

# MID-POINT LINE ALGORITHM (contd\*)

LECTURE 4

Equation of a line:

Implicit form  $\left\{ \begin{array}{l} y = mx + c \dots (i) \end{array} \right.$ , where  $m = \frac{dy}{dx}$

↓

$\Rightarrow y = \frac{dy}{dx} \cdot x + c \dots$  (multiply both sides with  $dx$ )

$\Rightarrow dx \cdot y = dy \cdot x + dx \cdot c \dots$  (take  $dx \cdot y$  to the right hand side)

$\Rightarrow \underbrace{dy \cdot x}_{\downarrow} - \underbrace{dx \cdot y}_{\downarrow} + \underbrace{dx \cdot c}_{\downarrow} = 0 \dots$

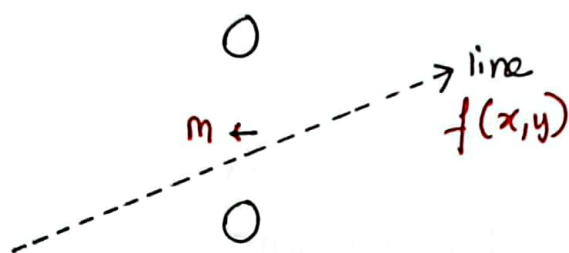
Explicit form  $\left\{ \right.$

$Ax + By + C = 0 \dots (ii)$

(replace,  
✓  $dy = A$   
✓  $-dx = B$   
✓  $dx \cdot c = C$ )

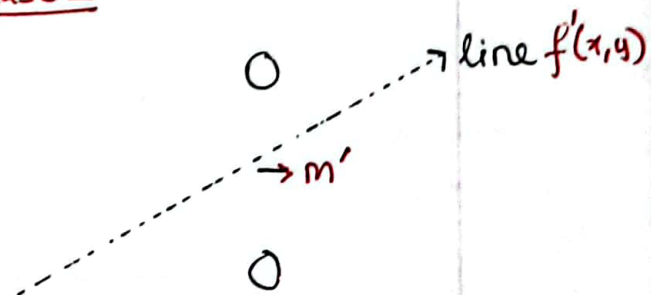
case 1

case 2



if we plug in the coordinates of  $M$  into the eqn of the line, the resulting value of the function will be -ve

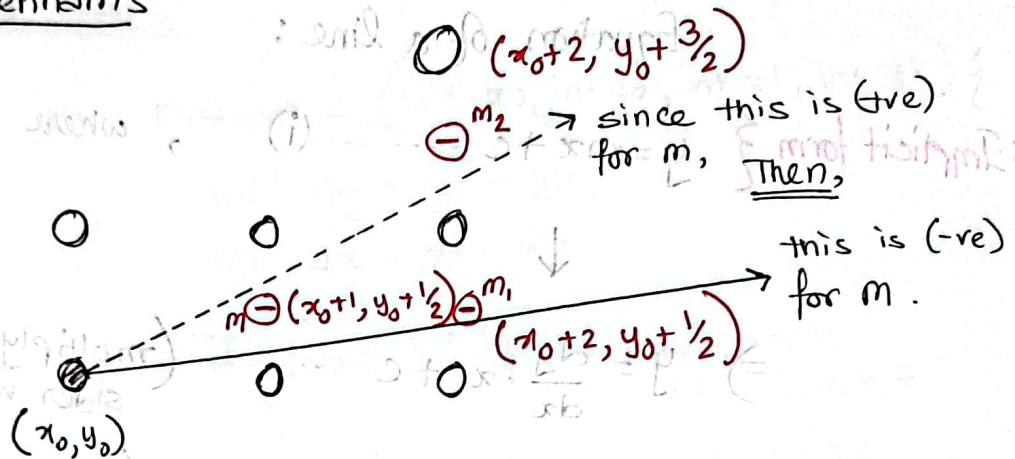
$f(M) = -ve$



if we plug in the coordinates of  $M'$  into the eqn of the line, the resulting value of the function will be +ve

