Homework #[01a]

① {How many significant figures are in each of these numbers? ⓐ 214 ⓑ 81.60 ⓒ 7.03 ⓓ 0.03 ⓔ 0.0086 ⓕ 3236 ⓖ 8700}

ⓐ $\boxed{\text{3 sigfigs by rule 1}}$

ⓑ $\boxed{\text{4 sigfigs by rules 1 and 3}}$

ⓒ $\boxed{\text{3 sigfigs by rules 1 and 2}}$

ⓓ $\boxed{\text{1 sigfig by rules 1 and 5}}$

ⓔ $\boxed{\text{2 sigfigs by rules 1 and 5}}$

ⓕ $\boxed{\text{4 sigfigs by rule 1}}$

ⓖ $\boxed{\text{2 sigfigs by rules 1 and 4b}}$

② {Write the following numbers in scientific notation: ⓐ 1.156 ⓑ 21.8 ⓒ 0.0068 ⓓ 328.65 ⓔ 0.219 ⓕ 444}

ⓐ $\boxed{1.156}$

ⓑ $\boxed{2.18 \times 10^{1}}$

ⓒ $\boxed{6.8 \times 10^{-3}}$

ⓓ $\boxed{3.2865 \times 10^{2}}$

ⓔ $\boxed{2.19 \times 10^{-1}}$

ⓕ $\boxed{4.44 \times 10^{2}}$

③ {What is the percent uncertainty in the measurement 5.48 ± 0.25 [m]?}

③ $$\boxed{U_p(m_{meas}) \equiv \frac{U}{V} \times 100\% = \frac{0.25\ [\cancel{m}]}{5.48\ [\cancel{m}]} \times 100\% = 4.6\%}$$

④ {Write the following numbers in both scientific notation in S.I. units and as full decimal numbers without prefixes or power-of-10 notation. ⓐ 286.6 [mm] ⓑ 85 [µV] ⓒ 760 [mg] ⓓ 62.1 [ps] ⓔ 22.5 [nm] ⓕ 2.50 [GV]}

ⓐ 2.866×10^{-1} [m]
0.2866 [m]

ⓑ 8.5×10^{-5} [V]
0.000085 [V]

ⓒ 7.6×10^{-4} [kg]
0.00076 [kg]

ⓓ 6.21×10^{-11} [s]
$0.\underbrace{0000000000}_{\text{10 zeros}}621$ [s]

ⓔ 2.25×10^{-8} [m]
0.0000000225 [m]

ⓕ 2.50×10^{9} [V]
2,500,000,000 [V]

⑤ {Determine the conversion factor between ⓐ $\left[\frac{km}{h}\right]$ and $\left[\frac{mi}{h}\right]$ ⓑ $\left[\frac{m}{s}\right]$ and $\left[\frac{ft}{s}\right]$ ⓒ $\left[\frac{km}{h}\right]$ and $\left[\frac{m}{s}\right]$.}

ⓐ $$1 = 0.6214\left[\frac{mi}{km}\right] \cdot \frac{\frac{1}{[h]}}{\frac{1}{[h]}} = 0.6214\ \frac{\left[\frac{mi}{h}\right]}{\left[\frac{km}{h}\right]}$$

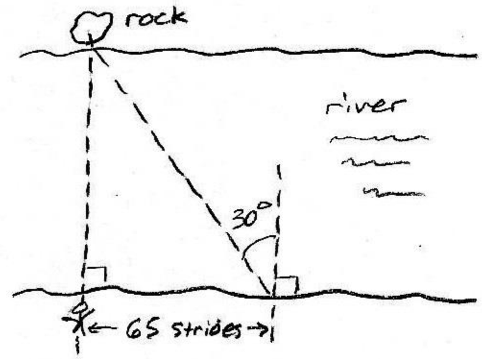
ⓑ $$3.281\left[\frac{ft}{m}\right] \cdot \frac{\left[\frac{1}{s}\right]}{\left[\frac{1}{s}\right]} = 3.281\ \frac{\left[\frac{ft}{s}\right]}{\left[\frac{m}{s}\right]}$$

ⓒ $$1 \cdot 1 \cdot 1 = \left(\frac{1000\ [m]}{1\ [km]}\right) \cdot \left(\frac{1\ [h]}{60\ [\cancel{min}]}\right) \cdot \left(\frac{1\ [\cancel{min}]}{60\ [s]}\right) = \frac{1000\ [m]}{1\ [km]} \cdot \frac{1\ [h]}{3600\ [s]}$$

or $$1 = 0.278\ \frac{\left[\frac{m}{s}\right]}{\left[\frac{km}{h}\right]}$$

(continued)

(6) Jim stands beside a wide river and wonders how wide it is. He spots a large rock on the far bank directly across from him. He then walks upstream 65 strides and judges that the angle between him and the rock is 30° downstream from his new position if he faces straight out at the water. If the river's banks are approximately parallel, and if Jim measures his stride to be about 0.8 [m] long, how wide is the river?



$$\boxed{\begin{aligned} w = s\cdot\tan(\theta) &= (52\,[m])\cdot\tan(60^\circ) \\ &= 52\,[m]\cdot 1.732 \\ &\approx 90\,[m] \end{aligned}}$$

(7) If you are driving 95 [km/h] along a straight road and you look to the side for 2.0 [s], how far do you travel forward on the road during this inattentive period?

