Practice modeling and simulation Home taken exam (re exam)

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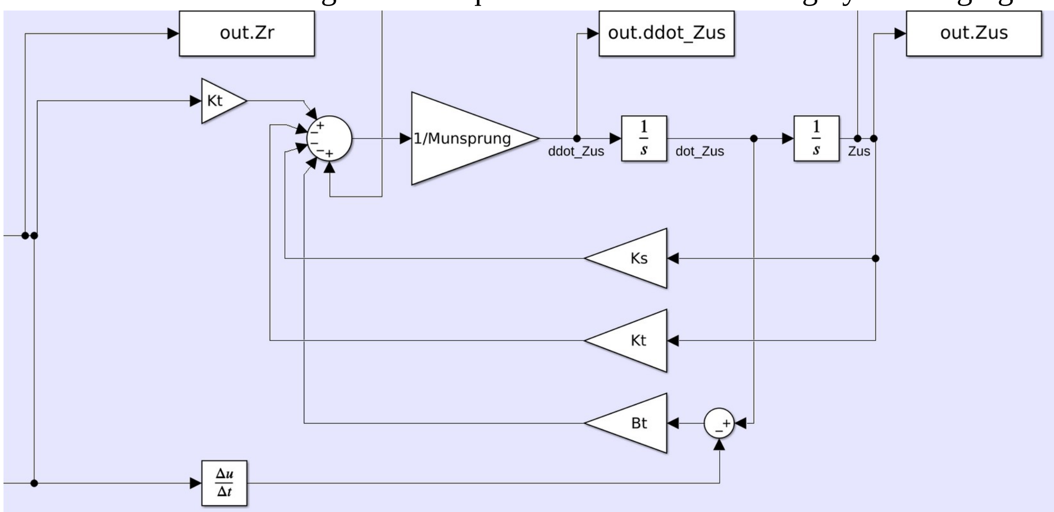
Q1/ to start simulating using all the parameters and variables the following notes need to be considered:
- the system input is Z_r which is the road displacement and the output is the sprung mass displacement which is the vertical displacement of the cart.
- given that the sprung subsystem is not directly connected to Z_r , then the inputs to the sprung subsystem in the simulation model will be taken from the unsprung subsystem.

Q2/ Starting to create the simulink model from the second equation since the input is direct and not dependent. Now to be able to build the simulink model rearrange the second equation to find $\ddot{z}_{us}$:

$$\ddot{z}_{us}=\frac{-z_{us}(K_s+K_t)+K_s\cdot z_s-B_t\cdot\dot{z}_{us}+B_t\cdot\dot{z}_r+K_t\cdot z_r}{M_{unsprung}}$$

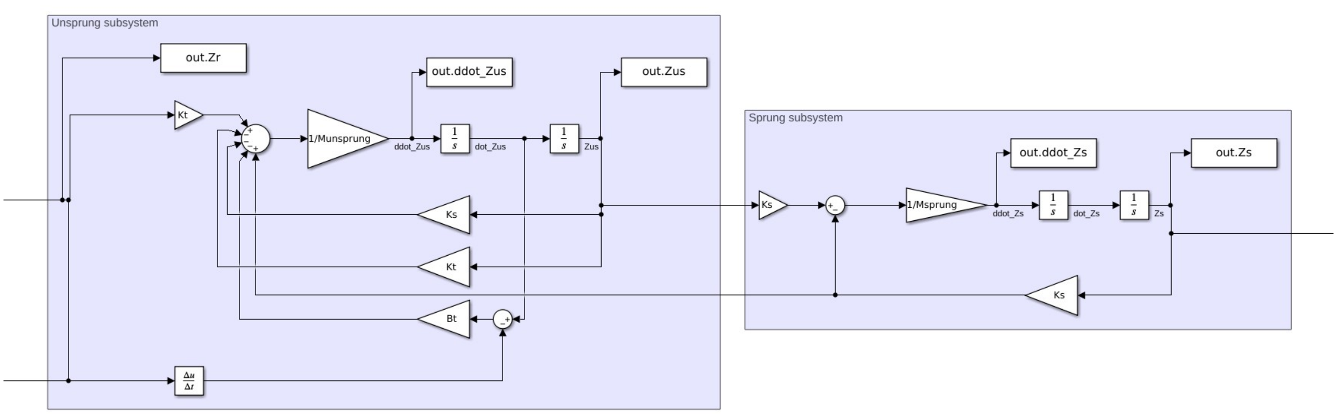
and in simulink the model will be as follows:
and after that start modeling the first equation in simulink starting by rearranging the equation for $\ddot{z}_s$



