

Solution of question 9.3.12

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Question: In an examination, 20 questions of true-false type are asked. Suppose a student tosses a fair coin to determine his answer to each question. If the coin falls heads, he answer true; if it falls tails, he answer false. Find the probability that he answers at least 12 questions correctly.

Solution: Let us define:

Parameter	Value	Description
n	20	number of Questions
p	0.5	probability of answering correct
q	0.5	probability of answering wrong
$\mu = np$	10	mean of the distribution
$\sigma^2 = npq$	5	variance of the distribution
Y	0,1,2,3,...,20	Number of correct answers

$$Y \sim \mathcal{N}(\mu, \sigma^2) \quad (1)$$

The CDF of Y :

$$F_Y(y) = 1 - \Pr(Y > y) \quad (2)$$

$$= 1 - \Pr\left(\frac{Y - \mu}{\sigma} > \frac{y - \mu}{\sigma}\right) \quad (3)$$

But,

$$\frac{Y - \mu}{\sigma} \sim \mathcal{N}(0, 1) \quad (4)$$

$$(5)$$

the Q-function is defined as:

$$Q(x) = \Pr(Y > x) \quad \forall x \in Y \sim \mathcal{N}(0, 1) \quad (6)$$

therefore the cdf will be:

$$F_Y(y) = \begin{cases} 1 - Q\left(\frac{y - \mu}{\sigma}\right), & y > \mu \\ Q\left(\frac{\mu - y}{\sigma}\right), & y < \mu \end{cases} \quad (7)$$

The probability of getting atleast 12 answers correct:
considering 0.5 as coorection term:

$$\Pr(Y > 12.5) = 1 - F_Y(12.5) \quad (8)$$

$$= Q\left(\frac{12.5 - \mu}{\sigma}\right) \quad (9)$$

$$= Q\left(\frac{2.5}{\sqrt{5}}\right) \quad (10)$$

$$= Q(1.118) \quad (11)$$

Gaussian vs Binomial graph

