HACETTEPE UNIVERSITY
DEPARTMENT OF
GEOMATICS ENGINEERING

GMT202

ADJUSTMENT COMPUTATION & PARAMETER ESTIMATION

2021-2022 SPRING TERM

ASSIGNMENT 8

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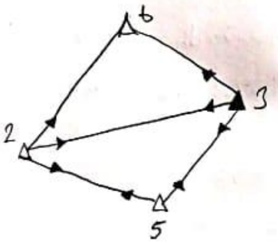
1)

Balance the Trigonometric Leveling Network given below using the indirect measures method.



NN	H_i
Exact Elevation	
3	1016,253
Approximate Heights	
2	1117,001
5	1047,644
6	1101,859

DN	BN	Vertical Angle Z_{i-j}	Instrumental Height (i)	Reflector Height (t)	S_{i-j}
2	3	102,92374	1,42	1,75	2194,193
	5	102,28561	1,42	1,81	1924,510
	6	102,51359	1,42	1,76	1875,414
3	2	97,08010	1,61	1,90	2194,200
	5	98,71777	1,61	1,87	1562,956
	6	96,35727	1,61	1,83	1495,632
5	2	97,70589	1,45	1,88	1924,500
	3	101,27326	1,45	1,82	1562,961

Numbers of measures (n) = 8

Unknown Numbers (u) = 3

$f = n - u \Rightarrow 8 - 3 = 5$ ✓ There is adjustment.

$$Z^{\circ}_{i-j} = \text{arc cot}\left\{\frac{1}{S_{i-j}} \cdot (H^{\circ}_2 - H^{\circ}_1 - K \cdot S^2_{i-j} - i + t)\right\} \quad K = \frac{1-k}{2 \cdot r} = \frac{1 - 0,13}{2 \cdot (6370000)} = 0,0000000683$$

$$a = \frac{\sin^2 \cdot Z^{\circ}_{1-2}}{S_{1-2} \cdot 100} \cdot \left(\frac{200}{\pi} \cdot 10000\right) \quad b = -a \quad -l = (Z^{\circ}_{1-2} - Z_{1-2}) \cdot 10000$$

DN	BN	S_{i-j}	$H^{\circ}_j - H^{\circ}_i$	$K \cdot S^2_{i-j}$	i	t	Z°_{i-j}	Z_{i-j}	a (cc/cm)	-l (cc)
2	3	2194,193	-100,748	0,3288	1,42	1,75	102,92100	102,92374	2,8953	-27,4
	5	1924,510	-69,357	0,2529	1,42	1,81	102,28878	102,28561	3,3037	31,7
	6	1875,414	-15,142	0,2402	1,42	1,76	102,51060	102,51359	3,3943	-29,9
3	2	2194,200	100,748	0,3288	1,61	1,90	97,08010	97,08010	2,8953	0
	5	1562,956	31,391	0,1668	1,61	1,87	98,71777	98,71777	4,0715	0
	6	1495,632	85,606	0,1528	1,61	1,83	96,35728	96,35727	4,2426	0
5	2	1924,500	69,357	0,2529	1,45	1,88	97,70083	97,70589	3,3037	-50,6
	3	1562,961	-31,391	0,1668	1,45	1,82	101,27016	101,27326	4,0715	-31,0

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②

$vz_{i-j} = a \cdot dh_i + b \cdot dh_j - l$

dh3 is known

$vz_{2-3} = 2,8953 \cdot dh_2 - \cancel{2,8953 \cdot dh_3} - 27,4$
$vz_{2-5} = 3,3037 \cdot dh_2 - 3,3037 \cdot dh_5 + 31,7$
$vz_{2-6} = 3,3943 \cdot dh_2 - 3,3943 \cdot dh_6 - 29,9$
$vz_{3-2} = \cancel{2,8953 \cdot dh_3} - 2,8953 \cdot dh_2 + 0,0$
$vz_{3-5} = \cancel{4,0715 \cdot dh_3} - 4,0715 \cdot dh_5 + 0,0$
$vz_{3-6} = \cancel{4,2426 \cdot dh_3} - 4,2426 \cdot dh_6 + 0,0$
$vz_{5-2} = 3,3037 \cdot dh_5 - 3,3037 \cdot dh_2 - 50,6$
$vz_{5-3} = 4,0715 \cdot dh_5 - \cancel{4,0715 \cdot dh_3} - 31,00$

$\Rightarrow$

$vz_{2-3} = 2,8953 \cdot dh_2 - 27,4$
$vz_{2-5} = 3,3037 dh_2 - 3,3037 dh_5 + 31,7$
$vz_{2-6} = 3,3943 \cdot dh_2 - 3,3943 \cdot dh_6 - 29,9$
$vz_{3-2} = -2,89 \cdot dh_2 + 0,0$
$vz_{3-5} = -4,0715 \cdot dh_5 + 0,0$
$vz_{3-6} = -4,2426 \cdot dh_6 + 0,0$
$vz_{5-2} = 3,3037 \cdot dh_5 - 3,3037 \cdot dh_2 - 50,6$
$vz_{5-3} = 4,0715 \cdot dh_5 - 31,00$

