(Under)graduate Research Project:
Investigations on the Dimensions of Spline Spaces

Monitored by Simon Foucart

Objective: Produce a software to compute dimension formulas for multivariate spline spaces.

Prerequisite: Linear Algebra, Programming, Interest in Mathematics, Academic Credential.

Description: For a tetrahedral partition Δ of a polyhedral domain $\Omega \subseteq \mathbb{R}^3$, the space of $\mathcal{C}^r$ splines of degree $\leq d$ in 3 variables over Δ is defined by

$$\mathcal{S}_d^r(\Delta) := \big\{ s \in \mathcal{C}^r(\Omega) : \ s|_T \text{ trivariate polynomial of degree } \leq n \text{ for each tetrahedron } T \in \Delta \big\}.$$

Determining the dimension of this space is an unsettled problem, but, for a fixed partition Δ, a formula of the type

$$\dim \mathcal{S}_d^r(\Delta) = \sum_{k=0}^{k^\star} a_k \binom{d+3-k}{3}$$

can be predicted from the computations of only a finite values of the dimensions. This project aims at cataloging such formulas for a number of partitions Δ and at making their productions completely automated (as well as reliable). To do so, the disparate codes for the calculation of individuals $\dim \mathcal{S}_d^r(\Delta)$ and for the subsequent formula generation need to be merged. They also need to be sped up, inasmuch as possible. The completed project will pave the way to the experimental discovery of a formula for $\dim \mathcal{S}_d^r(\Delta)$ valid in the generic case for all partitions Δ. While enhancing programming skills, the project will provide exposure to the fields of Approximation Theory and Algebraic Geometry.

Outcome: A stand-alone piece of software and a formal mathematical report.

To Apply: Contact Prof. Foucart at `foucart@math.drexel.edu` to schedule a face-to-face appointment. Funding for this research experience is not available at the moment.

References

[1] P. Alfeld, Trivariate MDS code, `http://www.math.utah.edu/~pa/3DMDS/`

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[3] L. J. Billera and L. L. Rose, Gröbner basis methods for multivariate splines, in: Mathematical methods in computer aided geometric design, T. Lyche and L. L. Schumaker (eds.), 1989, 93–104.

[4] S. Foucart and T. Sorokina, On the dimension of multivariate spline spaces, especially on Alfeld splits, Preprint.

[5] M.-J. Lai and L. L. Schumaker, Spline functions on triangulations, Cambridge University Press, Cambridge, 2007.