Modeling and Robust Control of Ball Plate System using different control methodologies

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***Abstract*—The ball plate system is a popular nonlinear control problem and various control system strategies have been put forward to approach the problem. This paper on the ball plate system focuses on the analysis and comparison of two strategies of control, Proportional Integral Derivative (PID) and Linear Quadratic Regulator (LQR). The comparison between these strategies is based on the performance in trajectory tracking of a circle and fixed point tracking, and how the system reacts to noise and external disturbance in each case. The control schemes are simulated in MATLAB after modelling the system.**

***Index Terms*—Ball plate system, PID, LQR, trajectory tracking, fixed point tracking**

I. Introduction

The ball plate system is a very popular educational model to teach and validate various control strategies. The control objective of the ball and plate problem is to balance a ball or to make it trace a desired trajectory on a flat plate, solely by tilting the plate relative to the horizontal plane. This system is of particular interest to the control community because it allows the user to study and validate a wide class of both linear and nonlinear control schemes before applying them to real-life applications that exhibit similar dynamics. The ball moves on a plate fixed with motors that control the inclination of the plate based on the control input obtained and the ball has 2DOF of motion (along the X and Y-axis). The control algorithm is first verified using the derived nonlinear simulation model in MATLAB for given methods.

The ball and plate system is an extension of the Ball and Beam system [1]. It is considered a benchmark model of a driftless nonholonomic system. A state feedback controller using the pole assignment method was designed in [2] to maintain a desired position of the sphere over the plane. A computer vision technique allowed us to measure the two-dimensional position of the sphere over the plane in real-time. [3] considered the feedback delay in the control loop for determining the state-space model of the Ball-Plate System, and secondly, based on the geometric method and the state feedback control an observer is synthesized. The ball and plate system is used as a nonlinear, uncertain, and MIMO system to verify the effectiveness of the proposed controller. The invasive weed optimization (IWO) method, which is one of the metaheuristic optimization algorithms, is used to obtain the optimal parameters of the proposed controller [4]. In [5] the authors discuss the use of PD position controller and PD angle controller to control the ball and plate system. Design and Implementation of a Ball-Plate Control System using computer vision as a feedback sensor and Python Script for Educational Purposes in STEM Technologies is proposed in [6]. In their paper, Mujadin et al. [7] developed the ball plate system control using a resistive touch screen as the sensor position of the ball. Cheng et al. [8] present a skillful robotic wrist system using a visual servo control technique to demonstrate the dexterity of the mechanical wrist from the viewpoint of table tennis whereas the ball and plate system is chosen as the basic stage of this work. Dusek et al. [9] present the modeling of the ball and plate systems based on first principles which consider the balance of torques and forces. Ibrahim Mustafa Mehedi et al. [10] presented fractional order controller design for the beam and ball system. [11] discusses an efficient method to improve the balancing and tracking of the trajectory of the BOPS based on machine learning (ML) algorithm with the Pseudo proportional-derivative (PPD) controller. Various control methods are used to maintain the position of ball on plate such as, Fuzzy control [12], PID [13], SMC [14], LQR [15]. We present the analysis and comparison of two such methods: PID and LQR.

The BPS is marginally stable since it contains a double pole in the origin. This results in the system having only non-decaying oscillatory components in its step response when introduced to a feedback loop. To gain stability and enhance the performance of the system a PID controller is introduced. Values of K_p, K_i and K_d determine the control output that is required to attain a stable system response and are established through an iterative process in MATLAB. The Linear Quadratic Regulator (LQR) is a well-known method that provides optimally controlled feedback gains to enable the closed-loop stable and high-performance design of systems.

Control theory and its applications are crucial when operating within the area of dynamic systems. The system should be able to recover and compensate for disturbances and external actions imposed on a given system being inherently unstable or semi-stable. Based on the simulation, a comparison of the given three strategies is also made based on their response to external disturbances.

This paper is organized as follows. Section [II] presents the modeling of ball plate system including the system description, mathematical model and state space vector description. Section [III] provides description of control methodologies used for both PID and LQR. Section [IV] presents the results of our simulation and discusses our observation for both methods. Section [V] shows how our work differs from any other proposed schemes and finally, Section [VI] concludes the paper.

