HACETTEPE UNIVERSITY
DEPARTMENT OF
GEOMATICS ENGINEERING

GMT202

ADJUSTMENT COMPUTATION & PARAMETER ESTIMATION

2021-2022 SPRING TERM

ASSIGNMENT 4

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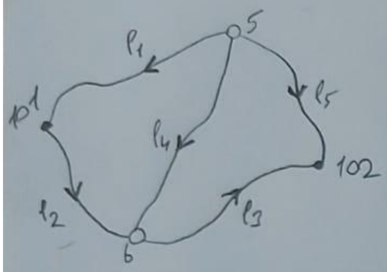
To perform a weighted least-square adjustment the below levelling net, the levelling observation at the know fixed heights of the benchmark ie. 101 and 102 are given. Please form;

a) a linearited observation equations

b) the coefficient matrix of the linearized observation eq. (jacobian matrix, design matrix). the weight of (ℓ) and the reduced observation vector (ℓ)

c) Estimate m_e, δx, $x \pm m_x$, $v \pm m_v$, $\ell \mp m_\ell$ and query it. $\hat{\ell} = f(x_0 + \delta x)$



Benchmark	Fixed (known) height, H (meter)
101	2200,116
102	2422,628

$n=5$, $u=2$ $\quad f = n - u = \underline{3}$ There is adjustment

i	Observed height difference in meter	Line height in km
ℓ_1	31.784	0.7
ℓ_2	117.134	0.8
ℓ_3	105.388	1.5
ℓ_4	148.914	1.2
ℓ_5	254.288	1.1

$H_5^0 = H_{101} - \ell_1 = 2,168,332\,m$

$H_6^0 = H_{101} + \ell_2 = 2.317,250\,m$

a)

$\hat{\ell} = \ell_1 + v_1 = H_{101} - H_5 \Rightarrow v_1 = -\delta H_5 - [\ell_1 - (H_{101} - H_5)] = 0$

$\hat{\ell} = \ell_2 + v_2 = H_6 - H_{101} \Rightarrow v_2 = \delta H_6 - [\ell_2 - (H_6^0 - H_{101})] = 0$

$\hat{\ell} = \ell_3 + v_3 = H_{102} - H_6 \Rightarrow v_3 = -\delta H_6 - [\ell_3 - (H_{102} - H_6^0)] = 0,01$

$\hat{\ell} = \ell_4 + v_4 = H_6 - H_5 \Rightarrow v_4 = \delta H_6 - \delta H_5 - [\ell_4 - (H_6^0 - H_5^0)] = -0,004$

$\hat{\ell} = \ell_5 + v_5 = H_{102} - H_5 \Rightarrow v_5 = -\delta H_5 - [\ell_5 - (H_{102} - H_5^0)] = -0,008$

b) $v_1 = -\delta H_5$

$v_2 = \delta H_6$

$v_3 = -\delta H_6 - 1$

$v_4 = \delta H_6 - \delta H_5 + 0{,}4$

$v_5 = -\delta H_5 + 0{,}8$

$$\Rightarrow \underbrace{\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix}}_{v} = \underbrace{\begin{bmatrix} -1 & 0 \\ 0 & 1 \\ 0 & -1 \\ -1 & 1 \\ -1 & 0 \end{bmatrix}}_{A} \cdot \underbrace{\begin{bmatrix} \delta H_5 \\ \delta H_6 \end{bmatrix}}_{\delta x} - \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \\ -0{,}4 \\ -0{,}8 \end{bmatrix}}_{\ell}$$

$P_i = \dfrac{S_0^2}{S_i^2}$ (If S_0 is not given, 1 is taken). $\quad s_i = \sqrt{\ell_i}$

$$P_i = \begin{bmatrix} 1/0{,}7 & 0 & 0 & 0 & 0 \\ 0 & 1/0{,}8 & 0 & 0 & 0 \\ 0 & 0 & 1/1{,}5 & 0 & 0 \\ 0 & 0 & 0 & 1/1{,}2 & 0 \\ 0 & 0 & 0 & 0 & 1/1{,}1 \end{bmatrix}$$

c) $\delta x = (A^T \cdot P \cdot A)^{-1} \cdot (A^T \cdot P \cdot \ell)$

$$= \begin{bmatrix} 0{,}3426 & 0{,}1038 \\ 0{,}1038 & 0{,}3951 \end{bmatrix} \cdot \begin{bmatrix} 1{,}0606 \\ -1 \end{bmatrix}$$

$$\delta x = \begin{bmatrix} 0{,}2596 \\ -0{,}2850 \end{bmatrix}$$

$m_{R_i} = \mp \dfrac{m_0}{\sqrt{P_i}} = \dfrac{0{,}5233}{\sqrt{P_i}} \Rightarrow m_{R_i}$

