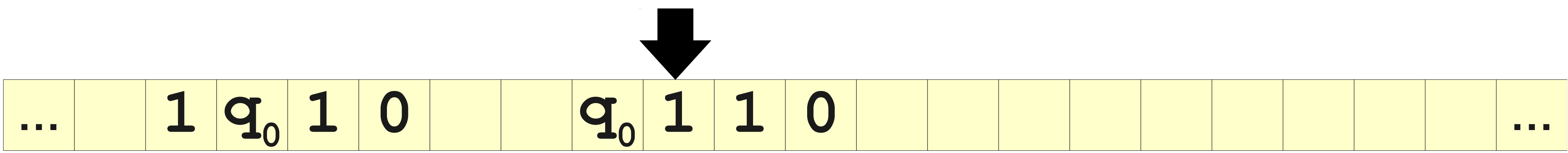

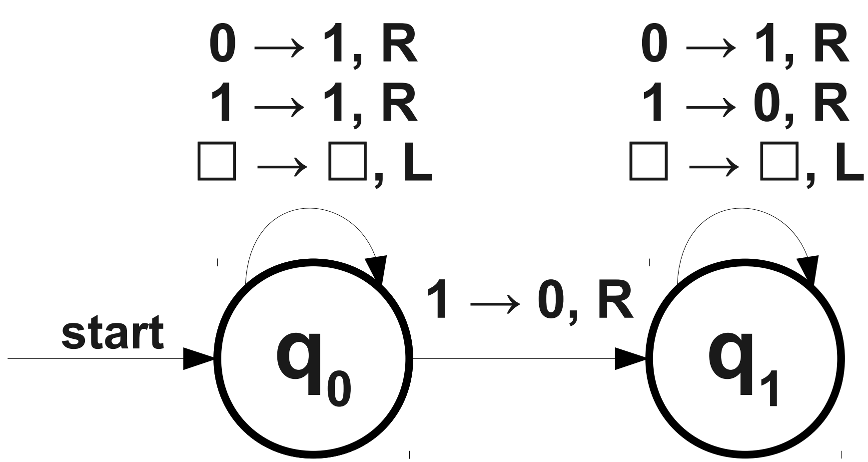

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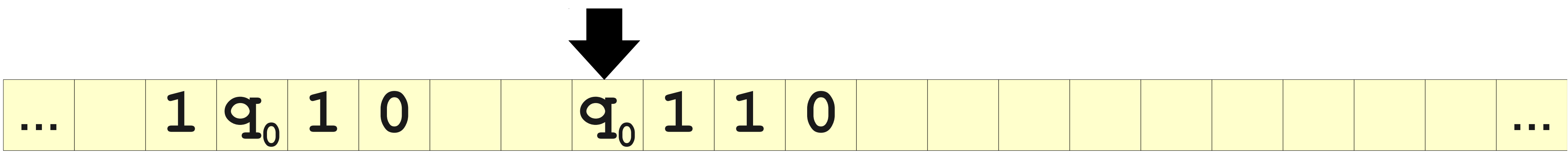

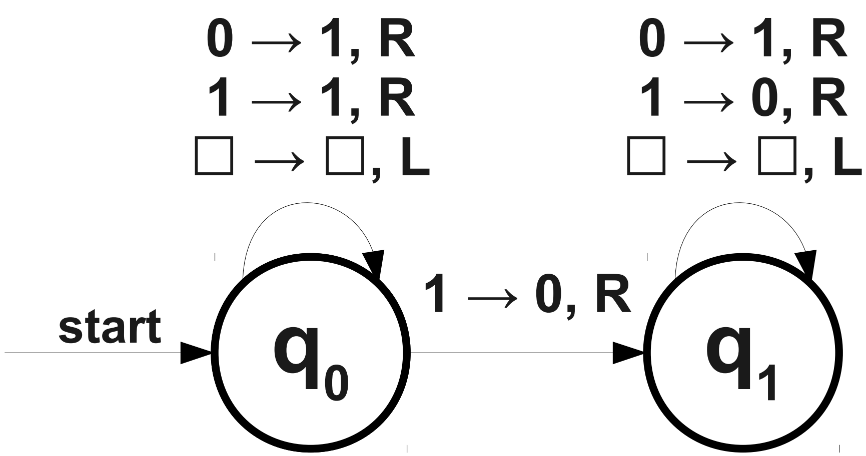

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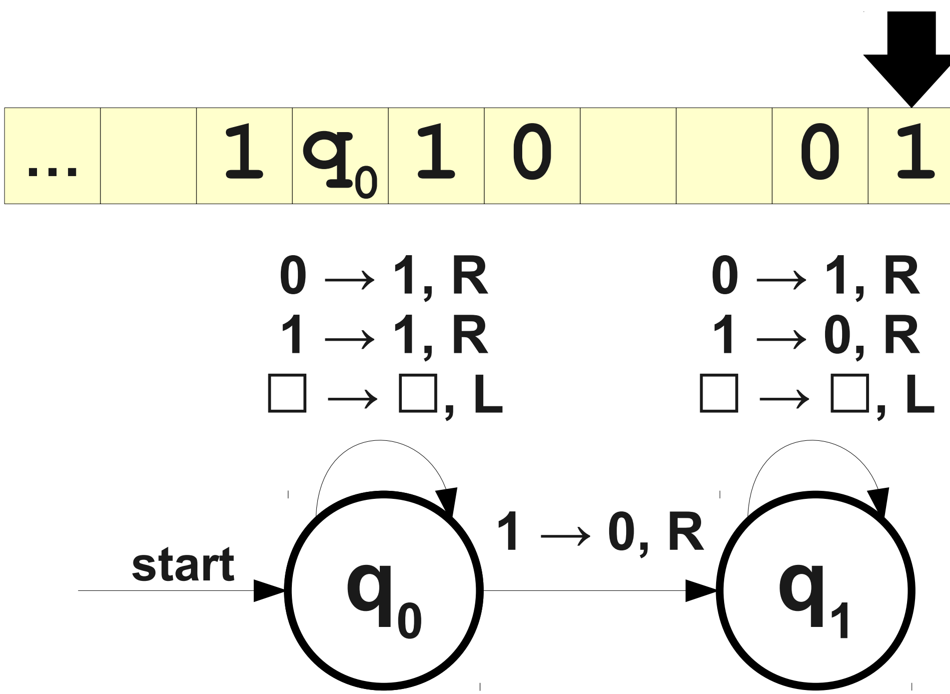

