Section 1.4

Measures of Variability

Objectives

- Compute the range, variance, and standard deviation.

Range

Range = Maximum Data Value − Minimum Data Value

Example:

The following data were collected from samples of call lengths (in minutes) observed for two different mobile phone users. Calculate the range of each data set.

a. 2, 25, 31, 44, 29, 14, 22, 11, 40

b. 2, 2, 44, 2, 2, 2, 2, 2

c. What could be misleading about using the range as a measurement?

Variance

Population Variance	Sample Variance
$$\sigma^2 = \frac{\sum(x_i - \mu)^2}{N}$$	$$s^2 = \frac{\sum\left(x_i - \bar{x}\right)^2}{n-1}$$
• x_i is the i^{th} value in the population • μ is the population mean • N is the number of values in the population	• x_i is the i^{th} data value • $\bar{x}$ is the sample mean • n is the number of data values in the sample

Standard Deviation

- The **standard deviation** is a measure of how much we might expect a typical member of the data set to differ from the mean. It is the square root of the variance.

Population Standard Deviation	Sample Standard Deviation
$\boldsymbol{\sigma = \sqrt{\frac{\sum(x_i - \mu)^2}{N}}}$	$s = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n-1}}$

Note: $\sum(x_i - \bar{x})^2 = \sum(x_i)^2 - \frac{(\sum x_i)^2}{n}$

Example:

You and your friends just measured the heights of your dogs (in millimeters). The heights (at the shoulders) are: 600 mm, 470mm, 170mm, 430mm and 300 mm. Compute the standard deviation of these heights.

Solution:

The mean is 394 millimeters.

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
600mm	600-394=206	$(206)^2 = 42{,}436$
470mm	470-394=76	$(76)^2 = 5{,}776$
170mm	170-394=-224	$(-224)^2 = 50{,}176$
430mm	430-394=36	$(36)^2 = 1{,}296$
300mm	300-394=-94	$(-94)^2 = 8{,}836$
Total:		108,520

- The variance is :

$$s^2 = \frac{108{,}520}{5-1} = 27{,}130$$

- The standard deviation is:

$$s = \sqrt{27{,}130} = 164.71$$

- The standard deviation is 164.71 millimeters.

Properties of the Standard Deviation

- *s* measures spread about the mean. Use *s* to describe the spread of a distribution only when you use the mean to describe the center.
- $s = 0$ only when there is no spread. This happens only when all observations have the same value. So standard deviation zero means no spread at all. Otherwise, $s > 0$. As the observations become more spread out about their mean, *s* gets larger.

Example:

Mark is looking into investing a portion of his recent bonus into the stock market. While researching different companies, he discovers the following standard deviations of one year of daily stock closing prices.

Profacto Corporation: Standard deviation of stock prices = $1.02

Yardsmoth Company: Standard deviation of stock prices = $9.67

What do these two standard deviations tell you about the stock prices of these companies?