$f'(M') = +ve$

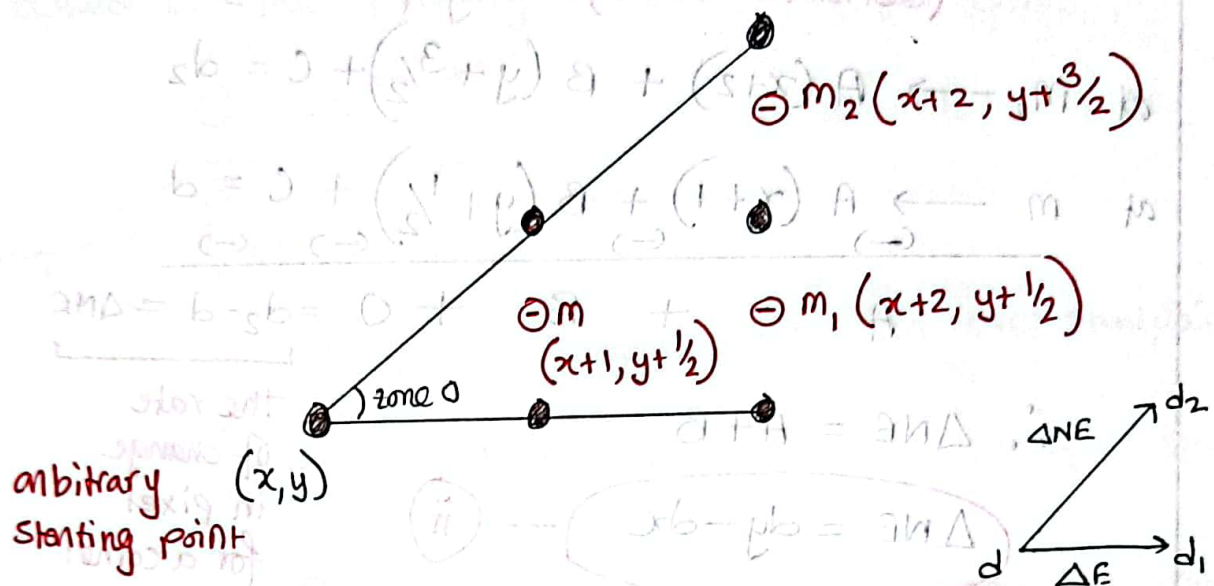
# Bresenham's



we will be considering the rate of change (in the deviation from the midpoint)

(ii) to further locate the pixels in the path.

## Designing the Algo for Zone 0.



(deviation at  $m_1$ , solving for  $\Delta E$ )

$$\text{at } m_1, \rightarrow A(x+2) + B(y+\frac{1}{2}) + C = d_1$$

$$\text{at } m, \rightarrow A(x+1) + B(y+\frac{1}{2}) + C = d$$

$$\lim_{\Delta x \rightarrow 0} (A(x+1) + B(y+\frac{1}{2}) + C) - (A(x+2) + B(y+\frac{1}{2}) + C) = d - d_1 \approx \Delta E$$

$$\therefore A = \Delta E$$

$$\text{meaning, } \Delta E = dy$$

(i)

the rate of change of pixel for a horizontal movement.



(deviation at  $m_2$ , solving for  $\Delta NE$ )

$$\text{at } m_2 \rightarrow A(x+2) + B(y+3/2) + C = d_2$$

$$\text{at } m \rightarrow A(x+1) + B(y+1/2) + C = d$$

$$A + B + 0 = d_2 - d = \Delta NE$$

$$\therefore \Delta NE = A + B$$

$$\Delta NE = dy - dx$$

ii

the rate of change in pixel for a corner movement.

we still need to solve for the initial deviation /  $d_{init}$  at  $m$ , for starting points  $(x_0, y_0)$

$$\text{at } m, A(x_0+1) + B(y_0+1/2) + C = d_{init}$$

$$\Rightarrow Ax_0 + By_0 + C + A + B/2 = d_{init}$$

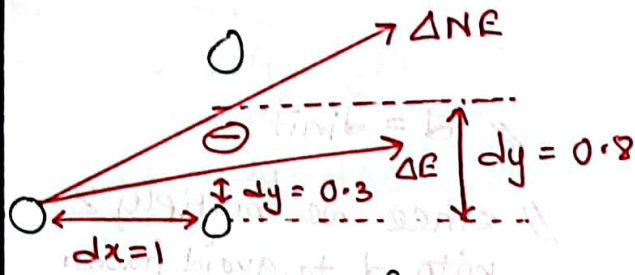
$$\Rightarrow d_{init} = A + B/2$$

since  $(x_0, y_0)$  is a coordinate from the line  $Ax_0 + By_0 + C = 0$

$$\Rightarrow d_{init} = dy - dx/2$$

iii

Ø Now, we need figure out if we'll move to  $\Delta E / \Delta NE$  based on the polarity of the value for  $d_{init}$ :



Ø for  $\Delta NE$ , where line goes over midpoint.

$$d_{init} = dy - \frac{dx}{2} = 0.8 - \frac{1}{2} = 0.3 (+ve)$$

Ø for  $\Delta E$ , where line goes below midpoint.

$$d_{init} = 0.3 - \frac{1}{2} = -0.2 (-ve)$$

\*

∴ We can conclude that for a +ve value for  $d$  the movement will be  $\Delta NE$ ,

AND,

for a -ve value for  $d$  the movement will be  $\Delta E$ .

