Solution to 12.13.3.82

Sameer kendal - EE22BTECH11044

Question: Two dice are thrown. If it is known that sum of the numbers on the dice was less than 6,the probability of getting a sum 3, is

A) $\frac{1}{18}$

B) $\frac{5}{18}$

C) $\frac{1}{5}$

D) $\frac{2}{5}$

Solution: Let random variables such that

parameters	value	description
X	$1 \leq X \leq 6$	outcome of the first die
Y	$1 \leq Y \leq 6$	outcome of the second die

Consider a random variable W such that

$$W = X + Y; \tag{1}$$

W can take values from {2 to 12},

$$p_X(k) = \begin{cases} \frac{1}{6}, & 1 \leq k \leq 6 \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

$$p_X(k) = p_Y(k) \tag{3}$$

PMF of W using z-transform:
applying the z-transform on both the sides

$$z\{W\} = z\{X + Y\} \tag{4}$$

$$M_W(z) = M_{X+Y}(z) \tag{5}$$

Using the expectation operator:

$$E[z^{-W}] = E[z^{-X-Y}] \tag{6}$$

$$= E[z^{-X}] \cdot E[z^{-Y}] \tag{7}$$

$$= \left(\sum_{i=1}^{6} p_X(i) \cdot z^{-i} \right) \cdot \left(\sum_{j=1}^{6} p_Y(j) \cdot z^{-j} \right) \tag{8}$$

Extracting the PMF by considering the defenition of z-transform

$$M_W(z) = p_W(0) + p_W(1)z + ... + p_W(k)z^k + ... \tag{9}$$

$$= \frac{1}{36} \left(z^{-1} + ... + z^{-6} \right) \cdot \left(z^{-1} + ... + z^{-6} \right) \tag{10}$$

$$\begin{aligned} &= \frac{1}{36}(z^{-2} + 2z^{-3} + 3z^{-4} + 4z^{-5} + 5z^{-6} + 6z^{-7} \\ &\quad + 5z^{-8} + 4z^{-9} + 3z^{-10} + 2z^{-11} + z^{-12}) \end{aligned} \tag{11}$$

Defined for all values $2 \leq k \leq 12$
From (11) ,

$$p_W(k = 3) = \frac{2}{36} \tag{12}$$

$$p_W(k < 6) = p_W(k = 2) + p_W(k = 3) + p_W(k = 4) + p_W(k = 5) \tag{13}$$

$$= \frac{1}{36} + \frac{2}{36} + \frac{3}{36} + \frac{4}{36} \tag{14}$$

$$= \frac{10}{36} \tag{15}$$

We know,

$$p_W(k = 3|k < 6) = \frac{p_W((k = 3)(k < 6))}{p_W(k < 6)} \tag{16}$$

$$= \frac{\frac{2}{36}}{\frac{10}{36}} \tag{17}$$

$$= \frac{2}{10} \tag{18}$$

$$= \frac{1}{5} \tag{19}$$

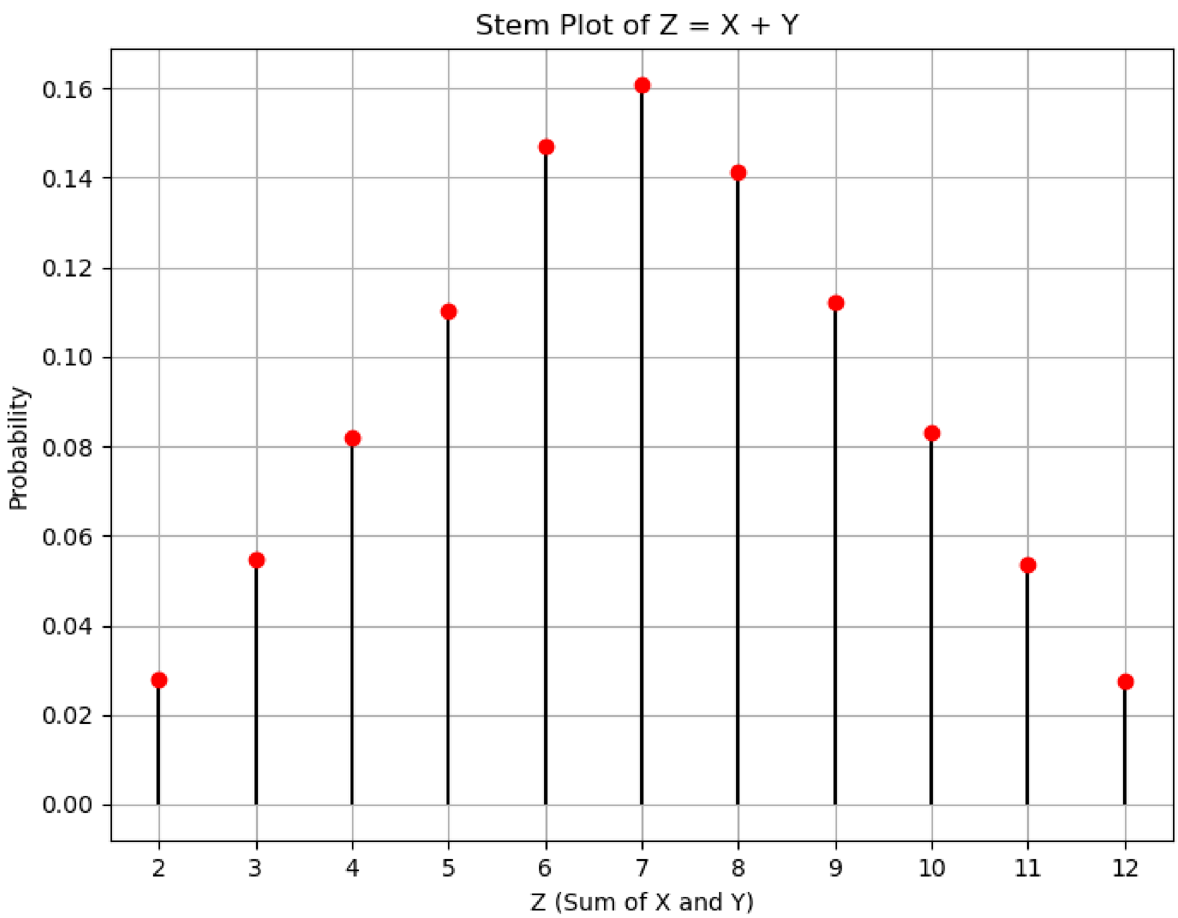

Fig. 4. Stem plot for P(Z)