HACETTEPE UNIVERSITY
DEPARTMENT OF
GEOMATICS ENGINEERING

GMT202

ADJUSTMENT COMPUTATION & PARAMETER ESTIMATION

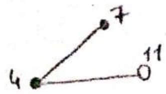
2021-2022 SPRING TERM

ASSIGNMENT 7

ABDULSAMET TOPTAŞ - 21905024

(1)

Adjust the GPS network given below using the indirect measures method. Take the mean error of the unit measure as $m_0 = \pm 2$ cm.



NN	x (m)	y (m)	z (m)
	Exact Coordinates		
4	3710709,539	3084028,627	4157648,644
7	3710479,640	3084171,030	4157677,581
	Approximate Coordinates		
11	3710442,600	3084257,800	4157623,100

DN	BN	ΔX (m)	ΔY (m)	ΔZ (m)	$m_{\Delta x}$ (cm)	$m_{\Delta y}$ (cm)	$m_{\Delta z}$ (cm)	$r_{\Delta x \Delta y}$ =	0,2
7	4	229,897	-142,404	-28,937	1,2	2,4	1,3	$r_{\Delta x \Delta z}$ =	0,4
11	4	266,878	-229,233	25,477	2,3	1,5	1,0	$r_{\Delta x \Delta y}$ =	0,3

Number of measures = n = 2 bais × 3 (coordinate differene) = 6

Unknown Number = u = 3 (coordinates of point)

Degrees of Freedom f = n − u = 6 − 3 > 0 ✓ Ther is adjustment

$\vec{\Delta X}_{7-4} + V_{x_{7-4}} = X_4 - X_7$ $\qquad V_{x_{7-4}} = X_4 - X_7 - \Delta X_{7-4}$

$\Delta Y_{7-4} + V_{y_{7-4}} = Y_4 - Y_7$ $\qquad V_{y_{7-4}} = Y_4 - Y_7 - \Delta Y_{7-4}$

$\Delta Z_{7-4} + V_{z_{7-4}} = Z_4 - Z_7$ $\qquad V_{z_{7-4}} = Z_4 - Z_7 - \Delta Z_{7-4}$

$\vec{\Delta X}_{11-4} + V_{x_{11-4}} = X_4 - X_{11}$ $\qquad V_{x_{11-4}} = X_4 - X_{11} - \Delta X_{11-4}$

$\vec{\Delta Y}_{11-4} + V_{y_{11-4}} = Y_4 - Y_{11}$ $\qquad V_{y_{11-4}} = Y_4 - Y_{11}$

$\vec{\Delta Z}_{11-4} + V_{z_{11-4}} = Z_4 - Z_{11}$ $\qquad V_{z_{11-4}} = Z_4 - Z_{11} - \Delta Z_{11-4}$

Approximate values

$X_4 = X^0_4 + dX_4$ $\qquad Y_4 = Y^0_4 + dY_4$ $\qquad Z_4 = Z^0_4 + dZ_4$

$X_7 = X^0_7 + dX_7$ $\qquad Y_7 = Y^0_7 + dY_7$ $\qquad Z_7 = Z^0_7 + dZ_7$

$X_{11} = X^0_{11} + dX_{11}$ $\qquad Y_{11} = Y^0_{11} + dY_{11}$ $\qquad Z_{11} = Z^0_{11} + dZ_{11}$

Abdulsamet TOPÇU
21905024

c) I will edit the above equations.

$$Vx_{7-4} = x^0_4 + dx_4 - x^0_7 - dx_7 - \Delta x_{7-4} \qquad Vx_{7-4} = dx_4 - dx_7 + x^0_4 - x^0_7 - \Delta x_{7-4}$$

$$Vy_{7-4} = y^0_4 + dy_4 - y^0_7 - dy_7 - \Delta y_{7-4} \qquad Vy_{7-4} = dy_4 - dy_7 + y^0_4 - y^0_7 - \Delta y_{7-4}$$

$$Vz_{7-4} = z^0_4 + dz_4 - z^0_7 - dz_7 - \Delta z_{7-4} \qquad Vz_{7-4} = dz_4 + dz_4 + z^0_4 - z^0_7 - \Delta z_{7-4}$$

$$Vx_{11-4} = x^0_4 + dx_4 - x^0_{11} - dx_{11} - \Delta x_{11-4} \qquad Vx_{11-4} = dx_4 - dx_{11} + x^0_4 - x^0_{11} - \Delta x_{11-4}$$

