CMSC 474, Game Theory

3b. More about Normal-Form Games

Dana Nau

University of Maryland

2nd part of Chapter 3,
plus related topics

What we covered last time

- Minimax regret
- Dominant strategies
    - Prisoner's Dilemma, Which Side of the Road, Matching Pennies
    - Comparison to the Chocolate Dilemma survey
    - Iterated elimination of strictly dominated strategies (IESDS)
    - Iterated elimination of weakly dominated strategies (IEWDS)
- *p*-beauty contest
    - Some reasons for not choosing a Nash equilibrium

Outline

- Rationalizability
- Common knowledge
- Correlated equilibrium
- Trembling-hand perfect equilibrium
- ε-Nash equilibrium
- Evolutionarily stable strategies

- After class, I'll post Homework 3
  - Due 1 week from today

Rationalizability

- A strategy is **rationalizable** if a *perfectly rational agent* could justifiably play it against *perfectly rational opponents*
  - The formal definition is complicated
- Informally:
  - A strategy for agent *i* is rationalizable if it's a best response to strategies that *i* could *reasonably* believe the other agents have
  - To be reasonable, *i*'s beliefs must take into account *i*'s knowledge of
    - the other agents' knowledge of *i*'s rationality,
    - their knowledge of *i*'s knowledge of *their* rationality,
    - their knowledge of *i*'s knowledge of their knowledge of *i*'s rationality,
    - and so on *ad infinitum*
- **Rationalizable strategy profile**: consists only of rationalizable strategies

Rationalizability

	Left	Right
Left	1, 1	0, 0
Right	0, 0	1, 1

- Every Nash equilibrium is composed of rationalizable strategies

Example: Which Side of the Road

- For Agent 1, the pure strategy s_1 = *Left* is rationalizable because
  - s_1 = *Left* is 1's best response if 2 uses s_2 = *Left*,
  - 1 can reasonably believe 2 would rationally use s_2 = *Left*, because
    - s_2 = *Left* is 2's best response if 1 uses s_1 = *Left*,
    - 1 can believe that 2 can reasonably believe 1 would rationally use s_1 = *Left*, because
      - s_1 = *Left* is 1's best response if 2 uses s_2 = *Left*,
      - 1 can reasonably believe 2 would rationally use s_2 = *Left*, because
        - … and so on, *ad infinitum* …

Rationalizability

	Heads	Tails
Heads	1, –1	–1, 1
Tails	–1, 1	1, –1

- Some rationalizable strategies are not part of any Nash equilibrium

Example: Matching Pennies

- For Agent 1, the pure strategy *Heads* is rationalizable because
  - *Heads* is 1's best response if 2 uses *Heads*,
  - and 1 can reasonably believe 2 would rationally use *Heads*, because
    - *Heads* is 2's best response if 1 uses *Tails*,
    - and 2 can reasonably believe 1 would rationally use *Tails*, because
      - *Tails* is 1's best response if 2 uses *Tails*,
      - and 1 can reasonably believe 2 would rationally use *Tails*, because
        - … and so on, *ad infinitum* …

Strategies that Aren't Rationalizable

Prisoner's Dilemma

- Strategy *C* isn't rationalizable for agent 1
- Not a best response to any of agent 2's strategies
  - *D* will always give 1 a bigger payoff

	C	*D*
C	3, 3	0, 5
D	5, 0	1, 1

The 3x3 game we used earlier

- *M* is not a rationalizable strategy for agent 1
- It's a best response if 2's strategy is *R*
- But 1 cannot reasonably believe that 2 would rationally play *R*
  - *R* isn't 2's best response to any of agent 1's strategies
  - *L* and *C* will always give 2 a bigger payoff

	L	*C*	*R*
U	3, 1	0, 1	0, 0
M	1, 1	1, 1	5, 0
D	0, 1	4, 1	0, 0

Strategies that Aren't Rationalizable

Prisoner's Dilemma

- Strategy *C* isn't rationalizable for agent 1
- Not a best response to any of agent 2's strategies
  - *D* will always give 1 a bigger payoff

