CMSC 474, Game Theory

9. Social Choice

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Social Choice

- Suppose you're voting in an election, and there are 4 candidates: *a*, *b*, *c, d*

- Suppose that out of 100 voters,
  - 8: first choice is *a*
  - 44: first choice is *b*
  - 46: first choice is *c*
  - 2: first choice is *d*

➢ Who should win?

Simple Plurality

- **Simple plurality** (First Past the Post):
  - Each voter votes for one candidate; highest number of votes wins

- Suppose that out of 100 voters,
  - 8: 1st choice *a*, 2nd choice *b*
  - 44: 1st choice *b*
  - 46: 1st choice *c*
  - 2: 1st choice *d*, 2nd choice *c*
- Each votes for 1st choice
  - *c* wins

- If *a* and *d* weren't available, the votes would be
  - *b*: 44+8 = 52
  - *c*: 46+2 = 48
  - *b* would win
- Who *should* win?
  - How to vote if you prefer *a*?
  - How to vote if you prefer *d*?

Simple Plurality

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- Suppose that out of 100 voters,
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  - 2: 1st choice *d*, 2nd choice *c*
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- If *a* and *d* weren't available, the votes would be
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- Who *should* win?
  - How to vote if you prefer *a*?
  - How to vote if you prefer *d*?

- "Spoiler" candidates in US presidential elections
  - 2000: Bush (Rep), Gore (Dem), Nader (Green)
  - 1980: Reagan (Rep), Carter (Dem), Anderson (Independent)
  - 1912: Taft (Rep), Roosevelt (Rep), Wilson (Dem)

Runoff Method

- Each voter votes for one candidate
- If no candidate has a majority
  - Hold an election between top two candidates

- 100 voters
  - 8: 1^{st} choice a, 2^{nd} choice b
  - 44: 1^{st} choice b
  - 46: 1^{st} choice c
  - 2: 1^{st} choice d, 2^{nd} choice c
- Each votes for 1^{st} choice
  - No majority
  - Top two choices: b and c

- Runoff election
  - b: 44+8 = 52
  - c: 46+2 = 48
  - b wins

Rank-Order Voting Systems

- Rather than voting for a single candidate, each voter specifies a total ordering of the candidates

- 100 voters

8: 1st choice a	8: $a > b > c > d$
44: 1st choice b	22: $b > a > c > d$
	22: $b > c > a > d$
46: 1st choice c	23: $c > b > d > a$
	23: $c > d > b > a$
2: 1st choice d	2: $d > c > b > a$

- Many voting methods that use this

Hare System

- Australia uses the **Hare system** (Instant Runoff Voting)
- Each voter specifies a total ordering of the candidates
    - 8: $a > b > c > d$
    - 22: $b > a > c > d$
    - 22: $b > c > a > d$
    - 23: $c > b > d > a$
    - 23: $c > d > b > a$
    - 2: $d > c > b > a$

- loop until one candidate has a majority:
    - remove the candidate with the smallest number of 1^{st}-choice votes
    - recount the votes using the remaining preferences

Hare System

Initial orderings:	After eliminating d:	After eliminating a:
8: $a > b > c > d$	8: $a > b > c$	8: $b > c$
22: $b > a > c > d$	22: $b > a > c$	22: $b > c$
22: $b > c > a > d$	22: $b > c > a$	22: $b > c$
23: $c > b > d > a$	23: $c > b > a$	22: $c > b$
23: $c > d > b > a$	23: $c > b > a$	22: $c > b$
2: $d > c > b > a$	2: $c > b > a$	2: $c > b$
d has 2 votes	a has 8 votes	b has 52 votes
a has 8 votes	b has 44 votes	c has 48 votes
b has 44 votes	c has 48 votes	
c has 46 votes		b has majority

Borda Count

- Translate each voter's preferences into utility values
  - ➢ If there are n alternatives,
    - 1st one gets n points,
    - 2nd one gets n–1 points,
    - …
    - nth one gets 1 point
- Compute total number of points for each alternative:
  - ➢ Highest number => winner