$m_0 = \pm \sqrt{\dfrac{v^T P v}{n-u}}$

$m_0 = \pm 0{,}5233$

$$m_{R_i} = \begin{bmatrix} 0{,}4378 & 0 & 0 & 0 & 0 \\ 0 & 0{,}4681 & 0 & 0 & 0 \\ 0 & 0 & 0{,}641 & 0 & 0 \\ 0 & 0 & 0 & 0{,}5732 & 0 \\ 0 & 0 & 0 & 0 & 0{,}5488 \end{bmatrix}$$

$Q_x = (A^T P A)^{-1}$

$m_x = \pm m_0 \sqrt{Q_x \text{ diag}}$

$$Q_x = \begin{bmatrix} 0{,}3426 & 0{,}1018 \\ 0{,}1038 & 0{,}3951 \end{bmatrix}$$

$$m_x = \begin{bmatrix} 0{,}3063 \\ 0{,}3289 \end{bmatrix}$$

$m_v = \pm m_0 \cdot \sqrt{Q_{vv} \text{ diag}}$

$Q_{vv} = Q_{\ell\ell} - Q_{\tilde{\ell}\tilde{\ell}}$

$v = A \cdot \delta x - \ell$

$$v = \begin{bmatrix} -0{,}2596 \\ -0{,}2850 \\ -0{,}7150 \\ -0{,}1446 \\ 0{,}5404 \end{bmatrix}$$

*(Calculated using Matlab).

$$m_{\hat{\ell}} = \mp m_0 \sqrt{Q_{\hat{\ell}\hat{\ell}} \, diag}$$

$$Q_{\hat{\ell}\hat{\ell}} = A \cdot Q_x \cdot A^T$$

$$Q_{\hat{\ell}\hat{\ell}} = \begin{bmatrix} 0,3426 & -0,1038 & 0,1038 & 0,2388 & 0,3426 \\ & 0,3951 & -0,3951 & 0,2913 & -0,1038 \\ & & 0,3951 & -0,2913 & 0,1038 \\ & & & 0,5301 & 0,2388 \\ & & & & 0,3426 \end{bmatrix}$$

$$m_{\hat{\ell}} = \begin{bmatrix} 0,3063 \\ 0,3289 \\ 0,3289 \\ 0,3810 \\ 0,3063 \end{bmatrix}$$

$$\hat{\ell} = l + v$$

$$\hat{\ell} = \begin{bmatrix} -0,2596 \\ -0,2850 \\ 0,2850 \\ -0,5446 \\ -0,2596 \end{bmatrix}$$

*(Calculated using Matlab)

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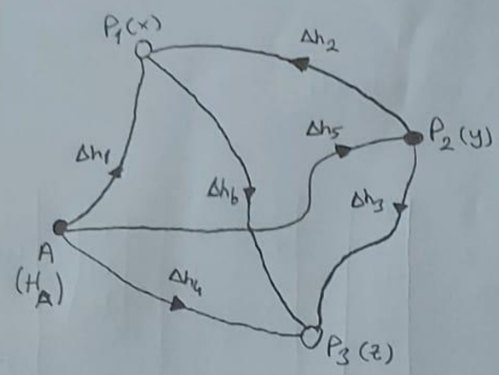
Balance the leveling grid given below using the indirect measures method. $H_A = 80,673$ m.



i	Δh_i	S_i (km)
1	43,156	0,65
2	19,218	0,80
3	33,524	1,00
4	57,440	1,40
5	23,962	1,50
6	14,267	1,95

Number of Measures = $n = 6$

Unknown Numbers = $u = 3$

Degrees of Freedom = $f = n - u = \underline{6-3 > 0}$

So, There is adjustment.

