1. (a) If the graph of f is shifted 3 units upward, its equation becomes $y = f(x) + 3$.

   (b) If the graph of f is shifted 3 units downward, its equation becomes $y = f(x) - 3$.

   (c) If the graph of f is shifted 3 units to the right, its equation becomes $y = f(x - 3)$.

   (d) If the graph of f is shifted 3 units to the left, its equation becomes $y = f(x + 3)$.

   (e) If the graph of f is reflected about the x-axis, its equation becomes $y = -f(x)$.

   (f) If the graph of f is reflected about the y-axis, its equation becomes $y = f(-x)$.

   (g) If the graph of f is stretched vertically by a factor of 3, its equation becomes $y = 3f(x)$.

   (h) If the graph of f is shrunk vertically by a factor of 3, its equation becomes $y = \frac{1}{3}f(x)$.

2. (a) To obtain the graph of $y = 5f(x)$ from the graph of $y = f(x)$, stretch the graph vertically by a factor of 5.

   (b) To obtain the graph of $y = f(x - 5)$ from the graph of $y = f(x)$, shift the graph 5 units to the right.

   (c) To obtain the graph of $y = -f(x)$ from the graph of $y = f(x)$, reflect the graph about the x-axis.

   (d) To obtain the graph of $y = -5f(x)$ from the graph of $y = f(x)$, stretch the graph vertically by a factor of 5 and reflect it about the x-axis.

   (e) To obtain the graph of $y = f(5x)$ from the graph of $y = f(x)$, shrink the graph horizontally by a factor of 5.

   (f) To obtain the graph of $y = 5f(x) - 3$ from the graph of $y = f(x)$, stretch the graph vertically by a factor of 5 and shift it 3 units downward.

3. (a) (graph 3) The graph of f is shifted 4 units to the right and has equation $y = f(x-4)$.

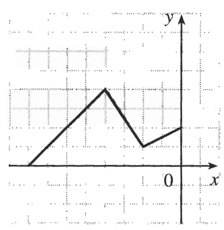
(b) (graph 1) The graph of f is shifted 3 units upward and has equation $y = f(x) + 3$.

(c) (graph 4) The graph of f is shrunk vertically by a factor of 3 and has equation $y = \frac{1}{3}f(x)$.

(d) (graph 5) The graph of f is shifted 4 units to the left and reflected about the x-axis. Its equation is $y = -f(x+4)$.

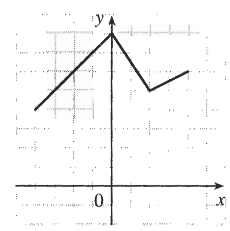
(e) (graph 2) The graph of f is shifted 6 units to the left and stretched vertically by a factor of 2. Its equation is $y = 2f(x+6)$.

4. (a) To graph $y = f(x+4)$ we shift the graph of f, 4 units to the left.



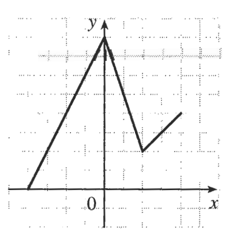
The point $(2, 1)$ on the graph of f corresponds to the point $(2-4, 1) = (-2, 1)$.

(b) To graph $y = f(x) + 4$ we shift the graph of f, 4 units upward.



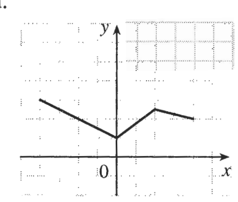
The point $(2, 1)$ on the graph of f corresponds to the point $(2, 1+4) = (2, 5)$.

(c) To graph $y = 2f(x)$ we stretch the graph of f vertically by a factor of 2.



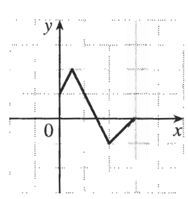
The point $(2, 1)$ on the graph of f corresponds to the point $(2, 2 \cdot 1) = (2, 2)$.

(d) To graph $y = -\frac{1}{2}f(x) + 3$, we shrink the graph of f vertically by a factor of 2, then reflect the resulting graph about the x-axis, then shift the resulting graph 3 units upward.



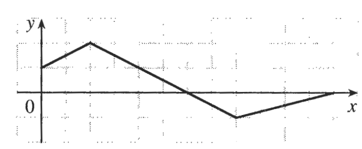
The point $(2, 1)$ on the graph of f corresponds to the point $\left(2, -\frac{1}{2} \cdot 1 + 3\right) = (2, 2.5)$.

5. (a) To graph $y = f(2x)$ we shrink the graph of f horizontally by a factor of 2.



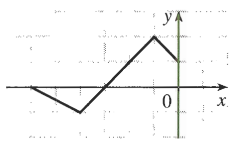
The point $(4, -1)$ on the graph of f corresponds to the point $\left(\frac{1}{2} \cdot 4, -1\right) = (2, -1)$.

(b) To graph $y = f\left(\frac{1}{2}x\right)$ we stretch the graph of f horizontally by a factor of 2.



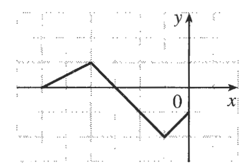
The point $(4, -1)$ on the graph of f corresponds to the point $(2 \cdot 4, -1) = (8, -1)$.

(c) To graph $y = f(-x)$ we reflect the graph of f about the y-axis.



The point $(4, -1)$ on the graph of f corresponds to the point $(-1 \cdot 4, -1) = (-4, -1)$.

(d) To graph $y = -f(-x)$ we reflect the graph of f about the y-axis, then about the x-axis.



The point $(4, -1)$ on the graph of f corresponds to the point $(-1 \cdot 4, -1 \cdot -1) = (-4, 1)$.