8: $a > b > c > d$
22: $b > a > c > d$
22: $b > c > a > d$
23: $c > b > d > a$
23: $c > d > b > a$
2: $d > c > b > a$

Points:

	4	3	2	1
8	a	b	c	d
22	b	a	c	d
22	b	c	a	d
23	c	b	d	a
23	c	d	b	a
2	d	c	b	a

Totals:

a 190 points
b 319 points
c 316 points
d 175 points

b wins

Condorcet Winner

- A **Condorcet winner** is a candidate w such that wins one-on-one comparisons to all other candidates
  - For every candidate $v \neq w$, a majority prefers w to v

8: $a > b > c > d$

22: $b > a > c > d$

22: $b > c > a > d$

23: $c > b > d > a$

23: $c > d > b > a$

2: $d > c > b > a$

- All pairs of candidates:
  - a vs. b: 8 prefer a, 92 prefer $\boldsymbol{b}$
  - a vs. c: 30 prefer $\boldsymbol{a}$, 70 prefer c
  - a vs. d: 52 prefer $\boldsymbol{a}$, 48 prefer d
  - b vs. c: 52 prefer $\boldsymbol{b}$, 48 prefer c
  - b vs. d: 75 prefer $\boldsymbol{b}$**,** 25 prefer c
- Condorcet winner: b
- At most one Condorcet winner
  - if x wins all one-on-one comparisons, every $y \neq x$ loses at least one of them
- Sometimes no Condorcet winner

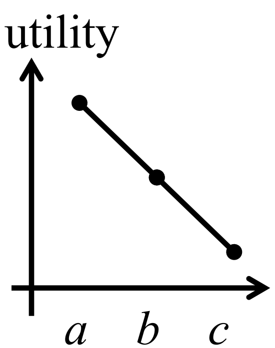
Condorcet's Paradox

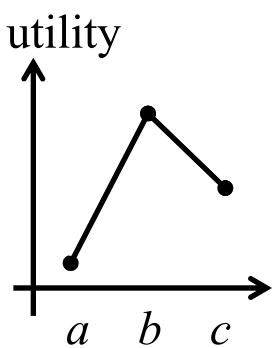
- Consider an election with three candidates: *a, b,* and *c*
    - 1/3 prefer $a > b > c$
    - 1/3 prefer $b > c > a$
    - 1/3 prefer $c > a > b$
  - ➢ How to choose a winner?
    - If *a* wins, 2/3 would have preferred *c* to *a*
    - If *b* wins, 2/3 would have preferred *a* to *b*
    - If *c* wins, 2/3 would have preferred *b* to *c*
- **Condorcet cycle**: for every candidate *x*, there's another candidate *y* such that a majority of the voters would prefer *y* to *x*

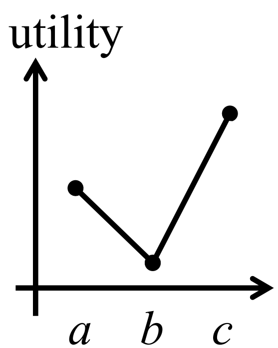
Peaks in Preferences

- Suppose the alternatives have a natural linear ordering $a < b < c$
  - cost, time, risk, return on investment, left-to-right politics, …

- 1/3 prefer $a > b > c$ 1/3 prefer $b > c > a$ 1/3 prefer $c > a > b$
  - ➢ Graph preferences as utilities:



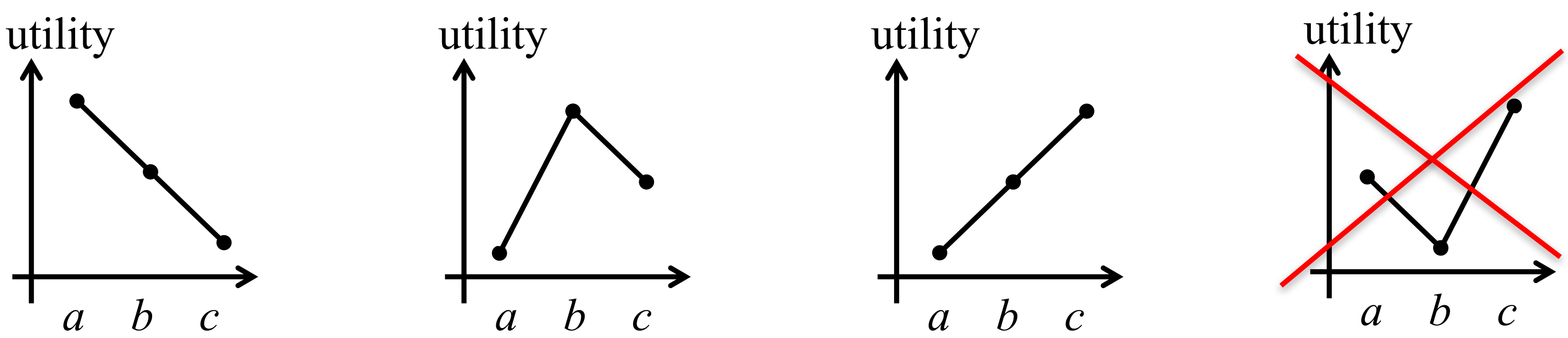




- The third set of preferences has two **peaks**
  - ➢ Rather than b, these voters prefer things on both sides of it
- Suggests something might be a little odd
  - ➢ Maybe some other ordering matters more?

Single-Peaked Preferences

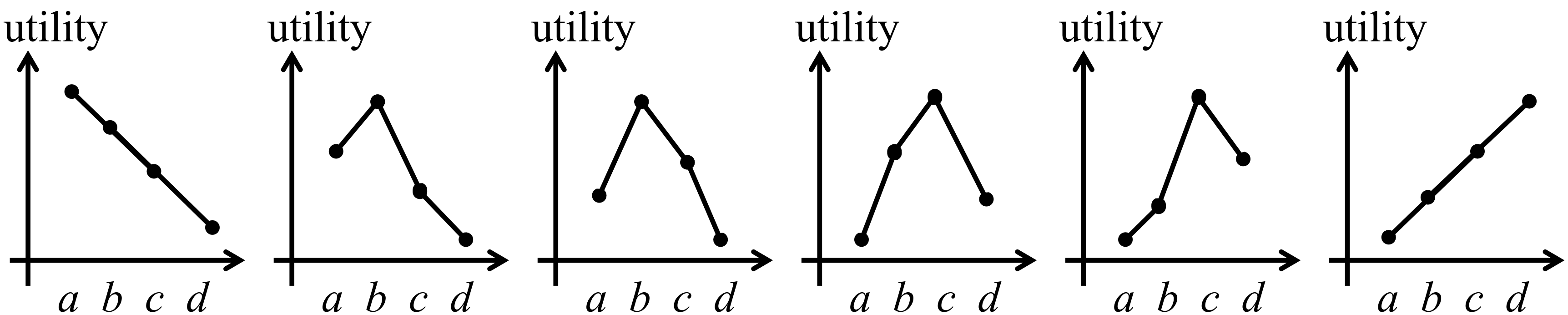
- The voters' preferences are **single-peaked** if no voter ranks a choice lower than its two nearest neighbors



- If there exists a linear ordering of the alternatives such that the voters' preferences are single-peaked, then there's no Condorcet cycle
- If there's no Condorcet cycle
  - Then there's a Condorcet winner (unless two candidates are tied with each other)

Condorcet Winner

8 prefer	22 prefer	22 prefer	23 prefer	23 prefer	2 prefer
$a>b>c>d$	$b>a>c>d$	$b>c>a>d$	$c>b>d>a$	$c>d>b>a$	$d>c>b>a$



