HACETTEPE UNIVERSITY
DEPARTMENT OF
GEOMATICS ENGINEERING

GMT202

ADJUSTMENT COMPUTATION & PARAMETER ESTIMATION

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ASSIGNMENT 3

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A functional model in matrix format is given below. The data for the Stochastic model of this model are given in the table.
Since the antecedent mean squared error is $s_0 = \pm 1,6$ mm, balance these measures with different sensitivities using the indirect measures method.

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \underbrace{\begin{bmatrix} -0,0639 & 0,9979 \\ -0,9902 & -0,1398 \\ 0,9747 & 0,2232 \end{bmatrix}}_{\#\ A\ \#} \cdot \begin{bmatrix} dx \\ dy \end{bmatrix} - \underbrace{\begin{bmatrix} 1,58 \\ -3,26 \\ 5,99 \end{bmatrix}}_{\#\ l\ \#}$$

$m_1 = \pm 0,94$ mm	$r_{ij} = 0,8$
$m_2 = \pm 0,69$ mm	
$m_3 = \pm 0,90$ mm	

Making a adjustment decision
(Dengeleme kararının verilmesi)

Number of measures $n = 3$
(Ölçü Sayısı)

Unknown Number $U = 2$
(Bilinmeyen Sayısı)

Degrees of freedom = Excess Number of measures
(Serbestlik Derecesi) (Fazla ölçü Sayısı)

$= f = n - u = 1 > 0$

= There is adjustment.
(Dengeleme var)

Stochastic model Weights are different and correlated
(Stokastik model Ağırlıklar farklı ve korelasyon var).

$$\underline{K}_{ee} = \begin{bmatrix} m_1^2 & r_{12} \cdot m_1 \cdot m_2 & r_{13} \cdot m_1 \cdot m_3 \\ r_{12} \cdot m_1 \cdot m_2 & m_2^2 & r_{23} \cdot m_2 \cdot m_3 \\ r_{13} \cdot m_1 \cdot m_3 & r_{23} \cdot m_2 \cdot m_3 & m_3^2 \end{bmatrix}$$

$$\underline{K}_{ee} = \begin{bmatrix} 0,94^2 & 0,8 \times 0,94 \times 0,69 & 0,8 \times 0,94 \times 0,90 \\ 0,8 \times 0,94 \times 0,69 & 0,69^2 & 0,8 \times 0,69 \times 0,90 \\ 0,8 \times 0,94 \times 0,90 & 0,8 \times 0,69 \times 0,90 & 0,90^2 \end{bmatrix}$$

$$\underline{K}_{ee} = \begin{bmatrix} 0,8836 & 0,5189 & 0,6768 \\ 0,5189 & 0,4761 & 0,4968 \\ 0,6768 & 0,4968 & 0,8100 \end{bmatrix}$$

$K_{ee} = s_0^2 \cdot \underline{Q}_{ee}$ $\qquad \underline{Q}_{ee} = \dfrac{\underline{K}_{ee}}{m_0^2}$

$$\underline{Q}_{ee} = \frac{1}{1.6^2} \cdot \begin{bmatrix} 0,8836 & 0,5189 & 0,6768 \\ 0,5189 & 0,4761 & 0,4968 \\ 0,6768 & 0,4968 & 0,8100 \end{bmatrix} \Rightarrow \underline{Q}_{ee} = \begin{bmatrix} 0,3452 & 0,2027 & 0,2644 \\ 0,2027 & 0,1860 & 0,1941 \\ 0,2644 & 0,1941 & 0,3164 \end{bmatrix}$$

$$P_{ll} = Q_{ll}^{-1} = \left[\begin{array}{ccc|ccc} 0,3452 & 0,2027 & 0,2644 & 1 & 0 & 0 \\ 0,2027 & 0,1860 & 0,1941 & 0 & 1 & 0 \\ 0,2644 & 0,1941 & 0,3164 & 0 & 0 & 1 \end{array}\right] = \begin{bmatrix} 10,0255 & -6,0669 & -4,6560 \\ -6,0669 & 18,6130 & -6,3487 \\ -4,6560 & -6,3487 & 10,9460 \end{bmatrix}$$

P_{ll}

* (Calculated using Matlab)

Establishing and Solving Normal Equations
(Normal Denklemlerin Kurulması ve Çözümü)

$A^T \Rightarrow a_{ij} \Rightarrow a_{ji}$:

$$A^T = \begin{bmatrix} -0,6939 & -0,9902 & 0,9747 \\ 0,9979 & -0,1398 & 0,2232 \end{bmatrix}_{2\times3} \qquad P = \begin{bmatrix} 10,0255 & -6,0669 & -4,6560 \\ -6,0669 & 18,6130 & -6,3487 \\ -4,6560 & -6,3487 & 10,9460 \end{bmatrix}_{3\times3}$$

$$A^T \cdot P = \begin{bmatrix} 0,8343 & -24,2240 & 17,2608 \\ 9,8177 & -10,0771 & -1,3178 \end{bmatrix}_{2\times3} \Rightarrow \text{(Calculated using Matlab and checked manually with calculator)}$$