6. The graph of $y = f(x) = \sqrt{3x - x^2}$ has been shifted 2 units to the right and stretched vertically by a factor of 2. Thus, a function describing the graph is

$$y = 2f(x-2) = 2\sqrt{3(x-2) - (x-2)^2} = 2\sqrt{3x - 6 - (x^2 - 4x + 4)} = 2\sqrt{-x^2 + 7x - 10}$$

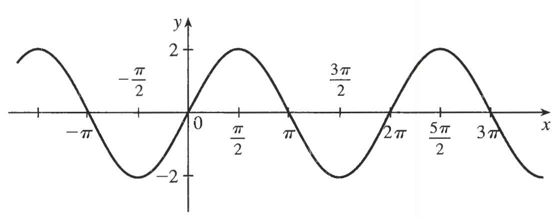
7. The graph of $y = f(x) = \sqrt{3x - x^2}$ has been shifted 4 units to the left, reflected about the x-axis, and shifted downward 1 unit. Thus, a function describing the graph is

$$y = \underbrace{-1 \cdot}_{\substack{\text{reflect} \\ \text{about } x\text{-axis}}} \quad f \underbrace{(x+4)}_{\substack{\text{shift} \\ \text{4 units left}}} \quad \underbrace{-\ 1}_{\substack{\text{shift} \\ \text{1 unit left}}}$$

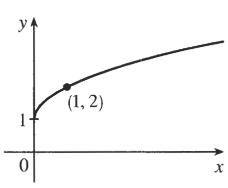
This function can be written as

$$y = -f(x+4) - 1 = -\sqrt{3(x+4) - (x+4)^2} - 1 = -\sqrt{3x + 12 - (x^2 + 8x + 16)} - 1 = -\sqrt{-x^2 - 5x - 4} - 1$$

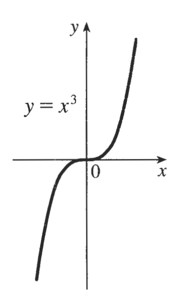
8. (a) The graph of $y = 2\sin x$ can be obtained from the graph of $y = \sin x$ by stretching it vertically by a factor of 2.

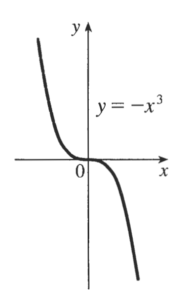


(b) The graph of $y = 1 + \sqrt{x}$ can be obtained from the graph of $y = \sqrt{x}$ by shifting it upward 1 unit.

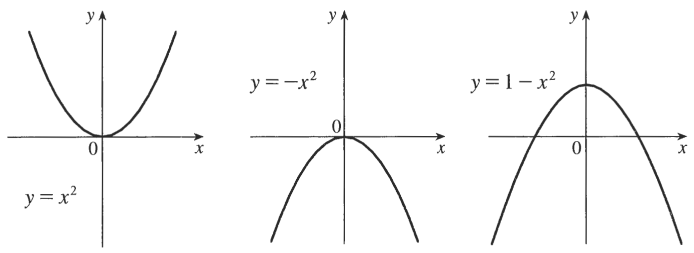


9. $y = -x^3$: Start with the graph of $y = x^3$ and reflect about the x-axis. Note: Reflecting about the y-axis gives the same result since substituting $-x$ for x gives us $y = (-x)^3 = -x^3$.

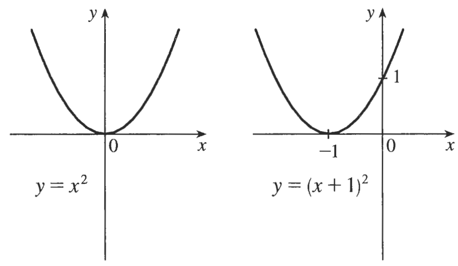




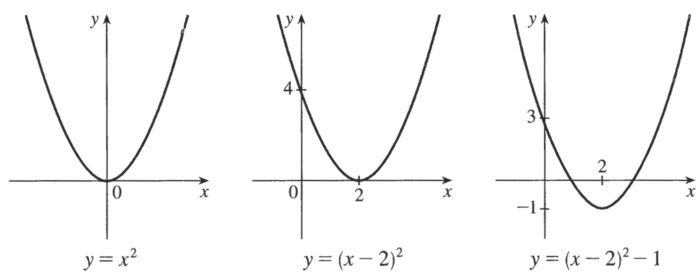
10. $y = 1 - x^2 = -x^2 + 1$: Start with the graph of $y = x^2$, reflect about the x-axis, and then shift 1 unit upward.



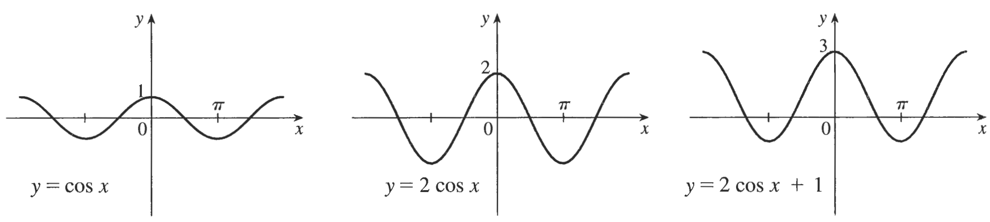
11. $y = (x + 1)^2$: Start with the graph of $y = x^2$ and shift 1 unit to the left.



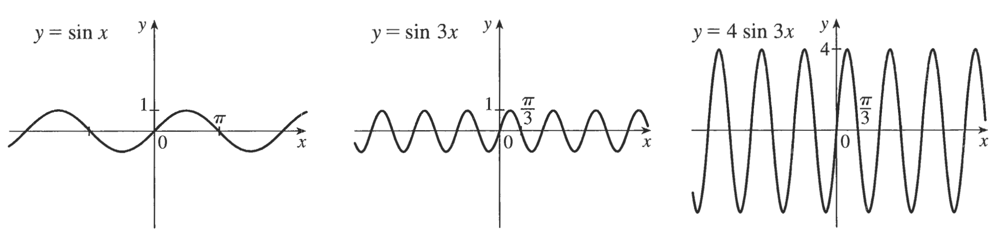
12. $y = x^2 - 4x + 3 = (x^2 - 4x + 4) - 1 = (x - 2)^2 - 1$: Start with the graph of $y = x^2$, shift 2 units to the right, and then shift 1 unit downward.