II. Modeling

A. System Description

The ball-plate system developed and modeled in this case is a 2DoF system, having a simple plate and a ball placed over it. The inclination in both dimensions is provided by two motors, mounted in perpendicular directions such that the inclination provided by one motor doesn't effect the motion in other direction. The inclination of the plate in both the directions can be utilized to balance the ball at the desired position on the plate, as well as make the ball follow a trajectory on it.

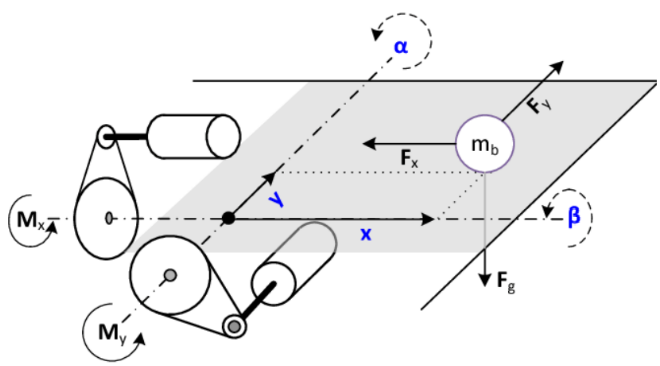


Fig. 1: Free Body Diagram of System.

B. System Equations

The dynamic equations of the system are derived in this section. Certain assumptions are made regarding the system which simplify the model: (1) There is no slipping of ball on the plate (2) Friction everywhere else is neglected (3) The ball is homogenous and symmetrical (4) The ball and the plate are in contact all the time. Lagrangian method is used to derive these equations. The Euler-Lagrange equation for the ball and plate system is:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial q_i}\right) - \frac{\partial L}{\partial q_i} = Q_i \tag{1}$$

Where q_i represents the i direction coordinate and Q_i stands for the composite force in the i direction. L is defined as the Lagrangian of the system.

The system can be described as having 4 degrees of freedom. Two, of the ball in the x and y directions, and two of the plate inclination, with rotational axis of the plate in both of these directions. Table I describes the variables of the system:

TABLE I: System Variables and Constants

Variable	Description
x_b	x coordinate of ball on plate
y_b	y coordinate of ball on plate
$\dot{x}_b$	translational velocity of ball along x-axis
$\dot{y}_b$	translational velocity of ball along y-axis
α	angle of inclinatin of plate along x-axis
β	angle of inclinatin of plate along y-axis
$\dot{\alpha}$	angular speed of plate along x-axis
$\dot{\beta}$	angular speed of plate along y-axis
m_b	mass of ball
I_b	Moment of Inertia of ball
I_b	Moment of Inertia of plate
M	Mass of plate
w_x	angular speed of ball along x-axis
w_y	angular speed of ball along y-axis
T_x	Torque applied on the plate along x-axis
T_y	Torque applied on the plate along y-axis
r_b	radius of ball

The inclination is with respect to the horizontal plane and the coordinates are according to the position of the ball on the plate with respect to the centre of the plate. Then, the kinetic energy of the system (T) can be defined as the sum of kinetic energies of the ball and the plate:

$$T = T_b + T_p \tag{2}$$

The kinetic energy of the ball is the sum of both translational and rotational energies with respect to its centre of mass:

$$T_b = \frac{1}{2}m_b\left(\dot{x}_b^2 + \dot{y}_b^2\right) + \frac{1}{2}I_b\left(w_x^2 + w_y^2\right) \tag{3}$$

Here, we can replace the values of w_x and w_y as $\dot{x}_b = w_x r_b$ and $\dot{y}_b = w_y r_b$. The kinetic energy of the plate is the rotational energy of a square plate with a point mass (ball assumed as a point mass) attached at coordinates (xb, yb) with respect to its centre of mass:

$$T_p = \frac{1}{2}\left(I_b + I_p\right)\left(\dot{\alpha}^2 + \dot{\beta}^2\right) + \frac{1}{2}m_b(x_b\dot{\alpha} + y_b\dot{\alpha})^2 \tag{4}$$