	C	*D*
C	3, 3	0, 5
D	5, 0	1, 1

The 3x3 game we used earlier

- *M* is not a rationalizable strategy for agent 1
- It's a best response if 2's strategy is *R*
- But 1 cannot reasonably believe that 2 would rationally play *R*
  - *R* isn't 2's best response to any of agent 1's strategies
  - *L* and *C* will always give 2 a bigger payoff

	L	*C*	*R*
U	3, 1	0, 1	0, 0
M	1, 1	1, 1	5, 0
D	0, 1	4, 1	0, 0

Rationalizability

- The formal definition of rationalizability is complicated because of the infinite regress
  - But we can say some intuitive things about rationalizable strategies
- Nash equilibrium strategies are always rationalizable
  - So the set of rationalizable strategies (and strategy profiles) is always nonempty
- A strategy that's strictly dominated will never be a best response to any strategy profile of the other agents
  - So if a strategy doesn't survive IESDS, it isn't rationalizable
- https://www.youtube.com/watch?v=ED9gaAb2BEw

Rationalizability

- In two-player games:
  - Perform IESDS
  - The rationalizable strategies are the ones that remain afterwards

- In *n*-player games:
  - Iteratively remove all strategies that are never a best response to any strategy profile by the other agents
    - this removes strictly dominated strategies
    - it may also remove some others
  - The rationalizable strategies are the ones that remain afterwards

Common Knowledge

- Rationalizability is closely related to the idea of ***common knowledge***
- Important for Nash equilibria
  - Can't expect agents to converge on a Nash equilibrium unless they all have rational preferences and their rationality is common knowledge
- The book mentions common knowledge in at least 5 different places, but doesn't define what it means

- Definition is analogous to the definition of rationalizability
- Among a group of individuals, a property *p* is *common knowledge* if
  - They all know *p*
  - They all know that they all know *p*
  - They all know that they all know that they all know *p*
  - …

Example of Common Knowledge

- On an island, there are k people who have blue eyes, and the rest of the people have green eyes. There is at least one blue-eyed person on the island ($k \geq 1$).
- If a person ever knows they have blue eyes, they must leave the island at dawn the next day. Each person can see every other person's eye color, but there are no mirrors, and there is no discussion of eye color.
  - So nobody knows their own eye color.
- At some point, an outsider comes to the island, calls all the people together, and makes the following public announcement: "at least one of you has blue eyes". This makes the fact common knowledge:
  - they all know it, they all know that they all know it, and so on.
- Everyone on the island (including the outsider) is truthful, and can do arbitrarily complex logical reasoning. This also is common knowledge.

- **Poll 3.4:** What will happen?

Example of Common Knowledge

- http://en.wikipedia.org/wiki/Common_knowledge_(logic)#Example

- On the *k*th dawn after the announcement, all the blue-eyed people will leave the island.
- Case $k = 1$. Let's call the blue-eyed person A.
  - Before the announcement, A didn't know anyone had blue eyes.
  - After the announcement, A knows at least one person has blue eyes.
  - A can see that it isn't anyone else, so it must be A.
  - So A leaves at the 1st dawn.

Example of Common Knowledge

- Case $k = 2$. Let's call the blue-eyed people *A* and *B*.
  - Before the announcement, *B* knows the island has at least one blue-eyed person, but doesn't know **whether they all know**.
    - If *B* didn't have blue eyes then *A* wouldn't see anyone with blue eyes, and wouldn't know whether anyone on the island is blue-eyed.
  - After the announcement, *B* knows that they all know, **including *A***.
  - *B* knows that **if** *B* doesn't have blue eyes, then *A* will know *A*'s eyes must be blue, so *A* will leave at the 1st dawn.
  - When that doesn't happen, *B* infers that *A* can't be the only blue-eyed person.
  - *B* sees no blue-eyed people other than *A*, so *B* infers that the other blue-eyed person is *B*.
  - Thus *B* leaves at the 2nd dawn.
  - Using similar reasoning, *A* also leaves at the 2nd dawn.

