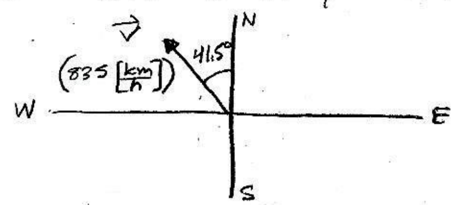
Homework #[01b]

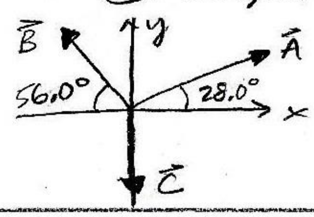
① An airplane is traveling 835 [km/h] in a direction 41.5° west of north.
ⓐ Find the components of the velocity vector in the northerly and westerly directions
ⓑ How far north and how far west has the plane traveled after 1.75 [h] ?



ⓐ
$$v_{WEST} \equiv -v_x = |\vec{v}| \sin(\theta) = 553 \left[\frac{km}{h}\right] \text{ in the westerly direction}$$
$$v_{NORTH} \equiv +v_y = |\vec{v}| \cos(\theta) = 625 \left[\frac{km}{h}\right] \text{ in the northerly direction}$$

ⓑ
$$\Delta x_{WEST} = |\vec{v}| \sin(\theta) t = 968 \text{ [km] westwards}$$
$$\Delta y_{NORTH} = |\vec{v}| \cos(\theta) t = 1090 \text{ [km] northwards}$$

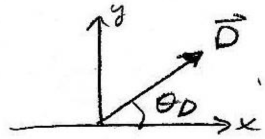
② Three vectors are shown in the diagram. Their magnitudes are given in arbitrary units. Determine the sum of the three vectors. Give the resultant in terms of ⓐ components ⓑ magnitude and angle with the +x axis:



$A \equiv |\vec{A}| = 44.0$
$B \equiv |\vec{B}| = 26.5$
$C \equiv |\vec{C}| = 31.0$

ⓐ The resultant is $\vec{D} = D_x \hat{x} + D_y \hat{y}$ where,
$$D_x = A\cos(\theta_A) - B\cos(\theta_B) + 0 = 24.0$$
$$D_y = A\sin(\theta_A) + B\sin(\theta_B) - C = 11.6$$

ⓑ Defining θ_D as the angle between $\vec{D}$ and the +x axis:



then $\theta_D = \tan^{-1}\left(\frac{D_y}{D_x}\right)$

$\theta_D = 25.8°$

and the magnitude of $\vec{D}$ is
$$D \equiv |\vec{D}| = \sqrt{D_x^2 + D_y^2} = 26.7$$
where we used D_x and D_y from (a) in both D and θ_D

③ Estimate by what factor a person can jump farther on the Moon as compared to the Earth if the takeoff speed and angle are the same. The acceleration due to gravity on the Moon is one-sixth what it is on Earth.

$$\frac{\Delta x_{☾}}{\Delta x_{\oplus}} = \frac{g_{\oplus}}{\left(\frac{g_{\oplus}}{6}\right)} = 6$$

(4) {A ball thrown horizontally at 12.2 [m/s] from the roof of a building lands 21.0 [m] from the base of the building. How high is the building?}

$$\boxed{h \equiv |y - y_0| = \left| \frac{a_y}{2}\left(\frac{x}{v_{x0}}\right)^2 \right| = 14.5 \text{ [m]}}$$

(5) {A shot-putter throws the "shot" (mass 7.3 [kg]) with an initial speed of 14.4 [m/s] at a 34.0° angle to the horizontal. Calculate the horizontal distance traveled by the shot if it leaves the athlete's hand at a height of 2.10 [m] above the ground.}

$$\boxed{x = x_0 + v_0 \cos(\theta) t_- = 22.3 \text{ [m]}}$$

(6) {A passenger on a boat moving at 1.70 [m/s] on a still lake walks up a flight of stairs at a speed of 0.60 [m/s], as in the diagram. The stairs are angled at 45° above the direction of motion. What is the velocity of the passenger relative to the water?}

$$\vec{v}_{P,W} = \left(|\vec{v}_{P,B}|\cos(\theta) + v_{B,W_x}\right)\hat{x} + \left(|\vec{v}_{P,B}|\sin(\theta)\right)\hat{y}$$