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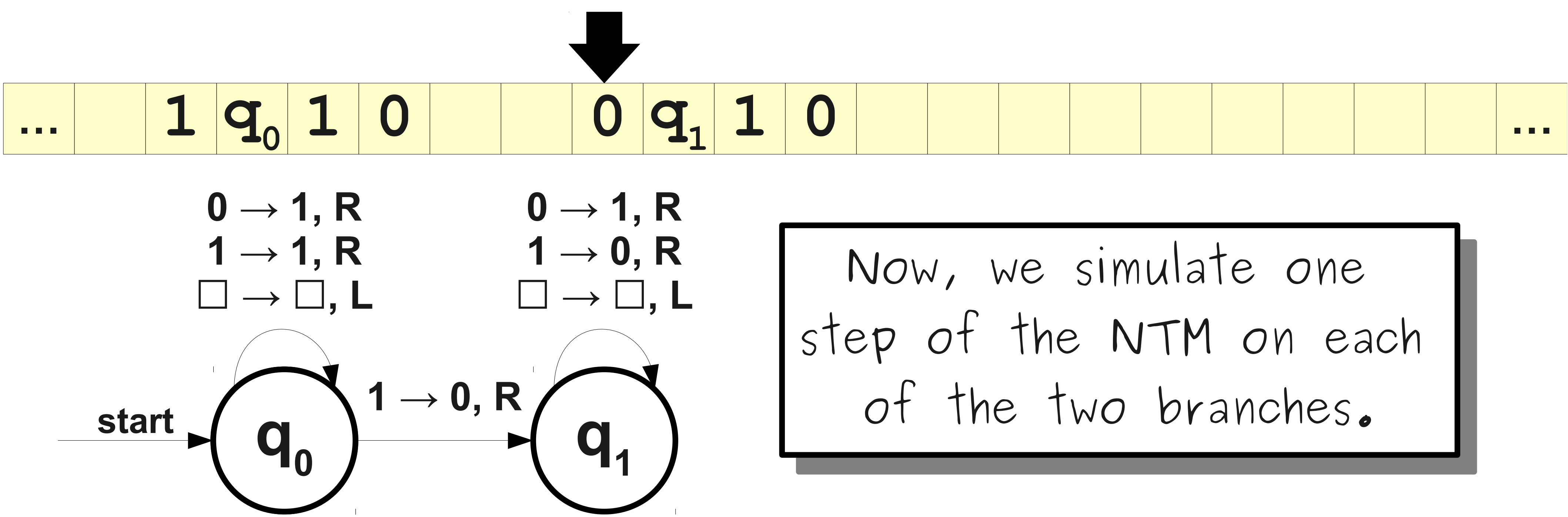

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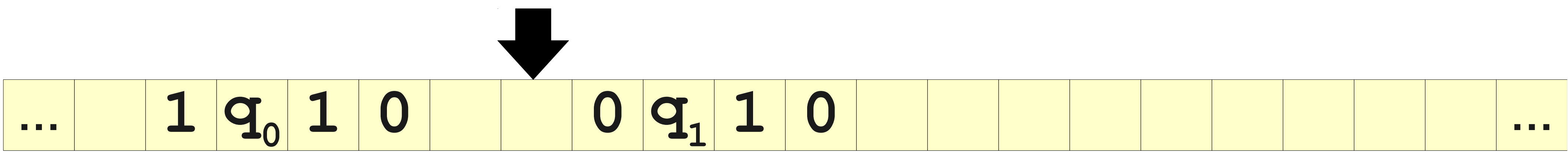

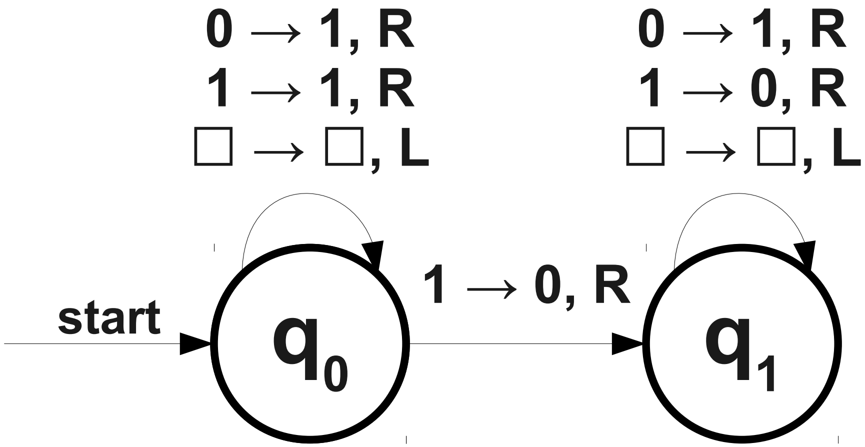

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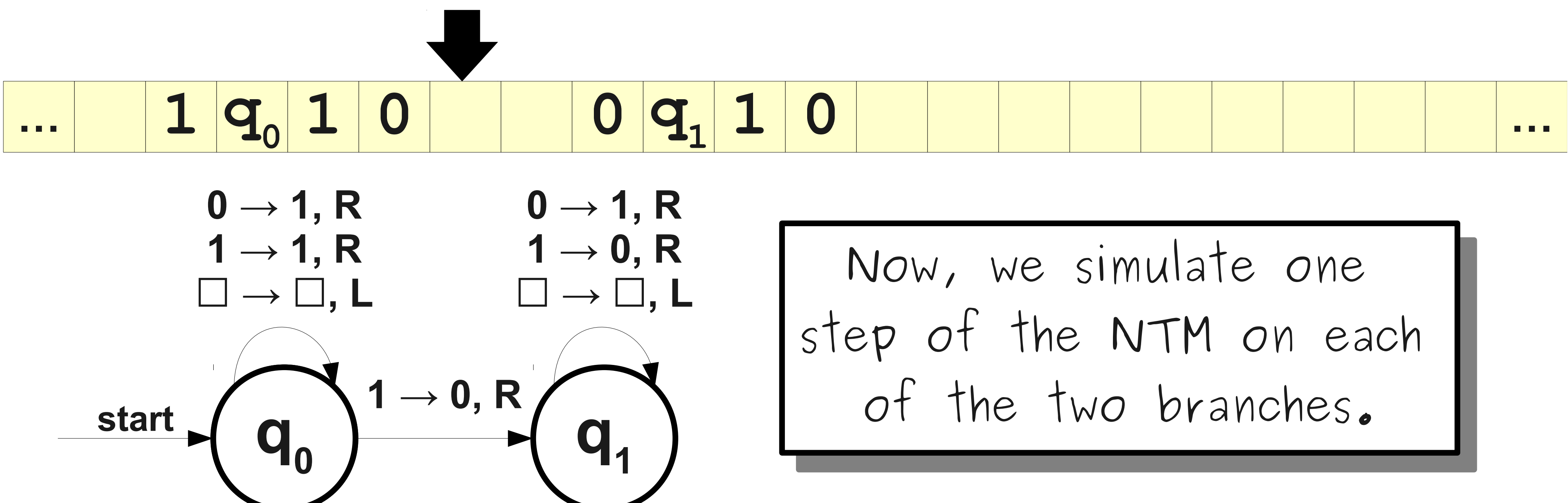

