HACETTEPE UNIVERSITY
DEPARTMENT OF
GEOMATICS ENGINEERING

GMT202

ADJUSTMENT COMPUTATION & PARAMETER ESTIMATION

2021-2022 SPRING TERM

ASSIGNMENT 1

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Expand the below non-linear functions around the apriori values of $x_0 = 3$ and $y_0 = 7$ using Taylor series up to second degree term

① $f(x,y) = \sqrt{(x-5)^2 + (y-2)^2}$

② $g(x,y) = \tan^{-1}\left(\frac{x-5}{y-2}\right)$

③ $f(t) = \cos t \quad (t_0 = 60°)$

Solution ①

$f(x,y) \Rightarrow$ ① $f(x_0,y_0)$ + ② $\frac{\partial f}{\partial x}\Big|_{x_0y_0} \frac{(x-x_0)}{1!}$ + ③ $\frac{\partial f}{\partial y}\Big|_{x_0y_0} \frac{(y-y_0)}{1!}$ + ④ $\frac{\partial^2 f}{\partial x^2}\Big|_{x_0y_0} \frac{(x-x_0)^2}{2!}$

$x_0 = 3$, $y_0 = 7$

+ ⑤ $\frac{\partial^2 f}{\partial y^2}\Big|_{x_0y_0} \frac{(y-y_0)^2}{2!}$ + ⑥ $\frac{\partial^2 f}{\partial x \partial y}\Big|_{x_0y_0} \frac{(x-x_0)(y-y_0)}{2!}$

① $f(x,y) = \sqrt{(x-5)^2 + (y-2)^2} \Rightarrow \sqrt{(-2)^2 + (5)^2} \Rightarrow \underline{\sqrt{29}}$

② $\frac{\partial f}{\partial x}\Big|_{x_0y_0} \frac{(x-x_0)}{1!} \Rightarrow \frac{x-5}{\sqrt{(x-5)^2+(y-2)^2}} \cdot (x-3) \Rightarrow \underline{\frac{-2}{\sqrt{29}} \cdot (x-3)}$

③ $\frac{\partial f}{\partial y}\Big|_{x_0y_0} \frac{(y-y_0)}{1!} \Rightarrow \frac{y-7}{\sqrt{(x-5)^2+(y-2)^2}} \cdot (y-7) \Rightarrow \underline{\frac{5}{\sqrt{29}} \cdot (y-7)}$

④ $\frac{\partial^2 f}{\partial x^2}\Big|_{x_0y_0} \frac{(x-x_0)^2}{2!} \Rightarrow \frac{(y-2)^2}{[(x-5)^2+(y-2)^2]^{3/2}} \cdot \frac{(x-3)^2}{2} \Rightarrow \underline{\frac{25}{58\sqrt{29}} \cdot (x-3)^2}$

⑤ $\frac{\partial^2 f}{\partial y^2}\Big|_{x_0y_0} \frac{(y-y_0)^2}{2!} \Rightarrow \frac{(x-5)^2}{[(x-5)^2+(y-2)^2]^{3/2}} \cdot \frac{(y-7)^2}{2} \Rightarrow \underline{\frac{4}{58\sqrt{29}} \cdot (y-7)^2}$

⑥ $\frac{\partial^2 f}{\partial x \partial y}\Big|_{x_0y_0} \frac{(x-x_0)(y-y_0)}{2!} \Rightarrow \frac{-(x-5)(y-2)}{[(x-5)^2+(y-2)^2]^{3/2}} \cdot \frac{(x-3)(y-7)}{2} \Rightarrow \underline{\frac{10}{58\sqrt{29}} \cdot (x-3)(y-7)}$

$= \sqrt{29} + \left[\frac{-2}{\sqrt{29}}(x-3)\right] + \left[\frac{5}{\sqrt{29}}(y-7)\right] + \left[\frac{25}{58\sqrt{29}} \cdot (x^2-6x+9)\right] + \left[\frac{4}{58\sqrt{29}}(y^2-14y+49)\right] +$

$\left[\frac{10}{58\sqrt{29}}(xy-7x-3y+21)\right]$

$\Rightarrow \frac{25x^2+4y^2+10xy-336x+204y+631}{58\sqrt{29}} \Rightarrow \underline{\sqrt{29}\,\frac{(25x^2+4y^2+10xy-336x+204y+631)}{1682}}$

Result

$(\sqrt{29})$

Solution (2)

