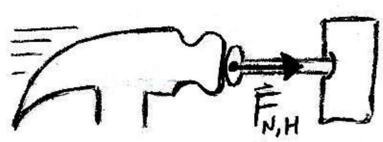
Newton's Third Law:

> If one object exerts a force on a second object, the second object exerts an equal force in the opposite direction on the first object. (s.t. both forces lie in same line)

("strong form" of Newton's 3rd Law)

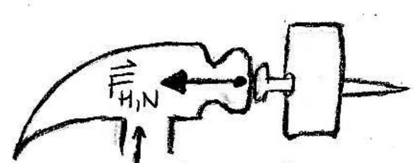
• **ex**: if a hammer hits a nail,

(the nail moves due to the force of the hammer on it)



$\vec{F}_{N,H}$

(force on Nail due to Hammer)



$\vec{F}_{H,N}$

(force on Hammer due to Nail)

(the hammer is decelerated to rest by the force of the nail on the hammer)

• at all times, we have:

(13)

• Expressed quantitatively:

$$\vec{F}_{H,N} = -\vec{F}_{N,H}. \tag{14}$$

• In general, for objects 1 and 2:

$$\boxed{\vec{F}_{2,1} = -\vec{F}_{1,2}} \quad \leftarrow \text{(Newton's 3rd Law (strong form))} \tag{15}$$

• CAUTION:

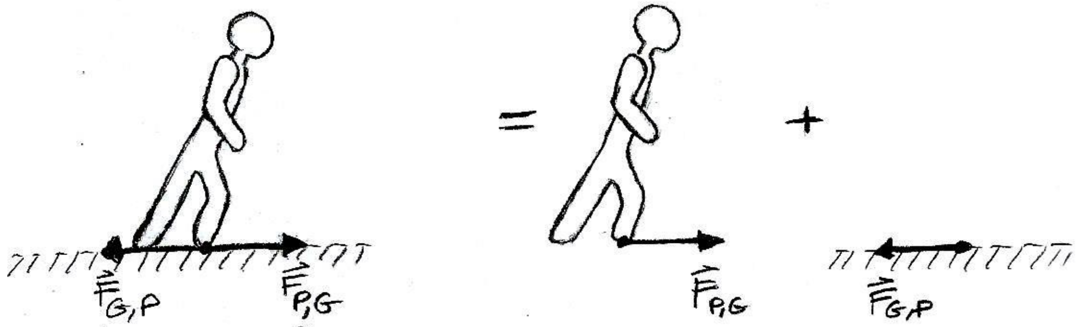
> Newton's-3rd-law force pairs $\vec{F}_{2,1}$, $\vec{F}_{1,2}$ act on different objects.

(see (13) for example)

• This is important because $\Sigma\vec{F}$ in Newton's 2nd law only refers to forces acting on one object (the object whose mass is m in $\Sigma\vec{F} = m\vec{a}$)

• Note: • if diagram is clear, we can omit double subscripts,
• can use one subscript to label what caused the force.

• **ex** Person walking on Ground:



(notice we don't use a negative sign because we've drawn the vectors pointing in opposite directions)

(Force on Ground due to Person)

(Force on Person due to Ground)

• But it is still true that $\vec{F}_{G,P} = -\vec{F}_{P,G}$

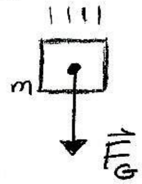
• The diagram means the negative sign is there

• each object can be evaluated in terms of external forces acting on it

• Drawn separately, we could abbreviate as:
$\vec{F}_G \equiv \vec{F}_{P,G}$ and $\vec{F}_P \equiv \vec{F}_{G,P}$

Weight: $\equiv$ $\boxed{\text{Force on an object due to gravity}}$

- Imagine an object <u>falling</u> near Earth's surface: (free-fall):



$$\downarrow \vec{a} = \vec{g} \quad ; \quad g \equiv |\vec{g}| \approx 9.80 \left[\tfrac{m}{s^2}\right] \tag{16}$$

