Hybrid Indices Report (DEJD Variations)

Report Date : 30 Aug 2024

Product Information

Product Specification :
https://docs.google.com/spreadsheets/d/13ppWafDpjXYBNgWxm9WejV2oxwoQHRv2DVsyBZRbhjw/edit gid=427402466#gid=427402466

Demo Launch: 26 August 2024 3:29pm GMT+8 (7:29am UTC)

Background & Introduction

The Hybrid indices are DEX versions of the Crash/Boom indices. These indices will look a little better, since in between jumps, its a vol index, and be much more well-behaved for option pricing. The two types of Hybrid Indices are Vol-and-Crash and Vol-and-Boom.

Index Construction

For information about the index construction, please refer to the Double Exponential (DEX) Index Validation Report as of 22 May 2023 (link.

The following is a summary of the construction

DEX (DEJD)

Recall that DEX is given as

$$S_t = S_{t-1} \exp\left[\left(r - d - \frac{\sigma^2}{2} - \lambda\alpha\right) dt + \sigma\sqrt{dt}W + \sum_{i=1}^{N(\lambda dt)} J_i\right]$$

Where:

- S - Spot;
- r - Risk-free rate;
- d - Dividend rate;
- N - Poisson process with intensity (mean) λ, that is $P(N(\lambda) = k) = \frac{\lambda^k e^{-\lambda}}{k!}$;
- λ - Expected of number of jumps per annum;
- J - Random jump size with the double exponential distribution as detailed below;
- α - The drift correction from the jump terms, i.e $E(e^J - 1)$;
- σ - Volatility;
- W - A normally distributed random sample.

The density of J (double exponential distribution) is given as:

$$w(J) = \frac{q_-}{\eta_-} e^{\frac{J}{\eta_-}} \mathbf{1}_{J<0} + \frac{q_+}{\eta_+} e^{-\frac{J}{\eta_+}} \mathbf{1}_{J\geq 0}$$

Where:

- $\mathbf{1}$ is the indicator function;
- $q_\pm$ - The probability of a positive/negative jump ($q_- + q_+ = 1$);
- $\eta_\pm$ - The expected size of a positive/negative jump.

Defining Vol over CB

For these indices, we modify the jump distribution to get a one-sided distribution, by setting one of $q_\pm = 0$. We will have two types of indices:

Vol-and-Crash

We set $q_+ = \eta_+ = 0$, and $q_- = 1$, $\eta_- > 0$:

$$w(J) = \frac{1}{\eta_-} e^{\frac{J}{\eta_-}} \mathbf{1}_{J<0}$$

Vol-and-Boom

We set $q_- = \eta_- = 0$, and $q_+ = 1$, $\eta_+ > 0$:

$$w(J) = \frac{1}{\eta_+} e^{-\frac{J}{\eta_+}} \mathbf{1}_{J\geq 0}$$

Parameters

Index	Number of Jumps per Hour λ	Average Positive Jump Size η_+	Average Negative Jump Size η_-	Probability of Positive Jump q_+	Probability of Negative Jump q_-	Volatility	Percentage Spread
Vol-and-Crash 400	9, which is 78840 per year	0.00%	0.20%	0%	100%	20%	0.01%
Vol-and-Crash 550	6.55, which is about 57338 per year	0.00%	0.20%	0%	100%	20%	0.01%
Vol-and-Crash 750	4.8, which is 42048 per year	0.00%	0.20%	0%	100%	20%	0.01%

Index	Number of Jumps per Hour λ	Average Positive Jump Size η_+	Average Negative Jump Size η_-	Probability of Positive Jump q_+	Probability of Negative Jump q_-	Volatility	Percentage Spread
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Vol-and-Boom 550	6.55, which is about 57338 per year	0.20%	0.00%	100%	0%	20%	0.01%
Vol-and-Boom 750	4.8, which is 42048 per year	0.20%	0.00%	100%	0%	20%	0.01%

Model Validation

Summary

For the validation of the Hybrid indices, we cover the below areas and conclude the outcomes.

Note that this validation covers the data from 20 August 2024 to 29 August 2024.

This index was released on Demo at 26 August 2024.

More details can be found in respective section.

Section	Area	Validation	Outcome	Pass/Fail
1	Check the moment of the feed data	We compute the moment of the feed data, and check whether it is matching with the true distribution.	We checked the moments of the feed data vs simulation for 3 sets of data, Dev data only, Demo data only and Dev + Demo data. The moments are match up to acceptable margins.	**Passed**
2	Backward engineering the parameters.	Here we check the parameters backward engineering from the feed data moment. We do not proceed with the MLE method as the implementation is very difficult to implement	The result looks good with set initial condition and boundary condition, even with only 9 days of data.	**Passed**
3	Convergence of the feed data moment	The convergence speed of the first 3 moments are checked.	The moments have not converged fully yet, but are closed. Similar tests with the DEX indices suggest it will	**Passed**

Section	Area	Validation	Outcome	Pass/Fail
			converge with 2 weeks+ of data	
4	Concentration Risk Stress Testing	Few strategy testings on simulation data is run to ensure that there is no potential exploitation and the reasonableness of the spread.	The strategy testing does not discover any potential exploitation. The strategy testing result also shows that the Product Quants suggested spreads cover this strategy very well, as the returns are very low.	**Passed**
5	Correlation between variations	This is to test the pairwise correlations, which should be almost zero	Correlations between tickly data and hourly data do not have significantly high correlation.	**Passed**

Section 1

In here we are checking the moments of the real feed data. The validation steps are:

1. Obtain the feed data from Metabase.
2. Compute the feed data moments (Mean, Volatility, Skewness & Kurtosis).
3. Run the simulation and compute the moments.
4. Compare (2) & (3)

Conclusion:

1. Mean - Acceptable as the abs difference is low.
2. Volatility - Acceptable as both abs and rel difference are low.
3. Skewness - Acceptable as rel difference is low.
4. Kurtosis - Acceptable.

