Double Exponential Jump Diffusion (DEJD) Validation Report

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Squad Members

Area	Person In Charge
Squad Leader	Magdalena (magdalena.nitefor@deriv.com)
Project Manager	Maria Semashko (maria.semashko@deriv.com)
Product Owner	Antoine Mille (antoine.mille@deriv.com) Alassane (alassane@deriv.com)
Backend	Afshin Paydar (afshin.paydar@deriv.com)
Model Validation	Kuan Lim (kuanlim@deriv.com)

Product Information

Product Description

Product Specification : https://docs.google.com/document/d/1aYXK3aZ6B-3d5jDWAhqvO60UAigiXHJE/edit

Perl Code (29 March 2023): https://github.com/regentmarkets/perl-Feed-Index-JumpDiffusion/tree/108c50f135b866e6972357c348949778a42dd743

A Double Exponential Jump Diffusion process simulates the financial markets with constant volatilities and can make large jumps. It is composed of geometric brownian motion to which we add Symmetric or Asymmetric jumps with Exponentially distributed sizes.

The index is generated every second.

The Jump index mimics a typical market where shocks can occur from news events. For example, the spot jumping downwards represents a bad news event, while a jump upwards represents a shock from a good news event. Moreover, this model allows us to easily create indices that have an asymmetry between the positive and negative jumps (e.g. more positive jumps but small jump size and less negative jumps but big jump size)).

There will be two asymmetic indices :

- DEX D900
- DEX U900

Construction

The spot price at time t is defined based on the spot price the timestep before following :

$$S_t = S_{t-1} exp\left[(r - d - \frac{\sigma^2}{2} - \lambda\alpha)dt + \sigma\sqrt{dt}W + \sum_{i=1}^{N(\lambda dt)} J_i\right]$$

where

$$
\begin{aligned}
S &= \text{Spot Price} \\
r &= \text{Interest Rate} \\
d &= \text{Dividend Rate} \\
\sigma &= \text{Volatility} \\
N &= \text{Poisson Process with constant mean } \lambda dt \\
\lambda &= \text{Poisson Process mean} \\
dt &= \frac{1}{365 * 86400} \\
J &= \text{Random jump size from double expontial jump distribution } w \\
\alpha &= E(e^J - 1) \\
W &= \text{Random number sample from normal distribution}
\end{aligned}
$$

The jump size J follows double exponential jump distribution w:

$$w(J) = \frac{q_-}{\eta_-} e^{\frac{J}{\eta_-}} 1_{(J<0)} + \frac{q_+}{\eta_+} e^{-\frac{J}{\eta_+}} 1_{(J\geq 0)}$$

where

$$
\begin{aligned}
q_+(q_-) &= \text{Probability of positive/negative jump} \\
\eta_+(\eta_-) &= \text{Expected size of positive(negative) jump}
\end{aligned}
$$

For DEX D900 and DEXU900, the parameters are:

Index	Number of Jumps per Hour λ	Average Positive Jump Size η_+	Average Negative Jump Size η_-	Probability of Positive Jump q_+	Probability of Negative Jump q_-	Volatility
DEX U900	20, which is 175200 per year	0.30%	0.04%	20%	80%	25%
DEX D900	20, which is 175200 per year	0.04%	0.30%	80%	20%	25%

Model Validation

Summary

For the validation of DEX indices, we cover the below areas and conclude the outcomes. More details can be found in respective section.

Section	Area	Validation	Outcome	Passed?
1	$\alpha = E(e^J - 1)$ derivation and computation	• To check derivation of the drift adjustment for the jump process, namely α • To check the α is computed correctly, which involves the Moment Generating Function of the Double Exponential Jump Distribution	The formula in Perl are correct.	**Passed**
2	For the Double Jump Exponetial Jump process, the implementation is different from the specification.	In Perl code, firstly a random number from Uniform Distribution is generated. If the random number is below the probability of positive jump, the system will generate another random number from the exponential distribution with $\lambda = -\frac{1}{\eta_+}$, otherwise $\lambda = \frac{1}{\eta_-}$. This is different from	Upon running the simulation based on these 2 methods, the moments are matching.	**Passed**

		the specification which does not mention there will be 2 random numbers.		
3	Check the moment of the feed data	We compute the moment of the feed data, and check whether it is matching with the true distribution.	We checked the moments of the feed data vs simulation. The moments are matching.	**Passed**
4	Backward engineering the parameters.	Here we check the parameters backward engineering from the feed data moment. We do not proceed with the MLE method as the implementation is very difficult to implement	The result looks fine with the set initial condition and boundary condition.	**Passed**
5	Convergence of the feed data moment	The convergence speed of the first 3 moments are checked.	It takes about 4 days to converge to the true moments. The result is fine.	**Passed**

Section 1

The derivation of the drift adjustment for the jump process is correct, which is $\alpha = E(e^J - 1)$. The detailed derivation is in Appendix.

