

(Under)graduate Research Project: Investigations in Computational Geometric Analysis

Monitored by Simon Foucart

Objective: Produce a software to compute geometric quantities of normed spaces.

Prerequisite: Linear Algebra, Programming, Interest in Mathematics, Academic Credential.

Description: Given an n -dimensional subspace of a normed space X , its projection constant and ℓ_∞ -condition number are defined by

$$\begin{aligned} p(V, X) &:= \inf \{ \|P\|, P : X \twoheadrightarrow V \text{ projection} \}, \\ \kappa_\infty(V) &:= \inf \{ \|T\| \cdot \|T^{-1}\|, T : \ell_\infty^n \rightarrow V \}. \end{aligned}$$

The exact values of these quantities are usually hard to determine, so they are only known in very few cases. However, these and other intermediate quantities can be computed by solving optimization problems — sometimes even linear optimization problems. This project aims at implementing modern optimization techniques to efficiently carry out the computations, and then at cataloging the values of $p(V)$ and $\kappa_\infty(V)$ for a number of spaces V . The first step consists in improving and unifying two existing MATLAB codes designed separately for $p(V)$ and for $\kappa_\infty(V)$. The second step consists in producing a practical graphical user interface. The completed project will pave the way to the experimental discovery of formulas for $p(V)$ and $\kappa_\infty(V)$ in specific cases. While enhancing programming skills, the project will provide exposure to the fields of Approximation Theory, Geometry of Banach Spaces, and Optimization.

Outcome: A stand-alone piece of software and a formal mathematical report.

To Apply: Contact Prof. Foucart at foucart@math.drexel.edu to schedule a face-to-face appointment. Funding for this research experience is not available at the moment.

References

- [1] S. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge University Press, Cambridge, 2004.
- [2] E.W. Cheney and K.H. Price, Minimal projections, 261–289. In: A. Talbot (ed.), *Approximation Theory*, Academic Press, London, 1970.
- [3] S. Foucart, Allometry constants of finite-dimensional spaces: theory and computations, *Numerische Mathematik*, 112/4, 535–564, 2009.
- [4] M. Grant, S. Boyd, and Y. Ye, *cvx: MATLAB software for disciplined convex programming*, <http://www.stanford.edu/~boyd/cvx/>
- [5] MOSEK Optimization Software, <http://mosek.com/>