SECTION 2.3 Counting Techniques

CALCULATING CLASSICAL PROBABILITY

- When the outcomes of an experiment are equally likely, the probability that an event A occurs is given by

$$P(A) = \frac{n(A)}{N}$$

where $n(A)$ is the number of outcomes in A and N is the number of outcomes in the sample space.

- In this case, computing probability reduces to counting outcomes.

Example:

1) Beck is allergic to peanuts. At a large dinner party one evening, he notices that the cheesecake options on the dessert table contain the following flavors: 10 slices of chocolate, 12 slices of caramel, 12 slices of peanut butter chocolate, and 8 slices of strawberry. Assume that the desserts are served to guests at random.

   a. What is the probability that Beck's cheesecake contains peanuts?

   b. What is the probability that Beck's dessert does not contain chocolate?

2) For a school fundraiser, 1000 raffle tickets are sold for $5 each. Each ticket is assigned a three-digit number using the digits 0-9. What is the probability that the winning ticket will be one with three repeating digits?

3) If you roll one six-sided die twice, what is the probability that the second number rolled is at least as large as the first?

FUNDAMENTAL COUNTING PRINCIPLE

- Consider a set of k elements (or objects). If the first element can be selected in n_1 ways, the second element can be selected in n_2 ways,… the k^{th} element can be selected in n_k ways, then the number of ways to select an ordered array of k elements is $n_1 n_2 n_3 \dots n_k$.

Example:

1) Kilby begins her first year in an online degree program in July. The first semester she will randomly be assigned to one section for each of four different core courses. If there are 8 English I sections, 12 College Algebra sections, 11 American History sections, and 5 Physical Science sections, how many different options are there for Killy's schedule for her first semester?

2) The governing board of the local charity, Mission Stateville, is electing a new vice president and secretary to replace outgoing board members. If the board consists of 11 members who don't already hold an office, in how many different ways can the two positions be filled if no one may hold more than one office?

3) Consider a beginning archer who only manages to hit the target 50% of the time. What is the probability that in three shots, the archer will hit the target all three times?

Example:

4) When ordering his new computer, Joe must choose one of 5 monitors, one of 4 printers, and one of 6 scanners. Joe likes 2 of the monitors, 1of the printers, and 3 of the scanners. If his wife randomly chooses his computer system for him, what is the probability that she will choose a system that makes him happy?

PERMUTATIONS AND COMBINATIONS

Consider a group of n elements (or objects).

- Any <u>ordered</u> subset of *k* elements is called a permutation of size *k*. The number of permutations of size *k* from these *n* elements is given by

$$P_{k,n} = P(k,n) = \frac{n!}{(n-k)!}$$

- Any <u>unordered</u> subset of *k* elements is called a combination of size *k*. The number of combinations of size *k* from these *n* elements is given by

$$C_{k,n} = C(k,n) = \binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

Example:

1) Given a group of three friends, Harry, Heather, and Sheena:

a) How many ways can you arrange the way they stand in line for the movies?

b) How many ways can you choose two of them to ride in a car together?

2) A class of 18 fifth graders is holding elections for class president, vice president, and secretary. In how many different ways can the officers be elected?

3) Maya has a bag of 15 blocks, each of which is a different color including red, blue, and yellow. Maya reaches into the bag and pulls out 3 blocks. What is the probability that the blocks she has chosen are red, blue, and yellow?