$\Delta h_1 + v_1 = H_{P_1} - H_A$ $\quad$ $\Delta h_1 + v_1 = x - H_A$ $\quad$ $v_1 = x - H_A - \Delta h_1$

$\Delta h_2 + v_2 = H_{P_1} - H_{P_2}$ $\quad$ $\Delta h_2 + v_2 = x - y$ $\quad$ $v_2 = x - y - \Delta h_2$

$\Delta h_3 + v_3 = H_{P_3} - H_{P_2}$ $\quad$ $\Delta h_3 + v_3 = z - y$ $\quad$ $v_3 = z - y - \Delta h_3$

$\Delta h_4 + v_4 = H_{P_3} - H_A$ $\quad$ $\Delta h_4 + v_4 = z - H_A$ $\quad$ $v_4 = z - H_A - \Delta h_4$

$\Delta h_5 + v_5 = H_{P_2} - H_A$ $\quad$ $\Delta h_5 + v_5 = y - H_A$ $\quad$ $v_5 = y - H_A - \Delta h_5$

$\Delta h_6 + v_6 = H_{P_3} - H_{P_1}$ $\quad$ $\Delta h_6 + v_6 = z - y$ $\quad$ $v_6 = z - y - \Delta h_6$

Approximate values;

$x = x_0 + dx \quad y = y_0 + dy \quad z = z_0 + dz$

$x_0 = H_A + \Delta h_1 \quad x_0 = 80,673 + 43,156 = 123,829$ m

$y_0 = H_A + \Delta h_5 \quad y_0 = 80,673 + 23,962 = 104,635$ m

$z_0 = H_A + \Delta h_4 \quad z_0 = 80,673 + 57,440 = 138,115$ m

$v_1 = x - H_A - \Delta h_1 \quad v_1 = dx + x_0 - H_A - \Delta h_1$

$v_2 = x - y - \Delta h_2 \quad v_2 = dx - dy + x_0 - y_0 - \Delta h_2$

$v_3 = z - y - \Delta h_3 \quad v_3 = -dy + dz + z_0 - y_0 - \Delta h_3$

$v_4 = z - H_A - \Delta h_4 \quad v_4 = dz + z_0 - H_A - \Delta h_4$

$v_5 = y - H_A - \Delta h_5 \quad v_5 = dy + y_0 - H_A - \Delta h_5$

$v_6 = z - x - \Delta h_6 \quad v_6 = -dx + dz + z_0 - x_0 - \Delta h_6$

The above values are in mm order.

$v_1 = dx + 123,829 - 80,673 - 43,156$ $\quad v_1 = dx$

$v_2 = dx - dy + 123,829 - 104,635 - 19,218$ $\quad v_2 = dx - dy - 24$

$v_3 = -dy + dz + 138,115 - 104,635 - 33,524$ $\quad v_3 = -dy + dz - 46$

$v_4 = dz + 138,115 - 80,673 - 57,440$ $\quad v_4 = dz$

$v_5 = dy + 104,635 - 80,673 - 23,962$ $\quad v_5 = dy$

$v_6 = -dx + dz + 138,115 - 123,829 - 14,267$ $\quad v_6 = -dx + dz + 17$

The above values are in mm order.

$$\left.\begin{aligned}
v_1 &= 1.dx + 0.dy + 0.dz + 0\\
v_2 &= 1.dx - 1.dy + 0.dz - 24\\
v_3 &= 0.dx - 1.dy + 1.dz - 46\\
v_4 &= 0.dx + 0.dy + 1.dz + 0\\
v_5 &= 0.dx + 1.dy + 0.dz + 0\\
v_6 &= -1.dx + 0.dy + 1.dz + 17
\end{aligned}\right\}$$

Let's write the above equations in the format $v = A.x - \ell$ $\Rightarrow$

$$\begin{bmatrix} v_1\\ v_2\\ v_3\\ v_4\\ v_5\\ v_6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0\\ 1 & -1 & 0\\ 0 & -1 & 1\\ 0 & 0 & 1\\ 0 & 1 & 0\\ -1 & 0 & 1 \end{bmatrix} . \begin{bmatrix} dx\\ dy\\ dz \end{bmatrix} - \begin{bmatrix} 0\\ 24\\ 46\\ 0\\ 0\\ -17 \end{bmatrix}$$

$$P_i = \begin{bmatrix} 1/0,65 & 0 & 0 & 0 & 0 & 0\\ 0 & 1/0,80 & 0 & 0 & 0 & 0\\ 0 & 0 & 1/1,00 & 0 & 0 & 0\\ 0 & 0 & 0 & 1/1,40 & 0 & 0\\ 0 & 0 & 0 & 0 & 1/1,50 & 0\\ 0 & 0 & 0 & 0 & 0 & 1/1,95 \end{bmatrix} \qquad P_i = \frac{1}{S_j\,(km)}$$

