FWC Assignment-1

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1) Write the vector equation of the line passing through $(1, 2, 3)$ and perpendicular to plane $\mathbf{r} \cdot (\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}) = 0$.

2) In the interval $\pi/2 < x < \pi$,find the value of x such that $\begin{pmatrix} 2\sin x & 3 \\ 1 & 2\sin x \end{pmatrix}$ is singular.

3) Find the solution of the differential equation $\frac{dy}{dx} = x^3 e^{-2y}$.

4) Write the integrating factor of the differential equation
$$\sqrt{x}\frac{dy}{dx} + y = e^{-2\sqrt{x}} \quad (1)$$
.

5) Write the direction ratio's of vector $3\mathbf{a} + 2\mathbf{b}$ where $\mathbf{a} = \mathbf{i} + \mathbf{j} - 2\mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$.

6) Find the projection of the vector $\mathbf{a} = 2\mathbf{i}+3\mathbf{j}+2\mathbf{k}$ on the vector $\mathbf{b} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$.

7) Evaluate $\int_0^{\pi/4} log\,(1 + \tan x)\,dx$.

8) Find $\int \frac{x}{(x^2 + 1)(x - 1)}$

OR

Find $\int_0^{\frac{1}{\sqrt{2}}} \frac{\sin^{-1} x}{(1 - x^2)^{\frac{3}{2}}}$.

9) Four cards are drawn successively with replacement from a well shuffled deck of 52 cards.What is the probability that

   a) all the four cards are spades ?

   b) only 2 cards are spades?

OR

A pair of dice is thrown four times. If getting a doublet is considered a success,find the probablity distribution of number of successes. Hence find the mean of distribution.

10) Prove that $[\mathbf{a}, \mathbf{b} + \mathbf{c}, \mathbf{d}] = [\mathbf{a}, \mathbf{b}, \mathbf{d}] + [\mathbf{a}, \mathbf{c}, \mathbf{d}]$.

11) Find the shortest distance between the following line: $\mathbf{r} = 2\mathbf{i} - 5\mathbf{j} + \mathbf{k} + \lambda\,(3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k})$ and $\mathbf{r} = 7\mathbf{i} - 6\mathbf{k} + \lambda\,(\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$.

12) Prove that $2\tan^{-1}(\frac{1}{2}) + \tan^{-1}(\frac{1}{7}) = \sin^{-1}(\frac{31}{25\sqrt{2}})$.

OR

Solve for x : $\tan^{-1}(\frac{1-x}{1+x}) = \frac{1}{2}\tan^{-1} x, x > 0$.

13) For what value of λ the function defined by
$$f(x) = \begin{cases} \lambda\left(x^2 + 2\right), & if x \leq 0 \\ 4x + 6, & if x > 0 \end{cases} \quad (2)$$
is continuous at $x = 0$? Hence check the differentiability of $f(x)$ at $x = 0$.

14) If x=$ae^t(\sin t + \cos t)$ and $y = ae^t(\sin t - \cos t)$, prove that $\frac{dy}{dx} = \frac{x+y}{x-y}$.

15) If y= Ae^{mx} + Be^{nx} ,show that $\frac{d^2y}{dx^2} - (m + n)\frac{dy}{dx} + mny = 0$.

16) Find $\int \frac{x+3}{\sqrt{5 - 4x - 2x^2}}$.

17) Evaluate: $\int_{-2}^{2} \frac{x^2}{1+5^x}dx$.

18) If
$$f(x) = \begin{cases} \frac{\sin(a+1)x + 2\sin x}{x}, & x < 0 \\ 2, & x = 0 \\ \frac{\sqrt{1+bx}-1}{x}, & x > 0 \end{cases} \quad (3)$$
is continuous at $x = 0$, then find the values of a and b.

19) A typist charges ₹145 for typing 10 English and 3 Hindi pages, while charges for typing 3 English and 10 Hindi pages are ₹180.

Using matrices, find the charges of typing one English and one English page separately. However typist charged only ₹2 per page from a poor student Shyam for 5 Hindi pages. How much less was charged from this peeo boy? Which values are reflected in this problem?

20) Solve for x: $\tan^{-1}(x-1)+\tan^{-1}(x+1)=\tan^{-1}(3x)$.

OR

Prove that $\tan^{-1}\left(\frac{6x-8x^3}{1-12x^2}\right)+\tan^{-1}\left(\frac{4x}{1-4x^2}\right)=\tan^{-1}2x; |2x|<\frac{1}{\sqrt{3}}$.

21) Using the method of integration, find the area of the triangular region whose vertices are (2,-2), (4,-3) and (1,2).

22) Using properties of determinants, prove that

$$\begin{vmatrix}(x+y)^2 & zx & zy\\ zx & (z+y)^2 & xy\\ zy & xy & (z+x)^2\end{vmatrix}=2xyz\,(x+y+z)^3 \tag{4}$$

OR

If $A=\begin{pmatrix}1&0&2\\0&2&1\\2&0&3\end{pmatrix}$ and $A^3-6A^2+7A+kI_3=O$ find k.

23) A retired person wants to invest an amount of ₹50,000. His broker recommends investing in the type of bonds 'A' and 'B' yielding 10% and 9% return respectively on the invested amount. He decides to invest at least ₹20,000 in bond 'A' and at least ₹10,000 in bond 'B'. He also wants to invest at least as much in bond 'A' as in bond 'B'. Solve this linear programming problem graphically to maximise his returns.

24) Find the equations of the plane which contains the line of intersection of the planes

$$\mathbf{r}\cdot(\mathbf{i}-2\mathbf{j}+3\mathbf{k})-4=0 \tag{5}$$

$$\mathbf{r}\cdot(-2\mathbf{i}+\mathbf{j}+\mathbf{k})+5=0 \tag{6}$$

and whose intercept on x-axis is equal to that of y-axis.

25) Prove that $y=\frac{4\sin\theta}{2+\cos\theta}-\theta$ is an increasing function of θ on $\left[0,\frac{\pi}{2}\right]$.

OR

Show that semi-vertical angle of a cone of a maximum volume and given slant height is $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$.

26) Let $A=R\times R$ and $*$ be a binary operation on A defined by $(a,b)*(c,d)=(a+c,b+d)$ Show that * is commutative and associative. Find the identity elemeny for * on A. Also find the inverse of every element $(a,b)\in A$.

27) Three numbers are selected at random (without replacement) from first six positive integers. Let X denote the largest of the three numbers obtained. Find the probability distribution of X. Also, find the mean and variance of the distribution.