Classical Mechanics

Physics:

> Physics ≡ Quantitative description of everything in the "physical world."

Main Areas:

Classical Physics (pre-1900)

- (our focus) → Mechanics (motion, fluids, sound)
- Optics (light) } Electromagnetism
- Electricity } Electromagnetism
- Magnetism } Electromagnetism
- Thermodynamics
- Astronomy (involves all above)

Modern Physics (1900-present)

- Special and General Relativity
- Quantum Mechanics
  - atomic physics
  - physical chemistry
  - nuclear + particle physics
- Astrophysics/Cosmology

} Quantum Field Theories like
- string theory
- loop quantum gravity

- Biophysics (involves quantum)
- Computational Physics
  - classical computing (transistors)
  - quantum computing

} Applications and Extension to Complex Systems

The Four Forces:

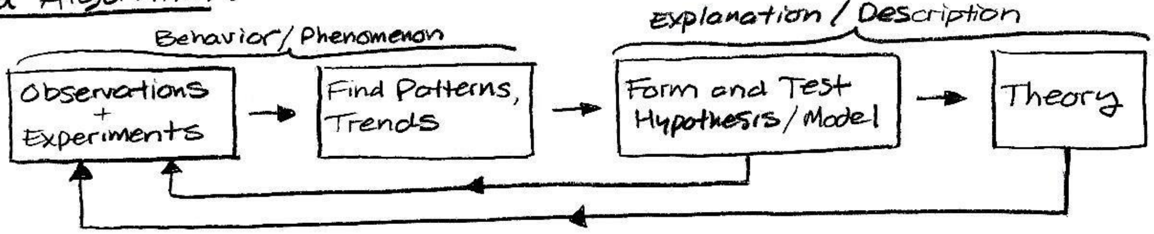
relative strength of forces ↑

Force	Theory	Mediator	Range [m]
Strong	Chromodynamics	Gluon	$\sim 10^{-15}$ (atomic nucleus-sized)
Electromagnetic	Electrodynamics	Photon	∞
Weak	Flavordynamics	W and Z (intermediate vector bosons)	$\sim 10^{-18}$ (0.1% of a proton "diameter")
Gravitational	General Relativity (Geometrodynamics)	Graviton	∞

The Scientific Method: ≡

> An algorithm (recipe of steps) for
> - making clear observations of some phenomenon
> - and creating a model (mathematical construct) to reliably describe the phenomenon

- Simplified Algorithm:



- Important Ingredients:
  - Data ≡ emperical (experimentally obtained) facts (• These are quantitative observations • Must be reproducible)
  - Observation ≡ qualitative description (• used before any testing or data • used while and after data is obtained + analyzed)
  - Analysis ≡ quantitative exploration of data (can yield new observations + hypotheses)
  - Hypothesis ≡ tentative explanation or theoretical model (must design experiments to challenge the model)
  - Experiment ≡ test of a hypothesis or model (• involves measurements • generates data)
  - Law ≡ broad generalization accurately describing what happens (not. nec. why) (• physics laws are descriptive • political laws are prescriptive)
  - Theory ≡ broad, detailed model accurately describing a phenomenon ("principles" are for more specific models)

(No particular order; is a nonlinear process)

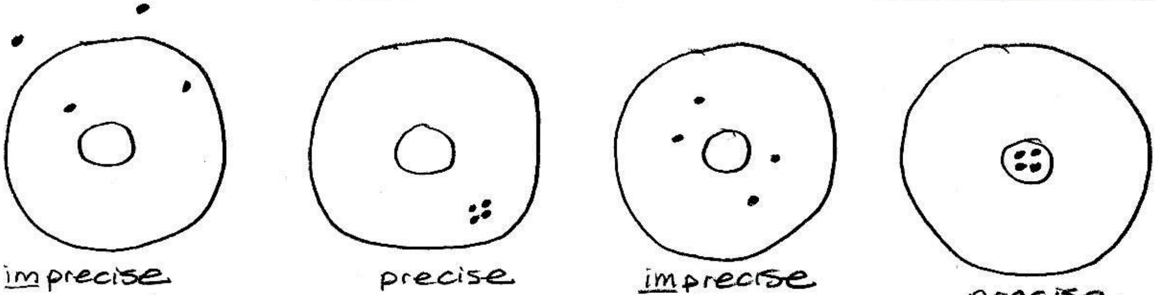
- CAUTION: Tests cannot "prove" theories. (Theories are just working explanations, usually only accurate under certain conditions)