- Newton's 2nd Law: for the falling object

$$\Sigma \vec{F} = m\vec{a} \tag{17}$$

- Only <u>1 force</u> acting: $\longrightarrow$

$$\vec{F}_G = m\vec{a} \tag{18}$$

- And from (16) $\vec{a} = \vec{g}$:

$$\boxed{\vec{F}_G = m\vec{g}} \quad \text{(Force due to } \underline{\text{Gravity}} \text{ on an object of mass m)} \tag{19}$$

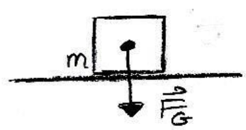
  - <u>Direction</u> of $\vec{F}_G$ is toward the center of the Earth
  - <u>Magnitude</u> of $\vec{F}_G$ is

$$\boxed{|\vec{F}_G| = mg} \quad \text{(The object's "weight" on/near Earth's surface)} \tag{20}$$

- <u>CAUTION</u>:
  - object does not have to be falling to have $\vec{F}_G$ acting on it
  - falling let us <u>isolate</u> the mass all except for $\vec{F}_G$

The Normal Force:

- Imagine an object <u>at rest</u> on the ground on Earth:



- gravity still acts on object, so it still feels

$$\vec{F}_G = m\vec{g} \tag{21}$$

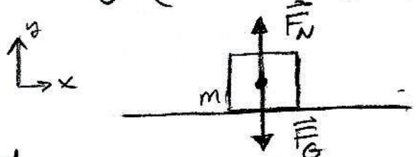
- But since its velocity is constant ($\vec{v}_i = \vec{0}$, $\vec{v}_f = \vec{0}$), it has no acceleration!

$$\vec{a} = \vec{0} \tag{22}$$

- There must be at least <u>one more force</u> that opposes $\vec{F}_G$ to cause $\vec{a} = 0$.
- We call it:

$$\boxed{\vec{F}_N \equiv \text{the normal force}} \tag{23}$$

- $\vec{F}_N$ is a <u>contact force</u>, acting <u>perpendicularly</u> (the meaning of "normal") to the common surface of contact.



(the normal force $\vec{F}_N$ is exerted by the ground on the object) (24)

- Apply Newton's 2nd law to the object:

$$\Sigma \vec{F} = m\vec{a} \tag{25}$$

$$\vec{F}_N + \vec{F}_G = \vec{0} \quad (\vec{a} = \vec{0})$$

- Solve for $\vec{F}_N$, and use fact: $\vec{F}_G = |\vec{F}_G|(-\hat{y}) = mg(-\hat{y}) \rightarrow$

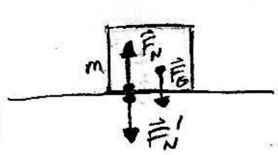
$$\boxed{\vec{F}_N = -\vec{F}_G = -\left[(mg)(-\hat{y})\right] = mg\hat{y}} \tag{26}$$

- Check:

$$\Sigma \vec{F} = \vec{F}_N + \vec{F}_G = +mg\hat{y} - mg\hat{y} = \vec{0} \checkmark \quad \text{since } m\vec{a} = \vec{0} \tag{27}$$

- <u>CAUTION</u>:
  - $\vec{F}_N$ and $\vec{F}_G$ are <u>not</u> a 3rd law pair since they act on <u>same object</u>
  - 3rd law pair is $\vec{F}_N$ acting on box, and $\vec{F}_N'$ acting on ground, due to box:

(shifted forces for visibility)



($\vec{F}_N$ and $\vec{F}_N'$ are both normal forces)

- By <u>3rd law</u>,

$$\vec{F}_N' = -\vec{F}_N \tag{28}$$

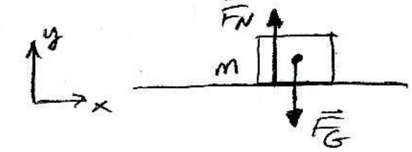
- So, putting (26) into (28):

$$\boxed{\vec{F}_N' = -mg\hat{y}} \tag{29}$$

ex: A box of mass 10.0 [kg] sits on a table on Earth.
(a) What is the weight of the box and the normal force exerted on it by the table?
(b) If you push down on the box with a force of 40.0 [N], what is the normal force on the box due to the table?
(c) If you pull upward on the box with a force of 40.0 [N], what is the normal force on the box due to the table?
In all cases here, suppose the box remained at rest.

