

## Lecture 9.2: Conversion of NFA/ $\epsilon$ -NFA to DFA

*Presenter: Sabrina Zaman Ishita (SZI)*

*Scribe: Sabrina Zaman Ishita (SZI)*

This is the continuation note on conversion of NFA/ $\epsilon$ -NFA to DFA.

To convert an NFA to DFA, we need to use:

$\text{Move}_{\text{NFA}}(S,a) \rightarrow$  the transition function from NFA

$\epsilon$ -Closure ( $s$ )  $\rightarrow$  where  $s$  is a single state from NFA

$\epsilon$ -Closure ( $S$ )  $\rightarrow$  where  $S$  is a set of states from NFA

By using these, we need to construct:

$S_{\text{DFA}} \rightarrow$  the set of states in the DFA

Initially,  $S_{\text{DFA}} \leftarrow \{ \}$

Add  $x$  to  $S_{\text{DFA}}$  where  $x$  is some set of NFA states

Example: “Add  $\{3, 5, 7\}$  to  $S_{\text{DFA}}$ ”

Here, 3, 5, and 7 –all are distinct states of NFA and they have been added to the DFA as a combined single state.

We’ll “mark” some of the states in the  $S_{\text{DFA}}$

Marked = “We’ve done this one” ( $\checkmark$ )

Unmarked = “Still need to do this one”

$\text{Move}_{\text{DFA}}(T,b) \rightarrow$  The transition function from DFA

To add an edge to the growing DFA

Set  $\text{Move}_{\text{DFA}}(T,b)$  to  $S$  (where  $S$  and  $T$  are sets of NFA states)



Where  $S$  and  $T$  are sets of NFA states which became single states in DFA.

The algorithm to convert an NFA to DFA is given below, after that an example workout is shown for your convenience.

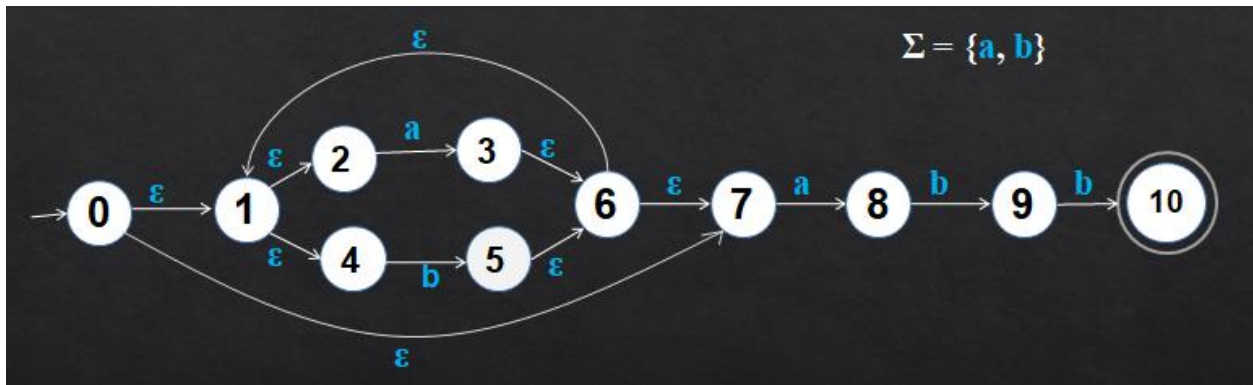
### Algorithm: NFA to DFA

```

SDFA = {}
Add  $\epsilon$ -Closure( $s_0$ ) to SDFA as the start state
Set the only state in SDFA to "unmarked"
while SDFA contains an unmarked state do
  Let T be that unmarked state
  Mark T
  for each a in  $\Sigma$  do
    S =  $\epsilon$ -Closure(MoveNFA(T, a))
    if S is not in SDFA already then
      Add S to SDFA (as an "unmarked" state)
    endIf
    Set MoveDFA(T, a) to S
  endFor
endWhile
for each S in SDFA do
  if any  $s \in \mathbf{S}$  is a final state in the NFA then
    Mark S as a final state in the DFA
  endIf
endFor

```

### Example:



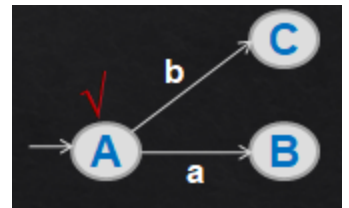
Let's work on the start state of the DFA first. The start state of the NFA is 0. It should be included in the start state of the DFA as well. All the other states reachable from 0 should also be included here since from state 0 you can go to those states with zero cost. So, the start state of the DFA is basically,

- Start state:  
 $\epsilon$ -Closure (0)  
 $= \{0, 1, 2, 4, 7\}$   
 $= \mathbf{A}$

Let's work with **A** now, we need to define transition functions for **A** with each input symbol from  $\Sigma=\{a,b\}$ . So, [**A** = { 0, 1, 2, 4, 7 }]

- $\text{Move}_{\text{DFA}}(\mathbf{A}, a)$   
 $= \varepsilon\text{-Closure}(\text{Move}_{\text{NFA}}(\mathbf{A}, a))$   
 $= \varepsilon\text{-Closure}(\{3, 8\})$   
 $= \{1, 2, 3, 4, 6, 7, 8\}$   
 $= \mathbf{B}$