The relative potential energy of the system is considered as the relative potential energy of the ball with respect to its position at origin at zero inclination, and the change in potential energy of the plate is neglected:

$$V = V_b = m_b gh = m_b g\left(x_b sin\alpha + y_b sin\beta\right) \tag{5}$$

Thus, Lagrangian of the system is:

$$L = T - V = T_b + T_p - V_b \tag{6}$$

Using equations 1-5 we derive the dynamical equations of the system:

$$\frac{\partial L}{\partial \dot{x}_b} = \left(m_b + \frac{I_b}{r_b^2}\right)(\dot{x}_b) \tag{7}$$

$$\frac{\partial L}{\partial x_b} = m_b(x_b\dot{\alpha}^2 + y_b\dot{\alpha}\dot{\beta}) - m_b g sin\alpha \tag{8}$$

$$\frac{\partial L}{\partial \dot{\alpha}} = (I_b + I_p)\dot{\alpha} + m_b(x_b^2\dot{\alpha} + x_b y_b\dot{\beta}) \tag{9}$$

$$\frac{\partial L}{\partial \alpha} = -m_b g x_b cos\alpha \tag{10}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_b}\right) - \frac{\partial L}{\partial x_b} = \left(m_b + \frac{I_b}{r_b^2}\right)\ddot{x}_b \\ -m_b(x_b\dot{\alpha}^2 + y_b\dot{\alpha}\dot{\beta}) + m_b g sin\alpha = 0 \tag{11}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\alpha}}\right) - \frac{\partial L}{\partial \alpha} = (I_b + I_p + m_b x_b^2)\ddot{\alpha} + \\ 2m_b x_b \dot{x}_b \dot{\alpha} + m_b x_b y_b \ddot{\beta} + m_b y_b \dot{x}_b \dot{\beta} \\ + m_b x_b \dot{y}_b \dot{\beta} + m_b g x_b cos\alpha = T_x \tag{12}$$

Similarly, equations can be obtained for y-direction.

These equations can be manipulated to represent the system in vector form. Henceforth, the way in which these vectors are written determine the way the system is modeled and controlled.

Approach 1

The system inputs are taken as plate angles rather than the torques on the plate. In a way, this approach limits the system to just the segment of interest i.e., the position of the ball. The state vector and dynamic equation of the system are:

$$X = \begin{bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \quad ; \quad \dot{X} = \begin{bmatrix} \dot{x} \\ \frac{x_b\dot{\alpha}^2 + y_b\dot{\alpha}\dot{\beta} - gsin\alpha}{C_1} \\ \dot{y} \\ \frac{y_b\dot{\beta}^2 + x_b\dot{\alpha}\dot{\beta} - gsin\beta}{C_1} \end{bmatrix} \tag{13}$$

Where, $C_1 = 1 + \frac{I_b}{m_b r_b^2}$

It is evident from these equations that the non-linearity in this system is too complex to be worked upon. Thus the system is linearized and the equations are decoupled in x and y directions to apply individual control laws in independent directions x and y. Such approach is relevant when the control input is directly the angle of the plate, and hence the hardware implementation of the system involves servo motors whose angles directly map on to the angle of inclination of the plates.

Approach 2

In this approach, the entire system, including the plate is modeled. Thus the state vector and the dynamic equation of the system are:

$$X = \begin{bmatrix} x \\ \dot{x} \\ \alpha \\ \dot{\alpha} \\ y \\ \dot{y} \\ \beta \\ \dot{\beta} \end{bmatrix} ; \dot{X} = \begin{bmatrix} \dot{x} \\ \frac{x_b\dot{\alpha}^2 + y_b\dot{\alpha}\dot{\beta} - gsin\alpha}{C_1} \\ \dot{\alpha} \\ \frac{(T_x - 2m_b x_b \dot{x}_b \dot{\alpha} - m_b x_b y_b \ddot{\beta} - m_b y_b \dot{x}_b \dot{\beta} - m_b x_b \dot{y}_b \dot{\beta} - m_b g x_b cos\alpha)}{C_2 + m_b x_b^2} \\ \dot{y} \\ \frac{y_b\dot{\beta}^2 + x_b\dot{\alpha}\dot{\beta} - gsin\beta}{C_1} \\ \dot{\beta} \\ \frac{(T_y - 2m_b y_b \dot{y}_b \dot{\beta} - m_b x_b y_b \ddot{\alpha} - m_b x_b \dot{y}_b \dot{\alpha} - m_b y_b \dot{x}_b \dot{\alpha} - m_b g y_b cos\beta)}{C_2 + m_b y_b^2} \end{bmatrix} \tag{14}$$

