

Solution to 12.13.3.82

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Question: Two dice are thrown. If it is known that sum of the numbers on the dice was less than 6, the probability of getting a sum 3, is

A) $\frac{1}{18}$

B) $\frac{5}{18}$

C) $\frac{1}{5}$

D) $\frac{2}{5}$

Solution: Let random variables such that

parameters	value	description
X	$1 \leq X \leq 6$	outcome of the first die
Y	$1 \leq Y \leq 6$	outcome of the second die

Consider a random variable W such that

$$W = X + Y; \quad (1)$$

W can take values from $\{2 \text{ to } 12\}$,

$$p_X(k) = \begin{cases} \frac{1}{6}, & 1 \leq k \leq 6 \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

$$p_X(k) = p_Y(k) \quad (3)$$

Since X and Y are independent events,

$$M_W(z) = P_{X_1}(z)P_{X_2}(z) \quad (4)$$

$$M_W(z) = \frac{1}{36} (z^{-1} + \dots + z^{-6}) \cdot (z^{-1} + \dots + z^{-6}) \quad (5)$$

$$= p_W(2) + p_W(3)z + \dots + p_W(k)z^k + \dots \quad (6)$$

$$= \frac{1}{36} (z^{-2} + 2z^{-3} + 3z^{-4} + 4z^{-5} + 5z^{-6} + 6z^{-7} + 5z^{-8} + 4z^{-9} + 3z^{-10} + 2z^{-11} + z^{-12}) \quad (7)$$

From (7) ,

$$\Pr(W = 3) = \frac{2}{36} \quad (8)$$

$$\Pr(W < 6) = \Pr(W = 2) + \Pr(W = 3) + \Pr(W = 4) + \Pr(W = 5) \quad (9)$$

$$= \frac{1}{36} + \frac{2}{36} + \frac{3}{36} + \frac{4}{36} \quad (10)$$

$$= \frac{10}{36} \quad (11)$$

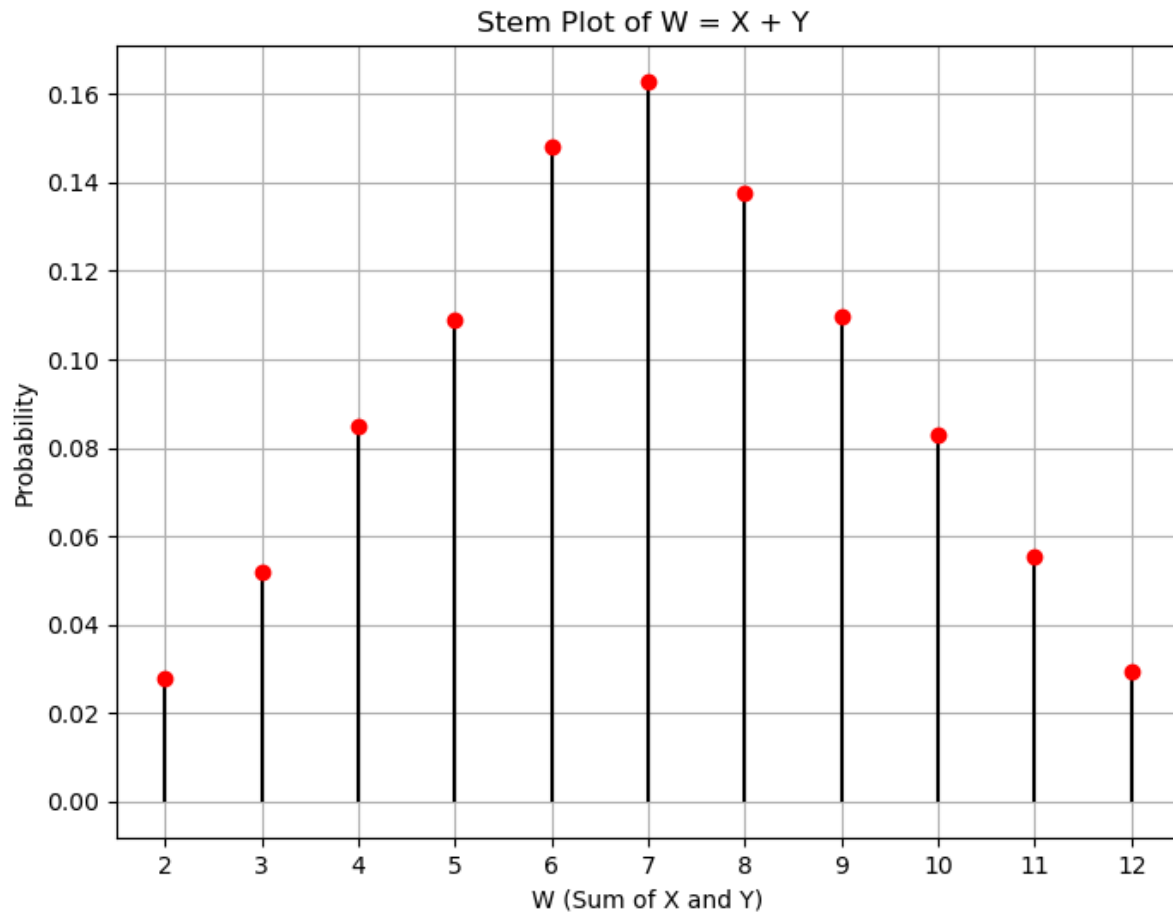


Fig. 4. Stem plot for $P(Z)$

We know,

$$\Pr(W = 3|W < 6) = \frac{\Pr((W = 3)(W < 6))}{\Pr(W < 6)} \quad (12)$$

$$= \frac{\frac{2}{36}}{\frac{10}{36}} \quad (13)$$

$$= \frac{2}{10} \quad (14)$$

$$= \frac{1}{5} \quad (15)$$