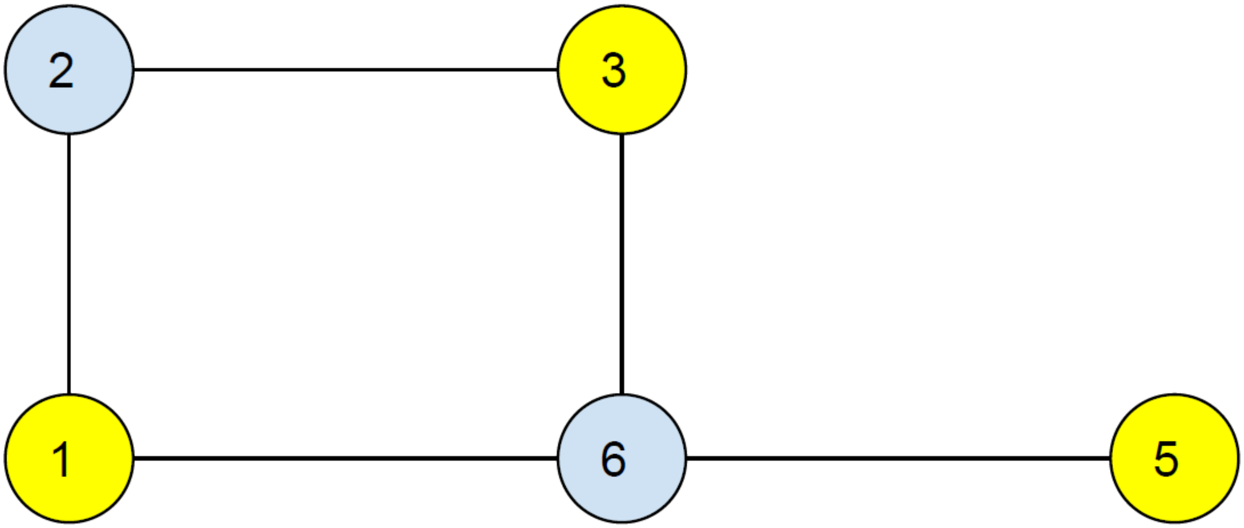
Write a flow ILP for the Steiner tree problem for the graph (see attached) with yellow terminals 1, 3, 5.



Solution:

$S = \{1, 3, 5\}$

X_e : whether edge e is chosen or not chosen.

Minimize: $\sum_{e \in E} c_e X_e$,

$\sum_{e \in (A, A')} X_e >= 1$, for all S separating some (a_p, b_p) pair.

$X_e >= 0$.

Flow: $f_{pq}^{13} >= 0 \ \forall \ (p, q) \in E, \ \forall \ R \in V$

$\sum f_{pr}^{13} = \sum f_{rq}^{13}$

Source:

1: $\sum f_{p1}^{13}$ (incoming) = $\sum f_{1q}^{13} - 1$(outgoing)

Destination:

3: $\sum f_{p3}^{13}$ (incoming) = $\sum f_{3q}^{13} + 1$(outgoing)

Therefore,

For vertex1:

$X_{1,2} + X_{1,6} >= 1$ (Separating vertex 1)

$X_{2,3} + X_{3,6} >= 1$ (separating vertex 3)

$x_{56} >= 1$ (Separating vertex 5)

Edge between vertices 1,2:

$X_{1,2} >= f_{1,2}^{1,3}$

$X_{1,2} >= f_{2,1}^{1,3}$

$X_{1,2} >= f_{1,2}^{1,5}$

$X_{1,2} >= f_{2,1}^{1,5}$

Edge between vertices 2,3:

$X_{2,3} >= f_{2,3}^{1,3}$

$X_{2,3} >= f_{3,2}^{1,3}$

$X_{2,3} >= f_{2,3}^{1,5}$

$X_{2,3} >= f_{3,2}^{1,5}$

Edge between vertices1,6:

$X_{1,6} >= f_{1,6}^{1,3}$

$X_{1,6} >= f_{6,1}^{1,3}$

$X_{1,6} >= f_{1,6}^{1,5}$

$X_{1,6} >= f_{6,1}^{1,5}$

Edge between vertices 3,6

$X_{3,6} >= f_{3,6}^{1,3}$

$X_{3,6} >= f_{6,3}^{1,3}$

$X_{3,6} >= f_{3,6}^{1,5}$

$X_{3,6} >= f_{6,3}^{1,5}$

For the edge between 6, 5

$X_{5,6} >= f_{6,5}^{1,5}$

$X_{5,6} >= f_{5,6}^{1,5}$