

Lighting/Illumination/Reflection Model



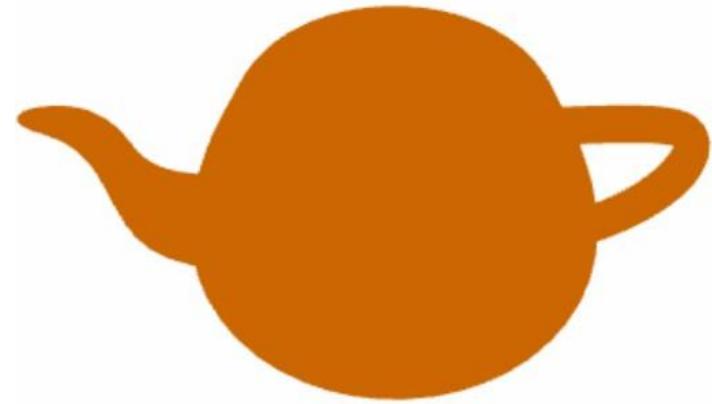
Some of the contents used in this lecture belong to...

- Dr Jon Shiach, Manchester Metropolitan University, “Direct Lighting Model”, <https://www.youtube.com/watch?v=7CdS8oOJtVA>
- Mr. Jacobson, University of Toronto, “WebGL Phong Shading”,
<http://www.cs.toronto.edu/~jacobson/phong-demo/> (Online demo)

Basic Terms

- **Illumination:** the transport of energy from light sources to surfaces & points
 - Local illumination
 - Global illumination
- **Lighting model or Illumination model:** Express the factors determining a surface's color or luminous intensity (outgoing or reflected light) at a particular 3D point

Effects of Lighting

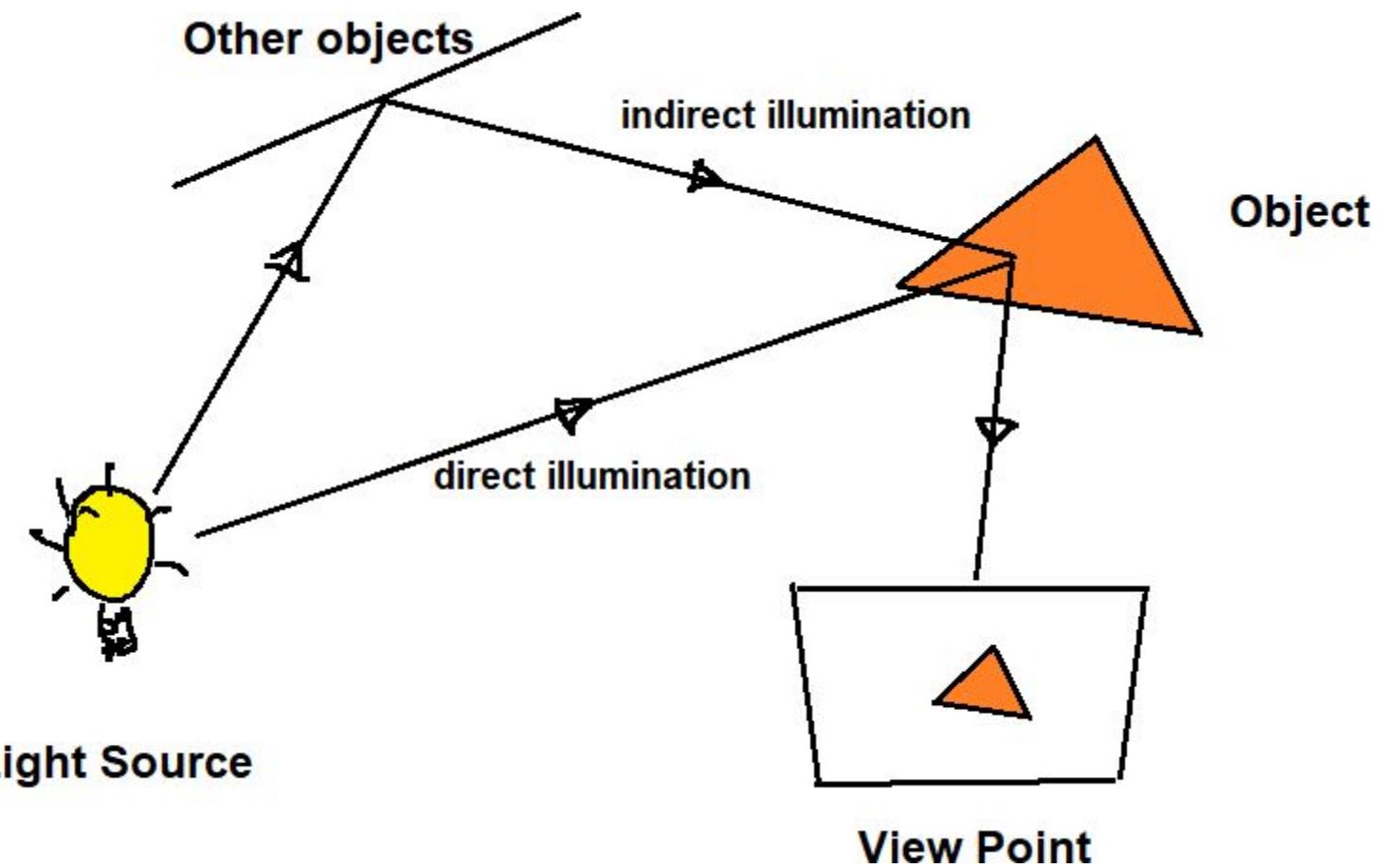


Pixel color only



Pixel colors with lighting

Illumination



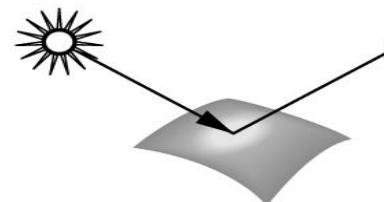
Components of Illumination

- Two components of illumination:
 1. Light sources
 2. Surface properties
- Light source described by a luminance/intensity ‘I’
 - Each color is described separately
 - $I = [I_r \ I_g \ I_b]$
- Types of Light Sources:
 1. Ambient Light
 2. Diffuse Light
 3. Spot Light

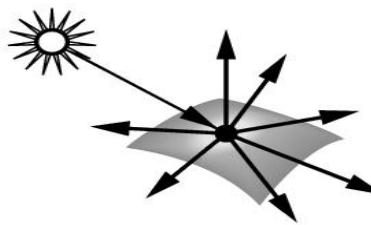
Types of Surface

Interaction between light and material can be classified as

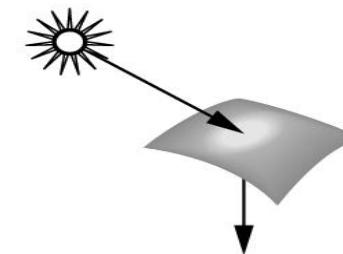
- **Specular surfaces** – Ideal mirror
- **Diffuse surfaces** – Reflected light is ideally reflected to all directions uniformly
- **Translucent surfaces** – Allow some lights to penetrate the surface (e.g. refraction in glass, water)



(a)

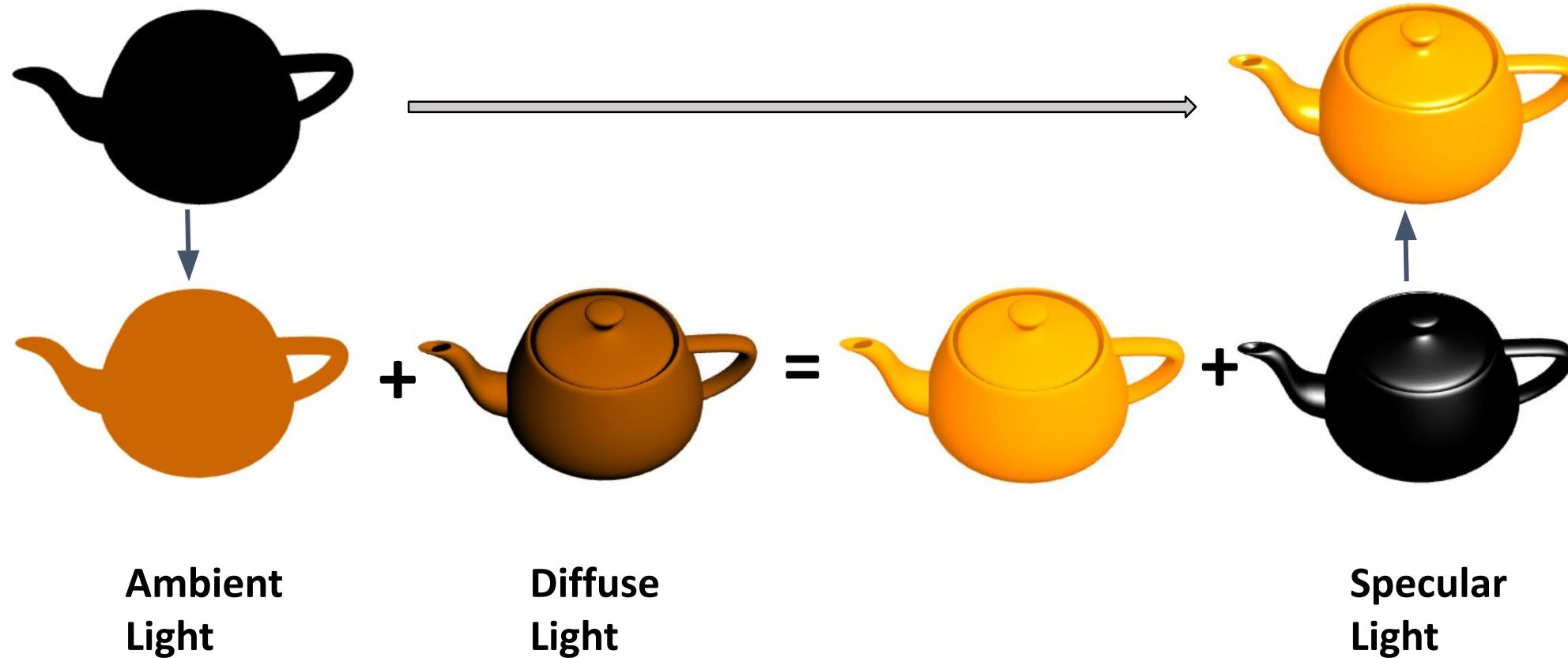


(b)



(c)

Phong's Reflection Model



Ambient Light

- No identifiable source or direction
- Product of multiple reflections of light from the many surfaces present in the environment
- Computationally inexpensive



Ambient Light

Categories:

1. Global ambient light

- Independent of light source
- Lights entire scene
- Example: reflection of sunlight from several surfaces

2. Local ambient light

- Contributed by additional light sources
- Can be different for each light and primary color
- Example: Reflection of fluorescent lamps from several surfaces

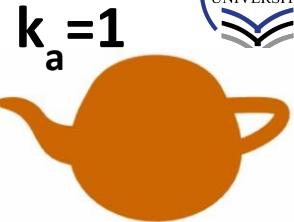
Ambient Reflection

- **Ambient reflection** is the reflection of light that does not come directly from a light source
- Even in a darkened room, we can make out the edges of objects – this is because of light bouncing off of objects
- Since Phong is a **direct** lighting model, we assume that ambient light falls equally on all objects, i.e.,

$$A = I_a k_a$$

where I_a is the intensity of the ambient light and $k_a \in [0, 1]$ is the **ambient coefficient**