I rewrote it with unknowns for $v = A \cdot x - l$

$vz_{2-3} = 2,8953 \cdot dh_2 + 0 \cdot dh_5 + 0 \cdot dh_6 - 27,4$
$vz_{2-5} = 3,3037 \cdot dh_2 - 3,3037 \cdot dh_5 + 0 \cdot dh_6 + 31,7$
$vz_{2-6} = 3,3943 \cdot dh_2 + 0 \cdot dh_5 - 3,3943 \cdot dh_6 - 29,9$
$vz_{3-2} = -2,8953 \, dh_2 + 0 \cdot dh_5 + 0 \cdot dh_6 + 0,0$
$vz_{3-5} = 0 \cdot dh_2 - 4,0715 \cdot dh_5 + 0 \cdot dh_6 + 0,0$
$vz_{3-6} = 0 \cdot dh_2 + 0 \cdot dh_5 - 4,2426 \cdot dh_6 + 0,0$
$vz_{5-2} = -3,3037 \, dh_2 + 3,3037 \cdot dh_5 + 0 \cdot dh_6 - 50,6$
$vz_{5-3} = 0 \cdot dh_2 + 4,0715 \cdot dh_5 + 0 \cdot dh_6 - 31,0$

using $v = A \cdot dx - l$ format;

$$\begin{bmatrix} vz_{2-3} \\ vz_{2-5} \\ vz_{2-6} \\ vz_{3-2} \\ vz_{3-5} \\ vz_{3-6} \\ vz_{5-2} \\ vz_{5-3} \end{bmatrix} = \begin{bmatrix} 2,8953 & 0 & 0 \\ 3,3037 & -3,3037 & 0 \\ 3,3943 & 0 & -3,3943 \\ -2,8953 & 0 & 0 \\ 0 & -4,0715 & 0 \\ 0 & 0 & -4,2426 \\ -3,3037 & 3,3037 & 0 \\ 0 & 4,0715 & 0 \end{bmatrix} \begin{bmatrix} dh_2 \\ dh_5 \\ dh_6 \end{bmatrix} - \begin{bmatrix} 27,4 \\ -31,7 \\ 29,9 \\ 0 \\ 0 \\ 0 \\ 50,6 \\ 31,10 \end{bmatrix}$$

$$\underline{N} = \underline{A}^T \underline{A} = \begin{bmatrix} 50,1154 & -21,8287 & -11,5215 \\ -21,8287 & 54,9834 & 0,0000 \\ -11,5215 & 0,0000 & 29,5212 \end{bmatrix} \qquad \underline{n} = \underline{A}^T \underline{l} = \begin{bmatrix} -91,32 \\ 398,06 \\ -101,30 \end{bmatrix}$$

$$\underline{Q}_{xx} = \underline{N}^{-1} = \begin{bmatrix} 0,0271 & 0,0107 & 0,0106 \\ 0,0107 & 0,0225 & 0,0042 \\ 0,0106 & 0,0042 & 0,0380 \end{bmatrix} \qquad \underline{x} = \underline{Q}_{xx} \cdot \underline{n} = \begin{bmatrix} dh_2 \\ dh_5 \\ dh_6 \end{bmatrix} = \begin{bmatrix} 0,74 \\ 7,53 \\ -3,14 \end{bmatrix} \text{cm}$$

The Exact value of the Unknown;

$$\begin{bmatrix} H_2 \\ H_5 \\ H_6 \end{bmatrix} = \begin{bmatrix} H_2^\circ \\ H_5^\circ \\ H_6^\circ \end{bmatrix} + \begin{bmatrix} dh_2 \\ dh_5 \\ dh_6 \end{bmatrix} \qquad \begin{bmatrix} H_2 \\ H_5 \\ H_6 \end{bmatrix} = \begin{bmatrix} 1117,001 \\ 1047,644 \\ 1101,859 \end{bmatrix} + \begin{bmatrix} 0,74 \\ 7,53 \\ -3,14 \end{bmatrix} = \begin{bmatrix} 1117,0084 \\ 1047,7193 \\ 1101,8276 \end{bmatrix}$$

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(1)