$$Vy_{11-4} = y^0_4 + dy_4 - y^0_{11} - dy_{11} - \Delta y_{11-4} \qquad Vy_{11-4} = dy_4 - dy_{11} + y^0_4 - y^0_{11} - \Delta y_{11-4}$$

$$Vz_{11-4} = z^0_4 + dz_4 - z^0_{11} - dz_{11} - \Delta z_{11-4} \qquad Vz_{11-4} = dz_4 - dz_{11} + z^0_4 - z^0_{11} - \Delta z_{11-4}$$

$$-l_1 = x^0_4 - x^0_7 - \Delta x_{7-4} = 0{,}2 \text{ cm}$$
$$-l_2 = y^0_4 - y^0_7 - \Delta y_{7-4} = 0{,}1 \text{ cm}$$
$$-l_3 = z^0_4 - z^0_7 - \Delta z_{7-4} = 0{,}0 \text{ cm}$$
$$-l_4 = x^0_4 - x^0_{11} - \Delta x_{11-4} = 6{,}1 \text{ cm}$$
$$-l_5 = y^0_4 - y^0_{11} - \Delta y_{11-4} = 6{,}0 \text{ cm}$$
$$-l_6 = z^0_4 - z^0_{11} - \Delta z_{11-4} = 7{,}1 \text{ cm}$$

Points 4 and 7 are fixed points. No corrections are made to these points. I will discard the unknowns of these points dx_4, dy_4, dz_4 and dx_7, dy_7, dz_7 from the correction equations and arrange them.

$$Vx_{7-4} = 0{,}2 \qquad Vx_{7-4} = \cancel{0.dx_{11}} + \cancel{0.dy_{11}} + \cancel{0.dz_{11}} + 0{,}2$$
$$Vy_{7-4} = 0{,}1 \qquad Vy_{7-4} = \cancel{0.dx_{11}} + \cancel{0.dy_{11}} + \cancel{0.dz_{11}} + 0{,}1$$
$$Vz_{7-4} = 0{,}0 \qquad Vz_{7-4} = \cancel{0.dx_{11}} + \cancel{0.dy_{11}} + \cancel{0.dz_{11}} + 0{,}0$$
$$Vx_{11-4} = -dx_{11} + 6{,}1 \qquad Vx_{11-4} = -dx_{11} + \cancel{0.dy_{11}} + \cancel{0.dz_{11}} + 6{,}1$$
$$Vy_{11-4} = -dy_{11} + 6{,}0 \qquad Vy_{11-4} = \cancel{0.dx_{11}} - dy_{11} + \cancel{0.dz_{11}} + 6{,}0$$
$$Vz_{11-4} = -dz_{11} + 7{,}1 \qquad Vz_{11-4} = \cancel{0.dx_{11}} + \cancel{0.dy_{11}} - dz_{11} + 7{,}1$$

Write $V = A.x - l$ format;

$$\begin{bmatrix} Vx_{7-4} \\ Vy_{7-4} \\ Vz_{7-4} \\ Vx_{11-4} \\ Vy_{11-4} \\ Vz_{11-4} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} dx_{11} \\ dy_{11} \\ dz_{11} \end{bmatrix} - \begin{bmatrix} -0{,}2 \\ -0{,}1 \\ -0{,}1 \\ -6{,}1 \\ -6{,}0 \\ -7{,}1 \end{bmatrix}$$

Abdulsamet TOPTAŞ
21905024

(3)

Stochastic Model

$$K_{7-4} = \begin{bmatrix} m^2_{\Delta X_{7-4}} & r_{\Delta X_{7-4}\Delta Y_{7-4}} \cdot m_{\Delta X_{7-4}} \cdot m_{\Delta Y_{7-4}} & r_{\Delta X_{7-4}\Delta Z_{7-4}} \cdot m_{\Delta X_{7-4}} \cdot m_{\Delta Z_{7-4}} \\ r_{\Delta X_{7-4}\Delta Y_{7-4}} \cdot m_{\Delta X_{7-4}} \cdot m_{\Delta Y_{7-4}} & m^2_{\Delta Y_{7-4}} & r_{\Delta Y_{7-4}\Delta Z_{7-4}} \cdot m_{\Delta Y_{7-4}} \cdot m_{\Delta Z_{7-4}} \\ r_{\Delta X_{7-4}\Delta Z_{7-4}} \cdot m_{\Delta X_{7-4}} \cdot m_{\Delta Z_{7-4}} & r_{\Delta Y_{7-4}\Delta Z_{7-4}} \cdot m_{\Delta Y_{7-4}} \cdot m_{\Delta Z_{7-4}} & m^2_{\Delta Z_{7-4}} \end{bmatrix}$$

