SECTION 2.1

SAMPLES SPACES AND EVENTS

Objectives

- Introduce the sample space of a probability event, events and operations on them.

Sample Space

- A **(probability) experiment** (or *trial*) is any process with a result determined by chance. Each individual result that is possible for a probability experiment is an **outcome**.
- The **sample space** of an experiment, denoted by S, is the set of all possible outcomes of that experiment.
- An **event** is a subset of outcomes from the sample space S.
- An event is simple if it consists of exactly one outcome and compound if it consists of more than one outcome.

Examples

1) Flipping a coin: $S = \{H, T\}$, where H is head, and T is tail.

   Event A of getting a head: $A = \{H\}$

2) Tossing a dice: $S = \{1, 2, 3, 4, 5, 6\}$

   Event O of getting an odd number: $O = \{1, 3, 5\}$

3) Examining product quality: $S = \{G, D\}$, where G is good, and D is defective.

4) Drawing a raffle ticket

5) Tossing a coin, then rolling a die:

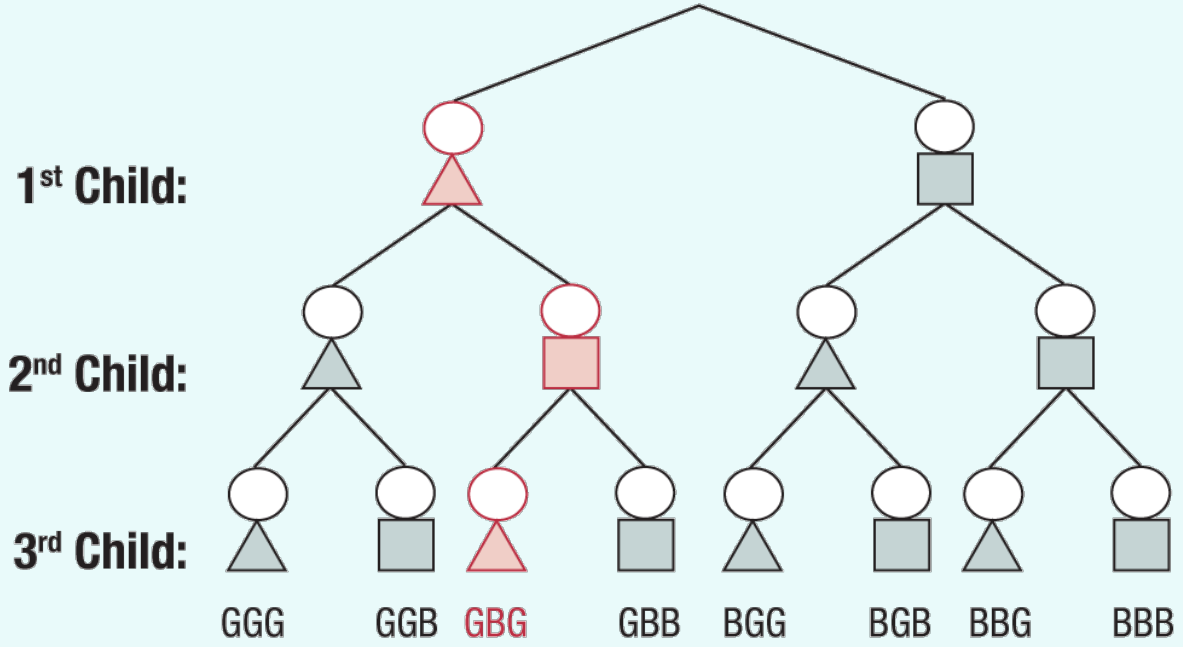
   $$S = \{H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6\}$$

   Event B of tossing a tail then rolling an even number: $B = \{T2, T4, T6\}$

Examples

6) Consider a family with three children. Use a tree diagram to find the sample space for the gender of each child in regard to birth order.