Also, the formula involves the moment generating function of the double exponential jump distribution, the derivation is also in Appendix.

The formula is correct in perl, hence the validation is passed.

Section 2

In here we use two methods to check the jump distribution without the diffusion part.

1) With 2 random numbers, which same as perl code. 2) With only 1 random number, the formula is shown in Appendix.

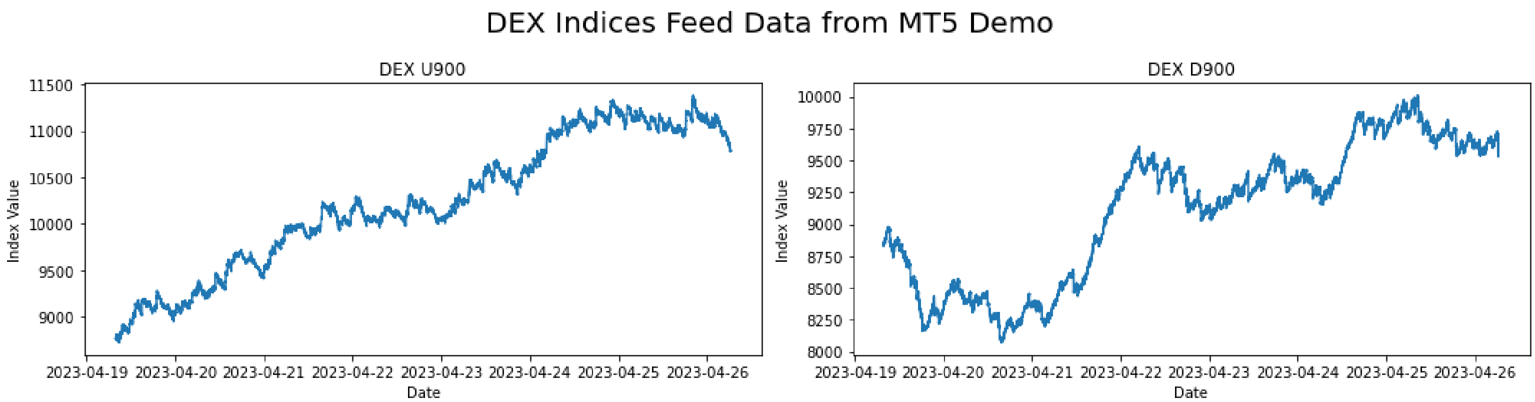
Running a simulation (DEX U900) based on these 2 methods, the moments (Mean, Volatility, Skewness and Kurtosis) are matching.

