# SECTION 2.5

## Independent events

# Independent Events

- Two events A and B are independent if one event happening does not affect the probability of the other happening, i.e $P(A|B) = P(A)$.
- Multiplication Rule for Independent Events:

A and B are independent if and only if

$$P(A \cap B) = P(A) \cdot P(B)$$

$A_1, A_2, \ldots, A_k$ are independent if and only if

$$P(A_1 \cap A_2 \ldots \cap Ak) = P(A_1) \cdot P(A_2) \cdots P(A_k)$$

**Example:**

An oil exploration company currently has two active projects, one in Asia and the other in Europe. Let A be the event that the Asian project is successful, and B be the event that the European project is successful. Suppose that A and B are independent events with $P(A) = 0.4$ and $P(B) = 0.7$.

a) If the Asian project is not successful, what is the probability that the European project is also not successful? Explain your reasoning.

b) What is the probability that at least one of the two projects will be successful?

c) Given that at least one of the two projects is successful, what is the probability that only the Asian project is successful?

**Example:**

Consider randomly selecting a single individual and having that person test drive 3 different vehicles. Define events $A_1$, $A_2$, and $A_3$ by

A1 = likes vehicle #1 A2 – likes vehicle #2

A3 = likes vehicle #3

Suppose that $P(A_1) = 0.55, P(A_2) = 0.65, P(A_3) = 0.70,$

$P(A_1 \cup A_2) = 0.80, P(A_2 \cap A_3) = 0.40,$ and $P(A_1 \cup A_2 \cup A_3) = 0.88$

a) What is the probability that the individual likes both vehicle #1 and vehicle #2?

b) Determine and interpret $P(A_2|A_3)$.

c) Are $A_2$ and $A_3$ independent events? Answer in two different ways.

d) If you learn that the individual did not like vehicle #1, what now is the probability that he/she liked at least one of the other two vehicles?

# Practice Problems

1) The probabilities for blood type are as follows: P(O) = 0.44, P(A) = 0.42, P(B) = 0.10, and P(AB) = 0.04.

   a) Two people are selected simultaneously and at random from all people in the United States. What is the probability that both have blood type O?

   b) Three people are chosen simultaneously and at random. What is the probability that all three have blood type B?

2) The Art Competition has entries from three painters: Pam, Pia and Pablo.

   - Pam put in 15 paintings, 4% of her works have won First Prize.
   - Pia put in 5 paintings, 6% of her works have won First Prize.
   - Pablo put in 10 paintings, 3% of his works have won First Prize.

A First Prize painting is selected. What is the probability that it is Pam's painting?