Corrections $\underline{v} = \underline{A} \cdot \underline{x} - \underline{l}$ (cc = second)

$$
\begin{bmatrix} vZ_{2-3} \\ vZ_{2-5} \\ vZ_{2-6} \\ vZ_{3-2} \\ vZ_{3-5} \\ vZ_{3-6} \\ vZ_{5-2} \\ vZ_{5-3} \end{bmatrix}
=
\begin{bmatrix} 2,8953 & 0 & 0 \\ 3,3037 & -3,3037 & 0 \\ 3,3943 & 0 & -3,3943 \\ -2,8953 & 0 & 0 \\ 0 & -4,0715 & 0 \\ 0 & 0 & -4,2426 \\ -3,3037 & 3,3037 & 0 \\ 0 & 4,0715 & 0 \end{bmatrix}
\cdot
\begin{bmatrix} 0,74 \\ 7,53 \\ -3,14 \end{bmatrix}
-
\begin{bmatrix} 27,4 \\ -31,7 \\ 29,9 \\ 0 \\ 0 \\ 0 \\ 50,6 \\ 31,10 \end{bmatrix}
=
\begin{bmatrix} -25,24 \\ 9,28 \\ -16,70 \\ -2,13 \\ -30,66 \\ 13,36 \\ -28,14 \\ -0,30 \end{bmatrix}
$$

Adjusted Measures $\hat{\underline{Z}}_{i-j} = \underline{Z}_{i-j} + v\underline{Z}_{ij}$

$$
\begin{bmatrix} \hat{Z}_{2-3} \\ \hat{Z}_{2-5} \\ \hat{Z}_{2-6} \\ \hat{Z}_{3-2} \\ \hat{Z}_{3-5} \\ \hat{Z}_{3-6} \\ \hat{Z}_{5-2} \\ \hat{Z}_{5-3} \end{bmatrix}
=
\begin{bmatrix} Z_{2-3} \\ Z_{2-5} \\ Z_{2-6} \\ Z_{3-2} \\ Z_{3-5} \\ Z_{3-6} \\ Z_{5-2} \\ Z_{5-3} \end{bmatrix}
+
\begin{bmatrix} vZ_{2-3} \\ vZ_{2-5} \\ vZ_{2-6} \\ vZ_{3-2} \\ vZ_{3-5} \\ vZ_{3-6} \\ vZ_{5-2} \\ vZ_{5-3} \end{bmatrix}
\qquad
\begin{bmatrix} \hat{Z}_{2-3} \\ \hat{Z}_{2-5} \\ \hat{Z}_{2-6} \\ \hat{Z}_{3-2} \\ \hat{Z}_{3-5} \\ \hat{Z}_{3-6} \\ \hat{Z}_{5-2} \\ \hat{Z}_{5-3} \end{bmatrix}
=
\begin{bmatrix} 102,92373 \\ 102,28561 \\ 100,51359 \\ 97,08010 \\ 98,71777 \\ 96,35727 \\ 97,70589 \\ 101,27326 \end{bmatrix}
+
\begin{bmatrix} -25,24 \\ 9,28 \\ -16,70 \\ -2,13 \\ -30,66 \\ 13,36 \\ -28,14 \\ -0,30 \end{bmatrix}
=
\begin{bmatrix} 102,92121 \\ 102,28653 \\ 100,51192 \\ 97,07988 \\ 98,71470 \\ 96,35861 \\ 97,70308 \\ 101,27323 \end{bmatrix}
$$

Control of Adjusted Measures ;

$$\hat{\underline{Z}}_{i-j} = \underline{Z}_{i-j} + \underline{vZ}_{i-j} = \hat{\underline{Z}}_{i-j} = \operatorname{arccot}\left\{ \frac{1}{S_{i-j}} \cdot \left(H_j - H_i - K \cdot S_{i-j}^2 - i + t \right) \right\}$$

$$
\begin{bmatrix} 102,92121 \\ 102,28653 \\ 100,51192 \\ 97,07988 \\ 98,71470 \\ 96,35861 \\ 97,70308 \\ 101,27323 \end{bmatrix}
=
\begin{bmatrix} 102,92121 \\ 102,28653 \\ 100,51192 \\ 97,07988 \\ 98,71470 \\ 96,35861 \\ 97,70308 \\ 101,27323 \end{bmatrix}
\checkmark
$$

Square Mean Error ;

$$m_0 = \pm\sqrt{\frac{\underline{v}^T \underline{v}}{n-u}} = \pm\sqrt{\frac{2917,60}{8-3}} \boxed{\pm 24,16^{cc}}$$

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(5) Mean Error of Unknowns ;

$$\underline{Q}_{xx} = \underline{N}^{-1} = \begin{bmatrix} 0,0271 & 0,0107 & 0,0106 \\ 0,0107 & 0,0225 & 0,0042 \\ 0,0106 & 0,0042 & 0,0380 \end{bmatrix}$$

$$mH_2 = \pm m_0 \sqrt{q_{xx}} = \pm 24,16 \sqrt{0,0271} = \underline{\pm 3,97 \text{ cm}}$$

$$mH_5 = \pm m_0 \sqrt{q_{yy}} = \pm 24,16 \sqrt{0,0225} = \underline{\pm 3,62 \text{ cm}}$$

$$mH_6 = \pm m_0 \sqrt{q_{zz}} = \pm 24,16 \sqrt{0,0380} = \underline{\pm 4,71 \text{ cm}}$$

Mean Error of Measures ;

$$m_{\ell_i} = \pm \frac{m_0}{\sqrt{P_i}} \text{ cc}$$

Since the weights are taken equal, the mean error of the measures is equal to the mean square error.