to make it as follows:

$$\ddot{z}_s=\frac{K_s z_{us}-z_s K_s}{M_{sprung}}$$

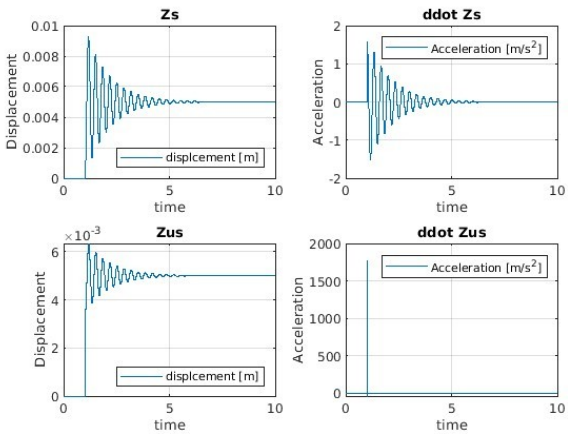
and modeling the equation in simulink will result in a model as in the following figure, where the “unsprung subsystem” represents the second equation and “Sprung subsystem” represents the first equation:



where all the triangles in the model refer to gain blocks, and the red borders in the first figure disappeared after assigning values to the parameters in the MATLAB script and running the simulation.

Q3/ Starting with the model verification assigned the values given to the parameters in the MATLAB script and multiplied the “Kt” and “Ks” by a 1000 to get both of them in the [N/m], instead of kilos, then change the solver type and fix the step size.

1- Now to give a road displacement of 5[mm] set the input of the simulation model “Zr” which is a “Step” block to the following parameters:
- Step time = 1
- Initial value = 0
- Final value = 0.005
where the final value represent the height of the bump in the road and it is 5[mm] and the output will be as shown in the following figures:



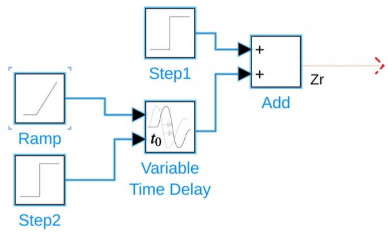
Where the “Zus” and “ddot Zus” in the bottom row are the displacement and acceleration of the unsprung mass respectively, and in this case it refers to the tire of the caster, and the plot is showing that the 5[mm] road bump will cause a maximum of 6.3[mm] displacement in the unsprung mass as an under-damped response since it keep oscillating for a while, and that represents the compression happening in the tire with acceleration of +1766[m/s^2], yet it is not showing the tire is going back to it’s original place, and that is because a step input stay constant after reaching it’s final value. So it is more similar to a change in the ground height than to a small road bump that goes up and goes back down.
The plots for “Zs” and “ddot Zs” represent the sprung displacement and acceleration respectively, and it shows that the 5[mm] road bump is resulting a maximum of 9[mm] displacement in the sprung mass which shows that when the cart is passing the bump it will jump vertically for an extra 4[mm] and it is happening in an acceleration equals a maximum of 1.56[m/s^2], where it is larger than the acceptable range of the payload vertical acceleration of +1.4 to -1.4. and from exploring other input values it shows that the maximum road bump the system can take is 4.4[mm] and it keeps the acceptable vertical acceleration range.

2- To simulate a bump with some inclination an input as a “Ramp” function should be used and to get a bump of flat-incline-flat structure, will use the following simulink blocks:

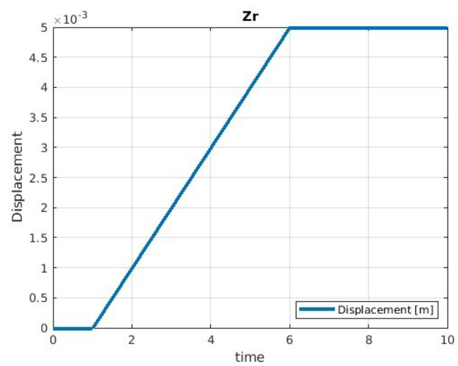
- Ramp:
  - Slope: 0.001
  - Start time: 1
  - Initial output: 0

- Step1:
  - Step time: 6
  - Initial value: 0
  - Final value: 0.005
- Step2:
  - Step time: 6
  - Initial value: 0
  - Final value: 10
- Variable Time Delay.
- Add.

