HACETTEPE UNIVERSITY
DEPARTMENT OF
GEOMATICS ENGINEERING

GMT202

ADJUSTMENT COMPUTATION & PARAMETER ESTIMATION

2021-2022 SPRING TERM

ASSIGNMENT 6

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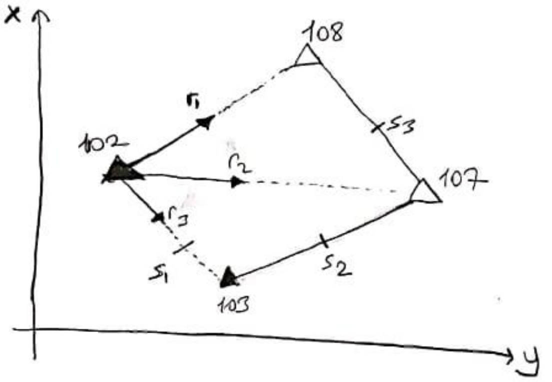
Adjust the mesh whose direction and side dimensions are given below, using the indirect measures method. For the directions, take the antecedent mean-square error as $s_0 = \pm 10^{cc}$.



NN	X (m)	Y (m)
Exact Coordinates		
102	7849,474	164,526
103	7731,373	608,285
Approximate Coordinates		
107	7969,948	719,676
108	8404,160	342,243

DN	BN	Side (m)	m_s (mm)
102	103	459,192	±3
103	107	263,297	±5
107	108	575,324	±4

DN	BN	Direction	m_d (cc)
102	108	0,00000	±10
	107	66,65613	±10
	103	96,81793	±10

Number of measures => $n = 6$ (3 directions and 3 side measures)

Number of unknowns => $u = 5$ (Two coordinate pairs and one routing unknown)

Degrees of Freedom => $f = n - u = 6 - 5 = 1 > 0$ => so, There is adjustment

Unknowns $= dz_{102}\ ,\ dx_{107}\ ,\ dy_{107}\ ,\ dx_{108}\ ,\ dy_{108}$

$$t^o_{12} = \arctan\left(\frac{y^o_2 - y^o_1}{x^o_2 - x^o_1}\right) \qquad s^o_{12} = \sqrt{(y^o_2 - y^o_1)^2 + (x^o_2 - x^o_1)^2}$$

$$a_{12} = \frac{\sin(t^o_{12})}{s^o_{12}} * \frac{\frac{200}{\pi} \cdot 10000}{100} \qquad b_{12} = -\frac{\cos(t^o_{12})}{s^o_{12}} * \frac{\frac{200}{\pi} \cdot 10000}{100} \qquad z^o_1 = \frac{[t_{12} - r_1]}{n}$$

DN	BN	Direction r_i (g)	t^o_{ik} (g)	s^o_{ik} (m)	$t^o_{ik} - r_i$	$-l_{ik}$ (cc) $t^o_{ik} - r_i - z^o_{102}$	a_{ik} cc/mm	b_{ik} cc/mm
102	108	0,00000	19,73894	582,460	19,73894	−8,8	0,3335	−1,0409
	107	66,65613	86,39556	568,072	19,73943	−3,9	1,0952	−0,2377
	103	96,81793	116,55902	459,206	19,74109	12,7	1,3397	0,3565
				$z^o_{102} = 19,73982$				

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- I will write the correction equations in the following format:

$v_{ik} = -dz_i + a_{ik} \cdot dx_i + b_{ik} \cdot dy_i - a_{ik} \cdot dx_k - b_{ik} \cdot dy_k - \ell_{ik}$

$v_{102-108} = -dz_{102} + 0{,}3335 \cdot dx_{102} - 1{,}0409 \cdot dy_{102} - 0{,}3335 \cdot dx_{108} + 1{,}0409 \cdot dy_{108} - 8{,}8$

$v_{102-107} = -dz_{102} + 1{,}0952 \cdot dx_{102} - 0{,}2377 \cdot dy_{102} - 1{,}0952 \cdot dx_{107} + 0{,}2377 \cdot dy_{107} - 3{,}9$

$v_{102-103} = -dz_{102} + 1{,}3397 \cdot dx_{102} + 0{,}3565 \cdot dy_{102} - 1{,}3397 \cdot dx_{103} - 0{,}3565 \cdot dy_{103} + 12{,}7$

- Points 102 and 103 are the reference points, I will subtract the coefficients for these points from the correction equations and rewrite the equations:

$v_{102-108} = -dz_{102} - 0{,}3335 \cdot dx_{108} + 1{,}0409 \cdot dy_{108} - 8{,}8$

$v_{102-107} = -dz_{102} - 1{,}0952 \cdot dx_{107} + 0{,}2377 \cdot dy_{107} - 3{,}9$

$v_{102-103} = -dz_{102} \quad \emptyset \quad \emptyset \quad +12{,}7$

- Let's rearrange these equations for the unknown and eliminate the dz_{102} routing unknown:

$v_{102-108} = -dz_{102} + 0 \cdot dx_{107} + 0 \cdot dy_{107} - 0{,}3335 \cdot dx_{108} + 1{,}0409 \cdot dy_{108} - 8{,}8$

$v_{102-107} = -dz_{102} - 1{,}0952 \cdot dx_{107} + 0{,}2377 \cdot dy_{107} + 0 \cdot dx_{108} + 0 \cdot dy_{108} - 3{,}9$

$v_{102-103} = -dz_{102} + 0 \cdot dx_{107} + 0 \cdot dy_{107} + 0 \cdot dx_{108} + 0 \cdot dy_{108} + 12{,}7$

	dz_{102}	dx_{107}	dy_{107}	dx_{108}	dy_{108}	$-\ell$
$v_{102-108}$ =	-1	0	0	-0,3335	1,0409	-8,8
$v_{102-107}$ =	-1	-1,0952	0,2377	0	0	-3,9
$v_{102-103}$ =	-1	0	0	0	0	12,7
Total	-3	-1,0952	0,2377	-0,3335	1,0409	0,00
	1	0,3651	-0,0792	0,1112	-0,3470	0,00

(divide $n=3$, $-n=-3$)

- Orientation unknown equation:

$1 \cdot dz_{102} + 0{,}3651 \cdot dx_{107} - 0{,}0792 \cdot dy_{107} + 0{,}1112 \cdot dx_{108} - 0{,}3470 \cdot dy_{108} = 0$

- dz_{102} routing unknown correction equations:

	dx_{107}	dy_{107}	dx_{108}	dy_{108}	$-\ell$
$v_{102-108}$ =	0,3651	-0,0792	-0,2223	0,6939	-8,8
$v_{102-107}$ =	-0,7301	0,1584	0,1112	-0,3470	-3,9
$v_{102-103}$ =	0,3651	-0,0792	0,1112	-0,3470	12,7

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- Correction equations for directions in the format $\underline{v} = \underline{A} \cdot \underline{x} - \underline{l}$

$$\begin{bmatrix} V_{102-108} \\ V_{102-107} \\ V_{102-103} \end{bmatrix} = \begin{bmatrix} 0,3651 & -0,0792 & -0,2223 & 0,6935 \\ -0,7301 & 0,1854 & 0,1112 & -0,3470 \\ 0,3651 & -0,0792 & 0,1112 & -0,3470 \end{bmatrix} \cdot \begin{bmatrix} dx_{107} \\ dy_{107} \\ dx_{108} \\ dy_{108} \end{bmatrix} - \begin{bmatrix} 8,8 \\ 3,9 \\ -12,7 \end{bmatrix}$$

$$S_{i2}^{o} = \sqrt{(y_2^o - y_1^o)^2 + (x_2^o - x_1^o)^2}$$

$$a_{12} = -\frac{(x_2^o - x_1^o)}{S_{12}^o} \qquad b_{12} = -\frac{(y_2^o - y_1^o)}{S_{12}^o} \qquad -l_{ik} = S_{ik}^o - S_{ik} \text{ unitary}$$

DN	BN	ΔX (m)	Δy (m)	S_{ik}^o (m)	S_{ik}	$-l_{ik}$ (mm)	a_{ik}	b_{ik}
102	103	-118, 101	443, 759	459, 206	459, 192	13,7	0,2572	-0, 9664
103	107	238, 575	111, 391	263, 298	263, 297	1.3	-0,9061	-0,4231
107	108	434, 212	-377, 433	575, 322	575,324	-1,7	-0,7547	0,6560