$(2\times3)\cdot(3\times3)$

$$N = A^T P A \Rightarrow N = \begin{bmatrix} 40,7380 & 8,0672 \\ 8,0672 & 10,9118 \end{bmatrix} \qquad n = A^T P l \Rightarrow n = \begin{bmatrix} 183,5611 \\ 40,4737 \end{bmatrix}$$

$$Q_{xx} = N^{-1} = \begin{bmatrix} 0,0288 & -0,0213 \\ -0,0213 & 0,1074 \end{bmatrix} \qquad x = N^{-1} \cdot n \Rightarrow x = \begin{bmatrix} dx \\ dy \end{bmatrix} = \begin{bmatrix} 4,42 \\ 0,44 \end{bmatrix}$$

The Exact Value of the Unknown (Bilinmeyenlerin Kesin Değeri)

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + \begin{bmatrix} dx \\ dy \end{bmatrix}$$

Calculation of Correction (Düzeltmelerin Hesabı) $\Rightarrow v = A \cdot x - l$

$$v = \begin{bmatrix} -0,6939 & 0,9979 \\ -0,9902 & -0,1398 \\ 0,9747 & 0,2232 \end{bmatrix} \cdot \begin{bmatrix} 4,42 \\ 0,44 \end{bmatrix} - \begin{bmatrix} 1,58 \\ -3,26 \\ 5,99 \end{bmatrix} \Rightarrow v = \begin{bmatrix} -1,42 \\ -1,18 \\ -1,58 \end{bmatrix}$$

Audit of Correction (Düzeltmelerin Denetimi)

$$A^T = \begin{bmatrix} -0,6939 & -0,9902 & 0,9747 \\ 0,9979 & -0,1398 & 0,2232 \end{bmatrix} \qquad P = \begin{bmatrix} 10,0255 & -6,0669 & -4,6560 \\ -6,0669 & 18,6130 & -6,3487 \\ -4,6560 & -6,3487 & 10,9460 \end{bmatrix} \qquad v = \begin{bmatrix} -1,42 \\ -1,18 \\ -1,58 \end{bmatrix}$$

$$A^T P v = \begin{bmatrix} 0,00 \\ 0,00 \end{bmatrix} \qquad v^T = \begin{bmatrix} -1,42 & -1,18 & -1,58 \end{bmatrix} \qquad -l^T = \begin{bmatrix} 1,58 & -3,26 & 5,99 \end{bmatrix}$$

$$v^T P v = \begin{bmatrix} 8,57 \end{bmatrix} \qquad -l^T P v = \begin{bmatrix} 8,57 \end{bmatrix} \qquad \text{* (Calculated using Matlab)}$$

$\underline{v}^T \underline{P} \underline{v} = \underline{l}^T \underline{P} \underline{l} - \underline{n}^T \underline{x}$ $\qquad \underline{n}^T \underline{x} = [829,15]$

$[8,75] = [837,90] - [829,15]$ $\qquad \underline{l}^T \underline{P} \underline{l} = [837,90]$

$\underline{x} = \begin{bmatrix} 4,42 \\ 0,44 \end{bmatrix}$ $\quad \underline{n}^T = [183,5611 \quad 40,4737]$ *(Calculated using Matlab)

Balanced Measures (Dengeli Ölçüler)

$\hat{\underline{l}}_i = \underline{l}_i + \underline{v}_i$

$$\begin{bmatrix} \hat{l}_1 \\ \hat{l}_2 \\ \hat{l}_3 \end{bmatrix} = \begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

Square Mean Error (Karesel Ortalama Hata)

$$m_0 = \pm\sqrt{\frac{\underline{v}^T \underline{P} \underline{v}}{n-u}} = \pm\sqrt{\frac{8.57}{3-2}} = \pm 2,93 \text{ mm}$$

Mean Error of Unknown (Bilinmeyenlerin Ortalama Hatası)

$$\underline{Q}_{xx} = N^{-1} = \begin{bmatrix} 0,0288 & -0,0213 \\ -0,0213 & 0,1073 \end{bmatrix}$$ Inverse weight matrix of unknowns (Bilinmeyenlerin ters ağırlık matisi)

$m_x = \pm m_0 \sqrt{q_{xx}} = \pm 2,93\sqrt{0,0288} = \pm 0,50$ mm

$m_y = \pm m_0 \sqrt{q_{yy}} = \pm 2,93\sqrt{0,1074} = \pm 0,96$ mm

Average errors of unknowns (Bilinmeyenlerin Ortalama Hataları)

