Analysis of Mfeat data set

Descriptive analysis

importing libraries

In [8]:

```
#Loading data
import pandas as pd
import pandas as pd
import numpy as np
import seaborn as sns #visualisation
import matplotlib.pyplot as plt #visualisation
from scipy import stats
import numpy as np
from sklearn.preprocessing import StandardScaler
from sklearn.model_selection import train_test_split
```

importing training data

In [9]:

```
data = pd.read_csv("data_train.csv")
```

Descriptice analysis

In [1357]:

```
description=data.describe()
description.to_csv("description.csv")
```

In [266]:

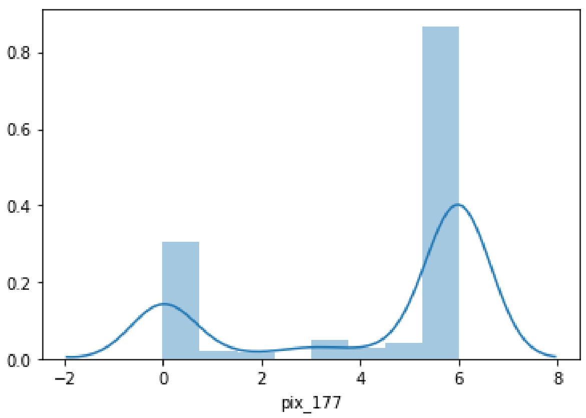
```
data2=data.groupby('class').mean()
data2.to_csv("description2.csv")
```

In [1358]:

```
sns.distplot(data['pix_177'])
```

Out[1358]:

```
<matplotlib.axes._subplots.AxesSubplot at 0xb909b8c8>
```



shapiro normality test

In [ ]:

```
from scipy.stats import shapiro
for fea in data.columns:
    print(str(fea)+","+str(shapiro(data[fea])[1]))
```

Plotting variables

In [1847]:

```
#defining function for plotting boxplots of variables
def draw_boxplot(df, variables, n_rows, n_cols):
    fig=plt.figure(figsize=(50, 50), dpi= 60, facecolor='w', edgecolor='k')
    for i, var_name in enumerate(variables):
        ax=fig.add_subplot(n_rows,n_cols,i+1)
        df.boxplot(var_name,ax=ax)
        ax.set_title(var_name+" Distribution")
    fig.tight_layout()  # Improves appearance a bit.
    fig.savefig("abc.png")
```

In [1849]:

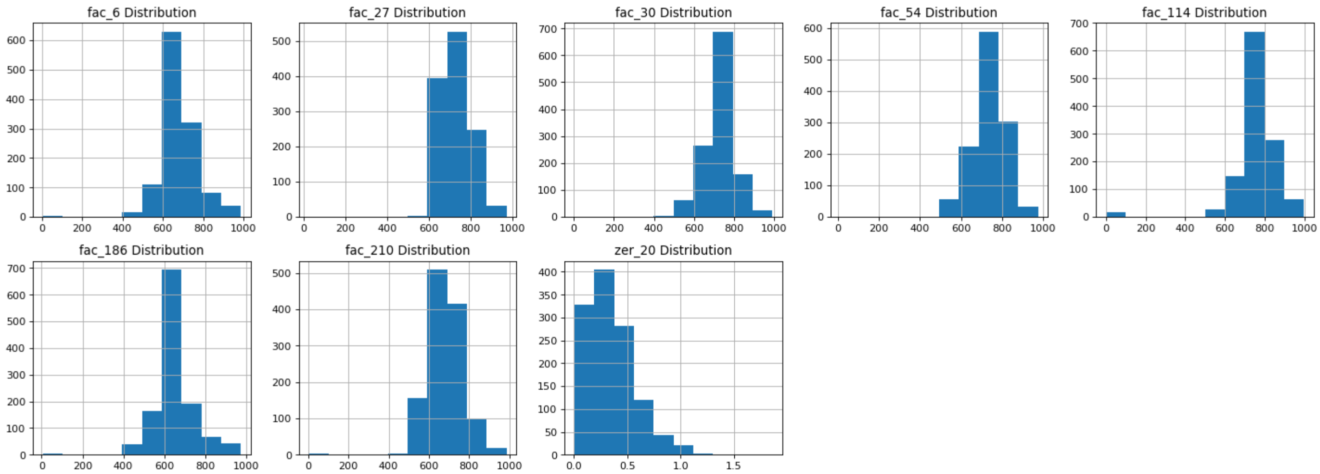
```
#defining function for plotting histograms of variables

def draw_histograms(df, variables, n_rows, n_cols):
    fig=plt.figure(figsize=(18, 16), dpi= 80, facecolor='w', edgecolor='k')
    for i, var_name in enumerate(variables):
        ax=fig.add_subplot(n_rows,n_cols,i+1)
        df[var_name].hist(bins=10,ax=ax)
        ax.set_title(var_name+" Distribution")
    fig.tight_layout()  # Improves appearance a bit.
    fig.savefig("abc.png")
```

In [594]:

```
variables=['fac_6','fac_27','fac_30','fac_54','fac_114','fac_186','fac_210','pix_1','pix_15','pix_2
2','pix_23','pix_24','pix_91','pix_105','pix_106','pix_121','pix_136','pix_211','pix_226','pix_240'
,'zer_1','zer_2','zer_8','zer_14','zer_20','zer_30','zer_32','zer_39','zer_44']
```

In [1852]:

```
draw_histograms(data,variables,5,5)
```



In [292]:

```
skew=data.skew()
skew.to_csv("skew.csv")
```

```
E:\CVM\GTU\WPy64-3741\python-3.7.4.amd64\lib\site-packages\ipykernel_launcher.py:2: FutureWarning: The signature of `Series.to_csv` was aligned to that of `DataFrame.to_csv`, and argument 'header' will change its default value from False to True: please pass an explicit value to suppress this warning.
```

In [295]:

```
median=data.median()
median=median.to_csv("median.csv")
```

```
E:\CVM\GTU\WPy64-3741\python-3.7.4.amd64\lib\site-packages\ipykernel_launcher.py:2: FutureWarning: The signature of `Series.to_csv` was aligned to that of `DataFrame.to_csv`, and argument 'header' will change its default value from False to True: please pass an explicit value to suppress this warning.
```

In [296]:

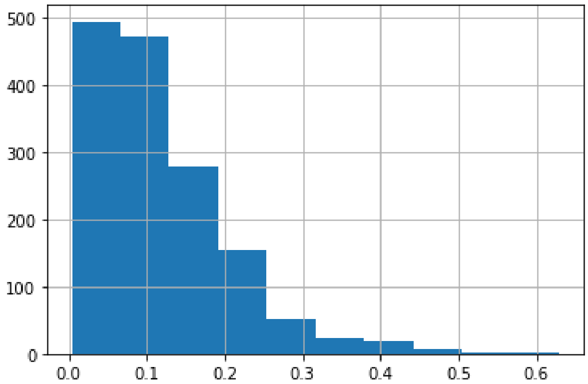
```
mode=data.mode()
mode=mode.to_csv("mode.csv")
```

In [1173]:

```
data['zer_8'].hist()
```

Out[1173]:

```
<matplotlib.axes._subplots.AxesSubplot at 0xb4c677c8>
```



Cleaning Data

Data cleanning (outliers)

In [10]:

```
def outliers_iqr(ys):
    quartile_1, quartile_3 = np.percentile(ys, [25, 75])
    iqr = quartile_3 - quartile_1
    lower_bound = quartile_1 - (iqr * 3)
    upper_bound = quartile_3 + (iqr * 3)
    return np.where((ys > upper_bound) | (ys < lower_bound))
```

In [11]:

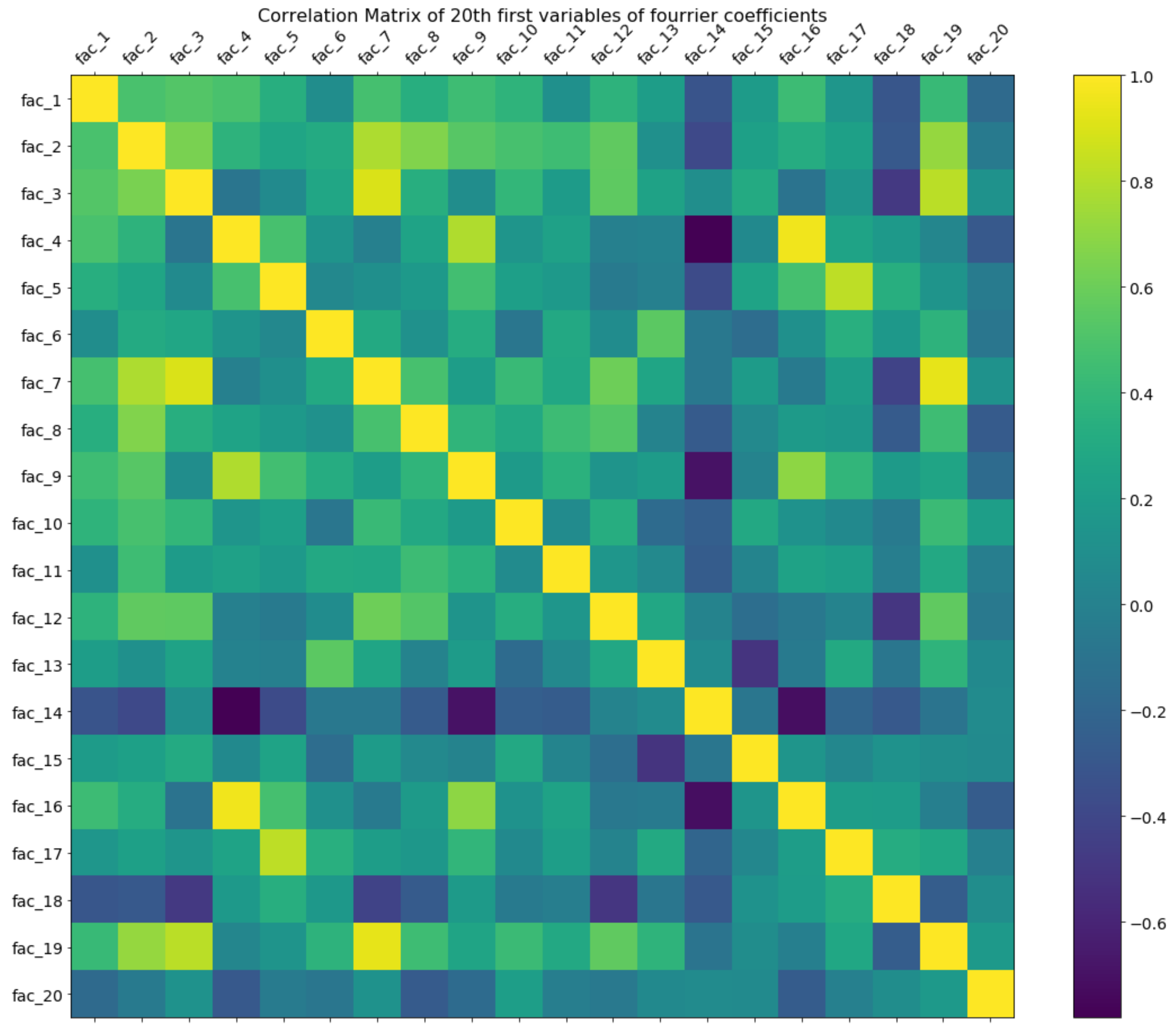
```
variables=['fac_6','fac_27','fac_30','fac_54','fac_114','fac_186','fac_210','zer_20']
index=[]
for var in variables:
    outliers=outliers_iqr(data[var])[0]
    index=np.concatenate((outliers, index), axis=0)
outliers=np.unique(index)
outlier_indexes=outliers.astype(int)
data.drop(outlier_indexes,inplace=True)
```

```
data.drop(outlier_indexes,inplace=True)
```

Correlation Matrix

In [628]:

```
df=data.iloc[:,range(0,20)]
f = plt.figure(figsize=(19, 15))
plt.matshow(df.corr(), fignum=f.number)
plt.xticks(range(df.shape[1]), df.columns, fontsize=14, rotation=45)
plt.yticks(range(df.shape[1]), df.columns, fontsize=14)
cb = plt.colorbar()
cb.ax.tick_params(labelsize=14)
plt.title('Correlation Matrix of 20th first variables of fourrier coefficients', fontsize=16);
f.savefig("correlation.png")
```