a: • diagram:

(we show forces shifted for visibility)



• Force due to gravity ("weight force"):

$$\vec{F}_G = -mg\,\hat{y} \qquad (1)$$

• acceleration of box at rest:

$$\vec{a} = \vec{0} \qquad (2)$$

• Newton's 2^{nd} law:

$$\sum \vec{F} = m\vec{a} \qquad (3)$$

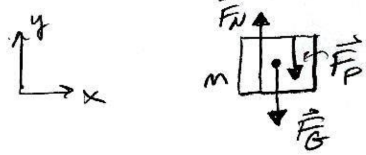
$$\vec{F}_N + \vec{F}_G = \vec{0}$$

$$\vec{F}_N = -\vec{F}_G = +mg\,\hat{y} \qquad (4)$$

(a)

$$\boxed{\begin{aligned} &|\vec{F}_G| = mg = (10.0\,[\text{kg}])\left(9.80\left[\tfrac{\text{m}}{\text{s}^2}\right]\right) = 98.0\,[\text{N}] = \left(\begin{smallmatrix}\text{weight}\\ \text{of box}\end{smallmatrix}\right) \quad (5)\\ \text{and}\quad &\vec{F}_N = (98.0\,[\text{N}])\,\hat{y} = \left(\begin{smallmatrix}\text{normal force on box}\\ \text{due to table}\end{smallmatrix}\right) \quad (6)\end{aligned}}$$

b: • diagram:



• weight force is still:

$$\vec{F}_G = -mg\,\hat{y} \qquad (7)$$

• still at rest:

$$\vec{a} = \vec{0} \qquad (8)$$

• Given push:

$$\vec{F}_P = |\vec{F}_P|(-\hat{y})\;;\; |\vec{F}_P| \equiv 40.0\,[\text{N}] \qquad (9)$$

• Newton's 2^{nd} law:

$$\sum \vec{F} = m\vec{a}$$

$$\vec{F}_N + \vec{F}_G + \vec{F}_P = \vec{0}$$

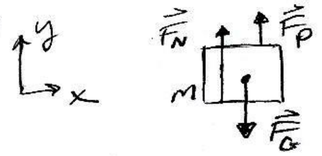
$$\vec{F}_N = -\vec{F}_G - \vec{F}_P = -(-mg\hat{y}) - (|\vec{F}_P|(-\hat{y}))$$

(pushing down causes $\vec{F}_N$ to increase, but weight is the same)

(b)

$$\boxed{\begin{aligned}\vec{F}_N &= +mg\hat{y} + |\vec{F}_P|\,\hat{y} = (mg + |\vec{F}_P|)\,\hat{y}\\ &= (98.0\,[\text{N}] + 40.0\,[\text{N}])\,\hat{y} = (138\,[\text{N}])\,\hat{y}\end{aligned}} \qquad (10)$$

c: • diagram:



• weight force is still:

$$\vec{F}_G = -mg\,\hat{y} \qquad (11)$$

• still at rest:

$$\vec{a} = \vec{0} \qquad (12)$$

• Given pull:

$$\vec{F}_P = |\vec{F}_P|(+\hat{y})\;;\; |\vec{F}_P| \equiv 40.0\,[\text{N}] \qquad (13)$$

• Newton's 2^{nd} law:

$$\sum \vec{F} = m\vec{a}$$

$$\vec{F}_N + \vec{F}_G + \vec{F}_P = \vec{0} \qquad (14)$$

$$\vec{F}_N = -\vec{F}_G - \vec{F}_P = -(-mg\hat{y}) - (|\vec{F}_P|\,\hat{y})$$

($\vec{F}_N$ is not always the same strength as weight)