Where, $C_2 = I_b + I_p$ The control inputs are torques on plates applied by the motors. This approach is thus relevant to those hardware implementations in which electrical command given to the motors (as voltage or current) directly generates proportional amount of torque.

III. Control Methodology

The control methodologies adopted for both the approaches to model the system are different. While in the first approach, focus is on the position of the ball and the plates are not monitored, the second approach requires monitoring and control of every aspect, namely- the angular velocities as well as angle of inclinations of plate in both directions in addition to the state of the ball.

A. Method 1: Using PID

PID control has been implemented to control the position of the ball.The system is dynamically represented as shown in [13]. The first step is to linearize these equations and decouple the system after making certain assumptions. These decoupled equations are:

$$\ddot{x}_b = -Csin\alpha \quad ; \quad \ddot{y}_b = -Csin\beta \tag{15}$$

where $C = g/C1$. Thus the system can be represented in state space form as:

$$\dot{X} = AX + BU \tag{16}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad ; \quad B = \begin{bmatrix} 0 & 0 \\ -C & 0 \\ 0 & 0 \\ 0 & -C \end{bmatrix} \quad ; \quad U = \begin{bmatrix} sin\alpha \\ sin\beta \end{bmatrix}$$

For small values of alpha and beta, the values $sin\alpha$ and $sin\beta$ can be approximated as α and β.

PID Controller Design

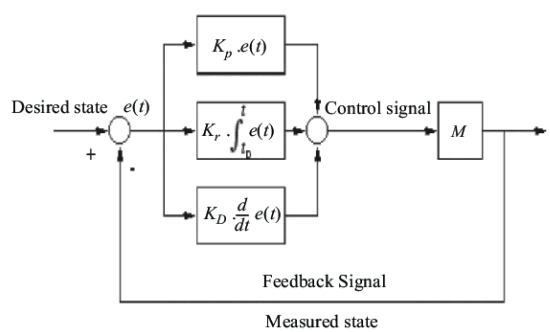


Fig. 2: Block Diagram of PID Controller

The desired position of the ball is fed as an input to the system. The error is the difference between the desired position and the current position of the ball, which is minimised through the control input to the system represented by the equation:

$$K_p e(t) + K_i \int_{t_0}^{t} e(t)\,dt + K_d \frac{d}{dt} e(t) \tag{17}$$

Here, the constants K_p, K_i and K_d are the parameters that need to be tuned to obtain the desired response. Thus with this control law, the overall transfer function of the system can be represented as:

$$T(s) = C(s)G(s) = (K_p + sK_d + \frac{K_i}{s})(\frac{1}{s^2}) \tag{18}$$

From the condition of stability (all poles of this transfer function lie on the left hand side of the complex plane), the following condition has been obtained for the controller: $K_pK_d > K_i$. Table II lists all the parameters describing the system.

TABLE II: System parameters for PID controller

Parameter	Description	Value
m_b	mass of ball	4
I_b	Moment of Inertia of ball	5
I_p	Moment of Inertia of plate	6
K_p	Proportional gain constant	7
K_d	Derivative gain constant	8
K_i	Integral Gain constant	9
e_t	Error between desired and current posiition	
$\frac{d}{dt}e(t)$	Error between desired and current velocities	
$\int_{t_0}^{t} e(t)\,dt$	Error integrated over time	

This control law is applied separately in both directions x and y to minimise the individually determined errors in x and y directions respectively. The selection of K_p, K_i and K_d values give minimum settling time of the system response and negligible steady state error and are selected by hit and trial method.

The above control law is applied to control the position of ball in two scenarios: (i.) Fixed point tracking and (ii.) Trajectory tracking.