We also note that the jumps are in the right direction, and the sample so far has longer times between jumps than expected on average.

```
5          0.00002535
7         -0.00006489
8         -0.00002839
9         -0.00002535
10         0.00004461
              ...
1231250   -0.00003051
1231251    0.00004331
1231252    0.00004134
1231256   -0.00003051
1231257   -0.00002461
Name: return, Length: 713097, dtype: float64
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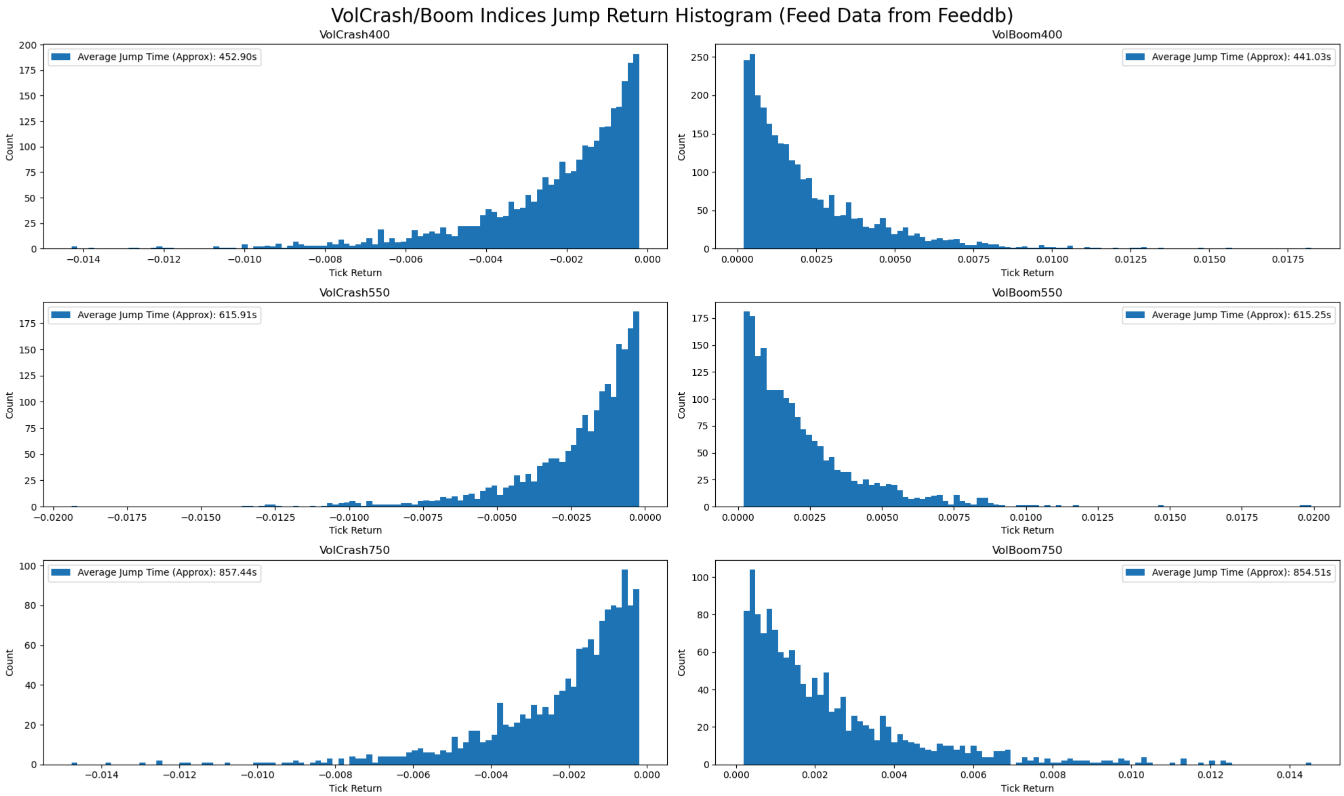
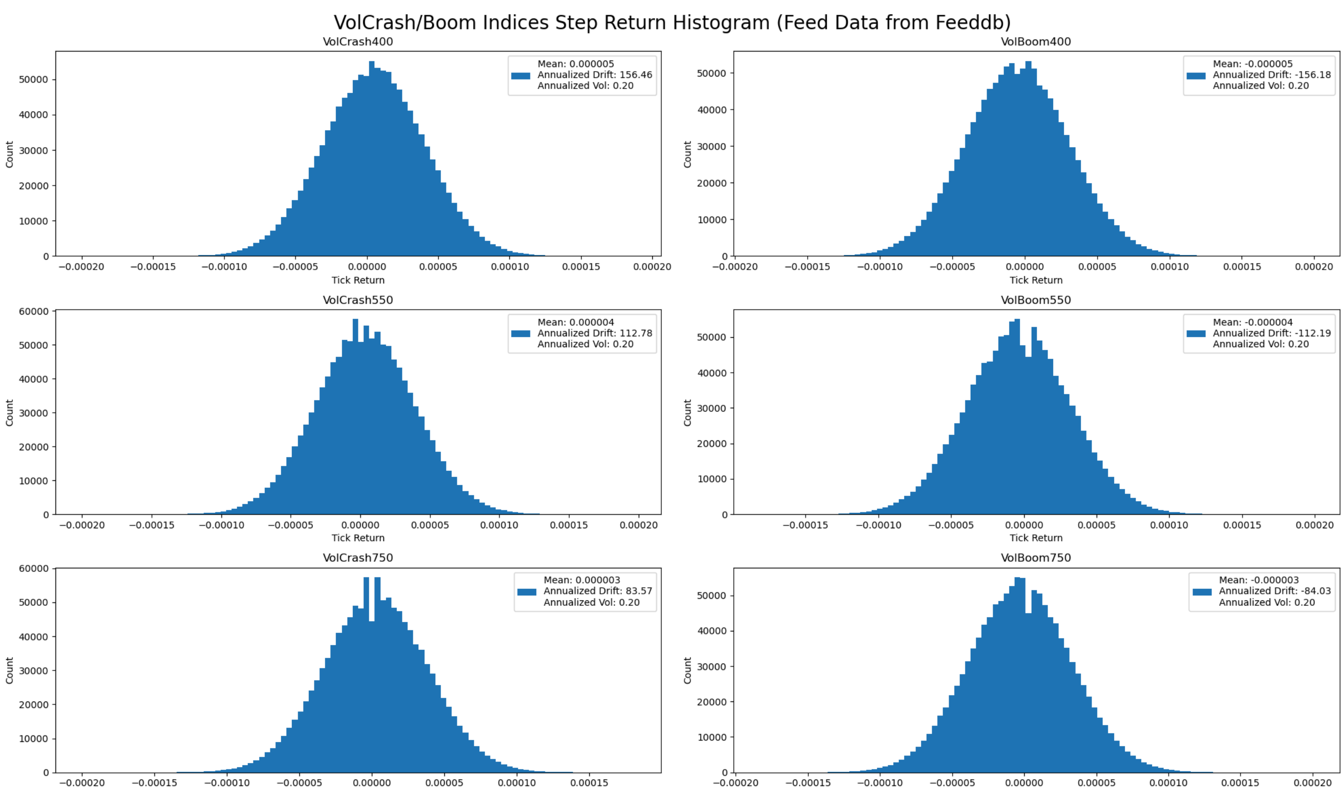


And the step distributions are normal, biased in the right direction (Opposite of jumps), with the correct volatility.



Moment Matching on Demo Data only

Feed Data Moments (Demo Data Only)

	Mean	Volatility	Skewness	Kurtosis
VolCrash400	-0.000000187	0.84337779	-40.256888378	2262.596197806
VolCrash550	-0.000000135	0.742422885	-46.674946046	3033.43002554
VolCrash750	0.000000355	0.556750364	-50.997719114	3942.82898582
VolBoom400	-0.00000028	0.803840048	43.447797721	2841.732981484
VolBoom550	0.000000055	0.681293004	38.139488686	1932.944987681
VolBoom750	-0.000000009	0.630178538	52.039863128	3828.183651538

True Moments

	Mean	Volatility	Skewness	Kurtosis
VolCrash400	-0.000000011	0.818974969	-38.688266564	2122.276346075
VolCrash550	-0.000000008	0.706190806	-43.885624299	2791.859200819
VolCrash750	-0.000000006	0.613501426	-49.084362442	3594.354169577
VolBoom400	-0.000000011	0.818974969	38.688266564	2122.276346075
VolBoom550	-0.000000008	0.706190806	43.885624299	2791.859200819
VolBoom750	-0.000000006	0.613501426	49.084362442	3594.354169577

Difference (abs)

	Mean	Volatility	Skewness	Kurtosis
VolCrash400	-0.000000177	0.024402821	-1.568621814	140.319851731
VolCrash550	-0.000000127	0.036232079	-2.789321747	241.570824721
VolCrash750	0.000000361	-0.056751063	-1.913356671	348.474816243
VolBoom400	-0.000000269	-0.015134921	4.759531156	719.456635408
