1. Supplementary Information

Title: Multi-path Multiple Traveling Purchasers' Problems Using a Novel Variable-length Quantum-inspired Genetic Algorithm

Appendix A. Supplemental Material

This appendix provides the instances of the realistic MTPP used in the study. These instances serve as representative examples of the model to evaluate the performance of the proposed algorithm in the main body of the report.

Realistic MTPP Instance 1

Instance 1 features ten markets with three routes and three vehicles, three products, and five purchasers. The market M_0 is the depot, and the market M_9 is considered the selling point; no products are available in the depot and the selling point.

Table A1 shows the price and availability of the three products P_0 , P_1 , and P_2 in those ten markets (M). The demand of the three products is shown in Table A2 for each of the five purchasers (T). Table A3 presents the 10-market 4D cost matrix with three routes and three vehicles.

Table A1: Product Price and Availability in Realistic MTPP Instance 1

Market		Price		Availability			
M	P_0	P_1	P_2	P_0	P_1	P_2	
0	0	0	0	0	0	0	
1	18	14	17	19	16	24	
2	14	11	13	12	12	16	
3	10	17	11	17	20	16	
4	13	19	15	23	17	11	
5	14	15	17	11	24	12	
6	12	16	18	24	14	11	
7	15	14	11	14	19	16	
8	12	19	15	21	19	16	
9	0	0	0	0	0	0	

Table A2: Product Demand in Realistic MTPP Instance 1

T	P_0	P_1	P_2
0	11	14	18
1	12	19	15
2	16	13	19
2 3	10	12	14
4	14	14	11

Table A3: 10-market 4D Cost Matrix in Realistic MTPP Instance 1

-i/j	0	1	2	3	4	5	6	7	8	9
		/ 18 56 96 \	/ 84 50 54	83 58 49	/ 86 83 34 \	/ 96 49 89	(54 13 58)	/ 96 46 38	/ 51 21 71 \	48 24 84
0	∞	75 77 73	43 26 92	50 30 18	66 24 77	70 79 77	23 76 70	90 42 16	52 39 30	66 72 18
		79 55 21	79 67 45	\ 59 89 15 <i>\</i>	72 41 16	27 42 51	92 76 42	51 80 26	60 69 93	37 78 84
	(48 81 84)	,	/ 51 72 34	22 31 20	/ 17 78 61	$\int 37 \ 63 \ 12 \$	/ 98 85 13 \	/ 36 48 68	/ 46 46 47	/ 18 78 74 \
1	44 89 35	∞	79 59 50	49 69 68	66 60 59	40 48 97	30 83 15	21 59 54	82 38 24	72 51 73
	31 70 66		54 60 13	51 86 60	18 59 80	29 34 46	86 35 91	79 98 69	39 12 19	24 90 54
	93 23 54	/ 36 70 66 \		96 33 79	/ 17 18 49	/ 37 81 61	(53 74 69)	$\int 67 \ 65 \ 17$	/ 26 39 68	$(25 \ 36 \ 92)$
2	71 54 49	64 42 95	∞	33 73 80	90 21 50	58 25 51	48 37 65	77 47 80	22 70 19	98 61 49
	24 43 31	17 40 20		$\begin{pmatrix} 49 & 75 & 34 \end{pmatrix}$	72 28 74	86 11 77	22 61 18	13 25 53	25 62 80	$\begin{pmatrix} 40 & 64 & 62 \end{pmatrix}$
	(36 12 23)	/ 17 34 53	(58 52 81 \		$\int 36 \ 59 \ 38$	68 74 74	/ 96 30 31	83 50 81	$\int 65 \ 55 \ 90 \ $	(13 36 48)
3	95 47 52	69 53 49	33 94 39	∞	19 69 69	93 78 38	13 22 59	95 38 28	32 98 93	96 98 23
	51 79 15	93 24 60	\ 58 56 85 \		$(20 \ 35 \ 31)$	19 64 91	58 43 37	94 66 35	26 24 57	89 18 76
	85 89 29	78 39 68	43 33 10	(83 89 23)		/ 54 48 22 \	/ 11 70 96	$\int 61 \ 47 \ 27$	/ 83 21 48	$(36 \ 21 \ 53)$
4	89 20 56	46 43 88	93 94 31	76 25 42	∞	93 11 25	20 36 55	86 81 12	21 60 92	55 68 37
	51 38 11	18 75 76	12 20 16	63 74 87		30 34 21	92 71 78	73 17 75	17 45 96	\ 95 56 35 <i>\</i>
	95 92 54	(19 74 22)	$(26 \ 93 \ 75)$	(11 69 19)	$\int 39 \ 57 \ 78$		$\int 34 \ 50 \ 57$	$(82 \ 95 \ 19)$	(27 83 38 \	$(24 \ 23 \ 35)$
5	12 63 59	76 47 10	81 48 51	35 83 32	63 93 87	∞	32 33 31	88 15 52	37 92 22	61 23 50
	23 66 84	95 24 63	66 79 85	80 51 30	77 63 35		98 76 84	$\begin{pmatrix} 62 & 41 & 23 \end{pmatrix}$	91 91 63	20 58 11
	$\int 73 \ 14 \ 27$	70 81 45	$(80\ 85\ 57)$	$\int 32 \ 67 \ 60$	$\int 92 \ 56 \ 89$	$\int 92 \ 71 \ 18$		$(21 \ 83 \ 57)$	79 44 71	$\int 52 \ 24 \ 47$
6	25 83 14	83 38 97	13 53 26	94 47 35	19 18 84	84 85 81	∞	38 55 96	66 32 33	63 30 40
	48 62 55	\ 54 75 87 <i>\)</i>	89 31 30	\ 14 10 43 /	19 14 45	90 31 97		86 84 51	15 22 48	$\begin{pmatrix} 42 & 54 & 42 \end{pmatrix}$
	75 39 48	(94 69 91)	$\int 95 \ 85 \ 53$	$\int 66 \ 16 \ 35$	(54 86 91)	74 61 17	(25 81 46)		$\int 24 \ 25 \ 21$	$\int 56 74 77$
7	43 26 71	86 52 78	68 28 81	25 75 13	21 94 64	56 93 74	62 49 79	∞	60 10 60	91 81 56
	30 36 80	39 50 57	\ 59 57 78 <i>\ \</i>	28 63 71 /	66 18 65	22 95 95	\ 98 80 92 \		52 51 70	\ 53 30 63 <i>\</i>
	$\int 22 \ 92 \ 85$	(68 74 47)	$\int 75 89 82$	$\int 37 \ 37 \ 24$	$\int 75 \ 44 \ 21$	(79 64 52)	(85 94 47)	$(20 \ 52 \ 62)$		$\int 36 \ 55 \ 91$
8	20 79 20	92 78 28	77 66 34	95 23 17	74 25 59	22 43 68	20 34 85	12 71 58	∞	75 79 58
	74 17 53	$\setminus 69 75 39 /$	80 42 57	93 59 48	94 93 42	97 10 73	88 66 20	11 18 50		19 44 98
	$\int 13 \ 88 \ 70$	(98 12 69)	(86 12 31)	$(27 \ 11 \ 91)$	$\int 23 \ 14 \ 82$	85 31 15	(85 72 82)	$(21 \ 72 \ 54)$	(49 23 82)	
9	35 79 84	22 67 65	31 50 22	35 68 90	63 97 81	69 81 96	84 49 13	58 76 58	75 14 27	∞
	13 19 69	18 70 89	86 79 81	13 10 10	95 31 35	92 20 78	50 27 20	66 17 85	11 34 18	

Realistic MTPP Instance 2

Instance 2 features six markets with three routes and three vehicles, two products, and three purchasers. The market M_0 is the depot, and the market M_5 is considered the selling point; no products are available in the depot and the selling point.

Table A4 shows the price and availability of the two products P_0 and P_1 in those six markets (M). The demand of the three products is shown in Table A5 for each of the three purchasers (T). Table A6 presents the 6-market 4D cost matrix with three routes and three vehicles.

