**Question – 1 Solution:**

1. **General Integer Linear Programming for Minimum Vertex Cover**

* A decision variable xa is taken ∀ a ∈ Vertices V
* xa = 1 -> we place the ‘a’ vertex into the cover

xa = 0 -> we don’t place the ‘a’ vertex into the cover

* We used Linear cost function and linear constraints on the variables xa

To formulate Weighted Vertex Cover Problem

Weights (W) = (w1, w2, ……., wa) are the weights for the vertices.

* Minimize the total weights of the vertices in the Cover which subject to the vertices in C form a cover
* Total Weight = Σ a ∈ C weight (V)

= Σ a ∈ V weight (V) \* xa

Instead of taking the sum over vertices in the cover, we can take the formulated variable weight multiplied by the vertex xa

* If a is in cover xa = 1 else xa = 0
* Constraints:

Vertices form a cover

a ∈ C or b ∈ C ∀ (a, b) ∈ Edges E

xa + xb >= 1 ∀ (a, b) ∈ Edges E

xa ∈ {0,1} ∀ a ∈ Vertices V

**Minimize Σ a ∈ V weight (V) \* xa subject to xa + xb >= 1 ∀ (a, b) ∈ Edges E and**

**xa ∈ {0,1} ∀ a ∈ Vertices V**

**Algorithm:**

**Begin: Weighted\_Vertex\_Cover\_LP(G)**

1. Generate the linear programming corresponding to graph G.
2. Minimize Σ a ∈ V weight (V) \* xa subject to xa + xb >= 1 ∀ (a, b) ∈ Edges E and

xa ∈ {0,1} ∀ a ∈ Vertices V

1. Solve the 0/1 Linear programming
2. Cover C <- {a ∈ Vertices V: xa = 1}
3. Return Cover C

**End**

0/1 Linear Programming is NP-Hard (not a feasible solution)

1. **Integer Linear Programming for the given graph**

1 2

6 3

5 4

1. Objective: Minimize Σ a ∈ V weight (V) \* xa

1 \* x1 + 2 \* x2 + 3 \* x3 + 4 \* x4 + 5 \* x5 + 6 \* x6

1. Subject to:

x1 + x2 >= 1

x2 + x3 >= 1

x3 + x4 >= 1

x3 + x6 >= 1

x4 + x6 >= 1

x4 + x5 >= 1

x1 + x6 >= 1

x5 + x6 >= 1

Such that x2 = x4 = x6 = 1

x1 = x3 = x5 = 0

the above can be written in matrix form as where variables = vertices = columns in A

and constraints = edges = rows in A

AX >= b

A = X = B =

1. Considering the greedy approach, we get x1 = x3 = x5 = x4 = 1 and x2 = x6 = 0 so the sub-optimal cost is 13 whereas optimal x2 = x4 = x6 = 1 and x1 = x3 = x5 = 0 cost is 12

**Sub-optimal value <= 2 \* optimal value**

13 <= 2\*12

13 <= 24

**Question 2 Solution:**

1. **General ILP for Minimum Dominating Set**

* a decision variable xa, ∀ a ∈ V (vertices)
  + 1. xa = 1 => we place a in the cover
    2. xa = 0 => we don’t place a in cover
* We then formulate the weighted vertex cover using cost functions and linear constraints on the variables xa as Weights (W) = (w1, w2, ……., wa)
* Minimize total weight of vertices in V subject to vertices in D form a dominant set of total weight = Σa∈D da
* Instead of taking the weight for every vertex we take the formulated variable weight multiplied by vertex xa (if it is in xa we take xa = 1 and 0 otherwise)
* Minimize Σa∈D Wa . xa,subject to the vertices form a cover if
  + 1. a ∈ Dominant Set or all the adjacent vertices in Dominant set , ∀ (a, b) ∈ E
    2. Implies that xa = 1, xa ∈ Dominant Set, xa + Σj∈N(a) xj ≥ 1, N = {Neighboring vertices of i}
    3. Xa + Σj∈N(a) xj ≥ 1, ∀ neighboring edges ∈ E 🡪 this condition is to ensure the domination
* Xa ∈ {0, 1}, ∀ a ∈ V

**Algorithm:**

**Begin:** Weighted\_Dominating\_Set\_LP(G)

1. while there exist, non-dominated vertices do

randomly select a vertex a from all non-dominated vertices;

select a vertex u ∈ N[a] with the highest degree.

W := W ∪ {a};

1. Generate LP corresponding to G
   * + Minimize Σa∈D Wa . xa
     + Subject to xa + Σj∈N(a) xj ≥ 1, N = {Neighboring vertices of a}
     + xa ∈ {0, 1}, ∀ a ∈ V -> this is not a linear constraint and that’s why it is called as integer linear programming
2. Solve the linear programming problem
3. Dominating Set 🡨 {a ∈ V: xa = 1}
4. return Dominating Set

**End**

**Given graph:**

1 2

6 3

5 4

Objective: We want to minimize the number of vertices in the dominating set. So, the objective is to minimize 1 . x1 + 2 . x2 + 3 . x3 + 4 . x4 + 5 . x5 + 6 . x6

Each vertex needs to be in the set or have a neighbor in the set. So, we get a constraint for each vertex of the form Σa∈N(v) xa ≥ 1, for all v ∈ V.

Subject to

x1 + x2 + x6 >= 1, x1 + x2 + x3 >= 1, x2 + x3 + x4 + x6 >= 1, x3 + x4 + x5 + x6 >= 1, x4 + x5 + x6 >= 1, x1 + x3 + x4 + x5 + x6 >= 1

xa ∈ {0, 1}

if we represent this as a matrix form, we get AX >= B

A = X = B =

Solving the Dominating set problem, we get {1, 6}

thus, we have the integer constraints as x1 = x6 = 1 and x2 = x3 = x4 = x5 = 0

Considering the greedy approach (starting with the vertex having minimal weight), we get x1 = x4 = 1 and x2 = x3 = x5 = x6 = 0 so, optimal cost is 5.