Mean Error of Measures (Ölçülerin Ortalama Hatası)

$$\underline{P} = \begin{bmatrix} 10,0255 & -6,0669 & -4,6560 \\ -6,0669 & 18,6130 & -6,3487 \\ -4,6560 & -6,3487 & 10,9460 \end{bmatrix}$$

$$m_{l_i} = \pm \frac{m_0}{\sqrt{P_i}}$$

$$m_{l_1} = \pm \frac{m_0}{\sqrt{P_1}} = \pm \frac{2,93}{\sqrt{10,0255}} = \pm 9,28 \text{ mm}$$

$$m_{l_2} = \pm \frac{m_0}{\sqrt{P_2}} = \pm \frac{2,93}{\sqrt{18,6130}} = \pm 12,64 \text{ mm}$$

$$m_{l_3} = \pm \frac{m_0}{\sqrt{P_3}} = \pm \frac{2,93}{\sqrt{10,9460}} = \pm 9,69 \text{ mm}$$

Dengeli ölçülerin ortalama hatası (Average Error of Balanced Measures)

$\underline{Q}_{\hat{l}\hat{l}} = \underline{A} \cdot \underline{Q}_{xx} \cdot \underline{A}^T$ ⇒ Covariance matrix of Balanced Measures
(Dengeli ölçülerin kovaryans matrisi)

$\underline{Q}_{xx} = \underline{N}^{-1} = \begin{bmatrix} 0,0288 & -0,0213 \\ -0,0213 & 0,1074 \end{bmatrix}$ $\quad \underline{A} = \begin{bmatrix} -0,0639 & 0,9979 \\ -0,9902 & -0,1398 \\ 0,9747 & 0,2232 \end{bmatrix}$

$\underline{A}^T = \begin{bmatrix} -0,0639 & 0,9902 & 0,9747 \\ 0,9979 & -0,1398 & 0,2232 \end{bmatrix}$

$\underline{Q}_{\hat{l}\hat{l}} = \underline{A} \cdot \underline{Q}_{xx} \cdot \underline{A}^T \Rightarrow \underline{Q}_{\hat{l}\hat{l}} = \begin{bmatrix} 0,1097 & 0,0077 & 0,0017 \\ 0,0077 & 0,0244 & -0,0235 \\ 0,0017 & -0,0235 & 0,0234 \end{bmatrix}$

$m_{\hat{l}_i} = \pm m_0 \cdot \sqrt{\underline{Q}_{\hat{l}_i\hat{l}_i}}$ ⇒ Average Error of Balanced Measures

$m_{\hat{l}_1} = \pm m_0 \cdot \sqrt{\underline{Q}_{\hat{l}_1\hat{l}_1}} \Rightarrow \pm 2,93 \cdot \sqrt{0,1097} = \pm 0,97\,mm$

$m_{\hat{l}_2} = \pm m_0 \cdot \sqrt{\underline{Q}_{\hat{l}_2\hat{l}_2}} \Rightarrow \pm 2,93 \cdot \sqrt{0,0244} = \pm 0,46\,mm$

$m_{\hat{l}_3} = \pm m_0 \sqrt{\underline{Q}_{\hat{l}_3\hat{l}_3}} \Rightarrow \pm 2,93 \cdot \sqrt{0,0234} = \pm 0,45\,mm$

Düzeltmelerin ortalama hatası (Average Error of Corrections)

$\underline{Q}_{vv} = \underline{Q}_{ll} - \underline{Q}_{\hat{l}\hat{l}}$ ⇒ Düzeltmelerin kovaryans matrisi
(Covariance matrix of Corrections)

$\underline{Q}_{vv} = \underline{P}^{-1} - \underline{Q}_{\hat{l}\hat{l}}$

$\underline{Q}_{vv} = \begin{bmatrix} 0,3452 & 0,2027 & 0,2644 \\ 0,2027 & 0,1860 & 0,1941 \\ 0,2644 & 0,1941 & 0,3164 \end{bmatrix} - \begin{bmatrix} 0,1097 & 0,0077 & 0,0017 \\ 0,0077 & 0,0244 & -0,0235 \\ 0,0017 & -0,0235 & 0,0234 \end{bmatrix}$

$\underline{Q}_{vv} = \begin{bmatrix} 0,2354 & 0,1950 & 0,2686 \\ 0,1950 & 0,1616 & 0,2176 \\ 0,2686 & 0,2176 & 0,2930 \end{bmatrix}$

$m_{v_i} = \pm m_0 \cdot \sqrt{\underline{Q}_{v_iv_i}}$ ⇒ Average Error of Corrections (Düzeltmelerin ortalama hatası)

$m_{v_1} = \pm m_0 \cdot \sqrt{\underline{Q}_{v_1v_1}} = \pm 2,93 \cdot \sqrt{0,2354} = \pm 1,42\,mm$

$m_{v_2} = \pm m_0 \sqrt{\underline{Q}_{v_2v_2}} = \pm 2,93 \cdot \sqrt{0,1616} = \pm 1,18\,mm$

$m_{v_3} = \pm m_0 \cdot \sqrt{\underline{Q}_{v_3v_3}} = \pm 2,93 \cdot \sqrt{0,2930} = \pm 1,58\,mm$

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