

Lecture 14.2: CFG Definitions, Terminologies, Examples-2

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In this lecture, you will learn some more terminologies related to context-free grammar.

Recall, the following grammar you learned in the previous lecture for valid mathematical expressions:

$$\begin{aligned}
 E &\rightarrow (E) \\
 E &\rightarrow E + E \\
 E &\rightarrow E - E \\
 E &\rightarrow E \times E \\
 E &\rightarrow E \div E \\
 E &\rightarrow \text{Num} \\
 \text{Num} &\rightarrow 0|1|2|3|4|5|6|7|8|9
 \end{aligned}$$

Derivation, Sentential form and Sentence

You should already have a basic idea about the term **derivation**. Recall, we usually say S derives a string w if we can reach to w starting from S by a number of replacements or substitutions. Let's see an alternative derivation of the expression $4 \times 3 - 2$:

$4 \times 3 - 2$	Derivation Step
E	<i>Sentential Form</i>
$\Rightarrow E - E$	<i>Sentential Form</i>
$\Rightarrow E \times E - E$	<i>Sentential Form</i>
$\Rightarrow \text{Num} \times \text{Num} - \text{Num}$	<i>Sentential Form</i>
$\Rightarrow 4 \times 3 - 2$	<i>Sentential Form</i>
	<i>Sentence</i>

In the above Example we say:

- E derives the string $4 \times 3 - 2$ and we express it as $E \xrightarrow{*} 4 \times 3 - 2$.
- A **sentential form** is any string derivable from the start symbol. Thus, in the derivation of $4 \times 3 - 2$; each intermediate line i.e. $E - E$, $E \times E - E$, $\text{Num} \times \text{Num} - \text{Num}$ etc. are all sentential forms along with E and $4 \times 3 - 2$ themselves.
- A **sentence** is a sentential form consisting only of terminals such as $4 \times 3 - 2$ in the above example.

Summary

We can express the derivation of w from a start symbol S as $S \Rightarrow u_1 \Rightarrow u_2 \dots \Rightarrow u_{n-1} \Rightarrow w$ or simply $S \xrightarrow{*} w$, where $S, u_1, u_2 \dots, u_{n-1}, w$ each of this is a sentential form and w is the sentence.

Now, we can define the language of a CFG formally as $L = \{w \in \Sigma^* \mid S \xrightarrow{*} w\}$.