$$K_{7-4} = \begin{bmatrix} 1,2^2 & 0,2 \times 1,2 \times 2,4 & 0,4 \times 1,2 \times 1,3 \\ 0,2 \times 1,2 \times 2,4 & 2,4^2 & 0,3 \times 2,4 \times 1,3 \\ 0,4 \times 1,2 \times 1,3 & 0,3 \times 2,4 \times 1,3 & 1,3^2 \end{bmatrix}$$

$$K_{7-4} = \begin{bmatrix} 1,44 & 0,58 & 0,62 \\ 0,58 & 5,76 & 0,94 \\ 0,62 & 0,94 & 4,96 \end{bmatrix} \qquad K_{11-4} = \begin{bmatrix} 5,29 & 0,69 & 0,92 \\ 0,69 & 2,25 & 0,45 \\ 0,92 & 0,45 & 1,00 \end{bmatrix}$$

$$K_{ll} = \left[\begin{array}{ccc|ccc} 1,44 & 0,58 & 0,62 & 0 & 0 & 0 \\ 0,58 & 5,76 & 0,94 & 0 & 0 & 0 \\ 0,62 & 0,94 & 4,69 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 5,29 & 0,69 & 0,92 \\ 0 & 0 & 0 & 0,69 & 2,25 & 0,45 \\ 0 & 0 & 0 & 0,92 & 0,45 & 1,00 \end{array}\right]$$

I will calculate weights for each using the formulas below:

$$K_{7-4} = m_0^2 \cdot Q_{7-4} \qquad Q_{7-4} = \frac{K_{7-4}}{m_0^2} \qquad P_{7-4} = Q^{-1}_{7-4} \quad \text{weight matrix}$$

$$Q_{7-4} = \begin{bmatrix} 1,44 & 0,58 & 0,62 \\ 0,58 & 5,76 & 0,94 \\ 0,62 & 0,94 & 4,69 \end{bmatrix} / 2^2 = \begin{bmatrix} 0,36 & 0,14 & 0,16 \\ 0,14 & 1,44 & 0,23 \\ 0,16 & 0,23 & 0,42 \end{bmatrix}$$

$$P_{7-4} = \begin{bmatrix} 3,33 & -0,15 & -1,15 \\ -0,15 & 0,77 & -0,37 \\ -1,15 & -0,37 & 3,00 \end{bmatrix} \qquad P_{11-4} = \begin{bmatrix} 0,91 & -0,12 & -0,78 \\ -0,12 & 1,97 & -0,77 \\ -0,78 & -0,77 & 5,07 \end{bmatrix}$$

$$P = \left[\begin{array}{ccc|ccc} 3,33 & -0,15 & -1,15 & 0 & 0 & 0 \\ -0,15 & 0,77 & -0,37 & 0 & 0 & 0 \\ -1,15 & -0,37 & 3,00 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0,91 & -0,12 & -0,78 \\ 0 & 0 & 0 & -0,12 & 1,97 & -0,77 \\ 0 & 0 & 0 & -0,78 & -0,77 & 5,07 \end{array}\right] \qquad \underline{N} = \underline{A}^T \underline{P} \underline{A} = \begin{bmatrix} 0,91 & -0,12 & -0,78 \\ -0,12 & 1,57 & -0,77 \\ -0,78 & -0,77 & 5,07 \end{bmatrix}$$

$$\underline{Q}_{xx} = \underline{N}^{-1} = \begin{bmatrix} 1,32 & 0,17 & 0,23 \\ 0,17 & 0,56 & 0,11 \\ 0,23 & 0,11 & 0,25 \end{bmatrix} \qquad \underline{x} = \underline{Q}_{xx} \cdot \underline{n} = \begin{bmatrix} dx_{11} \\ dy_{11} \\ dz_{11} \end{bmatrix} = \begin{bmatrix} 6,1 \\ 6,0 \\ 7,1 \end{bmatrix} \text{cm}$$

$$\underline{n} = \underline{A}^T \underline{P} \underline{l} = \begin{bmatrix} -0,74 \\ 5,58 \\ 26,57 \end{bmatrix}$$