And as shown in the following figure:



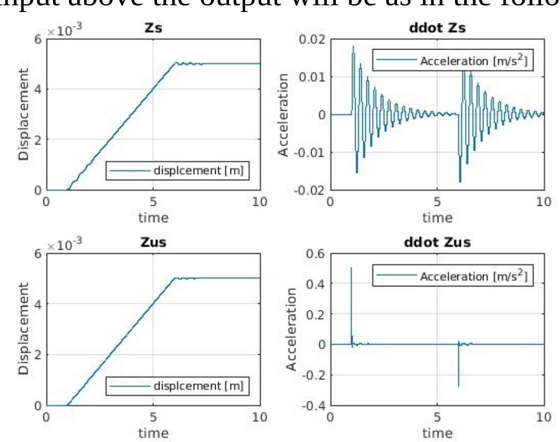
and the wave to be used as an input will be as the following plot:



where this wave got designed and implemented in the way it is for the following considerations:

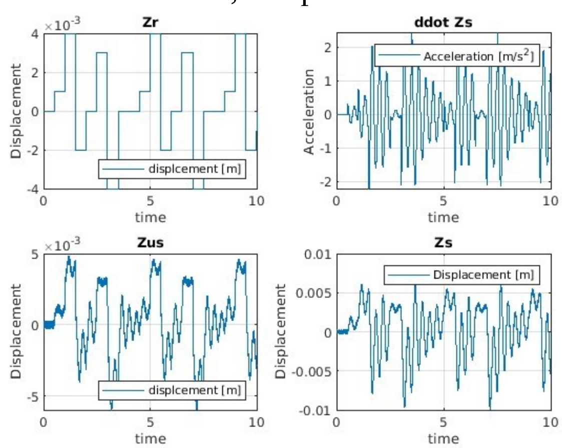
- the maximum value of the wave is 5[mm] to make the results comparable to the previous scenario.
- The slope of the line is inclined in the middle part to represent the inclination of the road bump, in contrast to the vertical line in the previous scenario.
- The inclination is delayed for 1[sec] to represent the first flat part of the bump and the ramp with the slop for 5[secs] to represent the inclination, and then a horizontal line to represent the second flat part of the bump.

and when the system get the input above the output will be as in the following sub figures:

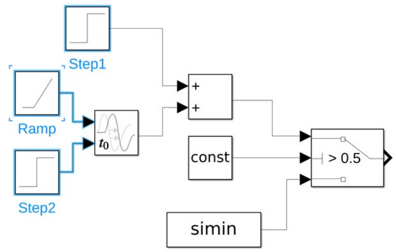


where in the bottom row of the figure the values of the variables ‘Zus’ and ‘ddot Zus’ are plotted, and they represent the unsprung displacement ‘Zus’ which keep increasing gradually to a maximum of 5[mm] with small oscillations after the changes of the slope, with positive acceleration ‘ddot Zus’ of 0.499[m/s^2] in the beginning of the slope representing the increase in the compression of the tires and a negative acceleration of -0.27[m/s^2] in the end of the slope represent returning the tire to it’s original shape. And comparing the results to the previous scenario shows a significant decrease in the acceleration of the unsprung mass as a response to the bumps.
In the first row of the figure above the ‘Zs’ and ‘ddot Zs’ are plotted and the displacement ‘Zs’ is represented with a gradually increasing vibrating line with maximum 5[mm] with some oscillation on the points of change in the road slope, representing the increase in the height of the sprung mass, and the ‘ddot Zs’ shows that the maximum vertical acceleration of the sprung mass is 0.018[m/s^2] which also much less than the acceleration in the previous scenario and meets the requirements of the hospital carts movements.
I think both of the scenarios outputs are correct and expected because an inclination in a road bump will of course make the displacement and vertical acceleration of a payload decrease, because the sudden change in the road height will get the payload to be jolted and other parts to be more extended, and the root reason for that is the earth gravity and the sudden change in it or the gradual change in case of the inclination.
I can conclude from that is there are ways to improve the performance of the caster, one is changing the environment such as inclining the flat bumps in the hospital.

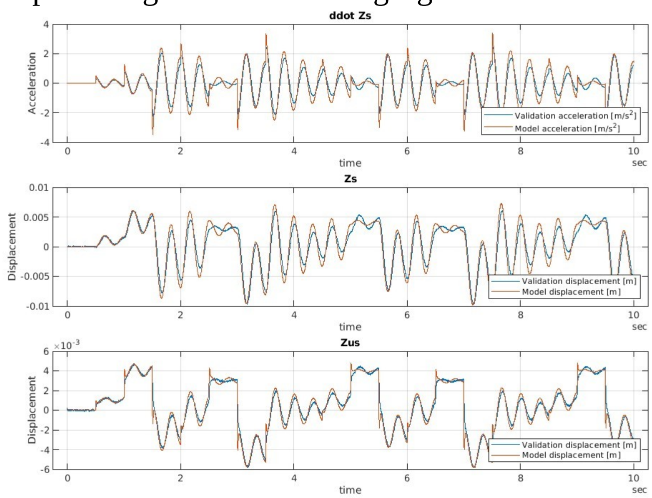
Q4/ 1- To validate the model, starting from importing the data will use “readtable” function since it is recommended by the MATLAB documentation, and plot the data which shows the following plots:



in the bottom row of the figure the two plots are shown for “Zus” and “Zs”, does not show the data in a clear and smooth line so for that can use the moving average filter and the command “smooth()” in MATLAB.
To compare the validation data with the output of the model using the same input data, but to run the same model with out a conflict with the verification input block, a switch has been included: and now the input of the system is as shown in the figure:



where the "const" block is the variable that sets the threshold of the switch, and "simin" is the block that imports the validation data to the simulink model from the workspace.
Now plotting the outputs of the validation data after filtering it, along the output of the simulink model running on the validation input data gives the following figures:



the figure shows differences between the validation data and the model output but it falls in an acceptable range because in reality the system parts will not be as perfect as the simulation, due to different environmental factors, frictions between parts of the cart and the road in addition to slight changes in the parameters resulting from wear and tear or aging.

2- to make a sensitivity analysis will make changes to the following variables and record the change in the output of the system:

- Msprung = 275 kg → 247.5 kg
- Munsprung = 15 Kg → 13.5 Kg
- Ks = 140000 N/m → 126000 N/m
- Kt = 300000 N/m → 270000 N/m
- Bt = 5000 Ns/m → 5500 Ns/m

running 4 tests while changing one parameter at a time, using the input data from the validation data sheet, shows that each parameter has different effect on the output variables, and as follows:

- Changing Msprung will mainly affect "ddot Zs" since the change mentioned above will increase the maximum of it from 3.4[m/s^2] to 4.3[m/s^2] also affect "Zs" and "Zus" slightly but leaves "ddot Zus" the same. So decreasing the sprung mass will be one solution to get the payload in the acceptable range of acceleration
- The change in Munsprung changes ddot Zus in a significant amount taking it from a maximum of "1910[m/s^2]" to "2065[m/s^2]" but this is the only output variable it affects.
- The change in "Ks" changes the output Zs from a maximum of 9.8[mm] to 11[mm] and slightly affect "ddot Zs" and "Zus" but leaving "ddot Zus" the same.

- The change in "Kt" by the amount mentioned above, affects all the four measured variables slightly, but no one is significantly changed.
- The change in "Bt" has a big effect on "ddot Zus" since it increases it's maximum from 1910[m/s^2] to 2041[m/s^2], and changes other output variables in a small value.

Comparing these results will give the following conclusions:

- changing the suspension spring stiffness or the sprung mass will affect the sprung outputs of the system.
- Changes to the damping coefficient of the tyre or the Munsprung will change the unsprung outputs of the system.
- The tyre spring stiffness does not have a significant effect on any of the outputs.