- k_a is set to provide the right amount of ambient light for a scene, e.g., $k \rightarrow 1$ for bright scenes and $k \rightarrow 0$ for dark or nighttime scenes.



$$k_a = 0$$

Ambient Reflection Coefficient

- Effect of adding ambient light to the diffuse light reflected by a sphere
- Diffuse source intensity is 1.0
Diffuse reflection coefficient is 0.4
Ambient source intensity is 1.0
- Moving from left to right the ambient reflection coefficient takes on values: 0.0, 0.1, 0.3, 0.5, and 0.7
 - Too little ambient light makes shadows too deep and harsh
 - Too much makes the picture look washed out and bland

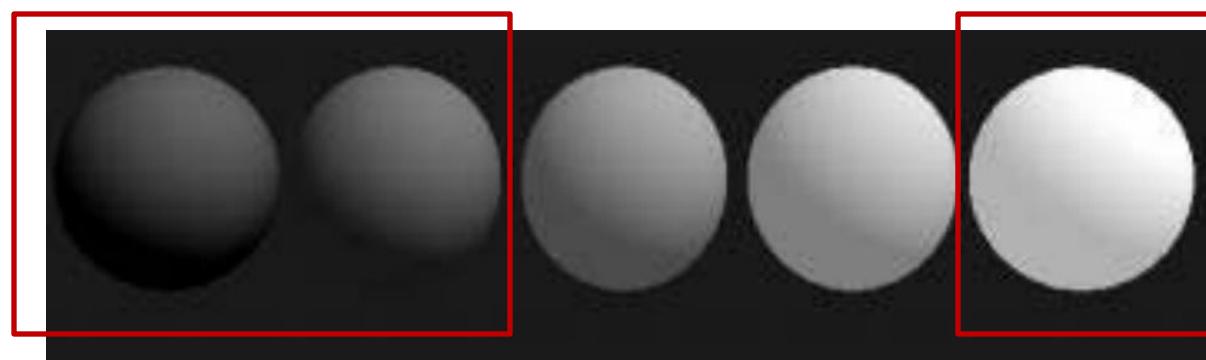
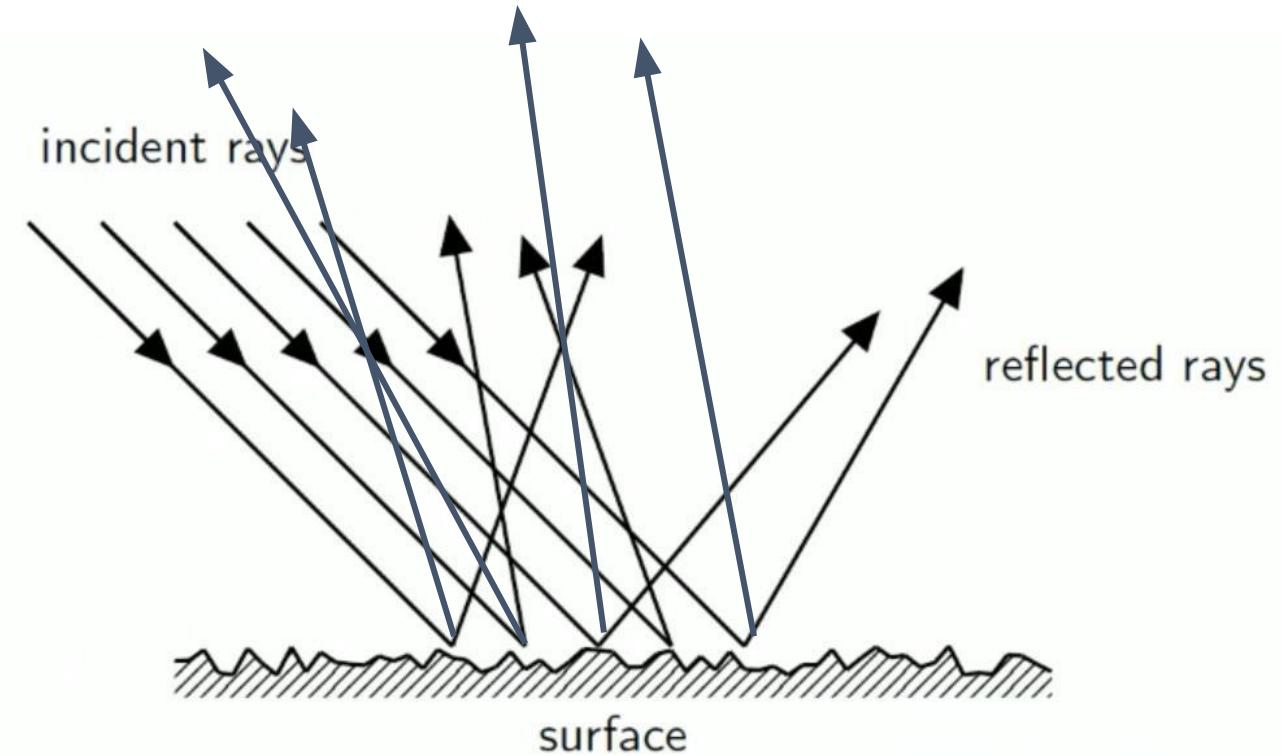


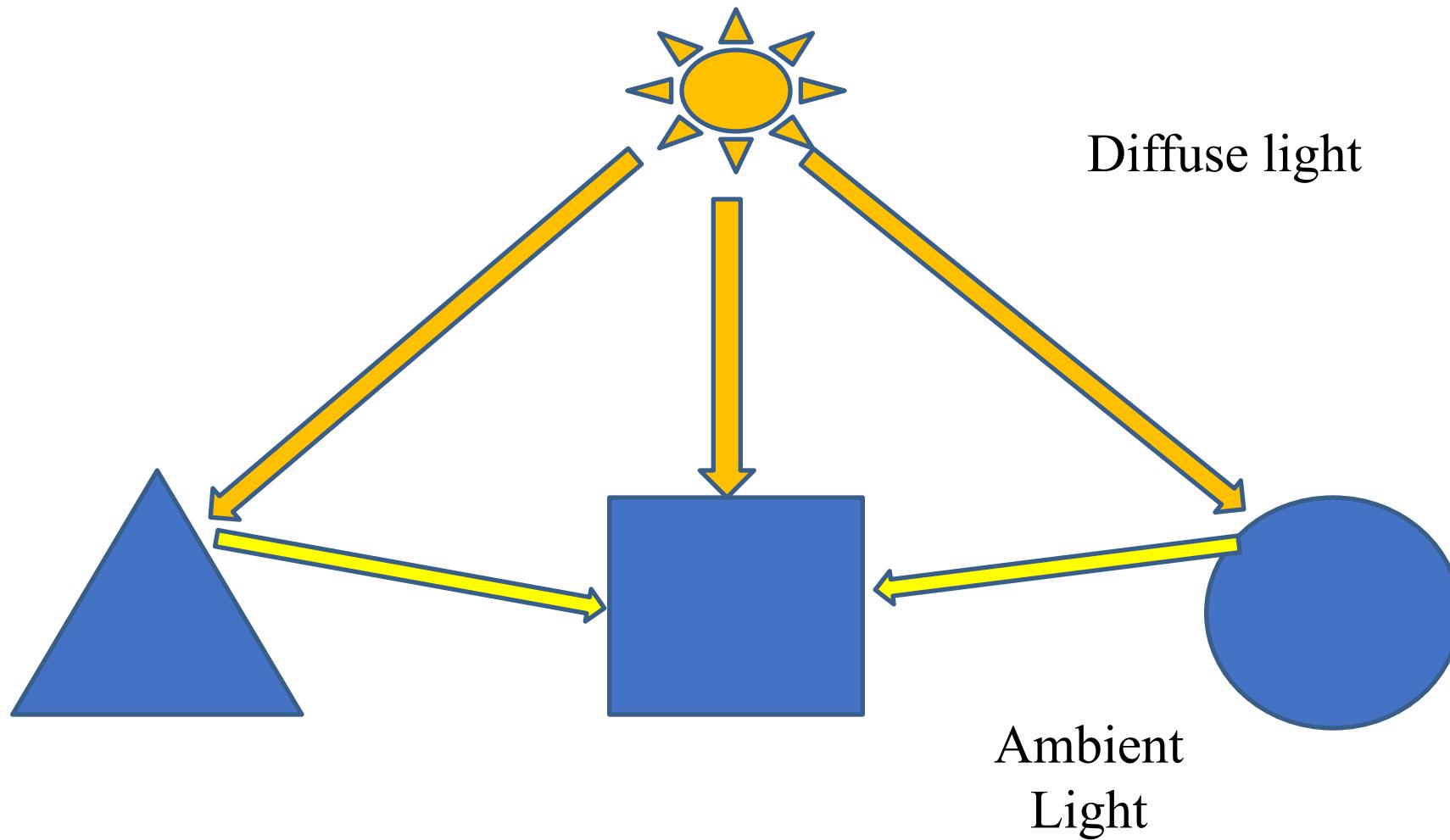
Figure 8.16. On the effect of ambient light.

Diffuse Reflection

- Result of irregular reflection of light
- Light is scattered in all direction. Not all are visible
- Does Not depend on the position of viewpoint

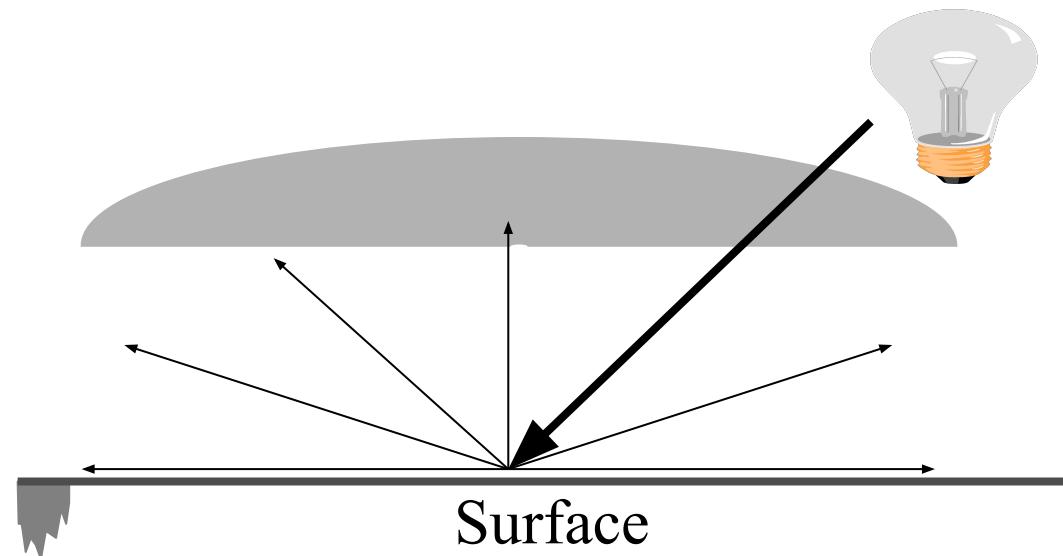


Ambient Light vs Diffuse Light



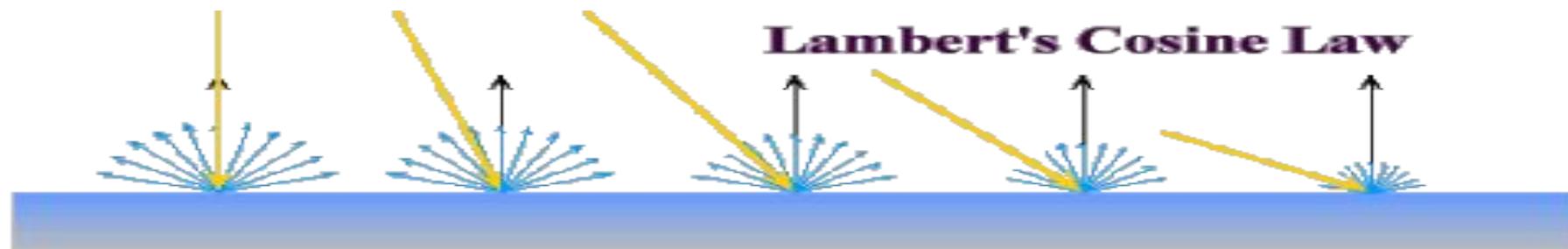
Ideal Diffuse Reflection

- Assumes surface reflects **equally in all directions**.
- An ideal diffuse surface is, at the microscopic level, a very rough surface.
 - Example: chalk, clay, some paints

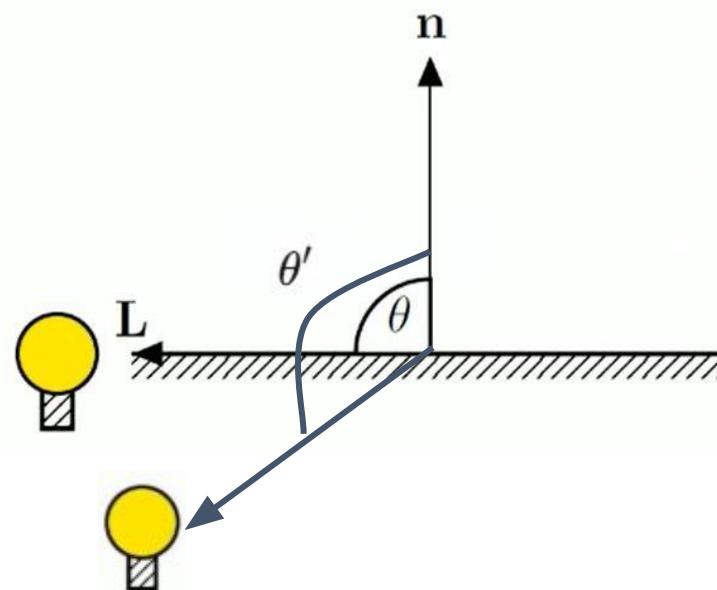
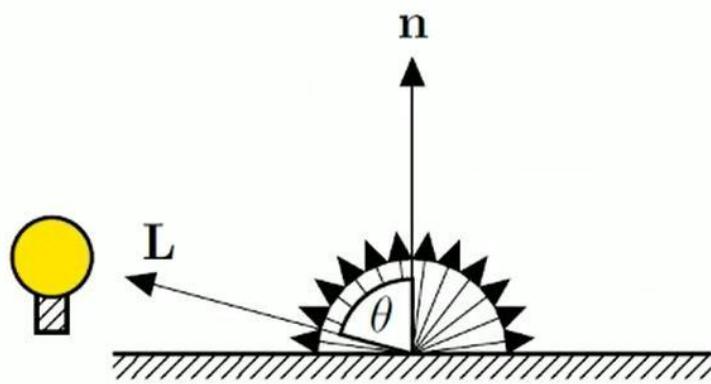
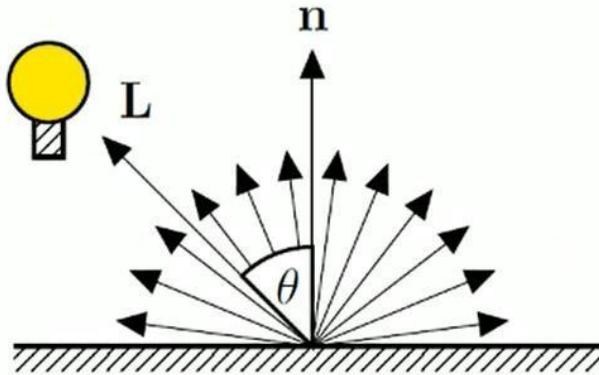
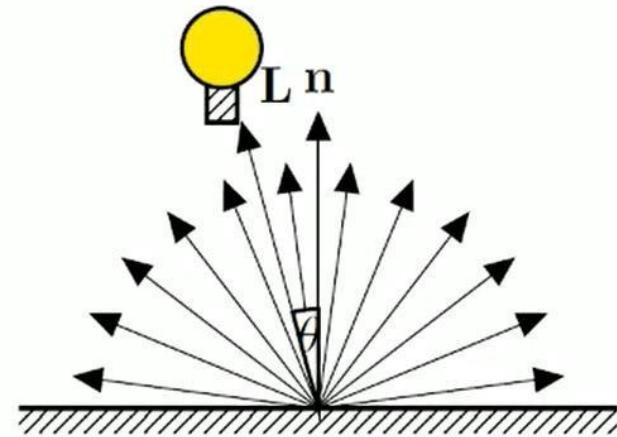


Ideal Diffuse Reflection

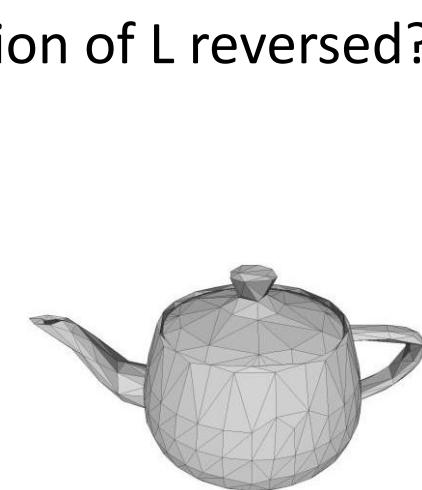
- Ideal diffuse reflectors reflect light according to Lambert's cosine law.
- Lambert's law determines how much of the incoming light energy is reflected. **The reflected intensity is independent of the viewing direction.**
- But reflected light intensity depends on incident angle of light.



Diffuse Reflection (Phong's Model)



- Light reflected equally in all direction.
- Magnitude of reflection depends in incident angle.
- Direction of L reversed? Why?



Diffuse Reflection - Phong's Model

- Phong's diffuse reflection model depends upon the position of the light source relative to the surface

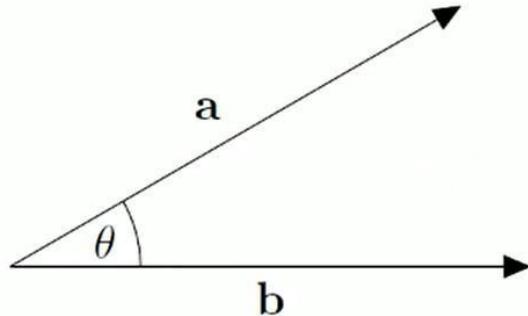


$$D = I_p k_d \max[\cos(\theta), 0]$$

where

- I_p is the intensity of the point light source
- $k_d \in [0, 1]$ is the **diffuse coefficient**
- θ is the angle between the lighting vector and the surface normal
- The $\max[\cos(\theta), 0]$ is used so that no light is reflected if the light source is behind the surface

Diffuse Reflection



- The definition of the dot product is

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos(\theta)$$

- If \mathbf{L} and \mathbf{n} are unit vectors then

$$\mathbf{L} \cdot \mathbf{n} = \cos(\theta)$$

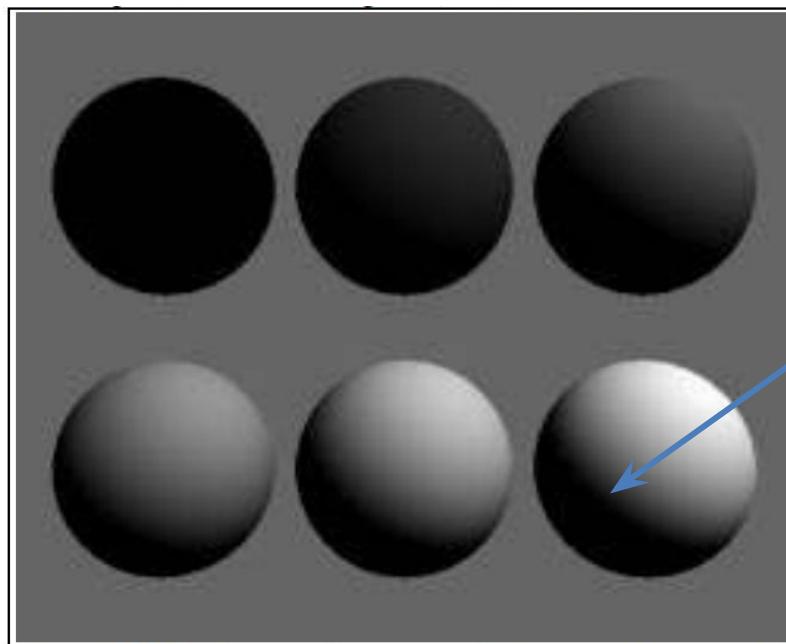
so we can replace the expensive cosine calculation by an easy dot product, i.e,

$$D = I_p k_d \max(\mathbf{L} \cdot \mathbf{n}, 0)$$

Here \mathbf{L} and \mathbf{n} are both unit vectors

Diffuse Reflection Coefficient

- $I_d = \max \{I_s k_d \cos\theta, 0\}$
- Source intensity is 1.0
- Background intensity is 0.4
- Sphere reflecting diffuse light, for six reflection coefficients: 0, 0.2, 0.4, 0.6, 0.8, and 1.