Dropping high correlated features

In [12]:

```
df=data
# Create correlation matrix
corr_matrix = df.corr().abs()

# Select upper triangle of correlation matrix
upper = corr_matrix.where(np.triu(np.ones(corr_matrix.shape), k=1).astype(np.bool))

# Find index of feature columns with correlation greater than 0.95
to_drop = [column for column in upper.columns if any(upper[column] > 0.95)]
```

In [13]:

```
data.drop(data[to_drop], axis=1,inplace=True)
```

Dropping variables with low variance

In [14]:

```
variables2=['pix_1','pix_15','pix_22','pix_23','pix_24','pix_91','pix_105','pix_106','pix_121','pix
_136','pix_211','pix_226','pix_240','zer_1','zer_2','zer_8','zer_32']
```

In [15]:

```
data.drop(data[variables2], axis=1,inplace=True)
```

Exploratory analysis

AFM analysis

In [16]:

```
import prince
```

In [17]:

```
X = data.drop(['class'],axis=1)
columns=data.columns.get_values()
index=['classe {}'.format(i+1) for i in range(10)]
```

```
E:\CVM\GTU\WPy64-3741\python-3.7.4.amd64\lib\site-packages\ipykernel_launcher.py:2: FutureWarning:
The 'get_values' method is deprecated and will be removed in a future version. Use '.to_numpy()' o
r '.array' instead.
```

In [18]:

```
#Construct groups of variables
group1=data.columns.get_values()[0:153]
group2=data.columns.get_values()[153:229]
group3=data.columns.get_values()[229:293]
group4=data.columns.get_values()[293:298]
group5=data.columns.get_values()[298:525]
group6=data.columns.get_values()[525:551]
```

```
E:\CVM\GTU\WPy64-3741\python-3.7.4.amd64\lib\site-packages\ipykernel_launcher.py:2: FutureWarning:
The 'get_values' method is deprecated and will be removed in a future version. Use '.to_numpy()' o
r '.array' instead.

E:\CVM\GTU\WPy64-3741\python-3.7.4.amd64\lib\site-packages\ipykernel_launcher.py:3: FutureWarning:
The 'get_values' method is deprecated and will be removed in a future version. Use '.to_numpy()' o
r '.array' instead.
  This is separate from the ipykernel package so we can avoid doing imports until
E:\CVM\GTU\WPy64-3741\python-3.7.4.amd64\lib\site-packages\ipykernel_launcher.py:4: FutureWarning:
The 'get_values' method is deprecated and will be removed in a future version. Use '.to_numpy()' o
r '.array' instead.
  after removing the cwd from sys.path.
E:\CVM\GTU\WPy64-3741\python-3.7.4.amd64\lib\site-packages\ipykernel_launcher.py:5: FutureWarning:
The 'get_values' method is deprecated and will be removed in a future version. Use '.to_numpy()' o
r '.array' instead.
  """
E:\CVM\GTU\WPy64-3741\python-3.7.4.amd64\lib\site-packages\ipykernel_launcher.py:6: FutureWarning:
The 'get_values' method is deprecated and will be removed in a future version. Use '.to_numpy()' o
r '.array' instead.

E:\CVM\GTU\WPy64-3741\python-3.7.4.amd64\lib\site-packages\ipykernel_launcher.py:7: FutureWarning:
```

```
The 'get_values' method is deprecated and will be removed in a future version. Use '.to_numpy()' o
r '.array' instead.
  import sys
```

In [19]:

```python
groupes={'groupe1':group1,'groupe2':group2,'groupe3':group3,'groupe4':group4,'groupe5':group5,'grou
pe6':group6}
```

In [20]:

```python
mfa = prince.MFA(
groups=groupes,
n_components=552,
n_iter=3,
copy=True,
check_input=True,
engine='auto',
random_state=42
)
```

In [21]:

```python
mfa = mfa.fit(X)
```

In [22]:

```python
mfa.row_coordinates(X)
```

Out[22]:

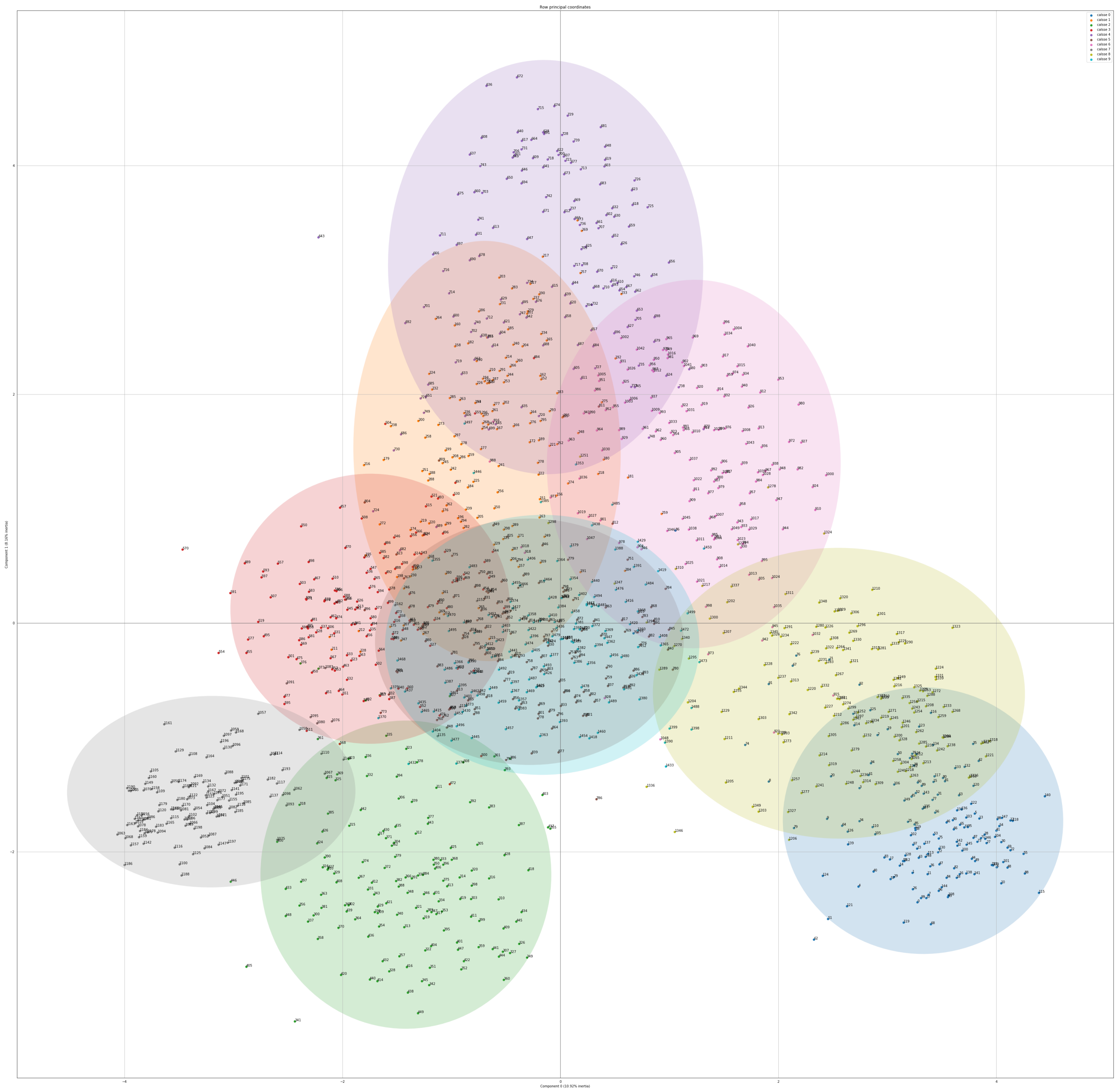
	0	1	2	3	4	5	6	7	8	9	...	541	542	
0	3.586230	-2.148888	0.058858	-0.156174	0.019948	0.550095	1.241363	-0.025500	-0.759428	0.092222	...	0.005527	-0.003577	0.00
1	3.351882	-1.972610	-0.510141	-0.518834	-0.095881	-0.890958	1.401210	0.687267	-0.058989	-0.211366	...	0.008154	0.001074	0.00
2	2.769339	-0.923711	-0.556986	0.557096	1.600492	0.892374	2.024398	-0.566913	-0.943459	-2.492594	...	0.006777	-0.009234	0.00
3	2.923882	-1.357988	1.010672	0.732647	-1.182839	-0.786635	0.976628	2.284944	-0.544354	0.645414	...	0.003017	-0.000613	0.00
4	3.084450	-2.000958	-0.706424	-0.293951	0.765419	-0.505314	1.841370	-0.294092	-0.658960	0.124180	...	-0.000771	-0.002269	0.00
...	...	...	...	...	...	...	...	...	...	...	...	...	...	
1495	-0.998496	-0.201105	-1.625644	0.543908	-0.375769	-1.290984	-1.854041	0.875811	-0.155084	0.447490	...	0.002806	0.010712	0.00
1496	-0.816900	-1.026531	-1.730549	0.800533	-0.616348	-1.434995	-0.097577	0.028516	0.072298	-0.226435	...	0.003489	-0.014714	0.00
1497	-0.994816	1.699053	-1.355912	-0.075675	2.116691	2.126245	-1.295645	-0.990727	-1.684412	0.486327	...	-0.004448	-0.011109	0.00
1498	-0.803576	-0.838592	-1.879528	2.000384	0.419841	-1.460013	-1.421362	-0.290801	-0.089692	-0.366525	...	-0.008254	0.003061	0.00
1499	1.040357	0.067146	-2.150715	1.787893	-0.245394	-0.935088	-0.474119	-0.697323	1.302906	-0.009137	...	-0.008391	-0.013112	0.00

1475 rows × 551 columns

In [1441]:

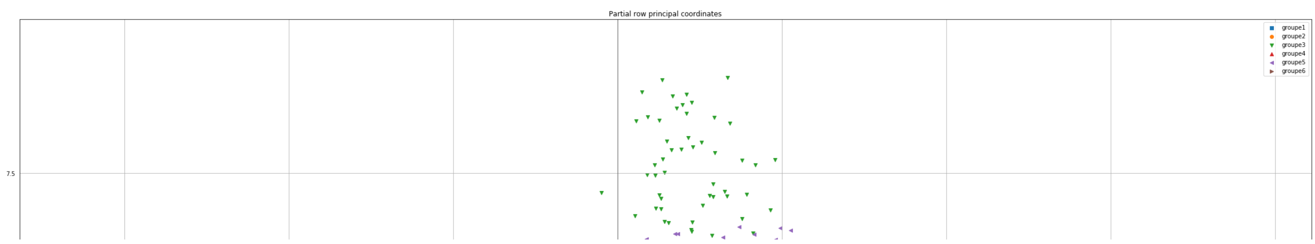
```python
#Plotting individuals in factor plan
ax = mfa.plot_row_coordinates(
X,
ax=None,
figsize=(60, 60),
x_component=0,
y_component=1,
labels=X.index,
```

