Homework #3

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Pledge: I pledge my honor that I have abided by the Stevens Honor System. **Signature:** Samir Banjara

Problem 1

(A) Find the probability of rolling two dice and not getting doubles.

Solution:

Given the set of possibilities of rolling two dice,

$$s = \begin{array}{l} \{(1,1),(2,1),(3,1),(4,1),(5,1),(6,1)\} \\ \{(1,2),(2,2),(3,2),(4,2),(5,2),(6,2)\} \\ \{(1,3),(2,3),(3,3),(4,3),(5,3),(6,3)\} \\ \{(1,4),(2,4),(3,4),(4,4),(5,4),(6,4)\} \\ \{(1,5),(2,5),(3,5),(4,5),(5,5),(6,5)\} \\ \{(1,6),(2,6),(3,6),(4,6),(5,6),(6,6)\} \end{array}$$

There are 30 rolls out of 36 that are not an ordered double. Thus, the probability is $\frac{30}{36}$.

(B) Given that every fifth person in line will get a coupon for a free box of popcorn at the movies, what is the probability that you don't get a coupon when you're in line?

Solution:

Every 5th person in line gets a coupon. That means 1 person out of 5 gets a coupon. The probability of not getting a coupon is then:

$$1 - \frac{1}{5} = \frac{4}{5}$$

Problem 2

Insulin pens used to administer a patient's insulin at hospitals have a malfunction rate of 9%. This means that out of a box of 200 pens, 18 are defective. Find the probability of randomly selecting 3

defective insulin pens in a row from a brand-new box of 200 pens, if a defective pen is immediately discarded.

Solution:

The probability of selecting a defective pen out of a new box of 200 is $\frac{18}{200}$. Since the pen is discarded, the probability of picking another defective pen is:

$$\frac{17}{199}$$

For the third pen:

$$\frac{16}{198}$$

Thus, the total probability is:

$$\left(\frac{18}{200}\right)\left(\frac{17}{199}\right)\left(\frac{16}{198}\right) = 0.000621 \times 100\% = 0.0621\%$$

Problem 3

The following table displays the breakdown of attendees at an International Biology conference by country and their role in the company they were representing.

	Canada	France	South Korea	UK	US
CEO	138	45	4	19	117
Director	8	4	25	6	63
Partner	23	7	3	20	103
Chairman	12	9	3	9	62
Other	112	146	154	143	2103

A random attendee is selected for an interview.

a) What is the probability that a Partner is selected, given that the attendee is from South Korea?

Solution:

The conditional probability is:

$$P(\text{Partner}|\text{South Korea}) = \frac{3}{189} = \frac{1}{63}$$

b) What is the probability that a Canadian is selected, given that the attendee is a director of the company?

Solution:

The conditional probability is:

$$P(\text{Canadian}|\text{Director}) = \frac{8}{106}$$

c) What is the probability that a director is selected, given that the attendee is Canadian?

Solution:

The conditional probability is:

$$P(\text{Director}|\text{Canadian}) = \frac{8}{293}$$

d) What is the probability that a CEO is selected, given that the attendee is from the continent of North America?
Solution:
The conditional probability is:

$$P(\text{CEO}|\text{North America}) = \frac{255}{2741}$$

Problem 4

Every 6 months, university email requires that a new 5-digit password be set up. No digits are allowed to be repeated and it must be different from your last two passwords. If you let your computer randomly choose a 5-digit code for you with no repeating digits, what is the probability that it will choose one of the last 2 passwords you've had? Round your answer to five decimal places.
   Solution:
The total number of possible passwords is:

$$10 \times 9 \times 8 \times 7 \times 6 = 30240$$

The probability that 2 of the computer-generated passwords match your last 2 is:

$$\frac{2}{30240} = 0.00007$$

Problem 5

Virginia's Veggie Café offers 5 types of homemade bread, 10 toppings, and 6 different condiments. How many different super sandwiches can be made if a super sandwich consists of 6 different toppings and 2 different condiments?
   Solution:
The total number of combinations is:

$$5 \times \binom{10}{6} \times \binom{6}{2} = 5 \times 210 \times 15 = 15750$$

Problem 6

Ashley's Internet service is terribly unreliable. On any given day, there is a 15% chance that her Internet connection will be lost. What is the probability that her Internet service is not broken for five days in a row?
   Solution:
The probability that the Internet works on a given day is:

$$P(\text{Works}) = 1 - 0.15 = 0.85$$

The probability that it works for five days in a row is:

$$(0.85)^5$$

Problem 7

Because Tristan has diabetes, he must ensure his daily diet includes 2 vegetables, 3 fruits, and 2 breads. At the grocery store, he has a choice of 20 vegetables, 8 fruits, and 5 breads.

a) In how many ways can he make up his daily requirements if he doesn't eat 2 helpings of the same thing?

Solution:

The number of combinations is:

$$\binom{20}{2} \times \binom{8}{3} \times \binom{5}{2} = 106400$$

b) What's the probability that a random choice from his possibilities would yield either carrots or spinach?

Solution:

The number of ways to include carrots or spinach is:

$$P(A) + P(B) - P(A \cap B) = 19 + 19 - 1 = 37$$

The probability is:

$$\frac{20720}{106400} \approx 0.1947 \times 100\% = 19.47\%$$