MA 225 - Infinite Series **HW1**

Name (Printed): ____________________

Pledge and Sign: ____________________

- **All explanations and answers must be clearly and neatly written. Explain each step in your solution. Your solutions should make very clear to the instructor that you understand all of the steps and the logic behind the steps.**
- **You are allowed to discuss the homework problems with other students. However, you are not allowed to copy solutions from other students or other sources including AI chatbots. You should list at the end of the problem set the sources you consulted and people you worked with on this assignment.**
- **The final document should be saved and submitted as a single .pdf file, and please be sure all problem solutions are presented starting from the first to the last (that is, the first solution must correspond to problem 1 and the second to problem 2 and so on). Be sure you upload high quality scans of your solutions.**
- **Typed submissions (for example in LaTeX) will be positively considered in the grade. Overleaf is an easy avenue to start learning LaTeX. See the tutorials at https://www.overleaf.com/learn**
- **Honor code applies fully. Your submission should reflect your own understanding of the material. It is prohibited to post the following problems on any website/forum or any other online means.**
- **Legibility, organization of the solution, and clearly stated reasoning where appropriate are all important. Points will be deducted for sloppy work or insufficient explanations.**

1. Determine whether each sequence is convergent or divergent. If convergent find the limit.

(a) [4 pts.] $\lim_{n\to\infty} ne^{-n}$

(b) [3 pts.] $\lim_{n\to\infty} \dfrac{\sin^2 n}{n+3}$

(c) [3 pts.] $\lim_{n\to\infty} \dfrac{5n^2+2}{3n^2+3}$

2. (a) [7 pts.] Express the repeating decimal $0.45454545\ldots$ as a geometric series.

(b) [3 pts.] Use the sum formula for a convergent geometric series to express this decimal as a rational number, e.g. as a quotient of two integers.

3. (a) [5 pt] Prove that if $\displaystyle\sum_{n=0}^{\infty} a_n$ converges and $\displaystyle\sum_{n=0}^{\infty} b_n$ diverges, then $\sum_{n=0}^{\infty}(a_n+b_n)$ diverges.

[Hint: To derive a contradiction assume $\displaystyle\sum_{n=0}^{\infty}(a_n+b_n)$ converges and consider

$\displaystyle\sum_{n=0}^{\infty}(a_n+b_n) - \sum_{n=0}^{\infty} a_n$.]

(b) [5 pt] Is the series $\displaystyle\sum_{n=0}^{\infty} \frac{n+(-1)^n}{n^2}$ convergent or divergent? Explain!