- I will write the correction equations in the following format:

$$V_{S_{12}} = a_{12} \cdot dx_1 + b_{12} \cdot dy_1 - a_{12} \cdot dx_2 - b_{12}\, dy_2 - l_{12} = 0$$

$$V_{S102-103} = +0,2572 \cdot dx_{102} - 0,9664 \cdot dy_{102} - 0,2572 \cdot dx_{103} + 0,9664 \cdot dy_{103} + 13,7$$

$$V_{S103-107} = -0,9061 \cdot dx_{103} - 0,4231 \cdot dy_{103} + 0,9061 \cdot dx_{107} + 0,4231 \cdot dy_{107} + 1,3$$

$$V_{S107-108} = -0,7547 \cdot dx_{107} + 0,6560 \cdot dy_{107} + 0,7547 \cdot dx_{108} - 0,6560 \cdot dy_{108} - 1,7$$

- Points 102 and 103 are the reference points. I will subtract the coefficients for these points from the correction equations and rewrite the equations:

$$V_{S102-103} = \cancel{} \quad \cancel{} \quad \cancel{} \quad \cancel{} + 13,7$$

$$V_{S103-107} = +0,9061 \cdot dx_{107} + 0,4231 \cdot dy_{107} \quad \cancel{} - \cancel{} + 1,3$$

$$V_{S107-108} = -0,7547 \cdot dx_{107} + 0,6560 \cdot dy_{107} + 0,7547 \cdot dx_{108} - 0,6560 \cdot dy_{108} - 1,7$$

- Let's rearrange these equations according to the unknowns:

$$\begin{array}{lccccc} & dx_{107} & dy_{107} & dx_{108} & dy_{108} & -l \\ V_{S102-103} = & 0 & 0 & 0 & 0 & 13,7 \\ V_{S103-107} = & 0,9061 & 0,4321 & 0 & 0 & 1,3 \\ V_{S107-108} = & -0,7547 & 0,6560 & 0,7547 & -0,6560 & -1,7 \end{array}$$

- Let me write these equations in the format $\underline{v} = \underline{A} \cdot \underline{x} - \underline{l}$

$$\begin{bmatrix} V_{S102-103} \\ V_{S103-107} \\ V_{S107-108} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0,9061 & 0,4321 & 0 & 0 \\ -0,7547 & 0,6560 & 0,7547 & -0,6560 \end{bmatrix} \cdot \begin{bmatrix} dx_{107} \\ dy_{107} \\ dx_{108} \\ dy_{108} \end{bmatrix} - \begin{bmatrix} -13,7 \\ -1,3 \\ 1,7 \end{bmatrix}$$

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* calculated by Matlab

Functional Model

- I will combine the $\underline{v} = \underline{A} \cdot \underline{x} - \underline{l}$ matrices I wrote for directions and sides.

$$\begin{bmatrix} v_{102-108} \\ v_{102-107} \\ v_{102-103} \\ v_{s102-103} \\ v_{s103-107} \\ v_{s107-108} \end{bmatrix} = \begin{bmatrix} 0,3651 & -0,0792 & -0,2223 & 0,6939 \\ -0,7301 & 0,1584 & 0,1112 & -0,3470 \\ 0,3651 & -0,0792 & 0,1112 & -0,3470 \\ 0 & 0 & 0 & 0 \\ 0,9661 & 0,4321 & 0 & 0 \\ -0,7547 & 0,6560 & 0,7547 & -0,6560 \end{bmatrix} \cdot \begin{bmatrix} dx_{107} \\ dy_{107} \\ dx_{108} \\ dy_{108} \end{bmatrix} - \begin{bmatrix} 3,8 \\ 3,9 \\ -12,7 \\ -13,7 \\ -1,3 \\ 1,7 \end{bmatrix}$$

Stochastic Model : Using the definition of weight $\boxed{P = \frac{s_0^2}{m_i^2} = \frac{10^2}{m_i^2}}$

$$\left.\begin{aligned} P_{d1} &= \frac{s_0^2}{m_{d_1}^2} = \frac{10^2}{10^2} = 1 \\ P_{d2} &= \frac{s_0^2}{m_{d_2}^2} = \frac{10^2}{10^2} = 1 \\ P_{d3} &= \frac{s_0^2}{m^2} = \frac{10^2}{10^2} = 1 \end{aligned}\right\} \text{unitless}$$

$$P_{s_1} = \frac{s_0^2}{m_{s_1}^2} = \frac{10^2}{3^2} = 11,11 \ cc^2/mm^2$$

$$P_{s_2} = \frac{s_0^2}{m_{s_2}^2} = \frac{10^2}{5^2} = 4,00 \ cc^2/mm^2$$

$$P_{s_3} = \frac{s_0^2}{m_{s_3}^2} = \frac{10^2}{4^2} = 6,25 \ cc^2/mm^2$$

$$\underline{P} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 11,11 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4,00 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6,25 \end{bmatrix}$$

