R2_Samir_Banjara

Ch 1.1

State the order of the given ordinary differential equation. Determine whether the equation is linear or nonlinear

Problem 4

$$\frac{d^2u}{dr^2} + \frac{du}{dr} + u = \cos(r + u)$$

Solution: Second order non-linear equation

Problem 5

$$\frac{d^2y}{dx^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

Solution: Second order non-linear equation

Problem 32

Find the values of m so that the function $y = e^{mx}$ is a solution of the given differential equation,

$$5y' = 2y$$

Solution: First find the derivative of the given function $y = e^{mx}$

$$\frac{d}{dx}(y) = \frac{d}{dx}(e^{mx})$$
$$y' = me^{mx}$$

Substitute y and y' into the given differential equation $5y' = 2y$

$$5(me^{mx}) = 2(e^{mx})$$
$$5m = 2$$
$$m = \frac{2}{5}$$

Thus, $y = e^{\frac{2}{5}x}$ is a solution to $5y' = 2y$

Ch 2.1

Find the critical points and phase portrait of the given autonomous first-order differential equation. Classify each critical point as asymptotically stable, unstable, or semi-stable. By hand, sketch typical solution curves in the regions in the xy-plane determined by the graphs of the equilibrium solutions.

Problem 26

$$\frac{dy}{dx} = y(2-y)(4-y)$$

Solution:

Find Critical Points

Seprate RHS to 0 to finnd critical points,

$$y(2-y)(4-y) = 0$$

Critical points: $y = 0$, $y = 2$, $y = 4$

Classifiy critical points

To classifiy critical points we must first look at the phase line and extrapolate it into a phase portrait.

Interval between critical points: $(-\infty, 0), (0, 2), (2, 4), (4, \infty)$

For $y \in (-\infty, 0)$

Test value $y = -1$,

$$-1(2+1)(4+1) = -15$$

Decreasing.

For $y \in (0, 2)$

Test value $y = 1$,

$$1(2-1)(4-1) = 3$$

Increasing.

For $y \in (2, 4)$

Test value $y = 3$,

$$3(2-3)(4-3) = -3$$

Decreasing.

For $y \in (4, \infty)$

Test value $y = 5$,

$$5(2-5)(4-5) = 15$$

Increasing.

Phase Line & Point

ATTACHED AS IMAGE

Stability

For $y = 0$ the solution moves away from $y = 0$ from both sides, thus, unsatble. For $y = 2$ the solution moves towards $y = 2$ from both sides, thus stable. For $y = 4$ the solution moves away from $y = 4$ from both sides. thus unstable.

Ch 2.2

Solve the given differential equation by separation of varibles.

Problem 2

$$\frac{dy}{dx} = (x+1)^2$$

Solution: Separate y variables into one side and x into another.

$$dy = (x+1)^2 dx$$

Take the integral of both sides

$$\begin{aligned} \int dy &= \int (x+1)^2 \, dx \\ y &= \frac{1}{3}(x+1)^3 + C \\ &= \frac{x^3}{3} + x^{2+x} + C \end{aligned}$$

Problem 7

$$\frac{dy}{dx} = \left(e^{3x+2y}\right)$$

Solution:

$$\frac{dx}{dy} = e^{3x} \cdot e^{2y} \cdot dx$$

$$\frac{1}{e^{2y}} dy = e^{3x} dx$$

Take the integral of both sides.

$$\int \frac{1}{e^{2y}}\, dy = \int e^{3x}\, dx$$

Part 1:

$$\int \frac{1}{e^{2y}}\, dx = -\frac{1}{2} e^{-2y} + C$$

Part 2:

$$\int e^{3x}\, dx = \frac{1}{3} e^{3x} + C$$

So we now have,

$$-\frac{1}{2} e^{-2y} = \frac{1}{3} e^{3x} + C$$

Find y

$$-\frac{1}{2} e^{-2y} = \frac{1}{3} e^{3x} + C$$

$$e^{-2y} = -\frac{2}{3} e^{3x} + C$$

$$\ln(e^{-2y}) = \ln\left(-\frac{2}{3} e^{3x} + C\right)$$

$$-2y = \ln\left(-\frac{2}{3} e^{3x} + C\right)$$

$$y = -\frac{1}{2} \ln\left(-\frac{2}{3} e^{3x} + C\right)$$

Final explicit solution:

$$y = -\frac{1}{2} \ln\left(-\frac{2}{3} e^{3x} + C\right)$$