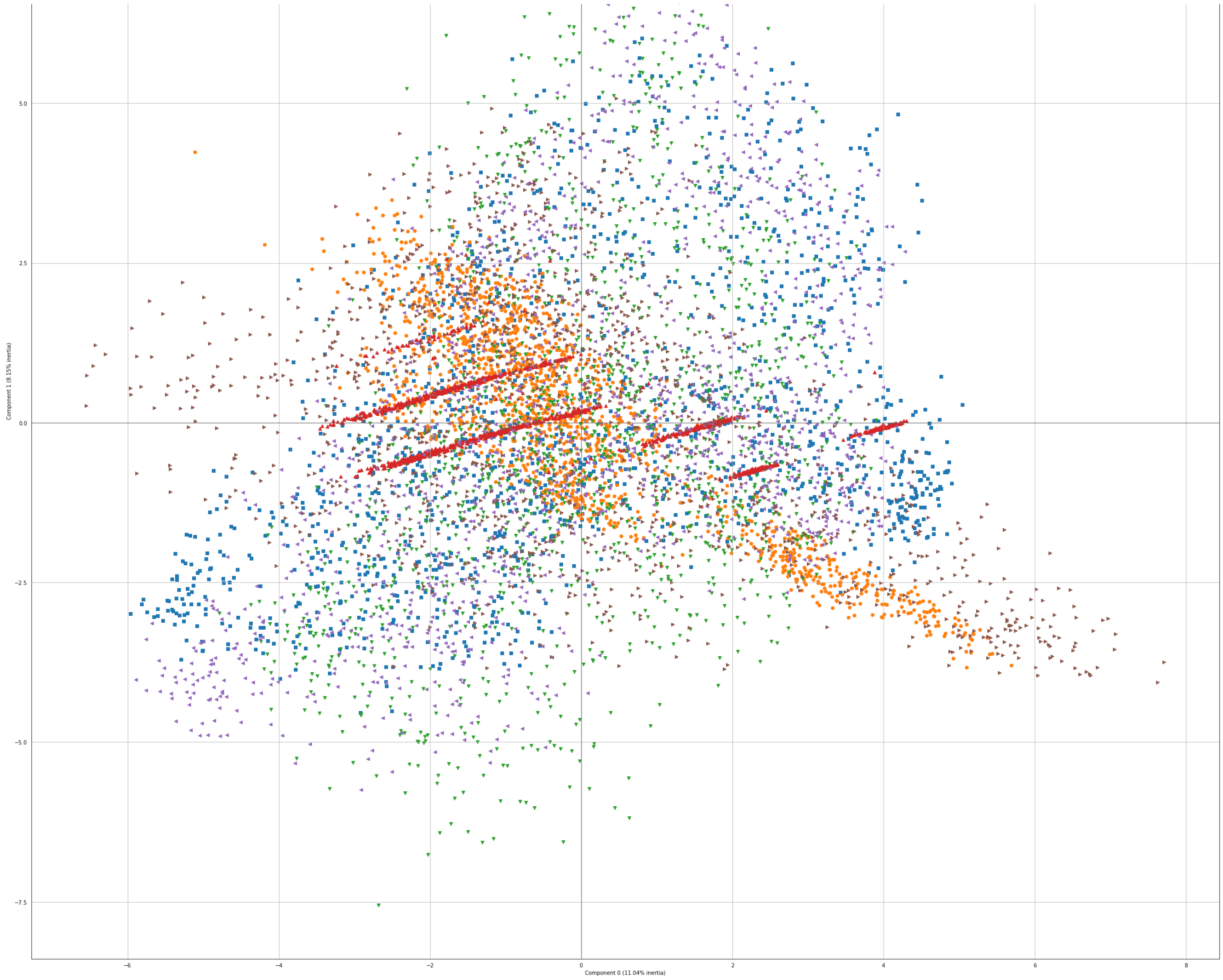
```
color_labels=['calsse {}'.format(t) for t in data['class']],
ellipse_outline=False,
ellipse_fill=True,
show_points=True
)
```



In [1716]:

```
#plotting groups of variables in factor plan
ax = mfa.plot_partial_row_coordinates(
X,
ax=None,
figsize=(40, 40),
x_component=0,
y_component=1,
)
ax.get_figure().savefig('mfa_partial_row_coordinates.svg')
```





In [24]:

```
mfa.explained_inertia_[1:5]
```

Out[24]:

```
[0.08217937708154475,
 0.05961432873508772,
 0.05164220121032245,
 0.04380062719065025]
```

In [25]:

```
mfa.eigenvalues_[1:5]
```

Out[25]:

```
[2.91070069225868, 2.1114721731898367, 1.8291084229501207, 1.5513687303650898]
```

In [815]:

```
for name, fa in sorted(mfa.partial_factor_analysis_.items()):  # doctest: +ELLIPSIS
    print('{} eigenvalues: {}'.format(name, fa.eigenvalues_))
```

```
groupe1 eigenvalues: [31.652894448704345, 22.186686645308537, 17.370605332140986,
11.824414087733315, 9.322442930738104]
groupe2 eigenvalues: [11.826189853131451, 5.201117259313143, 4.826680457949398,
2.7460448926675713, 2.5228997397450064]
groupe3 eigenvalues: [5.634611681148499, 5.191973620618263, 3.60443355550322, 3.296018299990814, 2
.6702615969627854]
groupe4 eigenvalues: [3.249032035312228, 1.0867312262405882, 0.6024743026216429,
0.0576321840053602 6, 0.004130251820181291]
groupe5 eigenvalues: [38.27200100014378, 23.38902551814938, 21.518856530621235,
```

```
15.165299776777033, 13.106170412401877]
groupe6 eigenvalues: [6.946679426942888, 5.035272546562144, 3.128178821658927, 2.4879890692192514, 2.006571668634843]
```

K-means analysis

In [1918]:

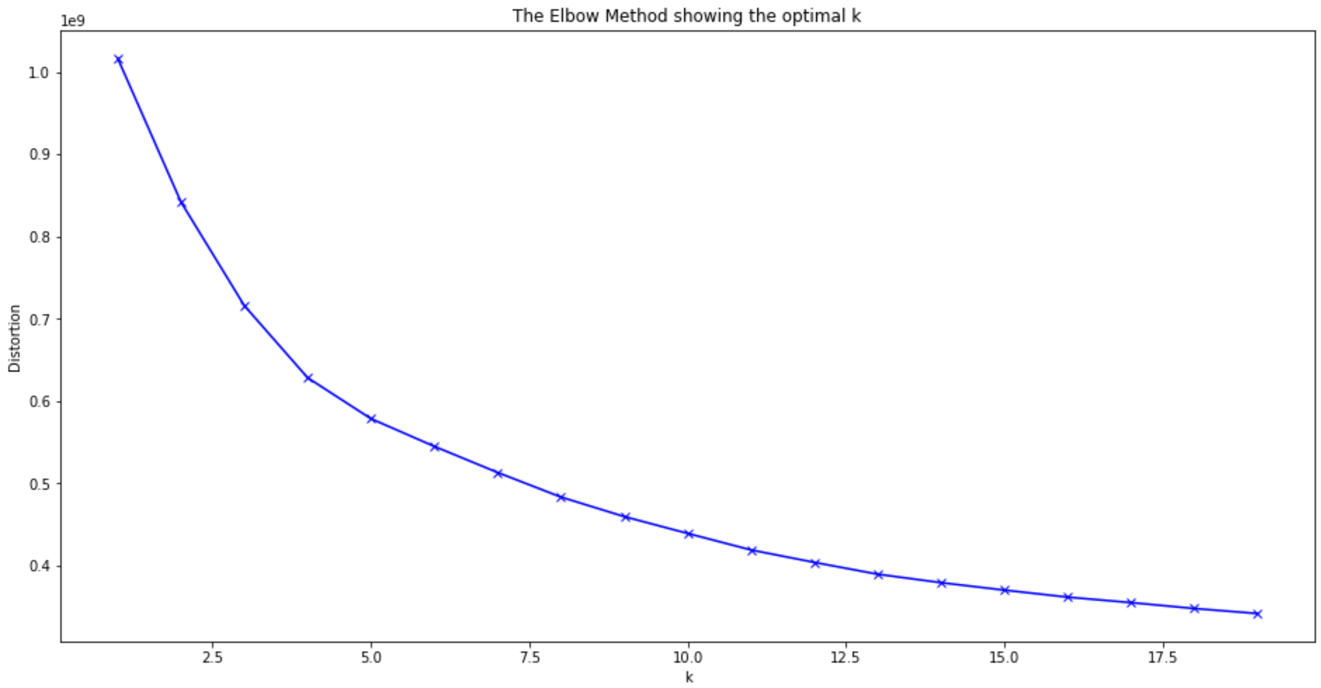
```python
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
%matplotlib inline
from sklearn.cluster import KMeans
from sklearn import datasets
#we took one group or all variables
#df=data.loc[:,group7]
df=data.drop(['class'],axis=1)
```