Measurement : ≡ Assignment of a quantitative value (a number) to an observed phenomenon based on a comparison to a chosen standard value assigned to the same kind of phenomenon under standardized repeatable conditions.

- Units : the standard value used as a comparison in a measurement (more on this soon)

Precision vs. Accuracy :

- accurate ≡ close to "ideal" value
- precise ≡ repeated measurements are close to each other



imprecise and innaccurate — precise but innaccurate — imprecise yet accurate (even though average is bull's eye!) — precise and accurate

- Calibration ≡ careful use of standards + device specifications to achieve best precision and accuracy

Uncertainty :

- No measurement is ever perfect
  - everything in the universe affects it + we can't get or use all that info
- All measurements must include a statement of uncertainty

ex : • Width of a board :

$$W_{meas} = \underbrace{8.8}_{\text{measured value in [cm]}} \underbrace{\pm 0.1 \text{ [cm]}}_{\text{estimated uncertainty}}$$

(• units : cm = centimeters
• units apply to all numbers in measurement, unless specified otherwise)

- So we are certain the actual width W is :

$$8.7 \text{ [cm]} \leq W \leq 8.9 \text{ [cm]}$$

- Uncertainty in a given value with no quoted uncertainty is : assumed to be one or a few units in the last digit specified.

Percent Uncertainty :

- given measurement :

$$M_{meas} = V \pm \mathcal{U}$$

where $V \equiv$ measured value in specified units
$\mathcal{U} \equiv$ uncertainty value in the same units

- percent uncertainty is :

$$\boxed{\mathcal{U}_p(M_{meas}) \equiv \frac{\mathcal{U}}{V} \times 100\%}$$

- ex : • given $W_{meas} = 8.8 \pm 0.1$ [cm] ,

$$\mathcal{U}_p(W_{meas}) = \frac{0.1\,[\cancel{cm}]}{8.8\,[\cancel{cm}]} \times 100\% \approx 1\% \leftarrow$$

(So W_{meas} has approximately a 1% uncertainty)

Significant Figures $\equiv$ $\boxed{\text{number of reliably known digits in a number}}$ "sigfigs"

- Rules:

	Type of Digits	Significant?	ex:
①	All nonzero digits	yes	$\underbrace{314}_{\text{sig}}0$
②	Sandwiched zeros (Sandwiched by nonzero digits)	yes	$2\underbrace{00}_{\text{sig}}2$, $3\underbrace{0}_{\text{sig}}1.2\underbrace{00}_{\text{sig}}5$
③	Trailing zeros to right of decimal point	yes	$4.3\underbrace{000}_{\text{sig}}$
④	Trailing zeros *not* to right of decimal point		
④a	left of *explicit* decimal pt.	yes	$5\underbrace{00}_{\text{sig}}.$
④b	*no explicit* decimal point	no	$5\underbrace{\boxed{00}}_{\text{not sig}}$
⑤	Placeholder zeros on the left	no	$\underbrace{\boxed{0}}_{\text{not sig}}.5$, $\underbrace{\boxed{00}}_{\text{not sig}}1.23$, $\underbrace{\boxed{0.00}}_{\text{not sig}}34$
⑥	Exact numbers have *infinite significant figures*	yes	$\pi \equiv \underbrace{3.1415....}_{\text{sig}}$, $c \equiv \underbrace{299792458.\bar{0}}_{\text{sig}} \left[\frac{m}{s}\right]$
⑦	Scientific notation *powers of 10* like 10^n	no	$\underbrace{3.4}_{\text{sig}} \times \underbrace{\boxed{10^4}}_{\text{not sig}} = \underbrace{34}_{\text{sig}}\underbrace{\boxed{000}}_{\text{not sig}}$, $\underbrace{3.40}_{\text{sig}} \times \underbrace{\boxed{10^4}}_{\text{not sig}} = \underbrace{340}_{\text{sig}}\underbrace{\boxed{00}}_{\text{not sig}}$, $\underbrace{3.4000}_{\text{sig}} \times \underbrace{\boxed{10^4}}_{\text{not sig}} = \underbrace{34000}_{\text{sig}}$