$$P_i = \begin{bmatrix} 1,54 & 0 & 0 & 0 & 0 & 0\\ 0 & 1,25 & 0 & 0 & 0 & 0\\ 0 & 0 & 1,00 & 0 & 0 & 0\\ 0 & 0 & 0 & 0,71 & 0 & 0\\ 0 & 0 & 0 & 0 & 0,67 & 0\\ 0 & 0 & 0 & 0 & 0 & 0,51 \end{bmatrix} \qquad \underline{N} = \underline{A}^T \underline{P}\, \underline{A} = \begin{bmatrix} 3,30 & -1,25 & -0,51\\ -1,25 & 2,92 & -1,00\\ -0,51 & -1,00 & 2,23 \end{bmatrix}$$

$$\underline{n} = \underline{A}^T \underline{P}\, \underline{\ell} = \begin{bmatrix} 38,72\\ -76,00\\ 37,28 \end{bmatrix} \qquad \underline{Q}_{xx} = N^{-1} = \begin{bmatrix} 0,44 & 0,26 & 0,22\\ 0,26 & 0,56 & 0,31\\ 0,22 & 0,31 & 0,64 \end{bmatrix} \qquad \underline{x} = \underline{Q}_{xx} . n = \begin{bmatrix} dx\\ dy\\ dz \end{bmatrix} = \begin{bmatrix} 5,12\\ -20,94\\ 8,52 \end{bmatrix} mm$$

The Exact Value of the unknown

$$\begin{bmatrix} x\\ y\\ z \end{bmatrix} = \begin{bmatrix} x_0\\ y_0\\ z_0 \end{bmatrix} + \begin{bmatrix} dx\\ dy\\ dz \end{bmatrix} \qquad \begin{bmatrix} x\\ y\\ z \end{bmatrix} = \begin{bmatrix} 123,829\\ 104,635\\ 138,113 \end{bmatrix} + \begin{bmatrix} 5,12\\ -20,94\\ 8,52 \end{bmatrix} = \begin{bmatrix} 123,834\\ 104,614\\ 138,122 \end{bmatrix}$$

* (Calculated using Matlab).

Corrections $(v = A \cdot x - l)$

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 5{,}12 \\ -20{,}94 \\ 8{,}52 \end{bmatrix} - \begin{bmatrix} 0 \\ 24 \\ 46 \\ 0 \\ 0 \\ -17 \end{bmatrix} = \begin{bmatrix} 5.12 \\ 2.06 \\ -16.54 \\ 8.52 \\ -20.94 \\ 20.39 \end{bmatrix}$$

Balanced Measures $(\Delta \hat{h}_i = \Delta h_i + v_i)$

$$\begin{bmatrix} \Delta\hat{h}_1 \\ \Delta\hat{h}_2 \\ \Delta\hat{h}_3 \\ \Delta\hat{h}_4 \\ \Delta\hat{h}_5 \\ \Delta\hat{h}_6 \end{bmatrix} = \begin{bmatrix} \Delta h_1 \\ \Delta h_2 \\ \Delta h_3 \\ \Delta h_4 \\ \Delta h_5 \\ \Delta h_6 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{bmatrix}$$

$$\begin{bmatrix} \Delta\hat{h}_1 \\ \Delta\hat{h}_2 \\ \Delta\hat{h}_3 \\ \Delta\hat{h}_4 \\ \Delta\hat{h}_5 \\ \Delta\hat{h}_6 \end{bmatrix} = \begin{bmatrix} 43.156 \\ 19.218 \\ 33.524 \\ 57.440 \\ 23.962 \\ 14.267 \end{bmatrix} + \begin{bmatrix} 5.12 \\ 2.06 \\ -16.54 \\ 8.52 \\ -20.94 \\ 20.39 \end{bmatrix} = \begin{bmatrix} 43.161 \\ 19.220 \\ 33.507 \\ 57.449 \\ 23.941 \\ 14.287 \end{bmatrix}$$

Control of Balanced Measures

$\Delta h_1 + v_1 = H_{P_1} - H_A$ 43,161 = 43,161 ✓

$\Delta h_2 + v_2 = H_{P_1} - H_{P_2}$ 19,220 = 19,220 ✓

$\Delta h_3 + v_3 = H_{P_3} - H_{P_2}$ 33,507 = 33,507 ✓

$\Delta h_4 + v_4 = H_{P_3} - H_A$ 57,449 = 57,449 ✓

$\Delta h_5 + v_5 = H_{P_2} - H_A$ 23,941 = 23,941 ✓

$\Delta h_6 + v_6 = H_{P_3} - H_{P_1}$ 14,287 = 14,287 ✓

Square Mean Error

$$m_0 = \pm\sqrt{\frac{\underline{v}^T \underline{P} \underline{v}}{n-u}} = \pm\sqrt{\frac{876{,}79}{6-3}} = \underline{\pm 17{,}10 \text{ mm}}$$

