ASSIGNMENT-1

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Question : Suppose the equation of AB,BC and CA are respectively given by

$$\mathbf{n}_i^\top \mathbf{x} = c_i \qquad i = 1, 2, 3. \tag{1}$$

The equation of respective angle bisector are then given by

$$\frac{\mathbf{n}_i^\top \mathbf{x} - c_i}{\|\mathbf{n}_i\|} = \pm \frac{\mathbf{n}_j^\top \mathbf{x} - c_j}{\|\mathbf{n}_j\|} \qquad i \neq j \tag{2}$$

Substitute numerical values and find the equations of the angle bisectors of A, B and C.

Solution: Using the (1) to calculate the normal eqautions of AB,BC and CA :-

$$AB : \mathbf{n}_1^\top \mathbf{x} - c_1 = \begin{pmatrix} 7 & 5 \end{pmatrix} \mathbf{x} - 2 = 0, \tag{3}$$
$$BC : \mathbf{n}_2^\top \mathbf{x} - c_2 = \begin{pmatrix} 11 & 1 \end{pmatrix} \mathbf{x} + 38 = 0, \tag{4}$$
$$CA : \mathbf{n}_3^\top \mathbf{x} - c_3 = \begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} - 2 = 0. \tag{5}$$

Using the (2) to calculate the angle bisector of angle C :-

$$\frac{\mathbf{n}_i^\top \mathbf{x} - c_i}{\|\mathbf{n}_i\|} = \pm \frac{\mathbf{n}_j^\top \mathbf{x} - c_j}{\|\mathbf{n}_j\|} \tag{6}$$

Internal angular bisector can be evaluated by taking $'+'$ sign in the above equation.Taking $i = 2$ and $j = 3$:-

$$\frac{\mathbf{n}_2^\top \mathbf{x} - c_2}{\|\mathbf{n}_2\|} = \frac{\mathbf{n}_3^\top \mathbf{x} - c_3}{\|\mathbf{n}_3\|} \tag{7}$$

$$\frac{\begin{pmatrix} 11 & 1 \end{pmatrix} \mathbf{x} + 38}{\sqrt{\mathbf{n}_2^\top \mathbf{n}_2}} = \frac{\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} - 2}{\sqrt{\mathbf{n}_3^\top \mathbf{n}_3}} \tag{8}$$

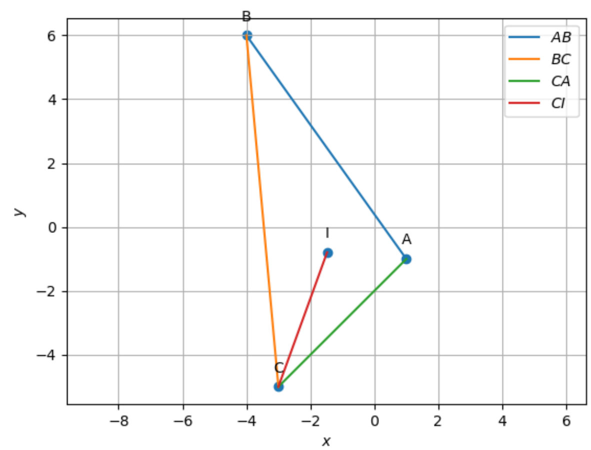

Fig. 0. Triangle generated using python

$$\implies \frac{\begin{pmatrix} 11 & 1 \end{pmatrix} \mathbf{x} + 38}{\sqrt{\begin{pmatrix} 11 & 1 \end{pmatrix} \begin{pmatrix} 11 \\ 1 \end{pmatrix}}} = \frac{\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} - 2}{\sqrt{\begin{pmatrix} 1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix}}} \tag{9}$$

$$\implies \frac{\begin{pmatrix} 11 & 1 \end{pmatrix} \mathbf{x}}{\sqrt{122}} - \frac{\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x}}{\sqrt{2}} = \frac{-2}{\sqrt{2}} - \frac{38}{\sqrt{122}} \tag{10}$$

$$\implies \begin{pmatrix} \frac{11 - \sqrt{61}}{\sqrt{122}} & \frac{1 + \sqrt{61}}{\sqrt{122}} \end{pmatrix} \mathbf{x} = -\frac{2\sqrt{61} + 38}{\sqrt{122}} \tag{11}$$

Hence, the (11) is the equation of internal angular bisector of angle C.