VolBoom550	0.000000063	-0.024897802	-5.746135613	-858.914213138
VolBoom750	-0.000000003	0.016677112	2.955500685	233.829481961

Difference (rel)

	Mean	Volatility	Skewness	Kurtosis
VolCrash400	16.633052851	0.029796785	0.040545156	0.066117616
VolCrash550	16.135919877	0.051306359	0.063558894	0.086526865
VolCrash750	-60.52850586	-0.092503554	0.038980982	0.096950606
VolBoom400	25.289494181	-0.018480322	0.1230226	0.339002334
VolBoom550	-7.971401745	-0.035256481	-0.130934348	-0.307649545
VolBoom750	0.54715106	0.027183493	0.060212673	0.065054658

Moment Matching on Dev Data only

Feed Data Moments (Dev Data Only)

	Mean	Volatility	Skewness	Kurtosis
VolCrash400	0.000000123	0.806588308	-36.816103272	1796.592217063
VolCrash550	-0.000000002	0.713066752	-46.217438899	3144.066566026
VolCrash750	0.000000012	0.611159733	-48.52777544	3486.17730592
VolBoom400	0.00000014	0.829999578	37.087205819	1862.012871363
VolBoom550	0.000000025	0.707227596	46.607430421	3453.012634576
VolBoom750	0.000000095	0.645112398	48.984240402	3335.910330168

True Moments

	Mean	Volatility	Skewness	Kurtosis
VolCrash400	-0.000000011	0.818974969	-38.688266564	2122.276346075
VolCrash550	-0.000000008	0.706190806	-43.885624299	2791.859200819
VolCrash750	-0.000000006	0.613501426	-49.084362442	3594.354169577
VolBoom400	-0.000000011	0.818974969	38.688266564	2122.276346075
VolBoom550	-0.000000008	0.706190806	43.885624299	2791.859200819
VolBoom750	-0.000000006	0.613501426	49.084362442	3594.354169577

Difference (abs)

	Mean	Volatility	Skewness	Kurtosis
VolCrash400	0.000000134	-0.012386661	1.872163292	-325.684129012
VolCrash550	0.000000006	0.006875946	-2.3318146	352.207365206
VolCrash750	0.000000018	-0.002341693	0.556587003	-108.176863657
VolBoom400	0.000000151	0.011024609	-1.601060745	-260.263474713
VolBoom550	0.000000033	0.00103679	2.721806122	661.153433757
VolBoom750	0.000000101	0.031610971	-0.100122041	-258.443839409

Difference (rel)