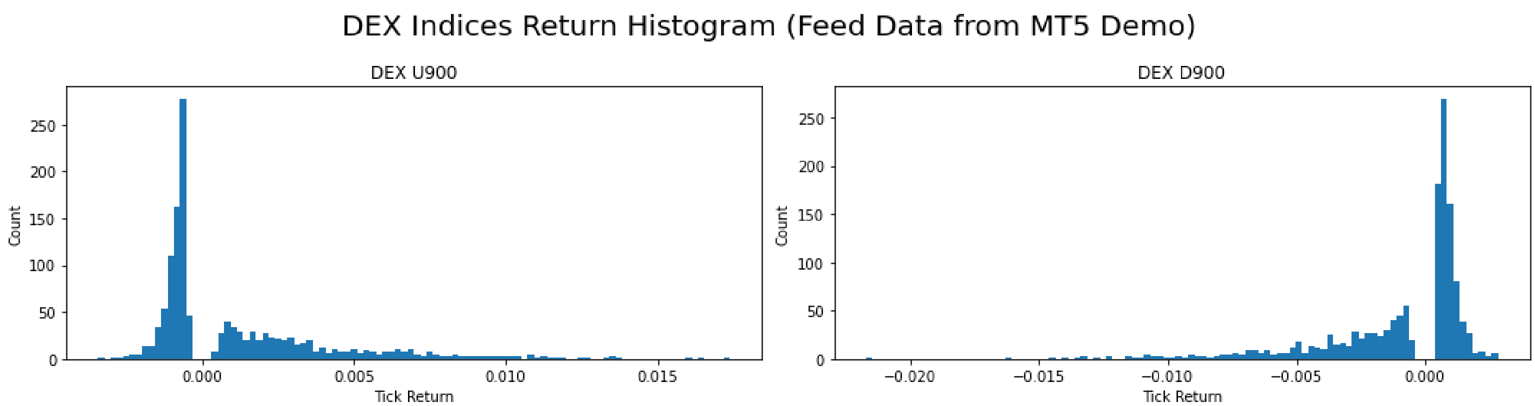
	Simulation based on Perl	Simulation based on MV logic	Difference %
Mean	-0.00028	-0.00028	-0.000984
Volatility	0.001944	0.001944	0.000391
Skewness	-3.939029	-3.937295	0.00044
Kurtosis	21.933217	21.917746	0.000706

Section 3

In here we are checking the moments of the real feed data. The Validation Steps are: 1) Obtain the feed data from MT5 demo accounts. 2) Compute the feed data moments (Mean, Volatility, Skewness & Kurtosis). 3) Run the simulation and compute the moments. 4) Compare (2) & (3)

Conclusion: 1) Mean - Acceptable as the abs difference is low. 2) Volatility - Acceptable as both abs and rel difference are low. 3) Skewness - Acceptable as rel difference is low. 4) Kurtosis - Acceptable.





Feed Data Moments

	Mean	Volatility	Skewness	Kurtosis
DEX U900	0.00000034	0.00015455	45.20972819	2953.52457144
DEX D900	0.00000013	0.00014861	-47.67062871	3541.44072733

Simulation Moments

	Mean	Volatility	Skewness	Kurtosis
DEX U900	-0.00000001	0.00015233	50.47877365	4086.6369544
DEX D900	0.00000006	0.00015115	-48.81241142	3631.46821956

Difference (abs)

	Mean	Volatility	Skewness	Kurtosis
DEX U900	0.00000035	0.00000221	-5.26904546	-1133.11238295
DEX D900	0.00000007	-0.00000254	1.14178271	-90.02749223

Difference (rel)

	Mean	Volatility	Skewness	Kurtosis
DEX U900	-34.82185545	0.01453387	-0.10438141	-0.27727258
DEX D900	1.03537906	-0.0168127	-0.02339124	-0.02479093

Section 4

Here we check the parameters backward engineering from the feed data moments.

The result is highly dependable on the initial and boundary condition. The difference in the results are acceptable.

Feed Data Params

	Vol	Lambda	Probability	Positive Eta	Negative Eta
DEX U900	0.254925	178651.44	0.20394	0.0028809	0.00040788
DEX D900	0.24166751	177918.00146253	0.76824	0.00040788	0.0028809

True Params

	Vol	Lambda	Probability	Positive Eta	Negative Eta
DEX U900	0.25	175200	0.2	0.003	0.0004
DEX D900	0.25	175200	0.8	0.0004	0.003

