```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from sklearn.model_selection import train_test_split
from sklearn.preprocessing import OneHotEncoder
from sklearn.metrics import confusion_matrix, precision_score, recall_score, f1_score
```

```
df = pd.read_csv("/content/drive/MyDrive/AI and ML/worksheet2/mnist_dataset.csv") # changed to read_csv and the correct file
# Step 2: Dataset Information
```

```
print(df.shape)
print(df.info())
```

```
(60000, 785)
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 60000 entries, 0 to 59999
Columns: 785 entries, label to pixel_783
dtypes: int64(785)
memory usage: 359.3 MB
None
```

Exercise Building a Softmax Regression for MNIST Digit Classification

Softmax function

```
def softmax(z):
  """
  Compute the softmax probabilities for a given input matrix.
  Parameters:
  z (numpy.ndarray): Logits (raw scores) of shape (m, n), where
  - m is the number of samples.
  - n is the number of classes.
  Returns:
  numpy.ndarray: Softmax probability matrix of shape (m, n), where
  each row sums to 1 and represents the probability
  distribution over classes.
  Notes:
  - The input to softmax is typically computed as: z = XW + b.
  - Uses numerical stabilization by subtracting the max value per row.
  """
# Your Code Here.
  z_shifted = z - np.max(z, axis=1, keepdims=True)
  exp_z = np.exp(z_shifted)
  return exp_z / np.sum(exp_z, axis=1, keepdims=True)
```

Softmax test function

```
z_test = np.array([[2.0, 1.0, 0.1], [1.0, 1.0, 1.0]])
softmax_output = softmax(z_test)
# Verify if the sum of probabilities for each row is 1 using assert
row_sums = np.sum(softmax_output, axis=1)
# Assert that the sum of each row is 1
assert np.allclose(row_sums, 1), f"Test failed: Row sums are {row_sums}"
print("Softmax function passed the test case!")
```

```
Softmax function passed the test case!
```

Prediction Function

```
def predict_softmax(X, W, b):
    """
    Predict the class labels for a set of samples using the trained softmax model.

    Parameters:
    X (numpy.ndarray): Feature matrix of shape (n, d), where n is the number of samples and d is the number of features.
    W (numpy.ndarray): Weight matrix of shape (d, c), where c is the number of classes.
    b (numpy.ndarray): Bias vector of shape (c,).

    Returns:
    numpy.ndarray: Predicted class labels of shape (n,), where each value is the index of the predicted class.
    """
```

```
    z = np.dot(X, W) + b  # Compute the scores (logits)
    y_pred = softmax(z)  # Get the probabilities using the softmax function

    # Assign the class with the highest probability
    predicted_classes = np.argmax(y_pred, axis=1)

    return predicted_classes
```

Test prediction case

```
# The test function ensures that the predicted class labels have the same number of elements as the
# input samples, verifying that the model produces a valid output shape.
# Define test case
X_test = np.array([[0.2, 0.8], [0.5, 0.5], [0.9, 0.1]]) # Feature matrix (3 samples, 2 features)
W_test = np.array([[0.4, 0.2, 0.1], [0.3, 0.7, 0.5]]) # Weights (2 features, 3 classes)
b_test = np.array([0.1, 0.2, 0.3]) # Bias (3 classes)
# Expected Output:
# The function should return an array with class labels (0, 1, or 2)
y_pred_test = predict_softmax(X_test, W_test, b_test)
# Validate output shape
assert y_pred_test.shape == (3,), f"Test failed: Expected shape (3,), got {y_pred_test.shape}"
# Print the predicted labels
print("Predicted class labels:", y_pred_test)
```

```
Predicted class labels: [1 1 0]
```

Loss function

```
def loss_softmax(y_pred, y):
    """
    Compute the cross-entropy loss.

    Parameters:
    y_pred (numpy.ndarray): Predicted probabilities of shape (n, c), where n is the number of samples and c is the number of
    y (numpy.ndarray): True labels (one-hot encoded) of shape (n, c).

    Returns:
    float: Cross-entropy loss.
    """
    epsilon = 1e-12  # To avoid log(0)
    y_pred = np.clip(y_pred, epsilon, 1.0 - epsilon)  # Prevent log(0) by clipping values
    n = y.shape[0]  # Number of samples
    loss = -np.sum(y * np.log(y_pred)) / n
    return loss
```

