UMass Boston
Department of Mathematics
Math 358 - Complex Analysis
Fall 2023
Homework #03 v.2

Due: Mon 10/16 at 2:30pm, in class

(1) Consider the power series

$$\sum_{n\geqslant 1}\frac{1}{n}z^n = z+\frac{z^2}{2}+\frac{z^3}{3}+\frac{z^4}{4}+\cdots$$

(a) Does the series converge for $z=-i$? (Note: changed from $z=i$ in first version)
(b) Does the series converge for $z=\frac{1}{2}(-1+i\sqrt{3})$?

(2) We are given a power series $\sum_{n\geqslant 0} a_n(z-2)^n$ that is convergent for $z=-1$. What can we conclude about the series

$$\sum_{n\geqslant 2023} a_n(-2-2i)^n?$$

Is it absolutely convergent, is it divergent, or could be either, depending on the series?

(3) (a) Let $z=-1-2i$. Compute $\operatorname{Exp}(z)$, $\operatorname{Log}(z)$, z^i, and i^z.
(b) Let $z=-1+ti$, with $t\in\mathbb{R}$. Compute $\operatorname{Exp}(z)$, $\operatorname{Log}(z)$, z^i, and i^z as functions of t.

(4) (a) Let $n>0$ be a positive real number. Compute n^{-2+i}.
(b) Show that the series

$$\sum_{n\geqslant 1} n^{-2+i}$$

is absolutely convergent.
(c) Let $z=x+iy\in\mathbb{C}$ such that $x>1$. Determine whether the series

$$\sum_{n\geqslant 1}\frac{1}{n^z}$$

is convergent or divergent.

(5) The Fibonacci sequence is given by $F_0=0$, $F_1=1$, and $F_n=F_{n-1}+F_{n-2}$ for all $n\geqslant 2$, and the general term is

$$F_n=\frac{1}{\sqrt{5}}\Big(\phi^n-(-\phi)^{-n}\Big),$$

where $\phi=\frac{1+\sqrt{5}}{2}$. Consider the power series

$$\sum_{n\geqslant 0}F_nz^n = F_0+F_1z+F_2z^2+F_3z^3+\cdots = 0+z+z^2+2z^3+3z^4+5z^5+8z^6+\cdots$$

(a) Determine the radius of convergence of the power series.
(b) For z in the disk of convergence, determine the sum of the series.