In [1919]:

```python
#Looking for the optimal K
distortions = []
K = range(1,20)
for k in K:
    kmeanModel = KMeans(n_clusters=k)
    kmeanModel.fit(df)
    distortions.append(kmeanModel.inertia_)
```

In [1921]:

```python
#Looking for the optimal K
plt.figure(figsize=(16,8))
plt.plot(K, distortions, 'bx-')
plt.xlabel('k')
plt.ylabel('Distortion')
plt.title('The Elbow Method showing the optimal k')
plt.show()
```



In [1922]:

```python
#train the model with 5 or 10 classes
#kmeanModel = KMeans(n_clusters=5)
kmeanModel = KMeans(n_clusters=10)
kmeanModel.fit(df)
```

Out[1922]:

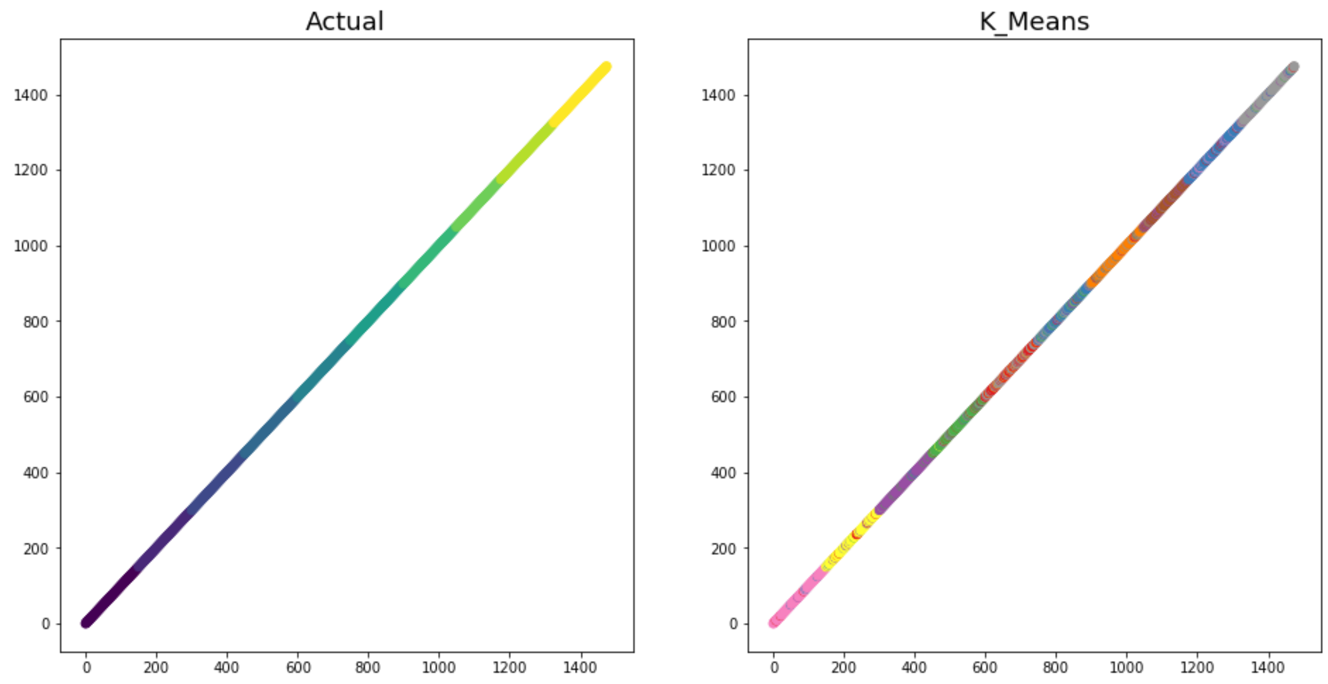
```
KMeans(algorithm='auto', copy_x=True, init='k-means++', max_iter=300,
       n_clusters=10, n_init=10, n_jobs=None, precompute_distances='auto',
       random_state=None, tol=0.0001, verbose=0)
```

In [1923]:

```
    #plot classification of kmean versus real data
    df['k_means']=kmeanModel.predict(df)
    df['target']=data['class']
    fig, axes = plt.subplots(1, 2, figsize=(16,8))
    axes[0].scatter(range(len(df)),range(len(df)), c=df['target'])
    axes[1].scatter(range(len(df)), range(len(df)), c=df['k_means'], cmap=plt.cm.Set1)
    axes[0].set_title('Actual', fontsize=18)
    axes[1].set_title('K_Means', fontsize=18)
```

Out[1923]:

```
Text(0.5, 1.0, 'K_Means')
```



Statistics modelling

Splitting train set and test set

In [1947]:

```
data = pd.read_csv("data_train.csv")
data, X_test, data_class, y_test = train_test_split(origin.drop(['class'],axis=1), origin['class'],
test_size=0.2, random_state=0)
```

reset indexes of data

In [1948]:

```
data['class']=data_class
data=data.reset_index(drop=True)
X_test=X_test.reset_index(drop=True)
```