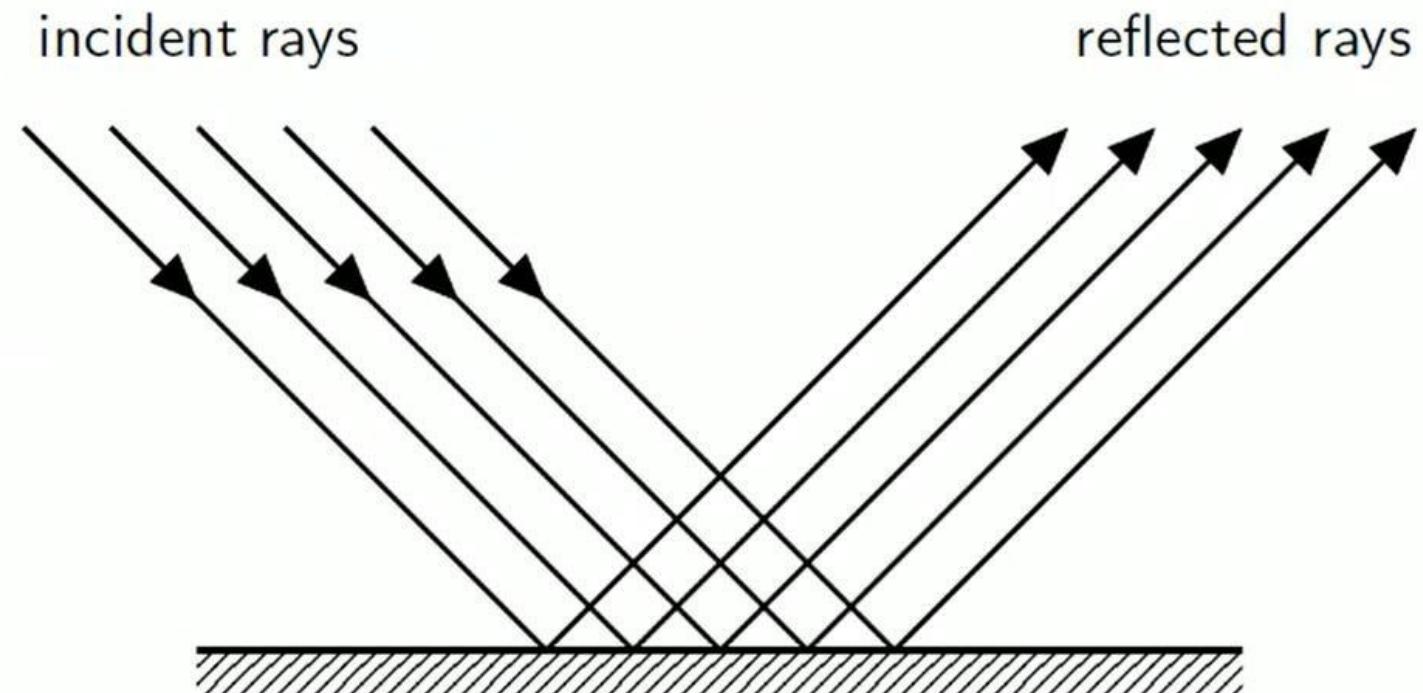


Angle θ between surface normal and incident light is $> 90^\circ$

- What is the ambient component here?
- What is the specular component?

Figure 8.11. Spheres with various reflection coefficients shaded with diffuse light.
 (file: fig8.11.bmp)

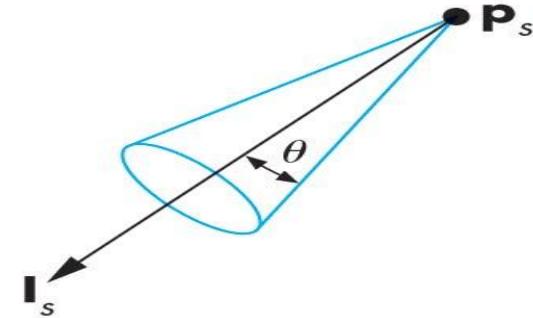
Specular Reflection





Spot Light

- *Spotlight* is a point source that emits light in restricted set of directions,
 - Requires color, point, direction, falloff parameter
 - usually direction boundary forms a cone shape.
 - Here θ is Cut Off Cone. No light is seen at points lying outside Cutoff angle.
- Intensity falls off directionally



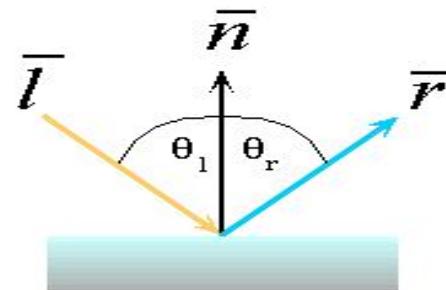
Ideal Specular Reflection

Reflection is only **at mirror angle**. An ideal mirror is a purely specular reflector.

View dependent reflection. That is, reflected light's intensity varies with viewer's position.

Intensity of reflected light is stronger near mirror angle and strongest at mirror angle.

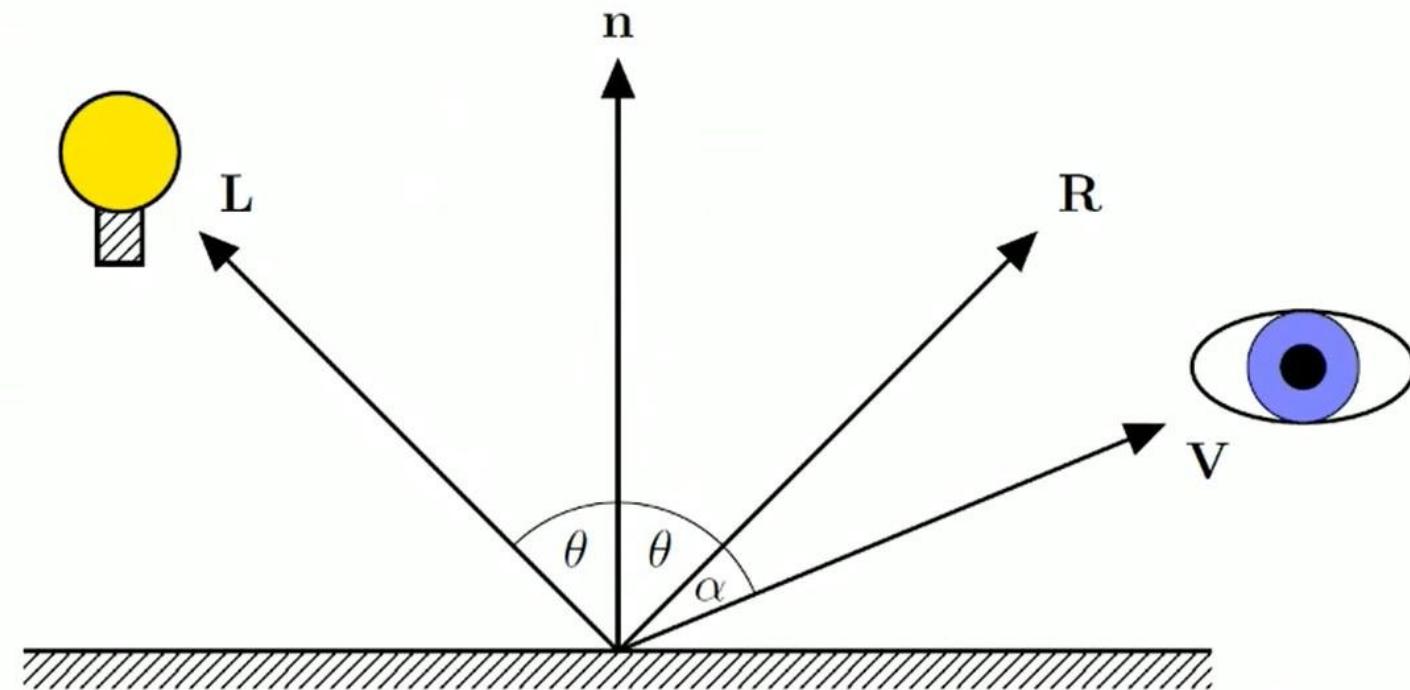
An Ideal specular reflection follows Snell's Law.



Snell's Laws:

- The incoming ray and reflected ray lie in a plane with the surface normal
- The angle that the reflected ray forms with the surface normal equals the angle formed by the incoming ray and the surface normal

Specular Reflection



Specular Reflection

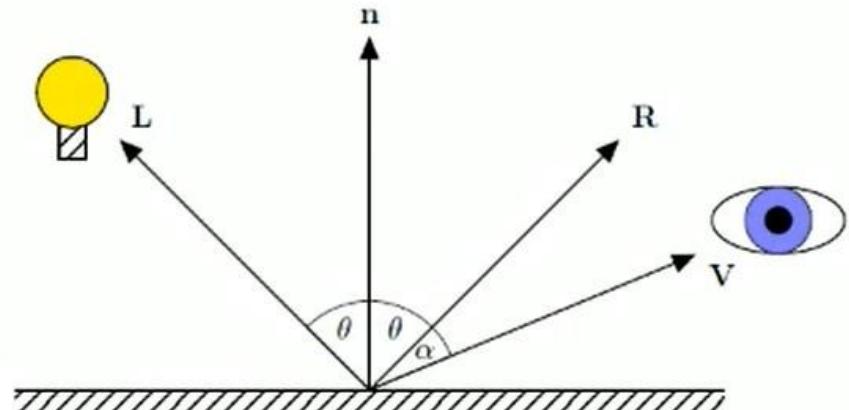


- Phong's specular model is

$$S = I_p k_s \cos^n(\alpha)$$

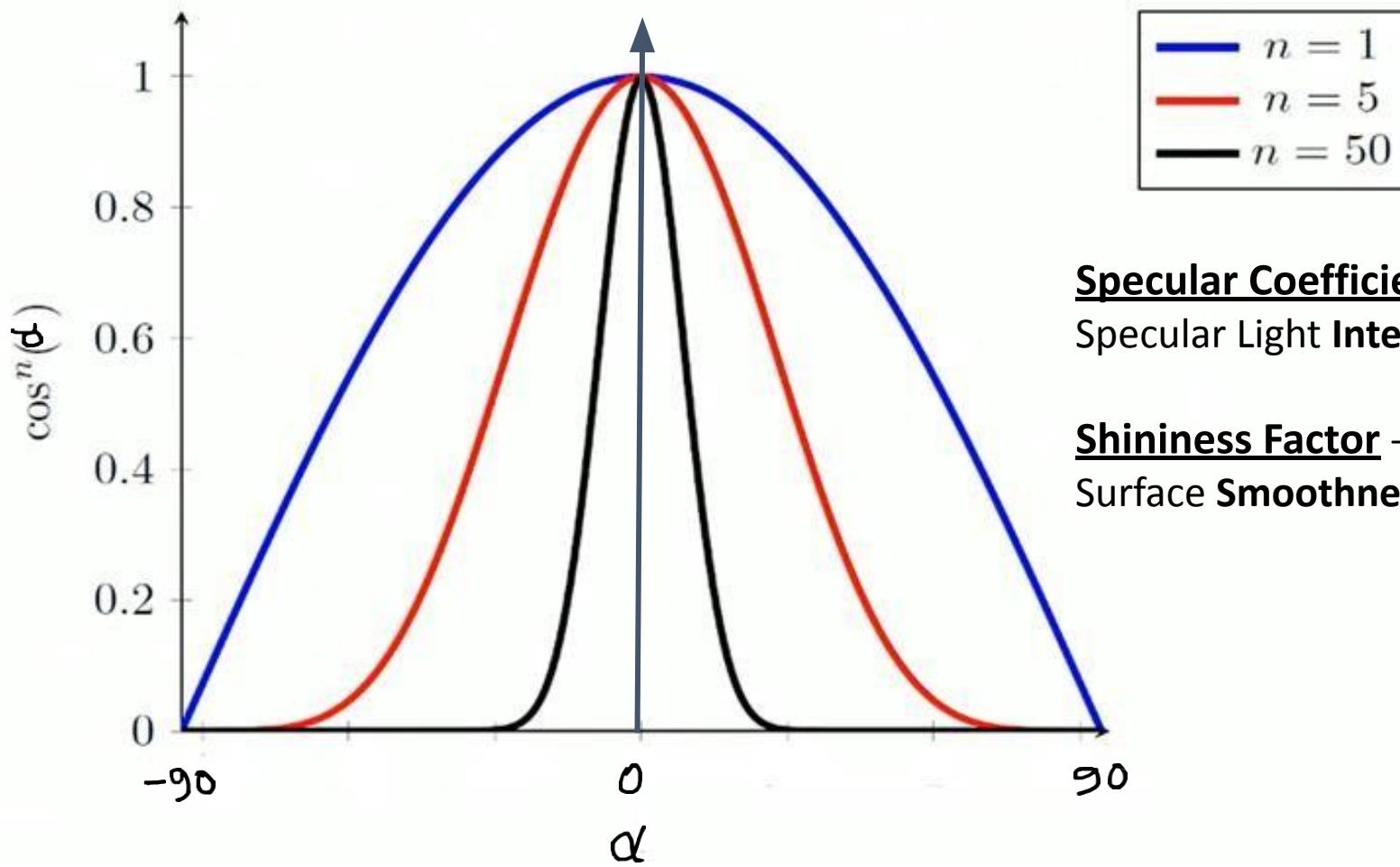
where

- $k_s \in [0, 1]$ is the **specular coefficient**
- n is the **specular exponent/Shininess**
- α is the angle between \mathbf{R} and \mathbf{V}
- The $\cos^n(\alpha)$ term determines the amount of light that is reflected



