Name: ***Solutions***

**1.** In each case find the limit, $\lim_{n\to\infty} a_n$ of the sequence $\{a_n\}_{n=1}^{\infty}$, or determine that it does not exist.

(a) $a_n = 5 - \dfrac{3}{n^2}$

(b) $a_n = 2 + (-1)^n$

(c) $a_n = \dfrac{3n^4 - 7n^2 + 5}{6 - 4n^4}$

(d) $\sqrt{\dfrac{2n+3}{3n+5}}$

(e) $a_n = \dfrac{n^2}{2^n}$

(f) $a_n = \left(1 + \dfrac{4}{n}\right)^n$

***Solution***:

(a) 5, since $\lim\limits_{n\to\infty} \dfrac{1}{n^p} = 0$ if $p > 0$

(b) DNE, the terms alternate between 1 and 3.

(c) $-\dfrac{3}{4}$. Divide the top and bottom by the highest power in the denominator, $n^4$, and use the $p$-test for sequences as stated in part (a).

(d) $\sqrt{\dfrac{2}{3}}$, the same as part (c).

(e) 0. Apply L'Hospital's Rule to $\lim\limits_{x\to\infty} \dfrac{x^2}{2^x}$ twice.

(f) $e^4$. Recall in fact that $e = \lim\limits_{n\to\infty} \left(1 + \dfrac{1}{n}\right)^n$. Therefore

$$\lim_{n\to\infty} \left(1 + \frac{4}{n}\right)^n = \left[\lim_{n\to\infty} \left(1 + \frac{4}{n}\right)^{n/4}\right]^4 = e^4.$$

Or equivalently, find $\lim\limits_{x\to\infty} \ln\left(1 + \dfrac{4}{x}\right)^x = \lim\limits_{x\to\infty} x \ln\left(1 + \dfrac{4}{x}\right) = \lim\limits_{x\to\infty} \dfrac{\ln(1 + \frac{4}{x})}{1/x} = 4$ by L'Hosptial's Rule and then use the fact $b = e^{\ln b}$ for any positive number $b$ and the corresponding limit law.