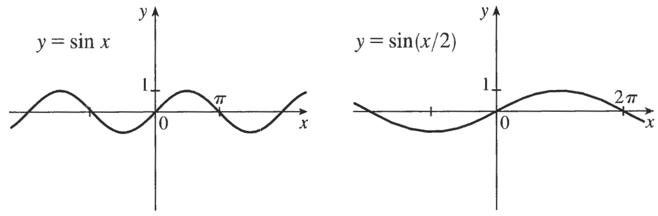
13. $y = 1 + 2\cos x$: Start with the graph of $y = \cos x$, stretch vertically by a factor of 2, and then shift 1 unit upward.



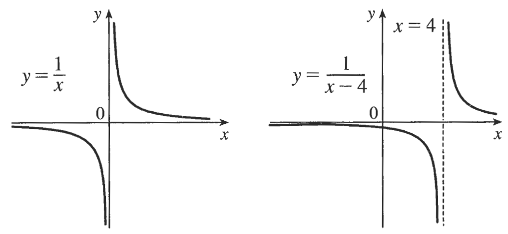
14. $y = 4\sin 3x$: Start with the graph of $y = \sin x$, compress horizontally by a factor of 3, and then stretch vertically by a factor of 4.



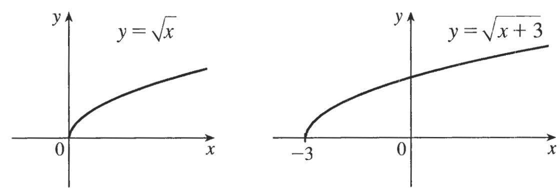
15. $y = \sin(x/2)$: Start with the graph of $y = \sin x$ and stretch horizontally by a factor of 2.



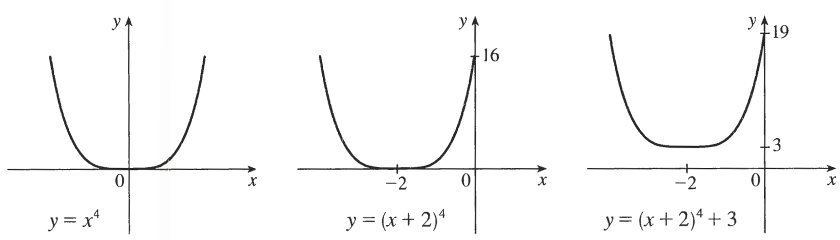
16. $y = 1/(x - 4)$: Start with the graph of $y = 1/x$ and shift 4 units to the right.



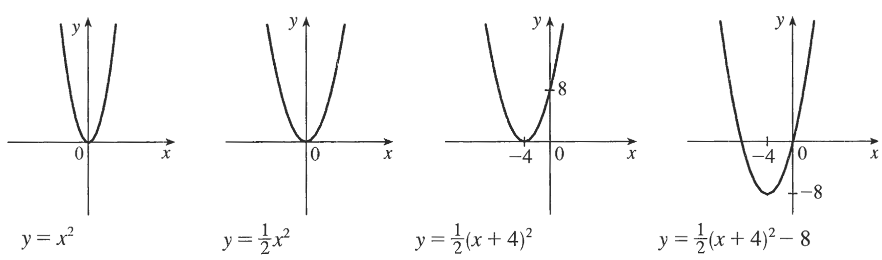
17. $y = \sqrt{x+3}$: Start with the graph of $y = \sqrt{x}$ and shift 3 units to the left.



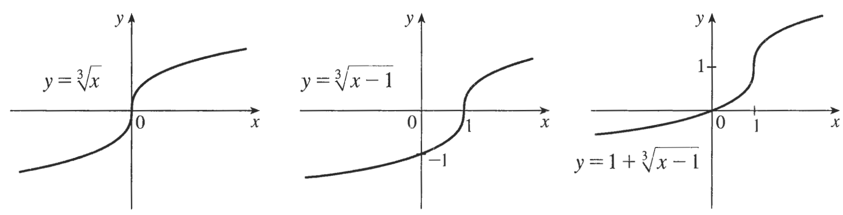
18. $y = (x + 2)^4 + 3$: Start with the graph of $y = x^4$, shift 2 units to the left, and then shift 3 units upward.



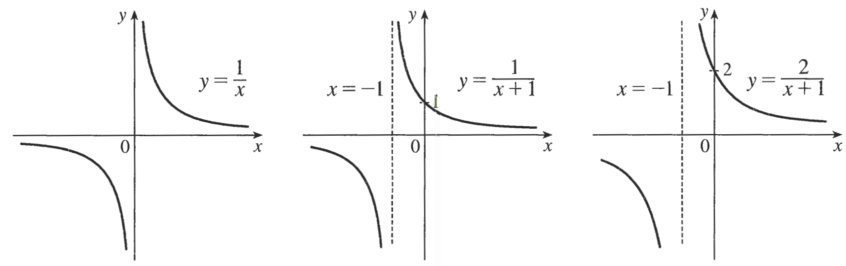
19. $y = \frac{1}{2}(x^2 + 8x) = \frac{1}{2}(x^2 + 8x + 16) - 8 = \frac{1}{2}(x + 4)^2 - 8$: Start with the graph of $y = x^2$, compress vertically by a factor of 2, shift 4 units to the left, and then shift 8 units downward.



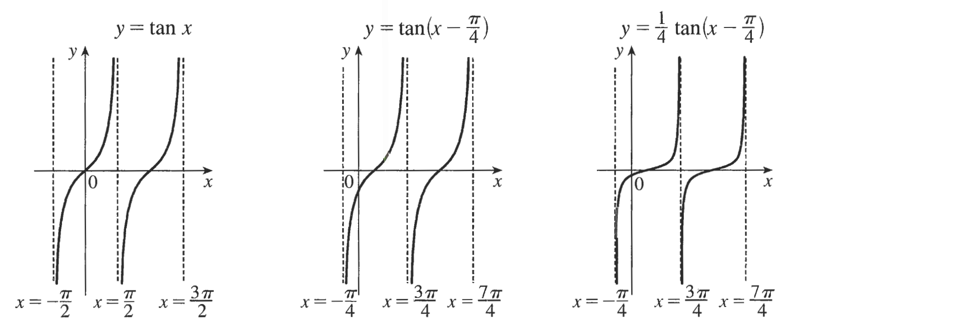
20. $y = 1 + \sqrt[3]{x-1}$: Start with the graph of $y = \sqrt[3]{x}$, shift 1 unit to the right, and then shift 1 unit upward.



