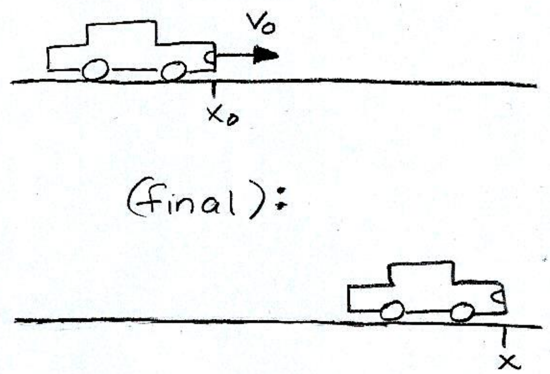
(Solution) Quiz [01a] Name:

While driving your car at initial velocity v_0, you suddenly slam on the breaks to come to a complete stop to avoid running a red light. Answer the following, show all your work, including a diagram depicting your car as a dot, and a coordinate system showing the +x direction as an arrow. Also show where 0 is, and where x_0 and x are by labeling them. Keep everything as symbols except for values of 0, box and label your answers for each part, and remember to write your name and circle your Recitation section on the front!

(initial):

(final):

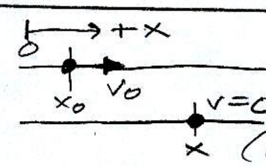


(a) If this process takes time T as you travel from x_0 to a stop at x, what is your acceleration a, assuming it remains constant, in terms of v_0 and T.
Note: in problems like this, we treat T as the symbolic value of t; i.e. $t=T$. It is understood that T contains things like sign and units unless we restrict its value explicitly, so you should not write units next to it.

(b) What is the displacement $x-x_0$ over which this happens in terms of v_0 and T? (Hint: a should not appear in your answer here)

(c) What is the technical term for the rate of change of velocity? (i.e., what do we call the change in velocity over the change in time in the limit where the change in time is infinitesimally small?)

(a)
- we're given v_0, $v=0$, $t=T$, x_0, x, and want a
- a kinematic equation involving a subset of these and a is:



$$v = v_0 + at \quad (1)$$

- plug in symbolic values:

$$0 = v_0 + aT \quad (2)$$

- solve for a:

$$\text{(a)}\ \boxed{a = -\frac{v_0}{T}} \quad (3)$$

(b)
- we want $x-x_0$, and have v_0, $v=0$, $t=T$, $a=-\frac{v_0}{T}$, and actually all kinematic equations but (1) will let us solve for $x-x_0$:

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$
$$x - x_0 = v_0T + \tfrac{1}{2}\left(-\frac{v_0}{T}\right)T^2 = v_0T - \frac{v_0T}{2} = \frac{v_0T}{2} \quad (4)$$

$$v^2 = v_0^2 + 2a(x-x_0)$$
$$0 = v_0^2 + 2\left(-\frac{v_0}{T}\right)(x-x_0)$$
$$\frac{2v_0}{T}(x-x_0) = v_0^2$$
$$x - x_0 = \frac{v_0T}{2} \quad (5)$$

$$x = x_0 + \tfrac{1}{2}(v_0+v)t$$
$$x - x_0 = \tfrac{1}{2}(v_0+0)T = \frac{v_0T}{2} \quad (6)$$

$$x = x_0 + vt - \tfrac{1}{2}at^2$$
$$x - x_0 = 0T - \tfrac{1}{2}\left(-\frac{v_0}{T}\right)T^2 = \frac{v_0T}{2} \quad (7)$$

- so

$$\text{(b)}\ \boxed{x - x_0 = \frac{v_0T}{2}} \quad (8)$$

(c)

$$\boxed{\lim_{\Delta t\to 0}\left(\frac{\Delta v}{\Delta t}\right) \equiv \frac{dv}{dt} \equiv a \equiv \text{acceleration}} \quad (9)$$