

ASSIGNMENT 6

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Question #1

$$\lim_{(x, y) \rightarrow (0, 0)} \frac{x^2 \cdot y^4}{x^4 + y^8}$$

Solution :

Find two different paths to approach the point that gives different values for the limit.

Along : $x = 0$

$$\begin{aligned} \lim_{(0, y) \rightarrow (0, 0)} \frac{x^2 \cdot y^4}{x^4 + y^8} &= \frac{0^2 \cdot y^4}{0^4 + y^8} \\ &= \frac{0}{y^8} \\ &= 0 \end{aligned}$$

Along : $x = y^2$

$$\begin{aligned}
 \lim_{(y^2, y) \rightarrow (0, 0)} \frac{x^2 \cdot y^4}{x^4 + y^8} &= \frac{(y^2)^4 \cdot y^4}{(y^2)^4 + y^8} \\
 &= \frac{y^4 \cdot y^4}{y^8 + y^8} \\
 &= \frac{y^8}{2y^8} \\
 &= \frac{1}{2}
 \end{aligned}$$

Thus, because the limits of the two paths $x = 0$ and $x = y^2$ are not equivalent, this function is divergent and the limit does not exist.