HACETTEPE UNIVERSITY
DEPARTMENT OF
GEOMATICS ENGINEERING

HACETTEPE UNIVERSITY
GEOMATICS ENGINEERING

GMT202

ADJUSTMENT COMPUTATION & PARAMETER ESTIMATION

2021-2022 SPRING TERM

ASSIGNMENT 5

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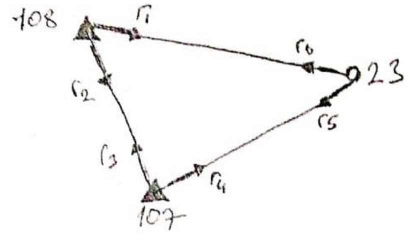
Adjust the network of directions given below using the indirect measures method.

NN	y (m)	x (m)
Exact Coordinates		
107	719,689	7969,933
108	342,246	8404,180
Approximate Coordinates		
23	638,765	8351,331

DN	BN	Direction
108	23	0,00000
	107	43,21580
107	108	0,00000
	23	32,24480
23	107	0,00000
	108	124,53835



Numbers of measures $n=6$

Number of unknowns $u = 2+3$ (1 coordinate pair and 3 routing unknowns)

Degrees of Freedom $f = n - u = 6-5 > 0$

There is adjustment

Coordinate Unknowns : dx_{23} , dy_{23}

Three Routing Unknowns : dz_{23} , dz_{107} , dz_{108}

(direction observation were made at points 23, 107 and 108)

$$t^o_{12} = \arctan\left(\frac{y^o_2 - y^o_1}{x^o_2 - x^o_1}\right) \qquad s^o_{12} = \sqrt{(y^o_2 - y^o_1)^2 + (x^o_2 - x^o_1)^2}$$

$$a_{12} = \frac{\sin(t^o_{12})}{s^o_{12}} \cdot \frac{\frac{200}{\pi} \cdot 10000}{100} \qquad b_{12} = -\frac{\cos(t^o_{12})}{s^o_{12}} \cdot \frac{\frac{200}{\pi} \cdot 10000}{100} \qquad z^o_1 = \frac{[t^o_{ik} - r_i]}{n}$$

DN	BN	Direction r_i (g)	t^o_{ik} (g)	s^o_{ik} (m)	$t^o_{ik} - r_i$	$-l_{ik}$ (cc) $t^o_{ik} - r_i - z^o_{108}$	a_{ik} cc/cm	b_{ik} cc/cm
108	23	0,00000	111,22866	301,192	111,22866	−18	20,8088	3,7088
	107	43,21580	154,44796	575,355	111,23216	18	7,2587	8,3511
				$z^o_{108} = 111,23041$				

Let's write the correction equations for the directions observations at point 108.

$$V_{108-23} = -dz_{108} + 20,8088 \cdot dx_{108} + 3,7088 \cdot dy_{108} - 20,8088 \cdot dx_{23} - 3,7088 \cdot dy_{23} - 18$$

$$V_{108-107} = -dz_{108} + 7,2587 \cdot dx_{108} + 8,3511 \cdot dy_{108} - 7,2587 \cdot dx_{107} - 8,3511 \cdot dy_{107} + 18$$

Points 107 and 108 are the reference points, Let's subtracts the coefficients of these points from the correction equations and rewrite the equations according to the unknowns.

$$V_{108-23} = -dz_{108} - 20{,}8088 \cdot dx_{23} - 3{,}7088 \cdot dy_{23} - 18$$
$$V'_{108-107} = -dz_{108} + 0 \cdot dx_{23} + 0 \cdot dy_{23} + 18$$

Total	-2	-20,8088	-3,7088	0,00	Let's divide n=2 -n=-2
	1	10,4044	1,8544	0,00	

Direction unknown equation at point 108.

$$1 \cdot dz_{108} + 10{,}4044 \cdot dx_{23} + 1{,}8544 \cdot dy_{23} = 0$$

Let's write the elimination of the (dz_{108}) rotating unknown from the correction equations below.

$$V_{108-23} = -10{,}4044 \cdot dx_{23} - 1{,}8544 \cdot dy_{23} - 18$$
$$V_{108-107} = +10{,}4044 \cdot dx_{23} + 1{,}8544 \cdot dy_{23} + 18$$