	Mean	Volatility	Skewness	Kurtosis
VolCrash400	-12.63373267	-0.015124591	-0.048390984	-0.153459812
VolCrash550	-0.697079209	0.009736669	0.053133905	0.126155132
VolCrash750	-2.964419425	-0.003816932	-0.011339396	-0.030096328
VolBoom400	-14.157979764	0.013461473	-0.041383626	-0.122634112
VolBoom550	-4.199148637	0.001468144	0.062020449	0.236814748
VolBoom750	-16.942385679	0.051525506	-0.002039795	-0.071902719

Moment Matching on Combined Data

Feed Data Moments (Combined)

	Mean	Volatility	Skewness	Kurtosis
VolCrash400	0.000000038	0.81692275	-37.885321411	1945.898718059
VolCrash550	-0.000000039	0.721300375	-46.378612028	3113.991845828
VolCrash750	0.000000106	0.596617203	-49.200410308	3604.150657233
VolBoom400	0.000000024	0.82285232	38.737420244	2110.117898942
VolBoom550	0.000000034	0.700154793	44.469879571	3080.057395177
VolBoom750	0.000000066	0.641019107	49.794649478	3464.439821177

True Moments

	Mean	Volatility	Skewness	Kurtosis
VolCrash400	-0.000000011	0.818974969	-38.688266564	2122.276346075
VolCrash550	-0.000000008	0.706190806	-43.885624299	2791.859200819
VolCrash750	-0.000000006	0.613501426	-49.084362442	3594.354169577
VolBoom400	-0.000000011	0.818974969	38.688266564	2122.276346075
VolBoom550	-0.000000008	0.706190806	43.885624299	2791.859200819
VolBoom750	-0.000000006	0.613501426	49.084362442	3594.354169577

Difference (abs)

	Mean	Volatility	Skewness	Kurtosis
VolCrash400	0.000000048	-0.002052219	0.802945153	-176.377628017
VolCrash550	-0.000000031	0.015109569	-2.492987729	322.132645009
VolCrash750	0.000000112	-0.016884223	-0.116047866	9.796487657
VolBoom400	0.000000035	0.003877351	0.04915368	-12.158447133
VolBoom550	0.000000042	-0.006036013	0.584255272	288.198194357
VolBoom750	0.000000072	0.027517681	0.710287036	-129.914348399

Difference (rel)

	Mean	Volatility	Skewness	Kurtosis
VolCrash400	-4.544311506	-0.002505839	-0.020754229	-0.083107758
VolCrash550	3.955747194	0.021395873	0.056806477	0.115382841