- $\text{Move}_{\text{DFA}}(\mathbf{A}, b)$   
 $= \varepsilon\text{-Closure}(\text{Move}_{\text{NFA}}(\mathbf{A}, b))$   
 $= \varepsilon\text{-Closure}(\{5\})$   
 $= \{1, 2, 4, 5, 6, 7\}$   
 $= \mathbf{C}$

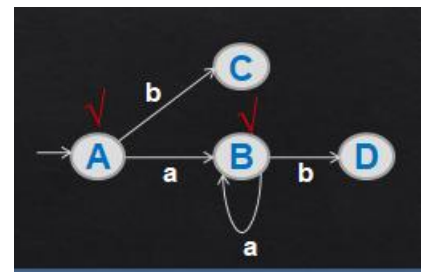


Let's work with **B** now,

**B**= {1, 2, 3, 4, 6, 7, 8}

- $\text{Move}_{\text{DFA}}(\mathbf{B}, a)$   
 $= \varepsilon\text{-Closure}(\text{Move}_{\text{NFA}}(\mathbf{B}, a))$   
 $= \varepsilon\text{-Closure}(\{3, 8\})$   
 $= \{1, 2, 3, 4, 6, 7, 8\}$   
 $= \mathbf{B}$

- $\text{Move}_{\text{DFA}}(\mathbf{B}, b)$   
 $= \varepsilon\text{-Closure}(\text{Move}_{\text{NFA}}(\mathbf{B}, b))$   
 $= \varepsilon\text{-Closure}(\{5, 9\})$   
 $= \{1, 2, 4, 5, 6, 7, 9\}$   
 $= \mathbf{D}$

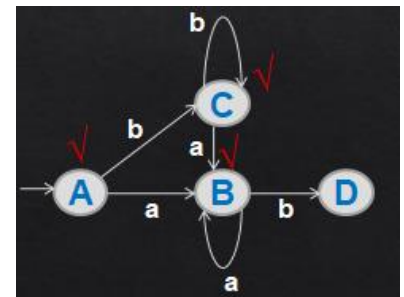


Let's work with **C** now,

**C** = {1, 2, 4, 5, 6, 7}

- $\text{Move}_{\text{DFA}}(\mathbf{C}, a)$   
 $= \varepsilon\text{-Closure}(\text{Move}_{\text{NFA}}(\mathbf{C}, a))$   
 $= \varepsilon\text{-Closure}(\{3, 8\})$   
 $= \{1, 2, 3, 4, 6, 7, 8\}$   
 $= \mathbf{B}$

- $\text{Move}_{\text{DFA}}(\mathbf{C}, b)$   
 $= \varepsilon\text{-Closure}(\text{Move}_{\text{NFA}}(\mathbf{C}, b))$   
 $= \varepsilon\text{-Closure}(\{5\})$   
 $= \{1, 2, 4, 5, 6, 7\}$   
 $= \mathbf{C}$

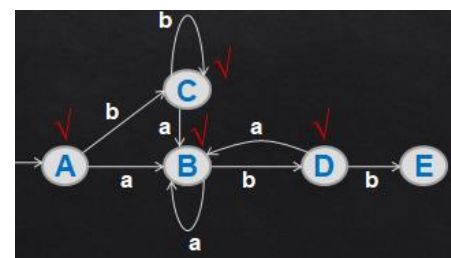


Let's work with **D** now,

**D** = {1, 2, 4, 5, 6, 7, 9}

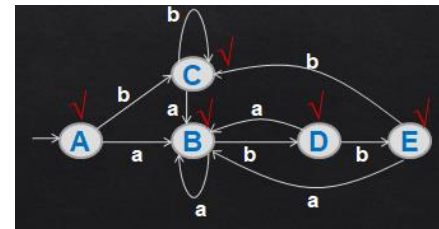
- $\text{Move}_{\text{DFA}}(\mathbf{D}, a)$   
 $= \varepsilon\text{-Closure}(\text{Move}_{\text{NFA}}(\mathbf{D}, a))$   
 $= \varepsilon\text{-Closure}(\{3, 8\})$   
 $= \{1, 2, 3, 4, 6, 7, 8\}$   
 $= \mathbf{B}$

- $\text{Move}_{\text{DFA}}(\mathbf{D}, b)$   
 $= \varepsilon\text{-Closure}(\text{Move}_{\text{NFA}}(\mathbf{D}, b))$   
 $= \varepsilon\text{-Closure}(\{5, 10\})$   
 $= \{1, 2, 4, 5, 6, 7, 10\}$   
 $= \mathbf{E}$



Let's work with **E** now,  $E = \{1, 2, 4, 5, 6, 7, 10\}$

- $\text{Move}_{\text{DFA}}(E, a)$   
 $= \epsilon\text{-Closure}(\text{Move}_{\text{NFA}}(E, a))$   
 $= \epsilon\text{-Closure}(\{3, 8\})$   
 $= \{1, 2, 3, 4, 6, 7, 8\}$   
 $= \mathbf{B}$
- $\text{Move}_{\text{DFA}}(E, b)$   
 $= \epsilon\text{-Closure}(\text{Move}_{\text{NFA}}(E, b))$   
 $= \epsilon\text{-Closure}(\{5\})$   
 $= \{1, 2, 4, 5, 6, 7\}$   
 $= \mathbf{C}$



Since, **E** contains the final state of the NFA (state 10), the final state of the DFA should be **E**. The final DFA is given below:

