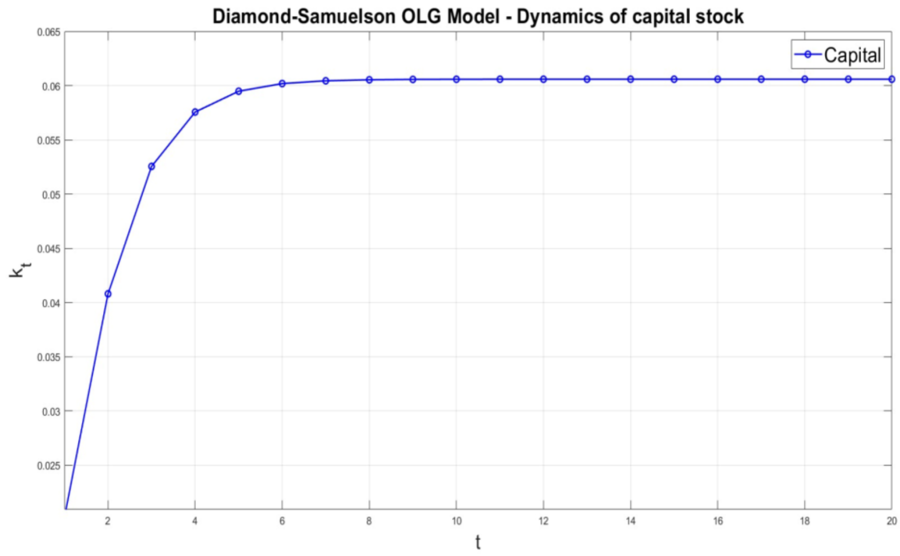
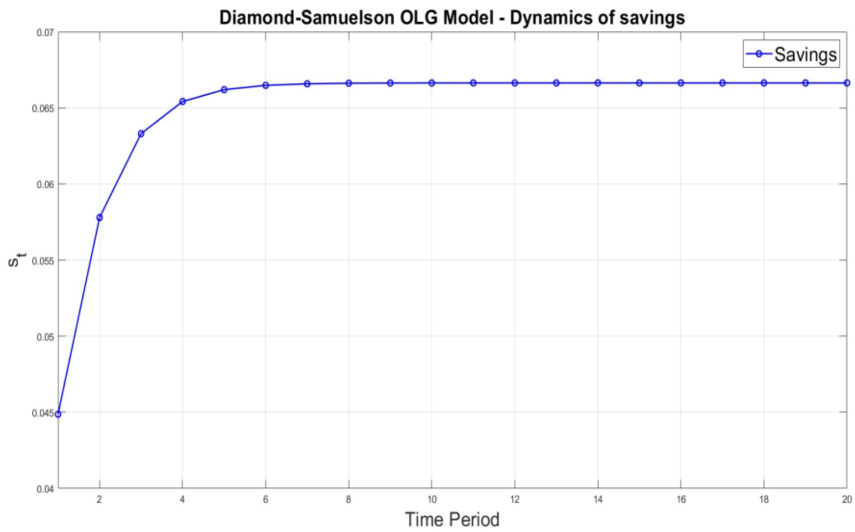
1. **Computing the steady state and the optimal dynamics of capital1, and plotting the results.**



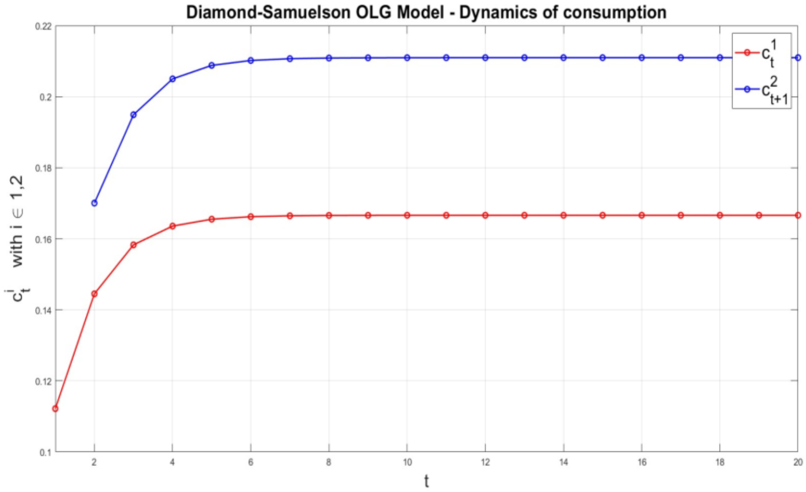
The Steady State Capital is **(k_ss = 0.0606)**:The steady-state value of capital indicates that the economy converges to a level where the capital stock per capita remains constant across generations.

