HACETTEPE UNIVERSITY
DEPARTMENT OF
GEOMATICS ENGINEERING

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GEOMATICS ENGINEERING

GMT202

ADJUSTMENT COMPUTATION & PARAMETER ESTIMATION

2021-2022 SPRING TERM

ASSIGNMENT 9

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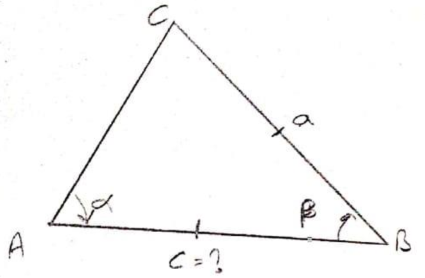
Given the two interior angles and the side length of a side opposide one of these angles, compute the side length and its mean square error of the side which is opposite to the third interior angle.



$c = ?$

$m_c = ?$

Given ; $\alpha = 57^g,5000 \pm 50^{cc}$

$\beta = 309,1780 \pm 60^{cc}$

$a = 257,50 \pm 0,15$ m

$r_{\alpha\beta} = 0,1$

$r_{\alpha a} = 0,05$

$r_{\beta a} = 0,12$

Unknowns ; $\Rightarrow c = ?$ $m_c = ?$

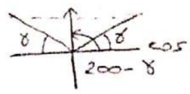
$321,77$m $\quad \pm 321,875$m

\# Result \#

$\boxed{321,77 \pm 321,875\,m}$

Solution

$$\frac{c}{\sin[200-(\alpha+\beta)]} = \frac{a}{\sin\alpha}$$



$$\sin[200-(\alpha+\beta)] = \sin(\alpha+\beta) \Rightarrow \frac{c}{\sin(\alpha+\beta)} = \frac{a}{\sin\alpha} \Rightarrow \boxed{c = \frac{a \cdot \sin(\alpha+\beta)}{\sin\alpha}} = 321,77\,m$$

$c = f(a, \alpha, \beta)$

$\boxed{\rho = \frac{200}{\pi} \cdot 100 \cdot 100 \qquad \rho = 636620 \text{ radian} \to cc \text{ unitless}}$

$$F = \begin{bmatrix} \frac{\partial f}{\partial a} & \frac{\partial f}{\partial \alpha} & \frac{\partial f}{\partial \beta} \end{bmatrix}$$

$$F = \begin{bmatrix} 1,250 & 190,593 & 63,067 \end{bmatrix}$$

$\downarrow 1 \quad \downarrow m \quad \downarrow m$

$$\frac{\partial f}{\partial a} = \frac{\sin(\alpha+\beta) \cdot \cancel{\sin\alpha}}{\sin^2\alpha} = \frac{\sin(\alpha+\beta)}{\sin\alpha} = \underline{1,250 \text{ unitless}}$$

$$F^T = \begin{bmatrix} 1,250 \\ 190,593 \\ 63,067 \end{bmatrix} \begin{matrix} \to 1 \\ \to m \\ \to m \end{matrix}$$

$$\frac{\partial f}{\partial \alpha} = \frac{a \cdot \cos(\alpha+\beta) \cdot \sin\alpha - \cos\alpha \cdot a \cdot \sin(\alpha+\beta)}{\sin^2\alpha} = \underline{190,593\,m}$$

$$\frac{\partial f}{\partial \beta} = \frac{a \cdot \cos(\alpha+\beta) \cdot \cancel{\sin\alpha}}{\sin^2\alpha} = \frac{a \cdot \cos(\alpha+\beta)}{\sin\alpha} = \underline{63,067\,m}$$

$$K_{xx} = \begin{bmatrix} a^2 & m_a \cdot \frac{m_\alpha}{\rho} \cdot r_{a\alpha} & m_a \cdot \frac{m_\beta}{\rho} \cdot r_{a\beta} \\ & \left(\frac{\alpha}{\rho}\right)^2 & m_\alpha \cdot \frac{m_\beta}{\rho} \cdot r_{\alpha\beta} \\ \text{Symmetric} & & \left(\frac{\beta}{\rho}\right)^2 \end{bmatrix} \Rightarrow K_x = \begin{bmatrix} 66306,250 & 0,000000707 & 0,000001696 \\ 0,000000707 & 0,000000008 & 0,000000001 \\ 0,000001696 & 0,000000001 & 0,000000002 \end{bmatrix}$$

as unit $\Rightarrow K_x = \begin{bmatrix} m^2 & m & m \\ m & 1 & 1 \\ m & 1 & 1 \end{bmatrix}$