Test case for loss function

```
# This test case Compares loss for correct vs. incorrect predictions.
# Expects low loss for correct predictions.
# Expects high loss for incorrect predictions.
# Define correct predictions (low loss scenario)
y_true_correct = np.array([[1, 0, 0], [0, 1, 0], [0, 0, 1]]) # True one-hot labels
y_pred_correct = np.array([[0.9, 0.05, 0.05],[0.1, 0.85, 0.05],[0.05, 0.1, 0.85]]) # High confidence in the correct class

# Define incorrect predictions (high loss scenario)
y_pred_incorrect = np.array([[0.05, 0.05, 0.9],[0.1, 0.05, 0.85], [0.85, 0.1, 0.05]])  # Highly confident in the wrong class
# Compute loss for both cases

loss_correct = loss_softmax(y_pred_correct, y_true_correct)
loss_incorrect = loss_softmax(y_pred_incorrect, y_true_correct)

# Validate that incorrect predictions lead to a higher loss
assert loss_correct < loss_incorrect, f"Test failed: Expected loss_correct < loss_incorrect, but got{loss_correct:.4f} >= {l

# Print results
print(f"Cross-Entropy Loss (Correct Predictions): {loss_correct:.4f}")
print(f"Cross-Entropy Loss (Incorrect Predictions): {loss_incorrect:.4f}")
```

```
Cross-Entropy Loss (Correct Predictions): 0.1435
Cross-Entropy Loss (Incorrect Predictions): 2.9957
```

Cost function

```
def cost_softmax(X, y, W, b):
    """
    Compute the softmax regression cost (cross-entropy loss).

    Parameters:
    X (numpy.ndarray): Feature matrix of shape (n, d), where n is the number of samples and d is the number of features.
    y (numpy.ndarray): True labels (one-hot encoded) of shape (n, c), where c is the number of classes.
    W (numpy.ndarray): Weight matrix of shape (d, c).
    b (numpy.ndarray): Bias vector of shape (c,).

    Returns:
    float: The softmax cost (cross-entropy loss).
    """
    n = X.shape[0]  # Number of samples
    z = np.dot(X, W) + b
    y_pred = softmax(z)
    cost = loss_softmax(y_pred, y)
    return cost
```

Test of cost function

```
# Example 1: Correct Prediction (Closer predictions)
X_correct = np.array([[1.0, 0.0], [0.0, 1.0]]) # Feature matrix for correct predictions
y_correct = np.array([[1, 0], [0, 1]]) # True labels (one-hot encoded, matching predictions)
W_correct = np.array([[5.0, -2.0], [-3.0, 5.0]]) # Weights for correct prediction
b_correct = np.array([0.1, 0.1]) # Bias for correct prediction
# Example 2: Incorrect Prediction (Far off predictions)
X_incorrect = np.array([[0.1, 0.9], [0.8, 0.2]]) # Feature matrix for incorrect predictions
y_incorrect = np.array([[1, 0], [0, 1]]) # True labels (one-hot encoded, incorrect predictions)
W_incorrect = np.array([[0.1, 2.0], [1.5, 0.3]]) # Weights for incorrect prediction
b_incorrect = np.array([0.5, 0.6]) # Bias for incorrect prediction
# Compute cost for correct predictions
cost_correct = cost_softmax(X_correct, y_correct, W_correct, b_correct)
# Compute cost for incorrect predictions
cost_incorrect = cost_softmax(X_incorrect, y_incorrect, W_incorrect, b_incorrect)
# Check if the cost for incorrect predictions is greater than for correct predictions
assert cost_incorrect > cost_correct, f"Test failed: Incorrect cost {cost_incorrect} is not greaterthan correct cost {cost_c
# Print the costs for verification
print("Cost for correct prediction:", cost_correct)
print("Cost for incorrect prediction:", cost_incorrect)
print("Test passed!")
```

```
Cost for correct prediction: 0.0006234364133349324
Cost for incorrect prediction: 0.29930861359446115
Test passed!
```