(i). Fixed point tracking can further be performed using two approaches, first, giving the final position of the ball and minimising the error between desired position and starting position in minimum time. The second approach is basically trajectory tracking where the desired trajectory is a straight line path from the starting position to the final position and then staying at the final position. This approach finds application in how quickly the ball should reach the final position and taking what path.

(ii). Trajectory tracking makes use of the underlying principles of fixed point tracking where the desired position is constantly changing point by point, and the error between the current position and the desired position at the current moment is minimised, eventually tracing a specific path. A circular trajectory has been traced in this case to show trajectory tracking capability.

B. Method 2 : Using LQR

Linear Quadratic Regulator is generally employed in applications where a stability problem needs to be addressed. The ball and plate system can also be analysed and presented as a stability problem where the ball needs to be balanced on a fixed position of the plate. The system is dynamically represented in [14] along with the 8-dimensional state vector. In order to simplify the system dynamics, linearization has been performed about the fixed point selected as origin. A fixed point of the system is a point where the system is stable if untouched. The horizontal position of the plate is stable when the ball is at origin (if the mass of the ball is not considerably less than the mass of the plate). When the ball is displaced from this position, system destabilizes and reaches the minimum energy point (where the vertical position of the ball is lower than its position at origin when the plate is horizontal, so that potential energy falls). In the neighbourhood of this fixed point, the system can be linearized. Hence, for the dynamical equations, Jacobian has been evaluated about the Zero state and following matrices are obtained:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -C & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -C_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -C & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -C_3 & 0 & 0 & 0 \end{bmatrix}; B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{C_2} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \frac{1}{C_2} \end{bmatrix}$$

$$\text{and Zero Vector } = X_0 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \tag{19}$$

Where $C_3 = \frac{m_b g}{C_2}$. The system can now be represented having the state space model:

$$\dot{X} = AX + BU \tag{20}$$

$$Y = CX + DU \tag{21}$$

where,

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}; D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$
$$Y = \begin{bmatrix} x_b \\ y_b \end{bmatrix} \text{ and } U = \begin{bmatrix} T_x \\ T_y \end{bmatrix} \tag{22}$$

LQR Controller Design

For this linearized state space system, the control input is proportional to the current state and is mathematically written as : $U = -KX$ where K is the gain matrix. The system eventually becomes: $\dot{X} = (A - BK)X$.

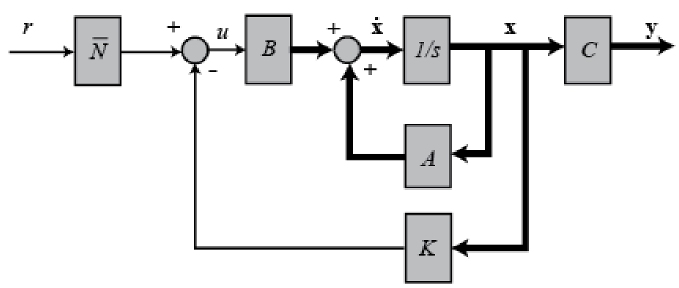


Fig. 3: Block Diagram ofLQR Controller

The response of this system is determined by the eigenvalues or poles. By controlling matrix K, the poles can be shifted to the left half of the complex plane to stabilize the system. Further, the position of these poles determine the response of the system, in terms of steady state error and settling time. One method to determine K is using manual pole placement. But it requires the intuition that a chosen eigenvalue will affect the system in the desired way. As its hard to establish a direct relation between the position of various poles and the effect it has on the system, a much more intuitive method has been adopted. Using LQR, the gain matrix K is determined according to the degree of control that needs to be applied in each dimension of the state space vector.