Example of Common Knowledge

- Case k=3. Let's call the blue-eyed people *A*, *B*, and *C*.
  - Before the announcement, *C* can infer that they all know there are blue-eyed people.
    - Even if *C* didn't have blue eyes, *A* and *B* would each see one other blue-eyed person.
  - But *C* doesn't know **whether they all know that they all know**.
    - For example, if *C* didn't have blue eyes, then (see case k=2 on the previous page) it would be possible for *B* to think that *A* doesn't know whether anyone has blue eyes.
  - After the announcement, *C* knows that if *A* and *B* are the **only** blue-eyed people then they'll leave at the 2nd dawn.
  - They don't do so, which tells *C* there must be a 3rd blue-eyed person. *C* sees only 2 blue-eyed people, and infers that *C* is the 3rd one.
  - Thus *C* leaves at the 3rd dawn.
  - Using similar reasoning, *A* and *B* also leave at the 3rd dawn.

Example of Common Knowledge

- For $k > 1$, the outsider is only telling the people what they already know: that there are blue-eyed people among them.
- But before the announcement, that isn't common knowledge.

  Either they don't all know that they all know,

  - or they don't all know that they all know that they all know,
  - or they don't all know that they all know that they all know that they all know, …

  - ➢ How many levels deep depends on how big k is
- Once it becomes common knowledge, each blue-eyed person has enough information to infer that if there were only k–1 blue-eyed people, they all would leave by the k–1$^{\text{th}}$ dawn.
- When this doesn't happen, each blue-eyed person knows there must be k blue-eyed people—and seeing only k–1 of them, realizes he/she must be the k^{th} one.

More about Common Knowledge

- Role of common knowledge in language and communication
  - Steven Pinker
  - http://www.youtube.com/watch?v=3-son3EJTrU

- Pinker uses the term “mutual knowledge” to mean what we call common knowledge
  - Game theorists use “mutual knowledge” for something else

Correlated Equilibrium (motivation)

Battle of the Sexes

- Pure-strategy equilibria (A,A) and (B,B)
  - Payoff profiles (2,1) and (1,2), “unfair”
- Mixed-strategy equilibrium (s_1, s_2)
  - $s_1 = \{(2/3, A), (1/3, B)\}$
  - $s_2 = \{(1/3, A), (2/3, B)\}$
  - Payoff profile (2/3, 2/3)
  - Fair, but Pareto dominated by (A,A) and (B,B)
- Flip a coin
  - Heads → both choose A; Tails → both choose B
    - Neither pure nor mixed
  - Payoff profile (1.5, 1.5)
  - Fair, Pareto optimal

$2/3 \times 1/3 = 2/9$ $2/3 \times 2/3 = 4/9$

	A	*B*
A	2, 1	0, 0
B	0, 0	1, 2

$1/3 \times 1/3 = 1/9$ $1/3 \times 2/3 = 2/9$

Correlated Equilibrium

- Let G be an n-player game
  - Random variables $\mathbf{v} = (v_1, \ldots, v_n)$ with domains $\mathbf{D} = (D_1, \ldots, D_n)$
  - Joint distribution $\pi(\mathbf{d}) = \pi(d_1, \ldots, d_n) = \Pr[v_1{=}d_1, \ldots, v_n{=}d_n]$
- Each agent i observes a **private signal**: the value of v_i
  - Each agent's strategy is a deterministic* mapping $\sigma_i : D_i \to A_i$
- Given a strategy profile $\boldsymbol{\sigma} = (\sigma_1, \ldots, \sigma_n)$, agent i's expected utility is

$$u_i(\boldsymbol{\sigma}) = \sum_{\mathbf{d}} \pi(\mathbf{d})\, u_i(\boldsymbol{\sigma}(\mathbf{d}))$$

$$u_i(\sigma_1, \ldots, \sigma_n) = \sum_{d_1,\ldots,d_n} \pi(d_1, \ldots, d_n)\, u_i(\sigma_1(d_1), \ldots, \sigma_n(d_n))$$

- $(\mathbf{v}, \pi, \boldsymbol{\sigma})$ is a **correlated equilibrium** if for every agent i and strategy σ_i',

$$u_i(\boldsymbol{\sigma}) \geq u_i(\sigma_i', \boldsymbol{\sigma}_{-i})$$

i.e., $u_i(\sigma_1, \ldots, \sigma_n) \geq u_i(\sigma_1, \ldots, \sigma_{i-1}, \sigma_i', \sigma_{i+1}, \ldots, \sigma_n)$

- The book says mixed strategies wouldn't give any greater generality. Why?