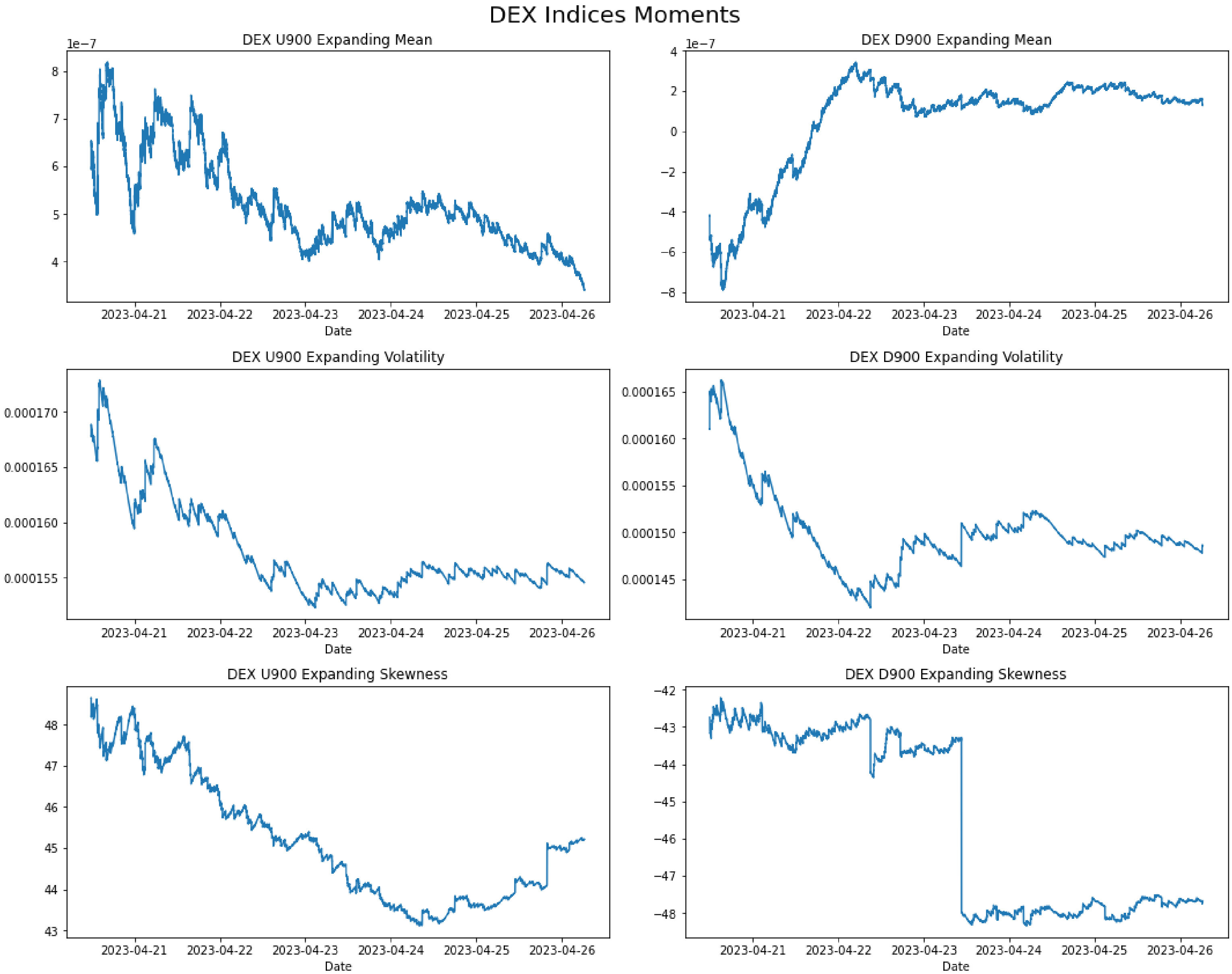
Difference (rel)

	Vol	Lambda	Probability	Positive Eta	Negative Eta
DEX U900	0.0197	0.0197	0.0197	-0.0397	0.0197
DEX D900	-0.03332998	0.01551371	-0.0397	0.0197	-0.0397

Section 5

We want to check the convergence speed of the DEX feed data in term of the moments.

Overall looks fine.



Appendix

Double Jump Drift Correction Derivation

We want to find the drift correction for $S = \sum_{i=1}^{N} J_i$ where $N \sim Poisson(\lambda dt)$ and J_i is i.i.d. That is, we want to find α such that:

$$S_t = S_{t-1} \exp[-\alpha \cdot \lambda dt + S]$$

has zero drift, i.e:

$$E\left[\frac{S_t}{S_{t-1}}\right] = E\left[e^{-\alpha \cdot \lambda dt + S}\right] = 1$$

So we condition over N

$$E\left[e^S\right] = E\left[E\left(e^S \mid N\right)\right]$$

Since N is fixed in this context, we can calculate $E\left(e^S \mid N\right)$:

$$E\left(e^S \mid N\right) = E\left[\exp\left(\sum_{i=1}^{N} J_i\right)\right] = E\left[\prod_{i=1}^{N} e^{J_i}\right] = \prod_{i=1}^{N} E\left[e^{J_i}\right]$$

