Solutions

Homework #[01a]

① { How many significant figures are in each of these numbers? ⓐ 214 ⓑ 81.60 ⓒ 7.03 ⓓ 0.03 ⓔ 0.0086 ⓕ 3236 ⓖ 8700 }

ⓐ 3 sigfigs by rule 1

ⓑ 4 sigfigs by rules 1 and 3

ⓒ 3 sigfigs by rules 1 and 2

ⓓ 1 sigfig by rules 1 and 5

ⓔ 2 sigfigs by rules 1 and 5

ⓕ 4 sigfigs by rule 1

ⓖ 2 sigfigs by rules 1 and 4b

② { Write the following numbers in scientific notation: ⓐ 1.156 ⓑ 21.8 ⓒ 0.0068 ⓓ 328.65 ⓔ 0.219 ⓕ 444 }

ⓐ 1.156

ⓑ 2.18×10^{1}

ⓒ 6.8×10^{-3}

ⓓ 3.2865×10^{2}

ⓔ 2.19×10^{-1}

ⓕ 4.44×10^{2}

③ { What is the percent uncertainty in the measurement 5.48 ± 0.25 [m]? }

$m_{meas} = V \pm u$; $V = 5.48$ [m] , $u = 0.25$ [m] (1)

• so percent uncertainty is:

$$\boxed{③ \quad u_p(m_{meas}) \equiv \frac{u}{V} \times 100\,\% = \frac{0.25\,\cancel{[m]}}{5.48\,\cancel{[m]}} \times 100\% = 4.6\,\%} \qquad (2)$$

④ { Write the following numbers in both scientific notation in S.I. units and as full decimal numbers without prefixes or power-of-10 notation. ⓐ 286.6 [mm] ⓑ 85 [μV] ⓒ 760 [mg] ⓓ 62.1 [ps] ⓔ 22.5 [nm] ⓕ 2.50 [GV] }

286.6×10^{-3} [m]

ⓐ 2.866×10^{-1} [m]
0.2866 [m]

85×10^{-6} [V]

ⓑ 8.5×10^{-5} [V]
0.000085 [V]

760×10^{-3} [g]

$7.6 \times 10^{-1}\,\cancel{[g]} \cdot \left(\frac{1\,[kg]}{1000\,\cancel{[g]}}\right)$

ⓒ 7.6×10^{-4} [kg]
0.00076 [kg]

62.1×10^{-12} [s]

ⓓ 6.21×10^{-11} [s]
$0.\underbrace{0000000000}_{10\text{ zeros}}621$ [s]

22.5×10^{-9} [m]

ⓔ 2.25×10^{-8} [m]
0.0000000225 [m]

ⓕ 2.50×10^{9} [V]
2,500,000,000 [V]

5 { Determine the conversion factor between ⓐ $\left[\frac{km}{h}\right]$ and $\left[\frac{mi}{h}\right]$? ⓑ $\left[\frac{m}{s}\right]$ and $\left[\frac{ft}{s}\right]$ ⓒ $\left[\frac{km}{h}\right]$ and $\left[\frac{m}{s}\right]$. }

a: • We need a conversion equation relating length in [km] and length in [mi]. From book's inside cover:

$$1\,[km] = 0.6214\,[mi] \tag{1}$$

• To get a conversion factor that transforms $\left[\frac{km}{h}\right]$ to $\left[\frac{mi}{h}\right]$, we want to multiply $\left[\frac{km}{h}\right]$ by something with [km] in the denominator (to cancel it), so divide both sides of (1) by 1 [km]:

$$1 = \frac{0.6214\,[mi]}{1\,[km]} \tag{2}$$

• Now we're done, but we could also use

$$\text{a}\quad \boxed{1 = 0.6214\left[\frac{mi}{km}\right]\cdot\frac{\frac{1}{[h]}}{\frac{1}{[h]}} = 0.6214\,\frac{\left[\frac{mi}{h}\right]}{\left[\frac{km}{h}\right]}} \tag{3}$$

check:

$$\cancel{\left[\frac{km}{h}\right]}\cdot 0.6214\,\frac{\left[\frac{mi}{h}\right]}{\cancel{\left[\frac{km}{h}\right]}} = 0.6214\left[\frac{mi}{h}\right] \checkmark \tag{4}$$