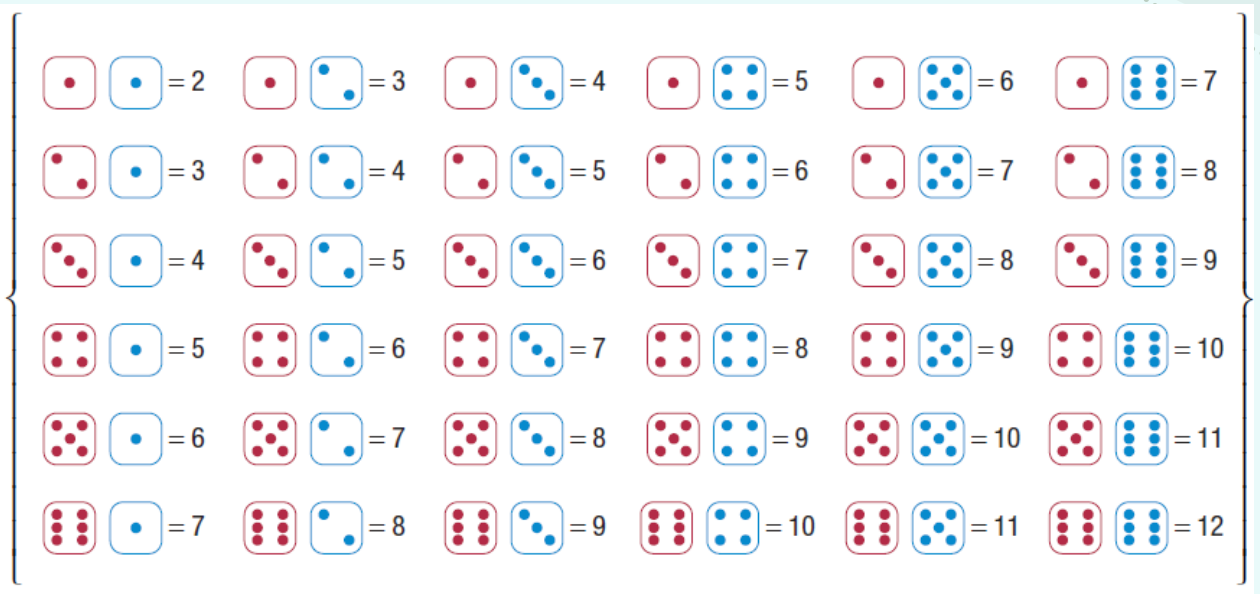
Sample space $S = \{GGG, GGB, GBG, GBB, BGG, BGB, BBG, BBB\}$



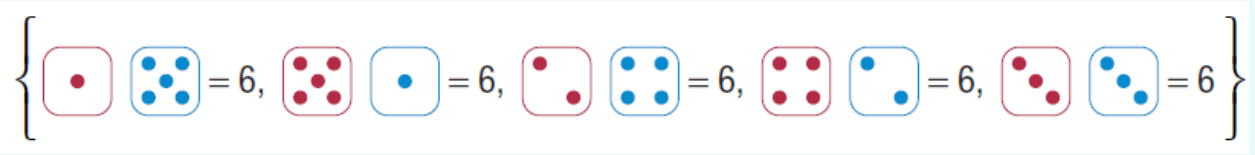
Examples

7) A red six-sided die and a blue six-sided die are rolled together.

