9 HW

Q1

Kira has two kids

- frank
  - frank has a daughter names Sarah
    - Sarah is mother of
    - Judy
    - and Ruben
- lola

Kira
Frank
Sarah
Judy
Ruben
Lola

Write down the relation $R = \{(a, b) \mid a \text{ is the parent of } b\}$

$$
\begin{aligned}
A &= \{\text{Kira, Frank, Sarah, Judy, Ruben, Lola}\} \\
A \times A &= (Kira, Kira), (Kira, Frank), (Kira, Sarah), (Kira, Judy), (Kira, Ruben) \\
&= (Frank, Frank), (Frank, Sarah), (Frank, Judy), (Frank, Ruben), \\
&= (Sarah, Sarah), (Sarah, Judy), (Sarah, Ruben), \\
&= (Judy, Judy), (Judy, Ruben) \\
&= (Ruben, Ruben), \\
&=
\end{aligned}
$$

As a matrix

$$A \times A = \left[\begin{matrix} (\text{Kira}, \text{Kira}) & (\text{Kira}, \text{Frank}) & (\text{Kira}, \text{Sarah}) & (\text{Kira}, \text{Judy} \\ (\text{Frank}, \text{Kira}) & (\text{Frank}, \text{Frank}) & (\text{Frank}, \text{Sarah}) & (\text{Frank}, \text{Jud} \\ (\text{Sarah}, \text{Kira}) & (\text{Sarah}, \text{Frank}) & (\text{Sarah}, \text{Sarah}) & (\text{Sarah}, \text{Jud} \\ (\text{Judy}, \text{Kira}) & (\text{Judy}, \text{Frank}) & (\text{Judy}, \text{Sarah}) & (\text{Judy}, \text{Judy} \\ (\text{Ruben}, \text{Kira}) & (\text{Ruben}, \text{Frank}) & (\text{Ruben}, \text{Sarah}) & (\text{Ruben}, \text{Jud} \\ (\text{Lola}, \text{Kira}) & (\text{Lola}, \text{Frank}) & (\text{Lola}, \text{Sarah}) & (\text{Lola}, \text{Judy} \end{matrix}\right.$$

$R = \{(\text{Kira}, \text{Frank}), (\text{Kira}, \text{Lola}), (\text{Frank}, \text{Sarah}), (\text{Sarah}, \text{Judy}), (\text{Sarah}, \text{Ruben})\}$
Relation R as a adjacency matrix:

$$R = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Question 2.1

- **Symmetric & AntiSymmetric & Transitive**

Let $R = \{\emptyset\}$ be a relation on a set A

Prove that

- if A is non-empty, the empty relation is not reflexive on A .
- The empty relation is symmetric and transitive for every set A
  Solution:

For a relation to be reflexive:

- For all elements in A, they should be related to themselves. xRx
  - Now in this case there are no elements in the Relation
    - and as A is non-empty no element is related to itself
      - *Note:* Would also hold for $A = \emptyset$
      - But in that case the empty relation would be reflexive.
    - hence the empty relation is not reflexive.

So, As A is not empty, there exists some element $a \in A$

- As R is empty
  - aRa does not hold
- Hence R is not reflexive
- Now for a set to be symmetric and transitive:
- As these are conditional statements
  - if the antecedent is false the statements would be true.
    - And as the relation is empty in both cases
      - the antecedent is false
    - hence the empty relation is symmetric and transitive.

Question 2.2

- **Reflexive** & **Symmetric** & **Transitive** & **Equivalence Relation**
  The complete relation $R = \mathbb{N} \times \mathbb{N}$
  defined on the natural numbers.

Imagine that set $\mathbb{N} = \{1, 2, 3, 4, 5. \cdots\} \quad \text{where } a \in \mathbb{N}$
then,

$$\mathbb{N} \times \mathbb{N} = \begin{array}{l} \{(1,1),(1,2),(1,3),(1,\cdots), \\ (2,1),(2,2),(2,3),(2,\cdots), \\ (3,1),(3,2),(3,3),(3,\cdots)\} \end{array}$$

Then the relation R is **Reflexive**,

- because diagonal entires exists for all elements $a \in A$
- (a, a)

The relation R is **Symmetric**,

- because $(a, b)(b, a)$ for all a and b

Transitive

- $(a, b)(b, c) \to (a, c)$

Question 2.3

- **Transitive**

The relation R on integers where aRb means $(a-2)<b$

a minus 2 is less than b

then b minus 2 is less than c

hence aRc hold true? yes Transitive

can aRb and bRa hold true? no, hence not symmetric
Definitely not reflexive

Question 2.4

- **Reflexive** & **Transitive**

The relation R on $\{w,x,y,z\}$ where
$R=\{(w,w),(x,y),(x,w),(x,x),(x,z),(y,y),(z,y),(z,z)\}$

Tabel! transitive and reflexive and symmetric tabel

Column 1	Column 2	Reflexive	Symmetric	Transitive
(w, w)	(w, w)	(w, w)		
(x, y)	(y, y)			(x, y)
(x, w)	(w, w)			(x, w)
(x, x)	(x, x)	(x, x)		
(x, z)	(z, z)			(x, z)
(x, z)	(z, y)			(x, y)
(y, y)	(y, y)	(y, y)		
(z, y)	(y, y)			(z, y)
(z, z)	(z, y)			(z, y)
(z, z)	(z, z)	(z, z)		
		Yes	No	Yes

Reflexive,

- Because it contains all (w, w), (x, x), (y, y), and (z, z)

Symmetric

- Does not contain

(w, x)	(x, w)	
(w, y)	(y, w)	
(w, z)	(z, w)	
(y, w)	(w, y)	
(y, x)	(x, y)	
(y, z)	(z, y)	
(x, w) (X)	(w, x) (Miss)	
(x, y) (X)	(y, x) (Miss)	
(x, z) (X)	(z, x) (Miss)	
(z, w)	(w, z)	
(z, x)	(x, z)	
(z, y) (X)	(y, z) (Miss)	

Transitive

Question 2.5

- **Reflexive** & **Symmetric**

aRb means $a^2 = b^2$

$a^2 = b^2$ is equivalent to $a = b$,

thus, it can only be reflexive, be the relation can only contian

$$\begin{bmatrix} (1,1) & & & & \\ & (2,2) & & & \\ & & (3,3) & & \\ & & & (4,4) & \\ & & & & (5,5) \end{bmatrix}$$

$1^2 = 1^2$
$2^2 = 2^2$
$3^2 = 3^2$
thus, the relation is reflexive.

is it antisymmetric?

- the definition for
  > - **Antisymmetric :**
  >   > - If $a = b$ whenever $(a, b) \in R$ and $(b, a) \in R$ implies $x = y$

Taking $a^2 = b^2$ we can say that the given relation for set of integers Z is reflexive because $a^2 = a^2$ and $b^2 = b^2$ will always hold true.

The relation is symmetric as well because $a^2 = b^2$ and $b^2 = a^2$

- so aRb and bRa holds

Not transitive.