DN	BN	Direction	t^0_{ik} (g)	S^0_{ik} (m)	$t^0_{ik} - r_i$	$-l_{ik}$ (cc) $= t^0_{ik} - r_i - z^0_{107}$	a_{ik} cc/cm	b_{ik} cc/cm
107	108	0,00000	354,44796	575,355	354,44796	15	-7,2587	-8,3511
	23	32,24480	386,68977	389,889	354,44497	-15	-3,3890	-15,9727
				$z^0_{107} = 354{,}44647$				

Let's write the correction equations for the direction observations at point 107.

$$V_{107-108} = -dz_{107} - \cancel{7{,}2587 \cdot dx_{107}} - \cancel{8{,}3511 \cdot dy_{107}} + \cancel{7{,}2587 \cdot dx_{108}} + \cancel{8{,}3511 \cdot dy_{108}} + 15$$
$$V_{107-23} = -dz_{107} - \cancel{3{,}3890 \cdot dx_{107}} - \cancel{15{,}9727 \cdot dy_{107}} + 3{,}3890 \cdot dx_{23} + 15{,}9727 \cdot dy_{23} - 15$$

Points 107 and 108 are the reference points. Let's subtract the coefficients of these points from the corrections equations and rewrite the equations according to the unknowns.

$$V_{107-108} = -dz_{107} + 0 \cdot dx_{23} + 0 \cdot dy_{23} + 15$$
$$V_{107-23} = -dz_{107} + 3{,}3890 \cdot dx_{23} + 15{,}9727 \cdot dy_{23} - 15$$

Total	-2	3,3890	15,9727	0,00	Let's divide
	1	-1,6945	-7,9863	0,00	n=2 -n=-2

Direction unknown equation at point 107

$$1 . dz_{107} - 1,6945 . dx_{23} - 7,9863 . dy_{23} = 0$$

Let's write the elimination form of the dz_{107} routing unknown from the correction equations below ;

$$V_{107-108} = -1,6945 . dx_{23} - 7,9863 . dy_{23} + 15$$

$$V_{107-23} = +1,6945 . dx_{23} + 7,9863 . dy_{23} - 15$$

DN	BN	Direction r_i (g)	t^o_{ik} (g)	s^o_{ik} (m)	$t^o_{ik} - r_i$	$-l_{ik}$ (cc) $t^o_{ik} - r_i - z^o_{107}$	a_{ik} cc/cm	b_{ik} cc/cm
23	107	0,00000	186,68977	389,889	186,68977	-3	3,3890	15,9727
	108	124,53835	311,22866	301,192	186,69031	3	-20,8088	-3,7088
				$z^o_{107} = 186,69004$				

Let's write the correction equations for the direction observations at point 23

$$V_{23-107} = -dz_{23} + 3,3890 . dx_{23} + 15,9727 . dy_{23} - \cancel{3,3890 . dx_{107}} - \cancel{15,9727 . dy_{107}} - 3$$

$$V_{23-108} = -dz_{23} - 20,8088 . dx_{23} - 3,7088 . dy_{23} + \cancel{20,8088 . dx_{108}} + \cancel{3,7088 . dy_{108}} + 3$$

Points 107 and 108 are the reference points. Let's subtract the coefficients of these points from the correction equations and rewrite the equations according to the unknowns.

$$V_{23-107} = -dz_{23} + 3,3890 . dx_{23} + 15,9727 . dy_{23} - 3$$

$$V_{23-108} = -dz_{23} - 20,8088 . dx_{23} - 3,7088 . dy_{23} + 3$$

Total	-2	-17,4189	12,2639	0,00	Let's divide n=2 -n=-2
	1	8,7099	6,1319	0,00	

Orientation unknown equation at point 23

$$1 . dz_{23} + 8,7099 . dx_{23} + 6,1319 . dy_{23} = 0$$

Let's write the elimination of the dz_{23} routing unknown from the correction equations below ;

$$V_{23-107} = +12,0989 . dx_{23} + 9,8407 . dy_{23} - 3$$

$$V_{23-108} = -12,0989 . dx_{23} - 9,8407 . dy_{23} + 3$$

Correction Equations

$$v_{108-23} = -10,4044 \cdot dx_{23} - 1,8544 \cdot dy_{23} - 18$$

$$v_{108-107} = +10,4044 \cdot dx_{23} + 1,8544 \cdot dy_{23} + 18$$

$$v_{107-108} = -1,6945 \cdot dx_{23} - 7,9863 \cdot dy_{23} + 15$$

$$v_{107-23} = +1,6945 \cdot dx_{23} + 7,9863 \cdot dy_{23} - 15$$

$$v_{23-107} = +12,0989 \cdot dx_{23} + 9,8407 \cdot dy_{23} - 3$$

$$v_{23-108} = -12,0989 \cdot dx_{23} - 9,8407 \cdot dy_{23} + 3$$

I will write the correction equations in the format $\boxed{\underline{V} = \underline{A} \cdot \underline{x} - \underline{l}}$