Implement optimization with gradient descent

Compute the gradients

```
def compute_gradient_softmax(X, y, W, b):
    """
    Compute the gradients of the cost function with respect to weights and biases.

    Parameters:
    X (numpy.ndarray): Feature matrix of shape (n, d).
    y (numpy.ndarray): True labels (one-hot encoded) of shape (n, c).
    W (numpy.ndarray): Weight matrix of shape (d, c).
    b (numpy.ndarray): Bias vector of shape (c,).

    Returns:
    tuple: Gradients with respect to weights (d, c) and biases (c,).
    """
    n, d = X.shape
    z = np.dot(X, W) + b
    y_pred = softmax(z)

    grad_W = np.dot(X.T, (y_pred - y)) / n  # Gradient with respect to weights
    grad_b = np.sum(y_pred - y, axis=0) / n  # Gradient with respect to biases
```

```
    return grad_W, grad_b
```

Test case for compute gradient softmax function

```
import numpy as np
# Define a simple feature matrix and true labels
X_test = np.array([[0.2, 0.8], [0.5, 0.5], [0.9, 0.1]]) # Feature matrix (3 samples, 2 features)
y_test = np.array([[1, 0, 0], [0, 1, 0], [0, 0, 1]]) # True labels (one-hot encoded, 3 classes)
# Define weight matrix and bias vector
W_test = np.array([[0.4, 0.2, 0.1], [0.3, 0.7, 0.5]]) # Weights (2 features, 3 classes)
b_test = np.array([0.1, 0.2, 0.3]) # Bias (3 classes)
# Compute the gradients using the function
grad_W, grad_b = compute_gradient_softmax(X_test, y_test, W_test, b_test)
# Manually compute the predicted probabilities (using softmax function)
z_test = np.dot(X_test, W_test) + b_test
y_pred_test = softmax(z_test)
# Compute the manually computed gradients
grad_W_manual = np.dot(X_test.T, (y_pred_test - y_test)) / X_test.shape[0]
grad_b_manual = np.sum(y_pred_test - y_test, axis=0) / X_test.shape[0]
# Assert that the gradients computed by the function match the manually computed gradients
assert np.allclose(grad_W, grad_W_manual), f"Test failed: Gradients w.r.t. W are not equal.\nExpected: {grad_W_manual}\nGot:
assert np.allclose(grad_b, grad_b_manual), f"Test failed: Gradients w.r.t. b are not equal.\nExpected: {grad_b_manual}\nGot:
# Print the gradients for verification
print("Gradient w.r.t. W:", grad_W)
print("Gradient w.r.t. b:", grad_b)
print("Test passed!")
```

```
Gradient w.r.t. W: [[ 0.1031051   0.01805685 -0.12116196]
 [-0.13600547  0.00679023  0.12921524]]
Gradient w.r.t. b: [-0.03290036  0.02484708  0.00805328]
Test passed!
```

Perform gradient descent

```
def gradient_descent_softmax(X, y, W, b, alpha, n_iter, show_cost=False):
    """
    Perform gradient descent to optimize the weights and biases.

    Parameters:
    X (numpy.ndarray): Feature matrix of shape (n, d).
    y (numpy.ndarray): True labels (one-hot encoded) of shape (n, c).
    W (numpy.ndarray): Weight matrix of shape (d, c).
    b (numpy.ndarray): Bias vector of shape (c,).
    alpha (float): Learning rate.
    n_iter (int): Number of iterations.
    show_cost (bool): Whether to display the cost at intervals.

    Returns:
    tuple: Optimized weights, biases, and cost history.
    """
    cost_history = []

    for i in range(n_iter):
        # Compute gradients
        grad_W, grad_b = compute_gradient_softmax(X, y, W, b)

        # Update weights and biases using the gradients
        W -= alpha * grad_W
        b -= alpha * grad_b

        # Compute and store cost
        cost = cost_softmax(X, y, W, b)
        cost_history.append(cost)

        # Print cost at regular intervals
        if show_cost and (i % 100 == 0 or i == n_iter - 1):
            print(f"Iteration {i}: Cost = {cost:.6f}")

    return W, b, cost_history
```

