



SECTION 2.2

AXIOMS, INTERPRETATIONS,
AND PROPERTIES OF
PROBABILITY



Objectives

- Learn the three axioms and properties of probability.
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Axioms of Probability

1) For any event A , $P(A) \geq 0$.

2) $P(S) = 1$.

3) If A_1, A_2, A_3, \dots is an infinite collection of disjoint events, then

$$P(A_1 \cup A_2 \cup A_3 \cup \dots) = \sum P(A_i)$$

Note: In our course we learn the **classical probability** (or *theoretical probability*) where all possible outcomes in the sample space are known and **equally likely** to occur.

Equally Likely Outcomes

- In our course we learn the **classical probability** (or *theoretical probability*) where all possible outcomes in the sample space are known and **equally likely** to occur.
- If a sample space contain N equally likely outcomes, then the probability for each outcome is $\frac{1}{N}$.

Properties of Probability

1) For any event A , $P(A) + P(A') = 1$. Thus $P(A) = 1 - P(A')$.

2) For any event A , $P(A) \leq 1$.

3) For any two events A and B ,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

4) For any three events A , B , and C ,

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

Example:

- 1) If you are worried that there is a 35% chance that you will fail your upcoming test, what is the probability that you will pass the test?
- 2) If there is a 5% chance that none of the items on a scratch-off lottery ticket will be a winner, what is the probability that at least one of the scratch-off items will win?
- 3) Roll a pair of standard six-sided dice. What is the probability that neither die is a three?
- 4) A wallet contains five \$10 bills, four \$5 bills, and six \$1 bills (nothing larger). If the bills are selected one by one in random order, what is the probability that at least two bills must be selected to obtain a first \$10 bill?

Example:

Ceresa is looking for a new condo to rent. Ceresa's realtor has provided her with the following list of amenities for available properties. The list contains the following.

- Close to the subway: 6 properties
- Low maintenance fee: 7 properties
- Green space: 5 properties
- Newly renovated: 2 properties
- Close to the subway and low maintenance fee: 2 properties
- Green space and newly renovated: 1 property

If Ceresa's realtor selects the first condo they visit at random, what is the probability that the property is either close to the subway or has a low maintenance fee?

Example:

A certain factory operates three different shifts. Over the last year, 200 accidents have occurred at the factory. Some of these can be attributed at least in part to unsafe working conditions, whereas the others are unrelated to working conditions. The accompanying table gives the percentage of accidents falling in each type of accident- shift category.

Suppose one of the 200 accident reports is randomly selected from a file of reports, and the shift and type of accident are determined.

- a) What are the simple events?
- b) What is the probability that the selected accident was attributed to unsafe conditions?
- c) What is the probability that the selected accident did not occur on the day shift?

		Unsafe Conditions	Unrelated to Conditions
Shift	Day	10%	35%
	Swing	8%	20%
	Night	5%	22%

Example:

Suppose that after a vote in the US senate on a proposed health care bill, the following table shows the breakdown of the votes by party.

Votes on Health Care Bill		
	Voted in Favor	Voted Against
Democrat	23	21
Republican	43	7
Independent	2	4

If a lobbyist stops a random senator after the vote, what is the probability that this senator will either be a Republican or have voted against the bill?

Practice Problems:

1) For a story she is writing in her high school newspaper, Grace surveys moviegoers selected at random as they leave the new feature *Mystery on Juniper Island*. She simply asks each moviegoer to rate the show using a thumbs-up or thumbs-down. The results of her survey are given in the table below.

Survey Results		
	Under 40 years old	40 years or older
Thumbs-up	23	40
Thumbs-down	19	11

What is the probability that one of Grace's survey respondents has either given a thumbs-up rating or is under 40 years old?

2) Roll a pair of dice. What is the probability of rolling either a total less than four or a total equal to ten?