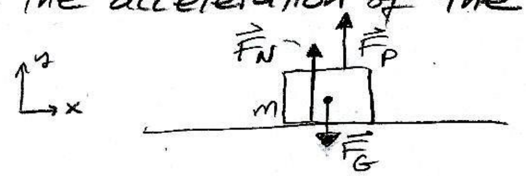
(pulling up causes $\vec{F}_N$ to decrease, but weight is the same)

(c)

$$\boxed{\begin{aligned}\vec{F}_N &= +mg\hat{y} - |\vec{F}_P|\,\hat{y} = (mg - |\vec{F}_P|)\,\hat{y}\\ &= (98.0\,[\text{N}] - 40.0\,[\text{N}])\,\hat{y} = (58.0\,[\text{N}])\,\hat{y}\end{aligned}} \qquad (15)$$

e4: { A box not at rest: If the pull force in the last example equals or exceeds the box's weight, it will accelerate. Suppose $F_P \equiv |\vec{F}_P| = 100.0\,[N]$ in the last example, part c, What is the acceleration of the box? }

- diagram:



- $\vec{F}_G$ is still

$$\vec{F}_G = -mg\hat{y} \qquad (1)$$

- but since $|\vec{F}_P| \geq |\vec{F}_G|$, then in general, $\vec{a} \geq \vec{0}$ (2)
- Pull is:

$$\vec{F}_P = F_P\hat{y}\,;\ F_P \equiv 100.0\,[N] \qquad (3)$$

- case 1: $\vec{a} = \vec{0}$
  - then!

$$\Sigma\vec{F} = m\vec{a}$$
$$\vec{F}_N + \vec{F}_G + \vec{F}_P = \vec{0}$$
$$\vec{F}_N = -\vec{F}_G - \vec{F}_P = +mg\hat{y} - F_P\hat{y}$$
$$\vec{F}_N = (mg - F_P)\hat{y} = (98.0\,[N] - 100\,[N])\hat{y}$$
$$\vec{F}_N = -2.00\,[N]\,\hat{y} \quad \text{when } \vec{a} = \vec{0} \qquad (4)$$

- but, the table can't pull the box down!
- Therefore, $\boxed{\vec{F}_N \text{ as an upward force has a minimum value at } \vec{F}_N = \vec{0}}$ $\Rightarrow \min(F_N) = 0$ (where $\vec{F}_N \equiv F_N\hat{y}$, and F_N is a scalar)
- Since we got $F_N = -2.00\,[N] < 0$, which is false, then the assumption that caused it ($\vec{a} = \vec{0}$) is also false, so:

$$\vec{a} \neq \vec{0} \qquad (5)$$

- case 2: $|\vec{a}| > 0$, $F_N = 0$:

$$\Sigma\vec{F} = m\vec{a}\,, \qquad \vec{a} \equiv a_y\hat{y}\,;\ a_y \equiv \text{scalar} \qquad (6)$$
$$\vec{F}_N + \vec{F}_G + \vec{F}_P = ma_y\hat{y}$$
$$0\cancel{\hat{y}} - mg\cancel{\hat{y}} + F_P\cancel{\hat{y}} = ma_y\cancel{\hat{y}}$$
$$-mg + F_P = ma_y$$
$$\frac{-mg + F_P}{m} = a_y$$

$$a_y = \frac{-mg + F_P}{m} = \frac{-98.0\,[N] + 100\,[N]}{10.0\,[kg]} = 0.20\left[\frac{m}{s^2}\right] \qquad (7)$$

so $$\boxed{\vec{a} = a_y\hat{y} = \left(0.20\left[\tfrac{m}{s^2}\right]\right)\hat{y}} \qquad (8)$$

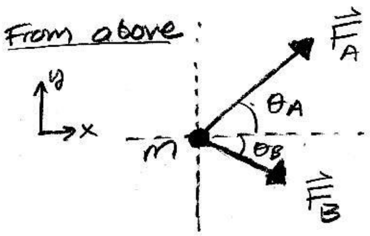
- this satisfies the assumptions, since $|\vec{a}| = 0.20\left[\frac{m}{s^2}\right] > 0$ ✓ and $F_N = 0$ ✓
- so be careful with normal forces!
  - try it first assuming no acceleration
  - if that gives nonsense (assuming your work has no errors), then try $|\vec{a}| > 0$ case