$$\underline{N} = \underline{A}^T \underline{P} \underline{A} = \begin{bmatrix} 7,6438 & -1,7347 & -3,6818 & 3,4745 \\ -1,7347 & 3,4435 & 3,1210 & -2,7724 \\ -3,6818 & 3,1210 & 3,6342 & -3,3260 \\ 3,4745 & -2,7724 & -3,3260 & 3,4122 \end{bmatrix}$$

$$\underline{n} = \underline{A}^T \underline{P} \underline{l} = \begin{bmatrix} -17,0137 \\ 5,7929 \\ 5,1599 \\ 2,1235 \end{bmatrix}$$

$$\underline{Q}_{xx} = N^{-1} = \begin{bmatrix} 0,8078 & -1,5232 & 2,2332 & 0,1166 \\ -1,5232 & 4,2160 & -5,6451 & -0,5260 \\ 2,2332 & -5,6451 & 10,2535 & 3,1338 \\ 0,1166 & -0,5260 & 3,1338 & 2,8016 \end{bmatrix}$$

$$\underline{x} = \underline{Q}_{xx} \cdot \underline{n} = \begin{bmatrix} dx_{107} \\ dy_{107} \\ dx_{108} \\ dy_{108} \end{bmatrix} = \begin{bmatrix} -10,8 \\ 20,1 \\ -11,1 \\ 17,1 \end{bmatrix} mm$$

Exact Value of the unknown

$$\begin{bmatrix} x_{107} \\ y_{107} \\ x_{108} \\ y_{108} \end{bmatrix} = \begin{bmatrix} x^0_{107} \\ y^0_{107} \\ x^0_{108} \\ y^0_{108} \end{bmatrix} + \begin{bmatrix} dx_{107} \\ dy_{107} \\ dx_{108} \\ dy_{108} \end{bmatrix}$$

$$\begin{bmatrix} x_{107} \\ y_{107} \\ x_{108} \\ y_{108} \end{bmatrix} = \begin{bmatrix} 7969,348 \\ 719,676 \\ 8404,160 \\ 342,243 \end{bmatrix} + \begin{bmatrix} -10,8 \\ 20,1 \\ -11,1 \\ 17,1 \end{bmatrix} = \begin{bmatrix} 7969,3372 \\ 719,6961 \\ 8404,1489 \\ 342,2601 \end{bmatrix}$$

* Calculated by Matlab

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ADJUSTMENT $\underline{v} = \underline{A} \cdot \underline{x} - \underline{l}$

$$\begin{bmatrix} v_{102-108} \\ v_{102-107} \\ v_{102-103} \\ v_{S102-103} \\ v_{S103-102} \\ v_{S102-108} \end{bmatrix} = \begin{bmatrix} 0,3651 & -0,0792 & -0,2223 & 0,6939 \\ -0,7301 & 0,1584 & 0,1112 & -0,3470 \\ 0,3651 & -0,0792 & 0,1112 & -0,3470 \\ 0 & 0 & 0 & 0 \\ 0,9661 & 0,4321 & 0 & 0 \\ -0,7547 & 0,6560 & 0,7547 & -0,6560 \end{bmatrix} \cdot \begin{bmatrix} -10,8 \\ 20,1 \\ -11,1 \\ 17,1 \end{bmatrix} - \begin{bmatrix} 8,8 \\ 3,9 \\ -12,7 \\ -13,7 \\ -1,3 \\ 1,7 \end{bmatrix} = \begin{bmatrix} 0,00 \\ 0,00 \\ 0,00 \\ 13,72 \\ 0,00 \\ 0,00 \end{bmatrix}$$

Adjusted Direction measures $\hat{\underline{r}}_i = \underline{r}_i + \underline{v}_i$

$$\begin{bmatrix} \hat{r}_1 \\ \hat{r}_2 \\ \hat{r}_3 \end{bmatrix} = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} + \begin{bmatrix} v_{102-108} \\ v_{102-107} \\ v_{102-103} \end{bmatrix} \qquad \begin{bmatrix} \hat{r}_1 \\ \hat{r}_2 \\ \hat{r}_3 \end{bmatrix} = \begin{bmatrix} 0,00000 \\ 66,65613 \\ 96,81793 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0,00000 \\ 66,65613 \\ 96,81793 \end{bmatrix}$$

