

## Lecture 5.3: Deterministic Finite Automata

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### Extended Transition Function

The extended transition function takes a state  $q$  and an input string  $w$  and yields the resulting state in which the processing of the string ends. The definition proceeds by induction over the length of  $w$ . The definition precedes by induction over length of  $w$ . It is sometimes represented as  $\hat{\delta}$  (delta hat) to distinguish it from the transition function.

**Induction basis:** When  $w$  is of length 0 or an empty string ( $\varepsilon$ ), it is defined as  $\hat{\delta}(q, \varepsilon) = q$

**Inductive Step:** In this step, we determine the transition of a string ( $w$ ) of length  $I+1$  from a string of length  $I$ . Let's say,  $w$  is of the form "va" where  $v$  is a string of length  $I$  and "a" is a symbol. Therefore,  $\hat{\delta}(q, va) = \hat{\delta}(\hat{\delta}(q, v), a)$

### Example

Let's consider the DFA that accepts strings with two consecutive 1s and we have to determine  $\hat{\delta}(q_0, 110)$ .

Transition table:

	0	1
$\rightarrow q_0$	$q_0$	$q_1$
$q_1$	$q_0$	$q_2$
$*q_2$	$q_2$	$q_2$

$$\begin{aligned}
& \delta(q_0, 110) \\
&= \delta(\delta(q_0, 11), 0) \\
&= \delta(\delta(\delta(q_0, 1), 1), 0) \\
&= \delta(\delta(\delta(\delta(q_0, \varepsilon), 1), 1), 0) \\
&= \delta(\delta(\delta(\delta(q_0, 1), 1), 0)) \quad [\text{The basis of the induction}] \\
&= \delta(\delta(q_1, 1), 0) \\
&= \delta(q_2, 0) \\
&= q_2
\end{aligned}$$

### Language of a DFA in terms of the Extended Transition Function

The language of a DFA  $(Q, \Sigma, \delta, q_0, F)$  is a set that contains strings  $w$  such that  $\delta(q_0, w) \in F$