Hw_3_Samir_Banjara

Pledge: I pledge my honor that I have abided by the Stevens Honor System. Signature: Samir Banjara

Problem 1:

A) Find the probability of rolling two dice and not getting doubles.

Solution: Given the set of possibilities of rolling two dice,

$$s = \begin{matrix} \{(1,1),(2,1),(3,1),(4,1),(5,1),(6,1)\} \\ \{(1,2),(2,2),(3,2),(4,2),(5,2),(6,2)\} \\ \{(1,3),(2,3),(3,3),(4,3),(5,3),(6,3)\} \\ \{(1,4),(2,4),(3,4),(4,4),(5,4),(6,4)\} \\ \{(1,5),(2,5),(3,5),(4,5),(5,5),(6,5)\} \\ \{(1,6),(2,6),(3,6),(4,6),(5,6),(6,6)\} \end{matrix}$$

30 rolls out of 36 are not an ordered double. Thus the probability is $\frac{30}{36}$.

B) Given that every fifth person in line will get a coupon for a free box of popcorn at the movies, what is the probability that you don't get a coupon when you're in line?

Solution: Every 5^{th} person in line gets a coupon. That means 1 person out of 5 gets a coupon. The probability of not getting a coupon is then $1 - \frac{1}{5} = \frac{4}{5}$

Problem 2:

Insulin pens used to administer a patient's insulin at hospitals have a malfunction rate of 9%. This means that out of a box of 200 pens, 18 are defective in some way and must be thrown away. Find the probability of randomly selecting 3 defective insulin pens in a row from a brand-new box of 200 pens, if a defective pen is immediately discarded.

Solution: Probability of randomly selecting a defective pen out of a brand new box of 200 is $\frac{18}{200}$. And since we are discarding the pen, the probability of picking another defective pen is $\left(\frac{18-1}{200-1}\right) = \left(\frac{17}{199}\right)$. And the probability that the third pen will be defective is $\left(\frac{16}{198}\right)$. Then the probability that each pen picked up in a row that will be defective is,

$$\left(\frac{18}{200}\right)\left(\frac{17}{199}\right)\left(\frac{16}{198}\right) = 0.000621 \times 100\% = 0.0621\%$$

Problem 3:

The following table displays the breakdown of attendees at an International Biology conference by country and their role in the company they were representing.

	Canada	France	South Korea	UK	US
CEO	138	45	4	19	117
Director	8	4	25	6	63
Partner	23	7	3	20	103
Chairman	12	9	3	9	62
Other	112	146	154	143	2103

A random attendee is selected for an interview.

a) What is the probability that a Partner is selected, given that the attendee is from South Korea?

Solution: Given two events A and B, the conditional probability of A given B is given by $P(A|B) = \frac{P(A \cap B)}{P(B)}$. Given there is $A = 3$ Partners who are Korean, and $B = 189$ total Korea participants, thus, the probability is given by,

$$\frac{3}{189} = \frac{1}{63}$$

b) What is the probability that a Canadian is selected, given that the attendee is a director of the company?

Solution: Given there is $A = 8$ Canadian's who are Directors, and $B = 106$ total participants who are directors, thus, the probability is given by,

$$\frac{8}{106}$$

c) What is the probability that a director is selected, given that the attendee is Canadian?

Solution: Given there is $A = 8$ Canadian's who are Directors, and $B = 293$ total participants who are Canadian, thus, the probability is given by,

$$\frac{8}{293}$$

d) What is the probability that a CEO is selected, given that the attendee is from the continent of North America?

Solution: The total number of North American CEO's is 255 and the total number of North American Attendants is 2741, the probability that they are a CEO given that they are from North America is $\frac{255}{2741}$

Problem 4:

Every 6 months, university email requires that a new 5-igit password be set up. No digits are allowed to be repeated and it must be different from your last two passwords. If you let your computer randomly choose a 5-digit code for you with no repeating digits, what is the probability that it will choose one of the last 2 passwords you've had? Round your answer to five decimal places.