Scientific Notation:

- represent a number as: $\boxed{N \times 10^n}$; where
  - N contains all significant digits
  - N only has one sig. digit left of decimal
  - ex: $1.0000 \le |N| \le 9.9999$

Significance Spectrum:

← Most significant   Least Significant →

$$\underbrace{\underset{\substack{\uparrow\\ \text{most sig}}}{5}.80024140070\!\!\!\underset{\substack{\uparrow\\ \text{least sig}}}{}}_{\text{all sig}} \times \underbrace{\boxed{10^{12}}}_{\text{not sig}}$$

Sigfigs Indicate Precision:

$$\boxed{\text{more sigfigs} \Rightarrow \text{more precise}}$$

Counting Significant Figures:

- ex:

Sample number	significant figures
0.3004	4
3405	4
3.004	4
3400	2
3.040	4
3400.	4
0.03400	4
3.400×10^3	4
340.5	4
3.4×10^3	2

Significant Figures in Operations:

- Addition and subtraction: answer must not have more decimal places than the input with the fewest decimal places

ex: $3.1 + 4.500 = 7.6$

- Multiplication and Division: answer must have no more digits than the input of fewest significant figures

ex: $3 \times 30. = 90$ ← (not 90., since it must have only 1 sigfig due to 3)

- Rounding:
  - Given a number to be rounded, ex: x.xxkdxx
    - where k is the digit to be kept (based on above rules) (but possibly altered)
    - and d is the digit to dropped,
  - the rules are:

① if $d < 5$ ($d=0,1,2,3,4$), drop d and all digits to its right

② if $d > 5$ ($d=6,7,8,9$), set $k' = k+1$, drop d and all digits to its right

③ if $d = 5$, $\left\{\begin{array}{l} \text{if } k \text{ odd, set } k' = k+1 \\ \text{if } k \text{ even, set } k' = k \end{array}\right\}$, drop d and all digits to its right

why? keeps the changes to numbers zero on average, since dropping $d=0$ is no change to it, and k is even or odd with equal probability

- ex
  - if total number of digits to keep is 3, then given: 4.7350,
    - $k=3$, $d=5$
    - so $k'=4 \rightarrow \boxed{4.74}$
  - same deal, but given: 4.7450
    - $k=4$, $d=5$
    - so $k'=4 \rightarrow \boxed{4.74}$
  - same deal, but given: 4.7950
    - $k=9$, $d=5$
    - so $k'=10 \rightarrow \boxed{4.80}$

- ex: sum:

$$\begin{array}{r|l} 12.34 & 02 \\ 9.34 & \\ +\ 5.80 & 5 \\ \hline 27.48 & 52 \end{array}$$

9.34 ← least precise (fewest decimal places)
↓
- number of decimal places in answer can be no more than 2
↓
- so $k=8$, $d=5$, so $k'=8$

$\boxed{27.48}$

- Note: don't use number of sigfigs for addition or subtraction (use rule above)

- Intermediate Steps: in compound operations, don't round until the end

- ex: $\dfrac{12.3 + 9.56 - 13.7}{4.08}$
  - First:

$$\begin{array}{r|l} 12.3 & \\ +\ 9.5 & 6 \\ -13.7 & \\ \hline 8.1 & 6 \end{array}$$

} → This step can yield at most one decimal place...