Abdulsamet TOPTAŞ
21905024

④ The exact value of the unknown

$$\begin{bmatrix} X_{11} \\ Y_{11} \\ Z_{11} \end{bmatrix} = \begin{bmatrix} X^{0}_{11} \\ Y^{0}_{11} \\ Z^{0}_{11} \end{bmatrix} + \begin{bmatrix} dX_{11} \\ dY_{11} \\ dZ_{11} \end{bmatrix} \qquad \begin{bmatrix} dX_{11} \\ dY_{11} \\ dZ_{11} \end{bmatrix} = \begin{bmatrix} 3710442{,}600 \\ 3084257{,}800 \\ 4157623{,}100 \end{bmatrix} + \begin{bmatrix} 6{,}1 \\ 6{,}0 \\ 7{,}1 \end{bmatrix} = \begin{bmatrix} 3710442{,}661 \\ 3084257{,}860 \\ 4157623{,}171 \end{bmatrix}$$

Adjustment ; $\underline{V} = \underline{A} \cdot \underline{x} - \underline{l}$

$$\begin{bmatrix} V_{X7-4} \\ V_{Y7-4} \\ V_{Z7-4} \\ V_{X11-4} \\ V_{Y11-4} \\ V_{Z11-4} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 6{,}1 \\ 6{,}0 \\ 7{,}1 \end{bmatrix} - \begin{bmatrix} -0{,}2 \\ -0{,}1 \\ -0{,}1 \\ -6{,}1 \\ -6{,}0 \\ -7{,}1 \end{bmatrix} = \begin{bmatrix} 0{,}2 \\ 0{,}1 \\ 0{,}0 \\ 0{,}0 \\ 0{,}0 \\ 0{,}0 \end{bmatrix}$$

Adjusted Measures ;

$$\begin{bmatrix} \Delta\hat{X}_{7-4} \\ \Delta\hat{Y}_{7-4} \\ \Delta\hat{Z}_{7-4} \\ \Delta\hat{X}_{11-4} \\ \Delta\hat{Y}_{11-4} \\ \Delta\hat{Z}_{11-4} \end{bmatrix} = \begin{bmatrix} \Delta X_{7-4} \\ \Delta Y_{7-4} \\ \Delta Z_{7-4} \\ \Delta X_{11-4} \\ \Delta Y_{11-4} \\ \Delta Z_{11-4} \end{bmatrix} + \begin{bmatrix} V_{X7-4} \\ V_{Y7-4} \\ V_{Z7-4} \\ V_{X11-4} \\ V_{Y11-4} \\ V_{Z11-4} \end{bmatrix} \qquad \begin{bmatrix} \Delta\hat{X}_{7-4} \\ \Delta\hat{Y}_{7-4} \\ \Delta\hat{Z}_{7-4} \\ \Delta\hat{X}_{11-4} \\ \Delta\hat{Y}_{11-4} \\ \Delta\hat{Z}_{11-4} \end{bmatrix} = \begin{bmatrix} 225{,}897 \\ -142{,}404 \\ -28{,}937 \\ 266{,}878 \\ -229{,}233 \\ 25{,}473 \end{bmatrix} + \begin{bmatrix} 0{,}2 \\ 0{,}1 \\ 0{,}0 \\ 0{,}0 \\ 0{,}0 \\ 0{,}0 \end{bmatrix} = \begin{bmatrix} 229{,}899 \\ -142{,}403 \\ -28{,}937 \\ 266{,}878 \\ -229{,}233 \\ 25{,}473 \end{bmatrix}$$

Control of Adjusted Measures ;

$$\begin{bmatrix} \Delta X_{7-4} + V_{X7-4} \\ \Delta Y_{7-4} + V_{Y7-4} \\ \Delta Z_{7-4} + V_{Z7-4} \\ \Delta X_{11-4} + V_{X11-4} \\ \Delta Y_{11-4} + V_{Y11-4} \\ \Delta Z_{11-4} + V_{Z11-4} \end{bmatrix} = \begin{bmatrix} X_4 - X_7 \\ Y_4 - Y_7 \\ Z_4 - Z_7 \\ X_4 - X_{11} \\ Y_4 - Y_{11} \\ Z_4 - Z_{11} \end{bmatrix} \qquad \begin{bmatrix} 229{,}899 \\ -142{,}403 \\ -28{,}937 \\ 266{,}878 \\ -229{,}233 \\ 25{,}473 \end{bmatrix} = \begin{bmatrix} 229{,}899 \\ -142{,}403 \\ -28{,}937 \\ 266{,}878 \\ -229{,}233 \\ 25{,}473 \end{bmatrix} \checkmark$$