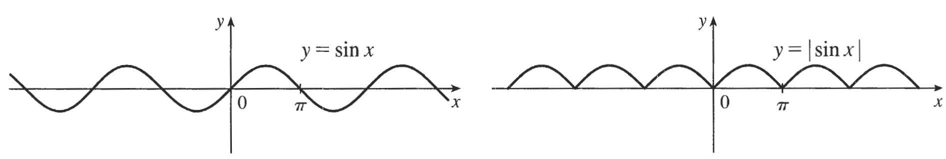
21. $y = 2/(x+1)$: Start with the graph of $y = 1/x$, shift 1 unit to the left, and then stretch vertically by a factor of 2.



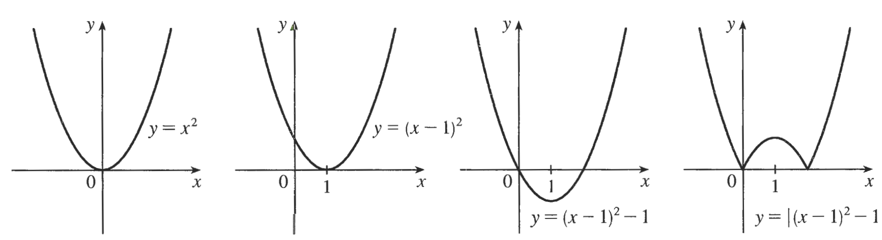
22. $y = \frac{1}{4}\tan(x - \frac{\pi}{4})$: Start with the graph of $y = \tan x$, shift $\frac{\pi}{4}$ units to the right, and then compress vertically by a factor of 4.



23. $y = |\sin x|$: Start with the graph of $y = \sin x$ and reflect all the parts of the graph below the x-axis about the x-axis.



24. $y = |x^2 - 2x| = |x^2 - 2x + 1 - 1| = |(x-1)^2 - 1|$: Start with the graph of $y = x^2$, shift 1 unit right, shift 1 unit downward, and reflect the portion of the graph below the x-axis about the x-axis.

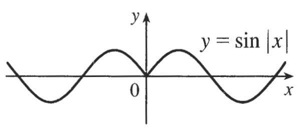


25. This is just like the solution to Example 4 except the amplitude of the curve (the 30°N curve in Figure 9 on June 21) is $14 - 12 = 2$. So the function is $L(t) = 12 + 2\sin\left[\frac{2\pi}{365}(t - 80)\right]$. March 31 is the 90th day of the year, so the model gives $L(90) \approx 12.34$ h. The daylight time (5:51 AM to 6:18 PM) is 12 hours and 27 minutes, or 12.45 h. The model value differs from the actual value by $\frac{12.45 - 12.34}{12.45} \approx 0.009$, less than 1%.

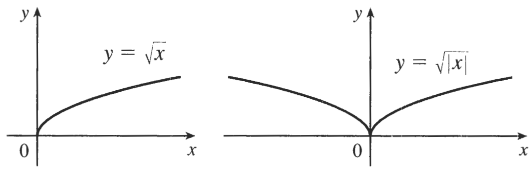
26. Using a sine function to model the brightness of Delta Cephei as a function of time, we take its period to be 5.4 days, its amplitude to be 0.35 (on the scale of magnitude), and its average magnitude to be 4.0. If we take $t = 0$ at a time of average brightness, then the magnitude (brightness) as a function of time t in days can be modeled by the formula $M(t) = 4.0 + 0.35\sin\left(\frac{2\pi}{5.4}t\right)$.

27. (a) To obtain $y = f(|x|)$, the portion of the graph of $y = f(x)$ to the right of the y-axis is reflected about the y-axis.

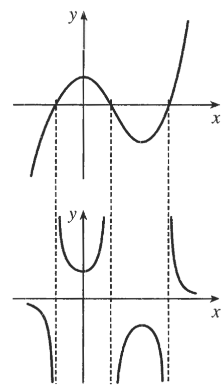
(b) $y = \sin|x|$



(c) $y = \sqrt{|x|}$



28. The most important features of the given graph are the x-intercepts and the maximum and minimum points. The graph of $y = 1/f(x)$ has vertical asymptotes at the x-values where there are x-intercepts on the graph of $y = f(x)$. The maximum of 1 on the graph of $y = f(x)$ corresponds to a minimum of $1/1 = 1$ on $y = 1/f(x)$. Similarly, the minimum on the graph of $y = f(x)$ corresponds to a maximum on the graph of $y = 1/f(x)$. As the values of y get large (positively or negatively) on the graph of $y = f(x)$, the values of y get close to zero on the graph of $y = 1/f(x)$.



29. $f(x) = x^3 + 2x^2$; $g(x) = 3x^2 - 1$. $D = \mathbb{R}$ for both f and g.

$(f + g)(x) = (x^3 + 2x^2) + (3x^2 - 1) = x^3 + 5x^2 - 1,\ D = \mathbb{R}.$

$(f - g)(x) = (x^3 + 2x^2) - (3x^2 - 1) = x^3 - x^2 + 1,\ D = \mathbb{R}.$

$(fg)(x) = (x^3 + 2x^2)(3x^2 - 1) = 3x^5 + 6x^4 - x^3 - 2x^2,\ D = \mathbb{R}.$

$\left(\dfrac{f}{g}\right)(x) = \dfrac{x^3 + 2x^2}{3x^2 - 1},\ D = \left\{x \mid x \neq \pm\dfrac{1}{\sqrt{3}}\right\}$ since $3x^2 - 1 \neq 0$.

