Assignment 9

Question 1

Find The maximum value of the function :
$f(x, y, z) = x + 3y + 5z$ Subject to the constraint :
$x^2 + y^2 + z^2 = 35$

1. Get gradient and set it equal to $(0, 0, 0)$

$\nabla f = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right\rangle$ $\nabla f = \langle 1, 3, 5 \rangle$ So, gradient at $(0, 0, 0)$ is given by $\nabla = f(0, 0, 0) = (1, 3, 5)$

2. Make a hessian matrix, and study the eigen values.

$$H_f = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Thus since the hessian matrix of the given function is a zero matrix, it is singular, and it inconclusive.

3. Lets try, $f(-1, -3, -5)$ and $f(1, 3, 5)$

So the min is at -35 and max is at 35

Question 2

Evaluate the double integral : $\int\int_D \frac{2y}{x^2+1}\,dA$ where :

$d = \{(x,y) \mid 0 \le x \le 1, \quad 0 \le y \le \sqrt{x}\}$

1. $\int_0^1 \frac{2y}{x^2+1}\,dx$
   - Take constant out
     - $2y\int_0^2 \frac{1}{x^2+1}\,dx$
   - $\int \frac{1}{1+x^2\,dx} = \arctan(x)$ identity
     - $2y[\arctan(x)]\,|_0^1 = \frac{2y\pi}{4} = \frac{y\pi}{2}$
2. $\int_0^{\sqrt{x}} \frac{\pi y}{2}\,dy$
   - take constant out
     - $\frac{\pi}{2}\int_0^{\sqrt{x}} y\,dy$
   - Power rule: $\int x^a\,dx = \frac{x^{a+1}}{a+1}$
     - $\frac{\pi}{2}\left[\frac{y^2}{2}\right]_0^{\sqrt{x}} = \frac{\pi}{2}\left[\frac{x}{2}\right] = \frac{\pi x}{4}$

Thus,

$$\int\int_D \frac{2y}{x^2 + 1}\, dA = \frac{\pi x}{4}$$