Written Assignment

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1. Show that the following is an equation of a sphere. Determine the center of the sphere and its radius.

$$x^2 + y^2 + z^2 - 4x + 2y = 10$$

   - Rewrite the given equation in the form of an equation of a sphere by completing the squares.

$$(x^2 - 4x + 4x\,) + \,(y^2 + 2y + 1\,) + \,(z^2\,) = 10$$

$$(x^2 - 2\,)^2 + \,(y^2 + 1\,)^2 + z^2 = 15$$

   - *Center* : $(2, -1, 0\,)$
   - *Radius* : $\sqrt{15}$

2. Determine if the points $P(0,2,0)$, $Q(1,2,3)$, $R(0,0,-2)$ be on the same line (are co-linear). Briefly explain how, you came to this conclusion.

   - We can say *P, Q,* and Q are co-linear if the largest length of *PQ,* and *PR.* and QR is equal to the sum of the other two. We can determine length of the vector by using the distance formula.

$$\sqrt{(x_2 - x_1\,)^2 + \,(y_2 - y_1\,)^2 + \,(z_2 - z_1\,)^2} = |P_1P_2\,|$$

$$|\overrightarrow{PQ}\,| = \sqrt{(0-1\,)^2 + \,(2-2)\,)^2 + \,(0-3\,)^2} = \sqrt{10}$$

$$|\overrightarrow{PR}\,| = \sqrt{(0-0\,)^2 + \,(2-0)\,)^2 + \,(0+2\,)^2} = \sqrt{8}$$

$$|\overrightarrow{QR}\,| = \sqrt{(1-0\,)^2 + \,(2-0)\,)^2 + \,(3-2\,)^2} = \sqrt{10}$$

   - Because the sum of any two lengths are not equal to the third. Points P, Q, and R are not co-linear.
     - $|\overrightarrow{QR}\,| + |\overrightarrow{PR}\,| \neq |\overrightarrow{PQ}\,|$
     - $|\overrightarrow{PR}\,| + |\overrightarrow{PQ}\,| \neq |\overrightarrow{QR}\,|$
     - $|\overrightarrow{QR}\,| + |\overrightarrow{PQ}\,| \neq |\overrightarrow{PR}\,|$

Also, (more explaantions) for my sake in other subjects - If the cross product of the the vectors AB and AC is not the zero vector $\langle 0, 0, 0 \rangle$, then then the given points are not co-linear. (Corollary 10, pg 817, James Stewart Early Transcendental Ed: 8th)

$$\overrightarrow{PQ} \times \overrightarrow{PR}$$

$$\overrightarrow{PQ} = (1-0)i + (2-2)j + (3-0)k = i + 3k$$

$$\overrightarrow{PR} = (0-0)i + (0-2)j + (-2-0)k = -2j + -2k$$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} i & j & k \\ 1 & 0 & 3 \\ 0 & -2 & -2 \end{vmatrix}$$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} 0 & 3 \\ -2 & -2 \end{vmatrix} i - \begin{vmatrix} 1 & 3 \\ 0 & -2 \end{vmatrix} j - \begin{vmatrix} 1 & 0 \\ 0 & -2 \end{vmatrix} k$$

$$6i - (-2j) - (-2k) = \langle 6, -2, -2 \rangle$$

$\therefore$given points are not co-linear, because we did not attain a zero vector

- Also, if $P(0,2,0)$, $Q(1,2,3)$, $R(0,0,-2)$ are co-linear then $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ are proportional.

$$\overrightarrow{PQ} = \lambda \cdot \overrightarrow{PR}$$