Vector Bootcamp (part 1)

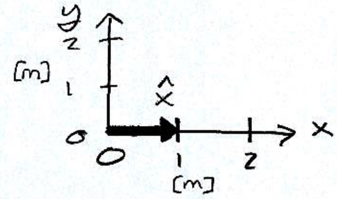
Vector:

- Cartesian form: $\boxed{\vec{v} \equiv v_x\hat{x} + v_y\hat{y}}$ (1)
- v_x and v_y are scalar components of $\vec{v}$ in each direction:

(scalars are numbers!) $v_x \equiv$ x component of $\vec{v} \equiv$ signed amount of $\vec{v}$ in x direction
$v_y \equiv$ y component of $\vec{v} \equiv$ signed amount of $\vec{v}$ in y direction (2)

- v_x and v_y can have any sign or be 0
- $\hat{x}$ and $\hat{y}$ are unit vectors (vectors of length 1)

($\hat{x}$ points in the +x direction
$\hat{y}$ points in the +y direction)



(3)

{① Make a vector with scalar component −3 in the x direction, and scalar component +4 in the y direction.}

{② Make a vector with x component 0 and y component −4.1.}

{③ Make a vector with x component a and y component b, where a and b are both symbolic variables.}

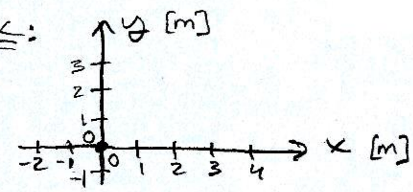
{④ Make a vector with x component 1 and y component 0.}

Coordinate System:

- coordinates are things like x and y
- you should indicate the origin (0,0) as well

a set of perpendicular lines with arrows showing the positive direction for each coordinate, with labels!

ex:



Length of a Vector:

- length of a vector in Cartesian form is its magnitude:

$$\boxed{|\vec{v}| \equiv \sqrt{v_x^2 + v_y^2}} \qquad (4)$$

- note: unit vectors do not appear in $|\vec{v}|$; it only depends on scalars
- so $|\vec{v}|$ is also a scalar, but is nonnegative: $|\vec{v}| \geq 0$

{(5) Find the magnitude of each vector in problems (1)–(4). Are any of them unit vectors?}

Unit Vectors:

- vectors of magnitude 1
- most general form (in 2D): $\boxed{\hat{u} \equiv \cos(\theta)\hat{x} + \sin(\theta)\hat{y}}$ (here we stick to real-valued vectors) (5)
- A unit vector in the direction of $\vec{v}$ can be found as: $\boxed{\hat{v} \equiv \frac{\vec{v}}{|\vec{v}|} = \frac{v_x}{\sqrt{v_x^2+v_y^2}}\hat{x} + \frac{v_y}{\sqrt{v_x^2+v_y^2}}\hat{y}}$ (valid only for $\vec{v} \neq \vec{0}$) (6)
  - note: $\vec{0} \equiv 0\hat{x} + 0\hat{y}$

{(6) what is a unit vector in the direction of nonunit vector $\vec{c} \equiv c_x\hat{x} + c_y\hat{y}$?}

{(7) what is the unit vector in (5) when $\theta = \frac{\pi}{2}$?}

{(8) what is the unit vector in (5) when $\theta = 0$?}

{(9) what is the unit vector in (5) when $\theta = \frac{\pi}{4}$?}

{(10) Make a vector of magnitude 3 that points in the direction of $\vec{w} \equiv c\hat{x} + d\hat{y}$.}

{(11) Make a vector of magnitude π that points in the direction opposite to $\vec{b} \equiv 2\hat{x} - \hat{y}$}

Projections of a Vector:

- Scalar projections:

$$\vec{v} \equiv v_x \hat{x} + v_y \hat{y} \tag{7}$$

$$\boxed{\begin{aligned} v_x &\equiv \text{scalar projection of } \vec{v} \text{ onto } \hat{x} \\ v_y &\equiv \text{scalar projection of } \vec{v} \text{ onto } \hat{y} \end{aligned}} \quad \text{(are scalars)} \tag{8}$$