(i). Fixed Point Tracking: In order to stabilize the system to the fixed point, i.e. move the ball to the origin from any initial position, the following diagonal matrices have been selected:

$$Q = \begin{bmatrix} 1000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 100 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} ; R = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix} \tag{23}$$

Each diagonal element of Q matrix expresses the desire of the particular dimension to be aggressively stabilized, i.e. it determines the amount of penalty to be levied upon the system if the corresponding dimension is not stabilized. Each diagonal element of R matrix penalises the amount of control given to the system in the particular dimension. Thus, the quadratic cost function:

$$J(u) = \int_0^\infty (x^T Qx + u^T Ru + 2x^T Nu)dt \tag{24}$$

is minimised. The feedback gain matrix K, is found out by solving the ricatti equation:

$$A^T S + SA - (SB + N)R^{-1}(b^T S + N^T) + Q = 0 \tag{25}$$

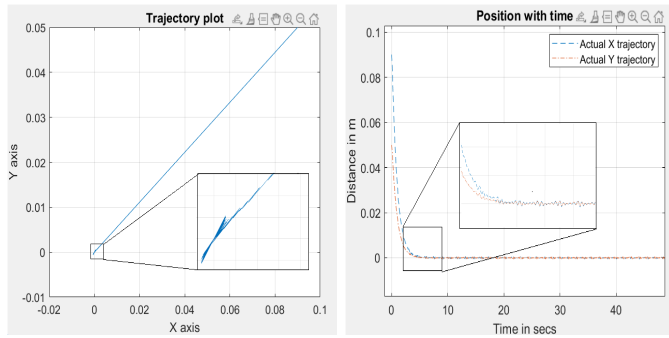
$$K = R^{-1}(b^T S + N^T) \tag{26}$$

In order to move the ball to any coordinate other than zero, the difference in the state of the desired position and the current position, referred to as the error, is fed to the controller which is then stabilized to the fixed point, i.e. made zero.

(ii). Trajectory tracking: The ball is made to trace a circular trajectory to exhibit trajectory tracking capability. This has been achieved by constantly minimising errors between desired positions changing with time and the current position, also eventually changing with time. The catch is diminishing the error between current position and the desired position before the next point of the trajectory is fed as input to the controller. Thus, it requires higher value of elements of Q matrix and lenient R matrix such that the response time becomes significantly smaller. The following Q and R matrices have been selected:

$$Q = \begin{bmatrix} 10^7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.01 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.01 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 10^7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.01 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.01 \end{bmatrix} ; R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{27}$$

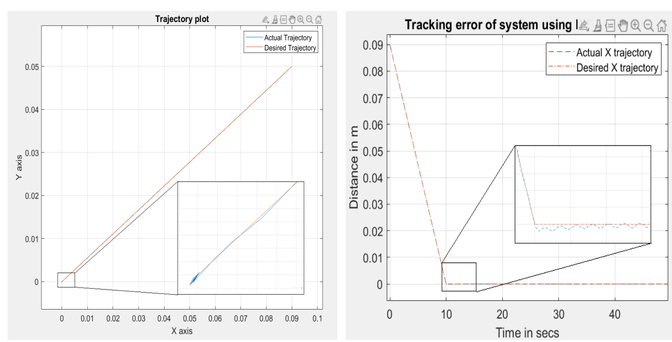
IV. Results and Discussion



(a) Trajectory Plot (b) Tracking position with time

Fig. 4: Fixed point tracking using PID−direct approach

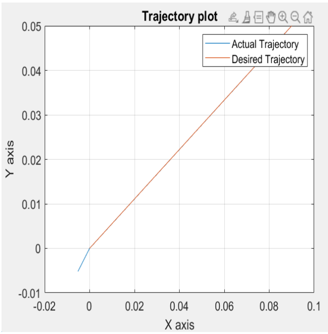
1.(i). Fixed Point Tracking using PID − Approach 1: Fig. 4 depicts the trajectory of the ball on the plate and its x and y coordinates with time. A small chattering is observed at steady state. These oscillations are of magnitude and order 5×10^{-4} or $0.5mm$. The initial point is $(0.09, 0.05)$m and final point is origin $(0, 0)$. The settling time is near 5 seconds.