$$\begin{bmatrix} v_{108-23} \\ v_{108-107} \\ v_{107-108} \\ v_{107-23} \\ v_{23-107} \\ v_{23-108} \end{bmatrix} = \underbrace{\begin{bmatrix} -10,4044 & -1,8544 \\ 10,4044 & 1,8544 \\ -1,6945 & -7,9863 \\ 1,6945 & 7,9863 \\ 12,0989 & 9,8407 \\ -12,0989 & -9,8407 \end{bmatrix}_{6\times2}}_{A} \cdot \begin{bmatrix} dx_{23} \\ dy_{23} \end{bmatrix} - \begin{bmatrix} 18 \\ -18 \\ -15 \\ 15 \\ 3 \\ -3 \end{bmatrix}$$

$$dx = \underbrace{(A^T A)}_{N}{}^{-1} \cdot \underbrace{A^T l}_{n}$$

$A = cc/cm$

$A^T = cc/cm$

$l = cc$

$x = cm$

$$\underline{N} = \underline{A}^T \underline{A} = \begin{bmatrix} 515,0117 & 303,7773 \\ 303,7773 & 328,1201 \end{bmatrix} \qquad \underline{n} = \underline{A}^T \underline{l} = \begin{bmatrix} -249,1780 \\ 227,1338 \end{bmatrix}$$

$$\underline{N} = \underline{A}^T \underline{A} = \frac{cc}{cm} \cdot \frac{cc}{cm} = \frac{cc^2}{cm^2} \qquad \underline{n} = \underline{A}^T \underline{l} = \frac{cc}{cm} \cdot cc = \frac{cc^2}{cm}$$

$$\underline{Q}_{xx} = \underline{N}^{-1} = \begin{bmatrix} 0,0043 & -0,0040 \\ -0,0040 & 0,0067 \end{bmatrix} \Rightarrow \frac{cm^2}{cc^2} \qquad \delta x = \frac{cm^{\cancel{2}}}{\cancel{cc^2}} \cdot \frac{\cancel{cc^2}}{\cancel{cm}} \qquad \boxed{\delta x = cm}$$

$$\underline{x} = \underline{Q}_{xx} \cdot \underline{n} = \begin{bmatrix} dx_{23} \\ dy_{23} \end{bmatrix} = \begin{bmatrix} -2,0 \\ 2,5 \end{bmatrix} \underline{cm}$$

The Exact Value of the Unknown

$$\begin{bmatrix} x_{23} \\ y_{23} \end{bmatrix} = \begin{bmatrix} x_{23}^0 \\ y_{23}^0 \end{bmatrix} + \begin{bmatrix} dx_{23} \\ dy_{23} \end{bmatrix} \Rightarrow \begin{bmatrix} x_{23} \\ y_{23} \end{bmatrix} = \overbrace{\begin{bmatrix} 8351,331 \\ 638,765 \end{bmatrix}}^{m} + \overbrace{\begin{bmatrix} -2,0 \\ 2,5 \end{bmatrix}}^{cm} = \begin{bmatrix} 8351,311 \\ 638,790 \end{bmatrix}$$

* Be careful with units when adding.

Adjustments

$$\underline{V} = \underline{A} \cdot \underline{x} - \underline{l} = \frac{cc}{\cancel{cm}} \cdot \cancel{cm} - cc$$

$$= \begin{bmatrix} -1,75 \\ 1,75 \\ -1,75 \\ 1,75 \\ -1,75 \\ 1,75 \end{bmatrix} \begin{matrix} v_{108-23} \\ v_{108-107} \\ v_{107-108} \\ v_{107-23} \\ v_{23-107} \\ v_{23-108} \end{matrix} = \begin{bmatrix} -10,4044 & -1,8544 \\ 10,4044 & 1,8544 \\ -1,6945 & -7,9863 \\ 1,6945 & 7,9863 \\ 12,0989 & 9,8407 \\ -12,0989 & -9,8407 \end{bmatrix} \cdot \begin{bmatrix} -2,0 \\ 2,5 \end{bmatrix} - \begin{bmatrix} 18 \\ -18 \\ -15 \\ 15 \\ 3 \\ -3 \end{bmatrix}$$