Average Error of Adjusted Measures ;

$$\underline{Q}_{\hat{l}\hat{l}} = \underline{A} \cdot \underline{Q}_{xx} \cdot \underline{A}^T = \begin{bmatrix} 0,23 & 0,16 & 0,16 & -0,23 & -0,13 & -0,13 & -0,16 & 0,13 \\ 0,16 & 0,31 & 0,11 & -0,16 & 0,16 & -0,09 & -0,31 & -0,16 \\ 0,16 & 0,11 & 0,51 & -0,16 & -0,09 & 0,40 & -0,11 & 0,09 \\ -0,23 & -0,16 & -0,16 & 0,23 & 0,13 & 0,13 & 0,16 & -0,13 \\ -0,13 & 0,16 & -0,09 & 0,13 & 0,37 & 0,07 & -0,16 & -0,37 \\ -0,13 & -0,09 & 0,40 & 0,13 & 0,07 & 0,68 & 0,09 & -0,07 \\ -0,16 & -0,31 & -0,11 & 0,16 & -0,16 & 0,09 & 0,31 & 0,16 \\ 0,13 & -0,16 & 0,09 & -0,13 & -0,37 & -0,07 & 0,16 & 0,37 \end{bmatrix}$$

$$m_{\hat{\ell}_i} = \pm m_0 \cdot \sqrt{\underline{Q}_{\hat{l}_i \hat{l}_i}}$$ Average of adjusted measures

$\underline{m_{\hat{\ell}_1} = 11,51^{cc}}$ $\underline{m_{\hat{\ell}_2} = 13,36^{cc}}$ $\underline{m_{\hat{\ell}_3} = 17,19^{cc}}$ $\underline{m_{\hat{\ell}_4} = 11,51^{cc}}$ $\underline{m_{\hat{\ell}_5} = 14,74^{cc}}$

$\underline{m_{\hat{\ell}_6} = 19,98^{cc}}$ $\underline{m_{\hat{\ell}_7} = 13,36^{cc}}$ $\underline{m_{\hat{\ell}_8} = 14,74^{cc}}$

Average Error of Corrections

$\underline{Q}_{vv} = \underline{Q}_{ll} - \underline{Q}_{\hat{l}\hat{l}}$ $\underline{Q}_{vv} = \underline{P}^{-1} - \underline{Q}_{\hat{l}\hat{l}}$ => Covariance matrix of corrections

$$\underline{P}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \underline{Q}_{vv} = \begin{bmatrix} 0,77 & 0,16 & 0,16 & -0,23 & -0,13 & -0,13 & -0,16 & 0,13 \\ 0,16 & 0,69 & 0,11 & -0,16 & 0,16 & -0,05 & -0,31 & -0,16 \\ 0,16 & 0,11 & 0,49 & -0,16 & -0,05 & 0,40 & -0,11 & 0,09 \\ -0,23 & -0,16 & -0,16 & 0,77 & 0,13 & 0,13 & 0,16 & -0,13 \\ -0,13 & 0,16 & -0,09 & 0,13 & 0,63 & 0,07 & -0,16 & -0,37 \\ -0,13 & -0,09 & 0,40 & 0,13 & 0,07 & 0,72 & 0,09 & -0,07 \\ -0,16 & -0,31 & -0,11 & 0,16 & -0,16 & 0,09 & 0,69 & 0,16 \\ 0,13 & -0,16 & 0,09 & -0,13 & -0,37 & -0,07 & 0,16 & 0,63 \end{bmatrix}$$

$$m_{v_i} = \pm m_0 \cdot \sqrt{\underline{Q}_{v_i v_i}}$$ Average error of corrections

*Calculated with Matlab

$\underline{m_{v_1} = 21,24^{cc}}$ $\underline{m_{v_4} = 21,24^{cc}}$ $\underline{m_{v_7} = 20,13^{cc}}$

$\underline{m_{v_2} = 20,13^{cc}}$ $\underline{m_{v_5} = 19,14^{cc}}$ $\underline{m_{v_8} = 19,14^{cc}}$

$\underline{m_{v_3} = 16,97^{cc}}$ $\underline{m_{v_6} = 13,58^{cc}}$

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