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| CSE331 | Automata and Computability |
| **Lecture 7: Nondeterministic Finite Automata (NFA)** | |
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## Nondeterministic Finite Automata (NFA)

Nondeterministic Finite Automata is a Finite State Machine that accepts or rejects a string according to the pattern it defines. Like a DFA, an NFA also consists of states and transitions. However, it has a lot less restrictions than a DFA. An NFA can have more than one transition on one input symbol. It can be in more than one state at any given point. In each step, whenever there are two or more transitions, it "copies" itself and each copy follows a transition. If there is no possible transition, the copy "dies".

## Formal Definition

An NFA is represented formally by a 5-tuple, $(Q, \Sigma, \delta, q_0, F)$, consisting of:

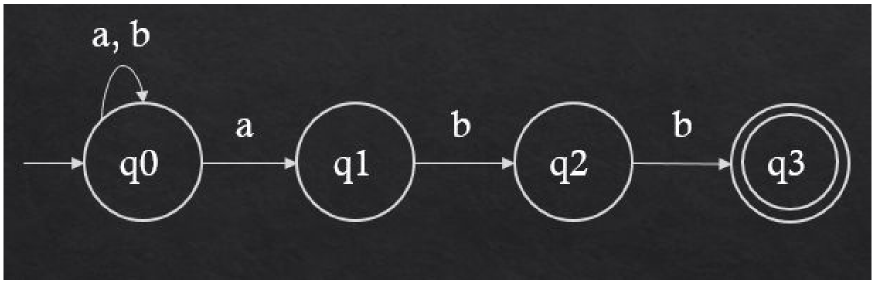
- a finite set of states, Q.
- a finite set of input symbols, $\Sigma$ .
- a transition function $\delta$ where $\delta$: $Q \times \Sigma \rightarrow P(Q)$ where P(Q) is the power set of Q.
- an initial (or start) state $q_0 \in Q$
- a set of states $F \subseteq Q$

## Acceptance of a String by an NFA

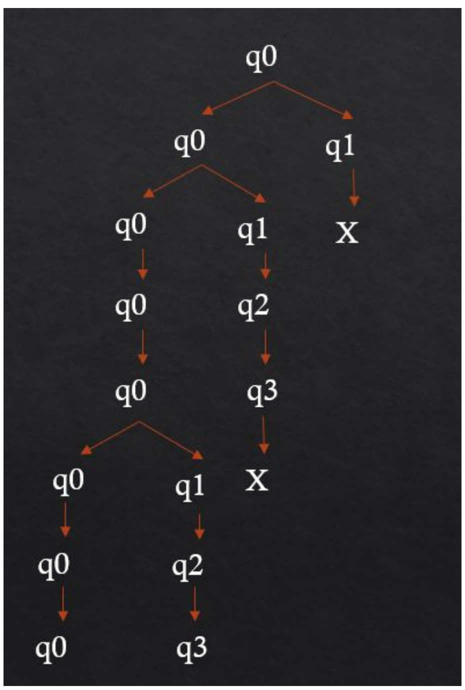
An NFA accepts the input string if there exists some choice of transitions that leads to the NFA ending in an accepting state. Thus, one accepting branch is enough for the overall NFA to accept, but every branch must reject for the overall NFA to reject.

**Example**

- L = {w | w ∈ (a, b)* and w ends with the substring abb}




Let's take a string *aabbabb* and simulate the NFA for the string. The states that it goes through can be demonstrated as a tree structure::




The last states the NFA is in after the processing of the string is done are q0 and q3.

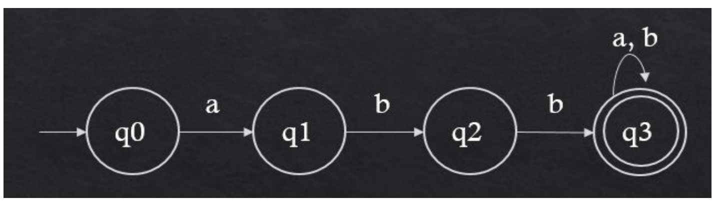
Therefore, the extended transition function δ(q0, *aabbabb*) = {q0, q3}. Since it contains the final state q3, the string *aabbabb* is accepted in the language.

The transition table for this NFA:

| | **a** | **b** |
|---|---|---|
| q0 | {q0, q1} | {q0} |
| q1 | ***Φ*** | {q2} |
| q2 | ***Φ*** | {q3} |
| q3 | ***Φ*** | ***Φ*** |

Here, the symbol, ***Φ***, denotes an empty i.e. there are not valid transitions.

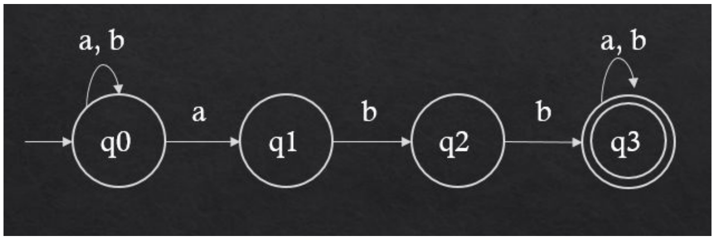
- L = {w | w ∈ (a, b)* and w starts with the substring abb}




The transition table:

| | **a** | **b** |
|---|---|---|
| q0 | {q1} | ***Φ*** |
| q1 | ***Φ*** | {q2} |
| q2 | ***Φ*** | {q3} |
| q3 | {q3} | {q3} |

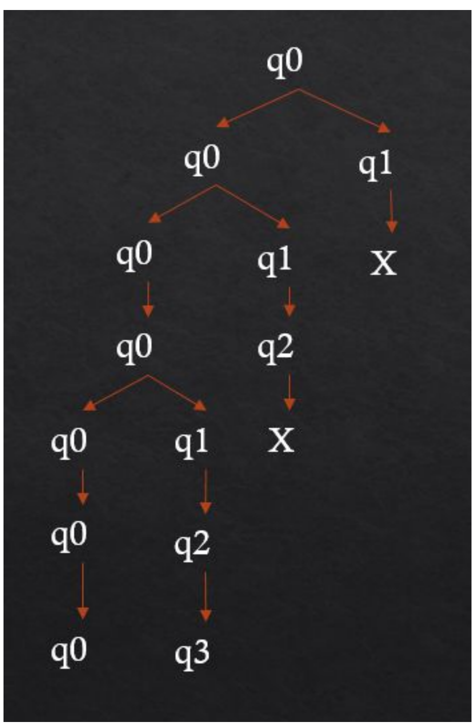
- L = {w | w ∈ (a, b)* and w contains the substring abb}




The transition table:

| | **a** | **b** |
|---|---|---|
| q0 | {q0, q1} | {q0} |
| q1 | ***Φ*** | {q2} |
| q2 | ***Φ*** | {q3} |
| q3 | {q3} | {q3} |

Processing the string *aabbabb* can be shown by the following diagram:

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