

1. Give a context-free grammar for each of the following languages.

- a) $L = \{w \mid w \text{ contains even number of } 0's\}$
- b) $L = \{w \mid w \text{ contains twice as many } 1s \text{ as } 0s\}$
- c) $L = \{w \mid w \text{ contains even number of } 0s \text{ and } 1s\}$
- d) $L = \{w \mid \text{where each } 0's \text{ is followed by at least as many } 1's\}$
- e) $L(G) = \{ a^i b^j c^k \mid i, j, k \geq 0 \text{ and } i=j \text{ or } i=k \}. \Sigma = \{a, b, c\}$
- f) $L(G) = \{ a^i b^j c^k \mid j > i+k \}. \Sigma = \{a, b, c\}$
- g) $L(G) = \{ a^n b^m \mid 0 < n < m < 3n \}. \Sigma = \{a, b\}$
- h) $L(G) = \text{set of all strings } w \text{ over } \{a, b\} \text{ such that } w \text{ is not palindrome.}$
- i) $L = \{w \mid w = w^R \text{ AND } |w| \text{ is even, } w \text{ is a palindrome}\}$
- j) $L(G) = \{ a^i b^j c^k \mid i, j, k \geq 0 \text{ and } i=j \text{ or } j=k \}. \Sigma = \{a, b, c\}$
- k) $L(G) = \{ a^n b^m c^m d^{2n} \mid n \geq 0, m > 0 \}$
- l) $L = \{w \mid w \text{ contains at least 4 a's}\}$

2. What does the following CFGs do?

- a) $S \rightarrow ZSZ \mid 0$
 $Z \rightarrow 0 \mid 1$
- b) $S \rightarrow 0E0 \mid 1E1 \mid \epsilon$
 $E \rightarrow 1E \mid 0E \mid \epsilon$
- c) $S \rightarrow AB$
 $A \rightarrow 0A1 \mid \epsilon$
 $B \rightarrow 1B \mid \epsilon$
- d) $S \rightarrow \epsilon \mid 1S1S1S0S \mid 1S1S0S1S \mid 1S0S1S1S \mid 0S1S1S1S$
- e) $S \rightarrow aSbb \mid aSb \mid \epsilon$

3. Convert the following Regular expressions to a CFG.

- a) $a(b \mid c^*)$
- b) $0^*1(0 + 1)^*$

- c) $(a + b)^*(a^* + (ba)^*)$
- d) $(a+b)^* aa (a+b)^*$
- e) $a^* + a(a|b)^*$

4. Consider the following context-free grammar $\Sigma=\{0,1\}$. Give leftmost and rightmost derivations for the following strings and check parse-tree ambiguity.

- a) $S \rightarrow 0A \mid 1B$
 $A \rightarrow 0AA \mid 1S \mid 1$
 $B \rightarrow 0S \mid 1BB \mid 0$

String: 001101

- b) $S \rightarrow A \ 1 \ B$
 $A \rightarrow 0A \mid \epsilon$
 $B \rightarrow 0B \mid 1B \mid \epsilon$

Strings: 10100, 0010101

- c) $D \rightarrow TL$
 $T \rightarrow c \mid Tc$
 $L \rightarrow L.V \mid V$
 $V \rightarrow a \mid b \mid 0 \mid 1 \mid Va \mid Vb \mid V0 \mid V1$

Strings: cabb0011.ab1 (Rightmost derivation)

- d) $S \rightarrow S + S$
 $S \rightarrow Sa \mid Sb \mid \epsilon$

String: abb + aab + baba

- e) $S \rightarrow SA \mid \epsilon$
 $A \rightarrow aa \mid ab \mid ba \mid bb$

String: aabbba

- f) $S \rightarrow aEbS$
 $S \rightarrow aEbScS \mid \epsilon$
 $E \rightarrow d$

String: adbadbadbcb

5. Are the following CFGs ambiguous? Are they inherently ambiguous? If not, then give its unambiguous representation.

a) $S \rightarrow Ab \mid AaB$

$A \rightarrow a \mid Aa$

$B \rightarrow b$

b) $S \rightarrow aS \mid aSbS \mid \epsilon$