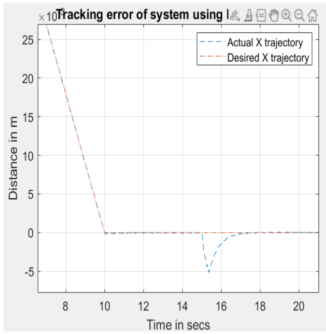
(a) Trajectory Plot (b) Tracking position with time

Fig. 5: Fixed point tracking using PID−trajectory approach

1.(ii). Fixed Point Tracking using PID − Approach 2: Here, the desired trajectory is a straight line leading to the final point origin $(0, 0)$ from initial point $(0.09, 0.05)$. As shown in Fig. 5, The ball traces the straight line path accurately with negligible error of magnitude and order 4×10^{-5} or $0.04mm$. Small chattering is also observed on the fixed point of order $\sim 10^{-4}$. Disturbance rejecton is depicted in Fig. 6. A disturbance in the control input causes the ball to leave its trajectory, which is corrected by controller.

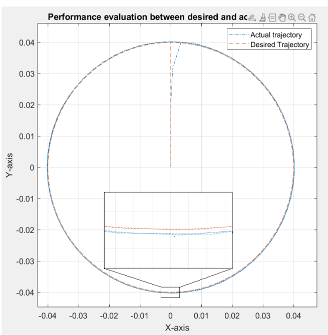


(a) Trajectory Plot

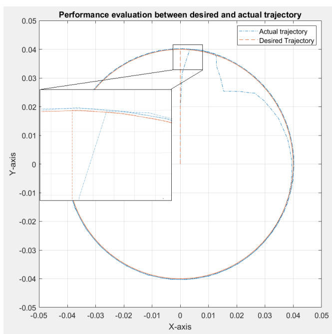


(b) Tracking position with time

Fig. 6: Disturbance Rejection while tracking fix-point using PID



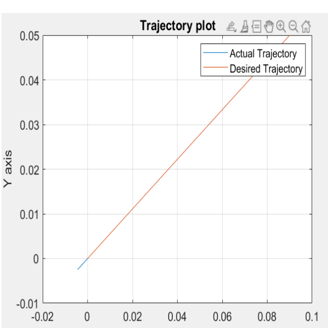
(a) Trajectory Plot — without any disturbance



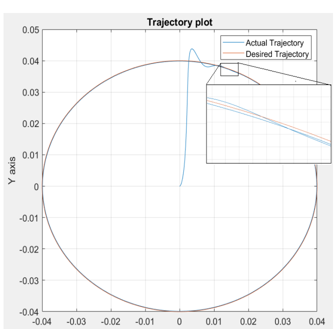
(b) Trajectory plot — with disturbance

Fig. 7: Circular Trajectory tracking using PID

1.(iii). Circular Trajectory Tracking using PID: Fig.7a demonstrates the path taken by the ball in order to trace a circle of radius $0.4m$. There is an almost constant error of magnitude and order 2.5×10^{-4} or $0.25mm$. Fig. 7b represents the path of the ball when its disturbed from its path.



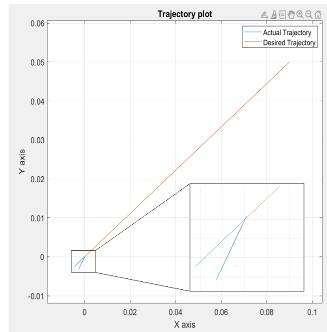
(a) Trajectory Plot — Fixed Point tracking



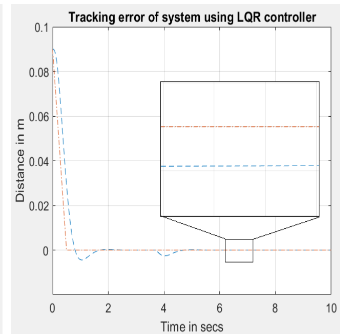
(b) Trajectory plot —Circular Trajectory tracking

Fig. 8: Controlling system using LQR

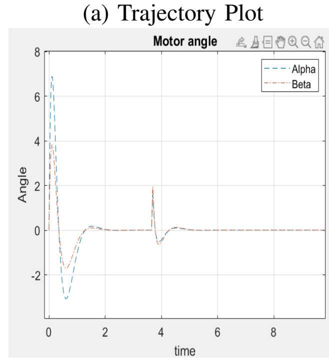
2.(i). Fixed Point tracking using LQR: The ball takes a straight line path from the initial point $(0.09, 0.04)m$ to the final point $(0, 0)$, Fig. 8a. An overshoot of magnitude $2.5 \times 10^{-3}m$ or $0.25cm$ is observed. The settling time is about 2 seconds. A constant steady state error of order $10^{-5}m$ is obtained. When disturbed slightly, the system stabilizes again as shown in Fig. 9c. Motor torques have been limited by keeping the system less agressive, as is interpretable from the response time.



