Solution to 11.16.3.24

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Question: If $\Pr(A+B) = \Pr(AB)$ for any two events A and B , then

A) $\Pr(A)$=$\Pr(B)$
B) $\Pr(A) > \Pr(B)$
C) $\Pr(A) < \Pr(B)$
D) none of these

Solution:

$$\Pr(A) + \Pr(B) - \Pr(AB) = \Pr(A+B) \tag{1}$$

$$\Pr(A) + \Pr(B) - \Pr(AB) = \Pr(AB) \tag{2}$$

$$[\Pr(A) - \Pr(AB)] + [\Pr(B) - \Pr(AB)] = 0 \tag{3}$$

But,

$$\Pr(A|B) = \frac{\Pr(AB)}{\Pr(B)} \tag{4}$$

Also,

$$0 \le \Pr(A|B) \le 1 \tag{5}$$

$$\implies 0 \le \frac{\Pr(AB)}{\Pr(B)} \le 1 \tag{6}$$

$$\implies 0 \le \Pr(AB) \le \Pr(B) \tag{7}$$

Similarly,

$$0 \le \Pr(AB) \le \Pr(A) \tag{8}$$

$$\therefore \Pr(A) - \Pr(AB) \ge 0 \tag{9}$$

$$\Pr(B) - \Pr(AB) \ge 0 \tag{10}$$

$$\implies \Pr(A) - \Pr(AB) = 0 \tag{11}$$

$$\implies \Pr(A) = \Pr(AB) \tag{12}$$

Also,

$$\Pr(B) - \Pr(AB) = 0 \tag{13}$$

$$\implies \Pr(B) = \Pr(AB) \tag{14}$$

From equations (12) and (14), it can be said that

$$\Pr(A) = \Pr(B) \tag{15}$$