Free-Body Diagrams:

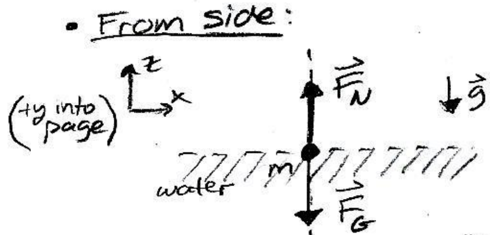
- To apply Newton's 2^{nd} law $\sum \vec{F} = m\vec{a}$ on object of mass m,
  - isolate the object of mass m
  - idealize it as a point-particle
  - Draw all forces acting on it (exclude its forces on other objects)
    - shift them a bit if necessary (but make a note)
  - Draw a clear + useful coordinate system

ex { A boat in still water (ignore friction or resistance from water) is pulled to the East by two people standing on North and South shores, each pulling with a taut rope on the front of the boat, attached to the same point. The person on the north shore pulls with a force of 40 [N] at an angle 45° north of east, and the person on the south shore pulls with 30 [N] at an angle 37° south of east. What is the total force on the boat? }

- Free-body diagram for boat:
  - From above:



  - From side:



(• If object is not moving in some direction, and we want some force in a perpendicular direction, we can ignore that other direction
• but only if no friction...)

- Notice we kept numbers aside:

$$|\vec{F}_A| = 40\,[N], \quad |\vec{F}_B| = 30\,[N]$$
$$\theta_A = 45° \text{ (above } +x) \quad \theta_B = 37° \text{ (below } +x) \qquad (1)$$

- Full-vector version: (state full definition of each vector in given variables)

$$\vec{F}_A = F_A\cos(\theta_A)\hat{x} + F_A\sin(\theta_A)\hat{y}, \quad F_A \equiv |\vec{F}_A| \qquad (2)$$
$$\vec{F}_B = F_B\cos(\theta_B)\hat{x} + F_B\sin(\theta_B)(-\hat{y}) \quad F_B \equiv |\vec{F}_B| \qquad (3)$$
$$\vec{a} \equiv a_x\hat{x} + a_y\hat{y} \;; \quad a_x, a_y \text{ gen. scalars} \qquad (4)$$

- 2^{nd} law:

$$\sum \vec{F} = m\vec{a} \qquad (5)$$

- Problem just asks for $\sum \vec{F}$:

$$\vec{F} \equiv \sum \vec{F} = \vec{F}_A + \vec{F}_B = F_A\cos(\theta_A)\hat{x} + F_A\sin(\theta_A)\hat{y} + F_B\cos(\theta_B)\hat{x} - F_B\sin(\theta_B)\hat{y} \qquad (6)$$

(group by unit vector)

Cartesian form

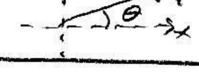
$$\vec{F} \equiv F_x\hat{x} + F_y\hat{y} = [F_A\cos(\theta_A) + F_B\cos(\theta_B)]\hat{x} + [F_A\sin(\theta_A) - F_B\sin(\theta_B)]\hat{y} \qquad (7)$$
$$= (52.2\,[N])\hat{x} + (10.2\,[N])\hat{y}$$

or also:

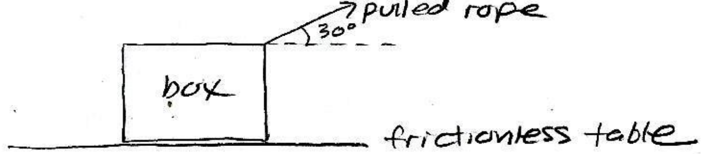
Polar form

$$|\vec{F}| = \sqrt{F_x^2 + F_y^2} = \sqrt{(52.2\,[N])^2 + (10.2\,[N])^2} = 53.2\,[N],$$
$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = 11.1° \text{ north of } +x \text{ axis, since } F_x > 0 \text{ and } F_y > 0$$

