SECTION 2.4

Conditional Probability

Conditional Probability

- Two events are **dependent** if one event happening affects the probability of the other event happening.
- **Conditional probability**, denoted P(A|B) and read "the probability of A given B," is the probability of event A occurring given that event B occurs first.

Example:

One card has already been chosen from a standard deck without replacement. What is the probability of now choosing a second card from the deck and it being red, given that the first card was a diamond?

Conditional Probability

For two dependent events, *A* and *B*, the conditional probability of *A* given that B has occurred is computed by the following formula.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Example:

Out of 300 applicants for a job, 212 are female and 110 are female and have a graduate degree.

a) What is the probability that a randomly chosen applicant has a graduate degree, given that she is female?

b) If 152 of the applicants have graduate degrees, what is the probability that a randomly chosen applicant is female, given that the applicant has a graduate degree?

Example

Suppose a marketing research firm has surveyed a panel of consumers to test a new product and produced the following cross tabulation indicating the number of panelists that liked the product, the number that did not like the product, and the number that were undecided.

Market Research Survey			
Age	**Like**	**Not Like**	**Undecided**
18-35	213	197	103
35-50	193	184	67
Over 50	144	219	83

If an individual is between 35 and 50 years old, what is the probability he or she will like the product?

Example

A computer software company receives hundreds of support calls each day. There are several common installation problems, call them A, B, c, and D. Several of these problems result in the same symptom, *lock up* after initiation. Suppose that the probability of a caller reporting the symptom *lock up* is 0.7 and the probability of a caller having problem A and a *lock up* is 0.6.

a) Given that the caller reports a lock up, what is the probability that the cause is problem A?

b) What is the probability that the cause of the malfunction is not problem A given that the caller is experiencing a lock up?

The Multiplication Rule for $P(A \cap B)$

For two dependent events, *A* and *B*, the probability of (*A* and *B*) is given by the following formula.

$$P(A \cap B) = P(A|B) \cdot P(B)$$

Example:

Assume that there are 17 newcomers and 24 long-term members in the Rotary Club. Two members are chosen at random each year to serve on the scholarship committee. What is the probability of choosing two members at random and the first being a newcomer and the second being a long-term member?

The Multiplication Rule for $P(A \cap B)$

The multiplication rule is most useful when the experiment consists of several stages in succession.

$$P(A_1 \cap A_2 \cap A_3) = P(A_1) \cdot P(A_2|A_1) \cdot P(A_3|A_1 \cap A_2)$$

Example

As part of an incentive, world Autos is offering any salesperson with at least 15 car sales in each month a chance to win one of three $50 gift cards. For every additional 5 cars, your name is entered again. The chart below shows the number of sales for each team member. Winning names are drawn without replacement from all entries.

a) Find the probability of S. Smith winning all three gift cards.
b) Find the probability of S. Smith winning the first gift card and M. Trent winning the second gift card.
c) Find the probability that all three winners are salespeople with only one entry in the drawing.

Monthly Sales at World Autos					
Salesperson	15 cars sold	20 cars sold	25 cars sold	30 cars sold	35 cars sold
B. Carple	x	x			
R. Henry	x				
M. Trent					
J. Owhan	x	x	x	x	
L. Prince	x	x	x		
S. Smith	x	x	x	x	x
H. Mowis	x				
T. Hilson	x				

The Law of Total Probability

- Let $A_1, \ldots, A_k$ be mutually exclusive and exhaustive events. Then for any other event B,

$$P(B) = P(B|A_1)P(A_1) + \cdots + P(B|A_k)P(A_k)$$

$$= \sum_{i=1}^{k} P(B|A_i)P(A_i)$$

Example:

Company A supplies 80% of widgets for a car shop and only 1% of their widgets turn out to be defective. Company B supplies the remaining 20% of widgets for the car shop and 3% of their widgets turn out to be defective. If a customer randomly purchases a widget from the car shop, what is the probability that it will be defective?

Bayes' Theorem

- Let $A_1, A_2, \ldots, A_k$ be a collection of *k* mutually exclusive and exhaustive events with *prior* probabilities $P(A_i)$ ($i = 1, \ldots, k$). Then for any other event B for which $P(B) > 0$, the *posterior* probability of A_j given that B has occurred is

$$P(A_j|B) = \frac{P(A_j \cap B)}{P(B)} = \frac{P(B|A_j)P(A_j)}{\sum_{i=1}^{k} P(B|A_i)P(A_i)} \qquad j=1,\ldots,k$$

Example

There is a drug test that is 98% accurate, meaning that 98% of the time, it shows a true positive result for someone using the drug, and 98% of the time, it shows a true negative result for nonusers of the drug. Assume 0.5% of people use the drug. If a person selected at random tests positive for the drug, determine the probability the person is actually a user of the drug.

Example

Assume that the chances of a person having a skin disease are 40%. Assuming that skin creams and drinking enough water reduces the risk of skin disease by 30% and prescription of a certain drug reduces its chance by 20%. At a time, a patient can choose any one of the two options with equal probabilities. It is given that after picking one of the options, the patient selected at random has the skin disease. Find the probability that the patient picked the option of skin screams and drinking enough water.

Example

A man is known to speak the truth 3/4 times. He draws a card and reports it is king. Find the probability that it is actually a king.