4 Assignment

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5:47pm

Question:

Find the distance between the plane passing through the points

$$P_1(1,\ 2,\ 3),\ \ P_2(2,\ 3,\ 5), P_3(3,\ 5,\ 7)$$

And

$$Q(2,\ -2,\ 2)$$

Explain how you found the answer using vector projections. Please draw a diagram to help with your solution.

Solution :

Distance D from Q to the plane is equal to the absolute value of the scalar projection of b onto the normal vector $n = \langle a, b, c \rangle$

Thus,

$$\begin{aligned} D &= |comp_n b| \\ &= \frac{n \cdot b}{n} \\ &= \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}} \end{aligned}$$

First let's find the equation of the plane:

$$
\begin{aligned}
a &= \vec{P_1 P_2} \\
&= (2-1)i + (3-2)j + (5-3)k \\
&= 1i + 1j + 2k \\
&= \langle 1, 1, 2 \rangle
\end{aligned}
$$

$$
\begin{aligned}
b &= \vec{P_1 P_3} \\
&= (3-1)i + (5-2)j + (7-3)k \\
&= 2i + 3j + 4k \\
&= \langle 2, 3, 4 \rangle
\end{aligned}
$$

Since both a and b lie on the same plane, their cross product $a \times b$ is orthogonal to the plane, and can be taken as normal vector. Thus,

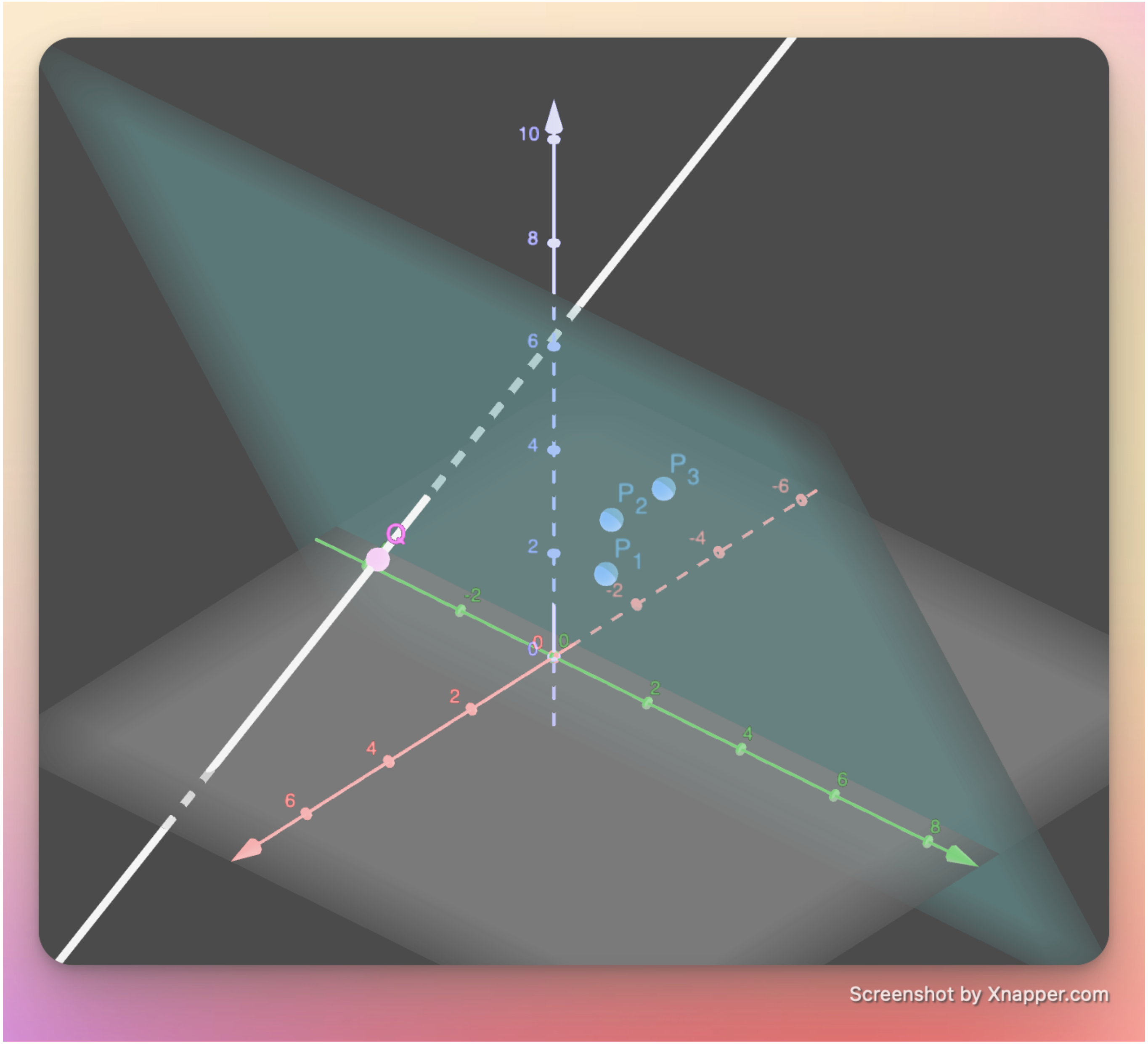
$$
\begin{aligned}
n = a \times b = &\begin{bmatrix} i & j & k \\ 1 & 1 & 2 \\ 2 & 3 & 4 \end{bmatrix} \\
&= (1 \cdot 4) - (2 \cdot 3)i - (1 \cdot 4) - (2 \cdot 2)j + (1 \cdot 3) - (1 \cdot 2)k \\
&= -2i - 0j + 1k
\end{aligned}
$$

With point $P_1(1, 2, 3)$ and normal vector n the equation of the plane is,

$$
-2x + 0y + 1z - 1 = 0
$$

Then the distance from point $Q(2, -2, 2)$ to the plane is,

$$
\frac{|-2(2) + 0(-2) + 1(2) - 1|}{\sqrt{(-2)^2 + (0)^2 + (1)^2}} = \frac{3}{\sqrt{5}}
$$



The white line that goes through point Q (pink), the shortest possible path, is the line perpendicular to the plane (green).

- Therefore, the distance D from point Q to the plane is along a line parallel to the normal vector.