$$= \left[\left(0.60 \left[\tfrac{m}{s}\right]\right)\cos(45°) + \left(1.70\left[\tfrac{m}{s}\right]\right)\right]\hat{x} + \left[\left(0.60\left[\tfrac{m}{s}\right]\right)\sin(45°)\right]\hat{y}$$

$$= \left(2.12\left[\tfrac{m}{s}\right]\right)\hat{x} + \left(0.424\left[\tfrac{m}{s}\right]\right)\hat{y}$$

or

$$|\vec{v}_{P,W}| = \sqrt{v_{P,W_x}^2 + v_{P,W_y}^2} = 2.17 \left[\tfrac{m}{s}\right],$$

$$\theta_{P,W} = \tan^{-1}\left(\frac{v_{P,W_y}}{v_{P,W_x}}\right) \simeq 11° \text{ above } +x\text{-direction}$$

7 Given the vector $\vec{v} \equiv a\hat{x} + b\hat{y} + c\hat{z}$, answer the following:

a) what are the components v_x, v_y, v_z of $\vec{v}$?

b) what is the magnitude of $\vec{v}$? (That is, find $|\vec{v}|$.)

c) what is the unit vector in the direction of $\vec{v}$?

a) Scalar components are the scalars that multiply the unit vector for their particular direction. So:

$$v_x = a \quad , \quad v_y = b \quad , \quad v_z = c$$

since then $\vec{v} = v_x\hat{x} + v_y\hat{y} + v_z\hat{z} = a\hat{x} + b\hat{y} + c\hat{z} = \vec{v}$ ✓

b)
$$|\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$$
$$= \sqrt{a^2 + b^2 + c^2}$$

c)
$$\hat{v} = \frac{a\hat{x} + b\hat{y} + c\hat{z}}{\sqrt{a^2+b^2+c^2}}$$
$$= \frac{a}{\sqrt{a^2+b^2+c^2}}\hat{x} + \frac{b}{\sqrt{a^2+b^2+c^2}}\hat{y} + \frac{c}{\sqrt{a^2+b^2+c^2}}\hat{z}$$

8 Given the two vectors:

$$\vec{A} \equiv a\hat{x} - b\hat{y} \quad \text{and} \quad \vec{B} \equiv c\hat{x} + d\hat{y}$$ (;)

a) Find $\vec{A} + \vec{B}$

b) Find $\vec{A} - \vec{B}$

c) Find $|\vec{A}|$, $|\vec{B}|$, and $|\vec{A}+\vec{B}|$

d) What is the scalar y-component of $\vec{C} \equiv \vec{A} - \vec{B}$?

a) $$\vec{A}+\vec{B} = (a+c)\hat{x} + (-b+d)\hat{y}$$

b) $$\vec{A}-\vec{B} = (a-c)\hat{x} + (-b-d)\hat{y}$$

c) $$|\vec{A}| = \sqrt{a^2+b^2} \quad , \quad |\vec{B}| = \sqrt{c^2+d^2}$$
$$|\vec{A}+\vec{B}| = \sqrt{(a+c)^2 + (-b+d)^2}$$

d) $$C_y = -b-d$$

(a) In the 2D kinematic equations for projectiles where $a_x = 0$ and a_y is constant, rewrite them without time.
That is:

(a) solve the x equation for t

(b) plug the solution for t from (a) into the y-direction kinematic equations

(c) what x-direction parameters have the ability to affect y-direction variables such as y and v_y?

(a) $\boxed{t = \dfrac{x - x_0}{v_{x0}}}$

(b)

$$v_y = v_{y0} + a_y \frac{x - x_0}{v_{x0}}$$

$$y = y_0 + v_{y0} \frac{x - x_0}{v_{x0}} + \frac{1}{2} a_y \left(\frac{x - x_0}{v_{x0}}\right)^2$$

$$v_y^2 = v_{y0}^2 + 2a_y(y - y_0)$$

$$y = y_0 + \frac{1}{2}(v_{y0} + v_y)\left(\frac{x - x_0}{v_{x0}}\right)$$

$$y = y_0 + v_y \frac{x - x_0}{v_{x0}} - \frac{1}{2} a_y \left(\frac{x - x_0}{v_{x0}}\right)^2$$

(c) x, x_0, and v_{x0} have the ability to affect y-direction variables such as y and v_y. This is because the same time t passes as the object moves in both directions.