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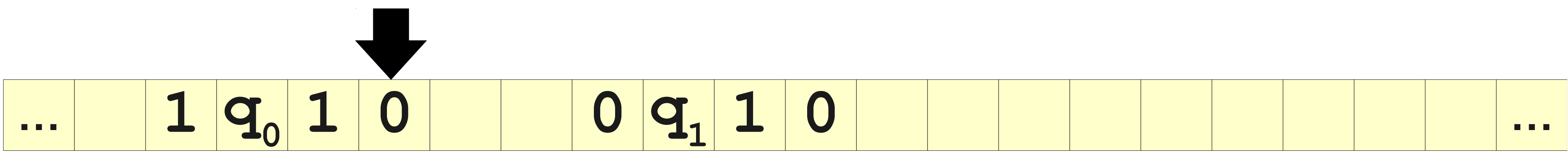

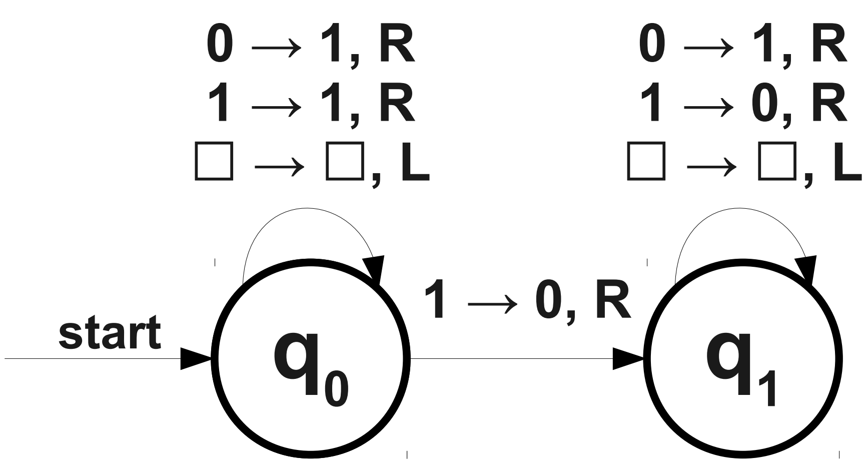

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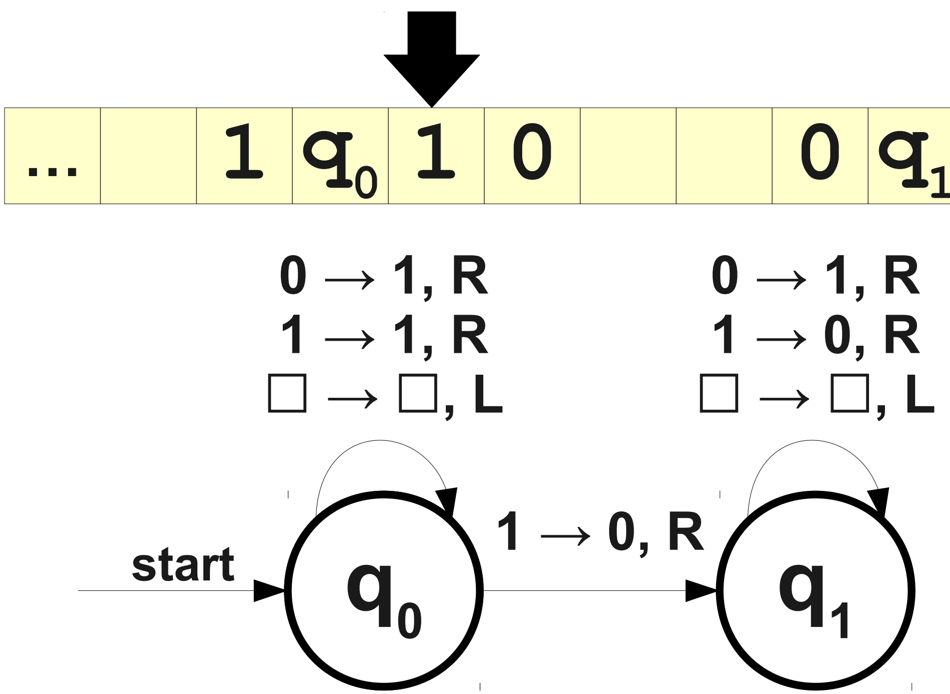

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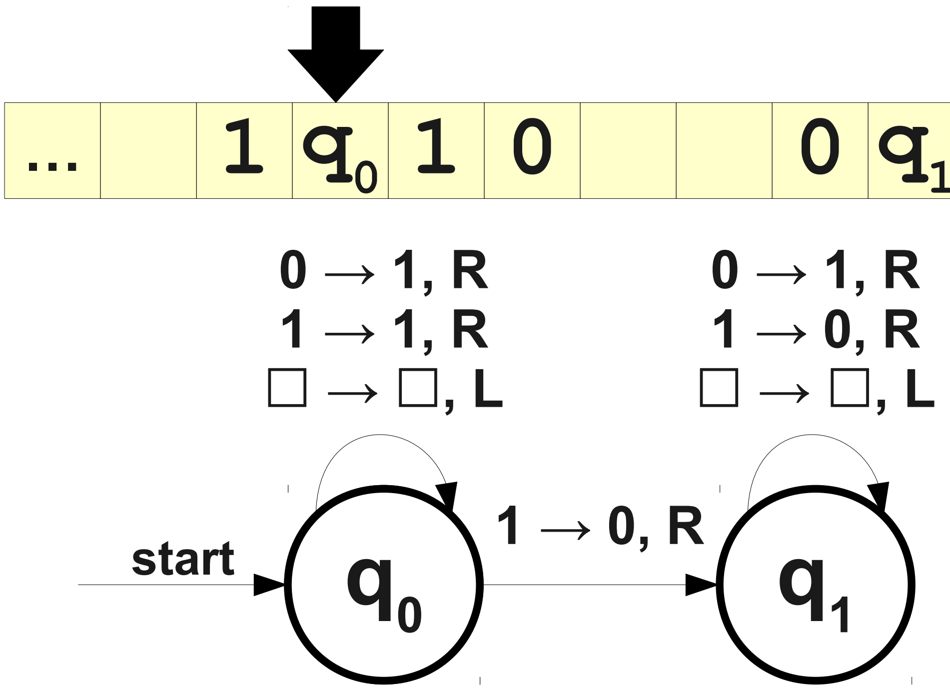