Preparing the dataset

Load and prepare mnist

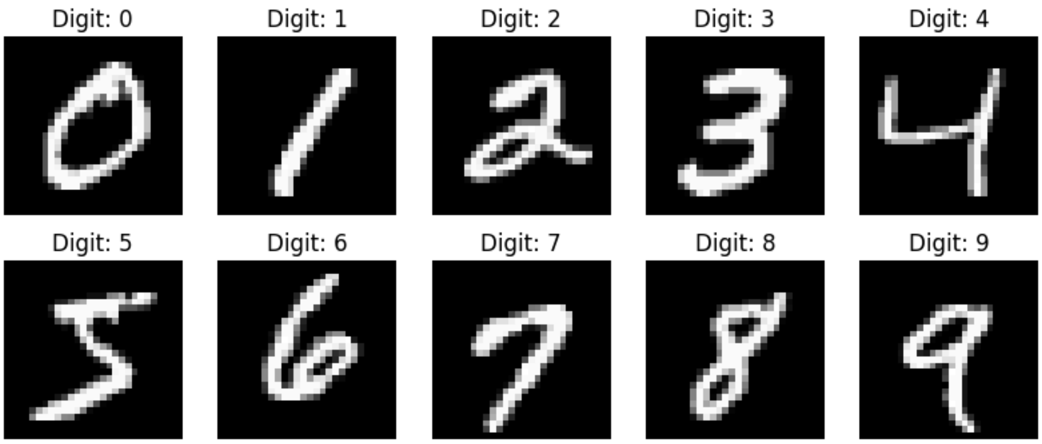
```
def load_and_prepare_mnist(csv_file, test_size=0.2, random_state=42):
    df = pd.read_csv(csv_file)
    y = df.iloc[:, 0].values
    X = df.iloc[:, 1:].values / 255.0
    X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=test_size, random_state=random_state)
    plot_sample_images(X, y)
    return X_train, X_test, y_train, y_test
```

Plot sample images

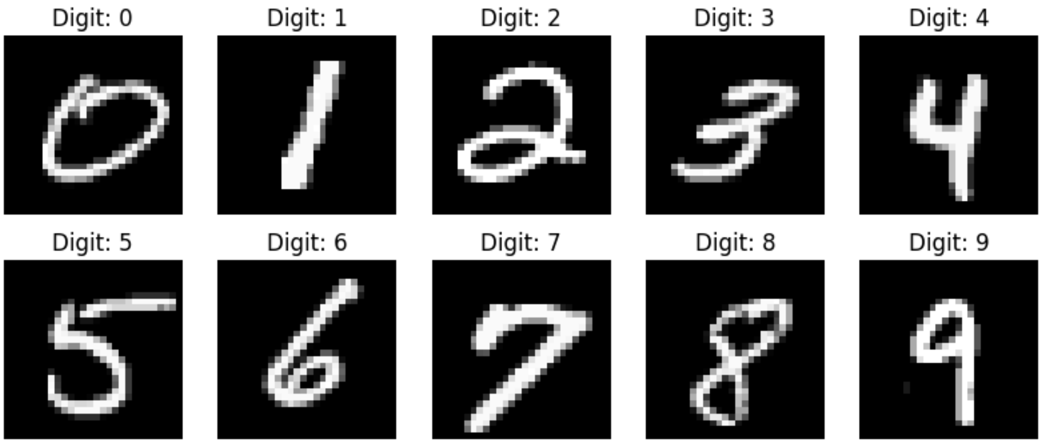
```
def plot_sample_images(X, y):
    plt.figure(figsize=(10, 4))
    unique_classes = np.unique(y)
    for i, digit in enumerate(unique_classes):
        index = np.where(y == digit)[0][0]
        image = X[index].reshape(28, 28)
        plt.subplot(2, 5, i + 1)
        plt.imshow(image, cmap='gray')
        plt.title(f"Digit: {digit}")
        plt.axis('off')
    plt.show()
```