$\underline{V} = cc$ units

*Calculated by Matlab

Adjusted Dimensions $\hat{\underset{\sim}{r}}_i = \underset{\sim}{r_i}^{g} + \underset{\sim}{v_i}^{cc}$

$$\begin{bmatrix}\hat r_1\\ \hat r_2\\ \hat r_3\\ \hat r_4\\ \hat r_5\\ \hat r_6\end{bmatrix} = \begin{bmatrix} r_1\\ r_2\\ r_3\\ r_4\\ r_5\\ r_6\end{bmatrix} + \begin{bmatrix} V_{108-23}\\ V_{108-107}\\ V_{107-108}\\ V_{107-23}\\ V_{23-107}\\ V_{23-108}\end{bmatrix} \Rightarrow \begin{bmatrix}\hat r_1\\ \hat r_2\\ \hat r_3\\ \hat r_4\\ \hat r_5\\ \hat r_6\end{bmatrix} = \begin{bmatrix}0{,}00000\\ 43{,}21580\\ 0{,}00000\\ 32{,}24480\\ 0{,}00000\\ 124{,}53835\end{bmatrix} + \begin{bmatrix}-1{,}75\\ 1{,}75\\ -1{,}75\\ 1{,}75\\ -1{,}75\\ 1{,}75\end{bmatrix} = \underbrace{\begin{bmatrix}-0{,}000175\\ 43{,}215975\\ -0{,}000175\\ 32{,}244975\\ -0{,}000175\\ 124{,}538525\end{bmatrix}}$$

$$= \begin{bmatrix}0{,}00000\\ 43{,}21615\\ 0{,}00000\\ 32{,}24515\\ 0{,}00000\\ 124{,}53870\end{bmatrix}$$

Control of adjusted direction dimensions

Orientation unknown equations

$$1 \cdot dz_{108} + 10{,}4044\, dx_{23} + 1{,}8544 \cdot dy_{23} = 0$$

$$1 \cdot dz_{107} - 1{,}6945 \cdot dx_{23} - 7{,}9863 \cdot dy_{23} = 0$$

$$1 \cdot dz_{23} + 8{,}7099\, dx_{23} + 6{,}1319 \cdot dy_{23} = 0$$

with matrix notation;

$$\begin{bmatrix} dz_{108}\\ dz_{107}\\ dz_{23}\end{bmatrix} = -\begin{bmatrix}10{,}4044 & 1{,}8544\\ -1{,}6945 & -7{,}9863\\ 8{,}7099 & 6{,}1319\end{bmatrix} \cdot \begin{bmatrix} dx_{23}\\ dy_{23}\end{bmatrix} = \begin{bmatrix}15{,}79\\ 16{,}73\\ 32{,}52\end{bmatrix} cc$$

$$\begin{bmatrix} z_{102}\\ z_{107}\\ z_{23}\end{bmatrix} = \begin{bmatrix} z^{0}_{102}\\ z^{0}_{107}\\ z^{0}_{23}\end{bmatrix} + \begin{bmatrix} dz_{108}\\ dz_{107}\\ dz_{23}\end{bmatrix} \qquad \begin{bmatrix} z_{102}\\ z_{107}\\ z_{23}\end{bmatrix} = \begin{bmatrix}111{,}23041\\ 354{,}44647\\ 186{,}69004\end{bmatrix} + \begin{bmatrix}15{,}79\\ 16{,}73\\ 32{,}52\end{bmatrix} = \begin{bmatrix}111{,}23199\\ 354{,}44814\\ 186{,}69329\end{bmatrix}$$

DN	BN	District from adjusted directions					From Adjusted coordinates District t_{ik}	Difference
		r_i (g)	v_i (cc)	$\hat r_i = \underline{r}_i + \underline{v}_i$	z	$t_{ik} = \hat{\underline{r}}_i + z_{102}$		
108	23	0,00000	-1,75	-0,00018	111,23199	111,23181	111,23181	0,00
	107	43,21580	1,75	43,21598	111,23199	154,44796	154,44796	0,00
107	108	0,00000	-1,75	-0,00018	354,44814	354,44796	354,44796	0,00
	23	32,24480	1,75	32,24498	354,44814	386,69312	386,69312	0,00
23	107	0,00000	-1,75	-0,00018	186,69329	186,69312	186,69312	0,00
	108	124,53835	1,75	124,53853	186,69329	311,23181	311,23181	0,00

* Calculated by Matlab

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Square Mean Error

$$m_0 = \pm\sqrt{\frac{v^T v}{n-u}} = \pm\sqrt{\frac{12{,}25}{6-5}} = \underline{\pm 3{,}5 \text{ cc}}$$

