Operations Analysis

Question:

In this exercise you will have to solve the following problem:
Given a budget of *b* dollars, find locations of ambulance stations that minimize the maximum response time to any location on a given map. Input to this problem will be a table, *r(a,b)*, with a response time of an ambulance station in location *a* to an emergency in location *b*, a vector *c(a)* with the cost of opening an ambulance station in location *a*, and a budget *b*.

Solution:

We will solve the problem into four parts named as assignment 1, assignment 2 and so on. The first step to solve the problem is to describe the problem, formulate the objective function and constraints along with the introduction of variables. The next step is to reformulate the problem so we can solve it using MATLAB. In the third step we will work with the most interesting part which is to write the matrices in the MATLAB program. The last step is to solve a small example.

Given Variables:

N: Size of the city

M: Number of ambulance stations

c: Cost vector

c(a): The cost of opening an ambulance station at location *a*

r: Distance matrix

r (a, b): The response time for an ambulance station located at *a* to respond to an emergency at location *b*

b: Budget in dollars

x: A binary vector indicating which stations to open

T: Maximum response time

Assignment 1:

In this assignment we will write the model based on the given information. We will also introduce new variables to build the objective function and we will write some limitations related to the objective function. To formulate the problem, we have introduced two new variables x_a and y_{ab} and let them

$$x_a = \begin{cases} 1 & if\ a\ is\ \ open \\ 0 & if\ a\ is\ \ close \end{cases}$$

$$y_{ab} = \begin{cases} 1 & if\ ambulance\ at\ location\ a\ serves\ location\ b \\ 0 & otherwise \end{cases}$$

Objective function: Is to minimize the maximum response time of ambulance to any location given on the map.

$$\min T \qquad (1) \text{ where } T = \max r(a,b)y_{ab}$$

$$\text{Subject to } T \geq r(a,b)y_{ab} \qquad (2)$$

$$\sum_{a=1}^{M} c(a)x_a \leq b \qquad (3)$$

$$\sum_{a=1}^{M} y_{ab} = 1 \qquad \forall\, b \qquad (4)$$

$$y_{\text{ab}} \leq x_a \qquad (5)$$

The Objective Function

We want to minimize the maximum response time to any location given on the map. For this we have introduced a new variable T. The new variable T is equal to the maximum response for an ambulance station located at a to respond to an emergency at location b times the ambulance station that serves this location. In this problem we want to reduce the response time in which an ambulance reaches the emergency location. We want the time between the ambulance station location and the emergency location to be minimum so, the ambulance can reach the emergency location as fast as possible.

Constraints

The constraints are our limitations and for the above objective function we have four constraints that are given above in the equation (2), (3), (4) and (5).

In our first constraint we have T greater than and equal to $r(a,b)y_{ab}$ or (the response time for an ambulance station located at a to respond to an emergency at location b multiply by the ambulance station located at location a can serve emergency at location b).So, the maximum of $r(a,b)y_{ab}$is obviously greater than or equal to $r(a,b)y_{ab}$ which is our first constraint from the objective function.

In the equation (3) we have the summation of $c(a)$ i.e. (cost of opening an ambulance station at location a) times x_a i.e. (the station is open or close) less than and equal to b (budget). Here we have a budget constraint which means the cost of opening the ambulance station should be in our budget. We cannot exceed from our budget.

In the equation (4) we have the summation of y_{ab}i.e. that is the ambulance station located at location *a* can serve emergency at location *b* equals to 1 for all the *b* i.e. the emergency location *b*. Here we have a limitation that one ambulance station can serves one location. Given there are many ambulance stations but only one station should serve one location that why it is equal to one.

In the last constraint we have y_{ab} (that is the ambulance station located at location a can serve emergency at location b) less than and equal to the x_a (that indicates the station is open or close). We let x_a equal to 1 if the station is open and 0 if the station is close. We also let y_{ab} equals to 1 if ambulance station at location a serves location b and for y_{ab} equals to zero it means the ambulance station at location a does not serve at location b. The meaning of this constraint is that if x_a is close it will not serve y_{ab} and if x_a is open it means it is one and it can serve to y_{ab} but it is also possible that x_a is open and it does not serve y_{ab}. So, y_{ab} can only be one if x_a is one but y_{ab} can be zero if x_a is one.