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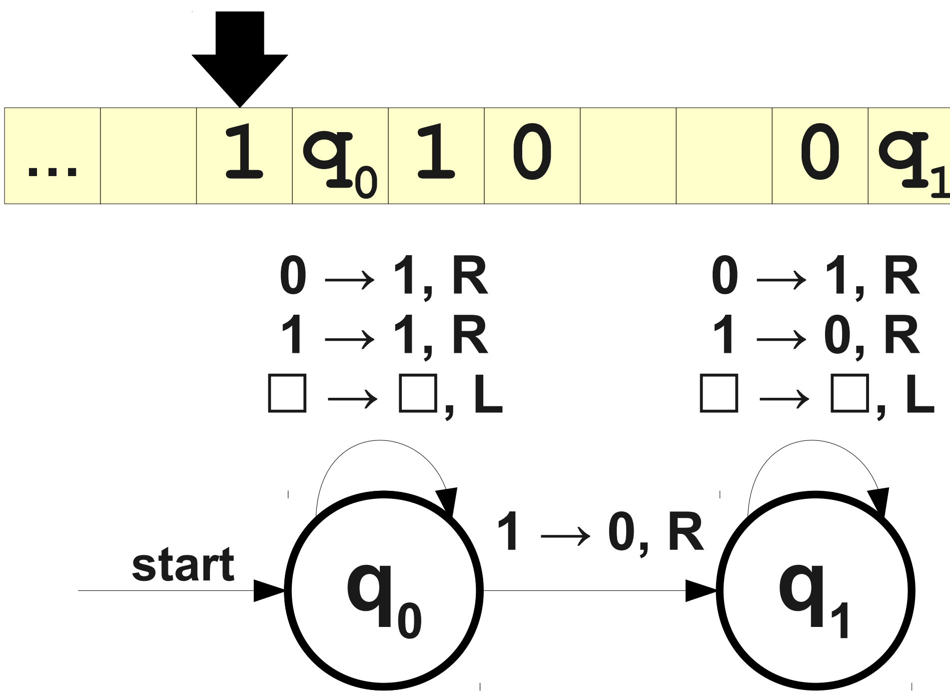

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- ***Theorem:*** For any NTM N, there exists a DTM D such that $\mathscr{L}(N) = \mathscr{L}(D)$.
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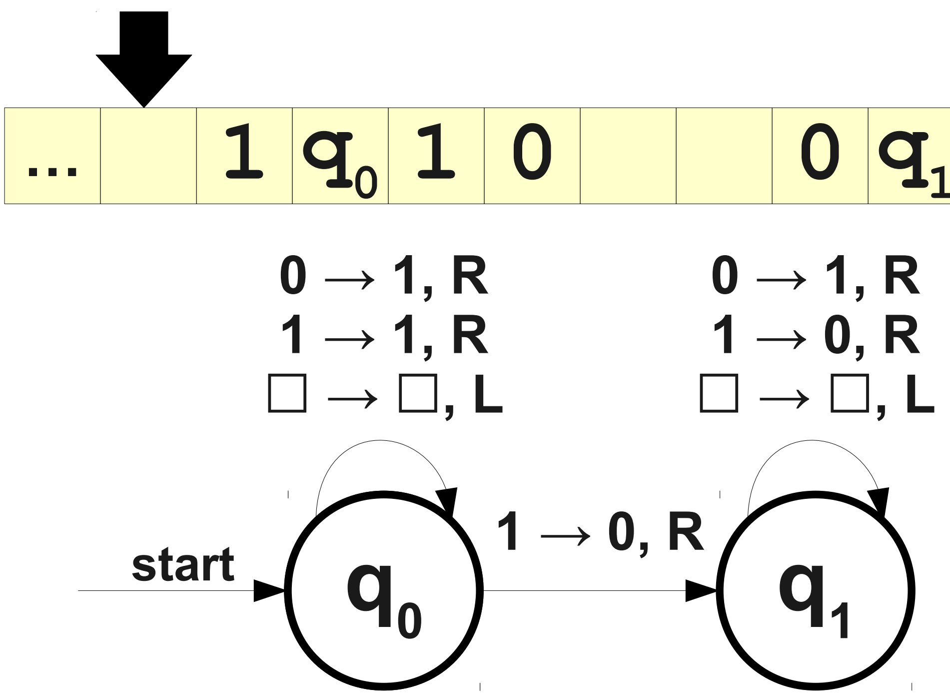

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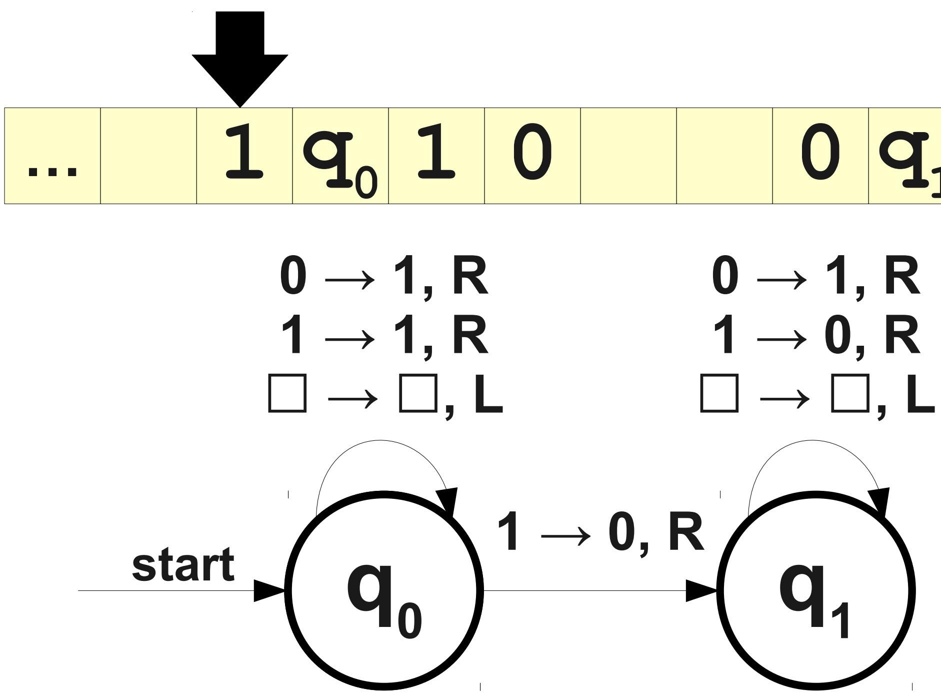