$g(x,y) = \tan^{-1}\left(\frac{x-5}{y-2}\right)$ ** $\boxed{x_0 = 3 \quad y_0 = 7}$

$$g(x,y) \Rightarrow \underset{①}{g(x_0,y_0)} + \frac{\partial f}{\partial x}\Big|_{x_0y_0} \underset{②}{\frac{(x-x_0)}{1!}} + \frac{\partial f}{\partial y}\Big|_{x_0y_0} \underset{③}{\frac{(y-y_0)}{1!}} +$$

$$\frac{\partial^2 f}{\partial x^2}\Big|_{x_0,y_0} \underset{④}{\frac{(x-x_0)^2}{2!}} + \frac{\partial^2 f}{\partial y^2}\Big|_{x_0y_0} \underset{⑤}{\frac{(y-y_0)^2}{2!}} + \frac{\partial^2 f}{\partial x \partial y}\Big|_{x_0y_0} \underset{⑥}{\frac{(x-x_0)(y-y_0)}{2!}}$$

① $g(x,y) \Rightarrow \tan^{-1}\left(\frac{x-5}{y-2}\right) \Rightarrow \tan^{-1}\left(\frac{-2}{5}\right) \Rightarrow \approx \underline{-0,3805}$

② $\frac{\partial f}{\partial x}\Big|_{x_0y_0} \frac{(x-x_0)}{1!} \Rightarrow \frac{y-2}{(x-5)^2+(y-2)^2} \cdot (x-3) \Rightarrow \underline{\frac{5}{29} \cdot (x-3)}$

③ $\frac{\partial f}{\partial y}\Big|_{x_0y_0} \frac{(y-y_0)}{1!} \Rightarrow \frac{-(x-5)}{(x-5)^2+(y-2)^2} \cdot (y-7) \Rightarrow \underline{\frac{2}{29} \cdot (y-7)}$

④ $\frac{\partial^2 f}{\partial x^2}\Big|_{x_0y_0} \frac{(x-x_0)^2}{2!} \Rightarrow \frac{-(y-2)(2x-10)}{\left[(x-5)^2+(y-2)^2\right]^2} \cdot \frac{(x-3)^2}{2} \Rightarrow \underline{\frac{20}{1682} \cdot (x-3)^2}$

⑤ $\frac{\partial^2 f}{\partial y^2}\Big|_{x_0y_0} \frac{(y-y_0)^2}{2!} \Rightarrow \frac{(x-5)(2y-4)}{\left[(x-5)^2+(y-2)^2\right]^2} \cdot \frac{(y-7)^2}{2} \Rightarrow \underline{\frac{-20}{1682} \cdot (y-7)^2}$

⑥ $\frac{\partial^2 f}{\partial x \partial y}\Big|_{x_0y_0} \frac{(x-x_0)(y-y_0)}{2!} \Rightarrow \frac{(x-5)^2-(y-2)^2}{\left[(x-5)^2+(y-2)^2\right]^2} \cdot \frac{(x-3)(y-7)}{2} \Rightarrow \underline{\frac{-21}{1682}(x-3)(y-7)}$

$$= -0,3805 + \left[\frac{5}{29}(x-3)\right] + \left[\frac{2}{29}(y-7)\right] + \left[\frac{20}{1682}(x^2-6x+9)\right] +$$

$$\left[\frac{-20}{1682}(y^2-14y+49)\right] + \left[\frac{-21}{1682}(xy-7x-3y+21)\right]$$

$$\Rightarrow -0,3805 + \frac{20x^2+317x-21xy+459y-20y^2-2923}{1682}$$

$$\Rightarrow \underline{\frac{20x^2-20y^2-21xy+317x+459y-3563}{1682}}$$

Result

Solution (3)

$f(t) = \cos t \quad (t_0 = 60°)$

$\frac{D}{180} = \frac{G}{200} = \frac{R}{\pi}$

$R = 60 \cdot \frac{\pi}{180}$

$\boxed{t_0 = 60 \cdot \frac{\pi}{180}}$

$f(t) \Rightarrow f(t_0) + \frac{\partial f}{\partial t}\Big|_{t_0} \cdot \frac{(t-t_0)}{1!} + \frac{\partial^2 f}{\partial t^2}\Big|_{t_0} \cdot \frac{(t-t_0)^2}{2!}$

$\Rightarrow \cos(60) + \left[-\sin(60) \cdot (t - 60 \cdot \frac{\pi}{180})\right] + \left[-\cos(60) - \frac{(t - 60 \cdot \frac{\pi}{180})^2}{2!}\right]$

$\Rightarrow \frac{1}{2} - \frac{\sqrt{3}}{2}(t - \frac{\pi}{3}) - \frac{1}{4}(t - \frac{\pi}{3})^2$

$\Rightarrow 0{,}5 + \left[(-0{,}866)(t - 1{,}047)\right] + \left[(-0{,}25) \cdot (t - 1{,}047)^2\right]$

Result

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