Solution: Given that we have to make a 5-digit password without repeating digits from the set of $\{0,1,2,3,4,5,6,7,8,9\}$. We can calculate how many digits can be a possible input in each index, until we are left with a 5 digit number.

$(10)_{n=0}$ digits can take the 0-th place. Tossing one out. Then $(9)_{n=1}$ digits can take the 1-st place. Then $(8)_{n=2}$ digits can take the 2-st place. Then $(7)_{n=3}$ digits can take the 3-st place. And finally, $(6)_{n=4}$ digits can take the 4-st place.

So the total number of possible passwords we could have is, $10 \times 9 \times 8 \times 7 \times 6 = 30240$.

The probability that 2 of the computer generated passwords are their previous password 2 password is $\frac{2}{30240} = 0.00007$

Problem 5:

Virginia's Veggie Café offers 5 types of homemade bread, 10 toppings, and 6 different condiments. How many different super sandwiches can be made if a super sandwich consists of 6 different toppings and 2 different condiments?

Solution: There are only a total of $\mathcal{C}_{1,5} = 5$ ways to combine the breads. Out of 10 toppings, the super sandwich has only 6, so there are $\mathcal{C}_{6,10} = 210$ combinations of toppings. And $\mathcal{C}_{2,6} = 15$ possible ways of combining the condiments. Thus the total number of sandwich combinations are $5 \times 210 \times 15 = 15,750$.

Problem 6:

Ashley’s Internet service is terribly unreliable. In fact, on any given day, there is a 15% chance that her Internet connection will be lost at some point that day. What is the probability that her Internet service is not broken for five days in a row?

Solution: Given there is a 15% chance that her wifi doesn't work, there is a 85% chance that is does work on a given day.
$P(Works) = 1 - 0.15 = 0.85$

$$(0.85) \times (0.85) \times (0.85) \times (0.85)\,(0.85) = (0.85)^5$$

Problem 7:

Because Tristan has diabetes, he must make sure that his daily diet includes 2 vegetables, 3 fruits, and 2 breads. At the grocery store, he has a choice of 20 vegetables, 8 fruits, and 5 breads.

a) In how many ways can he make up his daily requirements if he doesn’t like to eat 2 helpings of the same thing in one day?

Solution: Combinations of each staple in their diet:

- Vegetables: $\mathcal{C}_{2,20} = \dfrac{20!}{2!(20-2)!} = 56$
- Fruits: $\mathcal{C}_{3,8} = \dfrac{8!}{3!(8-3)!} = 190$
- Breads: $\mathcal{C}_{2,5} = \dfrac{5!}{2!(5-2)!} = 10$ So the number of combinations for his daily requirement without have the same helpings of the same thing is $106,400$

b) What's the probability that a random choice from his possibilities would yield either carrots or spinach in its menu, given that carrots and spinach are both vegetable choices at the grocery store?

Solution: Well we know that there is total number of 190 combinations for his veggies.

We can calculate the number of combinations of of 20 veggies.

Number of ways to choose 2 veggies which must include carrots: $\mathcal{C}(19,1) = 19$

Number of ways to choose 2 veggies which must include Spinach: $\mathcal{C}(19,1) = 19$

Number of way of choosing 2 veggies that are from carrots and Spinach $\mathcal{2},\mathcal{2} = 1$

Thus using *Inclusion-Exclusion Principle*, the number of vegetable combinations including Carrots or Spinach is,

$$P(A) + P(B) - P(A \cap B) = 19 + 19 - 1 = 37$$

Total combinations of daily serving that have carrots or vegetables, is $37 \times 56 \times 10 = 20720$.

Probability that his meal will have either carrots or veggies is $\dfrac{20720}{106400} \approx 0.1947 \times 100\% = 19.47\%$