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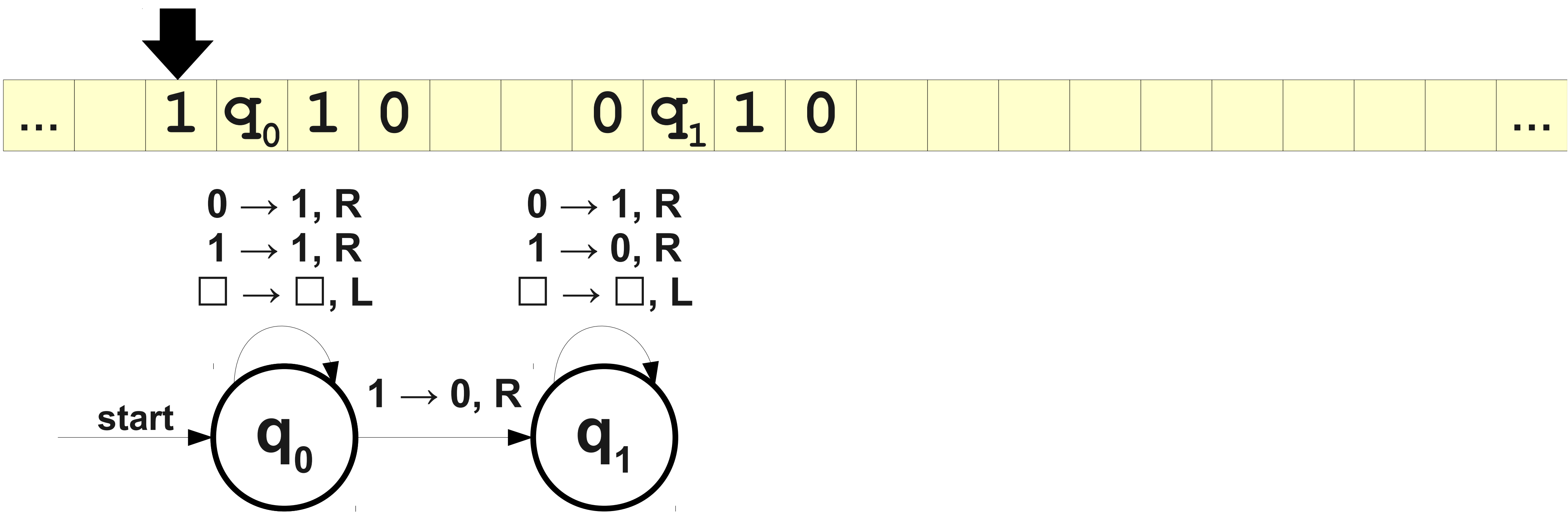

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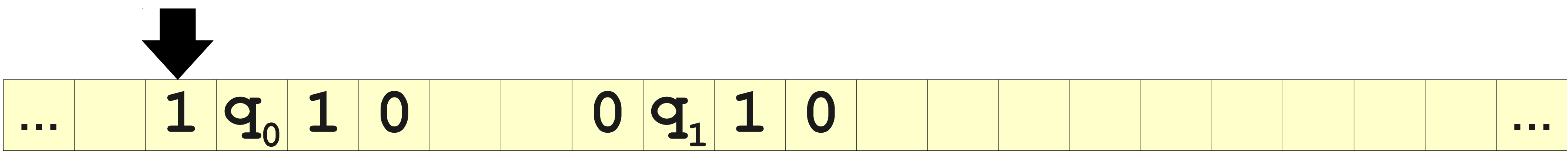

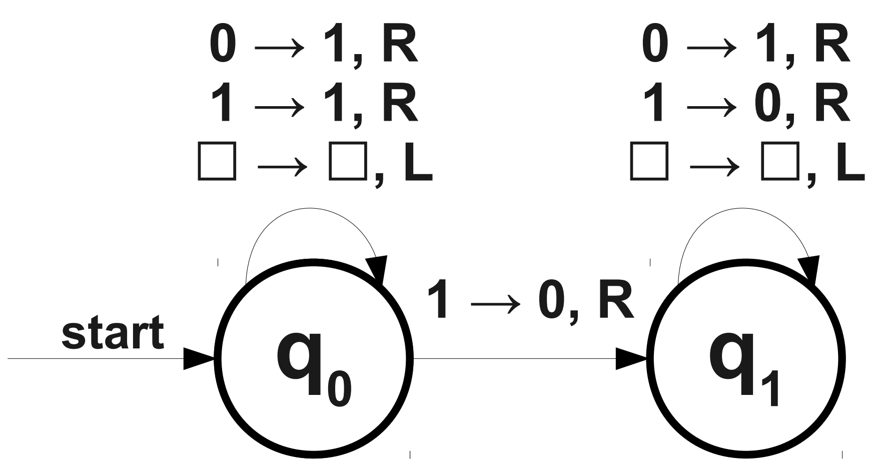

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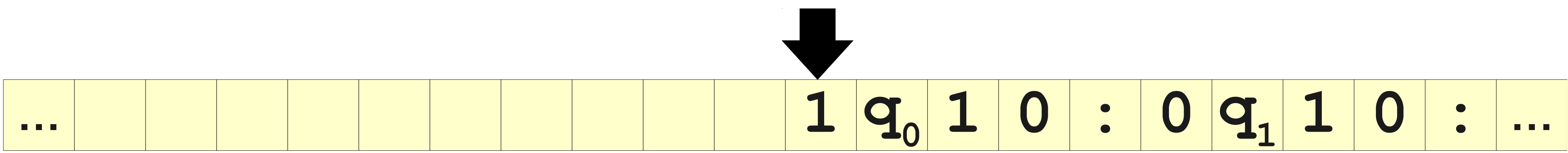