Control of Adjusted Direction measures

- Orientation unknown equation

$$1 \cdot dz_{102} + 0,3651 \cdot dx_{107} - 0,0792 \cdot dy_{107} + 0,1112 \cdot dx_{108} - 0,3470 \cdot dy_{108} = 0$$

- with Matrix Notation

$$dz_{102} = -\begin{bmatrix} 0,3651 & -0,0792 & 0,1112 & -0,3470 \end{bmatrix} \cdot \begin{bmatrix} dx_{107} \\ dy_{107} \\ dx_{108} \\ dy_{108} \end{bmatrix}$$

$$dz_{102} = -\begin{bmatrix} 0,3651 & -0,0792 & 0,1112 & -0,3470 \end{bmatrix} \cdot \begin{bmatrix} -10,8 \\ 20,1 \\ -11,1 \\ 17,1 \end{bmatrix} = 12,70^{cc}$$

$$z_{102} = z^0_{102} + dz_{102} = 19,74109 + 12,70^{cc}/10.000 = \boxed{19,74109}$$

DN	BN	District from adjusted directions					From Adjusted coordinates District t_{ik}	Difference
		r_i (g)	v_i (cc)	$\hat{r}_i = r_i + v_i$	z_{102}	$t_{ik} = \hat{r}_i + z_{102}$		
102	108	0,00000	0,00	0,00000	19,74109	19,74109	19,74109	0,00
	107	66,65613	0,00	66,65613	19,74109	86,39722	86,39722	0,00
	103	96,81793	0,00	96,81793	19,74109	116,55902	116,55902	0,00

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Adjusted Side Dimensions

$\hat{\underline{s}}_i = \underline{s}_i + \underline{v}_{s_i}$

$$\begin{bmatrix} \hat{s}_1 \\ \hat{s}_2 \\ \hat{s}_3 \end{bmatrix} = \begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix} + \begin{bmatrix} v_{s_{102-103}} \\ v_{s_{103-107}} \\ v_{s_{107-108}} \end{bmatrix} \qquad \begin{bmatrix} \hat{s}_1 \\ \hat{s}_2 \\ \hat{s}_3 \end{bmatrix} = \begin{bmatrix} 459,192 \\ 263,297 \\ 575,324 \end{bmatrix} + \begin{bmatrix} 13,72 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 459,206 \\ 263,297 \\ 575,324 \end{bmatrix}$$

Checking the Adjusted side Dimensions

DN	BN	Δx (m)	Δy (m)	From Adjusted Coordinates $\hat{s}_i = \sqrt{(\Delta x)^2 + (\Delta y)^2}$	From Adjusted side $\hat{s}_i = s_i + v_{s_i}$	$\underline{v}_{s_i}$ (mm)	$\underline{s}_i$ (m)
102	103	-118,101	443,759	459,206	459,206	13,72	459,192
103	107	238,564	111,411	263,297	263,297	0,00	263,297
107	108	434,212	-377,436	575,324	575,324	0,00	575,324

Square Mean Error

$$m_0 = \pm\sqrt{\frac{\underline{v}^T \underline{P} \underline{v}}{n-u}} = \pm\sqrt{\frac{2092,53}{6-5}} \neq \pm 45,7 \text{ mm}$$

Mean Error of Unknown

$$\underline{Q}_{xx} = \underline{N}^{-1} = \begin{bmatrix} 0,8078 & -1,5232 & 2,2332 & 0,1166 \\ -1,5232 & 4,2160 & -5,6451 & -0,5260 \\ 2,2332 & -5,6451 & 10,2535 & 3,1338 \\ 0,1166 & -0,5260 & 3,1338 & 2,8016 \end{bmatrix}$$

$$m_{x107} = \pm m_0 \cdot \sqrt{q_{xx_{107}}} = \pm 45,7 \cdot \sqrt{0,8078} = \pm 41,1 \text{ mm}$$

$$m_{y107} = \pm m_0 \cdot \sqrt{q_{yy_{107}}} = \pm 45,7 \cdot \sqrt{4,2160} = \pm 93,9 \text{ mm}$$

$$m_{x108} = \pm m_0 \sqrt{q_{xx_{108}}} = \pm 45,7 \cdot \sqrt{10,2535} = \pm 146,5 \text{ mm}$$

$$m_{y108} = \pm m_0 \sqrt{q_{yy_{108}}} = \pm 45,7 \cdot \sqrt{2,8016} = \pm 76,6 \text{ mm}$$