30. $f(x) = \sqrt{3 - x},\ D = (-\infty, 3]$; $g(x) = \sqrt{x^2 - 1},\ D = (-\infty, -1] \cup [1, \infty)$.

$(f + g)(x) = \sqrt{3 - x} + \sqrt{x^2 - 1},\ D = (-\infty, -1] \cup [1, 3]$, which is the intersection of the domains of f and g.

$(f - g)(x) = \sqrt{3 - x} - \sqrt{x^2 - 1},\ D = (-\infty, -1] \cup [1, 3].$

$(fg)(x) = \sqrt{3 - x} \cdot \sqrt{x^2 - 1},\ D = (-\infty, -1] \cup [1, 3].$

$\left(\dfrac{f}{g}\right)(x) = \dfrac{\sqrt{3 - x}}{\sqrt{x^2 - 1}},\ D = (-\infty, -1) \cup (1, 3]$. We must exclude $x = \pm 1$ since these values would make $\dfrac{f}{g}$ undefined.

31. $f(x) = x^2 - 1$, $D = \mathbb{R}$; $g(x) = 2x + 1$, $D = \mathbb{R}$.

(a) $(f \circ g)(x) = f(g(x)) = f(2x+1) = (2x+1)^2 - 1 = (4x^2 + 4x + 1) - 1 = 4x^2 + 4x$, $D = \mathbb{R}$.

(b) $(g \circ f)(x) = g(f(x)) = g(x^2 - 1) = 2(x^2 - 1) + 1 = (2x^2 - 2) + 1 = 2x^2 - 1$, $D = \mathbb{R}$.

(c) $(f \circ f)(x) = f(f(x)) = f(x^2 - 1) = (x^2 - 1)^2 - 1 = (x^4 - 2x^2 + 1) - 1 = x^4 - 2x^2$, $D = \mathbb{R}$.

(d) $(g \circ g)(x) = g(g(x)) = g(2x+1) = 2(2x+1) + 1 = (4x+2) + 1 = 4x + 3$, $D = \mathbb{R}$.

32. $f(x) = x - 2$; $g(x) = x^2 + 3x + 4$. $D = \mathbb{R}$ for both f and g, and hence for their composites.

(a) $(f \circ g)(x) = f(g(x)) = f(x^2 + 3x + 4) = (x^2 + 3x + 4) - 2 = x^2 + 3x + 2$.

(b) $(g \circ f)(x) = g(f(x)) = g(x-2) = (x-2)^2 + 3(x-2) + 4 = x^2 - 4x + 4 + 3x - 6 + 4 = x^2 - x + 2$.

(c) $(f \circ f)(x) = f(f(x)) = f(x-2) = (x-2) - 2 = x - 4$.

(d)
$$\begin{aligned}(g \circ g)(x) = g(g(x)) = g(x^2 + 3x + 4) &= (x^2 + 3x + 4)^2 + 3(x^2 + 3x + 4) + 4 \\ &= (x^4 + 9x^2 + 16 + 6x^3 + 8x^2 + 24x) + 3x^2 + 9x + 12 + 4 \\ &= x^4 + 6x^3 + 20x^2 + 33x + 32\end{aligned}$$

33. $f(x) = 1 - 3x$; $g(x) = \cos x$. $D = \mathbb{R}$ for both f and g, and hence for their composites.

(a) $(f \circ g)(x) = f(g(x)) = f(\cos x) = 1 - 3\cos x$.

(b) $(g \circ f)(x) = g(f(x)) = g(1 - 3x) = \cos(1 - 3x)$.

(c) $(f \circ f)(x) = f(f(x)) = f(1 - 3x) = 1 - 3(1 - 3x) = 1 - 3 + 9x = 9x - 2$.

(d) $(g \circ g)(x) = g(g(x)) = g(\cos x) = \cos(\cos x)$ [Note that this is *not* $\cos x \cdot \cos x$.]

34. $f(x) = \sqrt{x}$, $D = [0, \infty)$; $g(x) = \sqrt[3]{1-x}$, $D = \mathbb{R}$.

(a) $(f \circ g)(x) = f(g(x)) = f\left(\sqrt[3]{1-x}\right) = \sqrt{\sqrt[3]{1-x}} = \sqrt[6]{1-x}$.

The domain of $f \circ g$ is $\{x \mid \sqrt[3]{1-x} \geq 0\} = \{x \mid 1 - x \geq 0\} = \{x \mid x \leq 1\} = (-\infty, 1]$.

(b) $(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = \sqrt[3]{1 - \sqrt{x}}$.

The domain of $g \circ f$ is $\{x \mid x$ is in the domain of f and $f(x)$ is in the domain of $g\}$. This is the domain of f, that is, $[0, \infty)$.

(c) $(f \circ f)(x) = f(f(x)) = f(\sqrt{x}) = \sqrt{\sqrt{x}} = \sqrt[4]{x}$. The domain of $f \circ f$ is $\{x \mid x \geq 0$ and $\sqrt{x} \geq 0\} = [0, \infty)$.

(d) $(g \circ g)(x) = g(g(x)) = g\left(\sqrt[3]{1-x}\right) = \sqrt[3]{1 - \sqrt[3]{1-x}}$, and the domain is $(-\infty, \infty)$.

35. $f(x) = x + \dfrac{1}{x}$, $D = \{x \mid x \neq 0\}$; $g(x) = \dfrac{x+1}{x+2}$, $D = \{x \mid x \neq -2\}$

(a)
$$\begin{aligned}(f \circ g)(x) = f(g(x)) = f\left(\frac{x+1}{x+2}\right) &= \frac{x+1}{x+2} + \frac{1}{\dfrac{x+1}{x+2}} = \frac{x+1}{x+2} + \frac{x+2}{x+1} \\ &= \frac{(x+1)(x+1) + (x+2)(x+2)}{(x+2)(x+1)} = \frac{(x^2 + 2x + 1) + (x^2 + 4x + 4)}{(x+2)(x+1)} = \frac{2x^2 + 6x + 5}{(x+2)(x+1)}\end{aligned}$$

Since $g(x)$ is not defined for $x = -2$ and $f(g(x))$ is not defined for $x = -2$ and $x = -1$, the domain of $(f \circ g)(x)$ is $D = \{x \mid x \neq -2, -1\}$.

