MATHEMATICS

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1 Matrices

1. In the interval $\pi/2 < x < \pi$,find the value of x such that $\begin{pmatrix} 2\sin x & 3 \\ 1 & 2\sin x \end{pmatrix}$ is singular.

2. Express matrix $A = \begin{pmatrix} 2 & 4 & -6 \\ 7 & 3 & 5 \\ 1 & -2 & 4 \end{pmatrix}$ as the sum of a symmetric and skew-symmetric matrix.

3. If $A = \begin{pmatrix} 2 & 3 \\ 1 & -4 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -2 \\ -1 & 3 \end{pmatrix}$,verify that $(AB)^{-1} = B^{-1}A^{-1}$.

4. Using properties of determinants, solve for x: $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$.

5. A trust fund has ₹35,000 to be invested in two different type of bonds.The first bond pays 8% interest per annum which will be given to orphanage and second bond pays 10% interst per annum which will be given to an N.G.O(Cancer Aid Society).Using matrix multiplication , determine how to divide ₹35,000 among two type of bond if the trust fund obtains a an interest total of ₹3,200. What are the values reflected in question?

2 Probability

6. Four cards are drawn successively with replacement from a well shuffled deck of 52 cards.What is the probability that

   (a) all the four cards are spades ?

   (b) only 2 cards are spades?

7. A pair of dice is thrown four times. If getting a doublet is considered a success,find the probablity distribution of number of successes. Hence find the mean of distribution.

8. In answering a question on multiple choice test,a student either knows the answer or guesses. Let $\frac{3}{5}$ be the probability that he knows the answer and $\frac{2}{5}$ be the probability that he guesses.Assuming that a student who guesses at the answer will be correct with the probability of $\frac{1}{3}$. What is the probability that the student knows the answer given that he answered it correctly?

3 Algebra

9. Prove that
$2\tan^{-1}(\frac{1}{2}) + \tan^{-1}(\frac{1}{7}) = \sin^{-1}(\frac{31}{25\sqrt{2}})$.

10. Solve for x : $\tan^{-1}(\frac{1-x}{1+x}) = \frac{1}{2}\tan^{-1}x, x > 0$.

4 Vector

11. Write the vector equation of the line passing through $(1, 2, 3)$ and perpendicular to plane $\overrightarrow{r}.\left(\hat{i} + 2\hat{j} - 5\hat{k}\right) = 0$.

12. Write the direction ratio's of vector $3\overrightarrow{a} + 2\overrightarrow{b}$ where $\overrightarrow{a} = \hat{i} + \hat{j} - 2\hat{k}$ and $\overrightarrow{b} = 2\hat{i} - 4\hat{j} + 5\hat{k}$.

13. Find the projection of the vector $\overrightarrow{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on the vector $\overrightarrow{b} = 2\hat{i} + 2\hat{j} + \hat{k}$.

14. Prove that $[\overrightarrow{a}, \overrightarrow{b} + \overrightarrow{c}, \overrightarrow{d}] = [\overrightarrow{a}, \overrightarrow{b}, \overrightarrow{d}] + [\overrightarrow{a}, \overrightarrow{c}, \overrightarrow{d}]$.

15. Find the shortest distance between the following line: $\overrightarrow{r} = 2\hat{i} - 5\hat{j} + \hat{k} + \lambda\left(3\hat{i} + 2\hat{j} + 6\hat{k}\right)$ and
$\overrightarrow{r} = 7\hat{i} - 6\hat{k} + \lambda\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$.

16. Find the vector and cartesian equation of the planes passing through the line of intersection of the planes
$\overrightarrow{r}.\left(2\hat{i} + 2\hat{j} - 3\hat{k}\right) = 7$, $\overrightarrow{r}.\left(2\hat{i} + 5\hat{j} + 3\hat{k}\right) = 9$.
such that the intercepts made by the plane on x-axis and z-axis are equal.

5 Differentiation

17. Find the solution of the differential equation $\frac{dy}{dx} = x^3e^{-2y}$.

18. For what value of λ the function defined by

$$f(x) = \begin{cases} \lambda\left(x^2 + 2\right), & if x \leq 0 \\ 4x + 6, & if x > 0 \end{cases}$$

is continuous at $x = 0$? Hence check the differentiability of $f(x)$ at $x = 0$.

19. If x=$ae^t(\sin t + \cos t)$ and $y = ae^t(\sin t - \cos t)$, prove that $\frac{dy}{dx} = \frac{x + y}{x - y}$.

20. If y= $Ae^{mx} + Be^{nx}$,show that $\frac{d^2y}{dx^2} - (m + n)\frac{dy}{dx} + mny = 0$.

21. Solve the differential equation

$$\left(x\sin^2\frac{y}{x} - y\right)dx + xdy = 0$$

given $y = \frac{\pi}{4}$ when $x = 1$.

22. Solve the differential equation $\frac{dy}{dx} - 3y \cot x = \sin 2x$ given $y = 2$ when $x = \frac{\pi}{2}$.

23. Write the integrating factor of the differential equation

$$\sqrt{x}\frac{dy}{dx} + y = e^{-2\sqrt{x}} \tag{1}$$

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6 Integration

24. Evaluate $\int_0^{\pi/4} log\,(1 + \tan x)\, dx$.

25. Find $\int \frac{x}{\left(x^2+1\right)(x-1)}$

26. Find $\int_0^{\frac{1}{\sqrt{2}}} \frac{\sin^{-1} x}{\left(1-x^2\right)^{\frac{3}{2}}}$.

27. Find $\int \frac{x+3}{\sqrt{5-4x-2x^2}}$.

7 Linear forms

28. Using integration, find the area enclosed by parabola enclosed by $4y = 3x^2$ and the line $2y = 3x + 12$.

29. Find the area of region $\{(x, y) : y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\}$, using integration.

8 Functions

30. Show that the relation R in the set $A = \{1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : |a - b| \text{is divisible by } 2\}$ is an equivalence relation. Write all equivalence classes of R.

9 Optimization

31. A manufacturer produces nuts and bolts. It takes 2 hours work on machine A and 3 hour work on machine B to produce a package of nuts.It take 3hours work on machine$A$ and 2 hour work on machine$B$ to produce a package of bolts.He earns a profit of ₹24 per package on nuts and ₹18 per package on bolts.How many packages of each should be produced each day so as to maximize his profit, if he opeartes his machine for at most 10 hours a day. Make an L.P.P. from above and solve graphically?.

32. The sum of surfaces areas of a sphere and a cuboid with sides $\frac{x}{3}$,x and $2x$, is constant.Show that sum of their volumes is minimum if x is equal to three times the radius of sphere.