Table A4: Product Price and Availability in Realistic MTPP Instance 2

Market	Price		Availability		
M	P_0	P_1	P_0	P_1	
0	0	0	0	0	
1	17	19	10	13	
2	13	22	18	20	
3	18	17	17	22	
4	21	19	23	11	
5	0	0	0	0	

Table A5: Product Demand in Realistic MTPP Instance 2

T	P_0	P_1
0	13	10
1	13	13
2	12	10

Table A6: 6-market 4D Cost Matrix in Realistic MTPP Instance 2

$\overline{i/j}$	0	1	2	3	4	5
0	∞	144 280 768 675 231 146	$ \left(\begin{array}{cccc} 504 & 250 & 486 \\ 129 & 234 & 828 \end{array}\right) $	$ \left(\begin{array}{cccc} 83 & 174 & 196 \\ 450 & 270 & 36 \end{array}\right) $	$ \left(\begin{array}{cccc} 688 & 747 & 102 \\ 594 & 48 & 385 \end{array}\right) $	$ \left(\begin{array}{cccc} 672 & 196 & 534 \\ 280 & 316 & 616 \end{array}\right) $
1	$ \left(\begin{array}{cccc} 216 & 39 & 58 \\ 207 & 684 & 210 \end{array}\right) $	√ 711 330 63 / ∞	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	531 89 105 240 96 168 594 72 36	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	27 294 357 / 132 336 54 648 80 80
2	92 152 42 / 408 144 136	$\begin{pmatrix} 66 & 62 & 100 \\ 245 & 414 & 224 \end{pmatrix}$	180 621 465 /	111 234 168 / 37 441 24	$\left(\begin{array}{cccc} 279 & 490 & 462 \\ 196 & 765 & 91 \\ 240 & 222 & 75 \end{array}\right)$	560 79 483 / 36 288 68
2	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ \left(\begin{array}{cccc} 245 & 414 & 204 \\ 459 & 430 & 180 \\ 72 & 156 & 296 \end{array}\right) $	∞ / 279 115 270 \	$ \left(\begin{array}{ccc} 280 & 192 & 97 \\ 261 & 68 & 276 \end{array}\right) $	$ \begin{pmatrix} 240 & 332 & 75 \\ 516 & 280 & 728 \\ 80 & 760 & 126 \end{pmatrix} $	$ \begin{pmatrix} 63 & 472 & 162 \\ 632 & 490 & 207 \\ 192 & 66 & 237 \end{pmatrix} $
3	$\begin{pmatrix} 161 & 266 & 626 \\ 492 & 342 & 192 \\ 273 & 72 & 57 \end{pmatrix}$	$ \begin{pmatrix} 648 & 204 & 146 \\ 144 & 810 & 432 \end{pmatrix} $	$ \begin{pmatrix} 213 & 162 & 147 \\ 216 & 301 & 248 \end{pmatrix} $	∞	$ \begin{pmatrix} 270 & 486 & 144 \\ 450 & 297 & 468 \end{pmatrix} $	$ \left(\begin{array}{ccc} 192 & 36 & 231 \\ 198 & 146 & 400 \\ 98 & 375 & 34 \end{array}\right) $
4	$ \begin{pmatrix} 119 & 18 & 98 \\ 180 & 168 & 50 \\ 288 & 168 & 370 \end{pmatrix} $	$ \begin{pmatrix} 259 & 648 & 244 \\ 464 & 75 & 459 \\ 430 & 77 & 616 \end{pmatrix} $	$\begin{pmatrix} 424 & 592 & 345 \\ 48 & 185 & 130 \\ 198 & 183 & 54 \end{pmatrix}$	$ \begin{pmatrix} 201 & 390 & 102 \\ 693 & 188 & 240 \\ 13 & 25 & 212 \end{pmatrix} $	∞	$ \begin{pmatrix} 225 & 252 & 92 \\ 784 & 488 & 392 \\ 360 & 192 & 558 \end{pmatrix} $
5	$ \begin{pmatrix} 108 & 96 & 138 \\ 760 & 282 & 312 \\ 306 & 632 & 105 \end{pmatrix} $	$ \left(\begin{array}{cccc} 34 & 272 & 265 \\ 207 & 265 & 441 \\ 744 & 192 & 300 \right) $	$ \left(\begin{array}{cccc} 464 & 208 & 567 \\ 198 & 282 & 117 \\ 58 & 112 & 340 \end{array}\right) $	$ \left(\begin{array}{cccc} 245 & 70 & 44 \\ 252 & 249 & 40 \\ 356 & 290 & 88 \end{array}\right) $	$ \left(\begin{array}{ccc} 252 & 177 & 152 \\ 76 & 138 & 483 \\ 60 & 105 & 248 \end{array}\right) $	∞