Specular Reflection

Shininess



Specular Coefficient - Determines
Specular Light Intensity

Shininess Factor - Determines
Surface Smoothness



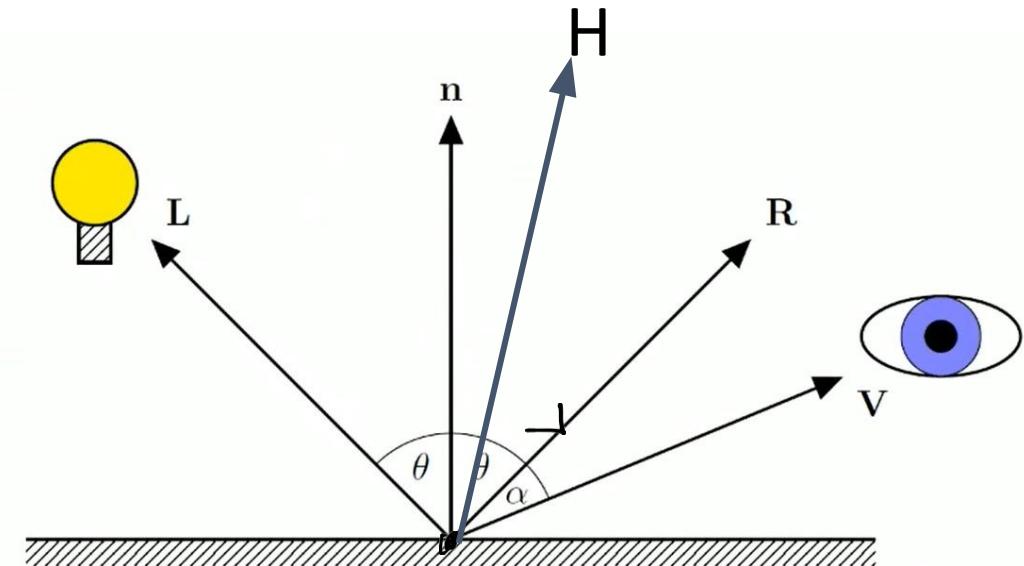
Specular Reflection

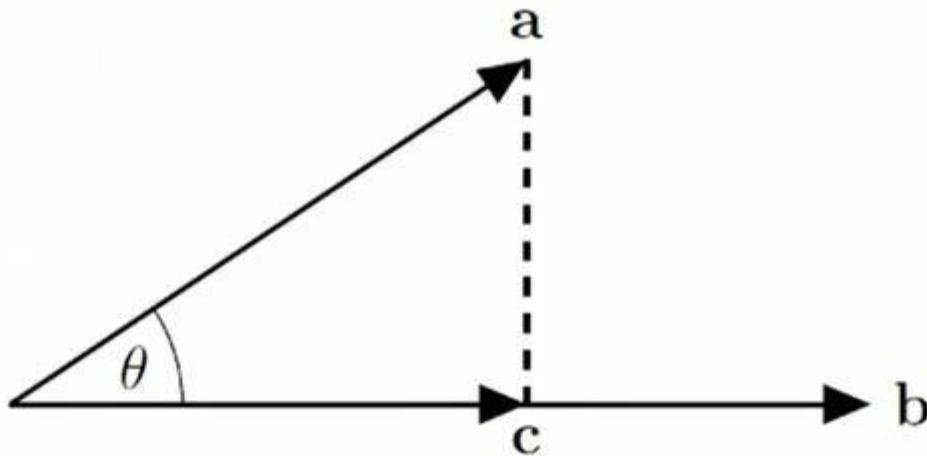


$$S = I_p k_s \cos^n(\alpha)$$

Calculate R, $\cos \alpha$

$\cos \alpha = V \cdot R$





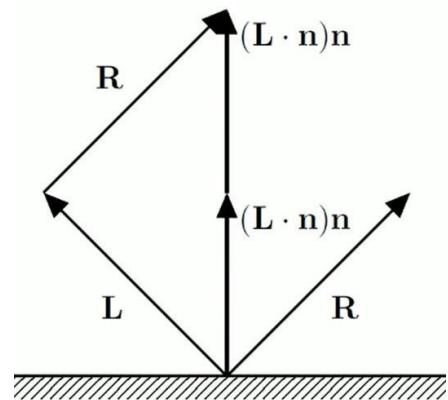
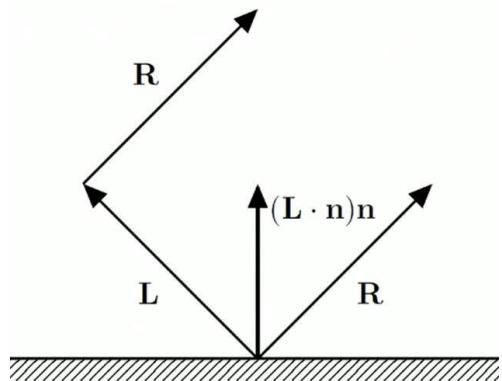
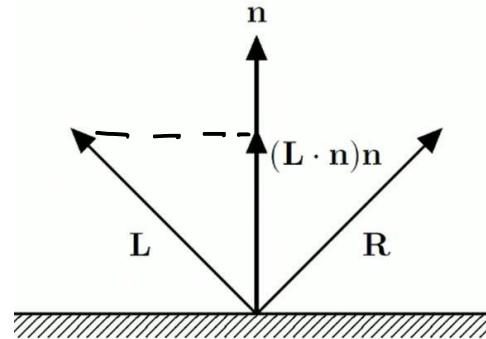
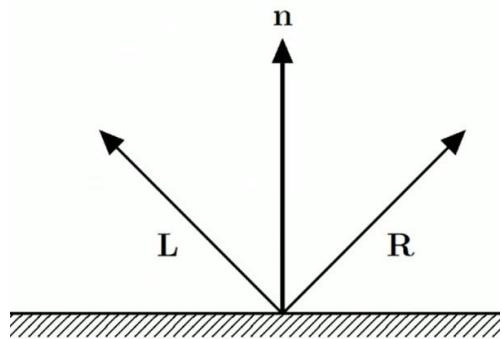
- Using the definition of a dot product

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos(\theta) = |\mathbf{a}| |\mathbf{b}| \frac{|\mathbf{c}|}{|\mathbf{a}|} = |\mathbf{b}| |\mathbf{c}|$$

$$\therefore |\mathbf{c}| = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{b}|}$$

- If \mathbf{b} is a unit vector then $|\mathbf{c}| = \mathbf{a} \cdot \mathbf{b}$ and

$$\mathbf{c} = (\mathbf{a} \cdot \mathbf{b})\mathbf{b}$$

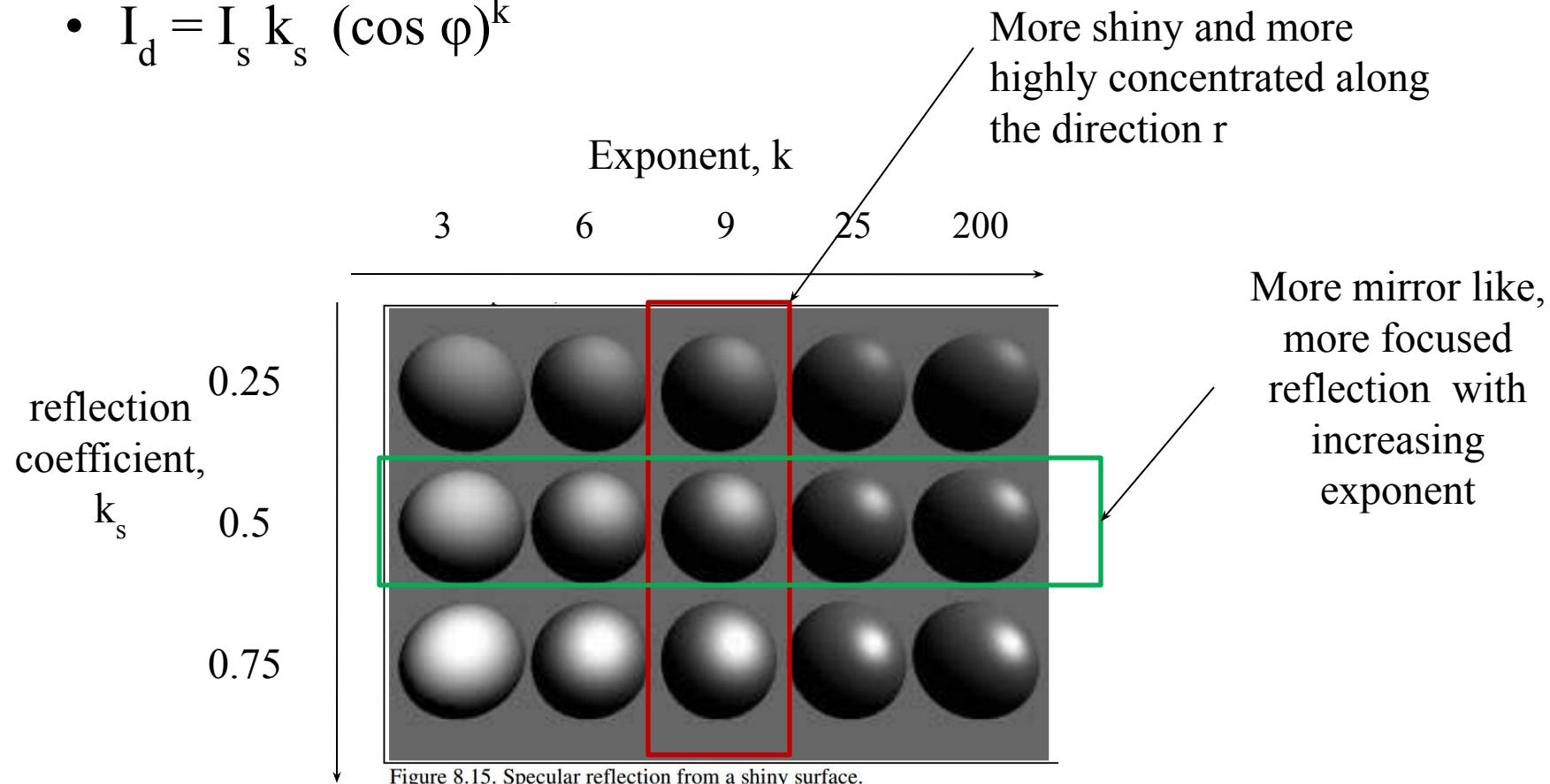


$$\begin{aligned} \mathbf{L} + \mathbf{R} &= 2(\mathbf{L} \cdot \mathbf{n})\mathbf{n} \\ \therefore \mathbf{R} &= 2(\mathbf{L} \cdot \mathbf{n})\mathbf{n} - \mathbf{L} \end{aligned}$$

