math625-assignment-2-samir-banjara

October 11, 2023

[1]:
```python
def false_position(f, a, b, TOL=1e-6, max_iter=1000):
    if f(a) * f(b) > 0:
        raise ValueError("The function must have different signs at a and b.")
    for i in range(max_iter):
        c = (a * f(b) - b * f(a)) / (f(b) - f(a))
        if f(c) == 0 or abs(f(c)) < TOL:
            break
        if f(a) * f(c) < 0:
            b = c
        else:
            a = c
    return c
```

[2]:
```python
# Example usage with the given function and interval:
# Define the function : f(x) = 230x^4 + 18x^3 + 9x^2 - 221x - 9
def f(x):
    return 230*x**4 + 18*x**3 + 9*x**2 - 221*x - 9
    # Find the root in the interval [-1, 0]
root = false_position(f, -1, 0)

 # Find the root in the interval [1, 0]
root2 = false_position(f, 0, 1)

# Print the results
print(f"Root: {root}")
print(f"f({root}) = {f(root)}")
print(f"Root2: {root2}")
print(f"f({root2}) = {f(root2)}")
```

```
Root: -0.040659284770926674
f(-0.040659284770926674) = -7.859050175085258e-07
Root2: 0.962398418572024
f(0.962398418572024) = -1.1896614182660414e-07
```

0.1 Lets try Newton's Method now

[3]:
```
def newton_method(p0, e, max_it, f, f_prime):
    for _ in range(max_it):
        p1 = p0 - f(p0) / f_prime(p0)
        if abs(p1 - p0) < e:
            return p1
        p0 = p1
    else:
        print('Warning. Max Iter Reached!')
        return p0
```

[4]:
```
def f(x):
    return 230*x**4 + 18*x**3 + 9*x**2 - 221*x - 9

def f_prime(x):
    return 920*x**3 + 54*x**2 + 18*x - 221

# Intervals
intervals = [(-1, 0), (0, 1)]
e = 1e-6
max_it = 50

roots = []

for a, b in intervals:
    p0 = (a + b) / 2  # Mid-point
    root = newton_method(p0, e, max_it, f, f_prime)
    roots.append(root)

print("Roots:", roots)
```

```
Roots: [-0.04065928831575899, -0.040659288315758865]
```

Lets try to experiment with Complex Numbers.

There is an issue with the use of absolute value of Real Numbers $\mathbb{R}$. We need to use the modulus of complex numbers defined as follows, Let z be a complex number represented in the standard form as $z = a + bi$ Where a and b are $\mathbb{R}$ real numbers, and i is the imaginary unit such that $i^2 = -1$

The modulus (or magnitude) of z, is the negative square root of the sum of squares of its real and imaginary parts, denoted, $|z| = \sqrt{a^2 + b^2}$

With the following properties: 1. $|z| \geq 0$ for all complex numbers z, and $|z| = 0$ if and only if $z = 0$. 2. $|z_1 z_2| = |z_1||z_2|$ for all complex number z_1 and z_2. 3. $|\hat{z}| = |z|$, where $\hat{z}$ is the complex conjugate of z. 4. $|z_1 + z_2| \leq |z_1| + |z_2|$ (Triangle Inequality) for all complex numbers z_1 and z_2

And the argument is defined as follows:

The argument of z, denoted by $\arg(z)$ or θ, is the angle (in radians) that the line segment joining the origin and the point representing z in the complex plane makes with the positive real axis. The angle is measured in the counter clockwise direction.

