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Option Pricing and Portfolio Optimization
Post Graduation Semester - 4

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Contents

1 Introduction

The concepts of portfolio optimization and option pricing, closely linked to statistical techniques, serve as foundations of modern investment theory in the constantly evolving world of financial markets. The theory of option pricing and portfolio construction is highly dependent on statistical models, procedures, and approaches. Incorporating statistics into finance improves our understanding of market dynamics and offers strong instruments for making choices in an unpredictable environment.

Within the context of contemporary financial econometrics, this study explores the underlying theories, empirical applications, and practical consequences of option pricing and portfolio optimization as they relate to statistics. It seeks to close the knowledge gap between theory and practice by offering an extensive understanding of these significant domains of finance.

Option Pricing

Option pricing theory, based on complex math and probability, deals with predicting the prices of options. The famous model by Black, Scholes, and Merton was the first widely used approach, assuming that asset prices move in a specific, predictable way called geometric Brownian motion. However, this model's assumptions often don't match real market behavior. To address this, more advanced models have been developed. These include stochastic volatility models, jump diffusion models, and Levy processes, which use more sophisticated statistical methods to more accurately capture market movements and improve option pricing.

Portfolio Optimization

Portfolio optimization, similarly, relies on statistical techniques to construct efficient investment portfolios. Modern portfolio theory, pioneered by Markowitz, utilizes statistical measures such as mean-variance analysis to quantify risk and return characteristics of assets. However, the practical implementation of portfolio optimization strategies requires addressing statistical challenges such as estimation error, parameter uncertainty, and non-normality of asset returns. This has led to the development of robust statistical methods, including Bayesian optimization, resampling techniques, and risk factor modeling, to enhance portfolio performance and mitigate the impact of statistical uncertainties.

Objectives

Against this statistical backdrop, this Project aims to achieve the following objectives: Statistical Examination: Conduct a comprehensive review of statistical methods employed in option pricing models and portfolio optimization techniques, highlighting their assumptions and statistical properties.

Practical Implications: Assess the practical relevance of statistical approaches in option pricing and portfolio optimization within a statistical framework.

Structure of the Project
The remainder of this project is organized as follows:

- Chapter 2 provides a detailed literature review of statistical methodologies in option pricing theory and portfolio optimization techniques used in the Data Analysis.
- Chapter 3 presents Analysis using Real Life Data including data pre-processing and statistical diagnostics. .
- Finally, Chapter 4 concludes the project by summarizing key findings.

By integrating statistical methodologies with option pricing and portfolio optimization, this project endeavors to advance our statistical understanding of financial markets, offering insights that are both statistically rigorous and practically relevant.

2 Literature Review of Statistical Methodologies in Option pricing and Portfolio optimization.

Return:

Revenue as a percentage of the initial investment is the measure of an investment's return. If one invests at time t_1 in an asset with price P_{t_1} and the price later at time t_2 is P_{t_2} , then the net return for the holding period from t_1 to t_2 is $(P_{t_2} - P_{t_1})/P_{t_1}$.

Future returns are unpredictable for the majority of assets, making them random variables. Naturally, investing aims to turn a profit. Both the quantity of assets retained and the change in prices determine the profit—or loss, in the event of a negative profit—from investing. Revenues that are high in comparison to the initial investment size attract the interest of investors. Because returns on assets are price changes represented as a percentage of the initial price, returns serve as an indicator of this.

Broadly, Returns can be measured in 3 ways.

- Net Returns
- Gross Returns
- Log Returns

But For our rest of the project we will mean log returns as returns.

Log Returns:

Let P_t be the price of an asset at time t. P_{t-1}, was the initial investment at the start of the holding period.

Continuously compounded returns, also known as log returns, are denoted by r_t and defined as

$$r_t = log\left(\frac{P_t}{P_{t-1}}\right)$$

Log returns are approximately equal to returns because if x is small, then $log(1+x) \approx x$ as can been seen in Figure 2.0.1 where $log(1 + x)$ is plotted. Notice in that figure that $log(1 + x)$ is very close to x if $|x| < 0.1$, e.g., for returns that are less than 10%.

One advantage of using log returns is simplicity of multi period returns. A k period log return is simply the sum of the single period log returns, rather than the product as for returns. To see this, note that

$$r_t(k) = log\left(\frac{P_t}{P_{t-1}}\right)\cdots\left(\frac{P_{t-k+1}}{P_{t-k}}\right)$$

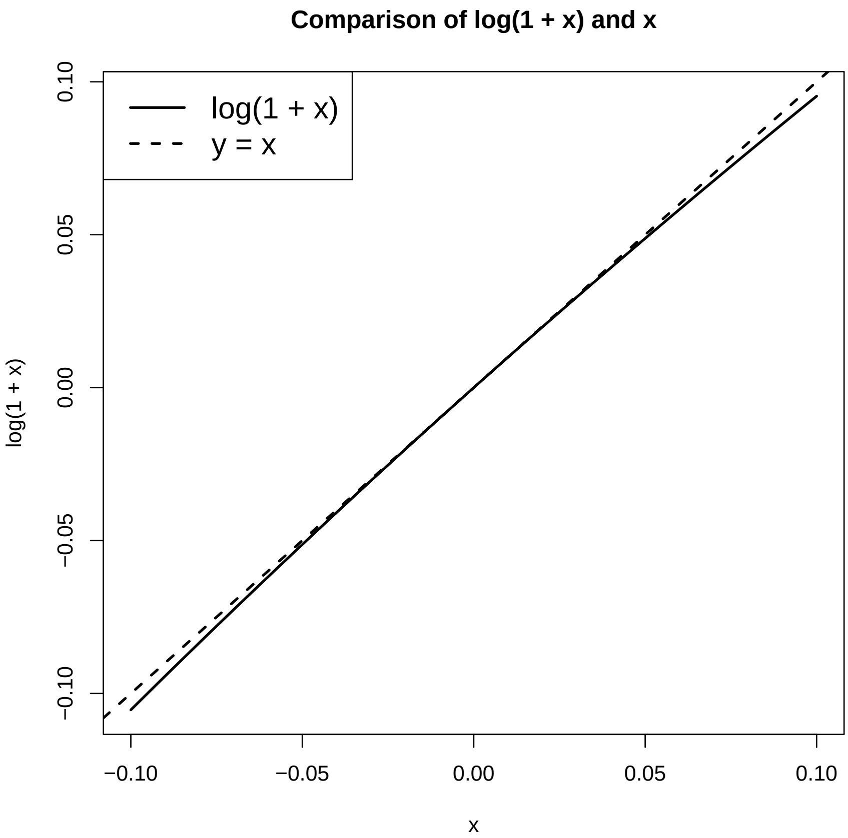

Figure 2.0.1: Comparison of functions log(1 + x) and x

$$= log\left(\frac{P_t}{P_{t-1}}\right) + \cdots + log\left(\frac{P_{t-k+1}}{P_{t-k}}\right)$$

$$= r_t + r_{t-1} + \cdots + r_{t-k+1}$$

Behavior Of Returns

Returns cannot be perfectly predicted, but rather they are random.This randomness implies that a return might be smaller than its expected value and even negative, which means that investing involves risk. The risk could be described by probability theory.Random phenomena do exhibit some regularities such as the law of large numbers and the central limit theorem. Application of probability to financial markets was undeveloped. In gambling, probabilities can be found by simple reasoning and an assumption of symmetry. This type of reasoning can't be applied to returns. *[3]*

University of Chicago economist Frank Knight made an important distinction between measurable uncertainty or risk proper (e.g., games of chance) where the probabilities are known and unmeasurable uncertainty where the probabilities are unknown.

Without any assumptions, finance would be in the realm of unmeasurable uncertainty with no way out. At time $t-1$, P_t and R_t would not only be unknown, but we would not know their probability distributions. Mathematical finance would not be possible in this case. However, we can estimate these probability distributions if we are willing to make the assumption that future returns will be similar to past returns, a condition called **stationarity**. With this assumption, the machinery of statistical inference can be applied and the probability distribution of P_t can be estimated from past data.

One of the major issues in finance is how we should model the probability distributions of returns. Specification of these distributions is essential for many purposes, for example, pricing of options.

A contentious question is whether returns are predictable using past returns or other data.

The "standard" model in many areas of finance, e.g., derivatives pricing, is the geometric random walk in discrete time or its analogue, geometric Brownian motion, in continuous time. This model says that returns are not predictable. However, empirical evidence cumulated over the past few decades shows that returns are at least to some extent predictable.

The Random Walk Model

i.i.d normal returns

Suppose that $R_1, R_2, ...$ are the returns from a single asset. A common model is that they are

1. mutually independent;
2. identically distributed, i.e., they have the same probability distributions and in particular the same mean and variance; and
3. normally distributed

There are two problems with this model.

First, because a normally distributed random variable can take any value between $-\infty$ and $+\infty$, the model implies the possibility of unlimited losses, but liability is usually limited ;since you can lose no more than your investment.

Second, multi period returns are not normally distributed because $\left(\frac{P_t}{P_{t-k}}\right) = \left(\frac{P_t}{P_{t-1}}\right)\cdots\left(\frac{P_{t-k+1}}{P_{t-k}}\right)$ is not normal – sums of normal's are normal but not so with products.

Now Suppose we assume log returns are i.i.d. Thus, we assume that $log\left(\frac{P_t}{P_{t-1}}\right)$ is $N(\mu,\sigma^2)$ so that $\left(\frac{P_t}{P_{t-1}}\right)$ is an exponential and therefore positive. This solves the first problem.

Also, $r_t(k) = log\left(\frac{P_t}{P_{t-1}}\right)\cdots\left(\frac{P_{t-k+1}}{P_{t-k}}\right) = log\left(\frac{P_t}{P_{t-1}}\right) + \cdots + log\left(\frac{P_{t-k+1}}{P_{t-k}}\right) = r_t + r_{t-1} + \cdots + r_{t-k+1}$

Since sums of normal random variables are themselves normal, the second problem is solved - normality of single-period log returns implies normality of multiple-period log returns.

Geometric random walks

Recall that

$$r_t(k) = r_t + r_{t-1} + \cdots + r_{t-k+1}$$
$$\implies \left(\frac{P_t}{P_{t-1}}\right) = e^{r_t + r_{t-1} + \cdots + r_{t-k+1}}$$

Taking $k = t$ we have,

$$P_t = P_0 e^{r_t + r_{t-1} + \cdots + r_{t-k+1}}$$

Therefore, if the log returns are assumed to be i.i.d. normal, then the process $P_t : t = 1, 2, ...$ is the exponential of a random walk. We call such a process a geometric random walk or an exponential random walk.

If $r_1, r_2, ...$ are i.i.d. $N(\mu,\sigma^2)$ then the process is called a log normal geometric random walk with parameters (μ,σ^2). As the time between steps becomes shorter and the step sizes shrink in the appropriate way, a random walk converges to Brownian motion and a geometric random walk converges to geometric Brownian motion; Geometric Brownian motion is the "standard model" for stock prices used in option pricing.

The effect of the drift μ

The geometric random walk model implies that future price changes are independent of the past and therefore not possible to predict but it does not imply that one cannot make money in the stock market. Quite to the contrary, since μ is generally positive, there is an upward trend to the random walk. It is only the future deviations from the trend that cannot be predicted. The trend itself can be predicted once μ is estimated. If the log return is $N(\mu,\sigma^2)$, then the return has a log normal distribution.

The median of the log normal distribution is e^{μ} and its expected value is $e^{\mu+\frac{\sigma^2}{2}}$ which is larger than the median because of right skewness. Using these facts, general formulas for the median price and expected price after k years are $P_0 e^{k\mu}$ and $P_0 e^{k\mu + k\sigma^2/2}$, respectively, where P_0 is the price at time 0.

Portfolio Theory

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Portfolio Theory answers the question "How should we invest our wealth?"
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Portfolio theory is based upon two principles.

1. Maximize the expected return
2. Minimize the risk

We define Risk to be the standard deviation of the return, though we are ultimately concerned with the probabilities of large losses.

These goals are somewhat at odds because riskier assets generally have a higher expected return, since investors demand a reward for bearing risk.

The difference between the expected return of a risky asset and the risk-free rate of return is called the **Risk Premium**.

Without risk premiums, few investors would invest in risky assets. Nonetheless, there are optimal compromises between expected return and risk.

Objective: Through this project we would discuss how to maximize expected return subject to an upper bound on the risk, or to minimize the risk subject to a lower bound on the expected return for the assets selected using usual procedures. We would also discuss the reduction of risk by diversifying the portfolio of assets held.

Derivative: A derivative is a financial instrument whose value is derived from the value of some underlying instrument such as interest rate, foreign exchange rate, or stock price.

Options are traded both on exchanges and in the over-the-counter market. There are two types of option.

Call Option:
Call options are financial contracts that give the buyer the right—but not the obligation—to buy a stock, bond, commodity, or other asset or instrument at a specified price within a specific period.

Put Option:
A put option (or "put") is a contract giving the option buyer the right, but not the obligation, to sell-or sell short-a specified amount of an underlying security at a predetermined price within a specified time frame.

In finance, the style or family of an option is the class into which the option falls, usually defined by the dates on which the option may be exercised. The vast majority of options are either European or American (style) options.

The key difference between American and European options relates to when the options can be exercised:

A **European option** may be exercised only at the expiration date of the option, i.e. at a single pre-defined point in time.

An **American option** on the other hand may be exercised at any time before the expiration date.

Maturity:
The date in the contract is known as the expiration date or maturity.

Strike price:
This predetermined price at which the buyer of the put option can sell the underlying security is called the strike price.

European options are generally easier to analyze than American options, and some of the properties of an American option are frequently deduced from those of its European counterpart. Many complex types of derivatives cannot be priced by a simple formula in closed form such as that of Black and Scholes for a European call. Rather, these options must be priced numerically, for example, using computer simulation.

The Law of One Price
The law of one price states that if two financial instruments have exactly the same payoffs, then they have the same price. This principle is used to price options. To valuate an option, one can find a portfolio or a self-financing trading strategy with a known price and that has exactly the same payoffs as the option. The price of the option is then known, since it must be the same as the price of the portfolio or self-financing[1] trading strategy.

Arbitrage:
Arbitrage means making risk-free profit with no invested capital. "Risk-free profit" means no matter what happens in market you will make money.

Example. If someone take a loan for one year from a bank at 2% and use the money to purchase one year treasury bills paying 3% then they would make guaranteed profit without investing their own money.

The law of one price is equivalent to stating that the market is free of arbitrage opportunities.

2.1 Option Pricing:

To find Option Premium value we have used Black Scholes Formula

Black-Scholes Formula
Let S_0 be the current stock price (we have switched notation from P_0), let K be the exercise price, let r be the continuously compounded interest rate, let σ be the volatility, and let T be the expiration date of an European call option. Then by Black-Scholes Formula the calls price at time 0 is

$$C = \Phi(d_1)S_0 - \Phi(d_2)Kexp(-rT)$$

where Φ is the standard normal cumulative distribution function,
$d_1 = \frac{log(S_0/K)+(r+\sigma^2/2)T}{\sigma\sqrt{T}}$ and $d_2 = d_1 - \sigma\sqrt{T}$

[1] A trading strategy is "self-financing" if it requires no investment other than the initial investment and allows no withdrawals of cash.

2.2 Portfolio Theory

One Risky Asset and One Risk-Free Asset

Example. Suppose we have one risky asset e.g., a mutual fund. Assume that the expected return is 0.1 and the standard deviation of the return is 0.5. Assume that there is also one risk-free asset, e.g., a 30-day Treasury-bill and the return is 0.05. The standard deviation of the return is 0 by definition of "risk-free".

We are faced with the problem of constructing an investment portfolio that we will hold for one time period which is called the holding period and which could be a day, a month, a quarter, a year, 10 years, and so forth. At the end of the holding period we might want to readjust the portfolio, so for now we are only looking at returns over one time period. Suppose that a fraction w of our wealth is invested in the risky asset and the remaining fraction 1 − w is invested in the risk-free asset. Then the expected return is

$$E(R) = w(0.1) + (1 - w)(0.05) = .05 + .95w$$

The variance of the return is

$$\sigma_R^2 = w^2(.5) + (1 - w)^2(0)^2 = w^2(0.5)^2$$

So, To decide what proportion w of one's wealth to invest in the risky asset one either chooses the expected return E(R) one wants or the amount of risk σ_R with which one is willing to live.

Although σ is a measure of risk, a more direct measure of risk is actual monetary loss.

`Finding an optimal portfolio can be achieved in two steps:`

1. `Finding the ``optimal'' portfolio of risky assets, called the ``tangency portfolio,''`
2. `Finding the appropriate mix of the risk-free asset and the tangency portfolio.`

Estimating $E(R)$ and σ_R

If returns on the asset are assumed to be stationary, then we can take a time series of past returns and use the sample mean and standard deviation. Whether the stationarity assumption is realistic is always debatable. If we think that $E(R)$ and σ_R will be different from the past, we could subjectively adjust these estimates upward or downward according to our opinions, but we must live with the consequences if our opinions prove to be incorrect. Another question is how long a time series to use, that is, how far back in time one should gather data. A long series, say 10 or 20 years, will give much less variable estimates. However, if the series is not stationary but rather has slowly drifting parameters, then a shorter series (maybe 1 or 2 years) will be more representative of the future. Almost every time series of returns is nearly stationary over short enough time periods.

The value of the risk-free rate, μ_f , will be known .

Risk-Efficient Portfolios with N Risky Assets

Assume that we have N risky assets and that the return on the i^{th} risky asset is R_i and has expected value μ_i. [3]

Define, $\boldsymbol{R} = \begin{pmatrix} R_1 \\ R_2 \\ \vdots \\ R_N \end{pmatrix}$

Then $\boldsymbol{\mu} = E(\boldsymbol{R}) = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_N \end{pmatrix}$

Let, Ω_{ij} be the covariance between R_i and R_j .
Also, let $\sigma_i = \sqrt{\Omega_{ij}}$ be the standard deviation of R_i.
Finally, let $\boldsymbol{\Omega} = COV(\boldsymbol{R}) = (\Omega_{ij})_{NXN}$
Let, $\boldsymbol{\omega} = (\omega_1, \omega_2, \ldots, \omega_N)$ be a vector of portfolio weights. The expected return on a portfolio with weights ω is

$$\sum_{i=1}^{N} \omega_i \mu_i = \boldsymbol{\omega^T \mu} \tag{2.1}$$

Suppose there is a target value, μ_P , of the expected return on the portfolio. When $N = 2$ the target, μ_P , is achieved by only one portfolio and its ω_1 value solves

$$\mu_P = \omega_1\mu_1 + \omega_2\mu_2 = \mu_2 + \omega_1(\mu_1 - \mu_2)$$

For $N \geq 3$, there will be an infinite number of portfolios achieving the target, μ_P . *The one with the smallest variance is called the "efficient" portfolio.* Our first goal is to find the efficient portfolio.
Clearly,

$$V(R) = V(\omega_1 R_1 + \omega_2 R_2 + \ldots + \omega_N R_N)$$

$$= V(\omega^T R) = \sum_{i=1}^{N}\sum_{j=1}^{N} \omega_i\omega_j\Omega_{ij}$$

$$= \boldsymbol{\omega}^T\boldsymbol{\Omega\omega} \tag{2.2}$$

Thus, given a target μ_P , the efficient portfolio minimizes (2.2) subject to

$$\boldsymbol{\omega}^T\boldsymbol{\mu} = \mu_P$$

$$\boldsymbol{\omega}^T\boldsymbol{1} = 1$$

Let's denote the weights of the efficient portfolio by $\boldsymbol{\omega}_{\mu_P}$.
To find $\boldsymbol{\omega}_{\mu_P}$, form the Lagrangian

$$L(\boldsymbol{\omega}, \delta_1, \delta_2) = \boldsymbol{\omega}^T\boldsymbol{\Omega\omega} + \delta_1(\mu_P - \boldsymbol{\omega}_{\mu_P}^T\boldsymbol{\mu}) + \delta_2(1 - \boldsymbol{\omega}^T\boldsymbol{1})$$

where δ_1 and δ_2 are Lagrange multipliers.

The solution to is $\omega_{\mu_P} = 1/2\boldsymbol{\Omega}^{-1}(\delta_1\boldsymbol{\mu} + \delta_2\mathbf{1}) = \boldsymbol{\Omega}^{-1}(\lambda_1\boldsymbol{\mu} + \lambda_2\mathbf{1})$
where λ_1 and λ_2 are new Lagrange multipliers: $\lambda_1 = 1/2\delta_1$ and $\lambda_2 = 1/2\delta_2$. Thus, $\omega_{\mu P} = \lambda_1\boldsymbol{\Omega}^{-1}\boldsymbol{\mu} + \lambda_2\boldsymbol{\Omega}^{-1}\mathbf{1}$, where λ_1 and λ_2 are yet to be determined scalar quantities. We need to use the constraints and to findλ_1 and λ_2. Using these constraints imply the equation.

$$\mu_P = \boldsymbol{\mu}^T\boldsymbol{\omega}_{\mu_P} = \lambda_1\boldsymbol{\mu}^T\boldsymbol{\Omega}^{-1}\mu + \lambda_2\boldsymbol{\mu}^T\boldsymbol{\Omega}^{-1}\mathbf{1}, \tag{2.3}$$

and

$$1 = \mathbf{1}^T\boldsymbol{\omega}_{\mu_P} = \lambda_1\mathbf{1}^T\boldsymbol{\Omega}^{-1}\boldsymbol{\mu} + \lambda_2\mathbf{1}^T\boldsymbol{\Omega}^{-1}\mathbf{1}. \tag{2.4}$$

These are equations in λ_1 and λ_2, since all other quantities in these equations are known. We introduce simpler notation for the coefficients:

$$A = \boldsymbol{\mu}^T\boldsymbol{\Omega}^{-1}\mathbf{1} = \mathbf{1}^T\boldsymbol{\Omega}^{-1}\boldsymbol{\mu}, \tag{2.5}$$

$$B = \boldsymbol{\mu}^T\boldsymbol{\Omega}^{-1}\boldsymbol{\mu}, \tag{2.6}$$

$$C = \mathbf{1}^T\boldsymbol{\Omega}^{-1}\mathbf{1} \tag{2.7}$$

Then (2.3) and (2.4) can be rewritten as

$$\mu_P = B\lambda_1 + A\lambda_2 \tag{2.8}$$

$$1 = A\lambda_1 + C\lambda_2 \tag{2.9}$$

. Let $D = BC - A^2$ be the determinant of this system of linear equations. The solution to (2.8) and (2.9) is $\lambda_1 = \frac{-A+C\mu_P}{D}$ and $\lambda_2 = \frac{B-A\mu_P}{D}$
It follows after some algebra that

$$\omega_{\mu P} = \boldsymbol{g} + \mu_P\boldsymbol{h}$$

where

$$g = \frac{B\boldsymbol{\Omega}^{-1}\mathbf{1} - A\boldsymbol{\Omega}^{-1}\boldsymbol{\mu}}{D} = B/D\boldsymbol{\Omega}^{-1}1 - A/D\boldsymbol{\Omega}^{-1}\boldsymbol{\mu}$$

and

$$h = \frac{C\boldsymbol{\Omega}^{-\mathbf{1}}\boldsymbol{\mu} - A\boldsymbol{\Omega}^{-\mathbf{1}}\mathbf{1}}{D} = C/D\boldsymbol{\Omega}^{-\mathbf{1}}\boldsymbol{\mu} - A/D\boldsymbol{\Omega}^{-\mathbf{1}}\mathbf{1}$$

Note **g** and **h** are fixed vectors do depend on the fixed vector $\boldsymbol{\mu}$ and the fixed matrix $\boldsymbol{\Omega}$ but they do not depend on μ_P which will be varied. Also, the scalars A, C, and D are functions of $\boldsymbol{\mu}$ and $\boldsymbol{\Omega}$ so they are also fixed, that is, independent of μ_P . The target expected return, μ_P , can be varied over some range of values, e.g.,

$$min_{i=1,...,N}\mu_i \leq \ \mu_P \ \leq max_{i=1,...,N}\mu_i$$

or

$$\mu_{min} \leq \mu_P \leq max_{i=1,...,N}\mu_i \tag{2.10}$$

where μ_{min} is the expected return of minimum variance portfolio given in equation in (2.12) the next section. As μ_P varies over the range (2.10), we get a locus $\boldsymbol{\omega}_{\mu_P}$ of efficient portfolios called the "efficient frontier."

We can illustrate the efficient frontier by the following algorithm.

1. Vary μ_P along a grid. For each value of μ_P on this grid, compute σ_{μ_P} by:
   (a) computing $\boldsymbol{\omega}_{\mu_P} = \boldsymbol{g} + \mu_P \boldsymbol{h}$; and
   (b) then computing $\sigma_{\mu_P} = \sqrt{\boldsymbol{\omega}^T \boldsymbol{\Omega} \boldsymbol{\omega}_{\boldsymbol{\mu_P}}}$.
2. Plot the values (μ_P, σ_{μ_P}). The values (μ_P, σ_{μ_P}) with $\mu_P \geq \mu_{min}$ are the efficient frontier. The other values of (μ_P, σ_{μ_P}) lie below the efficient frontier and are (very) inefficient portfolios.

The minimum variance portfolio

The efficient portfolio with expected return equal to μ_P has weights

$$\boldsymbol{\omega}_{\mu_P} = \boldsymbol{g} + \boldsymbol{h}\mu_P$$

The variance of the return on the portfolio $R_P = \boldsymbol{\omega}_{\mu_P}^T \boldsymbol{R}$ is

$$\begin{aligned} Var(R_P) &= (\boldsymbol{g} + \boldsymbol{h}\mu_P)^T \boldsymbol{\Omega}(\boldsymbol{g} + \boldsymbol{h}\mu_P) \\ &= \boldsymbol{g^T \Omega g} + 2\boldsymbol{g^T \Omega h}\mu_P + \boldsymbol{h^T \Omega h}\mu_P^2. \end{aligned} \tag{2.11}$$

To find the minimum variance portfolio we minimize this quantity over μ_P by solving

$$0 = \frac{d}{d\mu_p} Var(R_P) = 2\boldsymbol{g^T \Omega h} + 2\boldsymbol{h^T \Omega h}\mu_P$$

The solution is the expected return of the minimum variance portfolio given by

$$\mu_{min} = -\frac{\boldsymbol{g^T \Omega h}}{\boldsymbol{h \Omega h}} \tag{2.12}$$

Plugging μ_{min} into (2.11) , and calling the portfolio R_{min}, we find that the smallest possible variance of a portfolio is

$$Var(R_{min}) = \boldsymbol{g}^T \boldsymbol{\Omega g} - \frac{(\boldsymbol{g^T \Omega h})^2}{\boldsymbol{h^T \Omega h}}$$

If one wants to avoid short selling[2], then one must impose the additional constraints that $\omega_i \geq 0$ for $i = 1, ..., N$. Minimization of portfolio risk subject to $\omega^T \mu = \mu_P$, $\omega^T 1 = 1$

[2]Selling Short means one sells a stock without owning it.