Example: Battle of the Sexes

- Model the coin-flip as two random variables
  - $\mathbf{v} = (v_1, v_2)$
  - v_1 = signal to player 1
  - v_2 = signal to player 2
  - Domains $D_1 = D_2 = \{\text{H, T}\}$
  - Joint distribution: both players always see the same result
    - π(Heads, Heads) = π(Tails, Tails) = ½
    - π(Heads, Tails) = π(Tails, Heads) = 0
  - Strategy profile $\boldsymbol{\sigma} = (\sigma_1, \sigma_2)$
    - σ_1(Heads) = σ_2(Heads) = A
    - σ_1(Tails) = σ_2(Tails) = B
- $(\mathbf{v}, \pi, \boldsymbol{\sigma})$ is a correlated equilibrium
  - What happens if either player switches to a different strategy?

	A	B
A	2, 1	0, 0
B	0, 0	1, 2

Correlated Equilibrium

Theorem. For every Nash equilibrium $\mathbf{s} = (s_1, \ldots, s_n)$, there's a corresponding correlated equilibrium $\boldsymbol{\sigma} = (\sigma_1, \ldots, \sigma_n)$

- "Corresponding" means they produce the same distribution on outcomes

Basic idea of the proof: for each s_i, set up v_i and σ_i to mimic s_i

- signals $v_1, \ldots, v_n$ independently distributed
- Each v_i has domain A_i and probability distribution s_i
- $\sigma_i(a) = a$ for every a in A_i (i.e., if i sees signal a then i chooses action a)
- When the agents play the strategy profile $\boldsymbol{\sigma}$, the distribution over outcomes is identical to that under $\mathbf{s}$
- No agent i can benefit by deviating from σ_i, so $\boldsymbol{\sigma}$ is a correlated equilibrium

---

- There also are correlated equilibria that aren't equivalent to Nash equilibria
  - e.g., Battle of the Sexes

Trembling-Hand Perfect Equilibrium

- Similar to a Nash equilibrium, but robust against slight errors or "trembles"
  - Small perturbations of the agents' strategies

- Let $\mathbf{s} = (s_1, \ldots, s_n)$ be a mixed strategy profile for a game G
  - $\mathbf{s}$ is a **trembling-hand perfect equilibrium**
    if there exists a sequence of fully mixed strategy profiles $\mathbf{s}^0, \mathbf{s}^1, \ldots,$ with the following properties:
    - $\lim_{k\to\infty} \mathbf{s}^k = \mathbf{s}$
    - for every $\mathbf{s}^k$, every strategy s_i in $\mathbf{s}$ is a best response to $\mathbf{s}_{-i}^k$
- If you play your Nash equilibrium strategy s_i and the other players play $\mathbf{s}_{-i}^k$ instead of $\mathbf{s}_{-i}$, you still are doing the best that you can do
- The topic is complicated, and I won't say any more about it

every action has nonzero probability

ε-Nash Equilibrium

- Idea: an agent might gain by changing strategy, but the gain would be very small
- Let $\varepsilon > 0$. A strategy profile $\mathbf{s} = (s_1, \ldots, s_n)$ is an **ε-Nash equilibrium** if for every agent i and for every strategy $s_i' \neq s_i$,

  $$u_i(s_i, \mathbf{s}_{-i}) \geq u_i(s_i', \mathbf{s}_{-i}) - \varepsilon$$

- ε-Nash equilibria exist for every $\varepsilon > 0$
  - Every Nash equilibrium is an ε-Nash equilibrium,
  - Every Nash equilibrium is surrounded by a region of ε-Nash equilibria
- Computationally useful
  - Iterative improvement algorithms can stop when they get close
  - Finite-precision computers generally can't get closer than $\varepsilon \approx$ the machine precision
- But conceptually messy