(b)
$$(g \circ f)(x) = g(f(x)) = g\left(x + \frac{1}{x}\right) = \frac{\left(x + \dfrac{1}{x}\right) + 1}{\left(x + \dfrac{1}{x}\right) + 2} = \frac{\dfrac{x^2 + 1 + x}{x}}{\dfrac{x^2 + 1 + 2x}{x}} = \frac{x^2 + x + 1}{x^2 + 2x + 1} = \frac{x^2 + x + 1}{(x+1)^2}$$

Since $f(x)$ is not defined for $x = 0$ and $g(f(x))$ is not defined for $x = -1$, the domain of $(g \circ f)(x)$ is $D = \{x \mid x \neq -1, 0\}$.

(c) $(f\circ f)(x)=f(f(x))=f\left(x+\dfrac{1}{x}\right)=\left(x+\dfrac{1}{x}\right)+\dfrac{1}{x+\frac{1}{x}}=x+\dfrac{1}{x}+\dfrac{1}{\frac{x^2+1}{x}}=x+\dfrac{1}{x}+\dfrac{x}{x^2+1}$

$$=\frac{x(x)\left(x^2+1\right)+1\left(x^2+1\right)+x(x)}{x(x^2+1)}=\frac{x^4+x^2+x^2+1+x^2}{x(x^2+1)}$$

$$=\frac{x^4+3x^2+1}{x(x^2+1)},\qquad D=\{x\mid x\neq 0\}$$

(d) $(g\circ g)(x)=g(g(x))=g\left(\dfrac{x+1}{x+2}\right)=\dfrac{\dfrac{x+1}{x+2}+1}{\dfrac{x+1}{x+2}+2}=\dfrac{\dfrac{x+1+1(x+2)}{x+2}}{\dfrac{x+1+2(x+2)}{x+2}}=\dfrac{x+1+x+2}{x+1+2x+4}=\dfrac{2x+3}{3x+5}$

Since $g(x)$ is not defined for $x=-2$ and $g(g(x))$ is not defined for $x=-\frac{5}{3}$,

the domain of $(g\circ g)(x)$ is $D=\left\{x\mid x\neq -2,-\frac{5}{3}\right\}$.

36. $f(x)=\dfrac{x}{1+x},\ D=\{x\mid x\neq -1\};\quad g(x)=\sin 2x,\ D=\mathbb{R}$.

(a) $(f\circ g)(x)=f(g(x))=f(\sin 2x)=\dfrac{\sin 2x}{1+\sin 2x}$

Domain: $1+\sin 2x\neq 0\ \Rightarrow\ \sin 2x\neq -1\ \Rightarrow\ 2x\neq \dfrac{3\pi}{2}+2\pi n\ \Rightarrow\ x\neq \dfrac{3\pi}{4}+\pi n$ (n an integer).

(b) $(g\circ f)(x)=g(f(x))=g\left(\dfrac{x}{1+x}\right)=\sin\left(\dfrac{2x}{1+x}\right)$. Domain: $\{x\mid x\neq -1\}$

(c) $(f\circ f)(x)=f(f(x))=f\left(\dfrac{x}{1+x}\right)=\dfrac{\dfrac{x}{1+x}}{1+\dfrac{x}{1+x}}=\dfrac{\left(\dfrac{x}{1+x}\right)\cdot(1+x)}{\left(1+\dfrac{x}{1+x}\right)\cdot(1+x)}=\dfrac{x}{1+x+x}=\dfrac{x}{2x+1}$

Since $f(x)$ is not defined for $x=-1$, and $f(f(x))$ is not defined for $x=-\frac{1}{2}$,

the domain of $(f\circ f)(x)$ is $D=\{x\mid x\neq -1,-\frac{1}{2}\}$.

(d) $(g\circ g)(g)=g(g(x))=g(\sin 2x)=\sin(2\sin 2x)$. Domain: $\mathbb{R}$

37. $(f\circ g\circ h)(x)=f(g(h(x)))=f(g(x-1))=f(2(x-1))=2(x-1)+1=2x-1$

38. $(f\circ g\circ h)(x)=f(g(h(x)))=f(g(1-x))=f((1-x)^2)=2(1-x)^2-1=2x^2-4x+1$

39. $(f\circ g\circ h)(x)=f(g(h(x)))=f(g(x^3+2))=f[(x^3+2)^2]$

$$=f(x^6+4x^3+4)=\sqrt{(x^6+4x^3+4)-3}=\sqrt{x^6+4x^3+1}$$

40. $(f\circ g\circ h)(x)=f(g(h(x)))=f(g(\sqrt[3]{x}))=f\left(\dfrac{\sqrt[3]{x}}{\sqrt[3]{x}-1}\right)=\tan\left(\dfrac{\sqrt[3]{x}}{\sqrt[3]{x}-1}\right)$

41. Let $g(x)=x^2+1$ and $f(x)=x^{10}$. Then $(f\circ g)(x)=f(g(x))=f(x^2+1)=(x^2+1)^{10}=F(x)$.

42. Let $g(x)=\sqrt{x}$ and $f(x)=\sin x$. Then $(f\circ g)(x)=f(g(x))=f\left(\sqrt{x}\right)=\sin\left(\sqrt{x}\right)=F(x)$.

43. Let $g(x)=\sqrt[3]{x}$ and $f(x)=\dfrac{x}{1+x}$. Then $(f\circ g)(x)=f(g(x))=f(\sqrt[3]{x})=\dfrac{\sqrt[3]{x}}{1+\sqrt[3]{x}}=F(x)$.

44. Let $g(x)=\dfrac{x}{1+x}$ and $f(x)=\sqrt[3]{x}$. Then $(f\circ g)(x)=f(g(x))=f\left(\dfrac{x}{1+x}\right)=\sqrt[3]{\dfrac{x}{1+x}}=G(x)$.