- Our previous example (Condorcet winner *b*) is single-peaked

Condorcet Methods

- A **Condorcet method** is any voting method that chooses the Condorcet winner if it exists
- Examples
  - **Black method**
    - Choose Condorcet winner if it exists
    - Otherwise use Borda count
  - **Copeland method**
    - Choose candidate with highest score, where

      score = # of pairwise victories – # of pairwise defeats
- Several others

Discussion

- We discussed
  - Plurality vote
  - Runoff method
  - Hare system
  - Borda count
  - Condorcet methods
- In the example, plurality chose *c*; all of the others chose *b*
- There are cases where all five would choose different winners
  - http://www.eprisner.de/MAT107/Voting/Voting1.html

What's Fair?

- How do we decide whether a voting method's choices are fair?
- Arrow's criteria
  - **Unanimity**. If every voter prefers alternative X over alternative Y, then the method prefers X over Y
  - **Independence of Irrelevant Alternatives**. If some of the voters' preferences change but their preferences between X and Y remain unchanged, then the method's preference between X and Y will remain unchanged
  - **No dictators**. No single voter will always determine the method's preference

- 30: $b > a > c > d$
- 22: $b > c > a > d$
- 23: $c > b > d > a$
- 23: $c > d > b > a$
- 2: $d > c > b > a$
  - prefer $b > a$

- 30: $b > a > c > d$
- 22: $b > c > a > d$
- 23: $c > b > d > a$
- 25: $c > d > b > a$
  - still prefer $b > a$

Social Choice

- Arrow's criteria
  - **Unanimity**. If every voter prefers alternative X over alternative Y, then the method prefers X over Y
  - **Independence of Irrelevant Alternatives**. If some of the voters' preferences change but their preferences between X and Y remain unchanged, then the method's preference between X and Y will remain unchanged
  - **No dictators**. No single voter will always determine the method's preference

- **Arrow's Impossibility Theorem**: If there are more than 2 candidates, no rank-order voting system satisfies all three of the criteria.
  - For details (including a proof), download Arrows_Theorem.zip from the General Resources section of the Resources page on Piazza

Tactical Voting

- Change the outcome toward something that one prefers, by voting differently from one's true preferences
- e.g., Borda count: the voters who prefer $c > b$ can make c win

True preferences:

	4	3	2	1
8	*a*	*b*	*c*	*d*
22	*b*	*a*	*c*	*d*
22	*b*	*c*	*a*	*d*
23	*c*	*b*	*d*	*a*
23	*c*	*d*	*b*	*a*
2	*d*	*c*	*b*	*a*

a 190 points
b 342 points → *b* wins
c 293 points
d 175 points

Tactical voting:

	4	3	2	1
8	*a*	*b*	*c*	*d*
22	*b*	*a*	*c*	*d*
22	*b*	*c*	*a*	*d*
23	*c*	*d*	*a*	*b*
23	*c*	*d*	*a*	*b*
2	*c*	*d*	*a*	*b*

a 238 points
b 248 points
c 318 points → *c* wins
d 196 points

Tactical Voting

- Change the outcome toward something that one prefers, by voting differently from one's true preferences

- **Gibbard–Satterthwaite theorem**: if there are more than 2 candidates, every rank-order voting system has one of the following properties:
  - The system is dictatorial
    - there is a single individual who can choose the winner
  - There's is some candidate who can never win, regardless of voters' preferences
  - The rule is susceptible to tactical voting

Score Voting

- Also called **range voting**
- Not a rank-order method
  - each voter assigns a numeric score to each candidate
  - highest average score wins
- Satisfies all three of Arrow's criteria; Arrow's impossibility theorem doesn't apply
- Drawbacks
  - Doesn't satisfy Condorcet's criterion; majority may prefer a candidate other than the winner
  - Practical complication
  - Tactical voting has a bigger effect, e.g.,
    - true preferences: $c > b > d > a$
    - tactical (rank-order voting): $c > d > a > b$
    - tactical (range voting): 100 for c, 50 for d, 0 for a, 0 for b