Training of model

```
csv_file = "/content/drive/MyDrive/AI and ML/worksheet2/mnist_dataset.csv"
X_train, X_test, y_train, y_test = load_and_prepare_mnist(csv_file)
```



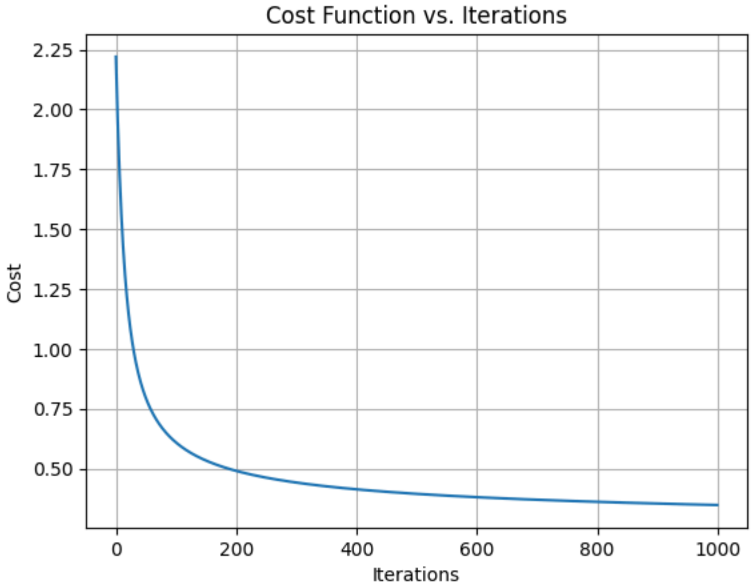

```
plot_sample_images(X_train, y_train)
```




```
# Check if y_train is one-hot encoded
if len(y_train.shape) == 1:
  encoder = OneHotEncoder(sparse_output=False) # Use sparse_output=False for newer versions of sklearn
  y_train = encoder.fit_transform(y_train.reshape(-1, 1)) # One-hot encode labels
  y_test = encoder.transform(y_test.reshape(-1, 1))
```

```
# Initialize parameters
W = np.random.randn(X_train.shape[1], y_train.shape[1]) * 0.01
b = np.zeros(y_train.shape[1])

# Train the model
alpha = 0.1
n_iter = 1000
W_opt, b_opt, cost_history = gradient_descent_softmax(X_train, y_train, W, b, alpha, n_iter, show_cost=True)


# Plot the cost history to visualize the convergence
plt.plot(cost_history)
plt.title("Cost Function vs. Iterations")
plt.xlabel("Iterations")
plt.ylabel("Cost")
plt.grid(True)
plt.show()
```

```
Iteration 0: Cost = 2.220247
Iteration 100: Cost = 0.608672
Iteration 200: Cost = 0.490325
Iteration 300: Cost = 0.441507
Iteration 400: Cost = 0.413312
Iteration 500: Cost = 0.394369
Iteration 600: Cost = 0.380494
Iteration 700: Cost = 0.369746
Iteration 800: Cost = 0.361090
Iteration 900: Cost = 0.353915
Iteration 999: Cost = 0.347891
```



Evaluate model

```
def evaluate_classification(y_true, y_pred):
    cm = confusion_matrix(y_true, y_pred)
    precision = precision_score(y_true, y_pred, average='weighted')
    recall = recall_score(y_true, y_pred, average='weighted')
    f1 = f1_score(y_true, y_pred, average='weighted')
    return cm, precision, recall, f1
```