Finding the tangency portfolio

We now remove the assumption that $\omega^T 1 = 1$. The quantity $1 - \omega^T 1$ is invested in the risk-free asset. The expected return is

$$\boldsymbol{\omega}^T \boldsymbol{\mu} + (1 - \boldsymbol{\omega}^T \mathbf{1})\mu_f \tag{2.13}$$

where μ_f is the return on the risk-free asset. Suppose the target expected return is μ_P . Then the constraint to be satisfied is that (2.13) is equal to μ_P . Thus, the Lagrangian function is

$$L = \boldsymbol{\omega}^{\boldsymbol{T}} \boldsymbol{\Omega} \boldsymbol{\omega} + \delta\mu_P - \boldsymbol{\omega}^{\boldsymbol{T}} \boldsymbol{\mu} - (1 - \boldsymbol{\omega}^{\boldsymbol{T}} \mathbf{1})\mu_f$$

.

Here δ is a Lagrange multiplier. The optimal weight vector, i.e., the vector of weights that minimizes risk subject to the constraint on the expected return, is

$$\omega_{\mu P} = \lambda \boldsymbol{\Omega}^{-1}(\boldsymbol{\mu} - \mu_f \mathbf{1}) \tag{2.14}$$

, where $\lambda = \delta/2$.

To find λ, we use the constraint

$$\boldsymbol{\omega}_{\mu_P}^T \boldsymbol{\mu} + (1 - \boldsymbol{\omega}_{\mu_P}^T \mathbf{1})\mu_f = \mu_P \tag{2.15}$$

Rearranging , (2.15) we get

$$\boldsymbol{\omega}_{\mu_P}^T (\boldsymbol{\mu} - \mu_f \mathbf{1}) = \mu_P - \mu_f.$$

Therefore, substituting we have

$$\lambda(\boldsymbol{\mu} - \mu_f \mathbf{1})^T \Omega^{-1} (\boldsymbol{\mu} - \mu_f \mathbf{1}) = \mu_P - \mu_f$$

,or

$$\lambda = \frac{(\mu_P - \mu_f)}{(\boldsymbol{\mu} - \mu_f \mathbf{1})^T \Omega^{-1} (\boldsymbol{\mu} - \mu_f \mathbf{1})}$$

Then substituting λ into (2.14) gives

$$\boldsymbol{\omega}_{\mu_P} = c_P \bar{\boldsymbol{\omega}}$$

where $c_P = \frac{(\mu_P - \mu_f)}{(\boldsymbol{\mu} - \mu_f \mathbf{1})^T \Omega^{-1} (\boldsymbol{\mu} - \mu_f \mathbf{1})}$ and

$$\bar{\omega} = \Omega^{-1}(\boldsymbol{\mu} - \mu_f \mathbf{1})$$

Note $(\boldsymbol{\mu} - \mu_f \mathbf{1})$ is the vector of excess returns, that is, the amount by which the expected returns on the risky assets exceed the risk-free return. The excess returns measure how much the market pays for assuming risk. $\bar{\omega}$ **is not quite a portfolio because these weights do not necessarily sum to one**. The tangency portfolio is a scalar multiple of $\bar{\omega}$

$$\boldsymbol{\omega_T} = \frac{\bar{\boldsymbol{\omega}}}{\mathbf{1^T}\bar{\boldsymbol{\omega}}}$$

So, $c_P(\mathbf{1^T}\boldsymbol{\omega})$tells us how much weight to put on the tangency portfolio,ω_T . The amount of weight to put on the risk-free asset is $= 1 - c_p(\bar{\boldsymbol{\omega}}^{\boldsymbol{T}}\mathbf{1})$.

3 Analysis using Real Life Data

3.1 Data Pre Processing

We have taken the data of closing price of 9 stocks,i.e. Sun Pharma, Cipla, ONGC , JSW, TCS , Maruti Suzuki,Hindustan Uniliver, Tata Power and Zomato over the time period 1 April 2022 to 1 April 2024 from the website of Bombay Stock Exchange; when the stock market was open.

```
## 'data.frame': 496 obs. of  10 variables:
##  $ Date              : Date, format: "2022-04-01" "2022-04-04" ...
##  $ Sun.Pharma        : num  254 255 260 260 257 ...
##  $ Cipla             : num  1015 1028 1021 1028 1037 ...
##  $ ONGC              : num  168 168 172 173 169 ...
##  $ JSW               : num  308 324 340 339 328 ...
##  $ TCS               : num  3757 3770 3814 3756 3685 ...
##  $ Maruti.Suzuki     : num  7692 7775 7760 7745 7637 ...
##  $ Hindustan.Uniliver: num  2080 2127 2142 2142 2165 ...
##  $ Tata.Power        : num  245 252 274 290 278 ...
##  $ Zomato            : num  84.3 86.2 83.8 84.5 82.5 ...
```

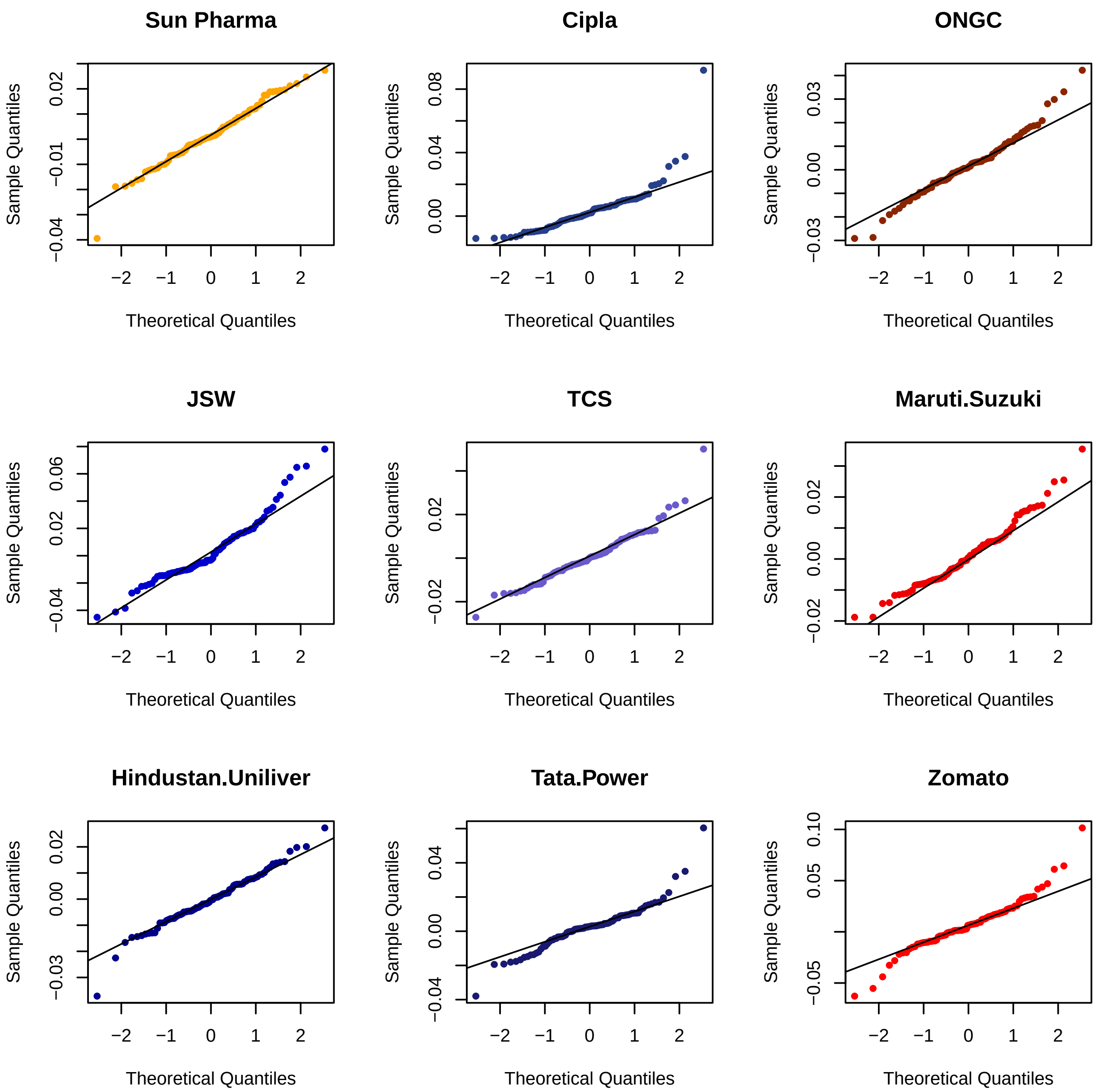
3.2 Finding Option Premium

We are to find value of option at maturity. We will be using **Black Scholes** Formula for that.

3.2.1 Normality Check of Log-Returns

To ensure applicability of Black Scholes Formula we need to check whether log returns follow normal distribution.