Steady-State Utility is (U_ss = -2.4145): Utility is negative because logarithmic utility is used, and c (consumption) values are less than 1.

2. **Computing the optimal dynamic convergence of consumption of young and old generation and savings, and plotting their optimal dynamics over the time horizon.**

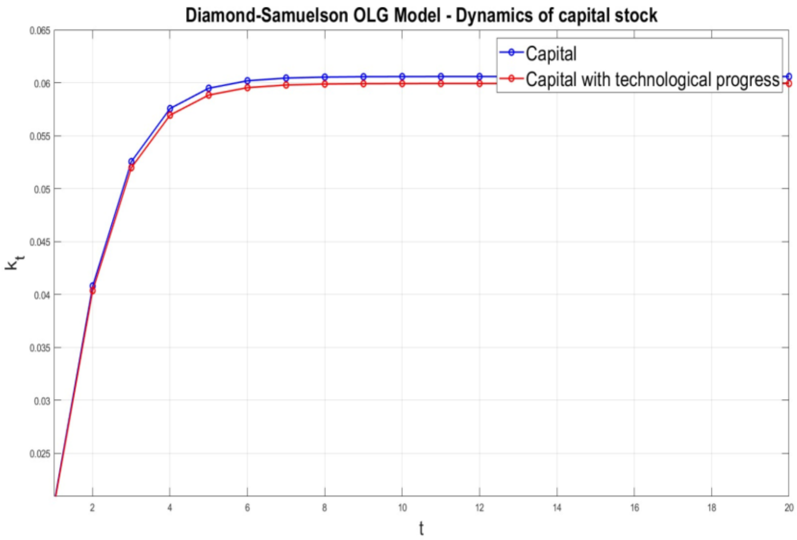


Savings Dynamics (st): The second plot illustrates the savings behavior. Savings stabilize at a steady-state value of 0.0666

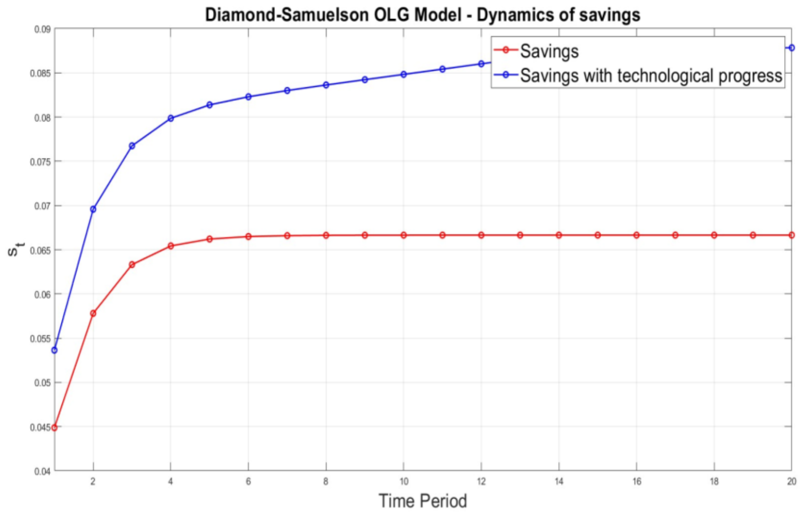


Consumption Dynamics: Ct1 representing young-age consumption, stabilizes at a steady-state value of 0.16661 Ct2 representing old-age consumption, converges to 0.21097. The steady-state values indicate an inter-generational consumption pattern where the older generation consumes more than the younger generation, which is consistent with savings made during their working life.

3. **Given the advance of new technologies (e.g., work automation), what we are likely to experience in the firms' market of advanced economies in future generations is a reduction in the share of labour in production in favour of an increase in TFP. Freely assuming credible values for these changes, show how these will affect capital and savings accumulation and plots together the results.**



In both cases, savings exhibit a rising trend initially and stabilize after a few periods, reaching a steady state. With technological progress, savings levels are consistently higher compared to the baseline case. Technological progress enhances (TFP), leading to higher output per worker.