b: • For $\left[\frac{m}{s}\right]$ to $\left[\frac{ft}{s}\right]$, start with conversion equation:

$$1\,[m] = 3.281\,[ft] \tag{5}$$

• to cancel [m], get it on bottom:

$$1 = \frac{3.281\,[ft]}{1\,[m]} \tag{6}$$

• again, can rewrite as

$$\text{b}\quad \boxed{3.281\left[\frac{ft}{m}\right]\cdot\frac{\left[\frac{1}{s}\right]}{\left[\frac{1}{s}\right]} = 3.281\,\frac{\left[\frac{ft}{s}\right]}{\left[\frac{m}{s}\right]}} \tag{7}$$

c: • For $\left[\frac{km}{h}\right]$ to $\left[\frac{m}{s}\right]$, first use the prefix definition to for length:

$$1\,[km] \equiv 1000\,[m] \tag{8}$$

• want to cancel [km], so put it on bottom in (8):

$$1 = \frac{1000\,[m]}{1\,[km]} \tag{9}$$

• For time, use definition of hour:

$$1\,[h] \equiv 60\,[min] \tag{10}$$

• get [h] on top to cancel it in $\left[\frac{km}{h}\right]$:

$$\frac{1\,[h]}{60\,[min]} = 1 \tag{11}$$

• use definition of minute:

$$1\,[min] \equiv 60\,[s] \tag{12}$$

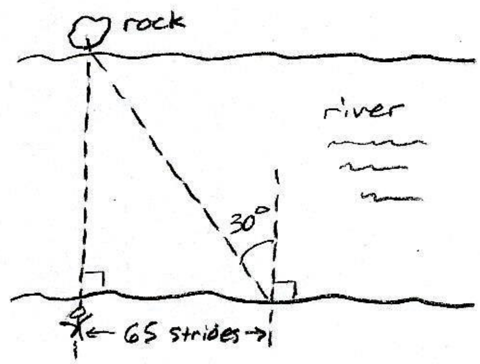
• get [min] on top:

$$\frac{1\,[min]}{60\,[s]} = 1 \tag{13}$$

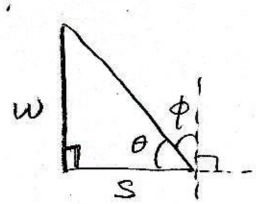
• Now combine all three conversion factors, from (9), (11), and (13):

$$\text{c}\quad \boxed{\begin{aligned} 1\cdot1\cdot1 &= \left(\frac{1000\,[m]}{1\,[km]}\right)\cdot\left(\frac{1\,[h]}{60\,\cancel{[min]}}\right)\cdot\left(\frac{1\,\cancel{[min]}}{60\,[s]}\right) = \frac{1000\,[m]}{1\,[km]}\cdot\frac{1\,[h]}{3600\,[s]} \\ \text{or}\quad 1 &= 0.278\,\frac{\left[\frac{m}{s}\right]}{\left[\frac{km}{h}\right]} \end{aligned}} \tag{14}$$

(6) Jim stands beside a wide river and wonders how wide it is. He spots a large rock on the far bank directly across from him. He then walks upstream 65 strides and judges that the angle between him and the rock is 30° downstream from his new position if he faces straight out at the water. If the river's banks are approximately parallel, and if Jim measures his stride to be about 0.8 [m] long, how wide is the river?



- The triangle this forms is:



(1)

where

$$\phi = 30^\circ \qquad (2)$$

- so by geometry:

$$\theta = 90^\circ - \phi = 60^\circ \qquad (3)$$

- And we know:

$$s = 65\,[\cancel{\text{strides}}] \cdot \left(\frac{0.8\,[\text{m}]}{1\,[\cancel{\text{stride}}]}\right) = 52\,[\text{m}] \qquad (4)$$

- we want width w, and by trigonometry:

$$\tan(\theta) = \frac{w}{s} \qquad (5)$$

- solve for w:

$$\boxed{\begin{aligned} w = s \cdot \tan(\theta) &= (52\,[\text{m}]) \cdot \tan(60^\circ) \\ &= 52\,[\text{m}] \cdot 1.732 \\ &\approx 90\,[\text{m}] \end{aligned}}$$