Q5/ To find the state space representation of the system that satisfies the two following equations:

$$\dot{x} = A\,x + B\,u$$

$$y = Cx + Du$$

will mark "Zr" as the system input and "Zs" as system output, and use the MATLAB function "linmod()" to find the representation of the simulink model, and running the the function will return the following values:

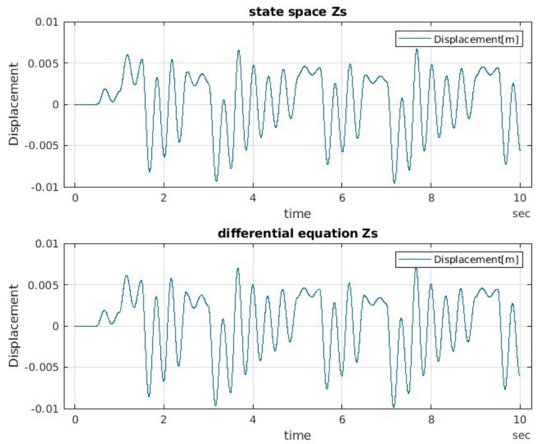
$$A = 1.0\text{e+}04 * \begin{bmatrix} 0 & 0 & 0 & 0 & 0.00010 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.00010 & 0 \\ 0.93333 & 0 & -2.93333 & -0.03666 & 0 \\ -0.05090 & 0 & 0.05090 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 20000 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 \end{bmatrix}$$

and using these matrices as parameters for the State space block in MATLAB with the road displacement data form the validation data sheet will give the following output:



and the plot confirms that the output of the simulink model does match the output of the state space model.

Q6/ To find the transfer function of the system after confirming the validity of the state space representation will use the function “ss2tf()” and input the matrices above to the function to get the matrices of the numerator and denominator which looks like the following:

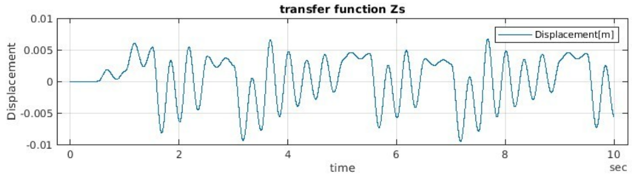
$$Numerator = 1.0e+07 * [0 \quad 0 \quad 0 \quad 0 \quad 1.01818 \quad 0]$$

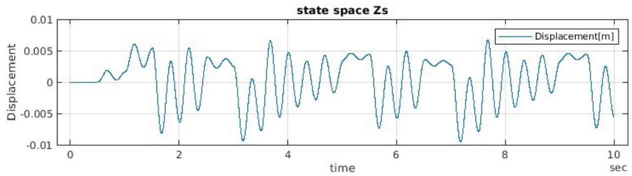
$$Denoimator = 1.0e+07 * [0.00000010 \quad 0.00003666 \quad 0.00298424 \quad 0.01866666 \quad 1.01818181 \quad 0]$$

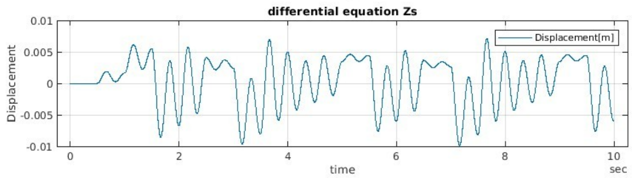
and putting these matrices in the form of a transfer function will be like the following:

$$\frac{Z_s(s)}{Z_r(s)} = \frac{1.0e+07 * 1.01818\, s}{1.0e+07 * (0.00000010\, s^5 + 0.00003666\, s^4 + 0.00298424\, s^3 + 0.01866666\, s^2 + 1.01818181\, s)}$$