VolCrash750	-18.87576712	-0.027521082	0.002364253	0.002725521
VolBoom400	-3.254263358	0.004734395	0.001270506	-0.005728965
VolBoom550	-5.241840956	-0.008547283	0.013313136	0.103228055
VolBoom750	-12.108085035	0.044853491	0.01447074	-0.036144003

Section 2

Here we check the parameters backward engineering from the feed data moments.

The result is highly dependable on the initial and boundary condition. The difference in the results are acceptable, within a 6% margin.

Feed Data Params

	Vol	Lambda	Probability	Positive Eta	Negative Eta
VolCrash400	0.19885	78386.67	0.0	0.0	0.0018915
VolCrash550	0.18915	54227.58545455	0.0	0.0	0.0019885
VolCrash750	0.19885	41806.224	0.0	0.0	0.0018915
VolBoom400	0.19885	78386.67	0.99425	0.0018915	0.0
VolBoom550	0.18915	54227.58545455	0.94575	0.0019885	0.0
VolBoom750	0.19885	41806.224	0.99425	0.0018915	0.0

True Params

	Vol	Lambda	Probability	Positive Eta	Negative Eta
VolCrash400	0.2	78840.0	0	0	0.002
VolCrash550	0.2	57338.18181818	0	0	0.002
VolCrash750	0.2	42048.0	0	0	0.002
VolBoom400	0.2	78840.0	1	0.002	0
VolBoom550	0.2	57338.18181818	1	0.002	0
VolBoom750	0.2	42048.0	1	0.002	0

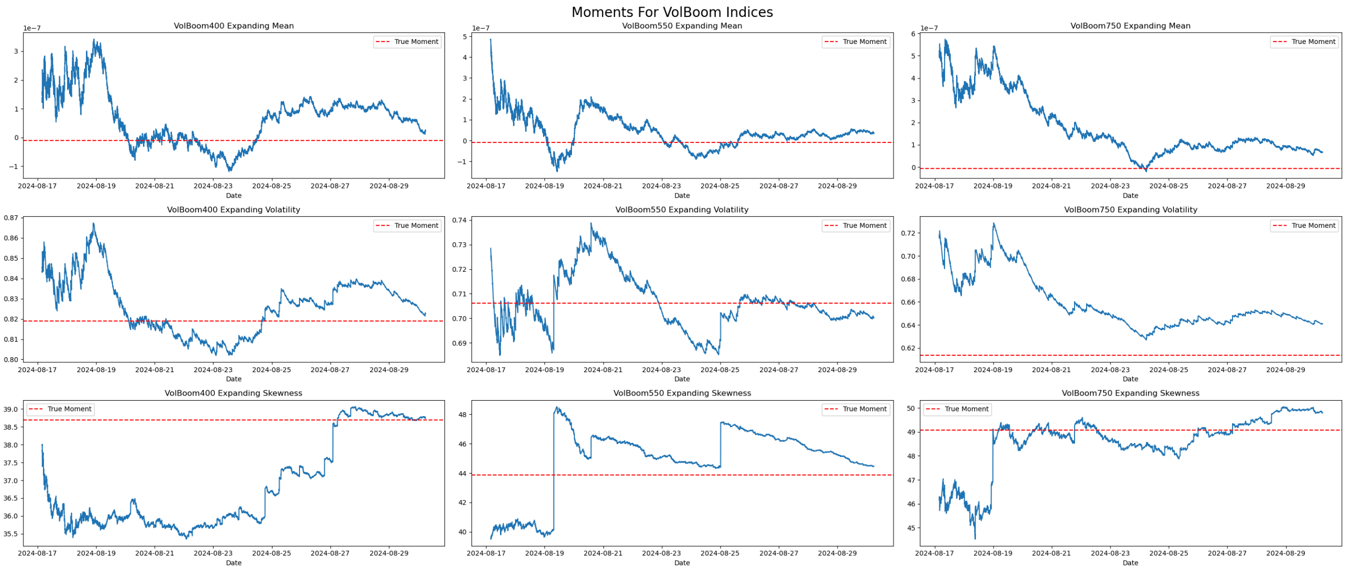
Difference (rel)

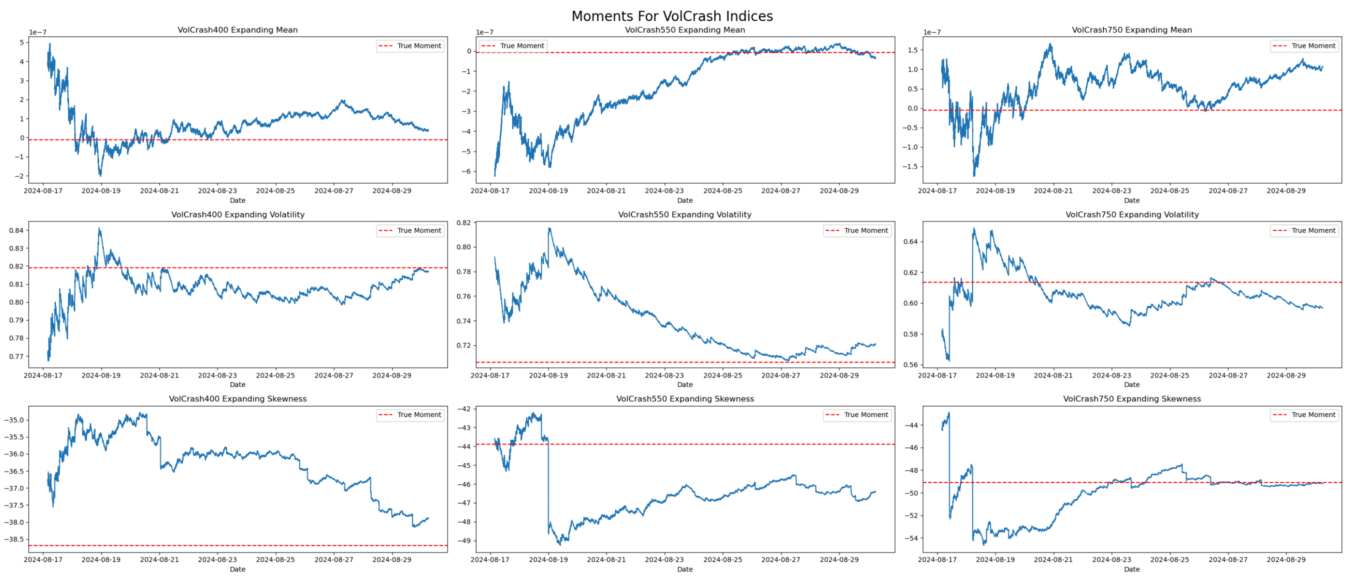
	Vol	Lambda	Probability	Positive Eta	Negative Eta
VolCrash400	-0.00575	-0.00575	NaN	NaN	-0.05425
VolCrash550	-0.05425	-0.05425	NaN	NaN	-0.00575
VolCrash750	-0.00575	-0.00575	NaN	NaN	-0.05425
VolBoom400	-0.00575	-0.00575	-0.00575	-0.05425	NaN
VolBoom550	-0.05425	-0.05425	-0.05425	-0.00575	NaN
VolBoom750	-0.00575	-0.00575	-0.00575	-0.05425	NaN

Section 3

We want to check the convergence speed of the feed data in term of the moments.

Overall looks fine, even with only 9 days of data.





Section 4

The simulation steps of the strategy testing are:

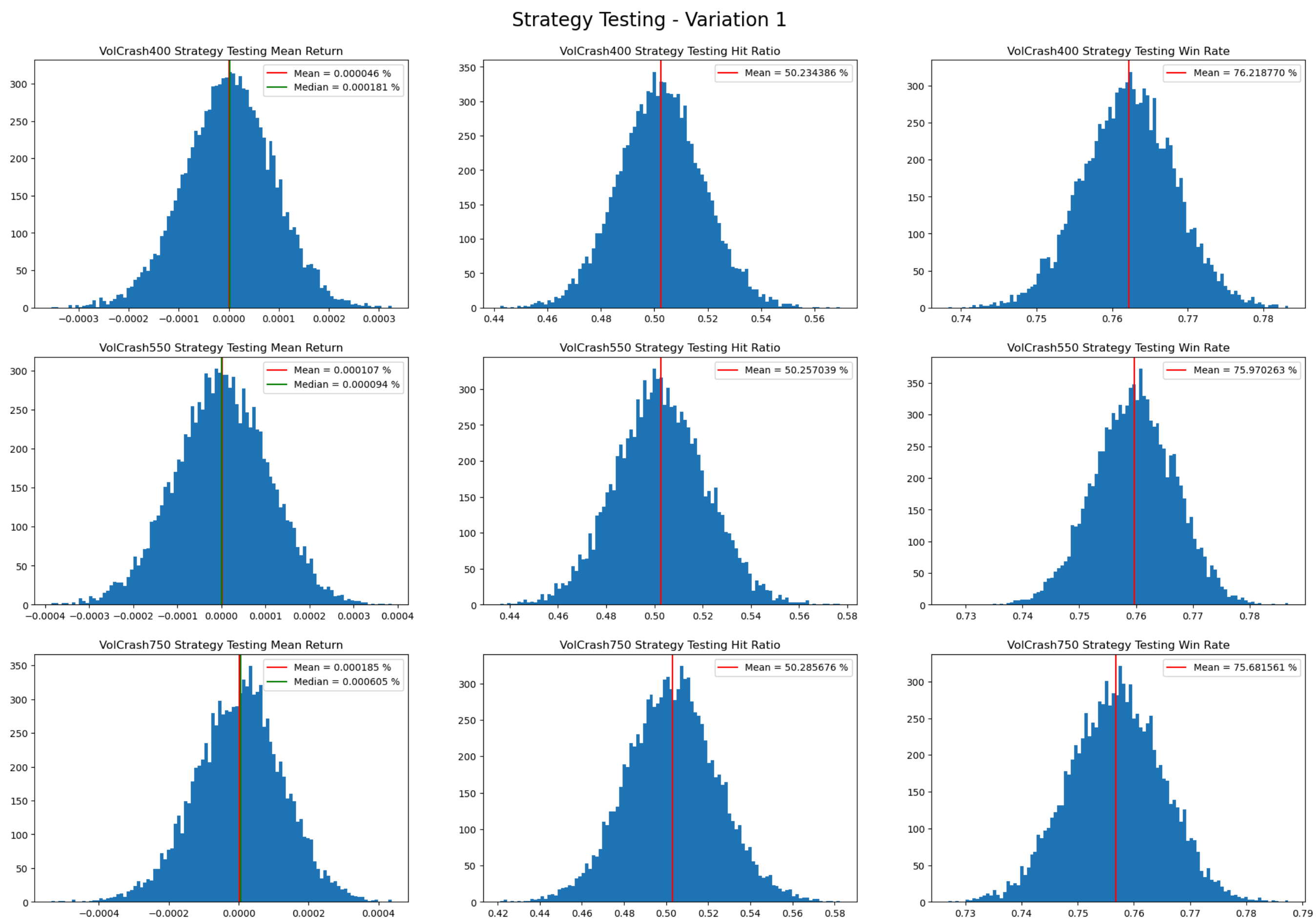
1. Run 3-months simulation for Vol Crash 400/550/750. Vol Boom indices are not run because results are equivalent.