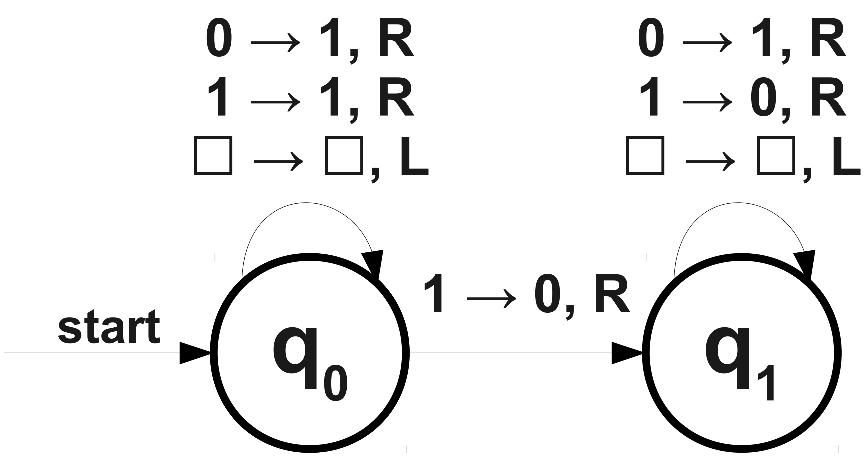

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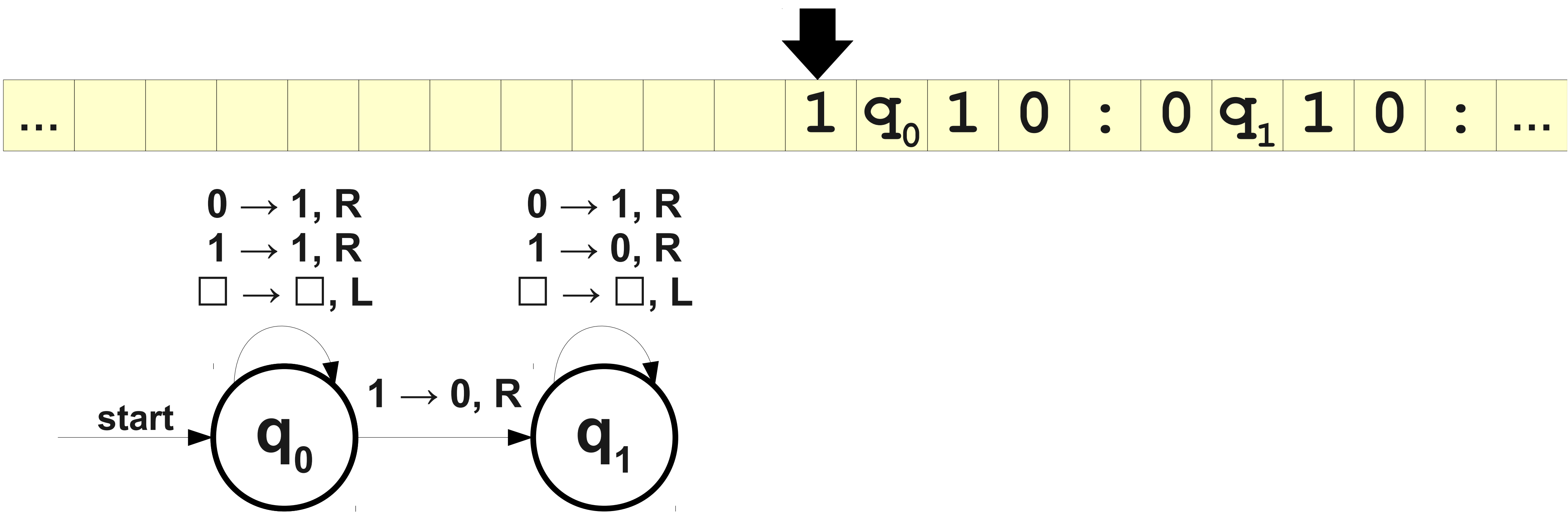

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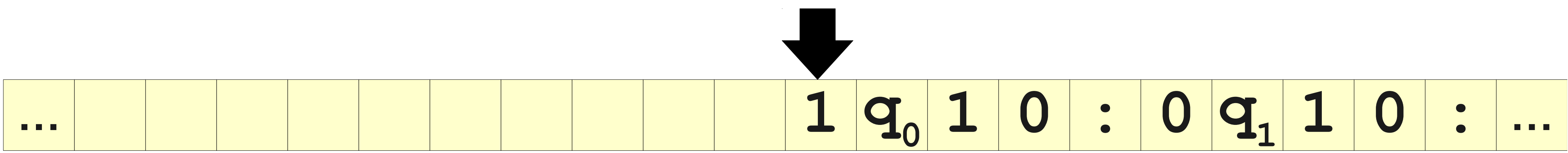

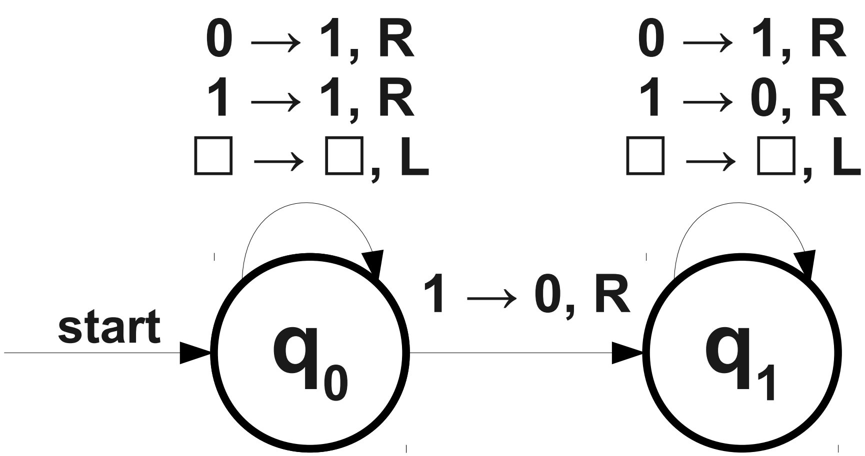

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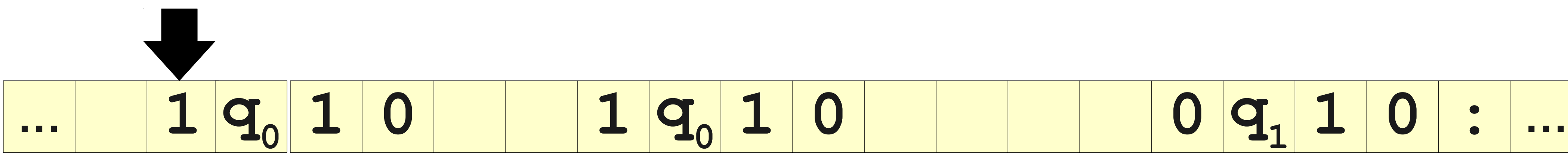

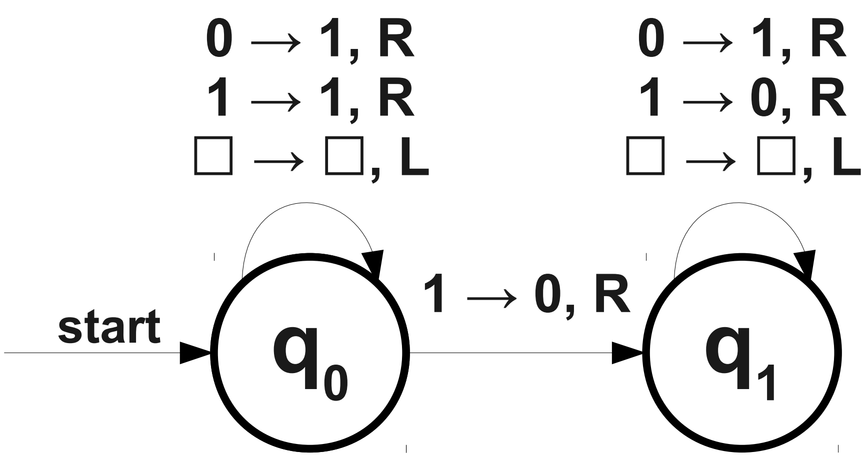

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Schematically

Workspace	Worklist of IDs

To simulate the NTM N with a DTM D, we construct D as follows:

- On input w, D converts w into an initial ID for N starting on w.
- While D has not yet found an accepting state:
  - D finds the next ID for N from the worklist.
  - D copies this ID once for each possible transition.
  - D simulates one step of the computation for each of these IDs.
  - D copies these IDs to the back of the worklist.