Square Mean Error ;

$$m_0 = \pm\sqrt{\frac{\underline{V}^T \underline{P}\,\underline{V}}{n-u}} = \pm\sqrt{\frac{0{,}14}{6-3}} = \pm\, \underline{0{,}121\ \text{cm}}$$

Abdulsamet TOPTAS
21905024

(5)

Mean Error of Unknowns ;

$$\underline{Q}_{xx} = \underline{N}^{-1} = \begin{bmatrix} 1,32 & 0,17 & 0,23 \\ 0,17 & 0,56 & 0,11 \\ 0,23 & 0,11 & 0,25 \end{bmatrix}$$

$m_x = \pm m_0 \sqrt{q_{xx}} = \pm 0,21 \sqrt{1,32} = \underline{\pm 0,24 \text{ cm}}$

$m_y = \pm m_0 \sqrt{q_{yy}} = \pm 0,21 \sqrt{0,56} = \underline{\pm 0,16 \text{ cm}}$

$m_z = \pm m_0 \sqrt{q_{zz}} = \pm 0,21 \sqrt{0,25} = \underline{\pm 0,11 \text{ cm}}$

Mean Error of Measures ; $m_{l_i} = \pm \dfrac{m_0}{\sqrt{P_i}}$ cm

$m_{\Delta x_{7-4}} = 0,12$ $\quad m_{\Delta x_{11-4}} = 0,22$

$m_{\Delta y_{7-4}} = 0,24$ $\quad m_{\Delta y_{11-4}} = 0,15$

$m_{\Delta z_{7-4}} = 0,12$ $\quad m_{\Delta z_{11-4}} = 0,09$

Average Error of Adjusted Measures ;

$$Q_{\hat{l}\hat{l}} = \underline{A} \cdot \underline{Q}_{xx} \cdot \underline{A}^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1,32 & 0,17 & 0,23 \\ 0 & 0 & 0 & 0,17 & 0,56 & 0,11 \\ 0 & 0 & 0 & 0,23 & 0,11 & 0,25 \end{bmatrix}$$

$m_{\Delta \hat{x}_{i-j}} = \pm m_0 \cdot \sqrt{Q_{\hat{l}_i\hat{l}_i}}$ mean Error of Adjusted Measures

$m_{\Delta \hat{x}_{7-4}} = 0$ $\quad m_{\Delta \hat{x}_{11-4}} = 0,24$

$m_{\Delta \hat{y}_{7-4}} = 0$ $\quad m_{\Delta \hat{y}_{11-4}} = 0,16$

$m_{\Delta \hat{z}_{7-4}} = 0$ $\quad m_{\Delta \hat{z}_{11-4}} = 0,11$

Abdulsamet TOPTAŞ
21905024

(b) Average Error of Corrections ;

$\underline{Q}_{vv} = \underline{Q}_{ll} - Q_{\hat{l}\hat{l}}$ covariance matrix of corrections

$\underline{Q}_{vv} = \underline{P}^{-1} - \underline{Q}_{\hat{l}\hat{l}}$

$$P^{-1} = \begin{bmatrix} 0{,}36 & 0{,}14 & 0{,}16 & 0 & 0 & 0 \\ 0{,}14 & 1{,}44 & 0{,}23 & 0 & 0 & 0 \\ 0{,}16 & 0{,}23 & 0{,}42 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1{,}32 & 0{,}17 & 0{,}23 \\ 0 & 0 & 0 & 0{,}17 & 0{,}56 & 0{,}11 \\ 0 & 0 & 0 & 0{,}23 & 0{,}11 & 0{,}25 \end{bmatrix}$$

$$\underline{Q}_{vv} = \begin{bmatrix} 0{,}36 & 0{,}14 & 0{,}16 & 0 & 0 & 0 \\ 0{,}14 & 1{,}44 & 0{,}23 & 0 & 0 & 0 \\ 0{,}16 & 0{,}23 & 0{,}42 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$m_{v_{i-j}} \mp m_0 \cdot \sqrt{\underline{Q}_{v_iv_i}}$ Average Error of Corrections

$m_{v7-4} = 0{,}13$ cm $\quad m_{v11-4} = 0$ cm

$m_{v7-4} = 0{,}25$ cm $\quad m_{v11-4} = 0$ cm

$m_{v7-4} = 0{,}14$ cm $\quad m_{1-4} = 0$ cm

Abdulsamet TOPTAŞ
21905024