Given n points in a rectilinear metric
Find: the smallest disc containing these points in O(n)

Solution:

Nimrod Megiddo has used prune-and-search techniques to find the minimal enclosing circle. This algorithm considers that each circle is defined by two or three of the given n points and notices the smallest of these points, which contain every point in O(n) time.

Prune-and-Search Algorithm

The prune-and-search algorithm is a technique of removing subsets of inputs without altering the solution. This algorithm can be applied where the answer is finally concluded by a small set of data, which is essential. The rest of the data is extra and not valid.

Instead of trying to locate the crucial data, we can identify the unwanted data and discard it.

Suppose our set has n input elements. We discard ¼ of them, which are outliers to the solution. We can recursively apply this method to the remaining items and finally reducing the input to $n \le 3$

Total Time Taken = $(n + 3n/4 + 9n/16 + \ldots.)$
$= n * (1 + 3/4 + 9/16 + \ldots.)$
$= n * (1/ (1-3/4))$ {GP sum of infinite series = $a/(1-r)$}
$= 4n$
So, this technique never exceeds less than 4n.

Minimum Enclosing Circle

This algorithm removes at least n/16 points in each iteration without affecting the solution. Like in prune and search algorithm. We eliminate n/16 points each time until we reach the base case which is n = 3

Total Time Taken = $(n + 15n/16 + 225n/256 + \ldots.)$
$= n * (1 + 15/16 + 225/256 + \ldots.)$
$= n *(1/ (1 – 15/16))$
$= 16n$
Time Complexity = O(n)

The following functions used for this algorithm:

Median (A, >): take elements A and compares them pairwise by the operator given and returns median

MEC-center (A, L): takes elements A and Line L and determines the side of the center of MEC position if it’s to the right or left of the Line L

1. Randomly pairing n points in set A. Here we get n/2 pairs
2. Finding the intersection of perpendicular bisectors with x-axis for each pair. Here we get n/2 bisectors
3. Using the median function, find the bisector with a median slope, which is M’.
4. Pair each bisector like this (slope >= M’, Slope < M’). Here we get n/4 convergence points, and these points are B.
5. Using median function again for points B for y-coordinates and let that be Y’.
6. Using the MEC-center function to find the location of the center with line y = Y’ (parallel to the x-axis), assume center C’ is above.
7. Say C has the points which are below the line y = Y’. So, C has n/8 points.
8. Finding the line L with slope M’ such that ½ points in C lie to the left of L and ½ to its right.
9. Using the MEC-center function to find the location of the center with line L. Assume C’ lies on L’s right.
10. Say D be the set of points which are lying left of line L. So, D has n/16 points.

Here we have found that center C’ should lie above Y’ and to the right of Line L and points in D are below Y’ and are to the left of line L.

Bisectors, having slope >= M’ in point D, never passes through the quadrant where C’ is there. The two points with that bisector have two points Px, Py where Px is nearer to C’ than Py. So Px can be discarded for each n/16 convergence point in D.

So, we can remove one point each time in A for n/16 intersections in D. this algorithm helps in finding the smallest disc using n points in linear time – O(n)