# Assignment 10

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## Question 1

Evaluate the double integral $\iint_D \ xy^2 \ dA$ Where $D$ is enclosed by $x = 0$ and $x = \sqrt{1 - y^2}$

### Solution

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**Intersection:** $0 = \sqrt{1-y^2} \, y = \pm 1$

**Type $II$ region:**

$D = \{(x, y) \mid -1 \leq y \leq 1, \quad 0 \leq x \leq \sqrt{1-y^2}\}$

Thus,

$$
\begin{aligned}
\iint_D xy^2 \, dA &= \int_{-1}^{1} \int_0^{\sqrt{1-y^2}} (xy^2) \, dx \, dy \\
&= y^2 \int_{-1}^{1} \left[ \frac{x^2}{2} \right]_{x=0}^{x=\sqrt{1-y^2}} dy \\
&= \int_{-1}^{1} \left( \frac{(1-y^2) \cdot y^2}{2} \right) dy \\
&= \frac{1}{2} \int_{-1}^{1} -y^4 + y^2 \, dy \\
&= \frac{1}{2} \left( -\int_{-1}^{1} y^4 \, dy + \int_{-1}^{1} y^2 \, dy \right) \\
&= \frac{1}{2} \left( \frac{-2}{5} + \frac{2}{3} \right) \\
&= \frac{2}{15}
\end{aligned}
$$

## Question 2

Set up the integral of the function $f(x, y, z) = x + y + z$ over the region $\mathbb{R}^3$ bounded by $x + 3y + z = 2$, $x = 3y$, $x = 0$, $z = 0$

### Solution

Determine the limits of integration for each variabe

From the equation $x = 3y$ we can focus on the region where $x \geq 0$

Next we need to find the limits for $y$ and $z$ at ta given value of $x$ we can rearrange the equation $x + 3y + z = 2$ to get $z = 2 - x - 3y$ Therefore, the limits for $y$ and $z$ are $0 \leq y \leq \frac{1}{3}$ and $0 \leq z \leq 2 - x - 3y$ respectively.

Finally we integrate $f(x, y, z) = x + y + z$ over the region $\mathbb{R}^3$ as follows:

$$
\begin{aligned}
V &= \int_{\mathbb{R}^3} f(x, y, z)\, dV \\
&= \int_{x=0}^{x=3} \int_{y=0}^{y=\frac{1}{3}x} \int_{z=0}^{z=2-x-3y} (x + y + z)\, dz,\ dy,\ dx
\end{aligned}
$$