Both exhibit similar initial trajectories, with capital stock increasing over time and stabilizing at a steady state. With technological progress, the steady-state level of capital stock is slightly higher than without it.

4. What is the level of capital and utility in steady state if a PAYG system with τ1 = 0.5 is in place?

Steady-State Capital: 0.0030,

Steady-State Utility: -5.7428

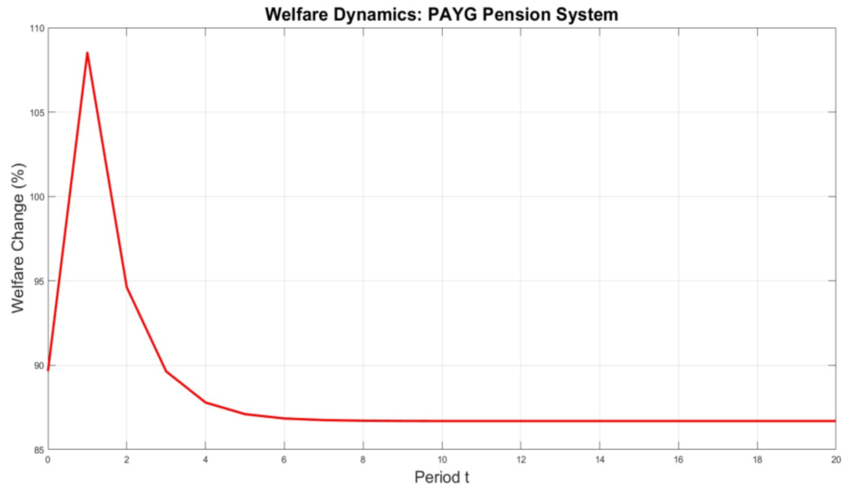
5. What happens to capital and utility if the replacement rate goes down to τ2 = 0.2? Compute the welfare change.

New Steady-State Capital (tau = 0.2): 0.0097,

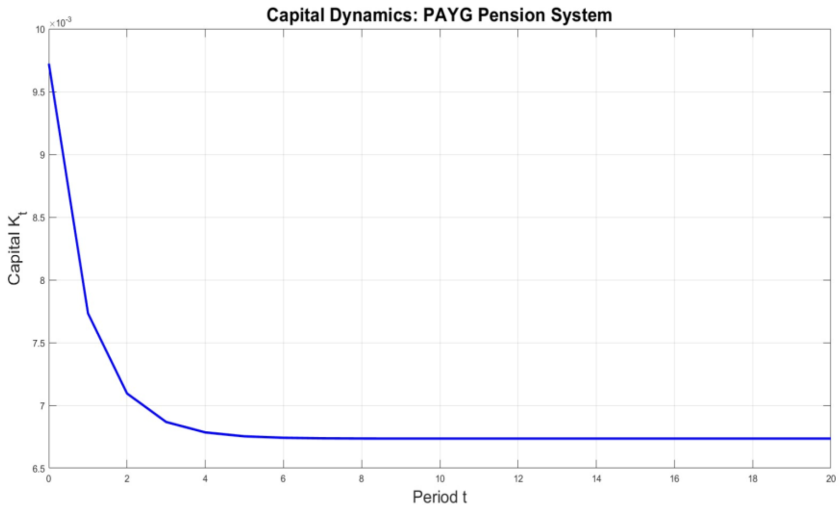
New Steady-State Utility (tau = 0.2): 4.8468,

Welfare Change (CEC): 89.6471%

6. **Plots the dynamic of capital and welfare assuming that there is a change in the pension system in which the replacement rate is increased from τ2 = 0.2 to τ1 = 0.5 and comments the welfare results.**



Welfare initially increases sharply for individuals in early generations after the reform. This is because the reform offers higher pensions (replacement rate of 0.5), improving consumption possibilities without requiring higher personal savings.



Initially, the capital stock declines significantly, reflecting the reduced incentives for savings due to the increased generosity of the PAYG pension system. This is expected as individuals rely more on pensions and less on private savings for future consumption. Over time, capital approaches a new steady state, where it stabilizes at a lower level compared to the initial steady state.