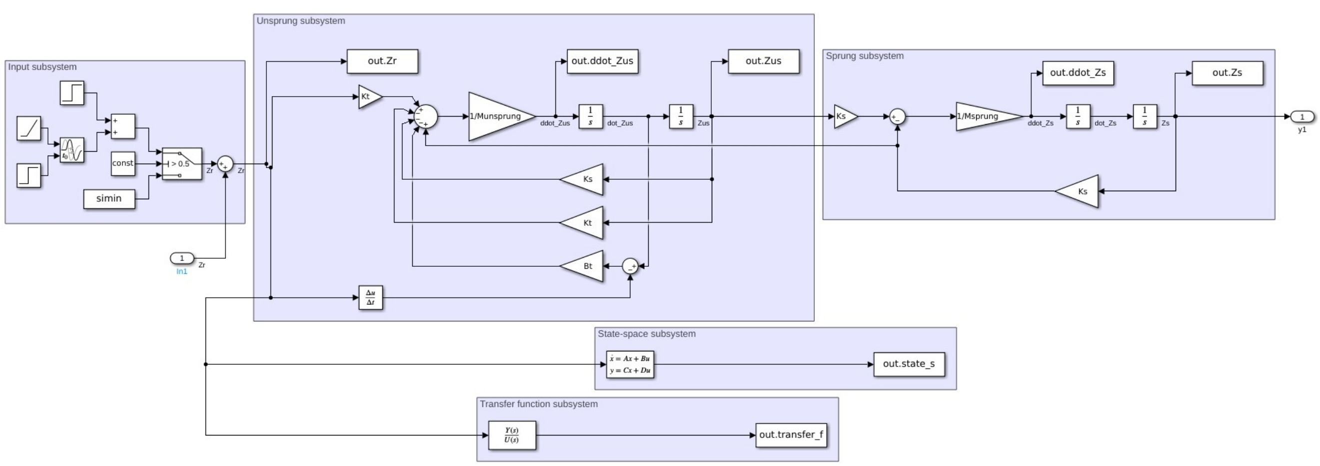
then giving this function as parameters to the “Transfer fcn” block of MATLAB, and the input is the road bump displacement date from the data sheet the output will be as follows compared to the outputs of the state space representation and the differential equation simulink model:



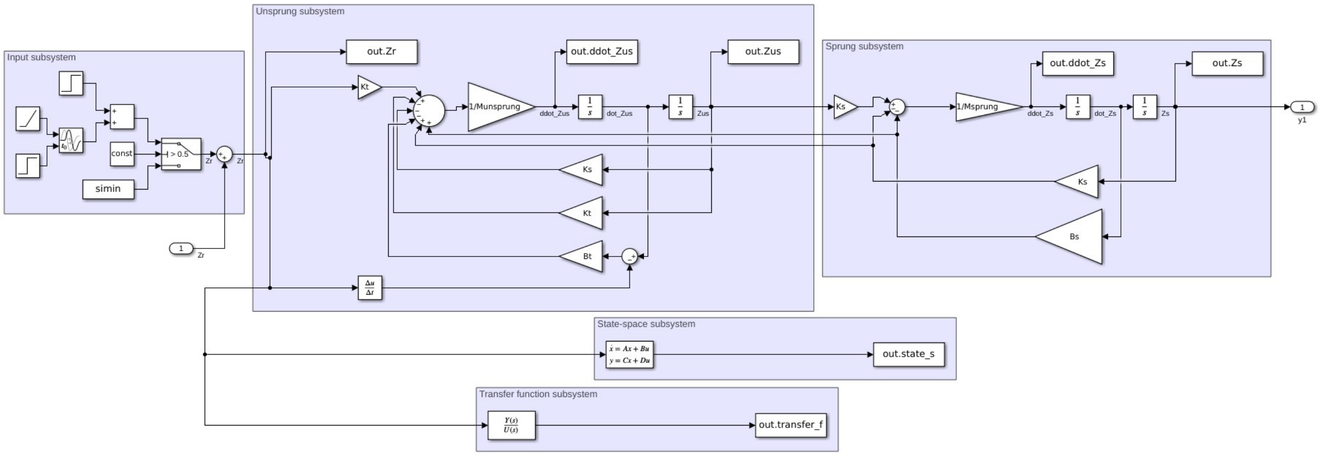




which shows that the transfer function gives the correct output. And the final system model will look as follows:



Q7/ Taking the suspension spring damping factor, means the simulink model will be looking as follows:



where “Bs” is the spring damping factor, and experimenting with few values shows that a value of 7000Ns/m will be the best, since it will be canceling the oscillation of the system while keeping the general system behavior without deformations, and looking back at the research question it shows that considering the damping factor of the suspension will get the model to work in the acceptable area of payload acceleration, since the maximum acceleration will be 1.18[m/s^2].