8.1|6 → But we keep the full result because we're not finished

- But we note the significance still
↓
- Digit rule from sum caused a number with 2 sigfigs, which affects division

- Then: $\dfrac{8.1\underline{6}}{4.08} = 2 \Rightarrow \boxed{2.0}$

- Exceptions:
  - significant figures are just guidelines...
  - check that the percent uncertainty of the answer isn't larger than that of the most uncertain input
    - if it is, retain just enough digits to achieve the maximum input uncertainty

Units ≡ The name of the standard amount of something assigned a value of 1

- *ex*: Units of Time:
  - Ancient definition of 1[s] ≡ "one second" is the amount of time between two heartbeats in a human
    - then all time measurements are compared to that
    - if some event lasts 5 heartbeats, that's five standard units of time, so it lasted 5[s].
    - heartbeats were not a reliable standard (not consistent, not precise)
  - Modern definition of 1[s] ≡ the time required for 9,192,631,770 oscillations of electromagnetic radiation emitted by cesium atoms changing between two particular states
    - very reliable and precise

Systems of Units:

- Different measurable quantities are generally related by a few base quantities
- All units used must be compatible, or they need conversion
- System of units ≡ compatible set of units
- *ex*:
  - SI ≡ "système International" ("MKS" for "meter-kilogram-second")
  - cgs ≡ "centimeter-gram-second"
  - BE ≡ "British engineering system" (foot, pound-force, second)
  - G ≡ "Gaussian" (used in electromagnetism sometimes)
  - HL ≡ "Heaviside-Lorentz" (used with elementary particles)

we'll use SI.

Base vs. Derived Quantities:

- Amazing Fact: Everything known can be expressed using no more than 7 "base quantities"
- *ex*: in SI:

Base Quantity	Unit	Unit Abbreviation
length	meter	m
time	second	s
mass	kilogram	kg
electric current	ampere	A
temperature	kelvin	K
amount of substance	mole	mol
luminous intensity	candela	cd

SI Supplementary Units

plane angle	radian	rad
solid angle	steradian	sr

- *ex*: (some)

Derived Quantities	Unit	Unit Abbreviation (if any) and Base Equiv.
area	square meter	m^2
volume	cubic meter	m^3
frequency	hertz	$Hz \equiv s^{-1}$
mass density (density)	kilogram per cubic meter	kg/m^3
speed, velocity	meter per second	m/s
acceleration	meter per second per second	m/s^2
force	newton	$N \equiv kg \cdot m/s^2$
work, energy, heat	joule	$N \cdot m = kg \cdot m^2/s^2$
electric charge	coulomb	$C \equiv A \cdot s$

Converting Units:

- **Purpose**:
  - To change between systems of units
  - To change magnitude notation
- **Rules/Steps**:
  ① only "do the math" when all units are in the same system, and it must be the system that applies to that equation (equations may need a different form in different systems)

  ② write the conversion equations (express the standard in one system as some amount in another system)

  ③ write 1 in a clever way ≡ conversion factor
- **ex**:
  - given $x = 21.5\,[in]$, we want it in [cm].
  - conversion equation:

(sometimes, we number equations to refer to them later, such as "(1)") →

$$1\,[in] = 2.54\,[cm] \qquad (1)$$

  - use (1) to get a conversion factor:

(divide both sides of (1) by 1[in]) →

$$\underbrace{\frac{1\,[\cancel{in}]}{1\,[\cancel{in}]}}_{} = \frac{2.54\,[cm]}{1\,[in]} \qquad (2)$$

$$1 = 2.54\left[\frac{cm}{in}\right] \leftarrow \text{(conversion factor)} \qquad (3)$$