Table A7: Product Price and Availability in Realistic MTPP Instance 3

Market		Price					Availability			
\overline{M}	P_0	P_1	P_2	P_3	P_4	P_0	P_1	P_2	P_3	P_4
0	0	0	0	0	0	0	0	0	0	0
1	30	22	26	18	10	21	18	24	6	0
2	25	16	24	20	23	12	10	11	22	12
3	34	25	30	12	17	20	8	22	13	32
4	14	17	24	20	15	34	12	0	18	10
5	11	14	10	18	8	10	13	25	0	6
6	13	10	9	15	6	26	20	33	12	24
7	10	13	8	7	12	15	0	7	16	8
8	9	14	22	16	11	23	5	12	8	20
9	0	0	0	0	0	0	0	0	0	0

Realistic MTPP Instance 3

Instance 3 features five markets with three routes and one vehicle, five products, and five purchasers. The market M_0 is the depot, and the market M_2 is considered the selling point; no products are available in the depot and the selling point. A real-life representation of google map data is shown in the Figure A1.

Table A7 shows the price and availability of the five products P_0 , P_1 , P_2 , P_3 , and P_4 in those five markets (M). The demand of the five products is shown in Table A8 for each of the five purchasers (T). Table A9 presents product purchase and loading time for a range of weights. Table A10 and A11 describes distance between different markets and travel time respectively.

Figure A1: A real-life representation of google map data with eight markets, one depot (Home), and one selling market (Work)

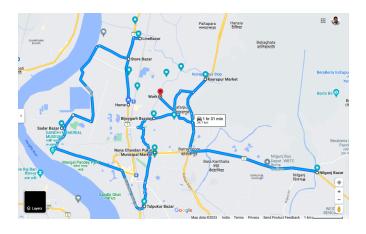


Table A8: Product Demand in Realistic MTPP Instance 3

\overline{T}	P_0	P_1	P_2	P_3	P_4
0	15	7	8	13	7
1	12	0	17	9	10
2	6	16	10	0	15
3	11	4	6	7	9
4	17	5	14	5	0

Table A9: Product Purchase and Loading Time in Realistic MTPP Instance 3

Weights (Kg)	Time (Minutes)
1 - 24	4
25 - 49	5
50 - 74	6
75 - 99	7
100 - 124	8

Table A10: 10-market 3D Distance (in Kilometers) Matrix in Realistic MTPP Instance 3

i/j	0	1	2	3	4	5	6	7	8	9
0	∞	(2.5 1.6 2.6)	(2.6 3.5 3.2)	(5.1 6.3 5.7)	(4.5 3.8 4.7)	(3 2.4 2.7)	(1.8 1.3 2)	(8.7 8.8 7.7)	(4 4.5 5.1)	(1.5 2.4 3.5)
1	$(2.5 \ 1.6 \ 2.6)$	∞	$(1 \ 1.4 \ 1.3)$	$(5.5 \ 5.8 \ 6.7)$	$(6 \ 6.3 \ 5.7)$	$(4.5 \ 4 \ 4.2)$	$(3.4 \ 3.5 \ 2.9)$	(8.8 10.3 8.2)	$(4.4 \ 3.2 \ 6.1)$	$(1.9 \ 2 \ 2.9)$
2	$(2.6 \ 3.5 \ 3.2)$	$(1 \ 1.4 \ 1.3)$	∞	$(6.4 \ 6.7 \ 7.6)$	$(7 \ 6.7 \ 6.9)$	$(5.2 \ 5.5 \ 6.6)$	$(5.7 \ 4.3 \ 4.4)$	(9.8 11.2 9.2)	$(5.4 \ 5.6 \ 7.8)$	$(3.7 \ 2.9 \ 4.2)$
3	$(5.1 \ 6.3 \ 5.7)$	$(5.5 \ 5.8 \ 6.7)$	$(6.4 \ 6.7 \ 7.6)$	∞ ′	$(5.4 \ 5.1 \ 6.8)$	$(5.1 \ 5.7 \ 5.9)$	$(6.5 \ 6.2 \ 8.2)$	$(10.8 \ 11.6 \ 11.9)$	(9 9.2 8.8)	$(6.7 \ 6.6 \ 6.8)$
4	$(4.5 \ 3.8 \ 4.7)$	(6 6.3 5.7)	(7 6.7 6.9)	$(5.4 \ 5.1 \ 6.8)$	` ∞ ′	$(2.1 \ 2.8 \ 3.2)$	(4.8 3.7 4.2)	(7.8 8.7 9.9)	(6 7.6 6.3)	$(5.8 \ 5.9 \ 5.3)$
5	(3 2.4 2.7)	(4.5 4 4.2)	$(5.2 \ 5.5 \ 6.6)$	$(5.1 \ 5.7 \ 5.9)$	$(2.1 \ 2.8 \ 3.2)$	· ∞	$(2 \ 2.5 \ 1.4)$	$(5.7 \ 7.1 \ 6.1)$	$(3.9 \ 4.6 \ 5.1)$	$(3.7 \ 3.2 \ 3.6)$
6	(1.8 1.3 2)	$(3.4 \ 3.5 \ 2.9)$	(5.7 4.3 4.4)	$(6.5 \ 6.2 \ 8.2)$	(4.8 3.7 4.2)	$(2\ 2.5\ 1.4)$	∞	(6.9 8 6.3)	$(2.7 \ 3.3 \ 3.4)$	(1.5 1.7 2)
7	(8.7 8.8 7.7)	(8.8 10.3 8.2)	(9.8 11.2 9.2)	(10.8 11.6 11.9)	(7.8 8.7 9.9)	$(5.7 \ 7.1 \ 6.1)$	$(6.9 \ 8 \ 6.3)$	` ∞ ´	$(7.3 \ 8.4 \ 7.3)$	$(6.9 \ 6.4 \ 6.8)$
8	$(4 \ 4.5 \ 5.1)$	(4.4 3.2 6.1)	$(5.4 \ 5.6 \ 7.8)$	(9 9.2 8.8)	(6 7.6 6.3)	(3.9 4.6 5.1)	$(2.7 \ 3.3 \ 3.4)$	$(7.3 \ 8.4 \ 7.3)$	· ∞	$(2.6 \ 3.3 \ 2.6)$
9	$(1.5 \ 2.4 \ 3.5)$	(1.9 2 2.9)	(3.7 2.9 4.2)	(6.7 6.6 6.8)	$(5.8 \ 5.9 \ 5.3)$	(3.7 3.2 3.6)	(1.5 1.7 2)	(6.9 6.4 6.8)	$(2.6 \ 3.3 \ 2.6)$	

