1 Exercises

Exercise 1. Consider the following recursive function:

```
def mystery(a, b):
    if b == 0:
        return 0
    if a == 0:
        return mystery(b - 1, a)
    return b + mystery(b, a - 1)
```

a. What is the value returned by the call `mystery(10, 0)`?

b. What is the value returned by the call `mystery(0, 10)`?

c. What is the value returned by the call `mystery(3, 7)`?

d. What is the value returned by the call `mystery(10, 3)`?

e. What is the value returned by the call `mystery(200, 300)`?

f. What does the function `mystery()` compute in general about a and b?

Exercise 2. Consider the function $S(n) = 1^2 + 2^2 + 3^2 + \cdots + n^2$, where n is a positive integer.

a. What is the value of $S(5)$?

b. Provide a recursive definition for $S(n)$.

c. Implement a function `S(n)` using recursion, such that it computes and returns $S(n)$

d. Trace the function call `S(5)`.

2 Solutions

Solution 1.

a. `0`

b. `0`

c. `21`

d. `30`

e. `60000`

f. The product ab.

Solution 2.

a. 55

b. $S(n) = \begin{cases} n^2 + S(n-1) & \text{if } n > 1, \text{ and} \\ 1 & \text{if } n = 1. \end{cases}$

c.

```
def S(n):
    if n == 1:
        return 1
    return n * n + S(n - 1)
```

d.

```
S(5)
  S(4)
    S(3)
      S(2)
        S(1)
          return 1
        return 2 * 2 + 1 = 5
      return 3 * 3 + 5 = 14
    return 4 * 4 + 14 = 30
  return 5 * 5 + 30 = 55
```