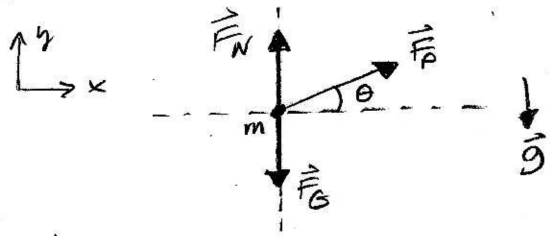
implies that we defined θ as:



ex: In the diagram below, if someone pulls a rope attached to a 10.0 [kg] box with a force of 40.0 [N] at 30° above the horizontal, (a) what is the acceleration of the box? (b) What is the normal force exerted on the box by the table? In both parts, assume a frictionless table.



- *Free-body diagram*:



(**Note**: • it's okay if forces are not exactly to scale; they are merely representative at this stage)

- *Define each Force*:

($F_{Ny} \equiv$ scalar) $\vec{F}_N \equiv$ normal force on box by table $\equiv F_{Ny}\hat{y}$ (1)

($F_P \equiv |\vec{F}_P|$) $\vec{F}_P \equiv F_P\cos(\theta)\hat{x} + F_P\sin(\theta)\hat{y} \equiv$ pull force   (2)

$$\vec{F}_G \equiv mg(-\hat{y}) = -mg\hat{y} \quad (3)$$

- *Newton's 2nd Law*:

$$\sum \vec{F} = m\vec{a} \quad ; \quad \vec{a} \equiv a_x\hat{x} + a_y\hat{y} \quad (4)$$

$$\vec{F}_N + \vec{F}_P + \vec{F}_G = m\vec{a} \quad (5)$$

- a: solve for $\vec{a}$:

$$\vec{a} = \frac{1}{m}\left(\vec{F}_N + \vec{F}_P + \vec{F}_G\right) \quad (6)$$

  - put (1–3) into (6):

$$\vec{a} = \frac{1}{m}\left(F_{Ny}\hat{y} + F_P\cos(\theta)\hat{x} + F_P\sin(\theta)\hat{y} - mg\hat{y}\right)$$

$$\vec{a} = \left[\frac{1}{m}F_P\cos(\theta)\right]\hat{x} + \frac{1}{m}\left[F_{Ny} + F_P\sin(\theta) - mg\right]\hat{y} \quad (7)$$

  - don't know F_{Ny} yet, so use fact that since box does not lift off table, then its vertical acceleration must be zero, so

$$a_y = 0 \quad \text{(since box doesn't lift off table)} \quad (8)$$

  - from (7) and (4):

$$a_y = \frac{1}{m}\left[F_{Ny} + F_P\sin(\theta) - mg\right] \quad (9)$$

  - plug (8) into (9):

$$0 = \frac{1}{m}\left[F_{Ny} + F_P\sin(\theta) - mg\right] \quad (10)$$

  - solve (10) for F_{Ny}:

$$F_{Ny} = mg - F_P\sin(\theta) \quad (11)$$

  - plug (11) into (7):

$$\vec{a} = \left[\frac{1}{m}F_P\cos(\theta)\right]\hat{x} + \frac{1}{m}\left[\cancel{mg} - \cancel{F_P\sin(\theta)} + \cancel{F_P\sin(\theta)} - \cancel{mg}\right]\hat{y}$$

$F_P = 40.0$ [N], $\theta = 30°$, $m = 10.0$ [kg] →

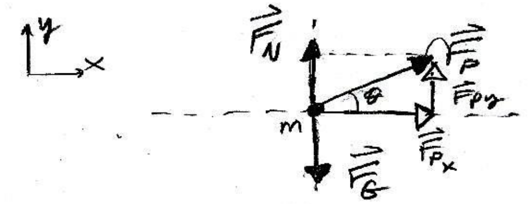
$$\text{(a)} \quad \boxed{\vec{a} = \left[\frac{1}{m}F_P\cos(\theta)\right]\hat{x} + 0\,\hat{y} = \left(3.46\left[\tfrac{m}{s^2}\right]\right)\hat{x}} \quad (12)$$