Assignment 2:

In this assignment we need to reformulate the problem so we can solve it using MATLAB. For this we will move all the variables from the right-hand side to the left-hand side. We will start with the first constraint and move the variable T to the other side. When we will move the variable the sign of variable will change and after that we will write it in matrix form.

$$T \geq r(a,b)y_{ab}$$

$$r(a,b)y_{ab} - T \;\leq\; 0$$

$$\begin{array}{ccccccccc}
y_{1\mathrm{a}} & y_{1\mathrm{b}} & y_{1\mathrm{c}} & y_{2\mathrm{a}} & y_{2\mathrm{b}} & y_{2\mathrm{c}} & x_1 & x_2 & T \\
r & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & r & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & r & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & r & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & r & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & r & 0 & 0 & -1
\end{array}$$

Starting with the second constraint we have no variable on the right-hand side. The given b is the budget which is constant so it will remain same. The constraint with its matrix is given below.

$$\sum_{a=1}^{M} c(a)x_a \leq b$$

y_{1a}	y_{1b}	y_{1c}	y_{2a}	y_{2b}	y_{2c}	x_1	x_2	T
0	0	0	0	0	0	c	c	0

Moving on to the next constraint which is given below with its matrix. The 1 on right-hand side is again constant and it will remain same. The b in the constraint is constant so, we have $y_{1a} + y_{2a} = 1$, $y_{1b} + y_{2b} = 1$ *and* $y_{1c} + y_{2c} = 1$. We can write this in following matrix form.

$$\sum_{a=1}^{M} y_{ab} = 1$$

y_{1a}	y_{1b}	y_{1c}	y_{2a}	y_{2b}	y_{2c}
1	0	0	1	0	0
0	1	0	0	1	0
0	0	1	0	0	1

For the next constraint we will move x_a on the left-hand side as it is variable, and we will write the constraint in the matrix form.

$$y_{ab} \leq x_a$$

$$y_{ab} - x_a \leq 0$$

y_{1a}	y_{1b}	y_{1c}	y_{2a}	y_{2b}	y_{2c}	x_1	x_2	T
1	0	0	0	0	0	−1	0	0
0	1	0	0	0	0	−1	0	0
0	0	1	0	0	0	−1	0	0
0	0	0	1	0	0	0	−1	0
0	0	0	0	1	0	0	−1	0
0	0	0	0	0	1	0	−1	0

Now we need to combine all the above matrices and built the giant matrix A.

y_{1a}	y_{1b}	y_{1c}	y_{2a}	y_{2b}	y_{2c}	x_1	x_2	T
0	0	0	0	0	0	c	c	0
1	0	0	1	0	0	0	0	0
0	1	0	0	1	0	0	0	0
0	0	1	0	0	1	0	0	0
1	0	0	0	0	0	−1	0	0
0	1	0	0	0	0	−1	0	0
0	0	1	0	0	0	−1	0	0
0	0	0	1	0	0	0	−1	0
0	0	0	0	1	0	0	−1	0
0	0	0	0	0	1	0	−1	0
r	0	0	0	0	0	0	0	−1
0	r	0	0	0	0	0	0	−1
0	0	r	0	0	0	0	0	−1
0	0	0	r	0	0	0	0	−1
0	0	0	0	r	0	0	0	−1
0	0	0	0	0	r	0	0	−1

Assignment 3:

MATLAB works with the minimization problem and our objective function is already in this form, so we do not need to convert our problem. The most interesting part in the MATLAB program is to write the matrices that we have built in the assignment 2. We have two matrices in our MATLAB program that is *A* matrix which consists of inequality constraints and *Aeq* matrix which consists of the equality constraints. To write the inequality and equality constraints we have used zeros (which fill the matrix with zeros), ones (to create an array of ones),repmat (repeat copies of array) ,eye (identity matrix) and kron (if *A* and *B* are matrices then kron will take all possible products between the elements of *A* and matrix *B*).

Along with the two matrices we also have some vectors in the code that are *lb* (lower bound), *ub* (upper bound), *b* (b-values of inequality constraints), *beq* (b-values of equality constraints), *f*(objective function) and *intcon* (a vector of positive values that contains the components that are integer valued). We have also call the intlinprog with all the outputs i.e. f, intcon, A, b, Aeq, beq, lb, ub and Intlinprog is a mixed integer programming solver in MATLAB.

In our command window we will call the program >> [x, fval] = labB (*N*,*M*,*seed*) and we will add the values for the *N* (Size of the city) **,** *M* (Number of ambulance stations) and for seed (random number generator) to get the optimal objective value.

Assignment 4:

In this example we have choose *N* = 6, *M* = 3 and the seed = 1 which is random number generator.

>> [x, fval] = labB (6,3,1)

LP: Optimal objective value is 3.980092.

Optimal solution found.