Problems with ε-Nash Equilibrium

- For every Nash equilibrium, there are ε-Nash equilibria that approximate it
- Converse isn't true
  - There are ε-Nash equilibria that aren't close to any Nash equilibrium
- Example: the game at right
  - Just one Nash equilibrium: (*D, R*)
  - Use IESDS to show it's the only one:
    - For agent 1, *D* dominates *U*, so remove *U*
    - Then for agent 2, *R* dominates *L*

	L	*R*
U	1, 1	0, 0
D	$1 + \varepsilon/2$, 1	500, 500

- (*D, R*) is an ε-Nash equilibrium
- There's another ε-Nash equilibrium: (*U, L*)
  - Neither agent can gain more than ε by deviating
  - But its payoffs aren't within ε of the Nash equilibrium

Problems with ε-Nash Equilibrium

- Some ε-Nash equilibria are very unlikely to arise
- Same example as before
  - Agent 1 might not care about a gain of ε/2, but might reason as follows:
    - *D* dominates *U*, so agent 2 may expect agent 1 to to play *D*
    - *R* is agent 2's best response to *D*
      - So agent 2 is likely to play *R*
    - If agent 2 plays *R*, agent 1 does *much* better by playing *D* rather than *U*
      - So choose *D*

	L	*R*
U	1, 1	0, 0
D	1 + ε/2, 1	500, 500

- In general, ε-approximation is much messier in games than in optimization problems

Evolutionary Stability

- Concept from evolutionary biology
- Population of various species (kinds of organisms)
  - Consider how their relative "fitness" causes their proportions of the population to grow or shrink
- Pure strategy ⇔ a particular species
  - Everything that might affect evolutionary fitness of this kind of organism
    - size, aggressiveness, sensory abilities, intelligence, …
- Mixed strategy = relative proportions of each species
- **Fitness** of a species = **expected payoff** from interacting with a random member of the population
  - High payoff ⇒ more likely to reproduce ⇒ proportion is likely to grow
  - Low payoff ⇒ less likely to reproduce ⇒ proportion is likely to shrink

Evolutionary Stability against Another Strategy

- Two species, *X* and *Y*
- Payoff matrix: payoffs when interacting each other
    - Symmetric 2-player game

	X	Y
X	a, a	b, c
Y	c, b	d, d

	X	Y
X	a	b
Y	c	d

Body-Size Game

- Source: http://www.cs.cornell.edu/home/kleinber/networks-book

- Two different sizes of beetles competing for food
    - Same size ⇒ equal shares
    - large vs small ⇒ large gets most of the food
- Large beetles get less fitness benefit from any given amount of food
    - Overhead of maintaining expensive metabolism

	small	*large*
small	5	1
large	8	3

Evolutionary Stability against Another Strategy

- Y **invades** X at level δ:
  - Infinite (or very large) population, all use strategy X
  - Change some fraction δ to strategy Y
  - $(1–\delta)$ = the fraction that uses X

	X	Y
X	a	b
Y	c	d

- Each species' fitness is its expected payoff:
  - $f_X = (1–\delta)a + \delta b$
  - $f_Y = (1–\delta)c + \delta d$
  - If $f_Y > f_X$, X's proportion will shrink and Y's will grow
  - If $f_X > f_Y$, X's proportion will grow and Y's will shrink

- X is **evolutionarily stable against** Y if $\exists\ \varepsilon > 0$ such that X will repel any invasion of Y at a level $< \varepsilon$
  - i.e., for all $\delta < \varepsilon$, $f_X > f_Y$

$$(1–\delta)a + \delta b > (1–\delta)c + \delta d$$

Evolutionary Stability against Another Strategy

- X is evolutionarily stable against Y if $\exists\, \varepsilon > 0$ such that for all $\delta < \varepsilon$, $f_X > f_Y$