Predict on the test set

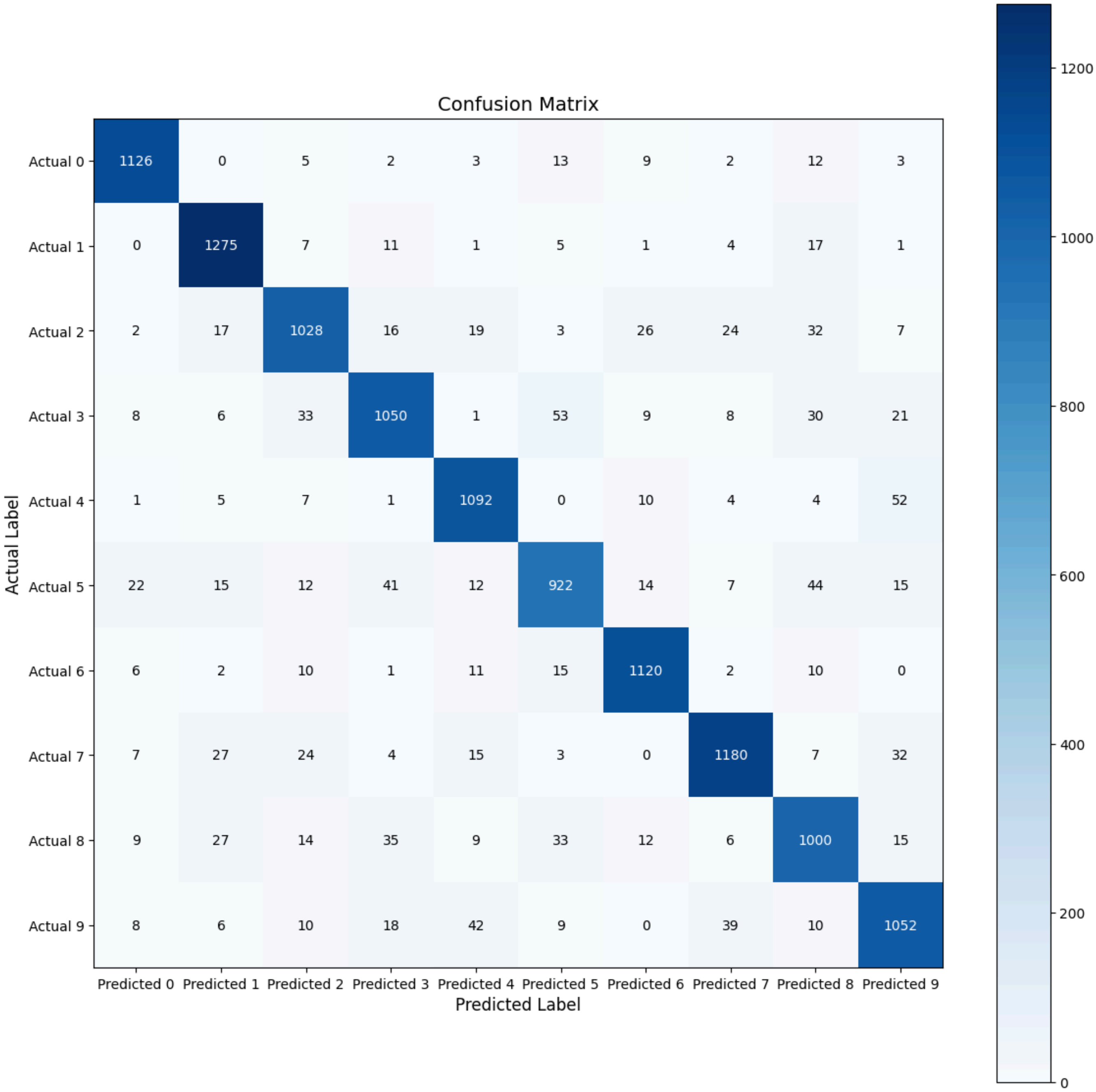
```
y_pred_test = predict_softmax(X_test, W_opt, b_opt)

# Evaluate the model
y_test_labels = np.argmax(y_test, axis=1)
cm, precision, recall, f1 = evaluate_classification(y_test_labels, y_pred_test)

# Print evaluation results
print("\nConfusion Matrix:")
print(cm)
print(f"Precision: {precision:.2f}")
print(f"Recall: {recall:.2f}")
```

```
print(f"F1-Score: {f1:.2f}")
```

```
Confusion Matrix:
[[1126    0    5    2    3   13    9    2   12    3]
 [   0 1275    7   11    1    5    1    4   17    1]
 [   2   17 1028   16   19    3   26   24   32    7]
 [   8    6   33 1050    1   53    9    8   30   21]
 [   1    5    7    1 1092    0   10    4    4   52]
 [  22   15   12   41   12  922   14    7   44   15]
 [   6    2   10    1   11   15 1120    2   10    0]
 [   7   27   24    4   15    3    0 1180    7   32]
 [   9   27   14   35    9   33   12    6 1000   15]
 [   8    6   10   18   42    9    0   39   10 1052]]
Precision: 0.90
Recall: 0.90
F1-Score: 0.90
```