$$\boxed{m_c^2 = \underbrace{KF}_{} = F \cdot K_x \cdot F^T}$$

$$= \underset{1\times3}{F} \cdot \underset{3\times3}{K_x} \cdot \underset{3\times1}{F^T}$$

$$m_c^2 = \sqrt{103603,51655\,m^2}$$

$$\boxed{m_c = 321,875\,m} \leftarrow \quad = 1\times1$$

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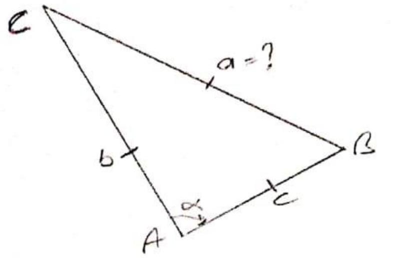
Given the two side lengths (b and c) and one interior angle (α) between these two sides of the below triangle, compute the other side length a and its mean square error. (Standard error, formal error, ma).



Given:
$\alpha = 110^g, 2080 \pm 40^{cc}$

$b = 280,50\ m \pm 20\ cm$

$c = 170,40\ m \pm 15\ cm$

$r_{\alpha b} = 0,2$

$r_{\alpha c} = 0,1$

$r_{bc} = 0,45$

Unknowns:

$a = ?$ $\quad ma = ?$

Solution

$a^2 = b^2 + c^2 - 2bc \cdot \cos\alpha = 151\,393,42\ m^2 \Rightarrow \boxed{a = 389,09\ m}$

$a = f(b, c, \alpha)$

$$2a \cdot \partial a = (2b - 2c \cdot \cos\alpha)\partial b + (2c - 2b \cdot \cos\alpha)\partial c + (2bc \cdot \sin\alpha)\partial\alpha$$

$$\partial a = \frac{(b - c \cdot \cos\alpha)}{a}\partial b + \frac{(c - b \cdot \cos\alpha)}{a}\partial c + \frac{(bc \cdot \sin\alpha)}{a}\partial\alpha$$

∂f

$$\underbrace{\frac{\partial f}{\partial b} = \frac{b - c \cdot \cos\alpha}{a}}_{0,921 \text{ unitless}} \qquad \underbrace{\frac{\partial f}{\partial c} = \frac{c - b \cdot \cos\alpha}{a}}_{0,767 \text{ unitless}} \qquad \underbrace{\frac{\partial f}{\partial \alpha} = \frac{bc \cdot \sin\alpha}{a}}_{109,27161\ \frac{1}{m} \Rightarrow 0,915\ cm}$$

$$F = \begin{bmatrix} \frac{\partial f}{\partial b} & \frac{\partial f}{\partial c} & \frac{\partial f}{\partial \alpha} \end{bmatrix} \Rightarrow F = \begin{bmatrix} \underset{1}{0,921} & \underset{1}{0,767} & \underset{cm}{0,915} \end{bmatrix}$$

$$F^T = \begin{bmatrix} 0,921 \\ 0,767 \\ 0,915 \end{bmatrix} \begin{matrix} \to 1 \text{ unitless} \\ \to 1 \text{ unitless} \\ \to cm \end{matrix}$$

$$\boxed{\begin{gathered} \rho = \frac{200}{\pi} \cdot 100 \cdot 100 \\ \rho = 636620 \quad \text{radian} \to cc \text{ unitless} \end{gathered}}$$

as unit $\quad Kx = \begin{bmatrix} cm^2 & cm^2 & cm \\ cm^2 & cm^2 & cm \\ cm & cm & 1 \end{bmatrix}$

$ma^2 = KF = F \cdot Kx \cdot F^T$

$$Kx = \begin{bmatrix} b^2 & mb \cdot mc \cdot r_{bc} & mb \cdot \frac{m\alpha}{\rho} \cdot r_{\alpha b} \\ & c^2 & mc \cdot \frac{m\alpha}{\rho} \cdot r_{\alpha c} \\ \text{symmetric} & & \left(\frac{\alpha}{\rho}\right)^2 \end{bmatrix} \Rightarrow Kx = \begin{bmatrix} 78680\,2500 & 135 & 0,000251327 \\ 135 & 290702500 & 0,000094248 \\ 0,000251327 & 0,000094248 & 0,000000042 \end{bmatrix}$$

$ma^2 = Kx = \sqrt{838415413,15544\ cm^2}$

$\boxed{ma = 28955,40387\ cm}$

$KF = F \cdot Kx \cdot F^T$

$= 1\times3 \cdot 3\times3 \cdot 3\times1$

$= 1\times1$

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