

Solution of question 9.3.12

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Question: In an examination, 20 questions of true-false type are asked. Suppose a student tosses a fair coin to determine his answer to each question. If the coin falls heads, he answer true; if it falls tails, he answer false. Find the probability that he answers at least 12 questions correctly.

Solution: Let us define:

Parameter	Value	Description
n	20	number of Questions
p	0.5	probability of answering correct
q	0.5	probability of answering wrong
$\mu = np$	10	mean of the distribution
$\sigma^2 = npq$	5	variance of the distribution
Y	0,1,2,3,...,20	Number of correct answers

The Gaussian distribution function is defined as:

$$p_Y(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad (x \in Y) \quad (1)$$

the Q-function from the gaussian distribution:

$$Q(x) = \int_x^\infty p_Y(t) dt \quad (2)$$

$$= \int_x^\infty \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(t-\mu)^2}{2\sigma^2}} dt \quad \text{from(1)} \quad (3)$$

The CDF of Y is given by:

$$F_Y(x) = \int_{-\infty}^x P_Y(t) dt \quad (4)$$

$$= 1 - \int_x^\infty P_Y(t) dt \quad (5)$$

$$= 1 - Q(x) \quad (6)$$

Probability of answering atleast 12 question correct is given by :

$$\Pr(Y \geq 12) = 1 - F_Y(11) \quad (7)$$

$$= Q(11) \quad \text{from(6)} \quad (8)$$

Gaussian vs Binomial graph