Mean Error of measures

$$\underline{P} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 11,11 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6,25 \end{bmatrix} \qquad m_i = \pm \frac{m_0}{\sqrt{\underline{P}_i}} \text{ mm}$$

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$m_{r1} = \pm \frac{45.7}{\sqrt{1}} = \pm 45{,}7^{cc}$ $\qquad$ $m_{s1} = \pm \frac{45{,}7}{\sqrt{11{,}11}} = \pm 13{,}7\ cm$

$m_{r2} = \pm \frac{45{,}7}{\sqrt{1}} = \pm 45{,}7^{cc}$ $\qquad$ $m_{s2} = \pm \frac{45{,}7}{\sqrt{4{,}00}} = \pm 22{,}9\ cm$

$m_{r3} = \pm \frac{45{,}7}{\sqrt{1}} = \pm 45{,}7^{cc}$ $\qquad$ $m_{s3} = \pm \frac{45{,}7}{\sqrt{6{,}25}} = \pm 18{,}3\ cm$

Avarage Error of Adjusted Measures

$$\underline{Q}_{\hat{l}\hat{l}} = \underline{A} \cdot \underline{Q}_{xx} \cdot \underline{A}^T = \begin{bmatrix} 0{,}6667 & -0{,}3333 & -0{,}3333 & 0 & 0 & 0 \\ -0{,}3333 & 0{,}6667 & -0{,}3333 & 0 & 0 & 0 \\ -0{,}3333 & -0{,}3333 & 0{,}6667 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0{,}25 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0{,}16 \end{bmatrix}$$

$m_i = \pm m_0 \cdot \sqrt{\underline{Q}_{\hat{l}\hat{l}}}$ Avarage error of adjusted measures

$m_{\hat{r}_1} = \pm 45{,}7 \cdot \sqrt{0{,}6667} = \pm 37{,}4\ cc$ $\qquad$ $m_{\hat{s}_1} = \pm 45{,}7 \cdot \sqrt{0{,}00} = \pm 0{,}0\ cm$

$m_{\hat{r}_2} = \pm 45{,}7 \cdot \sqrt{0{,}6667} = \pm 37{,}4\ cc$ $\qquad$ $m_{\hat{s}_2} = \pm 45{,}7 \cdot \sqrt{0{,}25} = \pm 22{,}9\ cm$

$m_{\hat{r}_3} = \pm 45{,}7 \cdot \sqrt{0{,}6667} = \pm 37{,}4\ cc$ $\qquad$ $m_{\hat{s}_3} = \pm 45{,}7 \cdot \sqrt{0{,}16} = \pm 18{,}3\ cm$

Average Error of Corrections

$\underline{Q}_{vv} = \underline{Q}_{ll} - \underline{Q}_{\hat{l}\hat{l}}$ Covariance matrix of corrections

$\underline{Q}_{vv} = \underline{P}^{-1} - \underline{Q}_{\hat{l}\hat{l}}$

$$\underline{P}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0{,}09 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0{,}25 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0{,}16 \end{bmatrix} \qquad \underline{Q}_{vv} = \begin{bmatrix} 0{,}3333 & 0{,}3333 & 0{,}3333 & 0 & 0 & 0 \\ 0{,}3333 & 0{,}3333 & 0{,}3333 & 0 & 0 & 0 \\ 0{,}3333 & 0{,}3333 & 0{,}3333 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0{,}09 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0{,}00 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0{,}00 \end{bmatrix}$$

* Calculated by Matlab

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$m_{v_i} = \pm m_0 \cdot \sqrt{Q_{v_i v_i}}$ Average errors of corrections

$m_{v_1} = \pm 45{,}7 \times \sqrt{0{,}3333} = \pm 26{,}4 \text{ mm}$

$m_{v_2} = \pm 45{,}7 \times \sqrt{0{,}3333} = \pm 26{,}4 \text{ mm}$

$m_{v_3} = \pm 45{,}7 \times \sqrt{0{,}3333} = \pm 26{,}4 \text{ mm}$

$m_{v_4} = \pm 45{,}7 \times \sqrt{0{,}0900} = \pm 13{,}7 \text{ mm}$

$m_{v_5} = \pm 45{,}7 \times \sqrt{0{,}0000} = \pm 0{,}0$

$m_{v_6} = \pm 45{,}7 \times \sqrt{0{,}0000} = \pm 0{,}0$

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