45. Let $g(t) = \cos t$ and $f(t) = \sqrt{t}$. Then $(f \circ g)(t) = f(g(t)) = f(\cos t) = \sqrt{\cos t} = u(t)$.

46. Let $g(t) = \tan t$ and $f(t) = \dfrac{t}{1+t}$. Then $(f \circ g)(t) = f(g(t)) = f(\tan t) = \dfrac{\tan t}{1 + \tan t} = u(t)$.

47. Let $h(x) = x^2$, $g(x) = 3^x$, and $f(x) = 1 - x$. Then

$(f \circ g \circ h)(x) = f(g(h(x))) = f(g(x^2)) = f\left(3^{x^2}\right) = 1 - 3^{x^2} = H(x)$.

48. Let $h(x) = |x|$, $g(x) = 2 + x$, and $f(x) = \sqrt[8]{x}$. Then

$(f \circ g \circ h)(x) = f(g(h(x))) = f(g(|x|)) = f(2 + |x|) = \sqrt[8]{2 + |x|} = H(x)$.

49. Let $h(x) = \sqrt{x}$, $g(x) = \sec x$, and $f(x) = x^4$. Then

$(f \circ g \circ h)(x) = f(g(h(x))) = f(g(\sqrt{x})) = f(\sec \sqrt{x}) = (\sec \sqrt{x})^4 = \sec^4(\sqrt{x}) = H(x)$.

50. (a) $f(g(1)) = f(6) = 5$ (b) $g(f(1)) = g(3) = 2$

(c) $f(f(1)) = f(3) = 4$ (d) $g(g(1)) = g(6) = 3$

(e) $(g \circ f)(3) = g(f(3)) = g(4) = 1$ (f) $(f \circ g)(6) = f(g(6)) = f(3) = 4$

51. (a) $g(2) = 5$, because the point $(2, 5)$ is on the graph of g. Thus, $f(g(2)) = f(5) = 4$, because the point $(5, 4)$ is on the graph of f.

(b) $g(f(0)) = g(0) = 3$

(c) $(f \circ g)(0) = f(g(0)) = f(3) = 0$

(d) $(g \circ f)(6) = g(f(6)) = g(6)$. This value is not defined, because there is no point on the graph of g that has x-coordinate 6.

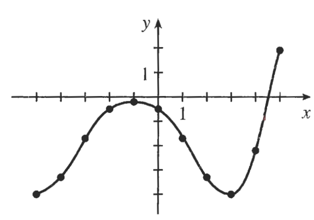
(e) $(g \circ g)(-2) = g(g(-2)) = g(1) = 4$

(f) $(f \circ f)(4) = f(f(4)) = f(2) = -2$

52. To find a particular value of $f(g(x))$, say for $x = 0$, we note from the graph that $g(0) \approx 2.8$ and $f(2.8) \approx -0.5$. Thus, $f(g(0)) \approx f(2.8) \approx -0.5$. The other values listed in the table were obtained in a similar fashion.

x	$g(x)$	$f(g(x))$
-5	-0.2	-4
-4	1.2	-3.3
-3	2.2	-1.7
-2	2.8	-0.5
-1	3	-0.2

x	$g(x)$	$f(g(x))$
0	2.8	-0.5
1	2.2	-1.7
2	1.2	-3.3
3	-0.2	-4
4	-1.9	-2.2
5	-4.1	1.9



53. (a) Using the relationship *distance* = *rate* · *time* with the radius r as the distance, we have $r(t) = 60t$.

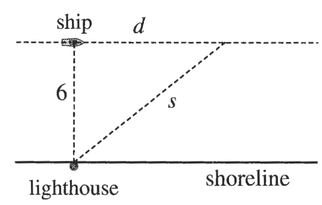
(b) $A = \pi r^2 \quad \Rightarrow \quad (A \circ r)(t) = A(r(t)) = \pi(60t)^2 = 3600\pi t^2$. This formula gives us the extent of the rippled area (in cm^2) at any time t.

54. (a) The radius r of the balloon is increasing at a rate of 2 cm/s, so $r(t) = (2\text{ cm/s})(t\text{ s}) = 2t$ (in cm).

(b) Using $V = \frac{4}{3}\pi r^3$, we get $(V \circ r)(t) = V(r(t)) = V(2t) = \frac{4}{3}\pi(2t)^3 = \frac{32}{3}\pi t^3$.
The result, $V = \frac{32}{3}\pi t^3$, gives the volume of the balloon (in cm^3) as a function of time (in s).

55. (a) From the figure, we have a right triangle with legs 6 and d, and hypotenuse s.

By the Pythagorean Theorem, $d^2 + 6^2 = s^2 \quad\Rightarrow\quad s = f(d) = \sqrt{d^2 + 36}$.



(b) Using $d = rt$, we get $d = (30\text{ km/hr})(t\text{ hr}) = 30t$ (in km). Thus,

$d = g(t) = 30t$.

(c) $(f \circ g)(t) = f(g(t)) = f(30t) = \sqrt{(30t)^2 + 36} = \sqrt{900t^2 + 36}$. This function represents the distance between the lighthouse and the ship as a function of the time elapsed since noon.

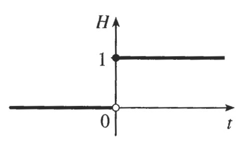
56. (a) $d = rt \quad\Rightarrow\quad d(t) = 350t$

(b) There is a Pythagorean relationship involving the legs with lengths d and 1 and the hypotenuse with length s:

$d^2 + 1^2 = s^2$. Thus, $s(d) = \sqrt{d^2 + 1}$.