2. Run the trading strategy on the simulation data.
   A. Define the jump size same as the negative eta.
   B. After each jump, wait for mean jump frequency (which is 400/550/750 seconds respectively). If there is no jump, enter into a short position.
   C. Enter into additional short position every mean jump frequency if there is no jump.
   D. Exit all positions in next jump.
3. Repeat the strategy for a sufficient number of times. In here, it is 10K.
4. Plot the average PnL%, Hit Ratio and Win Rate.

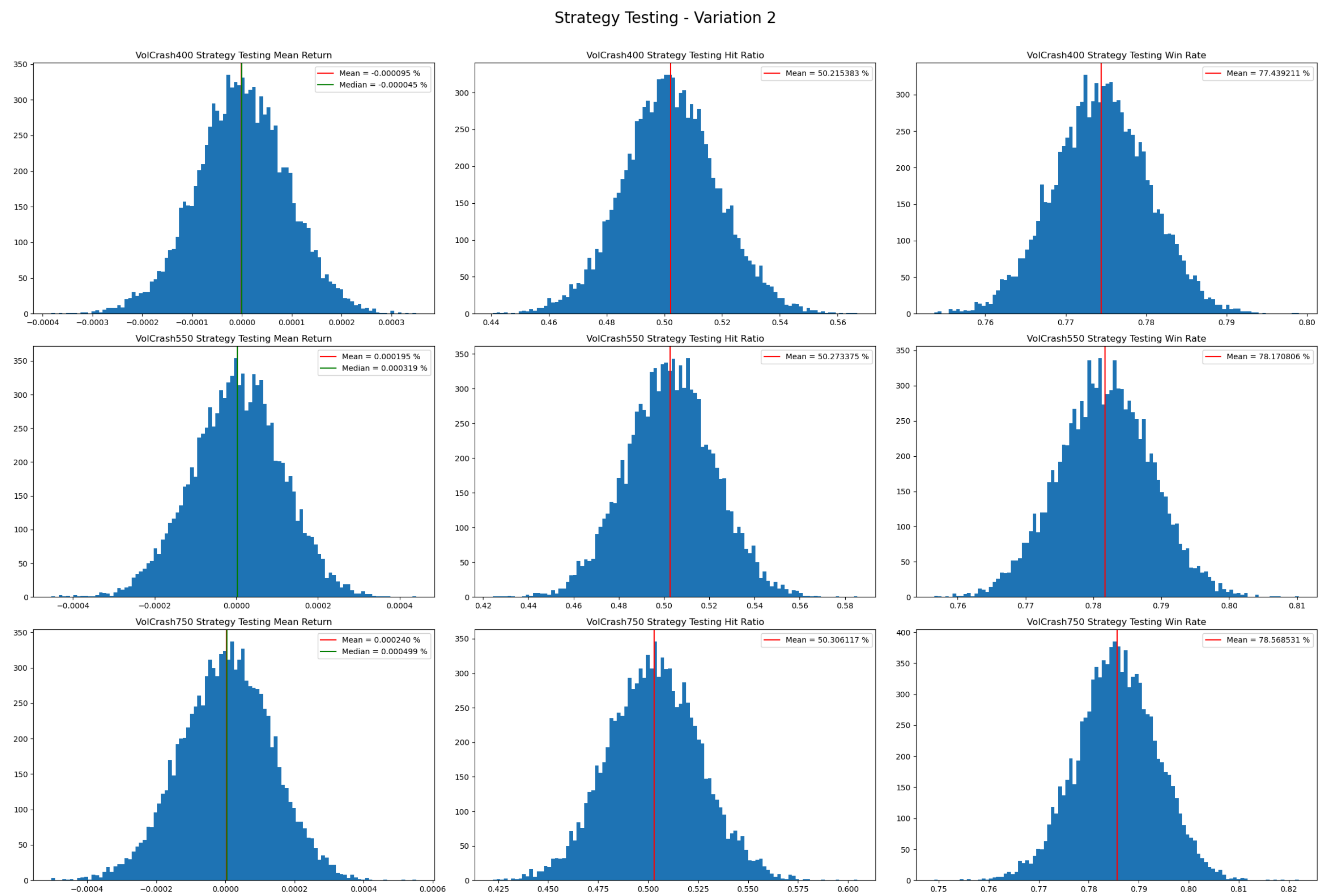
In total there are 3 variations of this strategies are run too, which is to:

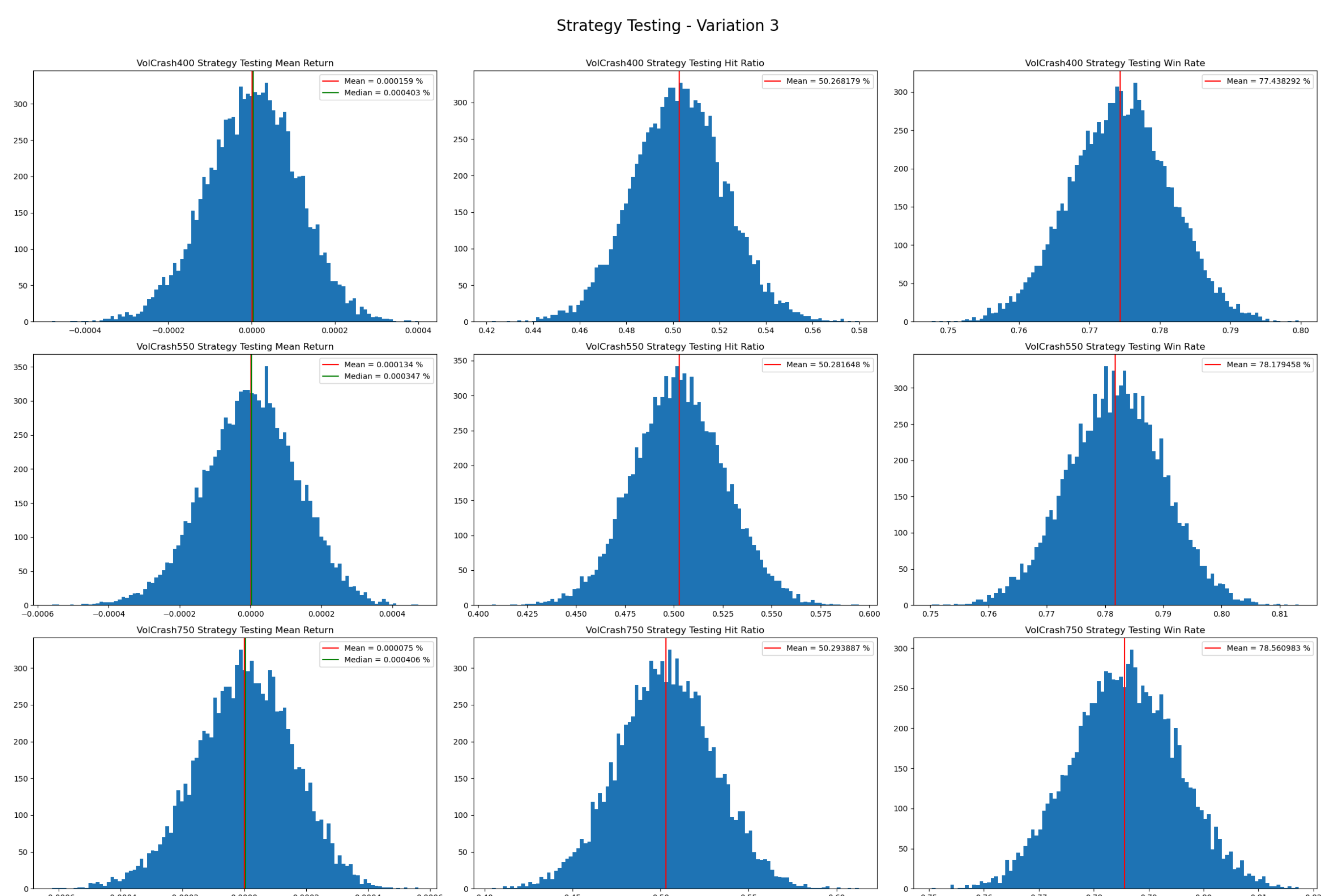
1. Waiting time and additional position frequency same as mean jump frequency.
2. Waiting time same as mean jump frequency but additional position frequency at 5 mins.
3. Double the waiting time and enter into additional positions every 5 mins.

Key points to conclude the strategy testing:

1. Mean Hit Ratio of 50% shows that the exploitation is not possible.
2. Mean Win Rate of 75% is expected due to the clear pattern of the index. They are higher than the DEX indices as VolC/B indices are more predictable, i.e we know the direction of the jumps always
3. Spread of 0.01% as proposed by Product Quants is sufficient for all the indices as it very comfortably covers the mean and median of the strategy PnL in %, which tend to be in the range of 0.0001%.




Strategy Testing - Variation 2
VolCrash400 Strategy Testing Mean Return
Mean = -0.000095 %
Median = -0.000045 %
VolCrash400 Strategy Testing Hit Ratio
Mean = 50.215383 %
VolCrash400 Strategy Testing Win Rate
Mean = 77.439211 %
VolCrash550 Strategy Testing Mean Return
Mean = 0.000195 %
Median = 0.000319 %
VolCrash550 Strategy Testing Hit Ratio
Mean = 50.273375 %
VolCrash550 Strategy Testing Win Rate
Mean = 78.170806 %