Mean Error of Unknowns

$$\underline{Q}_{xx} = N^{-1} = \begin{bmatrix} 0{,}0043 & -0{,}0040 \\ -0{,}0040 & 0{,}0067 \end{bmatrix} \quad \frac{cm^2}{cc^2}$$

$$m_x = \pm m_0 \sqrt{q_{xx}} = \pm 3{,}5\sqrt{0{,}0043} = \underline{\pm 0{,}2 \text{ cm}}$$

$$m_y = \pm \underbrace{m_0}_{cc} \underbrace{\sqrt{q_{yy}}}_{\frac{cm}{cc}} = \pm 3{,}5\sqrt{0{,}0067} = \underline{\pm 0{,}3 \text{ cm}}$$

$$\cancel{cc} \cdot \frac{cm}{\cancel{cc}}$$

Mean Error of Measures

$$m_{\ell_i} = \pm \frac{m_0}{\sqrt{P_i}}$$

Since the weights are equal in the direction nets, the mean errors of the measures are equal to the mean squared error.

Average Error of Adjusted Measures

$$\underline{Q}_{\hat{\ell}\hat{\ell}} = \underline{A} \cdot \underline{Q}_{xx} \cdot A^T = \begin{bmatrix} 0{,}3333 & -0{,}3333 & -0{,}1667 & 0{,}1667 & -0{,}1667 & 0{,}1667 \\ -0{,}3333 & 0{,}3333 & 0{,}1667 & -0{,}1667 & 0{,}1667 & -0{,}1667 \\ -0{,}1667 & 0{,}1667 & 0{,}3333 & -0{,}3333 & -0{,}1667 & 0{,}1667 \\ 0{,}1667 & -0{,}1667 & -0{,}3333 & 0{,}3333 & 0{,}1667 & -0{,}1667 \\ -0{,}1667 & 0{,}1667 & -0{,}1667 & 0{,}1667 & 0{,}3333 & -0{,}3333 \\ 0{,}1667 & -0{,}1667 & 0{,}1667 & -0{,}1667 & -0{,}3333 & 0{,}3333 \end{bmatrix}$$

$$1 = \frac{\cancel{cc}}{\cancel{cm}} \cdot \frac{\cancel{cm^2}}{\cancel{cc^2}} \cdot \frac{\cancel{cc}}{\cancel{cm}}$$

unitless

$m_{\hat{\ell}_i} = \pm m_0 \cdot \sqrt{\underline{Q}_{\hat{\ell}\hat{\ell}_i}}$ => mean error of adjusted measures

$$m_{\hat{\ell}_i} = \pm 3{,}5 \cdot \sqrt{0{,}3333} = \pm 2{,}02 \text{ cc}$$

Average Error of Corrections

$$\underline{Q}_{vv} = \underline{Q}_{\ell\ell} - \underline{Q}_{\hat{\ell}\hat{\ell}}$$

$\underline{Q}_{vv} = \underline{P}^{-1} - \underline{Q}_{\hat{\ell}\hat{\ell}}$ => unitless

$$\underline{P} = \underline{P}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\underline{Q}_{vv} = \begin{bmatrix} 0{,}6667 & -0{,}3333 & -0{,}1667 & 0{,}1667 & -0{,}1667 & 0{,}1667 \\ -0{,}3333 & 0{,}6667 & 0{,}1667 & -0{,}1667 & 0{,}1667 & -0{,}1667 \\ -0{,}1667 & 0{,}1667 & 0{,}6667 & -0{,}3333 & -0{,}1667 & 0{,}1667 \\ 0{,}1667 & -0{,}1667 & -0{,}3333 & 0{,}6667 & 0{,}1667 & -0{,}1667 \\ -0{,}1667 & 0{,}1667 & -0{,}1667 & 0{,}1667 & 0{,}6667 & -0{,}3333 \\ 0{,}1667 & -0{,}1667 & 0{,}1667 & -0{,}1667 & -0{,}3333 & 0{,}6667 \end{bmatrix}$$

$$m_{v_i} = \pm \overbrace{m_0 \cdot \sqrt{\underline{Q}_{v_i v_i}}}^{\text{Average error of corrections}}$$

$$m_{v_i} = \pm 3{,}5 \cdot \sqrt{0{,}6667}$$

$$\boxed{= \pm 2{,}86 \text{ cc}}$$

*Calculated by Matlab

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