By independence of the J_i. Since they are identically distributed, $E[e^{J_i}] = E[e^{J_1}]$, hence it is constant. We define the constant β such that:

$$e^\beta = E[e^{J_1}]$$

Then:

$$E\left[E\left(e^S \mid N\right)\right] = E\left[\prod_{i=1}^{N} E\left[e^{J_i}\right]\right] = E\left[\prod_{i=1}^{N} E\left[e^{J_1}\right]\right] = E\left[\prod_{i=1}^{N} e^\beta\right] = E\left[(e^\beta)^N\right]$$

We may rewrite this using the moment-generating funtion for N:

$$E\left[e^{\beta \cdot N}\right] = MGF_{N(\lambda dt)}(\beta) = \exp\left[\lambda dt(e^\beta - 1)\right]$$

Finally:

$$E\left[e^{-\alpha \cdot \lambda dt + S}\right] = e^{-\alpha \cdot \lambda dt + (e^\beta - 1)\lambda dt} = 1$$

So

$$\alpha = e^\beta - 1 = E[e^J - 1]$$

Moment Generating Function of Jump Size J

Here show the computation:

$$\begin{aligned}
E(e^{tJ}) &= \int_{-\infty}^{0} \frac{q_-}{\eta_-} e^{tJ} e^{\frac{J}{\eta_-}} dJ + \int_{0}^{\infty} \frac{q_+}{\eta_+} e^{tJ} e^{-\frac{J}{\eta_+}} dJ \\
&= \int_{-\infty}^{0} \frac{q_-}{\eta_-} e^{\frac{(t\eta_- + 1)J}{\eta_-}} dJ + \int_{0}^{\infty} \frac{q_+}{\eta_+} e^{-\frac{(1 - t\eta_+)J}{\eta_+}} dJ \\
&= \frac{q_-}{\eta_-} \times \frac{\eta_-}{t\eta_- + 1} \times e^{\frac{(t\eta_- + 1)J}{\eta_-}} \Big|_{-\infty}^{0} + \frac{q_+}{\eta_+} \times \frac{\eta_+}{-(1 - t\eta_+)} \times e^{-\frac{(1 - t\eta_+)J}{\eta_+}} \Big|_{0}^{\infty} \\
&= \frac{q_-}{t\eta_- + 1} - \frac{q_+}{-(1 - t\eta_+)} \\
&= \frac{q_-}{t\eta_- + 1} + \frac{q_+}{(1 - t\eta_+)}
\end{aligned}$$

Moments of Jump Size J

We can find the first moment using WolframAlpha:

$$\frac{d}{dt} E[e^{tJ}] = -\frac{q_- \eta_-}{(1 + \eta_- t)^2} + \frac{q_+ \eta_+}{(1 - \eta_+ t)^2}$$

And

$$E[X] = \frac{d}{dt}E[e^{tJ}]\Big|_{t=0} = -q_-\eta_- + q_+\eta_+$$

Furthermore

$$E[J^2] = \frac{d^2}{dt^2}E[e^{tJ}]\Big|_{t=0} = \frac{2q_-\eta_-^2}{(1+\eta_-t)^3} + \frac{2q_+\eta_+^2}{(1-\eta_+t)^3}\Big|_{t=0} = 2q_-\eta_-^2 + 2q_+\eta_+^2$$

$$E[J^3] = \frac{d^3}{dt^3}E[e^{tJ}]\Big|_{t=0} = -\frac{6q_-\eta_-^3}{(1+\eta_-t)^4} + \frac{6q_+\eta_+^3}{(1-\eta_+t)^4}\Big|_{t=0} = -6q_-\eta_-^3 + 6q_+\eta_+^3$$

$$E[J^4] = \frac{d^4}{dt^4}E[e^{tJ}]\Big|_{t=0} = \frac{24q_-\eta_-^4}{(1+\eta_-t)^5} + \frac{24q_+\eta_+^4}{(1-\eta_+t)^5}\Big|_{t=0} = 24q_-\eta_-^4 + 24q_+\eta_+^4$$

