

## Average growth rate

### Regression Equations:

We will solve estimating regression equations.

Let the variable  $y$  signify real GDP per capita. Assume that  $y(t)$  is a function of time:

$$y(t) = y_0 e^{gt}, \quad (1)$$

where  $y_0$  is  $y$  at time 0. The constant  $g$  is the (proportional) growth rate. To see this, differentiate both sides of the equation with respect to time, and denote the time derivative by a dot:

$$\dot{y} = y_0 g e^{gt} = gy.$$

The growth rate thus is:

$$\frac{\dot{y}}{y} = g.$$

Now take the natural logs of both sides of equation (1):

$$\ln y(t) = \ln y_0 + gt.$$

We transform this into a regression equation, where we try to ‘explain’ the growth of  $y$  by time,  $t$ . Adding an error term we have:

$$\ln y(t) = \beta_0 + \beta_1 t + \epsilon_t, \quad (2)$$

where,  $\beta_0 = \ln y_0$  and  $\beta_1$  is the growth rate  $g$ .

### Question:

Estimate equation (2) on the data for some country. Briefly explain the estimated growth rate.

### Solution:

Using Country Snapshot, I have chosen data of United States. The data is from year 1950 to 2014 and it consists of 65 observations. I have found the data on  $y = Y/pop$  that is the real GDP per capita and after taking natural log I have use the values for Input Y Range. For the Input X Range, I have use  $t$  which is the time from year 1 to year 65. After doing the regression analysis I have find the values of  $\beta_0$  that is the constant or the intercept term and  $\hat{\beta}_1$  that is the slope coefficient. The estimated growth rate  $\beta_1$  is equal to 0.020998 where as the  $\beta_0$  is equal to 9.605855.

$$\ln y(t) = 9.605855 + 0.020998t$$

The summary output is attached below.

SUMMARY OUTPUT								
<i>Regression Statistics</i>								
Multiple R	0.993123							
R Square	0.986292							
Adjusted R S	0.986075							
Standard Err	0.047176							
Observation	65							
ANOVA								
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>			
Regression	1	10.08854	10.08854	4533.018883	2.07803E-60			
Residual	63	0.140211	0.002226					
Total	64	10.22875						
	<i>Coefficients</i>	<i>andard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>	<i>Lower 95.0%</i>	<i>Upper 95.0%</i>
Intercept	9.605855	0.011839	811.3564	2.5012E-128	9.582196439	9.629514	9.582196439	9.62951416
t	0.020998	0.000312	67.3277	2.07803E-60	0.020375147	0.021622	0.020375147	0.02162165