Operations Analysis

Question:

In this exercise you will be responsible for planning the production in a factory.

The factory produces *P* different products using *R* different kinds of raw materials. To produce one unit of product *p* you will need b_{pr} units of raw material *r*. Each unit of raw material *r* costs $\$g_r$ and each unit of product *p* sells for $\$c_p$. You want to maximize the total profit, i.e. your revenue minus your costs. You need to decide how much to produce of each product and how much raw material of each type you need to buy.

To help you plan your production a team of experts has prepared *K* different scenarios for the demand of each product. Each scenario *k* is given a probability π_k and for each product *p* in each scenario *k* you also are given a demand of d_{kp} units of that product. You will have to maximize the expected outcome, i.e., the sum over all scenarios of the probability for that scenario times your profit in that scenario.

Remember that your purchase of raw materials and the amount of produced products must be the same for all scenarios. You can not sell more units of each product than the demand or the number of units you have produced, whichever the smallest. You can not produce or buy a negative number of units. All data can be assumed to be non-negative.

Solution:

The solution is divided into four parts named as assignment 1, assignment 2 and so on. In the first part we will explain each constraint and the objective function along with variables and problem description. In the next step we will modify the problem and formulate the maximization problem into a minimization problem. The third step is to reformulate the model and the constraints so the MATLAB can accept it. The last step is to solve a small instance using MATLAB.

Given Data:

P: Number of different products

R: Number of different types of raw material

b_{pr}: The amount of raw material used to produce one unit of product *p*

g_r: Cost price for one unit of raw material *r*

c_p: Selling price for one unit of product *p*

K: Number of scenarios

π_k: Probability of scenario k

d_{kp}: Demand of product p in scenario k

x_{kp}: The number units we sell in each scenario k

y_p: The number of units we produce for each product p

Assignment 1

In the assignment 1, we are supposed to maximize the total profits (revenue – expenses). We want to formulate an equation where we have the expected earning minus the costs that we want to maximize. For this we need to add the constraints with the inequalities.

The equation to maximize the profits:

$$\max \sum_{k=1}^{K} \sum_{p=1}^{P} \pi_k c_p x_{kp} - \sum_{p=1}^{P} y_p \sum_{r=1}^{R} b_{pr} g_r \quad (1)$$

$$\text{subject to} \quad x_{kp} = \min(y_p , d_{pk}) \qquad \forall\, pk$$

$$x_{kp} \geq 0 , y_p \geq 0$$

Objective Function:

The objective function for this problem is the profit which we want to maximize. The profit is revenue minus expenses. The revenue is the left-hand side of the equation (1) which consists three variables that are π_k , c_p and x_{kp}. The variable π_k is the probability given for k scenario and the probability is always between 0 and 1. c_p is the price of each product in dollars and x_{kp} is the number of units of product p that we want to sell in each scenario k. The right-hand side of the equation (1) is the cost that we want to subtract from the revenue, and it consists of y_p , b_{pr} and g_r. The cost is the summation of all the units produced from each product p times the summation of amount of raw material used to produce p units times the cost of each unit of raw material.

Constraints:

In the problem we are given two constraints that are mentioned below.

- You cannot sell more units of each product than the demand or the number of units you have produced.
- You cannot produce or buy negative number of units.

In the problem we are also given $\min(a\,,b)$ and we have used y_p and d_{pk} for the first limitation. The first constraint that is mentioned in the problem is that we cannot produce more than the demand and we cannot sell more than our production. We have to consider the demand before the production because we don't want to lose money by producing more than the demand. So, we will take the minimum of y_p (our production) and the minimum of the d_{pk}(demand) for the x_{kp}. The second limitation is also mentioned in the problem i.e. you cannot produce or buy negative numbers. As we know, we cannot produce or sell the negative number of units therefore both x and y are greater than zero. In reality, we can have negative profits but for this problem, we have assumed the data to be non-negative.

Assignment 2

In this assignment we need to substitute the minimum function that is given below with something else.

$$\min(a\,,b) = \begin{cases} a & \text{if } a \leq b \\ b & otherwise \end{cases}$$

We have substituted the min (a, b) with $\min(y_p\,, d_{pk})$.

$$\min(y_p\,, d_{pk}) = \begin{cases} y_p & \text{if } y_p \leq d_{pk} \\ d_{pk} & otherwise \end{cases}$$

The explanation of the minimum function; our production is less than demand therefore we will sell according to our production otherwise we might sell according to the demand.

$$\text{subject to} \quad 0 \leq x_{kp} \leq y_p$$

$$0 \leq x_{kp} \leq d_{kp}$$

In this constraint, we have lower bound of zero with the same reasoning that we cannot sell the negative number of units. The first upper bound means that we will sell less than the production which also means that we can sell what we have produced. The second upper bound shows that we cannot sell more than the demand. For example, if demand is for 10 units, then we cannot sell 11 units.

Assignment 3

We have formulated the maximization problem into the minimization problem using the concept -max f(x) = min(-f(x)).

$$min \sum_{p=1}^{P} y_p \sum_{r=1}^{R} b_{pr} g_r - \sum_{k=1}^{K} \sum_{p=1}^{P} \pi_k c_p x_{kp}$$

We need to reformulate the constraints as well so the MATLAB can accept it. The linear inequality constraints are given below:

$$x_{kp} - y_p \leq 0$$

$$x_{kp} - d_{kp} \leq 0$$

We have used the same model that we have created in the assignment 1, we just reformulated the model and the constraints so the MATLAB can accept it.

Assignment 4

In the MATLAB we have used the linprog function which is a linear programming solver. We will fill the values (4,3,2,1) where we have four kinds of raw material, three kinds of products and two different scenarios. When we run the code, we find the optimal solution that is given below. The code file has been attached with this document.

```
[x, y, fval] = labA(R,P,K,seed)

>> [x, y, fval] = labA(4,3,2,1)

Optimal solution found.

x =

    0.0000
    0.0000
   13.2317

y =

   11.9914
   17.7403
    8.6423
   19.7426


fval =

    7.7615
```

Using four kinds of raw material, three kinds of products and two different scenarios our expected profit is 7.7615.

Assignment 5

For the solved instance I have considered labA(3,4,7,8) i.e. we have three kinds of raw material, four kinds of products and seven different of scenarios we have the following output.

```
>> [x, y, fval] = labA(3,4,7,8)

Optimal solution found.

x =

         0

         0

   24.2349

         0

y =

   20.9810

   21.2052

   25.5878

fval =

   14.0394
```

Using three kinds of raw material, four kinds of products and seven different of scenarios our expected profit is 14.0394.