In general:

$$E[J^n] = n![(-1)^n q_-\eta_-^n + q_+\eta_+^n]$$

Moments of S

Using the same principle as earlier, we can calculate the MGF of S:

$$E[e^{tS}] = \exp\left[\lambda dt(E[e^tJ]-1)\right] = \exp\left[\lambda dt(\frac{q_-}{1+t\eta_-} + \frac{q_+}{1-t\eta_+} - 1)\right]$$

The first & second moments (Noting that $q_- + q_+ = 1$):

$$\begin{aligned}
E[S] &= \frac{d}{dt}E[e^{tS}]\Big|_{t=0} \\
&= \lambda dt\left(-\frac{q_-\eta_-}{(1+\eta_-t)^2} + \frac{q_+\eta_+}{(1-\eta_+t)^2}\right)\exp\left[\lambda dt(\frac{q_-}{1+t\eta_-} + \frac{q_+}{1-t\eta_+} - 1)\right]\Big|_{t=0} \\
&= \lambda dt(-q_-\eta_- + q_+\eta_+) = \lambda dt E[J]
\end{aligned}$$

$$\begin{aligned}
E[S^2] &= \frac{d^2}{dt^2}E[e^{tS}]\Big|_{t=0} \\
&= \Bigg[\lambda^2 dt^2\left(-\frac{q_-\eta_-}{(1+\eta_-t)^2} + \frac{q_+\eta_+}{(1-\eta_+t)^2}\right)^2 \\
&\quad + \lambda dt\left(\frac{2q_-\eta_-^2}{(1+\eta_-t)^3} + \frac{2q_+\eta_+^2}{(1-\eta_+t)^3}\right)\Bigg]\exp\left[\lambda dt(\frac{q_-}{1+t\eta_-} + \frac{q_+}{1-t\eta_+} - 1)\right]\Big|_{t=0} \\
&= \lambda^2 dt^2(-q_-\eta_- + q_+\eta_+)^2 + \lambda dt\left(2q_-\eta_-^2 + 2q_+\eta_+^2\right) \\
&= E[S]^2 + \lambda dt E[J^2]
\end{aligned}$$

We can convert to central moments ($E[(x-E(X))^n]$) for the second, third and fourth moments:

$$Var(S) = E[(S-E(S))^2] = E[S^2] - E[S]^2 = \lambda dt E[J^2]$$

$$E[(S-E(S))^3] = E[S^3] - 3E[S^2]E[S] + 2E[S]^3 = \lambda dt E[J^3]$$

In general:

$$E[(S-E(S))^n] = \lambda dt E[J^n]$$

Inverse CDF of DEJD with Single Random Number

To derive the inverse cdf of the Double Exponential Jump distribution, firstly we generate the random number from a uniform distribution between 0 and 1.

If the random number is less than the probability of negative jump, i.e. $r < q_-$:

$$F(X) = \int_{-\infty}^{x} \frac{q_-}{\eta_-} e^{\frac{J}{\eta_-}} dJ$$
$$r = q_- e^{\frac{J}{\eta_-}} \Big|_{-\infty}^{x}$$
$$r = q_- e^{\frac{x}{\eta_-}}$$

Then we will get:

$$x = \eta_- \times ln(\frac{r}{q_-})$$

If the random number is more the probability of negative jump, i.e. $r > q_-$:

$$F(X) = \int_{0}^{\infty} \frac{q_+}{\eta_+} e^{-\frac{J}{\eta_+}} dJ + \int_{-\infty}^{0} \frac{q_-}{\eta_-} e^{\frac{J}{\eta_-}} dJ$$
$$r = -q_+ e^{-\frac{J}{\eta_+}} \Big|_{0}^{x} + q_-$$
$$r = -q_+ e^{-\frac{x}{\eta_+}} + q_+ + q_-$$
$$r = -q_+ e^{-\frac{x}{\eta_+}} + q_+ + (1 - q_+)$$
$$r = -q_+ e^{-\frac{x}{\eta_+}} + 1$$

In result :

$$x = -\eta_+ \times ln(\frac{1-r}{q_+})$$

R&D effort needs to be in line with Deriv’s vision and mission as formulated by our CEO. Therefore all R&D projects are carefully selected by our C-Level senior management represented by JY and Rakshit and resources for the projects are only allocated after review and shortlisting based on their vision and priorities.

In line with the standards and criterias set out by the CEO, the Model Validation team has validated the product/indices as documented in this report.