Why All This Matters

- The equivalence of NTMs and DTMs should be absolutely astounding!
  - Nondeterministic TMs can magically guess a single correct option out of infinitely many options.
  - Deterministic TMs can just zip back and forth across the tape.
- This suggests that Turing machines are extremely powerful.

Just how powerful **are** Turing machines?

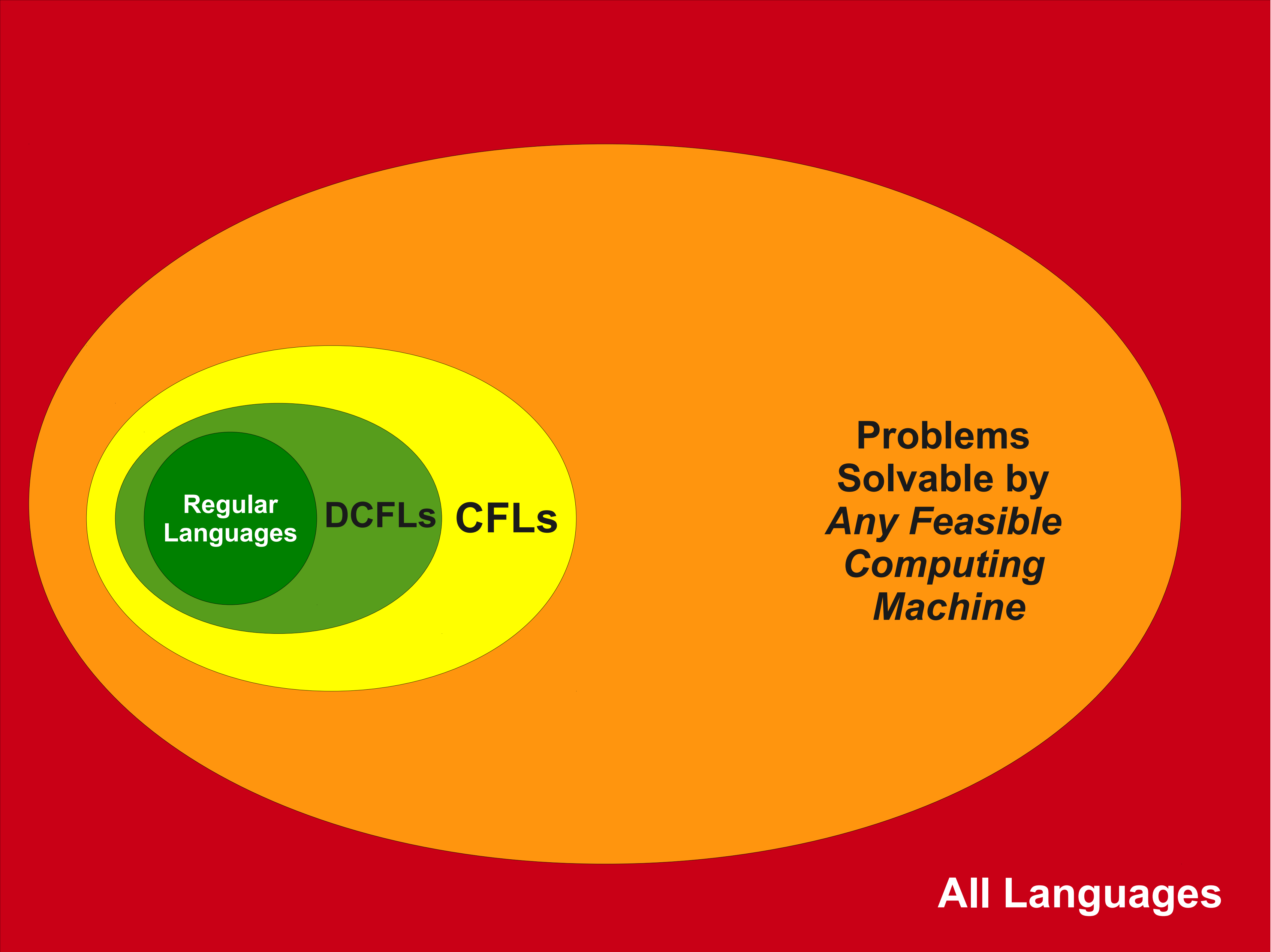
Effective Computation

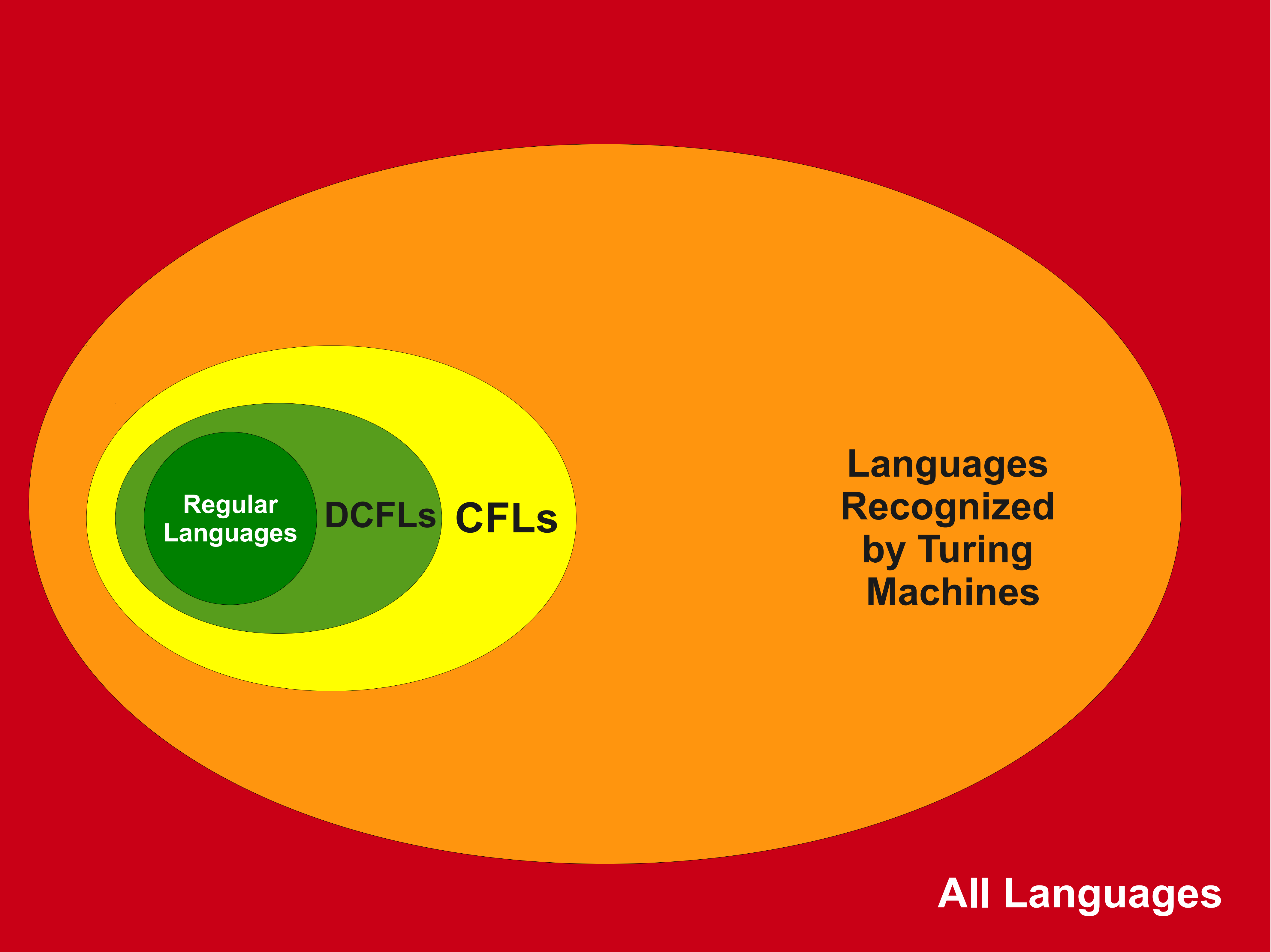
- An **effective method of computation** is a form of computation with the following properties:
  - The computation consists of a set of steps.
  - There are fixed rules governing how one step leads to the next.
  - Any computation that yields an answer does so in finitely many steps.