- b/: • Since $\vec{F}_N \equiv F_{Ny}\hat{y}$ in (1), then putting (11) into (1) gives:

$$\text{(b)} \quad \boxed{\vec{F}_N = \left(mg - F_P\sin(\theta)\right)\hat{y} = \left(78.0\,[N]\right)\hat{y}} \quad (13)$$

- Alternative solution:
  - write out separate scalar equations for each perpendicular direction

(really, this *is* what the full vector form is)



(• CAUTION: Don't treat $\vec{F}_P$, $\vec{F}_{Py}$, $\vec{F}_{Px}$ as separate forces! (you'd get $2\vec{F}_P$)
• That's why we use "empty" arrow tips on component vectors)

- Newton's 2nd law is now two separate equations:

$$\Sigma \vec{F}_x = m\vec{a}_x \quad ; \quad \Sigma \vec{F}_y = m\vec{a}_y \qquad (14)$$

- if $\Sigma \vec{F}_x \equiv (\Sigma F_x)\hat{x}$; $\Sigma \vec{F}_y \equiv (\Sigma F_y)\hat{y}$ (15)

where (ΣF_x) and (ΣF_y) are general scalars,
and

$$\vec{a}_x \equiv a_x \hat{x} \quad , \quad \vec{a}_y \equiv a_y \hat{y} \quad ; \quad a_x, a_y \text{ scalars} \qquad (16)$$

then (15) simplifies to two scalar equations:

$$(\Sigma F_x)\hat{x} = m a_x \hat{x} \quad ; \quad (\Sigma F_y)\hat{y} = m a_y \hat{y} \qquad (17)$$

$$\Downarrow \qquad\qquad \Downarrow$$

(CAUTION: scalar equations are 1-D vector eqns.)

$$\Sigma F_x = m a_x \quad ; \quad \Sigma F_y = m a_y \qquad (18)$$

- List component values separately:

(so here, F_{Px}, F_{Ny}, F_{Py}, F_{Gy} are general scalars, and can be negative (are *not* magnitudes) in general)

x-parts of forces	y-parts of forces:
$F_{Px} = F_P \cos(\theta)$; $F_P \equiv \lvert\vec{F}_P\rvert$	F_{Ny}
	$F_{Gy} = -mg$
	$F_{Py} = F_P \sin(\theta)$; $F_P \equiv \lvert\vec{F}_P\rvert$

(19)

- Newton's 2nd law separately: (put (19) into (18)):

$$\Sigma F_x = m a_x$$
$$F_{Px} = m a_x$$
$$F_P \cos(\theta) = m a_x \qquad (20)$$

$$\Sigma F_y = m a_y$$
$$F_{Ny} + F_{Gy} + F_{Py} = m a_y$$
$$F_{Ny} - mg + F_P \sin(\theta) = m a_y \qquad (21)$$

- Observe no vertical motion, so:

$$a_y = 0 \qquad (22)$$

- solve (20) for a_x:

$$a_x = \frac{1}{m} F_P \cos(\theta) \qquad (23)$$

ⓐ $$\boxed{\vec{a} = \vec{a}_x + \vec{a}_y = a_x \hat{x} + a_y \hat{y} = \frac{1}{m} F_P \cos(\theta)\,\hat{x} = \left(3.46 \left[\tfrac{m}{s^2}\right]\right)\hat{x}} \qquad (24)$$

- put (22) into (21), solve for F_{Ny}:

$$F_{Ny} - mg + F_P \sin(\theta) = 0$$
$$F_{Ny} = mg - F_P \sin(\theta)$$

ⓑ $$\boxed{\vec{F}_N = F_{Ny}\hat{y} = \left(mg - F_P \sin(\theta)\right)\hat{y} = \left(78.0\,[N]\right)\hat{y}} \qquad (25)$$