The *principal value* of the argument is given by: $\arg(z) = \arctan(\frac{b}{a})$

The quadrant in which z lies must be taken into account to determine the correct angle

1. if $a > 0$ and $b \geq 0$ then, $\arg(z) = \arctan(\frac{b}{a})$
2. if $a < 0$,then $\arg(z) = \arctan\left(\frac{b}{a}\right) + \pi$
3. if $a < 0$
4. if $a = 0$ and $b \neq 0$ then $\arg(z) = \frac{\pi}{2}$ for $b > 0$ and $\arg(z) = -\frac{\pi}{2}$ for $b < 0$

We need to modify the code, to reflect the solutions shown below.1. Replace all the absolute value computations with the modulus computations for complex numbers. we can still use `abs()`

2. Instead of checking if `f(a) * f(b) > 0` we can check if the argument of the two complex number is different by more than the `TOL`. This ensure they lie in different half-planes.

[5]:
```
def false_position_complex(f, a, b, TOL=1e-6, max_iter=1000):
    # Check if f(a) and f(b) lie in different half-planes
    if (f(a) * f(b).conjugate()).real >= 0:
        raise ValueError("The function must have different signs at a and b.")

    for i in range(max_iter):
        c = (a * f(b) - b * f(a)) / (f(b) - f(a))

        if f(c) == 0 or abs(f(c)) < TOL:
            break

        # Check if f(a) and f(c) lie in different half-planes
        if (f(a) * f(c).conjugate()).real < 0:
            b = c
        else:
            a = c

    return c
```

[6]:
```
# Example usage with a complex function and interval:
# Define the function: f(z) = z^2 + 1
def f(z):
    return z**2 + 1

# Find the root of the function in the interval [1-1j, 1+1j]
root = false_position_complex(f, 1-1j, 1+1j)

# Print the results
```

```python
# print(f"Root: {root}")
print(f"f({root}) = {f(root)}")
```

```
f((-4.3586274104249747e-07+0.9999998595050125j)) =
(2.8099014526272725e-07-8.717253596119342e-07j)
```

0.2 Question 2

Let $f(x) \in C\,[a, b]$ and let $p \in [a, b]$

1. Suppose that $f(p) \neq 0$ show that there is a $\delta > 0$ with $f(x) \neq 0$ for all $x \in [p - \delta, p + \delta]$
2. Suppose that $f(p) = 0$ and $k > 0$ is given, Show that there is a $\delta > 0$ with $f(x) \leq k$ for all $x \in [p - \delta, p + \delta]$

Solution:

1. Given that $f(x)$ is continuous on $[a, b]$ and $f(p) \neq 0$, by the property of continuity functions, $f(x)$ is continuous at $x = p$. This means that for any $\epsilon > 0$, there exists a $\delta > 0$ such that if $0 < |\,x - p\,| < \delta$ then $|\,f(x) - f(p)\,| < \epsilon$

Now choosing $\epsilon = \dfrac{|\,f(p)\,|}{2}$ This ensure that $f(x)$ does not change sign in the interval $(p - \delta, p + \delta)$, because the difference between $f(x)$ and $f(p)$ is less than half the magnitude of $f(p)$.

Given $|\,f(x) - f(p)\,| < \dfrac{|\,f(p)\,|}{2}$ this implies, $|\,f(x)\,| > \dfrac{|\,f(p)\,|}{2}$ since, $\dfrac{|\,f(p)\,|}{2} > 0$ Thus, it follows that there exists a $\delta > 0$ such that $f(x) \neq 0$ for all $x \in [p - \delta, p + \delta]$ *** 2. Given that $f(x)$ is continuous on $[a, b]$ and $f(p) = 0$, by the property of continuity functions, $f(x)$ is continuous at $x = p$. This means that for any $\epsilon > 0$, there exists a $\delta > 0$ such that if $0 < |\,x - p\,| < \delta$ then $|\,f(x) - f(p)\,| < \epsilon$

Choose $\epsilon = k$ Ensure that the difference between $f(x)$ and $f(p)$ is less than k Given $|\,f(x) - f(p)\,| < k$ and $f(p) = 0$, the inequality becomes $|f(x)| < k$, which implies $f(x) \leq k$ Thus, there exists a $\delta > 0$ such that $f(x) \leq k$ for all $x \in [p - \delta, p + \delta]$ ***