$$\boxed{\begin{aligned} D = \bar{s}\Delta t &= \left(95\left[\tfrac{km}{\cancel{h}}\right]\right)\left(2.0\,[\cancel{s}]\right)\cdot\left(\frac{1\,[\cancel{h}]}{3600\,[\cancel{s}]}\right) \\ &= 0.053\,[km]\cdot\left(\frac{1000\,[m]}{1\,[km]}\right) = 53\,[m] \end{aligned}}$$

(8) At highway speeds, a particular car can accelerate at 1.8 $\left[\frac{m}{s^2}\right]$. At this rate, how long does it take to accelerate from 65 $\left[\frac{km}{h}\right]$ to 120 $\left[\frac{km}{h}\right]$?

$$\boxed{\Delta t = \frac{V_f - V_i}{a} = \frac{(33.333 - 18.056)\left[\frac{m}{\cancel{s}}\right]}{1.8\left[\frac{m}{s^{\cancel{2}}}\right]} = 8.5\,[s]}$$

(9) A car slows down from 28 $\left[\frac{m}{s}\right]$ to rest in a distance of 88 [m]. What was its acceleration, assumed constant?

$$\boxed{\begin{aligned} a = \frac{V^2 - V_0^2}{2\Delta x} &= \frac{\left(0.0\left[\frac{m}{s}\right]\right)^2 - \left(28\left[\frac{m}{s}\right]\right)^2}{2\cdot(88\,[m])} = -4.45\left[\frac{m}{s^2}\right] \\ &\approx -4.4\left[\frac{m}{s^2}\right] \end{aligned}}$$

following strict rounding rules, since 4 is even

(continued)

(10) Determine the stopping distances for a car going at a constant initial speed of 95 [km/h] and human reaction time of 0.40 [s] for (a) an acceleration $a = -3.0 \left[\frac{m}{s^2}\right]$ (b) $a = -6.0 \left[\frac{m}{s^2}\right]$

(a) when $a = -3.0 \left[\frac{m}{s^2}\right]$,

$$x = x_0 + \frac{v^2 - v_0^2}{2a} = 126.6 \text{ [m]} \approx 130 \text{ [m]}$$

(b) when $a = -6.0 \left[\frac{m}{s^2}\right]$,

$$x = x_0 + \frac{v^2 - v_0^2}{2a} = 68.60 \text{ [m]} \approx 69 \text{ [m]}$$

(11) A stone is thrown vertically upward with a speed of $24.0 \left[\frac{m}{s}\right]$.
(a) How fast is it moving when it is at a height of 13.0 [m]?
(b) How much time is required to reach this height?
(c) Why are there two answers to (b)?

(a)
$$V_\pm = \pm\sqrt{v_0^2 + 2a(y - y_0)} = \pm 17.9 \left[\frac{m}{s}\right]$$

- so its speed is $s = |V_\pm| = 17.9 \left[\frac{m}{s}\right]$
- two answers here are V_+ for velocity at $y = 13.0$ [m] on trip upwards, and V_- for velocity at $y = 13.0$ [m] on trip downwards.

(b)
$$t_\pm = \frac{-v_0 \pm \sqrt{v_0^2 - 4\left(\frac{a}{2}\right)(y_0 - y)}}{2 \cdot \frac{1}{2} a}$$

$t_+ = 0.620$ [s]
$t_- = 4.28$ [s]

(c) t_+ corresponds to the time when the stone reaches $y = 13.0$ [m] on its upward trip,

t_- is the time when the stone reaches $y = 13.0$ [m] on its downward trip

There are two times because (b) is a 2^{nd}-order polynomial in t, so it has two roots.

Note, both times are "physical"; not every situation with multiple solutions has some that are nonphysical.

(continued)

(12) The acceleration due to gravity on the Moon is about one-sixth what it is on Earth. If an object is thrown vertically upward on the Moon, how many times higher will it go than it would on Earth, assuming the same initial velocity?

$$\boxed{\frac{\Delta y_{☾}}{\Delta y_{\oplus}} = \frac{3v_0^2}{g}\cdot\frac{1}{\left(\frac{v_0^2}{2g}\right)} = \frac{3\cancel{v_0^2}}{\cancel{g}}\frac{2\cancel{g}}{\cancel{v_0^2}} = 6}$$

So the object travels 6 times farther upward "on" the Moon than "on" Earth

(13) Suppose the position as a function of time for an object is known to be:

$$x(t) = C + A\cos(\omega t + \theta_0) + B\left(e^{-\alpha t} - 1\right) \qquad (i)$$

(a) what is the instantaneous velocity of this object as a function of t, if $C, A, \omega, \theta_0, B$, and α are all constant in time?

(b) what is an expression for the initial position at $t=0$, meaning $x_0 \equiv x(0)$?

(c) what is the initial velocity $v_0 \equiv v(0)$?

(a) $$\boxed{v(t) = -\left[A\omega\sin(\omega t+\theta_0) + B\alpha e^{-\alpha t}\right]}$$

(b) $$\boxed{x_0 \equiv x(0) = C + A\cos(\theta_0)}$$

(c) $$\boxed{v_0 \equiv v(0) = -\left[A\omega\sin(\theta_0) + B\alpha\right]}$$