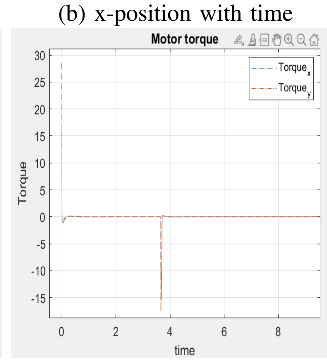
(a) Trajectory Plot



(b) x-position with time



(c) Plate Angles vs time



(d) Motor Torques vs time

Fig. 9: Disturbance Rejection while tracking fixed point using LQR

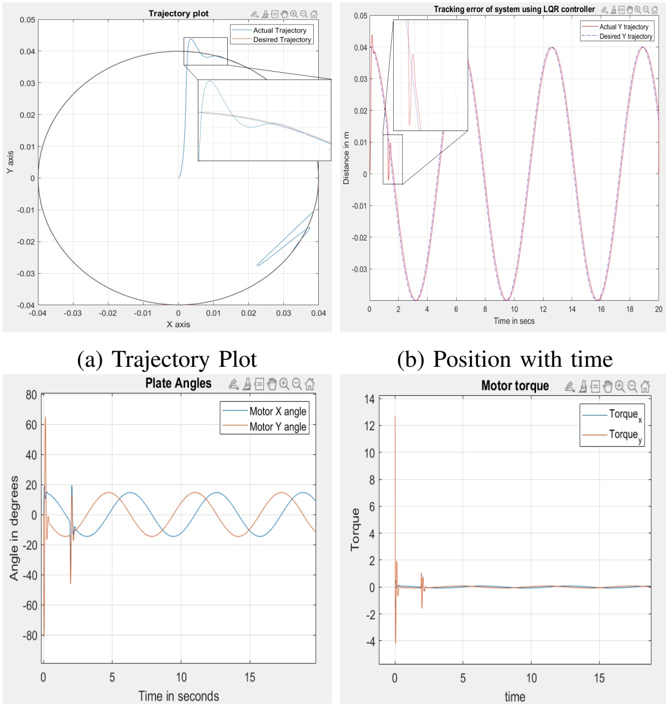
2.(i). Circular Trajectory tracking using LQR: Fig. 8b shows the trajectory plot of the ball following a circular trajectory of 0.04m. There is a constant error of order $10^{-4}m$ or $0.1mm$. Fig.10 illustrates the path of the ball when a random disturbance derails the ball from its desired trajectory. The ball quickly recovers and starts tracing the circle again. As this motion requires aggressive control of the ball, Plate angles and Motor torques show impulsive behaviour (Fig.10c and 10d).

V. Novelty in Work

This paper focuses on simple and widely popular techniques to robustly control a seemingly-complex system. Emphasis has been laid on accurately modeling the system using two separate approaches. Comparisons have been made not only between different methods of control, but also, different methods of modeling. Errors have been reduced to almost zero to obtain near-idle response of the system.

VI. Conclusion and Future Work

The control algorithms for PID and LQR were succesfully implemented for fixed point and trajectory tracking. The simulation results are promising and demonstrate that the ball



(a) Trajectory Plot

(b) Position with time

(c) Plate Angles vs time

(d) Motor Torques vs time

Fig. 10: Disturbance Rejection while tracking circular trajectory using LQR

would follow proper trajectory with minute errors in each case. However, based on the settling time, steady state error and oscillations in the response, its conclusive that LQR control performed better than PID control.

In the future, we aim to improve the response of the system. SMC, Fuzzy Control and techniques like Back-stepping, twisting and super-twisting can be employed. The use of Machine Learning algorithms to improve the response of the system can be analyzed. Further, Implementing the control on hardware system and observing the performance for each method will be carried out. The performance can also be judged on the application of avoiding hurdles while following a set trajectory.

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