(7) {If you are driving 95 [km/h] along a straight road and you look to the side for 2.0 [s], how far do you travel forward on the road during this inattentive period?}

- speed is constant here, so we just use average speed:

$$\bar{s} = \frac{D}{\Delta t} \qquad (1)$$

- Given

$$\bar{s} = 95\left[\frac{km}{h}\right], \quad \Delta t = 2.0\,[s] \qquad (2)$$

- Want "how far", so D; solve (1) for D:

$$\boxed{\begin{aligned} D = \bar{s}\Delta t &= \left(95\left[\frac{km}{\cancel{h}}\right]\right)\left(2.0\,[\cancel{s}]\right)\cdot\left(\frac{1\,[\cancel{h}]}{3600\,[\cancel{s}]}\right) \\ &= 0.053\,[km]\cdot\left(\frac{1000\,[m]}{1\,[km]}\right) = 53\,[m] \end{aligned}} \qquad (3)$$

(8) {At highway speeds, a particular car can accelerate at $1.8\left[\frac{m}{s^2}\right]$. At this rate, how long does it take to accelerate from $65\left[\frac{km}{h}\right]$ to $120\left[\frac{km}{h}\right]$?}

- Given acceleration: $a = 1.8\left[\frac{m}{s^2}\right]$ (1)
- Given initial and final velocities:

(using extra digits in intermediate steps)

$$V_i = 65\left[\frac{\cancel{km}}{\cancel{h}}\right]\cdot\left(\frac{1\,[\cancel{h}]}{3600\,[s]}\right)\left(\frac{1000\,[m]}{1\,[\cancel{km}]}\right) = 18.056\left[\frac{m}{s}\right] \qquad (2)$$

$$V_f = 120\left[\frac{\cancel{km}}{\cancel{h}}\right]\cdot\left(\frac{1000}{3600}\left[\frac{\cancel{h}\cdot m}{s\cdot\cancel{km}}\right]\right) = 33.333\left[\frac{m}{s}\right] \qquad (3)$$

- Assuming constant acceleration, $a = \bar{a}$, so

$$a = \bar{a} = \frac{\Delta V}{\Delta t} = \frac{V_f - V_i}{\Delta t} \qquad (4)$$

- solve (4) for Δt:

$$\boxed{\Delta t = \frac{V_f - V_i}{a} = \frac{(33.333 - 18.056)\left[\frac{m}{\cancel{s}}\right]}{1.8\left[\frac{m}{s^{\cancel{2}}}\right]} = 8.5\,[s]} \qquad (5)$$

(9) {A car slows down from $28\left[\frac{m}{s}\right]$ to rest in a distance of 88 [m]. What was its acceleration, assumed constant?}

- "slows down" ⟹ velocity changed ⟹ acceleration
- Given V_0, V, ΔX, want a:

$$V_0 = 28\left[\frac{m}{s}\right], \quad V = \underset{\text{"rest"}}{0.0}\left[\frac{m}{s}\right], \quad \Delta X = X - X_0 = 88\,[m] \qquad (1)$$

- (17c) from lecture has these variables, so solve it for a:

$$V^2 = V_0^2 + 2a(X - X_0) \qquad (2)$$

$$V^2 - V_0^2 = 2a\Delta X$$

$$\boxed{\begin{aligned} a = \frac{V^2 - V_0^2}{2\Delta X} &= \frac{\left(0.0\left[\frac{m}{s}\right]\right)^2 - \left(28\left[\frac{m}{s}\right]\right)^2}{2\cdot(88\,[m])} = -4.\overline{45}\left[\frac{m}{s^2}\right] \\ &\approx -4.4\left[\frac{m}{s^2}\right] \end{aligned}} \qquad (3)$$

following strict rounding rules, since 4 is even

(10) Determine the stopping distances for a car going at a constant initial speed of 95 [km/h] and human reaction time of 0.40 [s] for (a) an acceleration $a = -3.0 \left[\frac{m}{s^2}\right]$ (b) $a = -6.0 \left[\frac{m}{s^2}\right]$