  Case $a \neq c$: As $\delta \to 0$,

  $$f_X = (1-\delta)a + \delta b \to a$$

  $$f_Y = (1-\delta)c + \delta d \to c$$

	X	Y
X	a	b
Y	c	d

  There is an $\varepsilon > 0$ such that for all $\delta < \varepsilon$, $f_X > f_Y \Leftrightarrow a > c$

  - *i.e.*, X does better against X than Y does against X

  Case $a = c$:

  $$(1-\delta)a + \delta b > (1-\delta)b + \delta d \Leftrightarrow b > d$$

  - *i.e.*, X does better against Y than Y does against Y

- X is evolutionarily stable against Y iff either

  1. $a > c$, or
  2. $a = c$ and $b > d$

Evolutionary Stability against Another Strategy

- X is evolutionarily stable against Y if either
  1. $a > c$, or
  2. $a = c$ and $b > d$

	X	Y
X	a	b
Y	c	d

- **Body-size game**: two different sizes of beetles competing for food
  - Same size ⇒ equal shares
  - Large vs small ⇒ large gets most of the food
  - Larger beetles have more expensive metabolism

	small	*large*
small	5	1
large	8	3

  - 5 < 8, so *small* isn't evolutionarily stable against *large*

	large	*small*
large	3	8
small	1	5

- **Poll 3.5**: is *large* evolutionarily stable against *small*?

Evolutionarily Stable Strategies

- Mixed population ⇔ mixed strategy $s = \{(p,X), (1-p,Y)\}$
- s is an **evolutionarily stable strategy (ESS)** iff for *every* strategy $t \neq s$, either
  1. $u(s,s) > u(t,s)$, or
  2. $u(s,s) = u(t,s)$ and $u(s,t) > u(t,t)$
  - If $t = \{(q,X), (1-q,Y)\}$ then
    - $u(s,s) = p^2a + p(1-p)b + (1-p)pc + (1-p)^2d$
    - $u(s,t) = pqa + p(1-q)b + (1-p)qc + (1-p)(1-q)d$
    - $u(t,s) = qpa + q(1-p)b + (1-q)pc + (1-q)(1-p)d$
    - $u(t,t) = q^2a + q(1-q)b + (1-q)qc + (1-q)^2d$

for each action profile (x,y), $\Pr(x,y)u(x,y)$

	X	Y
X	a	b
Y	c	d

	s	t
s	$u(s,s)$	$u(s,t)$
t	$u(t,s)$	$u(t,t)$

- s is a **weak evolutionarily stable strategy** iff for every strategy $t \neq s$, either
  1. $u(s,s) > u(t,s)$, or
  2. $u(s,s) = u(t,s)$ and $u(s,t) \geq u(t,t)$
  - Includes cases where s and t have the same expected utility
    - In a mixture of s and t, neither will grow nor shrink

ESSs and Nash Equilibria

- s is an **evolutionarily stable strategy (ESS)** iff for *every* strategy $t \neq s$, either
  1. $u(s,s) > u(t,s)$, or
  2. $u(s,s) = u(t,s)$ and $u(s,t) > u(t,t)$

- Let G be a symmetric game

Theorem. If s is an ESS of G, then (s,s) is a Nash equilibrium of G.

Proof. Suppose s is an ESS. Then $u(t,s) \leq u(s,s)$ for every strategy $t \neq s$.
Thus s is a best response to s.
Thus in (s,s), each strategy is a best response to the other one.

Theorem. If (s,s) is a *strict* Nash equilibrium of G, then s is an ESS of G.