Visualizing the confusion matrix

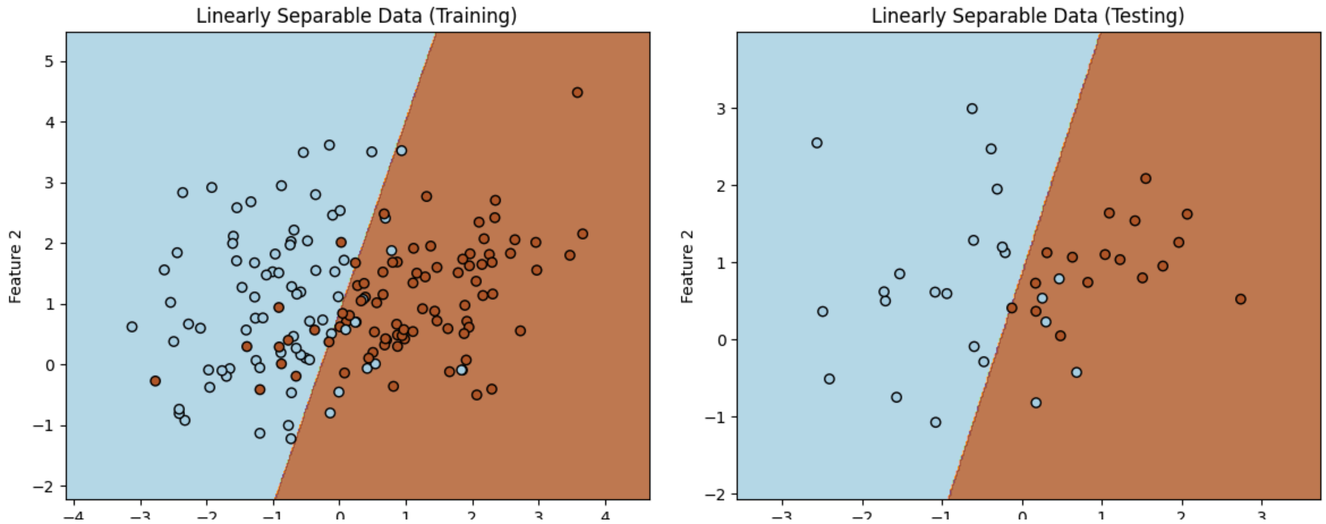
```
# Visualizing the Confusion Matrix
fig, ax = plt.subplots(figsize=(12, 12))
cax = ax.imshow(cm, cmap="Blues")  # Use a color map for better visualization

# Dynamic number of classes
num_classes = cm.shape[0]
ax.set_xticks(range(num_classes))
ax.set_yticks(range(num_classes))
ax.set_xticklabels([f"Predicted {i}" for i in range(num_classes)])
ax.set_yticklabels([f"Actual {i}" for i in range(num_classes)])

# Add labels to each cell in the confusion matrix
for i in range(cm.shape[0]):
    for j in range(cm.shape[1]):
        ax.text(j, i, cm[i, j], ha="center", va="center",
                color="white" if cm[i, j] > np.max(cm) / 2 else "black")

# Add grid lines and axis labels
ax.grid(False)
plt.title("Confusion Matrix", fontsize=14)
plt.xlabel("Predicted Label", fontsize=12)
plt.ylabel("Actual Label", fontsize=12)

# Adjust layout and show the plot
plt.tight_layout()
plt.colorbar(cax)
plt.show()
```