- vector projections:

$$\vec{v} \equiv \underbrace{v_x \hat{x}} + \underbrace{v_y \hat{y}} \tag{9}$$

$$\vec{v} \equiv \vec{v}_x + \vec{v}_y \tag{10}$$

(can also call $\vec{v}_x$ and $\vec{v}_y$ the <u>vector components</u> of $\vec{v}$)

(<u>Note</u>: could also have scalar and vector projections onto some arbitrary vector $\vec{a}$, so then replace $\hat{x}$ with $\hat{a}$ to get v_a and $\vec{v}_a \equiv v_a \hat{a}$. we'll learn how to find v_a later...)

$$\boxed{\begin{aligned} \vec{v}_x &\equiv v_x \hat{x} \equiv \text{vector projection of } \vec{v} \text{ onto } \hat{x} \\ \vec{v}_y &\equiv v_y \hat{y} \equiv \text{vector projection of } \vec{v} \text{ onto } \hat{y} \end{aligned}} \quad \text{(are vectors)} \tag{11}$$

{(12) what's the y-direction vector component of $\vec{c} \equiv c_x \hat{x} + c_y \hat{y}$?}

{(13) what's the x-direction vector component of $\vec{w} \equiv w_x \hat{x} + w_y \hat{y}$?}

{(14) Find the magnitudes of the vector components in (12) and (13).}

{(15) what's the y-direction vector component of $\vec{h} \equiv 7\hat{x} - \hat{y}$?}

{(16) what's the x-direction vector component of $\vec{g} \equiv 3\hat{y}$?}

{(17) Find the magnitude of the vector projections in (15) and (16).}

{(18) what's the scalar projection in the x direction of $\vec{w} \equiv w_x \hat{x} + w_y \hat{y}$?}

{(19) what's the y-direction scalar projection of $\vec{h} \equiv 7\hat{x} - \hat{y}$?}

Vector Addition:

- vectors add component-wise. If $\vec{a} \equiv a_x \hat{x} + a_y \hat{y}$ and $\vec{b} \equiv b_x \hat{x} + b_y \hat{y}$, then

$$\boxed{\begin{aligned} \vec{c} &\equiv \vec{a} + \vec{b} \\ &= \underbrace{(a_x + b_x)}\hat{x} + \underbrace{(a_y + b_y)}\hat{y} \\ &\equiv c_x \hat{x} + c_y \hat{y} \end{aligned}} \qquad \boxed{\begin{aligned} c_x &\equiv a_x + b_x \\ c_y &\equiv a_y + b_y \end{aligned}}$$

(12)

(13)

{(20) what is the scalar y component of $\vec{c}$ in (12) in terms of the scalar components of $\vec{a}$ and $\vec{b}$?}

{(21) what's the vector projection in the x direction of $\vec{c}$ in (12) in terms of the components of $\vec{a}$ and $\vec{b}$?}

{(22) what is the magnitude of $\vec{c}$ in (12) in terms of the scalar components of $\vec{a}$ and $\vec{b}$?}

{(23) what is the x-direction vector projection of the most general 2D unit vector $\hat{u} \equiv \cos(\theta)\hat{x} + \sin(\theta)\hat{y}$?}

{(24) Build the most general 2D vector possible by multiplying a general nonnegative number r times $\hat{u} \equiv \cos(\theta)\hat{x} + \sin(\theta)\hat{y}$. what is the magnitude of this vector? In what direction does it point?}

{(25) Build the most general 2D vector in a different way by defining two vectors: $\vec{A} \equiv r\cos(\theta)\hat{x}$ and $\vec{B} \equiv r\sin(\theta)\hat{y}$, and then defining the most general vector as $\vec{C} \equiv \vec{A} + \vec{B}$. what is $\vec{C}$ in terms of r and θ? How does $\vec{C}$ compare to $\vec{v}$ in (24)?}