  - Any computation that yields an answer always yields the correct answer.

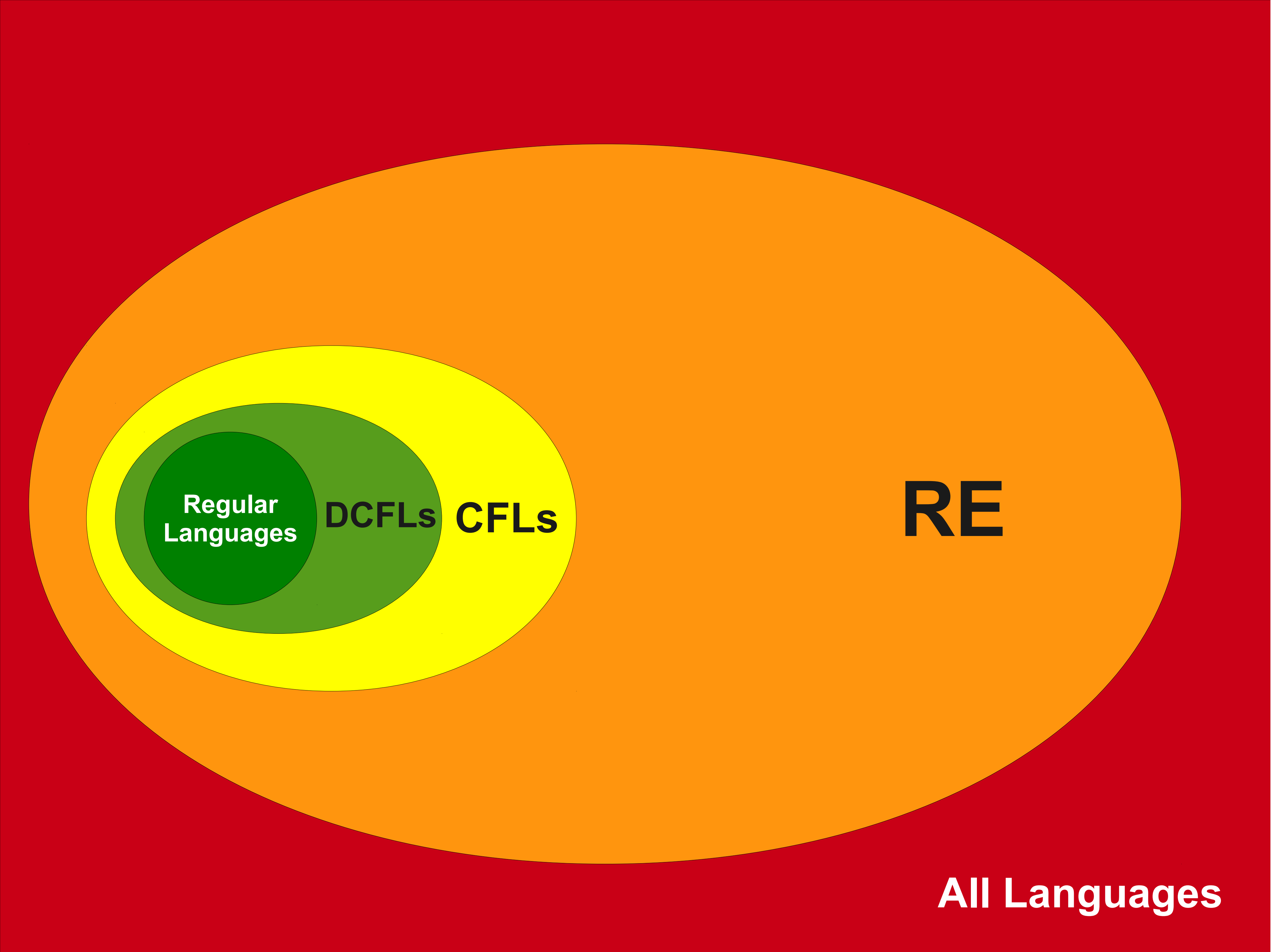
The **Church-Turing Thesis** states that

Every effective method of computation is either equivalent to or weaker than a Turing machine.

This is not a mathematical fact – it's a hypothesis about the nature of computation.


Regular Languages
DCFLs
CFLs
Problems Solvable by *Any Feasible Computing Machine*
All Languages


Regular Languages
DCFLs
CFLs
Languages Recognized by Turing Machines
All Languages


Regular Languages
DCFLs
CFLs
RE
All Languages

Next Time

- **Encodings**
  - How do we compute over arbitrary objects?
- **The Universal Turing Machine**
  - Can TMs compute over themselves?
- **The Limits of Turing Machines**
  - A language not in **RE**.