hw3

(1)

(a) To determine whether the series converges for , we can use the ratio test.

The ratio of consecutive terms is given by

Taking the limit as approaches infinity, we have

Since the limit is less than 1, the series converges for .

(b) To determine whether the series converges for , we can again use the ratio test.

The ratio of consecutive terms is given by

Since the ratio is a constant value less than 1, the series converges for .

(2)

Since is convergent for , it converges within a certain radius of convergence. Let's call this radius .

For , we can calculate the distance from using the distance formula:

Since , the series is absolutely convergent.

(3)

(a)

To compute , we can use the formula .

To compute , we can use , where is the principal value of the argument of .

To compute , where ,

(b)

To compute , we can again use the formula .

To compute , we can use , where is the principal value of the argument of .

To compute , where ,

(4)

(a)

We can represent as .

Using Euler's formula, , we have

(b)

To show that the series is absolutely convergent, we can use the comparison test.

Let's compare the series to the convergent series .

Using the inequality and , we have

Since the series converges, and we have a constant upper bound for the absolute value of each term in the series , we can conclude that the series is absolutely convergent.

(c)

To determine whether the series is convergent or divergent for , where , we can use the comparison test again.

Using the same inequality as in part (b), we have

Since , the series converges.

Therefore, by the comparison test, we can conclude that the series is convergent for , where .

(5)

(a)

To determine the radius of convergence of the power series , we can use the ratio test.

The ratio of consecutive terms is given by

Taking the limit as approaches infinity, we have

where is the modulus of .

Therefore, the radius of convergence of the power series is .

(b)

For in the disk of convergence, the sum of the series is given by

Since the Fibonacci sequence has the general term , we can substitute this expression into the series.

Using the formula for the sum of a geometric series, we can simplify this expression as

Therefore, the sum of the series is .