Solution of question 9.3.12

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Question: In an examination, 20 questions of true-false type are asked. Suppose a student tosses a fair coin to determine his answer to each question. If the coin falls heads, he answer true; if it falls tails, he answer false. Find the probability that he answers at least 12 questions correctly.
Solution: Let us define:

Parameter	Value	Description
n	20	number of Questions
p	0.5	probability of answering correct
q	0.5	probability of answering wrong
$\mu = np$	10	mean of the distribution
$\sigma^2 = npq$	5	variance of the distribution
Y	0,1,2,3,...,20	Number of correct answers

1) Gaussian:

$$Y \sim \mathcal{N}\left(\mu, \sigma^2\right) \tag{1}$$

The CDF of Y:

$$F_Y(y) = 1 - \Pr\left(Y > y\right) \tag{2}$$
$$= 1 - \Pr\left(\frac{Y-\mu}{\sigma} > \frac{y-\mu}{\sigma}\right) \tag{3}$$

But,

$$\frac{Y-\mu}{\sigma} \sim \mathcal{N}\left(0,1\right) \tag{4}$$
$$\tag{5}$$

the Q-function is defined as:

$$Q\left(x\right) = \Pr\left(Y > x\right) \; \forall x \in Y \sim \mathcal{N}(0,1) \tag{6}$$

therefore the cdf will be:

$$F_Y\left(y\right) = \begin{cases} 1 - Q\left(\frac{y-\mu}{\sigma}\right), & y > \mu \\ Q\left(\frac{\mu-y}{\sigma}\right), & y < \mu \end{cases} \tag{7}$$

The probability of getting atleast 12 answers correct:
considering 0.5 as coorection term:

$$\Pr\left(Y > 12.5\right) = 1 - F_Y\left(12.5\right) \tag{8}$$
$$= Q\left(\frac{12.5-\mu}{\sigma}\right) \tag{9}$$
$$= Q\left(\frac{2.5}{\sqrt{5}}\right) \tag{10}$$
$$= Q\left(1.118\right) \tag{11}$$
$$= 0.13178 \tag{12}$$

Gaussian vs Binomial

Number of questions answered correctly	Binomial	Gaussian
Atleast 12	0.2517	0.13178