Lineat separability and logistic regression

```
import numpy as np
import matplotlib.pyplot as plt
from sklearn.datasets import make_classification, make_circles
from sklearn.model_selection import train_test_split
from sklearn.linear_model import LogisticRegression

# Set random seed for reproducibility
np.random.seed(42)

# Generate linearly separable dataset
X_linear_separable, y_linear_separable = make_classification(
    n_samples=200, n_features=2, n_informative=2,
    n_redundant=0, n_clusters_per_class=1, random_state=42
)

# Split the data into training and testing sets
X_train_linear, X_test_linear, y_train_linear, y_test_linear = train_test_split(
```

```
        X_linear_separable, y_linear_separable, test_size=0.2, random_state=42
    )

    # Train logistic regression model on linearly separable data
    logistic_model_linear_separable = LogisticRegression()
    logistic_model_linear_separable.fit(X_train_linear, y_train_linear)

    # Generate non-linearly separable dataset (circles)
    X_non_linear_separable, y_non_linear_separable = make_circles(
        n_samples=200, noise=0.1, factor=0.5, random_state=42
    )

    # Split the data into training and testing sets
    X_train_non_linear, X_test_non_linear, y_train_non_linear, y_test_non_linear = train_test_split(
        X_non_linear_separable, y_non_linear_separable, test_size=0.2, random_state=42
    )

    # Train logistic regression model on non-linearly separable data
    logistic_model_non_linear_separable = LogisticRegression()
    logistic_model_non_linear_separable.fit(X_train_non_linear, y_train_non_linear)

    # Function to plot decision boundaries
    def plot_decision_boundary(ax, model, X, y, title):
        h = 0.02  # Step size in the mesh
        x_min, x_max = X[:, 0].min() - 1, X[:, 0].max() + 1
        y_min, y_max = X[:, 1].min() - 1, X[:, 1].max() + 1

        xx, yy = np.meshgrid(np.arange(x_min, x_max, h), np.arange(y_min, y_max, h))
        Z = model.predict(np.c_[xx.ravel(), yy.ravel()])
        Z = Z.reshape(xx.shape)

        ax.contourf(xx, yy, Z, alpha=0.8, cmap=plt.cm.Paired)
        ax.scatter(X[:, 0], X[:, 1], c=y, edgecolors="k", cmap=plt.cm.Paired)
        ax.set_title(title)
        ax.set_xlabel("Feature 1")
        ax.set_ylabel("Feature 2")

    # Create subplots
    fig, axes = plt.subplots(2, 2, figsize=(12, 10))

    # Plot decision boundary for linearly separable data (Training)
    plot_decision_boundary(axes[0, 0], logistic_model_linear_separable, X_train_linear, y_train_linear,
                           "Linearly Separable Data (Training)")

    # Plot decision boundary for linearly separable data (Testing)
    plot_decision_boundary(axes[0, 1], logistic_model_linear_separable, X_test_linear, y_test_linear,
                           "Linearly Separable Data (Testing)")

    # Plot decision boundary for non-linearly separable data (Training)
    plot_decision_boundary(axes[1, 0], logistic_model_non_linear_separable, X_train_non_linear,
                           y_train_non_linear, "Non-Linearly Separable Data (Training)")

    # Plot decision boundary for non-linearly separable data (Testing)
    plot_decision_boundary(axes[1, 1], logistic_model_non_linear_separable, X_test_non_linear,
                           y_test_non_linear, "Non-Linearly Separable Data (Testing)")

    plt.tight_layout()

    # Save the plots as PNG files
    plt.savefig("decision_boundaries.png")
    plt.show()
```


Linearly Separable Data (Training)
Feature 2
Linearly Separable Data (Testing)
Feature 2