3.2.1.1 Q-Q Plot



3.2.1.2 Kolmogorov-Smirnov Test

We do the Kolmogorov-Smirnov Test at 1% level of significance and get the following output

```
## The log returns of stock Sun.Pharma follow a normal distribution (fail to reject H0).
## The log returns of stock Cipla follow a normal distribution (fail to reject H0).
```

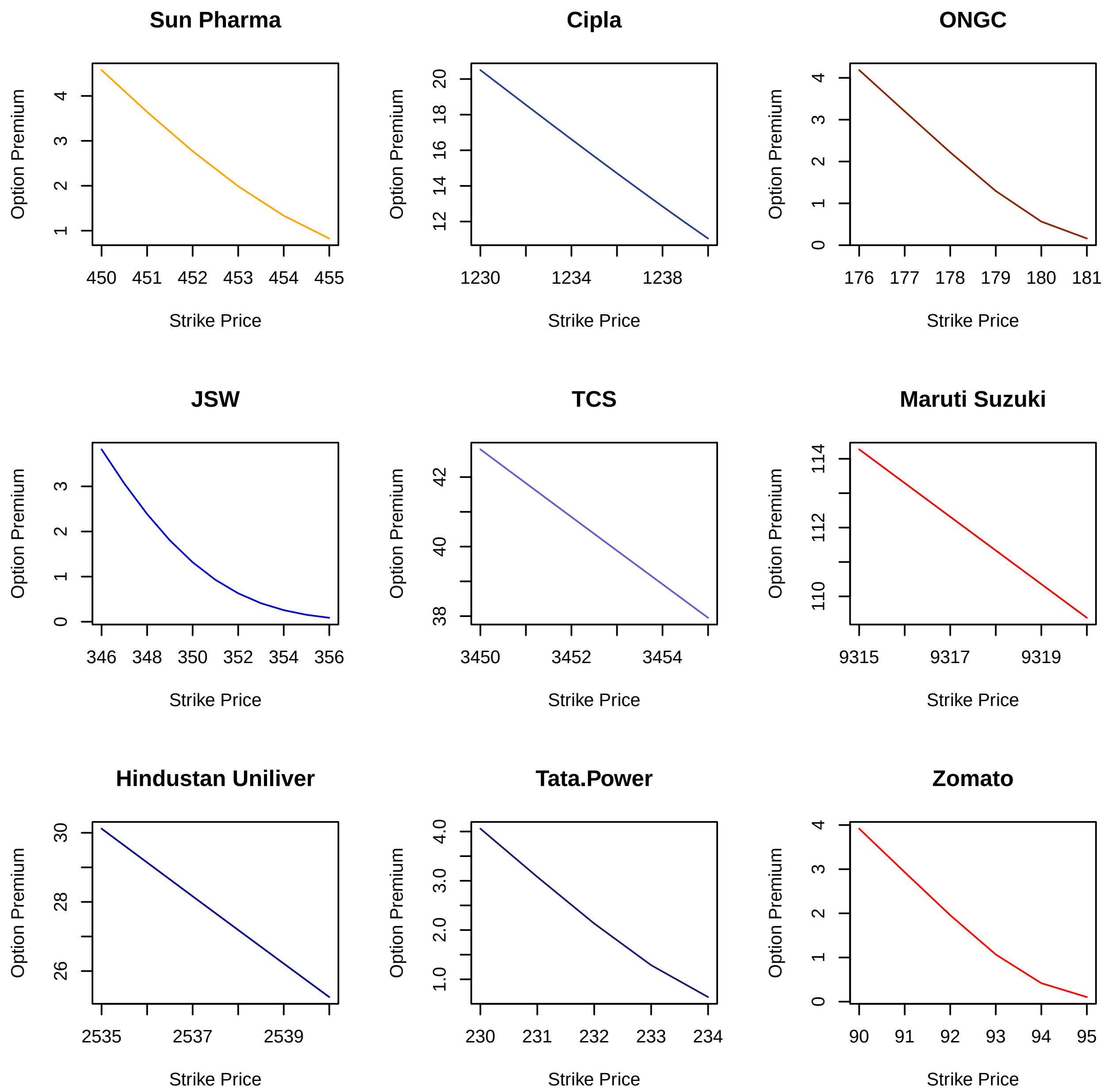
```
## The log returns of stock ONGC follow a normal distribution (fail to reject H0).
## The log returns of stock JSW follow a normal distribution (fail to reject H0).
## The log returns of stock TCS follow a normal distribution (fail to reject H0).
## The log returns of stock Maruti.Suzuki follow a normal distribution (fail to reject H0).
## The log returns of stock Hindustan.Uniliver follow a normal distribution (fail to reject H0).
## The log returns of stock Tata.Power follow a normal distribution (fail to reject H0).
## The log returns of stock Zomato follow a normal distribution (fail to reject H0).
```

Conclusion: So, we can see the log Returns follows normal distribution. Hence we can apply Black Sholes Formula in this data.

3.2.2 Change in Option Premium with changing parameters

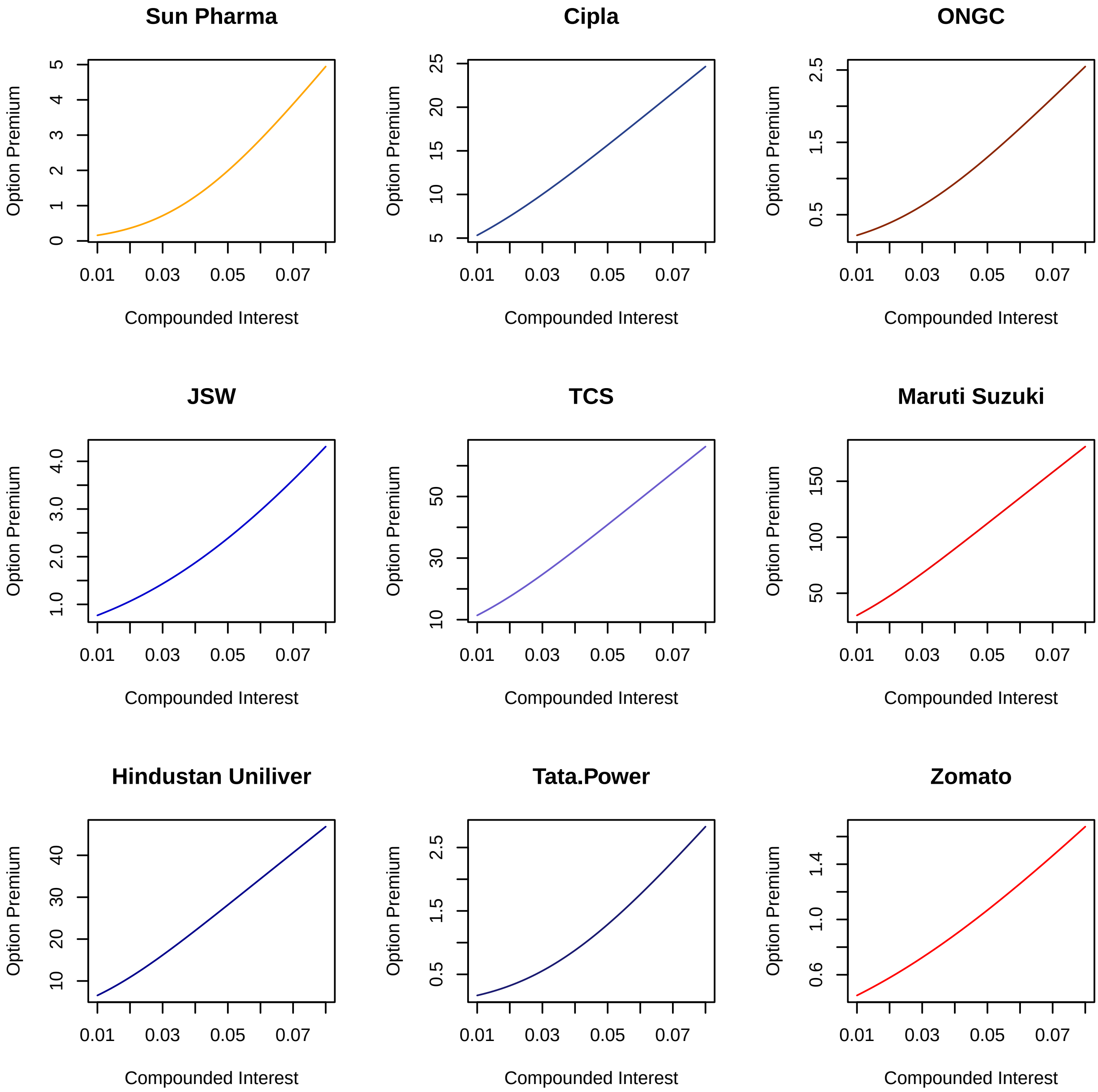
Now we will see the relation of various Parameters like Strike Price(K),Continuously Compounded interest (r),Maturity Date (T) with Option Premium.

