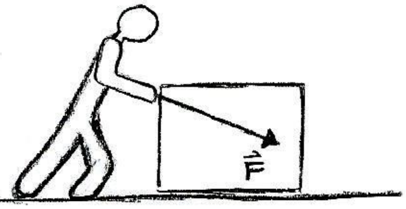
Dynamics: Newton's Laws of Motion

Force:

> Force is a push or pull on an object

← (qualitative definition)

- Two kinds:
  - contact force (pushing a cup with your hand, etc.)
  - noncontact force (gravity, etc.)
- Force is a vector:
  - has magnitude
  - has direction



Motion and Force:

> If an object is at rest, a force is needed to move it.

← (• To change velocity from 0 to nonzero, a force is needed • But force must be strong enough to overcome other forces...)

(• To change velocity (speed and/or direction), a force is needed) →

> If an object is in motion, a force is needed to change its speed or direction or both.

> To accelerate an object, a force is needed.

← (• Both observations above involve changes in velocity, which is acceleration:

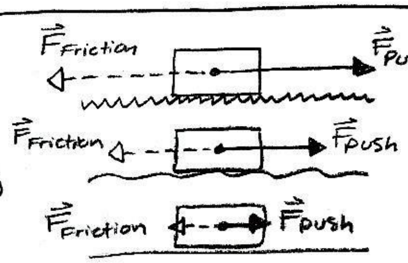
$$\vec{a} \equiv \lim_{\Delta t \to 0}\left(\frac{\Delta \vec{v}}{\Delta t}\right) \equiv \frac{d\vec{v}}{dt}$$
)

Galileo's Motion Hypothesis:

> The natural state of an object can be in motion with constant velocity or at rest.

(• Aristotle (384–322 B.C.) thought that the natural state of an object was at rest • thought force needed to maintain constant velocity of object)

- Galileo's Idealization:
  - treated friction as force
  - acknowledged ability of multiple forces acting on object simultaneously



(• pushing an object over increasingly smooth surfaces requires less force • In the ideal limit of no friction, no force is needed to maintain an initial velocity)

Newton's First Law of Motion:

- Law of inertia:

> inertia ≡ an object's resistance to changes in velocity

> Every object continues either in its state of rest or in uniform velocity in a straight line, as long as no net force acts on it.

(Isaac Newton (1642–1726))

(Newton's 1st law is the law of inertia)

Galilean-Newtonian Relativity:

- Inertial Reference Frame $\equiv$ **a reference frame with constant velocity**
- Noninertial Reference Frame $\equiv$ **a reference frame with changing velocity**
  ("accelerated reference frame")

(or just "frame") → • Recall, Reference Frame $\equiv$ a coordinate system attached to some object

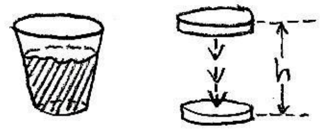
- Corollaries:
  - inertial frames feel no net force
  - noninertial frames do feel a net force

  (since force is needed to change velocity, then $\vec{a}=\vec{0} \Rightarrow \vec{F}_{net}=\vec{0}$, $\vec{a}\neq\vec{0} \Rightarrow \vec{F}_{net}\neq\vec{0}$)

- Galilean Relativity Principle:

  The laws of mechanics are the same in all inertial reference frames.

  - ex • in an airplane flying with constant $\vec{v}$, water stays in its glass, a dropped coin falls straight down, etc.



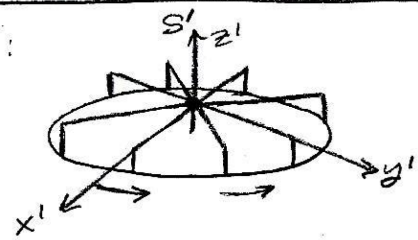
(in plane's frame)

(• In plane's frame, coin's path is a line
• In ground's frame, coin's path is a parabola)

(in ground's frame)

- Corollary: **All inertial frames are equally valid.** ← (It was really Einstein who noticed this, but it's relevant here)

- Noninertial Frames can Feel "Ficticious Forces":
  - ex: Merry-go-round:



(• All points in S' frame are continually changing direction $\Rightarrow$ changing $\vec{v}$
• Thus, S' is a noninertial frame
• You feel as if you're being pulled outward... but that's a "ficticious force")

(Note: Earth is rotating, so is also a noninertial frame, but the effects can often be ignored, so we'll treat it as inertial usually.)

  - The "outward pull" is really your inertial tendency to move in a line, but the merry-go-round keeps changing your direction by perpendicular contact force
  - So "ficticious forces" are real, but arise from accelerated motion.

- Consequence: **Newton's 1st law is only valid in inertial reference frames.**
  - ex: In a bus that suddenly brakes, all the bookbags slide forward!
    - The "ficticious force" that moved the bookbags is not a bookbag magnet, it is the acceleration of the bus frame (sudden drop in speed),

(1st law violated!) → • so bookbags didn't stay at rest (because braking bus is noninertial frame)
(1st law obeyed) → • but in ground frame, bookbags just kept going at same speed (ground is inertial frame)

Mass:

$$\boxed{\text{mass} \equiv \text{measure of an object's \underline{inertia}}}$$ ← (measures an object's resistance to changes in velocity)

- Corollary:
  - since changes in velocity (acceleration) require a force,
  - and since mass is resistance to changes in velocity,
  - then

> For a given change in velocity, and two objects with masses $m_1 < m_2$, it takes a stronger force to accelerate the larger mass m_2 than it takes for smaller mass m_1, to achieve the same $\vec{a}$.

