HACETTEPE UNIVERSITY
DEPARTMENT OF
GEOMATICS ENGINEERING

GMT202

ADJUSTMENT COMPUTATION & PARAMETER ESTIMATION

2021-2022 SPRING TERM

ASSIGNMENT 10

ABDULSAMET TOPTAŞ - 21905024

(1) Point coordinates in the ED-50 coordinate system are given in the table in both systems. It is desired to convert it to the ITRF-96 coordinate system with the help of common points. Apply the similarity transformation and calculate the transformation parameters. Calculate the coordinates of the new points in ITRF-96.

NN	ED-50 (m)		ITRF-96 (m)	
	Above (x)	Right (y)	Above (x)	Right (y)
8	54481,227	56219,662	40727,970	62084,098
9	54278,188	53056,137	40498,206	58921,596
10	55203,664	52952,417	41423,028	58810,095
12	54734,544	53754,865	40960,581	59616,631
16	54350,343	56110,555		
17	55800,011	53012,938		
18	54315,160	53205,945		

Number of Measures $\Rightarrow$ $n=4$ pairs $\times 2$ $\underline{n=8}$

Unknown parameter $\Rightarrow$ $\underline{u=4}$

Degrees of freedom $\Rightarrow$ $f = 8-4 = \boxed{4}$

Solution

$$Vx_p = X_o - b.y + a.x - X_p$$
$$Vy_p = y_o + b.x + a.y - y_p$$

$$Vx_8 = X_o - b.y_8 + a.x_8 - X_8$$
$$Vy_8 = y_o + b.x_8 + a.y_8 - y_8$$
$$Vx_9 = X_o - b.y_9 + a.x_9 - X_9$$
$$Vy_9 = y_o + b.x_9 + a.y_9 - y_9$$
$$Vx_{10} = X_o - b.y_{10} + a.x_{10} - X_{10}$$
$$Vy_{10} = y_o + b.x_{10} + a.y_{10} - y_{10}$$
$$Vx_{12} = X_o - b.y_{12} + a.x_{12} - X_{12}$$
$$Vy_{12} = y_o + b.x_{12} + a.y_{12} - y_{12}$$

$$\begin{bmatrix} Vx_8 \\ Vy_8 \\ Vx_9 \\ Vy_9 \\ Vx_{10} \\ Vy_{10} \\ Vx_{12} \\ Vy_{12} \end{bmatrix} = \begin{bmatrix} 1 & 0 & x_8 & -y_8 \\ 0 & 1 & y_8 & x_8 \\ 1 & 0 & x_9 & -y_9 \\ 0 & 1 & y_9 & x_9 \\ 1 & 0 & x_{10} & -y_{10} \\ 0 & 1 & y_{10} & x_{10} \\ 1 & 0 & x_{12} & -y_{12} \\ 0 & 1 & y_{12} & x_{12} \end{bmatrix} \cdot \begin{bmatrix} X_o \\ y_o \\ a \\ b \end{bmatrix} - \begin{bmatrix} x_8 \\ y_8 \\ x_9 \\ y_9 \\ x_{10} \\ y_{10} \\ x_{12} \\ y_{12} \end{bmatrix}$$

$$\begin{bmatrix} Vx_8 \\ Vy_8 \\ Vx_9 \\ Vy_9 \\ Vx_{10} \\ Vy_{10} \\ Vx_{12} \\ Vy_{12} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 54481,227 & -56219,662 \\ 0 & 1 & 56219,662 & 54481,227 \\ 1 & 0 & 54278,188 & -53056,137 \\ 0 & 1 & 53056,137 & 54278,188 \\ 1 & 0 & 55203,664 & -52952,417 \\ 0 & 1 & 52952,417 & 55203,664 \\ 1 & 0 & 54734,544 & -53754,865 \\ 0 & 1 & 53754,865 & 54734,544 \end{bmatrix}}_{A} \cdot \begin{bmatrix} X_o \\ y_o \\ a \\ b \end{bmatrix} - \begin{bmatrix} 40727,970 \\ 62084,098 \\ 40498,206 \\ 58921,596 \\ 41423,028 \\ 58810,095 \\ 40960,581 \\ 59616,631 \end{bmatrix}$$