Mean Error of Unknown

$$\underline{Q}_{xx} = \underline{N}^{-1} = \begin{bmatrix} 0{,}44 & 0{,}26 & 0{,}22 \\ 0{,}26 & 0{,}56 & 0{,}31 \\ 0{,}22 & 0{,}31 & 0{,}64 \end{bmatrix}$$

$$m_x = \pm m_0 \sqrt{q_{xx}} = \pm 17{,}10 \sqrt{0{,}44} = \pm 11{,}28 \text{ mm}$$

$$m_y = \pm m_0 \sqrt{q_{yy}} = \pm 17{,}10 \sqrt{0{,}56} = \pm 12{,}82 \text{ mm}$$

$$m_z = \pm m_0 \sqrt{q_{zz}} = \pm 17{,}10 \sqrt{0{,}64} = \pm 13{,}67 \text{ mm}$$

* (Calculated using Matlab).

Mean Error of Measures

$$m_{l_i} = \pm \frac{m_0}{\sqrt{P_i}} \text{ mm}$$

$m_{\Delta h_1} = 13{,}78$ $\qquad m_{\Delta h_4} = 20{,}23$

$m_{\Delta h_2} = 15{,}29$ $\qquad m_{\Delta h_5} = 20{,}94$

$m_{\Delta h_3} = 17{,}10$ $\qquad m_{\Delta h_6} = 23{,}87$

Average Error of Balanced Measures

$$\underline{Q}_{ll} = \underline{A} \cdot \underline{Q}_{xx} \cdot \underline{A}^T = \begin{bmatrix} 0,44 & 0,17 & -0,04 & 0,22 & 0,26 & -0,22 \\ 0,17 & 0,47 & 0,21 & -0,09 & -0,30 & -0,27 \\ -0,04 & 0,21 & 0,58 & 0,33 & -0,25 & 0,37 \\ 0,22 & -0,09 & 0,33 & 0,64 & 0,31 & 0,42 \\ 0,26 & -0,30 & -0,25 & 0,31 & 0,56 & 0,05 \\ -0,22 & -0,27 & 0,37 & 0,42 & 0,05 & 0,64 \end{bmatrix}$$

$m_{\Delta \hat{h}_i} = \pm m_0 \cdot \sqrt{\underline{Q}_{\hat{l}_i \hat{l}_i}}$ Average errors of balanced measures

$m_{\Delta \hat{h}_1} = 11{,}28$ mm $\qquad m_{\Delta \hat{h}_4} = 13{,}67$ mm

$m_{\Delta \hat{h}_2} = 11{.}78$ mm $\qquad m_{\Delta \hat{h}_5} = 12{.}82$ mm

$m_{\Delta \hat{h}_3} = 12{,}98$ mm $\qquad m_{\Delta \hat{h}_6} = 11{,}67$ mm

Average Error of Corrections

$\underline{Q}_{vv} = \underline{Q}_{ll} - \underline{Q}_{\hat{l}\hat{l}}$ Covariance matrix of corrections

$\underline{Q}_{vv} = \underline{P}^{-1} - \underline{Q}_{\hat{l}\hat{l}}$

$$P^{-1} = \begin{bmatrix} 0,65 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0,80 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.00 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1,40 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1,50 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1,95 \end{bmatrix}$$

* (Calculated using Matlab)

$$\underline{Q}_{vv} = \begin{bmatrix} 0,21 & -0,17 & 0,04 & -0,22 & 0,26 & -0,22 \\ -0,17 & 0,33 & -0,21 & 0,09 & 0,30 & 0,27 \\ -0,04 & -0,21 & 0,42 & -0,33 & 0,25 & -0,37 \\ -0,22 & 0,09 & -0,33 & 0,76 & -0,31 & -0,42 \\ -0,26 & 0,30 & 0,25 & -0,31 & 0,94 & -0,05 \\ 0,22 & 0,27 & -0,37 & -0,42 & -0,05 & 1,11 \end{bmatrix}$$

$m_{v_i} = \pm m_0 \cdot \sqrt{\underline{Q}_{v_i v_i}}$ Average errors of corrections

$m_{v_1} = 7{,}91$ mm $\qquad m_{v_4} = 14{.}91$ mm

$m_{v_2} = 9{,}75$ mm $\qquad m_{v_5} = 16{,}56$ mm

$m_{v_3} = 11{.}13$ mm $\qquad m_{v_6} = 19{,}57$ mm

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