Scaling and transforming data

Scaling and transforming data

In [1949]:

```
#transform train data and test data
from sklearn.model_selection import train_test_split
from sklearn.preprocessing import StandardScaler

droped_variables=variables2+to_drop
df=data.drop(['class'],axis=1)
df=df.drop(droped_variables,axis=1)
scaler = StandardScaler()
scaler.fit(df)
df = scaler.transform(df)
X_test=X_test.drop(droped_variables,axis=1)
scaler.fit(X_test)
X_test=scaler.transform(X_test)
```

RandoForest Modeling

In [1931]:

```
from sklearn.ensemble import RandomForestRegressor
from sklearn.model_selection import cross_val_score

score=[]
for i in [5,10,20,30,40,50,100]:
    regressor = RandomForestRegressor(n_estimators=i, random_state=0)
    model_cv = cross_val_score(regressor,df,data['class'],cv=5)
    score.append(model_cv.mean())
```

In [1950]:

```
score
```

Out[1950]:

```
(1200, 552)
```

In [1951]:

```
#Implement random forest regressor with best parameter
regressor = RandomForestRegressor(n_estimators=50, random_state=0)

regressor.fit(df, data['class'])

y_pred = regressor.predict(X_test)
y_pred=y_pred.astype(int)
```

In [1952]:

```
#Confusion Matrix

from sklearn.metrics import classification_report, confusion_matrix, accuracy_score

print(confusion_matrix(y_test,y_pred))
print(classification_report(y_test,y_pred))
print(accuracy_score(y_test, y_pred))
```

```
[[31  0  0  0  0  0  0  0  0  0]
 [ 1 28  2  0  0  0  0  0  0  0]
 [ 0  1 25  4  1  0  0  0  0  0]
 [ 0  0  5 31  0  0  0  0  0  0]
 [ 0  0  0  9 21  0  0  0  0  0]
 [ 0  0  0  1 11  6  0  0  0  0]
 [ 0  0  0  0  0  7 25  0  0  0]
 [ 0  0  1  1  1  4 20  4  0  0]
 [ 0  0  0  0  0  0  0  5 18  0]
 [ 0  0  0  0  1  1  0  1 13 21]]
              precision    recall  f1-score   support
```

```
           0       0.97      1.00      0.98        31
           1       0.97      0.90      0.93        31
           2       0.76      0.81      0.78        31
           3       0.67      0.86      0.76        36
           4       0.60      0.70      0.65        30
           5       0.33      0.33      0.33        18
           6       0.56      0.78      0.65        32
           7       0.40      0.13      0.20        31
           8       0.58      0.78      0.67        23
           9       1.00      0.57      0.72        37

    accuracy                           0.70       300
   macro avg       0.68      0.69      0.67       300
weighted avg       0.71      0.70      0.68       300

0.7
```

In [ ]:

```
#### SVM Modeling
```

In [1953]:

```
#Testing poly,rbf and linear kernels
#For rbf kernel, C parameter must be added and very between 0.1 to 100
#For poly kernel, degree must vary between 2 and 10

score=[]
for i in [1,2,3,5,10] :
    svclassifier = SVC(kernel='linear')

    model_cv = cross_val_score(svclassifier,df,data['class'],cv=5)

    score.append(model_cv.mean())
```

In [1954]:

```
max(score)
```

Out[1954]:

```
0.9858761595111185
```

In [1957]:

```
#implemnt classifier on train data
svclassifier = SVC(kernel='linear')
svclassifier.fit(df, data['class'])
y_pred = svclassifier.predict(X_test)
```

In [1958]:

```
from sklearn.metrics import classification_report, confusion_matrix, accuracy_score

print(confusion_matrix(y_test,y_pred))
print(classification_report(y_test,y_pred))
print(accuracy_score(y_test, y_pred))
```

```
[[31  0  0  0  0  0  0  0  0  0]
 [ 0 31  0  0  0  0  0  0  0  0]
 [ 0  0 31  0  0  0  0  0  0  0]
 [ 0  0  0 36  0  0  0  0  0  0]
 [ 0  0  0  0 29  0  1  0  0  0]
 [ 0  0  0  0  0 18  0  0  0  0]
 [ 0  0  0  0  0  0 32  0  0  0]
 [ 0  0  0  0  0  0  0 31  0  0]
 [ 0  0  0  0  0  0  1  0 22  0]
 [ 0  1  0  0  0  0  0  0  0 36]]
              precision    recall  f1-score   support

           0       1.00      1.00      1.00        31
           1       0.97      1.00      0.98        31
```

```
           1       0.97      1.00      0.98        31
           2       1.00      1.00      1.00        31
           3       1.00      1.00      1.00        36
           4       1.00      0.97      0.98        30
           5       1.00      1.00      1.00        18
           6       0.94      1.00      0.97        32
           7       1.00      1.00      1.00        31
           8       1.00      0.96      0.98        23
           9       1.00      0.97      0.99        37

    accuracy                           0.99       300
   macro avg       0.99      0.99      0.99       300
weighted avg       0.99      0.99      0.99       300

0.99
```

Implementing MLP classifier

In [1661]:

```
from sklearn.model_selection import cross_val_score
from sklearn.neural_network import MLPClassifier

score=[]
for i in [5,10,20,30,40,50,100,200]:
    mlp = MLPClassifier(hidden_layer_sizes=(i,i,i), max_iter=1000)
    model_cv = cross_val_score(mlp,df,data['class'],cv=5)
    score.append(model_cv.mean())
```

In [1662]:

```
score
```

Out[1662]:

```
[0.8793766302038897,
 0.95496498803918,
 0.970227147774749,
 0.9711510071408801,
 0.9778045539849852,
 0.9771140449006401,
 0.9779396688656433,
 0.9805044232654291]
```

In [1959]:

```
#implemnt classifier on train data
mlp = MLPClassifier(hidden_layer_sizes=(200,200,200), max_iter=1000)
mlp.fit(df, data['class'].values.ravel())
predictions = mlp.predict(X_test)
```