VolCrash750 Strategy Testing Mean Return
Mean = 0.000240 %
Median = 0.000499 %
VolCrash750 Strategy Testing Hit Ratio
Mean = 50.306117 %
VolCrash750 Strategy Testing Win Rate
Mean = 78.568531 %


Strategy Testing - Variation 3
VolCrash400 Strategy Testing Mean Return
Mean = 0.000159 %
Median = 0.000403 %
VolCrash400 Strategy Testing Hit Ratio
Mean = 50.268179 %
VolCrash400 Strategy Testing Win Rate
Mean = 77.438292 %
VolCrash550 Strategy Testing Mean Return
Mean = 0.000134 %
Median = 0.000347 %
VolCrash550 Strategy Testing Hit Ratio
Mean = 50.281648 %
VolCrash550 Strategy Testing Win Rate
Mean = 78.179458 %
VolCrash750 Strategy Testing Mean Return
Mean = 0.000075 %
Median = 0.000406 %
VolCrash750 Strategy Testing Hit Ratio
Mean = 50.293887 %
VolCrash750 Strategy Testing Win Rate
Mean = 78.560983 %

Section 5

In here we check the correlations between each class of indices. The correlation is important for the concentration risk diversification.

Overall, it looks fine.

For Vol Crash

Tickly

	VolCrash400	VolCrash550	VolCrash750
VolCrash400	1.00000000	0.00007735	0.00050582
VolCrash550	0.00007735	1.00000000	-0.00075818
VolCrash750	0.00050582	-0.00075818	1.00000000

Hourly

	VolCrash400	VolCrash550	VolCrash750
VolCrash400	1.00000000	0.05087994	-0.04683095
VolCrash550	0.05087994	1.00000000	0.02340763
VolCrash750	-0.04683095	0.02340763	1.00000000

For Vol Boom

Tickly

	VolBoom400	VolBoom550	VolBoom750
VolBoom400	1.00000000	-0.00033746	-0.00047900
VolBoom550	-0.00033746	1.00000000	0.00113286
VolBoom750	-0.00047900	0.00113286	1.00000000

Hourly

	VolBoom400	VolBoom550	VolBoom750
VolBoom400	1.00000000	-0.03760513	0.04019477
VolBoom550	-0.03760513	1.00000000	-0.01815376
VolBoom750	0.04019477	-0.01815376	1.00000000

Appendix

Drift