Sample space =



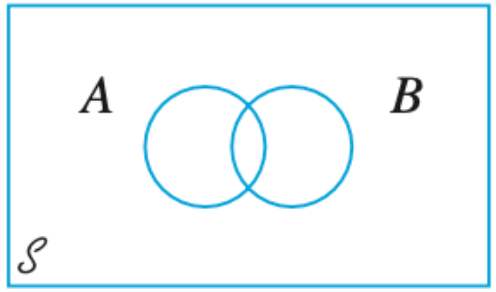
Event "the sum of the numbers rolled on the two dice equals 6":



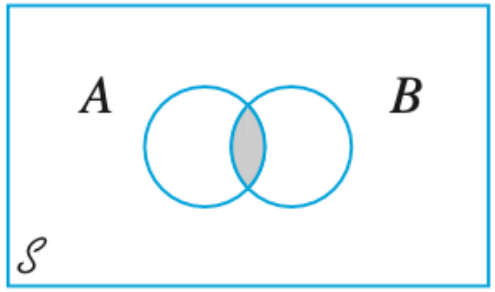
Some Relations from Set Theory

1. The complement of an event A, denoted by A', is the set of all outcomes in S that are not contained in A.
2. The union of two events A and B, denoted by $A \cup B$, is the event consisting of all outcomes that are either in A or in B.
3. The intersection of two events A and B, denoted by $A \cap B$, is the event consisting of all outcomes that are in both A and B.
4. The null event (consisting of no outcomes) is the empty subset of the sample space, denoted by $\emptyset$.
5. The sample space itself is also an event. It can be described as "the event that at least one outcome happens."
6. When $A \cap B = \emptyset$, A and B are said to be mutually exclusive or disjoint events.

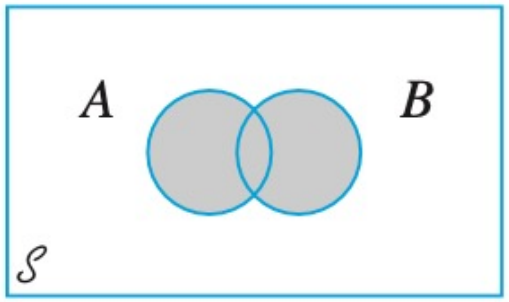
Venn Diagrams



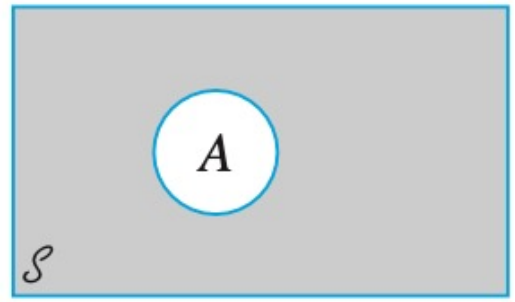
(a) Venn diagram of events A and B



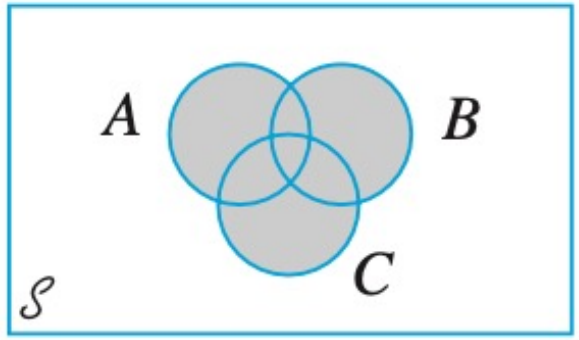
(b) Shaded region is $A \cap B$



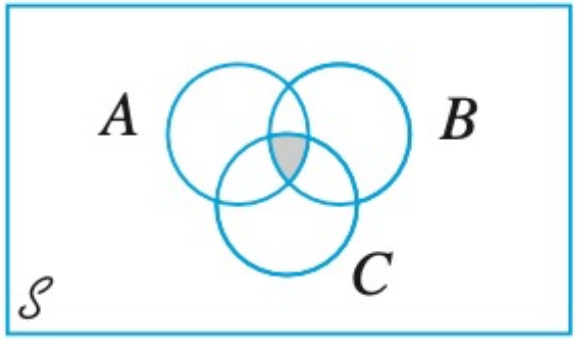
(c) Shaded region is $A \cup B$



(d) Shaded region is A'



(f) Shaded region is $A \cup B \cup C$



(g) Shaded region is $A \cap B \cap C$