In [1960]:

```
from sklearn.metrics import classification_report, confusion_matrix
print(confusion_matrix(y_test,predictions))
print(classification_report(y_test,predictions))
```

```
[[31  0  0  0  0  0  0  0  0  0]
 [ 0 31  0  0  0  0  0  0  0  0]
 [ 0  0 30  0  0  0  0  0  0  1]
 [ 0  0  0 36  0  0  0  0  0  0]
 [ 0  0  0  0 29  0  1  0  0  0]
 [ 0  0  0  0  0 18  0  0  0  0]
 [ 0  0  0  0  0  0 32  0  0  0]
 [ 0  0  0  0  0  0  0 31  0  0]
 [ 0  0  0  0  0  0  1  0 21  1]
 [ 0  1  0  0  0  1  0  0  0 35]]
              precision    recall  f1-score   support

           0       1.00      1.00      1.00        31
           1       0.97      1.00      0.98        31
```

```
           2       1.00      0.97      0.98        31
           3       1.00      1.00      1.00        36
           4       1.00      0.97      0.98        30
           5       0.95      1.00      0.97        18
           6       0.94      1.00      0.97        32
           7       1.00      1.00      1.00        31
           8       1.00      0.91      0.95        23
           9       0.95      0.95      0.95        37

    accuracy                           0.98       300
   macro avg       0.98      0.98      0.98       300
weighted avg       0.98      0.98      0.98       300
```

Generalization of SVM for all train data

In [1961]:

```
df = pd.read_csv("data_train.csv")
df.drop(outlier_indexes,inplace=True)
df.drop(droped_variables,axis=1,inplace=True)
Y=df['class']
df.drop(['class'],axis=1,inplace=True)
scaler = StandardScaler()
scaler.fit(df)
df = scaler.transform(df)
```

In [1962]:

```
svclassifier = SVC(kernel='linear')
svclassifier.fit(df, Y)
```

Out[1962]:

```
SVC(C=1.0, cache_size=200, class_weight=None, coef0=0.0,
    decision_function_shape='ovr', degree=3, gamma='auto_deprecated',
    kernel='linear', max_iter=-1, probability=False, random_state=None,
    shrinking=True, tol=0.001, verbose=False)
```

Produce predictions for test data

In [1966]:

```
df = pd.read_csv("data_test.csv")
df.drop(droped_variables,axis=1,inplace=True)
scaler = StandardScaler()
scaler.fit(df)
df = scaler.transform(df)
```

In [1967]:

```
y_pred = svclassifier.predict(df)
y_pred
```

Out[1967]:

```
array([1, 9, 1, 9, 6, 8, 0, 8, 4, 5, 6, 3, 2, 4, 1, 2, 7, 8, 3, 7, 0, 3,
       0, 2, 4, 4, 5, 0, 5, 5, 6, 4, 3, 6, 2, 2, 6, 8, 3, 1, 0, 7, 2, 9,
       4, 1, 2, 5, 4, 8, 3, 6, 1, 9, 5, 7, 1, 5, 6, 6, 7, 2, 9, 8, 8, 5,
       2, 2, 1, 8, 5, 5, 0, 4, 6, 4, 3, 7, 2, 2, 8, 9, 3, 9, 6, 3, 1, 0,
       2, 0, 2, 4, 9, 1, 5, 1, 5, 4, 3, 7, 9, 1, 3, 7, 6, 5, 0, 5, 9, 0,
       9, 6, 4, 6, 2, 9, 7, 7, 9, 7, 0, 9, 1, 5, 3, 9, 9, 6, 5, 6, 8, 6,
       1, 3, 2, 9, 2, 1, 4, 6, 8, 4, 7, 3, 5, 0, 8, 2, 4, 6, 8, 8, 1, 2,
       4, 9, 3, 8, 8, 9, 3, 6, 7, 6, 8, 0, 1, 7, 3, 9, 3, 0, 4, 5, 4, 2,
       5, 5, 9, 3, 6, 7, 8, 8, 4, 5, 2, 8, 9, 8, 3, 8, 6, 5, 0, 5, 9, 4,
       3, 0, 1, 5, 7, 0, 2, 0, 1, 2, 4, 2, 8, 1, 8, 7, 7, 8, 8, 8, 2, 4,
       4, 3, 9, 2, 4, 2, 3, 0, 1, 3, 0, 7, 0, 6, 9, 3, 6, 3, 3, 0, 6, 2,
       6, 9, 7, 7, 2, 1, 0, 7, 8, 6, 1, 7, 3, 6, 3, 0, 4, 3, 1, 9, 6, 7,
       5, 9, 6, 3, 4, 1, 8, 7, 0, 5, 5, 4, 2, 2, 3, 9, 0, 0, 4, 6, 1, 7,
       6, 4, 6, 8, 6, 6, 7, 4, 8, 2, 1, 1, 4, 4, 7, 4, 6, 0, 7, 1, 4, 5,
       4, 1, 6, 9, 1, 0, 2, 2, 6, 6, 5, 5, 3, 0, 8, 6, 4, 4, 3, 5, 9, 4,
```

```
       4, 1, 6, 9, 1, 0, 2, 2, 6, 6, 5, 5, 3, 0, 8, 6, 4, 4, 3, 5, 9, 4,
       1, 0, 7, 4, 7, 4, 3, 7, 4, 1, 6, 1, 7, 1, 0, 7, 7, 1, 9, 7, 9, 6,
       1, 6, 8, 5, 0, 0, 1, 1, 9, 5, 7, 7, 3, 1, 2, 2, 8, 3, 9, 0, 5, 4,
       9, 9, 5, 5, 9, 1, 0, 7, 6, 6, 0, 0, 8, 6, 2, 5, 8, 9, 8, 1, 7, 5,
       2, 6, 1, 4, 1, 2, 2, 3, 5, 0, 0, 5, 1, 7, 8, 6, 0, 3, 7, 3, 3, 2,
       8, 1, 3, 8, 9, 3, 0, 7, 0, 5, 5, 4, 1, 3, 8, 6, 5, 8, 4, 3, 1, 2,
       9, 0, 9, 2, 1, 8, 5, 7, 7, 0, 4, 5, 9, 9, 4, 3, 9, 9, 8, 3, 8, 0,
       2, 2, 4, 1, 2, 3, 5, 3, 8, 4, 0, 2, 8, 1, 0, 5, 0, 5, 1, 4, 8, 1,
       7, 9, 9, 7, 4, 9, 4, 7, 0, 3, 6, 2, 2, 8, 8, 9], dtype=int64)
```