Abdulsamet TOPTAŞ
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$$N = A^T A = \begin{bmatrix} 4 & 0 & 218697,623 & -215983,081 \\ 0 & 4 & 215983,081 & 218697,623 \\ 218697,623 & 215983,081 & 23626788659,969 & 0 \\ -215983,081 & 218697,623 & 0 & 23626788659,696 \end{bmatrix}$$

$$n = A^T l = \begin{bmatrix} 163609,785 \\ 2194432,420 \\ 21881060560,514 \\ 42565261110,794 \end{bmatrix} \quad Q_{xx} = N^{-1} = \begin{bmatrix} 792,49805 & 0 & -0,00734 & 0,00724 \\ 0 & 792,49805 & -0,00724 & -0,00734 \\ -0,00734 & -0,00724 & 0,000000342 & 0 \\ 0,00724 & -0,00734 & 0 & 0,000000342 \end{bmatrix}$$

$$x = Q_{xx} \cdot n = \begin{bmatrix} x_0 \\ y_0 \\ a \\ b \end{bmatrix} = \begin{bmatrix} -14238,6155 \\ 6311,5841 \\ 1,000212805 \\ -0,008426763 \end{bmatrix} m$$

$$\lambda = \sqrt{a^2 + b^2} = 1,000248303$$

$$\varepsilon = \tan^{-1}\frac{b}{a} = -0^g.5364$$

Corrections $v = A \cdot x - l$ $[v] = 0,00$?

$$\begin{bmatrix} VX_8 \\ VY_8 \\ VX_9 \\ VY_9 \\ VX_{10} \\ VY_{10} \\ VX_{12} \\ VY_{12} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 54481,227 & -56213,662 \\ 0 & 1 & 56213,662 & 54481,227 \\ 1 & 0 & 54278,188 & -53056,137 \\ 0 & 1 & 53056,137 & 54278,188 \\ 1 & 0 & 55203,664 & -52952,417 \\ 0 & 1 & 52952,417 & 55203,664 \\ 1 & 0 & 54734,546 & -53754,865 \\ 0 & 1 & 53754,865 & 54734,546 \end{bmatrix} \cdot \begin{bmatrix} -14238,6155 \\ 6311,5841 \\ 1,000212805 \\ -0,008426763 \end{bmatrix} - \begin{bmatrix} 40727,970 \\ 62084,098 \\ 40498,206 \\ 58921,596 \\ 41423,028 \\ 58810,095 \\ 40960,581 \\ 59616,631 \end{bmatrix} = \begin{matrix} (m) \\ \begin{bmatrix} -0,0029 \\ -0,0001 \\ 0,0199 \\ 0,0147 \\ -0,0032 \\ -0,0253 \\ -0,0138 \\ 0,0107 \end{bmatrix} \end{matrix}$$

Transformed coordinates and their Corrections ;

NN	ITRF-96 (m)		VX:	VY:	ITRF-96	
	Above (X)	Right (Y)	m	m	Above (X)	Right (Y)
8	40727,970	62084,098	-0,0029	-0,0001	40727,967	62084,098
9	40498,206	58921,596	0,0199	0,0147	40498,226	58921,611
10	41423,028	58810,095	-0,0032	-0,0253	41423,025	58810,070
12	40960,581	59616,631	-0,0138	0,0107	40960,567	59616,642

Abdulsamet TOPTAŞ
21905024

(3)

Control of Adjusted measures ;

$$\begin{bmatrix} X_8 + VX_8 \\ Y_8 + VY_8 \\ X_9 + VX_9 \\ Y_9 + VY_9 \\ X_{10} + VX_{10} \\ Y_{10} + VY_{10} \\ X_{12} + VX_{12} \\ Y_{12} + VY_{12} \end{bmatrix} - \begin{bmatrix} 1 & 0 & X_8 & -Y_8 \\ 0 & 1 & Y_8 & X_8 \\ 1 & 0 & X_9 & -Y_9 \\ 0 & 1 & Y_9 & X_9 \\ 1 & 0 & X_{10} & -Y_{10} \\ 0 & 1 & Y_{10} & X_{10} \\ 1 & 0 & X_{12} & -Y_{12} \\ 0 & 1 & Y_{12} & X_{12} \end{bmatrix} \cdot \begin{bmatrix} X_0 \\ Y_0 \\ a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Square Mean Error ;