- is a two-part problem:
  1. How far does the car go before the reaction happens?
  2. How far does it take to stop after reaction causes braking?
- For 1, reaction distance can be found from $\bar{s}$ and Δt:
  - Givens:

$$\bar{s} = 95 \left[\frac{km}{h}\right] \cdot \left(\frac{1000 [m]}{1 [km]}\right) \cdot \left(\frac{1 [h]}{3600 [s]}\right) = 26.39 \left[\frac{m}{s}\right] \quad (1)$$

$$\Delta t = 0.40 [s] \quad (2)$$

  - equation:, solve for D:

$$\bar{s} \equiv \frac{D}{\Delta t} \quad (3)$$

$$D = \bar{s} \Delta t = \left(26.39 \left[\frac{m}{s}\right]\right)(0.40 [s]) = 10.56 [m] \quad (4)$$

- For part 2, the starting position is D:

$$x_0 = D = 10.56 [m] \quad (5)$$

  - Givens:

$$v = 0.0 \left[\frac{m}{s}\right] \quad (6)$$

$$v_0 = \bar{s} = 26.39 \left[\frac{m}{s}\right] \quad (7)$$

$$a = \begin{cases} -3.0 \left[\frac{m}{s^2}\right] & ; (a) \\ -6.0 \left[\frac{m}{s^2}\right] & ; (b) \end{cases} \quad (8)$$

$$x_0 = D = 10.56 [m] \quad (9)$$

and want x (10)

- so, since (17c) has these, solve it for x:

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$v^2 - v_0^2 = 2a(x - x_0)$$

$$\frac{v^2 - v_0^2}{2a} = x - x_0$$

$$x = x_0 + \frac{v^2 - v_0^2}{2a} \quad (11)$$

- so

(a) when $a = -3.0 \left[\frac{m}{s^2}\right]$,

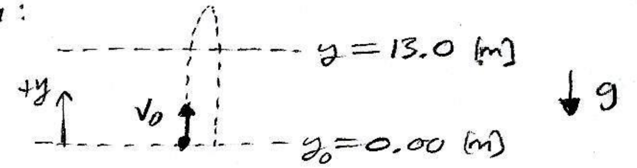
$$\boxed{x = x_0 + \frac{v^2 - v_0^2}{2a} = 126.6 [m] \approx 130 [m]} \quad (12)$$

(b) when $a = -6.0 \left[\frac{m}{s^2}\right]$,

$$\boxed{x = x_0 + \frac{v^2 - v_0^2}{2a} = 68.60 [m] \approx 69 [m]} \quad (13)$$

(11) A stone is thrown vertically upward with a speed of $24.0\ [\frac{m}{s}]$.
(a) How fast is it moving when it is at a height of $13.0\ [m]$?
(b) How much time is required to reach this height?
(c) Why are there two answers to (b)?

- Diagram:



- a: Givens:

$$y_0 = 0.00\ [m] \quad (1)$$
$$y = 13.0\ [m] \quad (2)$$
$$a = -g = -9.8\ [\tfrac{m}{s^2}] \quad (3)$$
$$v_0 = 24.0\ [\tfrac{m}{s}] \quad (4)$$

want v

- so solve (17c) from Lecture for v:

$$v^2 = v_0^2 + 2a(y - y_0)$$

(a)
$$\boxed{v_\pm = \pm\sqrt{v_0^2 + 2a(y-y_0)} = \pm\ 17.9\ [\tfrac{m}{s}]} \quad (5)$$

- so its speed is $s = |v_\pm| = 17.9\ [\frac{m}{s}]$
- two answers here are v_+ for velocity at $y = 13.0\ [m]$ on trip upwards, and v_- for velocity at $y = 13.0\ [m]$ on trip downwards.

- b: For the time, we have y_0, y, a, v_0, v, want t, so solve (17b) for t:

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2 \quad (6)$$
$$\tfrac{1}{2}at^2 + v_0 t + (y_0 - y) = 0 \quad (7)$$

(b)
$$t_\pm = \frac{-v_0 \pm \sqrt{v_0^2 - 4\left(\frac{a}{2}\right)(y_0 - y)}}{2 \cdot \frac{1}{2}a} \quad (8)$$

$$t_+ = 0.620\ [s]$$
$$t_- = 4.28\ [s]$$

(c) t_+ corresponds to the time when the stone reaches $y = 13.0\ [m]$ on its upward trip,

t_- is the time when the stone reaches $y = 13.0\ [m]$ on its downward trip

There are two times because (6) is a 2^{nd}-order polynomial in t, so it has two roots.