Here L and n are both unit vectors

Specular Reflection Coefficient

- The ambient and diffuse reflection coefficients are 0.1 and 0.4 for all spheres.
- $I_d = I_s k_s (\cos \varphi)^k$



Blinn and Torrence Variation

- Calculation of \mathbf{R} is computationally expensive. So in phong model the term $\mathbf{R} \cdot \mathbf{V}$ is sometimes replaced by $\mathbf{N} \cdot \mathbf{H}$, where \mathbf{H} is a unit vector that bisect the angle between \mathbf{L} and \mathbf{V} .
 - angle between \mathbf{N} and \mathbf{H} measures the falloff of intensity.
 - Though calculation of $\mathbf{N} \cdot \mathbf{H}$ is computationally inexpensive relative to $\mathbf{R} \cdot \mathbf{V}$, but $\mathbf{N} \cdot \mathbf{H}$ is not always equal to $\mathbf{R} \cdot \mathbf{V}$. In that case calculation of specular component will be approximate.
(See Solved Problem 11.11 in Schaum(2nd edition)))

Steps to Calculate H :

1. Add the light vector L and the view vector V : Here L and V are both unit vectors

$$L + V$$

2. Normalize the resulting vector:

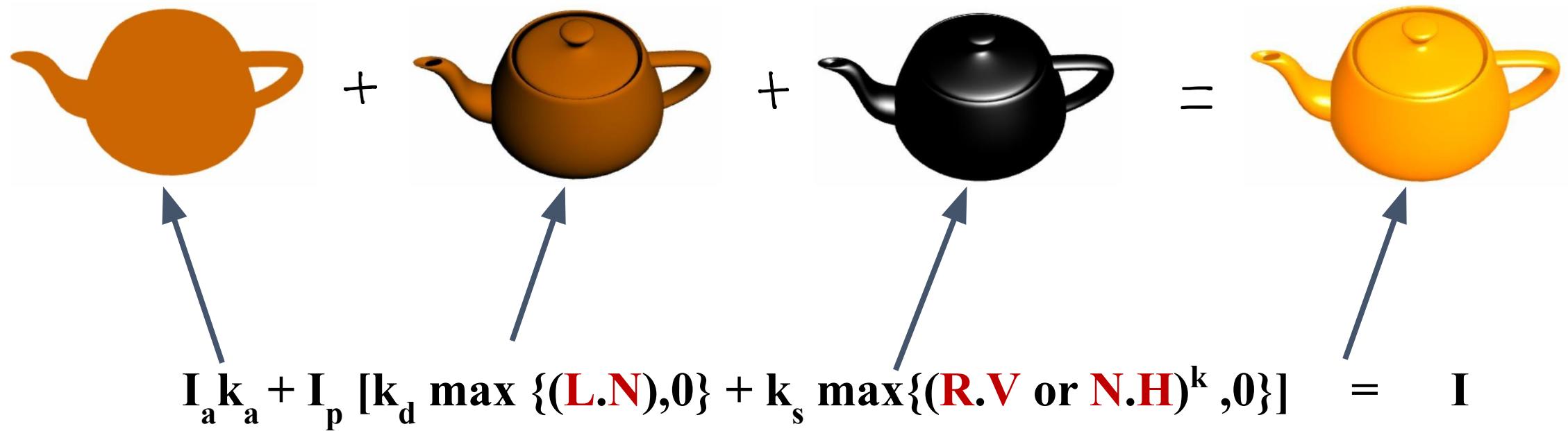
$$H = \frac{L + V}{\|L + V\|}$$

Where $\|L + V\|$ is the magnitude of the vector $L + V$, calculated as:

$$\|L + V\| = \sqrt{(L_x + V_x)^2 + (L_y + V_y)^2 + (L_z + V_z)^2}$$

After computing the magnitude, divide each component of $L + V$ by this magnitude to get the unit vector H .

Phong's Reflection Model



Here all are unit vectors

Mathematical Calculation of Phong Model

We need to incorporate the effect light present in environment. Thus total reflected light also includes ambient component.

$$\text{Ambient Component} = I_a k_a$$

Total reflected light intensity from Q,

$$\begin{aligned} I &= \text{Ambient Component} + \text{Diffuse Component} + \text{Specular Component} \\ &= I_a k_a + I_p k_d (\mathbf{L} \cdot \mathbf{N}) + I_p k_s (\mathbf{R} \cdot \mathbf{V})^k \end{aligned}$$

More specifically,

$$I = I_a k_a + I_p [k_d \max \{(\mathbf{L} \cdot \mathbf{N}), 0\} + k_s \max \{(\mathbf{R} \cdot \mathbf{V})^k, 0\}]$$

Additional Issues

When there are n light sources in the scene, their effects are cumulative: Intensity at Q,

$$I = I_a k_a + \sum_{(i=1 \text{ to } n)} I_{pi} \{ k_d (\mathbf{L} \cdot \mathbf{N}) + k_s (\mathbf{R} \cdot \mathbf{V})^k \}$$

The intensity of red, green and blue component of reflected light,

$$I_r = I_a k_{ar} + I_{pr} k_{dr} (\mathbf{L} \cdot \mathbf{N}) + I_{pr} k_s (\mathbf{R} \cdot \mathbf{V})^k$$

$$I_g = I_a k_{ag} + I_{pg} k_{dg} (\mathbf{L} \cdot \mathbf{N}) + I_{pg} k_s (\mathbf{R} \cdot \mathbf{V})^k$$

$$I_b = I_a k_{ab} + I_{pb} k_{db} (\mathbf{L} \cdot \mathbf{N}) + I_{pb} k_s (\mathbf{R} \cdot \mathbf{V})^k$$

k_s : coefficient for specular component which is same as the color of light source,
 not affected by surface color.

Attenuation

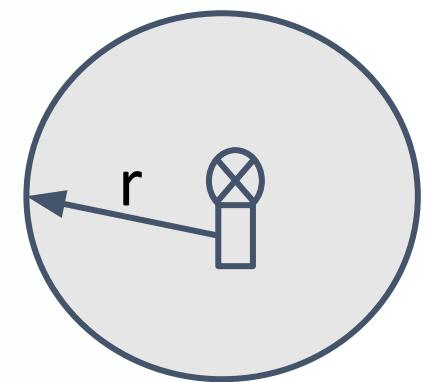
- **Attenuation** is the loss of light energy over space
- In Phong's model attenuation is account for by the variable f_{att} and applied to diffuse and specular components
- Theoretically is should follow the inverse square law, i.e.,

$$f_{att} = \frac{1}{d^2}$$

- In practice this removes too much light, Phong's model uses

$$f_{att} = \max \left(1 - \left(\frac{d}{r} \right)^2, 0 \right)$$

where r is the radius of the light source's sphere of influence



- Combining ambient, diffuse, specular and attenuation models results in Phong's model of reflection

$$I = I_a k_a + I_p f_{att} [k_d \max \{(\mathbf{L} \cdot \mathbf{N}), 0\} + k_s \max \{(\mathbf{R} \cdot \mathbf{V})^k, 0\}]$$

- For multiple light sources, the diffuse and specular components are calculated for each light source and added together, i.e.,

$$I = I_a k_a + \sum_{(i=1 \text{ to } n)} I_{pi} f_{att-i} \{k_d (\mathbf{L} \cdot \mathbf{N}) + k_s (\mathbf{R} \cdot \mathbf{V})^k\}$$

Exercise

A light source with **intensity 15** and **radius of influence 80** is located at **(4,2,3)** from which you are called to calculate the illumination of a point on the **xz** plane. The **camera is set at origin** and the light is reflected back from point **(0, 4)** of the plane. The **ambient, diffuse and specular coefficient** is given at **0.4, 0.2, 0.3**.

- a. For the above phenomenon, represent the **reflected ray R** in the unit vector.
- b. Calculate the **specular reflection intensity** for a **shininess factor of 99**.
- c. Calculate the **attenuation factor** for the given point in the above scenario.
- d. If the **ambient light** intensity is at **2**, calculate the **total reflected light intensity** at the given point according to **phong's model** with the **attenuation factor**.

Color

- Color is constructed by adding certain amounts of red, green, and blue light
- Light sources have three “types” of color:
- **ambient** = (I_{ar}, I_{ag}, I_{ab}) , **diffuse** = (I_{dr}, I_{dg}, I_{db}) , and **specular** = $(I_{spr}, I_{spg}, I_{spb})$
 - Ambient reflection coefficient : k_{ar} , k_{ag} , k_{ab}
 - Diffuse reflection coefficient : k_{dr} , k_{dg} , k_{db}
- Color of the specular component is often the same as that of the light source – mirror like, so the specular reflection coefficient:
 k_{sr} , k_{sg} , $k_{sb} = k_s$
 - e.g. **Glossy** red apple when illuminated by a yellow light is yellow rather than red

Color of an Object

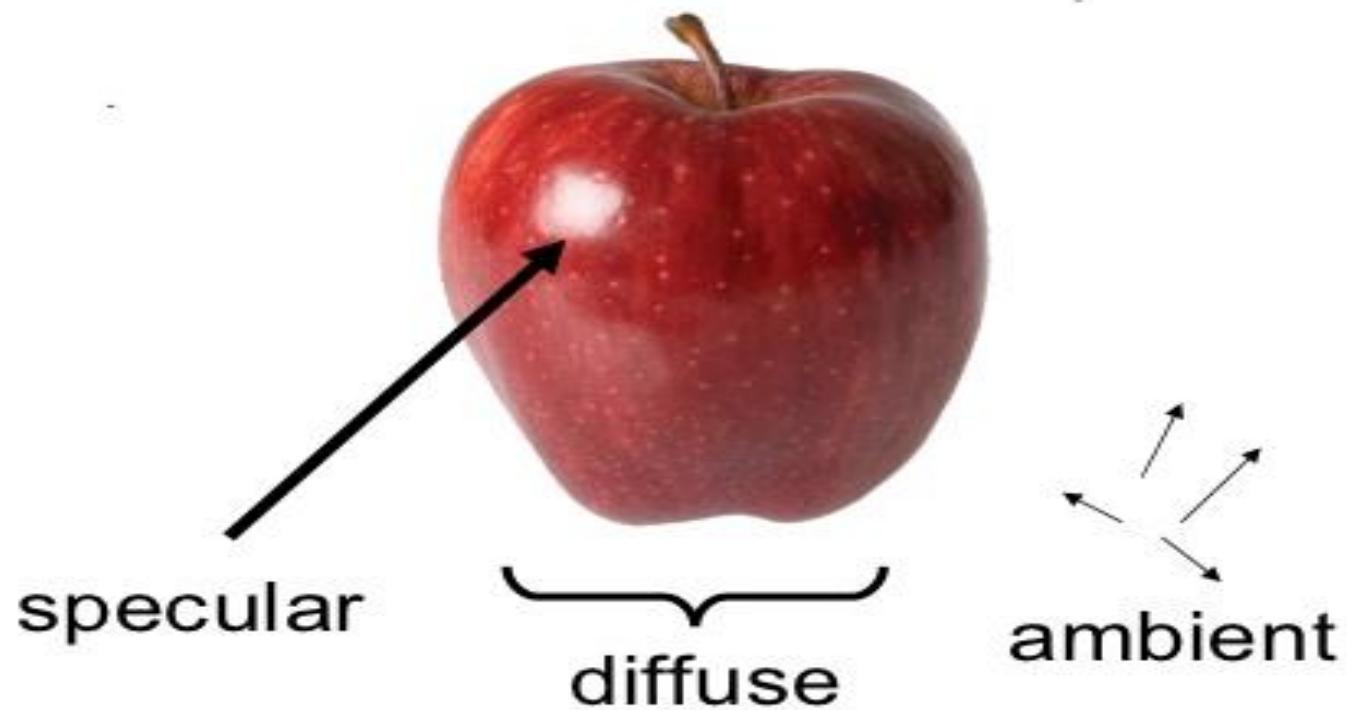


Image synthesis view:

“light, surface, and material interact to reflect light perceived as color,
modeled via simplifying assumptions”

Color of an Object

- By “color” of a surface we mean the color that is reflected from it when the illuminated by white light
- The ambient and diffuse reflection coefficients are based on the color of the surface itself.
- The specular reflection coefficient is based on the color of the light source (and the roughness property of the surface)

Color of an Object

- Color of a sphere is 30% red, 45% green, and 25% blue
- Light source in environment:

Monochrome light (Intensity : $I_{sr} = I_{sg} = I_{sb} = I_s$)

- Sphere's ambient and diffuse reflection coefficients:

$$k_{ar} = k_{dr} = 0.30k$$

$$k_{ag} = k_{dg} = 0.45k$$

$$k_{ab} = k_{db} = 0.25k$$

here, k is some scaling value

- The individual ambient and diffuse components have intensities

$$I_{ar} = 0.30k I_a, I_{dr} = 0.30k I_s \cos\theta$$

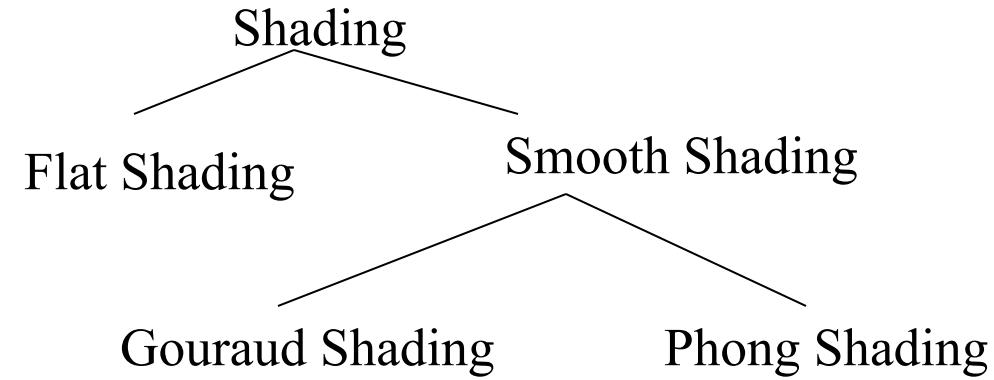
$$I_{ag} = 0.45k I_a, I_{dg} = 0.45k I_s \cos\theta$$

$$I_{ab} = 0.25k I_a, I_{db} = 0.25k I_s \cos\theta$$

- If the environment is uniformly lighted, $\theta = 0^\circ$ for sphere!!

Shading

- The process of assigning colors to pixels.



Shading Model

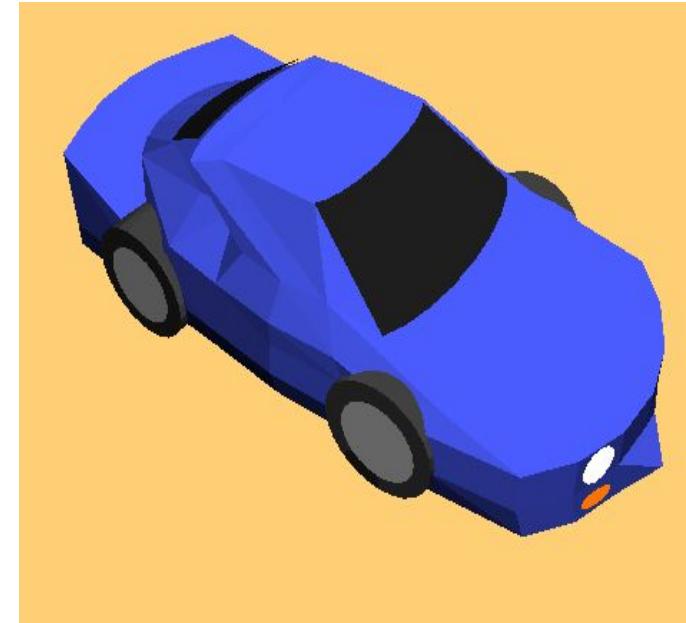
- Flat Shading
 - Compute Phong lighting once for entire polygon
- Gouraud Shading
 - Compute Phong lighting at the vertices and interpolate lighting values across polygon
- Phong Shading
 - Interpolate normals across polygon and perform Phong lighting across polygon

Flat Shading

- For each polygon
 - Determines a single intensity value at a chosen point on the polygon
 - Uses that value to shade the entire polygon
- Assumptions
 - Light source at infinity
 - Viewer at infinity
 - The polygon represents the actual surface being modeled

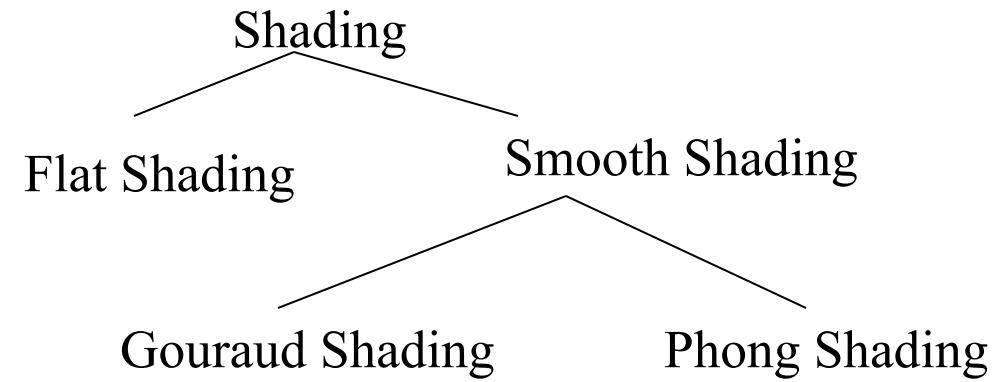
Problems of Flat Shading

- Specular highlights tends to get lost
- If chosen point on polygon is at location of the light source, then color of the polygon will be significantly distorted.



Shading

- The process of assigning colors to pixels.



Shading Model

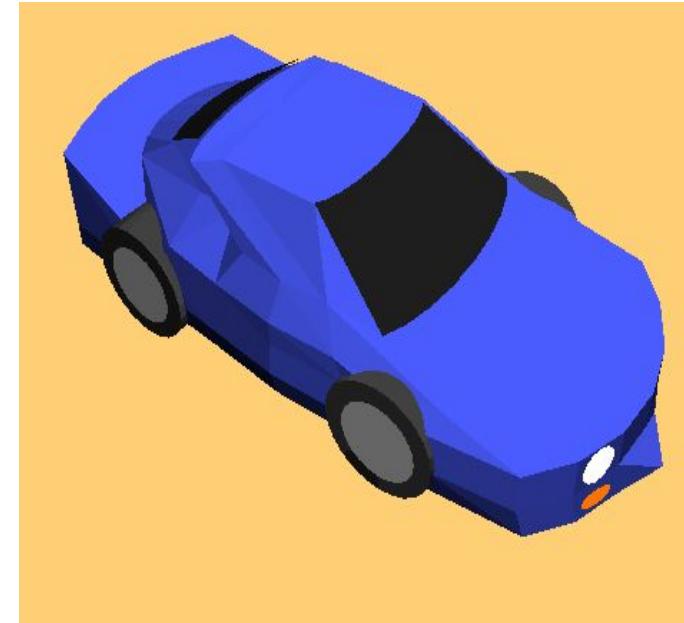
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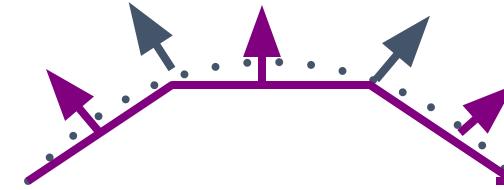
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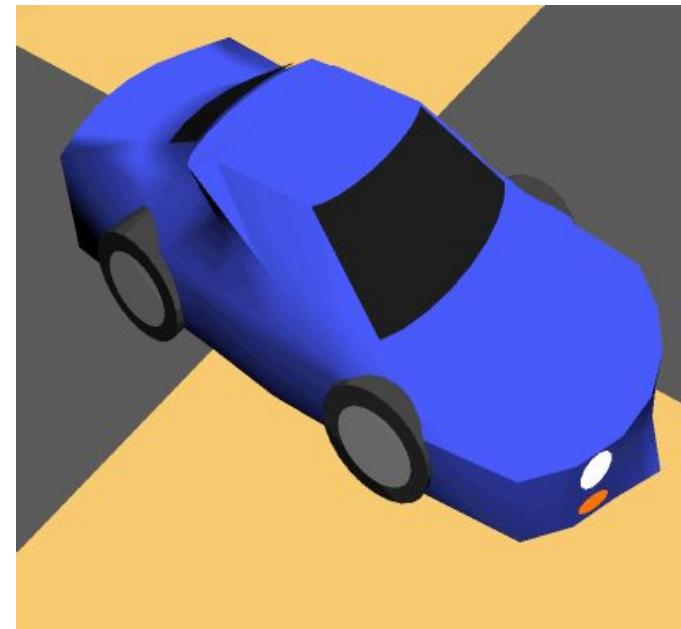
Smooth Shading

- Introduce vertex normals at each vertex
 - Used only for shading
 - Think of as a better approximation of the real surface that the polygons approximate
 - Finds color value for each point in the polygon individually
- Two types
 - Gouraud Shading
 - Phong Shading (do not confuse with Phong Lighting Model)

