

ASSIGNMENT-1

SAMEER KENDAL-EE22BTECH11044

Question : Suppose the equation of AB, BC and CA are respectively given by

$$\mathbf{n}_i^T \mathbf{x} = c_i \quad i = 1, 2, 3. \quad (1)$$

The equation of respective angle bisector are then given by

$$\frac{\mathbf{n}_i^T \mathbf{x} - c_i}{\|\mathbf{n}_i\|} = \pm \frac{\mathbf{n}_j^T \mathbf{x} - c_j}{\|\mathbf{n}_j\|} \quad i \neq j \quad (2)$$

Substitute numerical values and find the equations of the angle bisectors of A, B and C .

Solution: Using the (1) to calculate the normal equations of AB, BC and CA :-

$$AB : \mathbf{n}_1^T \mathbf{x} - c_1 = (7 \ 5) \mathbf{x} - 2 = 0, \quad (3)$$

$$BC : \mathbf{n}_2^T \mathbf{x} - c_2 = (11 \ 1) \mathbf{x} + 38 = 0, \quad (4)$$

$$CA : \mathbf{n}_3^T \mathbf{x} - c_3 = (1 \ -1) \mathbf{x} - 2 = 0. \quad (5)$$

Using the (2) to calculate the angle bisector of angle C :-

$$\frac{\mathbf{n}_i^T \mathbf{x} - c_i}{\|\mathbf{n}_i\|} = \pm \frac{\mathbf{n}_j^T \mathbf{x} - c_j}{\|\mathbf{n}_j\|} \quad (6)$$

Internal angular bisector can be evaluated by taking '+' sign in the above equation. Taking $i = 2$ and $j = 3$:-

$$\frac{\mathbf{n}_2^T \mathbf{x} - c_2}{\|\mathbf{n}_2\|} = \frac{\mathbf{n}_3^T \mathbf{x} - c_3}{\|\mathbf{n}_3\|} \quad (7)$$

$$\frac{(11 \ 1) \mathbf{x} + 38}{\sqrt{\mathbf{n}_2^T \mathbf{n}_2}} = \frac{(1 \ -1) \mathbf{x} - 2}{\sqrt{\mathbf{n}_3^T \mathbf{n}_3}} \quad (8)$$

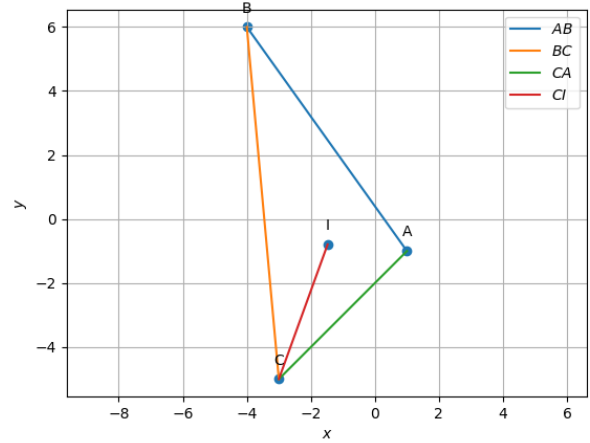


Fig. 0. Triangle generated using python

$$\Rightarrow \frac{(11 \ 1) \mathbf{x} + 38}{\sqrt{(11 \ 1) \begin{pmatrix} 11 \\ 1 \end{pmatrix}}} = \frac{(1 \ -1) \mathbf{x} - 2}{\sqrt{(1 \ -1) \begin{pmatrix} 1 \\ -1 \end{pmatrix}}} \quad (9)$$

$$\Rightarrow \frac{(11 \ 1) \mathbf{x}}{\sqrt{122}} - \frac{(1 \ -1) \mathbf{x}}{\sqrt{2}} = \frac{-2}{\sqrt{2}} - \frac{38}{\sqrt{122}} \quad (10)$$

$$\Rightarrow \left(\frac{11 - \sqrt{61}}{\sqrt{122}} \quad \frac{1 + \sqrt{61}}{\sqrt{122}} \right) \mathbf{x} = -\frac{2\sqrt{61} + 38}{\sqrt{122}} \quad (11)$$

Hence, the (11) is the equation of internal angular bisector of angle C .