Change in Option Premium with changing Strike price



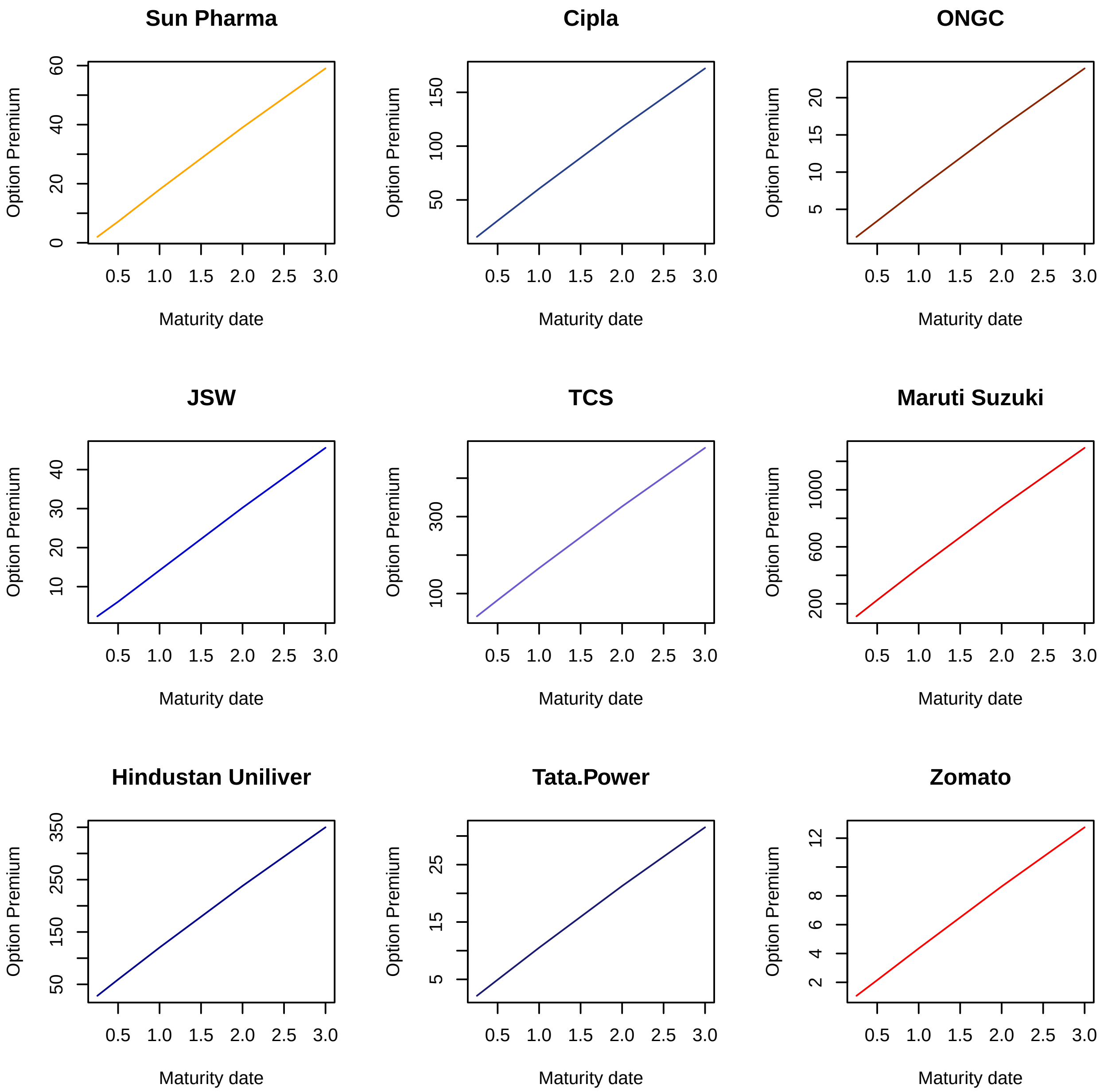
Interpretation: We can see the option premiums Decreases with increase in Strike Price as expected.

Change in Option Premium with changing Continuously Compounded interest



Interpretation: We can see the option premiums increases with increase in Continuously Compounded Interest.

Change in Option Premium with changing Maturity Date



Interpretation: We can see the option premiums increases with increase in Maturity Date.

3.2.3 Table

The following output gives us the option premiums of each stock with varying parameters.

```
## $Sun.Pharma
##    Strike Price Maturity Date Interest Rate Option Premium
## 1           450          0.25          0.01           1.05
## 2           450          0.25          0.02           1.72
## 3           450          0.50          0.01           2.11
## 4           450          0.50          0.02           3.75
## 5           453          0.25          0.01           0.16
## 6           453          0.25          0.02           0.36
## 7           453          0.50          0.01           0.71
## 8           453          0.50          0.02           1.67
## 9           455          0.25          0.01           0.03
## 10          455          0.25          0.02           0.08
## 11          455          0.50          0.01           0.27
## 12          455          0.50          0.02           0.80
##
## $Cipla
##    Strike Price Maturity Date Interest Rate Option Premium
## 1          1230          0.25          0.01           9.04
## 2          1230          0.25          0.02          11.71
## 3          1230          0.50          0.01          12.48
## 4          1230          0.50          0.02          17.84
## 5          1235          0.25          0.01           5.32
## 6          1235          0.25          0.02           7.50
## 7          1235          0.50          0.01           8.67
## 8          1235          0.50          0.02          13.44
## 9          1240          0.25          0.01           2.65
## 10         1240          0.25          0.02           4.17
## 11         1240          0.50          0.01           5.58
## 12         1240          0.50          0.02           9.52
##
## $ONGC
##    Strike Price Maturity Date Interest Rate Option Premium
## 1           176          0.25          0.01           2.44
## 2           176          0.25          0.02           2.88
## 3           176          0.50          0.01           2.90
## 4           176          0.50          0.02           3.75
## 5           179          0.25          0.01           0.22
## 6           179          0.25          0.02           0.39
## 7           179          0.50          0.01           0.57
## 8           179          0.50          0.02           1.08
## 9           181          0.25          0.01           0.00
## 10          181          0.25          0.02           0.01
## 11          181          0.50          0.01           0.06
## 12          181          0.50          0.02           0.20
##
## $JSW
##    Strike Price Maturity Date Interest Rate Option Premium
## 1           346          0.25          0.01           1.56
```

```
## 2          346          0.25          0.02            2.02
## 3          346          0.50          0.01            2.66
## 4          346          0.50          0.02            3.72
## 5          348          0.25          0.01            0.77
## 6          348          0.25          0.02            1.07
## 7          348          0.50          0.01            1.70
## 8          348          0.50          0.02            2.52
## 9          356          0.25          0.01            0.01
## 10         356          0.25          0.02            0.02
## 11         356          0.50          0.01            0.13
## 12         356          0.50          0.02            0.26
##
## $TCS
##    Strike Price Maturity Date Interest Rate Option Premium
## 1          3450          0.25          0.01           12.69
## 2          3450          0.25          0.02           19.07
## 3          3450          0.50          0.01           21.51
## 4          3450          0.50          0.02           35.56
## 5          3452          0.25          0.01           11.40
## 6          3452          0.25          0.02           17.49
## 7          3452          0.50          0.01           20.07
## 8          3452          0.50          0.02           33.80
## 9          3455          0.25          0.01            9.60
## 10         3455          0.25          0.02           15.23
## 11         3455          0.50          0.01           18.02
## 12         3455          0.50          0.02           31.22
##
## $Maruti.Suzuki
##    Strike Price Maturity Date Interest Rate Option Premium
## 1          9315          0.25          0.01           31.55
## 2          9315          0.25          0.02           49.14
## 3          9315          0.50          0.01           54.76
## 4          9315          0.50          0.02           93.68
## 5          9317          0.25          0.01           30.22
## 6          9317          0.25          0.02           47.50
## 7          9317          0.50          0.01           53.27
## 8          9317          0.50          0.02           91.88
## 9          9320          0.25          0.01           28.27
## 10         9320          0.25          0.02           45.08
## 11         9320          0.50          0.01           51.09
## 12         9320          0.50          0.02           89.18
##
## $Hindustan.Uniliver
##    Strike Price Maturity Date Interest Rate Option Premium
## 1          2535          0.25          0.01            7.80
## 2          2535          0.25          0.02           12.47
## 3          2535          0.50          0.01           14.00
## 4          2535          0.50          0.02           24.52
```

```
## 5          2537          0.25          0.01           6.55
## 6          2537          0.25          0.02          10.89
## 7          2537          0.50          0.01          12.57
## 8          2537          0.50          0.02          22.73
## 9          2540          0.25          0.01           4.92
## 10         2540          0.25          0.02           8.70
## 11         2540          0.50          0.01          10.57
## 12         2540          0.50          0.02          20.12
##
## $Tata.Power
##    Strike Price Maturity Date Interest Rate Option Premium
## 1           230          0.25          0.01           1.85
## 2           230          0.25          0.02           2.38
## 3           230          0.50          0.01           2.48
## 4           230          0.50          0.02           3.53
## 5           232          0.25          0.01           0.48
## 6           232          0.25          0.02           0.78
## 7           232          0.50          0.01           1.02
## 8           232          0.50          0.02           1.79
## 9           234          0.25          0.01           0.04
## 10          234          0.25          0.02           0.10
## 11          234          0.50          0.01           0.26
## 12          234          0.50          0.02           0.61
##
## $Zomato
##    Strike Price Maturity Date Interest Rate Option Premium
## 1            90          0.25          0.01           3.03
## 2            90          0.25          0.02           3.25
## 3            90          0.50          0.01           3.26
## 4            90          0.50          0.02           3.70
## 5            93          0.25          0.01           0.45
## 6            93          0.25          0.02           0.58
## 7            93          0.50          0.01           0.75
## 8            93          0.50          0.02           1.04
## 9            95          0.25          0.01           0.02
## 10           95          0.25          0.02           0.03
## 11           95          0.50          0.01           0.10
## 12           95          0.50          0.02           0.18
```

3.3 Portfolio Optimization

3.3.1 One Risky Asset(ONGC) and One Risk free Asset(SBI)

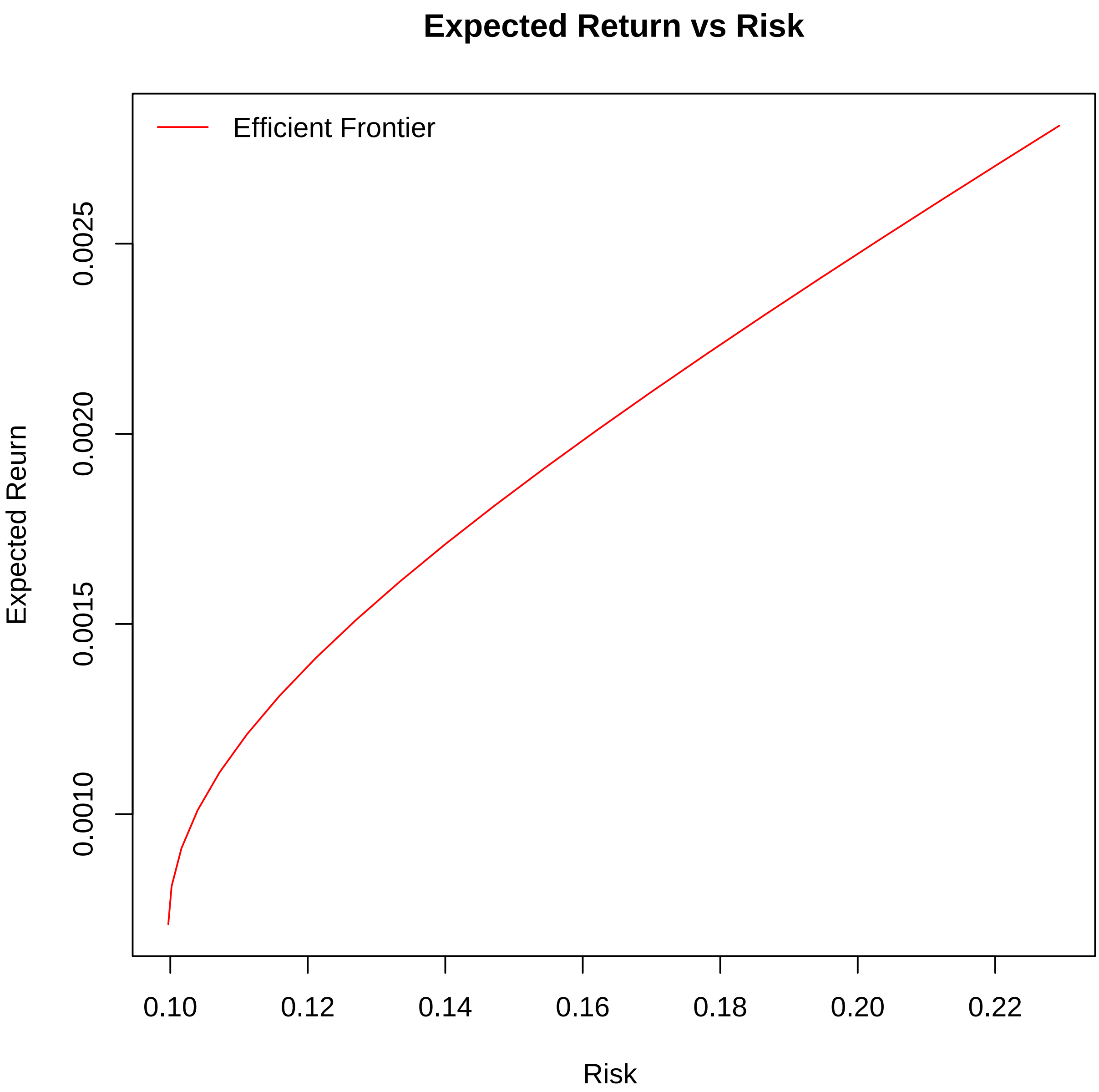
Assumption: We can't bear loss more than 10%. The probability that our expected return (loss in this case) is 0.1

As a risk free asset we have taken SBI savings account which gives returns of 2.7%.

```
## [1] "w: 0.361194629254572"
```

Interpretation: To ensure our return is not more than (-10%) with probability 0.1 we need to invest approximately 36% of our asset to ONGC stock and rest to SBI savings account.

3.3.2 To find the Portfolios



```
## [1] "Optical Allocation minimizing the variance with Expected return 10%:"
##              [,1]
##  [1,]  0.30771837
##  [2,]  0.23795388
##  [3,]  0.54903343
##  [4,] -0.04728974
##  [5,] -0.01406914
##  [6,]  0.27107113
##  [7,] -0.68121920
##  [8,]  0.15599921
##  [9,]  0.22080205
## [1] "The expected return of minimum variance portfolio is: 0.000710530479227121"
## [1] "Smallest possible variance of portfolio is 0.0099415294816141"
## [1] "Weights of Efficient Portfolio(w_mup):"
##              [,1]
##  [1,] 0.059434383
##  [2,] 0.052270508
##  [3,] 0.031671602
##  [4,] 0.001542320
##  [5,] 0.058510457
##  [6,] 0.064686996
##  [7,] 0.081880091
##  [8,] 0.007975864
##  [9,] 0.004460024
## [1] "The tangency portfolio is given by"
##             [,1]
##  [1,] 0.16398757
##  [2,] 0.14422146
##  [3,] 0.08738627
##  [4,] 0.00425547
##  [5,] 0.16143833
##  [6,] 0.17848025
##  [7,] 0.22591834
##  [8,] 0.02200650
##  [9,] 0.01230581
## [1] "The weight to put on risky asset: 0.362432244598202"
## [1] "The weight to put on risk free asset: 0.637567755401798"
```

4 Summarizing key findings

4.1 Option Pricing

- We have found that with increase in
  - Strike Price Option Premium decreases.
  - Continuously Compounded Interest Option Premium increases.
  - maturity date Option Premium increases.

4.2 Portfolio Optimization

- We have found that if we have Rs. 100 then we should invest approximately 36 rupees to risky asset and 64 to risk free asset (SBI savings account).
  - Out of that Rs. 36 we should invest in companies according to the following table

```
##                    Tangency_Portfolio
## Sun.Pharma                  5.9035525
## Cipla                       5.1919726
## ONGC                        3.1459058
## JSW                         0.1531969
## TCS                         5.8117799
## Maruti.Suzuki               6.4252889
## Hindustan.Uniliver          8.1330602
## Tata.Power                  0.7922339
## Zomato                      0.4430093
```

Notations

Symbol	Description
P_t	Price of an asset(Stock) at time "t"
r_t	Continuously Compounded Returns
S_t	Stock Price at time "t"
K	Exercise Price
r	Compounded Interest
σ	Standard Deviation/Volatility
T	Maturity Date/Expiration Date
C	Price of the option at time 0
Φ	Standard normal Cumulative Distribution
R	log Return
μ	Mean Return
R_i	Return of i^{th}risky asset
Ω_{ij}	Covariance between R_i and R_j
ω	Portfolio Weights
μ_p	Expected Portfolio of Return/Target Expected Value
δ_i	Lagrange Multipliers
ω_{μ_p}	Weights of Efficient Portfolio
μ_f	Value of the risk-free rate

R Code

- For Comparison of functions log(1 + x) and x

```
# Define a sequence of x values from -0.1 to 0.1
x <- seq(-0.1, 0.1, by=0.001)
# Calculate log(1 + x) for each x value
log_1_plus_x <- log(1 + x)
# Plot log(1 + x) vs x
plot(x, log_1_plus_x, type="l", lwd=2,xlab="x", ylab="log(1 + x)",
main="Comparison of log(1 + x) and x")
# Add the line y = x for comparison
abline(a=0, b=1, lwd=2, lty=2)
# Add a legend with reduced size
legend("topleft", legend=c("log(1 + x)", "y = x"), lty=c(1, 2), lwd=2, cex=1.4)
```

- Data Pre Processing

```
stock_price<-read.csv("C://Users//samim//Downloads//Stock_Prices.csv")
stock_price$Date=as.Date(stock_price$Date)
str(stock_price)
```

- Q-Q Plot

```
# Define a function to generate QQ plots
generate_qq_plot <- function(stock, stock_name, color)
{
        stock_Pt <- stock_price[[stock]][which(stock_price$Date >= "2023-04-03" &
        stock_price$Date <= "2023-08-11")]
        stock_Pt1 <- stock_price[[stock]][which(stock_price$Date >= "2023-03-31" &
        stock_price$Date < "2023-08-11")]
        qqnorm(log(stock_Pt/stock_Pt1), main = stock_name, col = color, pch = 20)
        qqline(log(stock_Pt/stock_Pt1))
}
# Plot QQ plots for each stock
par(mfrow=c(3,3))
generate_qq_plot("Sun.Pharma", "Sun Pharma", "orange")
generate_qq_plot("Cipla", "Cipla", "royalblue4")
generate_qq_plot("ONGC", "ONGC", "orangered4")
generate_qq_plot("JSW", "JSW", "blue3")
generate_qq_plot("TCS", "TCS", "slateblue")
generate_qq_plot("Maruti.Suzuki", "Maruti.Suzuki", "red2")
generate_qq_plot("Hindustan.Uniliver", "Hindustan.Uniliver", "blue4")
generate_qq_plot("Tata.Power", "Tata.Power", "midnightblue")
generate_qq_plot("Zomato", "Zomato", "red1")
```

- Kolmogorov Smirnov Test

```
for (i in 2:10)
{
        # Calculate log returns for the current stock column
        log_return <- function(stock, stock_price)
        {
        stock_Pt <- stock_price[which(stock_price$Date >= "2023-04-03" &
        stock_price$Date <= "2023-08-11"), stock]
        stock_Pt1 <- stock_price[which(stock_price$Date >= "2023-03-31" &
        stock_price$Date < "2023-08-11"), stock]
        log(stock_Pt / stock_Pt1)
```

```
        }
        log_returns <- log_return(i, stock_price)
         # Perform the test
        re <-ks.test(log_returns, "pnorm", mean =mean(log_returns), sd = sd(log_returns))
        # Print the result of the test
        if (re$p.value > 0.01) {
                cat("The log returns of stock", colnames(stock_price)[i],
                "follow a normal distribution (fail to reject H0).\n")
                }
        else
        {
        cat("The log returns of stock", colnames(stock_price)[i],
        "do not follow a normal distribution (reject H0).\n")
        }
}
```

- Table

```
# Define a function to calculate option premiums for different combinations of parameters for all stocks
calculate_option_premiums_all <- function(stocks, strike_prices_list, maturity_dates, interest_rates)
{
        all_premiums <- list()
        for (i in seq_along(stocks)) {
        premiums <- matrix(NA, nrow = length(strike_prices_list[[i]]) *
        length(maturity_dates) * length(interest_rates),ncol = 4,
        dimnames = list(NULL, c("Strike Price", "Maturity Date", "Interest Rate", "Option Premium")))
        counter <- 1
        for (strike in strike_prices_list[[i]])
        {
        for (maturity in maturity_dates)
        {
        for (rate in interest_rates)
        {
        stock <- stocks[i]
        stock_Pt <- stock_price[[stock]][which(stock_price$Date >= "2023-04-03" &
        stock_price$Date <= "2023-08-11")]
        stock_Pt1 <- stock_price[[stock]][which(stock_price$Date >= "2023-03-31" &
        stock_price$Date < "2023-08-11")]
        S0 <- stock_price[[stock]][which(stock_price$Date == "2023-08-14")]
        sig <- sqrt(var(log(stock_Pt/stock_Pt1)))
        T <- maturity
        r <- rate
        d1 <- (log(S0 / strike) + (r + sig^2/2) * T) / (sig * sqrt(T))
        d2 <- d1 - sig * sqrt(T)
        premium <- (pnorm(d1) * S0 - pnorm(d2) * strike * exp(-r * T))
        # Round premium to two decimal places
        premium <- round(premium, 2)
        premiums[counter, ] <- c(strike, maturity, rate,        premium)
        counter <- counter + 1
        }}}
        all_premiums[[stock]] <- as.data.frame(premiums)
        }
        return(all_premiums)
        }
# Set up parameters
strike_prices_list <- list(   "Sun.Pharma" = c(450, 453, 455),   "Cipla" = c(1230, 1235, 1240),   "ONGC" = c(176, 1
        "JSW" = c(346, 348, 356),   "TCS" = c(3450, 3452, 3455),   "Maruti.Suzuki" = c(9315, 9317, 9320),
         "Hindustan.Unilever" = c(2535, 2537, 2540),   "Tata.Power" = c(230,232,234),   "Zomato" = c(90, 93, 95) )
maturity_dates <- c(0.25, 0.5)
interest_rates <- seq(0.01, 0.02, by = 0.01)
stocks <- names(strike_prices_list)
# Calculate option premiums for all combinations for all stocks
all_premiums <- calculate_option_premiums_all(stocks, strike_prices_list, maturity_dates, interest_rates)
# Access premiums for all
all_premiums
```

- One Risky vs Risk Free Asset

```r
#Savings Account Return 2.70%
#Assumption we can bear only 10% loss
log_return_ONGC_1year<-{
ONGC_Pt <- stock_price$ONGC[which(stock_price$Date >= "2022-10-12" & stock_price$Date <= "2024-04-01")]
ONGC_Pt1 <- stock_price$ONGC[which(stock_price$Date >= "2022-10-11" & stock_price$Date < "2024-04-01")]
log(ONGC_Pt/ONGC_Pt1)
}
ER<-mean(log_return_ONGC_1year)
sigONGC<-sqrt(var(log_return_ONGC_1year))
SBI<-log(.027)
VR<-0.1
w<-(VR-SBI)/(sigONGC*qnorm(0.01)+(ER+SBI))
paste("w:" ,exp(w))
```

- Portfolio Optimization

```r
#To find the efficient Portfolio----------
#~R
log_return <- function(stock, stock_price) {
  stock_Pt <- stock_price[which(stock_price$Date >= "2022-10-12" & stock_price$Date <= "2024-04-01"), stock]
        stock_Pt1 <- stock_price[which(stock_price$Date >= "2022-10-11" & stock_price$Date < "2024-04-01"), stock]
        log(stock_Pt / stock_Pt1)
}
mean_ret<-colMeans(log_return(2:10,stock_price))#mu~
cov_mat<-cov(log_return(2:10,stock_price))*252 #omega_i_j
one_curl<-as.matrix(rep(1,9),nrow=9)
#-----------------------------------
library(MASS)
A <- t(mean_ret) %*% ginv(cov_mat) %*% one_curl
B <- t(mean_ret) %*% ginv(cov_mat) %*% mean_ret
C <- t(one_curl) %*% ginv(cov_mat) %*% one_curl
D <- B * C - A^2
g <- as.numeric(B/D) * (ginv(cov_mat) %*% one_curl) - as.numeric(A/D) * (ginv(cov_mat) %*% mean_ret)
h <- as.numeric(C/D) * (ginv(cov_mat) %*% mean_ret) - as.numeric(A/D) * (ginv(cov_mat) %*% one_curl)
mu_min=as.numeric((-t(g)%*%cov_mat%*%h)/(t(h)%*%cov_mat%*%h))
mu_p_values <- seq(mu_min, max(mean_ret), by = 0.0001)
sig_mup_values <- numeric(length(mu_p_values))
# Calculate minimum value of mean_ret
for (i in seq_along(mu_p_values)) {
mu_p <- mu_p_values[i]
if (mu_p >= mu_min) {
        w_p <- g + mu_p * h
# Calculate sig_mup
        sig_mup <- sqrt(t(w_p) %*% cov_mat %*% w_p)
# Store sig_mup value
        sig_mup_values[i] <- sig_mup
}
}
# Plot mu_p vs sig_mup
plot(sig_mup_values,mu_p_values, type = "l", ylab = "Expected Reurn", xlab = "Risk ",
main = "Expected Return vs Risk",col="red")
legend("topleft", legend = c("Efficient Frontier"), col = "red", lty = 1, bty = "n")
var_mu_min=as.numeric((t(g)%*%cov_mat%*%g)-((t(g)%*%cov_mat%*%h)^2/(t(h)%*%cov_mat%*%h)))
mu_f=log(0.027)
mu_p<-log(0.10)
cp=as.numeric((mu_p-mu_f)/t(mean_ret-as.matrix(mu_f*one_curl,nrow=9))%*%ginv(cov_mat)%*%(mean_ret-mu_f*one_curl))
wbar=ginv(cov_mat)%*%(mean_ret-mu_f*one_curl)
wt<-wbar/as.numeric(t(one_curl)%*%wbar)
w_mup=as.numeric(cp*(t(one_curl)%*%wbar))*wt
risky<-(cp*(t(one_curl)%*%wbar))
risk_free<-1-risky
paste("Optical Allocation minimizing the variance with Expected return 10%:")
print(w_p)
paste("The expected return of minimum variance portfolio is:",mu_min)
paste("Smallest possible variance of portfolio is",var_mu_min)
```

```
paste("Weights of Efficient Portfolio(w_mup):")
print(w_mup)
paste("The tangency portfolio is given by")
print(wt)
paste("The weight to put on risky asset:",risky)
paste("The weight to put on risk free asset:",risk_free)
```

- Summary

```
data.frame("Tangency_Portfolio"=wt*36,row.names = colnames(stock_price[2:10]))
```

Bibliography

[1] J. Hull. *Options, Futures, and Other Derivatives.* Prentice Hall, 2012.

[2] S.M. Ross. *An Elementary Introduction to Mathematical Finance.* Cambridge University Press, 2011.

[3] David Ruppert. *Statistics and finance: An introduction*, volume 27. Springer.