Note, both times are "physical"; not every situation with multiple solutions has some that are nonphysical.

(12) The acceleration due to gravity on the Moon is about one-sixth what it is on Earth. If an object is thrown vertically upward on the Moon, how many times higher will it go than it would on Earth, assuming the same initial velocity?

- 1 problem, 2 "planets". Base problem, irrespective of planet:
  - Given v_0, v, a, want $y-y_0$ (1)
  - Use (17c):
$$v^2 = v_0^2 + 2a(y-y_0) \tag{2}$$
  - use $\Delta y = y - y_0$, and $v = 0$ (at peak of trajectory),
$$0 = v_0^2 + 2a\Delta y \tag{3}$$
  - solve for Δy:
$$\Delta y = -\frac{v_0^2}{2a} \tag{4}$$
- On Earth:
$$a_\oplus = -g \tag{5}$$
so
$$\Delta y_\oplus = \frac{-v_0^2}{2(-g)} = \frac{v_0^2}{2g} \tag{6}$$
- On Moon:
$$a_☾ = -\frac{g}{6} \tag{7}$$
so
$$\Delta y_☾ = \frac{-v_0^2}{2\left(-\frac{g}{6}\right)} = 3\frac{v_0^2}{g} \tag{8}$$
- So the ratio of displacement "on" the Moon to displacement "on" Earth is:

$$\boxed{\frac{\Delta y_☾}{\Delta y_\oplus} = \frac{3v_0^2}{g}\cdot\frac{1}{\left(\frac{v_0^2}{2g}\right)} = \frac{3\cancel{v_0^2}}{\cancel{g}}\frac{2\cancel{g}}{\cancel{v_0^2}} = 6}$$

So the object travels 6 times farther upward "on" the Moon than "on" Earth

(13) Suppose the position as a function of time for an object is known to be:

$$x(t) = C + A\cos(\omega t + \theta_0) + B(e^{-\alpha t} - 1) \qquad (i)$$

(a) what is the instantaneous velocity of this object as a function of t, if $C, A, \omega, \theta_0, B$, and α are all constant in time?

(b) what is an expression for the initial position at $t=0$, meaning $x_0 \equiv x(0)$?

(c) what is the initial velocity $v_0 \equiv v(0)$?

- Instantaneous velocity is the time derivative of position:

$$\begin{aligned} v \equiv \frac{d}{dt}x(t) &= \frac{d}{dt}\left(C + A\cos(\omega t + \theta_0) + B(e^{-\alpha t} - 1)\right) \\ &= \frac{dC}{dt} + A\frac{d\cos(\omega t+\theta_0)}{dt} + B\left(\frac{de^{-\alpha t}}{dt} - \frac{d1}{dt}\right) \\ &= 0 + A\left[-\sin(\omega t+\theta_0)\cdot\omega\right] + B\left[e^{-\alpha t}\cdot(-\alpha) - 0\right] \\ &= -A\omega\sin(\omega t+\theta_0) - B\alpha e^{-\alpha t} \end{aligned} \qquad (1)$$

- so

(a) $$\boxed{v(t) = -\left[A\omega\sin(\omega t+\theta_0) + B\alpha e^{-\alpha t}\right]} \qquad (2)$$

- For x_0: from (i):

$$\begin{aligned} x_0 \equiv x(0) &= C + A\cos(\omega\cdot 0 + \theta_0) + B(e^{-\alpha 0} - 1) \\ &= C + A\cos(\theta_0) + B(1-1) \\ &= C + A\cos(\theta_0) \end{aligned}$$

- so

(b) $$\boxed{x_0 \equiv x(0) = C + A\cos(\theta_0)} \qquad (3)$$

- For v_0: from (2):

$$\begin{aligned} v_0 \equiv v(0) &= -\left[A\omega\sin(\omega 0 + \theta_0) + B\alpha e^{-\alpha 0}\right] \\ &= -\left[A\omega\sin(\theta_0) + B\alpha\right] \end{aligned} \qquad (4)$$

- so

(c) $$\boxed{v_0 \equiv v(0) = -\left[A\omega\sin(\theta_0) + B\alpha\right]} \qquad (5)$$