Unlike Crash/Boom indices, the jumps/steps aren't segregated, i.e a return can have both a step and a jump. That being the case, we notice from the plots that the drift seems to do the job of the small steps in the original CB, counteracting the jumps, so the relationship is more implicit. This is because of the jump drift correction $\lambda \alpha dt$ overpowering the vol drift correction $\frac{\sigma^2}{2} dt$:

VolBoom

Here,

$$\alpha = \mathbb{E}[e^J] - 1 = \frac{1}{1+\eta_+} - 1 < 0$$

Therefore the correction $e^{-\lambda\alpha dt} < 1$, counteracting the positive jumps. It dominates the vol drift iff:

$$\begin{aligned} |\lambda\alpha| &> \frac{\sigma^2}{2} \\ \lambda\left|\frac{1}{1+\eta_+} - 1\right| &> \frac{\sigma^2}{2} \\ \lambda\left(1 - \frac{1}{1+\eta_+}\right) &> \frac{\sigma^2}{2} \\ &\vdots \\ \eta_+ &> \frac{2\lambda}{2\lambda - \sigma^2} - 1 \end{aligned}$$

So we need this condition to hold if we want the Vol-and-Boom to work as expected, with negative drift and positive jumps This last quantity on the RHS is very close to 0 if λ is large. In all offered indices, the RHS is about 10^{-7}, and η_+ is significantly bigger, so the index runs as expected.

Vol-and-Crash

Here,

$$\alpha = \mathbb{E}[e^J] - 1 = \frac{1}{1-\eta_-} - 1 > 0$$

Therefore the correction $e^{-\lambda\alpha dt} > 1$, counteracting the negative jumps. With a similar process as above, we require:

$$\eta_- > 1 - \frac{2\lambda}{2\lambda + \sigma^2}$$

The RHS is also small if λ is sufficiently large. The RHS is about 10^{-7}, as and as with the Vol-and-Boom, the offered indices work as expected.

R&D effort needs to be in line with Deriv’s vision and mission as formulated by our CEO. Therefore all R&D projects are carefully selected by our C-Level senior management represented by JY and Rakshit and resources for the projects are only allocated after review and shortlisting based on their vision and priorities.

In line with the standards and criterias set out by the CEO, the Model Validation team has validated the product/indices as documented in this report.