Proof. Suppose (s,s) is a strict Nash equilibrium. Then s is the *only* best response s, i.e., for every $t \neq s$, $u(t,s) < u(s,s)$. Thus s satisfies condition (1) above.

	large	*small*
large	3	8
small	1	5

Hawk-Dove Game

- 2 animals contend for a piece of food
- Each animal may be either a hawk (H) or a dove (D)
  - Prize is worth 6 to each
  - Fighting has a cost of 5
- Hawk meets dove ⇒ hawk gets prize without a fight
  - Hawk gets 6, dove gets 0
- 2 doves meet ⇒ split the prize without a fight
  - Each dove gets 3
- 2 hawks meet ⇒ they fight, each has probability 0.5 of getting the prize
  - Hawk's expected payoff is –5 + 0.5(6) = –2

	H	D
H	–2	6
D	0	3

- Unique Nash equilibrium (s,s), where $s = \{(0.6, \text{H}), (0.4, \text{D})\}$
  - i.e., 60% hawks, 40% doves
- Is s an ESS?

Digression

- Let G be a symmetric game with two pure strategies X, Y
- Let s be a *fully mixed* strategy for G
  - i.e., X and Y both have nonzero probability
- Suppose (s,s) is a Nash equilibrium

	X	Y
X	a	b
Y	c	d

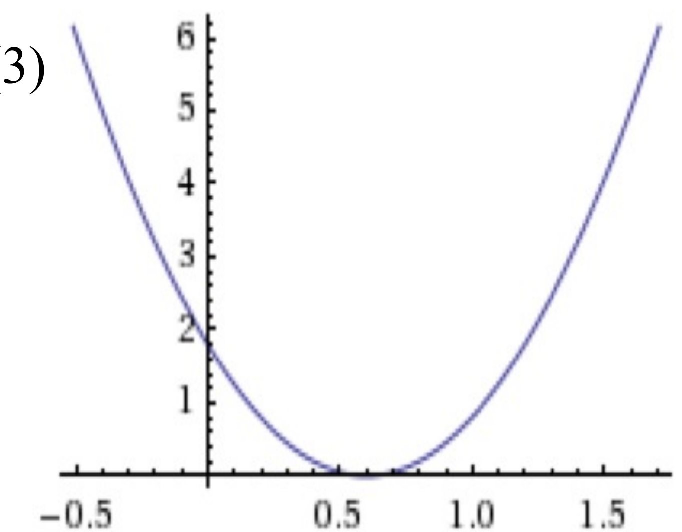
- **Poll 3.6:** Let t be any other strategy for G.
  What can we say about the relation between $u(s,s)$ and $u(t,s)$?

Hawk-Dove Game

	H	D
H	–2	6
D	0	3

	s	t
s	$u(s,s)$	$u(s,t)$
t	$u(t,s)$	$u(t,t)$

- Nash equilibrium (s, s), where $s = \{(0.6, \text{H}), (0.4, \text{D})\}$
- s is an ESS iff for every $t = \{(q,\text{H}), (1-q,\text{D})\}$, if $t \neq s$ then
    1. $u(s,s) > u(t,s)$, or
    2. $u(s,s) = u(t,s)$ and $u(s,t) > u(t,t)$
- Since s is fully mixed, $u(s,s) = u(t,s)$
- Next, show $u(s,t) > u(t,t)$:
    - $u(s,t) = 0.6q(-2) + 0.6(1-q)6 + 0.4q(0) + 0.4(1-q)(3)$
      $= 4.8 - 6q$
    - $u(t,t) = q^2(-2) + q(1-q)(6) + (1-q)q(0) + (1-q)^2(3)$
      $= 3 - 5q^2$
- Let $v = u(s,t) - u(t,t) = 5q^2 - 6q + 1.8$
    - Unique minimum: $v = 0$ when $q = 0.6$, i.e., $t = s$
    - If $t \neq s$ then $q \neq 0.6$, so $v > 0$
        - i.e., $u(s,t) > u(t,t)$



Summary

- Rationalizability
  - relation to IESDS
- Common knowledge
  - “island example”
  - role in language and communication
- Correlated equilibrium (e.g., Battle of the Sexes)
- Trembling-hand perfect equilibria
- Epsilon-Nash equilibria
- Evolutionary stability of one strategy against another
  - Body-Size game
- Evolutionarily stable strategy
  - Hawk-Dove game
  - Relation to Nash equilibria