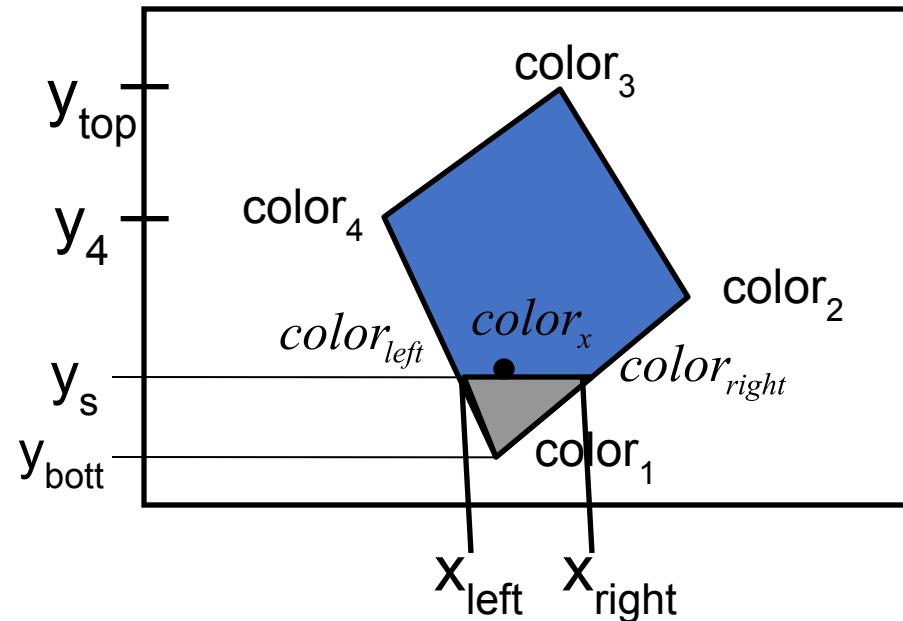


Gouraud Shading

- Most common approach
- Perform Phong lighting at the vertices
- Linearly interpolate the resulting colors over faces
 - Along edges
 - Along scanline



Gouraud Shading

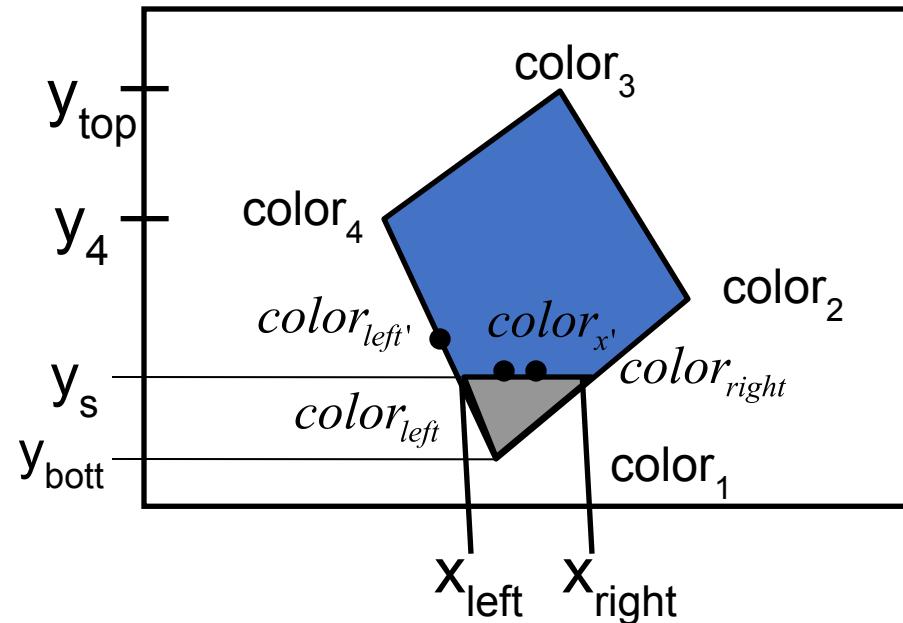


$$color_{left} = color_1 + (color_4 - color_1) \frac{y_s - y_{bott}}{y_4 - y_{bott}}$$

$$color_{right} = color_1 + (color_2 - color_1) \frac{y_s - y_{bott}}{y_2 - y_{bott}}$$

$$color_x = color_{left} + (color_{right} - color_{left}) \frac{x - x_{left}}{x_{right} - x_{left}}$$

Gouraud Shading



$$color_x = color_{left} + (color_{right} - color_{left}) \frac{x - x_{left}}{X_{right} - X_{left}}$$

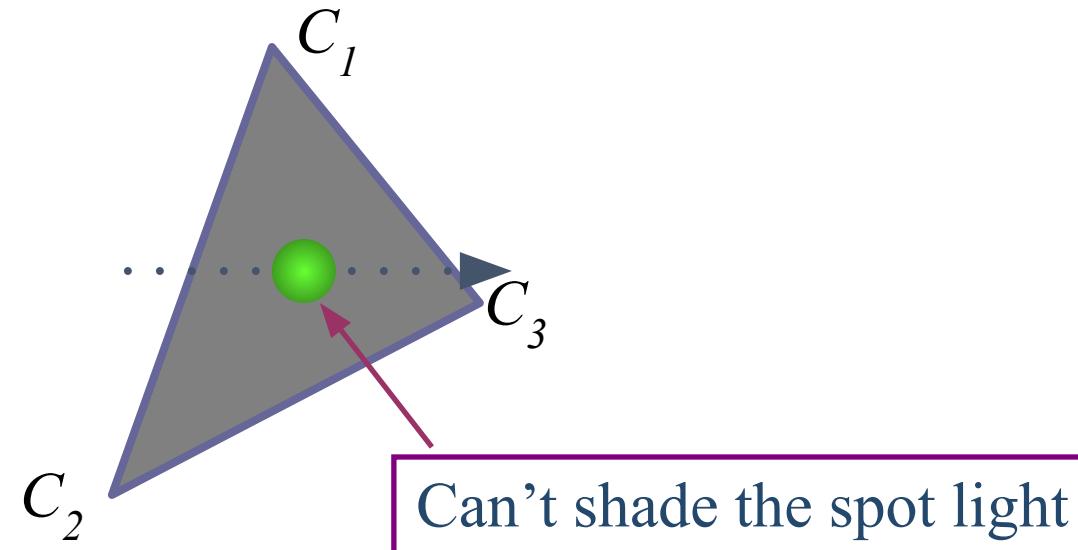
$$color_{x'} = color_x + K \Delta x$$

$$color_{left'} = color_{left} + K' \Delta y$$

Calculate the surface normals along the scan line and the edge using incremental approach

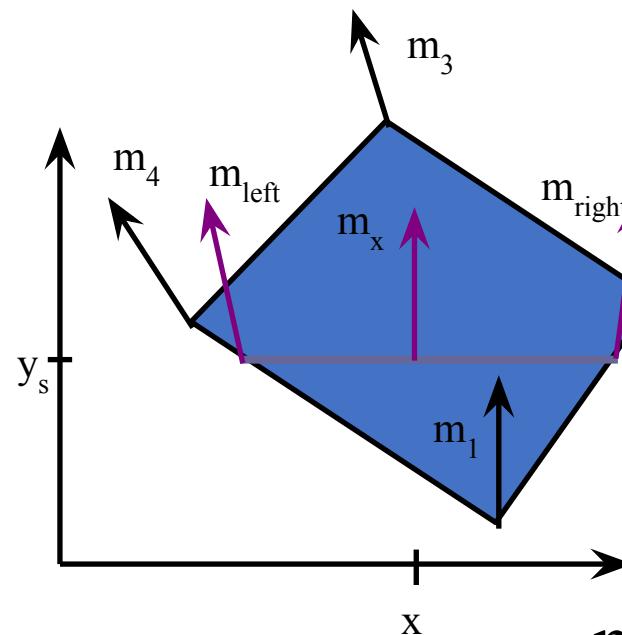
Problem of Gouraud Shading

- Often appears dull
- Lacks accurate specular component



Phong Shading

Interpolate normal vectors of face vertices at each pixel, then perform Phong lighting at each pixel.



$$m_{\text{left}} = m_1 + (m_4 - m_1) \frac{y_s - y_1}{Y_4 - y_1}$$

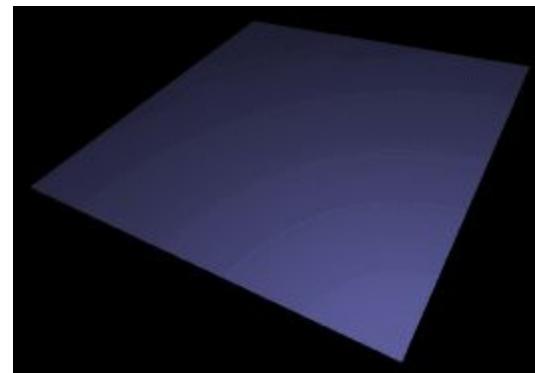
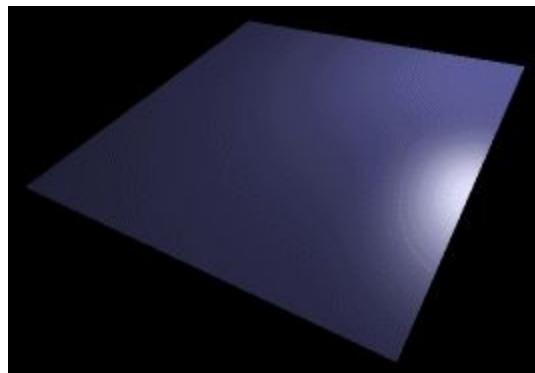
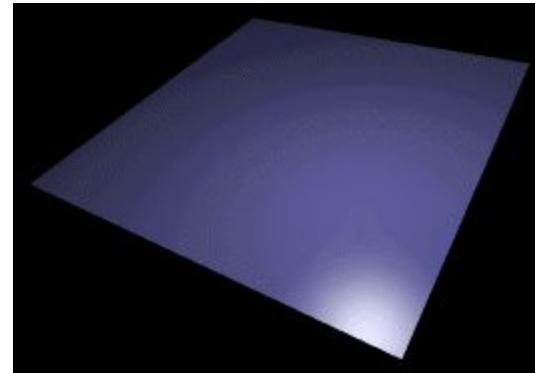
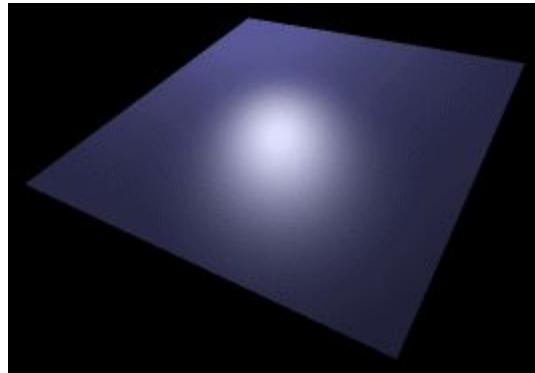
$$m_{\text{right}} = m_1 + (m_2 - m_1) \frac{y_s - y_1}{Y_2 - y_1}$$

$$m_x = m_{\text{left}} + (m_{\text{right}} - m_{\text{left}}) \frac{x - x_{\text{left}}}{x_{\text{right}} - x_{\text{left}}}$$

Calculate the surface normals along the scan line and the edge using incremental approach

Phong vs Gouraud Shading

- Phong shading is more smooth
- If a highlight does not fall on a vertex, Gouraud shading may miss it completely, but Phong shading does not.



Reference:

Computer Graphics: Principles and Practice: John F. Hughes, James D. Foley, Andries van Dam, Steven K. Feiner (2nd Edition)

Chapter: 16.1.1 - 16.1.4, 16.1.6, 16.2.1 - 16.2.5