- Units for Mass: $\boxed{\text{kilogram} \equiv \text{kg}}$ (S.I.)

- CAUTION:

mass ≠ weight

(an object's mass is the same on any planet (its inertia is an intrinsic property))

(an object's weight is the force it feels due to external gravitational sources (your weight on Moon is less than on Earth))

Newton's Second Law:

- Facts:
  - acceleration of an object requires a force on that object
  - multiple forces can act on an object simultaneously
  - force is a vector
  - by vector addition, the net force on an object is the sum:

$$\vec{F} \equiv \sum_{k=1}^{n} \vec{F}_k = \vec{F}_1 + \vec{F}_2 + \dots + \vec{F}_n \equiv \Sigma\vec{F} \qquad (1)$$

- Observations:
  - The acceleration of an object is proportional to the net force acting on it:

$$\vec{a} \propto \Sigma\vec{F} \qquad (2)$$

← (the harder the net push, the greater the acceleration)

  - The acceleration of an object is inversely proportional to the object's mass:

$$|\vec{a}| \propto \frac{1}{m} \qquad (3)$$

← (the larger the mass, the weaker the acceleration for a given net force applied)

- Combine (2) and (3):

$$\vec{a} = \frac{1}{m}\Sigma\vec{F} \qquad (4)$$

← ($\frac{1}{m}$ is the proportionality constant)

- rearrange (4):

$$\boxed{\Sigma\vec{F} = m\vec{a}} \qquad (5)$$

← (Newton's 2nd Law [for constant mass])

(sum of external forces acting on the object of mass m) | (the mass m and its acceleration $\vec{a}$, due to those external forces ΣF)

Newton's 2nd Law is 3 Equations:

- Full vector form: $$\sum \vec{F} = m\vec{a} \quad ; \quad \begin{cases} \sum \vec{F} \equiv (\sum F_x)\hat{x} + (\sum F_y)\hat{y} + (\sum F_z)\hat{z} \\ \vec{a} \equiv \quad a_x\hat{x} + a_y\hat{y} + a_z\hat{z} \end{cases} \quad (6)$$
- Component form:

$$\Downarrow$$

$$\boxed{\sum F_x = ma_x \; , \; \sum F_y = ma_y \; , \; \sum F_z = ma_z} \quad \text{(Newton's 2nd law writen as 3 scalar equations)} \quad (7)$$

- In 1D: can omit subscripts, since there's only 1 component!

$$\sum F = ma \quad (8)$$

  - (8) applies if motion is along a line
  - CAUTION: The single "scalar" equation (8) is really a vector equation since $\sum F$ and a are 1D vectors. Similarly in (7).
  - other notations: $$F \equiv \sum F = \sum_{k=1}^{n} F_k = F_{net} = F_{tot} = F_{sum} \quad (9)$$

Units of Force:

$$\boxed{\text{newton} \equiv N} \quad (S.I.) \quad (10)$$

- is a derived unit: $$1[N] \equiv 1\left[\frac{kg \cdot m}{s^2}\right] \quad (11)$$
- check with dimensional analysis: $$\underline{\sum F} = m\,a$$ $$[N] = [kg]\cdot\left[\frac{m}{s^2}\right] = \left[\frac{kg \cdot m}{s^2}\right] \; \checkmark \quad (12)$$

ex: {What average force is required to bring a 1500 [kg] car to rest from a speed of 100 [km/h] within a distance of 55 [m]?}

- Diagram:

- givens:

$$m = 1500\,[kg] \quad (1)$$

$$V_0 = 100\left[\frac{km}{h}\right]\cdot\left(\frac{1000\,[m]}{1\,[km]}\right)\cdot\frac{1\,[h]}{3600\,[s]} = 27.78\left[\frac{m}{s}\right] \quad (2)$$

$$V = 0\left[\frac{m}{s}\right] \quad (3)$$

$$x - x_0 = 55\,[m] \quad (4)$$

- We want: $(\sum F)_{avg}$.
- From Newton's 2nd law,

$$\sum F = ma \quad (5)$$

  - we have m, so we want a
  - Note: if we assume $a = \text{constant}$, then we can use kinematic eqns., and from (5), $\sum F$ is also constant, so it is its own average $\hookrightarrow (\sum F = (\sum F)_{avg})$
- So given $v_0, v, x - x_0$, and wanting a, use kinematic eqn:

$$v^2 = v_0^2 + 2a(x - x_0) \quad (6)$$

  - solve for a:

$$a = \frac{v^2 - v_0^2}{2(x - x_0)} \quad (7)$$

  - put (7) into (5):

$$\boxed{\sum F = \frac{m(v^2 - v_0^2)}{2(x - x_0)} = -10524\,[N] \approx -1.1 \times 10^4\,[N]} \quad (8)$$

  - the negative sign means that this force points in the $-x$ direction, which opposes V_0 and causes the deceleration

CAUTION: $\boxed{\text{Newton's 2nd law is only valid in inertial reference frames.}}$