$$m_0 = \pm\sqrt{\frac{\underline{v}^T \underline{v}}{n-u}} = \pm\sqrt{\frac{0,0016}{84}}$$

$$= \pm 0,02\ m$$

Mean Error of Unknowns ;

$$\underline{Q}_{xx} = \underline{N}^{-1} = \begin{bmatrix} 792,49805 & 0 & -0,00734 & 0,00724 \\ 0 & 792,49805 & -0,00724 & -0,00734 \\ -0,00734 & -0,00724 & 0,0000001342 & 0 \\ 0,00724 & -0,00734 & 0 & 0,0000001342 \end{bmatrix}$$

$$m_{X_0} = \pm m_0 \sqrt{q_{xx}} = \pm 0,02 \sqrt{792,49805} = \pm 0,56\ m$$

$$m_{Y_0} = \pm m_0 \sqrt{q_{yy}} = \pm 0,02 \sqrt{792,49805} = \pm 0,56\ m$$

$$m_a = \pm m_0 \sqrt{q_{aa}} = \pm 0,02 \sqrt{0,0000001342} = \pm 0,00000728\ m$$

$$m_b = \pm m_0 \sqrt{q_{bb}} = \pm 0,02 \sqrt{0,0000001342} = \pm 0,00000728\ m$$

Covariance Matrix of Corrections ; $\underline{Q}_W = \underbrace{\underline{I}}_{Q_{ll}} - \underbrace{\underline{A} \cdot \underline{Q}_{xx} \cdot \underline{A}^T}_{Q_{\hat{l}\hat{l}}}$

$$\begin{bmatrix} 0,081 & 0,000 & 0,020 & -0,1413 & 0,075 & 0,131 & -0,177 & 0,012 \\ 0,000 & 0,081 & 0,143 & 0,020 & -0,131 & 0,075 & -0,012 & -0,177 \\ 0,020 & 0,143 & 0,610 & 0,000 & -0,353 & -0,122 & -0,277 & -0,020 \\ -0,1413 & 0,020 & 0,000 & 0,610 & 0,122 & -0,353 & -0,020 & -0,277 \\ 0,075 & -0,131 & -0,353 & 0,122 & 0,566 & 0,000 & -0,288 & 0,009 \\ 0,131 & 0,075 & -0,122 & -0,353 & 0,000 & 0,566 & -0,009 & -0,288 \\ -0,177 & -0,012 & -0,277 & 0,020 & -0,288 & -0,009 & 0,742 & 0,000 \\ 0,012 & -0,177 & -0,020 & -0,277 & 0,009 & -0,288 & 0,000 & 0,742 \end{bmatrix}$$

Abdulsamet TOPTAŞ
2190524

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New Cordinates ;

$X_{16} = X_0 - b \cdot y_{16} + a \cdot x_{16}$ $\Rightarrow X_{16} = -14238,6155 - (-0,00842697b) \times 56510,555$
$+ (1.000212805) \times 54350,343 = 40596,136$ m

$y_{16} = y_0 + b \cdot x_{16} + a \cdot y_{16}$ $\Rightarrow y_{16} = 6311,5841 + (-0,00842697b3) \times 54350,343$
$+ (1,000212805) \times 56110,555 = 61976,071$ m

$X_{17} = X_0 - b \cdot y_{17} + a \cdot x_{17}$

$y_{17} = y_0 + b \cdot x_{17} + a \cdot y_{17}$

$X_{18} = X_0 - b \cdot y_{18} + a \cdot x_{18}$

$y_{18} = y_0 + b \cdot x_{18} + a \cdot y_{18}$

This is how we put the values in their place.

$a = 1,000212805$
$b = -0,00842697b3$
$X_0 = -14238,6155$
$y_0 = 6311,5841$
} m

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16	54350,343	56110,555	40596,136	61976,071
17	55800,011	53012,938	42020,009	58865,578
18	54315,160	53205,945	40536,468	59071,139

New Cordinates

Abdulsamet TOPTAŞ
21905024