  - multiply given number by 1 in a clever way by using (3) as 1:

$$x \cdot 1 = \left(21.5\,[\cancel{in}]\right)\cdot\left(2.54\left[\frac{cm}{\cancel{in}}\right]\right) = 54.6\,[cm] \qquad (4)$$

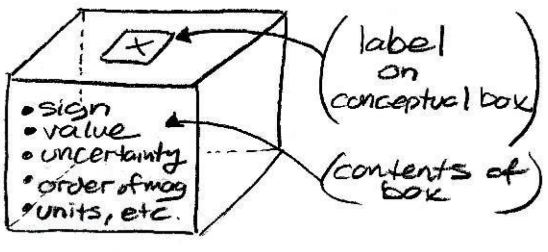
- **Note**: $1^{-1} = 1 = \frac{1\,[in]}{2.54\,[cm]} \approx 0.3937007874\left[\frac{in}{cm}\right]$ is also a valid conversion factor, but depends on situation...
- "clever" means use whichever "1" causes the "old" units to cancel, ↩ leaving only the "new" units.

Variables/Symbols:

$$\boxed{\text{variable} \equiv \text{symbol acting as a label on a conceptual box of related ideas}}$$

- **ex**:

$$\underbrace{x}_{\text{variable name or label}} = (\underbrace{-}_{\text{sign}}\underbrace{1.3}_{\text{main value}}\underbrace{\pm 0.2}_{\text{uncertainty}}) \times \underbrace{10^5}_{\text{order of magnitude}}\underbrace{[m]}_{\text{units}} \Rightarrow$$



- Easier to do algebra with just x
- Symbols are powerful:
  - Don't need to have specific values
  - Can represent a range of values (all at once!)
  - they visually and mathematically show us relationships between quantities
  - if we "crunch the math" from the start, we lose information about the relationships!
  - So:

$$\boxed{\text{Keep the problem in symbol form until the very end, and only then plug in the numbers, including both symbolic and numerical answers.}}$$

Prefixes: $\equiv$ abbreviation for a factor of a power of 10

- ex:

(these are most common, see book for more)

$\times 10^9$	giga	G
$\times 10^6$	mega	M
$\times 10^3$	kilo	k
$\times 10^0$	(uni)	
$\times 10^{-2}$	centi	c
$\times 10^{-3}$	milli	m
$\times 10^{-6}$	micro	μ
$\times 10^{-9}$	nano	n

- ex:

$1.21 \times 10^9 [W]$ ($W \equiv$ watts)

$= 1.21$ [gigawatts]

$= 1.21$ [GW]

Dimensional Analysis:

- to double-check your work,
- keep track of units and dimensions you expect to see at each stage
- dimension $\equiv$ the type of physical quantity (mass, length, etc.)
- ex:
  - velocity should have dimensions $\left[\frac{L}{T}\right] \equiv \left[\frac{\text{length}}{\text{time}}\right]$
  - so its SI units should be $\left[\frac{m}{s}\right]$
  - but if we get a "velocity" of v with units $\left[\frac{kg \cdot m}{s}\right]$, that has dimensions $\left[\frac{M \cdot L}{T}\right] \equiv \left[\frac{\text{mass} \cdot \text{length}}{\text{time}}\right] \neq \left[\frac{L}{T}\right]$,
  - so we'd know that v is incorrect
  - often indicates a simple algebra mistake

Rapid Estimation:

- aggressive rounding ex: $896.3 \rightarrow 10^3$, $\pi \rightarrow 3$ or 1
- simplified geometry ex:

- simplify!

Tips For Success:

- Do all your HW no matter what!
- These are not math problems (can't zoom through)
- Budget ~ 1 [hr] per problem
- Write out each step with narrating words
- Number equations + reference them by number
- Keep everything in symbols until the end
- Put answers in boxes, labeled as necessary
- Include both symbolic and numerical answers
- Compare your work to solutions
- Fix what you got wrong
- Keep all your notes and Homework, they will be valuable references.