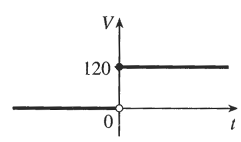
(c) $(s \circ d)(t) = s(d(t)) = s(350t) = \sqrt{(350t)^2 + 1}$

57. (a)



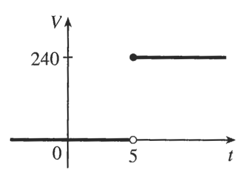
$$H(t) = \begin{cases} 0 & \text{if } t < 0 \\ 1 & \text{if } t \geq 0 \end{cases}$$

(b)



$$V(t) = \begin{cases} 0 & \text{if } t < 0 \\ 120 & \text{if } t \geq 0 \end{cases} \quad \text{so } V(t) = 120H(t).$$

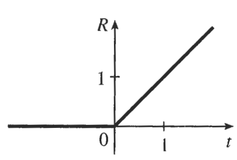
(c)



Starting with the formula in part (b), we replace 120 with 240 to reflect the different voltage. Also, because we are starting 5 units to the right of $t = 0$, we replace t with $t - 5$. Thus, the formula is $V(t) = 240H(t - 5)$.

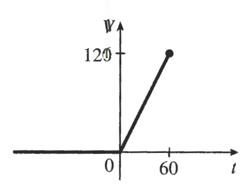
58. (a) $R(t) = tH(t)$

$$= \begin{cases} 0 & \text{if } t < 0 \\ t & \text{if } t \geq 0 \end{cases}$$



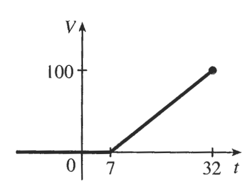
(b) $V(t) = \begin{cases} 0 & \text{if } t < 0 \\ 2t & \text{if } 0 \leq t \leq 60 \end{cases}$

so $V(t) = 2tH(t)$, $t \leq 60$.



(c) $V(t) = \begin{cases} 0 & \text{if } t < 7 \\ 4(t - 7) & \text{if } 7 \leq t \leq 32 \end{cases}$

so $V(t) = 4(t - 7)H(t - 7)$, $t \leq 32$.



59. If $f(x) = m_1 x + b_1$ and $g(x) = m_2 x + b_2$, then

$(f \circ g)(x) = f(g(x)) = f(m_2 x + b_2) = m_1(m_2 x + b_2) + b_1 = m_1 m_2 x + m_1 b_2 + b_1$.

So $f \circ g$ is a linear function with slope $m_1 m_2$.

60. If $A(x) = 1.04x$, then

$(A \circ A)(x) = A(A(x)) = A(1.04x) = 1.04(1.04x) = (1.04)^2 x$,

$(A \circ A \circ A)(x) = A((A \circ A)(x)) = A((1.04)^2 x) = 1.04(1.04)^2 x = (1.04)^3 x$, and

$(A \circ A \circ A \circ A)(x) = A((A \circ A \circ A)(x)) = A((1.04)^3 x) = 1.04(1.04)^3 x, = (1.04)^4 x$.

These compositions represent the amount of the investment after 2, 3, and 4 years.

Based on this pattern, when we compose n copies of A, we get the formula $\underbrace{(A \circ A \circ \cdots \circ A)}_{n\ A's}(x) = (1.04)^n x$.

61. (a) By examining the variable terms in g and h, we deduce that we must square g to get the terms $4x^2$ and $4x$ in h. If we let $f(x) = x^2 + c$, then $(f \circ g)(x) = f(g(x)) = f(2x+1) = (2x+1)^2 + c = 4x^2 + 4x + (1 + c)$. Since $h(x) = 4x^2 + 4x + 7$, we must have $1 + c = 7$. So $c = 6$ and $f(x) = x^2 + 6$.

(b) We need a function g so that $f(g(x)) = 3(g(x)) + 5 = h(x)$. But $h(x) = 3x^2 + 3x + 2 = 3(x^2 + x) + 2 = 3(x^2 + x - 1) + 5$, so we see that $g(x) = x^2 + x - 1$.

62. We need a function g so that $g(f(x)) = g(x+4) = h(x) = 4x - 1 = 4(x+4) - 17$. So we see that the function g must be $g(x) = 4x - 17$.

63. (a) If f and g are even functions, then $f(-x) = f(x)$ and $g(-x) = g(x)$.

(i) $(f+g)(-x) = f(-x) + g(-x) = f(x) + g(x) = (f+g)(x)$, so $f + g$ is an *even* function.

(ii) $(fg)(-x) = f(-x) \cdot g(-x) = f(x) \cdot g(x) = (fg)(x)$, so fg is an *even* function.

(b) If f and g are odd functions, then $f(-x) = -f(x)$ and $g(-x) = -g(x)$.

(i) $(f+g)(-x) = f(-x) + g(-x) = -f(x) + [-g(x)] = -[f(x) + g(x)] = -(f+g)(x)$, so $f + g$ is an *odd* function.

(ii) $(fg)(-x) = f(-x) \cdot g(-x) = -f(x) \cdot [-g(x)] = f(x) \cdot g(x) = (fg)(x)$, so fg is an *even* function.

64. If f is even and g is odd, then $f(-x) = f(x)$ and $g(-x) = -g(x)$. Now $(fg)(-x) = f(-x) \cdot g(-x) = f(x) \cdot [-g(x)] = -[f(x) \cdot g(x)] = -(fg)(x)$, so fg is an *odd* function.

65. We need to examine $h(-x)$.

$$h(-x) = (f \circ g)(-x) = f(g(-x)) = f(g(x)) \quad \text{[because } g \text{ is even]} \quad = h(x)$$

Because $h(-x) = h(x)$, h is an even function.

66. $h(-x) = f(g(-x)) = f(-g(x))$. At this point, we can't simplify the expression, so we might try to find a counterexample to show that h is not an odd function. Let $g(x) = x$, an odd function, and $f(x) = x^2 + x$. Then $h(x) = x^2 + x$, which is neither even nor odd.

Now suppose f is an odd function. Then $f(-g(x)) = -f(g(x)) = -h(x)$. Hence, $h(-x) = -h(x)$, and so h is odd if both f and g are odd.

Now suppose f is an even function. Then $f(-g(x)) = f(g(x)) = h(x)$. Hence, $h(-x) = h(x)$, and so h is even if g is odd and f is even.