Pseudo Code (MPL Zone 0):

drawline ( $x_0, y_0, x_1, y_1$ ) {

$$dy = y_1 - y_0$$

$$dx = x_1 - x_0$$

$$d = 2dy - dx$$

//  $d = d_{init}$ .

$$\Delta E = 2dy$$

// since we multiply 2 with  $d$  to avoid fraction

$$\Delta NE = 2(dy - dx)$$

// we need to multiply

$\Delta E$  &  $\Delta NE$  with 2 as well.

$$x = x_0$$

$$y = y_0$$

draw( $x, y$ );

while ( $x \leq x_2$ ) {

if ( $d \leq 0$ ) {

// for  $\Delta E$  movement only  $x$  increases.

$x++$

$$d += \Delta E$$

// add  $\Delta E$  to current value of  $d$

} else {

// for  $\Delta NE$  movement  $x, y$  both increase by 1.

$x++$

$y++$

$$d += \Delta NE$$

// add  $\Delta NE$  to current value of  $d$ .

}

draw( $x, y$ )

}

}



Q1)

Draw  $(30, 50)$  to  $(40, 54)$  using MPL

$\Delta y = 4$ ,  $\Delta x = 10$

$d = \Delta y - \Delta x / 2 = 2(\Delta y) - \Delta x = 8 - 10 = -2$

use this to avoid fraction.

$\Delta E = 2 \cdot \Delta y = +8$

$\Delta NE = 2(\Delta y - \Delta x) = 2(4 - 10) = -12$

$x$	$y$	$d$	$\Delta E / \Delta NE$	(PIXEL)
30	50	-2	$\Delta E$	$(30, 50)$
31	50	6	$\Delta NE$	$(31, 50)$
32	51	-6	$\Delta E$	$(32, 51)$
33	51	2	$\Delta NE$	$(33, 51)$
34	52	-10	$\Delta E$	$(34, 52)$
35	52	-2	$\Delta E$	$(35, 52)$
36	52	6	$\Delta NE$	$(36, 52)$
37	53	-6	$\Delta E$	$(37, 53)$
38	53	2	$\Delta NE$	$(38, 53)$
39	54	-10	$\Delta E$	$(39, 54)$
40	54	-2	$\Delta E$	$(40, 54)$

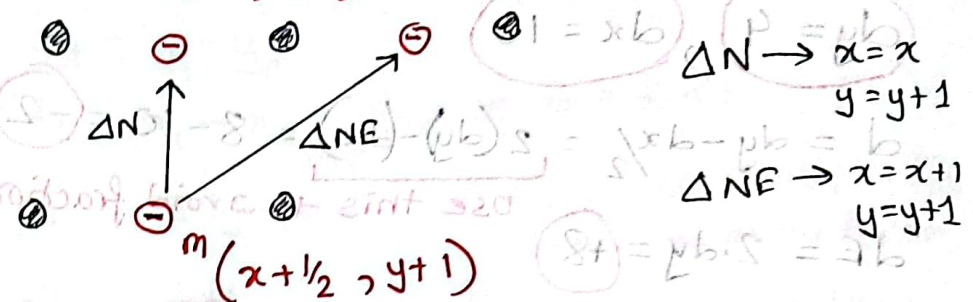
done

$x_b - y_b = \text{limit} \therefore$

# Ø Home Assignment Design for Zone 1 & Zone 5

## Zone 1

$$m_1(x+\frac{1}{2}, y+2) \quad m_2(x+\frac{3}{2}, y+2)$$



## Ø ΔN

at  $m_1$

$$A(x+\frac{1}{2}) + B(y+2) + C = d_1$$

at  $m$

$$\hookrightarrow A(x+\frac{1}{2}) + B(y+1) + C = d$$

$$0 + B + 0 = \Delta N$$

$$\therefore \Delta N = -dx \rightarrow \text{we'll use } [-2dx]$$

## Ø ΔNE

at  $m_2$

$$A(x+\frac{3}{2}) + B(y+2) + C = d_2$$

at  $m$

$$\hookrightarrow A(x+\frac{1}{2}) + B(y+1) + C = d$$

$$A + B = \Delta NE$$

$$\therefore \Delta NE = dy - dx \rightarrow \text{we'll use } [2(dy - dx)]$$

## Ø d<sub>init</sub>

at  $m$   
 $(x_0, y_0)$

$$A(x_0 + \frac{1}{2}) + B(y_0 + 1) + C = d_{init}$$

$$A/\frac{1}{2} + B = d_{init}$$

$$\therefore d_{init} = dy/\frac{1}{2} - dx \rightarrow \text{we'll use } [dy - 2dx]$$