Table A11: 10-market 3D Travel Time (in minutes) Matrix in Realistic MTPP Instance 3

i/j	0	1	2	3	4	5	6	7	8	9
0	∞	(6 4 7)	(5 10 9)	(10 12 13)	(10 11 12)	(6 6 9)	(577)	(13 18 21)	(12 13 10)	(6 8 12)
1	$(6\ 4\ 7)$	∞	$(3 \ 5 \ 4)$	$(12 \ 13 \ 18)$	(17 20 19)	(12 13 10)	(10 11 10)	$(23 \ 23 \ 23)$	$(13 \ 11 \ 17)$	$(7 \ 8 \ 12)$
2	$(5 \ 10 \ 9)$	$(3\ 5\ 4)$	∞	(17 18 21)	(19 22 23)	(12 10 20)	(18 13 12)	(21 18 21)	(12 13 20)	$(10 \ 10 \ 12)$
3	(10 12 13)	(12 13 18)	(17 18 21)	∞	(13 17 23)	(12 18 20)	$(20 \ 21 \ 22)$	$(21 \ 23 \ 21)$	(23 23 21)	(19 21 20)
4	$(10 \ 11 \ 12)$	(17 20 19)	(19 22 23)	(13 17 23)	∞	(9 11 10)	(13 13 10)	(21 22 18)	(12 18 21)	(17 17 18)
5	(669)	(12 13 10)	(12 10 20)	(12 18 20)	(9 11 10)	∞	(797)	(13 18 10)	(8 10 13)	(10 11 12)
6	(577)	(10 11 10)	(18 13 12)	(20 21 22)	(13 13 10)	(797)	` ∞ ´	(13 21 12)	(789)	(567)
7	(13 18 21)	$(23 \ 23 \ 23)$	(21 18 21)	(21 23 21)	(21 22 18)	(13 18 10)	$(13 \ 21 \ 12)$	· ∞	$(10 \ 20 \ 13)$	(13 17 18)
8	(12 13 10)	(13 11 17)	(12 13 20)	(23 23 21)	(12 18 21)	(8 10 13)	(789)	(10 20 13)	∞	(786)
9	(6 8 12)	(7 8 12)	(10 10 12)	(19 21 20)	(17 17 18)	(10 11 12)	$(5 \ 6 \ 7)$	(13 17 18)	(786)	

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